Wrapping light around a hair

University of Missouri-Rolla
Rolla, MO, 21 October 2004
and also....

at Harvard:

Jonathan Aschom
Mengyan Shen
Iva Maxwell
James Carey
Brian Tull
Dr. Yuan Lu
Dr. Richard Schalek

at Zhejiang University:

Dr. Sailing He
Dr. Jingyi Lou
Xuewen Chen
Liu Liu
Zhanghua Han

Dr. Ray Mariella (LLNL)
“I managed to illuminate the interior of a stream in a dark space. I have discovered that this strange arrangement offers one of the most beautiful, and most curious experiments that one can perform in a course on Optics.”

Daniel Colladon, Comptes Rendus, 15, 800–802 (1842)
D. Colladon, *La Nature*, 325 (1884)
Outline

• waveguiding
• nanowire fabrication
• optical properties
Waveguiding

two crossed planar waves...
Waveguiding

...cause an interference pattern
$E = 0$ on the nodal lines
Waveguiding

...satisfying boundary conditions for planar-mirror waveguide
Waveguiding

transverse standing wave, traveling along axis
Waveguiding

transverse standing wave, traveling along axis
Waveguiding

change angle of incident waves...
Waveguiding

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Waveguiding
change angle of incident waves...
boundary conditions only satisfied for certain $\theta$

standing wave in $y$-direction, traveling in $z$-direction
consider wave incident at angle $\theta$
Waveguiding
twice-reflected wave
self consistency:

\[ AC - AB = 2d \sin \theta = m\lambda \quad (m = 1, 2, \ldots) \]
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so:

\[ \sin \theta_m = m \frac{\lambda}{2d} \]
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so:

\[ \sin \theta_m = m \frac{\lambda}{2d} \]
number of modes:

\[ M = \frac{2d}{\lambda} \]
now consider a planar dielectric waveguide
rays incident at angle $\theta > \pi/2 - \theta_c$ are unguided
rays incident at angle $\theta < \frac{\pi}{2} - \theta_c$ are guided
rays incident at angle $\theta < \pi/2 - \theta_c$ are guided
self consistency:

\[ AC - AB = 2d \sin \theta - \frac{\phi_r}{\pi} \lambda = m\lambda \quad (m = 0, 1, 2\ldots) \]
self consistency:

\[ AC - AB = 2d \sin \theta - \frac{\varphi_r}{\pi} \lambda = m\lambda \quad (m = 0, 1, 2 \ldots) \]

so:

\[
\tan \left( \frac{\pi d}{\lambda} \sin \theta - m \frac{\pi}{2} \right) = \left( \frac{\sin^2 \left( \frac{\pi}{2} - \theta \right) - \theta_c}{\sin^2 \theta} - 1 \right)^{1/2}
\]
**Waveguiding**

**self consistency:**

\[ AC - AB = 2d \sin \theta - \frac{\varphi_r}{\pi} \lambda = m\lambda \quad (m = 0, 1, 2\ldots) \]

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Waveguiding

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Waveguiding

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number of modes:

\[ M = \frac{\sin\left(\frac{\pi}{2} - \theta_c\right)}{\lambda/2d} \]
number of modes:

\[ M = \frac{\sin(\pi/2 - \theta_c)}{\lambda/2d} \]

or:

\[ M = 2 \frac{d}{\lambda} (n_1^2 - n_2^2)^{1/2} \]
propagation constant of guided wave:

\[ \beta_m^2 = k^2 - k_y^2 = k^2 - \frac{m^2 \pi^2}{d^2} \]

group velocity:

\[ v_m = c \cos \theta_m \]
single mode condition for 600-nm light:

\[ M = \frac{2d}{\lambda} \quad 300 < d < 600 \text{ nm} \]

planar mirror

\[ M = 2 \frac{d}{\lambda} (n_1^2 - n_2^2)^{1/2} \quad d < 268 \text{ nm} \]

dielectric
Waveguiding

single mode condition for 600-nm light:

planar mirror

\[ M = \frac{2d}{\lambda} \quad \text{for} \quad 300 < d < 600 \text{ nm} \]

dielectric

\[ M = 2 \frac{d}{\lambda}(n_1^2 - n_2^2)^{1/2} \quad \text{for} \quad d < 268 \text{ nm} \]

can make \( d \) larger by making \( n_1 - n_2 \) smaller!
Vector potential obeys:

\[ \nabla^2 \vec{A} + \omega^2 \mu_0 \epsilon \vec{A} = -i \omega \mu_0 \nabla \epsilon \Phi \]
Waveguiding

Vector potential obeys:

$$\nabla^2 \vec{A} + \omega^2 \mu_0 \epsilon \vec{A} = 0$$
Waveguiding

Vector potential obeys:

$$\nabla^2 \vec{A} + \omega^2 \mu_0 \epsilon \vec{A} = 0$$

Substituting

$$\vec{A} = \hat{y}u(x,y)e^{-i\beta z}$$
Waveguiding

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Substituting

\[ \vec{A} = \hat{y} u(x,y) e^{-i\beta z} \]

yields:

\[ \nabla_T^2 u + [-\beta^2 + \omega^2 \mu \epsilon(r)] u = 0 \]
Waveguiding

Vector potential obeys:

$$\nabla^2 \mathbf{A} + \omega^2 \mu_0 \epsilon \mathbf{A} = 0$$

Substituting

$$\mathbf{A} = \hat{y} u(x,y) e^{-i\beta z}$$

yields:

$$\left( \nabla_T^2 u + [- \beta^2 + \omega^2 \mu \epsilon(r)] \right) u = 0$$

Compare to time-independent Schrödinger equation:

$$\nabla^2 \psi + \frac{2m}{\hbar^2} [E - V(r)] \psi = 0$$
Waveguiding
Waveguiding
Waveguiding
Waveguiding
Waveguiding
Waveguiding

single mode condition for 600-nm light:

\[ M = 2 \frac{d}{\lambda} (n_1^2 - n_2^2)^{1/2} \]

without cladding: \( d < 268 \text{ nm} \)

Add cladding with 0.4% index difference:

\( d < 5 \text{ \( \mu \)m} \)
Waveguiding

commercial single-mode fiber (Corning Titan®)

<table>
<thead>
<tr>
<th></th>
<th>core</th>
<th>cladding</th>
</tr>
</thead>
<tbody>
<tr>
<td>index</td>
<td>$n_1 = 1.468$</td>
<td>$n_2 = 1.462$</td>
</tr>
<tr>
<td>diameter:</td>
<td>8.3 µm</td>
<td>125.0 ± 1.0 µm</td>
</tr>
</tbody>
</table>

operating wavelength: $\lambda = 1310$ nm/1550 nm
Waveguiding

drawbacks of clad fibers:

• weak confinement
• no tight bending
• coupling requires splicing
Waveguiding
Outline

• waveguiding
• nanowire fabrication
• optical properties
Nanowire fabrication

two-step drawing process

standard
fiber
Nanowire fabrication

two-step drawing process
Nanowire fabrication

two-step drawing process

standard fiber

1-μm silica wire

drawing
Nanowire fabrication

two-step drawing process

standard fiber

1-µm silica wire

drawing

sapphire taper
Nanowire fabrication

two-step drawing process

standard fiber

1-μm silica wire

drawing

sapphire taper

silica wire
Nanowire fabrication

two-step drawing process

standard fiber

1-μm silica wire

drawing

flame

sapphire taper

silica wire
Nanowire fabrication

two-step drawing process

standard fiber

1-µm silica wire

drawing

flame

sapphire taper

silica wire
Nanowire fabrication

two-step drawing process

standard fiber

1-μm silica wire

flame

sapphire taper

silica wire

drawing
Nanowire fabrication
Nanowire fabrication

Nanowire fabrication
Nanowire fabrication
Nanowire fabrication
Nanowire fabrication

200 µm
Nanowire fabrication
Nanowire fabrication
Nanowire fabrication

50 µm
Nanowire fabrication

20 µm
Nanowire fabrication

10 µm

10 µm
Nanowire fabrication

6 µm
Nanowire fabrication

4 µm
Nanowire fabrication
Nanowire fabrication

312 nm

1 µm
# Waveguiding

## Specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter $D$</td>
<td>down to 20 nm</td>
</tr>
<tr>
<td>Length $L$</td>
<td>up to 90 mm</td>
</tr>
<tr>
<td>Aspect ratio $D/L$</td>
<td>up to $10^6$</td>
</tr>
<tr>
<td>Diameter uniformity $\Delta D/L$</td>
<td>$2 \times 10^{-6}$</td>
</tr>
</tbody>
</table>
Nanowire fabrication

\[ d = 260 \text{ nm} \]

\[ L = 4 \text{ mm} \]
Nanowire fabrication

240-nm wire
Nanowire fabrication

RMS roughness < 0.5 nm

20 nm
Nanowire fabrication

20 nm
Nanowire fabrication

measure tensile stress at breaking point
Nanowire fabrication

bend to breaking point

50 µm
Nanowire fabrication

bend to breaking point

50 \mu m
Nanowire fabrication

bend to breaking point

50 µm
Nanowire fabrication

minimum bending radius $R_{EB}$
gives tensile stress:

$$\sigma = \frac{ED}{2R_{EB}}$$

$E = $ Young’s modulus
$D = $ wire diameter
Waveguiding

tensile strength

![Graph showing tensile stress vs. nanowire diameter](image-url)
Nanowire fabrication
Nanowire fabrication
Nanowire fabrication
Outline

• waveguiding

• nanowire fabrication

• optical properties
Optical properties

coupling light into nanowires

diagram showing a fiber taper leading to a nanowire
Optical properties

coupling light into nanowires

objective

fiber taper

nanowire
Optical properties

coupling light into nanowires
Optical properties

280-nm nanowire

360 nm

450 nm
Optical properties
Optical properties
Optical properties

Poynting vector profile for 800-nm nanowire
Optical properties

Poynting vector profile for 800-nm nanowire
Optical properties

Poynting vector profile for 800-nm nanowire

evanescent wave
Optical properties

Poynting vector profile for 600-nm nanowire
Optical properties

Poynting vector profile for 500-nm nanowire
Optical properties

Poynting vector profile for 400-nm nanowire
Optical properties

Poynting vector profile for 300-nm nanowire
Optical properties

Poynting vector profile for 200-nm nanowire
Waveguiding

fraction of power carried in core

![Graph showing the fraction of power carried in core for different diameters.](attachment:image.png)
Optical properties
Optical properties

coupling light between nanowires

Diagram:
- Support
- Nanowire
- Fiber taper
Optical properties

coupling light between nanowires
Optical properties

coupling light between nanowires

- fiber taper
- light
- support
- nanowire
- fiber taper
Optical properties

coupling light between nanowires
Optical properties

coupling light between nanowires
Optical properties

“tunneling” of light
Optical properties

50 µm
Optical properties

50 μm
Optical properties

loss measurement

Optical properties

Loss measurement

Optical properties

loss measurement

Optical properties

loss measurement

Optical properties

loss measurement

Optical properties

loss measurement

![Graph showing optical loss vs. wire diameter. The graph has a logarithmic scale on the y-axis and a linear scale on the x-axis. Points are plotted along a line, with a label at 633 nm. The x-axis represents wire diameter in nm, ranging from 0 to 1200 nm. The y-axis represents optical loss in dB/mm, ranging from 0.0001 to 10.]({})

Optical properties

loss measurement

Optical properties

loss a single-mode diameter < 0.1 dB/mm

Optical properties
Optical properties
Optical properties

minimum bending radius: 5.6 µm
Optical properties

virtually no loss through 5 µm corner!
Optical properties

dispersion:

- modal dispersion
- material dispersion
- waveguide dispersion
- nonlinear dispersion
Optical properties

waveguide dispersion

Optics Express, 12, 1025 (2004)
Optical properties

waveguide dispersion

**Optics Express, 12, 1025 (2004)**
Optical properties

waveguide dispersion

Optics Express, 12, 1025 (2004)
Optical properties

waveguide dispersion

Optics Express, 12, 1025 (2004)
Optical properties

waveguide dispersion

![Graph showing waveguide dispersion](image)

Optics Express, 12, 1025 (2004)
Optical properties

waveguide dispersion

Optics Express, 12, 1025 (2004)
Optical properties

waveguide dispersion

Optics Express, 12, 1025 (2004)
Optical properties

nonlinear dispersion: \( n = n_0 + n_2 I \)
Optical properties

Nonlinear dispersion: \( n = n_o + n_2I \)
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nonlinear dispersion: \( n = n_0 + n_2 I \)
Optical properties

**nonlinear dispersion:** \[ n = n_o + n_2 I \]
Optical properties
Optical properties
Optical properties

self-phase modulation
Optical properties

self-phase modulation

![Graph showing optical properties and self-phase modulation](image-url)
Optical properties

self-phase modulation
• strong confinement

• very tight bending

• large evanescent wave
outlook

microphotonic components
Outlook

microphotonic components
microphotonic components
Outlook
Outlook
Outlook
Outlook

biosensor
Outlook

biosensor

receptor
Outlook

biosensor

Diagram of a biosensor with an input ('in') and an output ('out') arrow, and receptors indicated in the middle.
Outlook

biosensor

receptor

in

out
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