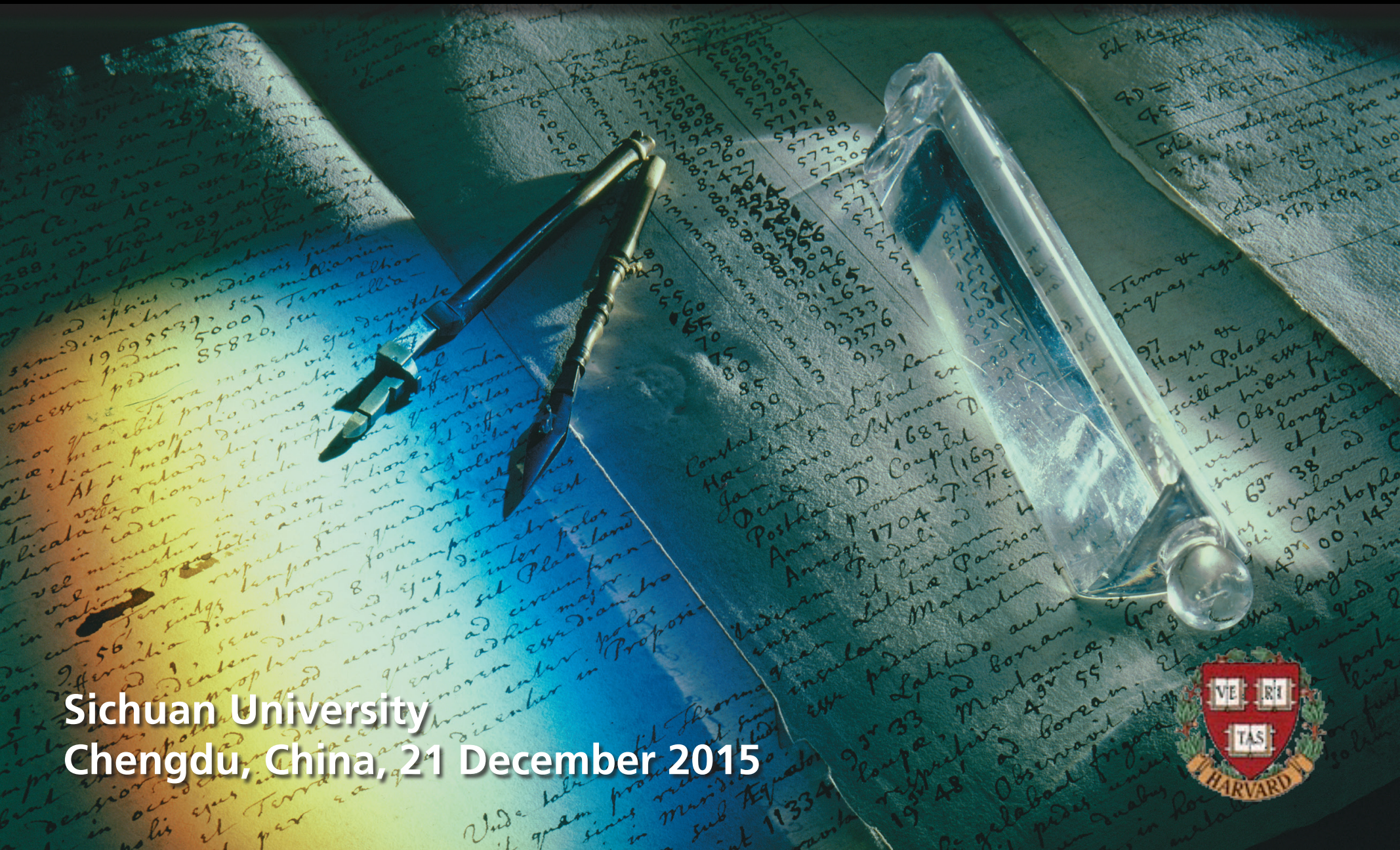
Less is more:
Extreme optics with zero refractive index



Sichuan University
Chengdu, China, 21 December 2015

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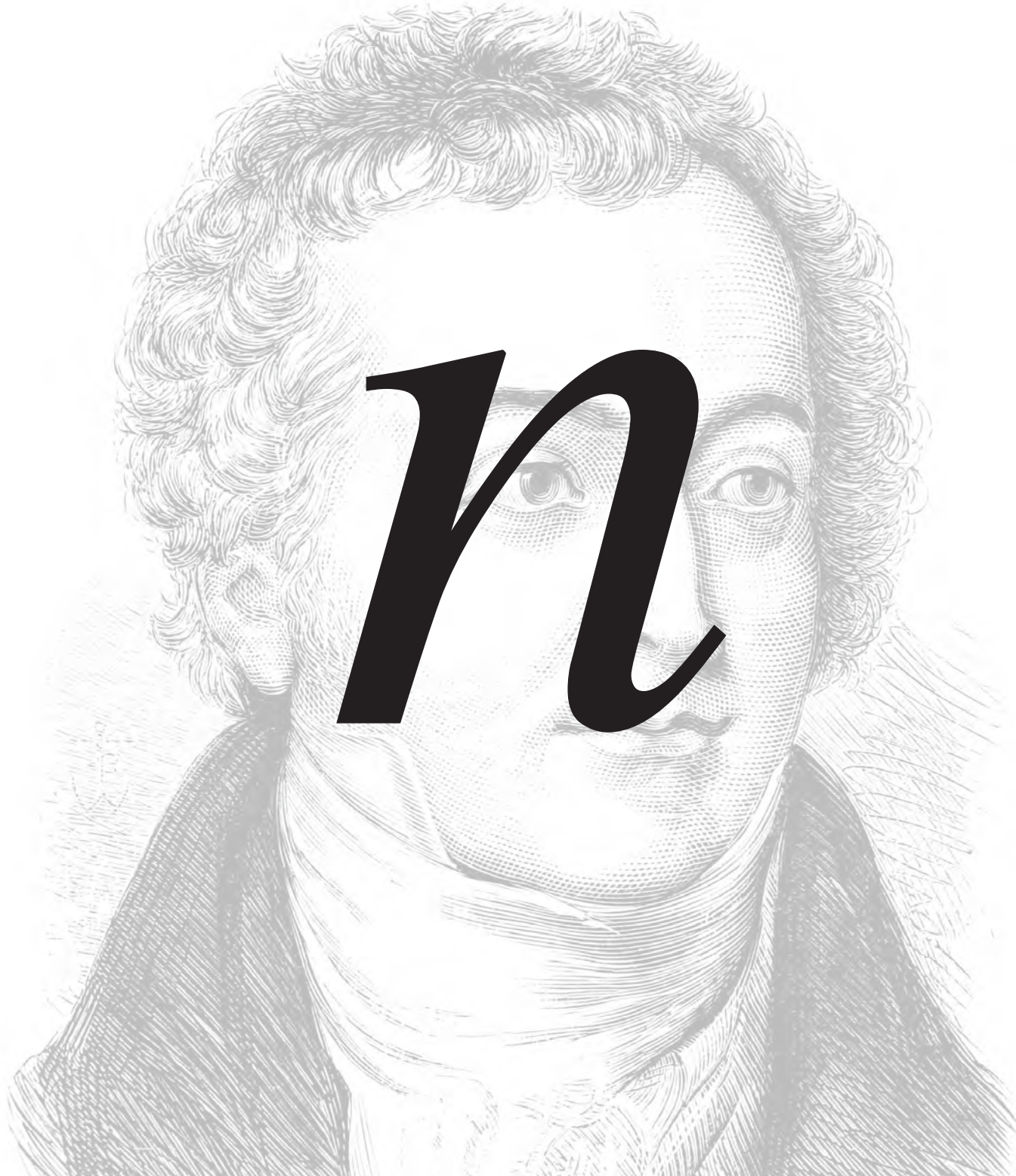

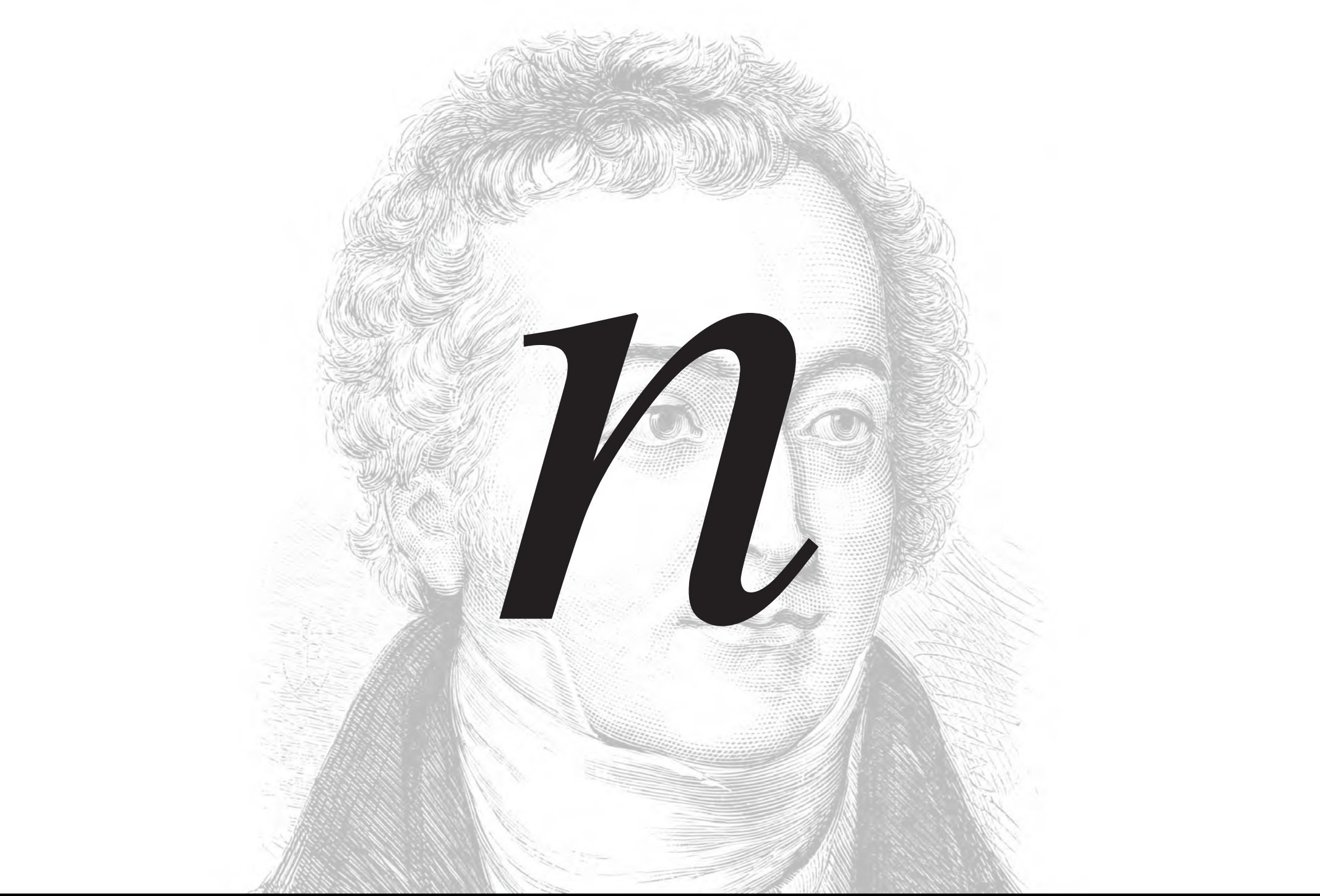

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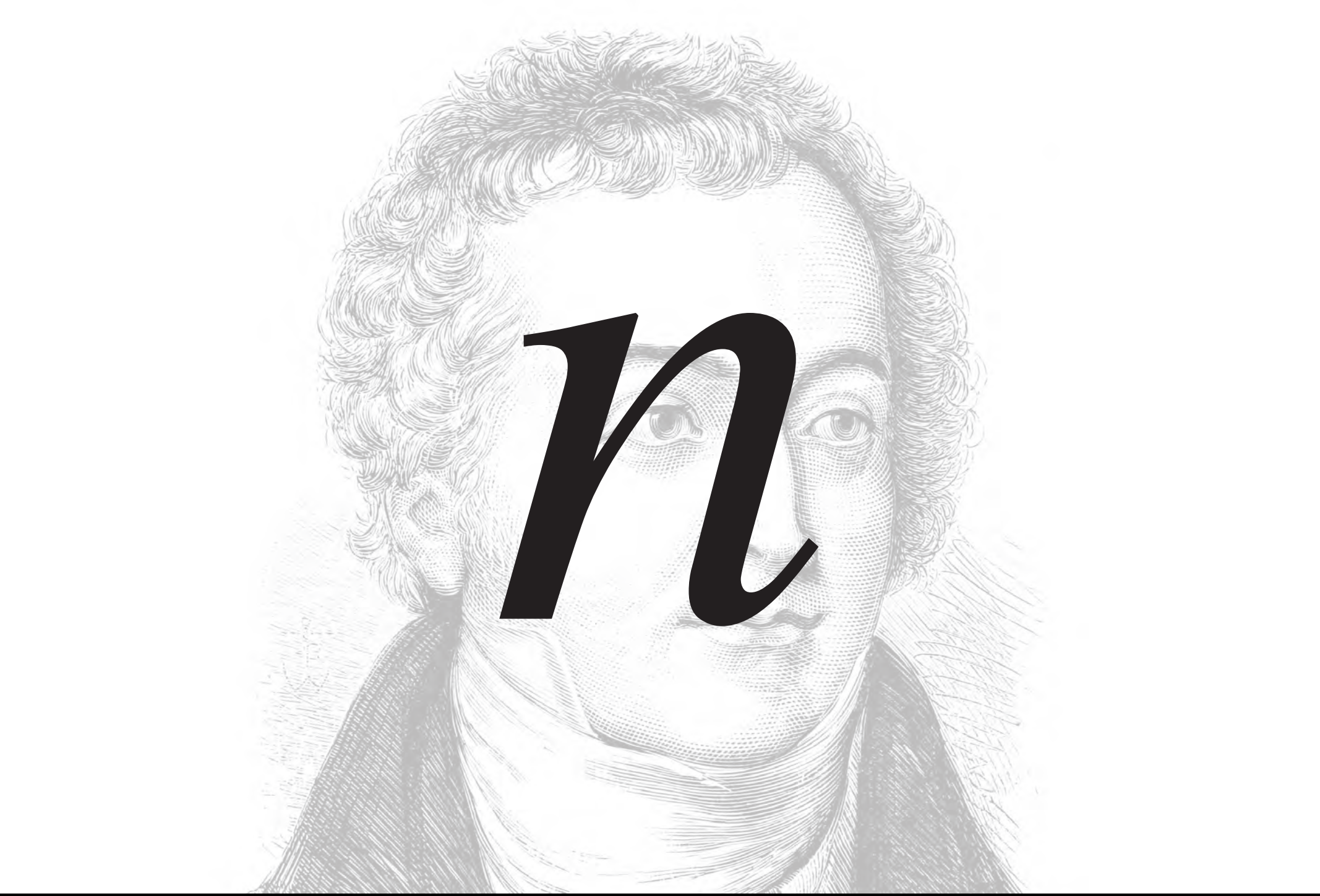

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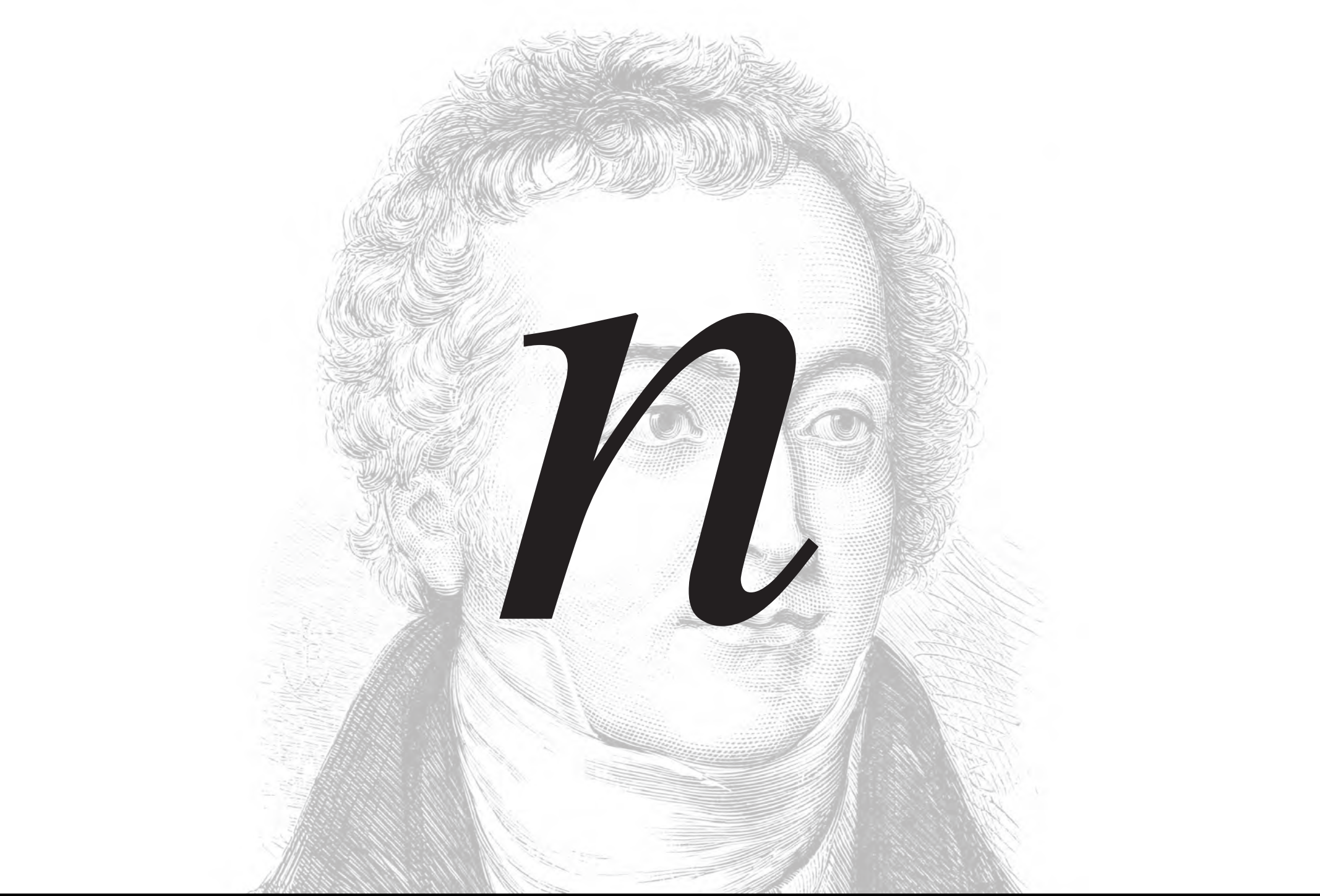

1 index

2 zero index

3 experiments

Propagation of EM wave

Propagation of EM wave

governed by wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

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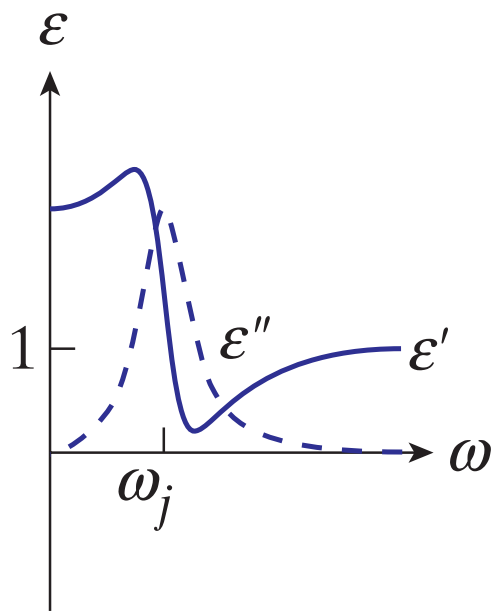
In dispersive media $n = n(\omega)$.

Dielectric constant

Lorentz oscillator



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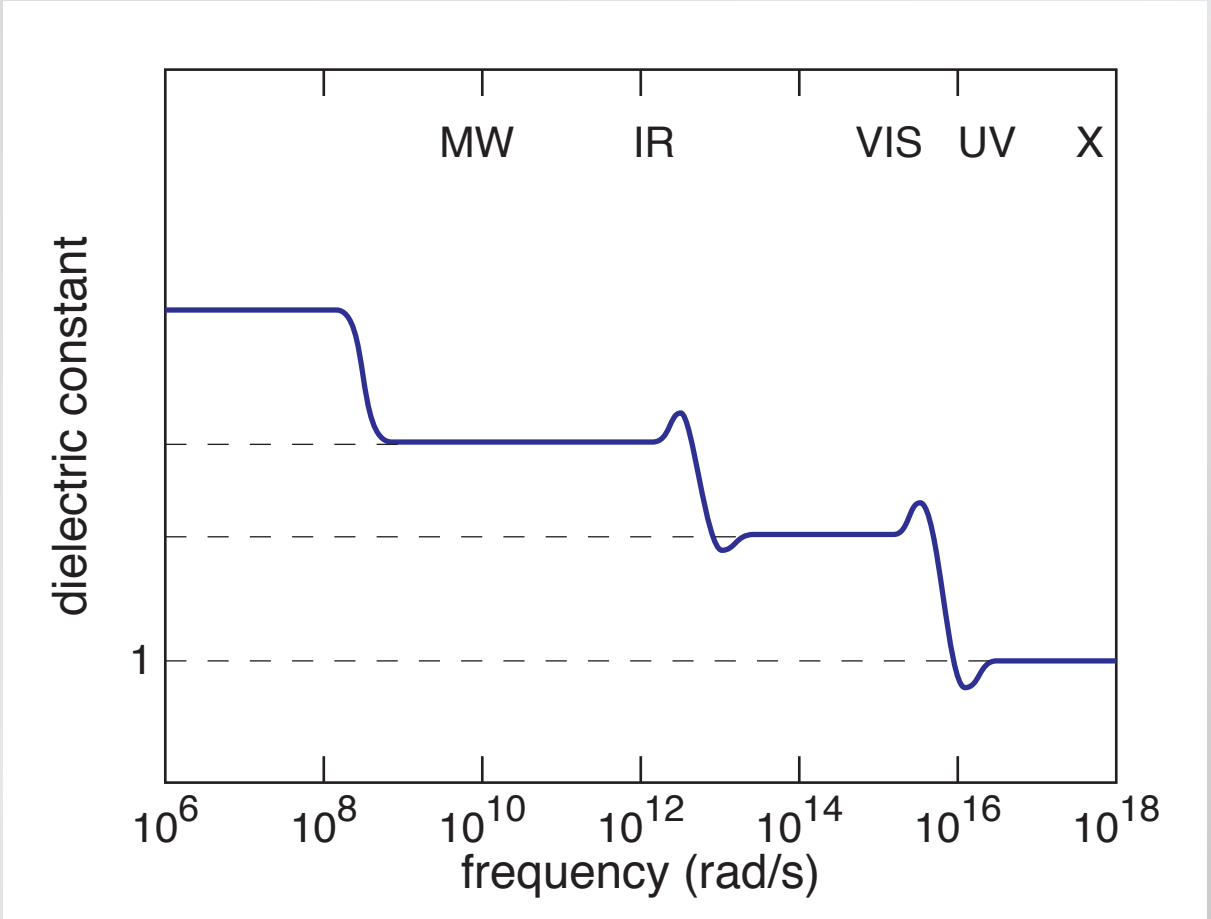

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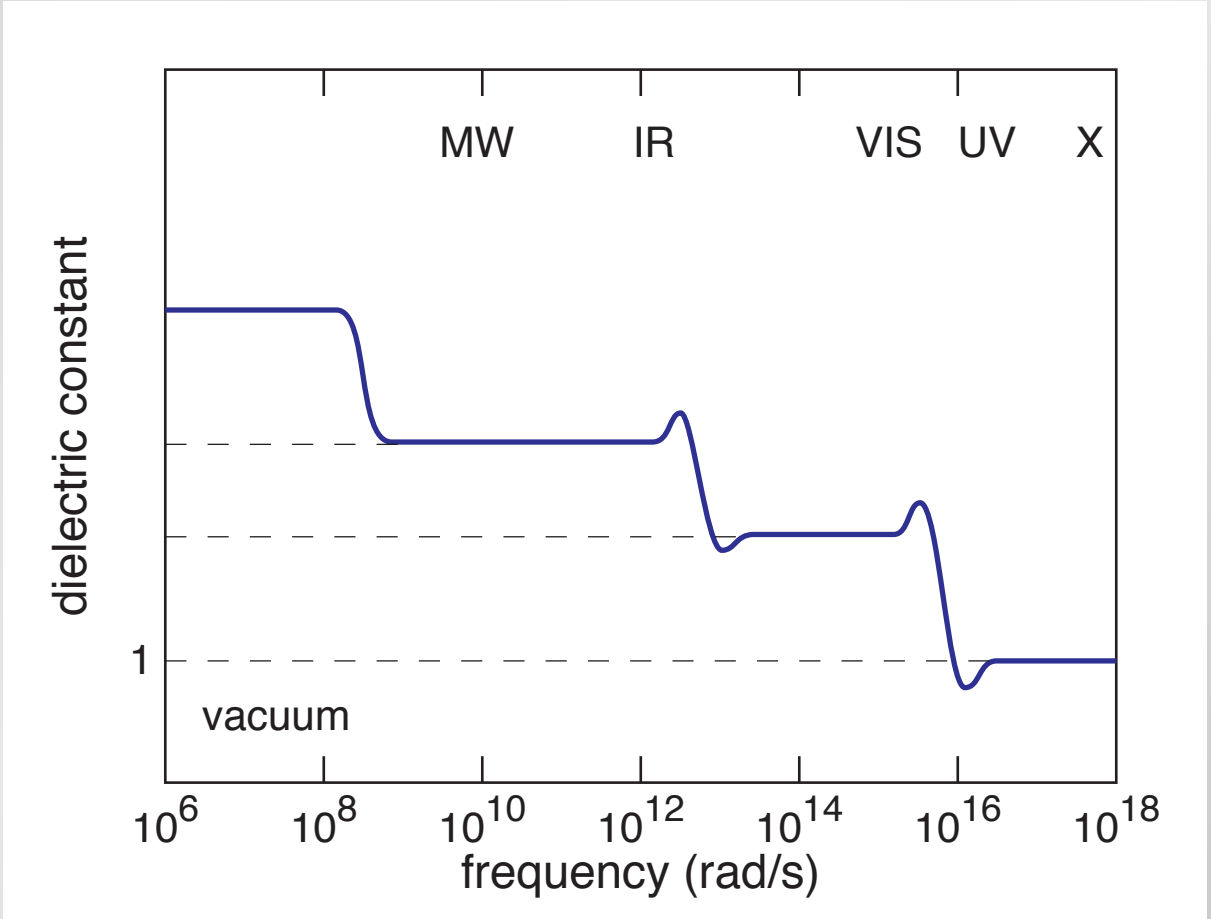

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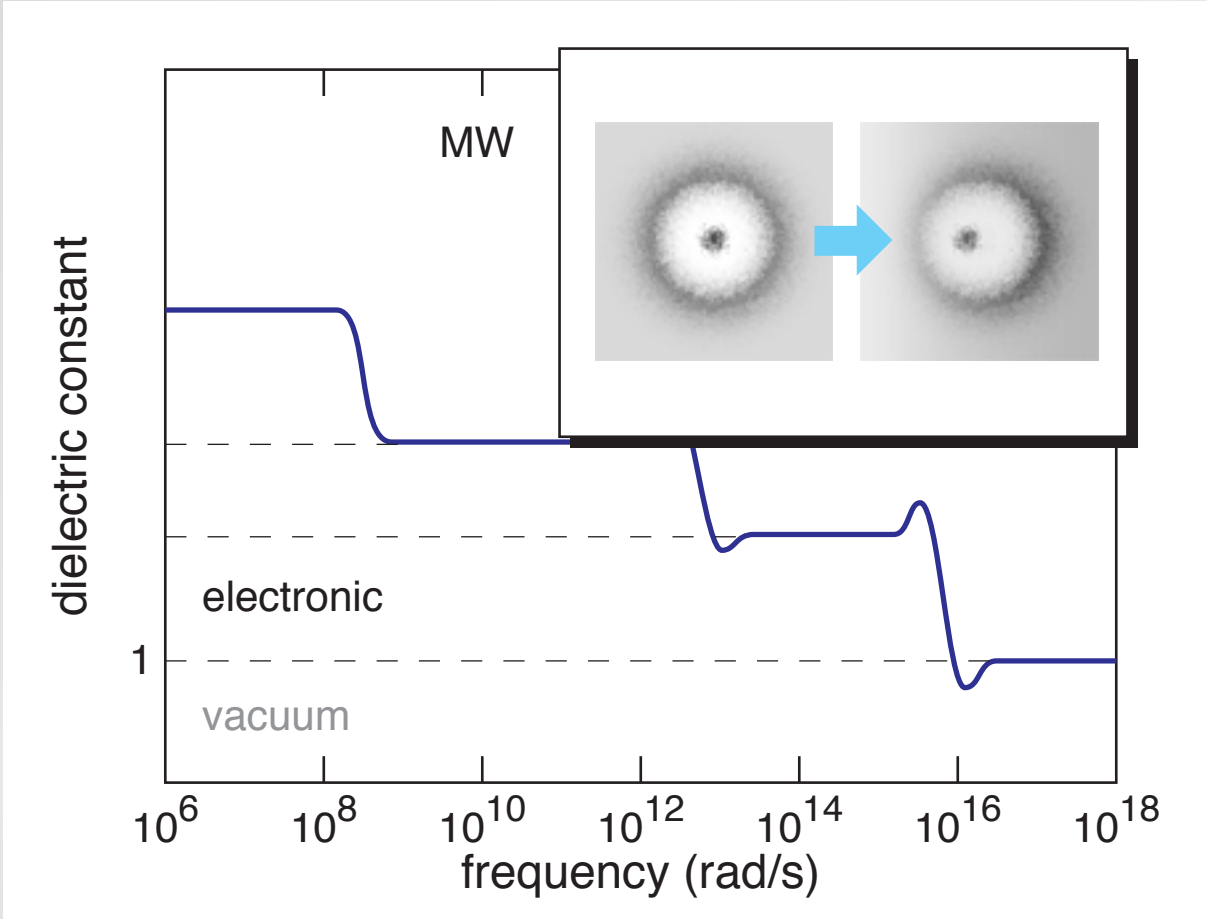

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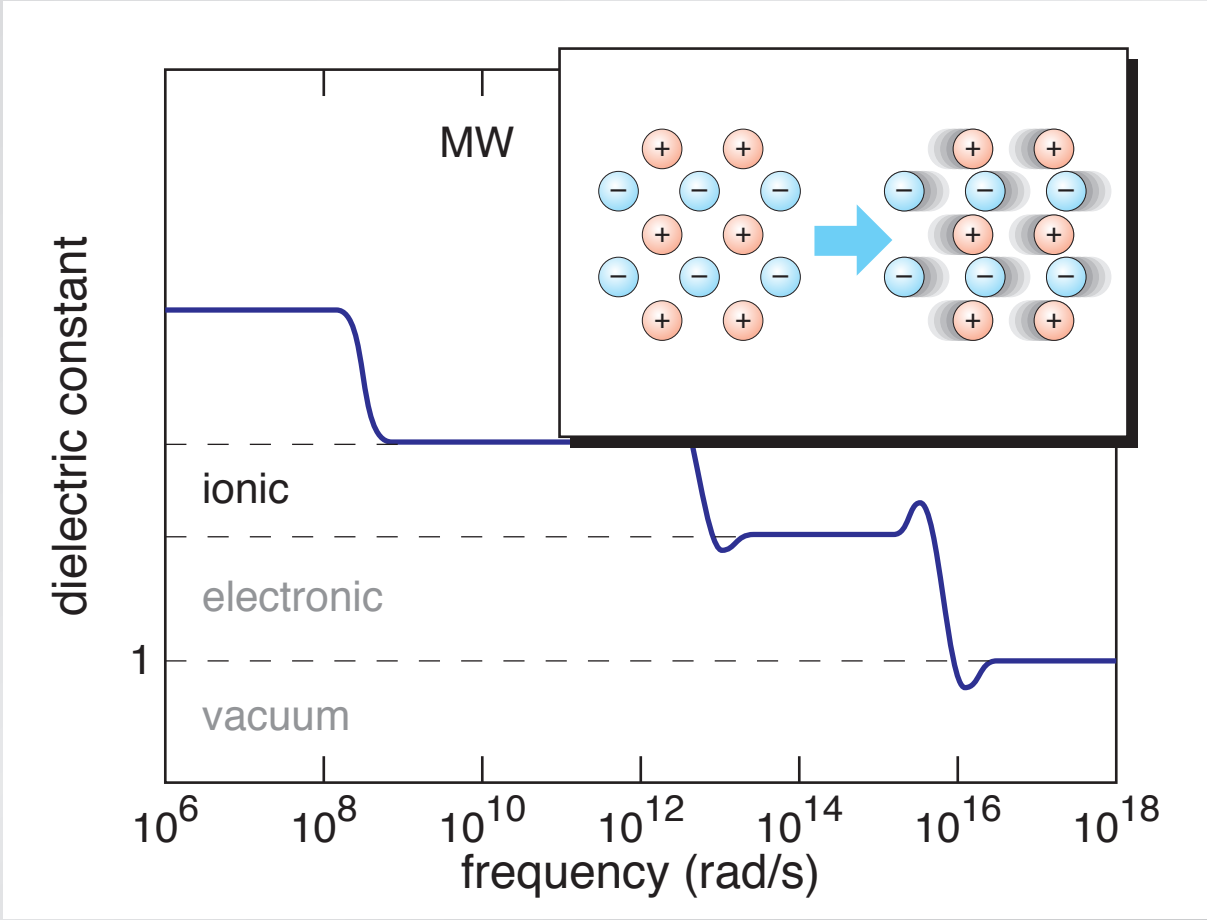

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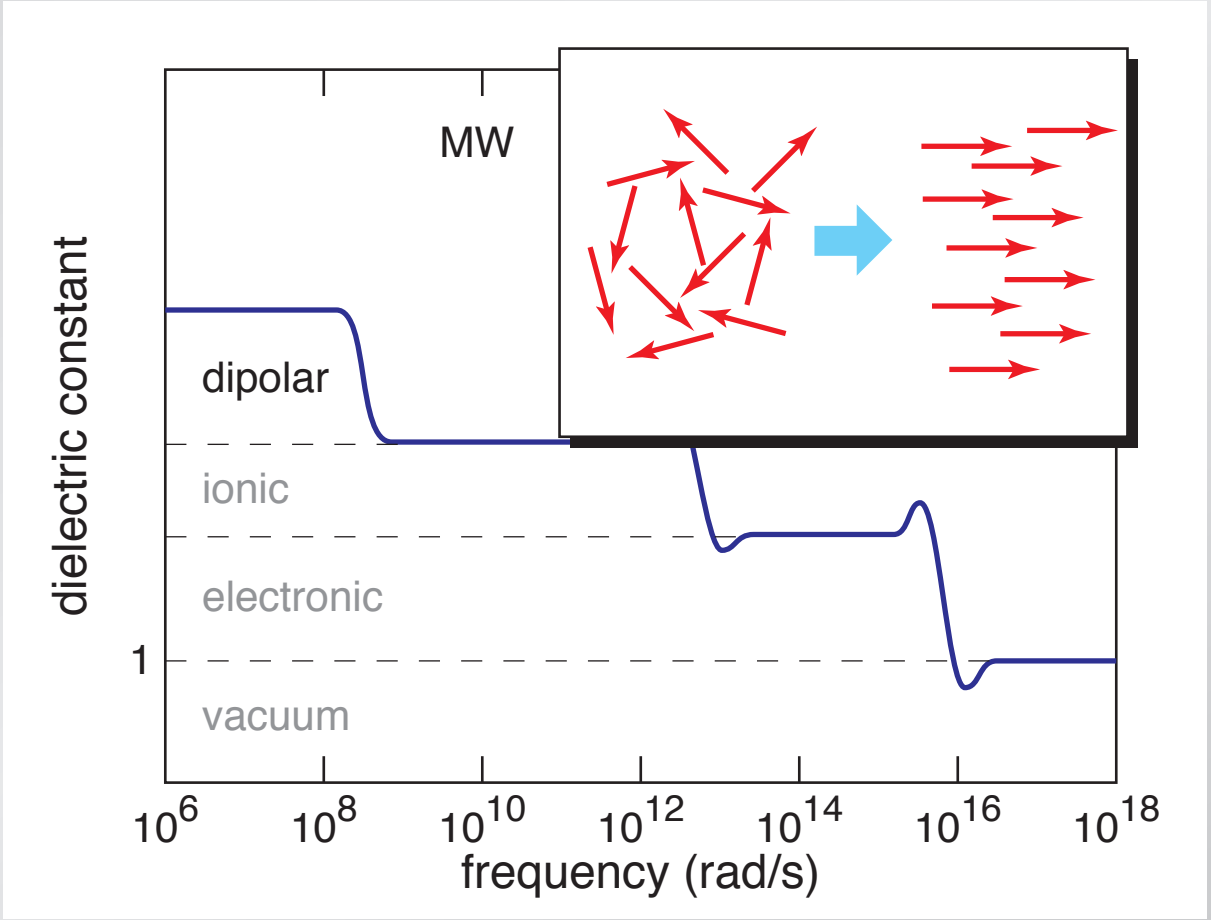

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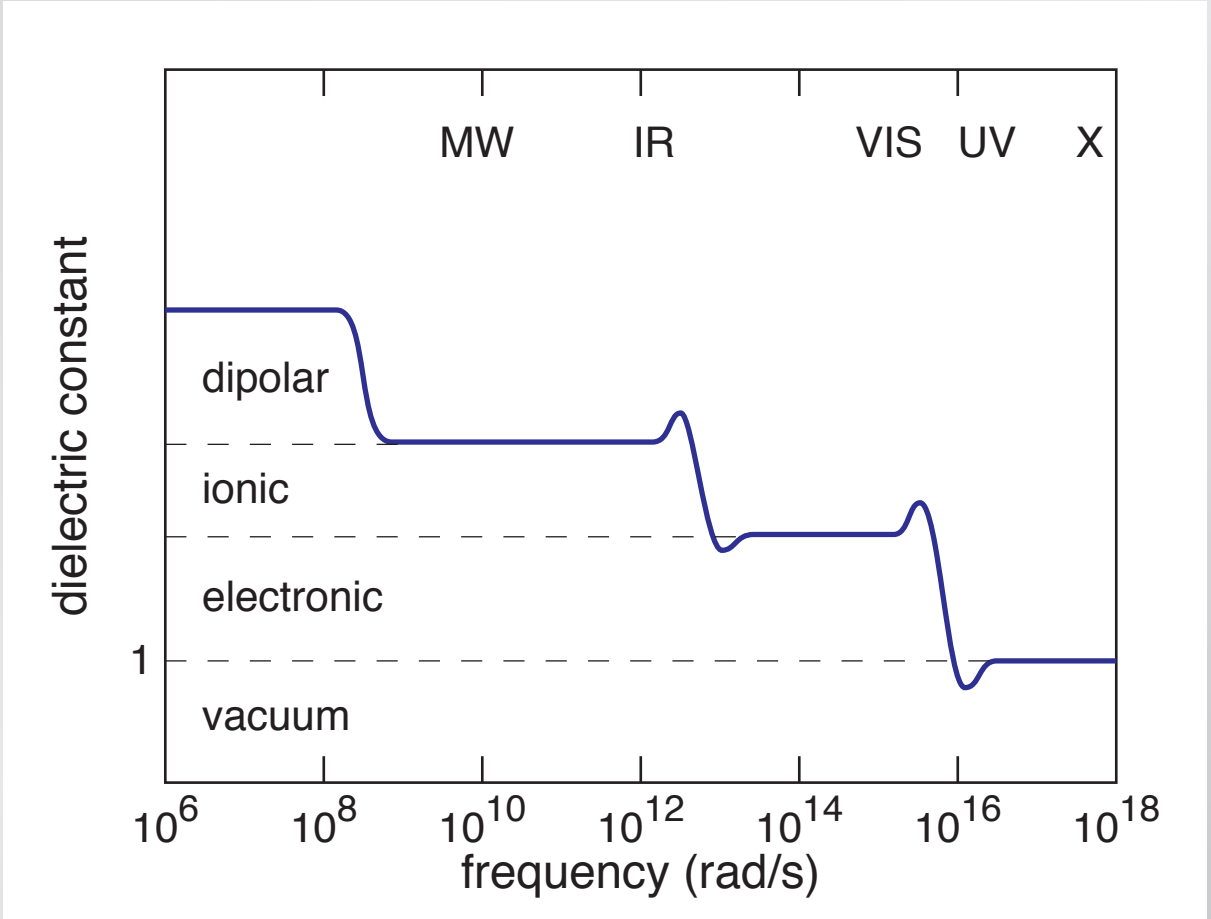
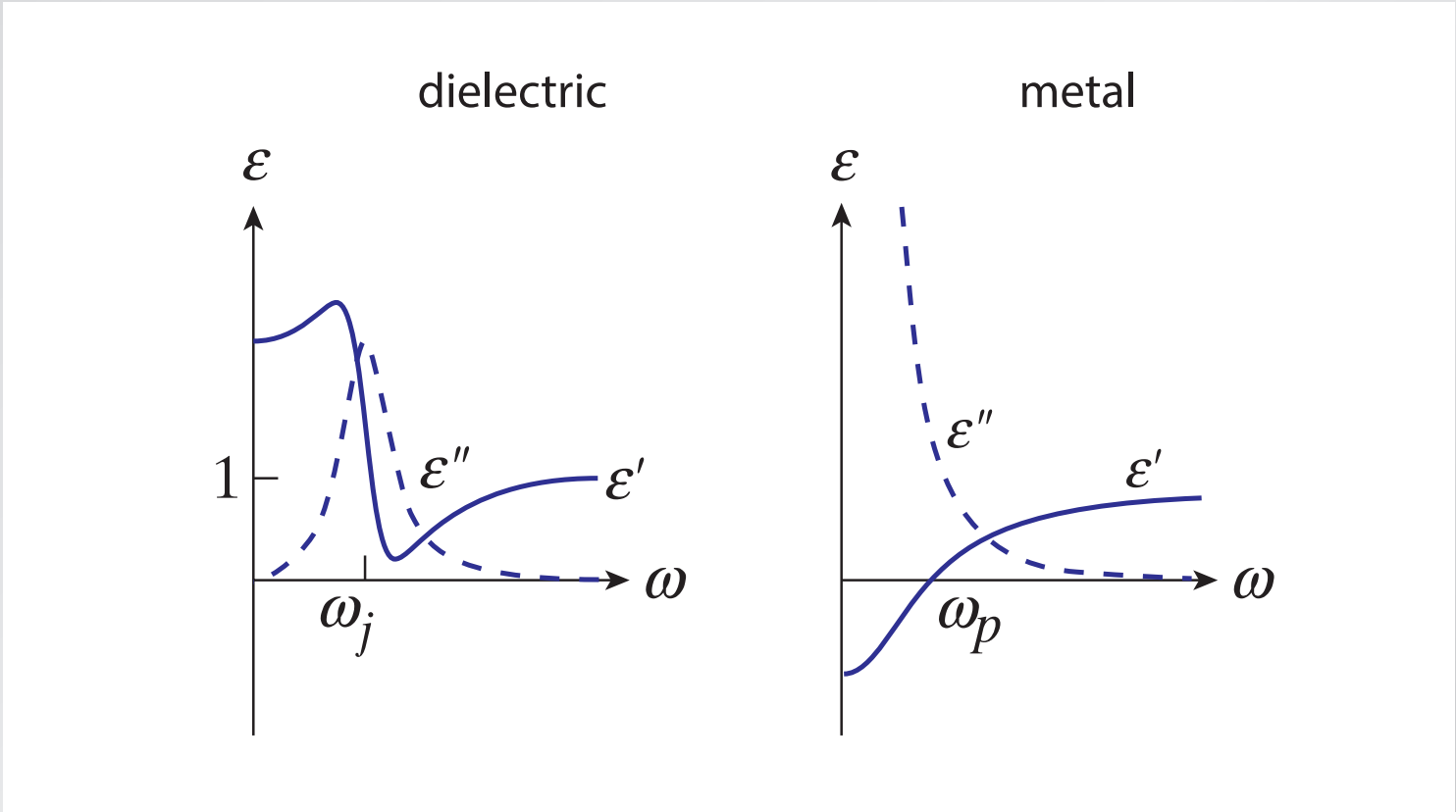
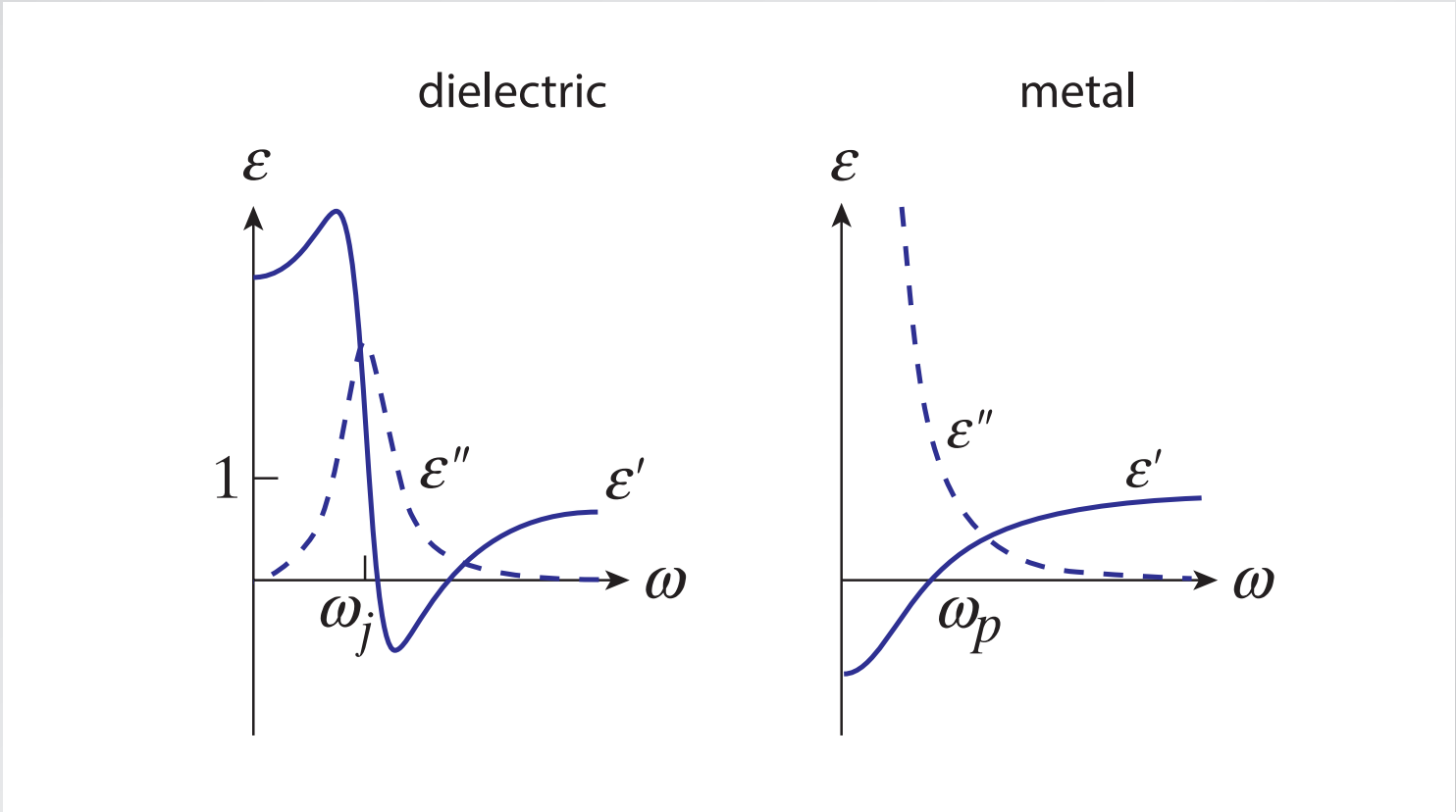


Lorentz and Drude models



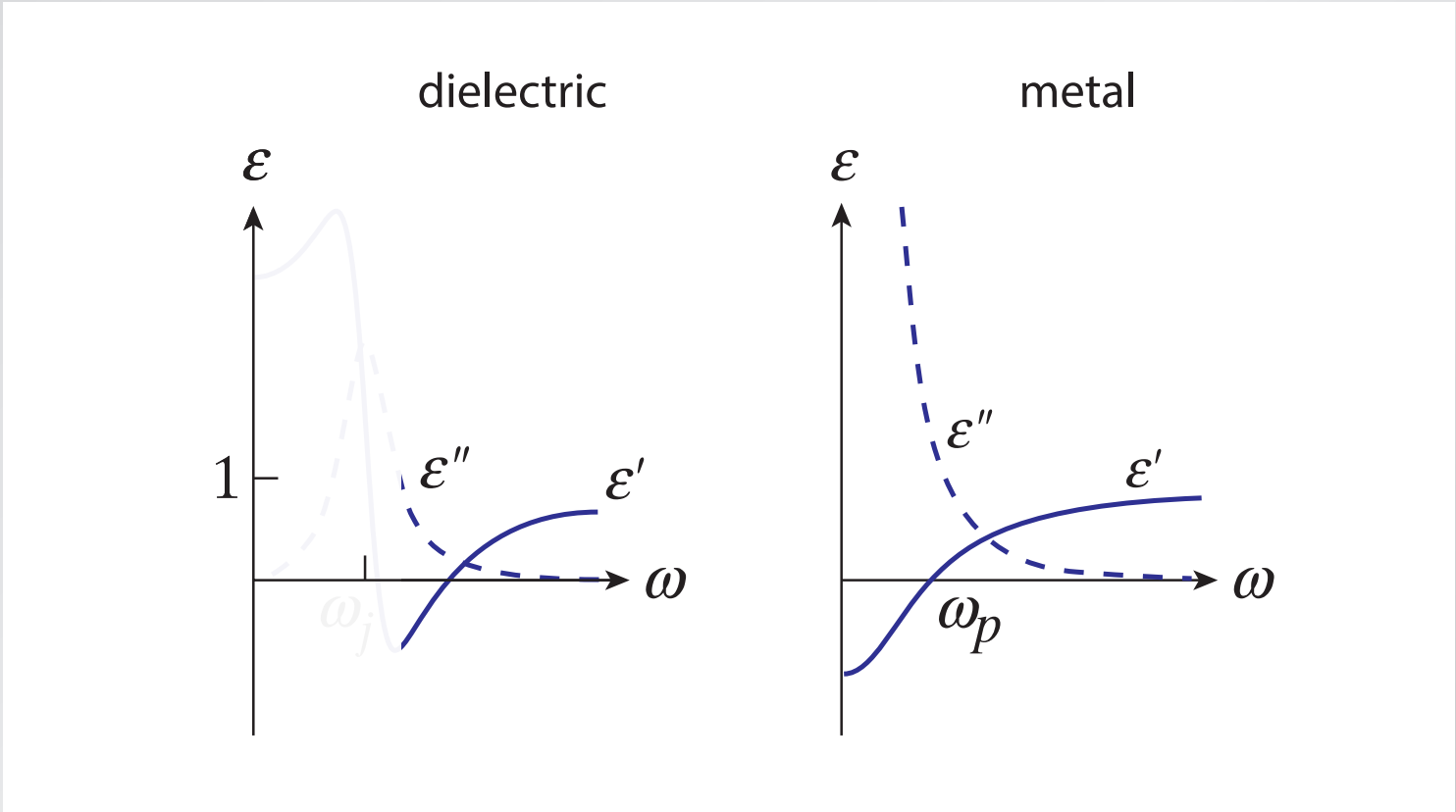
Lorentz and Drude models

for a strong (dielectric) resonance ε can become negative



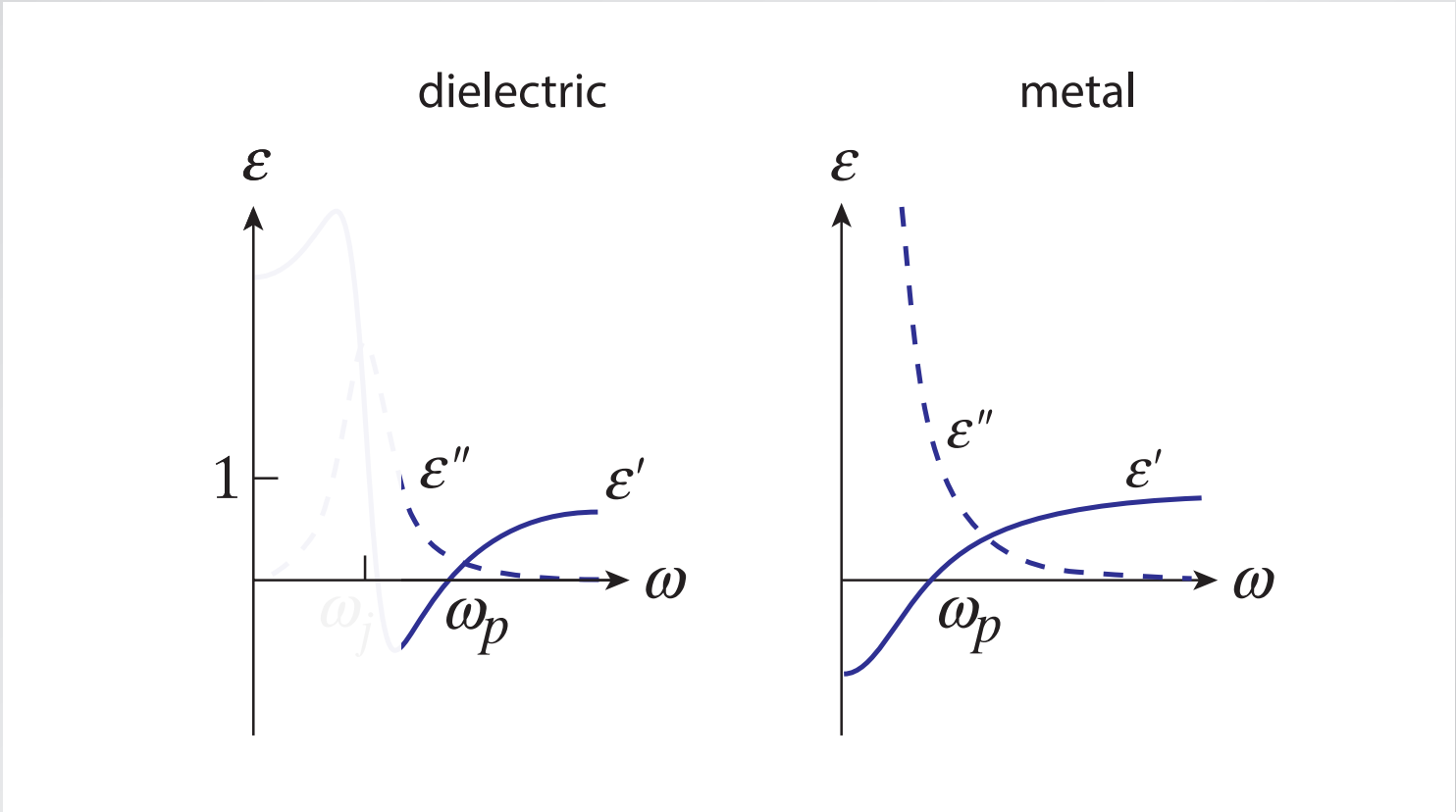
Lorentz and Drude models

valence electrons in dielectric then behave like a plasma



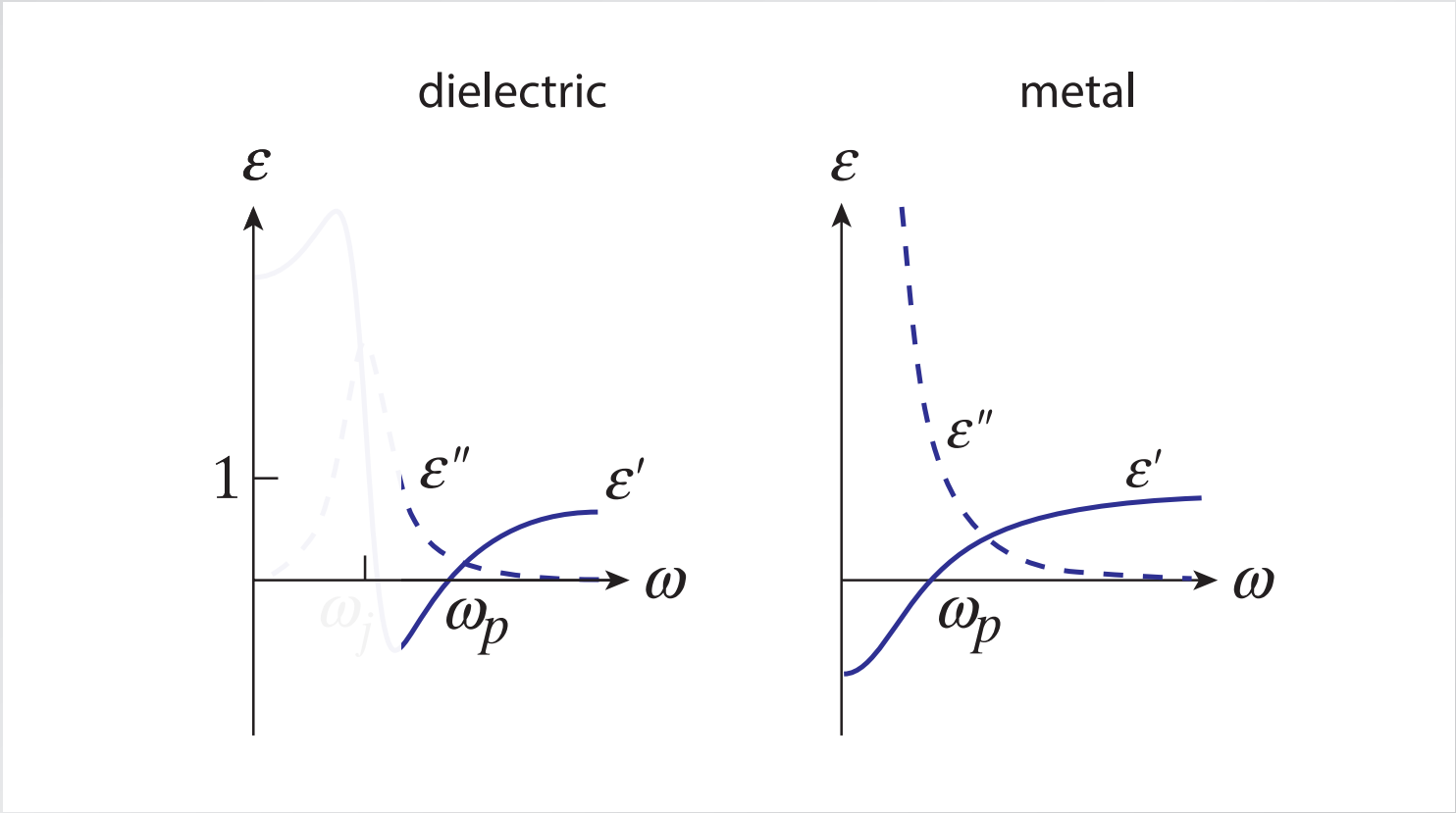
Lorentz and Drude models

with plasma frequency above the resonance



Lorentz and Drude models

(and far below the UV region)



Index also determined by magnetic response

$$n = \sqrt{\epsilon\mu}$$

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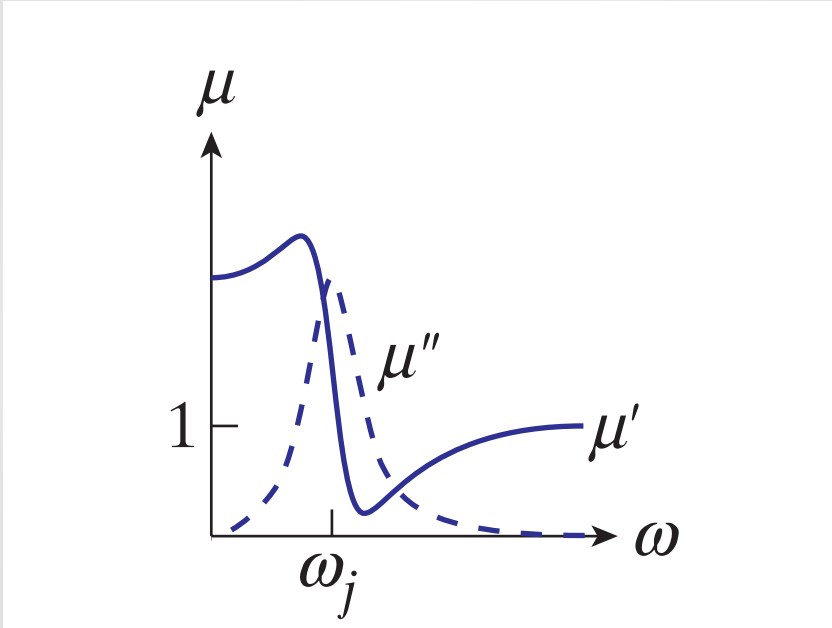
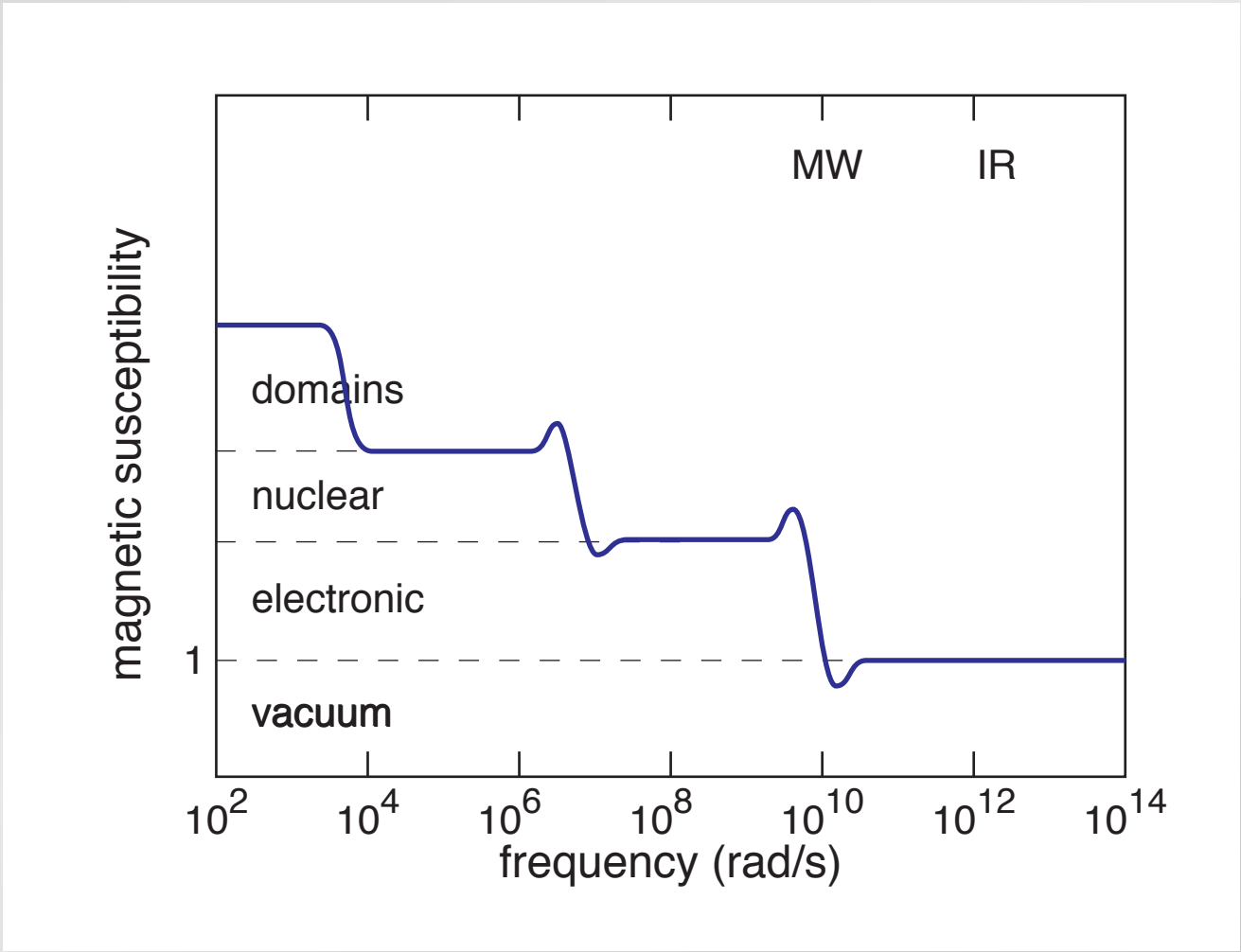
and magnetic response shows similar resonances

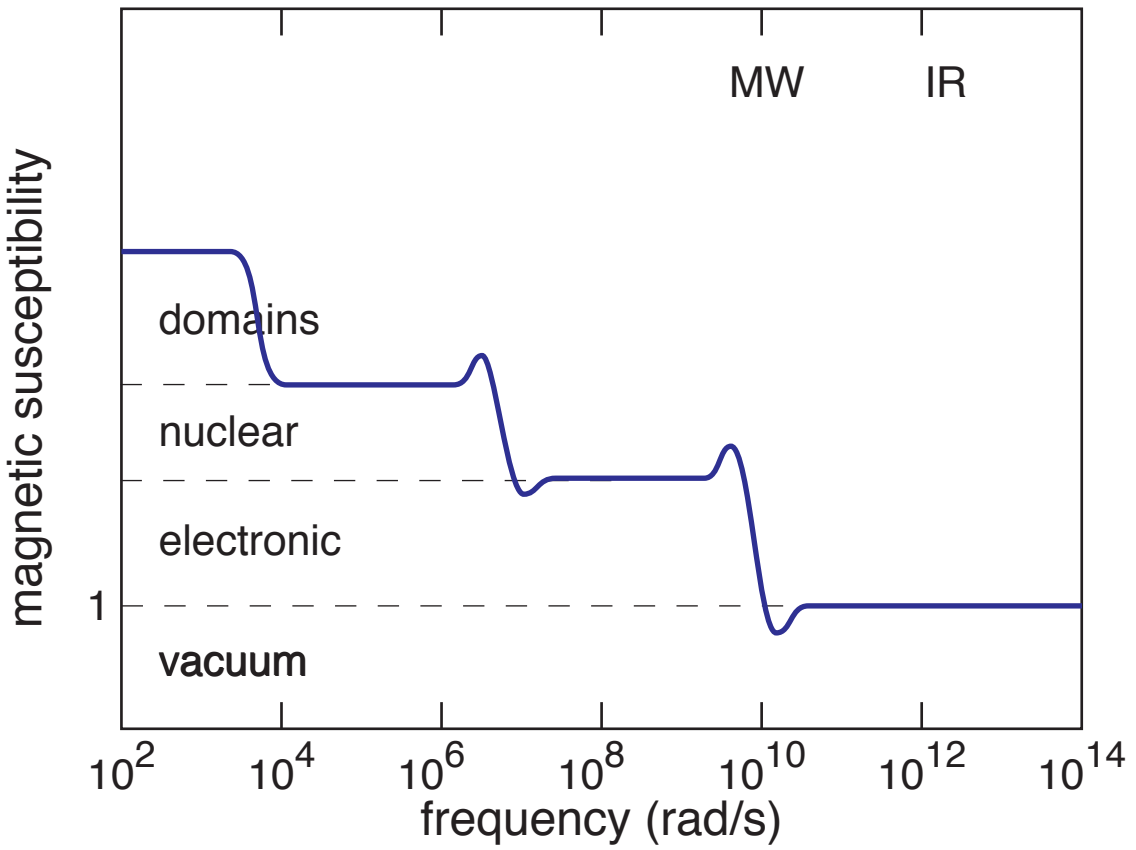


Magnetic response



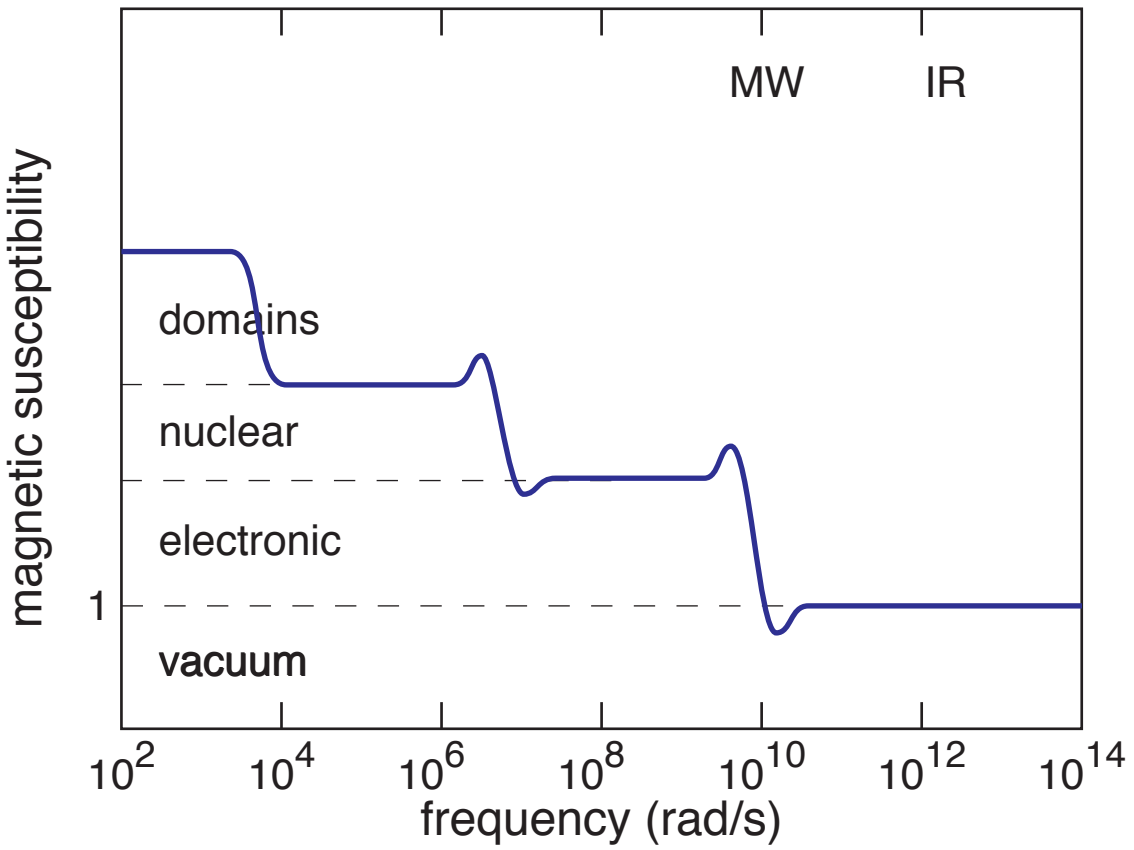
Magnetic response

but magnetic resonances occur below optical frequencies



Magnetic response

so, in optical regime, $\mu \approx 1$



Index of refraction

$$n = \sqrt{\epsilon\mu}$$

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What happens when $\mathrm{Re}\epsilon$ and/or $\mathrm{Re}\mu$ is negative?

Write complex quantities as

$$\varepsilon = |\varepsilon| e^{i\theta} \qquad \mu = |\mu| e^{i\phi}$$

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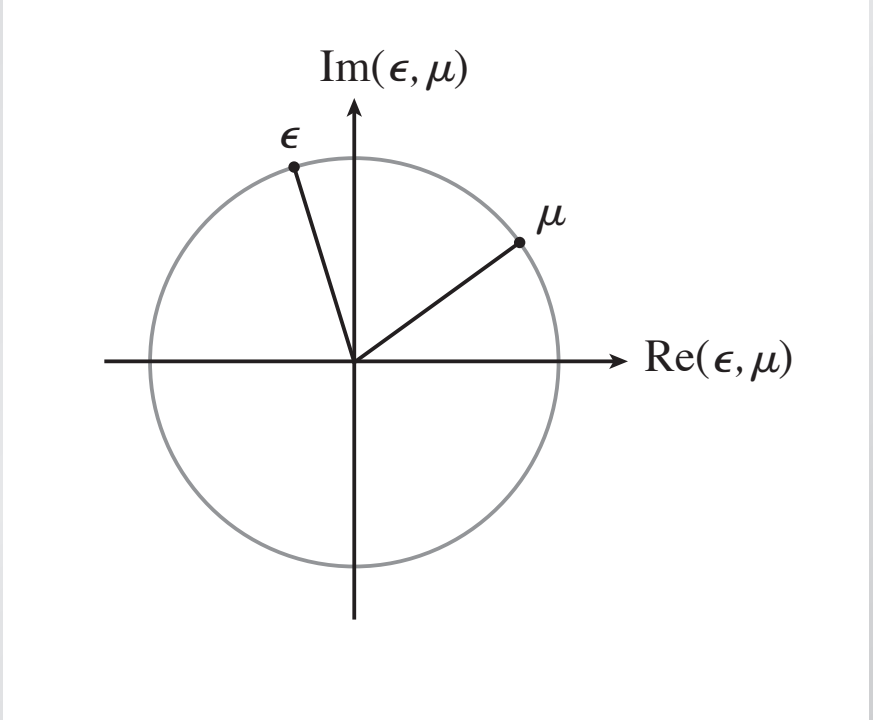

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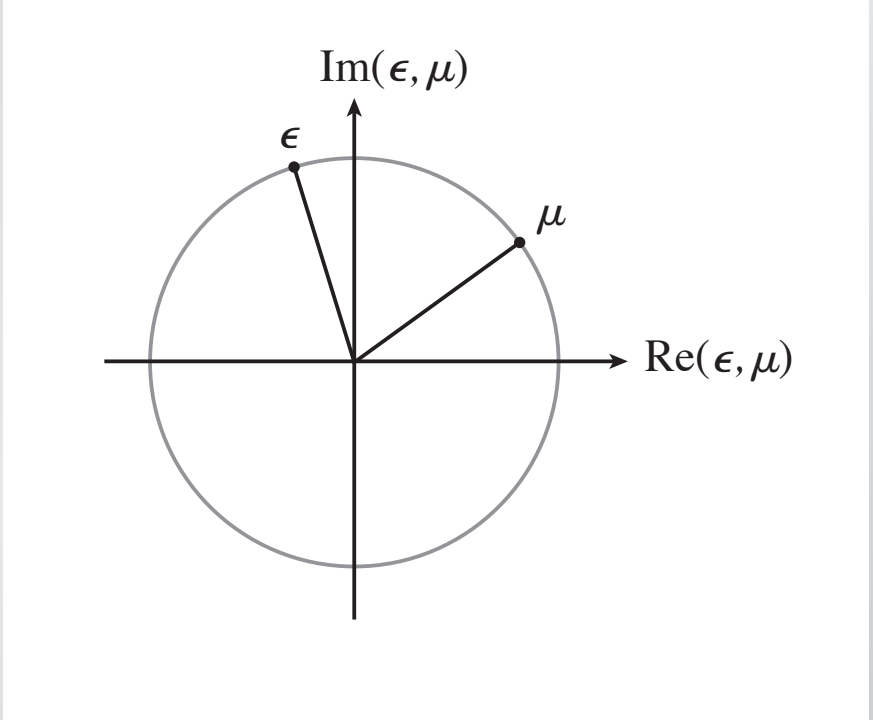

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$$n = \sqrt{|\varepsilon||\mu|}\, e^{i\frac{\theta+\phi}{2}}$$

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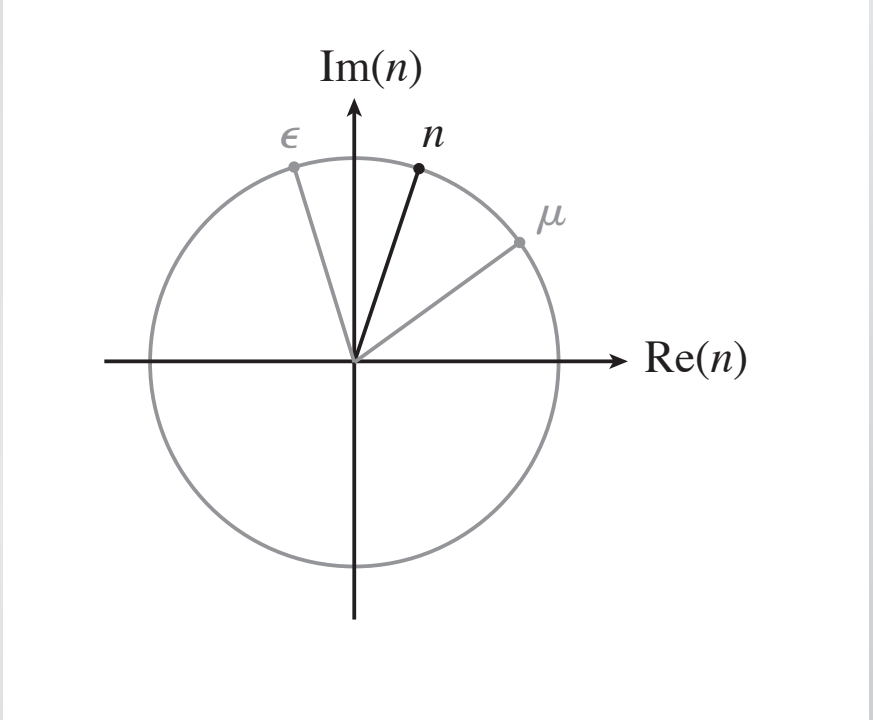

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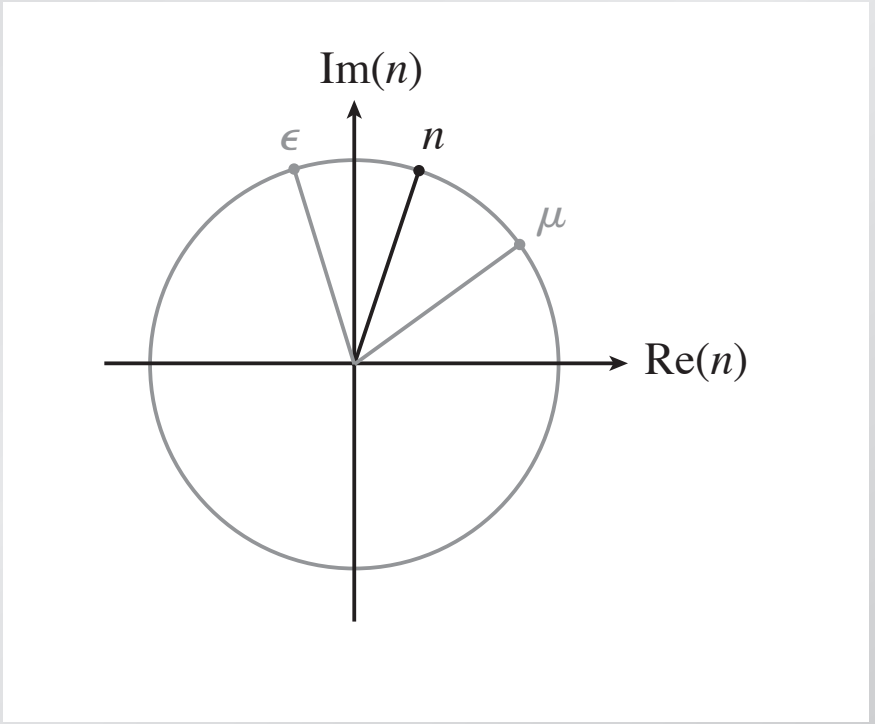

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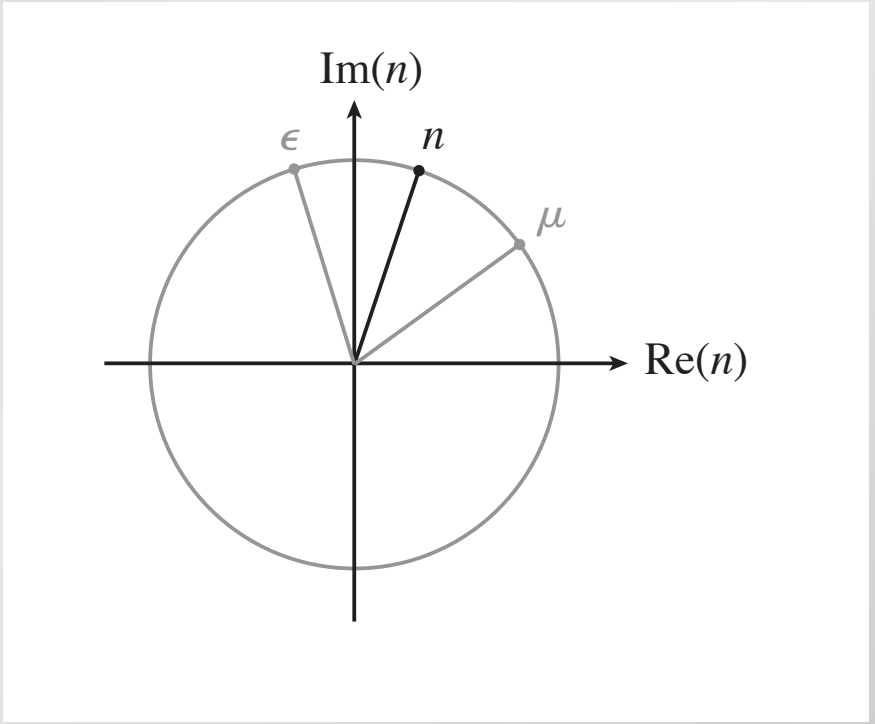

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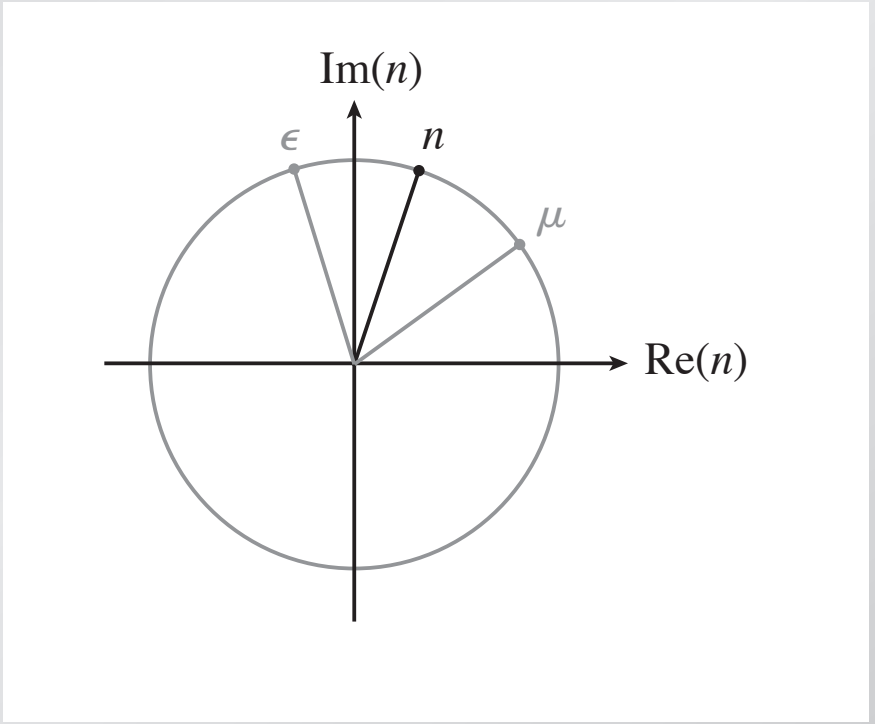

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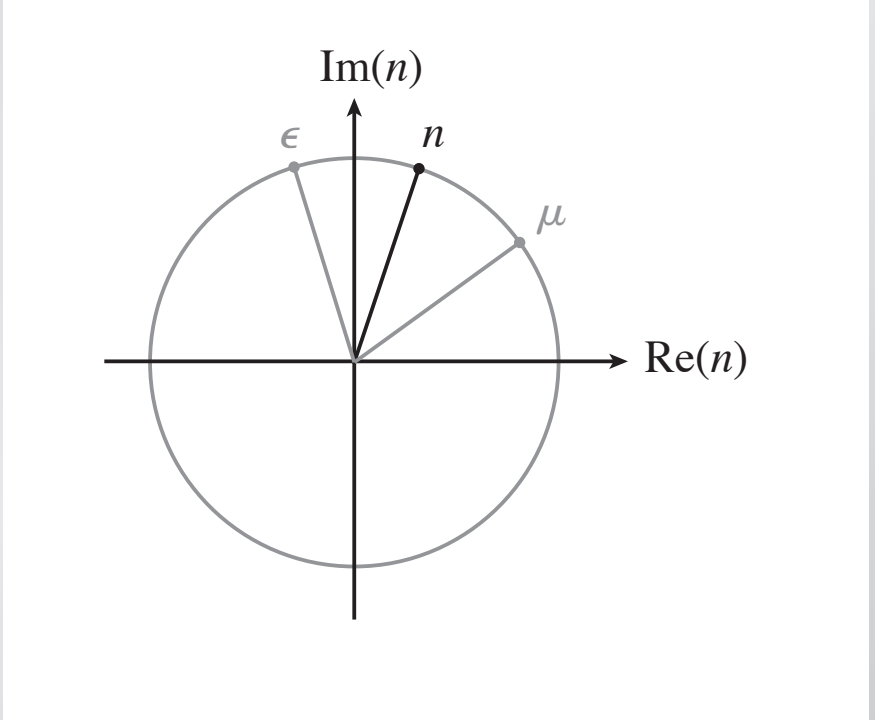

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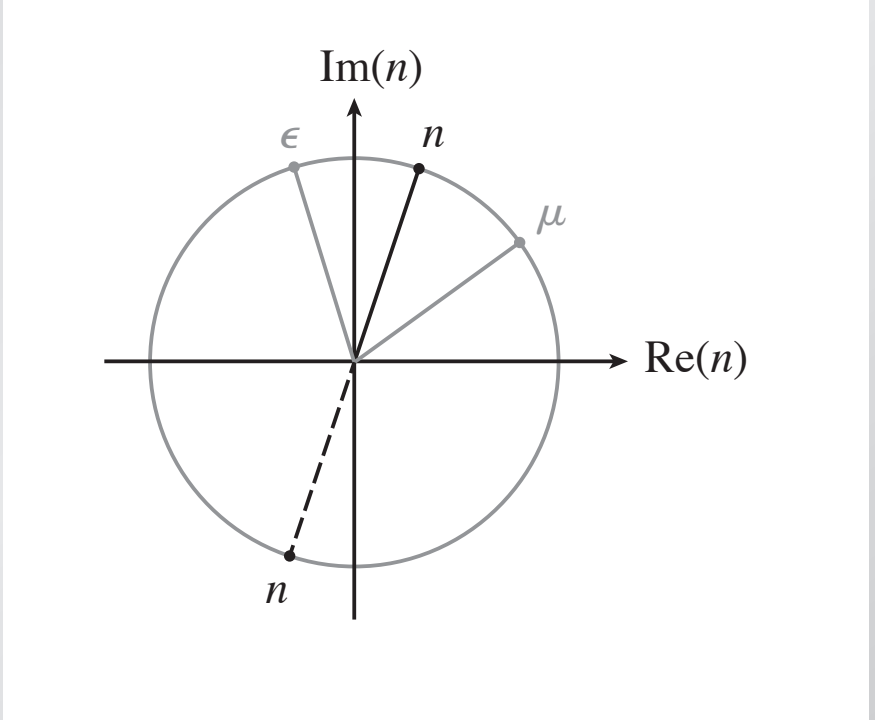

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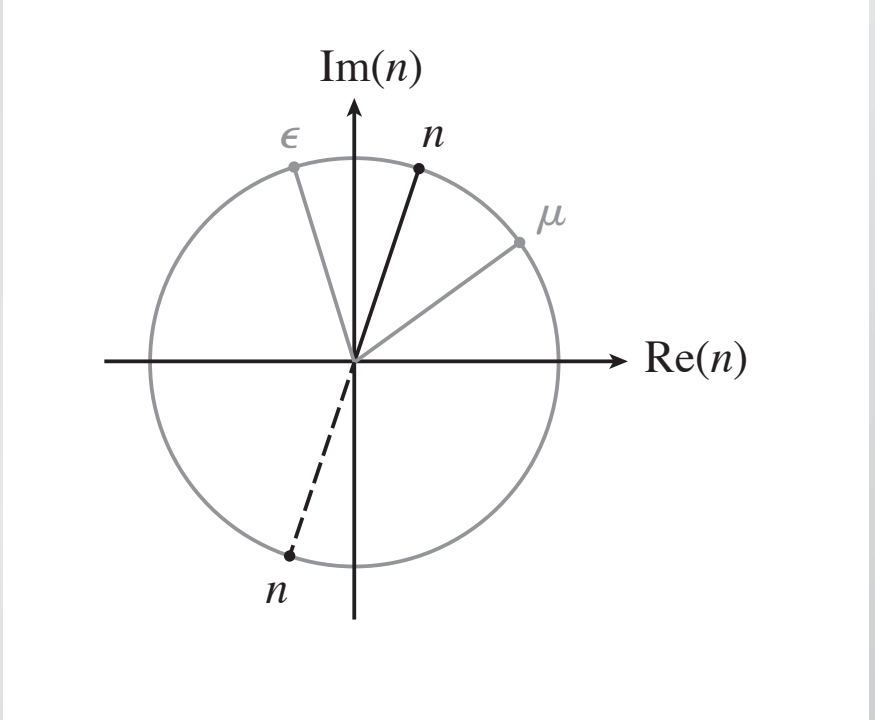

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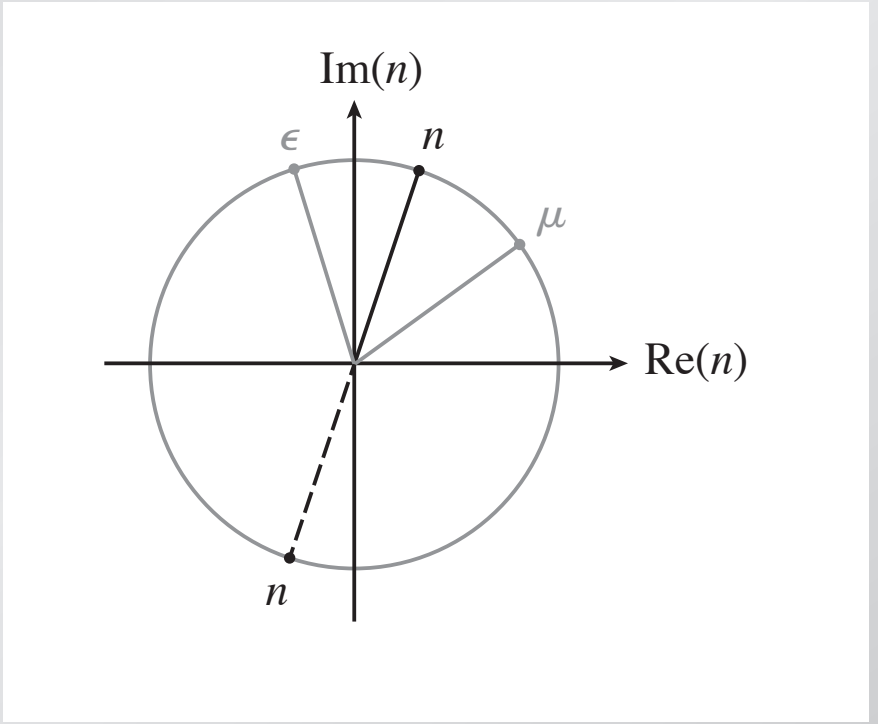

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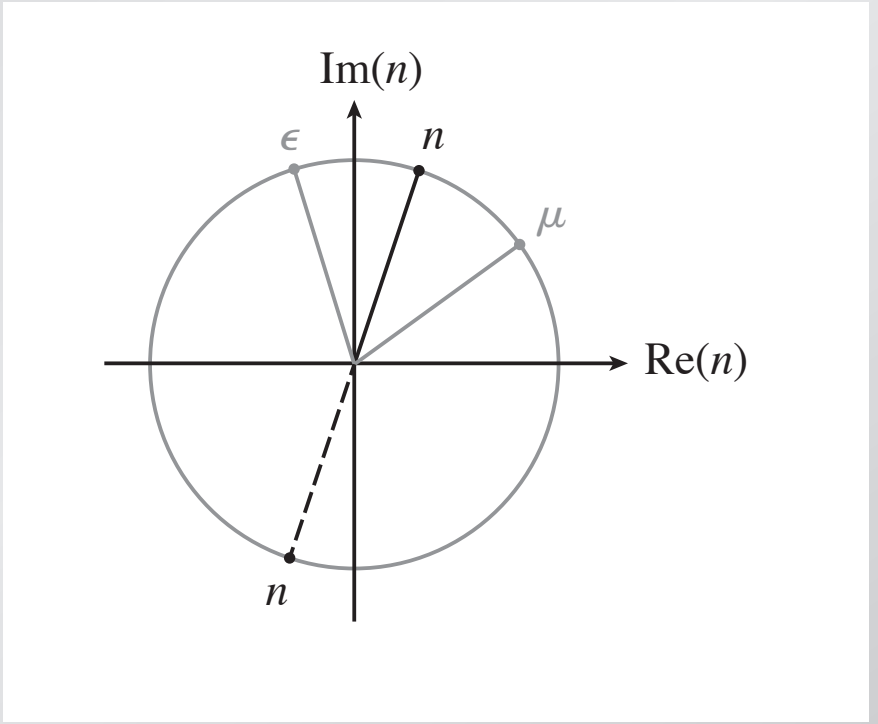

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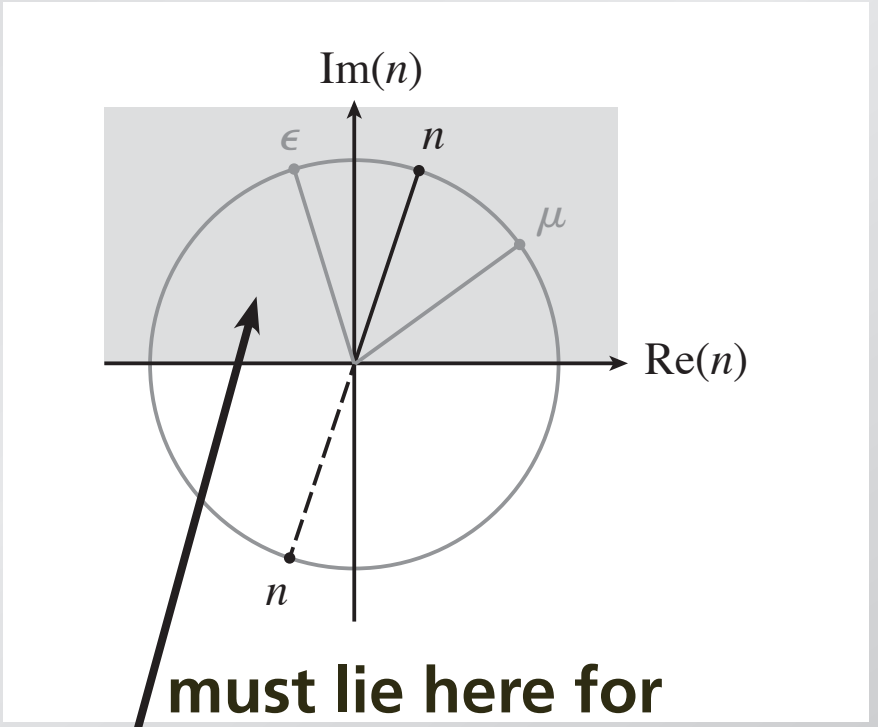

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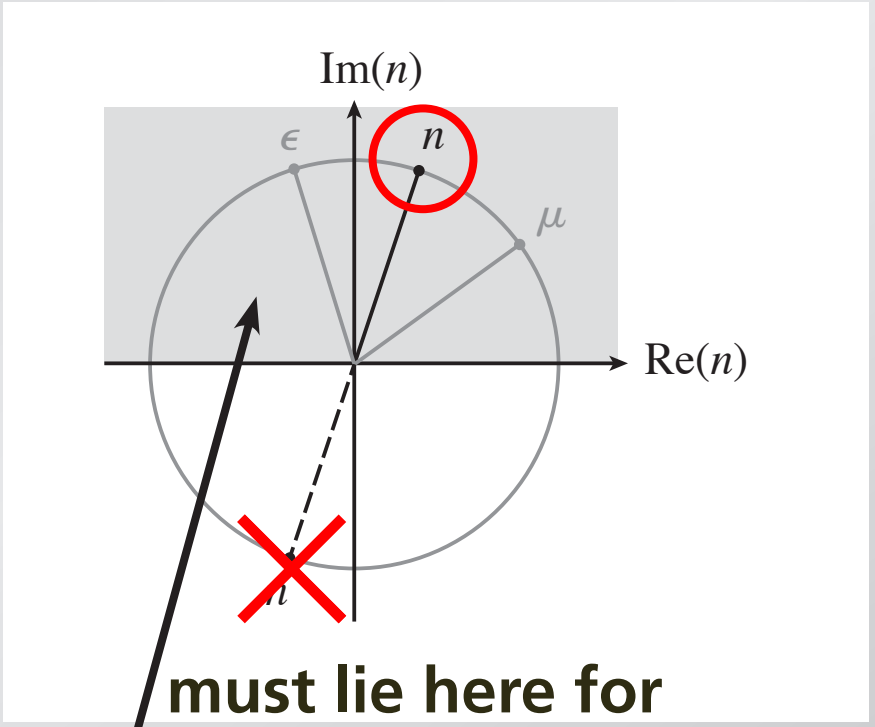

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Q: Is this the only possible solution?

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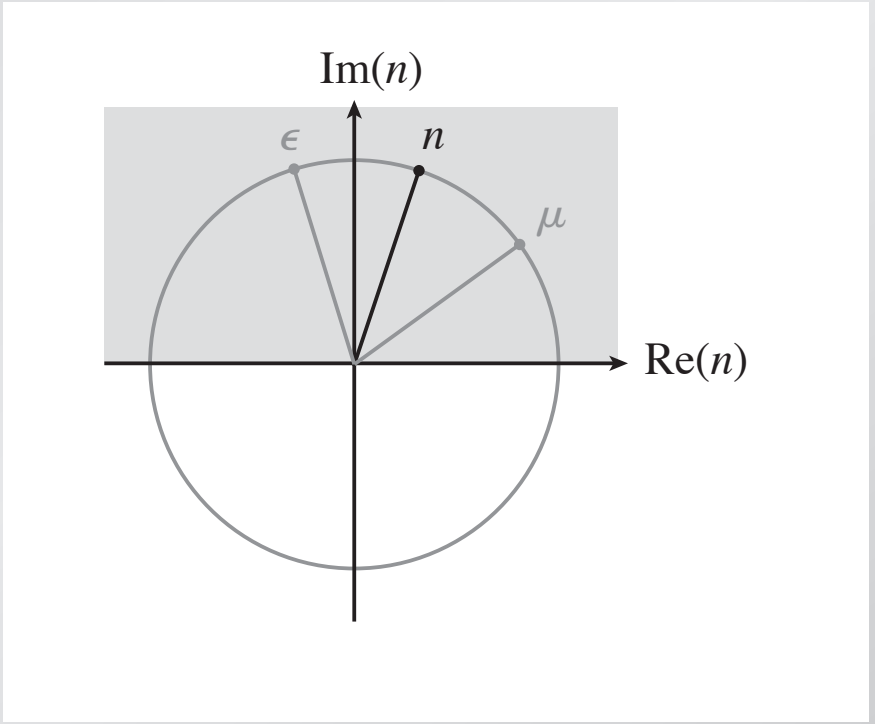
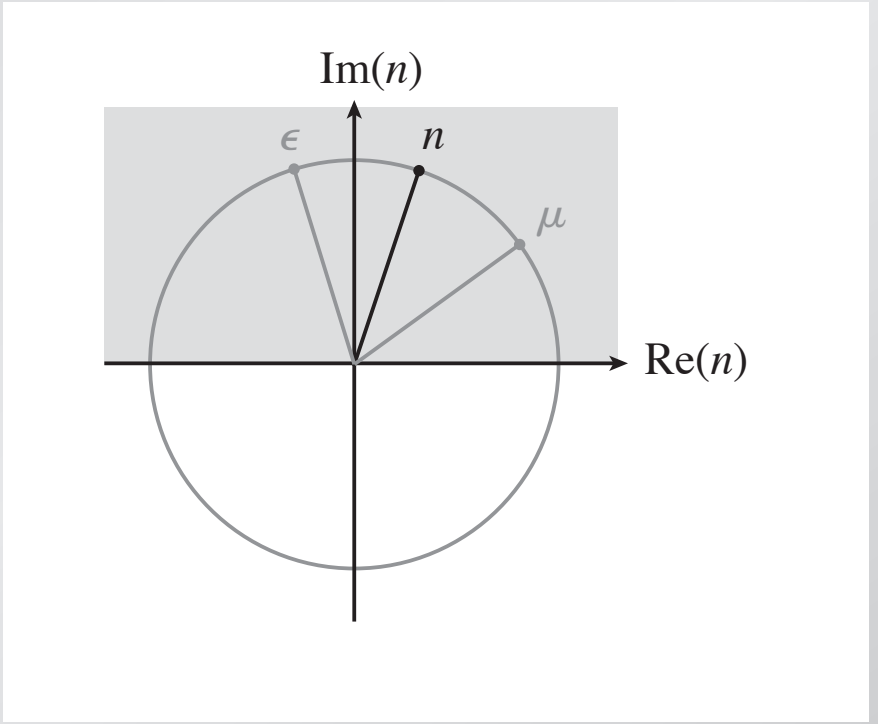


To find n (passive materials):

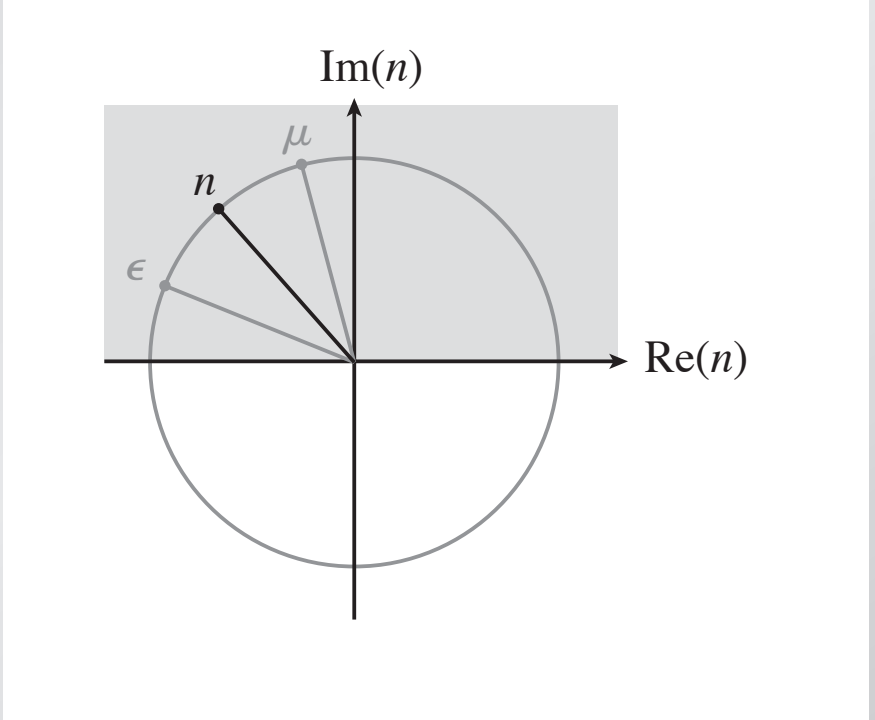
1. Draw line that bisects ϵ and μ

2. Choose upper branch



For certain values of ϵ and μ

we can get a *negative* $\mathrm{Re}(n)$!



Q: Must both $\mathrm{Re}\epsilon < 0$ and $\mathrm{Re}\mu < 0$ to get a negative index?

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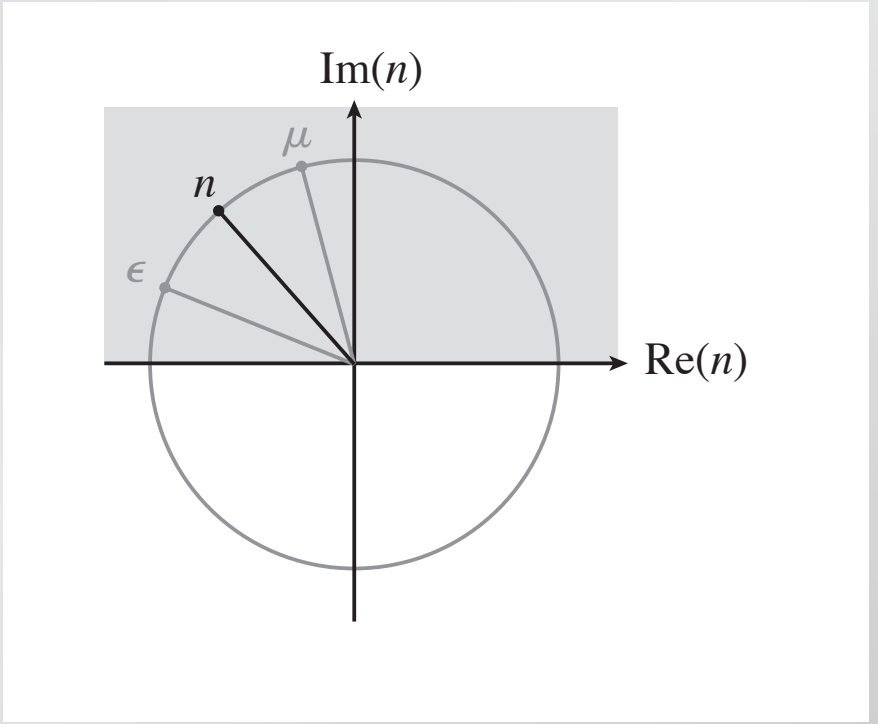

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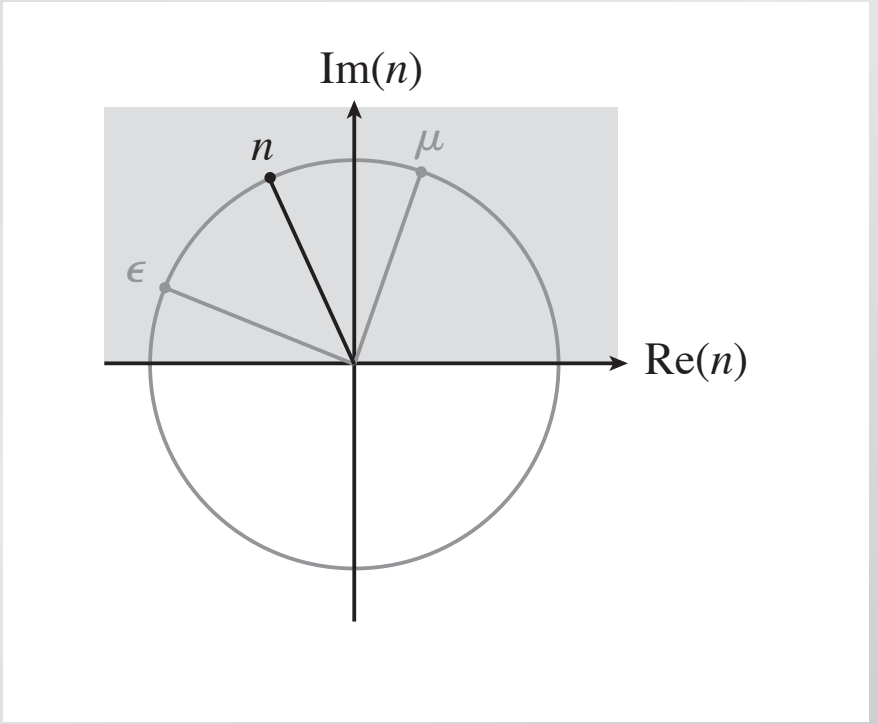
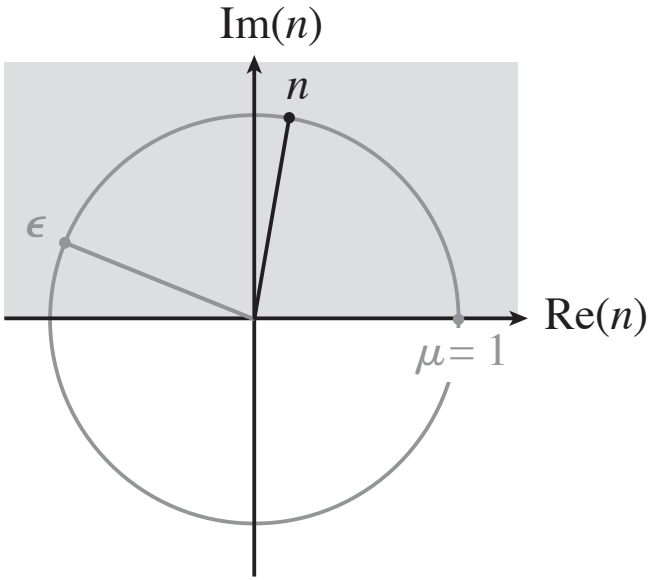


Note: need magnetic response

to achieve $n \leq 0$!



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Spatial and temporal dependence of wave component

$$E = E_o e^{i(kx - \omega t)} = E_o e^{-k''x} e^{i(k'x - \omega t)}$$

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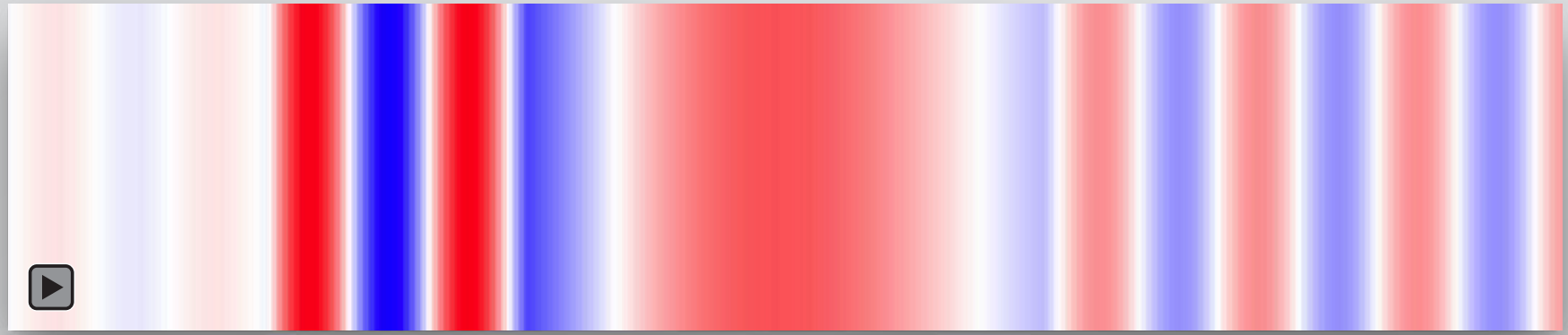
$$k = \frac{2\pi n}{\lambda_o} = \frac{2\pi(n' + in'')}{\lambda_o} = k' + ik''$$

Spatial and temporal dependence of wave component

$$E = E_o e^{i(kx - \omega t)} = E_o e^{-k''x} e^{i(k'x - \omega t)}$$

When $\mathrm{Re}(n) < 0$, $k' < 0$, and so phase velocity reversed!



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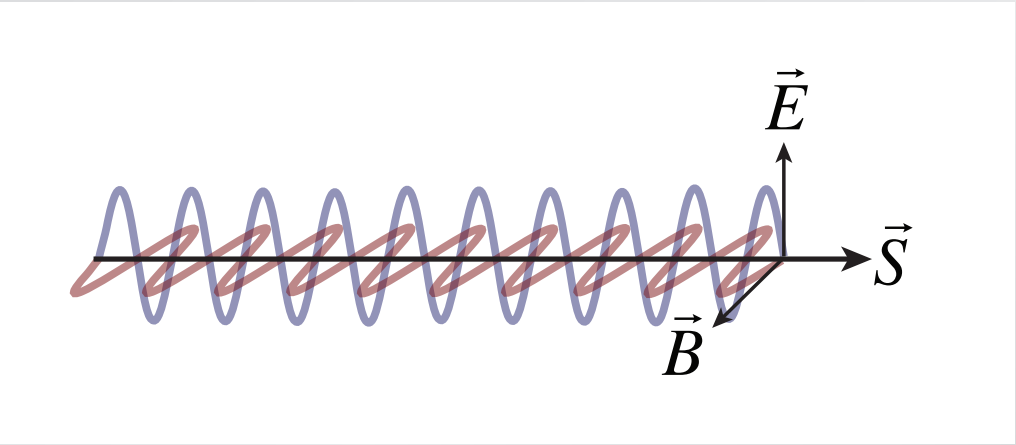

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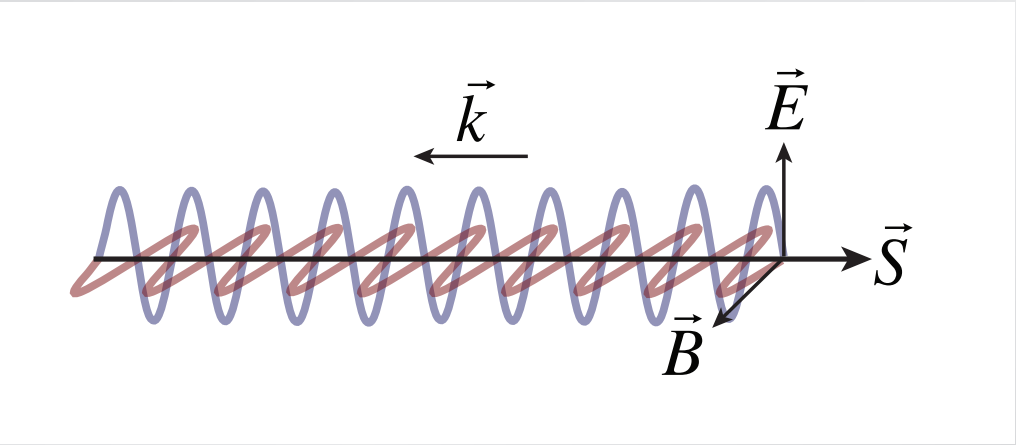
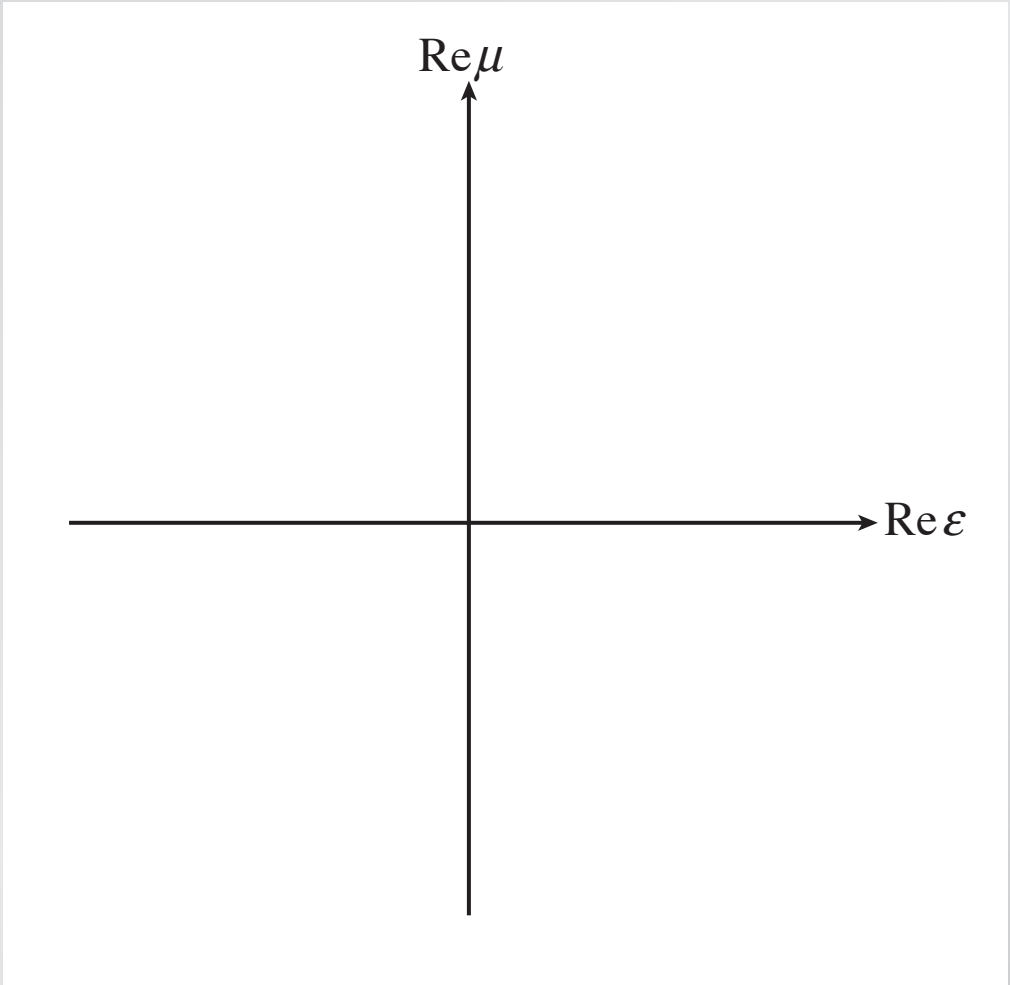


When $\mathrm{Re}(n) < 0$, $k' < 0$, and so phase velocity reversed!

classification of (non-lossy) materials



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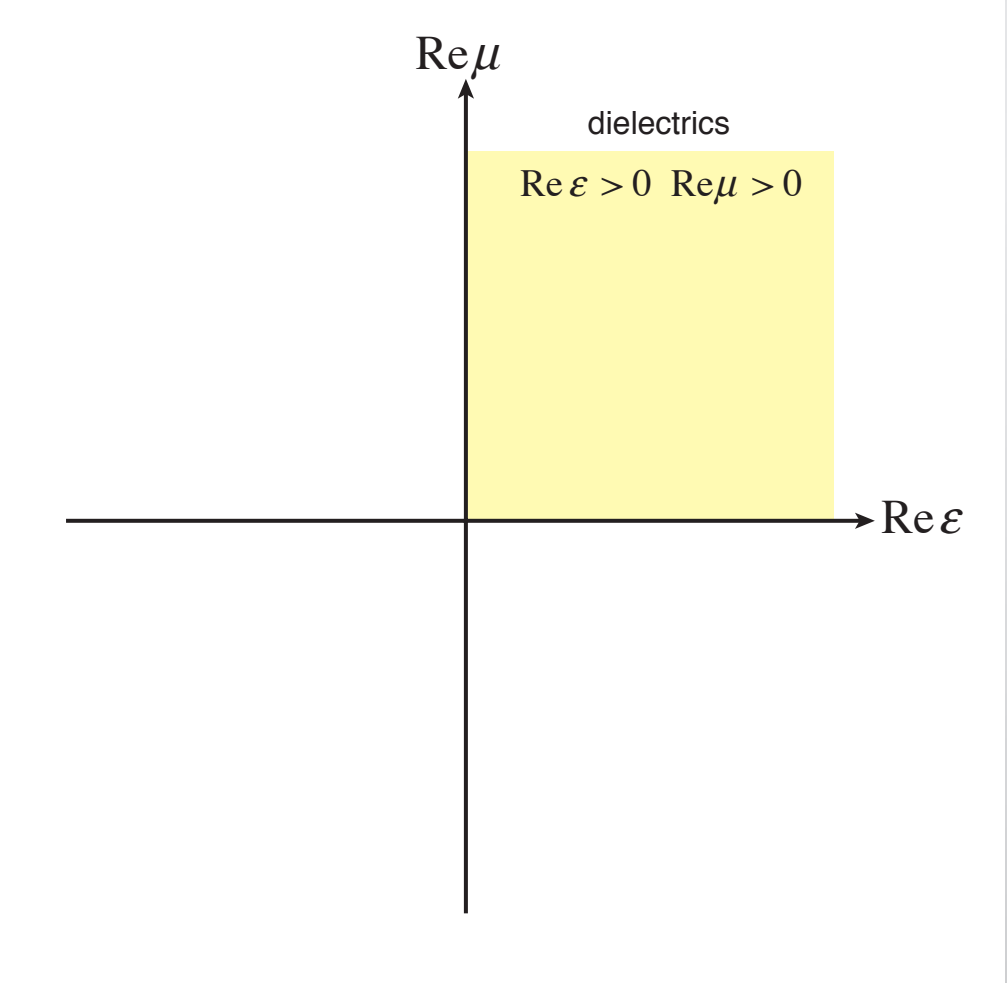

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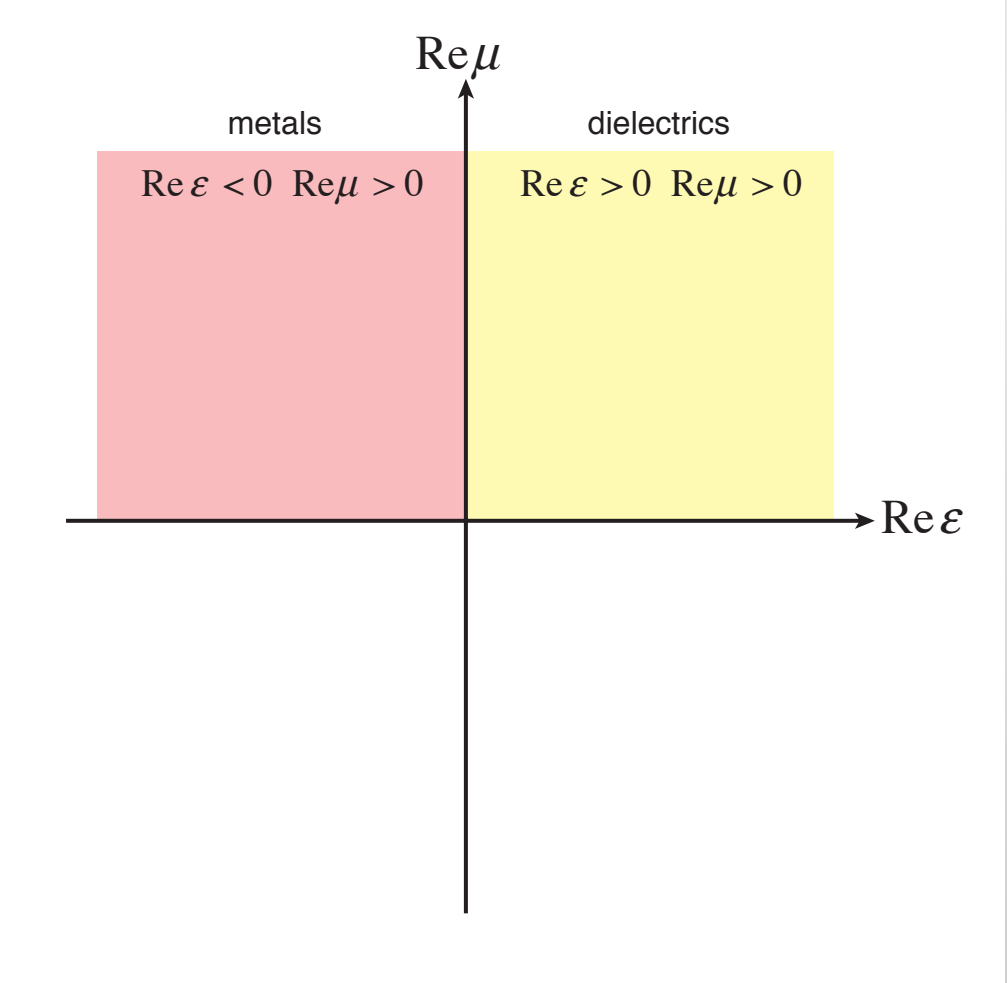

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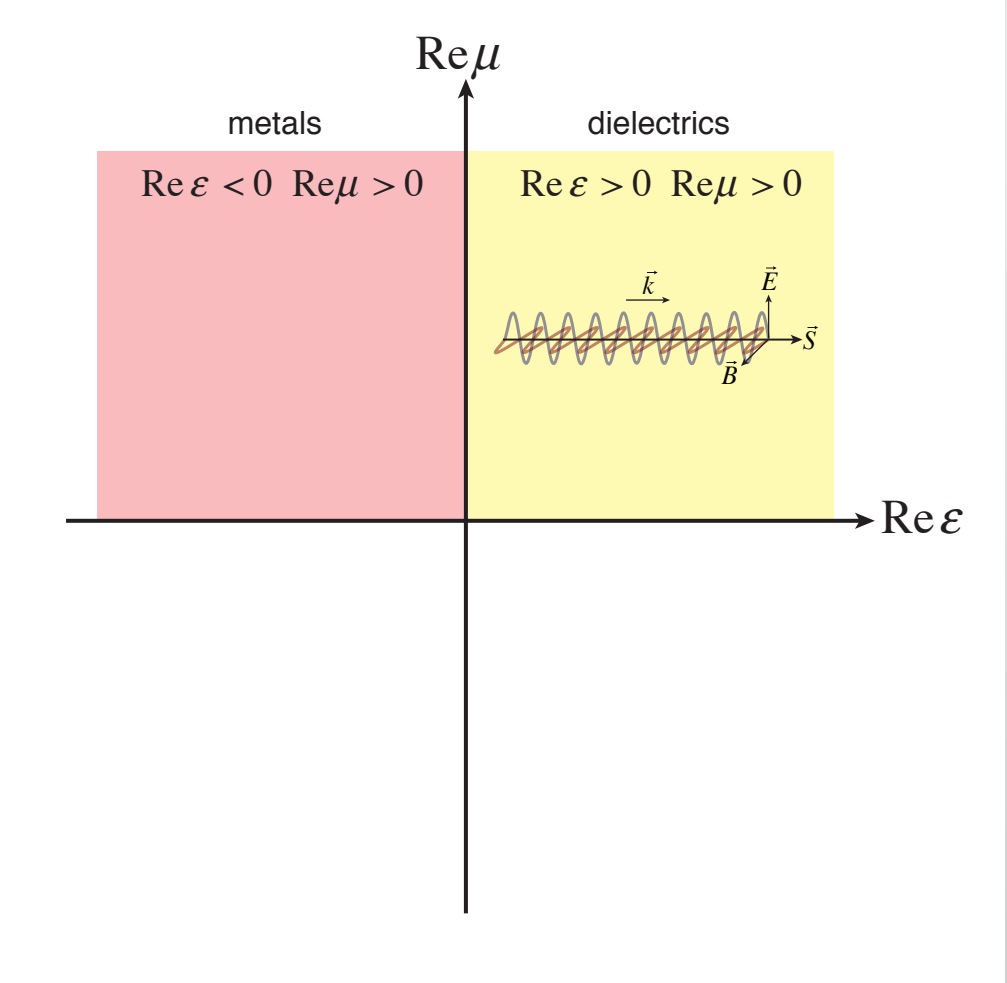

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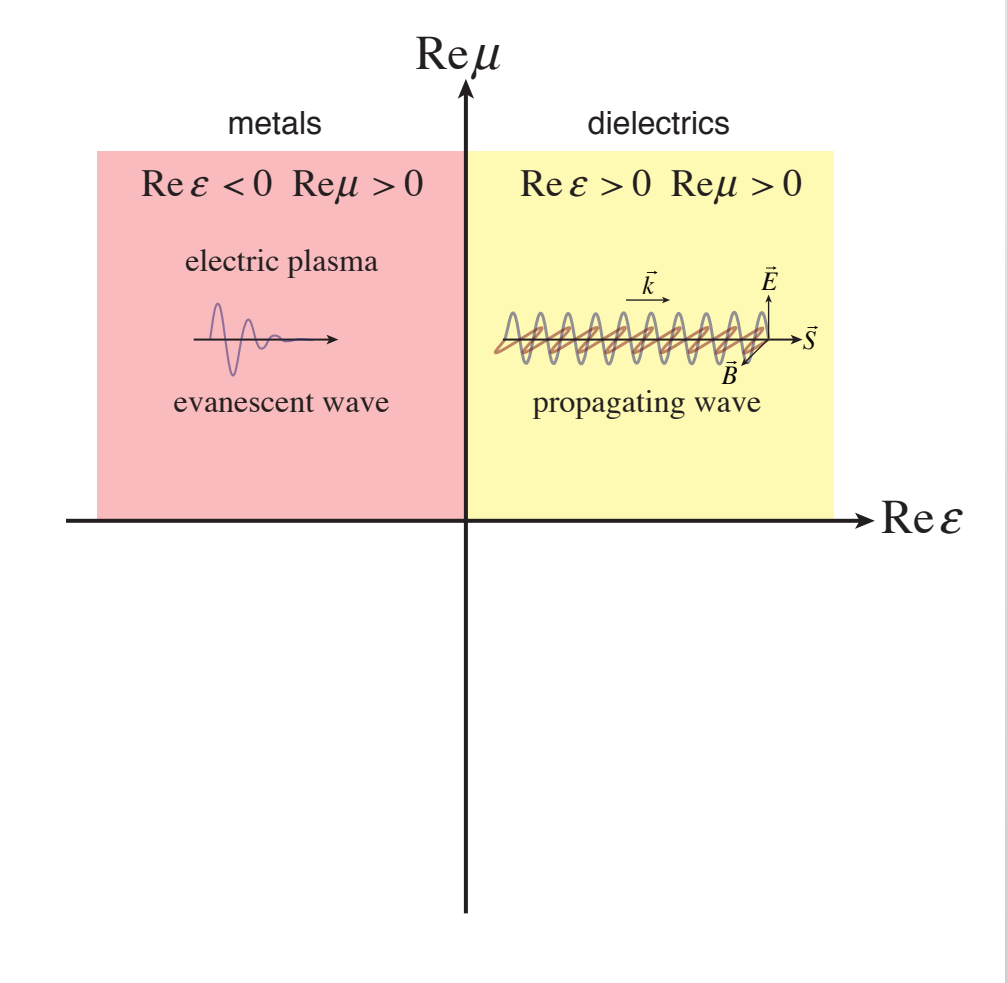

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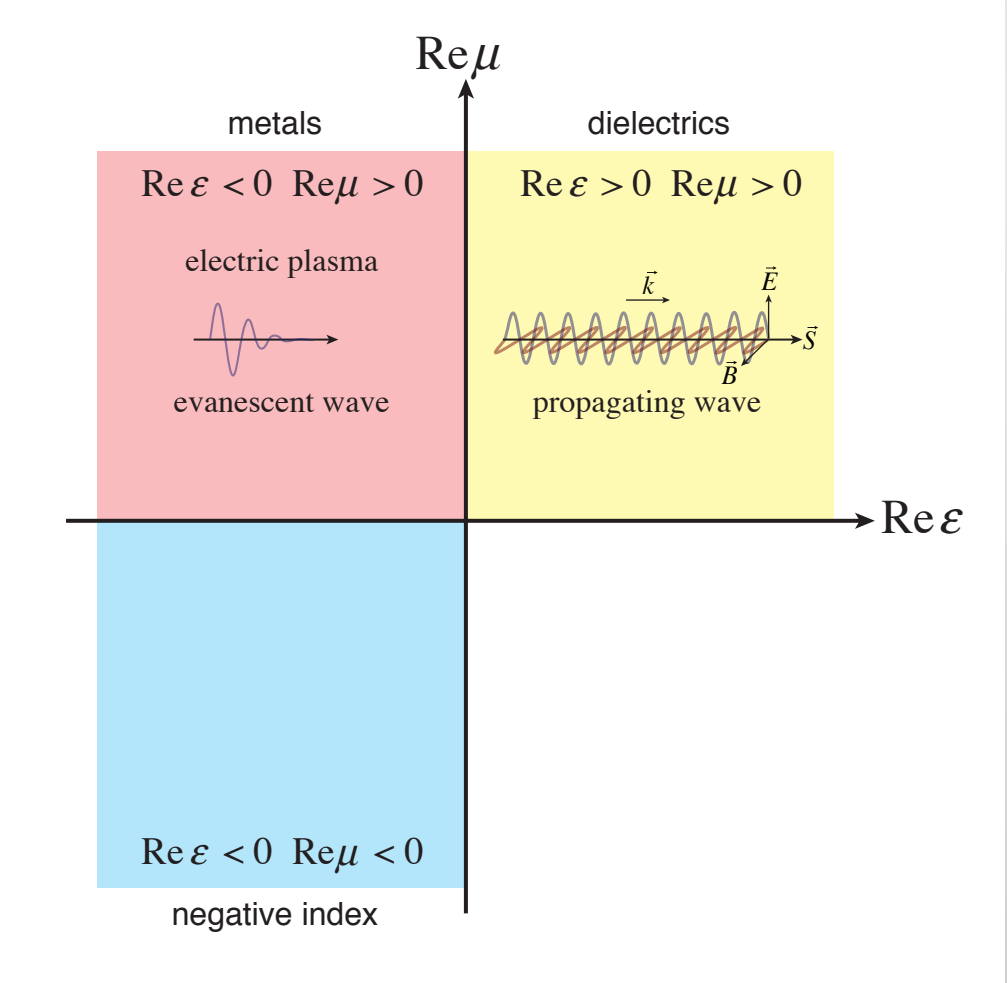

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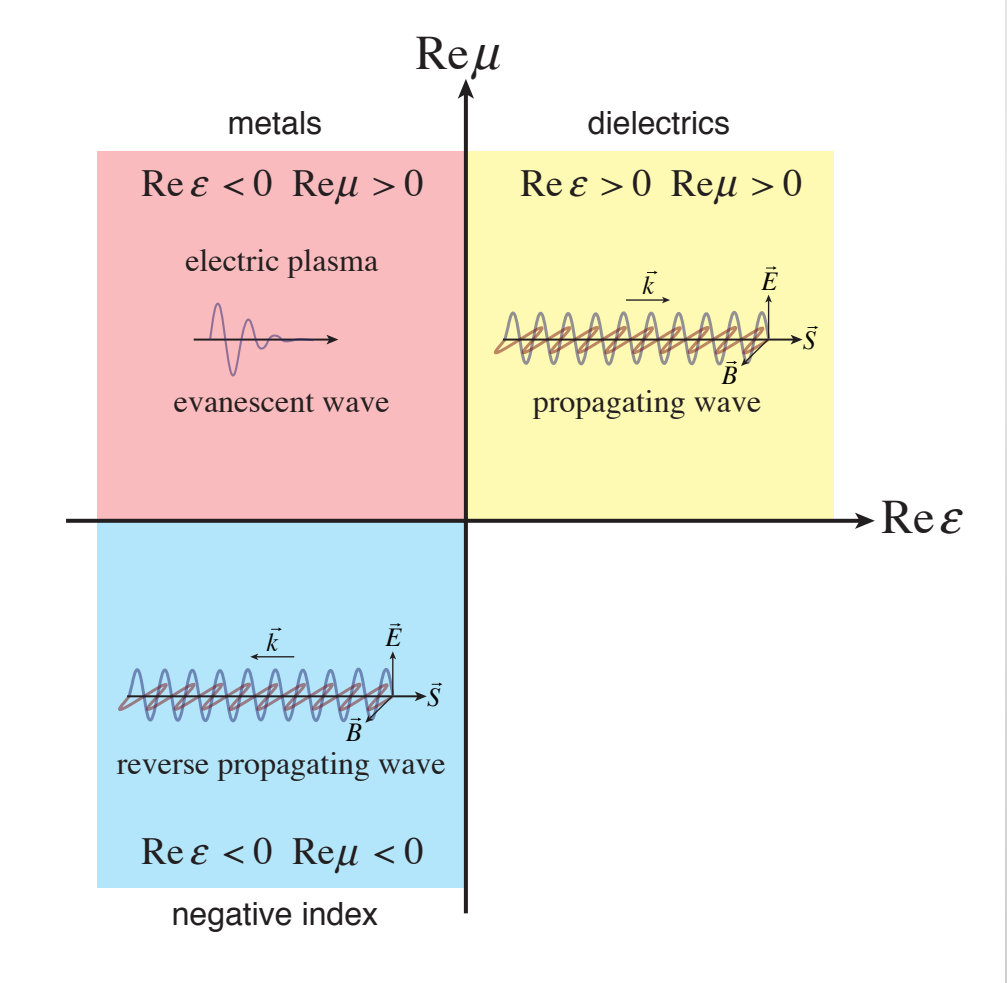

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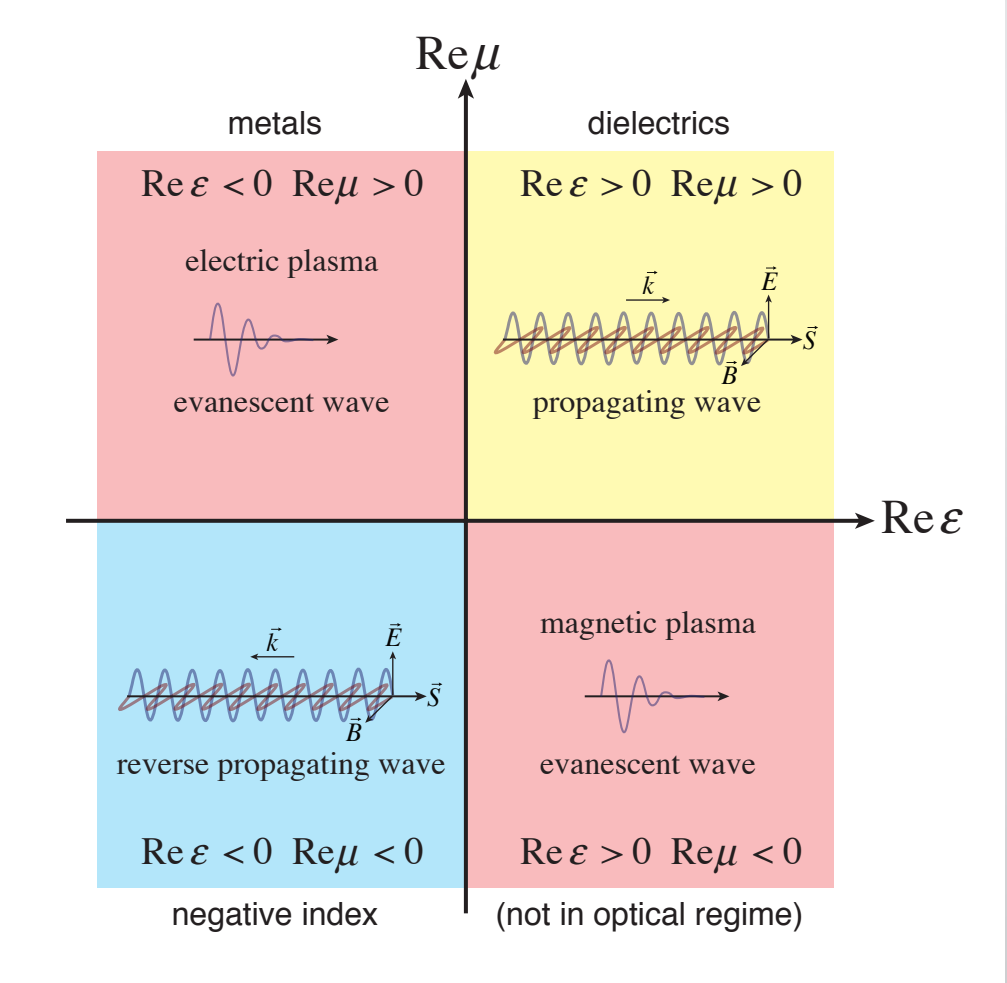
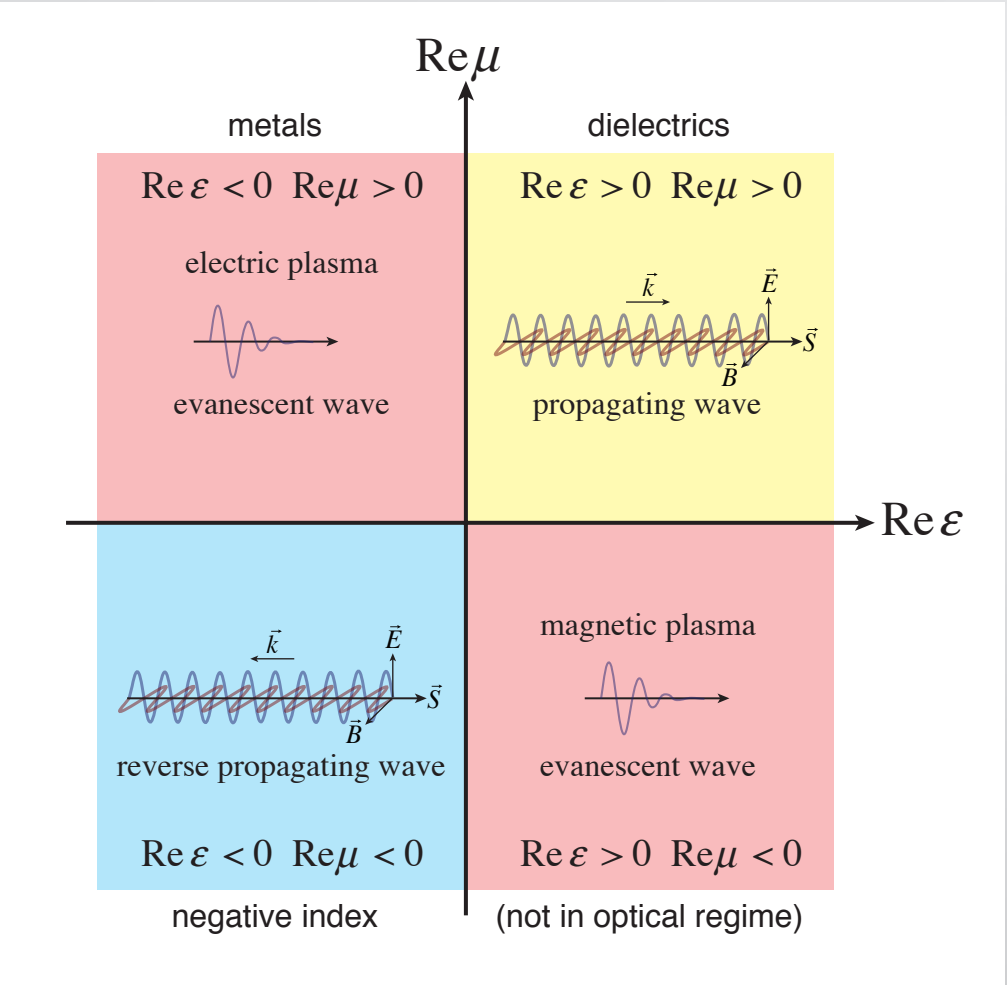


common materials very limited



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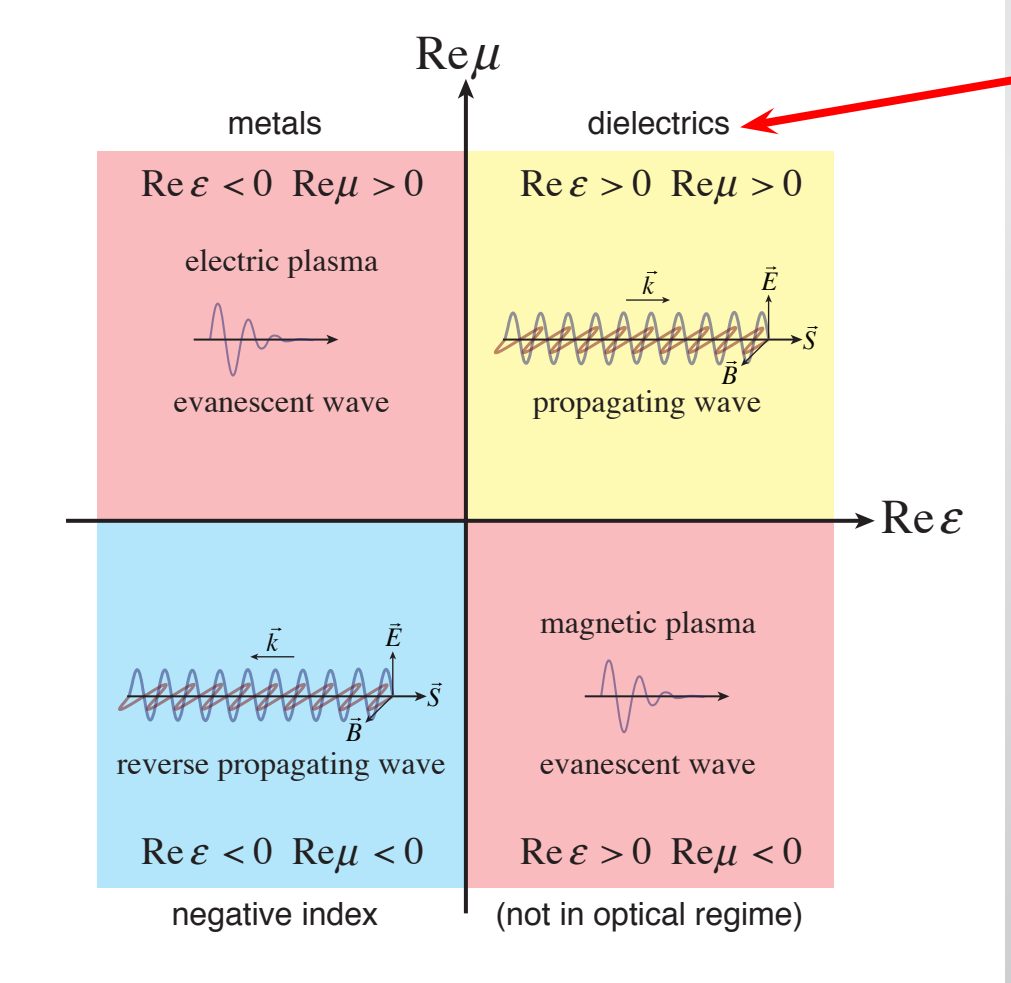
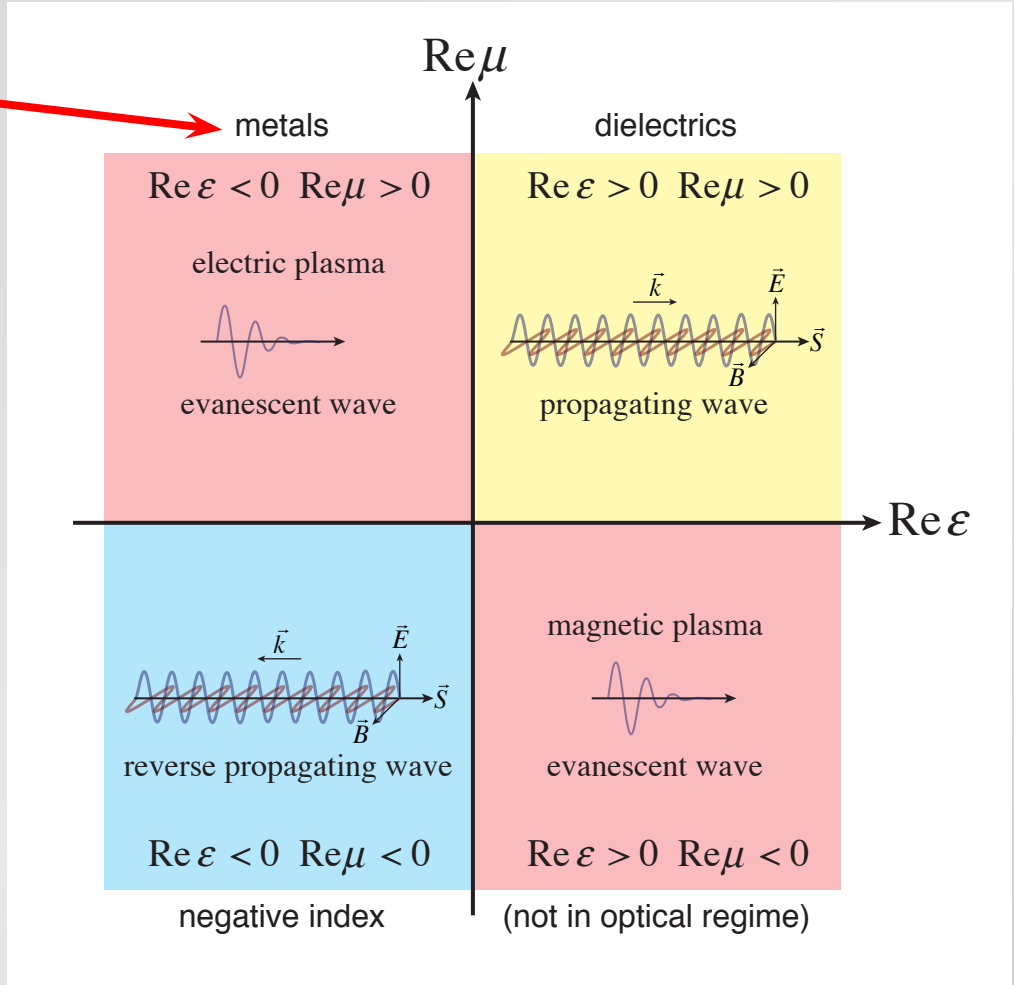


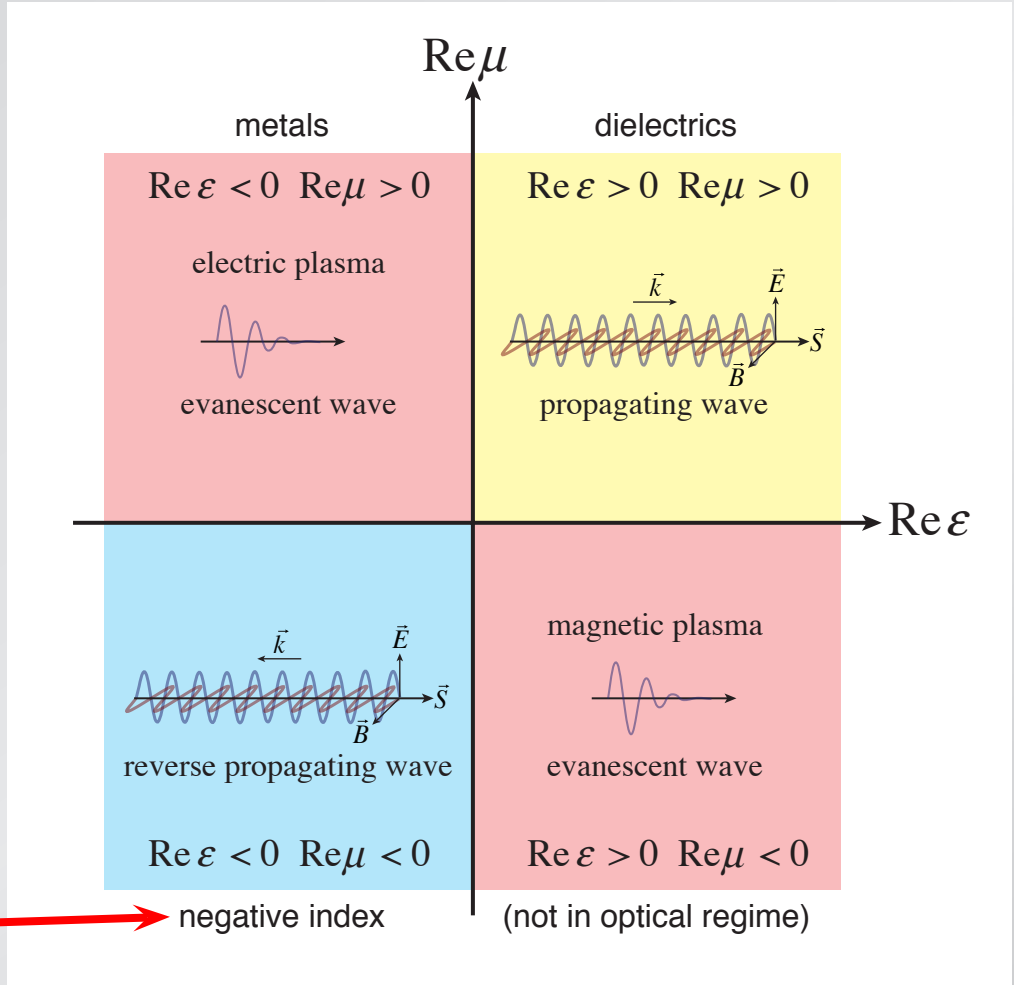
limited by diffraction

common materials very limited

lossy & no propagation

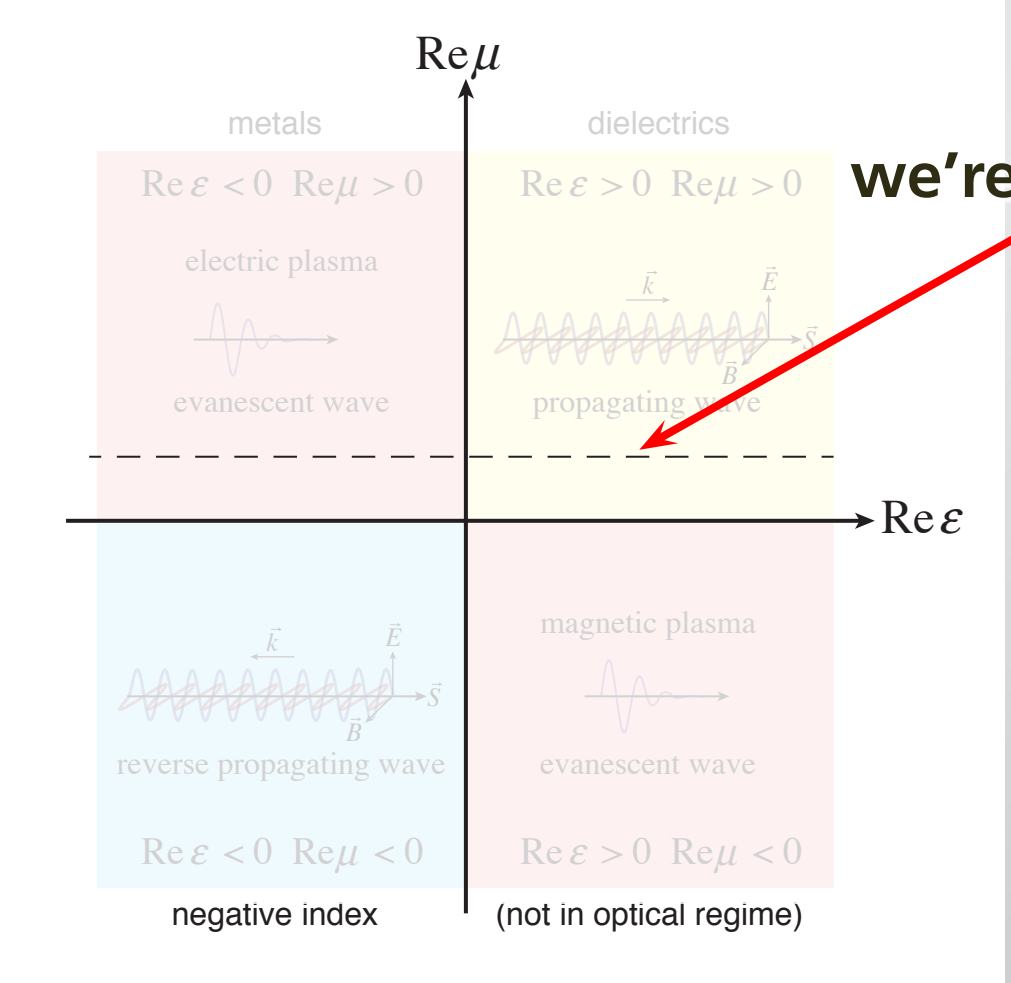


common materials very limited

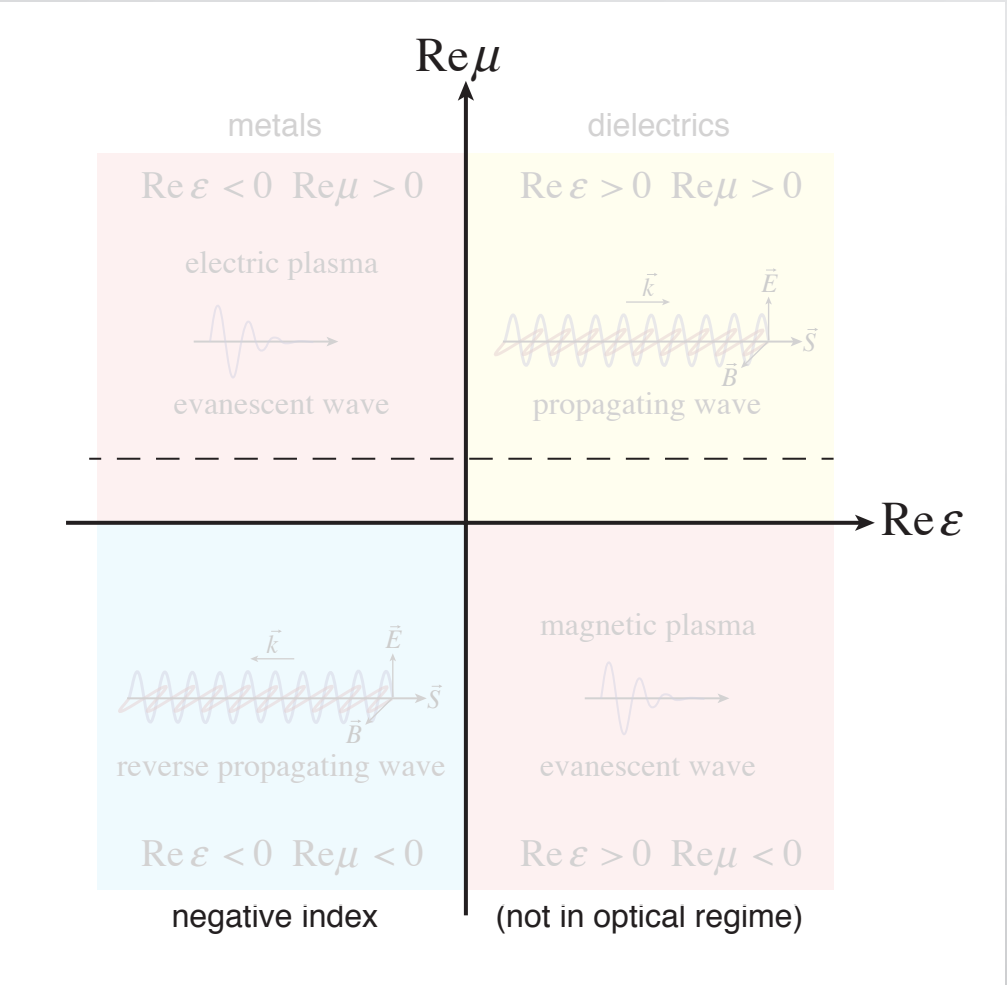


superlensing but…

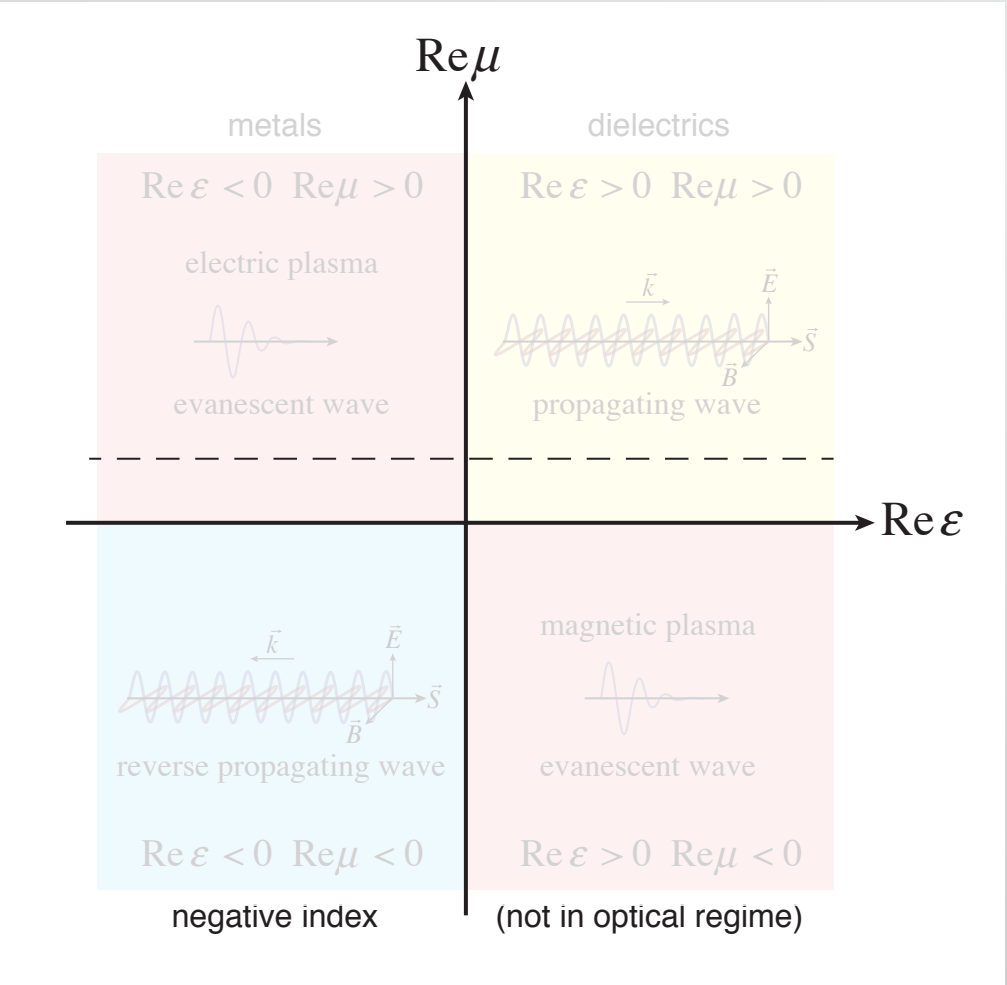
common materials very limited



What happens on the axes?



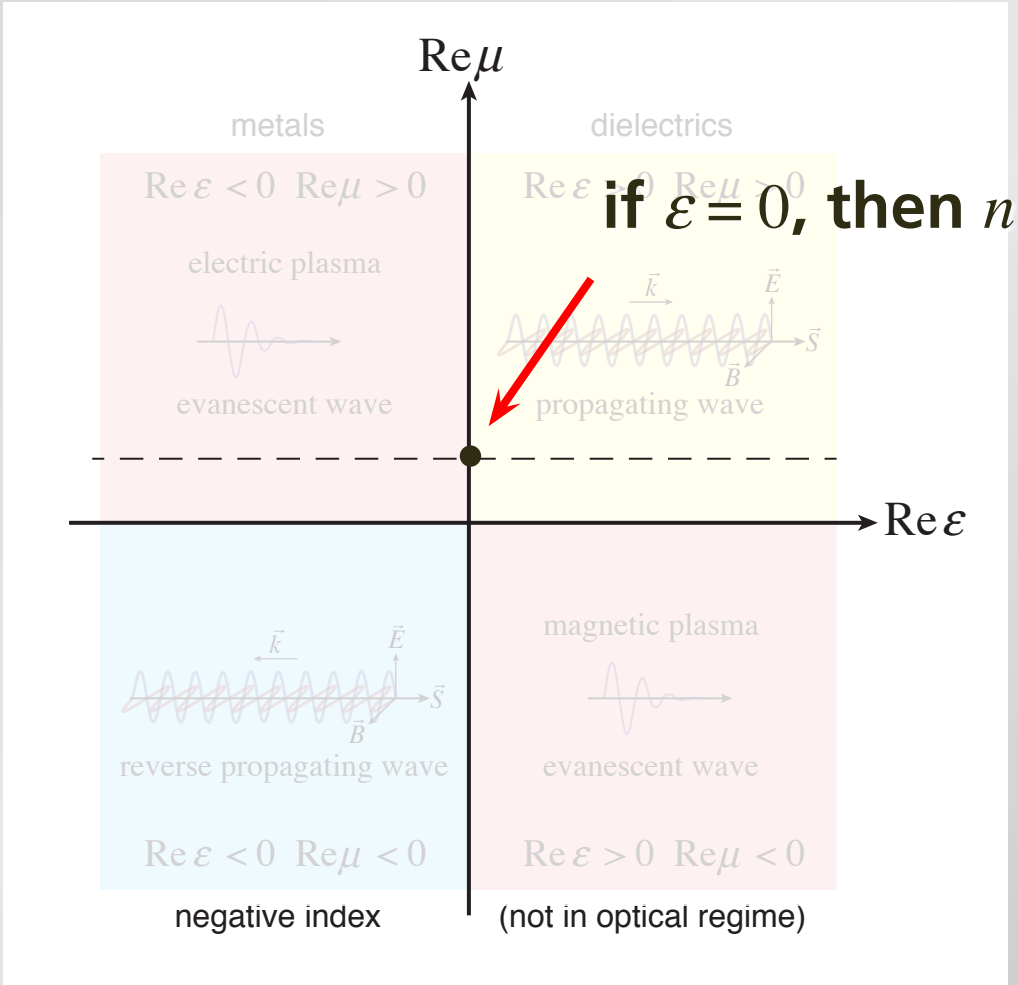
what if we let $\varepsilon = 0$?



1 index

2 zero index

what if we let $\varepsilon = 0$?



if $\varepsilon = 0$, then $n = 0$!

1 index

2 zero index

Q: If $n = 0$, which of the following is true?

1. the frequency goes to zero.
2. the phase velocity becomes infinite.
3. both of the above.
4. neither of the above.

1 index 2 zero index

wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

solution

$$\vec{E} = \vec{E}_o \, e^{i(kx - \omega t)}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}} c = \frac{1}{n} c$$

wave equation

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1 index

2 zero index

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solution

$$\vec{E} = \vec{E}_o \, e^{i(kx - \omega t)} \longrightarrow \vec{E} = \vec{E}_o \, e^{-i\omega t}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon \mu}} c = \frac{1}{n} c$$

1 index

2 zero index

wave equation

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$$\vec{E} = \vec{E}_o \, e^{i(kx - \omega t)} \longrightarrow \vec{E} = \vec{E}_o \, e^{-i\omega t}$$

where

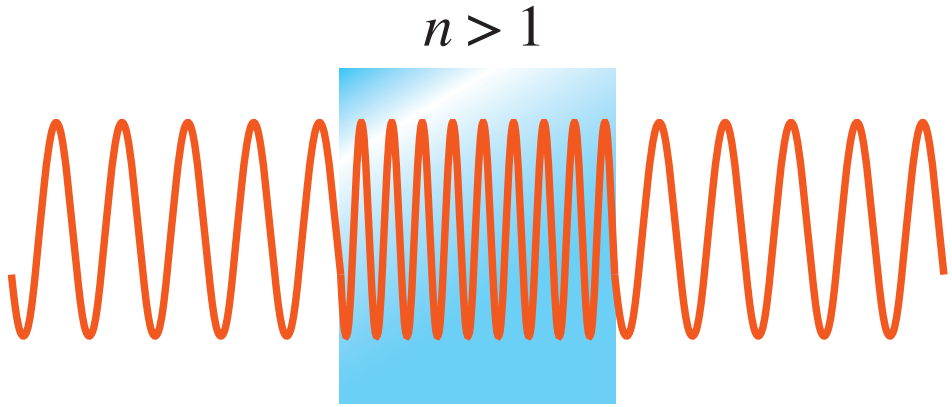
$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon \mu}} c = \frac{1}{n} c \longrightarrow \infty$$

1 index

2 zero index

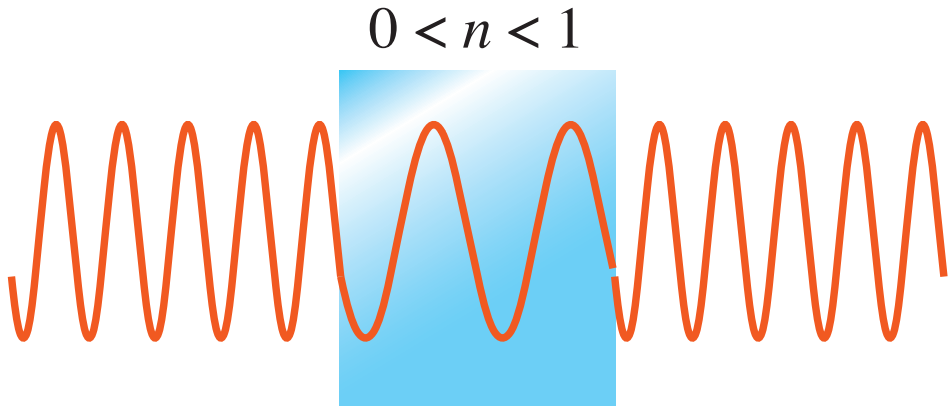
Q: If $n = 0$, which of the following is true?

1. the frequency goes to zero.
2. **the phase velocity becomes infinite.** ✔
3. both of the above.
4. neither of the above.



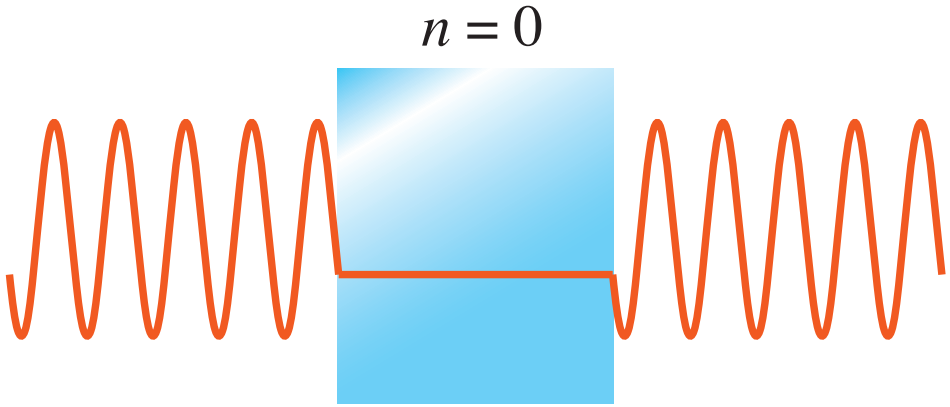
1 index

2 zero index



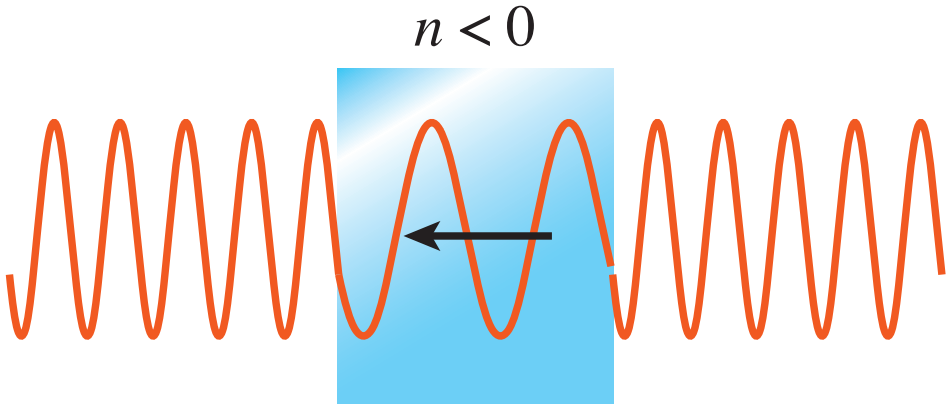
1 index

2 zero index



1 index

2 zero index

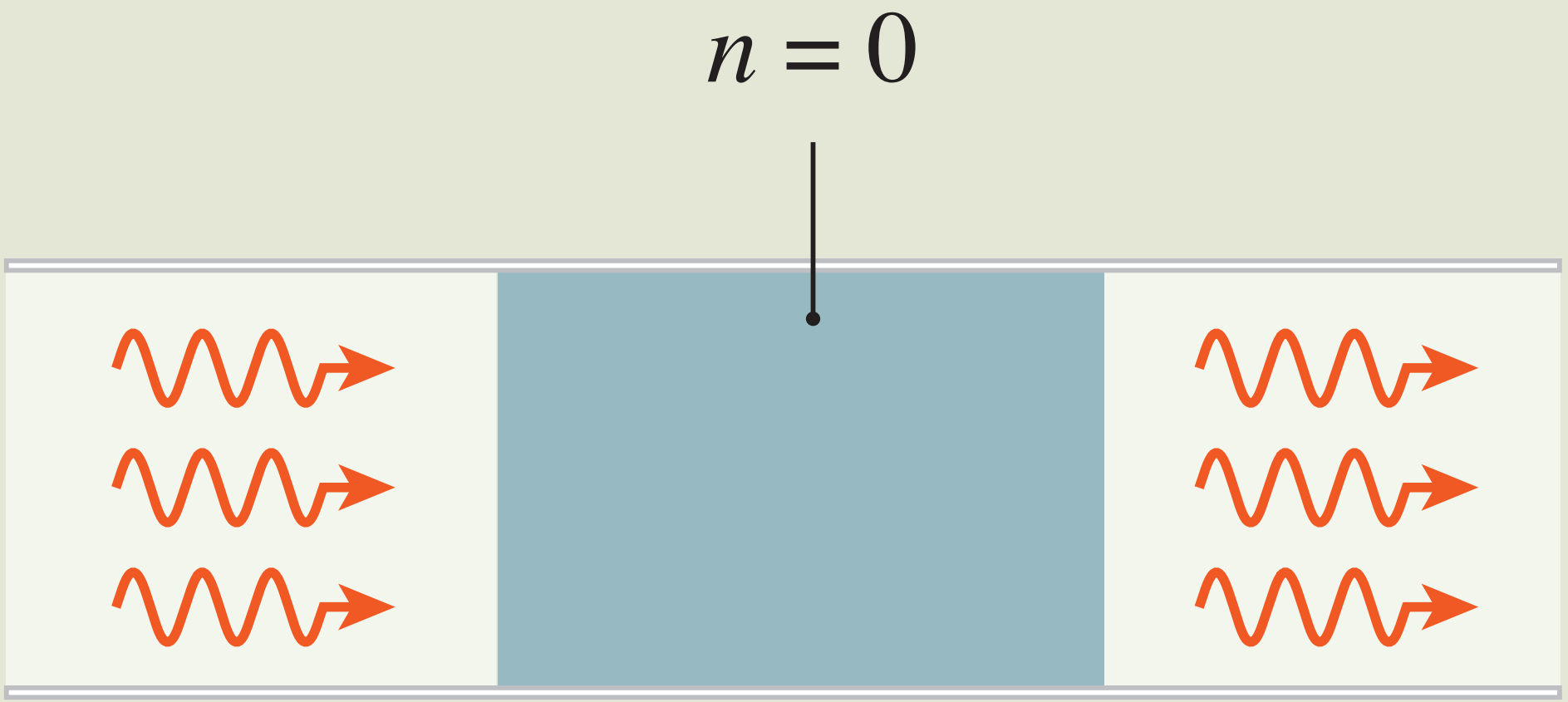


1 index

2 zero index

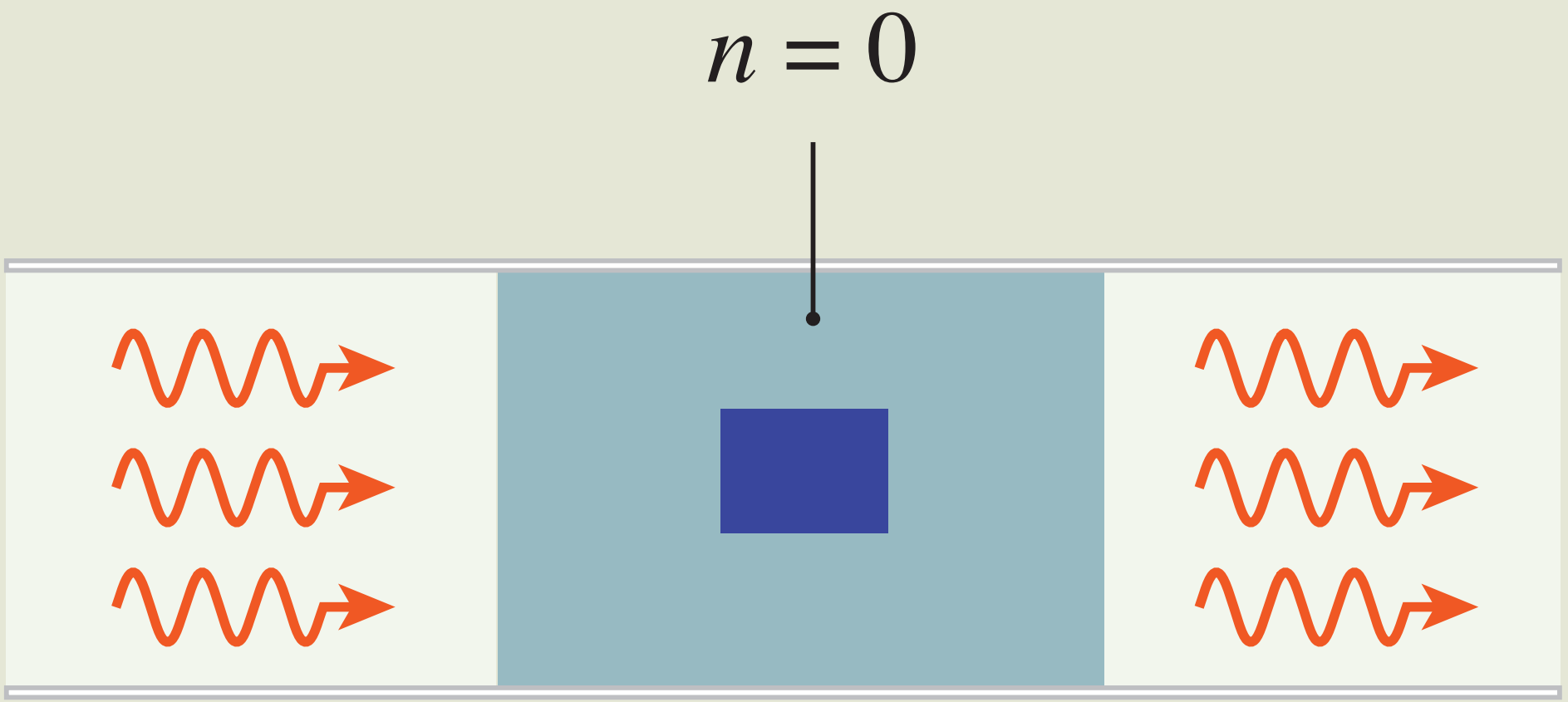
1 index

2 zero index



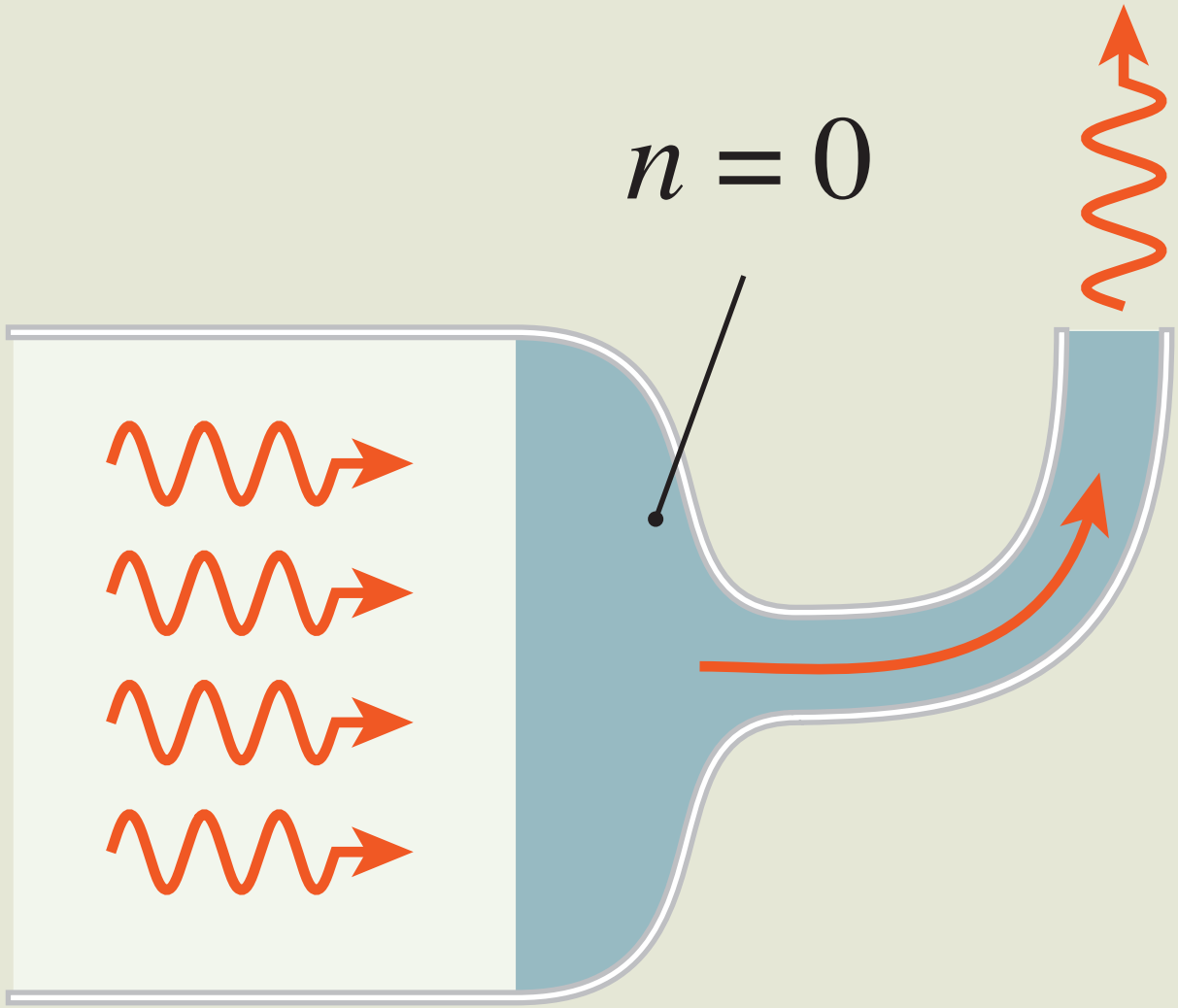
1 index

2 zero index



1 index

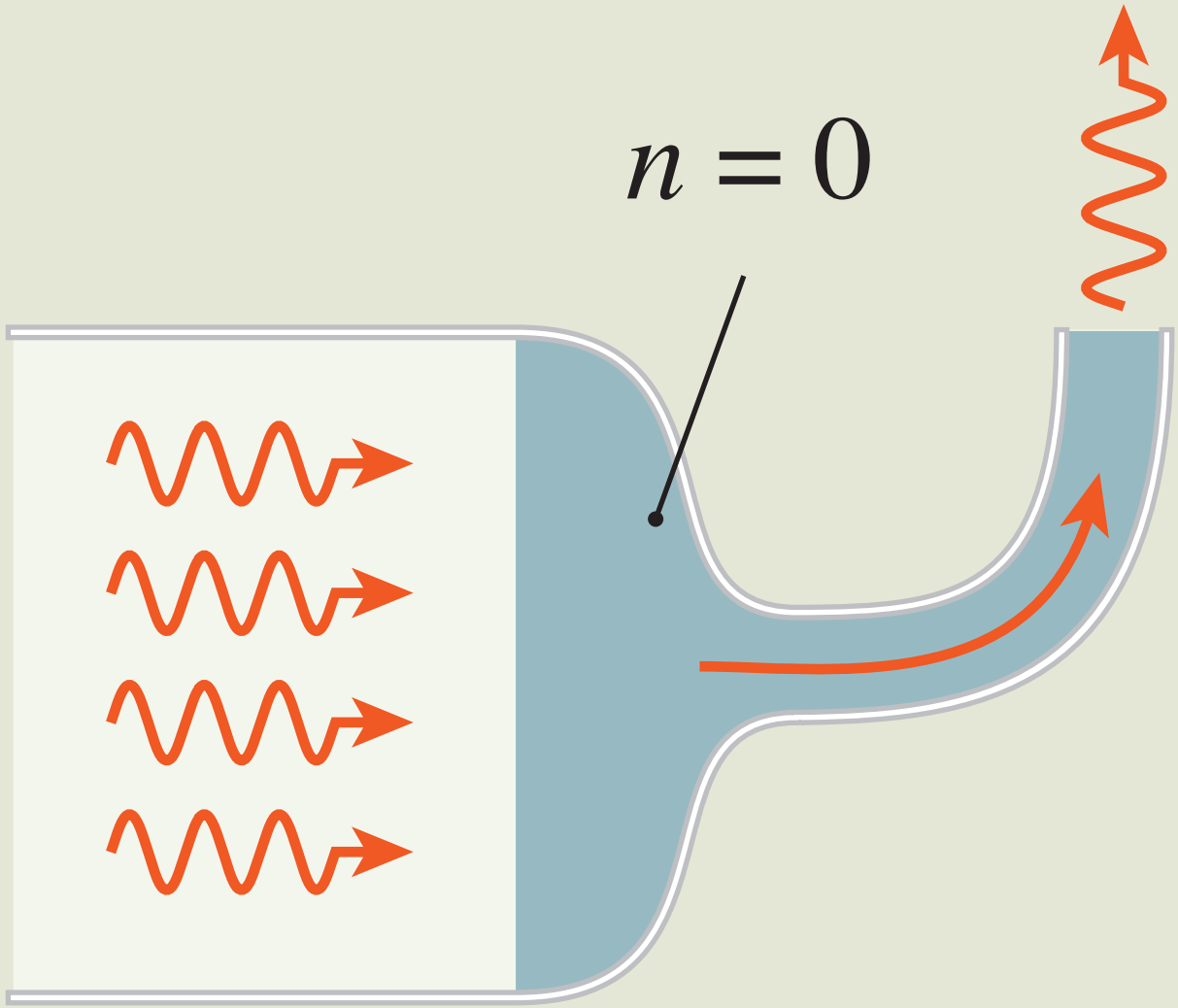
2 zero index



1 index

2 zero index

"tunneling with infinite decay length"



1 index

2 zero index

how?

$$n = \sqrt{\varepsilon\mu}$$

1 index

2 zero index

how?

$$n = \sqrt{\varepsilon\mu}$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

1 index

2 zero index

how?

$$n = \sqrt{\varepsilon\mu}$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

1 index

2 zero index

how?

$$\varepsilon \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

1 index

2 zero index

how?

$$\varepsilon \to 0 \qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to \infty$$

1 index

2 zero index

how?

$$\varepsilon \to 0 \qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \to 1$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to \infty$$

1 index

2 zero index

how?

$$\mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

1 index

2 zero index

how?

$$\mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to 0$$

1 index

2 zero index

how?

$$\mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \to -1$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to 0$$

1 index

2 zero index

how?

$$\varepsilon, \mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \;\to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where

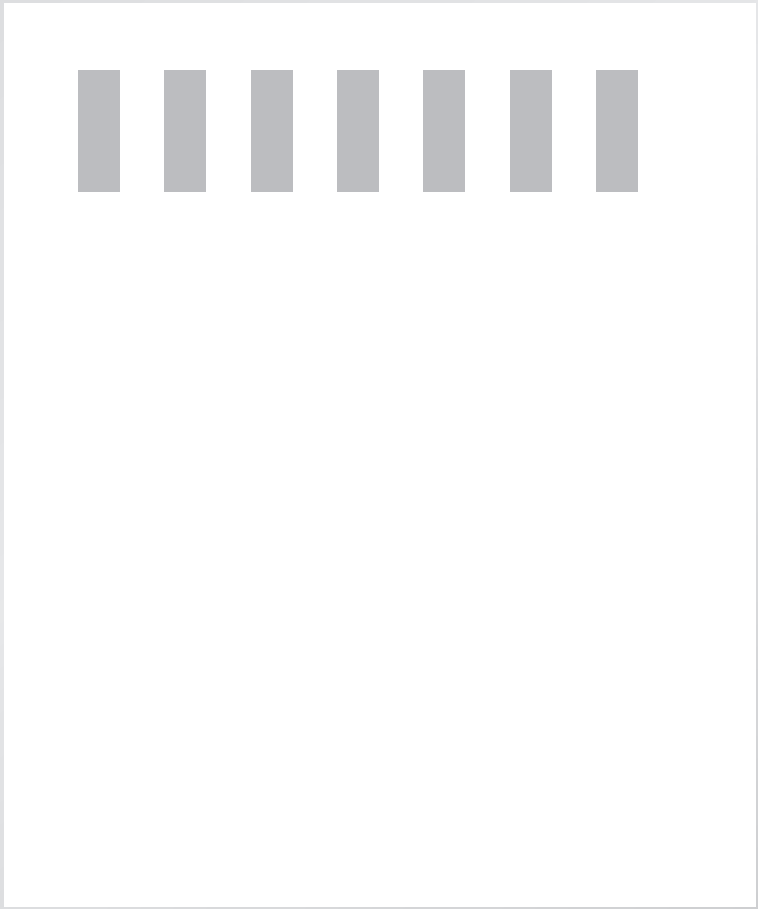
$$Z = \sqrt{\frac{\mu}{\varepsilon}} \qquad \textbf{finite!}$$

but $\mu \neq 1$ requires a magnetic response!

Engineering a magnetic response

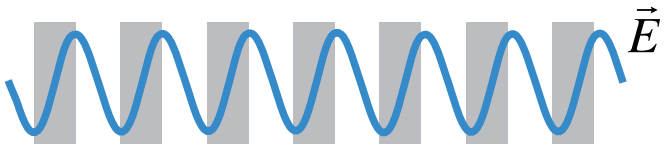
Engineering a magnetic response

use array of dielectric rods



Engineering a magnetic response

incident electromagnetic wave ($\lambda_{\text{eff}} \approx a$)

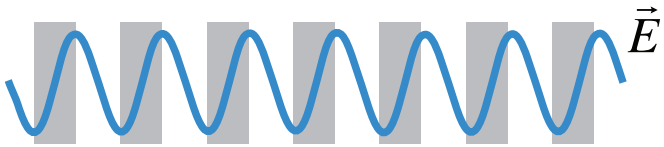


1 index

2 zero index

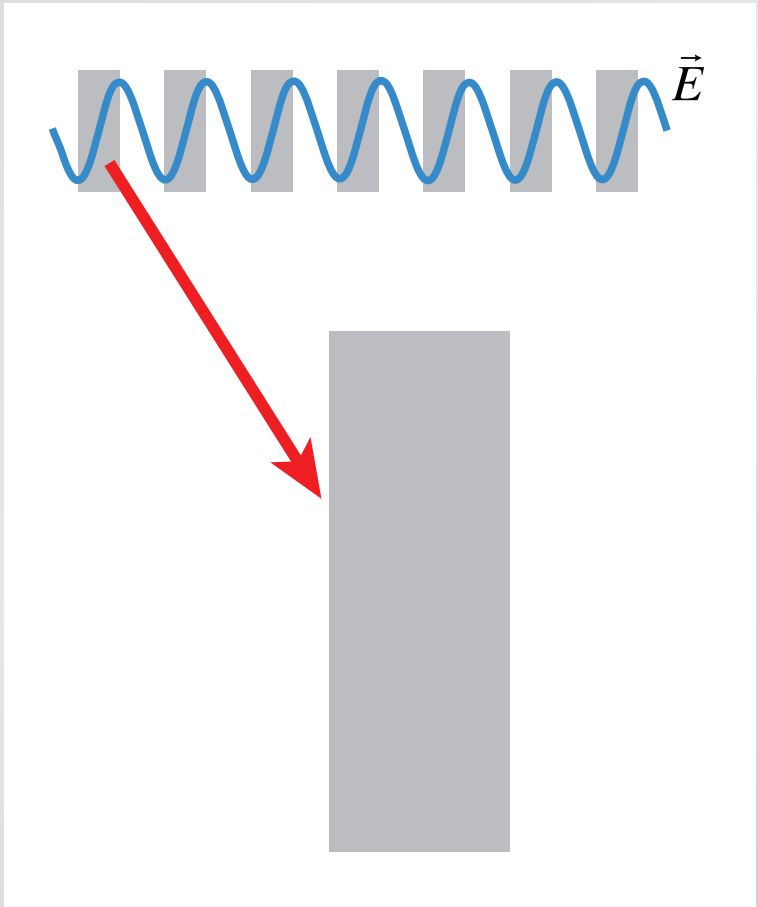
Engineering a magnetic response

produces an electric response…



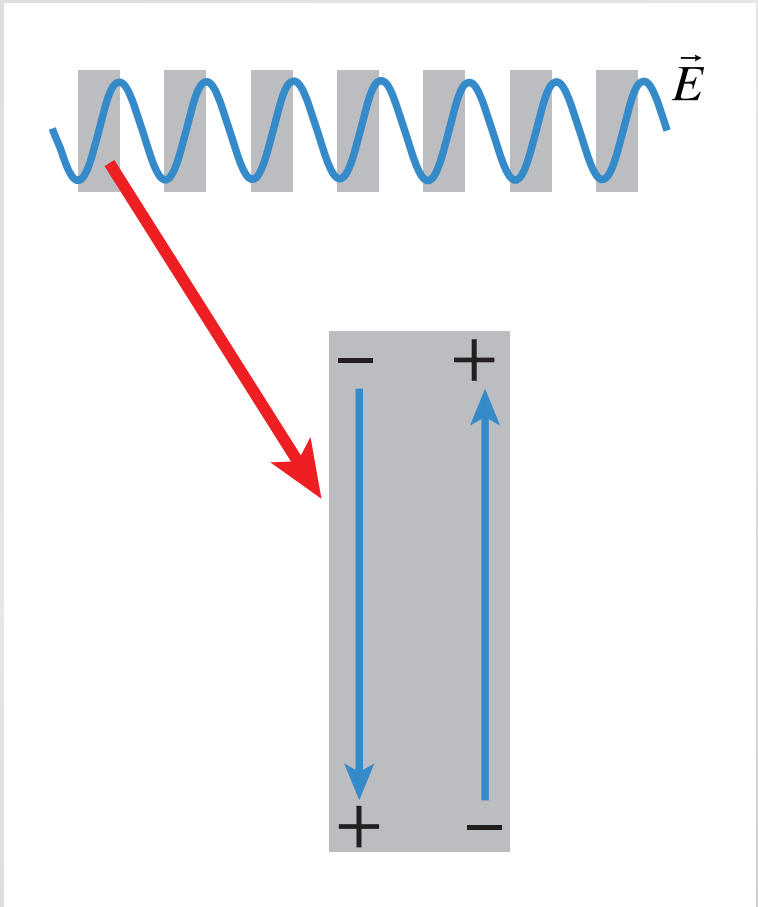
Engineering a magnetic response

… but different electric fields front and back…



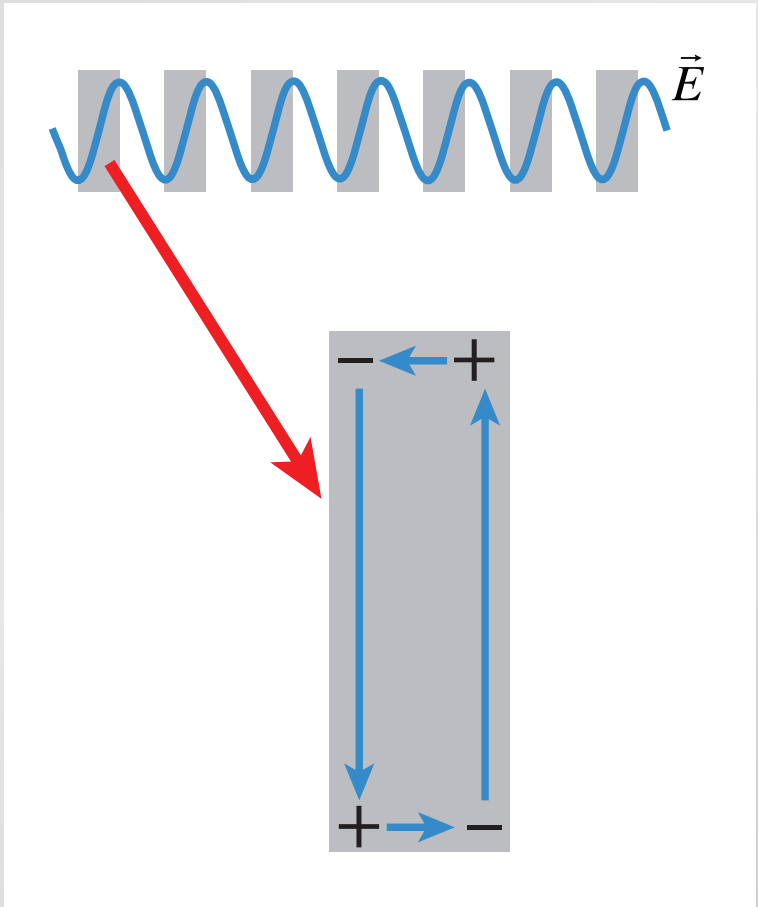
Engineering a magnetic response

…induce different polarizations on opposite sides…



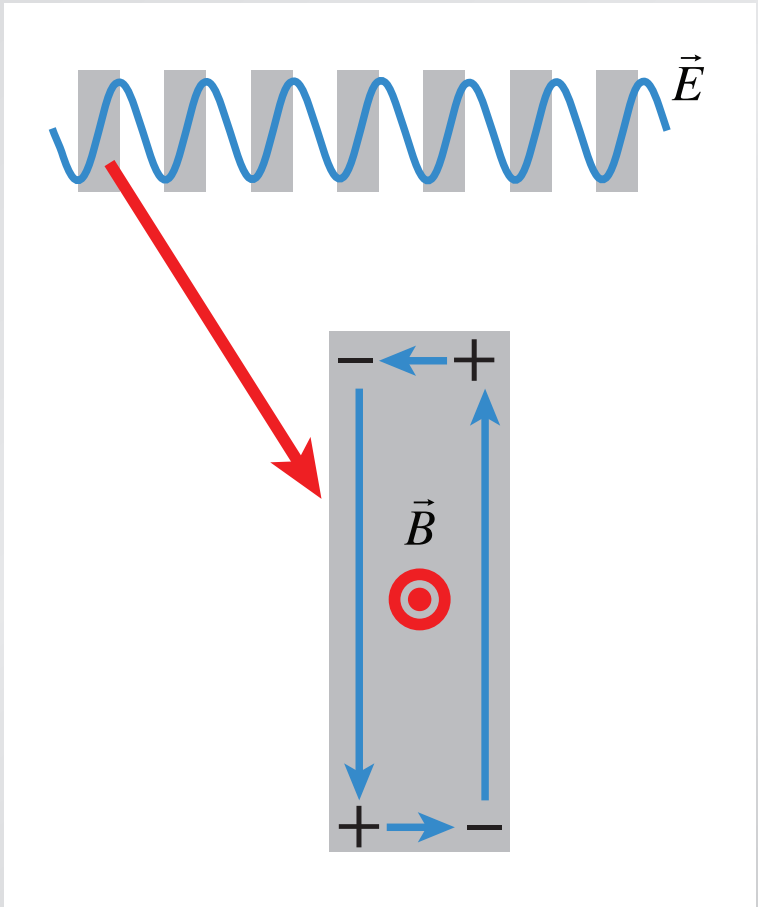
Engineering a magnetic response

…causing a current loop…



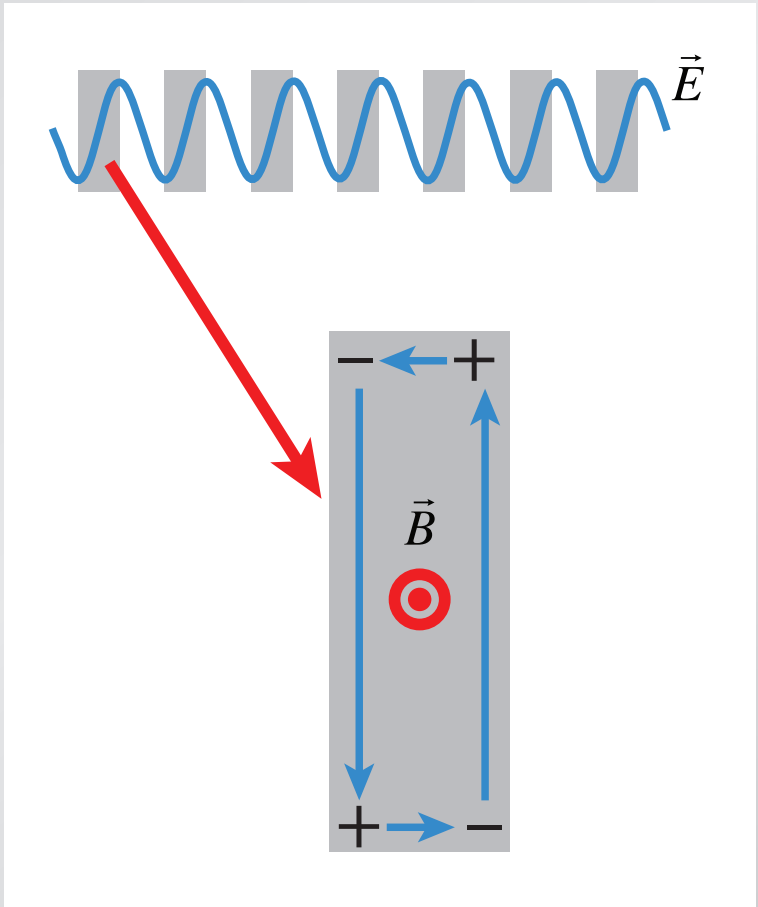
Engineering a magnetic response

…which, in turn, produces an induced magnetic field



Engineering a magnetic response

adjust design so electrical and magnetic resonances coincide



Engineering a magnetic response

adjustable parameters

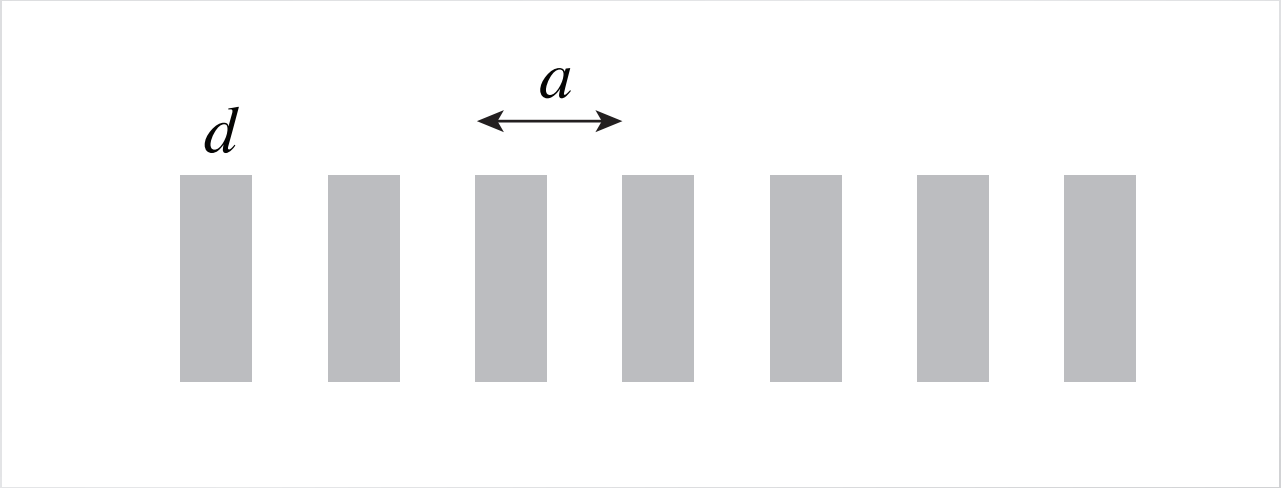
1 index 2 zero index 3 experiments

Engineering a magnetic response

adjustable parameters

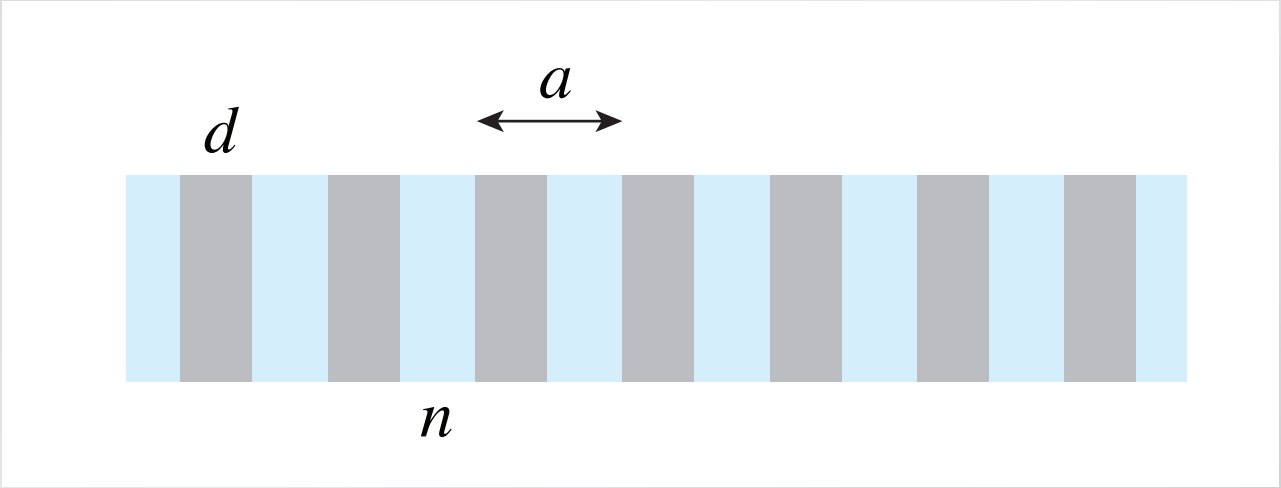
Engineering a magnetic response

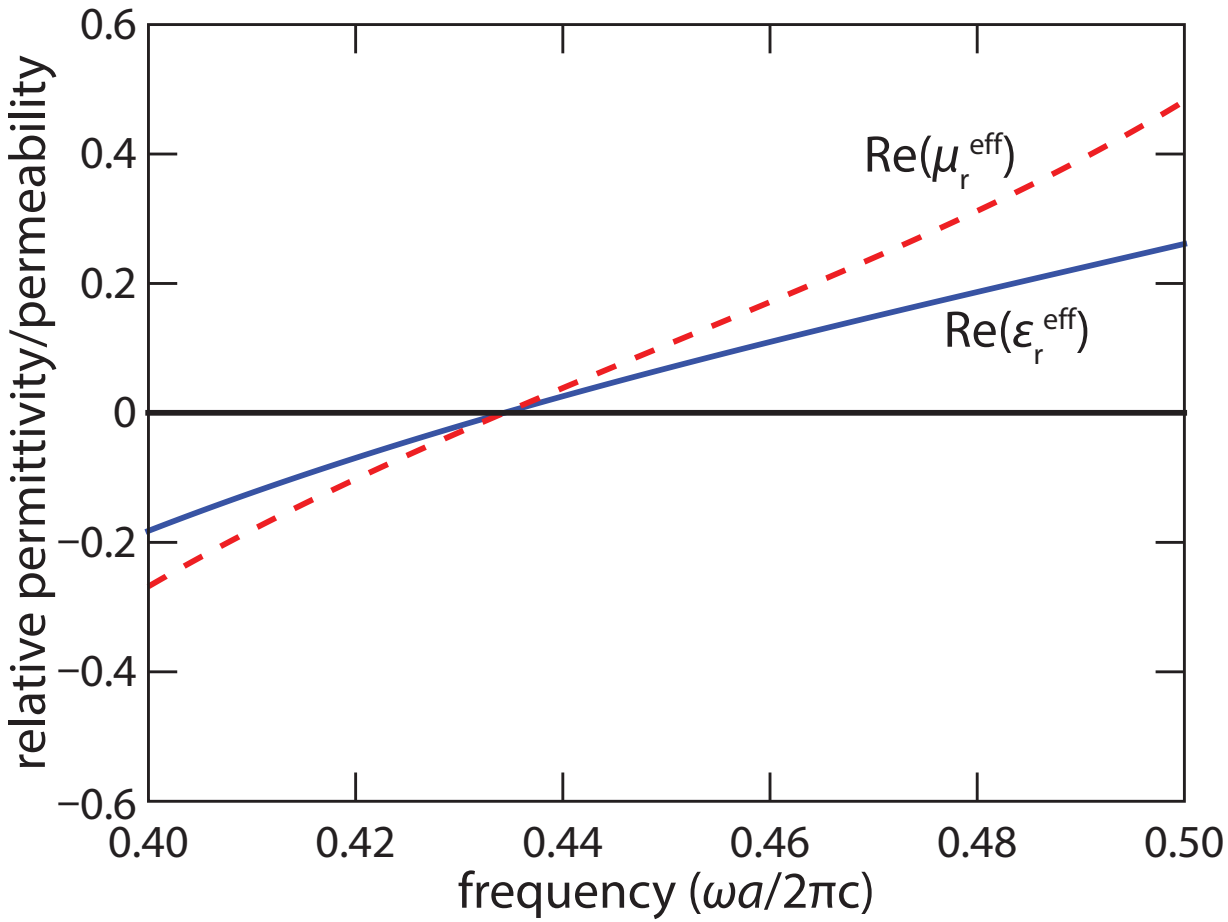
adjustable parameters



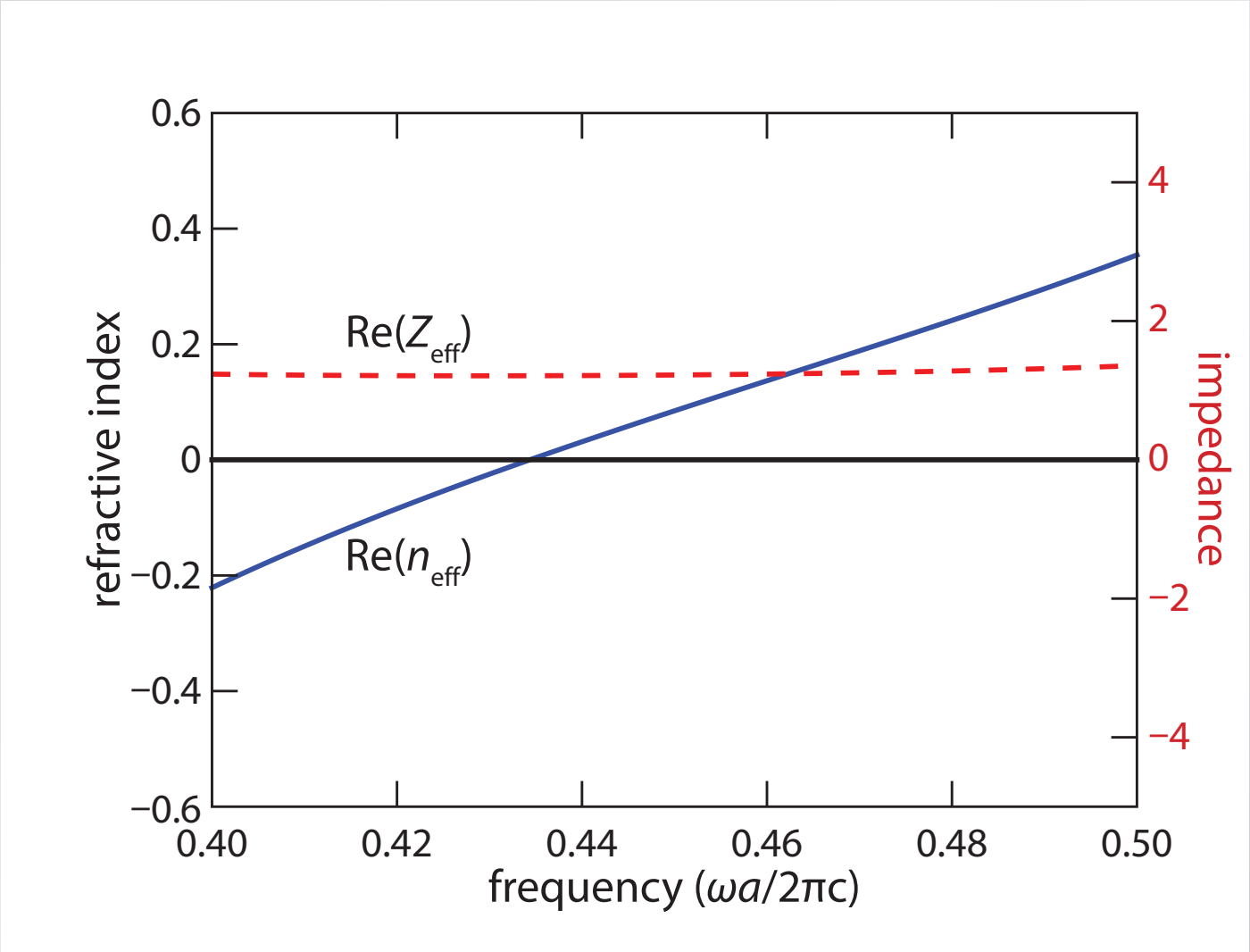
Engineering a magnetic response

adjustable parameters



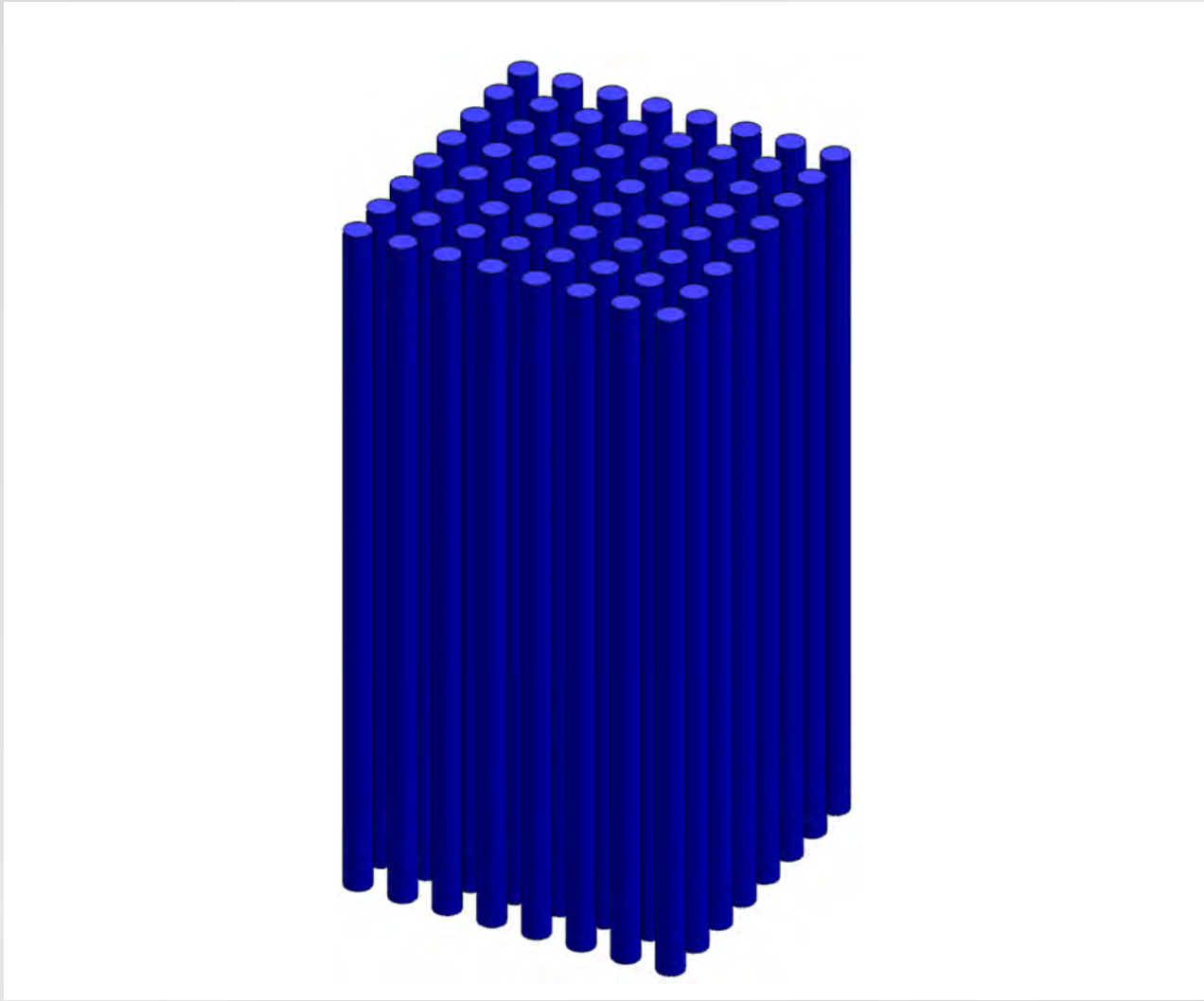


1 index 2 zero index 3 experiments



1 index 2 zero index 3 experiments

How to fabricate?



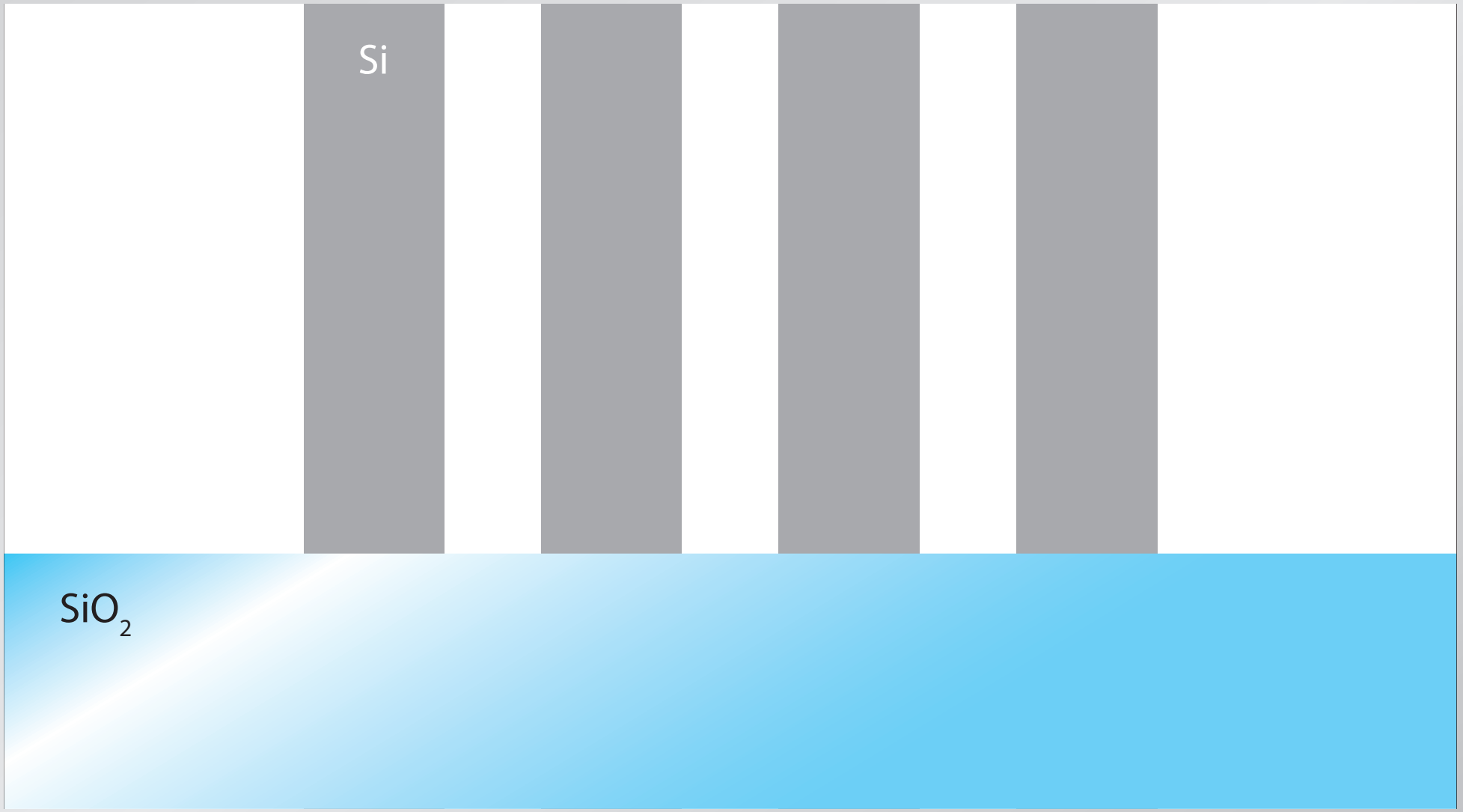
1 index

2 zero index

3 experiments

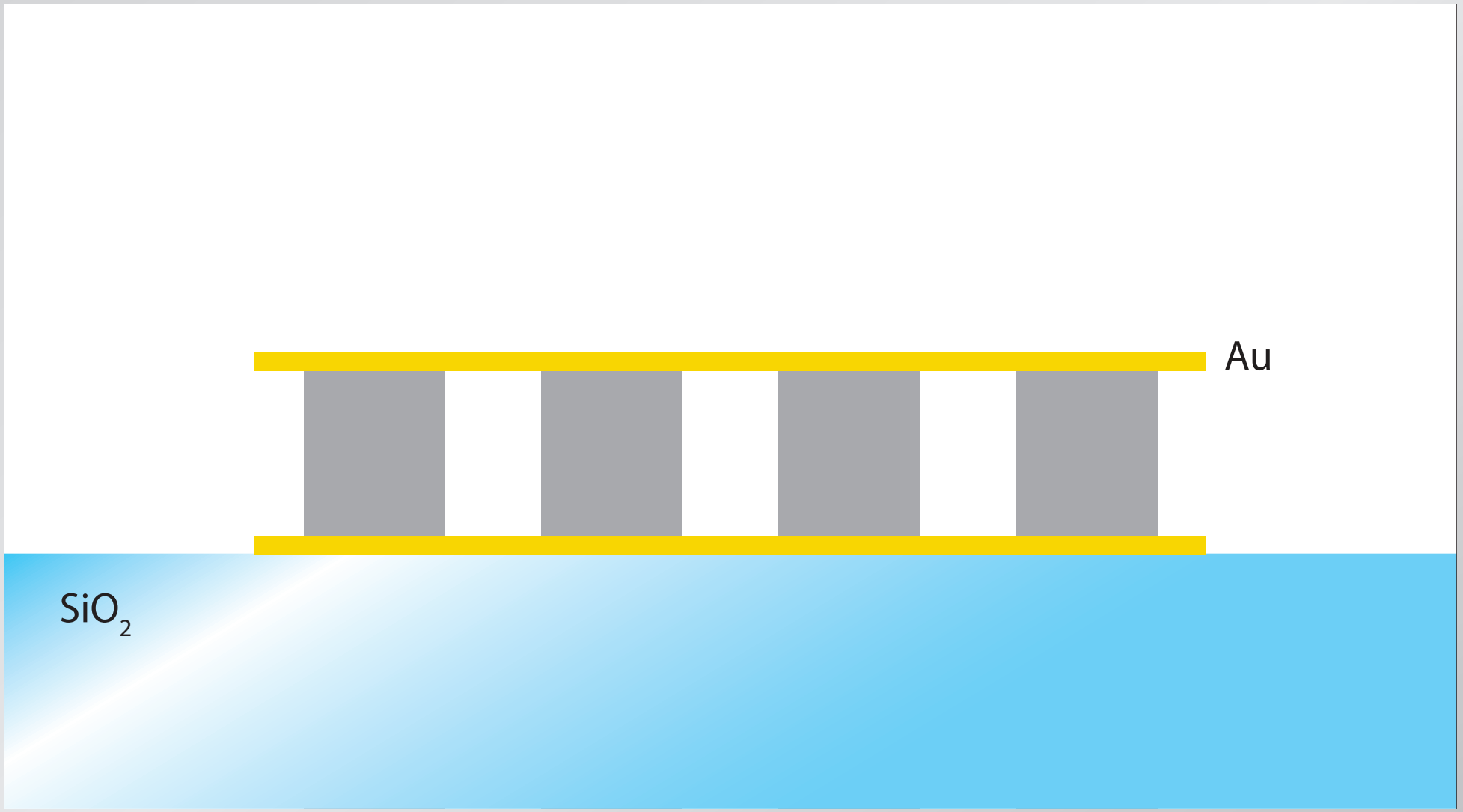
On-chip zero-index fabrication

On-chip zero-index fabrication

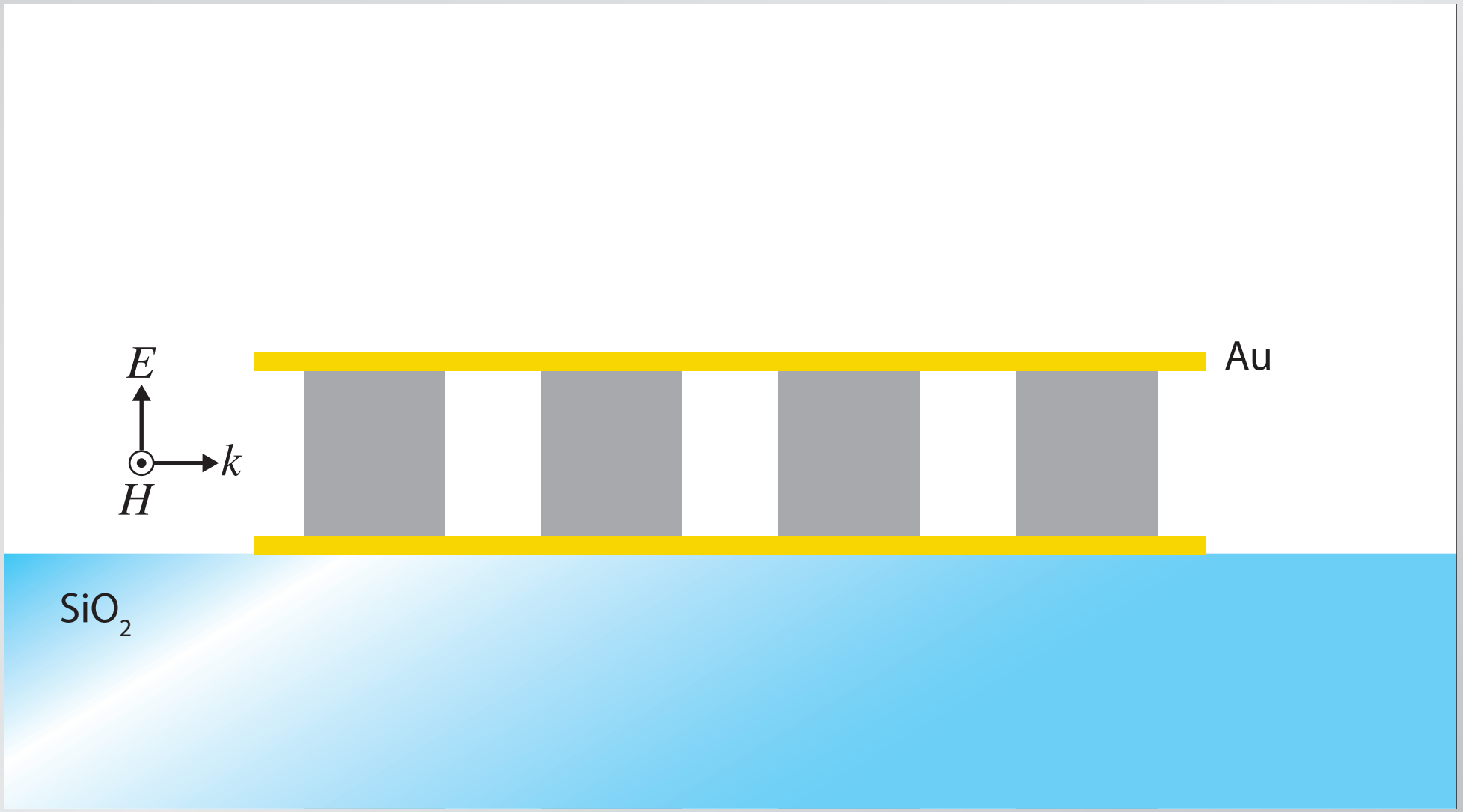


On-chip zero-index fabrication

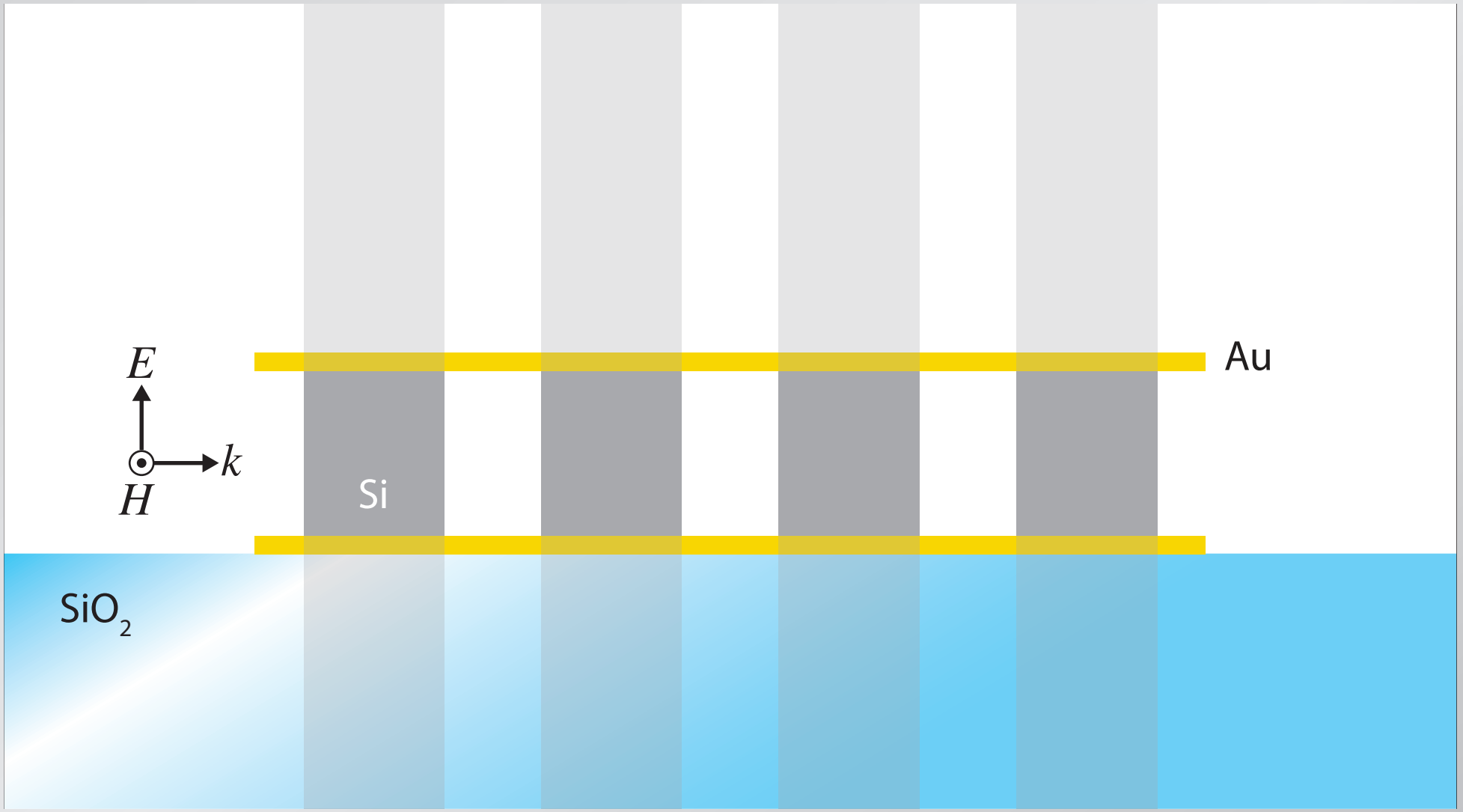
On-chip zero-index fabrication



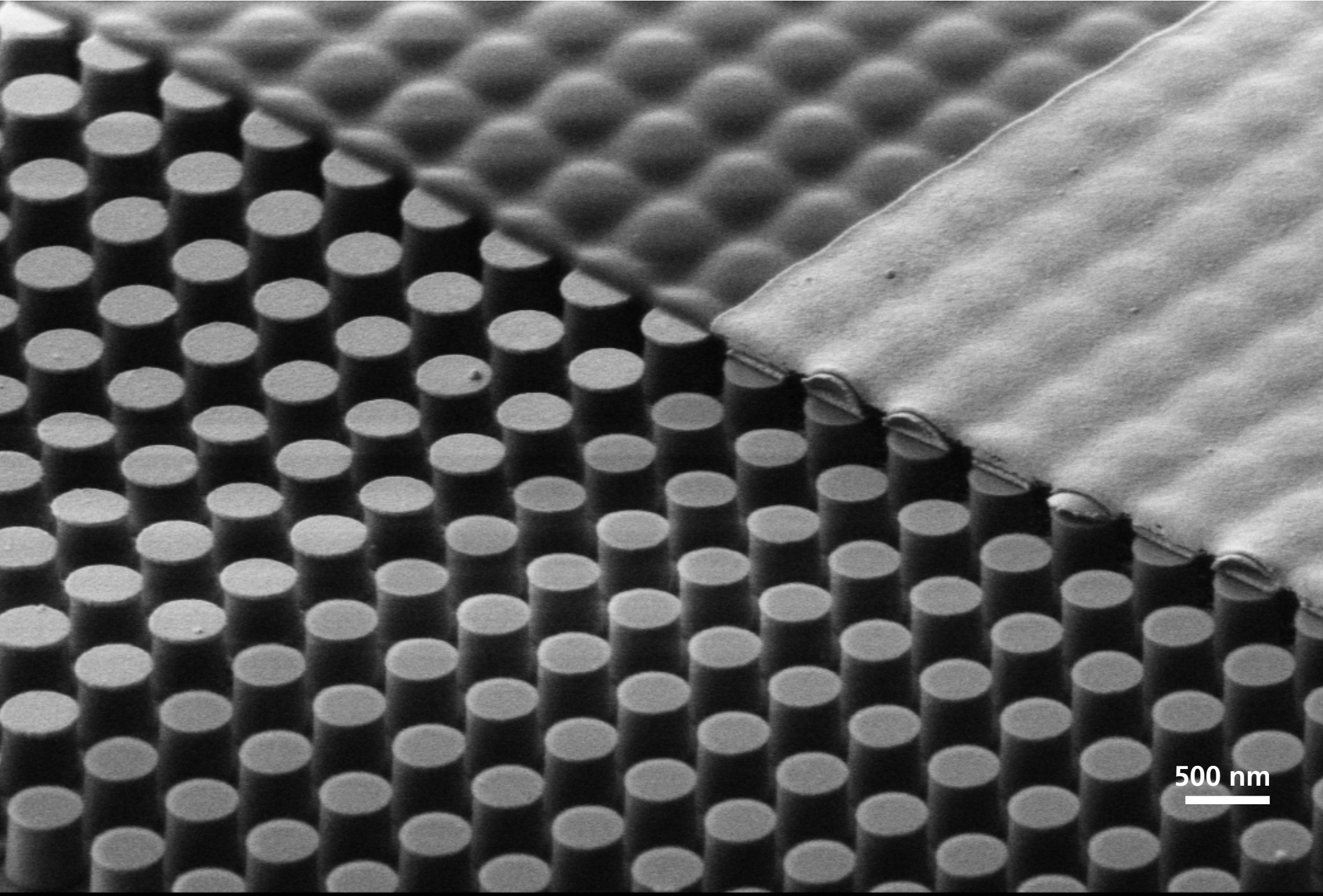
On-chip zero-index fabrication



On-chip zero-index fabrication



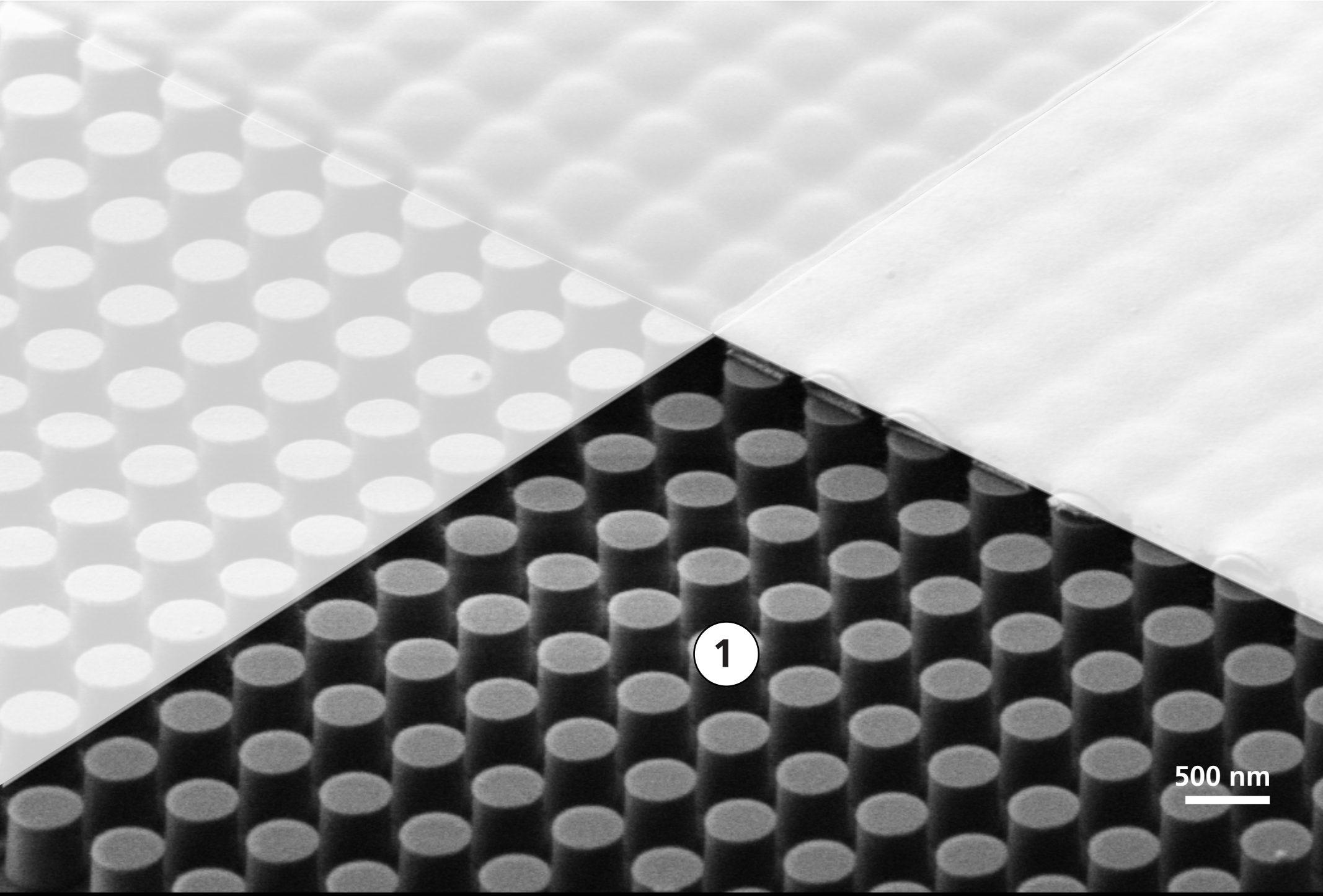
1 index 2 zero index 3 experiments



1 index

2 zero index

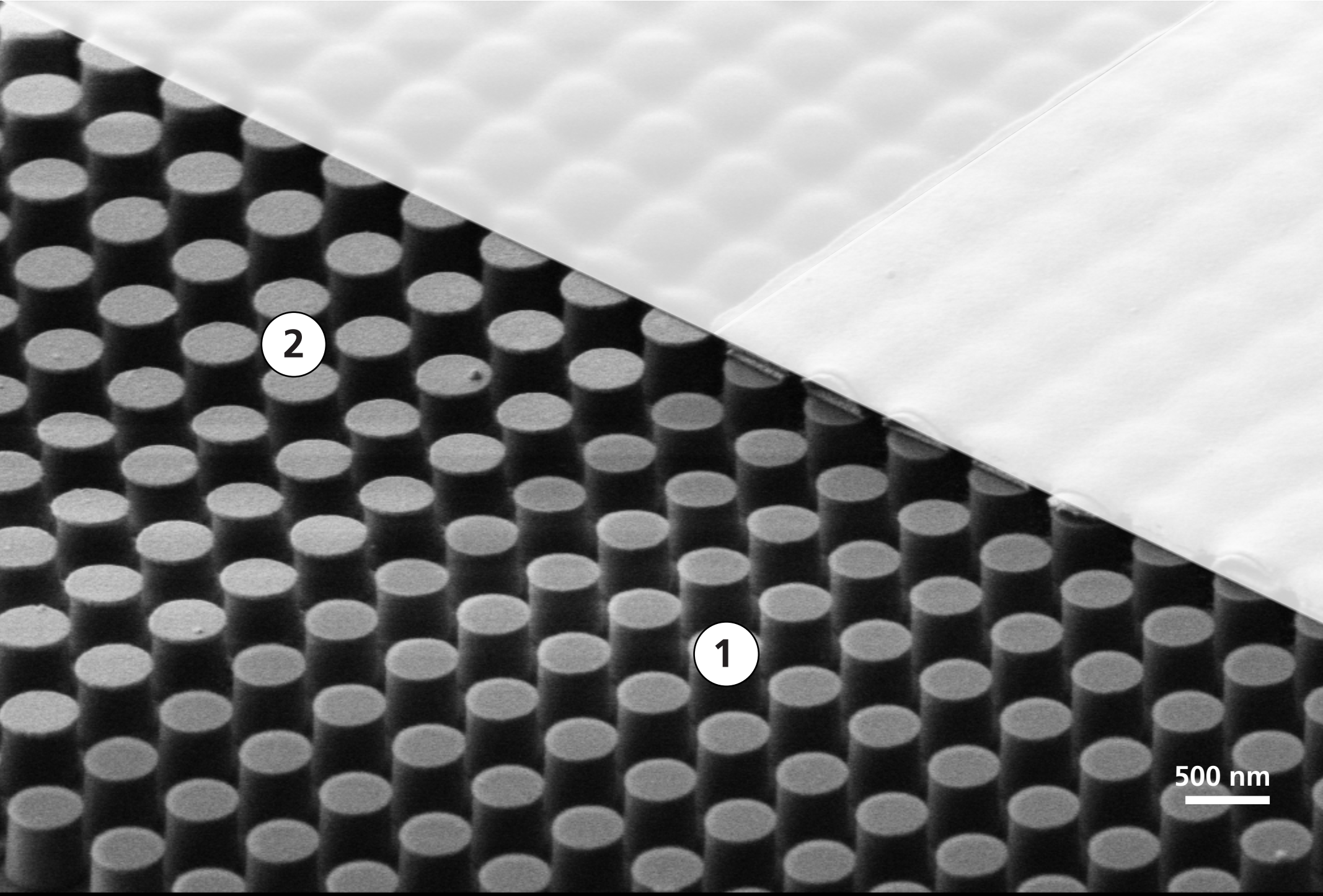
3 experiments



1 index

2 zero index

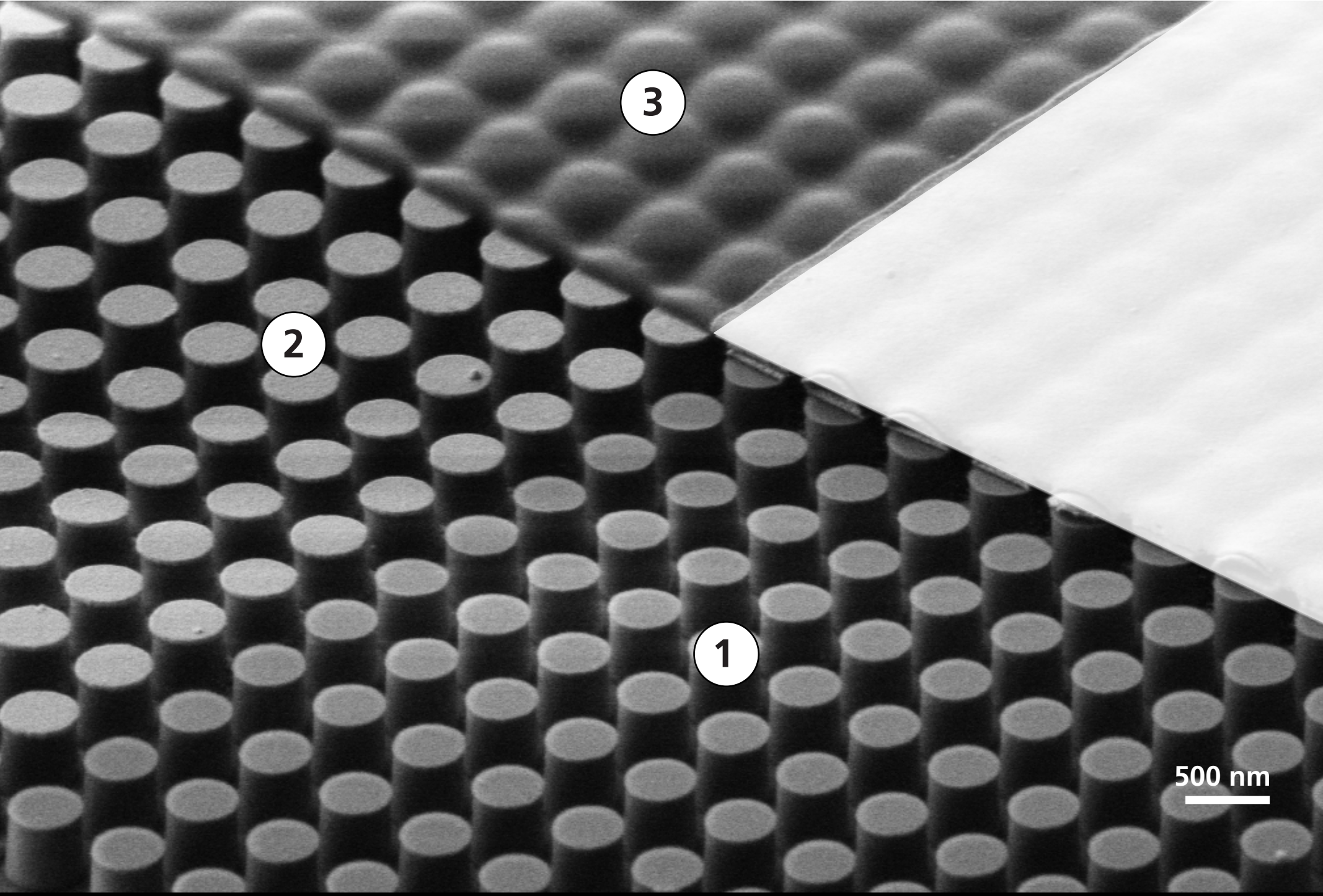
3 experiments



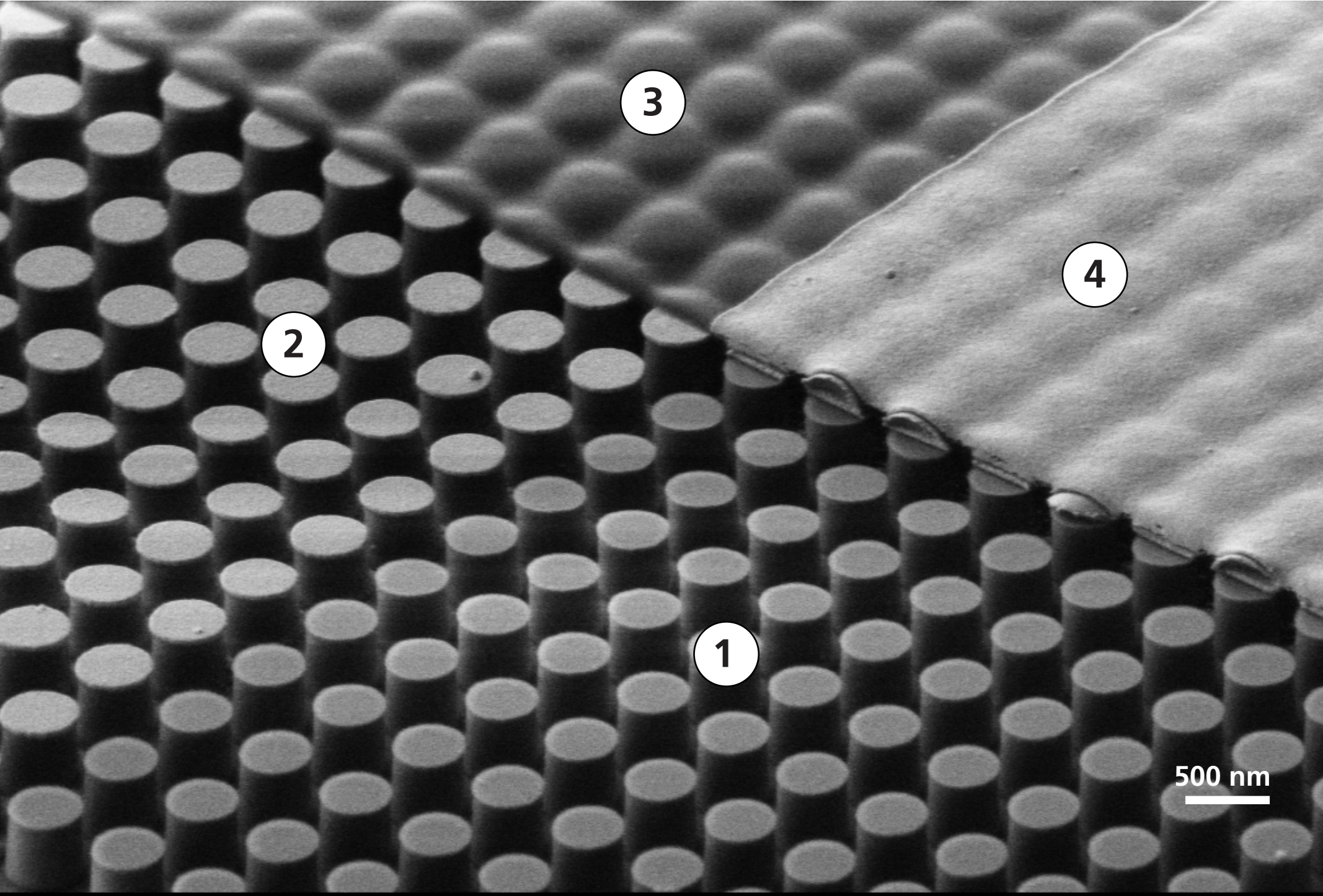
1 index

2 zero index

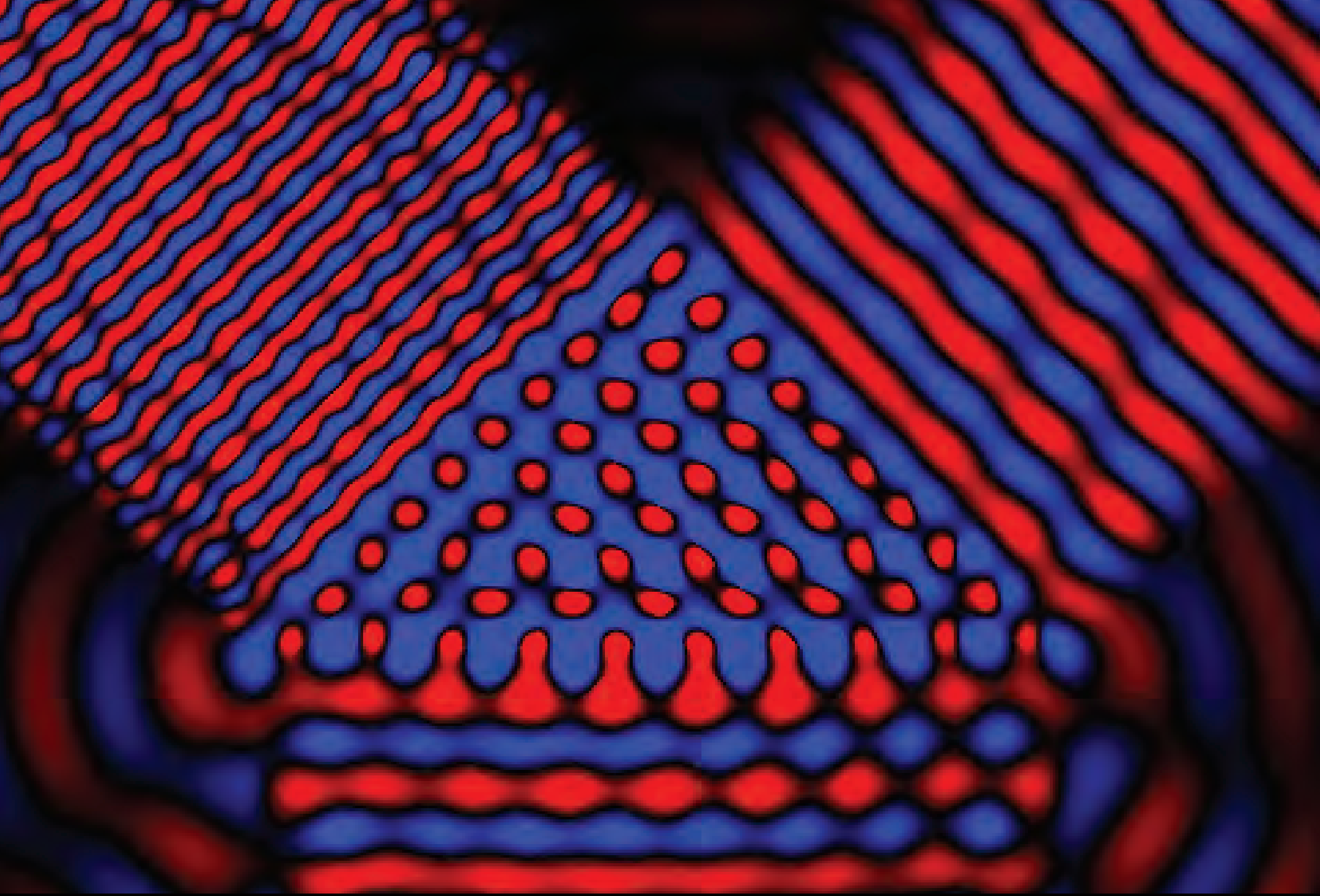
3 experiments



1 index 2 zero index 3 experiments

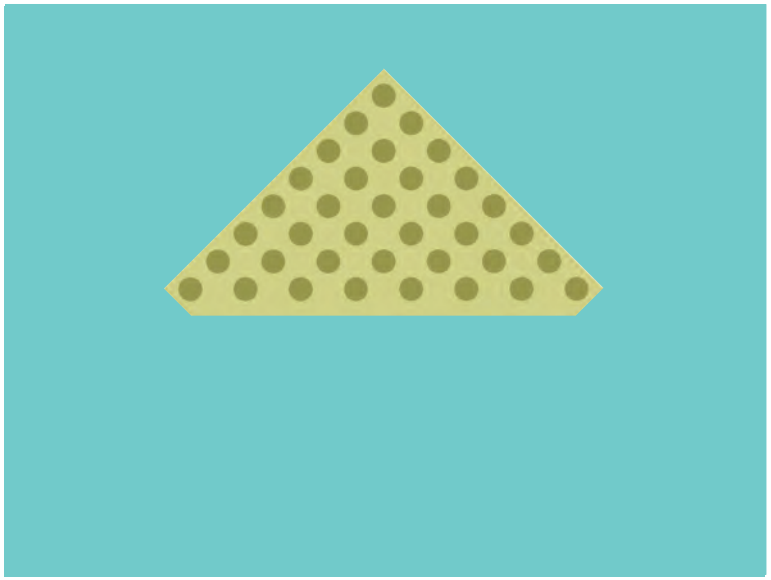


1 index 2 zero index 3 experiments

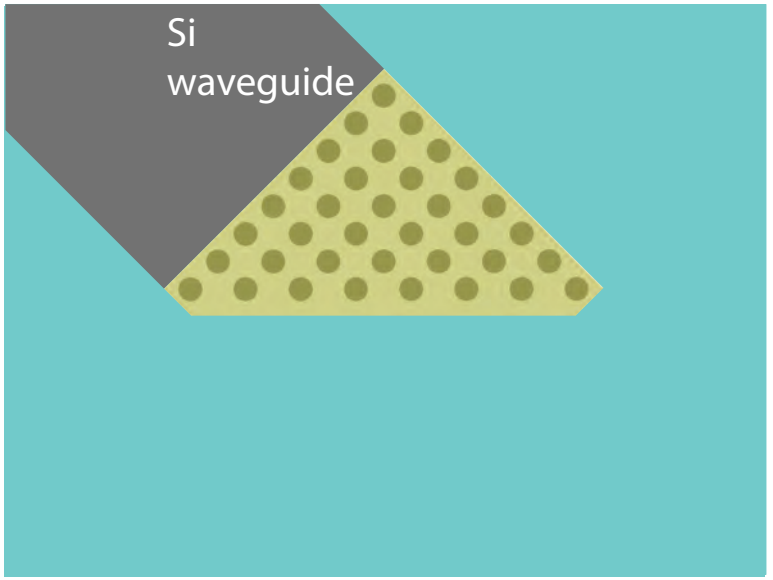

1 index
2 zero index
3 experiments

On-chip zero-index prism

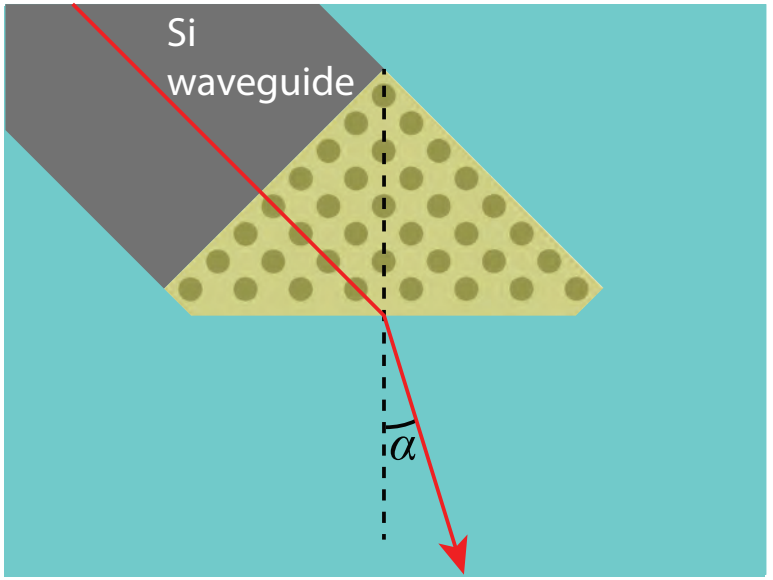
On-chip zero-index prism



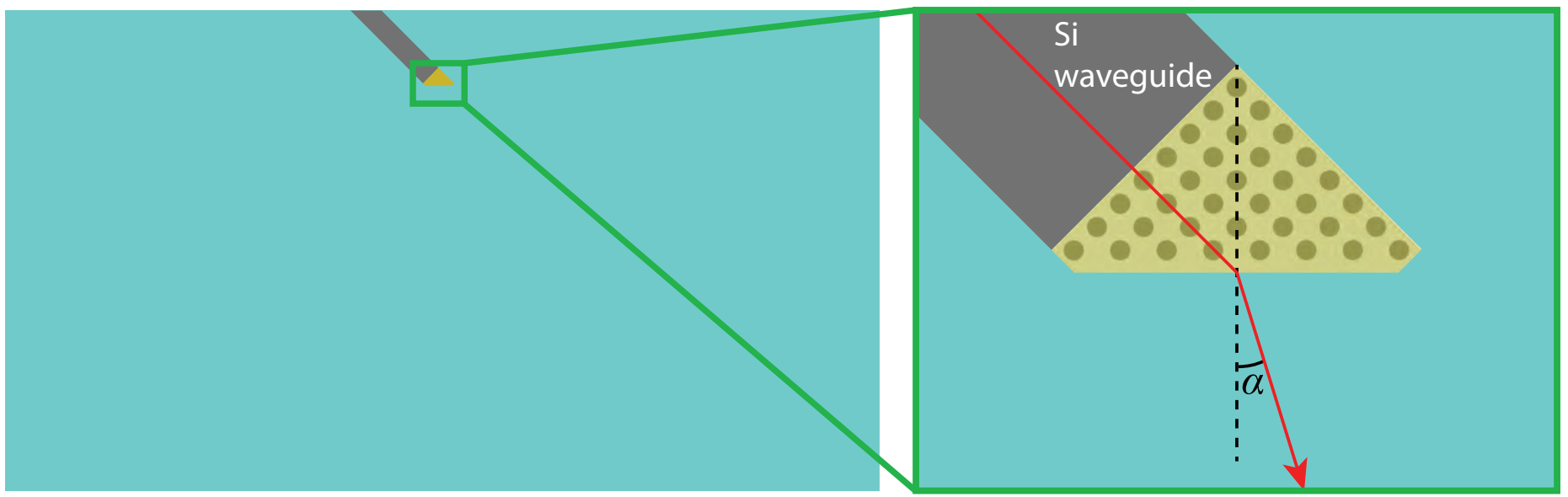
On-chip zero-index prism



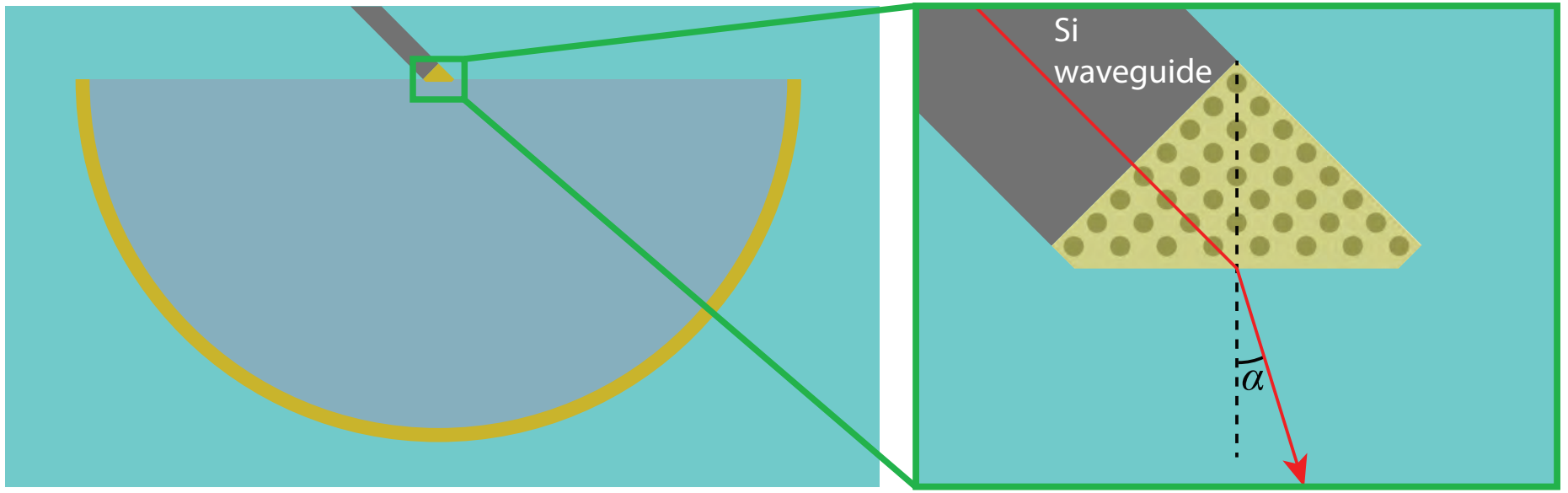
On-chip zero-index prism



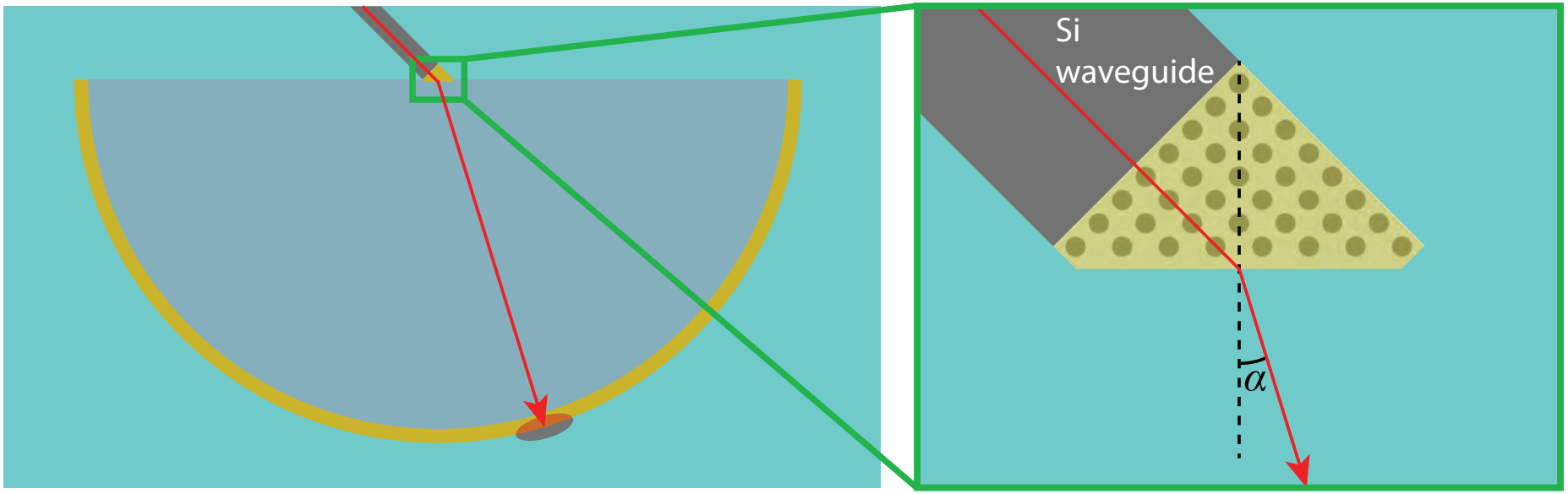
On-chip zero-index prism



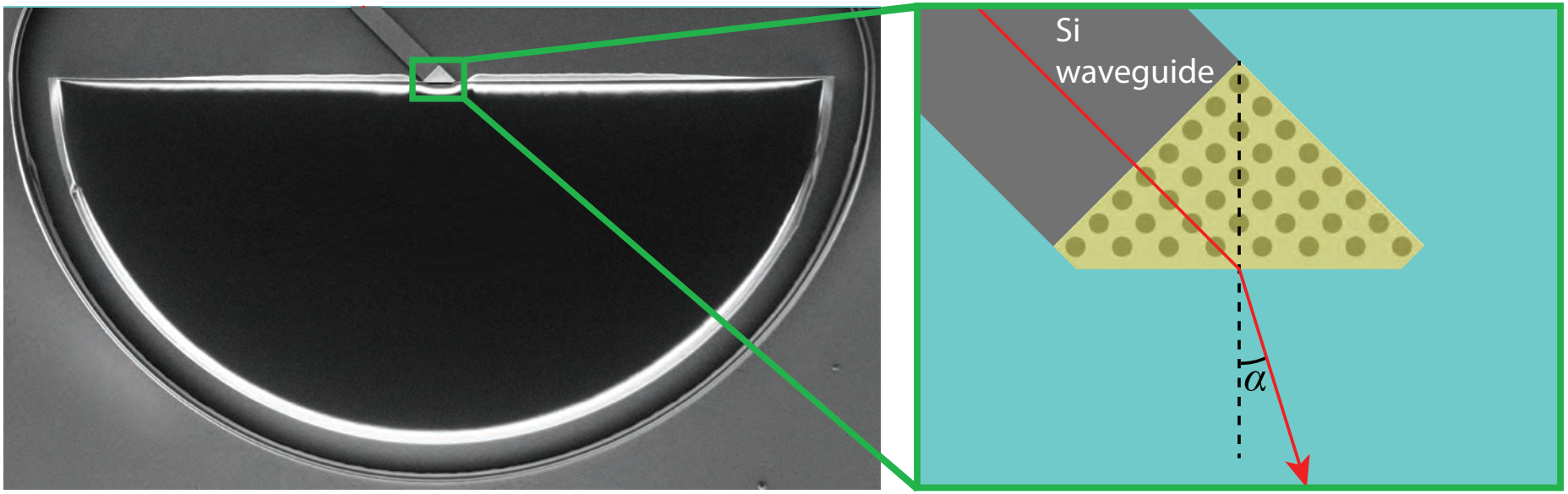
On-chip zero-index prism



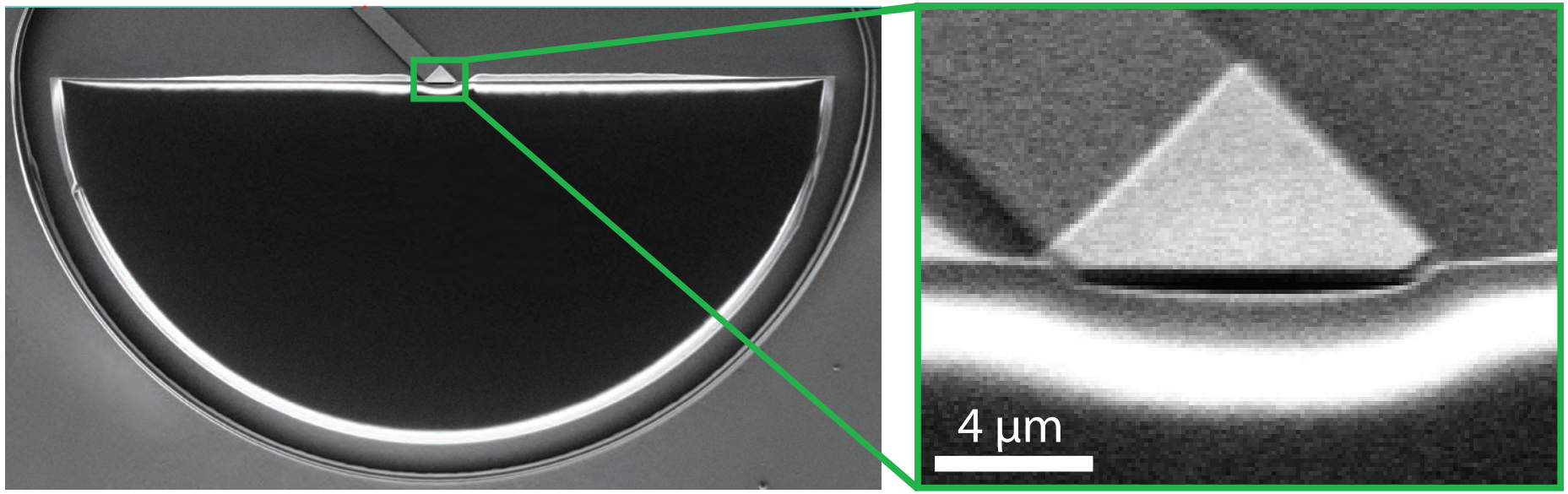
On-chip zero-index prism

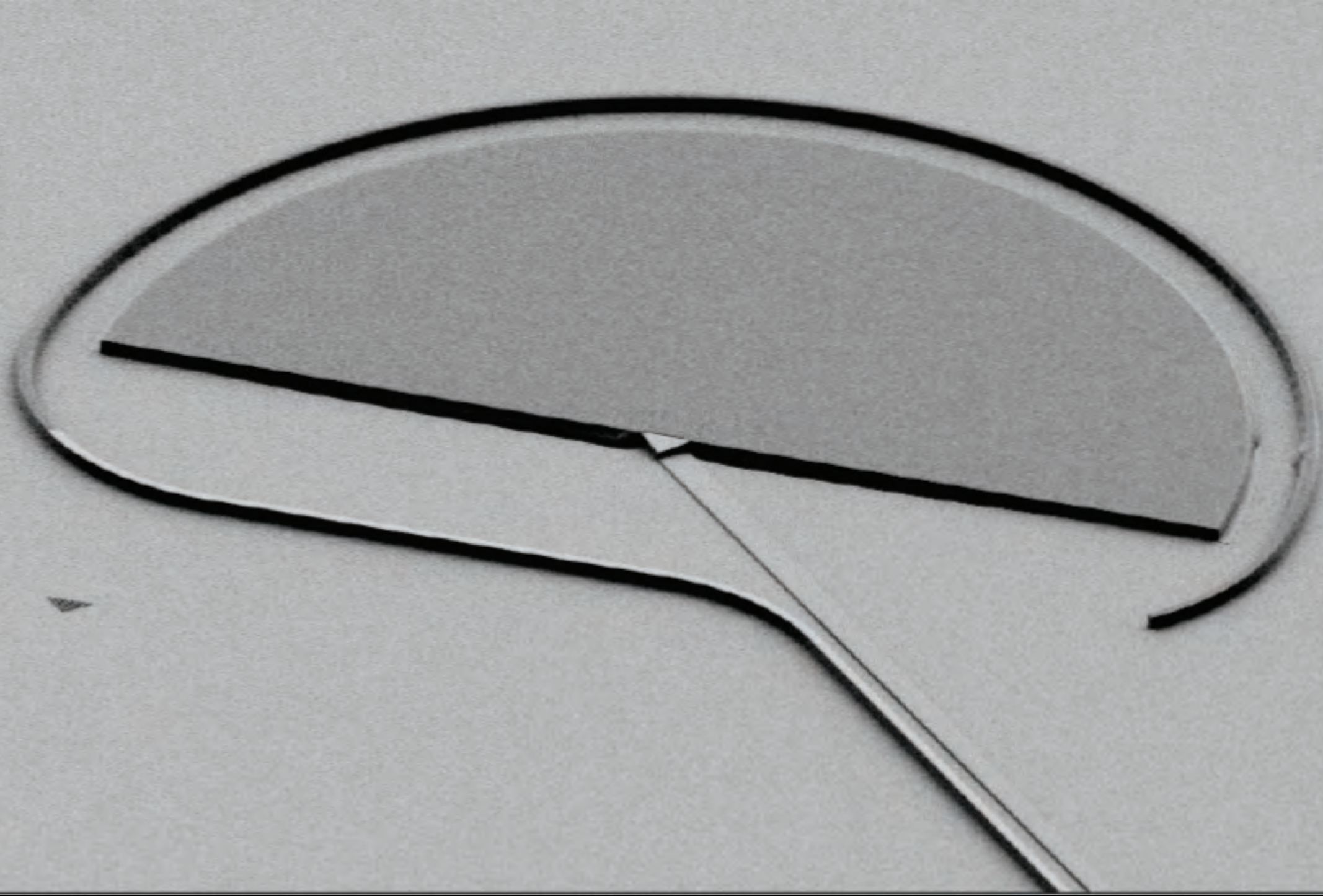


On-chip zero-index prism



On-chip zero-index prism

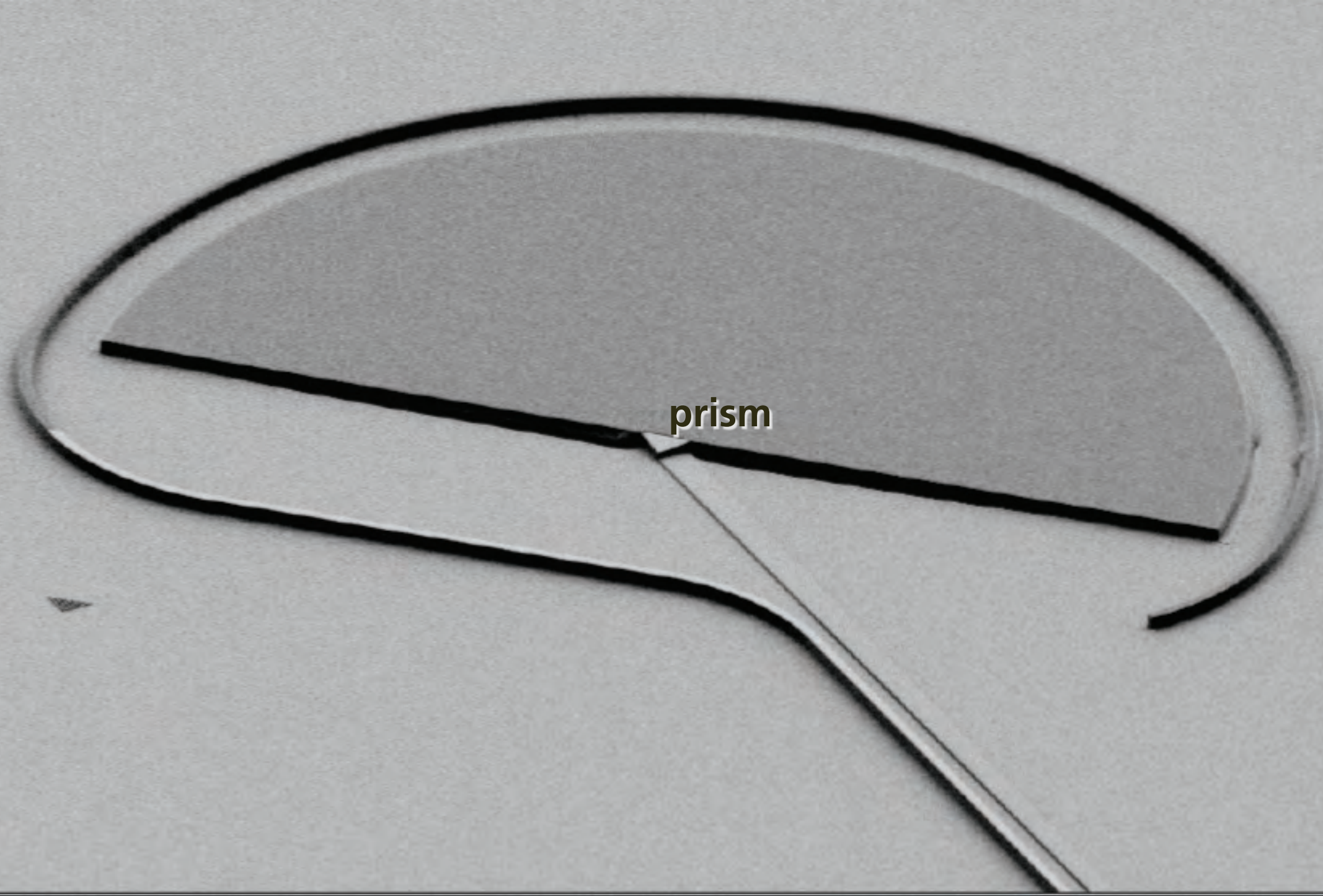




1 index

2 zero index

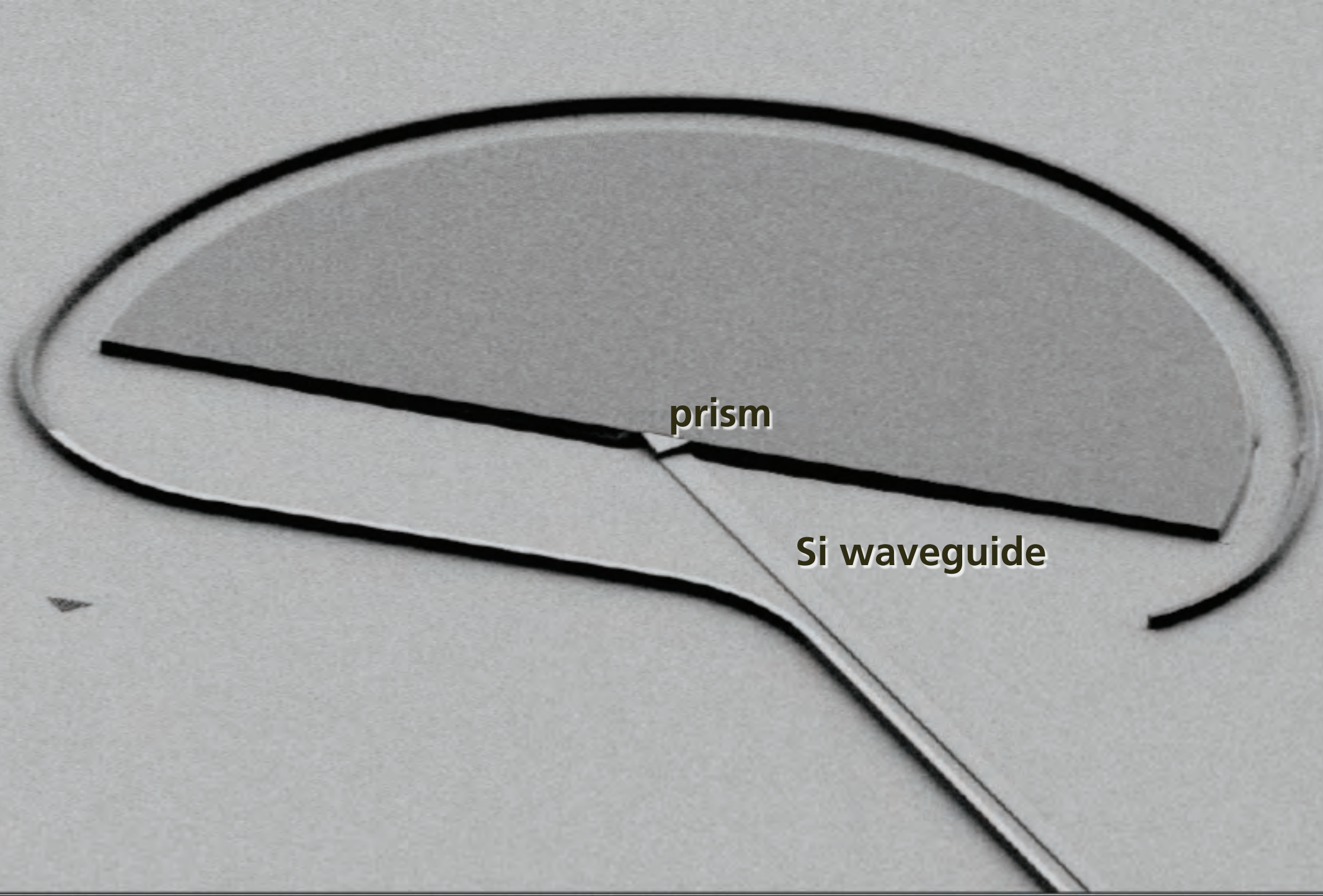
3 experiments



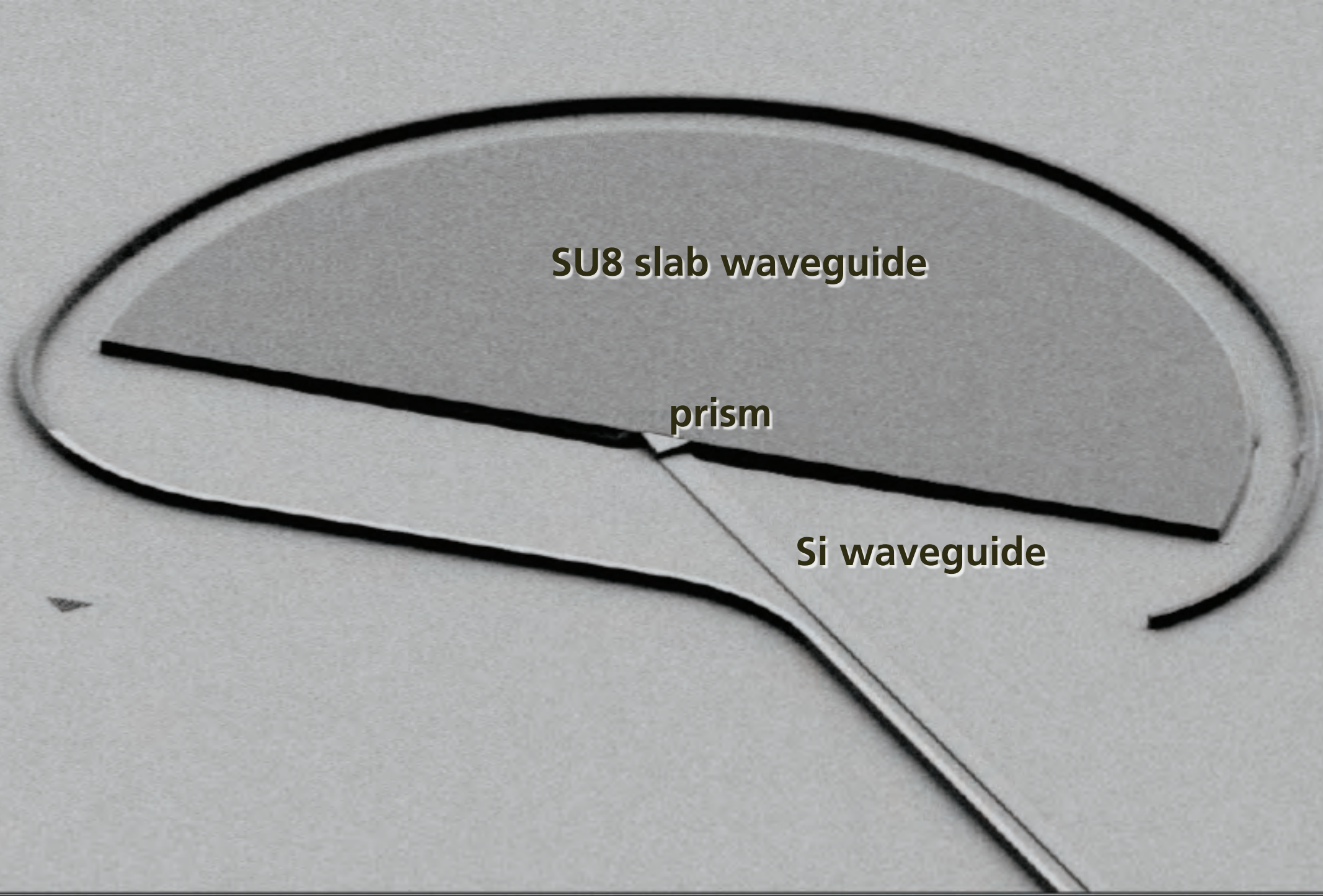
1 index

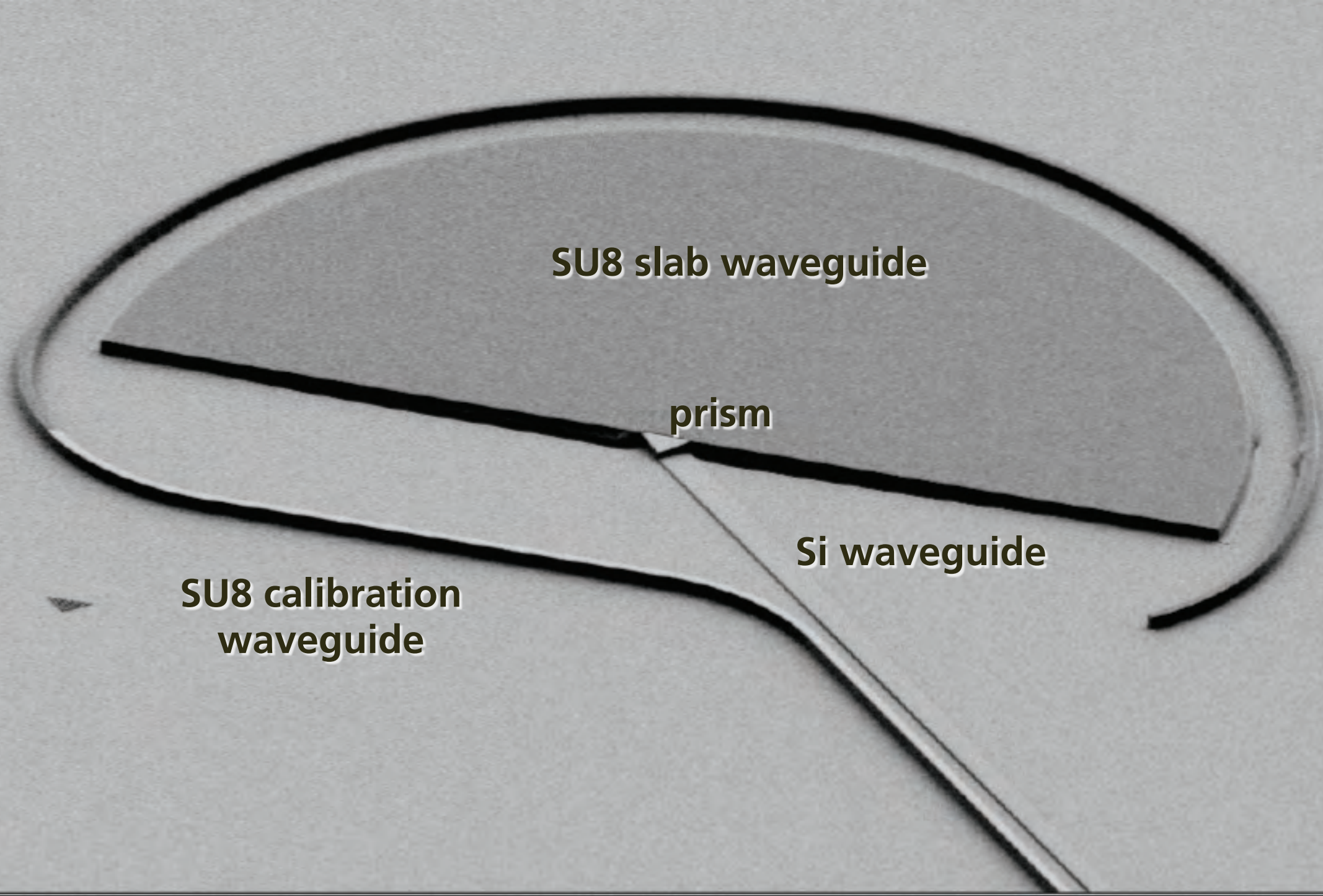
2 zero index

3 experiments

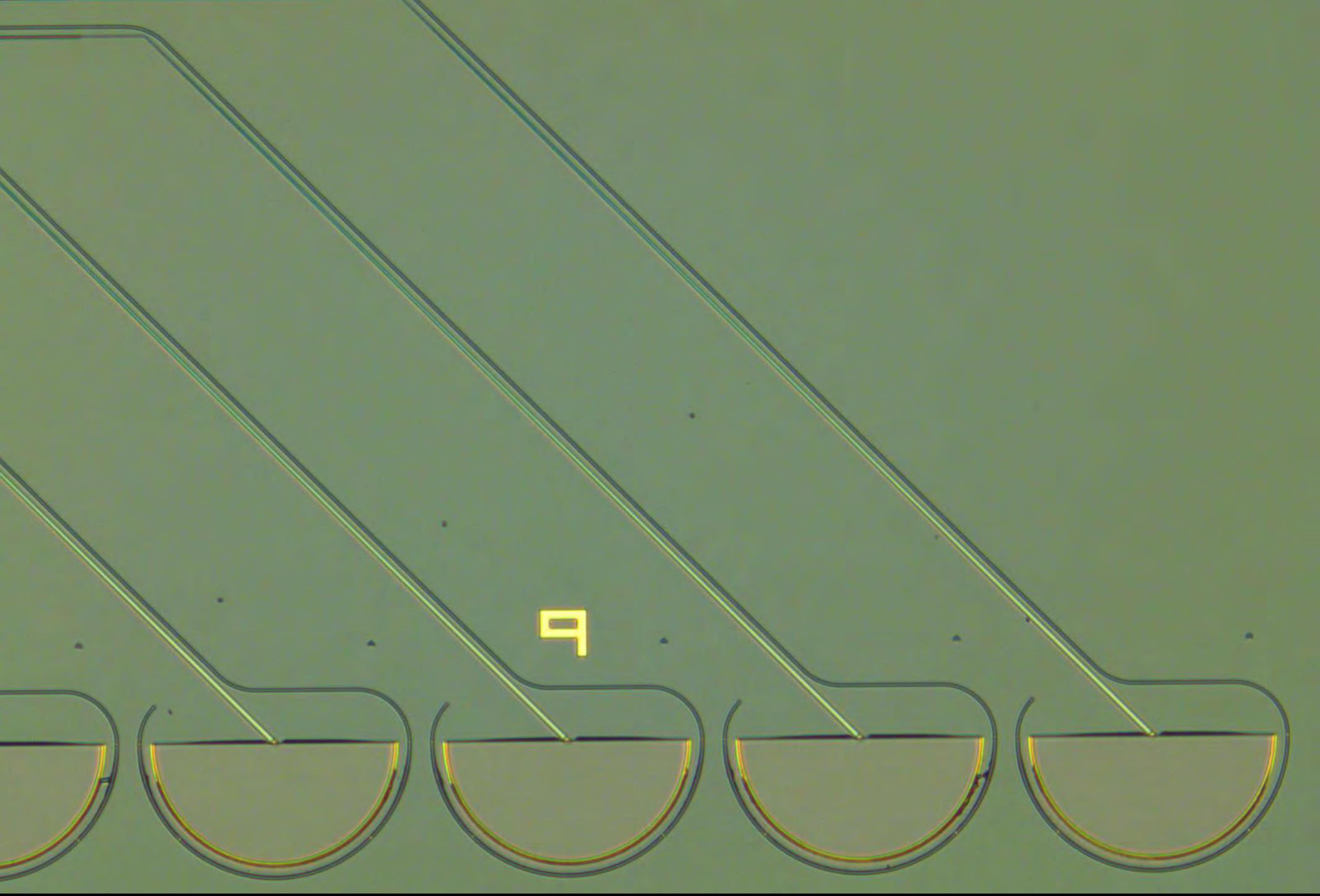


1 index 2 zero index 3 experiments


SU8 slab waveguide
prism
Si waveguide

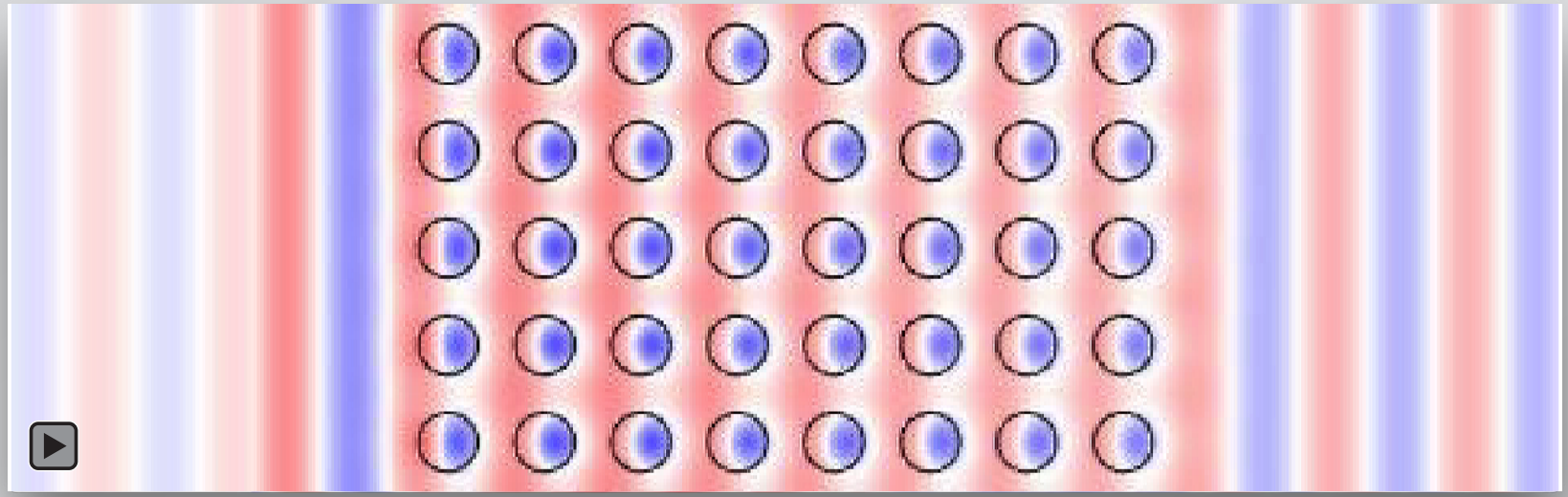


1 index 2 zero index 3 experiments

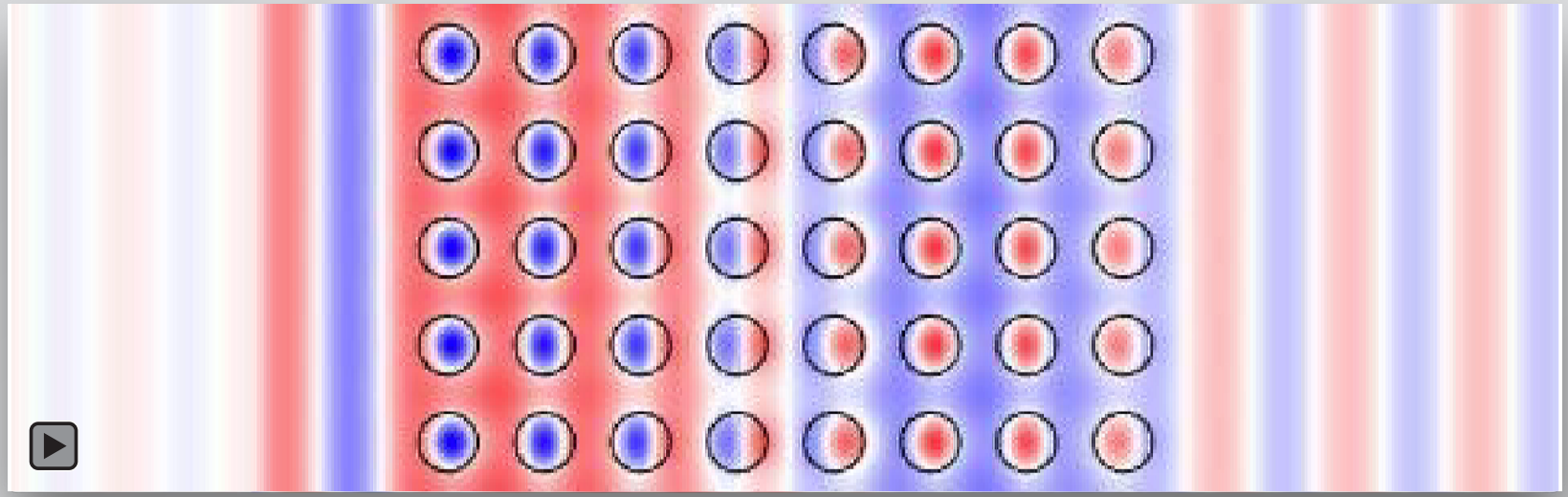


1 index 2 zero index 3 experiments

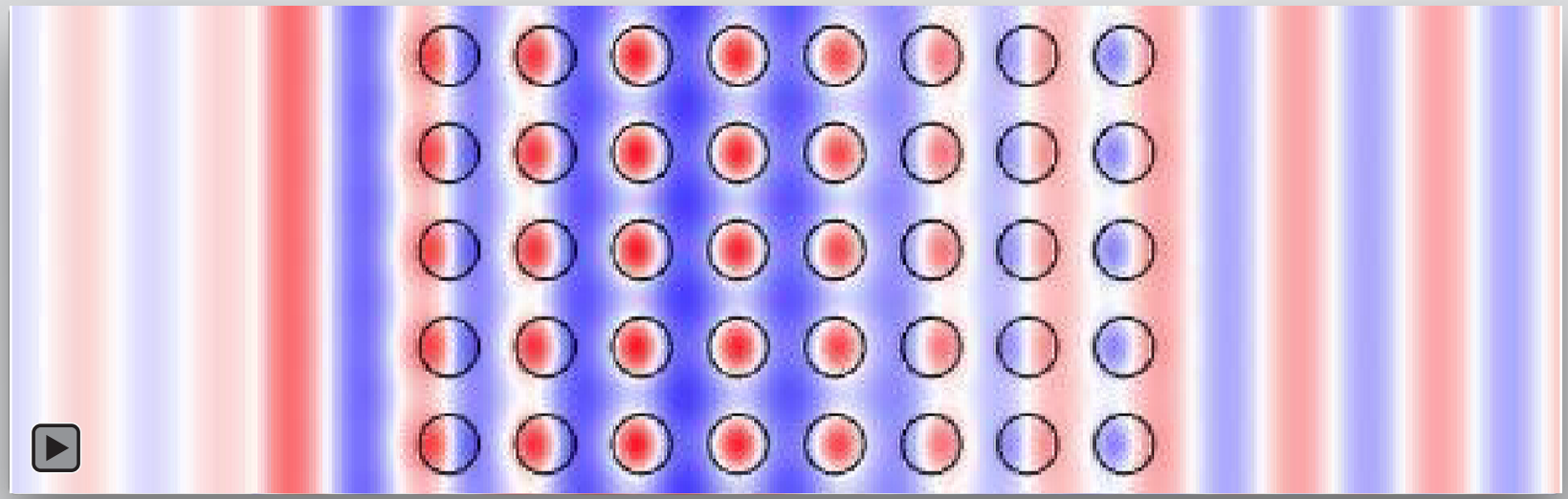
at design wavelength (1590 nm)



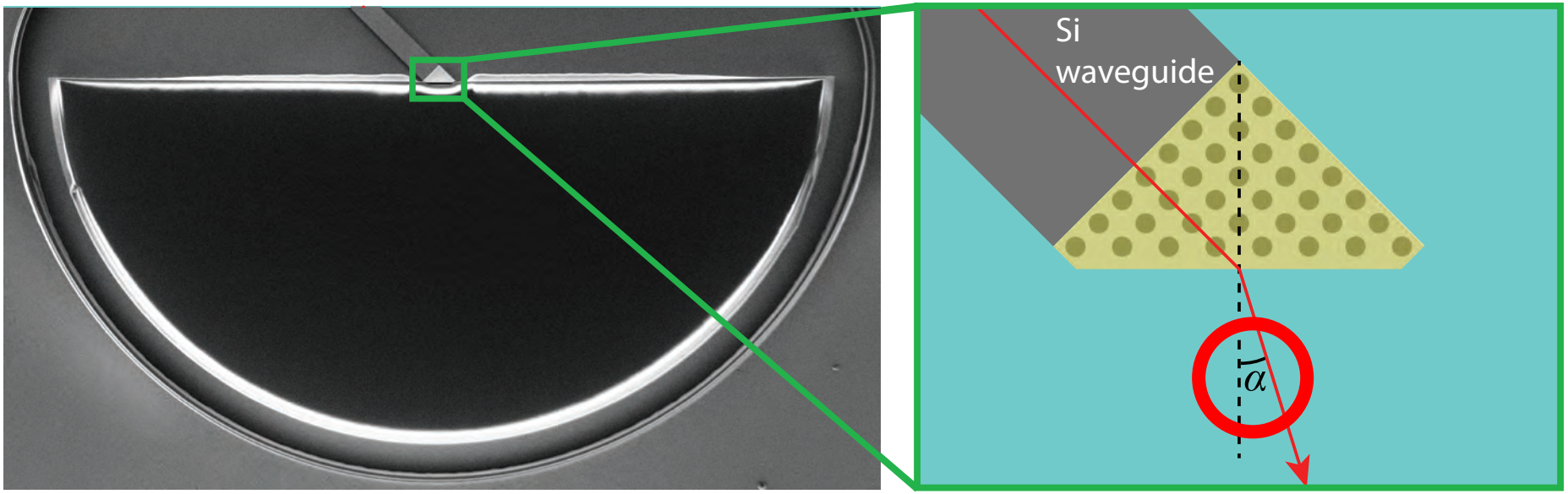
below design wavelength (1530 nm)

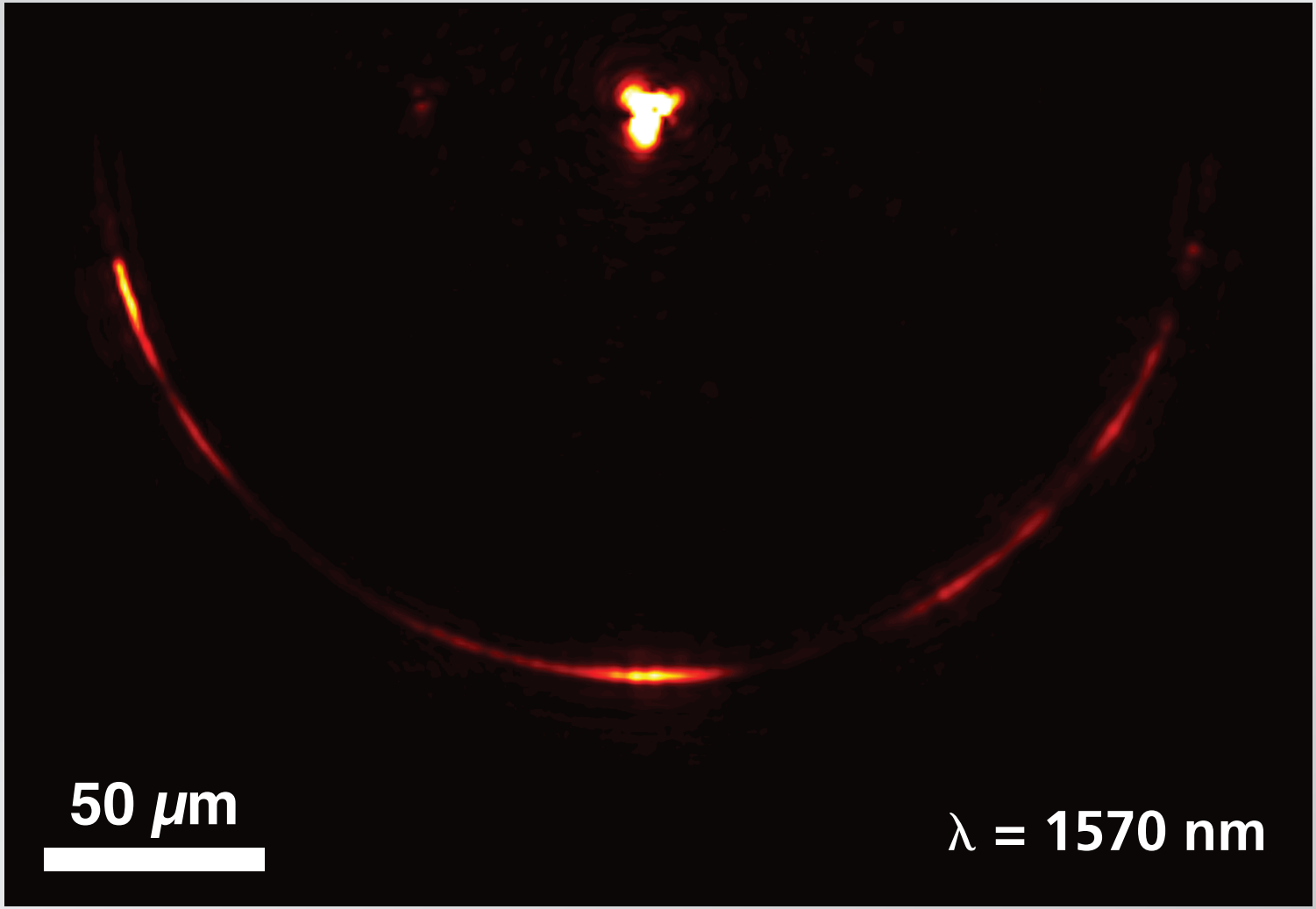


above design wavelength (1650 nm)

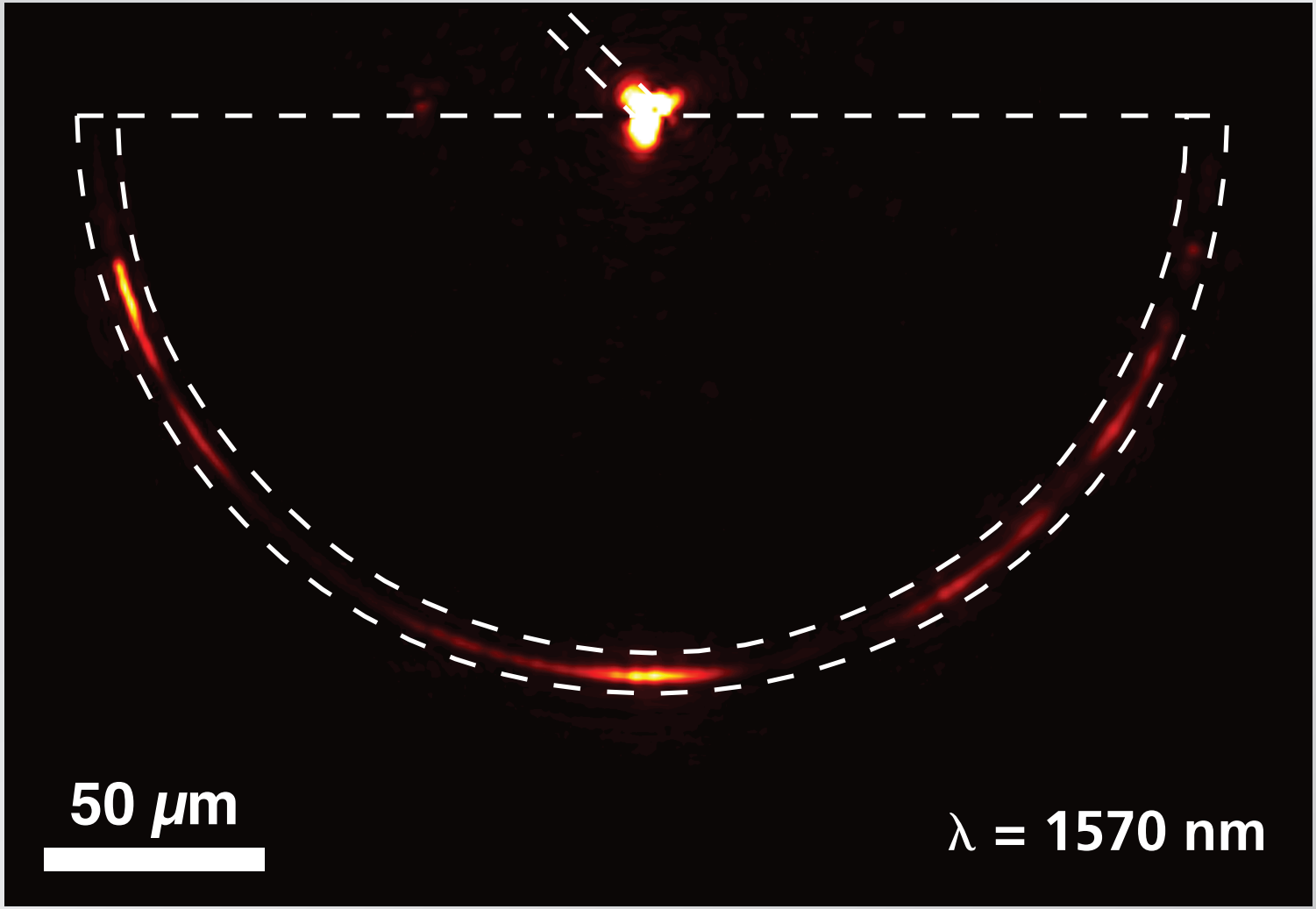


On-chip zero-index prism

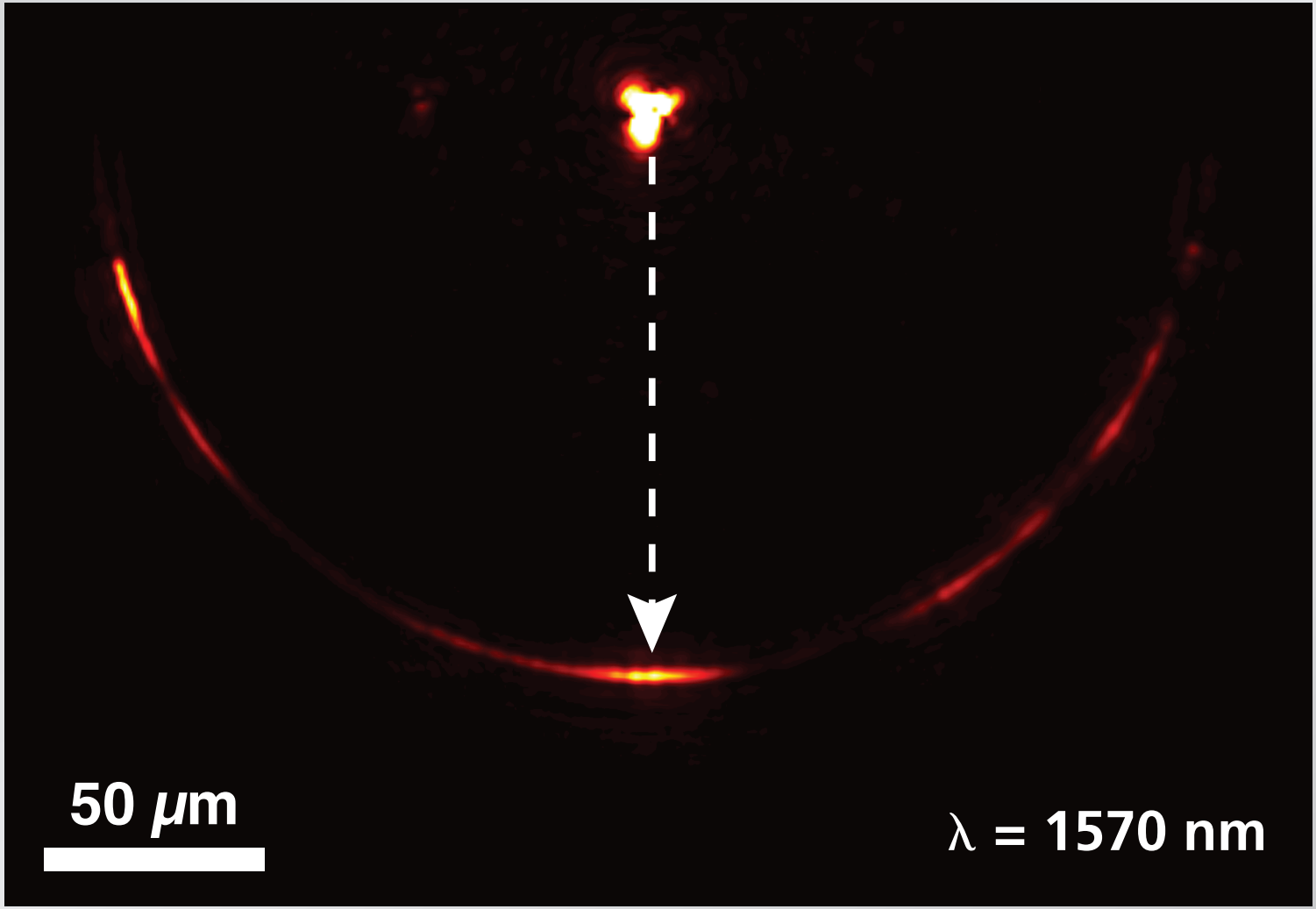




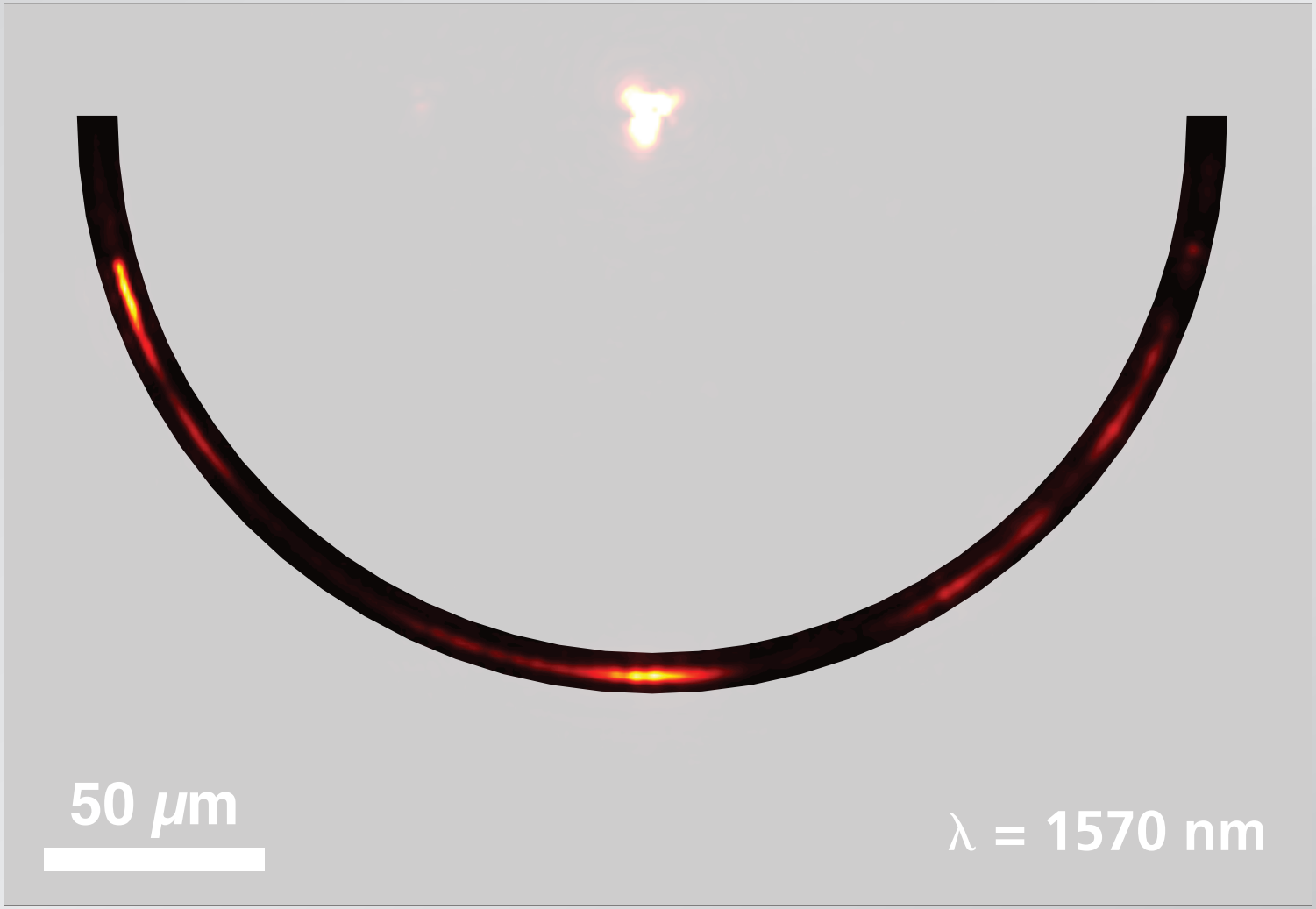
1 index 2 zero index 3 experiments



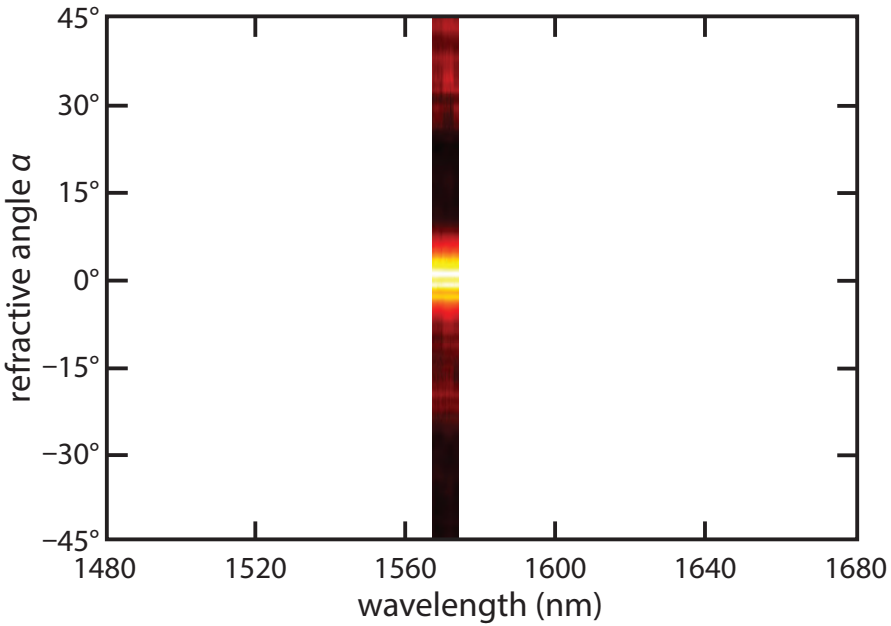
1 index 2 zero index 3 experiments



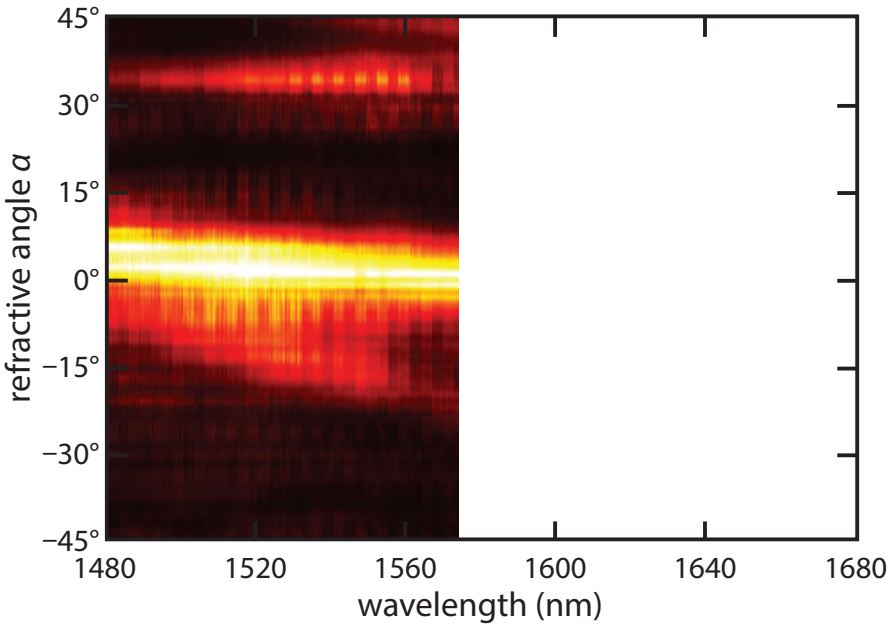
1 index 2 zero index 3 experiments


50 μm
λ = 1570 nm

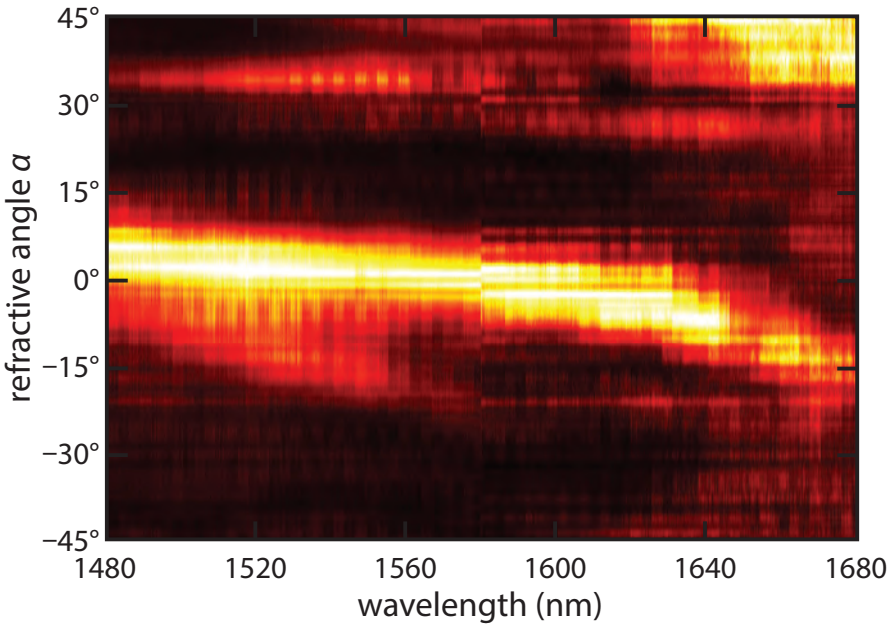
Wavelength dependence of refraction angle



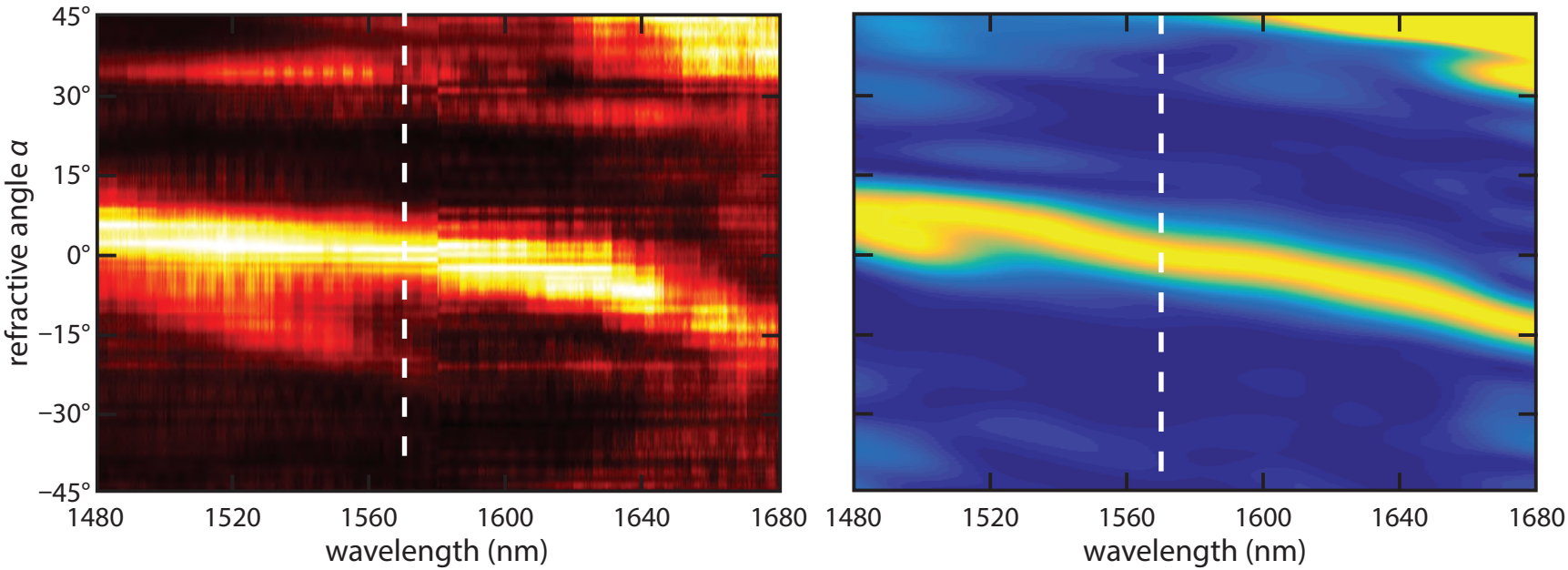
Wavelength dependence of refraction angle



Wavelength dependence of refraction angle



Wavelength dependence of refraction angle

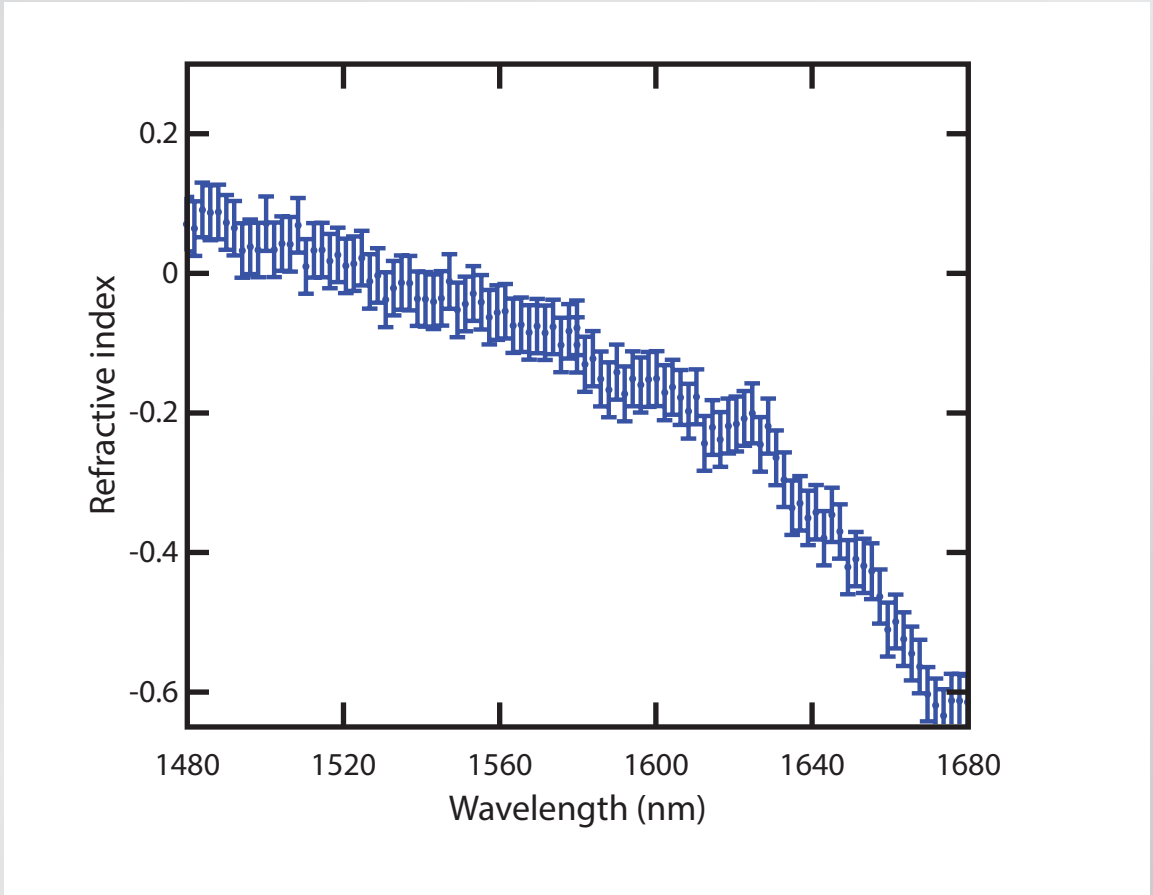


1 index

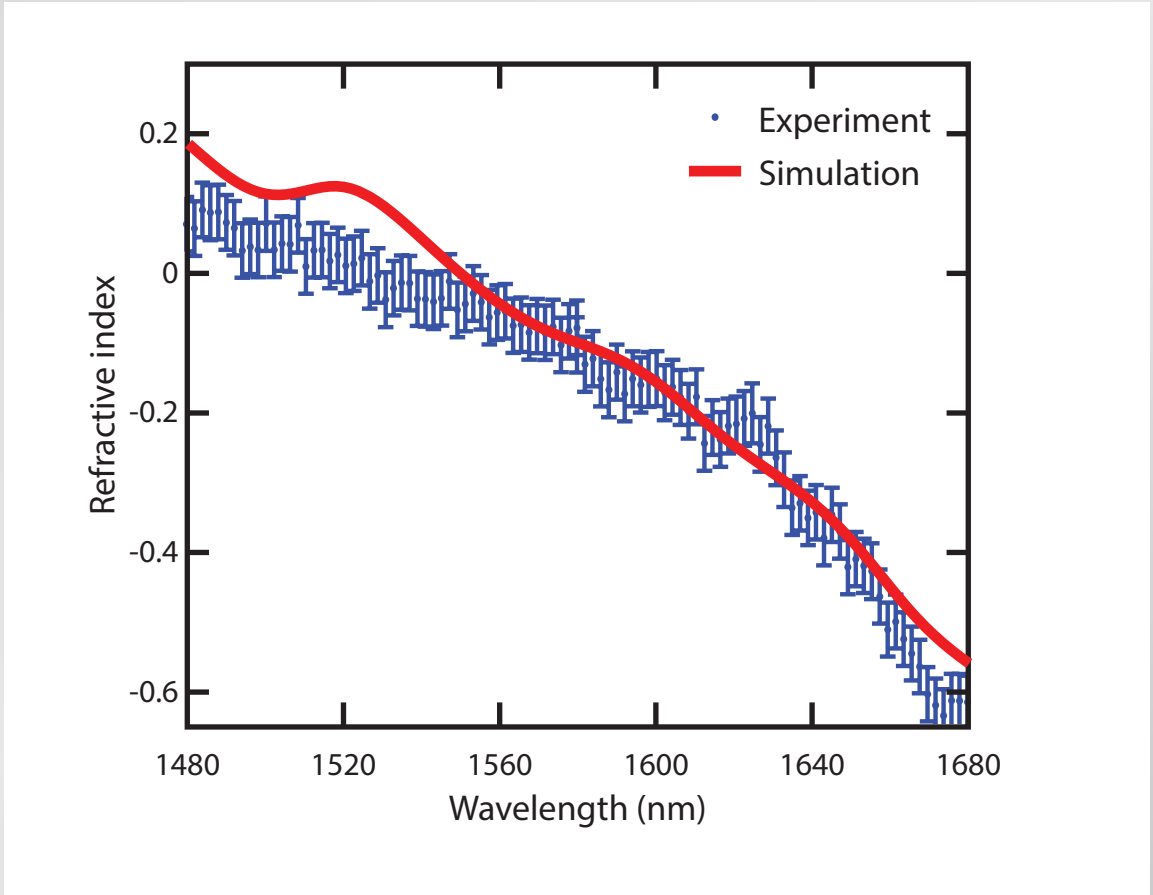
2 zero index

3 experiments

Wavelength dependence of index



Wavelength dependence of index



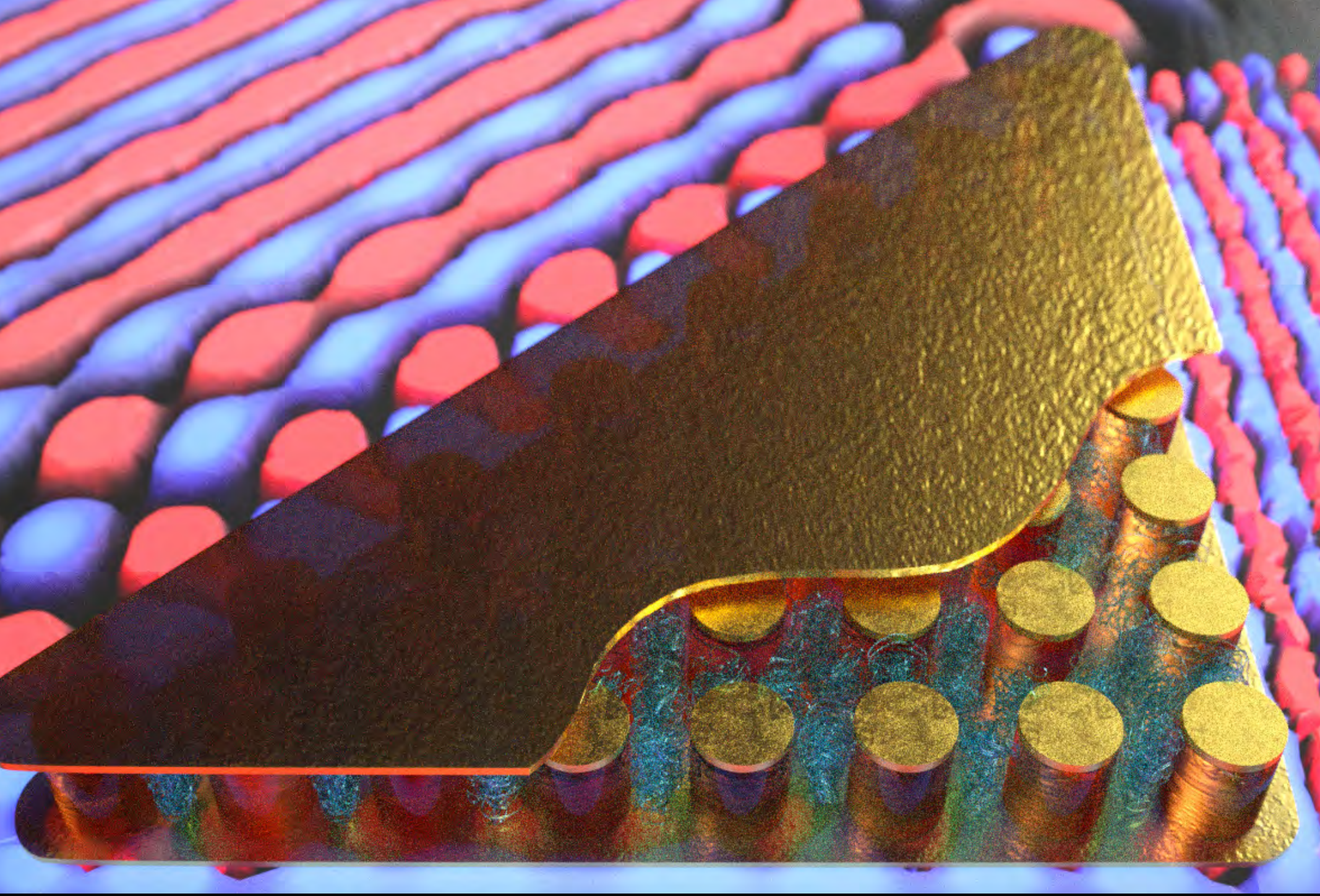
1 index

2 zero index

3 experiments

More extreme optics

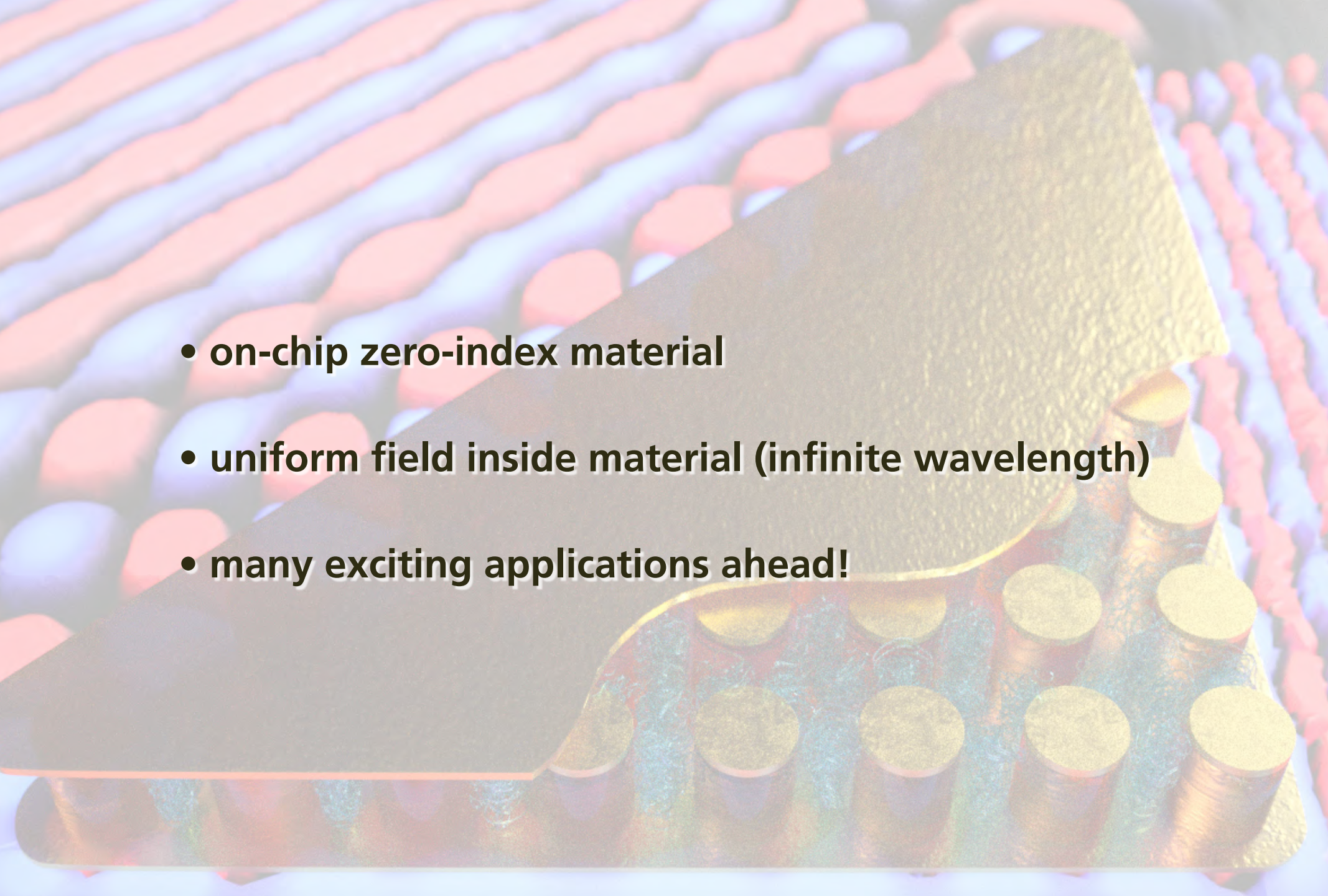
- suppressing losses
- beam steering & supercoupling
- nonlinear optics
- quantum optics



1 index

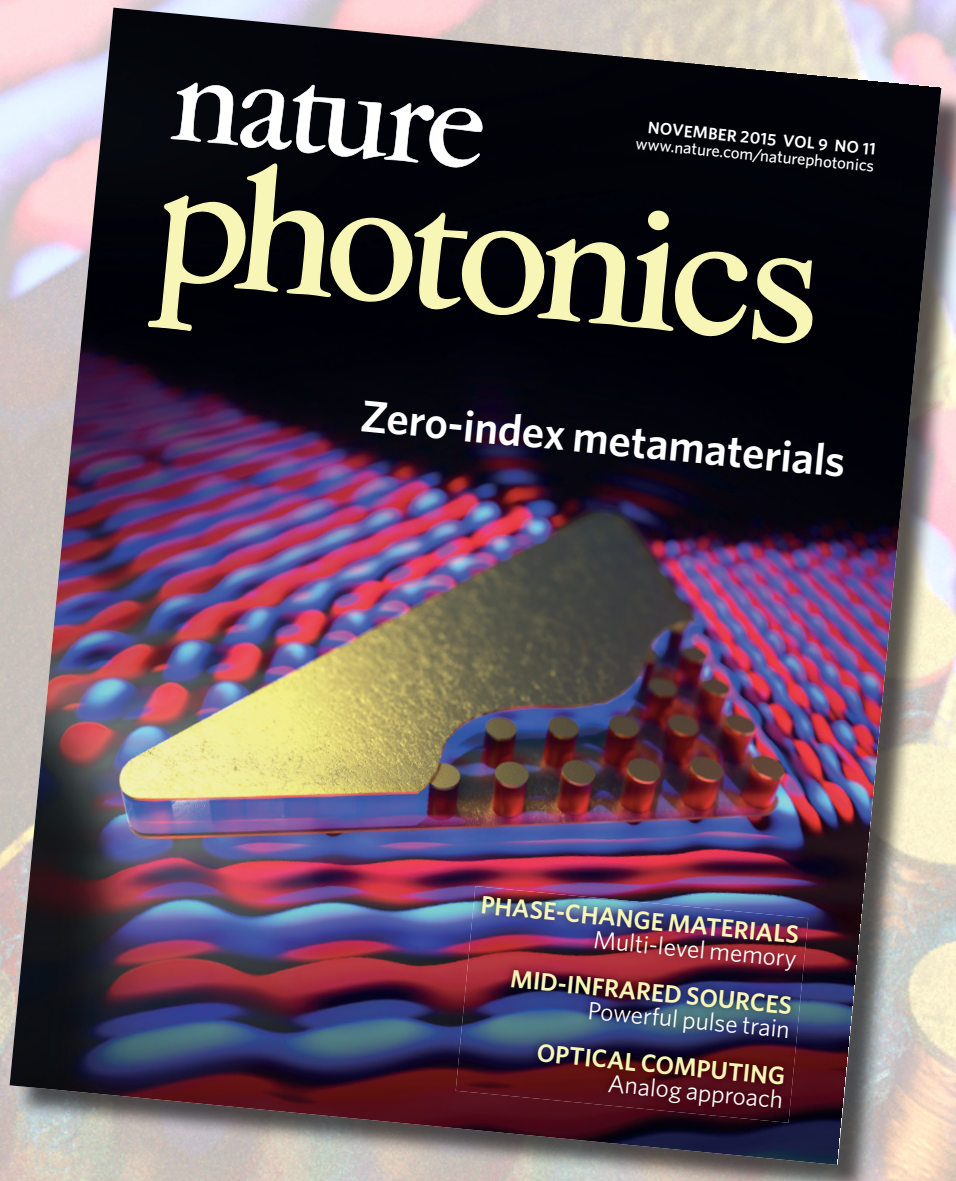
2 zero index

3 experiments



- on-chip zero-index material
- uniform field inside material (infinite wavelength)
- many exciting applications ahead!

More info: download paper!



1 index

2 zero index

3 experiments

The Team: Yang Li, Shota Kita,
Orad Reshef, Philip Muñoz, Daryl Vulis, Marko Lončar

Funding: National Science Foundation

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