Inverse Transformation Optics with Realistic Material Parameters

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Abstract We present a method to generate transformation functions based on a space of achievable material properties. To validate this approach, we consider the range of effective refractive index achievable using silver nanowires in a dielectric background. Given fabrication constraints, we generate a reduced cloaking transformation and confirm its performance using FDTD and FEM simulations. We explore conditions for finding appropriate mappings in restricted parameter spaces, and strategies for optimizing transformations to account for absorption and scattering.

Transformation optical designs provide unprecedented control of electromagnetic fields within optical devices. Typically, the device functionality is defined by a coordinate transformation, which is translated into the constitutive material properties of the device. While powerful and versatile, this method often results in extreme material requirements, which restricts the range of devices that can be realized. In order to achieve realistic designs, we present an inverse approach to transformation optics: rather than starting with a coordinate transformation, we consider the range of material properties that can be achieved experimentally. Within this parameter space, we can generate a customized coordinate transformation to achieve the desired functionality using a simple iterative method. The resulting design is guaranteed to be suitable for fabrication.

To verify our approach, we design a near-infrared invisibility cloak based on arrays of oriented silver nanowires embedded in a dielectric background, which can exhibit a wide range of anisotropic effective indices (Figure 1). This type of metamaterial has been used in designs for invisibility cloaks in the optical regime (1). However, such designs prescribe material properties that may be difficult to implement. For example, if we are restricted to a fixed nanowire aspect ratio (represented by the red line in Figure 1), we can no longer realize a cloak based on the proposed linear mapping.

Instead, we construct a custom mapping function based on the restricted parameter space (Figure 2). The mapping function connects the virtual coordinate r to the physical coordinate R, and is related to the anisotropic index within the cloak: $n_r = r/R$, and $n_\theta = \mathrm{d}r/\mathrm{d}R$. Starting from the boundary of the cloak where $R = r$ we select the structure that is index-matched to free space. This defines the slope of the mapping function, which we use to iteratively define material parameters in the interior of the cloaking shell. Eventually the mapping function reaches zero, which completes the cloaking transformation. In order to handle realistic materials, we also have to consider absorption loss. Typical transformation optical designs assume lossless materials or require sophisticated loss compensation strategies using active materials. Instead we extend our inverse method to include transformations of complex coordinates (2), allowing us to generate optimized cloaking transformations that incorporate the geometry-dependent anisotropic absorption in the metamaterial. We use finite-difference time-domain simulations to characterize the scattering properties of the resulting cloak, and compare the performance to standard transformation designs.

This investigation reveals an inherent non-uniqueness of coordinate transformations: there may exist multiple coordinate transformations within the given parameter space, which all qualify as invisibility cloaks.

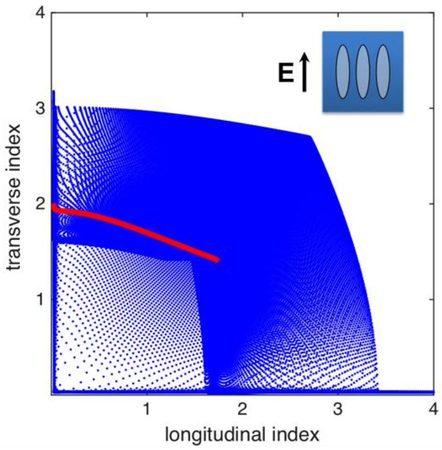

Figure 1. Effective anisotropic index in a nanowire metamaterial. The blue dots show the index values for longitudinal and transverse polarizations at a wavelength of 1064nm. Each dot corresponds to a different nanowire aspect ratio and filling fraction. The red line indicates structures with a fixed aspect ratio of 1:4.

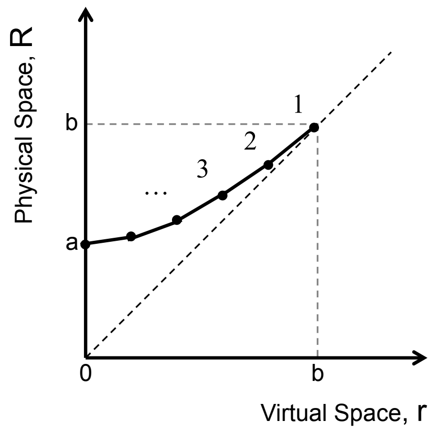

Figure 2. Generating a transformation function from material properties. The transformation function connects the virtual coordinate *r* with the physical coordinate *R*. First, the transformation is fixed at the boundary $R = r = b$. The structure at this point determines the slope in the transformation, which we use to calculate a new point. This process continues iteratively until $r = 0$.

Even in the case of severely restricted spaces, we are guaranteed to find a cloaking transformation if either component of the anisotropic index spans the range n = 0 to n = 1. By extension, unrestricted parameter spaces will necessarily contain an infinite number of cloaking transformations. We can leverage these additional degrees of freedom to improve the performance of inverse transformation optical designs. Using simulated annealing, we generate cloak designs that are optimized for easier fabrication, reduced loss, and impedance matching. Since the design process is based entirely on the optical properties of specific nanostructures, the resulting mapping is appropriate for implementations of transformation optics using realistic materials. This method allows for the design of transformation optical devices that directly incorporate the effects of optical absorption. In addition, the proposed framework enables easy comparison or combination of different metamaterial platforms within a single device.

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REFERENCES

1. Cai, W., U. Chettiar, A. Kildishev, and V. Shalaev, "Optical cloaking with metamaterials" *Nature Photonics*, Vol. 1, 224-227, 2007.
2. Han, T., C. Qiu, J. Hao, X. Tang and S. Zouhdi, "Gain-assisted transformation optics" *Optics Express*, Vol. 19, No. 9, 8610-8615, 2011.