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• optical properties of materials
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• optical properties of materials
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• engineering the index
Governed by wave equation

\[ \nabla^2 \vec{E} - \frac{\mu \varepsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \]
Propagating electromagnetic waves through a material is governed by the wave equation:

\[ \nabla^2 \vec{E} - \frac{\mu \varepsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \]

Solution:

\[ \vec{E} = \vec{E}_0 e^{i(kx - \omega t)} \]
Propagation of EM wave through medium

Governed by wave equation

\[
\nabla^2 \vec{E} - \frac{\mu \varepsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0
\]

Solution:

\[
\vec{E} = \vec{E}_o e^{i(kx - \omega t)}
\]

where

\[
\frac{\omega}{k} = \frac{1}{\sqrt{\varepsilon \mu}} c = \frac{1}{n} c
\]

and

\[
n = \sqrt{\varepsilon \mu}
\]
Propagation of EM wave through medium

Governed by wave equation

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Solution:

\[ \vec{E} = \vec{E}_o e^{i(kx - \omega t)} \]

where

\[ \frac{\omega}{k} = \frac{1}{\sqrt{\varepsilon \mu}} c = \frac{1}{n} c \]

and

\[ n = \sqrt{\varepsilon \mu} \].

In dispersive media \( n = n(\omega) \).
Propagation of EM wave through medium

Dielectric constant measures increase in capacitance

\[ \epsilon = \frac{C_d}{C_o} \]

\[ E = \frac{\Delta V}{d} \]

\[ Q_o \]
Propagation of EM wave through medium

Dielectric constant measures increase in capacitance

\[ \epsilon = \frac{C_d}{C_o} \]
Propagation of EM wave through medium

Dielectric constant measures increase in capacitance

\[ \epsilon = \frac{C_d}{C_o} \]
Propagation of EM wave through medium

Dielectric constant measures increase in capacitance

$$\epsilon = \frac{C_d}{C_0}$$

\[ E = \frac{\Delta V}{d} \]
Propagation of EM wave through medium

Electric field is sum of two contributions

\[ E = E_{\text{free}} + E_{\text{bound}} = \varepsilon E - P \]
Electric field is sum of two contributions

\[ E = E_{\text{free}} + E_{\text{bound}} = \epsilon E - P \]

Polarization \( P \) proportional to \( E \)

\[ P = \chi E \]
Propagation of EM wave through medium

Electric field is sum of two contributions

\[ E = E_{\text{free}} + E_{\text{bound}} = \varepsilon E - P \]

Polarization \( P \) proportional to \( E \)

\[ P = \chi E \]

so

\[ E = \varepsilon E - \chi E \]
Propagation of EM wave through medium

Electric field is sum of two contributions

\[ E = E_{\text{free}} + E_{\text{bound}} = \varepsilon E - P \]

Polarization \( P \) proportional to \( E \)

\[ P = \chi E \]

so

\[ E = \varepsilon E - \chi E \]

or

\[ \varepsilon = 1 + \chi \]
Propagation of EM wave through medium

Alternatively $\epsilon$ is measure of attenuation of electric field
Alternatively $\epsilon$ is measure of attenuation of electric field
Propagation of EM wave through medium

In vacuum:

\[ f \lambda = \frac{\omega}{k} = c \quad \Rightarrow \quad \omega = c k \]
Propagation of EM wave through medium

In medium:

\[ v = \frac{c}{\sqrt{\varepsilon(\omega)}} = \frac{c}{n} \quad \Rightarrow \quad \omega = \frac{c}{\sqrt{\varepsilon}} k \]
Propagation of EM wave through medium

In medium:

\[ v = \frac{c}{\sqrt{\varepsilon(\omega)}} = \frac{c}{n} \quad \Rightarrow \quad \omega = \frac{c}{\sqrt{\varepsilon}} k \]
Which charges participate?

- Valence electrons
- Ionic cores
- Free electrons
Dielectric function

Dielectric function

[Graph showing the dielectric constant as a function of frequency (rad/s). The x-axis represents frequency in powers of 10 from $10^6$ to $10^{18}$, and the y-axis represents the dielectric constant. The graph includes labels for MW, IR, VIS, UV, and X.]
Dielectric function

Dielectric constant vs. frequency (rad/s)

- Vacuum

Frequency range:
- Microwave (MW)
- Infrared (IR)
- Visible (VIS)
- Ultraviolet (UV)
- X-ray (X)
Dielectric function

Dielectric constant

10^6 10^8 10^10 10^12 10^14 10^16 10^18

frequency (rad/s)

MW

electronic

vacuum
Dielectric function

The diagram illustrates the dielectric constant as a function of frequency. The horizontal axis represents the frequency in radians per second, ranging from $10^6$ to $10^{18}$. The vertical axis shows the dielectric constant, with levels indicating ionic, electronic, and vacuum regions.

- **MW** (Microwave) region
- **vacuum**
- **ionic**
- **electronic**

The graph shows the behavior of the dielectric constant across different frequency ranges, highlighting the transition from ionic to electronic to vacuum behavior.
Dielectric function

- MW
- IR
- VIS
- UV
- X

Dielectric constant

- dipolar
- ionic
- electronic
- vacuum

Frequency (rad/s):

- $10^6$
- $10^8$
- $10^{10}$
- $10^{12}$
- $10^{14}$
- $10^{16}$
- $10^{18}$
Dielectric function

![Graph showing dielectric constant vs frequency (rad/s)]
Bound electrons

Electron on a string: \[ F_{binding} = - m_e \omega_o^2 x \]
Electron on a string:

\[ F_{\text{binding}} = - m_e \omega_0^2 x \]

\[ F_{\text{damping}} = - m_e \gamma \frac{dx}{dt} \]
Bound electrons

Electron on a string:

\[ F_{\text{binding}} = -m_e \omega_o^2 x \]

\[ F_{\text{damping}} = -m_e \gamma \frac{dx}{dt} \]

\[ F_{\text{driving}} = -eE = -eE_o e^{-i\omega t} \]
Electron on a string:

\[ F_{\text{binding}} = - m_e \omega_o^2 x \]

\[ F_{\text{damping}} = - m_e \gamma \frac{dx}{dt} \]

\[ F_{\text{driving}} = - eE = - e E_o e^{-i\omega t} \]

Equation of motion:

\[ m \frac{d^2x}{dt^2} = \sum F \]
Bound electrons

Electron on a string:

\[ F_{\text{binding}} = -m_e \omega_o^2 x \]

\[ F_{\text{damping}} = -m_e \gamma \frac{dx}{dt} \]

\[ F_{\text{driving}} = -eE = -eE_0 e^{-i\omega t} \]

Equation of motion:

\[ m \frac{d^2 x}{dt^2} = \sum F \]

\[ m \frac{d^2 x}{dt^2} + m\gamma \frac{dx}{dt} + m\omega_o^2 x = -eE \]
Bound electrons

Steady state: electron oscillates at driving frequency

\[ x(t) = x_0 e^{-i\omega t} \]
Bound electrons

Steady state: electron oscillates at driving frequency

\[ x(t) = x_o e^{-i\omega t} \]

\[ x_o = -\frac{e}{m \left( \omega_o^2 - \omega^2 \right) - i\gamma \omega} E_o \]
Bound electrons

Steady state: electron oscillates at driving frequency

\[ x(t) = x_o e^{-i\omega t} \]

\[ x_o = -\frac{e}{m (\omega_o^2 - \omega^2) - i\gamma\omega} E_o \]

Oscillating dipole

\[ p(t) = -e x(t) = \frac{e^2}{m (\omega_o^2 - \omega^2) - i\gamma\omega} E_o e^{-i\omega t} \]
Bound electrons

Steady state: electron oscillates at driving frequency

\[ x(t) = x_0 e^{-i\omega t} \]
\[ x_0 = -\frac{e}{m(\omega_0^2 - \omega^2) - i\gamma\omega} E_o \]

Oscillating dipole

\[ p(t) = -e x(t) = \frac{e^2}{m(\omega_0^2 - \omega^2) - i\gamma\omega} E_o e^{-i\omega t} \]

Polarization

\[ P(t) = \left( \frac{Ne^2}{m} \right) \sum_j \frac{f_j}{(\omega_j^2 - \omega^2 - i\gamma_j\omega)} E(t) \equiv \epsilon_0 \chi e E(t) \]
Bound electrons

Dielectric function

\[ \epsilon(\omega) \equiv 1 + \chi_e = 1 + \frac{Ne^2}{\varepsilon_0 m} \sum_j \frac{f_j}{(\omega_j^2 - \omega^2)} - i \gamma_j \omega \]
Dielectric function

\[ \epsilon(\omega) \equiv 1 + \chi_e = 1 + \frac{Ne^2}{\varepsilon_o m} \sum_j \frac{f_j}{(\omega_j^2 - \omega^2) - i\gamma_j\omega} \]

Q: For a single resonance, is the value of \( \epsilon(\omega) \) at high frequency

1. larger than,
2. the same as, or
3. smaller than the value at low frequency?
Bound electrons

Dielectric function

\[
\epsilon(\omega) \equiv 1 + \chi_e = 1 + \frac{Ne^2}{\varepsilon_0 m} \sum_j \frac{f_j}{(\omega_j^2 - \omega^2) - i\gamma_j \omega}
\]

Q: For a single resonance, is the value of \(\epsilon(\omega)\) at high frequency

1. larger than,

2. the same as, or

3. smaller than the value at low frequency? ✓
Bound electrons

Dielectric function

\[ \varepsilon(\omega) \equiv 1 + \chi_e = 1 + \frac{Ne^2}{\varepsilon_0 m} \sum_j \frac{f_j}{(\omega_j^2 - \omega^2)} - i \gamma_j \omega \]
Bound electrons

Amplitude of bound charge oscillation

\[ x_0 \]

\[ \omega \]

\[ \phi \]

\[ \omega_j \]

\[ \epsilon' \]

\[ \epsilon'' \]

\[ \omega_j \]
Bound electrons

Below resonance: bound charges keep up with driving field  \( \Rightarrow \) field attenuated, wave propagates more slowly
Bound electrons

At resonance: energy transfer from wave to bound charges $\Rightarrow$ wave attenuates (absorption)
Bound electrons

Above resonance: bound charges cannot keep up with driving field $\Rightarrow$ dielectric like a vacuum
Bound electrons

Dielectric function

\[ \epsilon(\omega) \equiv 1 + \chi_e = 1 + \frac{Ne^2}{\epsilon_0 m} \sum_j \frac{f_j}{(\omega_j^2 - \omega^2) - i\gamma_j \omega} \]
Free electrons

No binding: \[ F_{binding} \approx 0 \]
Free electrons

No binding: \( F_{\text{binding}} \approx 0 \)

Equation of motion: \( m \frac{d^2x}{dt^2} + m \gamma \frac{dx}{dt} = -eE \)
Free electrons

No binding: \[ F_{\text{binding}} \approx 0 \]

Equation of motion: \[ m \frac{d^2x}{dt^2} + m \gamma \frac{dx}{dt} = -eE \]

Solution: \[ x(t) = \frac{e}{m \omega^2 + i \gamma \omega} \frac{1}{E(t)} \] (no resonance)
Free electrons

No binding: \( F_{binding} \approx 0 \)

Equation of motion:
\[
m \frac{d^2 x}{dt^2} + m \gamma \frac{dx}{dt} = -eE
\]

Solution:
\[
x(t) = \frac{e}{m} \frac{1}{\omega^2 + i \gamma \omega} E(t) \quad \text{(no resonance)}
\]

Low frequency \( (\omega \ll \gamma) \) \( \Rightarrow \) current generated

\[
J = -Ne \frac{dx}{dt} = \frac{Ne^2}{m} \frac{1}{\gamma - i \omega} E \approx \frac{Ne^2}{m \gamma} E \equiv \sigma E
\]
Free electrons

\[ \omega \gg \gamma : \quad \sigma \text{ complex } \Rightarrow \quad J \text{ out of phase with } E \]
Free electrons

$\omega \gg \gamma \quad \sigma \text{ complex} \Rightarrow J \text{ out of phase with } E$

Dipole:

$$p(t) = -e x(t) = -\frac{e^2}{m} \frac{1}{\omega^2 + i\gamma \omega} E(t)$$
\( \omega \gg \gamma : \quad \sigma \text{ complex } \implies J \text{ out of phase with } E \)

**Dipole:**

\[
p(t) = -ex(t) = -\frac{e^2}{m} \frac{1}{\omega^2 + i\gamma \omega} E(t)
\]

**Polarization:**

\[
P(t) = -\frac{Ne^2}{m} \frac{1}{\omega^2 + i\gamma \omega} E(t) \equiv \varepsilon_o \chi_e E(t)
\]
Free electrons

\( \omega \gg \gamma : \quad \sigma \text{ complex} \Rightarrow J \text{ out of phase with } E \)

Dipole:

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p(t) = -ex(t) = -\frac{e^2}{m} \frac{1}{\omega^2 + i\gamma \omega} E(t)
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Polarization:

\[
P(t) = -\frac{Ne^2}{m} \frac{1}{\omega^2 + i\gamma \omega} E(t) \equiv \varepsilon_o x_e E(t)
\]

Dielectric function:

\[
\varepsilon(\omega) \equiv 1 + x_e = 1 - \frac{Ne^2}{m\varepsilon_o} \frac{1}{\omega^2 + i\gamma \omega} = \varepsilon'(\omega) + i\varepsilon''(\omega)
\]
Free electrons

\[ \epsilon(\omega) \equiv 1 + \chi_e = 1 - \frac{Ne^2}{m\varepsilon_o} \frac{1}{\omega^2 + i\gamma\omega} = \epsilon'(\omega) + i\epsilon''(\omega) \]
Free electrons

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**Little damping:** \( \gamma \approx 0 \quad \Rightarrow \quad \varepsilon'' = 0 \)
Free electrons

\[ \varepsilon(\omega) \equiv 1 + \chi_e = 1 - \frac{Ne^2}{m\varepsilon_o} \frac{1}{\omega^2 + i\gamma\omega} = \varepsilon'(\omega) + i\varepsilon''(\omega) \]

**Little damping:**  \( \gamma \approx 0 \Rightarrow \varepsilon'' = 0 \)

\[ \varepsilon'(\omega) = 1 - \frac{Ne^2}{m\varepsilon_o} \frac{1}{\omega^2} \equiv 1 - \frac{\omega_p^2}{\omega^2} \]
Free electrons

\[
\varepsilon(\omega) \equiv 1 + \chi_e = 1 - \frac{Ne^2}{m\varepsilon_0} \frac{1}{\omega^2 + i\gamma\omega} = \varepsilon'(\omega) + i\varepsilon''(\omega)
\]

Little damping: \(\gamma \approx 0 \implies \varepsilon'' = 0\)

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\varepsilon'(\omega) = 1 - \frac{Ne^2}{m\varepsilon_0} \frac{1}{\omega^2} \equiv 1 - \frac{\omega_p^2}{\omega^2}
\]
Free electrons

\[ \varepsilon(\omega) \equiv 1 + \chi_e = 1 - \frac{Ne^2}{m\varepsilon_0} \frac{1}{\omega^2 + i\gamma\omega} = \varepsilon'(\omega) + i\varepsilon''(\omega) \]

Add damping:

\[ \gamma \lesssim \omega_p \]

\[ E = \hbar \omega \]

\[ k = \sqrt{\varepsilon} \frac{\omega}{c} \]
Free electrons

Q: As the frequency is decreased to the plasma frequency...

1. the wavelength becomes very small
2. the wavelength becomes very large
3. the wavelength becomes infinite
4. the wavelength becomes zero

\[ E = \hbar \omega \]

\[ k = \sqrt{\varepsilon} \frac{\omega}{c} \]
Q: As the frequency is decreased to the plasma frequency...

1. the wavelength becomes very small
2. the wavelength becomes very large ✓
3. the wavelength becomes infinite
4. the wavelength becomes zero

![Graph showing the relationship between epsilon (ε), omega (ω), and the frequency of light (E = ħω).](image)
Q: When the wavelength becomes infinite, the index...

1. becomes very large
2. is zero
3. becomes smaller than 1 (but remains positive)
4. becomes negative
Q: When the wavelength becomes infinite, the index...

1. becomes very large
2. is zero ✔
3. becomes smaller than 1 (but remains positive)
4. becomes negative
Free electrons

Plasma acts like a high-pass filter

\[ E = \hbar \omega \]

\[ k = \sqrt{\varepsilon} \frac{\omega}{c} \]

\[ \omega > \omega_p \]

\[ \omega < \omega_p \]
Plasma acts like a high-pass filter

<table>
<thead>
<tr>
<th>log ( N ) (cm(^{-3} ))</th>
<th>( \omega_p ) (rad s(^{-1} ))</th>
<th>( \lambda_p )</th>
<th>Δω &gt; ω(_p)</th>
<th>Δω &lt; ω(_p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>( 6 \times 10^{15} )</td>
<td>330 nm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>( 6 \times 10^{13} )</td>
<td>33 µm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>( 6 \times 10^{11} )</td>
<td>3.3 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>( 6 \times 10^9 )</td>
<td>0.33 m</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Dielectrics as conductors

dielectric

metal

\[ \varepsilon^\prime \quad \varepsilon'' \quad \varepsilon^\prime \]

\[ \omega_j \quad \omega_p \]
for a strong (dielectric) resonance $\varepsilon$ can become negative
Dielectrics as conductors

valence electrons in dielectric then behave like a plasma
Dielectrics as conductors with plasma frequency above the resonance
Dielectrics as conductors

(and far below the UV region)
Magnetic response

Index also determined by magnetic response

\[ n = \sqrt{\epsilon \mu} \]
Magnetic response

Index also determined by magnetic response

\[ n = \sqrt{\epsilon \mu} \]

and magnetic response shows similar resonances
Magnetic response

but magnetic resonances occur below optical frequencies
Magnetic response

so, in optical regime, $\mu \approx 1$
Index of refraction

\[ n = \sqrt{\varepsilon \mu} \]

Both \( \varepsilon \) and \( \mu \) are complex and their real parts can be negative.
Index of refraction

\[ n = \sqrt{\varepsilon \mu} \]

Both \( \varepsilon \) and \( \mu \) are complex and their real parts can be negative.

What happens when \( \text{Re} \varepsilon \) and/or \( \text{Re} \mu \) is negative?
Write complex quantities as

\[ \varepsilon = |\varepsilon| e^{i\theta} \quad \mu = |\mu| e^{i\phi} \]
Write complex quantities as

\[ \mathcal{E} = |\mathcal{E}| e^{i\theta} \quad \mu = |\mu| e^{i\phi} \]
Write complex quantities as

\[ \varepsilon = |\varepsilon| e^{i\theta} \quad \mu = |\mu| e^{i\phi} \]

\[ n = \sqrt{|\varepsilon||\mu|} e^{\frac{i\theta+\phi}{2}} \]
Write complex quantities as

\[ \varepsilon = |\varepsilon| e^{i\theta} \quad \mu = |\mu| e^{i\phi} \]

\[ n = \sqrt{|\varepsilon||\mu|} e^{i\frac{\theta+\phi}{2}} \]
Q: Is this the only possible solution?

1. yes
2. no, there’s one more
3. there are many more
4. it depends
There is another root...
There is another root...

Can add $2\pi$ to exponent

$$e^{+i(\theta+\phi)} = e^{+i[\theta+\phi+2\pi]}$$
There is another root...

Can add $2\pi$ to exponent

$$e^{i(\theta+\phi)} = e^{i[\theta+\phi+2\pi]}$$

and so

$$n = \sqrt{\epsilon \mu} e^{i\left[\frac{\theta+\phi}{2}+\pi\right]}$$
There is another root…

Can add $2\pi$ to exponent

\[ e^{i(\theta+\phi)} = e^{i(\theta+\phi+2\pi)} \]

and so

\[ n = \sqrt{\varepsilon \mu} e^{i \left[ \frac{\theta + \phi}{2} + \pi \right]} \]
There is another root...

Can add $2\pi$ to exponent

\[ e^{+i(\theta+\phi)} = e^{+i[\theta+\phi+2\pi]} \]

and so

\[ n = \sqrt{\epsilon \mu} e^{i \left[ \frac{\theta+\phi}{2} + \pi \right]} \]

but...

\[ k = \frac{2\pi n}{\lambda_o} = \frac{2\pi (n' + in'')}{\lambda_o} = k' + ik'' \]
There *is* another root...

Can add \(2\pi\) to exponent

\[
e^{i(\theta+\phi)} = e^{i(\theta+\phi+2\pi)}
\]

and so

\[
n = \sqrt{\varepsilon/\mu} e^{i\left[\frac{\theta+\phi}{2}+\pi\right]}
\]

but...

\[
k = \frac{2\pi n}{\lambda_o} = \frac{2\pi(n'+in'')}{\lambda_o} = k'+ik''
\]

and

\[
E = E_0 e^{i(kx-\omega t)} = E_0 e^{-k''x} e^{i(k'x-\omega t)}
\]
There is another root…

Can add $2\pi$ to exponent

\[ e^{+i(\theta+\phi)} = e^{+i[\theta+\phi+2\pi]} \]

and so

\[ n = \sqrt{\varepsilon \mu e^{i\left[\frac{\theta+\phi+\pi}{2}\right]}} \]

but…

\[ k = \frac{2\pi n}{\lambda_o} = \frac{2\pi(n' + in'')}{\lambda_o} = k' + ik'' \]

and

\[ E = E_o e^{i(kx-\omega t)} = E_o e^{-k''x} e^{i(k'x-\omega t)} \]
There is another root...

Can add $2\pi$ to exponent

$$e^{i(\theta+\phi)} = e^{i(\theta+\phi+2\pi)}$$

and so

$$n = \sqrt{\varepsilon \mu} e^{i\left(\frac{\theta+\phi+\pi}{2}\right)}$$

but...

$$k = \frac{2\pi n}{\lambda_o} = \frac{2\pi(n' + in'')}{\lambda_o} = k' + ik''$$

and

$$E = E_0 e^{i(kx - \omega t)} = E_0 e^{-k'x} e^{i(k'x - \omega t)}$$

must lie here for passive material
There *is* another root...

Can add $2\pi$ to exponent

$$e^{+i(\theta+\phi)} = e^{+i[\theta+\phi+2\pi]}$$

and so

$$n = \sqrt{\varepsilon/\mu} e^{i \left[ \frac{\theta+\phi}{2} + \pi \right]}$$

but...

$$k = \frac{2\pi n}{\lambda_o} = \frac{2\pi (n' + in'')}{\lambda_o} = k' + ik''$$

and

$$E = E_o e^{i(kx-\omega t)} = E_o e^{-k''x} e^{i(k'x-\omega t)}$$
Q: Is this the only possible solution?

1. yes ✅
2. no, there’s one more
3. there are many more
4. it depends
To find $n$ (passive materials):

1. Draw line that bisects $\varepsilon$ and $\mu$
2. Choose upper branch
For certain values of $\epsilon$ and $\mu$ we can get a negative $\text{Re}(n)$!
Q: Must both $\text{Re}\varepsilon < 0$ and $\text{Re}\mu < 0$ to get a negative index?

1. yes

2. no
Q: Must both $\text{Re}\, \epsilon < 0$ and $\text{Re}\, \mu < 0$ to get a negative index?

1. yes
2. no ✔
Note: need magnetic response to achieve \( n \leq 0 \)!
Now remember

\[ k = \frac{2\pi n}{\lambda_o} = \frac{2\pi(n' + in'')}{\lambda_o} = k' + ik'' \]
Now remember

\[ k = \frac{2\pi n}{\lambda_o} = \frac{2\pi(n' + in'')}{\lambda_o} = k' + ik'' \]

Spatial and temporal dependence of wave component

\[ E = E_0 e^{i(kx-\omega t)} = E_0 e^{-k''x} e^{i(k'x-\omega t)} \]
Now remember

\[ k = \frac{2\pi n}{\lambda_o} = \frac{2\pi(n' + in'')}{\lambda_o} = k' + ik'' \]

Spatial and temporal dependence of wave component

\[ E = E_0 e^{i(kx - \omega t)} = E_0 e^{-k''x} e^{i(k'x - \omega t)} \]

When \( \text{Re}(n) < 0, \ k' < 0 \), and so phase velocity reversed!
classification of (non-lossy) materials
classification of (non-lossy) materials

\[ \text{Re}\,\varepsilon > 0 \quad \text{Re}\,\mu > 0 \]
classification of (non-lossy) materials
classification of (non-lossy) materials

- **Metals**: $\text{Re}\, \varepsilon < 0 \quad \text{Re}\, \mu > 0$
- **Dielectrics**: $\text{Re}\, \varepsilon > 0 \quad \text{Re}\, \mu > 0$
classification of (non-lossy) materials

- **Re \( \varepsilon \) > 0, Re \( \mu \) > 0**
  - Dielectrics

- **Re \( \varepsilon \) < 0, Re \( \mu \) > 0**
  - Metals
classification of (non-lossy) materials

- **Metals**
  - \( \text{Re } \varepsilon < 0 \), \( \text{Re } \mu > 0 \)
  - Electric plasma
  - Evanescent wave

- **Dielectrics**
  - \( \text{Re } \varepsilon > 0 \), \( \text{Re } \mu > 0 \)
  - Propagating wave
Index

classification of (non-lossy) materials

\[
\begin{align*}
\text{Re } \varepsilon &< 0 \quad \text{Re } \mu > 0 & \text{Re } \varepsilon &> 0 \quad \text{Re } \mu > 0 \\
\text{electric plasma} & & \text{propagating wave} \\
\text{evanescent wave} & & \\
\text{negative index} & & \\
\end{align*}
\]
classification of (non-lossy) materials

- **dielectrics**
  - \( \Re \varepsilon > 0 \), \( \Re \mu > 0 \)
  - electric plasma
  - evanescent wave

- **metals**
  - \( \Re \varepsilon < 0 \), \( \Re \mu > 0 \)
  - reverse propagating wave

- **negative index**
  - \( \Re \varepsilon < 0 \), \( \Re \mu < 0 \)
  - propagating wave
Index

classification of (non-lossy) materials

- Dielectrics: $\Re \varepsilon > 0$, $\Re \mu > 0$; electric plasma, propagating wave
- Metals: $\Re \varepsilon < 0$, $\Re \mu > 0$; magnetic plasma, evanescent wave
- Negative index: $\Re \varepsilon < 0$, $\Re \mu < 0$; reverse propagating wave
- (not in optical regime): $\Re \varepsilon > 0$, $\Re \mu < 0$; evanescent wave
Optical properties of materials

Key points

• optical properties arise from motion of charge

• optical properties depend strongly on driving frequency

• index positive, zero, imaginary, or even negative
Why did we do all this?
• optical properties of materials

• dispersion of pulses

• nonlinear optics

• waveguiding

• engineering the index
Pulse dispersion
Pulse dispersion
Pulse dispersion
Pulse dispersion
Pulse dispersion
Consider two propagating waves:

\[ y_1 = A \sin (k_1 x - \omega_1 t) \quad \text{and} \quad y_2 = A \sin (k_2 x - \omega_2 t) \]
Pulse dispersion

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propagating at speeds

\[ v_1 = \frac{\omega_1}{k_1} = f_1 \lambda_1 \quad \text{and} \quad v_2 = \frac{\omega_2}{k_2} = f_2 \lambda_2. \]
Pulse dispersion

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Superposition:

\[ y = A \left[ \sin (k_1 x - \omega_1 t) + \sin (k_2 x - \omega_2 t) \right] \]
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\[ v_1 = \frac{\omega_1}{k_1} = f_1 \lambda_1 \quad \text{and} \quad v_2 = \frac{\omega_2}{k_2} = f_2 \lambda_2 . \]

Superposition:

\[ y = A [\sin (k_1 x - \omega_1 t) + \sin (k_2 x - \omega_2 t)] \]

\[ \sin \alpha + \sin \beta = 2 \cos \left( \frac{\alpha - \beta}{2} \right) \sin \left( \frac{\alpha + \beta}{2} \right) \]
Pulse dispersion

\[ y = 2A \cos \frac{1}{2}[(k_1 - k_2)x - (\omega_1 - \omega_2)t] \sin \left[ \frac{k_1 + k_2}{2} x - \frac{\omega_1 + \omega_2}{2} t \right] \]
Pulse dispersion

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Let: \( k_1 - k_2 \equiv \Delta k \) and \( \omega_1 - \omega_2 \equiv \Delta \omega \)
Pulse dispersion

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Let: \( k_1 - k_2 \equiv \Delta k \) and \( \omega_1 - \omega_2 \equiv \Delta \omega \)

\[ \frac{k_1 + k_2}{2} \equiv k \] and \[ \frac{\omega_1 + \omega_2}{2} \equiv \omega \]
Pulse dispersion

\[
y = 2A \cos \left( \frac{1}{2}((k_1 - k_2)x - (\omega_1 - \omega_2)t) \right) \sin \left[ \frac{k_1 + k_2}{2} x - \frac{\omega_1 + \omega_2}{2} t \right]
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Let: \[ k_1 - k_2 \equiv \Delta k \quad \text{and} \quad \omega_1 - \omega_2 \equiv \Delta \omega \]

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and so:

\[
y = 2A \cos \left( \frac{1}{2}(x\Delta k - t\Delta \omega) \right) \sin (kx - \omega t)
\]
Pulse dispersion

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and so:

\[ y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta \omega) \sin (kx - \omega t) \]

traveling sine wave, with amplitude modulation.
Pulse dispersion

\[ y = 2A \cos \left( \frac{1}{2} (x \Delta k - t \Delta \omega) \right) \sin (kx - \omega t) \]

At \( t = 0 \):

\[ y = 2A \cos \left( \frac{1}{2} (x \Delta k) \right) \sin (kx) \]
Pulse dispersion

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carrier
Pulse dispersion

\[ y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta \omega) \sin (kx - \omega t) \]

At \( t = 0 \):

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carrier
Pulse dispersion

\[ y = 2A \cos \frac{1}{2} (x \Delta k - t \Delta \omega) \sin (kx - \omega t) \]

At \( t = 0 \):
\[ y = 2A \cos \frac{1}{2} (x \Delta k) \sin (kx) \]

envelope carrier
\[ y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta \omega) \sin (kx - \omega t) \]
Pulse dispersion

\[ y = 2A \cos \left( \frac{1}{2}(x \Delta k - t \Delta \omega) \right) \sin (kx - \omega t) \]

speed of carrier

\[ v_p = \frac{\omega}{k} = f\lambda \]
Pulse dispersion

\[ y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta \omega) \sin (kx - \omega t) \]

speed of carrier

\[ v_p = \frac{\omega}{k} = f\lambda \]

speed of envelope

\[ v_g = \frac{\Delta \omega}{\Delta k} = \frac{d\omega}{dk} \]
y = 2A \cos \left( \frac{1}{2} (x \Delta k - t \Delta \omega) \right) \sin (kx - \omega t)

speed of carrier (‘phase velocity’):

\[ v_p = \frac{\omega}{k} = f \lambda \]

speed of envelope (‘group velocity’):

\[ v_g = \frac{\Delta \omega}{\Delta k} = \frac{d \omega}{dk} \]
Pulse dispersion

let’s practice a bit!

(please complete worksheet)
For each wave, determine the wavevector $k$, the frequency $\omega$, and the propagation speed $v$:

$$k_1 = 8.0 \quad \text{and} \quad k_2 = \frac{7.2}{0.95} = 7.6 < k_1$$

$$\omega_1 = 8.0 \quad \text{and} \quad \omega_2 = 7.2$$

$$v_1 = \frac{\omega_1}{k_1} = 1. \quad \text{and} \quad v_2 = \frac{\omega_2}{k_2} = \frac{7.2}{2.57} = 0.95$$

Does the red get ahead of the blue or the other way around? Why?

Is the dispersion in the medium through which these waves propagate normal or anomalous? Why?
What is the phase velocity of the superposition of $y_1$ and $y_2$?

$$V_p = \frac{(\omega_1 + \omega_2)/2}{(k_1 + k_2)/2} = \frac{7.6}{7.8} = 0.98$$

What is the group velocity of the superposition of $y_1$ and $y_2$?

$$V_g = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{0.8}{0.4} = 2$$

Do the crests of the carrier wave travel forward or backward through the envelope? Does your prediction agree with your observation?
Pulse dispersion

2 sine waves, anomalous dispersion
Q: In the previous example, if we take $v = 1$ to mean $c$, then $v_g$ is “superluminal”. Is this possible?

1. no, neither $v_p$ nor $v_g$ can be larger than $c$.
2. no, only $v_p$ can be larger than $c$.
3. yes, $v_g$ can be larger than $c$ (but not $v_p$).
4. yes, both $v_p$ and $v_g$ can be larger than $c$. 
Pulse dispersion

\[ \frac{c}{\nu} \]

\[ \omega \]

\[ \omega_j \]

\[ \nu_p \]
Pulse dispersion
Pulse dispersion

\[ \omega \]

\[ \omega_j \]

\[ \omega_p \]

\[ \omega_g \]

\[ \frac{c}{v} \]

\[ v_E \]

\[ v_p \]

\[ v_g \]
Pulse dispersion

\[ \frac{c}{\nu} \]

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\[ v_s \]

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\[ 1 \]

\[ 0 \]
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4. yes, both \( v_p \) and \( v_g \) can be larger than \( c \). ✅
Pulse dispersion

\[ y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta \omega) \sin (kx - \omega t) \]

If no dispersion

\[ v_p = \frac{\omega_1}{k_1} = \frac{\omega_2}{k_2} \]
Pulse dispersion

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group velocity:

\[ v_g = \frac{\Delta \omega}{\Delta k} = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{\omega_1/k_1 - \omega_2/k_1}{1 - k_2/k_1} = \frac{v_p - \omega_2/k_1}{1 - k_2/k_1} \]
Pulse dispersion

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\[ v_g = \frac{v_p - \omega_2/k_2(k_2/k_1)}{1 - k_2/k_1} = v_p \frac{1 - k_2/k_1}{1 - k_2/k_1} = v_p \]
Pulse dispersion

\[ y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta \omega) \sin (kx - \omega t) \]

If no dispersion

\[ v_p = \frac{\omega_1}{k_1} = \frac{\omega_2}{k_2} \]

group and phase velocities are the same:

\[ v_g = v_p \]

and so the envelope and carrier travel together
Pulse dispersion

2 sine waves, no dispersion
Pulse dispersion

2 sine waves, normal dispersion
Pulse dispersion

$$y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta \omega) \sin (kx - \omega t)$$

Types of dispersion:

$$\frac{dn}{d\omega} > 0 \quad \text{normal dispersion}$$

$$\frac{dn}{d\omega} = 0 \quad \text{no dispersion}$$

$$\frac{dn}{d\omega} < 0 \quad \text{anomalous dispersion}$$
Pulse dispersion

\[ y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta \omega) \sin (kx - \omega t) \]

Types of dispersion:

\[ \frac{dn}{d\omega} > 0 \quad \text{normal dispersion} \quad \nu_g < \nu_p \]

\[ \frac{dn}{d\omega} = 0 \quad \text{no dispersion} \quad \nu_g = \nu_p \]

\[ \frac{dn}{d\omega} < 0 \quad \text{anomalous dispersion} \quad \nu_g > \nu_p \]
What is the relationship between phase and group velocities?
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We have: \[ v_g = \frac{d\omega}{dk} \quad \text{and} \quad k = nk_o \]
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\[ \frac{1}{v_g} = \frac{dk}{d\omega} = k_o \frac{dn}{d\omega} + n \frac{dk_o}{d\omega} = \]
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\[ = k_o \frac{dn}{d\omega} + \frac{n}{c} = k_o \frac{dn}{d\omega} + \frac{1}{v_p} \]
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or

$$\frac{1}{v_g} = k_o \frac{dn}{d\omega} + \frac{1}{v_p}$$
Pulse dispersion

relationship between phase and group velocities

\[
\frac{1}{v_g} = k_o \frac{dn}{d\omega} + \frac{1}{v_p}
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Pulse dispersion

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Pulse dispersion

relationship between phase and group velocities

\[ \frac{1}{v_g} = \frac{d}{d\omega} \left( k_o \frac{dn}{d\omega} \right) + \frac{1}{v_p} \]
Pulse dispersion

medium causes pulse to stretch
Pulse dispersion

medium causes pulse to stretch

compensate by rearranging spectral components!
Pulse dispersion compensation
Pulse dispersion compensation
Pulse dispersion compensation
Pulse dispersion compensation

How do these arrangements work?
Pulse dispersion compensation

How do these arrangements work?

(please complete worksheet)
Pulse dispersion compensation

Does path length difference compensate?
Does path length difference compensate?
Pulse dispersion compensation

Does path length difference compensate?
Pulse dispersion compensation

Does path length difference compensate?
Does path length difference compensate?

grating gives low frequency longer path length!
Pulse dispersion compensation

Does path length difference compensate?
Does path length difference compensate?
Pulse dispersion compensation

Does path length difference compensate?
Does path length difference compensate?
Pulse dispersion compensation

Does path length difference compensate?

...so prism gives low frequency shorter path length!
Pulse dispersion compensation

Consider traveling Gaussian pulse:

\[ y(t) = \exp \left[ -\frac{(x - v_g t)^2}{2\sigma_t^2} \right] \sin (kx - \omega t) \]
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Q: Can you tell if the medium is dispersive or not?
Pulse dispersion compensation

Consider traveling Gaussian pulse:

\[ y(t) = \exp\left[-\frac{(x-v_gt)^2}{2\sigma_t^2}\right]\sin (kx - \omega t) \]

Q: Can you tell if the medium is dispersive or not?

1. Yes, it is dispersive
2. No, it is not dispersive (pulse shape is constant)
3. Cannot tell
Consider traveling Gaussian pulse:

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A: Cannot tell (the medium is dispersive if \( v_g \neq \frac{\omega}{k} \))
Consider traveling Gaussian pulse:

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Q: Can you tell if the medium is dispersive or not?

A: Cannot tell (the medium is dispersive if \( v_g \neq \frac{\omega}{k} \))

...but Gaussian shape of pulse is constant!
Pulse dispersion compensation

Gaussian, no dispersion

\[ y(t) = \exp \left[ - \frac{(x - v_g t)^2}{2\sigma_t^2} \right] \sin (kx - \omega t) \]

\[ v_g = \frac{\omega}{k} \]
$$y(t) = \exp\left[-\frac{(x-v_g t)^2}{2\sigma_t^2}\right] \sin (kx - \omega t) \quad v_g \neq \frac{\omega}{k}$$
Pulse dispersion compensation
Pulse dispersion compensation

linear dispersion
Pulse dispersion compensation

linear dispersion
Pulse dispersion compensation

linear dispersion
Pulse dispersion compensation

Linear dispersion

\[ \frac{d\phi}{d\omega} = \text{constant} \]
Pulse dispersion compensation

only *nonlinear* dispersion changes pulse shape!

\[ \frac{d^2 \phi}{d \omega^2} \neq 0 \]
Pulse dispersion compensation

4 sine waves, no dispersion
Pulse dispersion compensation

4 sine waves, linear dispersion
Pulse dispersion compensation

4 sine waves, nonlinear dispersion
Pulse dispersion compensation

Write dispersion as Taylor series:
Pulse dispersion compensation

Write dispersion as Taylor series:

\[
\omega(k) = \omega_o + \left( \frac{d\omega}{dk} \right)_{k=k_o} (k-k_o) + \frac{1}{2} \left( \frac{d^2\omega}{dk^2} \right)_{k=k_o} (k-k_o)^2
\]
Pulse dispersion compensation

Write dispersion as Taylor series:

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\]

let

\[
u \equiv \left( \frac{d\omega}{dk} \right)_{k=k_0} \quad \text{and} \quad w \equiv \left( \frac{d^2\omega}{dk^2} \right)_{k=k_0}.
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Pulse dispersion compensation

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group velocity:

$$v_g = \frac{d\omega}{dk} = u + wk$$
Write dispersion as Taylor series:

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group velocity:

\[ v_g = \frac{d\omega}{dk} = u + wk \]

if \( w = 0 \), then group velocity and pulse shape constant!
Pulse dispersion compensation

So not path length but \( \frac{d^2 \phi}{d \omega^2} \) matters!
Pulse dispersion compensation

So not path length but $\frac{d^2 \phi}{d \omega^2}$ matters!

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Pulse dispersion compensation

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Pulse dispersion compensation

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Pulse dispersion compensation

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Dispersion of pulses

Key points

- frequency-dependence of phase velocity distorts pulses
- group velocity describes motion of envelope
- group and phase velocity can be positive or negative
- group and phase velocity can exceed $c$
• optical properties of materials
• dispersion of pulses
• nonlinear optics
• waveguiding
• engineering the index
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