Manipulating Light at the Nanoscale

NATO-ASI Summer school on Nano-Optics: Principles Enabling Basic Research And Applications
Centro Ettore Majorana
Erice, Italy, 8–9 July 2015
Manipulating Light at the Nanoscale

NATO-ASI Summer school on Nano-Optics: Principles Enabling Basic Research And Applications
Centro Ettore Majorana
Erice, Italy, 8–9 July 2015

@eric_mazur
for a copy of these slides:

http://ericmazur.com

Follow me! eric_mazur
Outline

• optical properties of materials
• dispersion of pulses
• nonlinear optics
• waveguiding
• engineering the index
Nonlinear optics

Linear optics:

\[ \vec{P} = \chi \vec{E} \]
Nonlinear optics

Linear optics:

\[ \vec{P} = \chi \vec{E} \]

Nonlinear polarization:

\[ P = \chi^{(1)}E + \chi^{(2)}E^2 + \chi^{(3)}E^3 + \ldots \]
Linear optics:

\[ \vec{P} = \chi \vec{E} \]

Nonlinear polarization:

\[ P = \chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \ldots \]

and so:

\[ P = P^{(1)} + P^{(2)} + P^{(3)} + \ldots \]
Nonlinear optics

Linear optics:

\[ \vec{P} = \chi \vec{E} \]

Nonlinear polarization:

\[ P = \chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \ldots \]

and so:

\[ P = P^{(1)} + P^{(2)} + P^{(3)} + \ldots \]

\[ P^{(2)} \approx P^{(1)} \quad \text{when} \quad E = E_{at} \approx \frac{e}{a}, \quad \text{and so} \quad \chi^{(n)} \approx \frac{\chi^{(1)}}{E_{at}^{n-1}}. \]
Nonlinear polarization can drive new field:

\[
\nabla^2 \vec{E} + \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{4\pi}{c^2} \frac{\partial^2 \vec{P}}{\partial t^2}
\]
Nonlinear optics

Nonlinear polarization can drive new field:

\[ \nabla^2 \mathbf{E} + \frac{n^2}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{4 \pi}{c^2} \frac{\partial^2 \mathbf{P}}{\partial t^2} \]

But even terms disappear in media with inversion symmetry!

\[ \mathbf{P}^{(2)} = \chi^{(2)} : \mathbf{E} \mathbf{E} \]
Nonlinear optics

Nonlinear polarization can drive new field:

\[ \nabla^2 \vec{E} + \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{4\pi}{c^2} \frac{\partial^2 \vec{P}}{\partial t^2} \]

But even terms disappear in media with inversion symmetry!

\[ \vec{P}^{(2)} = \chi^{(2)}:\vec{E}\vec{E} \]

Invert all vectors:

\[ -\vec{P}^{(2)} = \chi^{(2)}:(-\vec{E})(-\vec{E}) \]
Nonlinear optics

Nonlinear polarization can drive new field:

\[ \nabla^2 \vec{E} + \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{4\pi}{c^2} \frac{\partial^2 \vec{P}}{\partial t^2} \]

But even terms disappear in media with inversion symmetry!

\[ \vec{P}^{(2)} = \chi^{(2)} : \vec{E} \vec{E} \]

Invert all vectors:

\[ - \vec{P}^{(2)} = \chi^{(2)} : (-\vec{E})(-\vec{E}) \]

and so \( \chi^{(2)} = -\chi^{(2)} = 0 \).
Consider oscillating electric field:

\[ E(t) = E e^{i\omega t} + \text{c.c.} \]
Consider oscillating electric field:

\[ \mathbf{E}(t) = \mathbf{E} e^{i\omega t} + \text{c.c.} \]

Second-order polarization:

\[ P^{(2)}(t) = \chi^{(2)} \mathbf{E}^2(t) = \frac{1}{2} \chi^{(2)} \mathbf{E} \mathbf{E}^* + \frac{1}{4} [\chi^{(2)} \mathbf{E}^2 e^{-2\omega t} + \text{c.c.}] \]
Consider oscillating electric field:

\[ E(t) = E e^{i\omega t} + \text{c.c.} \]

Second-order polarization:

\[ P^{(2)}(t) = \chi^{(2)} E^2(t) = \frac{1}{2} \chi^{(2)} EE^* + \frac{1}{4} [\chi^{(2)} E^2 e^{2i\omega t} + \text{c.c.}] \]
Nonlinear optics

Consider oscillating electric field:

\[ E(t) = E e^{i\omega t} + \text{c.c.} \]

Second-order polarization:

\[ P^{(2)}(t) = \chi^{(2)} E^2(t) = \frac{1}{2} \chi^{(2)} E E^* + \frac{1}{4} [\chi^{(2)} E^2 e^{-2i\omega t} + \text{c.c.}] \]

Physical interpretation:
Nonlinear optics

Can also cause frequency mixing!
Can also cause frequency mixing! Let

\[ E(t) = E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t} \]
Nonlinear optics

Can also cause frequency mixing! Let

\[ E(t) = E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t} \]

Second-order polarization will contain terms with

\[ 2\omega_1 \text{ (SHG)}, 2\omega_2 \text{ (SHG)}, \omega_1 + \omega_2 \text{ (SFG)}, \omega_1 - \omega_2 \text{ (DFG)} \]
Nonlinear optics

Can also cause frequency mixing! Let

\[ E(t) = E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t} \]

Second-order polarization will contain terms with

\[
2\omega_1 \text{ (SHG)}, \, 2\omega_2 \text{ (SHG)}, \, \omega_1 + \omega_2 \text{ (SFG)}, \, \omega_1 - \omega_2 \text{ (DFG)}
\]

Physical interpretation:
Nonlinear optics

Linear response:

\[ \vec{P} = \chi \vec{E} \]
Nonlinear optics

Linear response:

\[ \vec{P} = \chi \vec{E} \]
Nonlinear optics

Nonlinear response:

\[ P^{(2)} = \chi^{(2)} E^2 \]
Nonlinear optics

Nonlinear response: \[ P^{(2)} = \chi^{(2)} E^2 \]
Nonlinear optics

Nonlinear response: \[ P^{(2)} = \chi^{(2)} E^2 \]
Nonlinear optics

Nonlinear response: \[ P^{(2)} = \chi^{(2)} E^2 \]
Nonlinear optics

Nonlinear response: \[ P^{(2)} = \chi^{(2)} E^2 \]
Nonlinear optics

Nonlinear response:

\[ P^{(2)} = \chi^{(2)}E^2 \]
Nonlinear optics

Nonlinear response:

\[ P^{(2)} = \chi^{(2)} E^2 \]
Nonlinear optics

Nonlinear response:

\[ P^{(2)} = \chi^{(2)} E^2 \]
Nonlinear optics

Nonlinear response:

\[ P^{(2)} = \chi^{(2)} E^2 \]
Nonlinear optics

Nonlinear response: \[ P^{(2)} = \chi^{(2)} E^2 \]
Nonlinear optics

Nonlinear response: \[ P^{(2)} = \chi^{(2)} E^2 \]
Nonlinear optics

Nonlinear response: \[ P^{(2)} = \chi^{(2)} E^2 \]
Nonlinear optics

Nonlinear response:

\[ P^{(2)} = \chi^{(2)} E^2 \]
Nonlinear optics

Nonlinear response:

\[ P^{(2)} = \chi^{(2)} E^2 \]
Nonlinear response:

\[ P^{(2)} = \chi^{(2)} E^2 \]
Nonlinear optics

Nonlinear response: \( P^{(2)} = \chi^{(2)} E^2 \)

Q: Silicon atoms are arranged in this way. Does bulk silicon generate second harmonic?

1. Yes, silicon is not centrosymmetric (as the unit cell shows)
2. No, the crystal as a whole is centrosymmetric
3. No, any radiation at the second harmonic is absorbed
4. Other
Nonlinear optics

Nonlinear response: \( P^{(2)} = \chi^{(2)} E^2 \)

Q: Silicon atoms are arranged in this way. Does bulk silicon generate second harmonic?

1. Yes, silicon is not centrosymmetric (as the unit cell shows)
2. No, the crystal as a whole is centrosymmetric ✓
3. No, any radiation at the second harmonic is absorbed
4. Other
Nonlinear optics
Third-order polarization: \[ P^{(3)}(t) = \chi^{(3)} E^3(t) \]
Nonlinear optics

Third-order polarization: \[ P^{(3)}(t) = \chi^{(3)} E^3(t) \]

3 frequencies, 3 terms + c.c.: complicated! In general

\[ \cos^3 \omega t = \frac{1}{4} \cos 3\omega t + \frac{3}{4} \cos \omega t \]
Nonlinear optics

Third-order polarization: \( P^{(3)}(t) = \chi^{(3)}E^3(t) \)

3 frequencies, 3 terms + c.c.: complicated! In general

\[
\cos^3 \omega t = \frac{1}{4} \cos 3\omega t + \frac{3}{4} \cos \omega t
\]

Intensity dependent term at fundamental frequency:

\[
P^{(3)}(t) = \chi^{(3)}E(t)E^*(t)E(t) = \chi^{(3)}I(t)E(t)
\]
Nonlinear optics

Third-order polarization: \[ P^{(3)}(t) = \chi^{(3)} E^3(t) \]

3 frequencies, 3 terms + c.c.: complicated! In general

\[ \cos^3 \omega t = \frac{1}{4} \cos 3\omega t + \frac{3}{4} \cos \omega t \]

Intensity dependent term at fundamental frequency:

\[ P^{(3)}(t) = \chi^{(3)} E(t) E^* (t) E(t) = \chi^{(3)} I(t) E(t) \]

and so \[ P = P^{(1)} + P^{(3)} = (\chi^{(1)} + \chi^{(3)} I) E \equiv \chi_{\text{eff}} E \]
Nonlinear optics

Third-order polarization: \[ P^{(3)}(t) = \chi^{(3)}E^3(t) \]

3 frequencies, 3 terms + c.c.: complicated! In general

\[ \cos^3 \omega t = \frac{1}{4} \cos 3\omega t + \frac{3}{4} \cos \omega t \]

Intensity dependent term at fundamental frequency:

\[ P^{(3)}(t) = \chi^{(3)}E(t)E^*(t)E(t) = \chi^{(3)}I(t)E(t) \]

and so

\[ P = P^{(1)} + P^{(3)} = \left( \chi^{(1)} + \chi^{(3)}I \right)E \equiv \chi_{\text{eff}}E \]

\[ n = \sqrt{\varepsilon} = \sqrt{1 + \chi_{\text{eff}}} \approx \sqrt{1 + \chi^{(1)}} + \frac{1}{2} \frac{\chi^{(3)}I}{\sqrt{1 + \chi^{(1)}}} = n_o + n_2I \]
Nonlinear optics

Intensity-dependent index of refraction:

\[ n = n_0 + n_2 I \]
Nonlinear optics

Intensity-dependent index of refraction:

\[ n = n_0 + n_2 I \]
Nonlinear optics

Intensity-dependent index of refraction:

\[ n = n_o + n_2 I \]
Intensity-dependent index of refraction:

\[ n = n_o + n_2I \]
Nonlinear optics

Intensity-dependent index of refraction:

\[ n = n_o + n_2 I \]
Phase:

\[ \frac{\phi}{2\pi} = \frac{nL}{\lambda} \]
Phase:

\[
\frac{\phi}{2\pi} = \frac{nL}{\lambda}
\]

\[
\phi = \frac{2\pi}{\lambda} L(n_0 + n_2I)
\]
Nonlinear optics

Phase:
\[ \frac{\phi}{2\pi} = \frac{nL}{\lambda} \]
\[ \phi = \frac{2\pi}{\lambda} L(n_o + n_2I) \]

Frequency change:
\[ \Delta \omega = -\frac{d\phi}{dt} \]
Nonlinear optics

Phase:
\[
\frac{\phi}{2\pi} = \frac{nL}{\lambda} \quad \phi = \frac{2\pi}{\lambda} L(n_o + n_2I)
\]

Frequency change:
\[
\Delta \omega = -\frac{d\phi}{dt}
\]

Q: Sketch the time dependence of the frequency shift for a Gaussian pulse and determine which is true (assume \( n_2 > 0 \)):

1. Leading edge is blue shifted, trailing edge red shifted
2. Leading and trailing edge blue shifted, center red shifted
3. Leading edge is red shifted, trailing edge blue shifted
4. Leading and trailing edge red shifted, center blue shifted
5. Other
Phase: \( \phi = \frac{nL}{\lambda} \)

Frequency change: \( \Delta \omega = -\frac{d\phi}{dt} \)

Q: Sketch the time dependence of the frequency shift for a Gaussian pulse and determine which is true (assume \( n_2 > 0 \)):

1. Leading edge is blue shifted, trailing edge red shifted
2. Leading and trailing edge blue shifted, center red shifted
3. Leading edge is red shifted, trailing edge blue shifted
4. Leading and trailing edge red shifted, center blue shifted
5. Other

\[ \frac{\phi}{2\pi} = \frac{nL}{\lambda} \quad \phi = \frac{2\pi}{\lambda} L(n_o + n_2I) \]
Nonlinear optics

Phase:
\[ \frac{\phi}{2\pi} = \frac{nL}{\lambda} \]
\[ \phi = \frac{2\pi}{\lambda} L(n_o + n_2I) \]

Frequency change:
\[ \Delta\omega = -\frac{d\phi}{dt} = \frac{-2\pi}{\lambda} Ln_2 \frac{dI}{dt} \]
Nonlinear optics
Intensity-dependent index of refraction:

\[ n = n_0 + n_2 I \]
Nonlinear optics

Intensity-dependent index of refraction:

\[ n = n_o + n_2 I \]
Nonlinear optics

self-focusing
Nonlinear optics

but susceptibility is complex!

<table>
<thead>
<tr>
<th>susceptibility</th>
<th>real part</th>
<th>imaginary part</th>
</tr>
</thead>
<tbody>
<tr>
<td>linear</td>
<td>refraction</td>
<td>absorption</td>
</tr>
<tr>
<td>nonlinear</td>
<td>SHG, SFG, DFG, THG,…</td>
<td>multiphoton absorption</td>
</tr>
</tbody>
</table>

\[ \alpha = \alpha_o + \beta I + \gamma I^2 + \ldots \]
Key points

• at high intensities, polarization no longer proportional to $E$

• nonlinearity can produce radiation at new frequencies

• nonlinearity causes index to depend on intensity of pulse
• optical properties of materials
• dispersion of pulses
• nonlinear optics
• waveguiding
• engineering the index
Waveguiding

two crossed planar waves...
Waveguiding

...cause an interference pattern
$E = 0$ on the nodal lines
...satisfying boundary conditions for planar-mirror waveguide
Waveguiding

transverse standing wave, traveling along axis
Waveguiding

transverse standing wave, traveling along axis
Waveguiding

change angle of incident waves...
Waveguiding

change angle of incident waves...
Waveguiding

change angle of incident waves...
Waveguiding

change angle of incident waves...
change angle of incident waves...
Waveguiding

change angle of incident waves...
Waveguiding

change angle of incident waves...
Waveguiding

change angle of incident waves...
Waveguiding

change angle of incident waves...
Waveguiding

change angle of incident waves…
Waveguiding

change angle of incident waves...
Waveguiding

change angle of incident waves...
Waveguiding

change angle of incident waves...
Waveguiding

change angle of incident waves...
Waveguiding

change angle of incident waves...
Waveguiding

change angle of incident waves...
Waveguiding

change angle of incident waves...
Waveguiding

change angle of incident waves...
boundary conditions only satisfied for certain $\theta$

standing wave in $y$-direction, traveling in $z$-direction
consider wave incident at angle $\theta$
twice-reflected wave
self consistency:

\[ AC - AB = 2d \sin \theta = m\lambda \quad (m = 1, 2, \ldots) \]
self consistency:

\[ AC - AB = 2d \sin \theta = m\lambda \quad (m = 1, 2, \ldots) \]

so:

\[ \sin \theta_m = m \frac{\lambda}{2d} \]
self consistency:

\[ AC - AB = 2d \sin \theta = m\lambda \quad (m = 1, 2, \ldots) \]

so:

\[ \sin \theta_m = m \frac{\lambda}{2d} \]
self consistency:

\[ AC - AB = 2d \sin \theta = m\lambda \quad (m = 1, 2, \ldots) \]

so:

\[ \sin \theta_m = m \frac{\lambda}{2d} \]
self consistency:

\[ AC - AB = 2d \sin \theta = m\lambda \quad (m = 1, 2, \ldots) \]

so:

\[ \sin \theta_m = m \frac{\lambda}{2d} \]
number of modes:

\[ M \equiv \frac{2d}{\lambda} \]
now consider a planar dielectric waveguide
rays incident at angle $\theta > \pi/2 - \theta_c$ are unguided
rays incident at angle $\theta < \frac{\pi}{2} - \theta_c$ are guided
Waveguiding

rays incident at angle $\theta < \pi/2 - \theta_c$ are guided
self consistency:

\[ AC - AB = 2d \sin \theta - \frac{\varphi_r}{\pi} \lambda = m\lambda \quad (m = 0, 1, 2\ldots) \]
self consistency:

\[ AC - AB = 2d \sin \theta - \frac{\varphi_r}{\pi} \lambda = m\lambda \quad (m = 0, 1, 2 \ldots) \]

so:

\[ \tan \left( \frac{\pi d}{\lambda} \sin \theta - m \frac{\pi}{2} \right) = \left( \frac{\sin^2 \left( \frac{\pi}{2} - \theta_c \right)}{\sin^2 \theta} - 1 \right)^{1/2} \]
self consistency:

\[
AC - AB = 2d \sin \theta - \frac{\varphi_r}{\pi} \lambda = m\lambda \quad (m = 0, 1, 2\ldots)
\]

so:

\[
\tan \left( \frac{\pi d}{\lambda} \sin \theta - m \frac{\pi}{2} \right) = \left( \frac{\sin^2 \left( \pi/2 - \theta_c \right)}{\sin^2 \theta} - 1 \right)^{1/2}
\]
self consistency:

\[ AC - AB = 2d \sin \theta - \frac{\varphi_r}{\pi} \lambda = m\lambda \quad (m = 0, 1, 2\ldots) \]

so:

\[ \tan \left( \frac{\pi d}{\lambda} \sin \theta - m\frac{\pi}{2} \right) = \left( \frac{\sin^2(\pi/2 - \theta_c)}{\sin^2 \theta} - 1 \right)^{1/2} \]
**Waveguiding**

**self consistency:**

\[ AC - AB = 2d \sin \theta - \frac{\varphi_r}{\pi} \lambda = m\lambda \quad (m = 0, 1, 2...) \]

**so:**

\[ \tan \left( \frac{\pi d}{\lambda} \sin \theta - m \frac{\pi}{2} \right) = \left( \frac{\sin^2 \left( \frac{\pi}{2} - \theta_c \right)}{\sin^2 \theta} - 1 \right)^{1/2} \]
**Waveguiding**

**self consistency:**

\[ AC - AB = 2d \sin \theta - \frac{\varphi_r}{\pi} \lambda = m\lambda \quad (m = 0, 1, 2...) \]

**so:**

\[ \tan \left( \frac{\pi d}{\lambda} \sin \theta - m \frac{\pi}{2} \right) = \left( \frac{\sin^2(\pi/2 - \theta_c)}{\sin^2 \theta} - 1 \right)^{1/2} \]
self consistency:

\[ AC - AB = 2d \sin \theta - \frac{\varphi_r}{\pi} \lambda = m\lambda \quad (m = 0, 1, 2...) \]

so:

\[
\tan \left( \frac{\pi d}{\lambda} \sin \theta - m \frac{\pi}{2} \right) = \left( \frac{\sin^2 \left( \frac{\pi}{2} - \theta_c \right)}{\sin^2 \theta} - 1 \right)^{1/2}
\]
number of modes:

\[ M = \frac{\sin\left(\frac{\pi}{2} - \theta_c\right)}{\lambda/2d} \]
number of modes:

\[ M \equiv \frac{\sin\left(\frac{\pi}{2} - \theta_c\right)}{\lambda/2d} \]

or:

\[ M \equiv 2 \frac{d}{\lambda} \left(\frac{n_1^2}{n_2^2} - 1\right)^{1/2} \]
propagation constant of guided wave:

\[ \beta_m^2 = k^2 - k_y^2 = k^2 - \frac{m^2 \pi^2}{d^2} \]

group velocity:

\[ v_m = c \cos \theta_m \]
single mode condition for 600-nm light:

planar mirror

\[ M = \frac{2d}{\lambda} \]

300 < d < 600 nm

dielectric

\[ M = 2 \frac{d}{\lambda} (n_1^2 - n_2^2)^{1/2} \]

d < 268 nm
Waveguiding

single mode condition for 600-nm light:

planar mirror

\[ M = \frac{2d}{\lambda} \quad 300 < d < 600 \text{ nm} \]

dielectric

\[ M = 2 \frac{d}{\lambda} \left( n_1^2 - n_2^2 \right)^{1/2} \quad d < 268 \text{ nm} \]

can make \( d \) larger by making \( n_1 - n_2 \) smaller!
Vector potential obeys:

$$\nabla^2 \vec{A} + \omega^2 \mu_0 \varepsilon \vec{A} = -i \omega \mu_0 \nabla \varepsilon \Phi$$
Waveguiding

Vector potential obeys:

\[ \nabla^2 \vec{A} + \omega^2 \mu_0 \varepsilon \vec{A} = 0 \]
Vector potential obeys:

\[ \nabla^2 \vec{A} + \omega^2 \mu_0 \epsilon \vec{A} = 0 \]

Substituting

\[ \vec{A} = \hat{y} u(x,y) e^{-i\beta z} \]
Waveguiding

Vector potential obeys:

$$\nabla^2 \vec{A} + \omega^2 \mu_0 \varepsilon \vec{A} = 0$$

Substituting

$$\vec{A} = \hat{y} u(x,y) e^{-i\beta z}$$

yields:

$$\nabla_T^2 u + [-\beta^2 + \omega^2 \mu \varepsilon(r)] u = 0$$
Vector potential obeys:

$$\nabla^2 \mathbf{A} + \omega^2 \mu_0 \epsilon \mathbf{A} = 0$$

Substituting

$$\mathbf{A} = \hat{y} u(x,y) e^{-i\beta z}$$

yields:

$$\nabla_T^2 u + [ - \beta^2 + \omega^2 \mu \epsilon(r) ] u = 0$$

Compare to time-independent Schrödinger equation:

$$\nabla^2 \psi + \frac{2m}{\hbar^2} [E - V(r)] \psi = 0$$
Waveguiding
Waveguiding

\[ m = 1 \]
Waveguiding
Waveguiding
Waveguiding
Waveguiding

$\infty \quad \infty$

$\infty \quad \infty$

$0 \quad \infty$

$-V \quad \infty$

$d \quad \infty$

$m = 1$
single mode condition for 600-nm light:

\[ M = 2 \frac{d}{\lambda} \left( n_1^2 - n_2^2 \right)^{1/2} \]

without cladding: \( d < 268 \text{ nm} \)
Waveguiding

single mode condition for 600-nm light:

\[ M = 2 \frac{d}{\lambda} (n_1^2 - n_2^2)^{1/2} \]

without cladding: \( d < 268 \text{ nm} \)

Add cladding with 0.4% index difference:

\( d < 5 \text{ µm} \)
commercial single-mode fiber (Corning Titan®)

- Core:
  - Index: \( n_1 = 1.468 \)
  - Diameter: \( 8.3 \, \mu m \)

- Cladding:
  - Index: \( n_2 = 1.462 \)
  - Diameter: \( 125.0 \pm 1.0 \, \mu m \)

Operating wavelength: \( \lambda = 1310 \, nm/1550 \, nm \)
drawbacks of clad fibers:

- weak confinement
- no tight bending
Waveguiding at the nanoscale
Waveguiding at the nanoscale

Poynting vector profile for 200-nm nanowire
Waveguiding at the nanoscale

300 nm
Waveguiding at the nanoscale
Waveguiding at the nanoscale
Waveguiding at the nanoscale
Waveguiding at the nanoscale
Waveguiding at the nanoscale
Waveguiding at the nanoscale
Waveguiding at the nanoscale
Waveguiding at the nanoscale

$L = 0 \mu m$
Waveguiding at the nanoscale

$L = 4 \mu m$
Waveguiding

Key points

• finite structures support a discrete set of modes

• each mode determined by boundary condition and extent

• each mode has unique field distribution

• modes unchanged as they propagate
• optical properties of materials
• dispersion of pulses
• nonlinear optics
• waveguiding
• engineering the index
Engineering the index

how to optimize manipulation of light at nanoscale?
common materials very limited

Engineering the index

- **Metals**
  - $\Re \varepsilon < 0$, $\Re \mu > 0$
  - Electric plasma
  - Evanescent wave

- **Dielectrics**
  - $\Re \varepsilon > 0$, $\Re \mu > 0$
  - Propagating wave

- **Negative Index**
  - $\Re \varepsilon < 0$, $\Re \mu < 0$
  - Magnetic plasma
  - Reverse propagating wave

- **(Not in optical regime)**
  - $\Re \varepsilon > 0$, $\Re \mu < 0$
Engineering the index

common materials very limited

- **Re** $\mu > 0$
- **Re** $\varepsilon < 0$
  - electric plasma
  - evanescent wave
  - magnetic plasma
  - reverse propagating wave

- **Re** $\mu < 0$
- **Re** $\varepsilon > 0$
  - propagating wave
  - evanescent wave
  - (not in optical regime)

limited by diffraction
Engineering the index

common materials very limited

lossy & no propagation

- Re $\varepsilon < 0$ Re $\mu > 0$
  - electric plasma
  - evanescent wave

- Re $\varepsilon > 0$ Re $\mu > 0$
  - propagating wave

- Re $\varepsilon < 0$ Re $\mu < 0$
  - magnetic plasma
  - reverse propagating wave

- Re $\varepsilon > 0$ Re $\mu < 0$
  - negative index

(evanescent wave)

(propagating wave)

(Re $\varepsilon$ vs. Re $\mu$)
common materials very limited

- metals
  - $\Re \varepsilon < 0$ $\Re \mu > 0$
  - electric plasma
  - reverse propagating wave
- dielectrics
  - $\Re \varepsilon > 0$ $\Re \mu > 0$
  - propagating wave
- magnetic plasma
  - $\Re \varepsilon > 0$ $\Re \mu < 0$
  - evanescent wave
- negative index
  - $\Re \varepsilon < 0$ $\Re \mu < 0$
  - (not in optical regime)

Engineering the index

superlensing

but…
Engineering the index

common materials very limited

we’re stuck here!
Engineering the index

\[ \Delta V \]

\[ \varepsilon \]
dielectric constant due to polarization of atoms
Engineering the index

metal-dielectric composite
Engineering the index

metal-dielectric composite
Engineering the index

polarization of metal particles increases dielectric constant
Engineering the index

provided $d \leq \lambda_{\text{eff}}$ can use effective dielectric constant

$$\Delta V$$

$\varepsilon$

$\varepsilon_{\text{eff}}$
can also do this with dielectric composite
what if we let $\varepsilon = 0$?
what if we let $\varepsilon = 0$?

if $\varepsilon = 0$, then $n = 0$!
Q: If $n = 0$, which of the following is true?

1. the frequency goes to zero.
2. the phase velocity becomes infinite.
3. both of the above.
4. neither of the above.
wave equation

\[ \nabla^2 \vec{E} - \frac{\mu \varepsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \]
wave equation

\[ \nabla^2 \vec{E} - \frac{\mu \epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \]

solution

\[ \vec{E} = \vec{E}_0 e^{i(kx - \omega t)} \]
wave equation

\[ \nabla^2 \vec{E} - \frac{\mu \varepsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \]

solution

\[ \vec{E} = \vec{E}_0 e^{i(kx - \omega t)} \]

where

\[ \frac{\omega}{k} = \frac{1}{\sqrt{\varepsilon \mu}} c = \frac{1}{n} c \]
wave equation

\[ \nabla^2 \vec{E} - \frac{\mu}{\epsilon} \frac{\vec{E}}{\alpha^2} = 0 \]

solution

\[ \vec{E} = \vec{E}_0 e^{i(kx - \omega t)} \]

where

\[ \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon \mu}} c = \frac{1}{n} c \]
wave equation

\[ \nabla^2 \vec{E} - \frac{\mu}{\epsilon_0 \omega^2} \vec{E} = 0 \]

solution

\[ \vec{E} = \vec{E}_0 e^{i(kx - \omega t)} \quad \rightarrow \quad \vec{E} = \vec{E}_0 e^{-i\omega t} \]

where

\[ \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon \mu}} \quad c = \frac{1}{n} c \]
wave equation

\[ \nabla^2 \vec{E} - \frac{\mu}{\epsilon_0^2} \vec{E} = 0 \]

solution

\[ \vec{E} = \vec{E}_0 e^{i(kx - \omega t)} \quad \rightarrow \quad \vec{E} = \vec{E}_0 e^{-i\omega t} \]

where

\[ \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon \mu}} \quad \frac{c}{n} = \frac{1}{n} c = \infty \]
Q: If \( n = 0 \), which of the following is true?

1. the frequency goes to zero.
2. the phase velocity becomes infinite. ✓
3. both of the above.
4. neither of the above.
$n > 1$
$0 < n < 1$
Zero index

\[ n = 0 \]
Zero index

$n < 0$
Zero index
$n = 0$
Zero index

\[ n = 0 \]
Zero index

$n = 0$
how?

\[ n = \sqrt{\varepsilon \mu} \]
how?

\[ n = \sqrt{\varepsilon \mu} \]

but \( \varepsilon \) and \( \mu \) also determine reflectivity

\[ r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \]
how?

\[ n = \sqrt{\varepsilon \mu} \]

but \( \varepsilon \) and \( \mu \) also determine reflectivity

\[ r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \]

where

\[ Z = \sqrt{\frac{\mu}{\varepsilon}} \]
Zero index

how?

\[ \varepsilon \to 0 \quad \quad n = \sqrt{\varepsilon \mu} \to 0 \]

but \( \varepsilon \) and \( \mu \) also determine reflectivity

\[ r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \]

where

\[ Z = \sqrt{\frac{\mu}{\varepsilon}} \]
how?

\[ \varepsilon \to 0 \quad n = \sqrt{\varepsilon \mu} \to 0 \]

but \( \varepsilon \) and \( \mu \) also determine reflectivity

\[ r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \]

where

\[ Z = \sqrt[2]{\frac{\mu}{\varepsilon}} \to \infty \]
how?

\[ \varepsilon \rightarrow 0 \quad \text{and} \quad n = \sqrt{\varepsilon \mu} \rightarrow 0 \]

but \( \varepsilon \) and \( \mu \) also determine reflectivity

\[ r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \rightarrow 1 \]

where

\[ Z = \sqrt{\frac{\mu}{\varepsilon}} \rightarrow \infty \]
Zero index

how?

\[ \mu \to 0 \quad n = \sqrt{\varepsilon \mu} \to 0 \]

but \( \varepsilon \) and \( \mu \) also determine reflectivity

\[ r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \]

where

\[ Z = \sqrt{\frac{\mu}{\varepsilon}} \]
Zero index

how?

\[ \mu \to 0 \quad \quad n = \sqrt{\epsilon \mu} \to 0 \]

but \( \epsilon \) and \( \mu \) also determine reflectivity

\[ r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \]

where

\[ Z = \sqrt{\frac{\mu}{\epsilon}} \to 0 \]
Zero index

how?

\[ \mu \to 0 \quad n = \sqrt{\varepsilon \mu} \to 0 \]

but \( \varepsilon \) and \( \mu \) also determine reflectivity

\[ r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \to -1 \]

where

\[ Z = \sqrt{\frac{\mu}{\varepsilon}} \to 0 \]
how?

\[ \varepsilon, \mu \to 0 \quad \Rightarrow \quad n = \sqrt{\varepsilon \mu} \to 0 \]

but \( \varepsilon \) and \( \mu \) also determine reflectivity

\[ r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \]

where

\[ Z = \sqrt{\frac{\mu}{\varepsilon}} \quad \text{finite!} \]
but $\mu \neq 1$ requires a magnetic response!
How can we produce coupled $E$ and $B$-fields?

LC circuit
How can we produce coupled $E$ and $B$-fields?

LC circuit
How can we produce coupled $E$ and $B$-fields?

LC circuit
How can we produce coupled $E$ and $B$-fields?

Engineering a magnetic response

LC circuit

1 GHz
How can we produce coupled $E$ and $B$-fields?

Engineering a magnetic response

LC circuit

split ring

1 GHz
How can we produce coupled $E$ and $B$-fields?

Engineering a magnetic response

LC circuit

split ring

1 GHz

10 THz
How can we produce coupled $E$ and $B$-fields?

- **LC circuit**
- **split ring**
- **wire pairs**

<table>
<thead>
<tr>
<th></th>
<th>1 GHz</th>
<th>10 THz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
How can we produce coupled $E$ and $B$-fields?

Engineering a magnetic response

LC circuit

split ring

wire pairs

1 GHz

10 THz

200 THz
Engineering a magnetic response

How can we produce coupled $E$ and $B$-fields?

- LC circuit
- Split ring
- Wire pairs

1 GHz
10 THz
200 THz

but... metallic losses & not easily made in 3D
instead, use array of dielectric rods
incident electromagnetic wave \((\lambda_{\text{eff}} \approx d)\)
Engineering a magnetic response

produces an electric response...
... but different electric fields front and back...
Engineering a magnetic response

...induce different polarizations on opposite sides...
Engineering a magnetic response

...causing a current loop...
Engineering a magnetic response

...which, in turn, produces an induced magnetic field
...which, in turn, produces an induced magnetic field

(but it’s still a dielectric, so there’s an electric response too!)
Engineering a magnetic response

adjust design so electrical and magnetic resonances coincide
Engineering a magnetic response

Adjust design so electrical and magnetic resonances coincide

(adjustable parameters: $n$, $d$, and $a$)
Zero-index materials
Zero-index materials

The graph shows the real part of the permittivity ($\text{Re}(\varepsilon_{\text{eff}})$) and permeability ($\text{Re}(\mu_{\text{eff}})$) as a function of frequency ($\omega a/2\pi c$). The data points indicate a trend where both $\text{Re}(\varepsilon_{\text{eff}})$ and $\text{Re}(\mu_{\text{eff}})$ increase with increasing frequency.
Zero-index materials

The figure shows the real parts of the effective refractive index ($\text{Re}(n_{\text{eff}})$) and the effective impedance ($\text{Re}(Z_{\text{eff}})$) as a function of frequency ($\omega a/2\pi c$). The frequency range is from 0.40 to 0.50, and the refractive index and impedance are plotted on a linear scale from -0.6 to 0.6 and from 0 to 4, respectively.
Zero-index materials

On-chip zero-index design

Si
Zero-index materials

On-chip zero-index design

Si

SiO₂
Zero-index materials

On-chip zero-index design

SiO$_2$
Zero-index materials

On-chip zero-index design

![Diagram of on-chip zero-index design](image)
Zero-index materials

On-chip zero-index design
Zero-index materials

On-chip zero-index design
Zero index at design wavelength (1590 nm)
Zero index

below design wavelength (1530 nm)
Zero index

above design wavelength (1650 nm)
Zero-index materials
Zero-index materials

1

500 nm
Zero-index materials
Zero-index materials
Zero-index materials
Zero-index materials
Zero-index materials

On-chip zero-index prism
Zero-index materials

On-chip zero-index prism
Zero-index materials

On-chip zero-index prism
Zero-index materials

On-chip zero-index prism
Zero-index materials

On-chip zero-index prism
Zero-index materials

On-chip zero-index prism
Zero-index materials

On-chip zero-index prism
Zero-index materials

On-chip zero-index prism
Zero-index materials

On-chip zero-index prism
Zero-index materials
Zero-index materials
Zero-index materials
Zero-index materials

- SU8 slab waveguide
- prism
- Si waveguide
Zero-index materials

- SU8 slab waveguide
- prism
- SU8 calibration waveguide
- Si waveguide
Zero-index materials
Zero-index materials

On-chip zero-index prism
Zero-index materials

\[ \lambda = 1570 \text{ nm} \]

50 \( \mu \text{m} \)
Zero-index materials

\[ \lambda = 1570 \text{ nm} \]
Zero-index materials

\[ \lambda = 1570 \text{ nm} \]
Zero-index materials

λ = 1570 nm

50 µm
Zero-index materials

Wavelength dependence of refraction angle
Zero-index materials

Wavelength dependence of refraction angle
Zero-index materials

Wavelength dependence of refraction angle
Zero-index materials

Wavelength dependence of refraction angle
Zero-index materials

Wavelength dependence of index

![Graph showing the wavelength dependence of index for zero-index materials. The x-axis represents wavelength in nm, ranging from 1480 to 1680, and the y-axis represents refractive index, ranging from -0.6 to 0.2.]
Zero-index materials

Wavelength dependence of index

![Graph showing wavelength dependence of index for zero-index materials, with data points for experiment and simulation. The x-axis represents wavelength in nm, ranging from 1480 to 1680, and the y-axis represents refractive index, ranging from -0.6 to 0.2. The graph includes a trend line for simulation and data points for experiment.]
unambiguous demonstration of on-chip zero-index material!
Q: What happens when a beam of light
at the wavelength for which \( n = 0 \)
strikes a side of a zero-index prism at
an angle away from the normal?

1. beats occur inside and around the prism.
2. the beam comes out at the same angle on the other facets.
3. the beam is perfectly reflected.
4. the beam is transmitted only for certain (nonzero) angles.
5. it couples perfectly, regardless of angle.
Q: What happens when a beam of light at the wavelength for which \( n = 0 \) strikes a side of a zero-index prism at an angle away from the normal?

1. beats occur inside and around the prism.
2. the beam comes out at the same angle on the other facets.
3. the beam is perfectly reflected. ✔
4. the beam is transmitted only for certain (nonzero) angles.
5. it couples perfectly, regardless of angle.
Key points

• tune optical properties using composite materials

• zero index requires a magnetic response

• produce magnetic response in dielectrics

• demonstrated on-chip impedance-matched $n = 0$
A very special thanks to
Phil Muñoz
Yang Li
Orad Reshef
Funding:

Air Force Office of Scientific Research
Natural Sciences and Engineering Research Council of Canada
Harvard Quantum Optics Center

for a copy of this presentation:

http://ericmazur.com

Follow me! eric_mazur