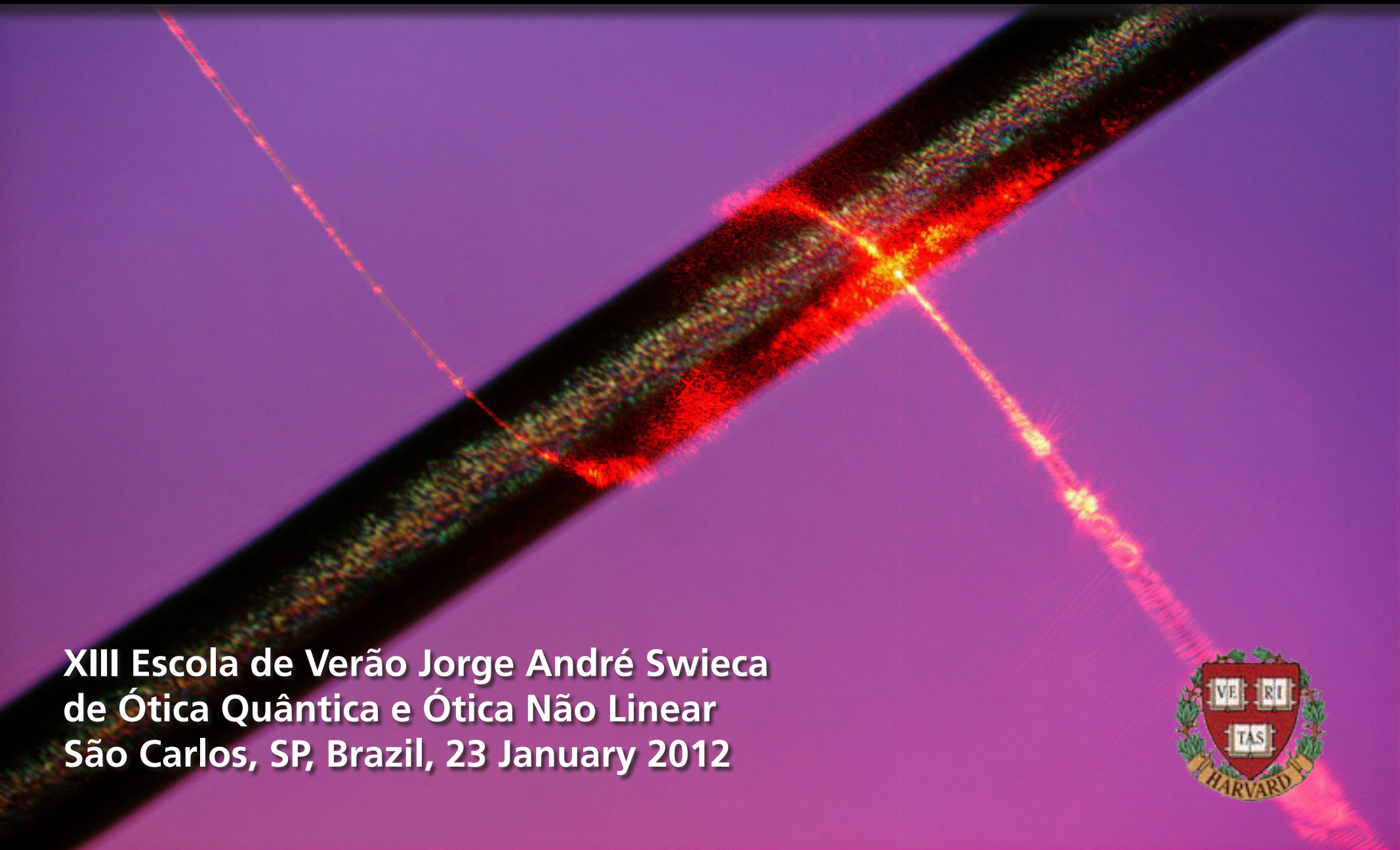
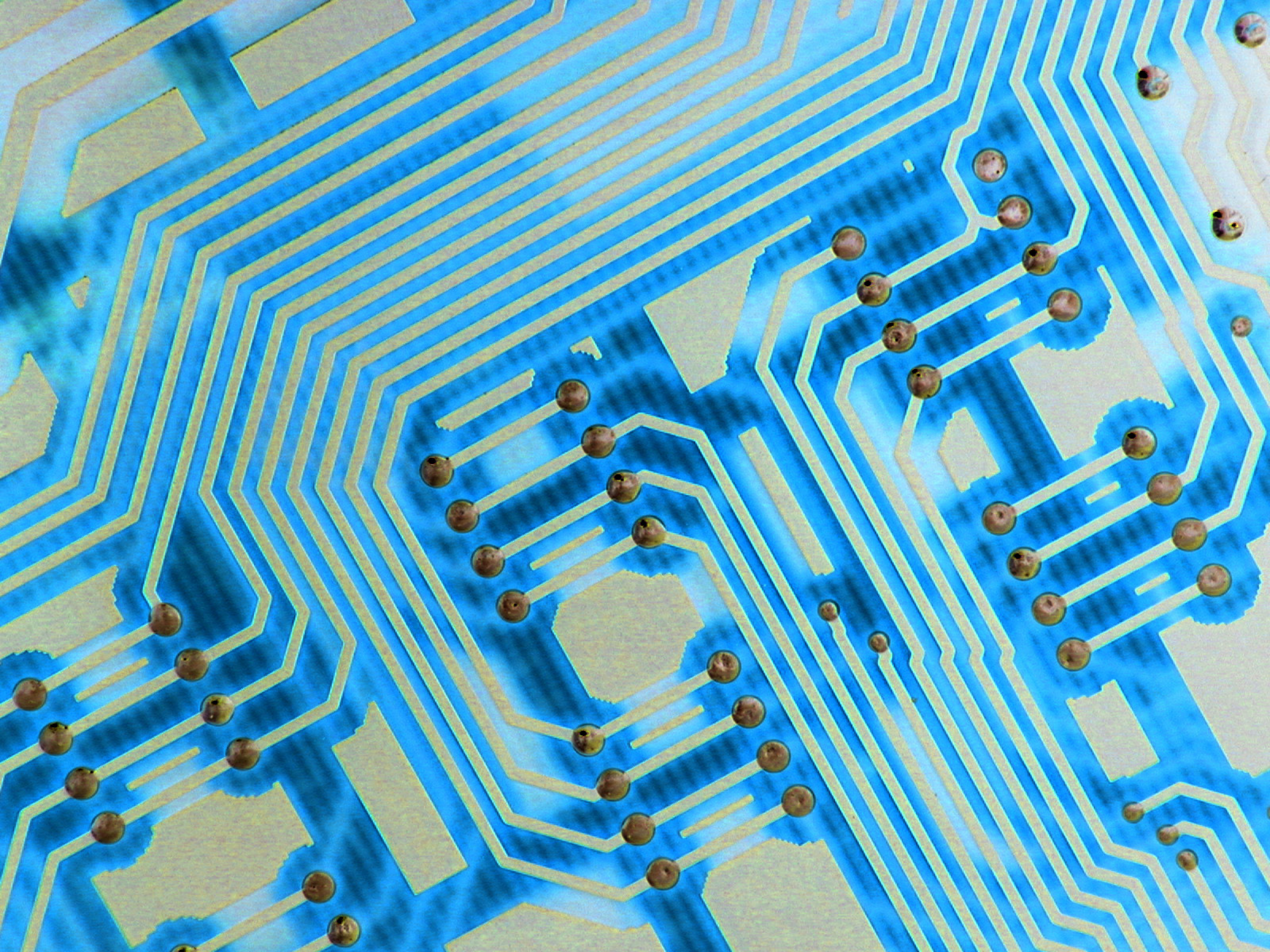
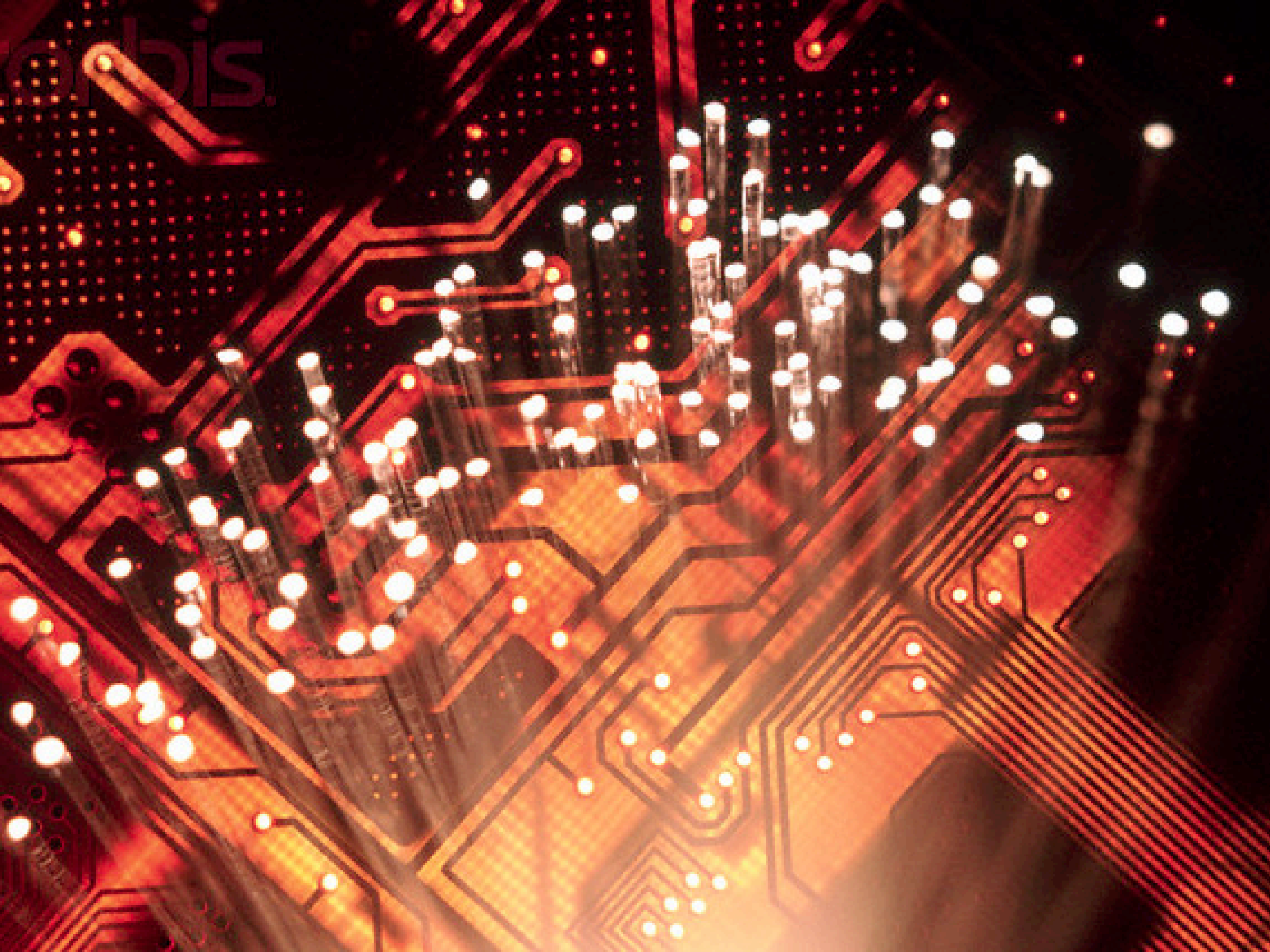
An introduction to femtosecond laser techniques: Part 1



XIII Escola de Verão Jorge André Swieca
de Ótica Quântica e Ótica Não Linear
São Carlos, SP, Brazil, 23 January 2012

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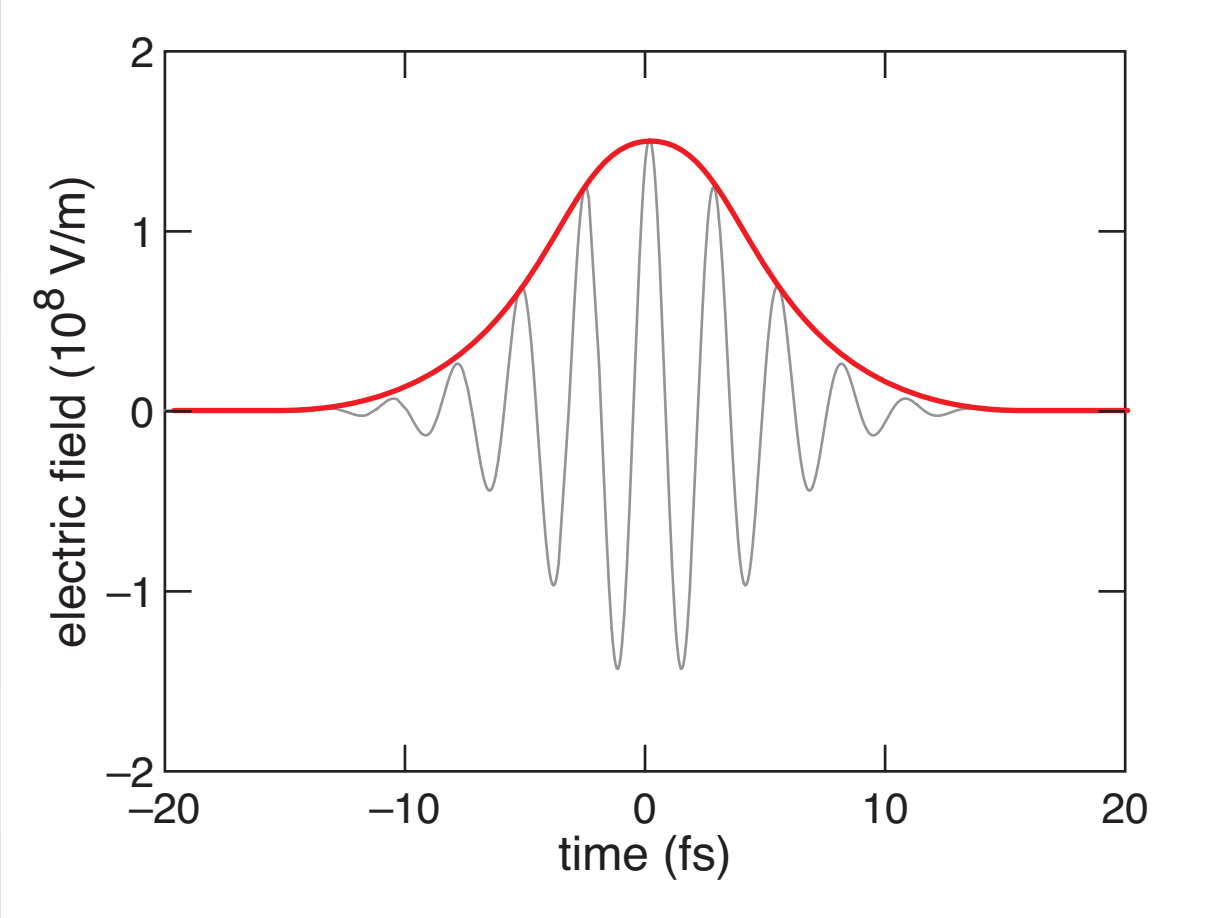
Introduction



Outline

- propagation of pulses
- nonlinear optics
- nanoscale optics
- nonlinear optics at the nanoscale

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Propagation of EM wave through medium

Governed by wave equation

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In dispersive media $n = n\,(\omega)$**.**

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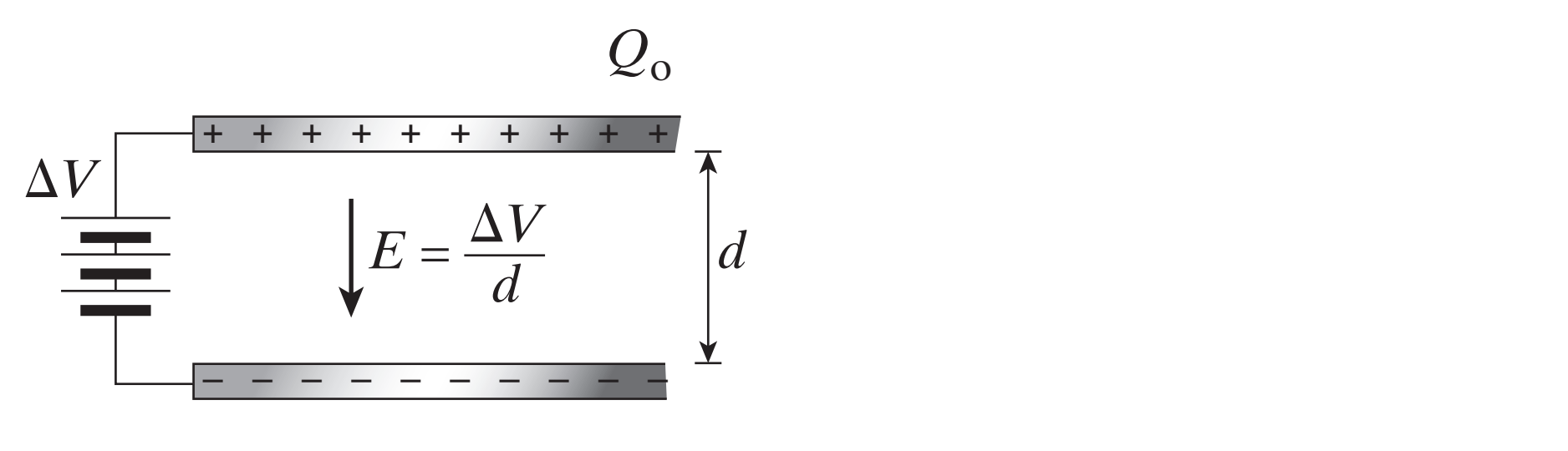
Dielectric constant measures increase in capacitance

$$\epsilon = \frac{C_d}{C_o}$$



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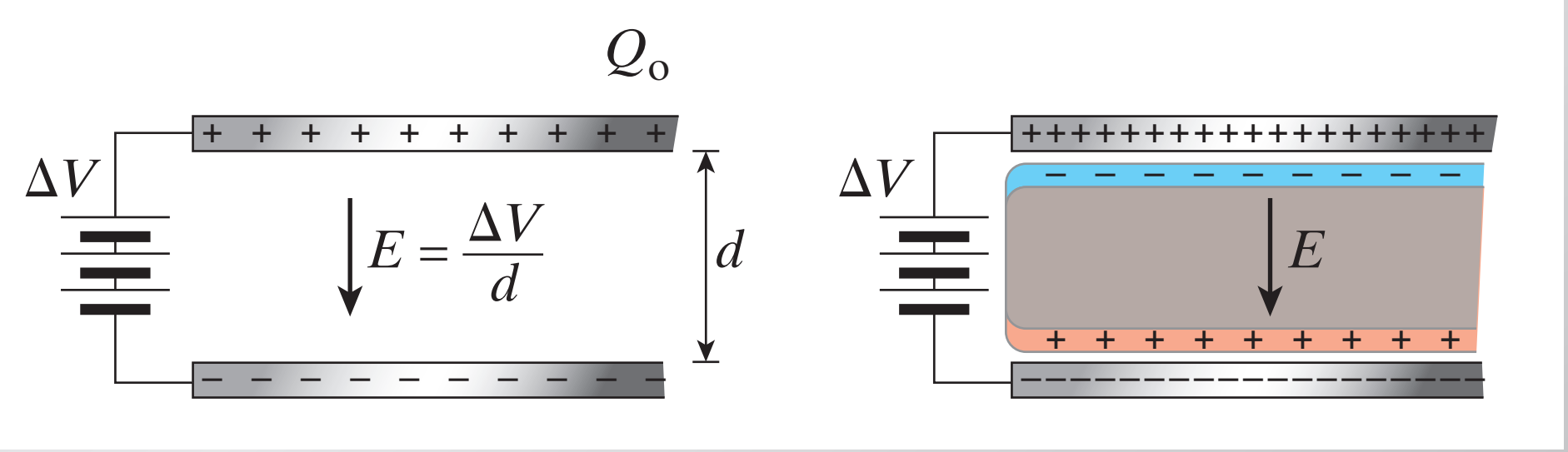

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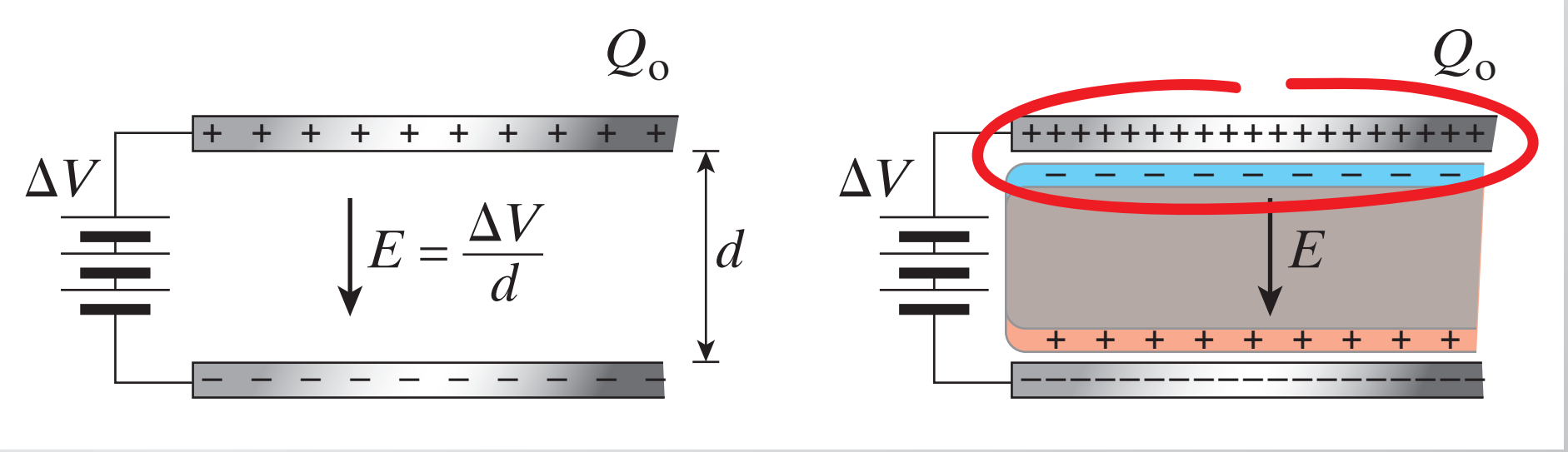

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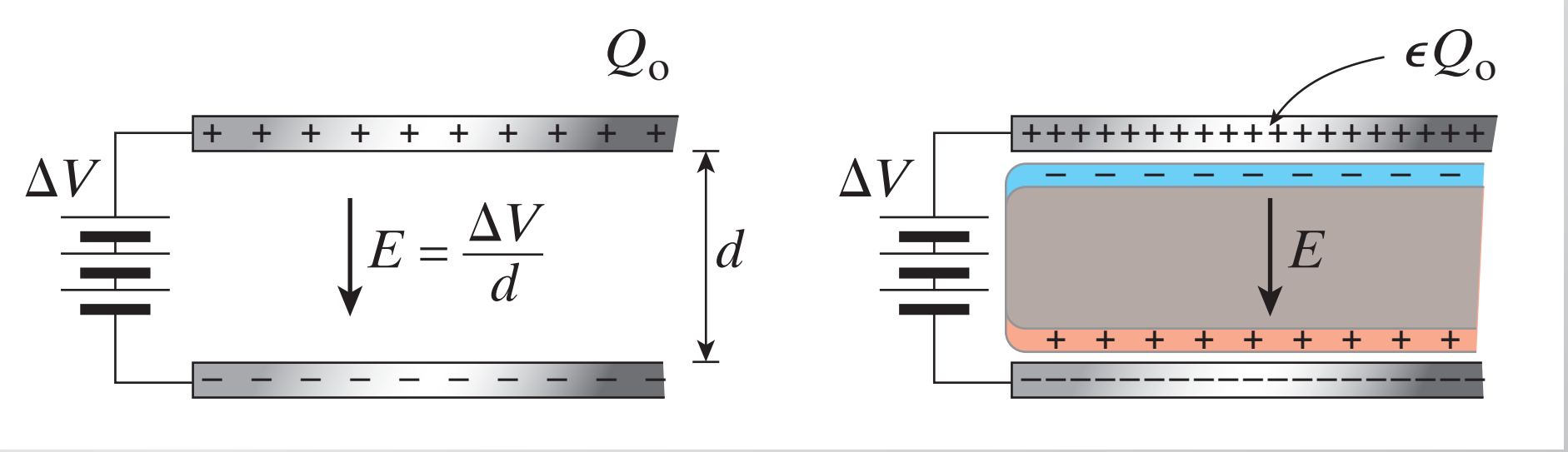
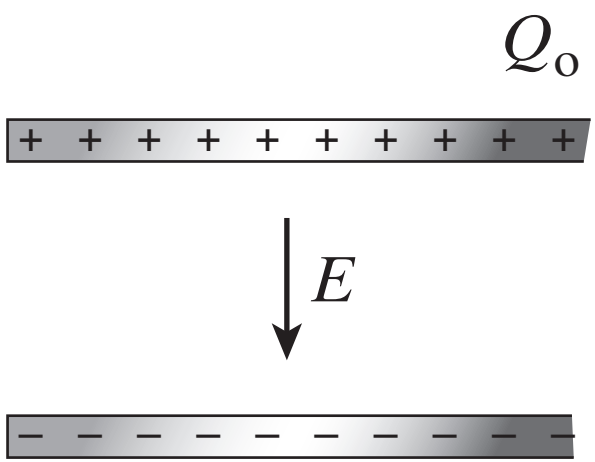


Propagation of EM wave through medium

Alternatively ϵ is measure of attentuation of electric field



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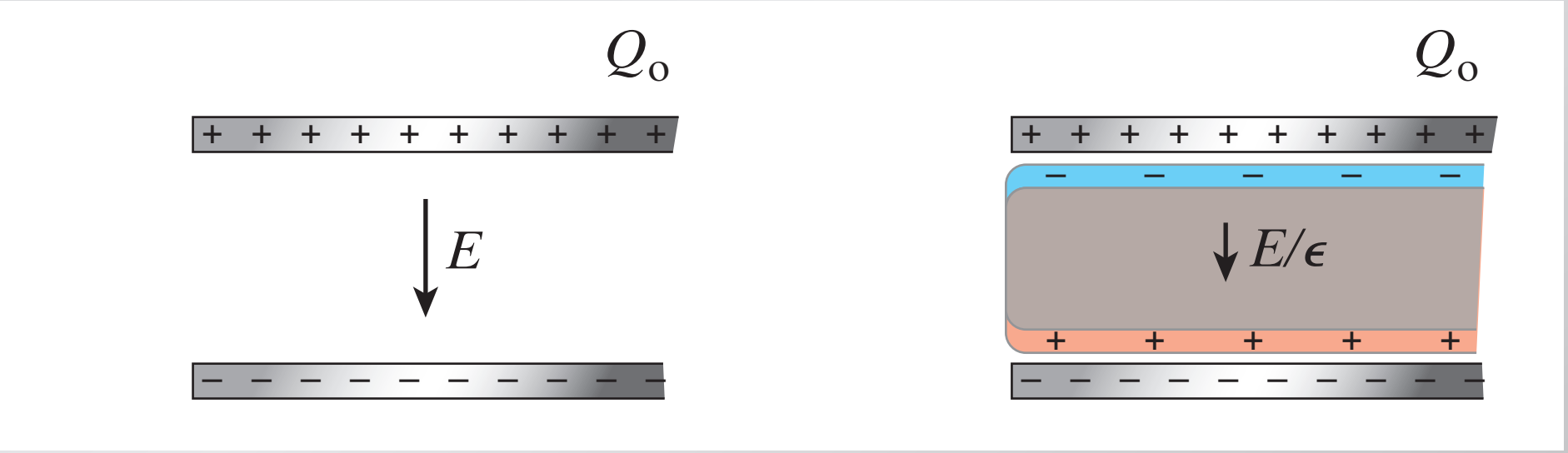
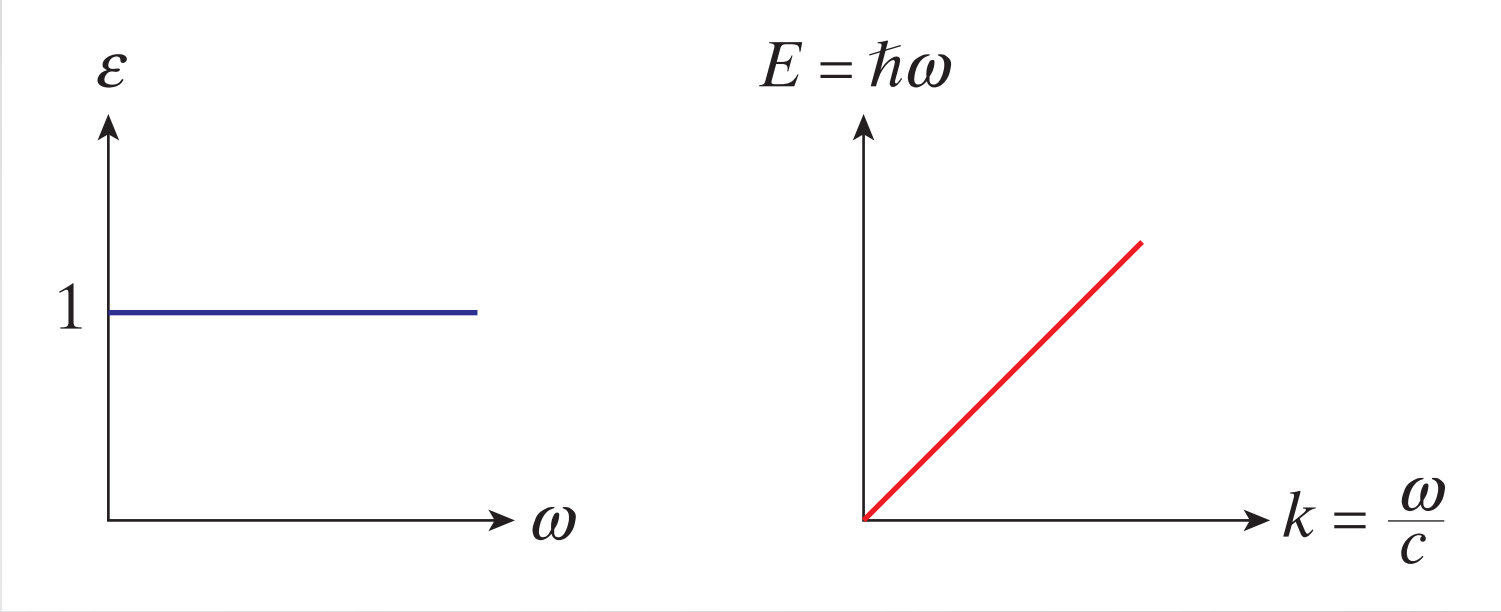


Propagation of EM wave through medium

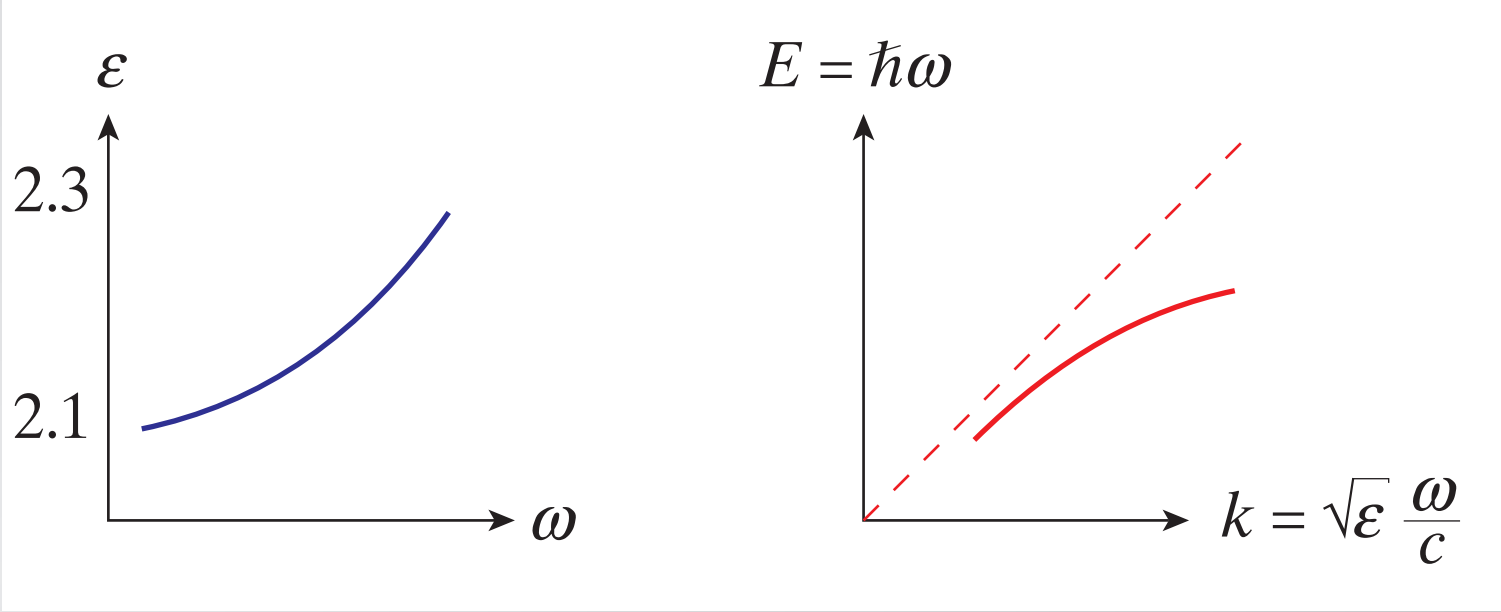
In vacuum: $$f\lambda = \frac{\omega}{k} = c \qquad \Rightarrow \qquad \omega = c\,k$$



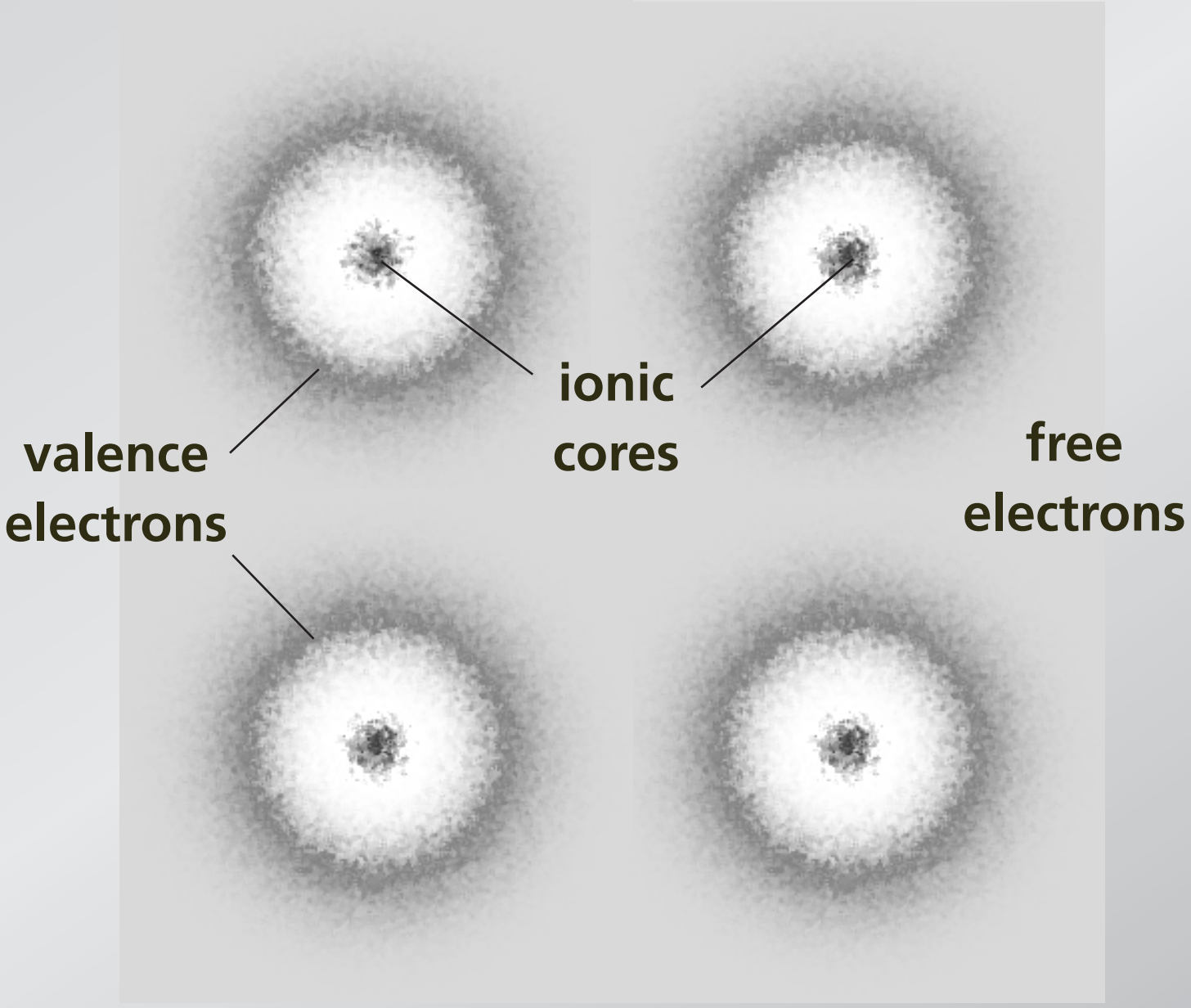
Propagation of EM wave through medium

In medium:

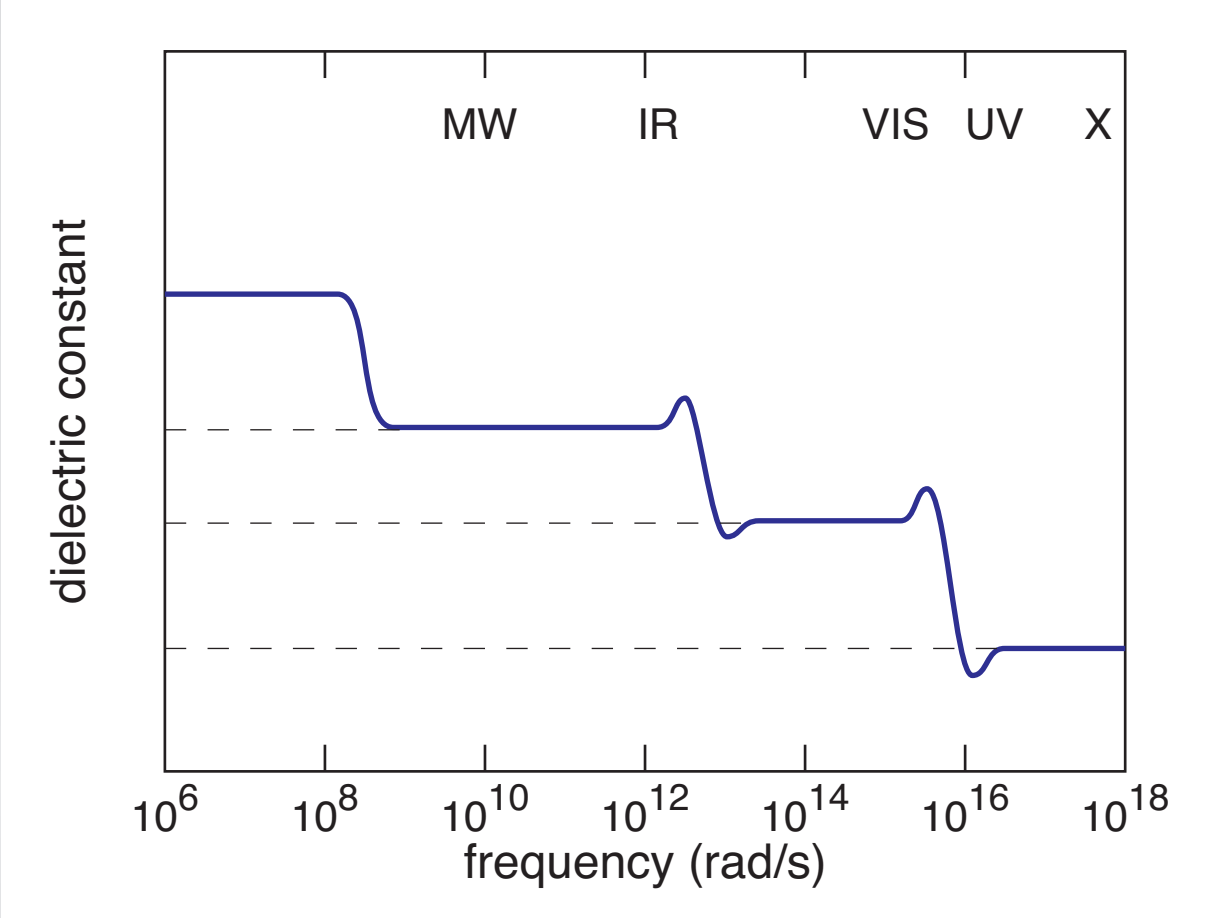
$$v = \frac{c}{\sqrt{\epsilon(\omega)}} = \frac{c}{n} \qquad \Rightarrow \qquad \omega = \frac{c}{\sqrt{\epsilon}} k$$



Which charges participate?



Dielectric function



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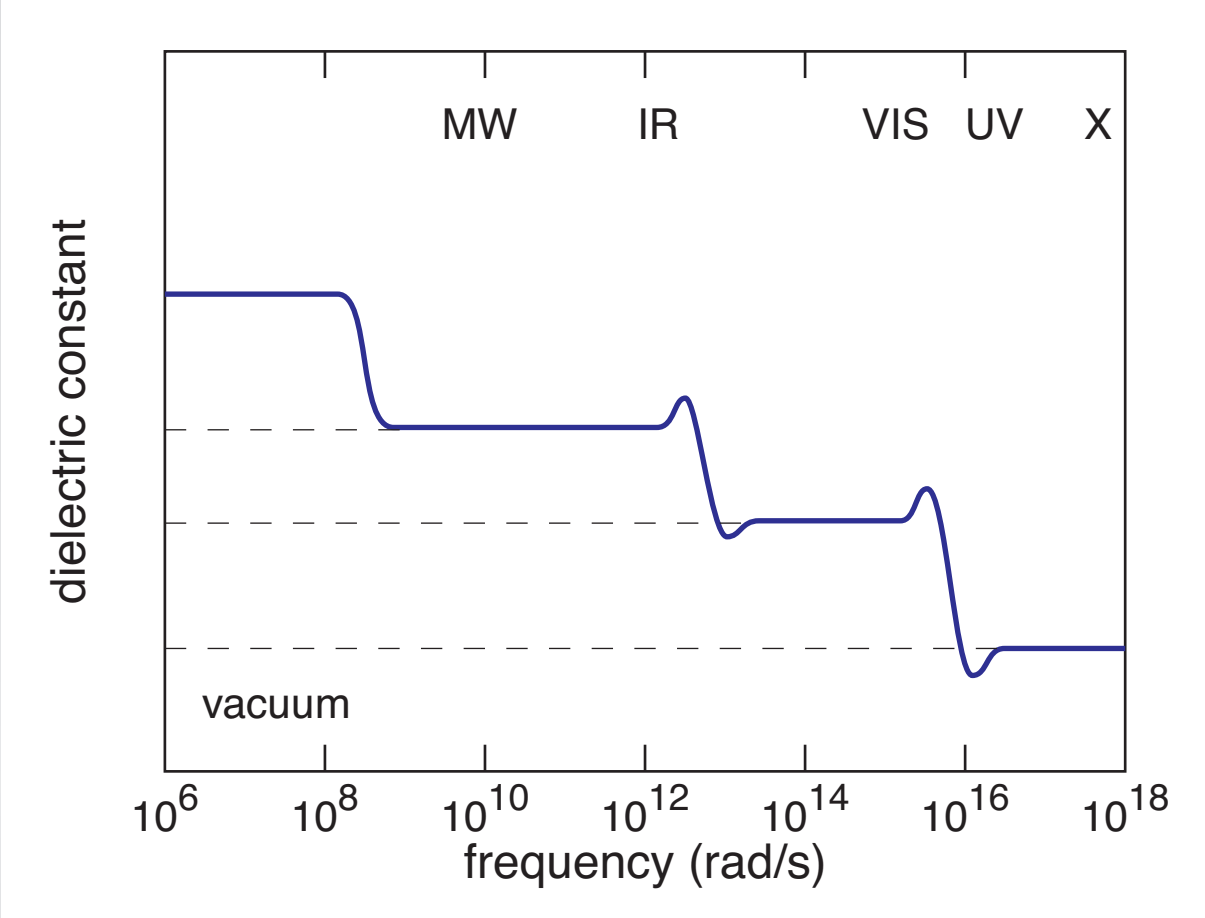

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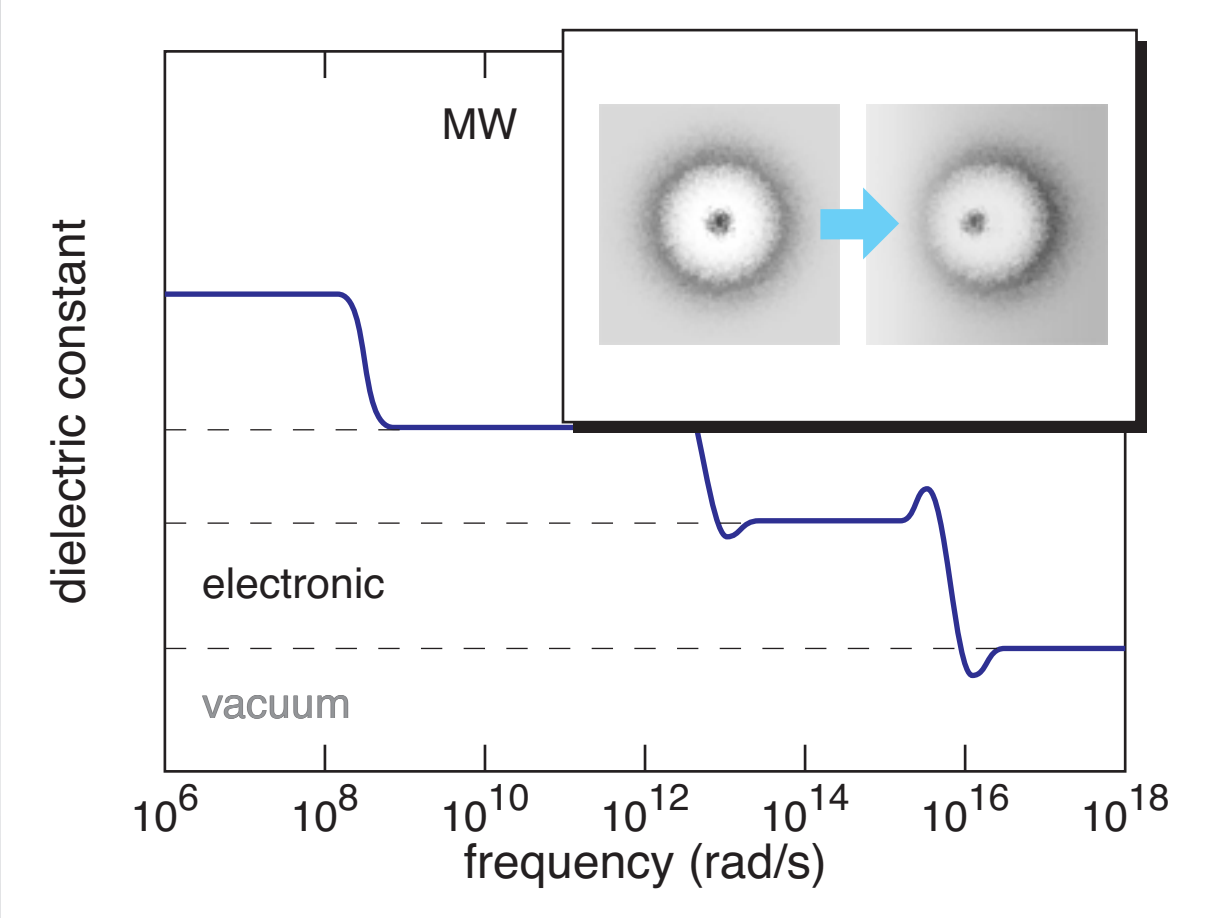

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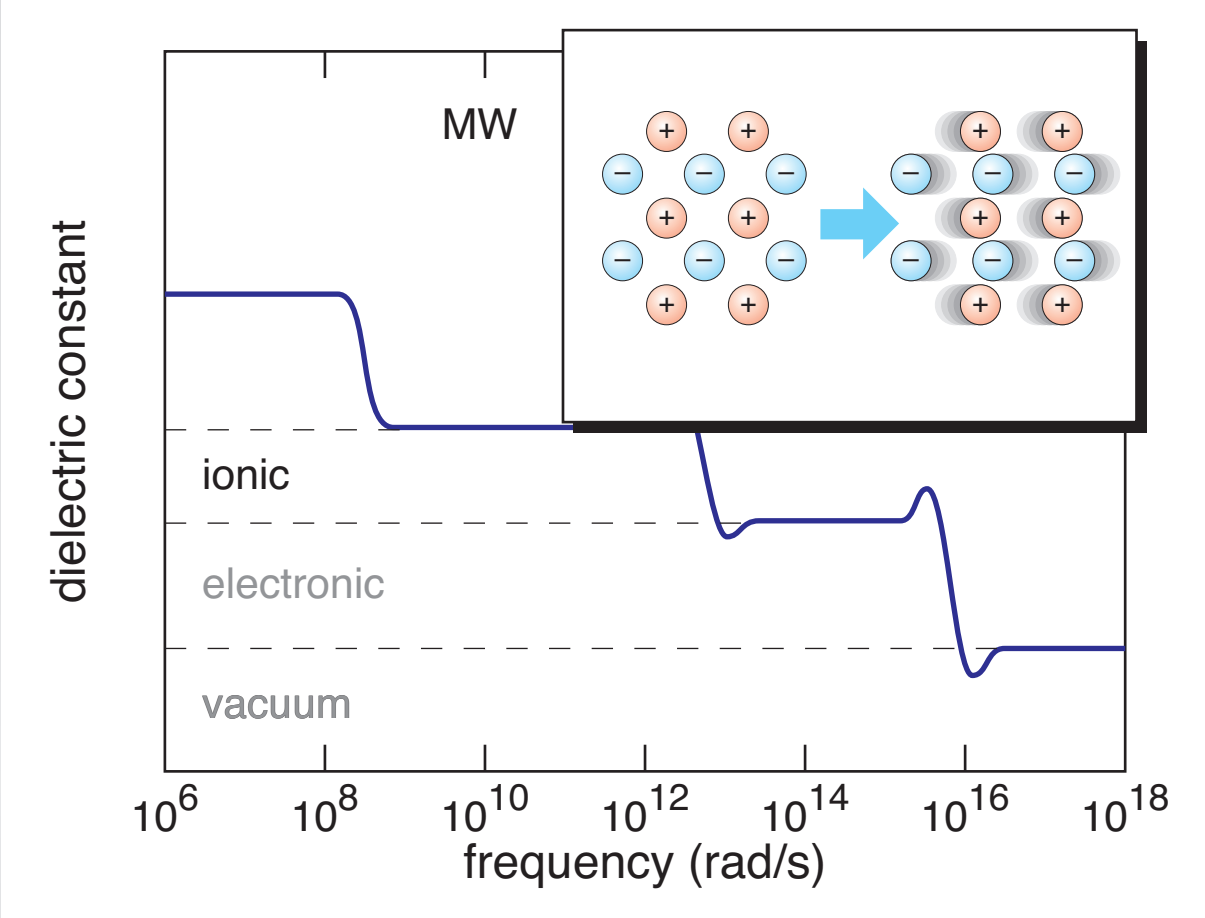

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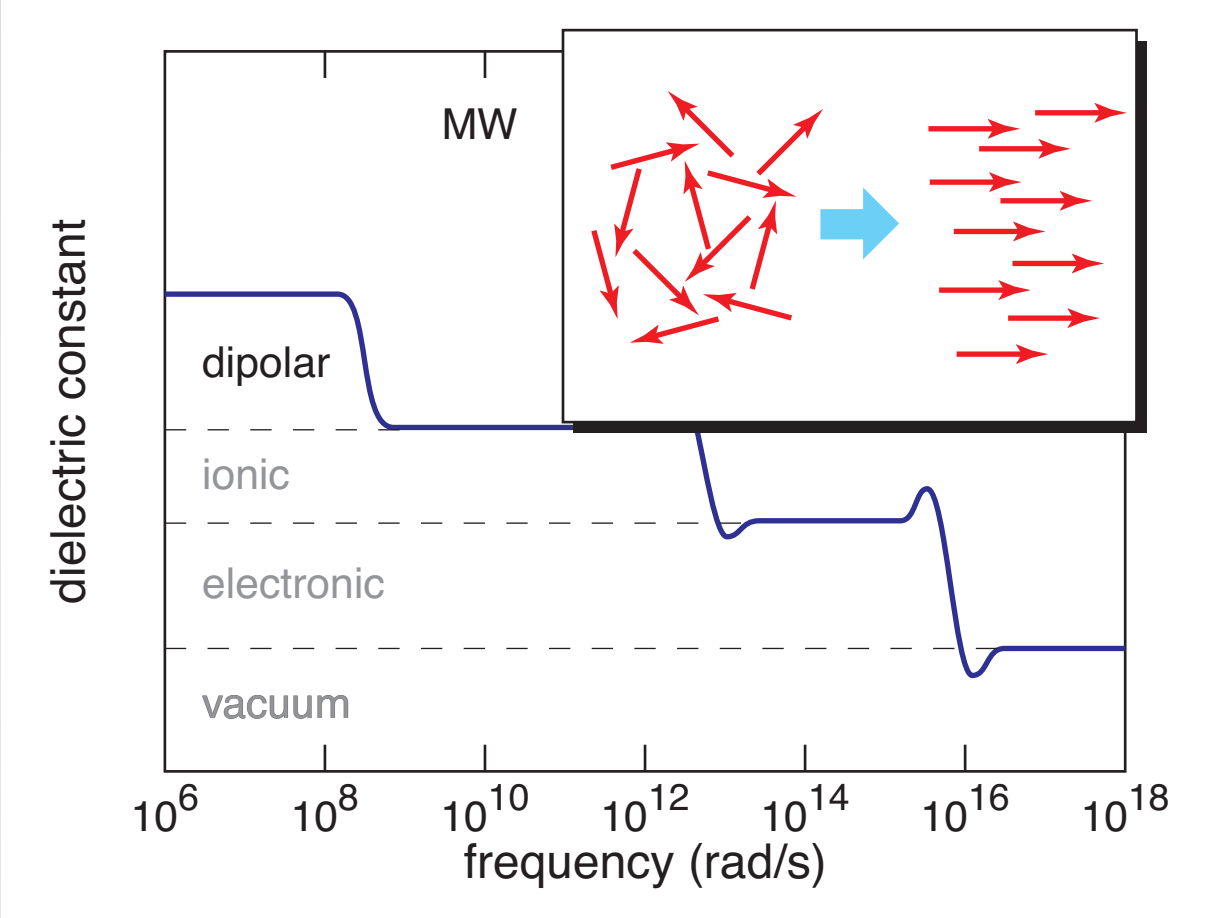

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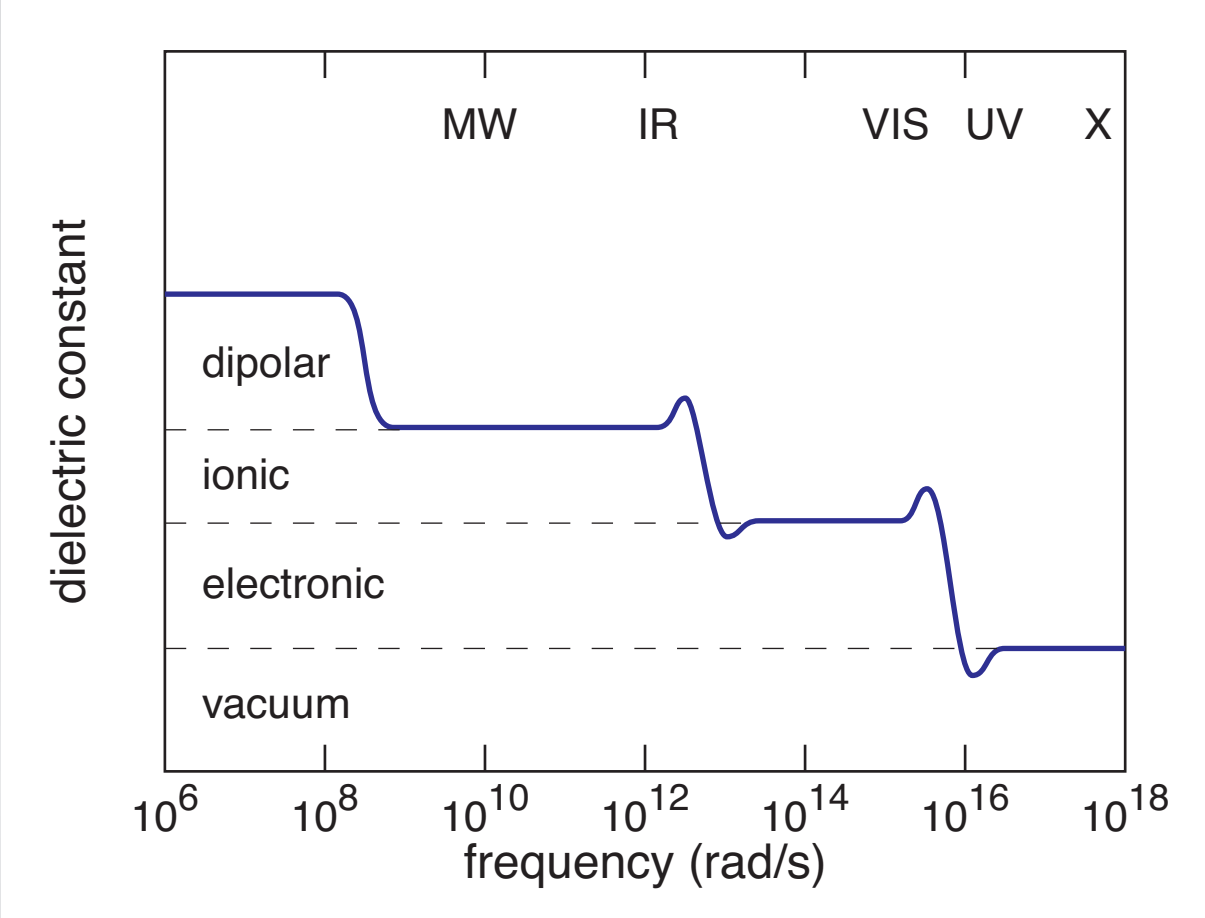

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Electron on a string: $$F_{binding} = -m_e \omega_o^2 x$$

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Steady state: electron oscillates at driving frequency

$$x(t) = x_o e^{-i\omega t}$$

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Polarization

$$P(t) = \left(\frac{Ne^2}{m}\right)\sum_j \frac{f_j}{(\omega_j^2 - \omega^2) - i\gamma_j\omega}E(t) \equiv \epsilon_o \chi_e E(t)$$

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Dielectric function

$$\epsilon(\omega) \equiv 1 + \chi_e = 1 + \frac{Ne^2}{\epsilon_o m} \sum_j \frac{f_j}{(\omega_j^2 - \omega^2) - i\gamma_j\omega}$$

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Q: For a single resonance, is the value of $\epsilon(\omega)$ at high frequency

1. **larger than,**
2. **the same as, or**
3. **smaller than the value at low frequency?**

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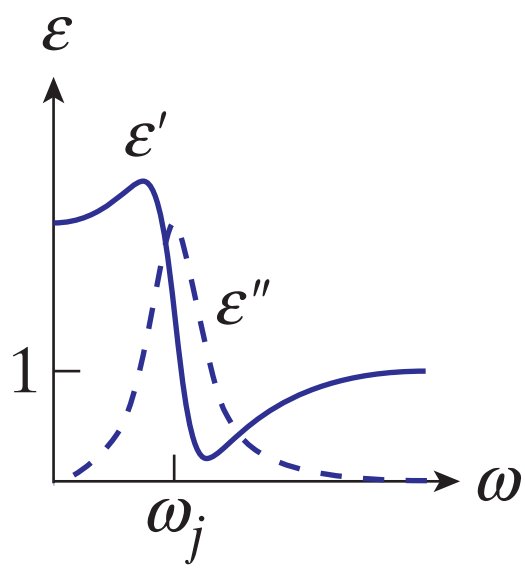
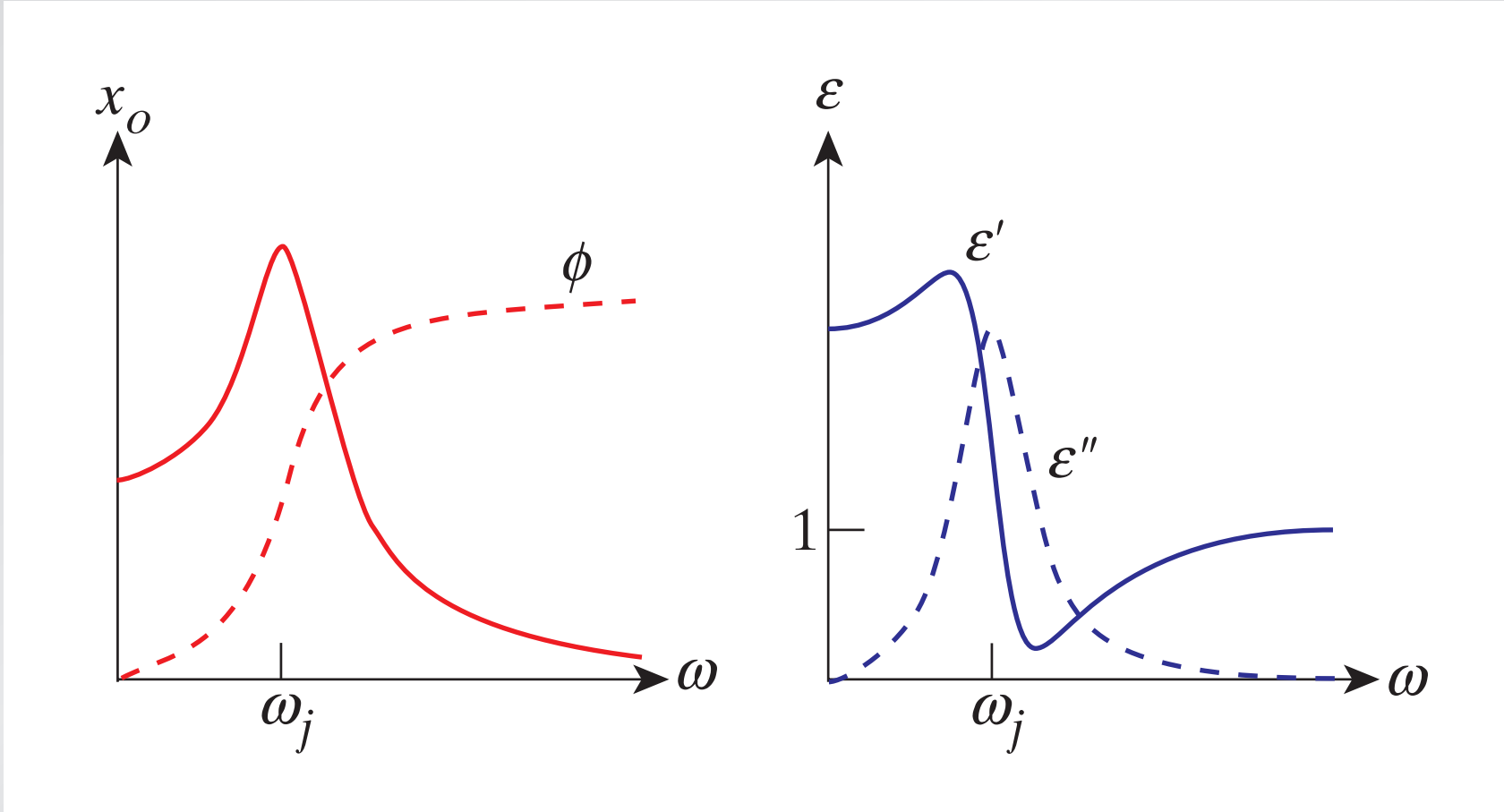


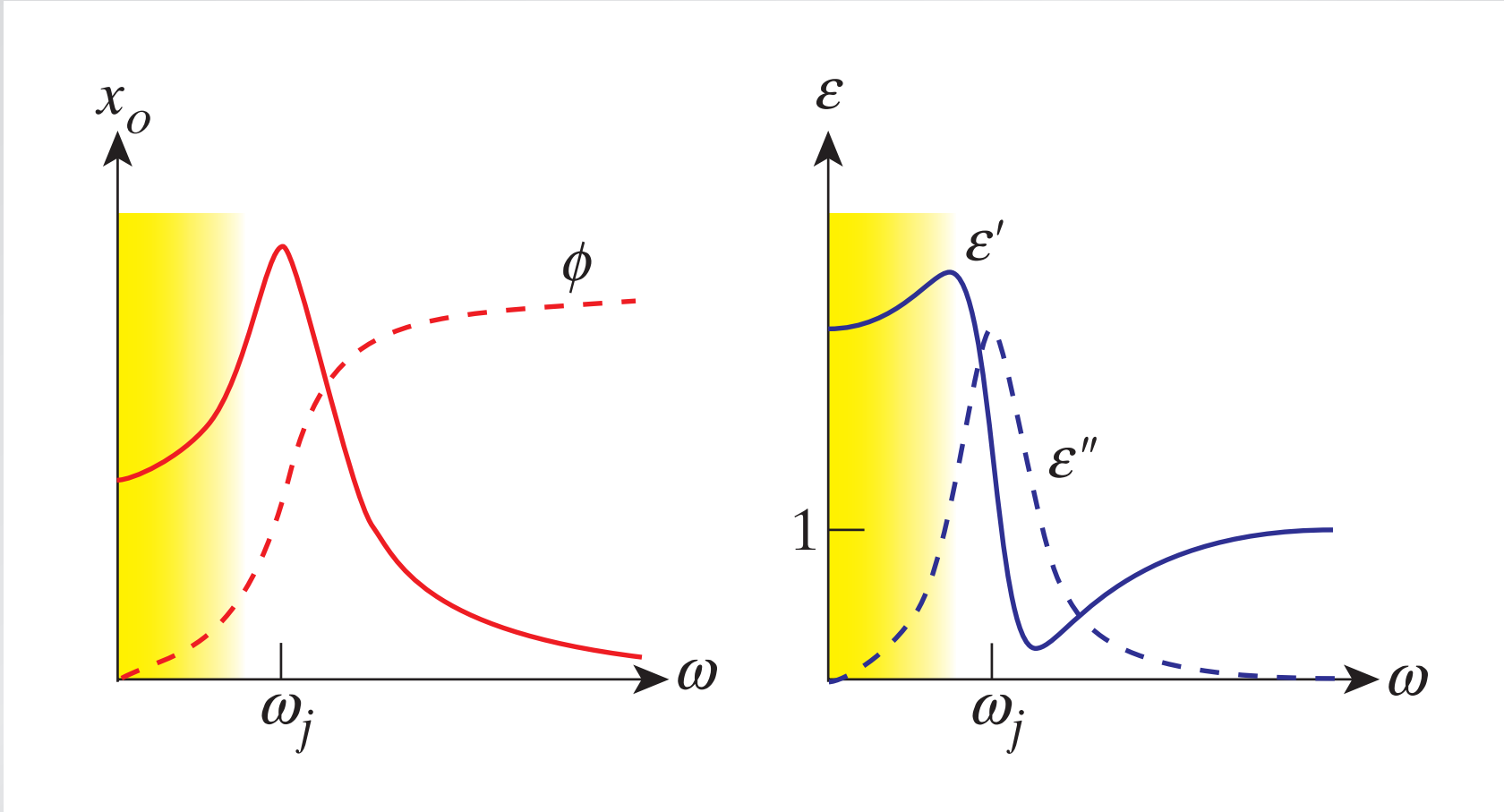
Bound electrons

Amplitude of bound charge oscillation



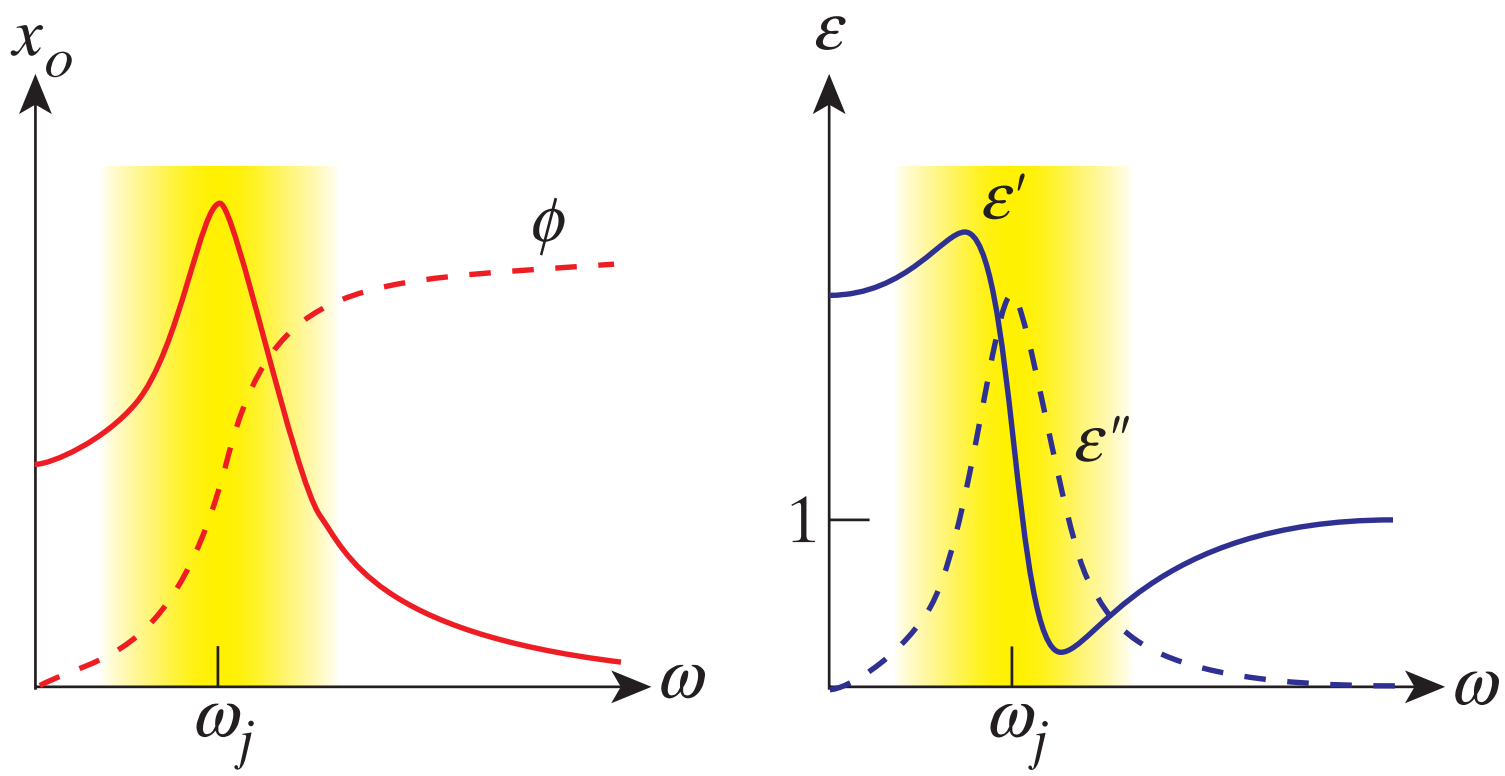
Bound electrons

Below resonance: bound charges keep up with driving field $\Rightarrow$ field attenuated, wave propagates more slowly



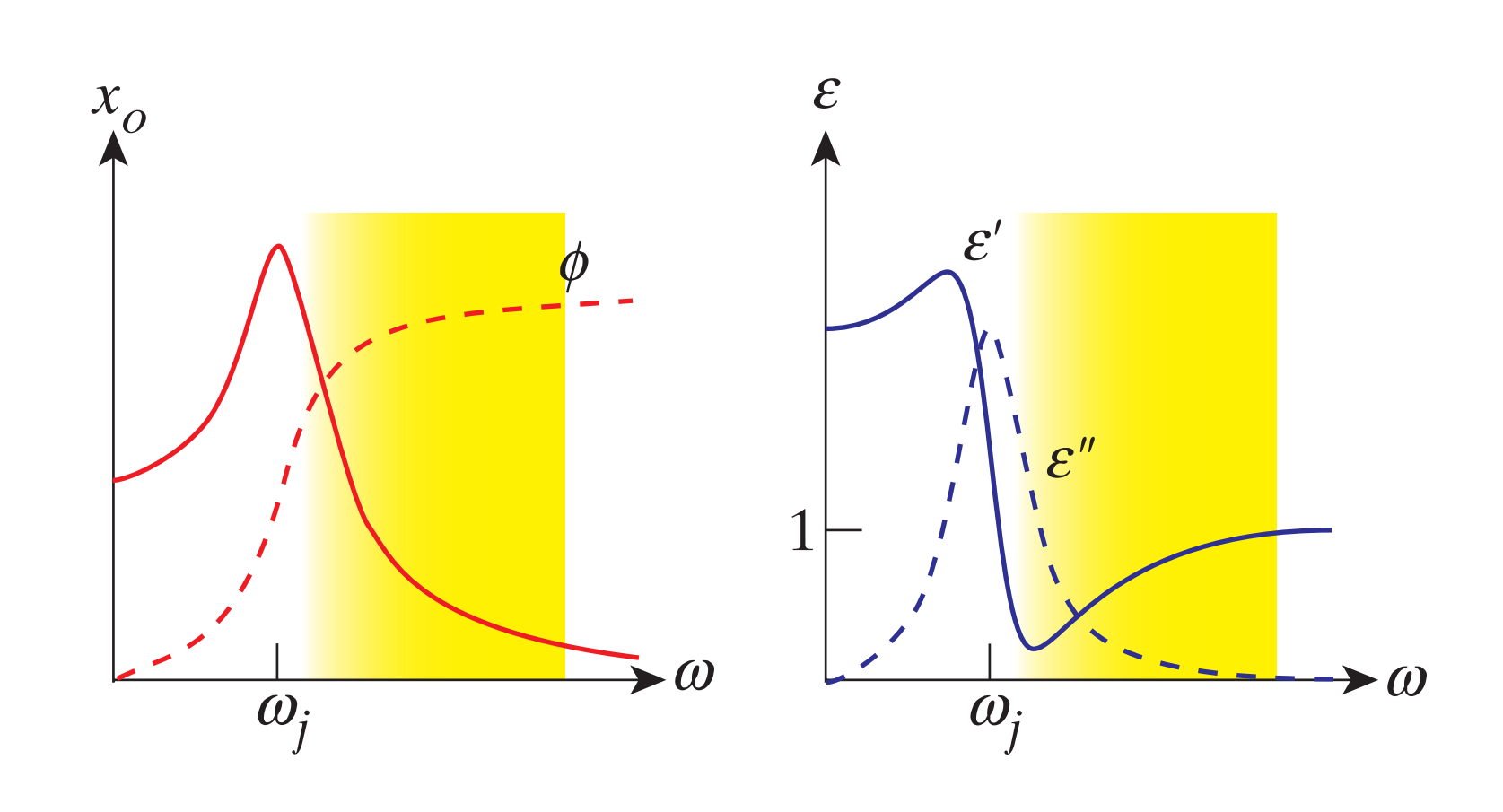
Bound electrons

At resonance: energy transfer from wave to bound charges $\Rightarrow$ wave attenuates (absorption)



Bound electrons

Above resonance: bound charges cannot keep up with driving field $\Rightarrow$ dielectric like a vacuum



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Solution: $$x(t) = \frac{e}{m}\frac{1}{\omega^2 + i\gamma\omega}E(t)$$ **(no resonance)**

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Low frequency ($\omega \ll \gamma$) $\Rightarrow$ current generated

$$J = -Ne\frac{dx}{dt} = \frac{Ne^2}{m}\frac{1}{\gamma - i\omega}E \approx \frac{Ne^2}{m\gamma}E \equiv \sigma E$$

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$\omega \gg \gamma$: σ complex $\Rightarrow$ J out of phase with E

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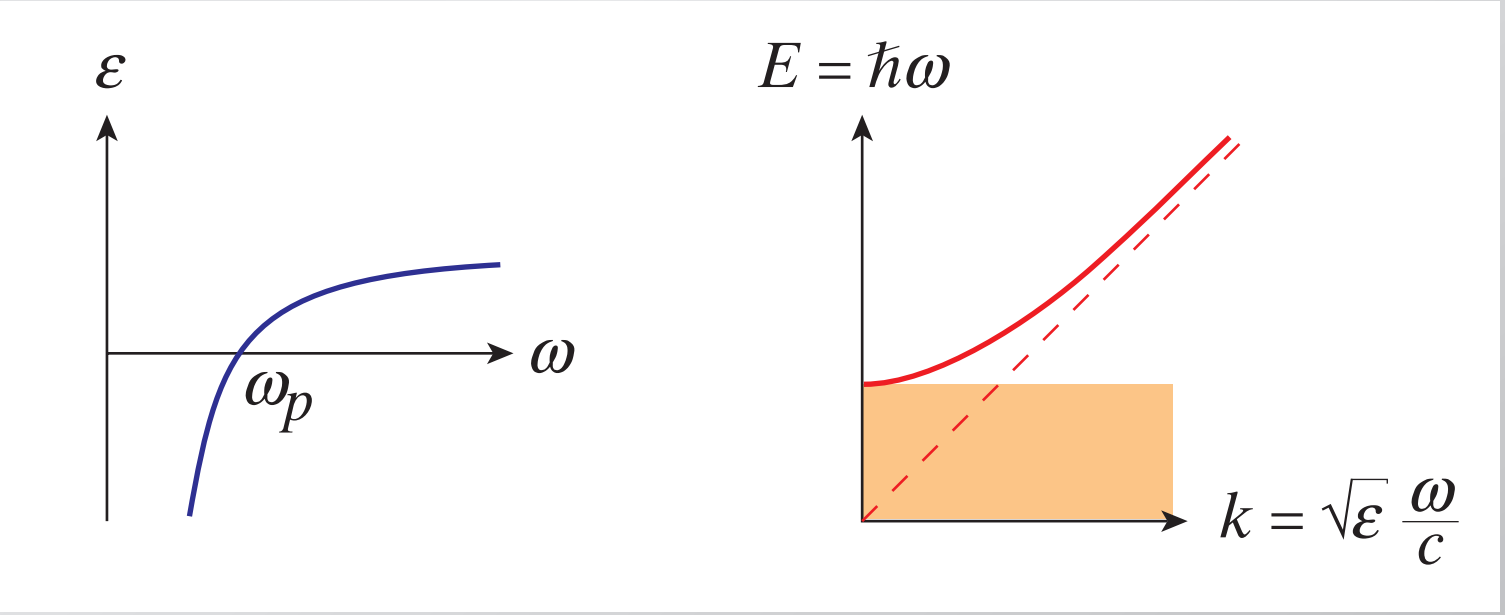

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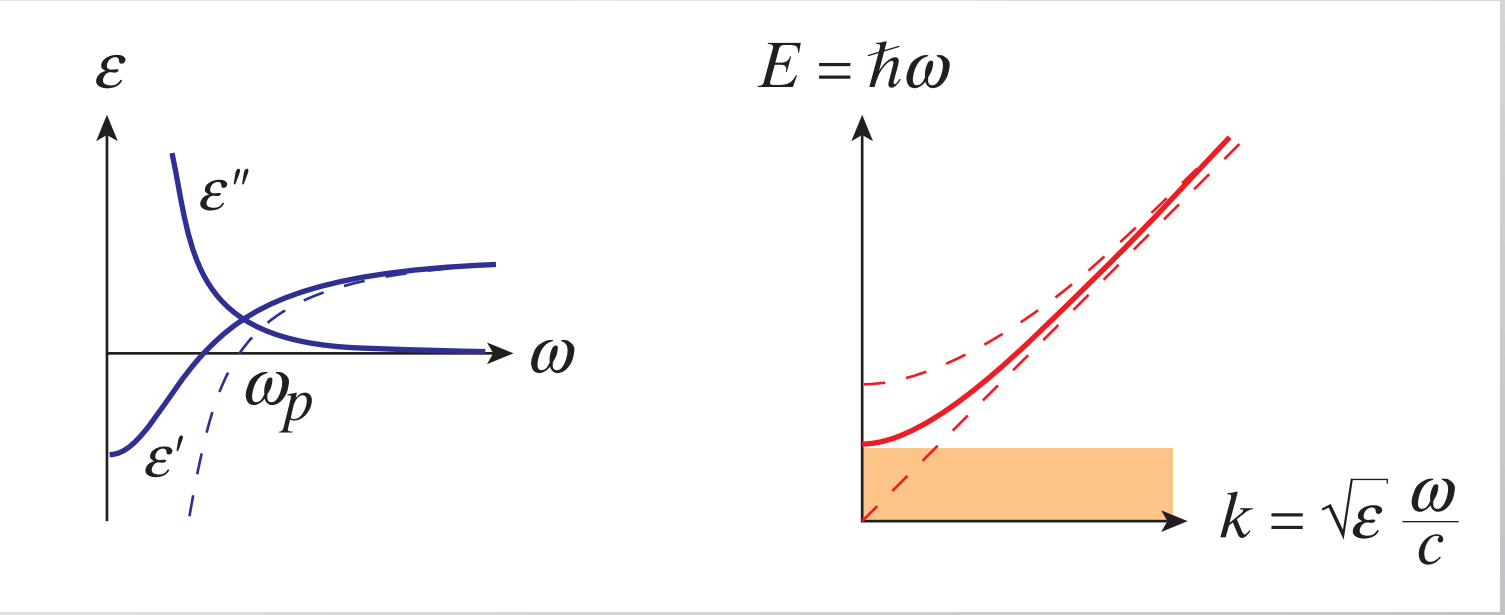
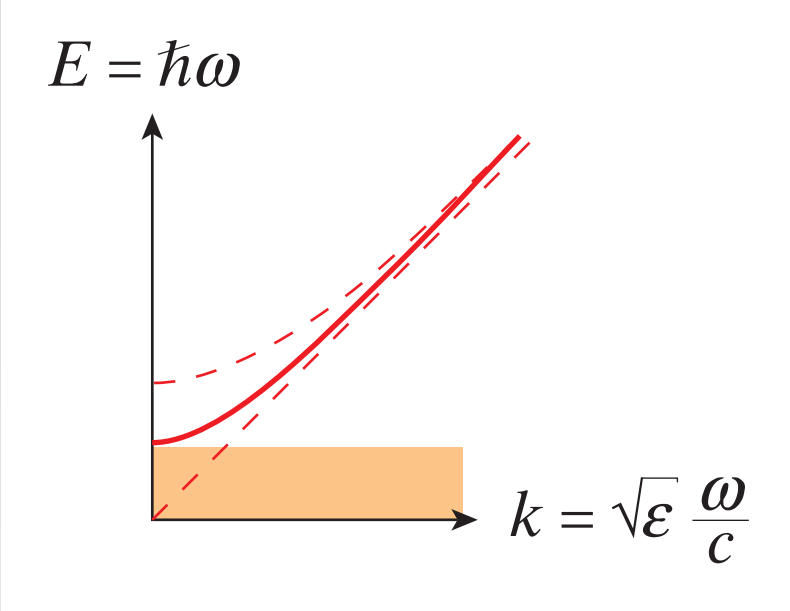
Add damping:

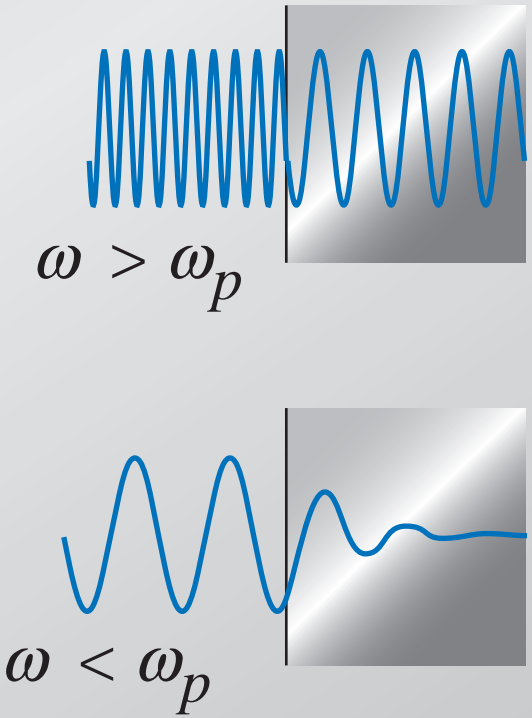
$$\gamma \lesssim \omega_p$$



Free electrons

Plasma acts like a high-pass filter





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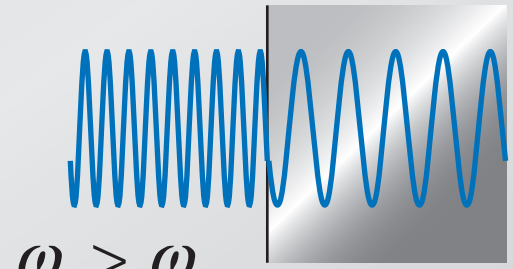
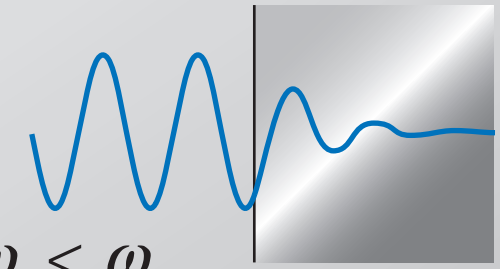
log N (cm^{-3})	ω_p (rad s^{-1})	λ_p
22	6×10^{15}	330 nm
18	6×10^{13}	33 μm
14	6×10^{11}	3.3 mm
10	6×10^{9}	0.33 m



$\omega > \omega_p$



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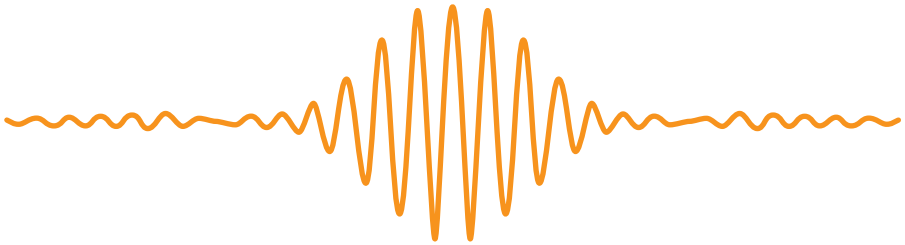
Pulse dispersion



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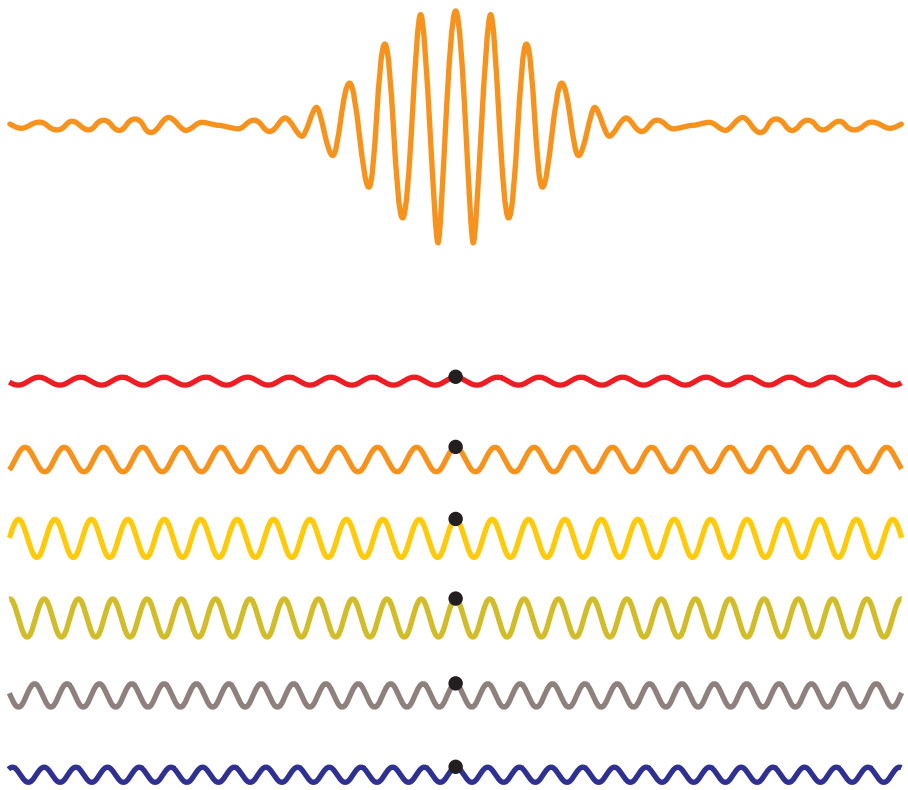

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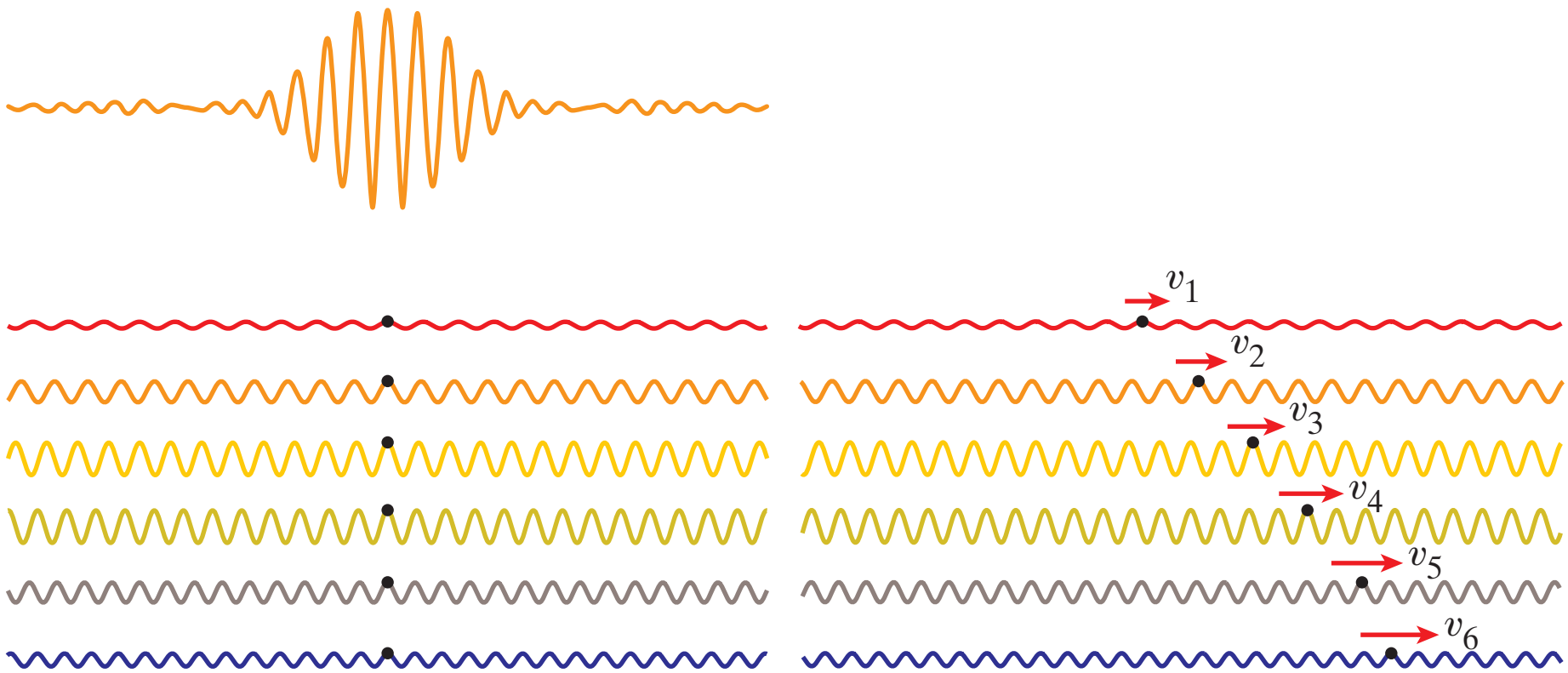

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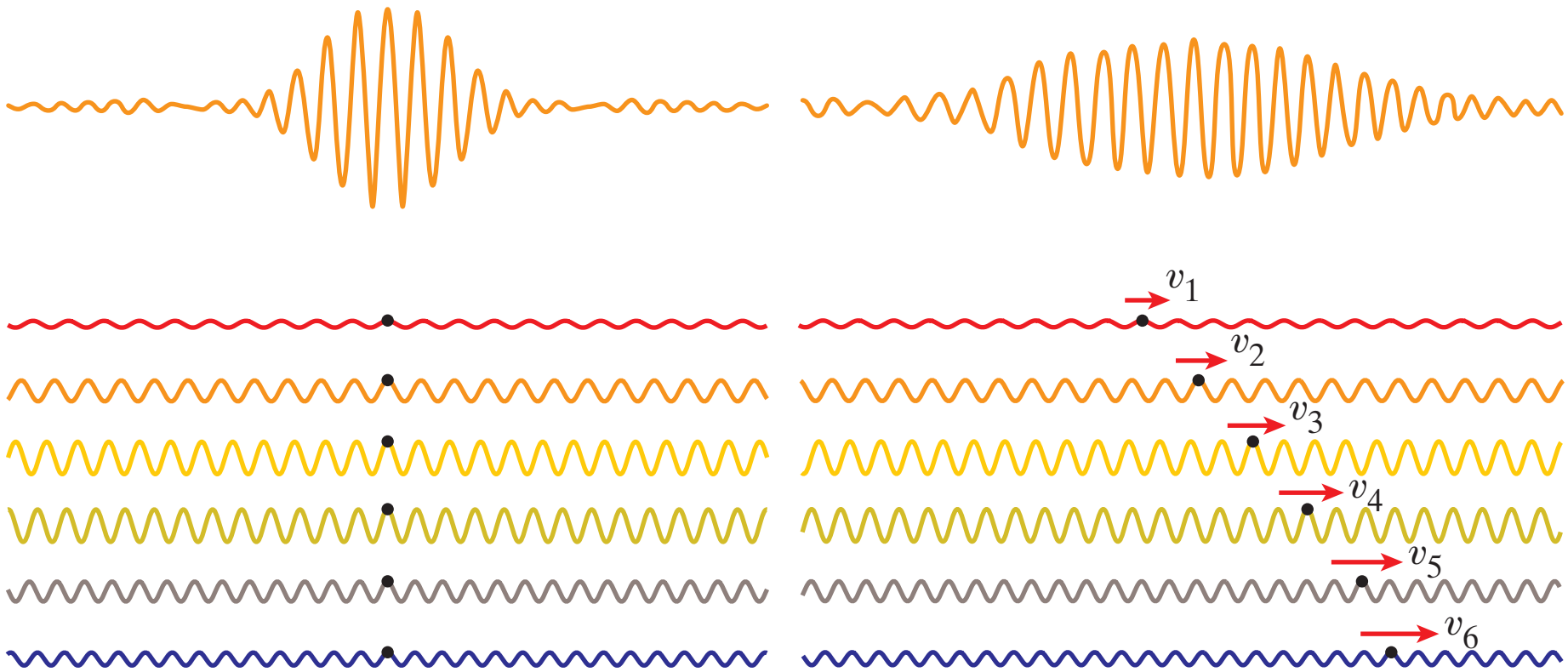
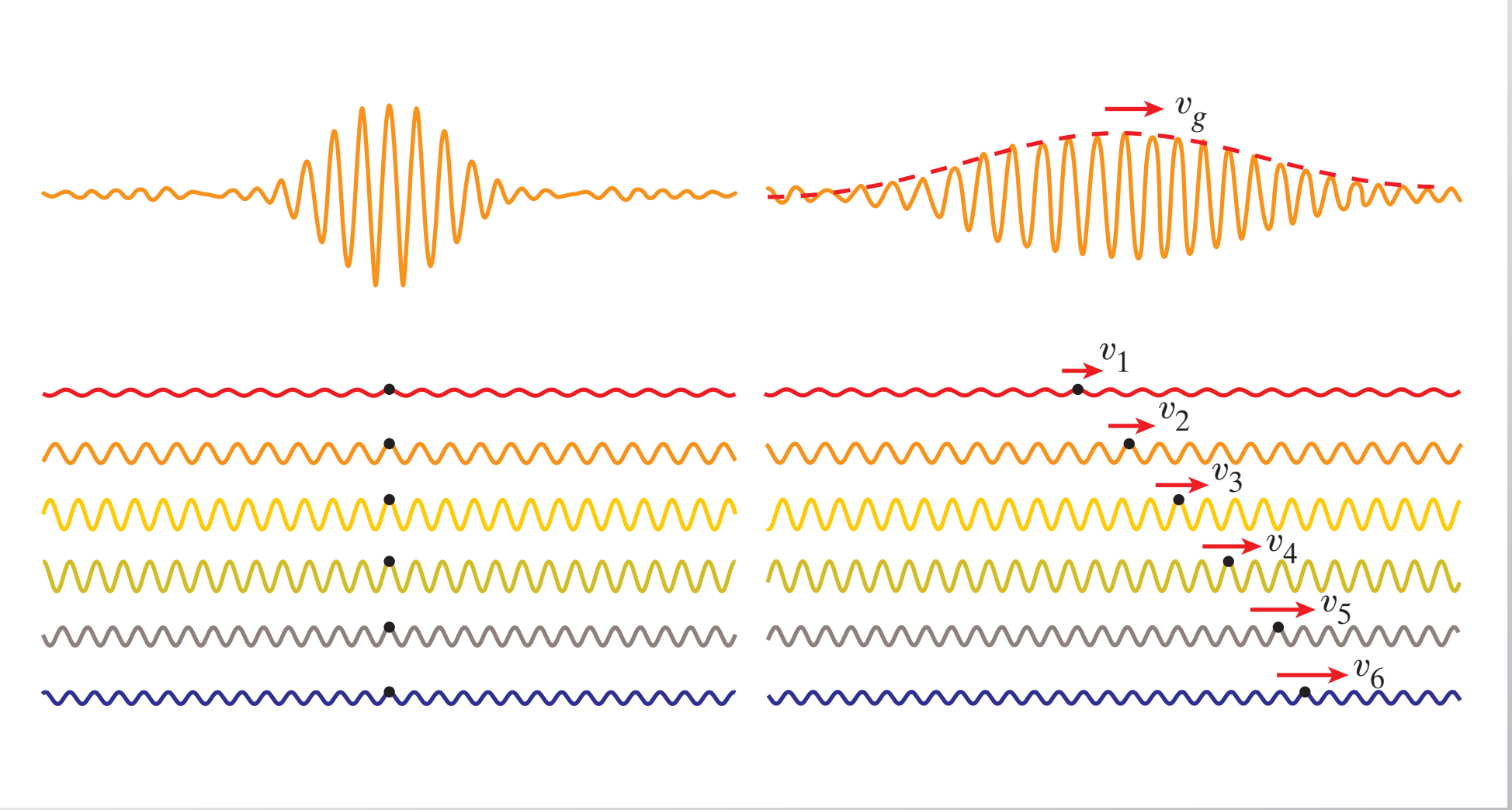


Pulse dispersion



Pulse dispersion

Consider two propagating waves:

$$y_1 = A \sin(k_1 x - \omega_1 t) \quad \textbf{and} \quad y_2 = A \sin(k_2 x - \omega_2 t)$$

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propagating at speeds

$$v_1 = \frac{\omega_1}{k_1} = f_1 \lambda_1 \quad \textbf{and} \quad v_2 = \frac{\omega_2}{k_2} = f_2 \lambda_2 \,.$$

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Superposition:

$$y = A[\sin(k_1 x - \omega_1 t) + \sin(k_2 x - \omega_2 t)]$$

Pulse dispersion

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Superposition:

$$y = A[\sin(k_1 x - \omega_1 t) + \sin(k_2 x - \omega_2 t)]$$

$$\sin\alpha + \sin\beta = 2\cos\left(\frac{\alpha - \beta}{2}\right)\sin\left(\frac{\alpha + \beta}{2}\right)$$

Pulse dispersion

$$y = 2A \cos \tfrac{1}{2}[(k_1 - k_2)x - (\omega_1 - \omega_2)t] \sin \left[\frac{k_1 + k_2}{2} x - \frac{\omega_1 + \omega_2}{2} t \right]$$

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Let: $k_1 - k_2 \equiv \Delta k$ **and** $\omega_1 - \omega_2 \equiv \Delta \omega$

Pulse dispersion

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$$\frac{k_1 + k_2}{2} \equiv k \quad \textbf{and} \quad \frac{\omega_1 + \omega_2}{2} \equiv \omega$$

Pulse dispersion

$$y = 2A\cos\tfrac{1}{2}[(k_1 - k_2)x - (\omega_1 - \omega_2)t]\sin\left[\frac{k_1 + k_2}{2}x - \frac{\omega_1 + \omega_2}{2}t\right]$$

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and so:

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Pulse dispersion

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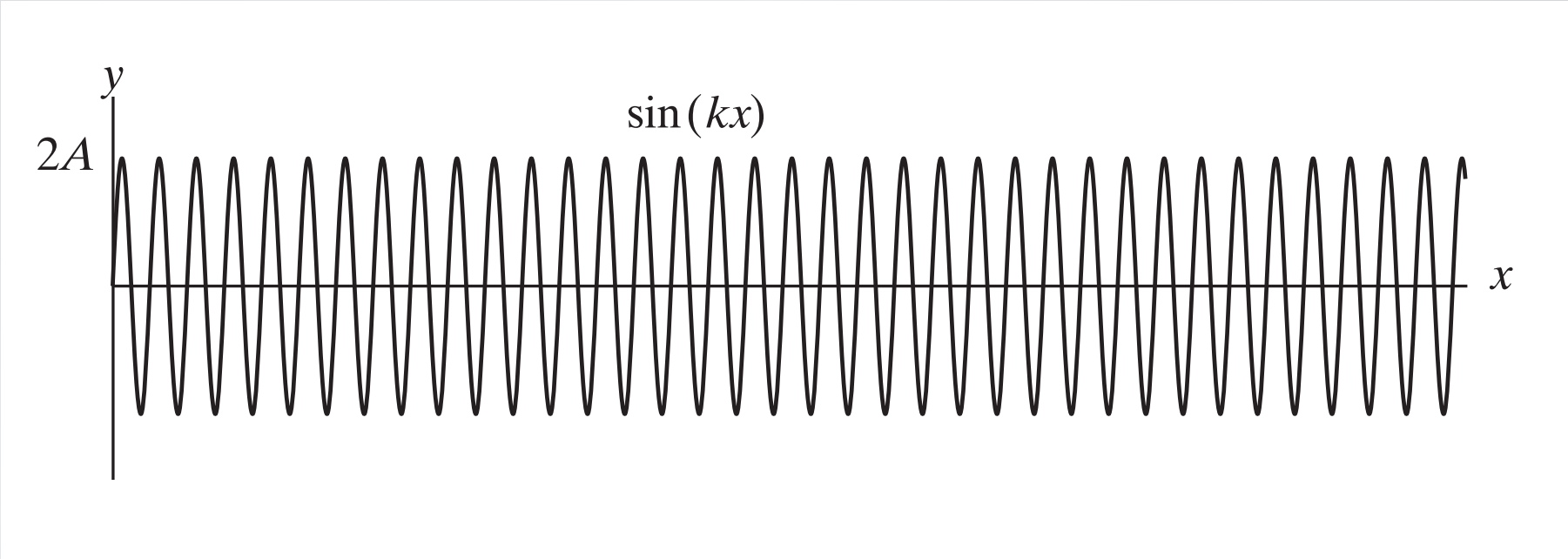
traveling sine wave, with amplitude modulation.

Pulse dispersion

$$y = 2A \cos \tfrac{1}{2}(x\Delta k - t\Delta\omega) \sin(kx - \omega t)$$

At $t = 0$**:**

$$y = 2A \cos \tfrac{1}{2}(x\Delta k) \sin(kx)$$



Pulse dispersion

$$y = 2A \cos \tfrac{1}{2}(x\Delta k - t\Delta\omega) \sin(kx - \omega t)$$

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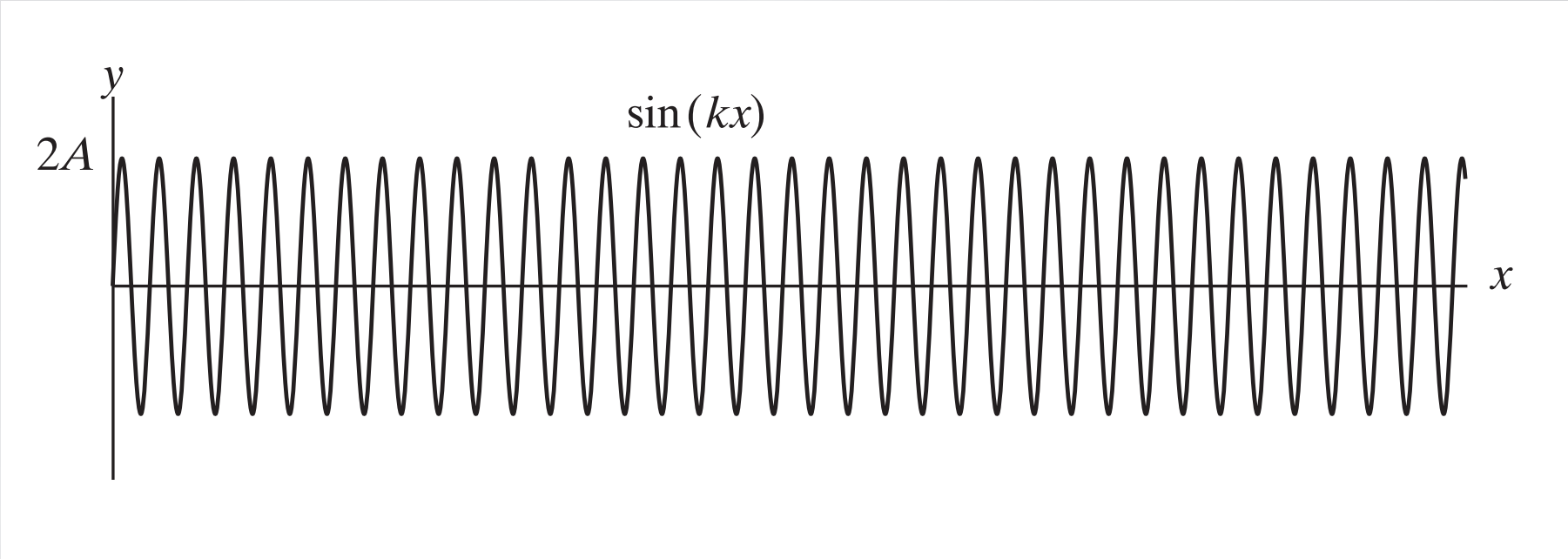
carrier



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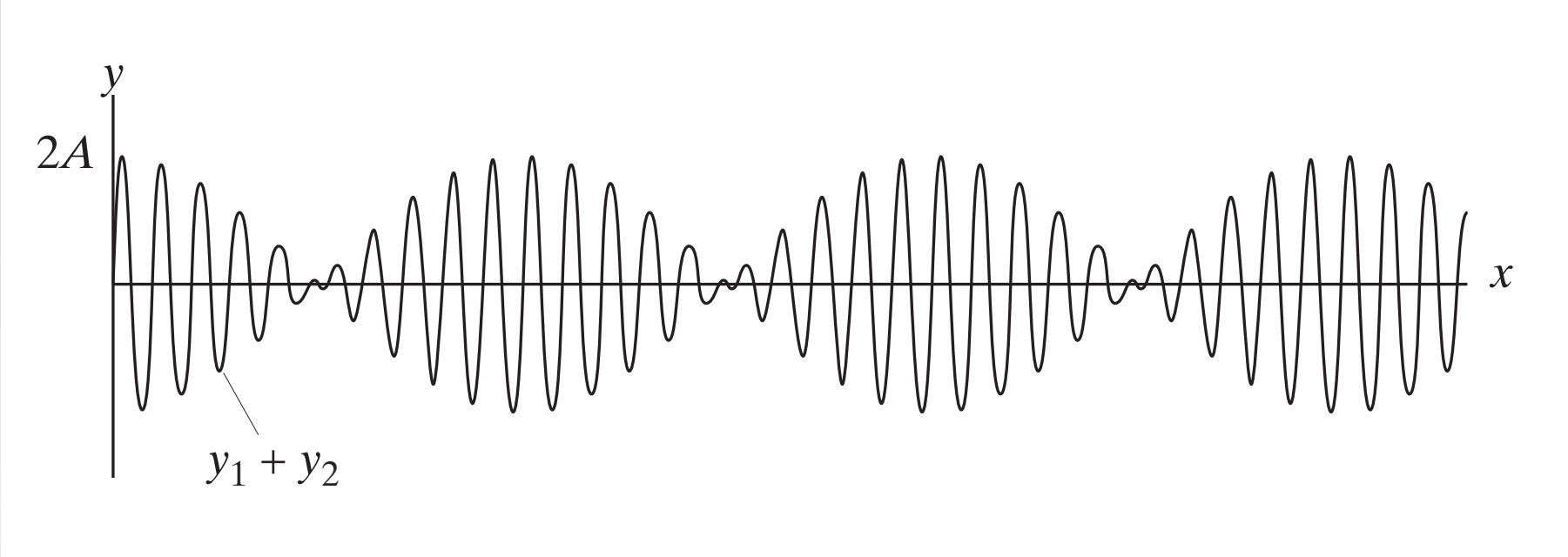

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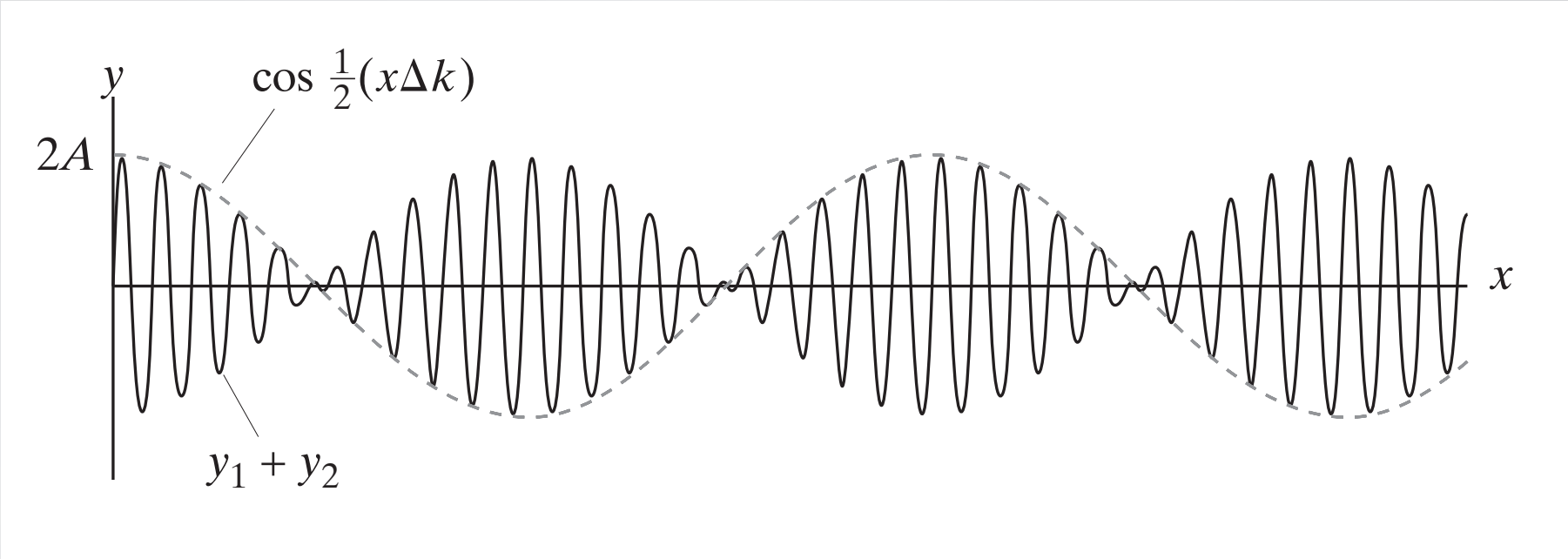
envelope **carrier**



Pulse dispersion

$$y = 2A \cos \tfrac{1}{2}(x\Delta k - t\Delta\omega) \sin (kx - \omega t)$$

Pulse dispersion

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speed of carrier

$$v_p = \frac{\omega}{k} = f\lambda$$

Pulse dispersion

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$$v_p = \frac{\omega}{k} = f\lambda$$

speed of envelope

$$v_g = \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk}$$

Pulse dispersion

$$y = 2A \cos \tfrac{1}{2}(x\Delta k - t\Delta\omega) \sin(kx - \omega t)$$

speed of carrier ('phase velocity'):

$$v_p = \frac{\omega}{k} = f\lambda$$

speed of envelope ('group velocity'):

$$v_g = \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk}$$

Pulse dispersion

let’s practice a bit!

(please complete worksheet)

Pulse dispersion

For each wave, determine the wavevector k, the frequency ω, and the propagation speed v:

$k_1 =$ 8.0 and $k_2 = \frac{7.2}{0.95} = 7.6 < k_1$

R B

$\omega_1 =$ 8.0 and $\omega_2 =$ 7.2

$v_1 = \frac{\omega_1}{k_1} = 1.$ and $v_2 = \frac{\omega_2}{k_2} = \frac{7.2}{7.57} = 0.95$

Does the red get ahead of the blue or the other way around? Why?

Is the dispersion in the medium through which these waves propagate normal or anomalous? Why?

Pulse dispersion

What is the phase velocity of the superposition of y_1 and y_2?

$$v_p = \frac{(\omega_1 + \omega_2)/2}{(k_1 + k_2)/2} = \frac{7.6}{7.8} = 0.98$$

What is the group velocity of the superposition of y_1 and y_2?

$$v_g = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{0.8}{0.4} = 2$$

Do the crests of the carrier wave travel forward or backward through the envelope? Does your prediction agree with your observation?

Pulse dispersion

$$y = 2A \cos \tfrac{1}{2}(x\Delta k - t\Delta\omega) \sin(kx - \omega t)$$

If no dispersion

$$v_p = \frac{\omega_1}{k_1} = \frac{\omega_2}{k_2}$$

Pulse dispersion

$$y = 2A \cos \tfrac{1}{2}(x\Delta k - t\Delta\omega) \sin(kx - \omega t)$$

If no dispersion $$v_p = \frac{\omega_1}{k_1} = \frac{\omega_2}{k_2}$$

group velocity:

$$v_g = \frac{\Delta\omega}{\Delta k} = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{\omega_1/k_1 - \omega_2/k_1}{1 - k_2/k_1} = \frac{v_p - \omega_2/k_1}{1 - k_2/k_1}$$

Pulse dispersion

$$y = 2A \cos \tfrac{1}{2}(x\Delta k - t\Delta\omega) \sin(kx - \omega t)$$

If no dispersion $$v_p = \frac{\omega_1}{k_1} = \frac{\omega_2}{k_2}$$

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$$v_g = \frac{v_p - \omega_2/k_2(k_2/k_1)}{1 - k_2/k_1} = v_p \frac{1 - k_2/k_1}{1 - k_2/k_1} = v_p$$

Pulse dispersion

$$y = 2A \cos \tfrac{1}{2}(x\Delta k - t\Delta\omega) \sin(kx - \omega t)$$

If no dispersion $$v_p = \frac{\omega_1}{k_1} = \frac{\omega_2}{k_2}$$

group and phase velocities are the same:

$$v_g = v_p$$

and so the envelope and carrier travel together

Pulse dispersion

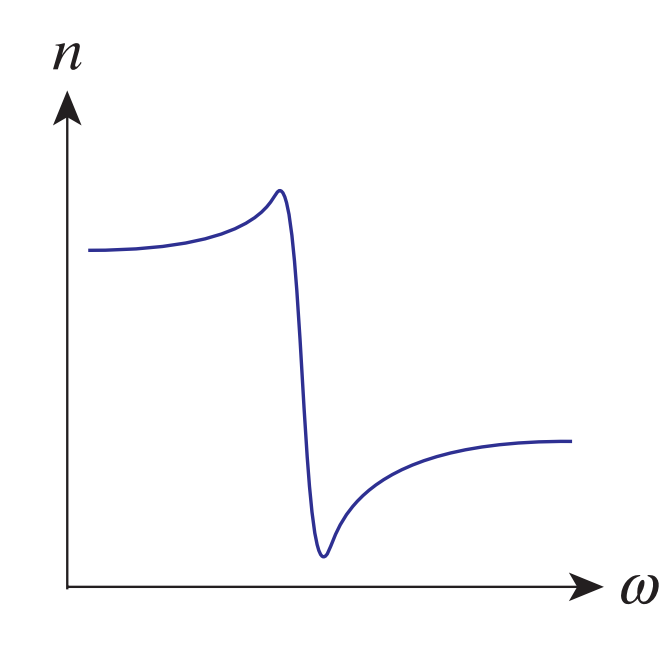
$$y = 2A \cos \tfrac{1}{2}(x\Delta k - t\Delta\omega) \sin(kx - \omega t)$$

Types of dispersion:

$\frac{dn}{d\omega} > 0$ **normal dispersion**

$\frac{dn}{d\omega} = 0$ **no dispersion**

$\frac{dn}{d\omega} < 0$ **anomalous dispersion**



Pulse dispersion

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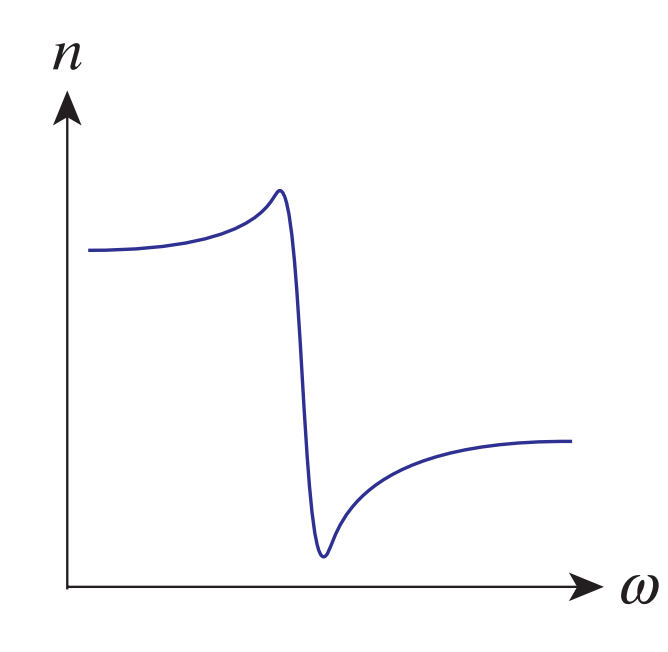
Types of dispersion:

$\frac{dn}{d\omega} > 0$ **normal dispersion** $v_g < v_p$

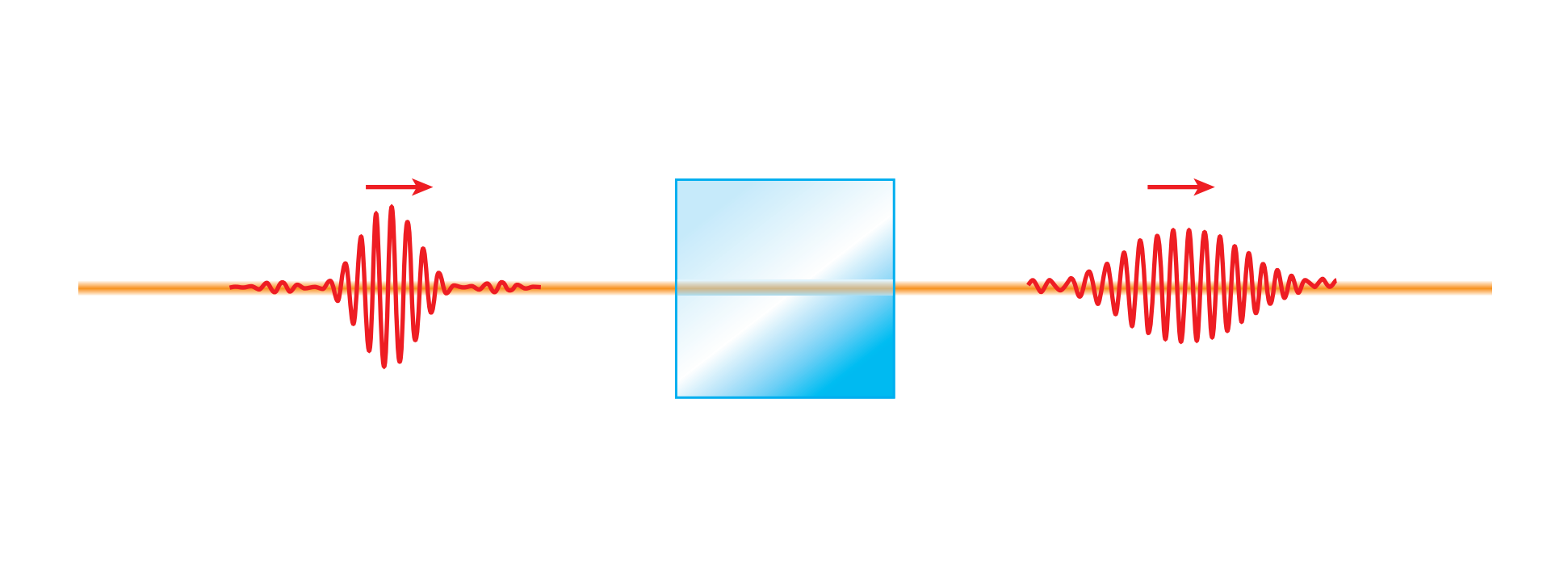
$\frac{dn}{d\omega} = 0$ **no dispersion** $v_g = v_p$

$\frac{dn}{d\omega} < 0$ **anomalous dispersion** $v_g > v_p$



Pulse dispersion

medium causes pulse to stretch



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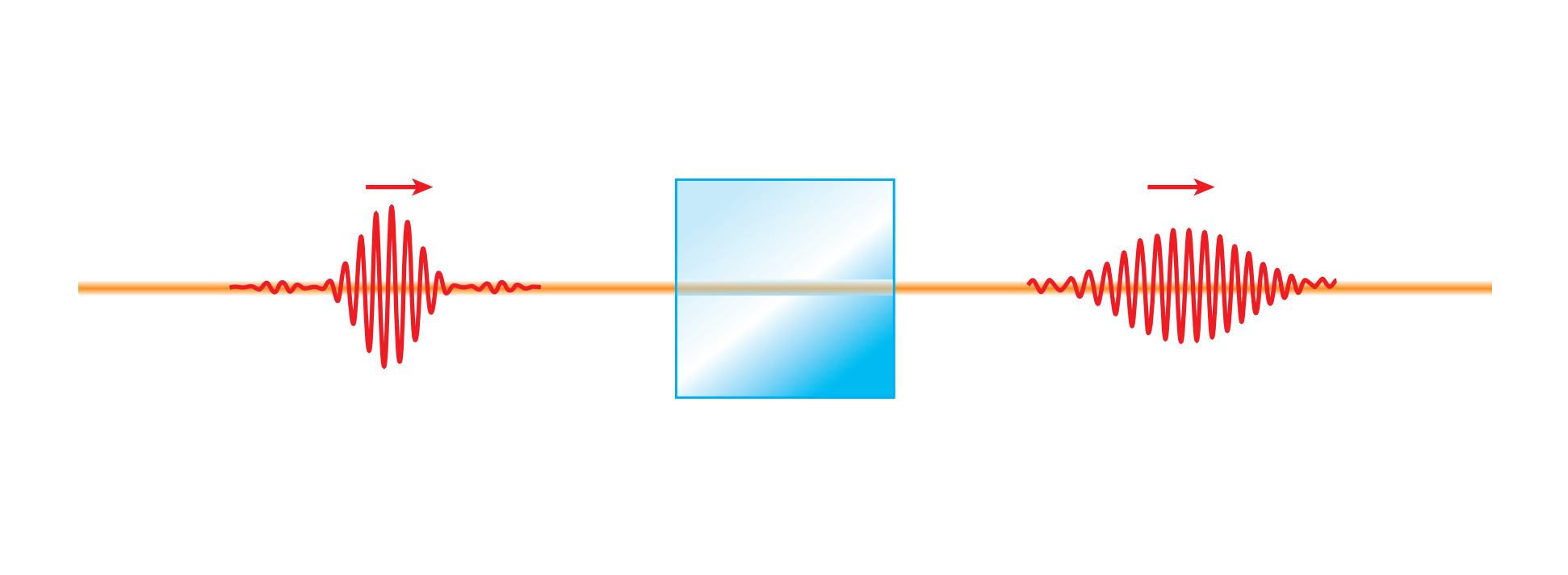
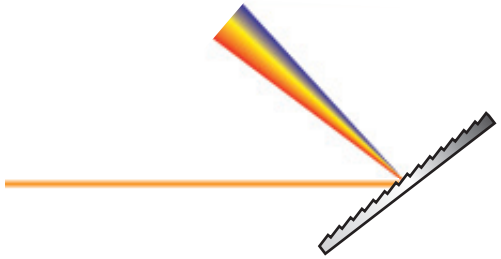
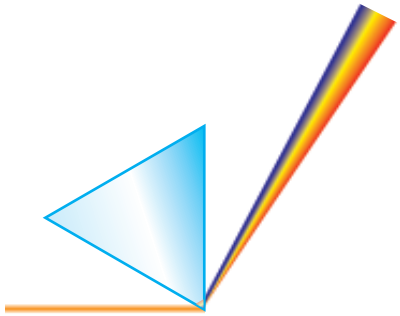


compensate by rearranging spectral components!

Pulse dispersion compensation





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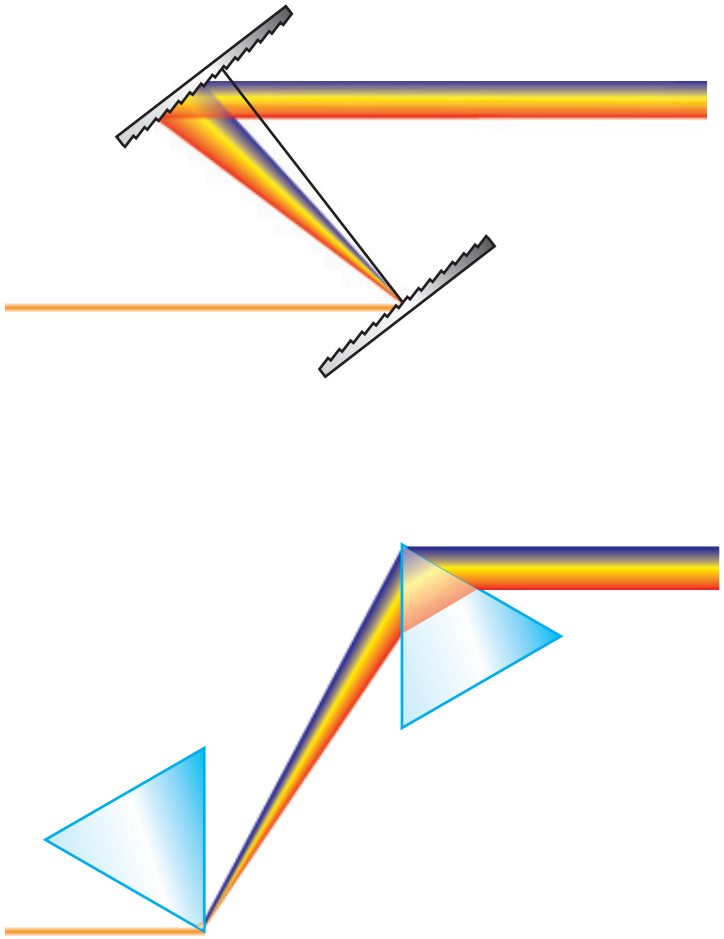

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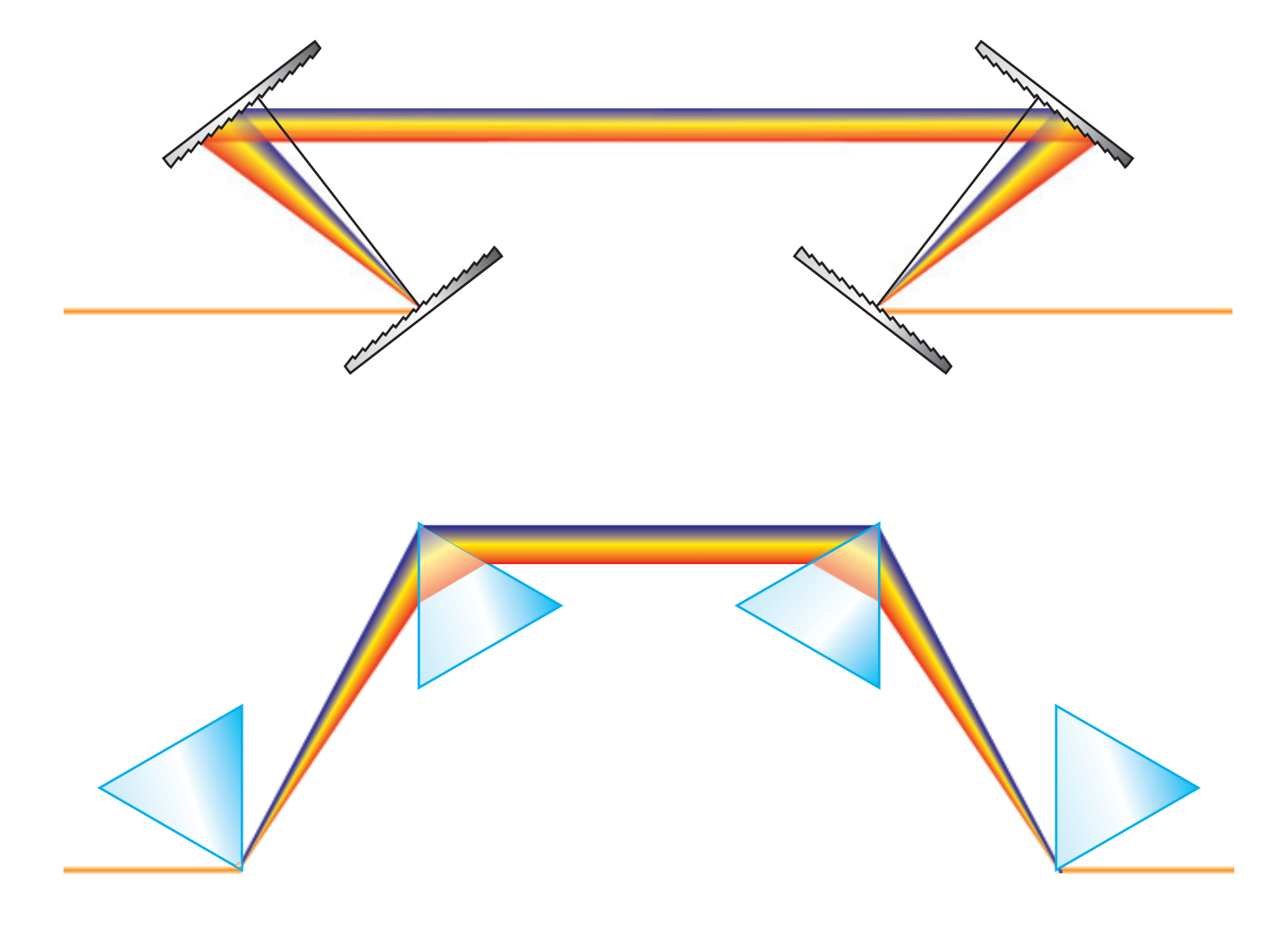

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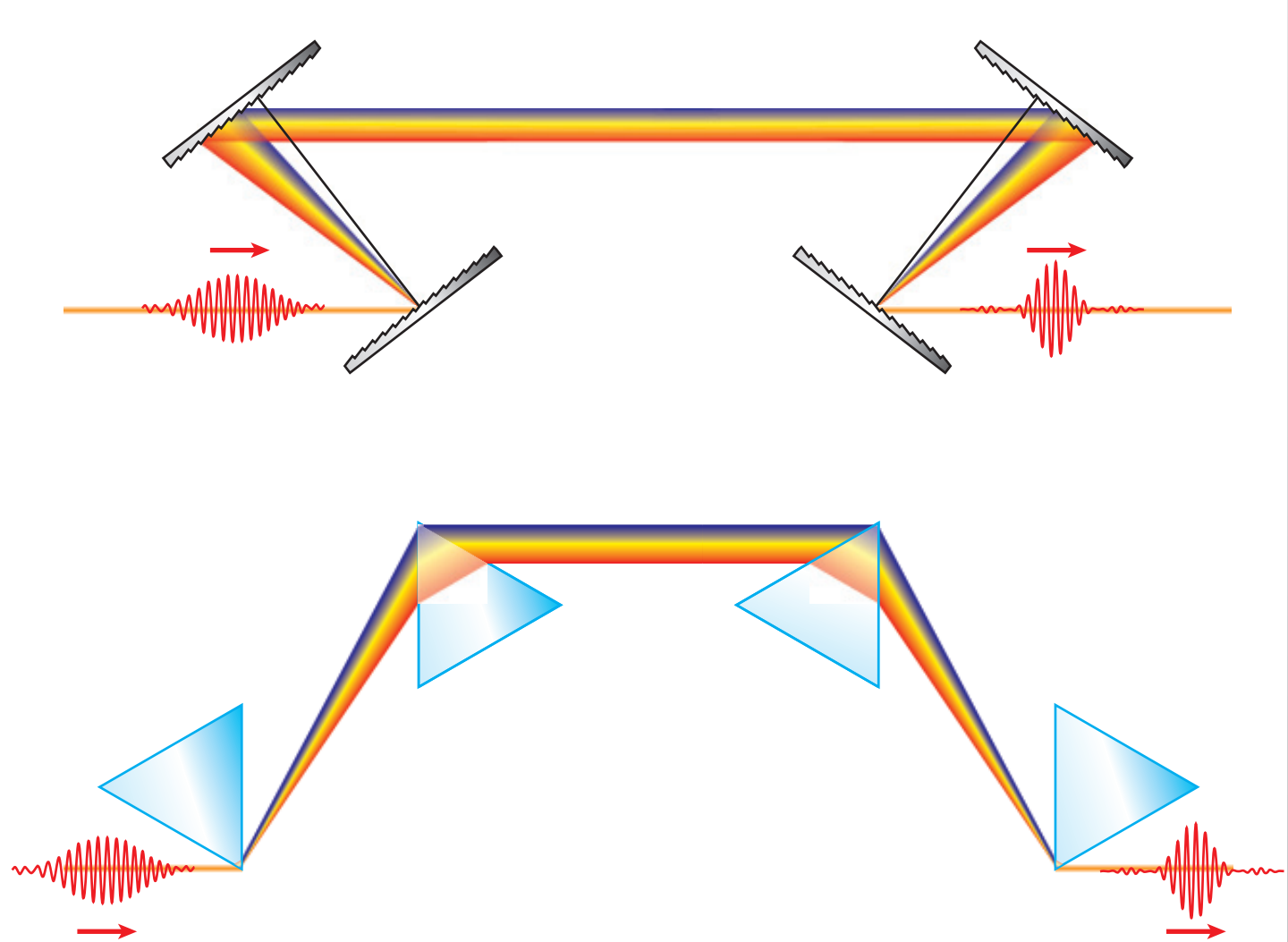

Pulse dispersion compensation

How do these arrangements work?

Pulse dispersion compensation

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(please complete worksheet)

Pulse dispersion compensation

Does path length difference compensate?

Pulse dispersion compensation

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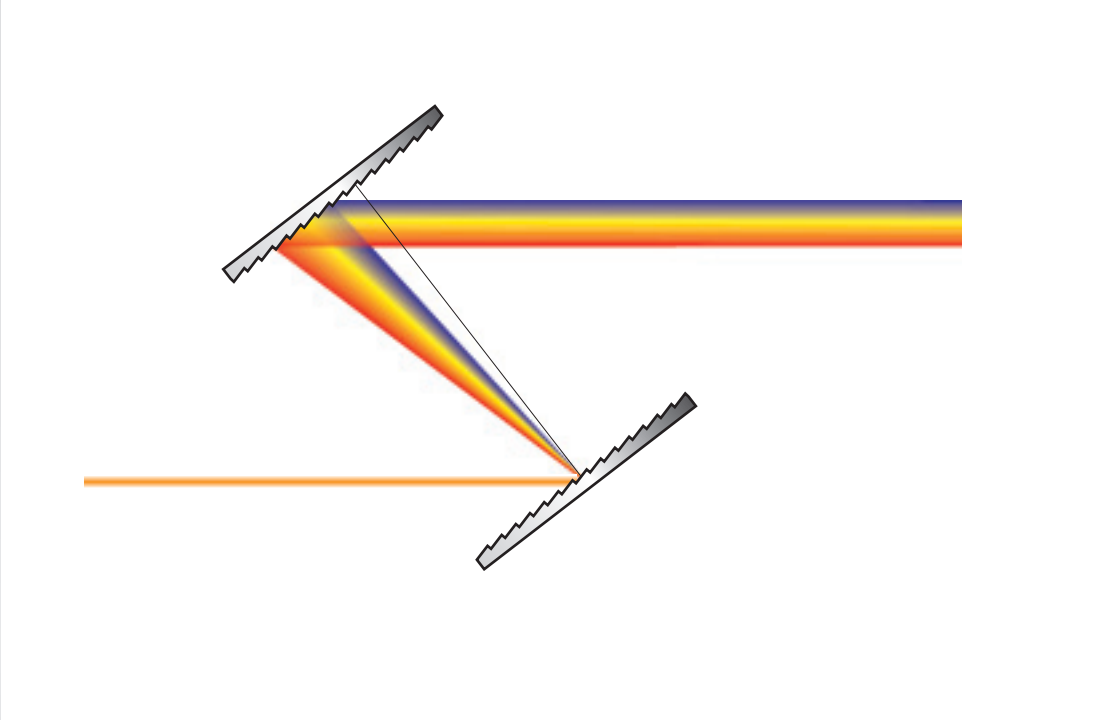
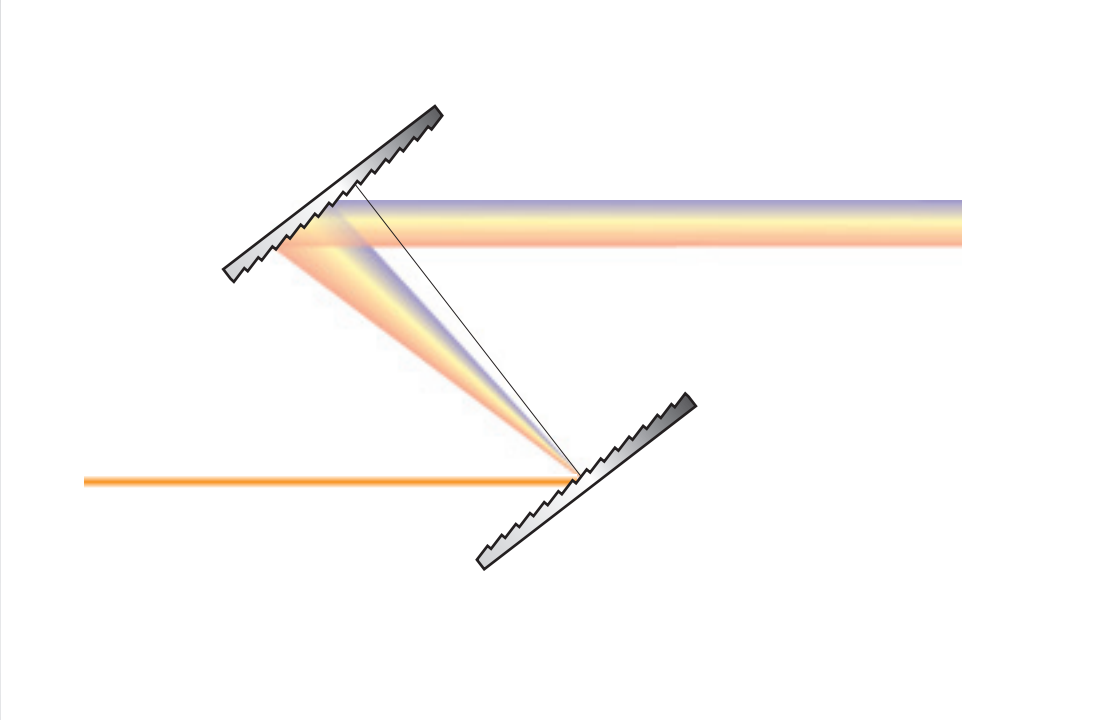


Pulse dispersion compensation

Does path length difference compensate?



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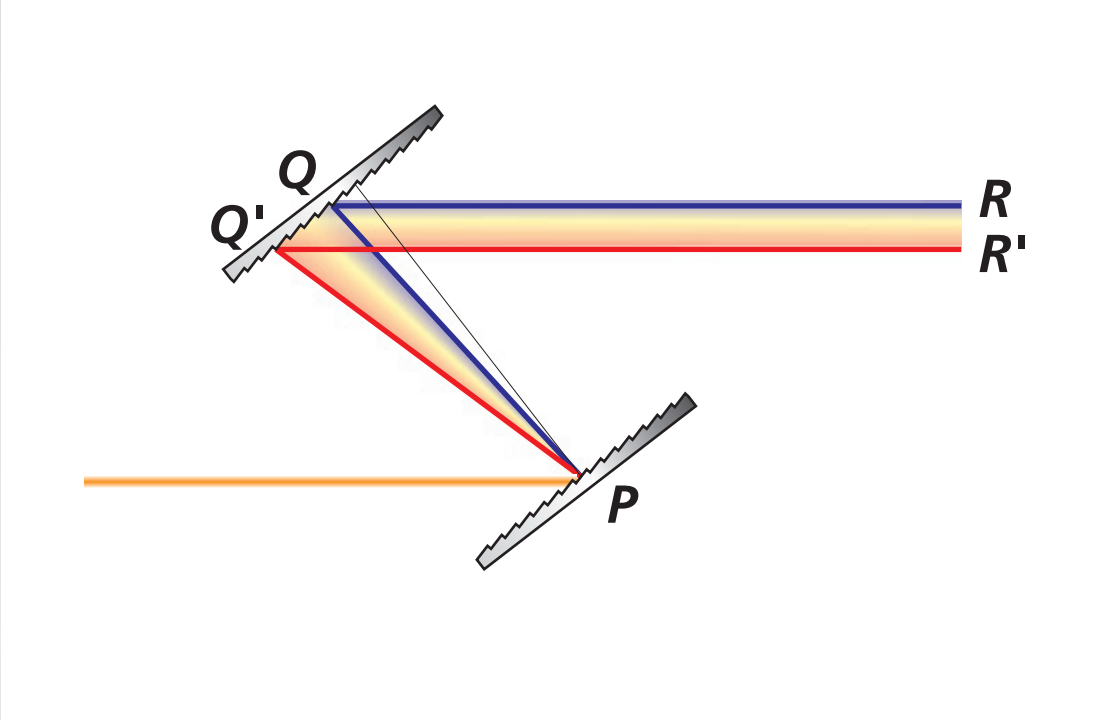

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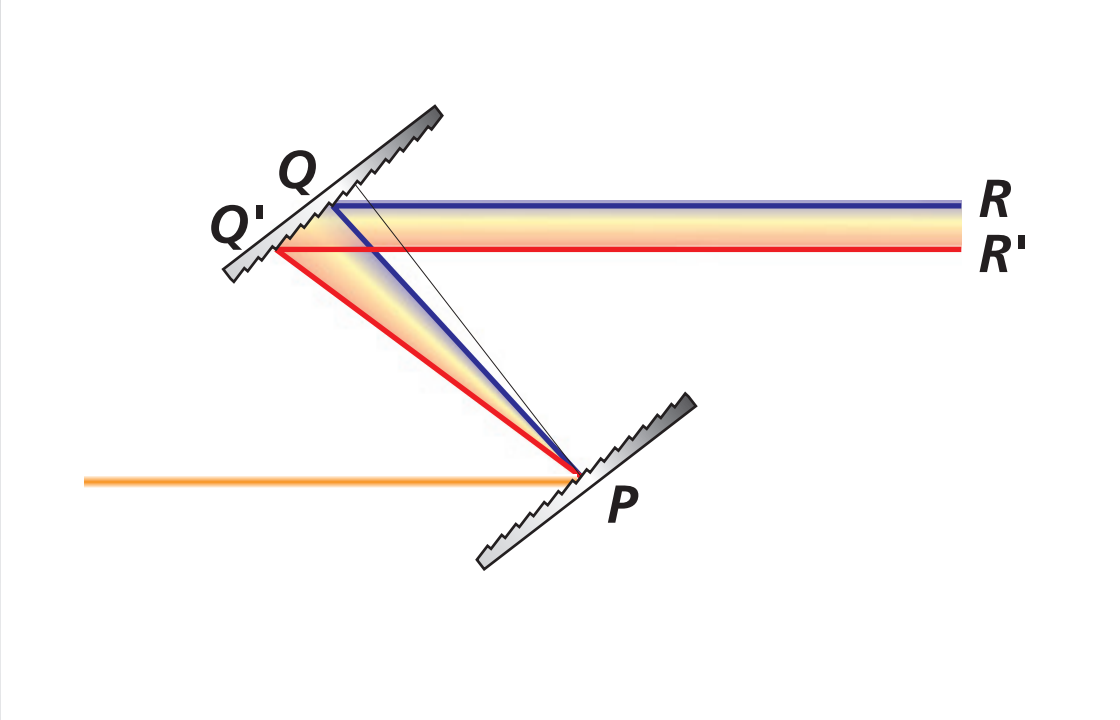

grating gives low frequency longer path length!

Pulse dispersion compensation

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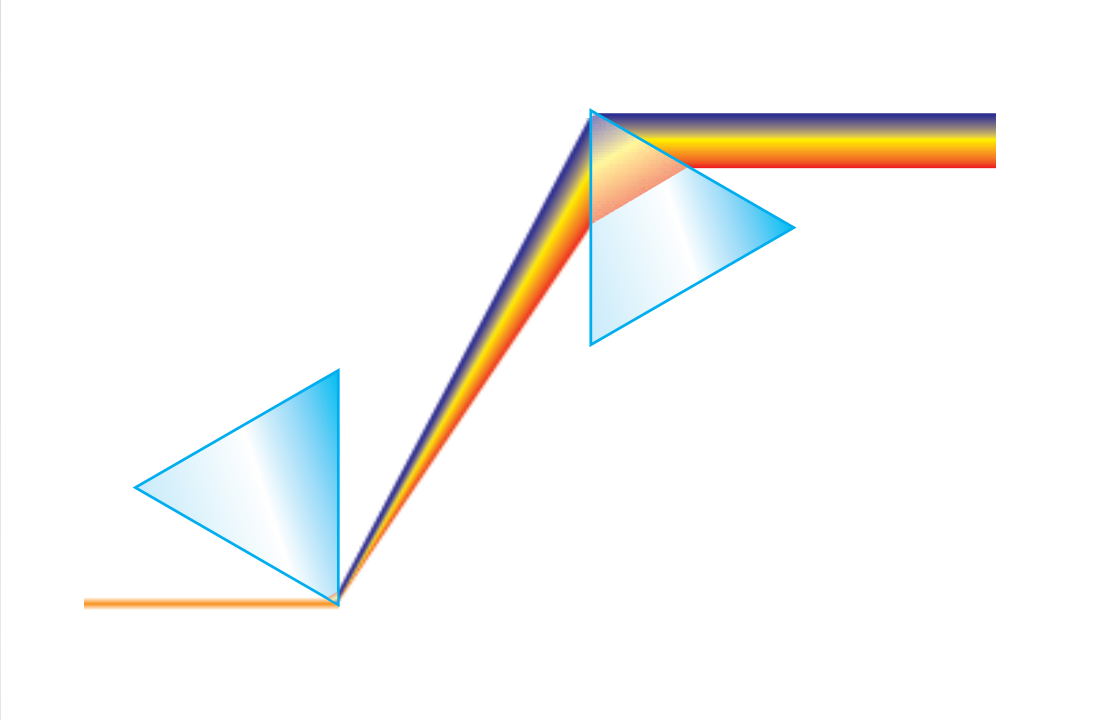

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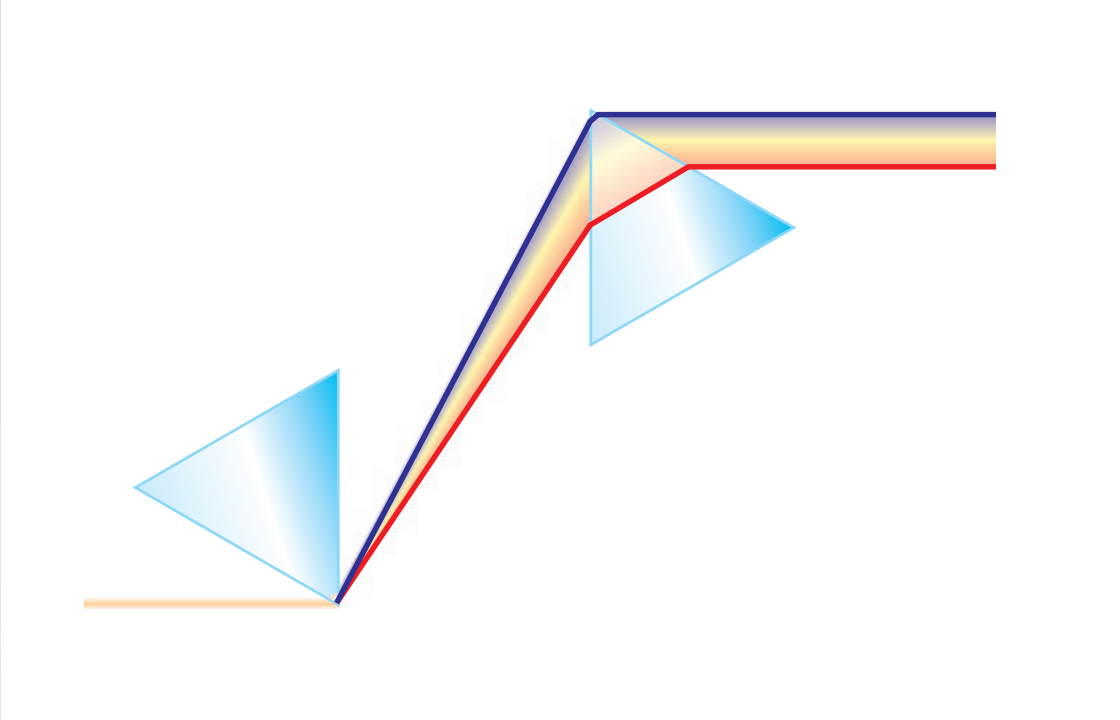

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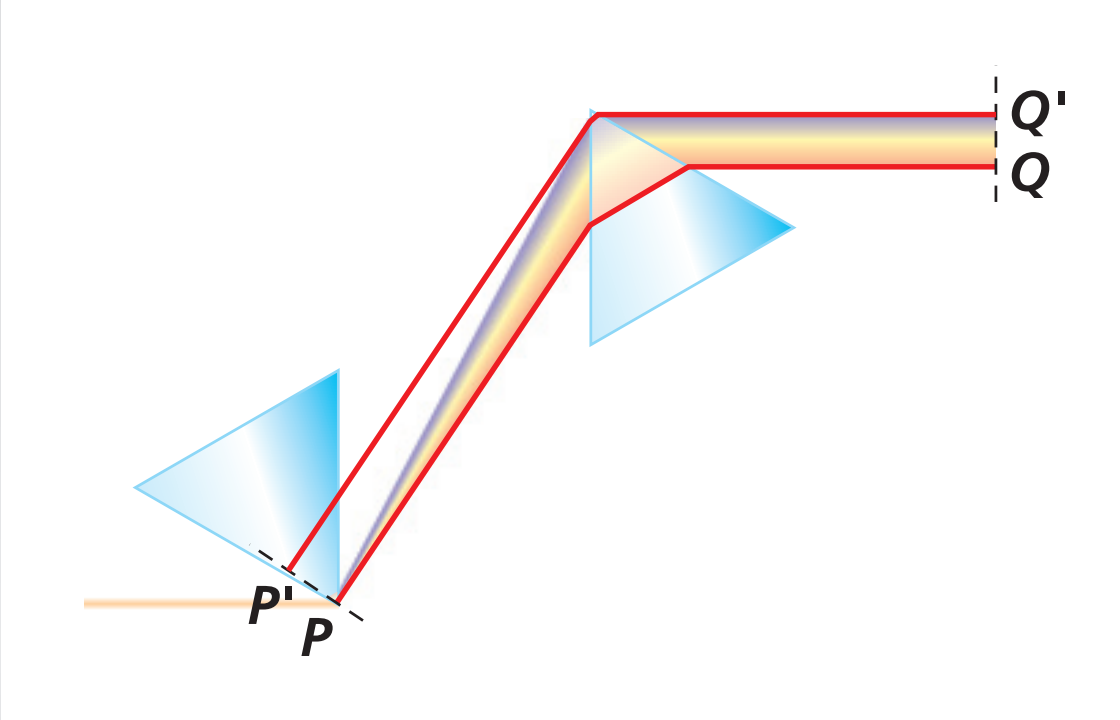

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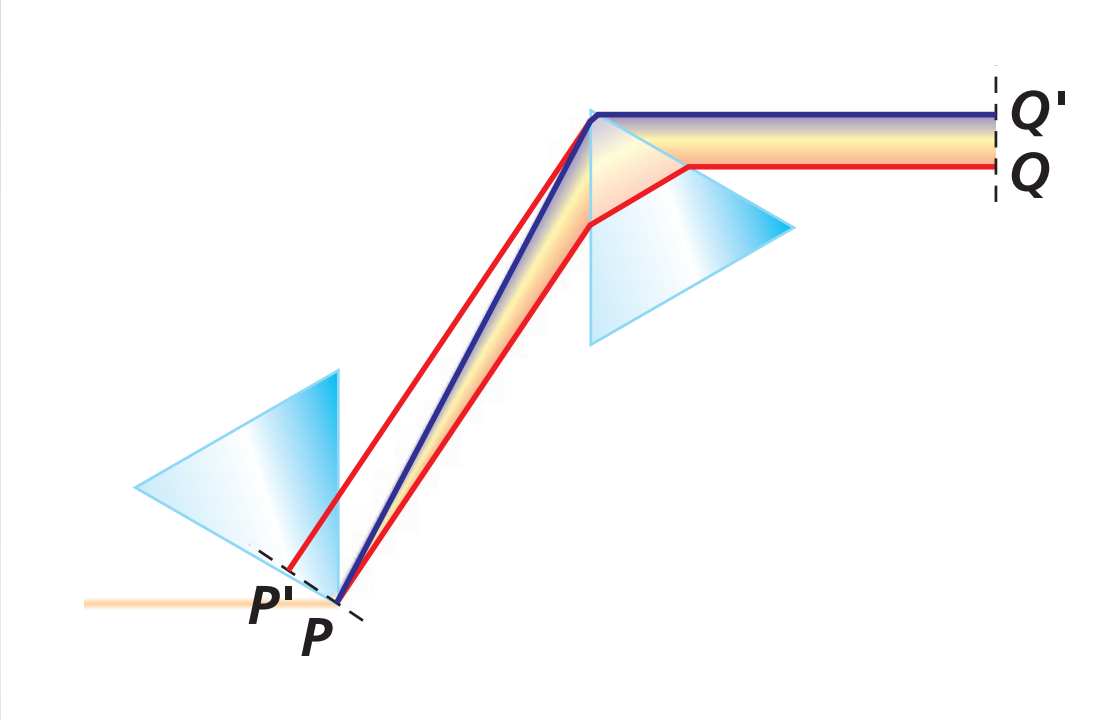

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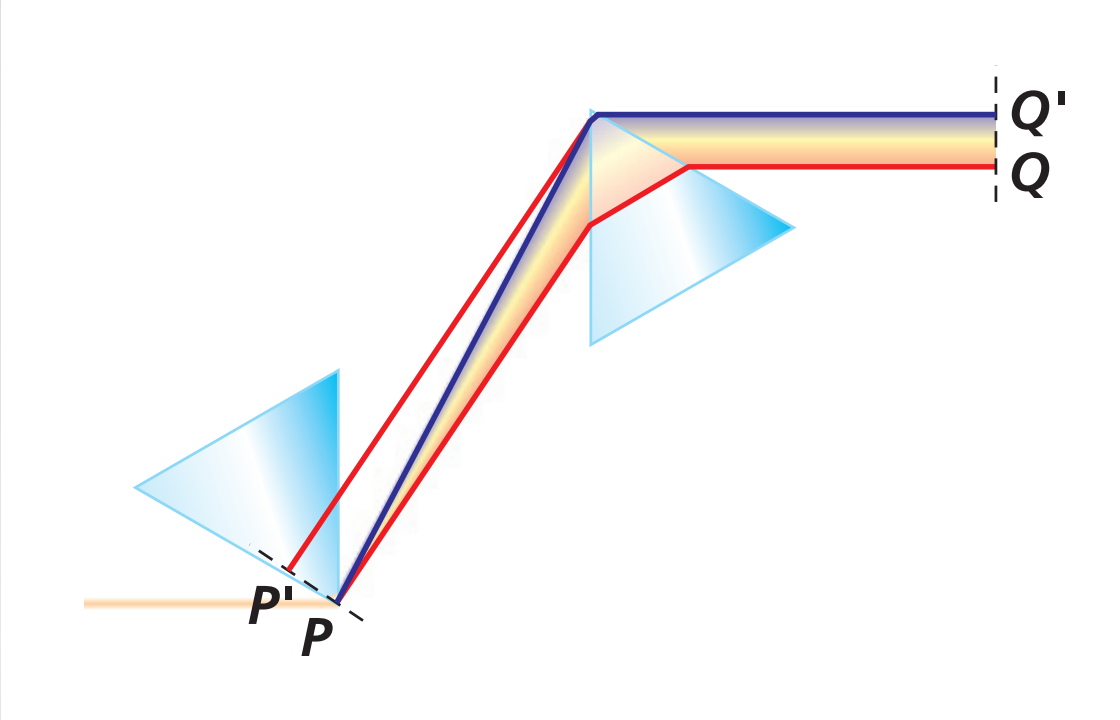

…so prism gives low frequency *shorter* path length!

Pulse dispersion compensation

consider traveling Gaussian pulse again:

$$y(t) = \exp\left[-\frac{(x - v_g t)^2}{2\sigma_t^2}\right]\sin(kx - \omega t)$$

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1. Yes, it is dispersive
2. No, it is not dispersive (pulse shape is constant)
3. Cannot tell

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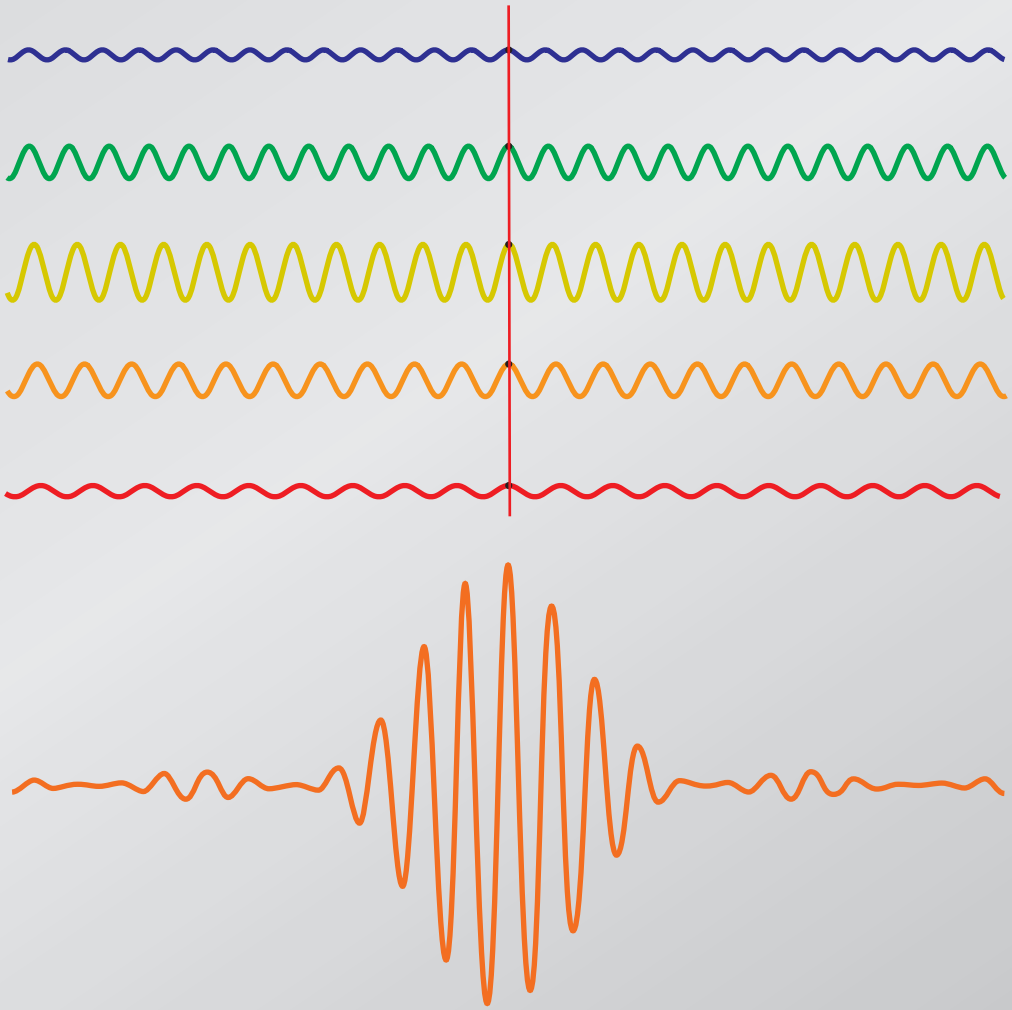
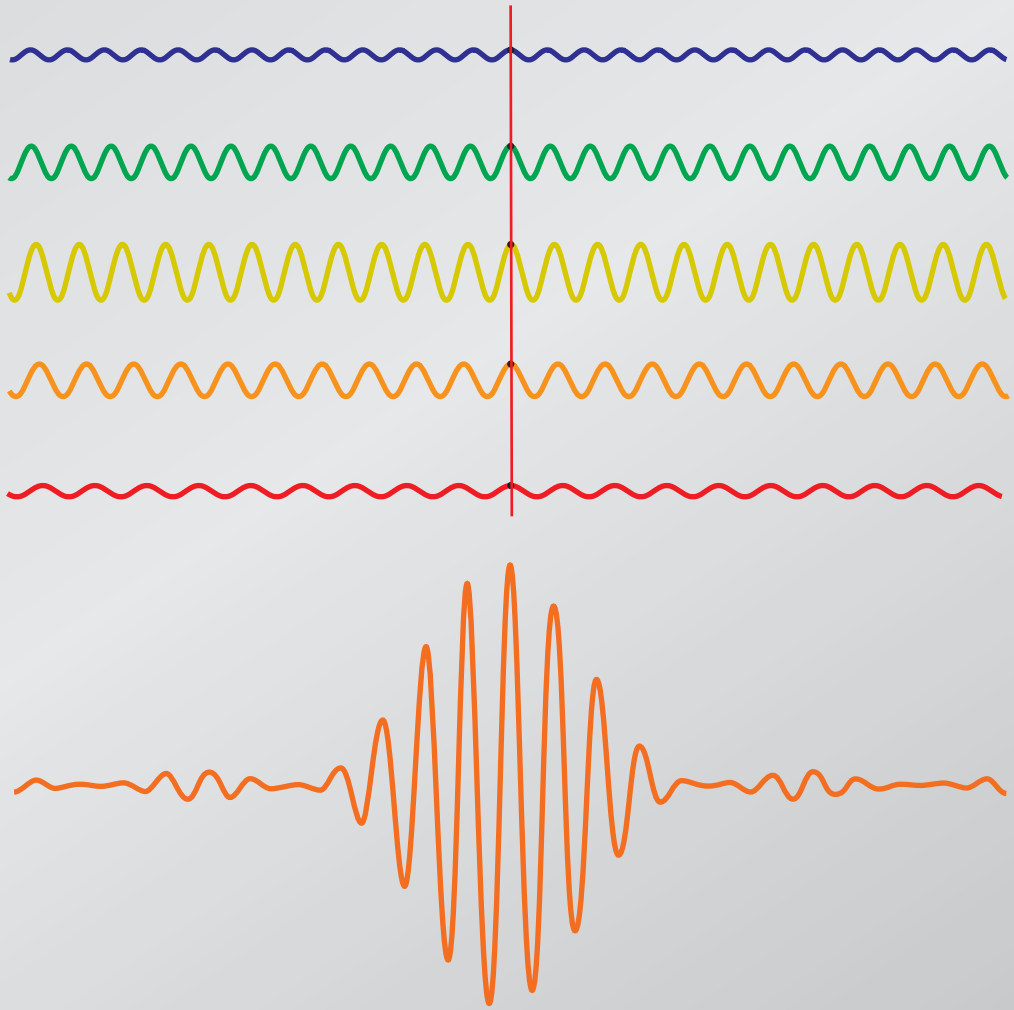
…but Gaussian shape of pulse is constant!

Pulse dispersion compensation



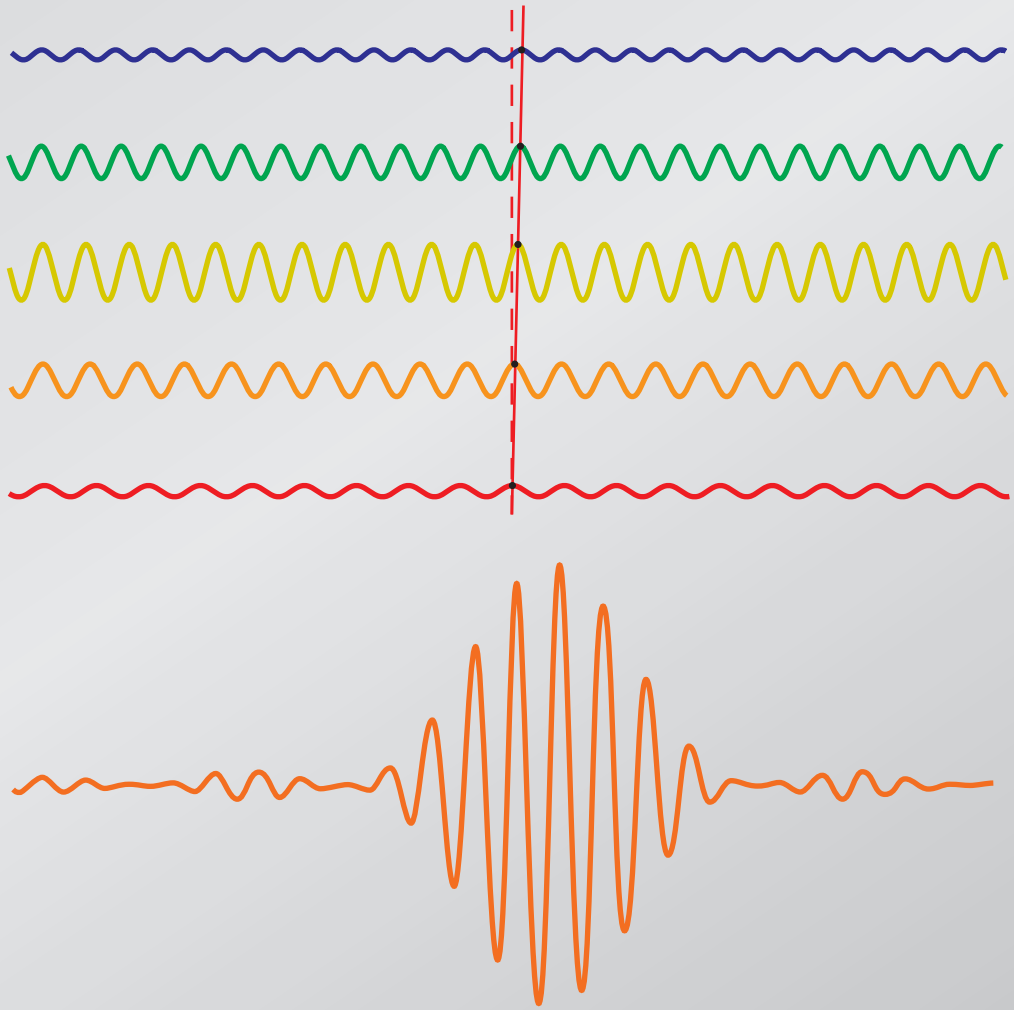
Pulse dispersion compensation

linear dispersion



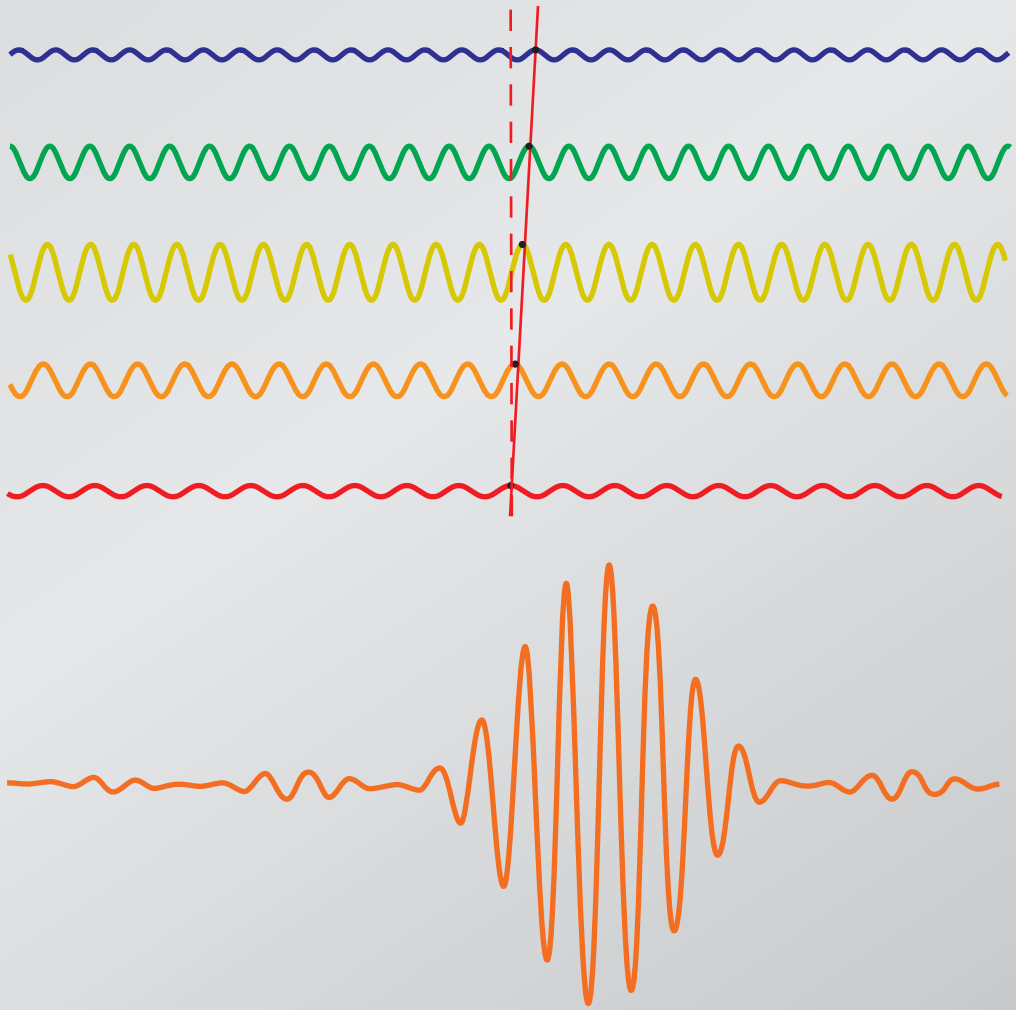
Pulse dispersion compensation

linear dispersion



Pulse dispersion compensation

linear dispersion



Pulse dispersion compensation

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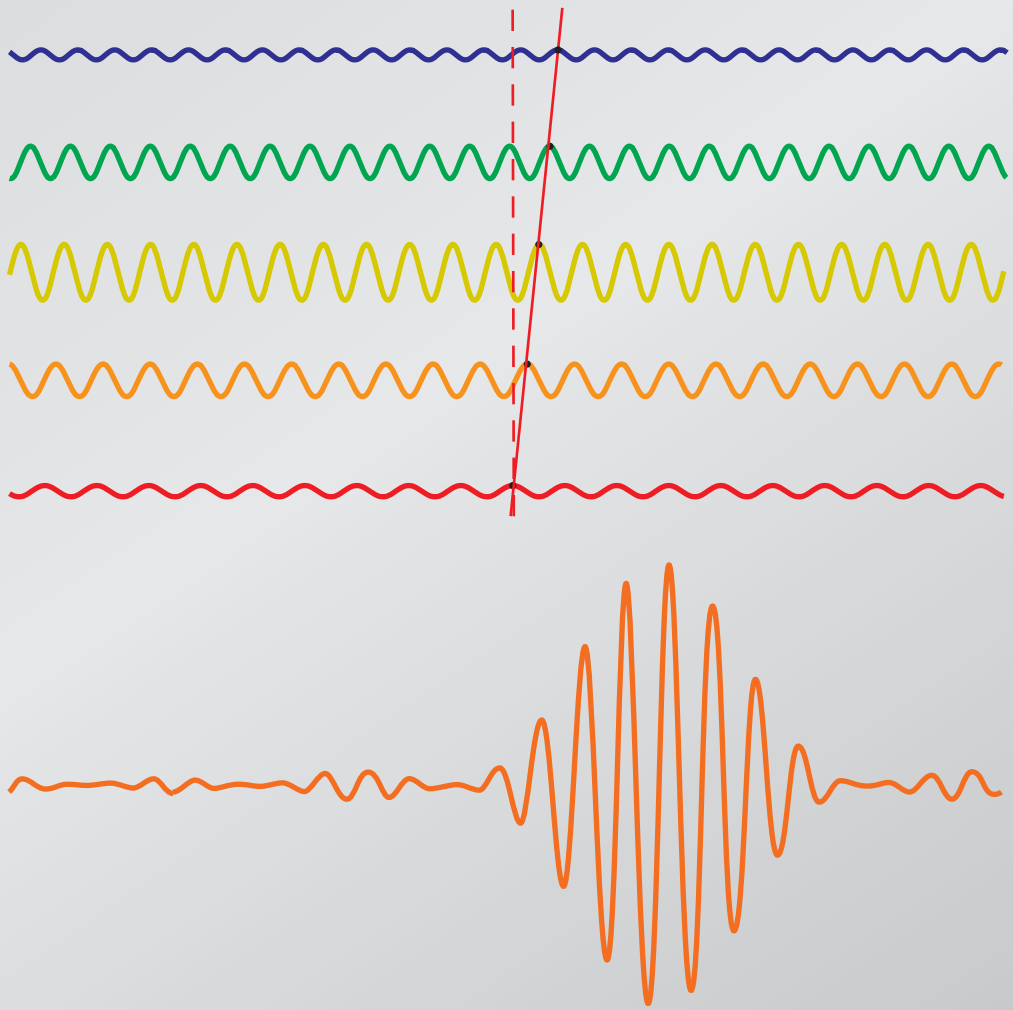

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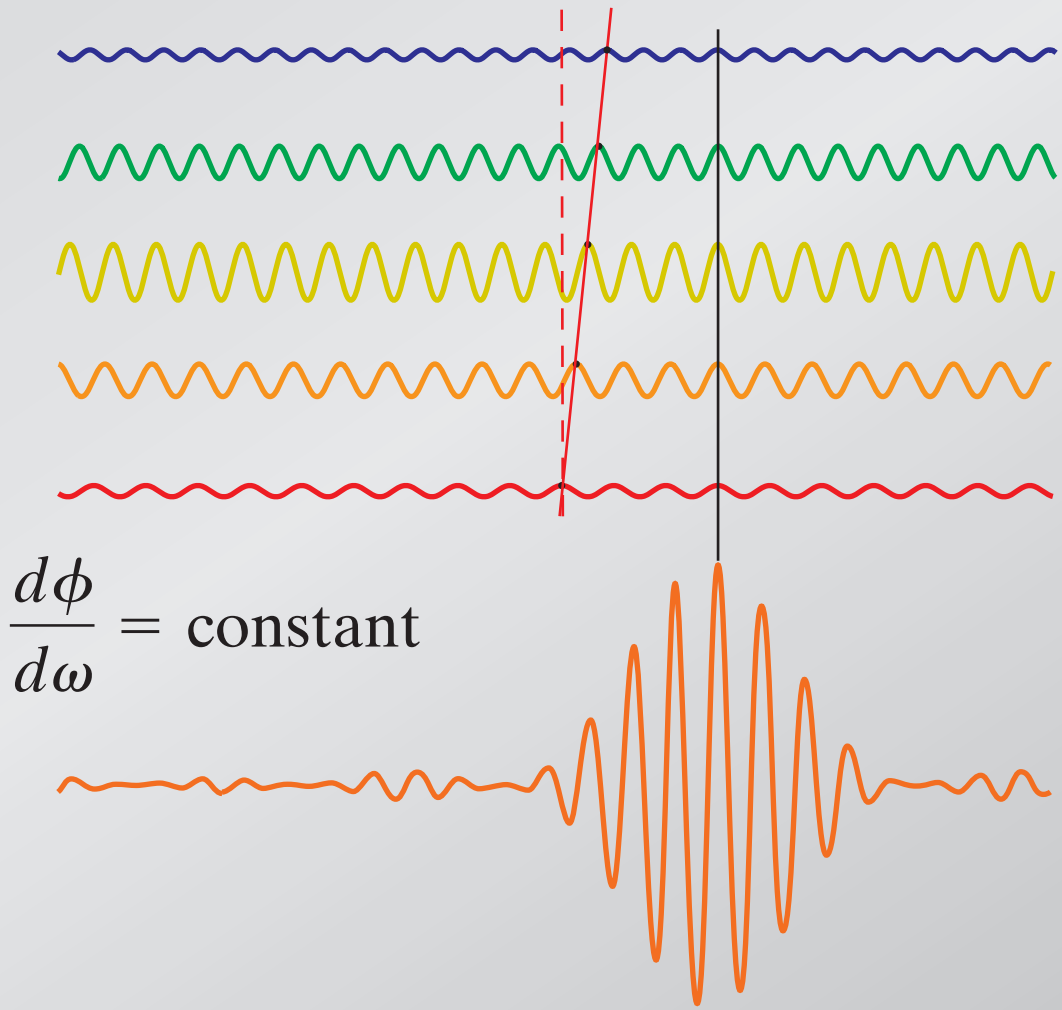
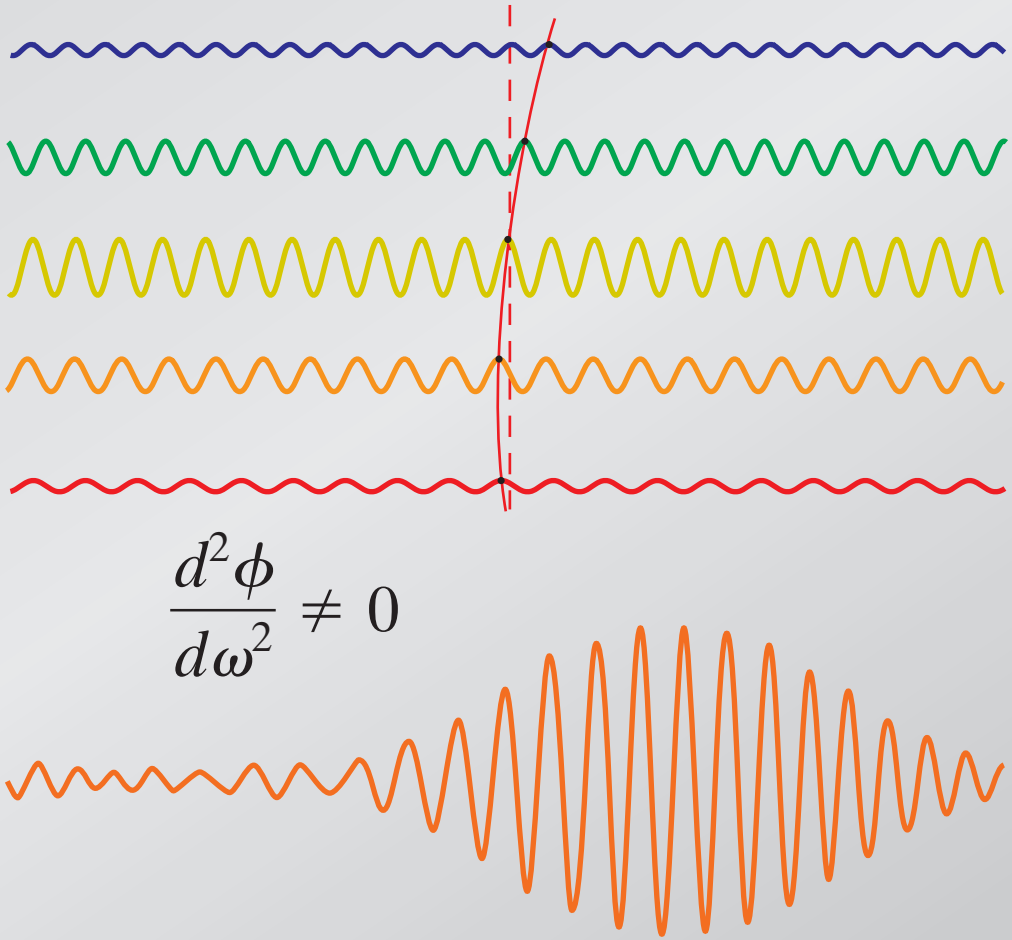


$$\frac{d\phi}{d\omega} = \text{constant}$$

Pulse dispersion compensation

only *nonlinear* dispersion changes pulse shape!



Pulse dispersion compensation

So not path length but $\frac{d^2\phi}{d\omega^2}$ matters!

Pulse dispersion compensation

So not path length but $\frac{d^2\phi}{d\omega^2}$ matters!

	$\frac{dl_{eff}}{d\omega}$	$\frac{d^2\phi}{d\omega^2}$
dispersion	+	+

Pulse dispersion compensation

So not path length but $\frac{d^2\phi}{d\omega^2}$ matters!

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Outline

- propagation of pulses
- nonlinear optics
- nanoscale optics
- nonlinear optics at the nanoscale

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$P^{(2)} \approx P^{(1)}$ **when** $E = E_{at} \approx \dfrac{e}{a}$**, and so** $\chi^{(n)} \approx \dfrac{\chi^{(1)}}{E_{at}^{n-1}}$**.**

Nonlinear optics

Nonlinear polarization can drive new field:

$$\nabla^2 \vec{E} + \frac{n^2}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 \vec{P}}{\partial t^2}$$

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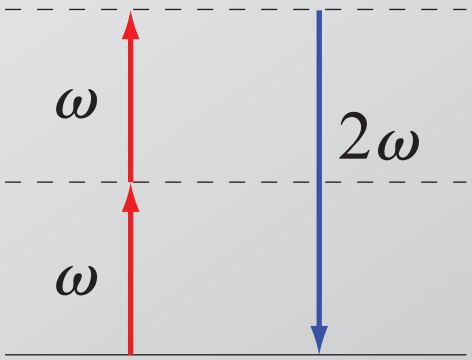

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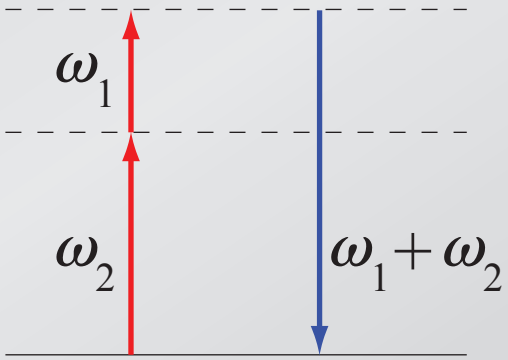

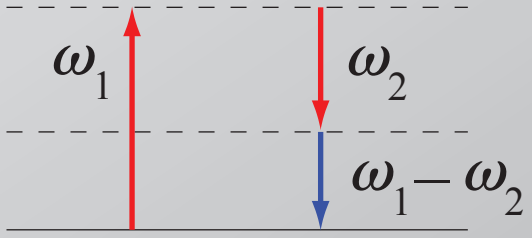

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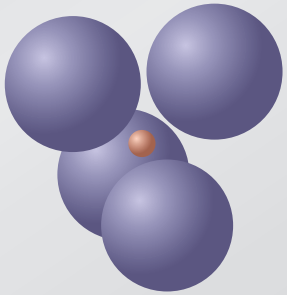

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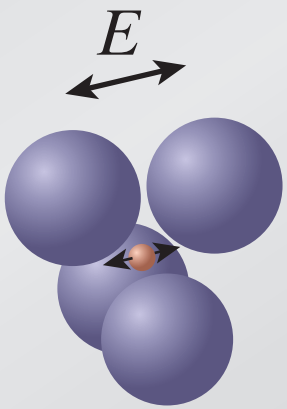

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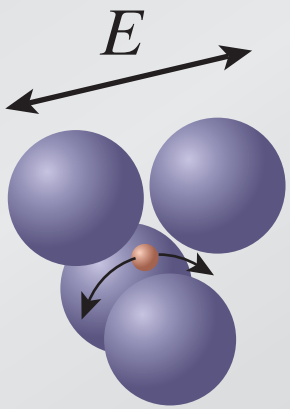

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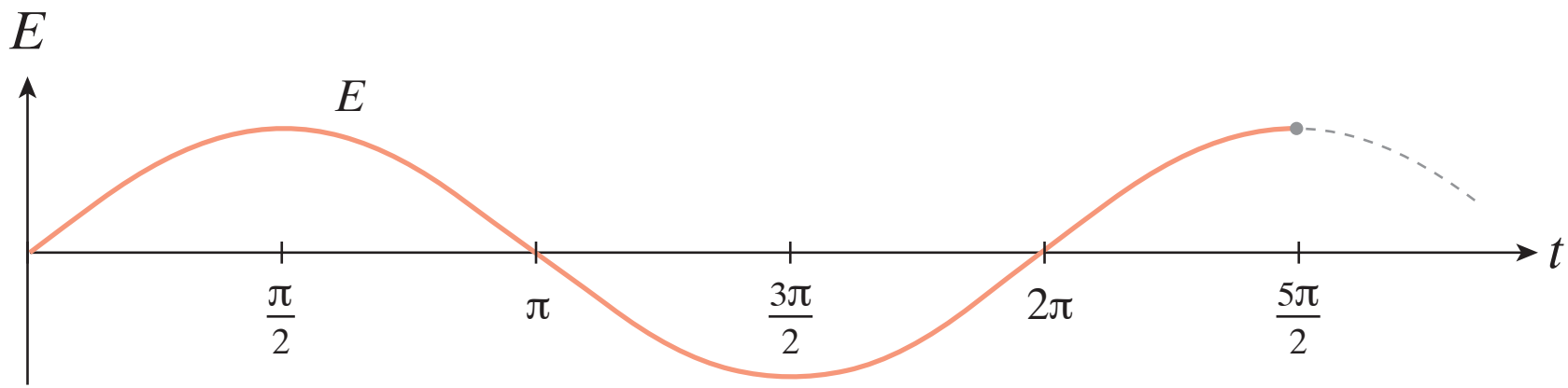

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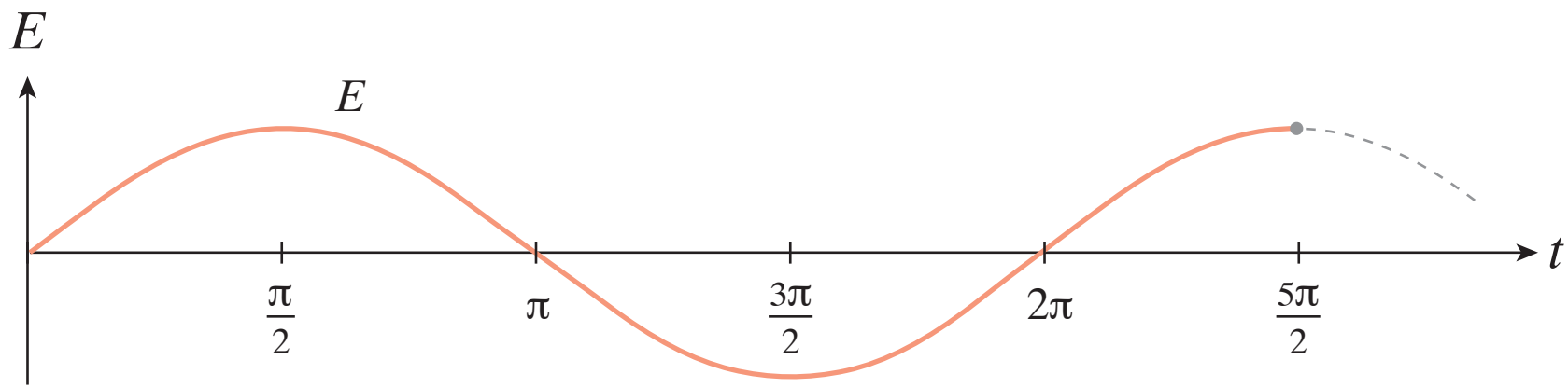

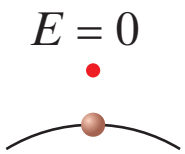

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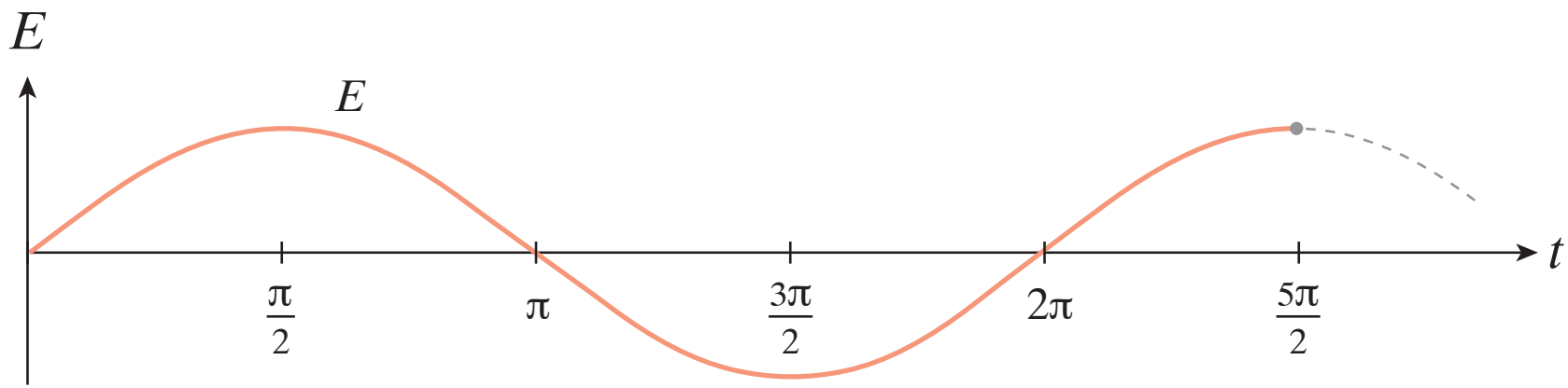

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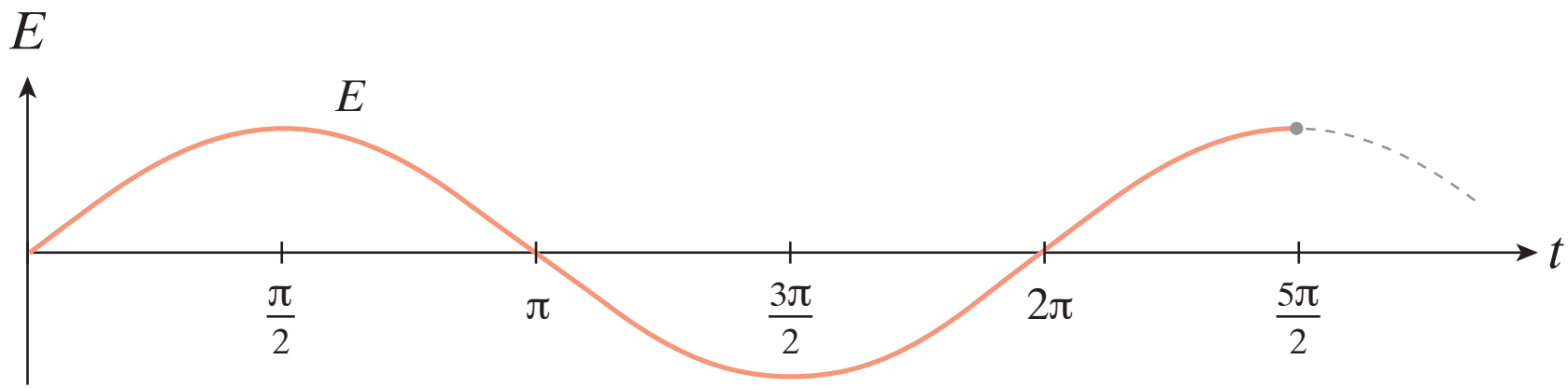

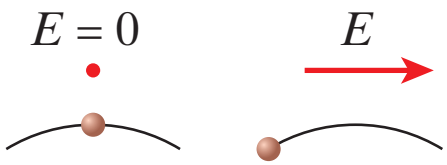

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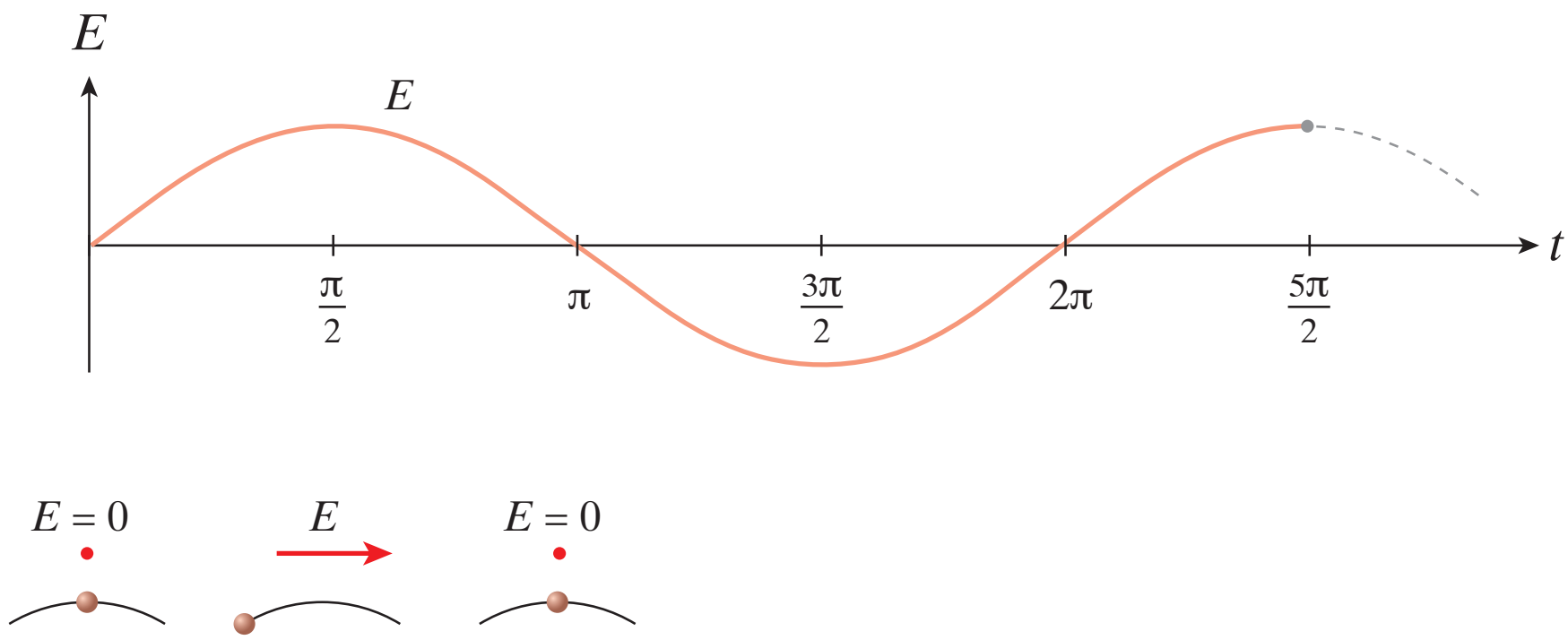

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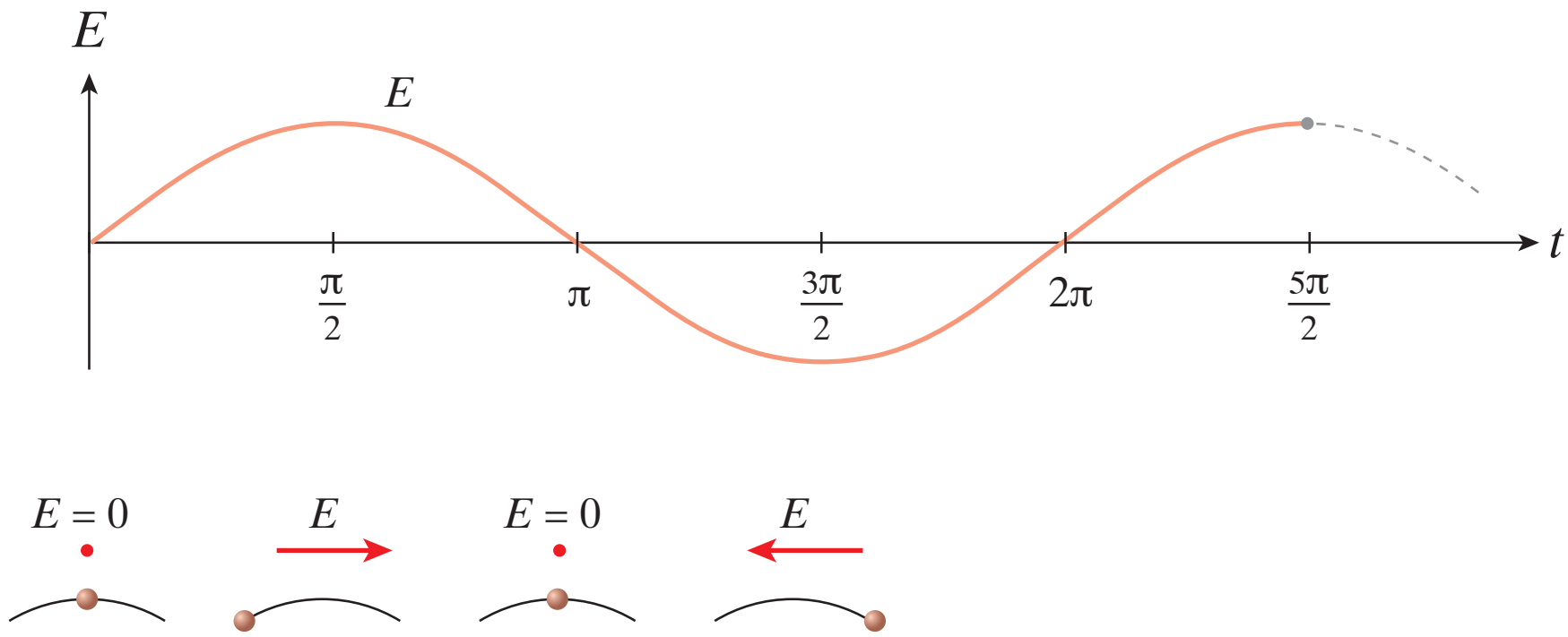

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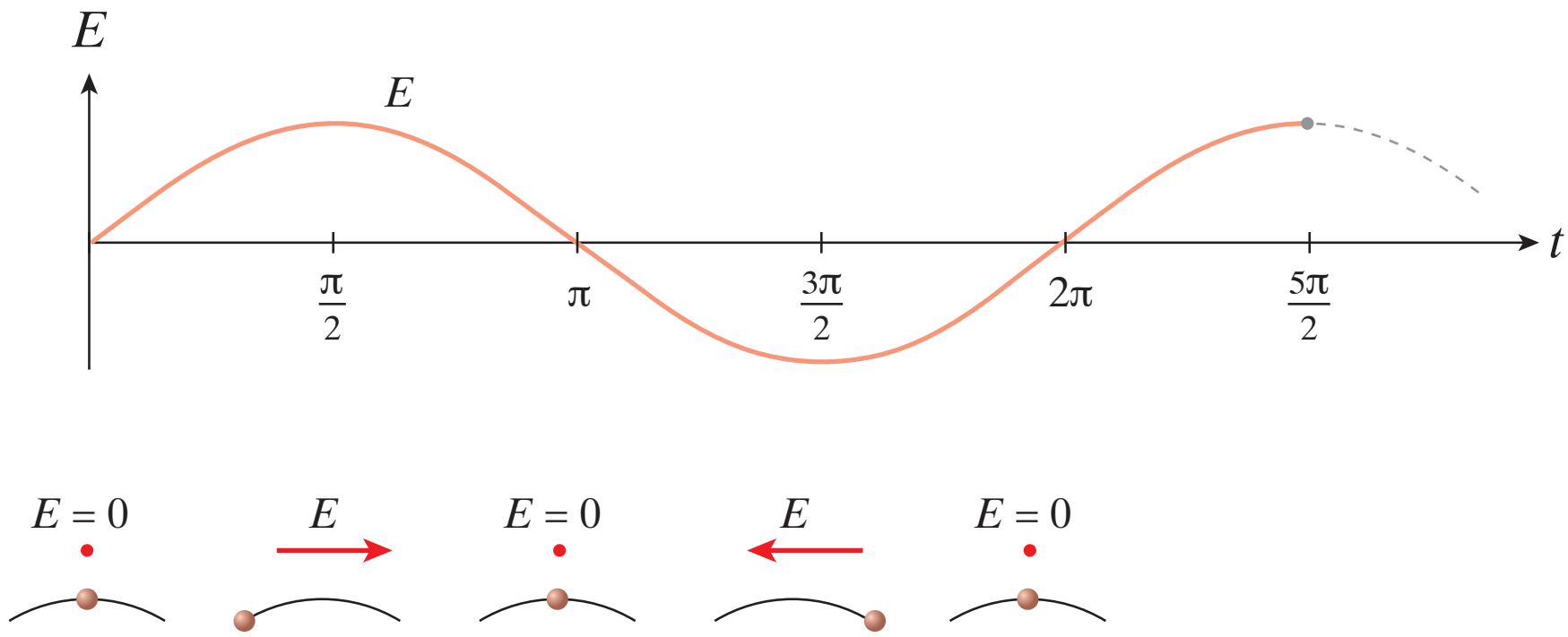

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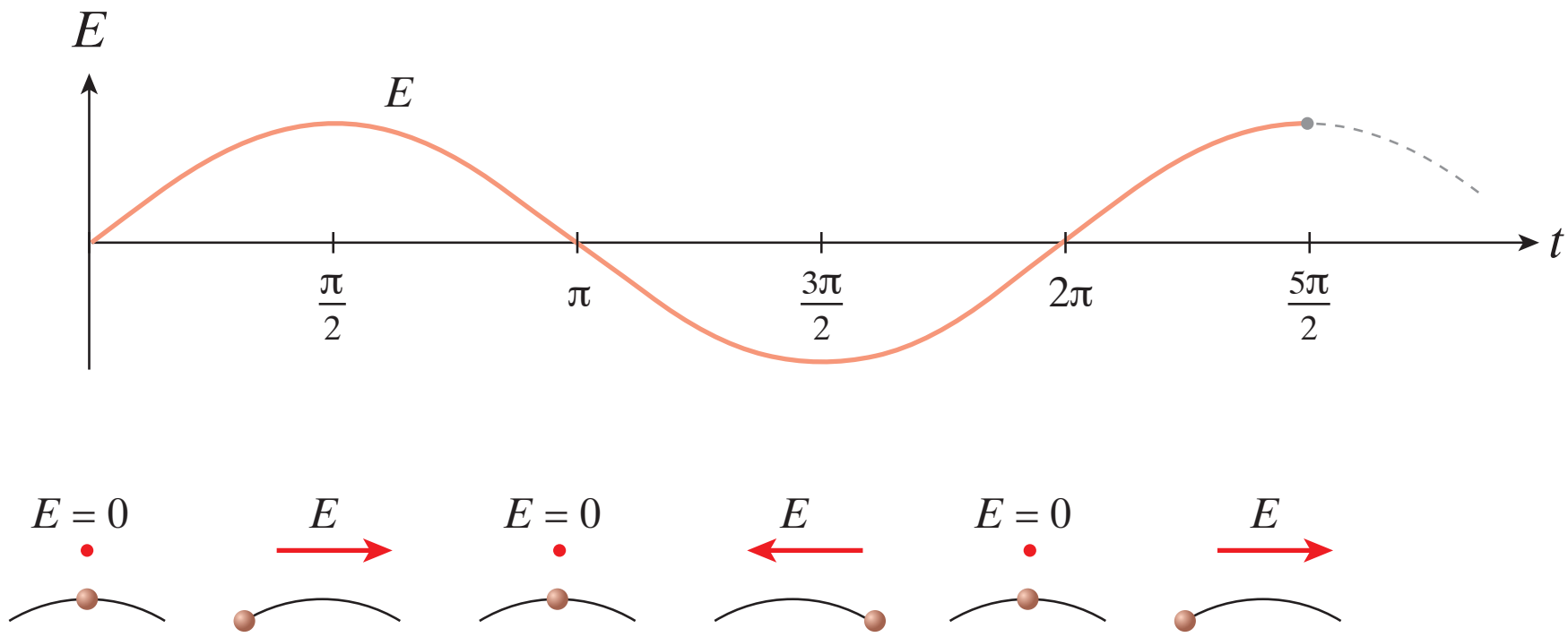

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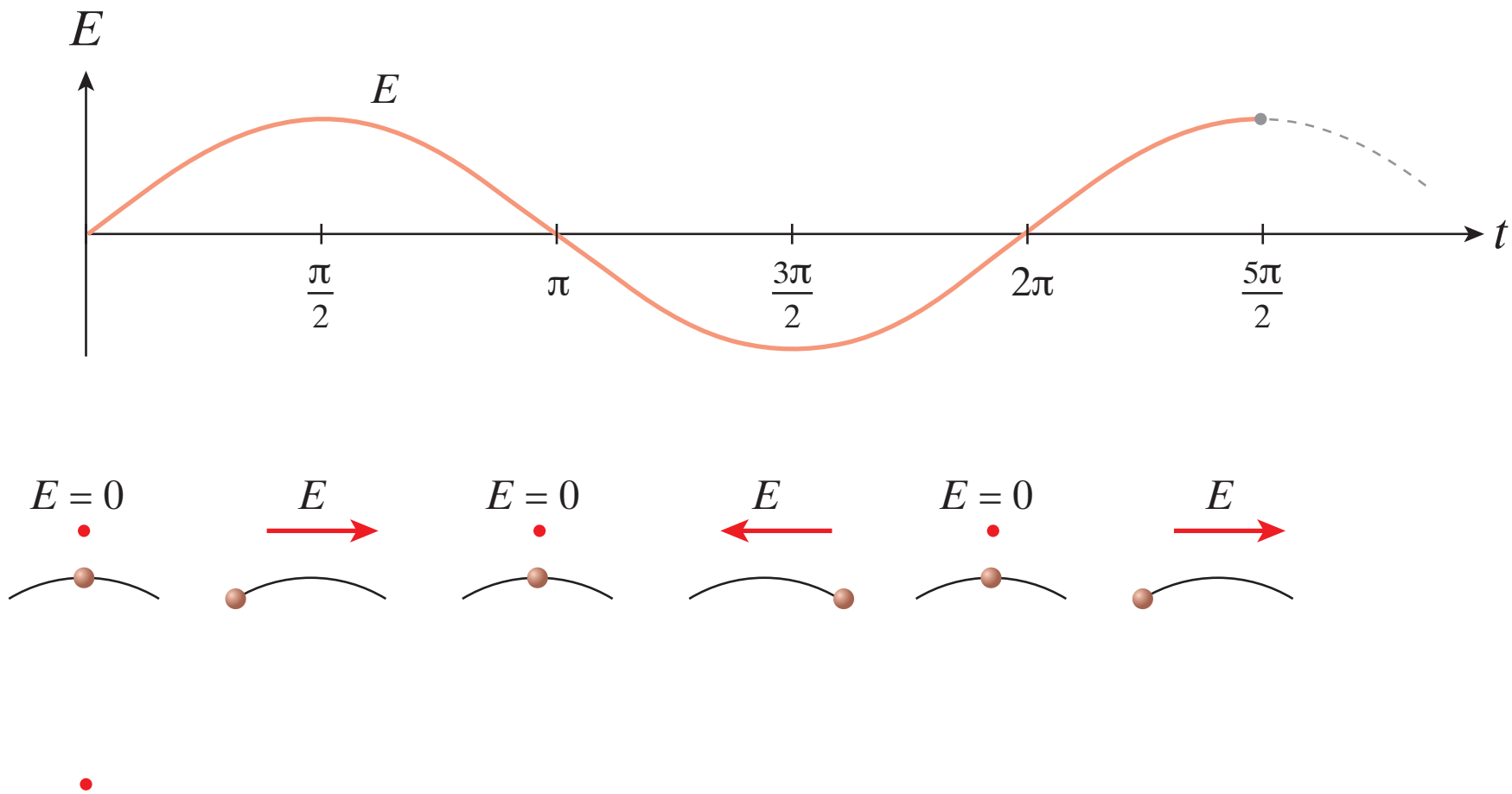

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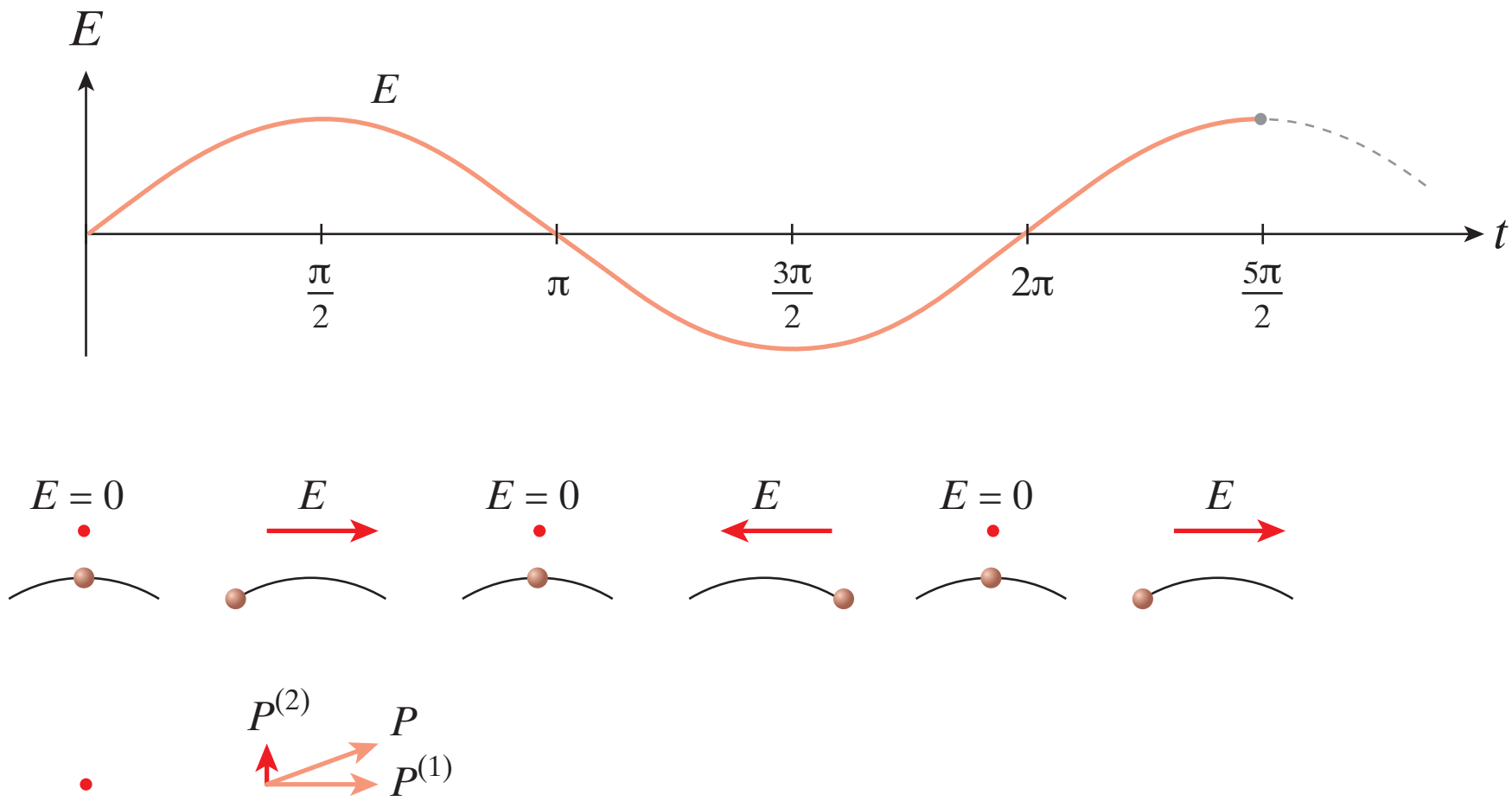

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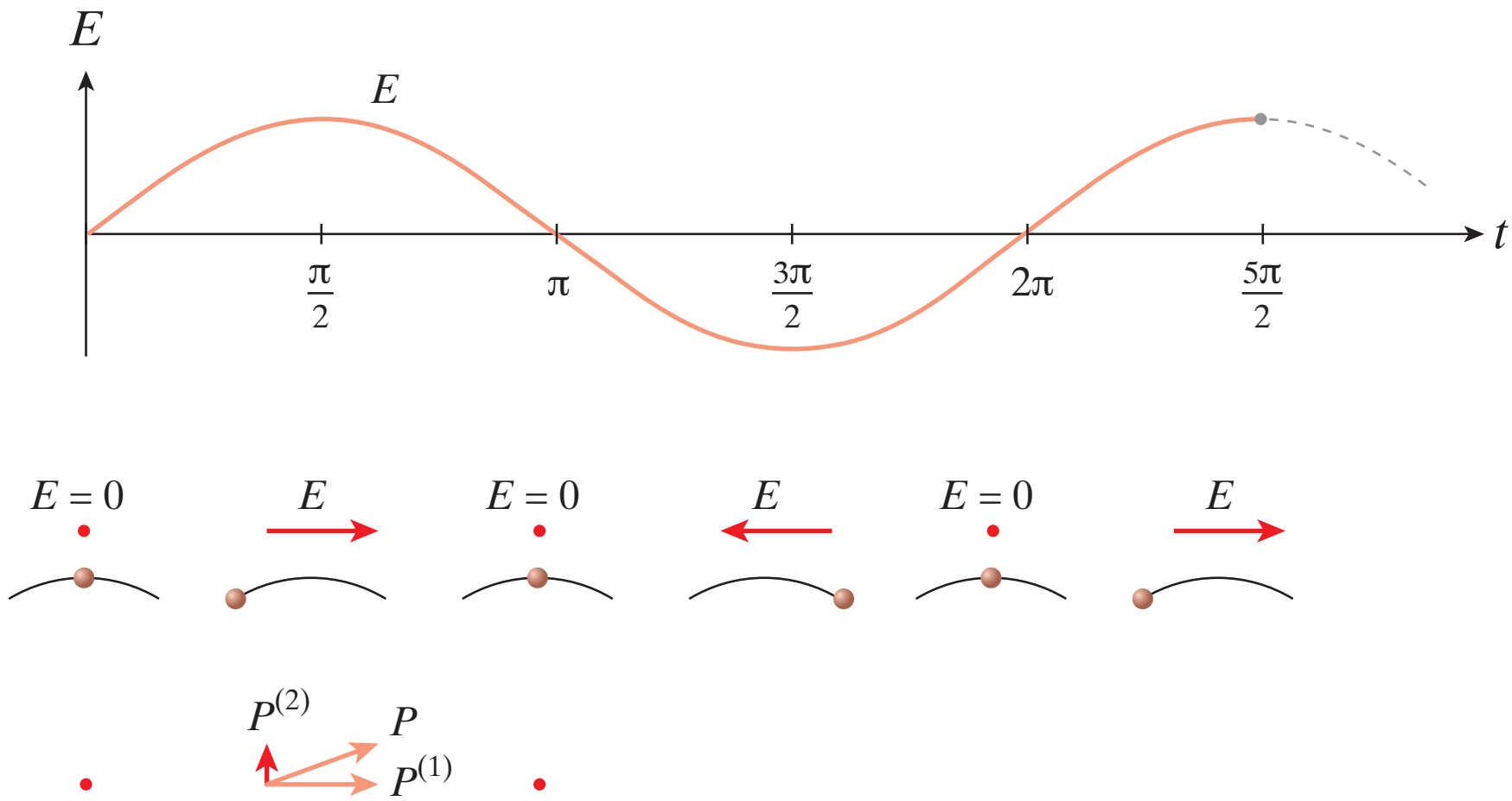

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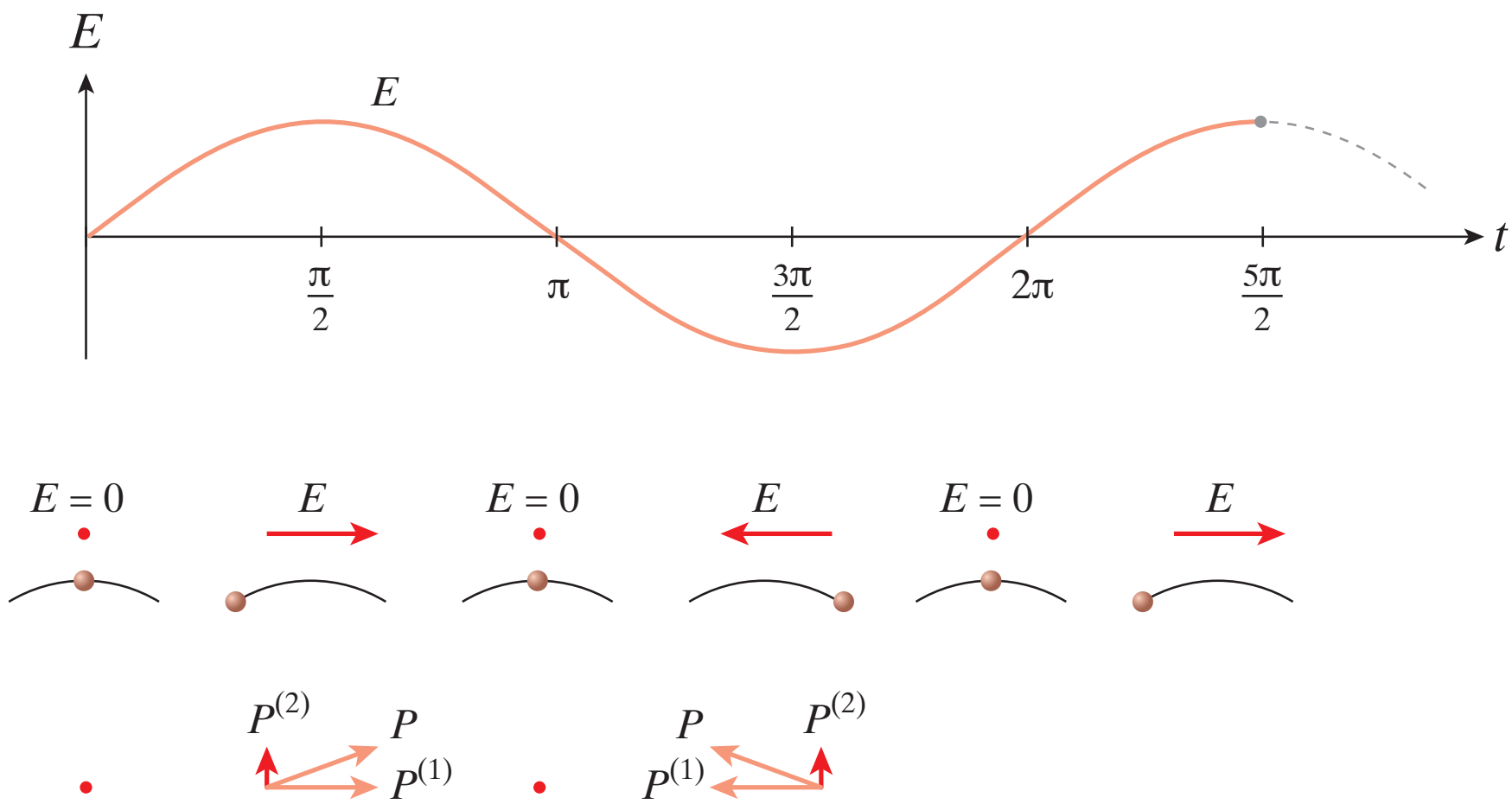

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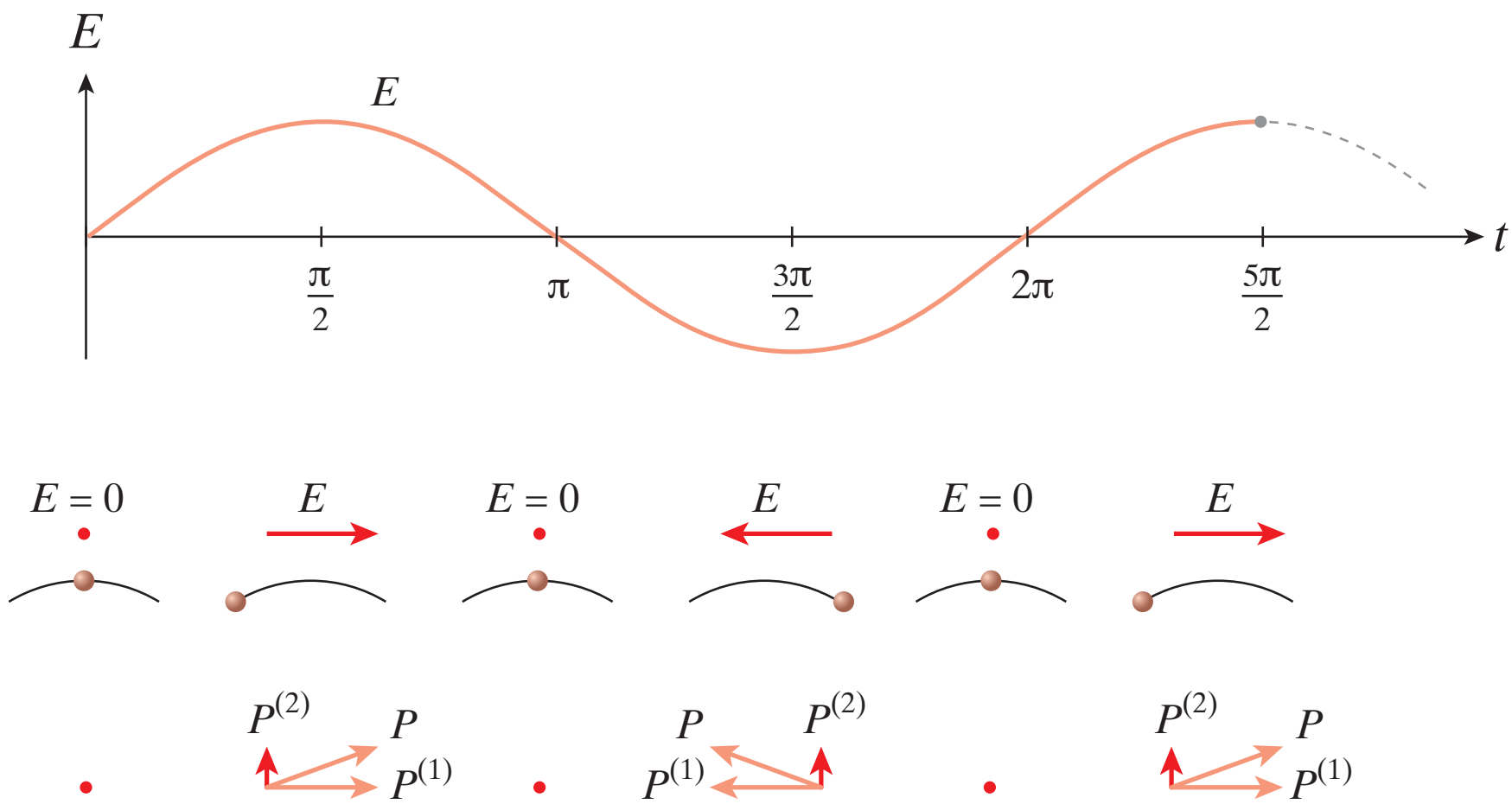

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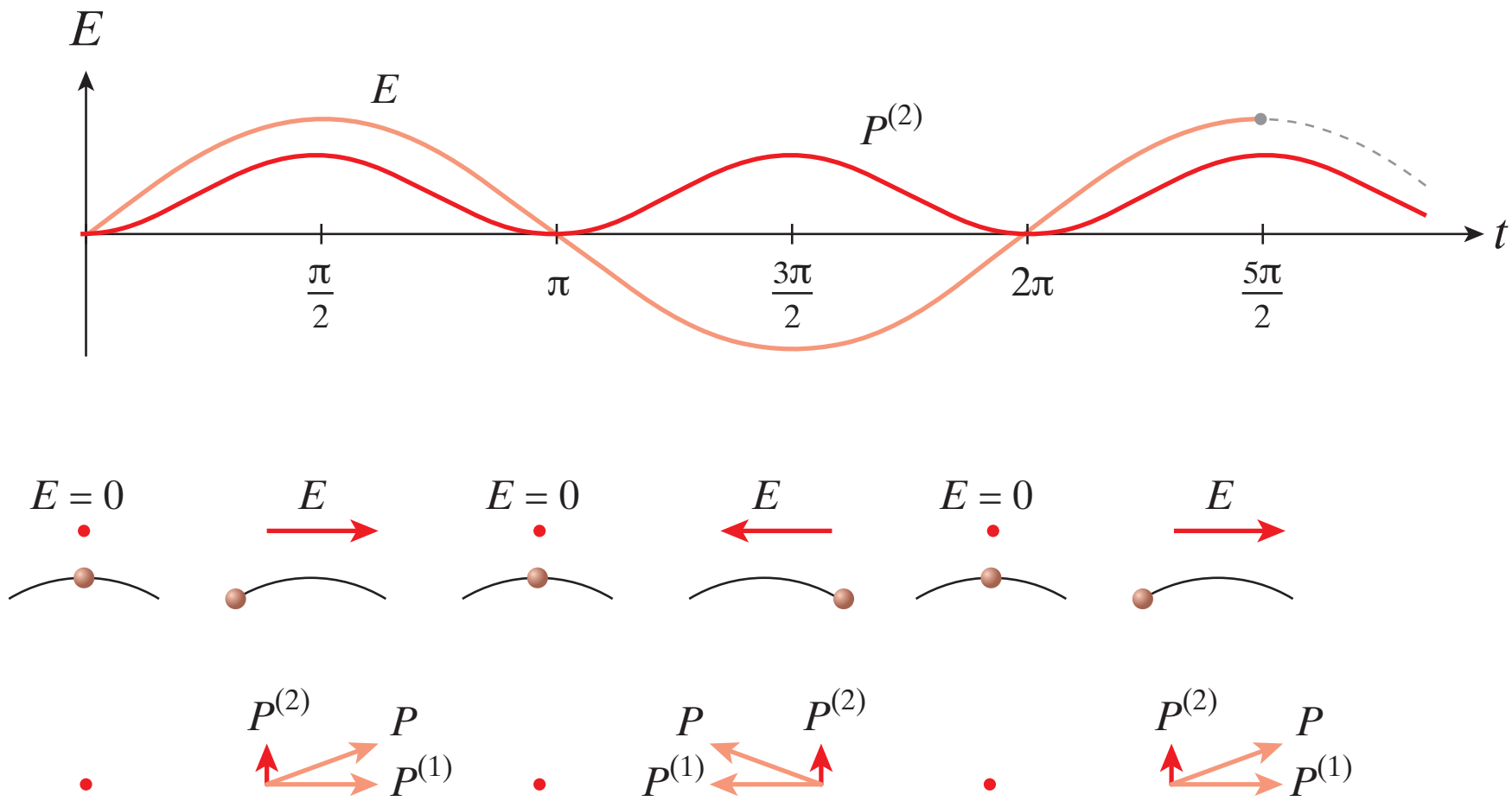

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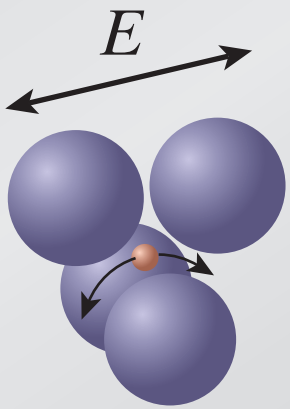

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$$n = \sqrt{\epsilon} = \sqrt{1 + \chi_{eff}} \approx \sqrt{1 + \chi^{(1)}} + \tfrac{1}{2} \frac{\chi^{(3)} I}{\sqrt{1 + \chi^{(1)}}} = n_o + n_2 I$$

Nonlinear optics

Intensity-dependent index of refraction:

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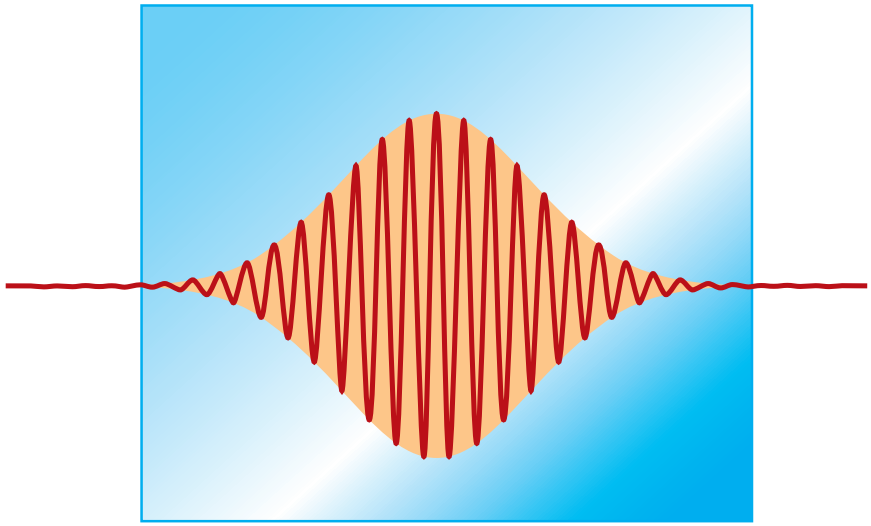

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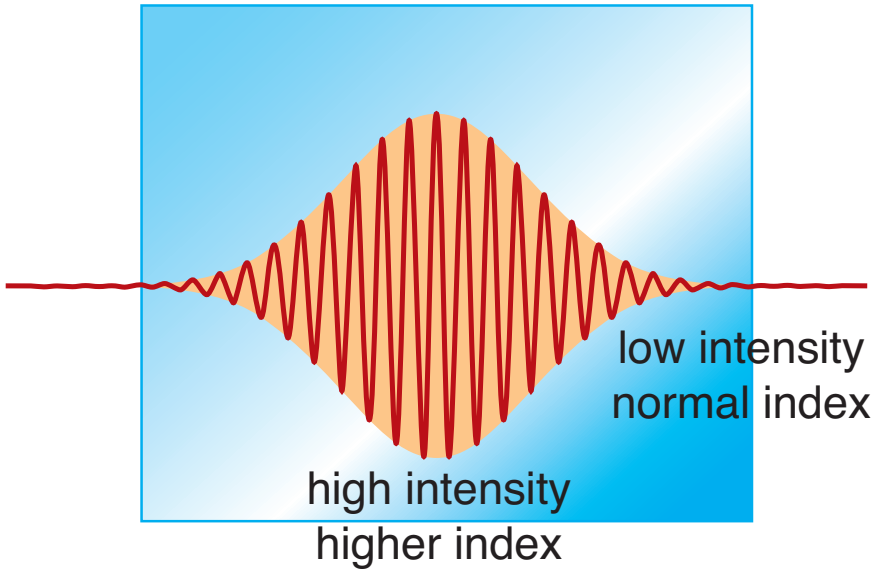

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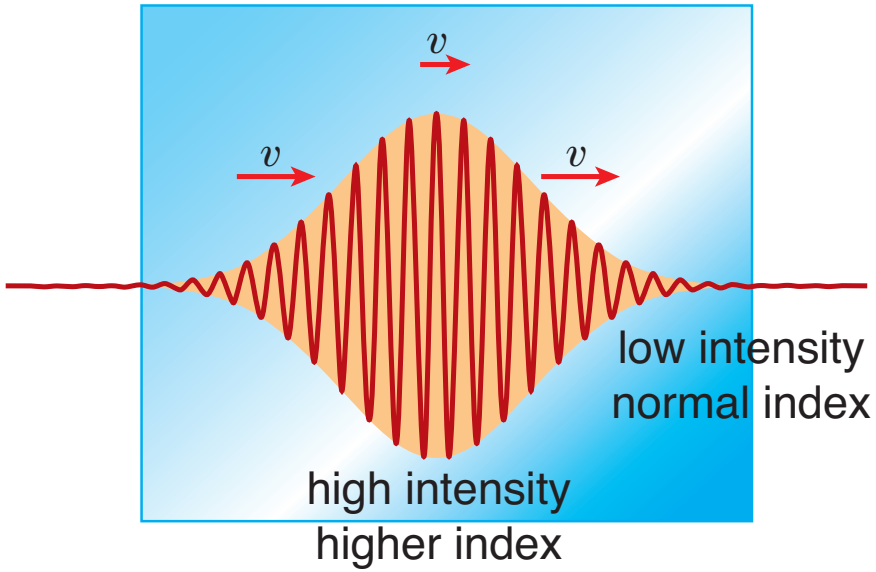

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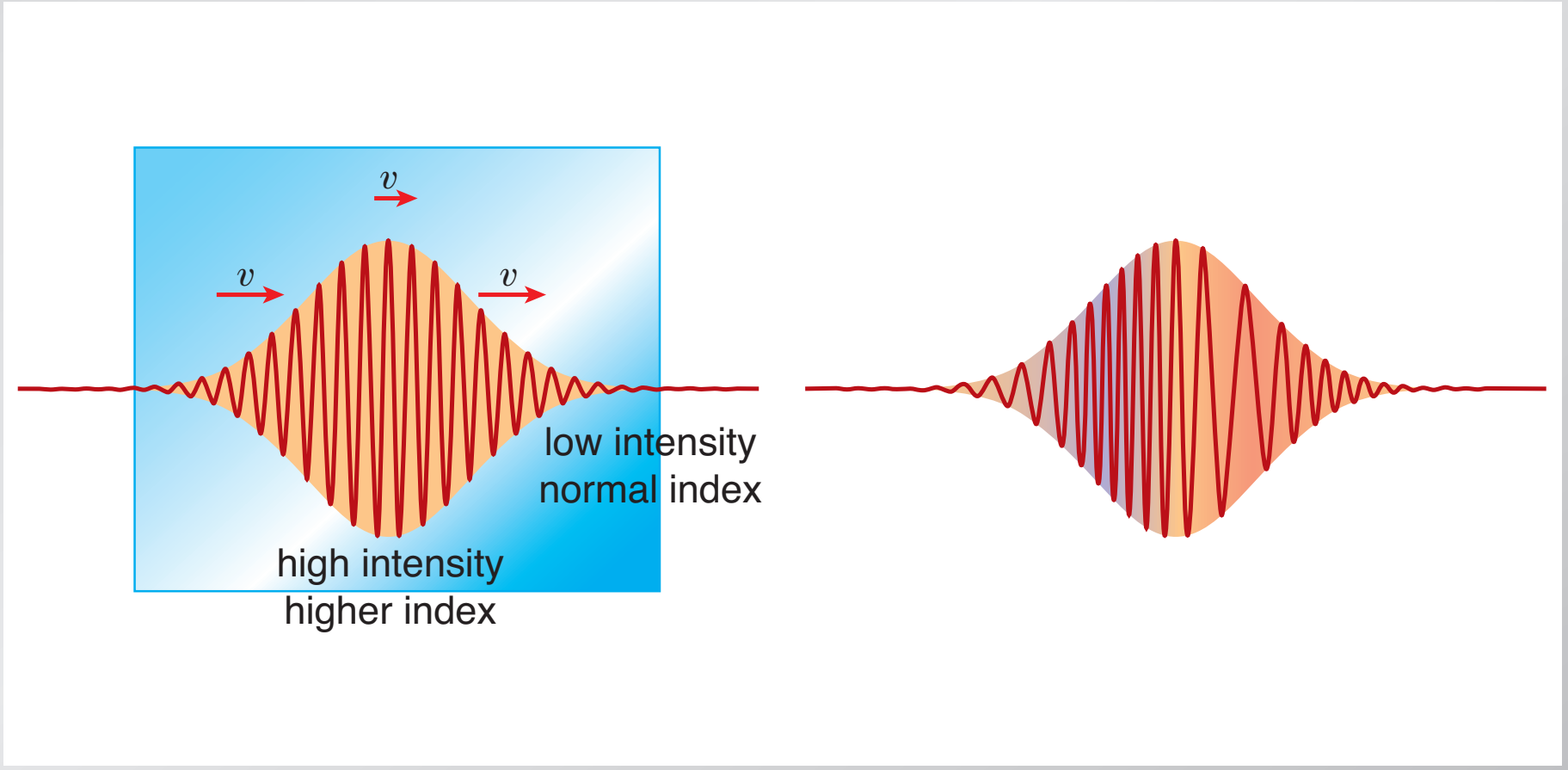

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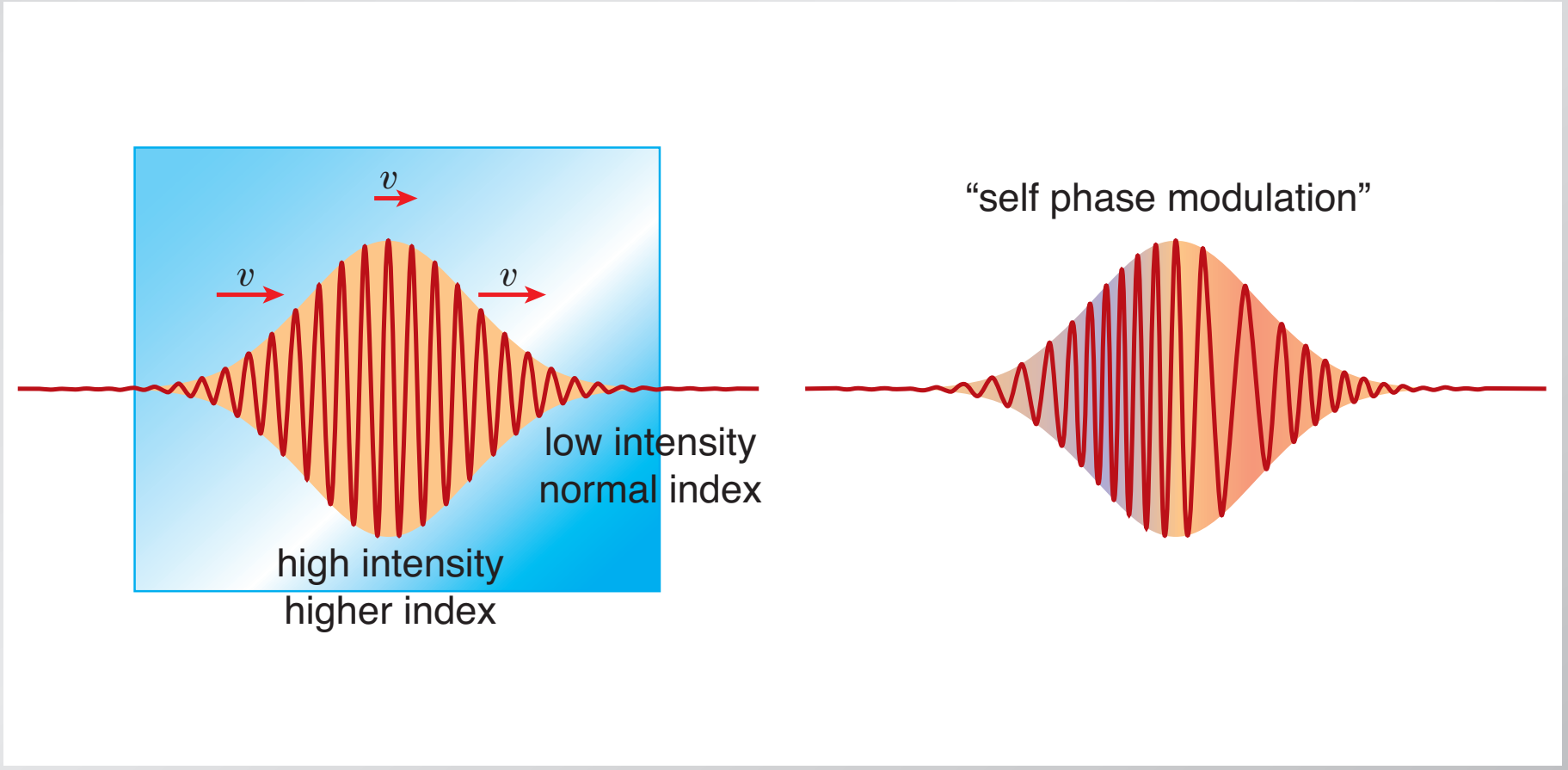

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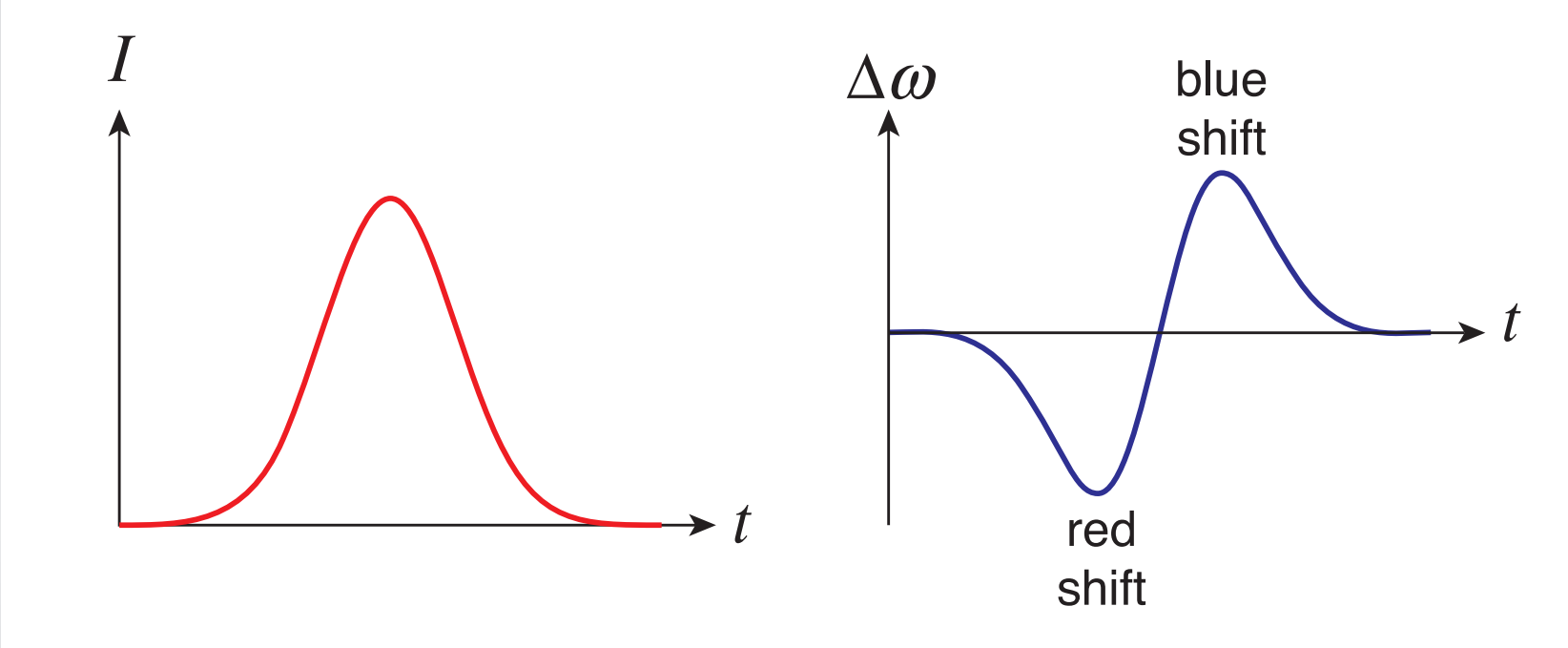

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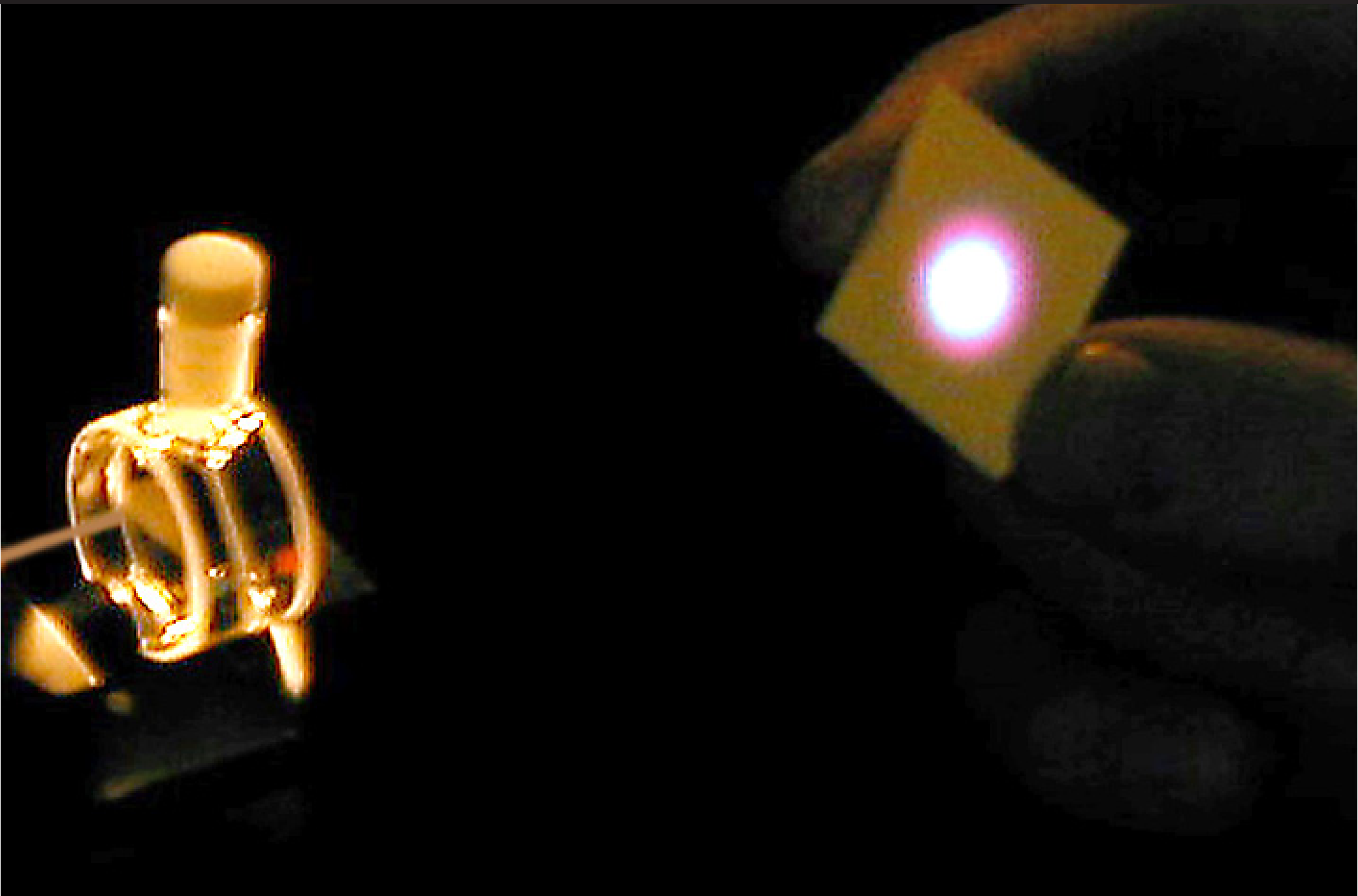

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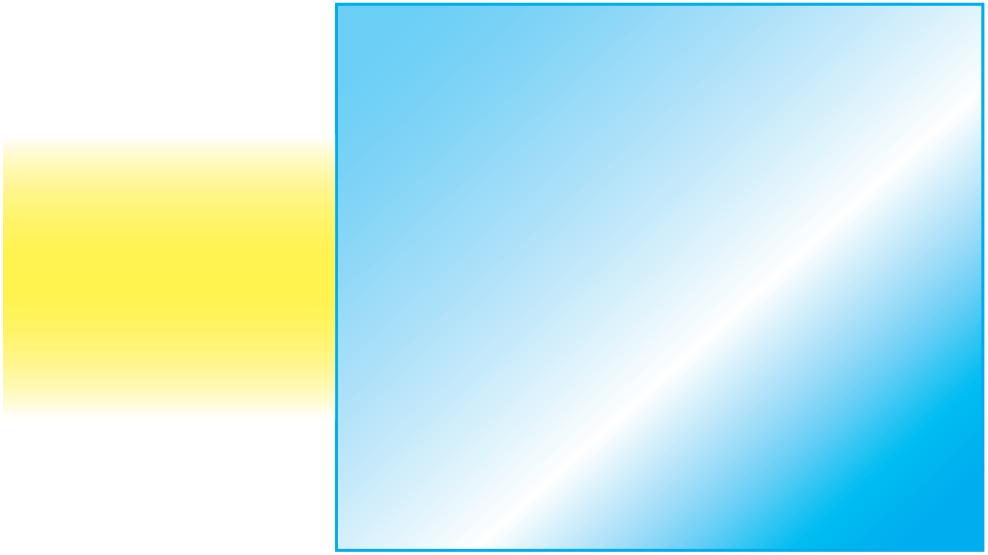

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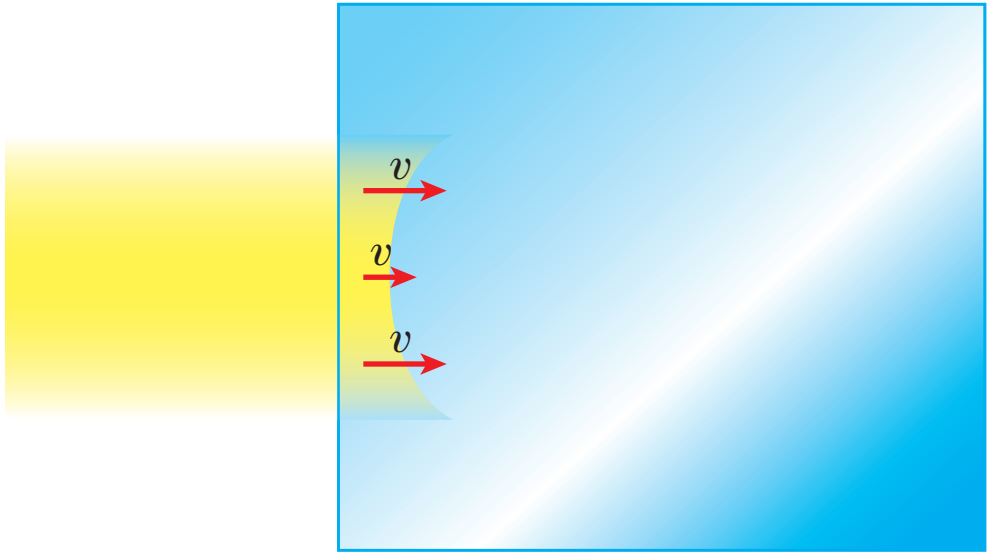
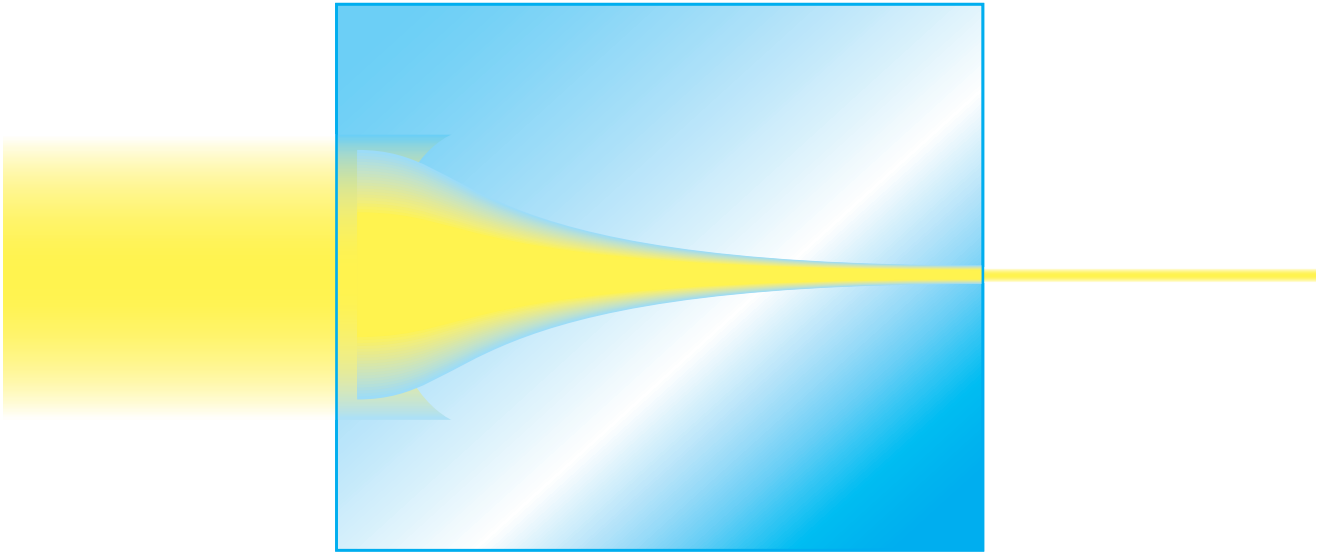


Nonlinear optics

self-focusing



Nonlinear optics

but susceptibility is complex!

susceptibility	real part	imaginary part
linear	refraction	absorption
nonlinear	SHG, SFG, DFG, THG,…	multiphoton absorption

$$\alpha = \alpha_o + \beta I + \gamma I^2 + \ldots$$

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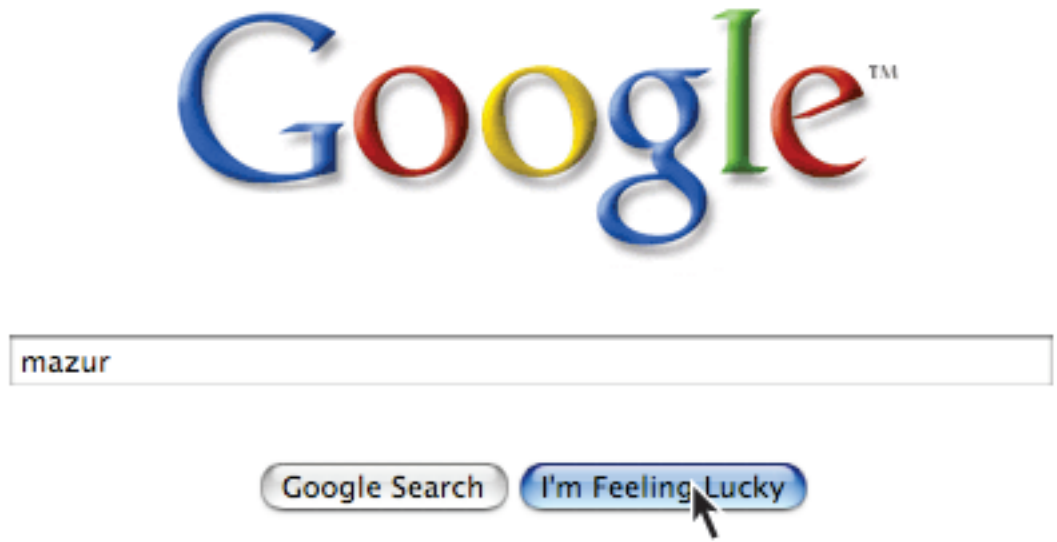
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