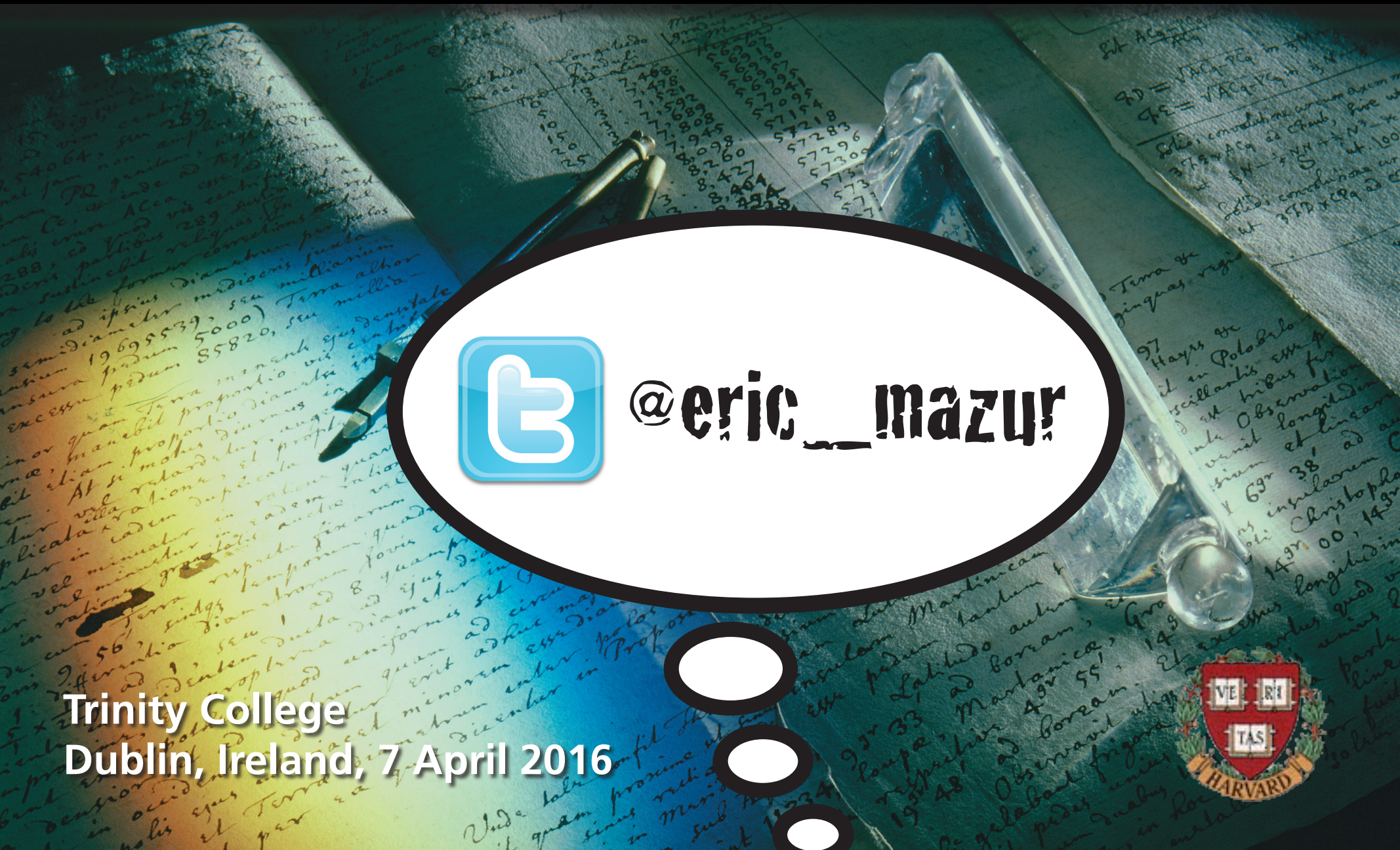
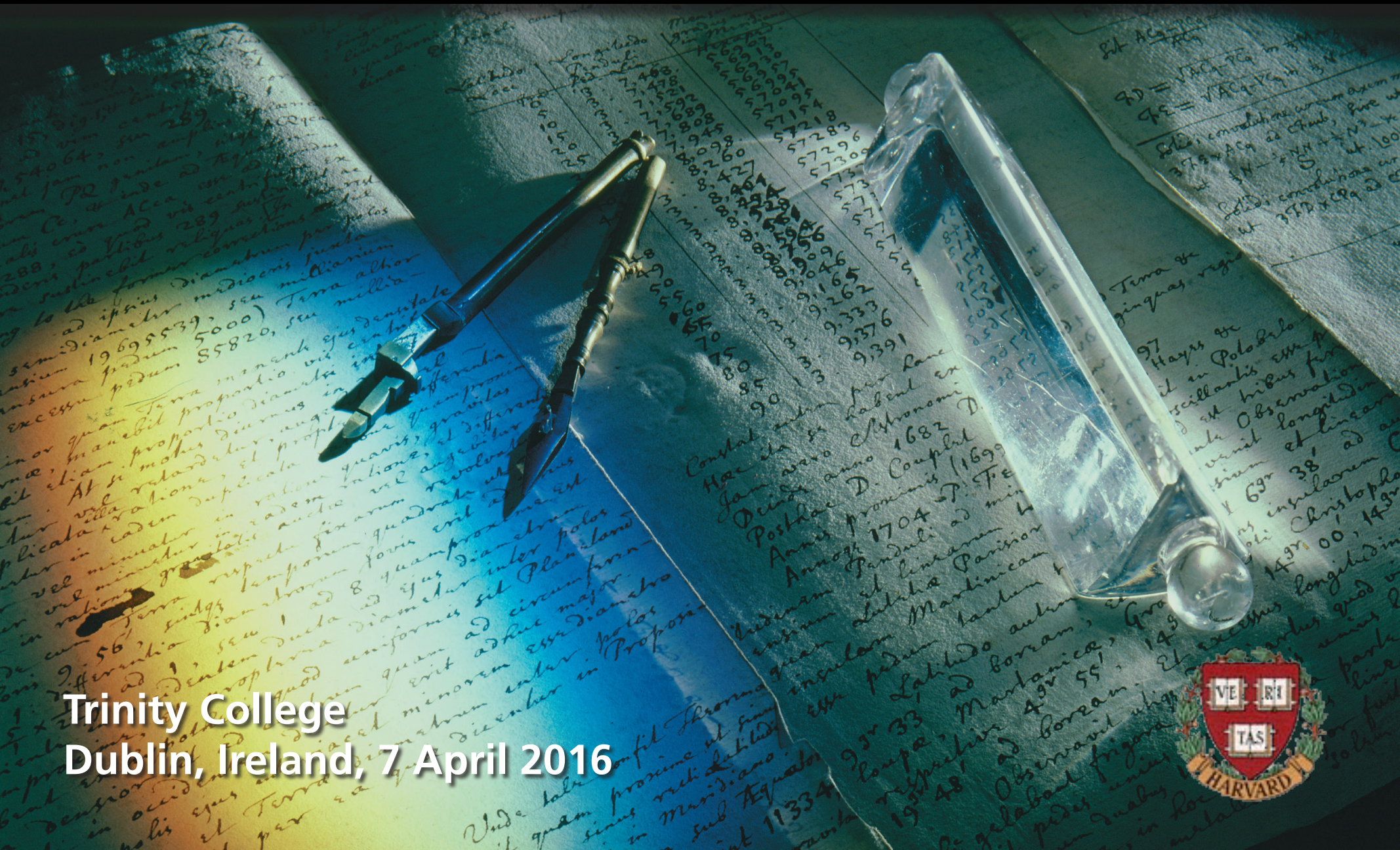
Less is more:
Extreme optics with zero refractive index



Trinity College
Dublin, Ireland, 7 April 2016

Less is more:
Extreme optics with zero refractive index

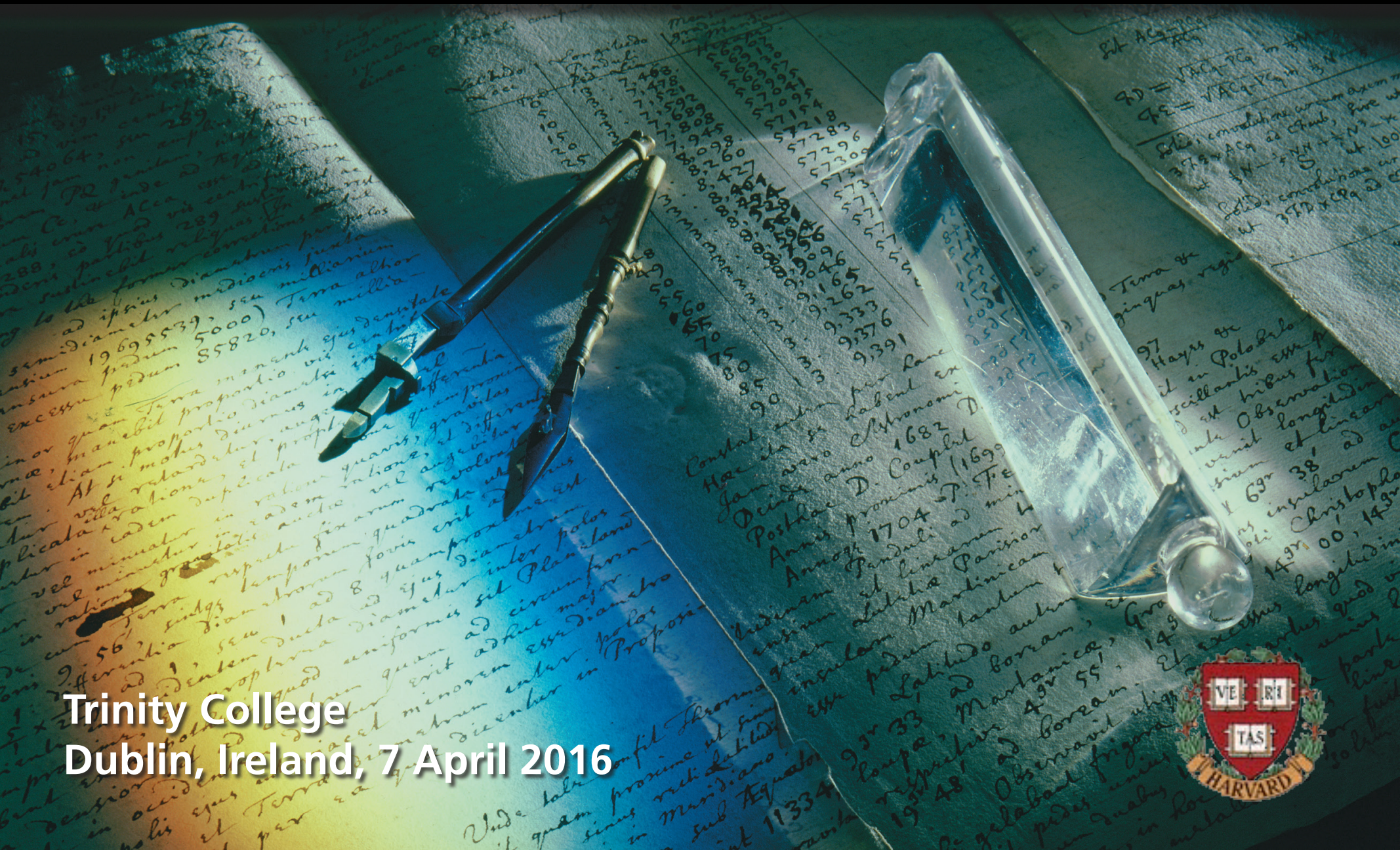


Trinity College
Dublin, Ireland, 7 April 2016

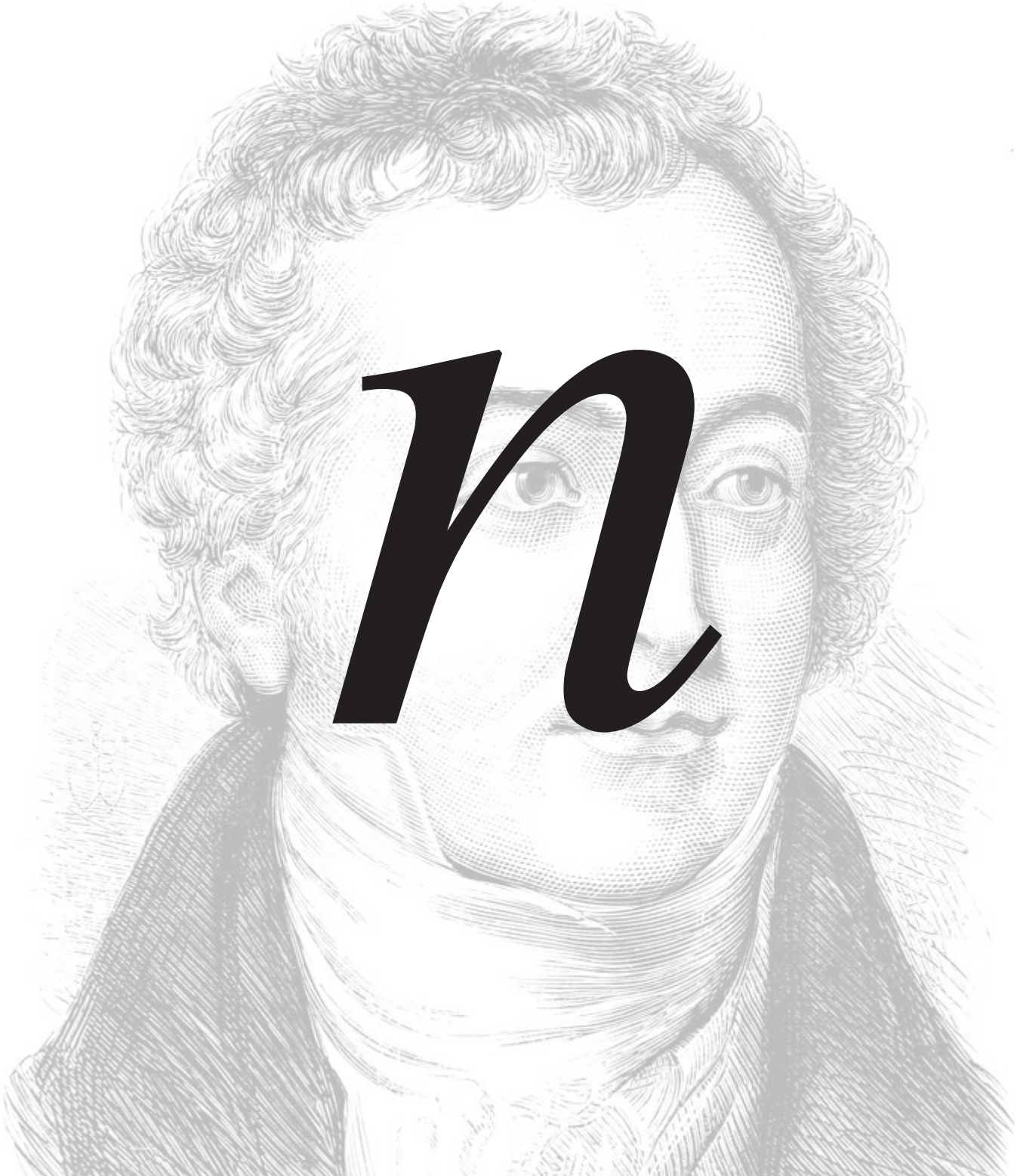
OSA® | 100

Reflecting a Century of Innovation

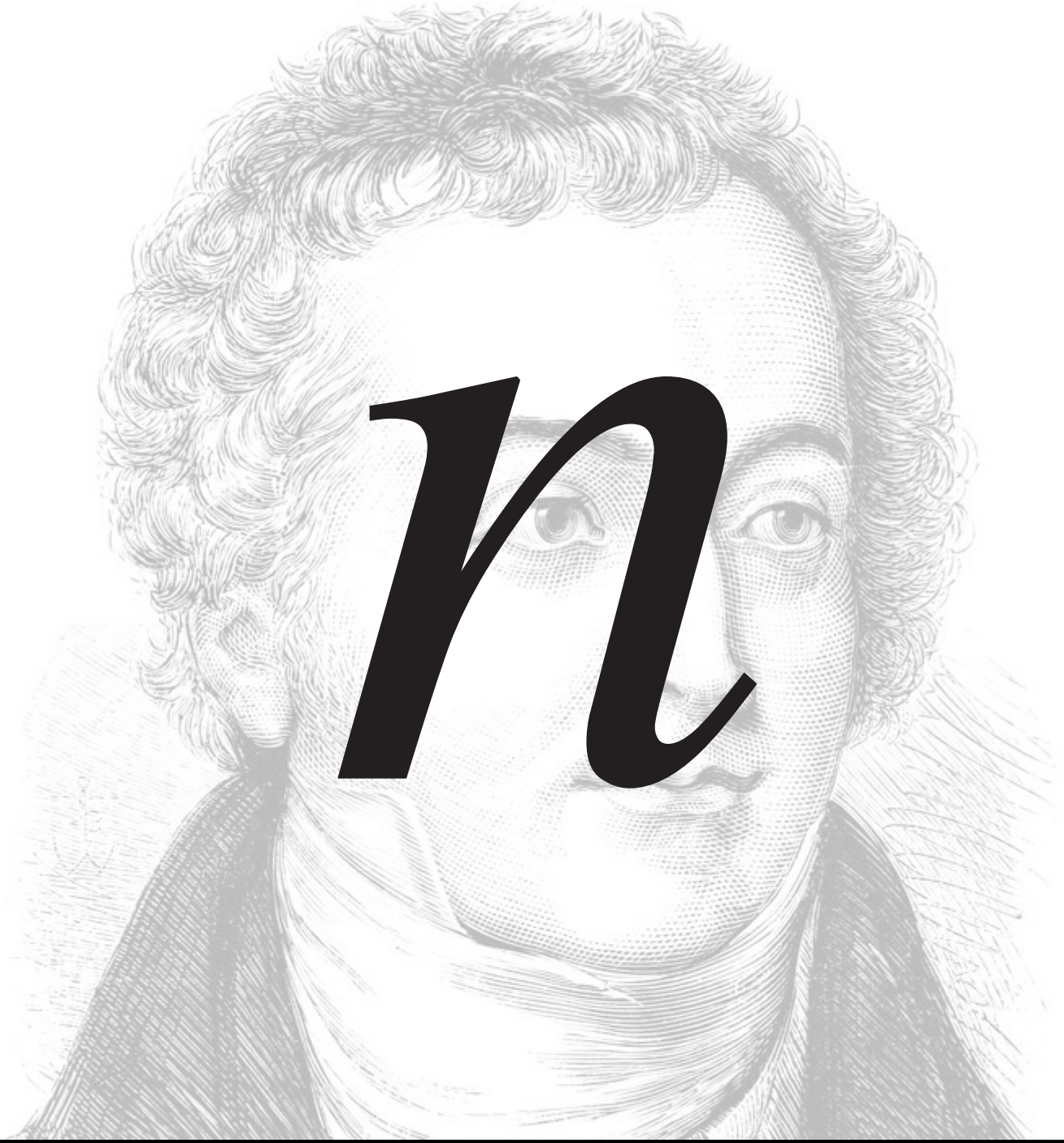
Less is more:
Extreme optics with zero refractive index



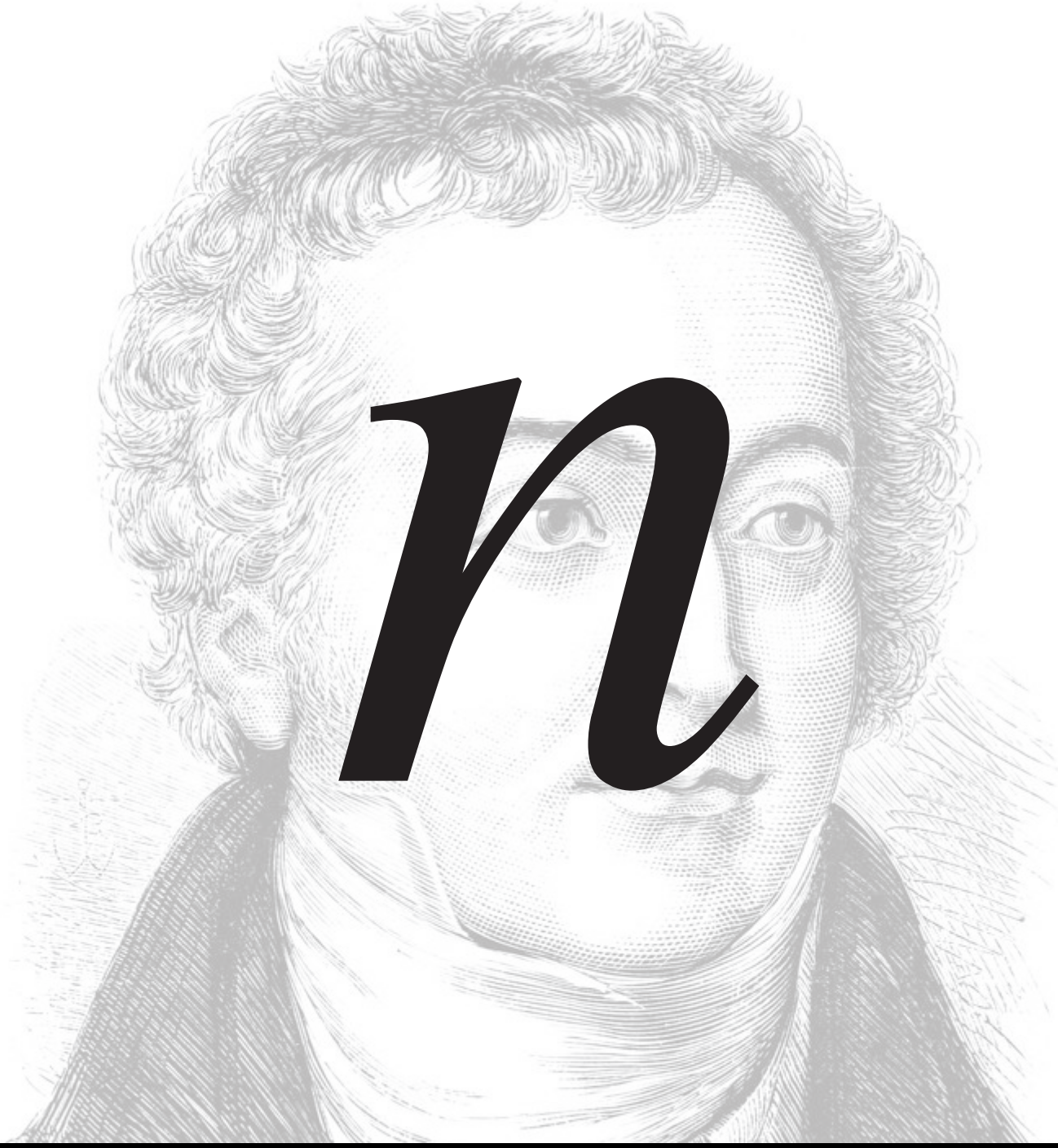
Trinity College
Dublin, Ireland, 7 April 2016



n

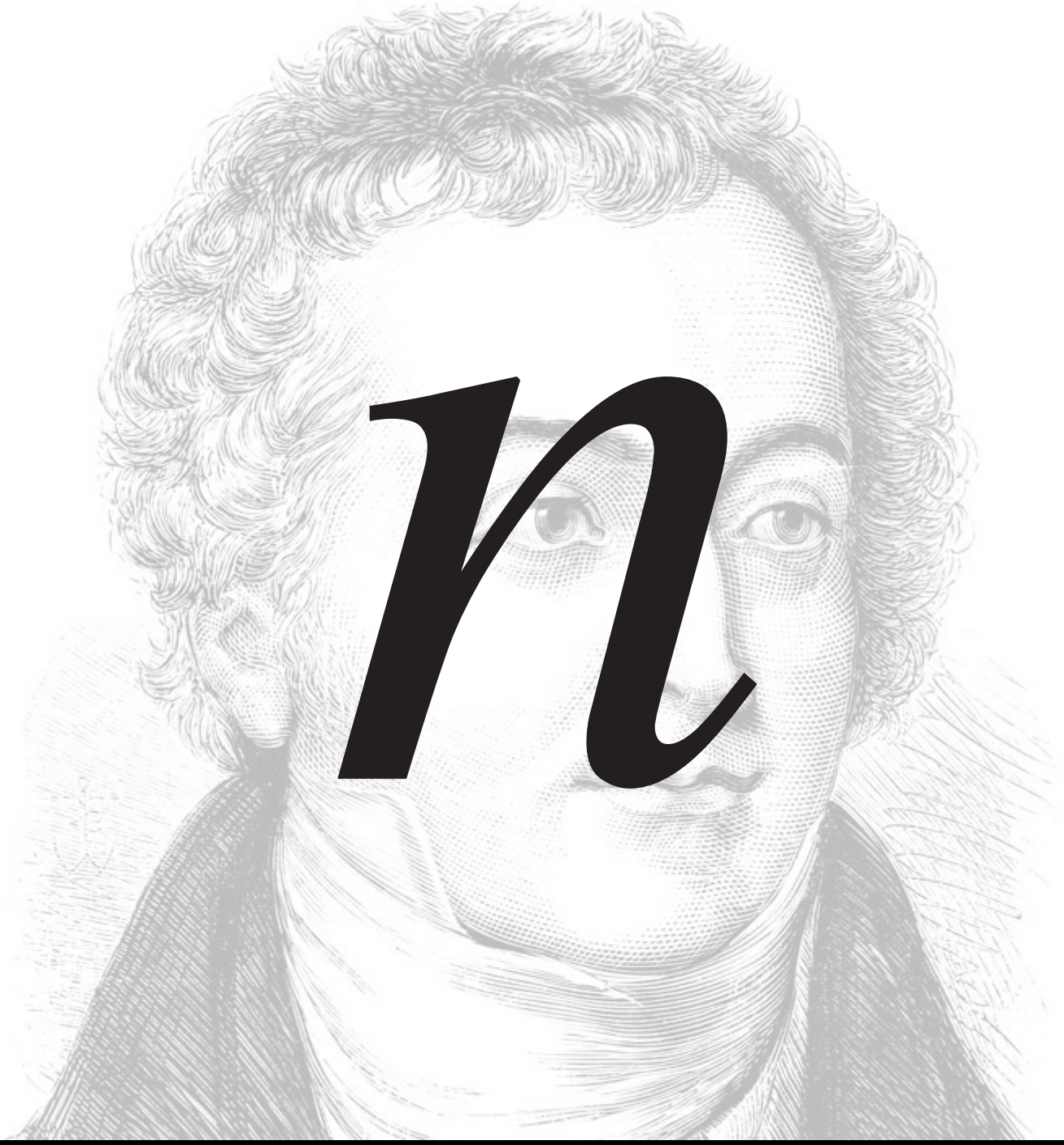


n



1 index

2 zero index



n

1 index

2 zero index

3 experiments

Propagation of EM wave

Propagation of EM wave

governed by wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

Propagation of EM wave

governed by wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

Solution:

$$\vec{E} = \vec{E}_o \, e^{i\,(kx - \omega t)}$$

Propagation of EM wave

governed by wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

Solution:

$$\vec{E} = \vec{E}_o\, e^{i\,(kx - \omega t)}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}}\, c = \frac{1}{n}\, c$$

Propagation of EM wave

governed by wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

Solution:

$$\vec{E} = \vec{E}_o \, e^{i\,(kx - \omega t)}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}}\, c = \frac{1}{n}\, c$$

and

$$n = \sqrt{\epsilon\mu}\ .$$

Propagation of EM wave

governed by wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

Solution:

$$\vec{E} = \vec{E}_o\, e^{i\,(kx - \omega t)}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}}\, c = \frac{1}{n}\, c$$

and

$$n = \sqrt{\epsilon\mu}\ .$$

In dispersive media $n = n\,(\omega)$.

Index of refraction

$$n = \sqrt{\epsilon\mu}$$

Index of refraction

$$n = \sqrt{\epsilon\mu}$$

So $n(\omega)$ determined by response of material to external fields

Index of refraction

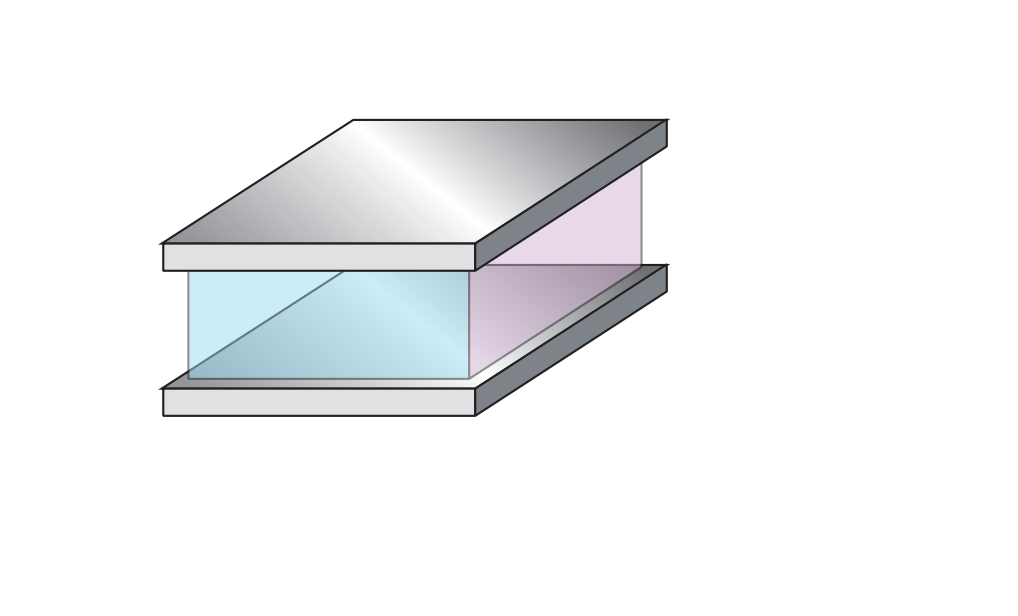
$$n = \sqrt{\epsilon\mu}$$

So $n(\omega)$ determined by response of material to external fields

Index of refraction

$$n = \sqrt{\epsilon\mu}$$

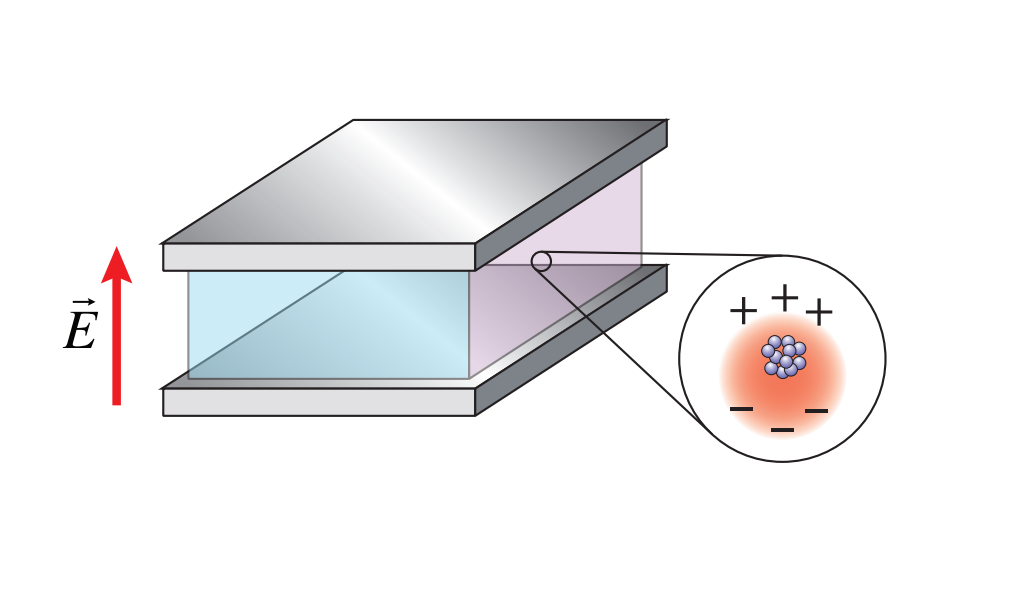
So $n(\omega)$ determined by response of material to external fields



Index of refraction

$$n = \sqrt{\epsilon\mu}$$

So $n(\omega)$ determined by response of material to external fields

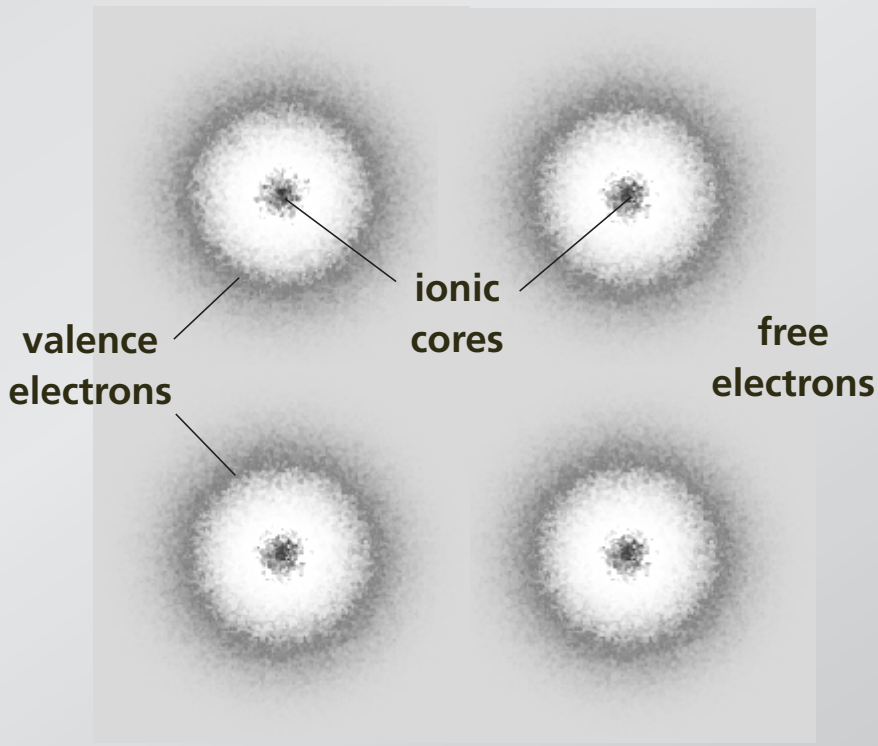


$\epsilon(\omega)$ measure of attenuation of electric field

Index of refraction

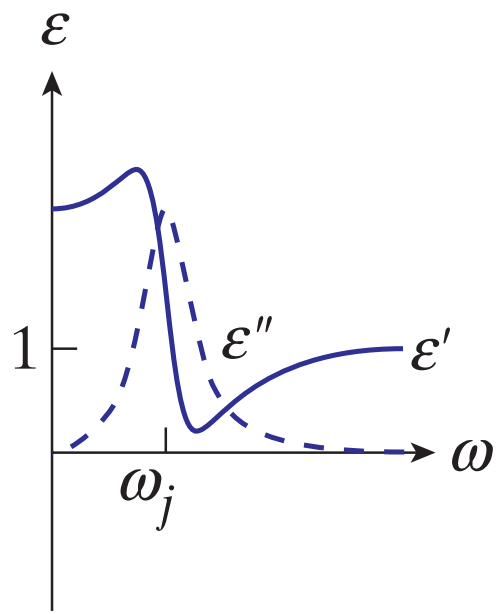
$$n = \sqrt{\epsilon\mu}$$

So $n(\omega)$ determined by response of material to external fields

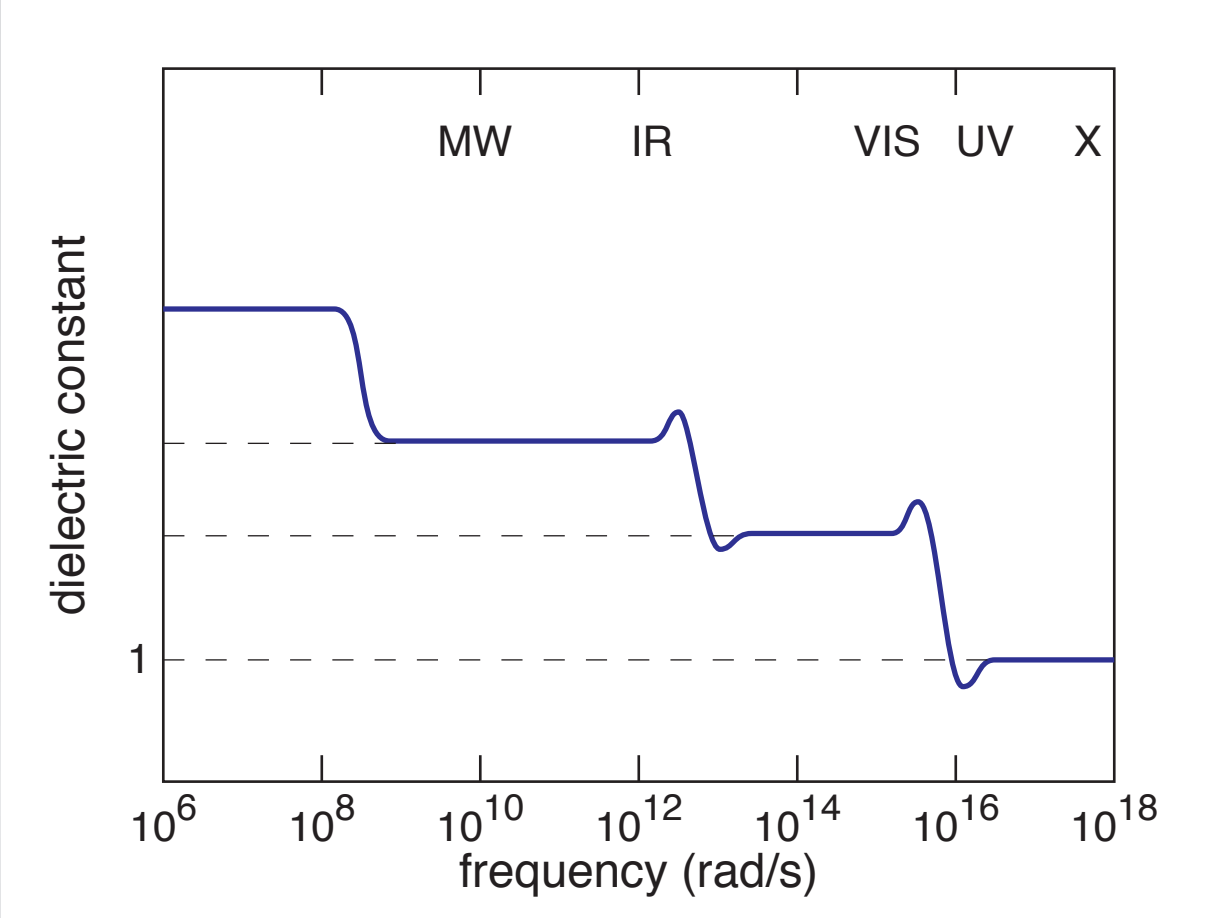


Dielectric constant

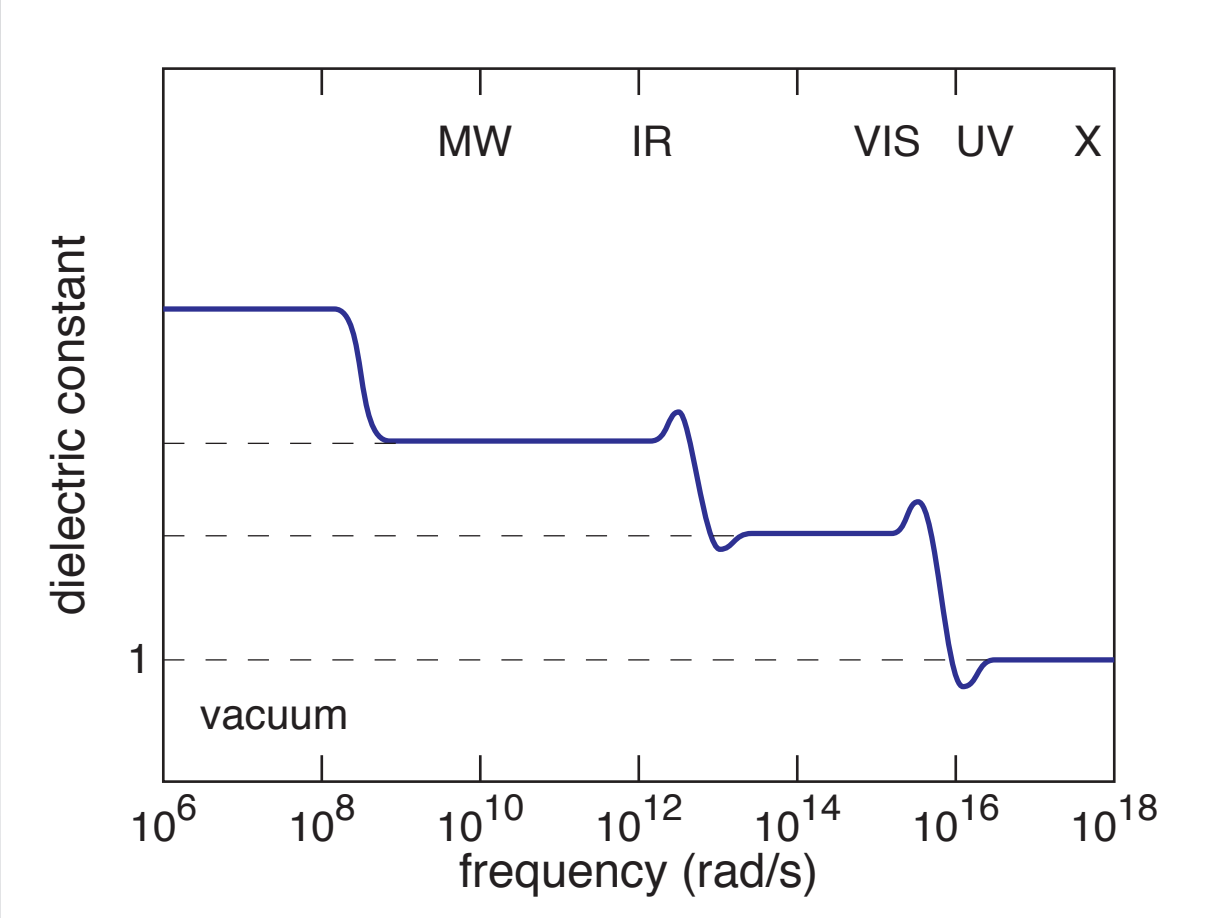
Lorentz oscillator



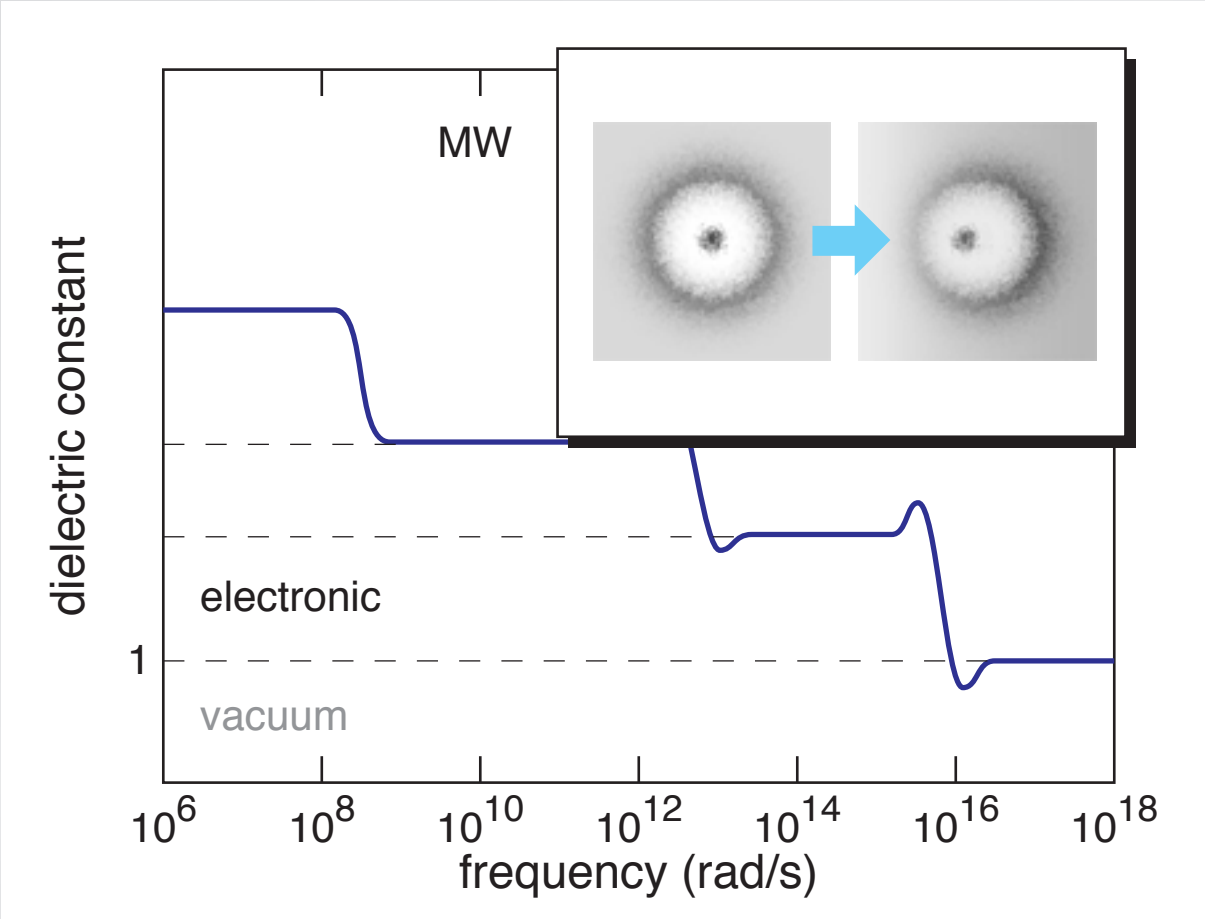
Dielectric constant



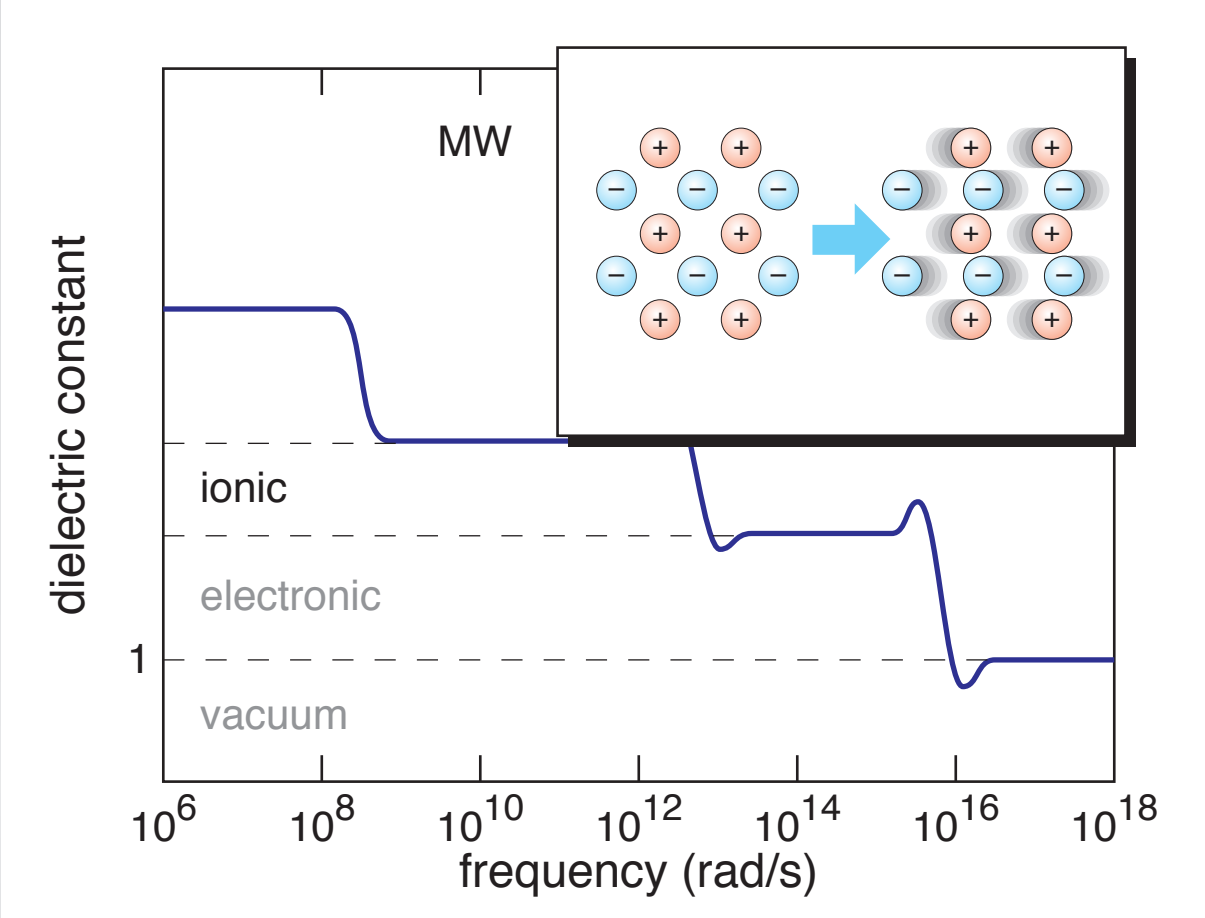
Dielectric constant



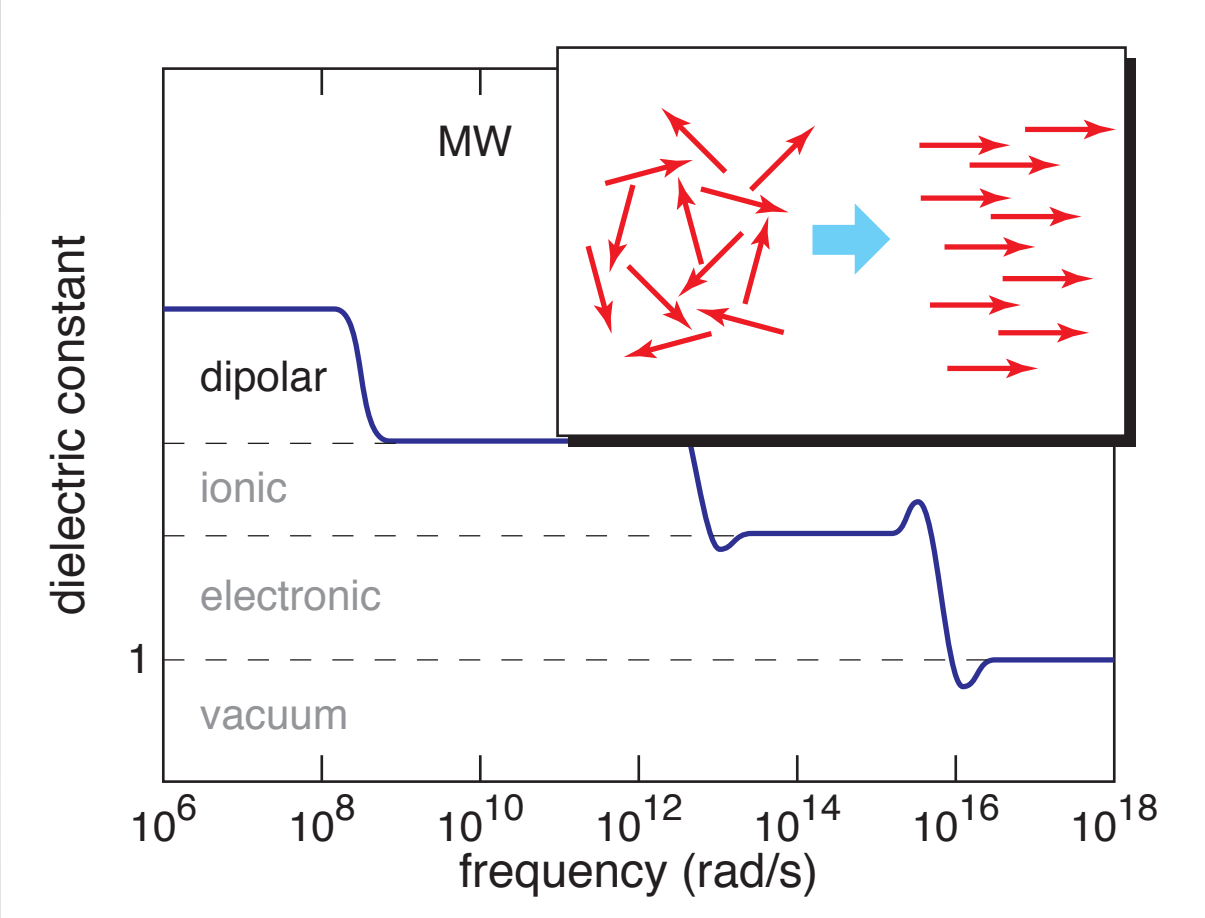
Dielectric constant



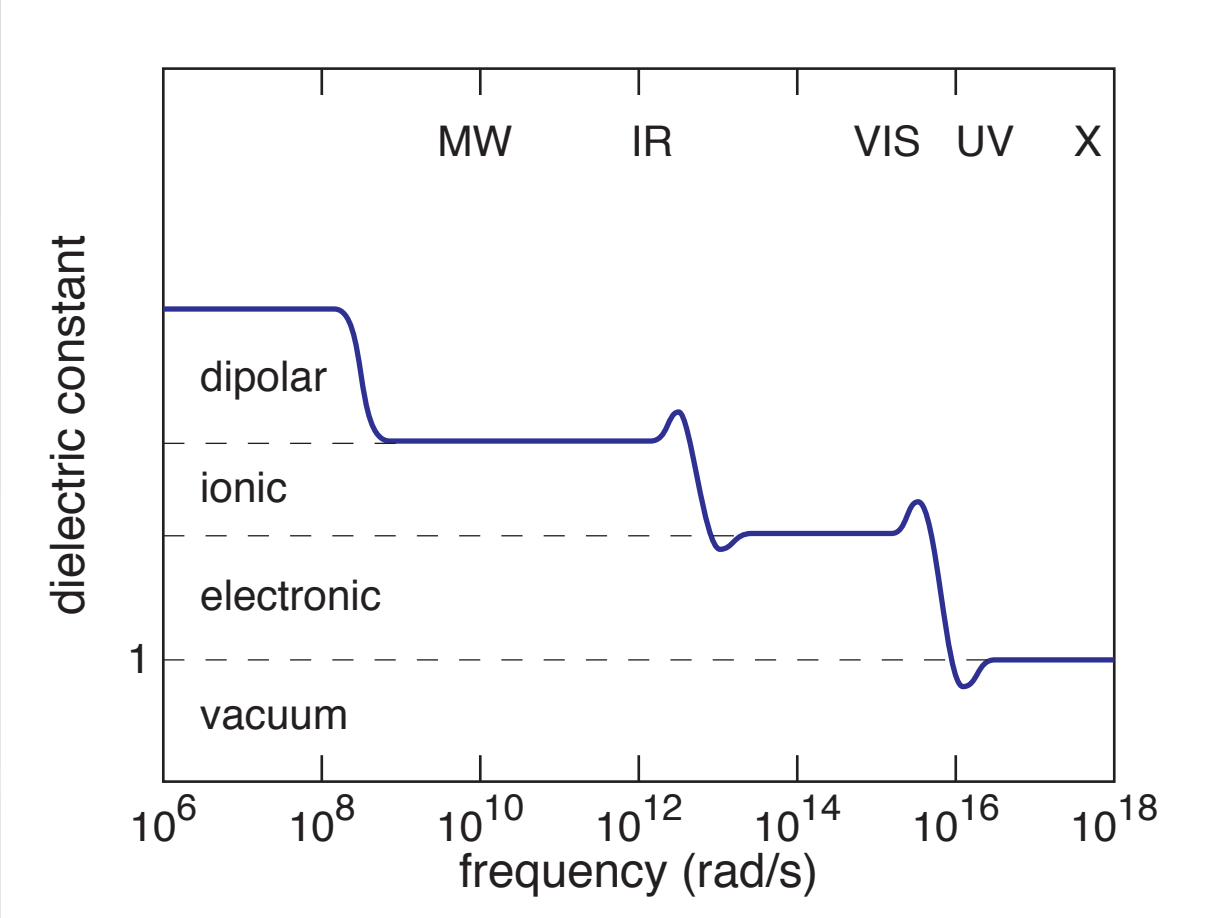
Dielectric constant



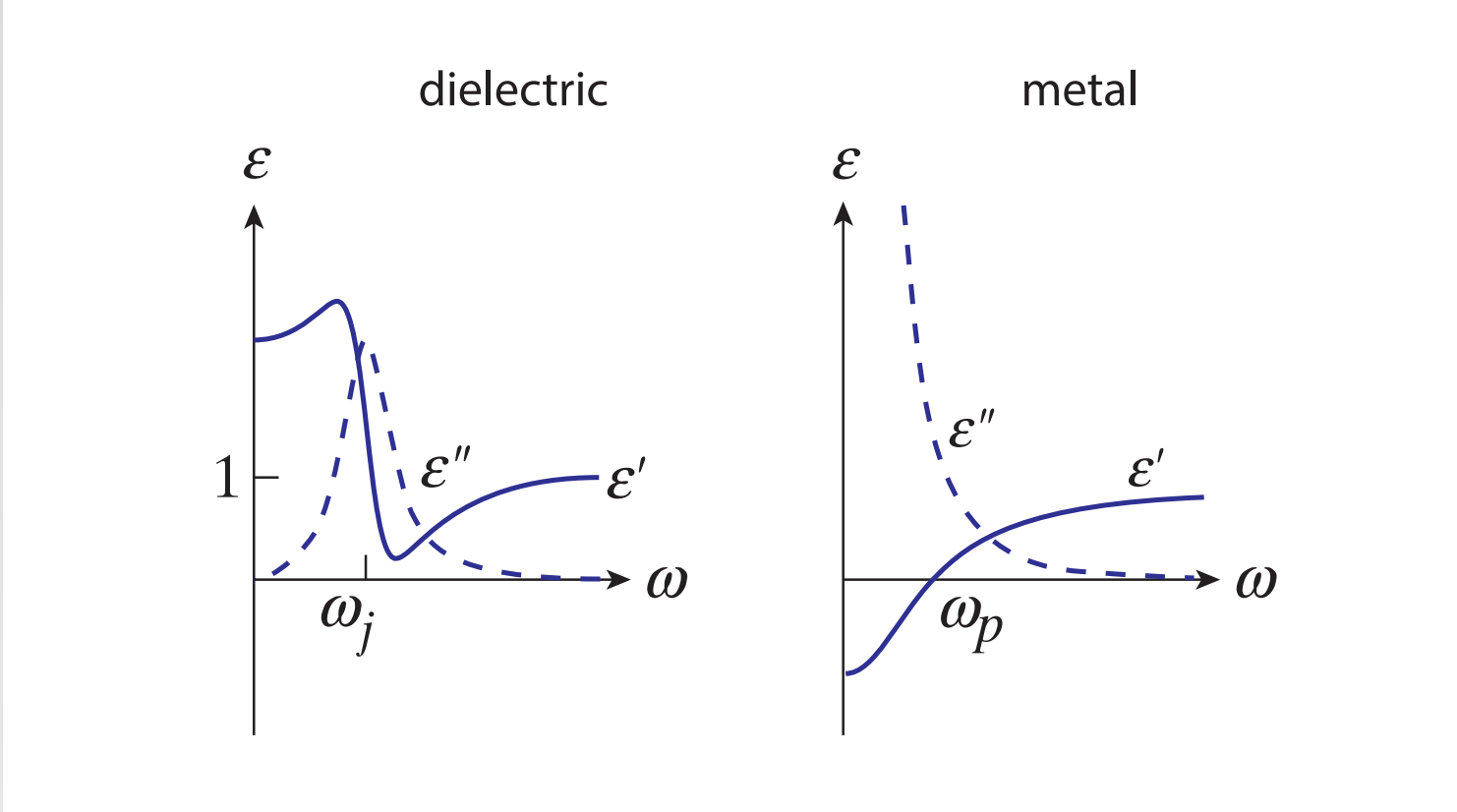
Dielectric constant



Dielectric constant

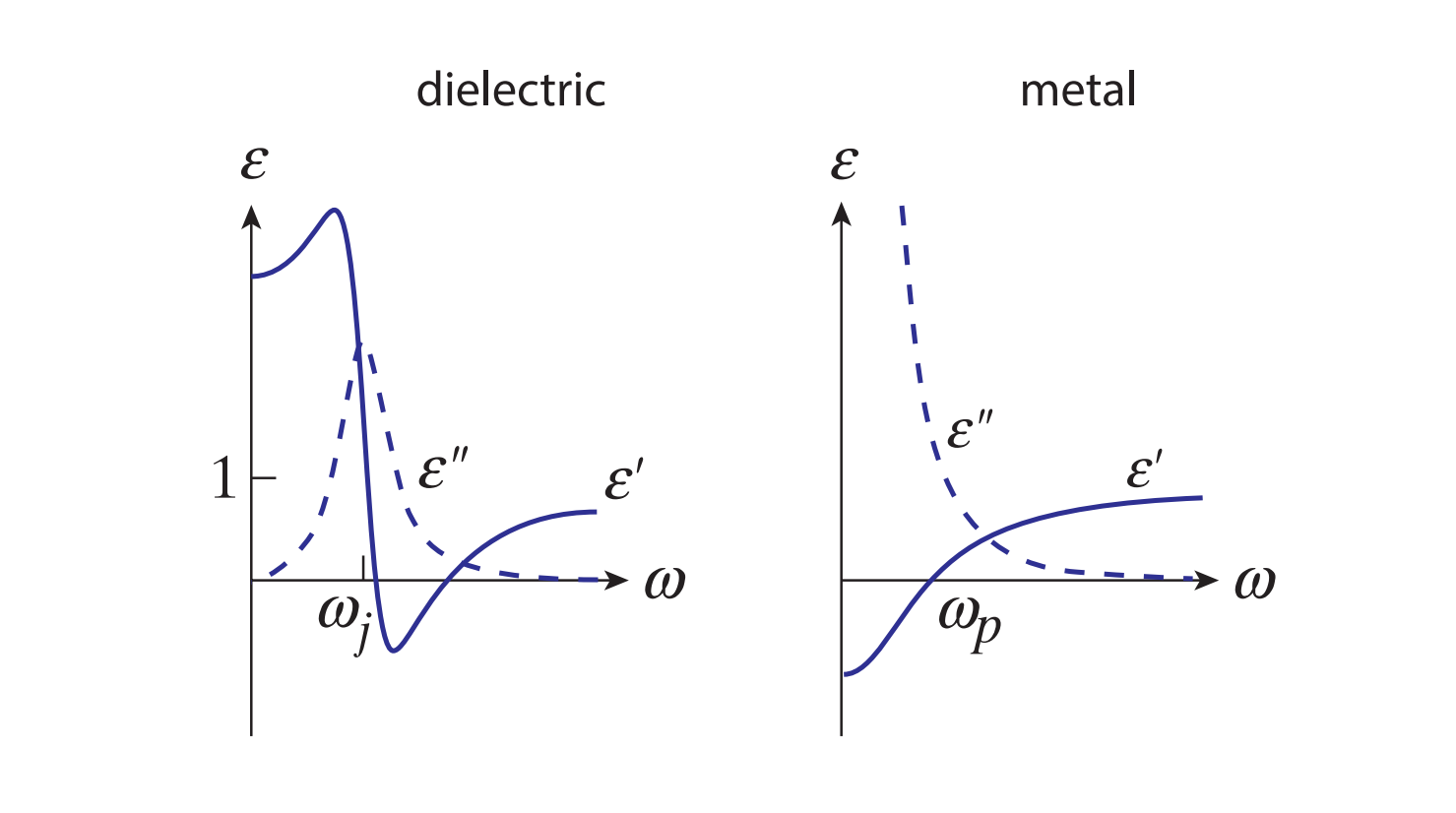


Lorentz and Drude models



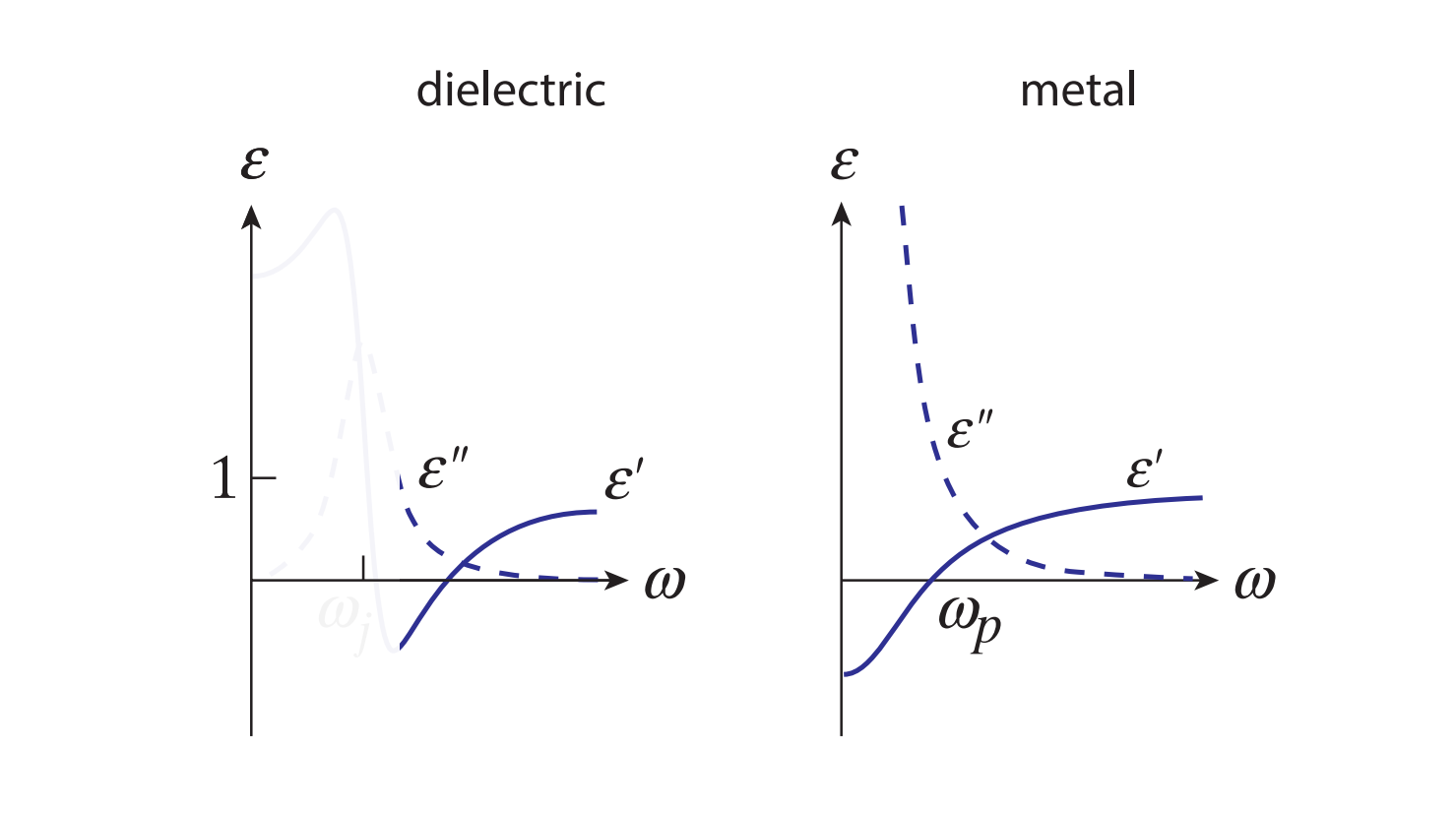
Lorentz and Drude models

for a strong (dielectric) resonance ε can become negative



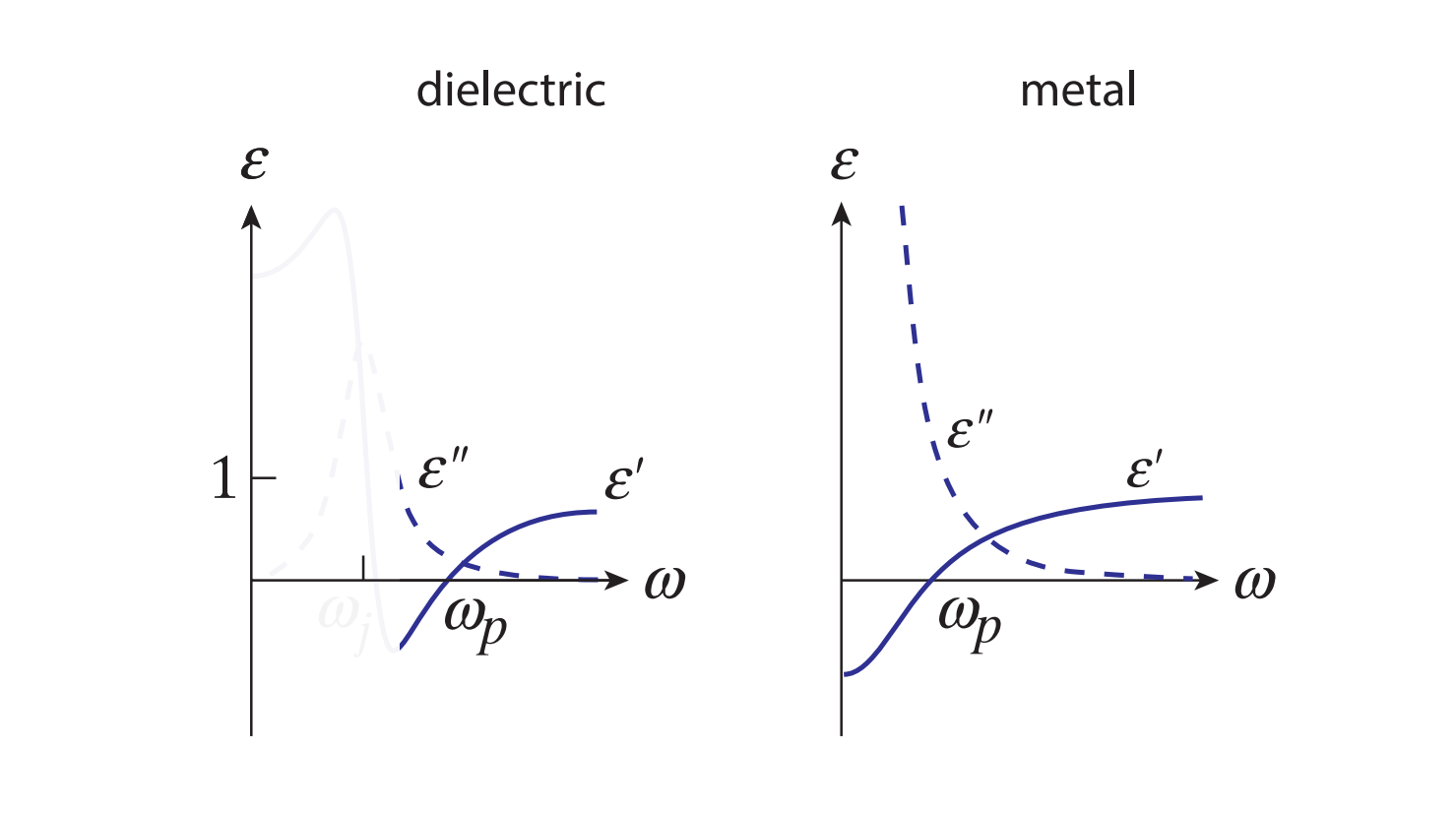
Lorentz and Drude models

valence electrons in dielectric then behave like a plasma



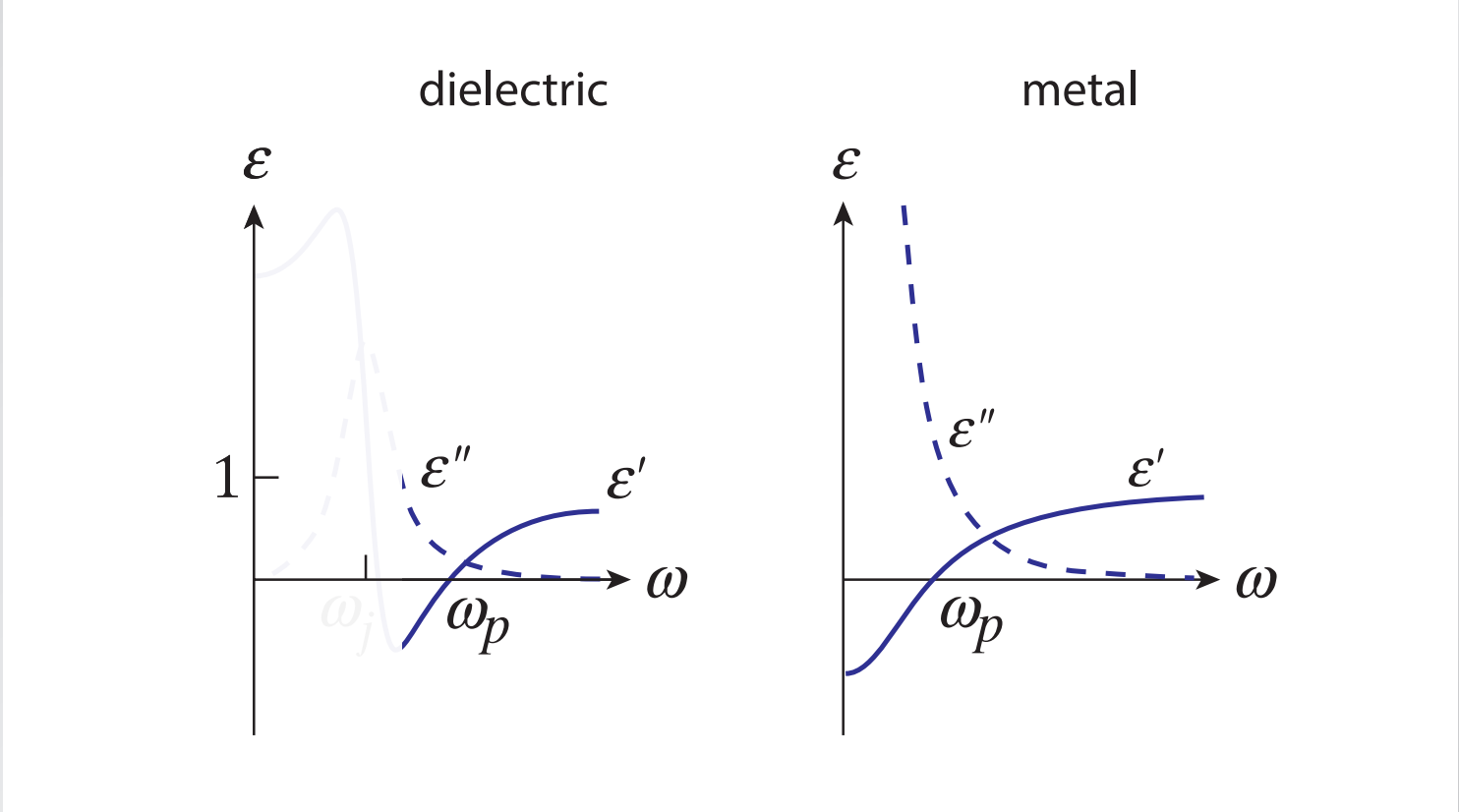
Lorentz and Drude models

with plasma frequency above the resonance



Lorentz and Drude models

(and far below the UV region)



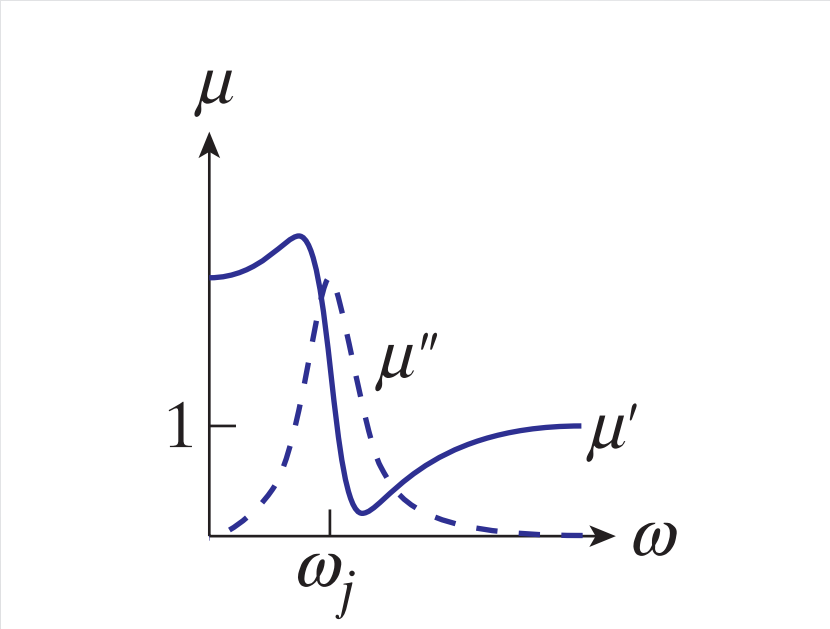
Index also determined by magnetic response

$$n = \sqrt{\epsilon\mu}$$

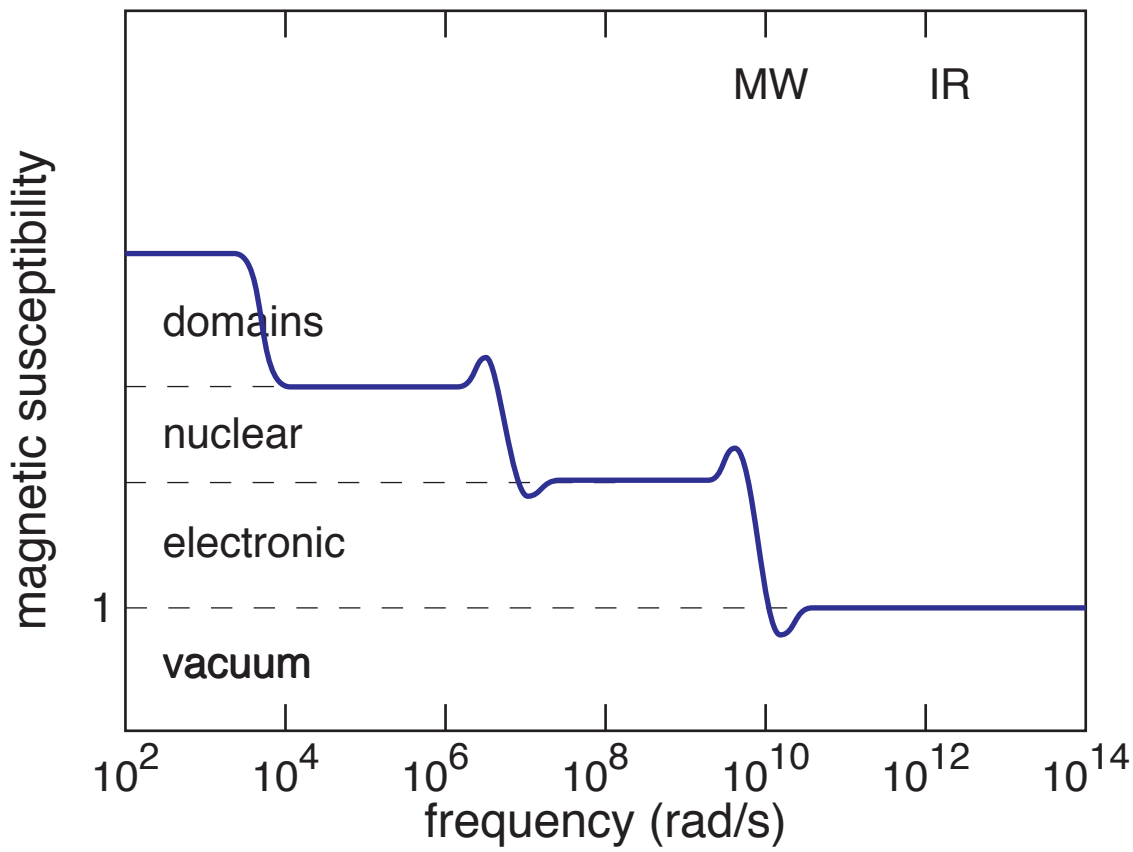
Index also determined by magnetic response

$$n = \sqrt{\epsilon\mu}$$

and magnetic response shows similar resonances

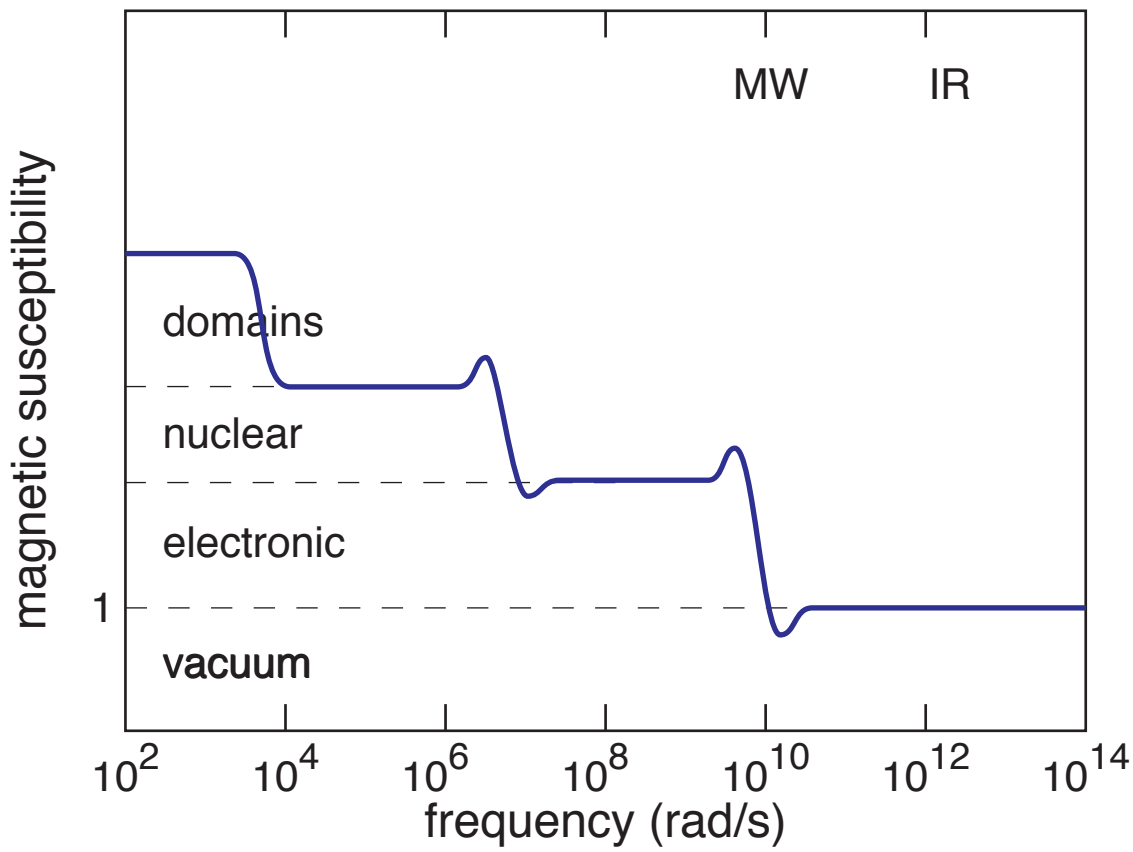


Magnetic response



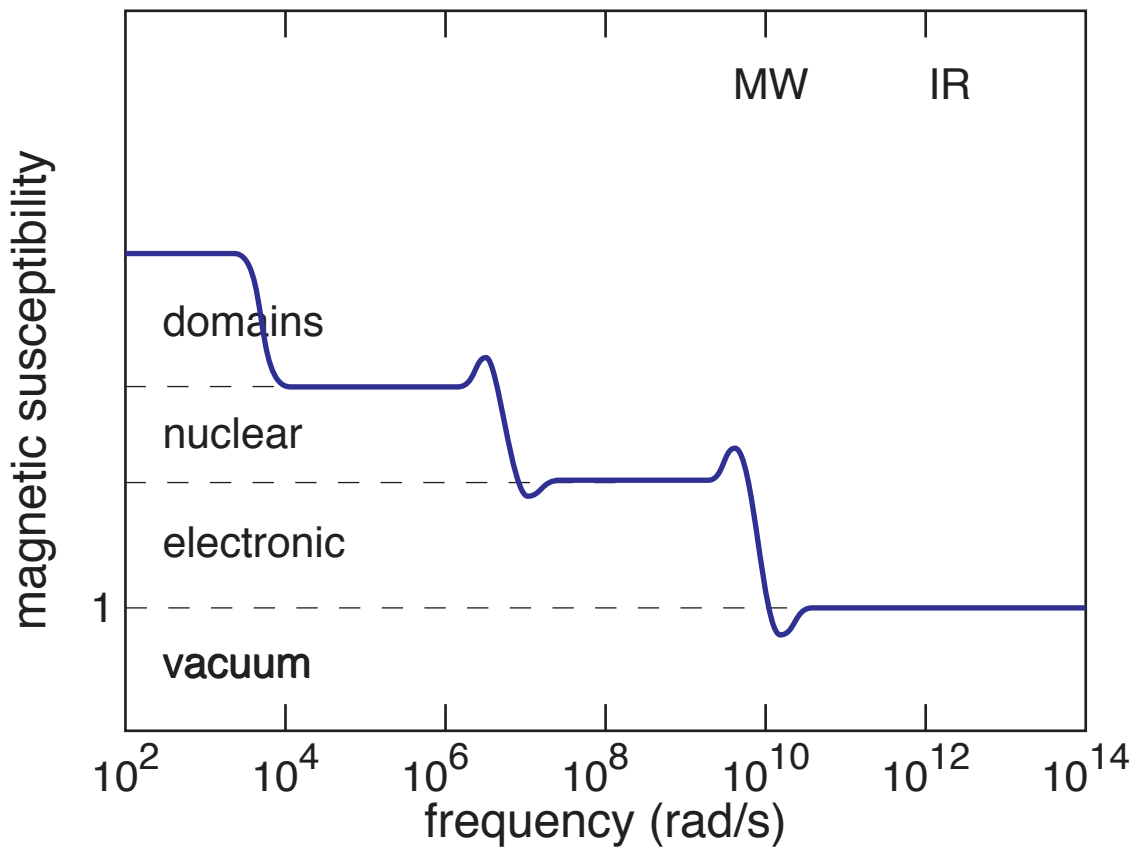
Magnetic response

but magnetic resonances occur below optical frequencies



Magnetic response

so, in optical regime, $\mu \approx 1$



Index of refraction

$$n = \sqrt{\epsilon\mu}$$

Both ϵ and μ are complex and their real parts can be negative.

Index of refraction

$$n = \sqrt{\epsilon\mu}$$

Both ϵ and μ are complex and their real parts can be negative.

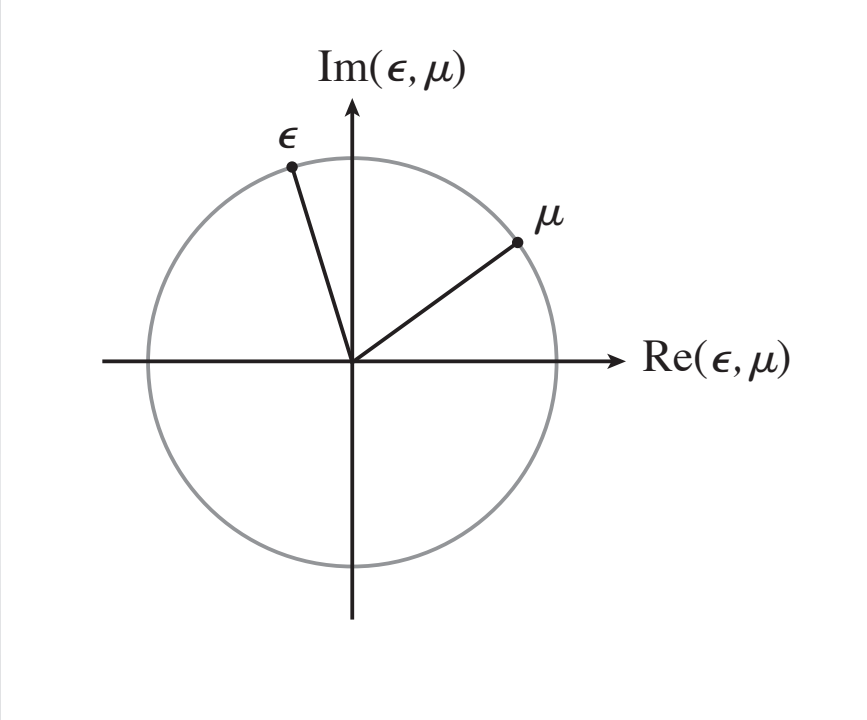
What happens when $\mathrm{Re}\epsilon$ and/or $\mathrm{Re}\mu$ is negative?

Write complex quantities as

$$\varepsilon = |\varepsilon| e^{i\theta} \qquad \mu = |\mu| e^{i\phi}$$

Write complex quantities as

$$\varepsilon = |\varepsilon| e^{i\theta} \qquad \mu = |\mu| e^{i\phi}$$

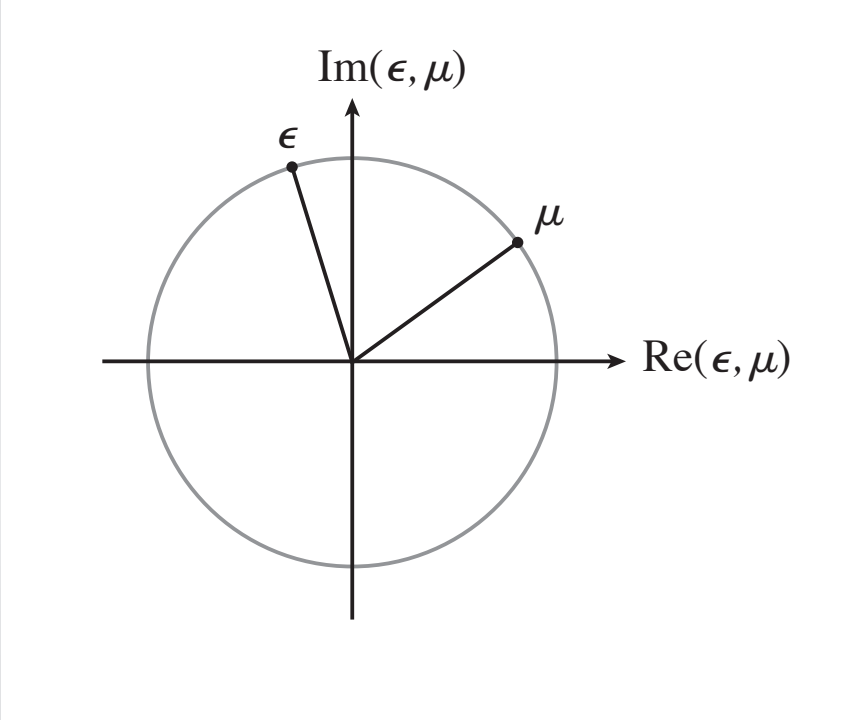


Write complex quantities as

$$\varepsilon = |\varepsilon| e^{i\theta} \qquad \mu = |\mu| e^{i\phi}$$

Index

$$n = \sqrt{|\varepsilon||\mu|}\, e^{i\frac{\theta+\phi}{2}}$$

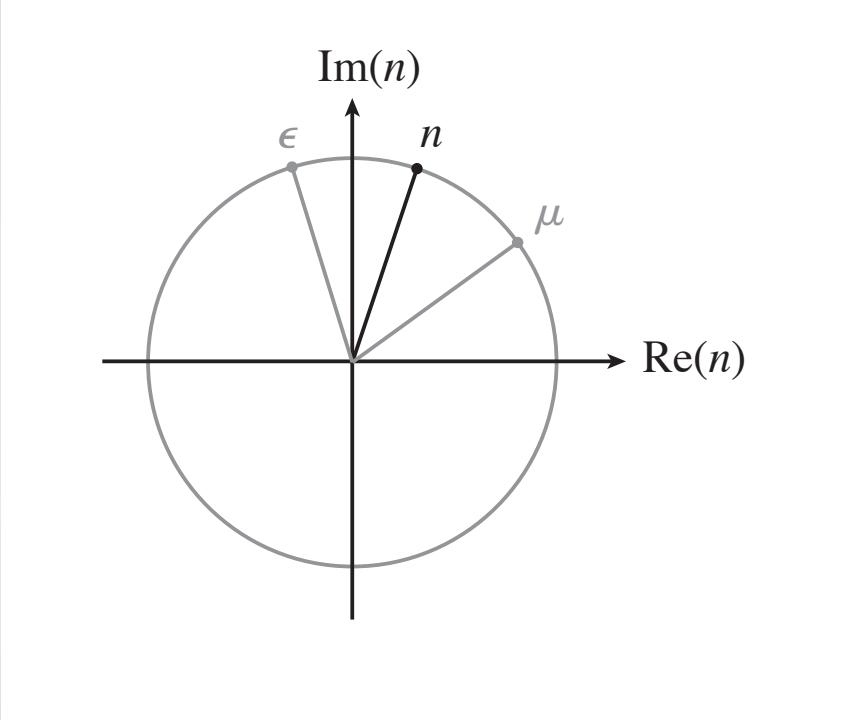


Write complex quantities as

$$\varepsilon = |\varepsilon| e^{i\theta} \qquad \mu = |\mu| e^{i\phi}$$

Index

$$n = \sqrt{|\varepsilon||\mu|}\, e^{i\frac{\theta+\phi}{2}}$$

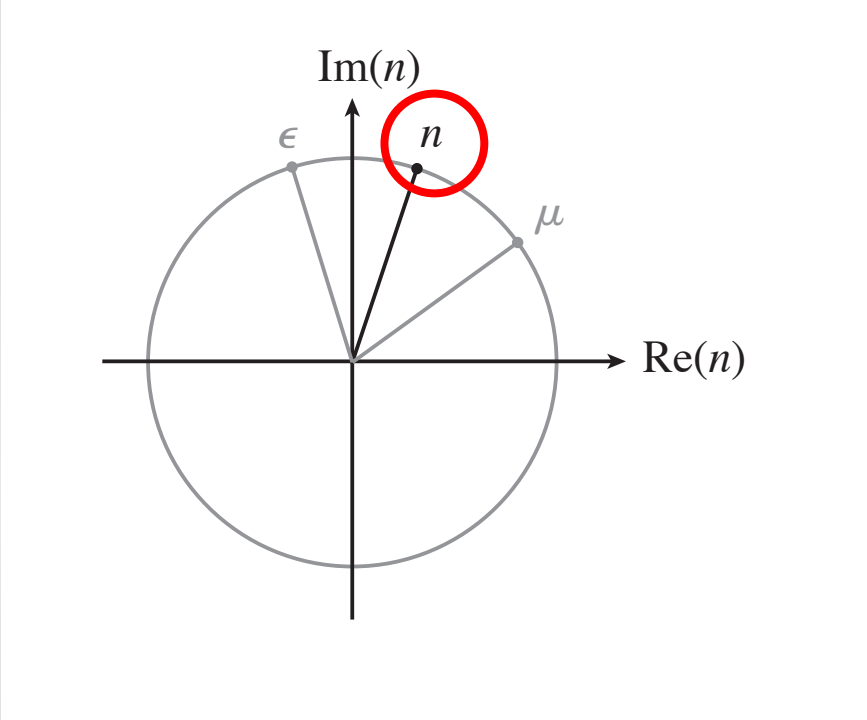


Write complex quantities as

$$\varepsilon = |\varepsilon| e^{i\theta} \qquad \mu = |\mu| e^{i\phi}$$

Index

$$n = \sqrt{|\varepsilon||\mu|}\, e^{i\frac{\theta+\phi}{2}}$$

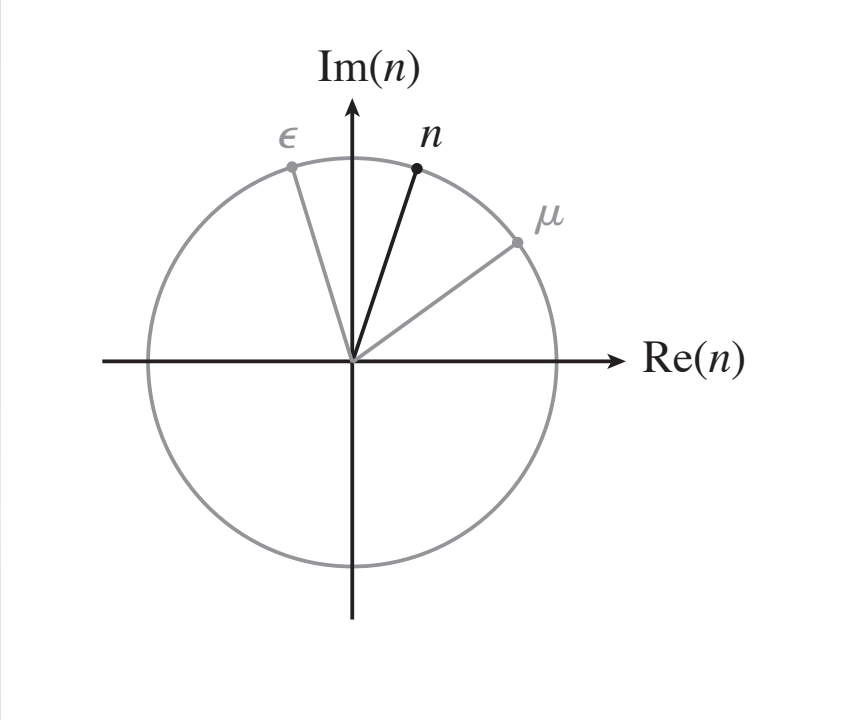


Q: Is this only possible value?

1. yes
2. no, there's one more
3. there are many more
4. it depends

1 index

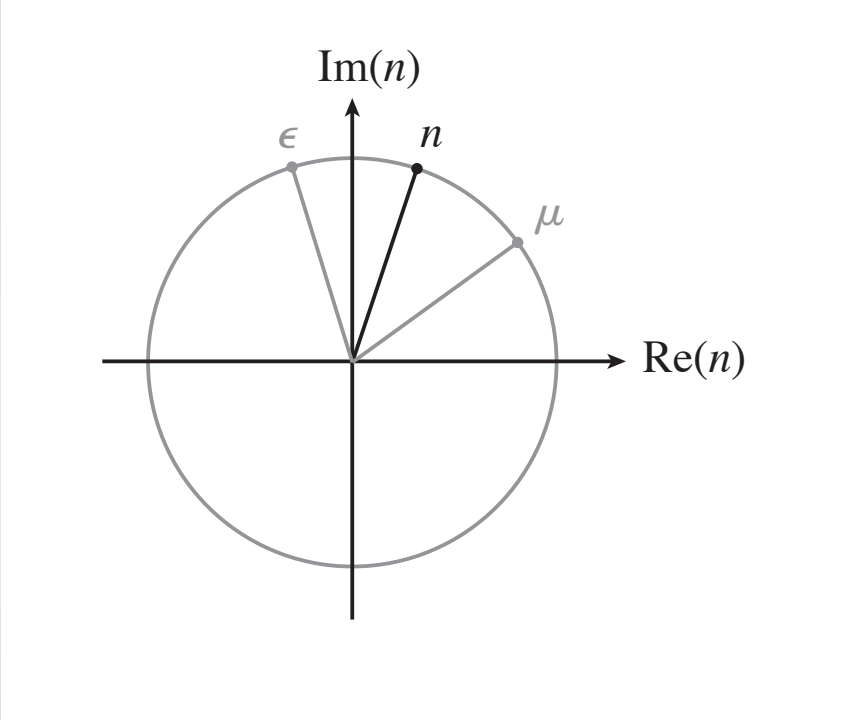
There *is* another root…



There *is* another root…

Can add 2π to exponent

$$e^{+i(\theta+\phi)} = e^{+i[\theta+\phi+2\pi]}$$



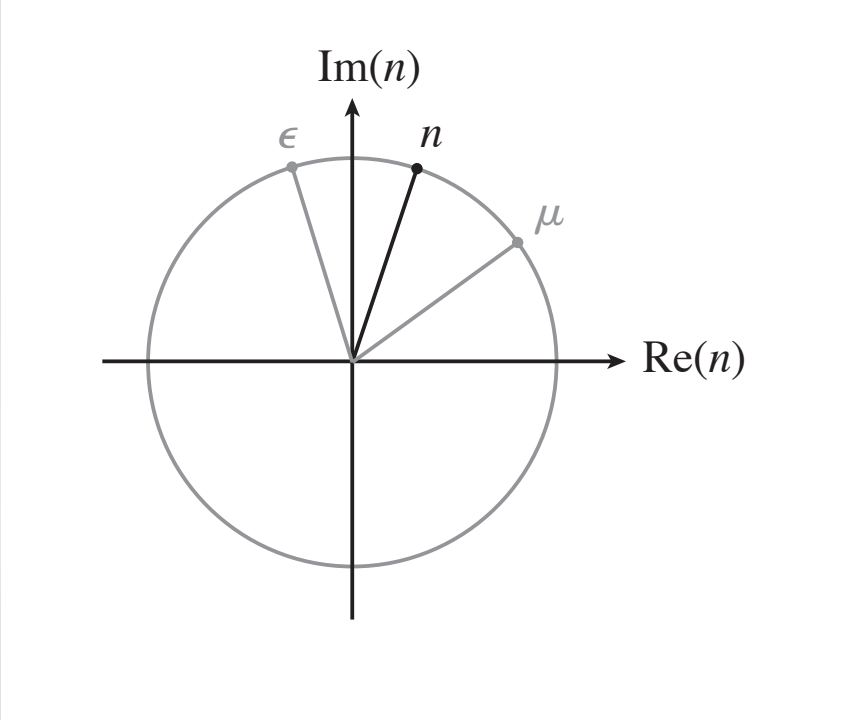
There *is* another root…

Can add 2π to exponent

$$e^{+i(\theta+\phi)} = e^{+i[\theta+\phi+2\pi]}$$

and so

$$n = \sqrt{|\varepsilon||\mu|}\, e^{i\left[\frac{\theta+\phi}{2}+\pi\right]}$$



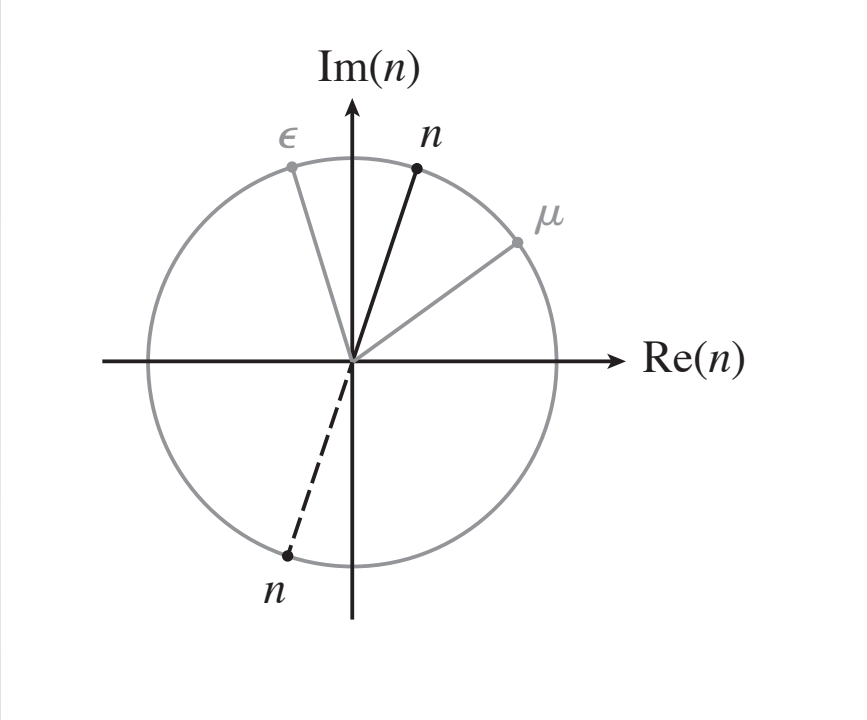
There *is* another root…

Can add 2π to exponent

$$e^{+i(\theta+\phi)} = e^{+i[\theta+\phi+2\pi]}$$

and so

$$n = \sqrt{|\varepsilon||\mu|}\, e^{i\left[\frac{\theta+\phi}{2}+\pi\right]}$$



There *is* another root…

Can add 2π to exponent

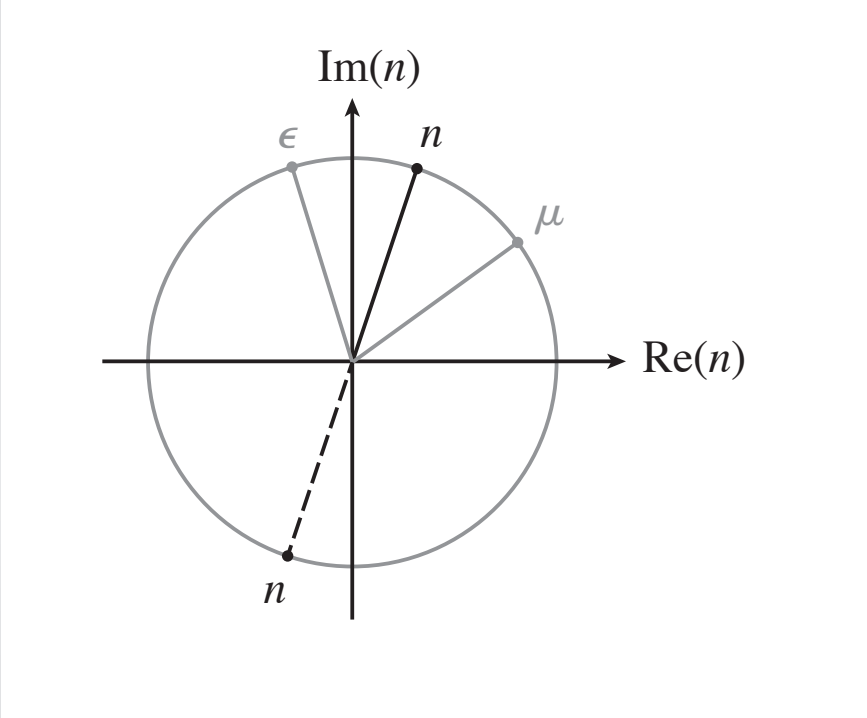
$$e^{+i(\theta+\phi)} = e^{+i[\theta+\phi+2\pi]}$$

and so

$$n = \sqrt{|\varepsilon||\mu|}\, e^{i\left[\frac{\theta+\phi}{2}+\pi\right]}$$

but…

$$k = \frac{2\pi n}{\lambda_o} = \frac{2\pi(n' + in'')}{\lambda_o} = k' + ik''$$



There *is* another root…

Can add 2π to exponent

$$e^{+i(\theta+\phi)} = e^{+i[\theta+\phi+2\pi]}$$

and so

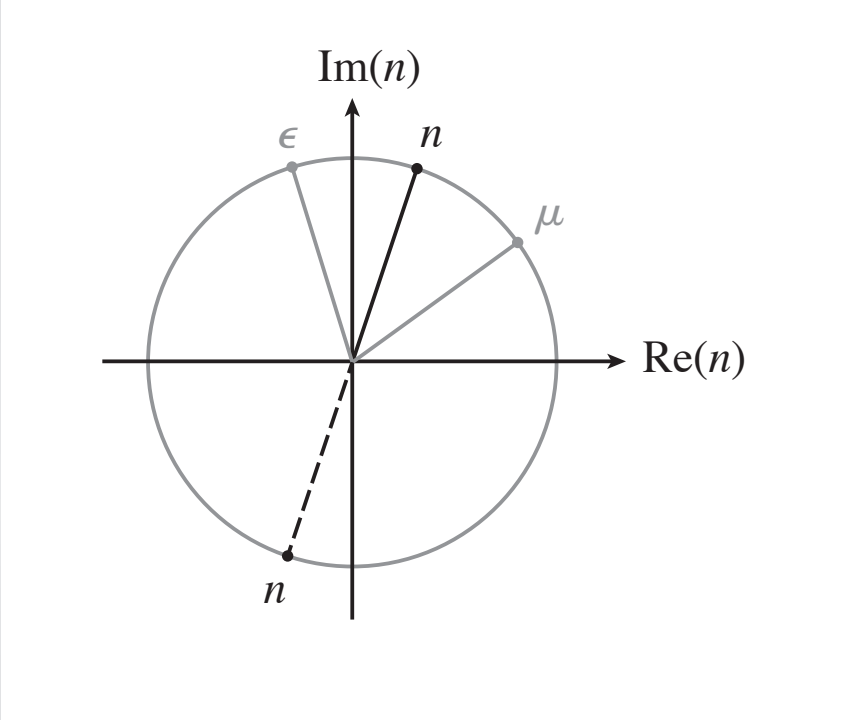
$$n = \sqrt{|\varepsilon||\mu|}\, e^{i\left[\frac{\theta+\phi}{2}+\pi\right]}$$

but…

$$k = \frac{2\pi n}{\lambda_o} = \frac{2\pi(n' + in'')}{\lambda_o} = k' + ik''$$

and

$$E = E_o e^{i(kx-\omega t)} = E_o e^{-k''x} e^{i(k'x-\omega t)}$$



There *is* another root…

Can add 2π to exponent

$$e^{+i(\theta+\phi)} = e^{+i[\theta+\phi+2\pi]}$$

and so

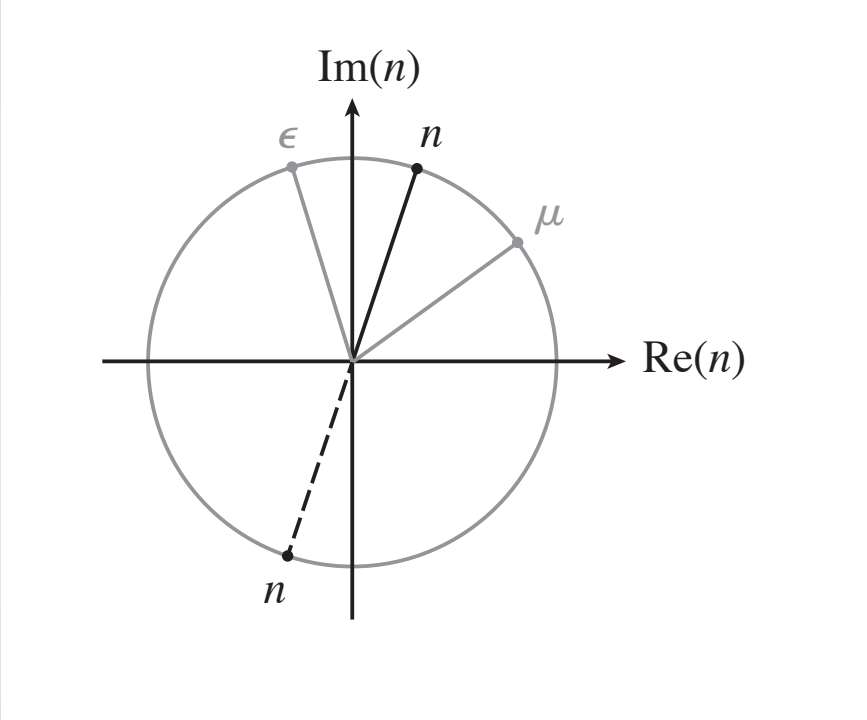
$$n = \sqrt{|\varepsilon||\mu|}\, e^{i\left[\frac{\theta+\phi}{2}+\pi\right]}$$

but…

$$k = \frac{2\pi n}{\lambda_o} = \frac{2\pi(n' + in'')}{\lambda_o} = k' + ik''$$

and

$$E = E_o e^{i(kx-\omega t)} = E_o e^{-k''x} e^{i(k'x-\omega t)}$$



There *is* another root…

Can add 2π to exponent

$$e^{+i(\theta+\phi)} = e^{+i[\theta+\phi+2\pi]}$$

and so

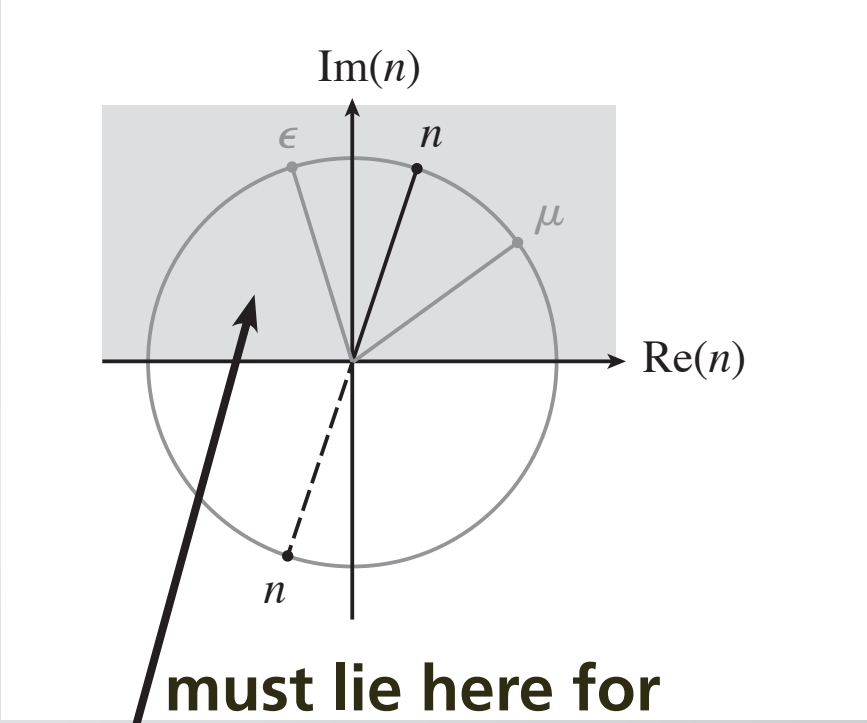
$$n = \sqrt{|\varepsilon||\mu|}\, e^{i\left[\frac{\theta+\phi}{2}+\pi\right]}$$

but…

$$k = \frac{2\pi n}{\lambda_o} = \frac{2\pi(n' + in'')}{\lambda_o} = k' + ik''$$

and

$$E = E_o e^{i(kx-\omega t)} = E_o e^{-k''x} e^{i(k'x-\omega t)}$$



must lie here for passive material

There *is* another root…

Can add 2π to exponent

$$e^{+i(\theta+\phi)} = e^{+i[\theta+\phi+2\pi]}$$

and so

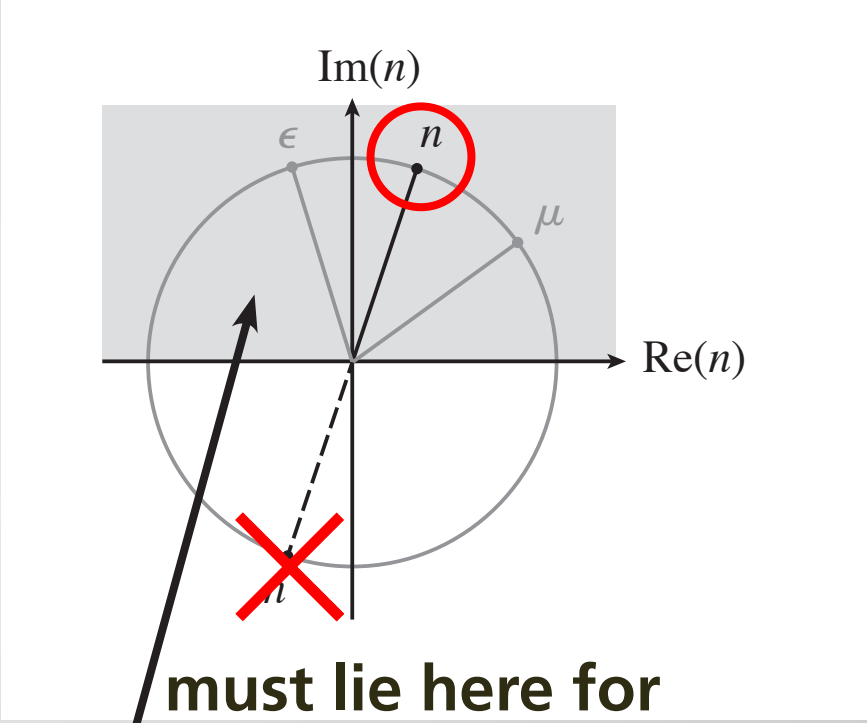
$$n = \sqrt{|\varepsilon||\mu|}\, e^{i\left[\frac{\theta+\phi}{2}+\pi\right]}$$

but…

$$k = \frac{2\pi n}{\lambda_o} = \frac{2\pi(n' + in'')}{\lambda_o} = k' + ik''$$

and

$$E = E_o e^{i(kx-\omega t)} = E_o e^{-k''x} e^{i(k'x-\omega t)}$$



must lie here for passive material

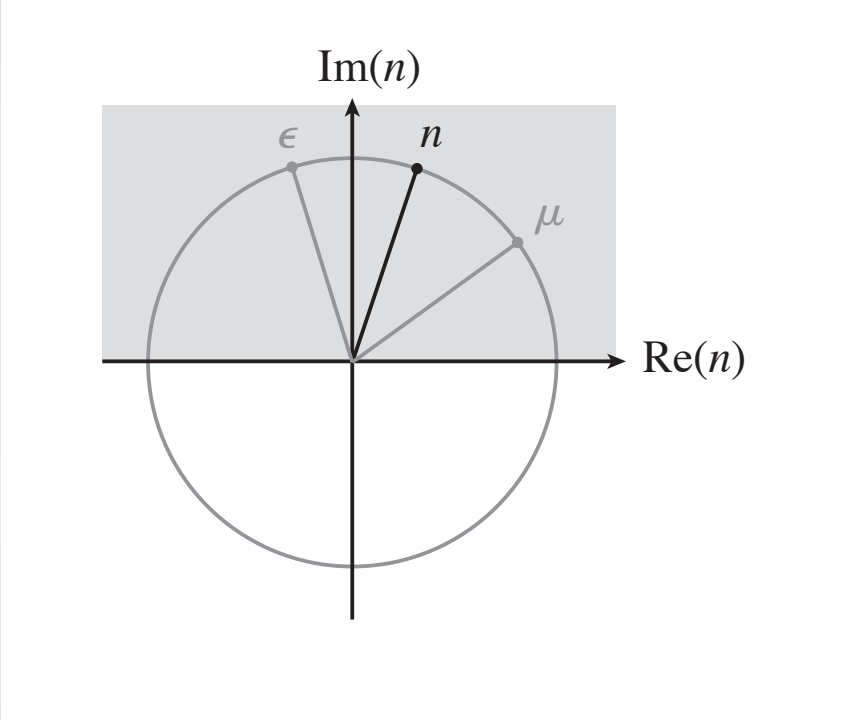
Q: Is this the only possible value?

1. yes ✔

2. no, there's one more

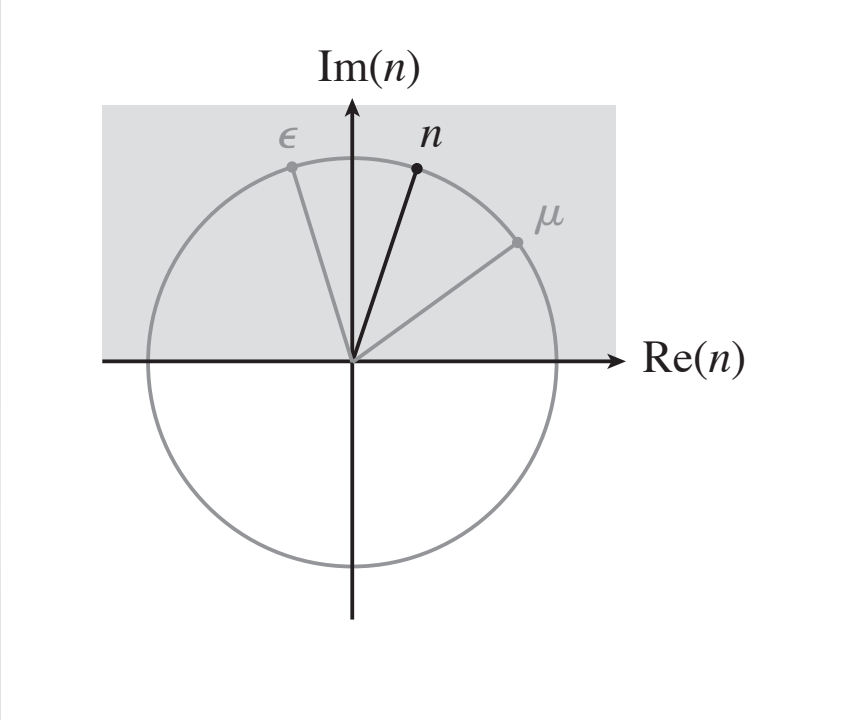
3. there are many more

4. it depends



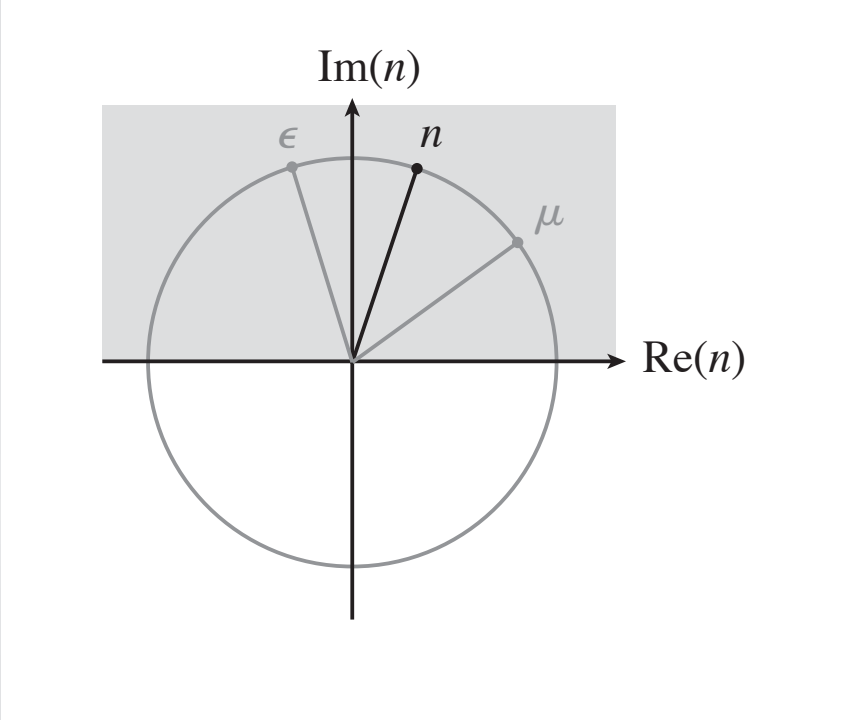
Q: Is this the only possible value?

1. yes ✔
2. no, there's one more ✔
3. there are many more
4. it depends



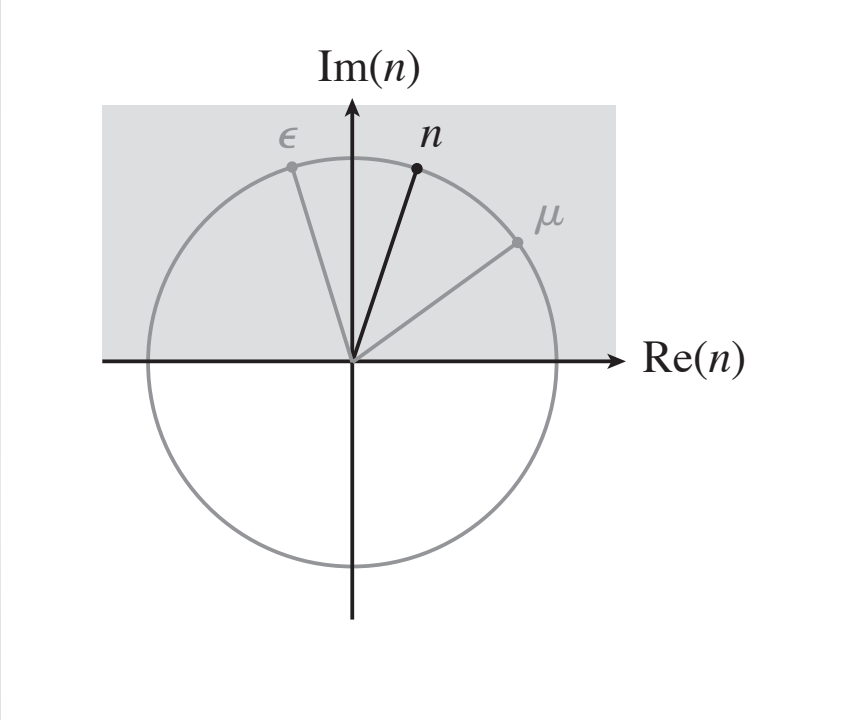
Q: Is this the only possible value?

1. yes ✔
2. no, there's one more ✔
3. there are many more
4. it depends ✔



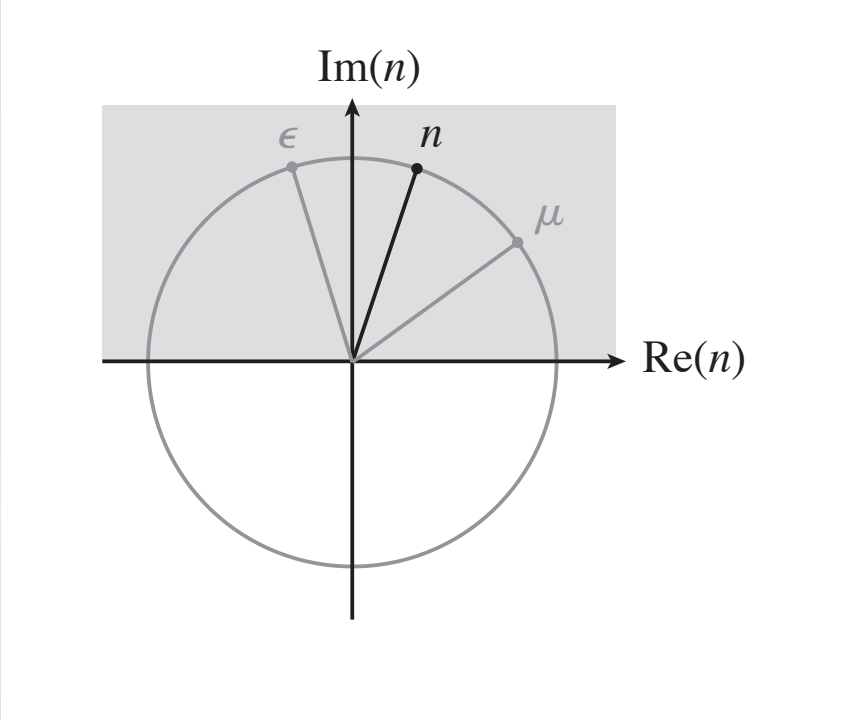
Q: Is this the only possible value?

1. yes ✔
2. no, there's one more ✔
3. there are many more ✔
4. it depends ✔



To find n (passive materials):

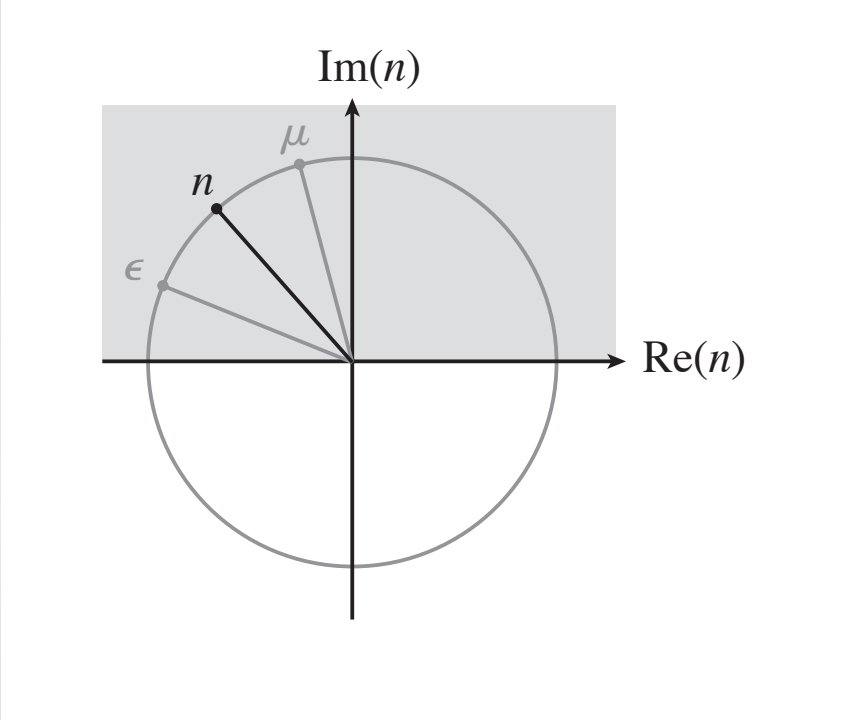
1. **Draw line that bisects ϵ and μ**
2. **Choose upper branch**



What happens when $\mathrm{Re}\epsilon$ and/or $\mathrm{Re}\mu$ is negative?

For certain values of ϵ and μ

we can get a *negative* $\mathrm{Re}(n)$!

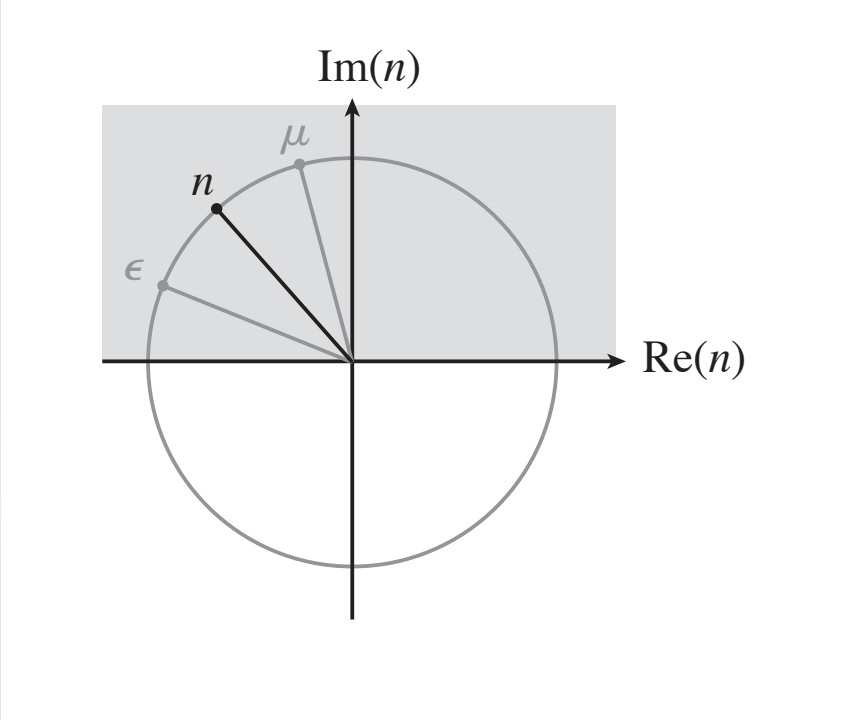


Q: Must both $\mathrm{Re}\epsilon < 0$ and $\mathrm{Re}\mu < 0$

to get a negative $\mathrm{Re}(n)$?

1. yes

2. no

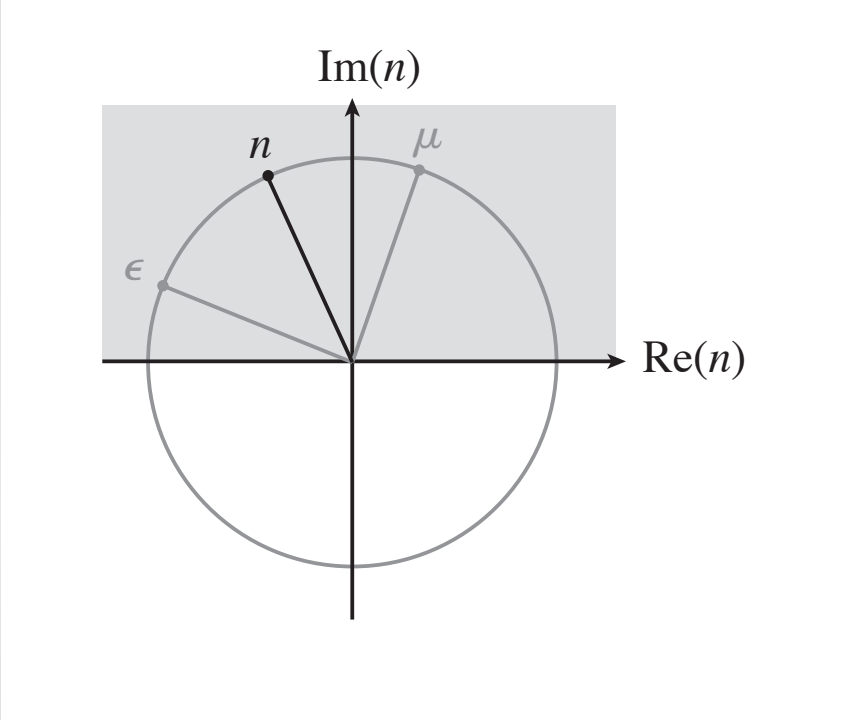


Q: Must both $\mathrm{Re}\epsilon < 0$ and $\mathrm{Re}\mu < 0$

to get a negative $\mathrm{Re}(n)$?

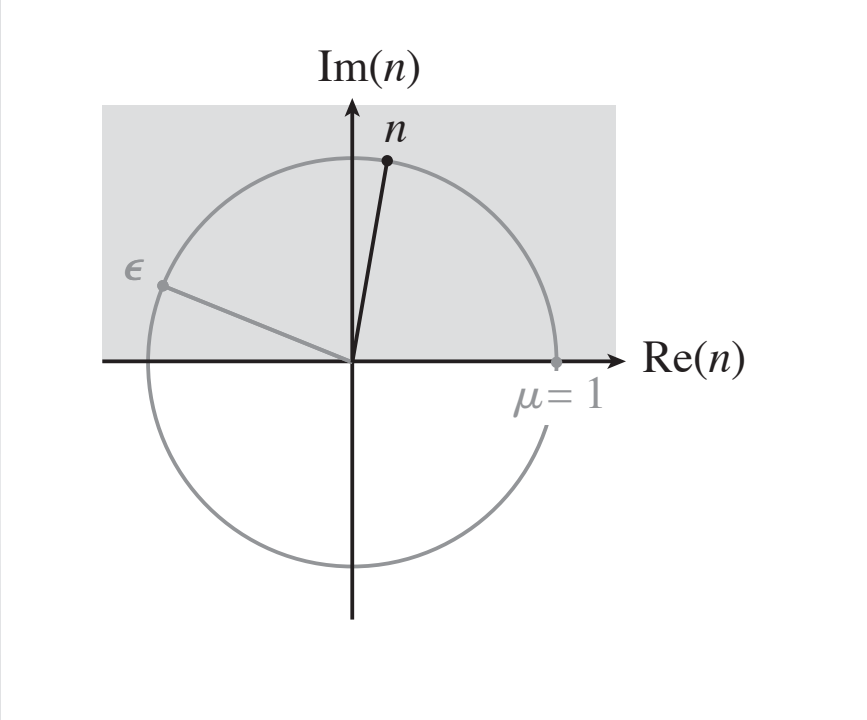
1. yes

2. no ✔



However, need magnetic response

to achieve $\mathrm{Re}(n) \leq 0$!



What happens when $\mathrm{Re}(n) < 0$?

What happens when $\mathrm{Re}(n) < 0$?

Remember

$$k = \frac{2\pi n}{\lambda_o} = \frac{2\pi(n' + in'')}{\lambda_o} = k' + ik''$$

$$E = E_o e^{i(kx - \omega t)} = E_o e^{-k''x} e^{i(k'x - \omega t)}$$

What happens when $\mathrm{Re}(n) < 0$?

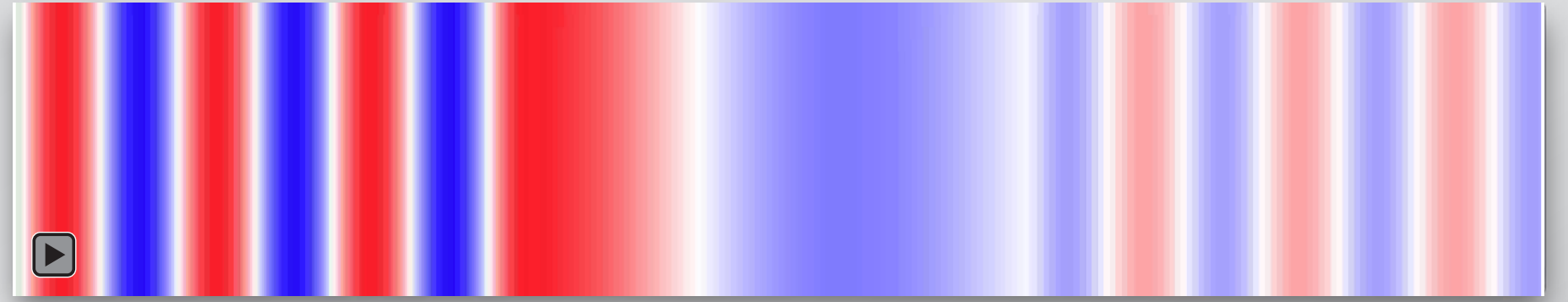
Remember

$$k = \frac{2\pi n}{\lambda_o} = \frac{2\pi(n' + in'')}{\lambda_o} = k' + ik''$$

$$E = E_o e^{i(kx - \omega t)} = E_o e^{-k''x} e^{i(k'x - \omega t)}$$

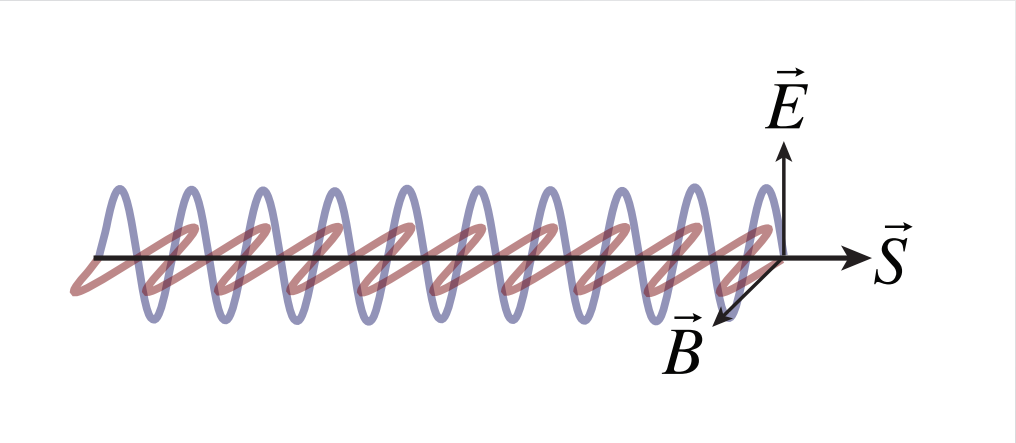
When $\mathrm{Re}(n) < 0$, $k' < 0$, and so phase velocity reversed!

What happens when $\mathrm{Re}(n) < 0$?



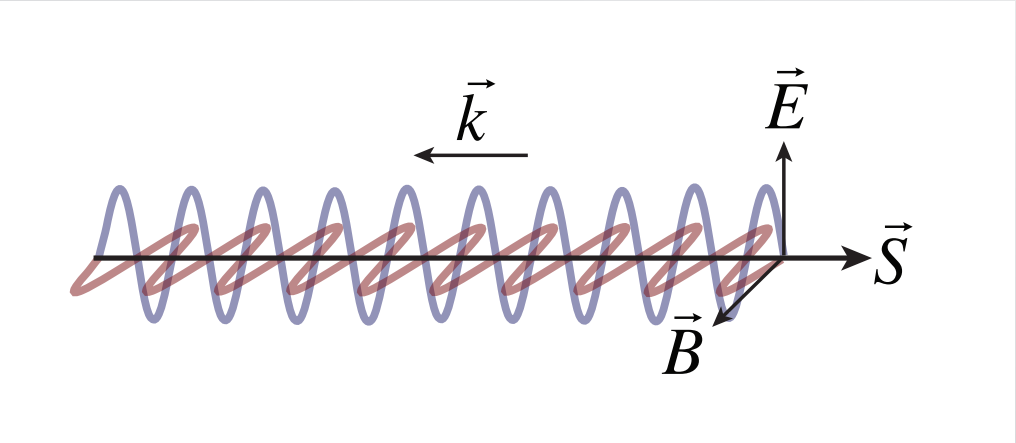
When $\mathrm{Re}(n) < 0$, $k' < 0$, and so phase velocity reversed!

What happens when $\mathrm{Re}(n) < 0$?



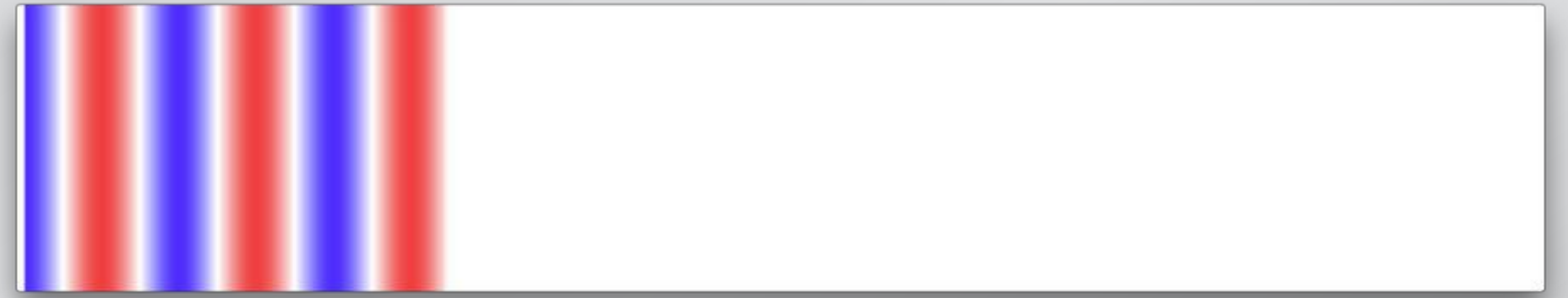
When $\mathrm{Re}(n) < 0$, $k' < 0$, and so phase velocity reversed!

What happens when $\mathrm{Re}(n) < 0$?

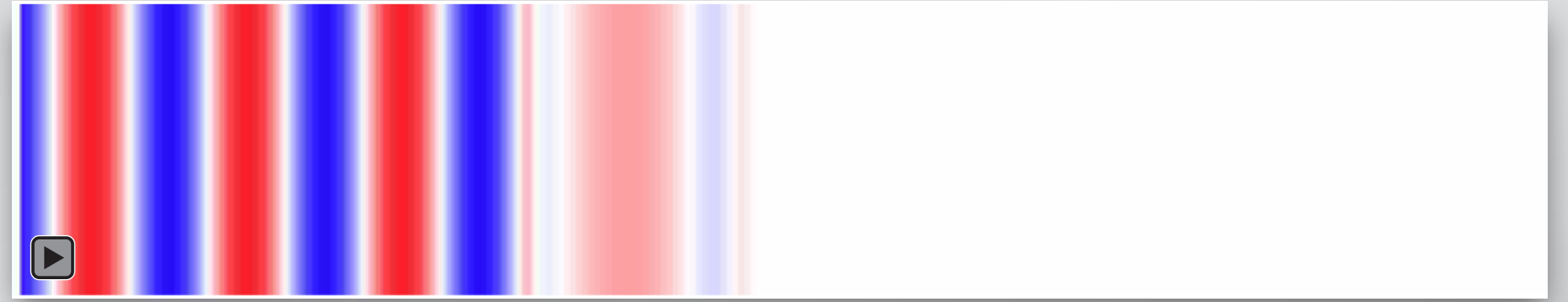


When $\mathrm{Re}(n) < 0$, $k' < 0$, and so phase velocity reversed!

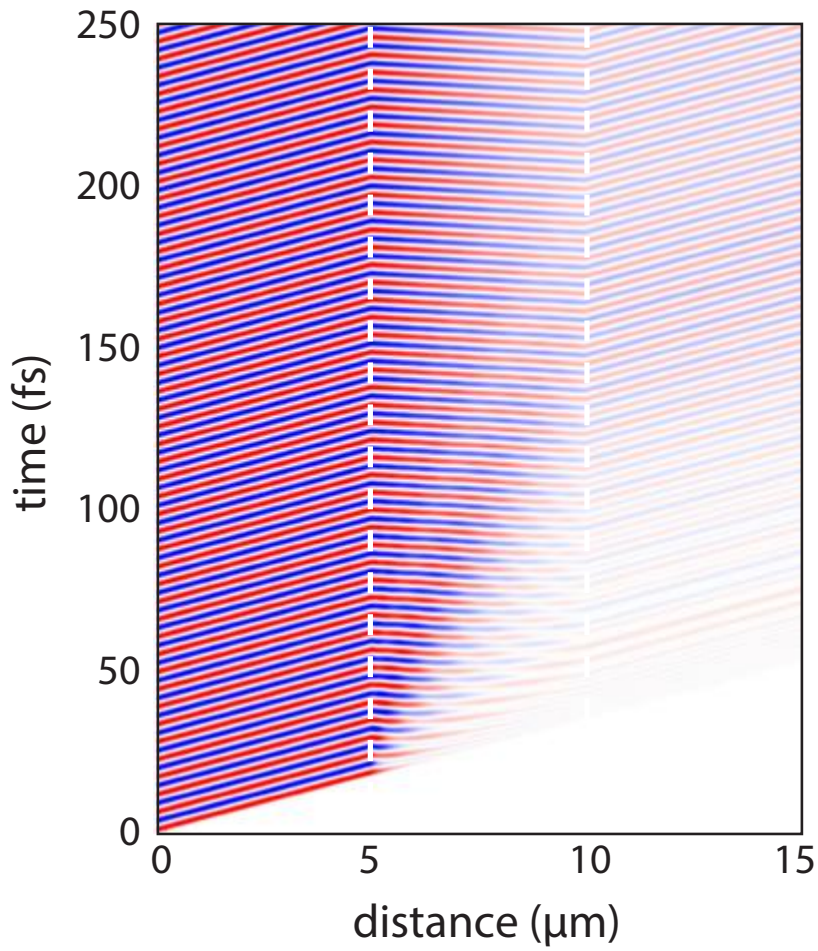
What about causality?



What about causality?

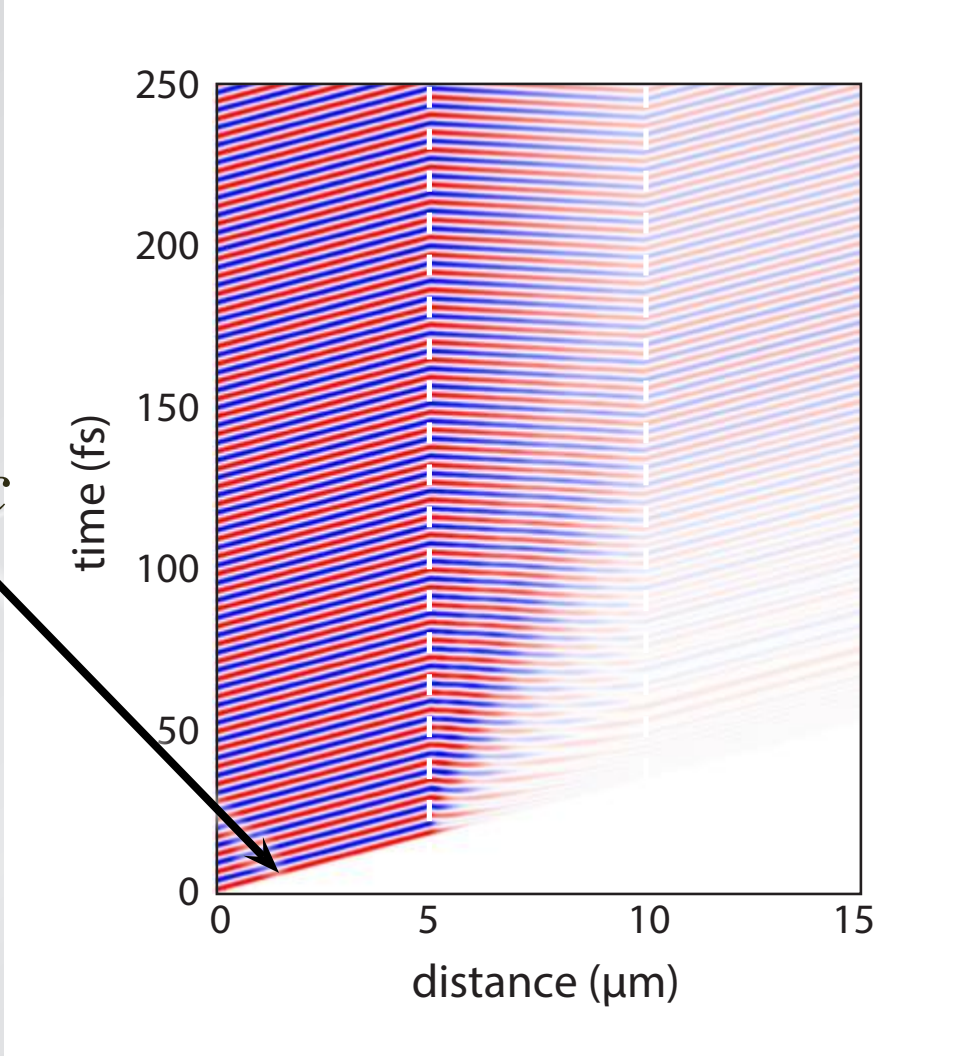


What about causality?

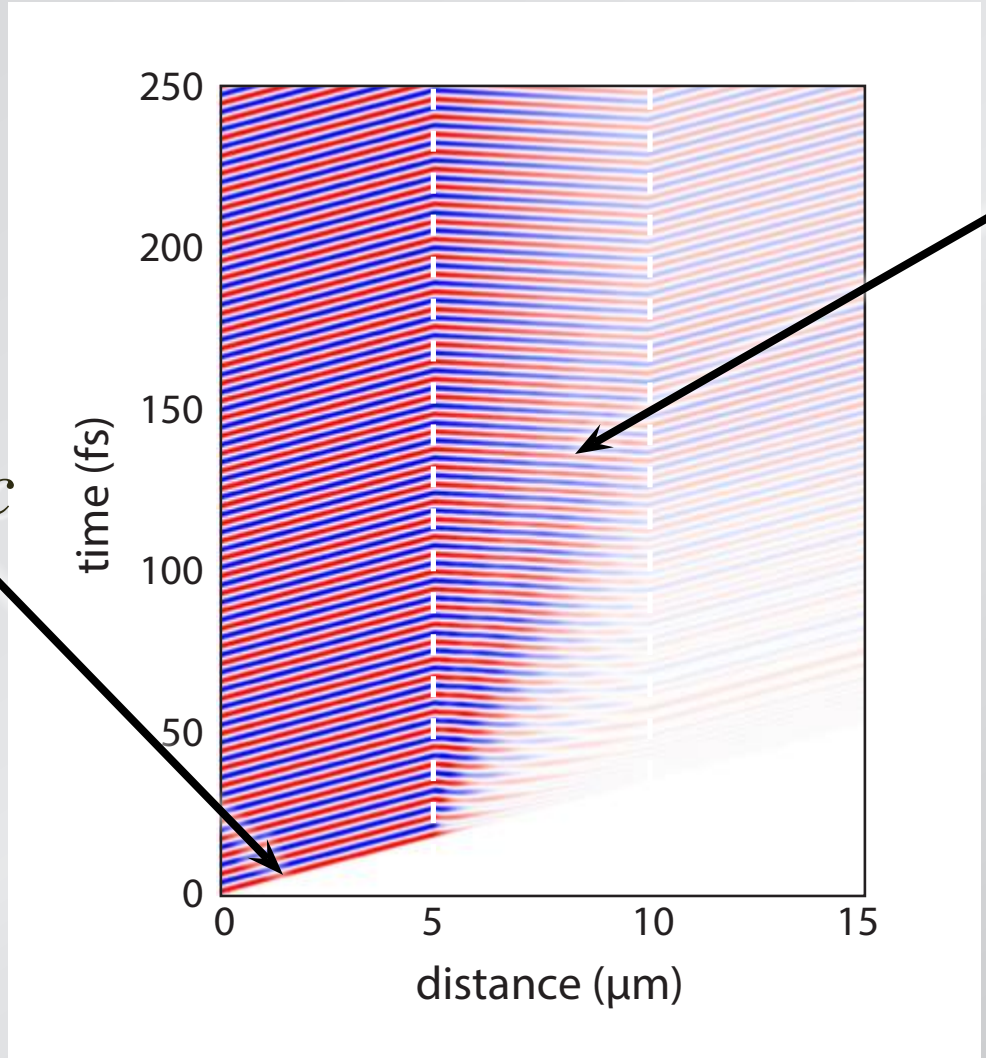


What about causality?

speed of light c



What about causality?



reverse phase propagation

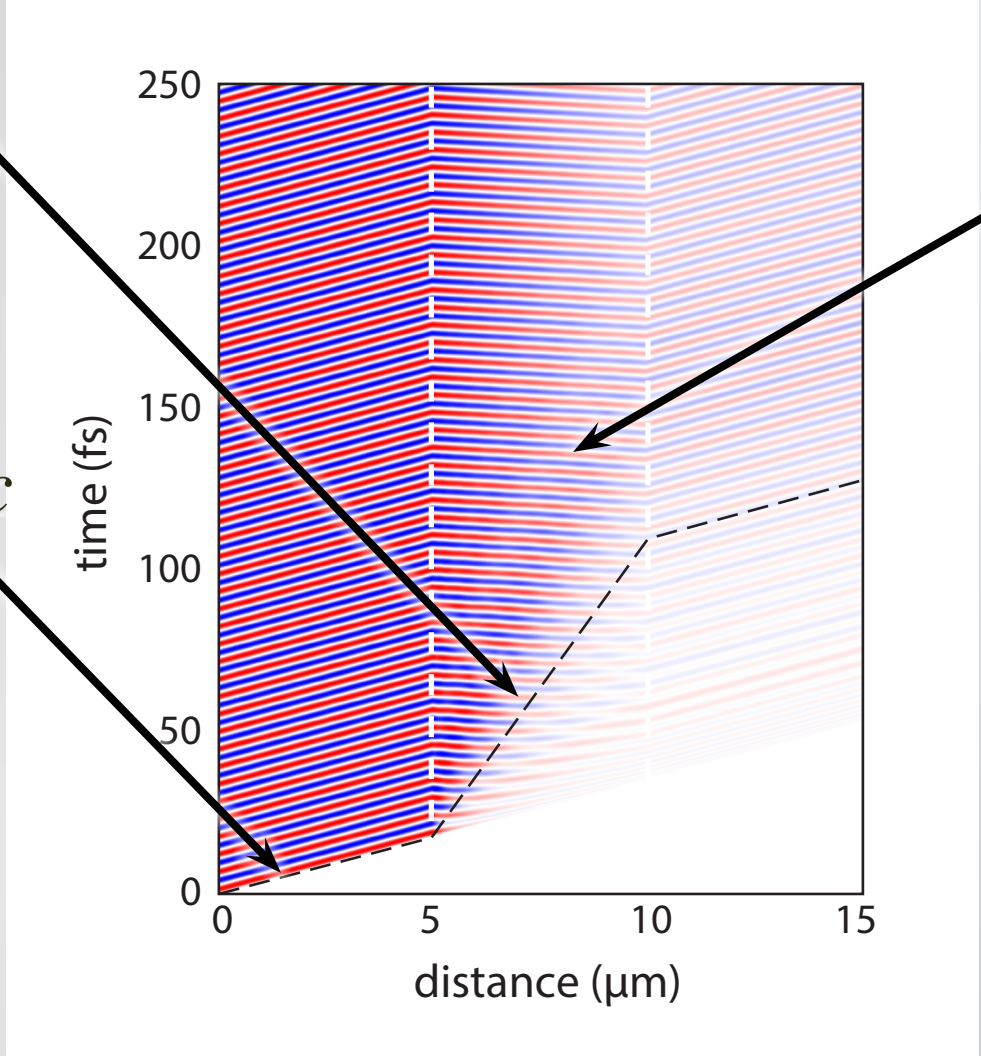
speed of light c

What about causality?

group velocity

$v_g < c$

speed of light c



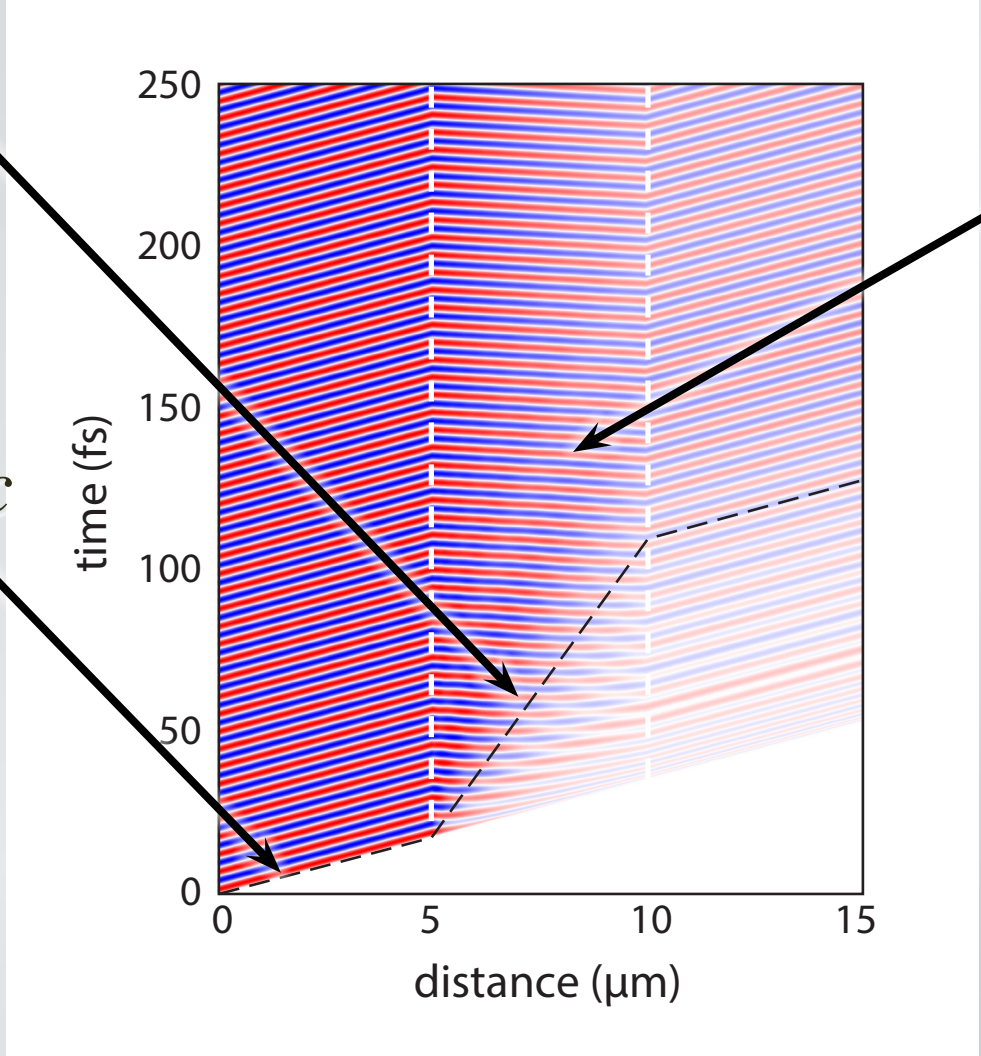
reverse phase propagation

What about causality?

group velocity

$v_g < c$

speed of light c



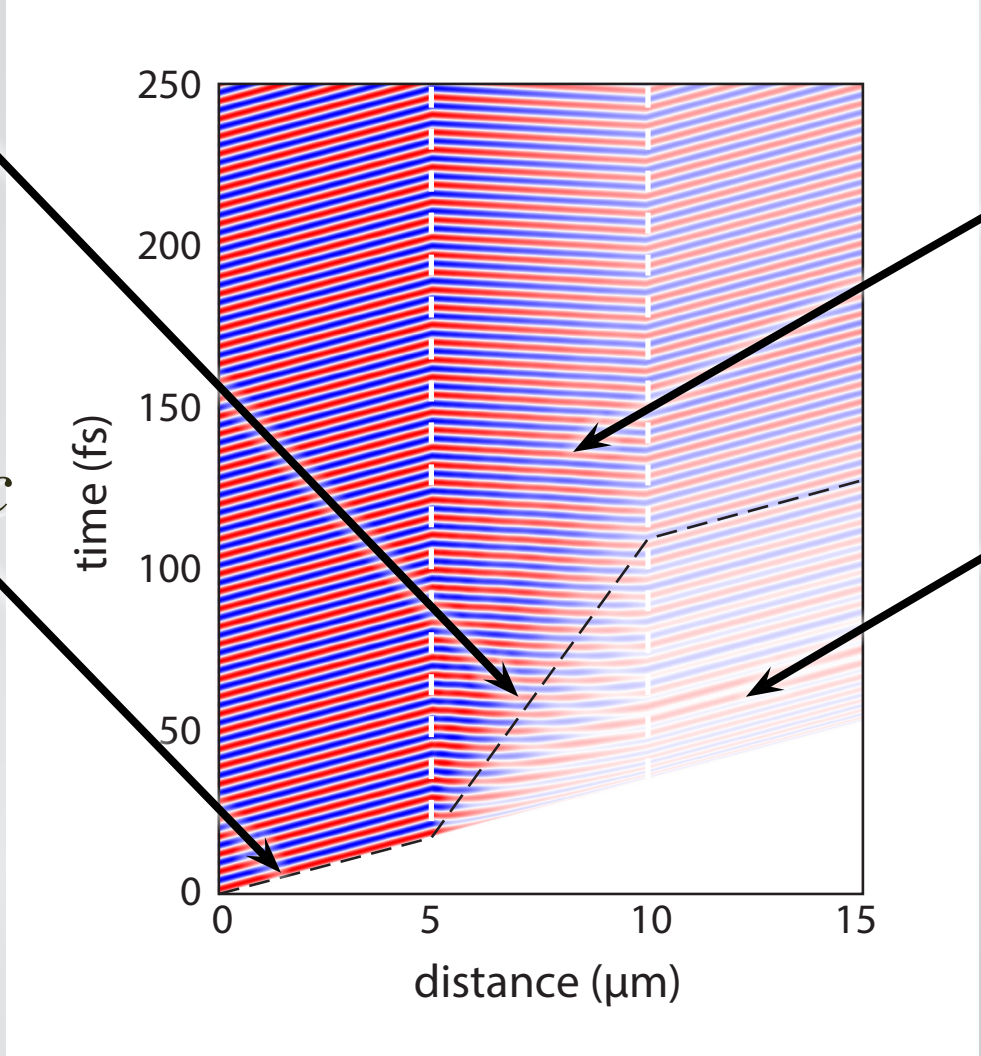
reverse phase propagation

What about causality?

group velocity

$v_g < c$

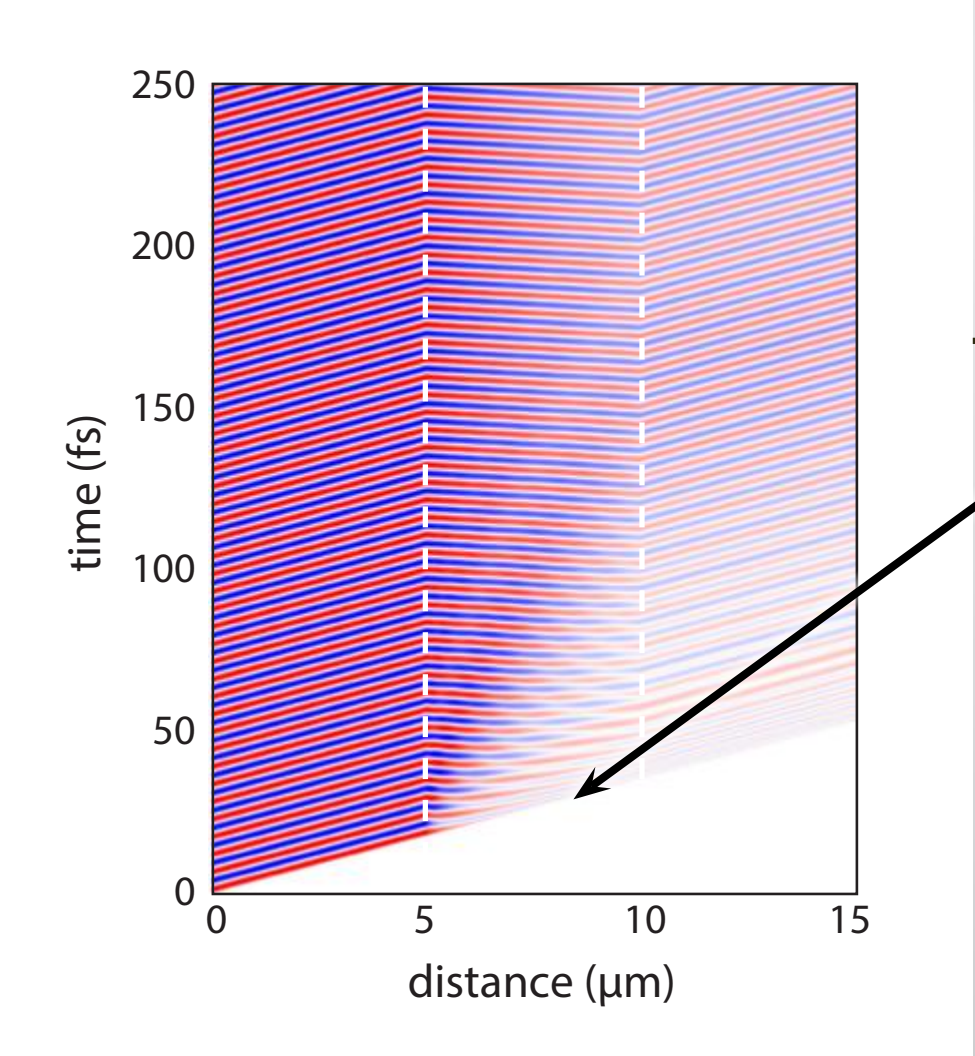
speed of light c



reverse phase propagation

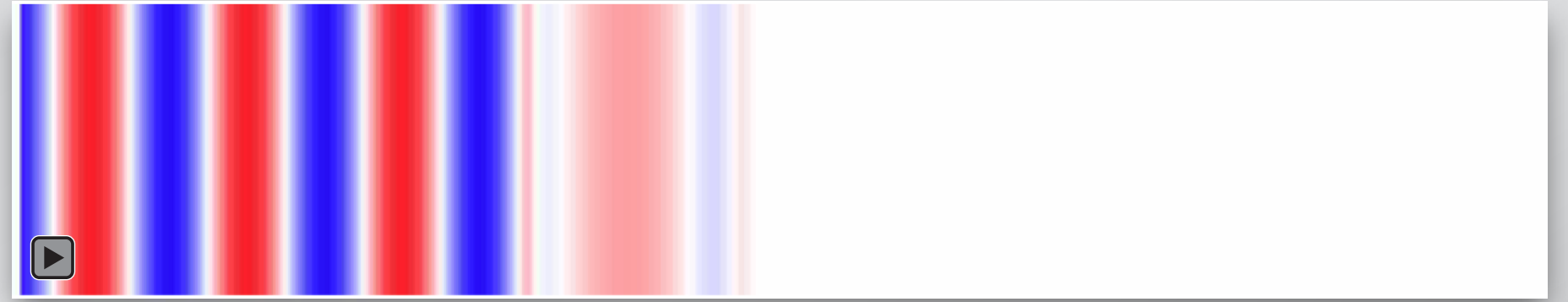
high-frequency precursors

What about causality?

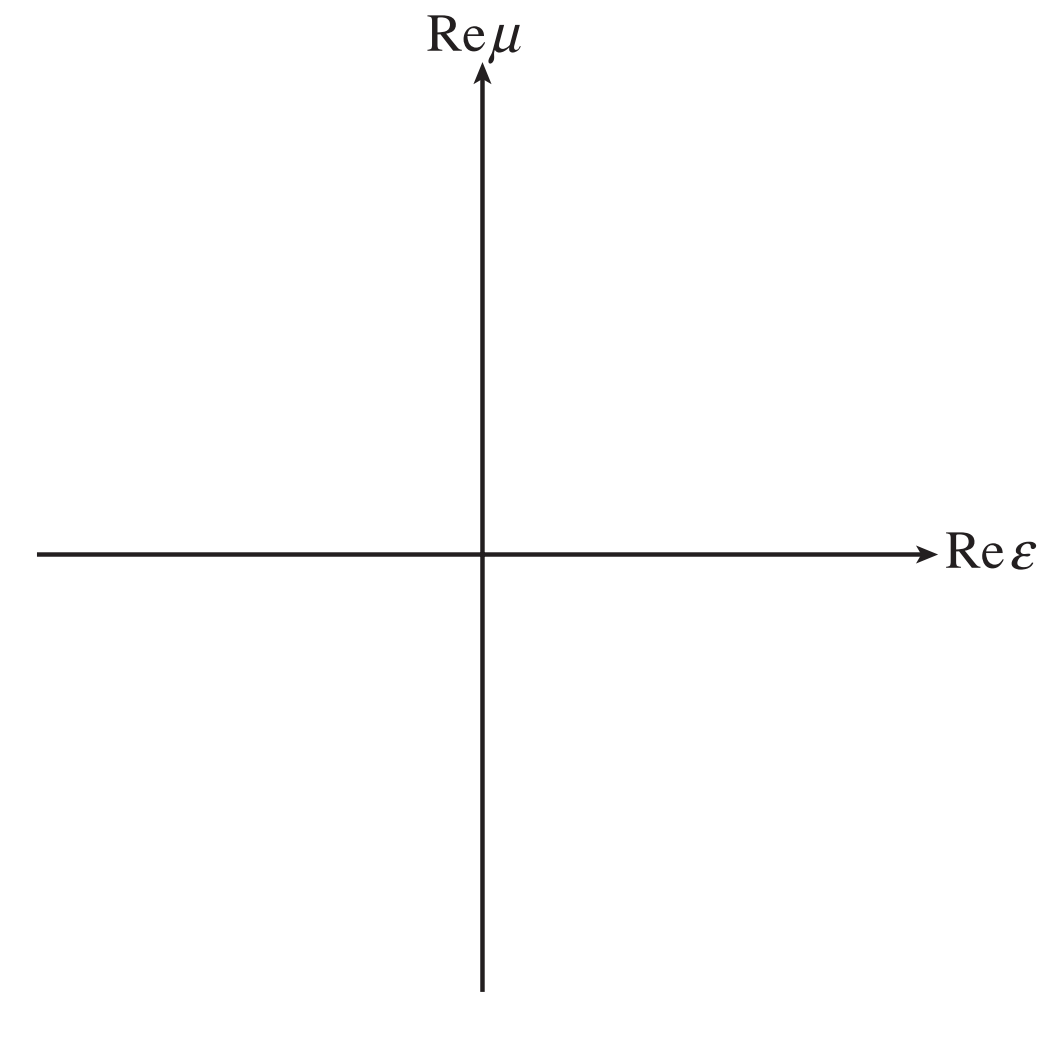


signal *always* travels at speed c!

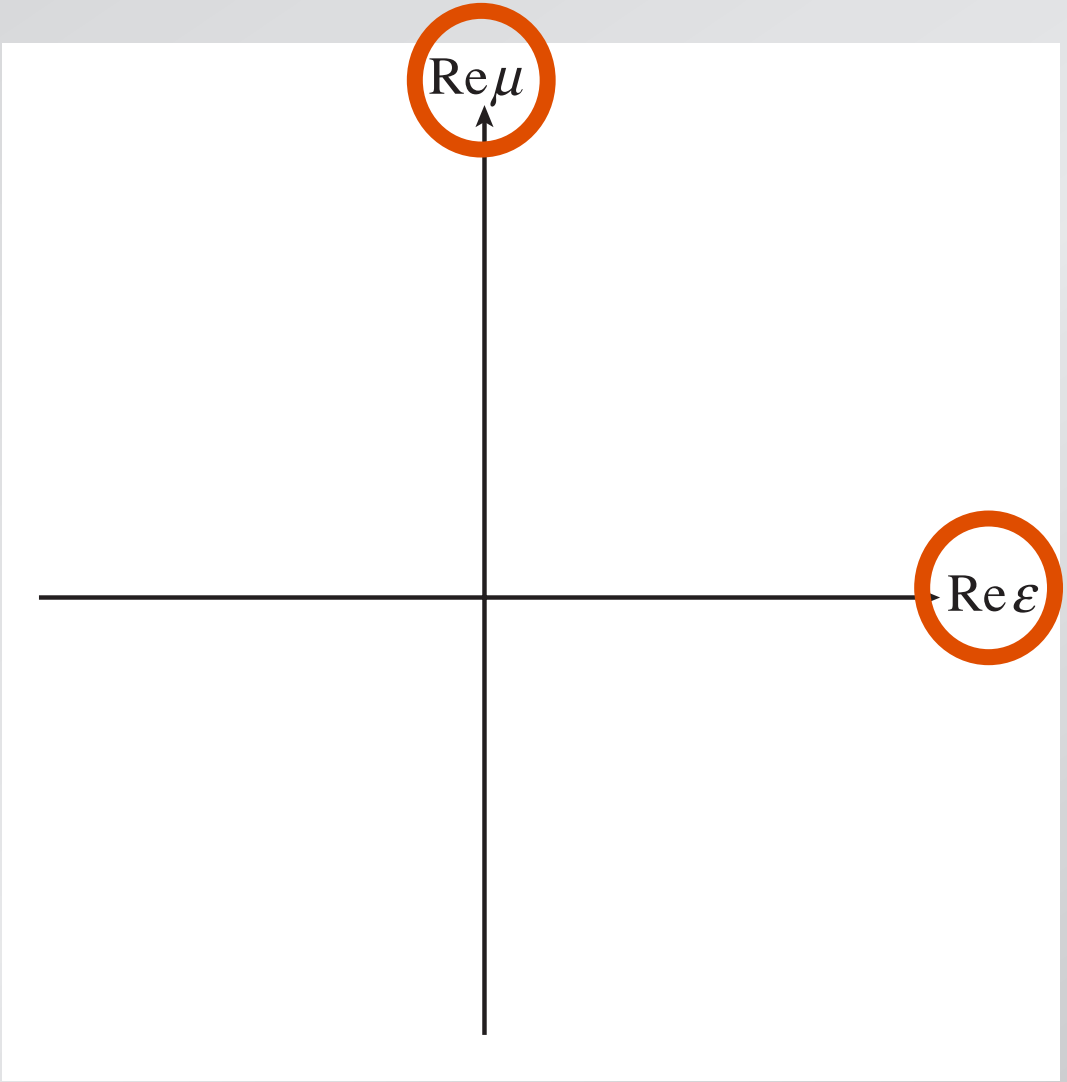
What about causality?



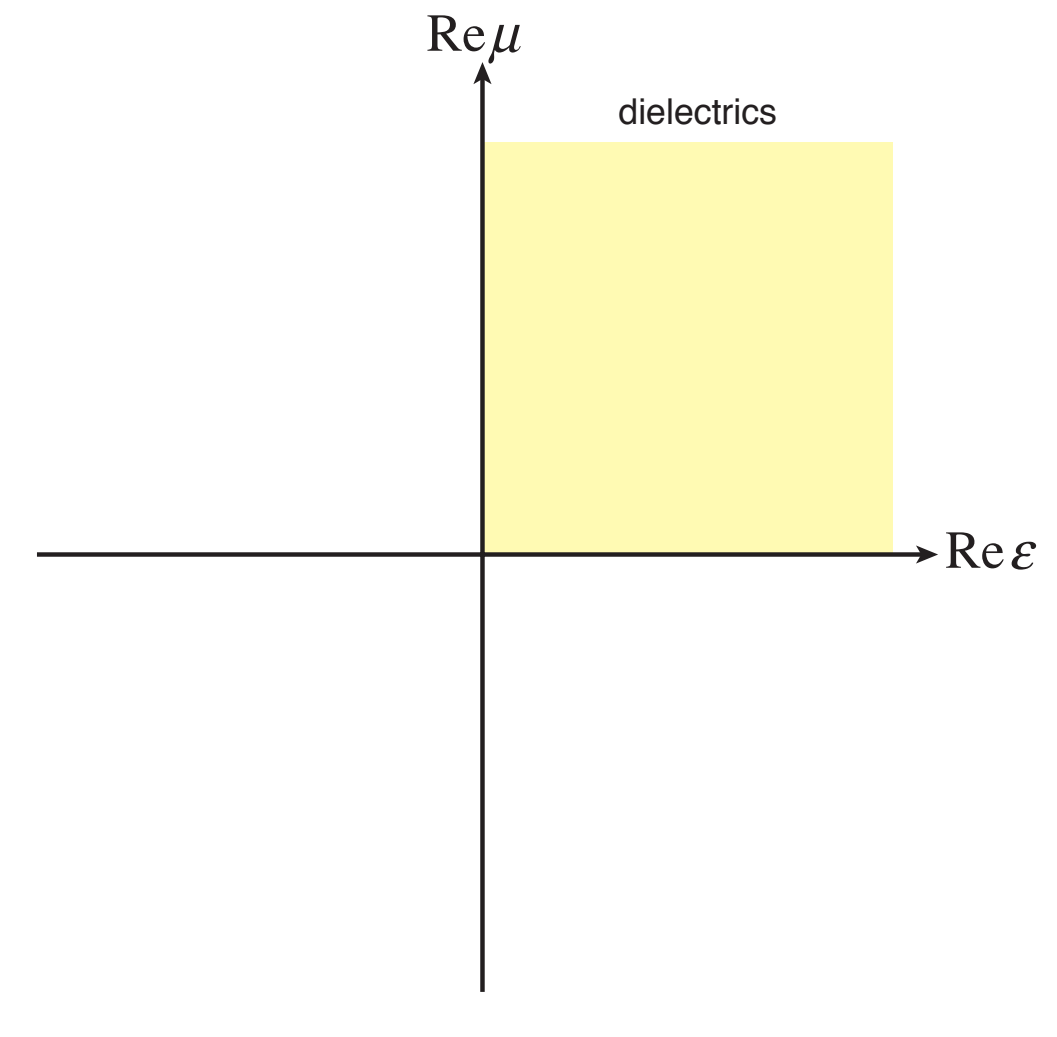
classification of (non-lossy) materials



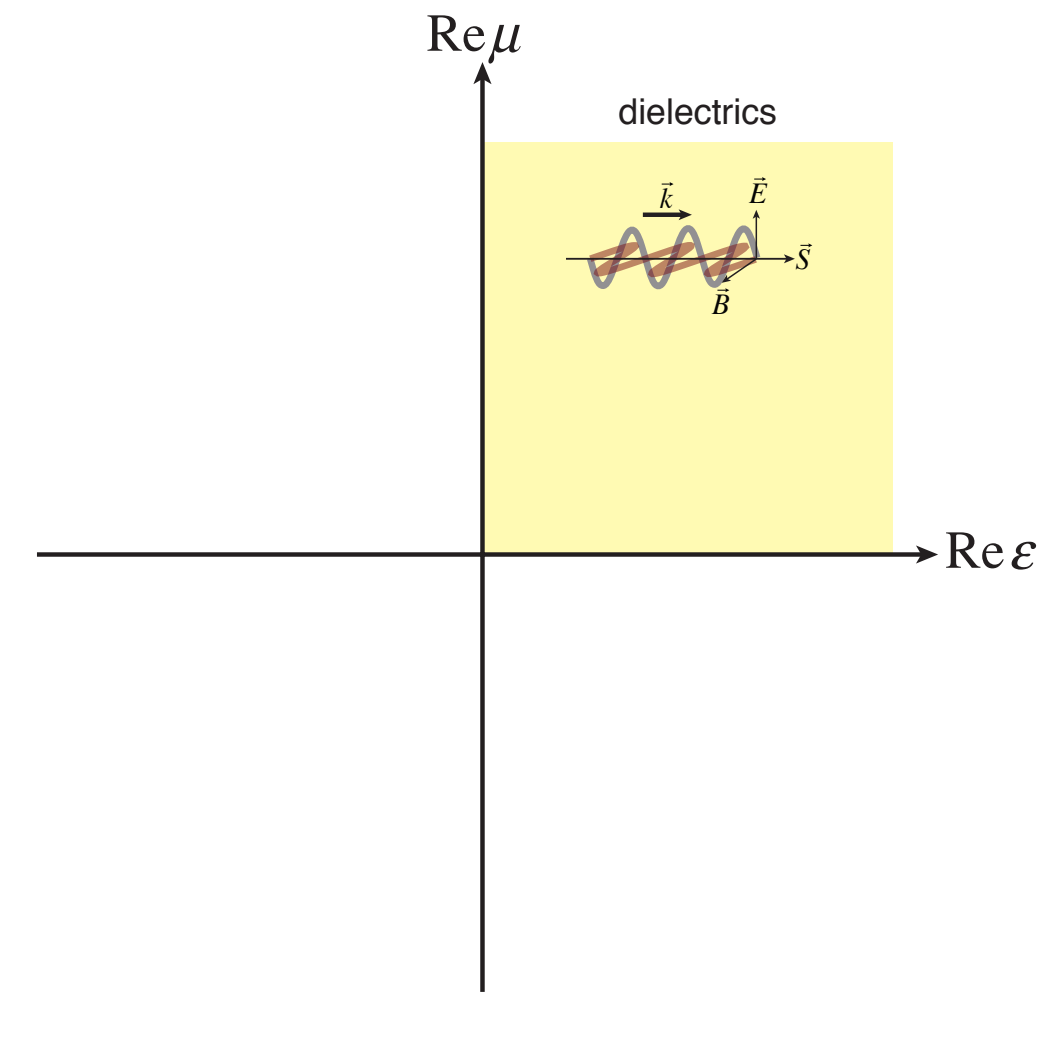
classification of (non-lossy) materials



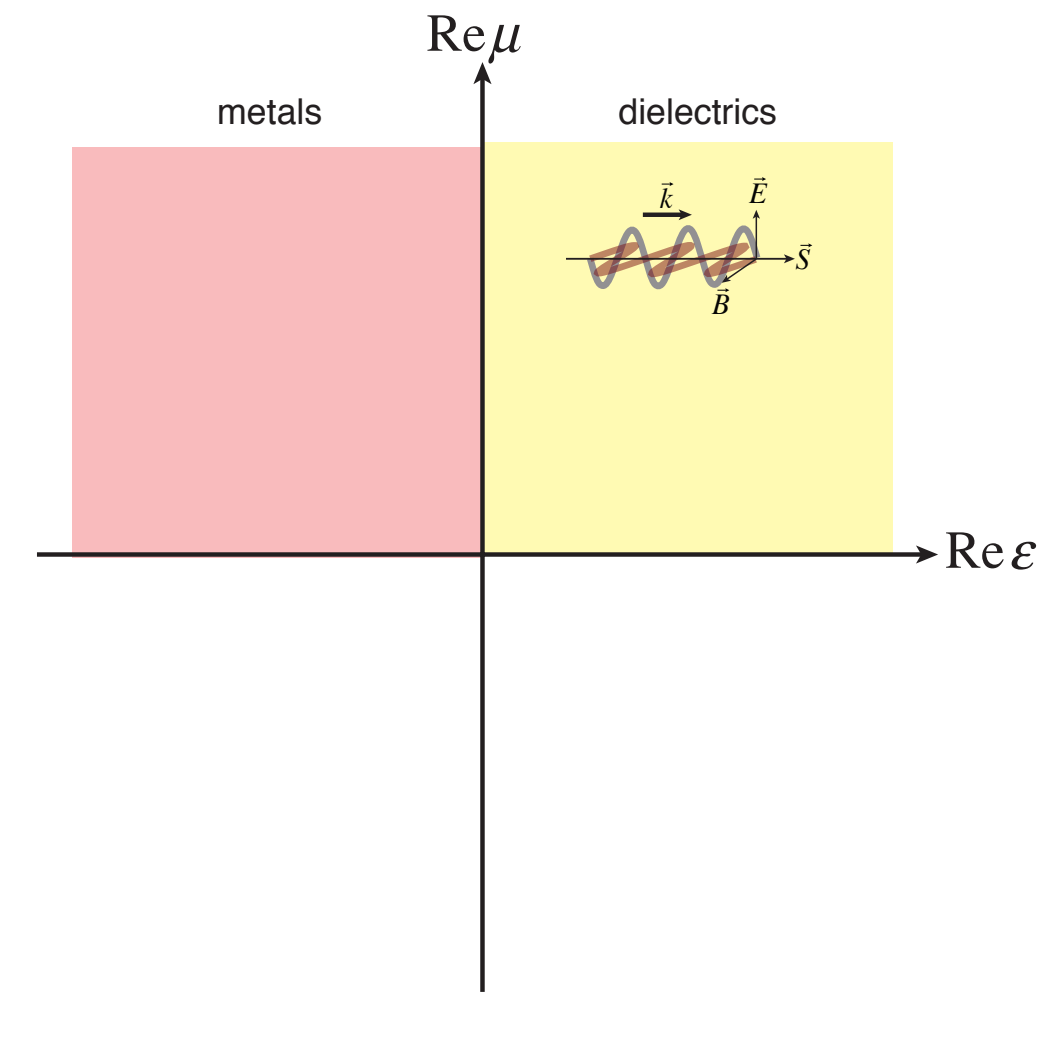
classification of (non-lossy) materials



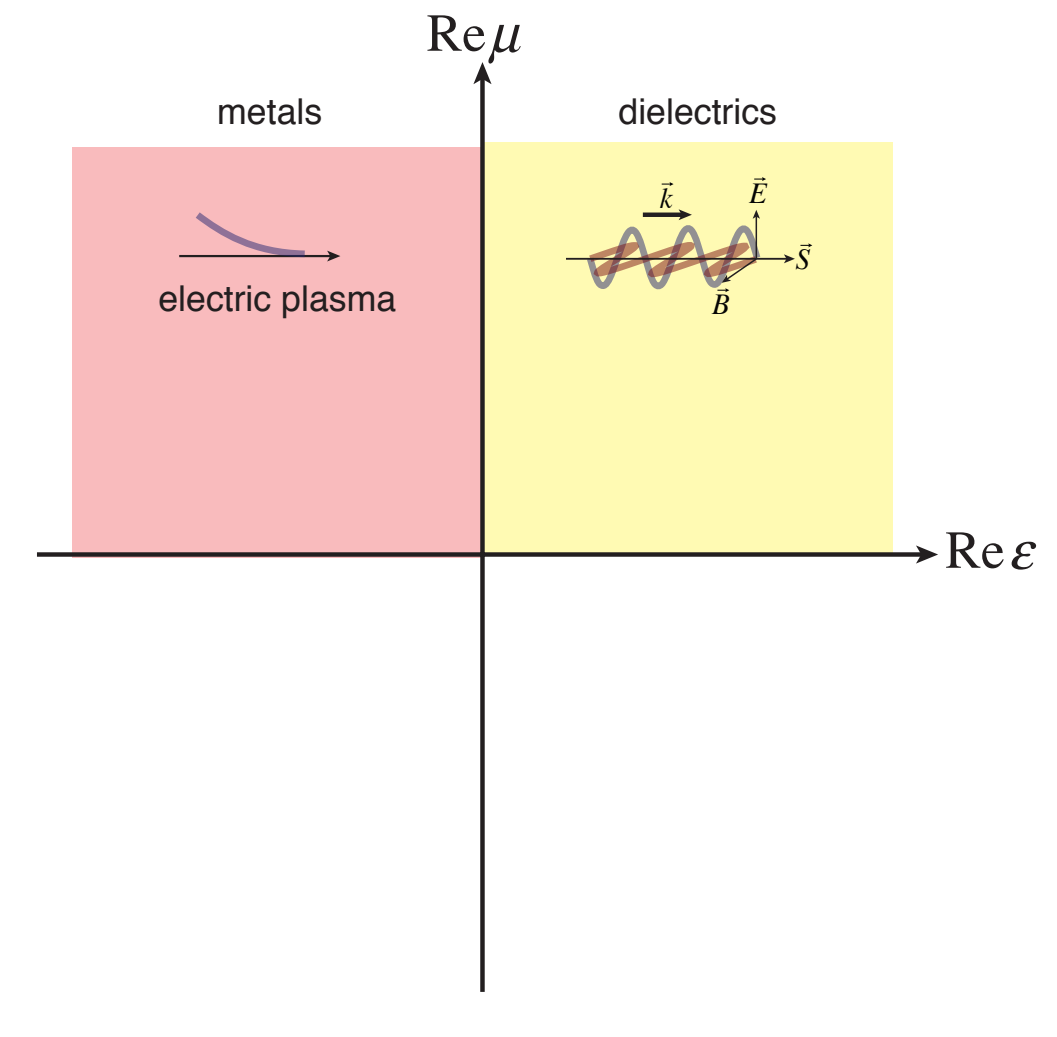
classification of (non-lossy) materials



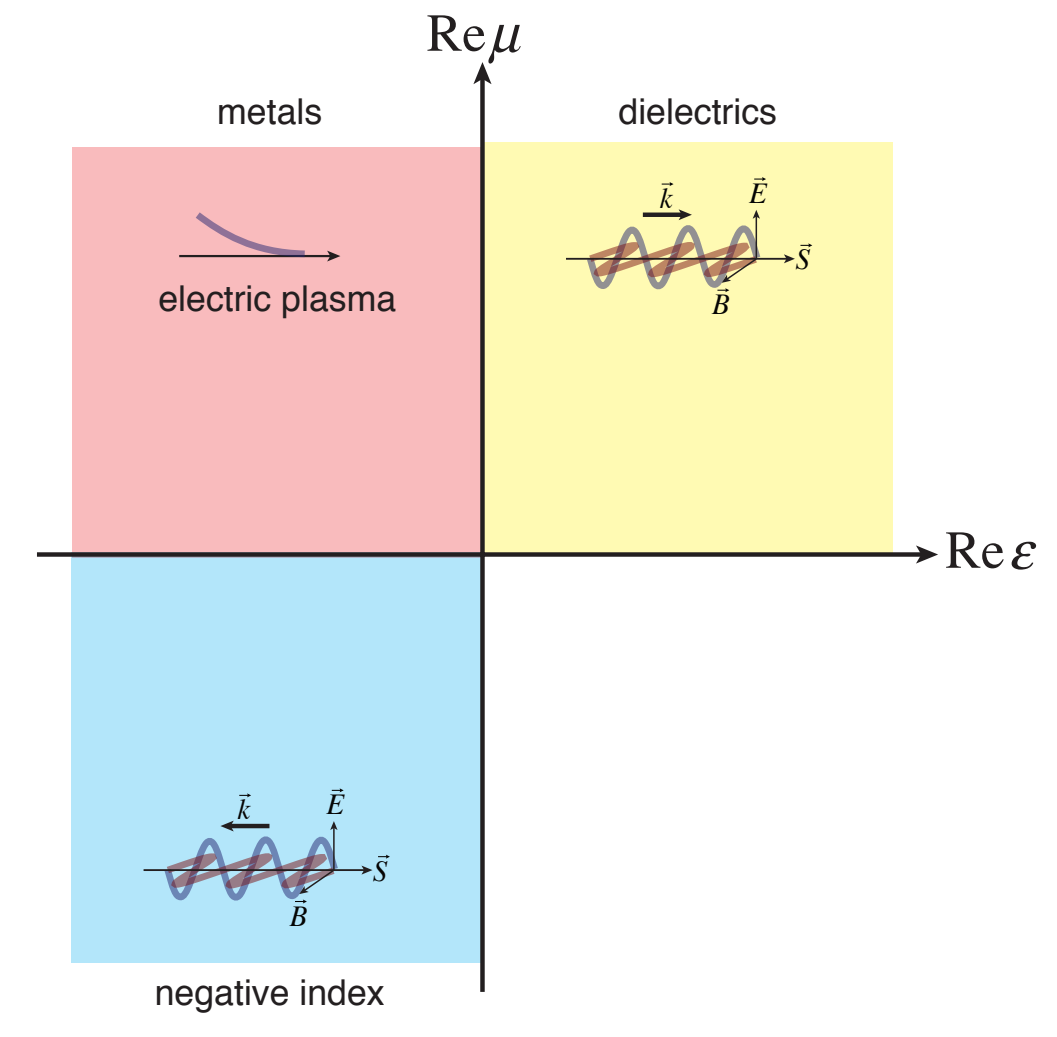
classification of (non-lossy) materials



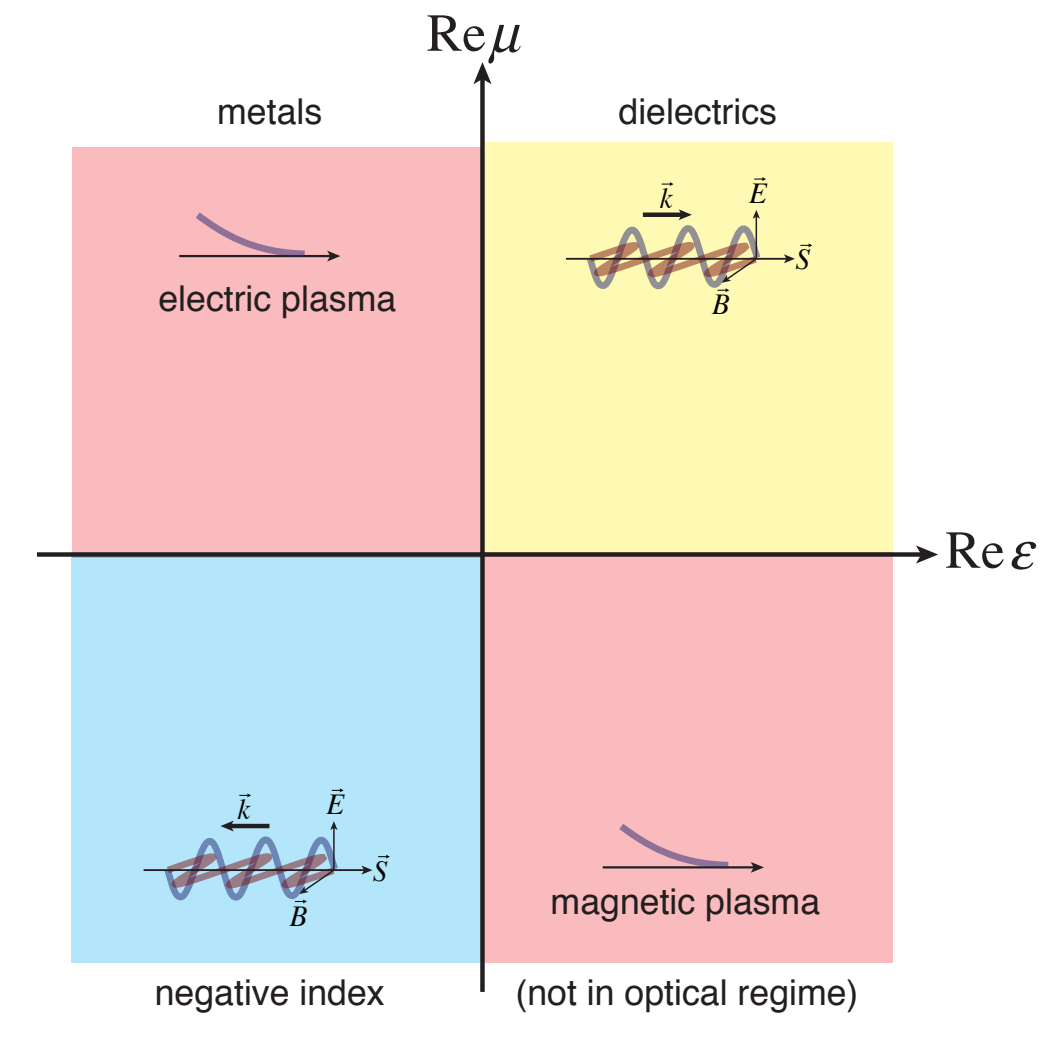
classification of (non-lossy) materials



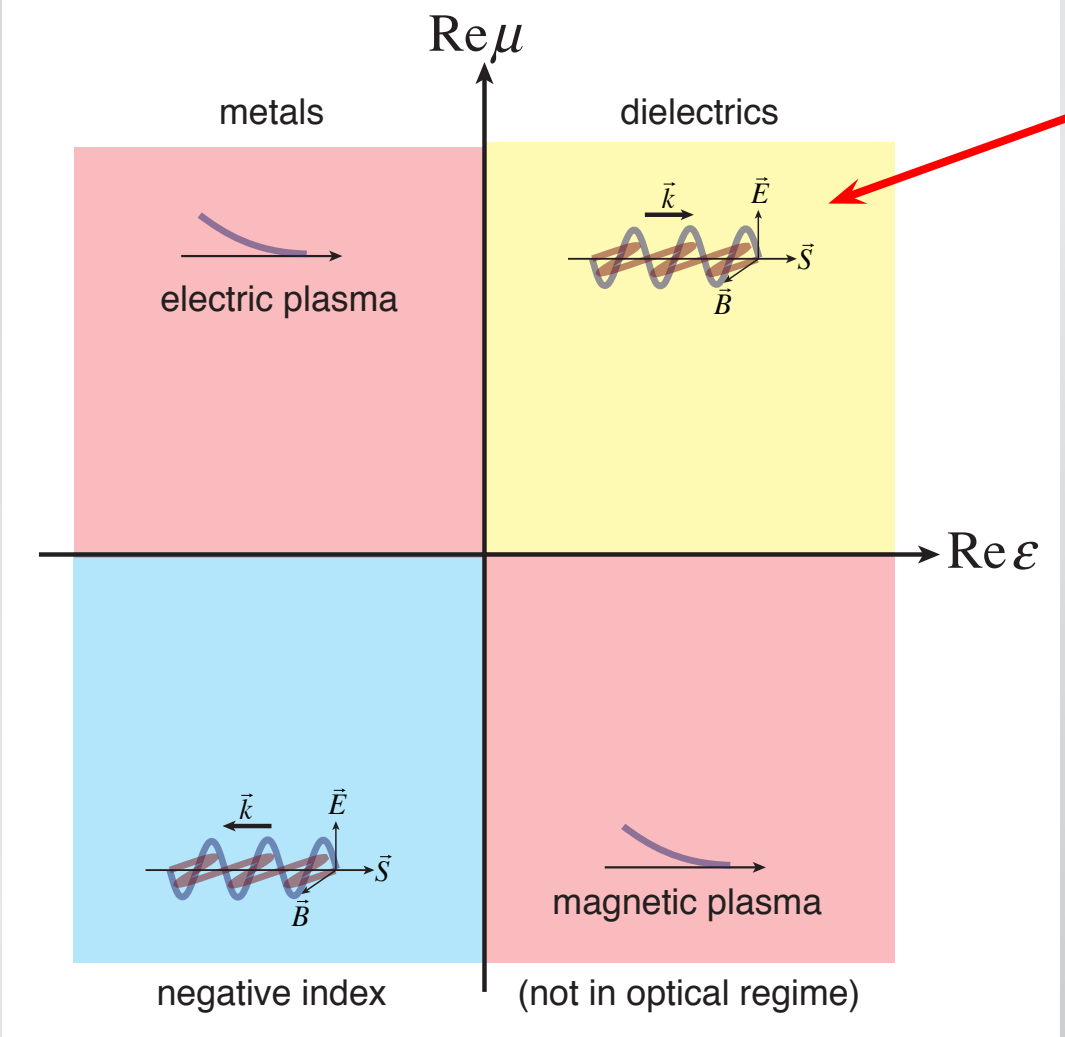
classification of (non-lossy) materials



classification of (non-lossy) materials



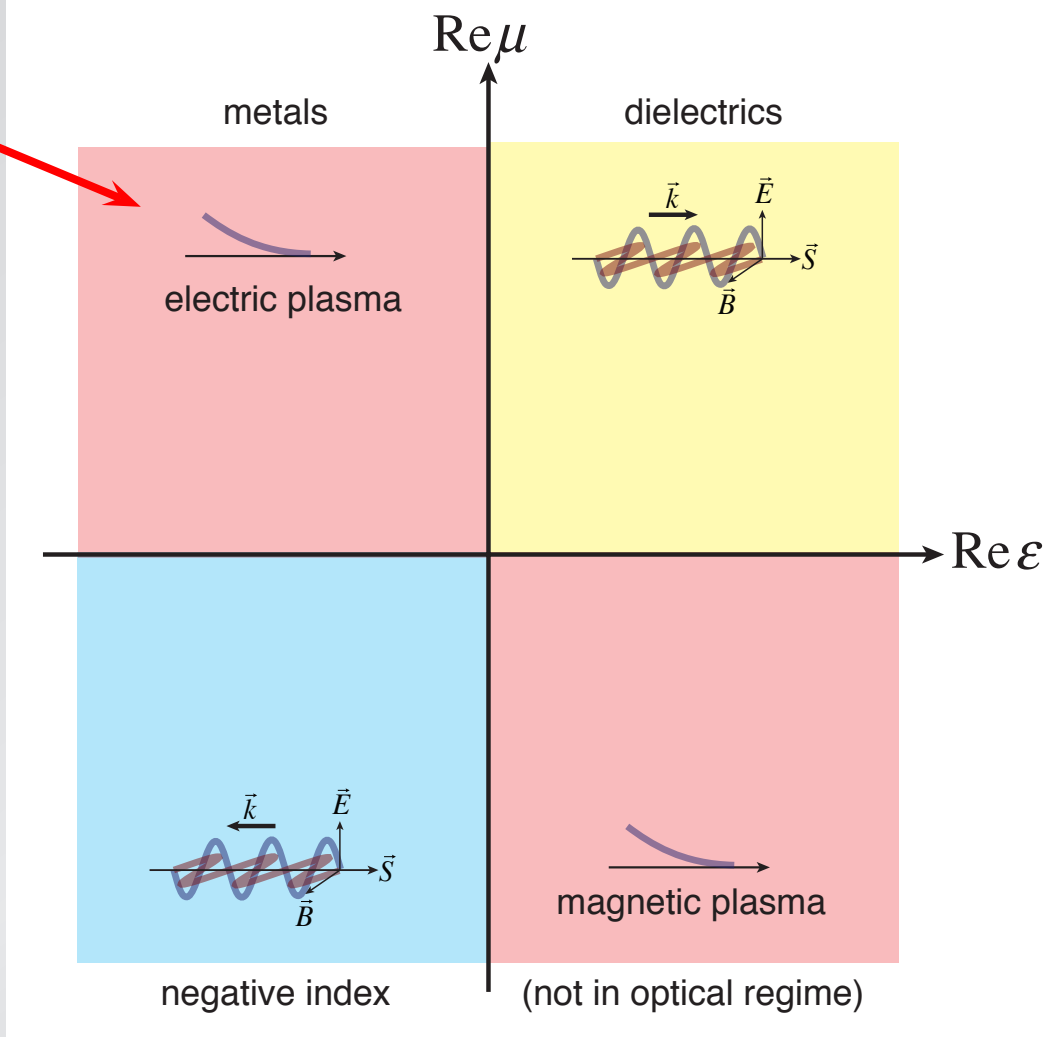
classification of (non-lossy) materials



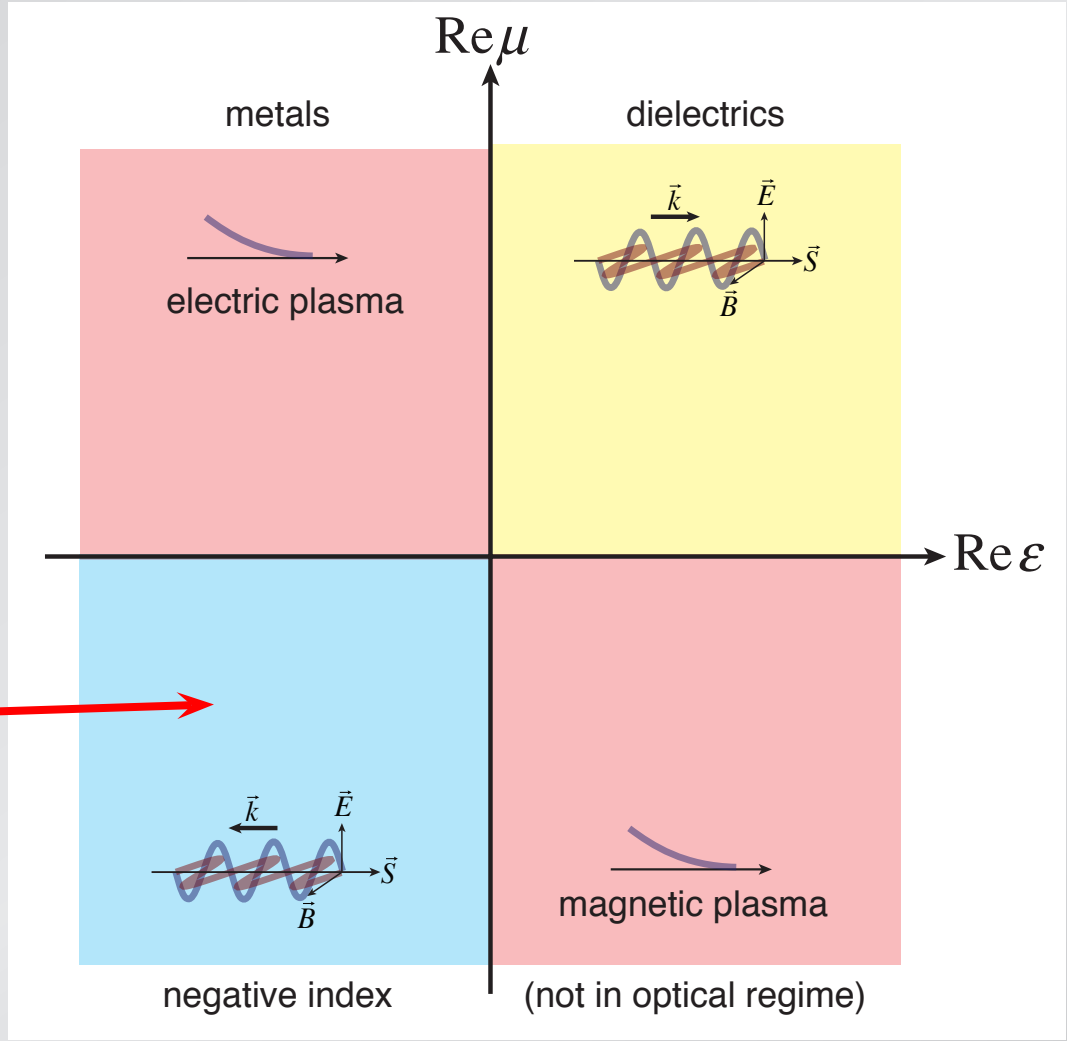
limited by diffraction

classification of (non-lossy) materials

no propagation

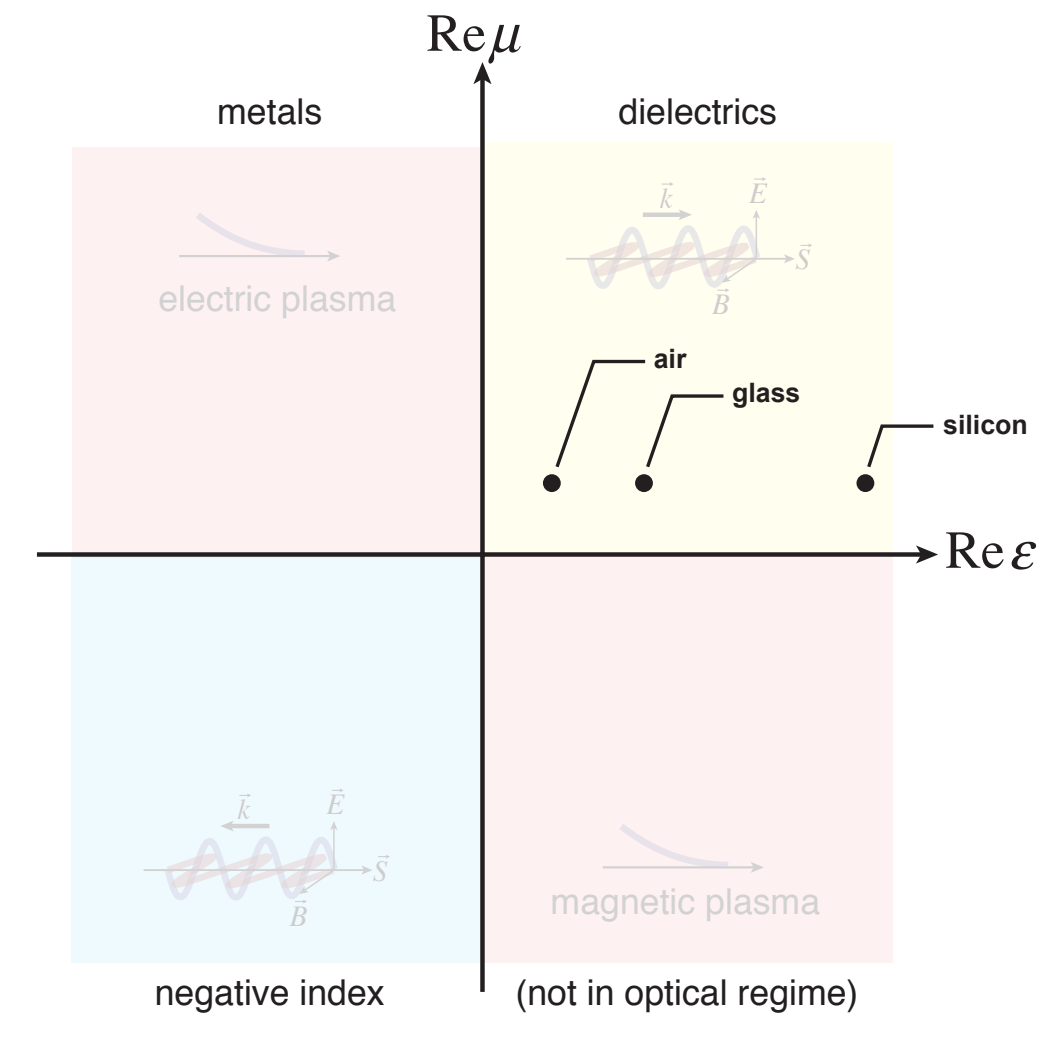


classification of (non-lossy) materials

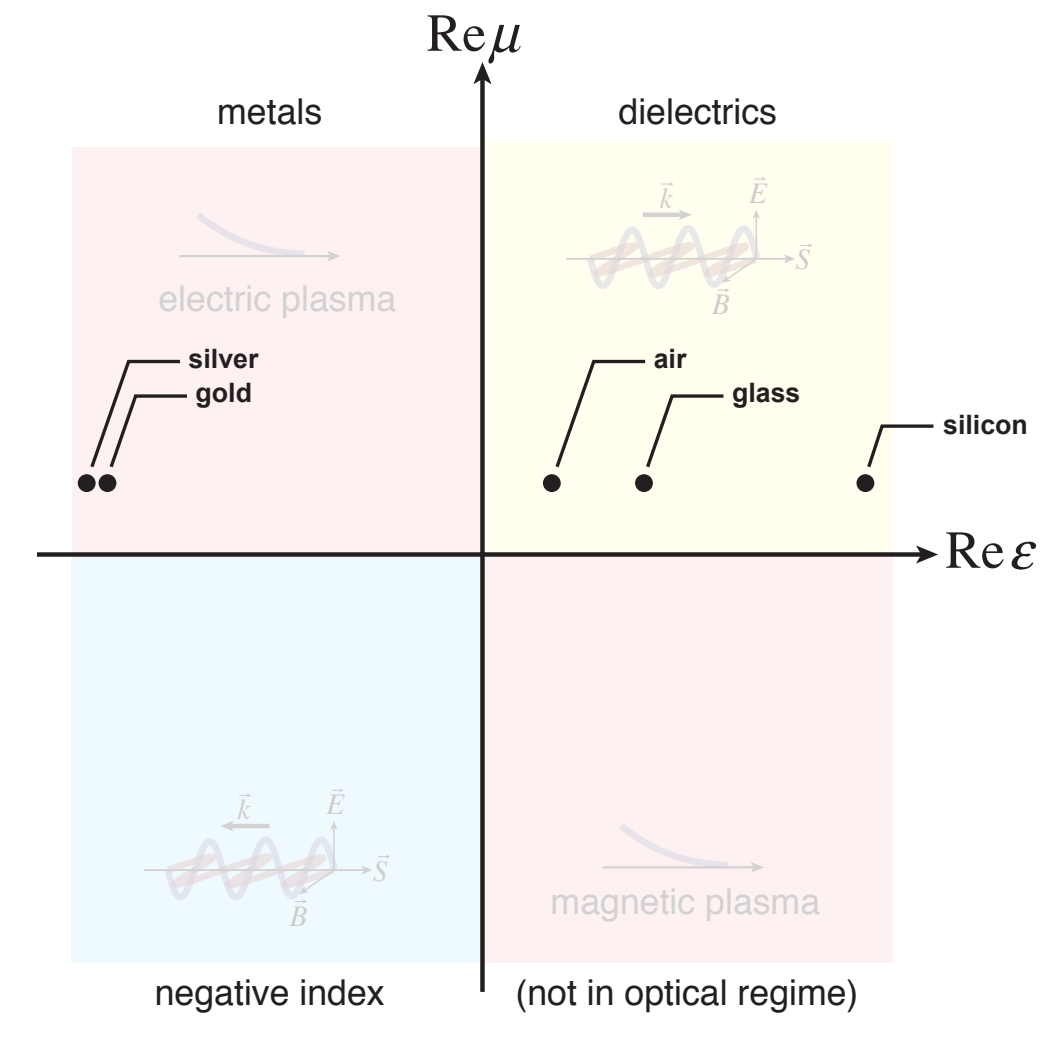


superlensing but…

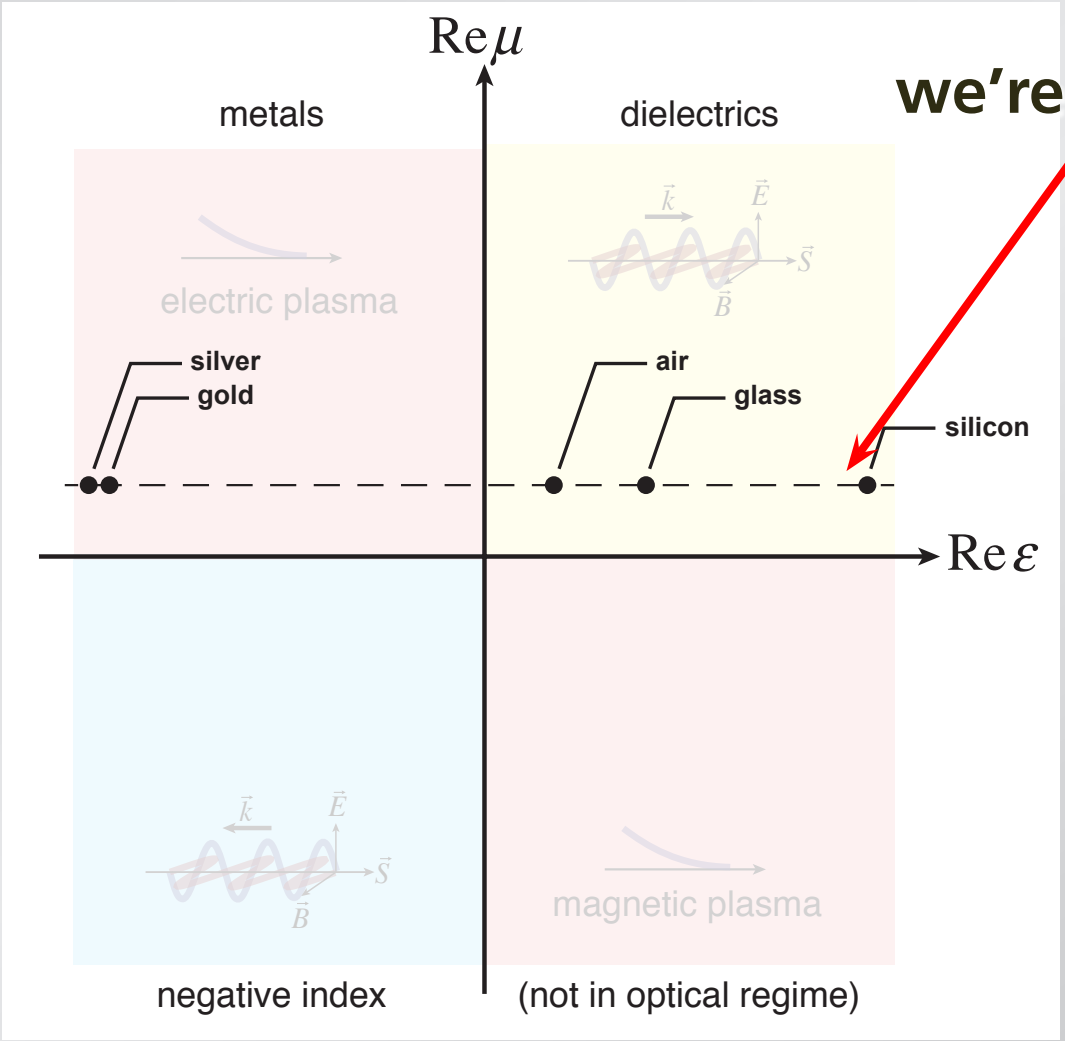
common materials very limited



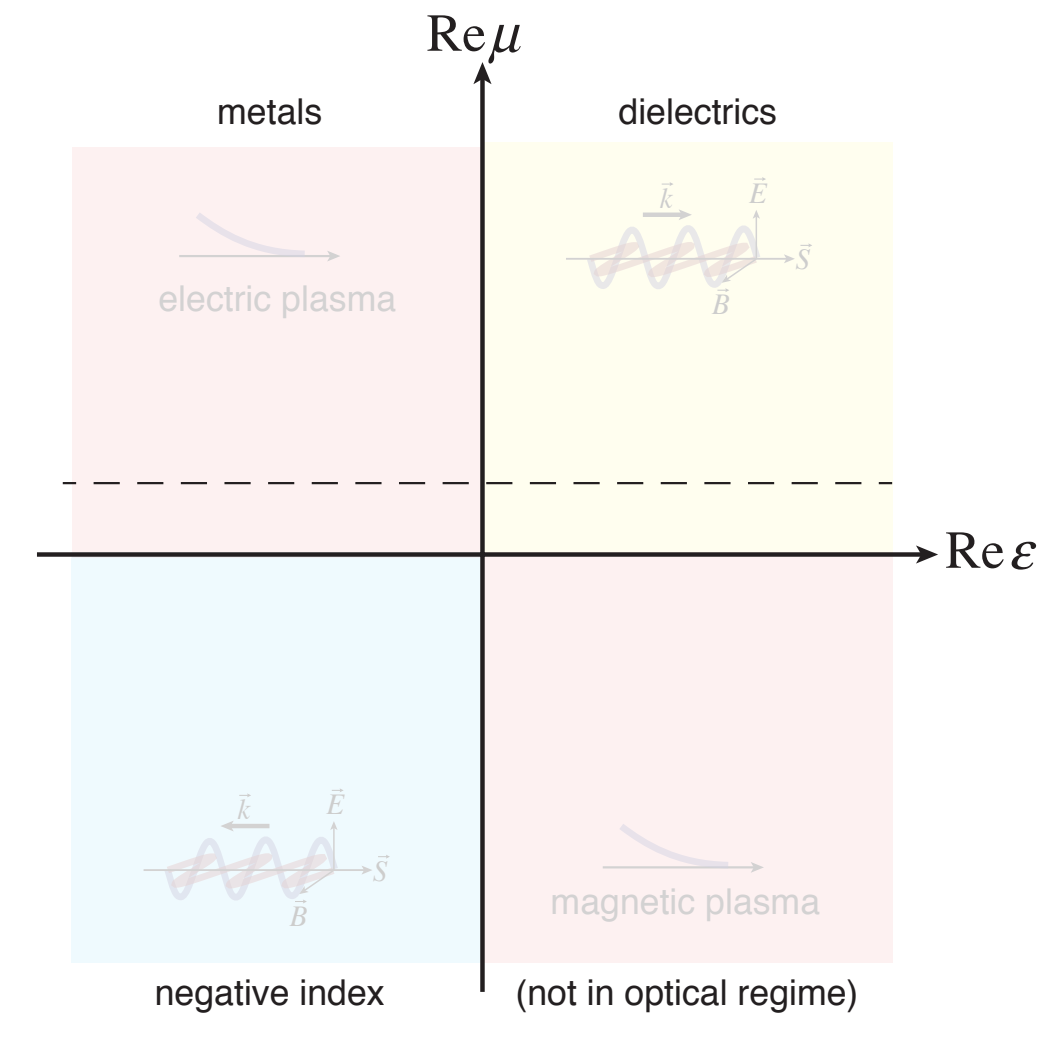
common materials very limited



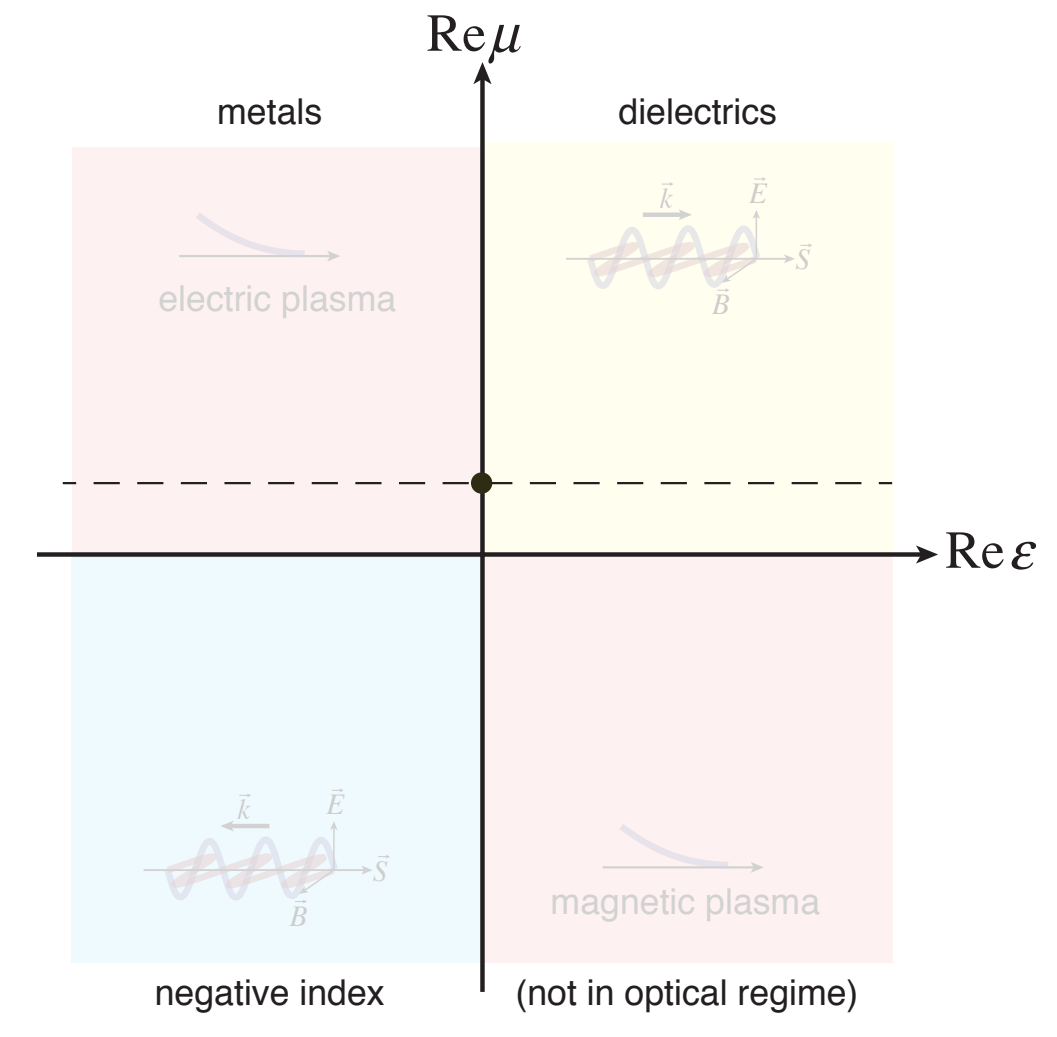
common materials very limited



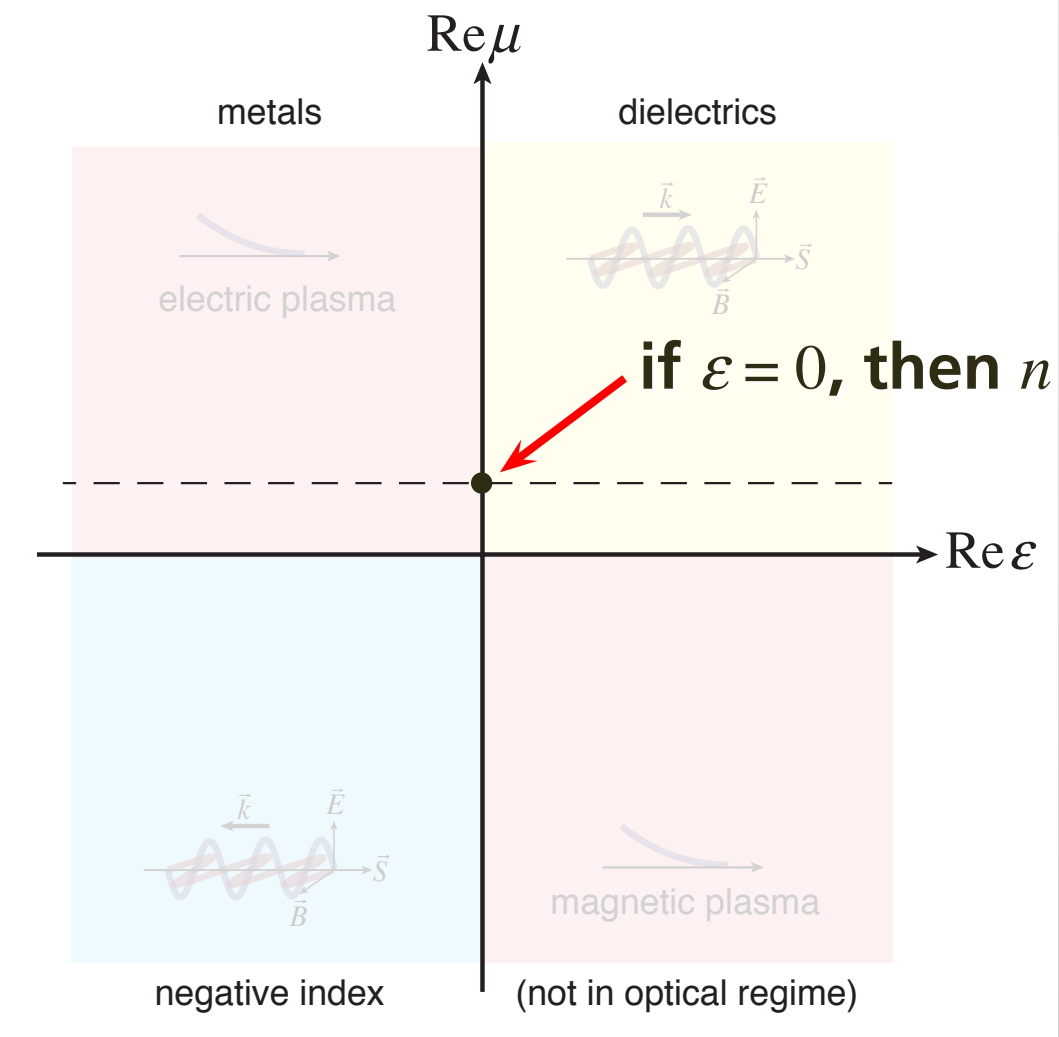
What happens on the axes?



what if we let $\varepsilon = 0$?



what if we let $\varepsilon = 0$?



Q: If $n = 0$, which of the following is true?

1. the frequency goes to zero.
2. the phase velocity becomes infinite.
3. both of the above.
4. neither of the above.

1 index

2 zero index

wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

solution

$$\vec{E} = \vec{E}_o \, e^{i\,(kx - \omega t)}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}}\, c = \frac{1}{n}\, c$$

wave equation

$$\nabla^2 \vec{E} - \cancel{\frac{\mu \epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}} = 0$$

solution

$$\vec{E} = \vec{E}_o \, e^{i\,(kx - \omega t)}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon \mu}} c = \frac{1}{n} c$$

1 index

2 zero index

wave equation

$$\nabla^2 \vec{E} - \cancel{\frac{\mu \epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}} = 0$$

solution

$$\vec{E} = \vec{E}_o \, e^{i(kx - \omega t)} \longrightarrow \vec{E} = \vec{E}_o \, e^{-i\omega t}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}} c = \frac{1}{n} c$$

1 index

2 zero index

wave equation

$$\nabla^2 \vec{E} - \cancel{\frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}} = 0$$

solution

$$\vec{E} = \vec{E}_o\, e^{i(kx - \omega t)} \longrightarrow \vec{E} = \vec{E}_o\, e^{-i\omega t}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}}\, c = \frac{1}{n}\, c \longrightarrow \infty$$

1 index

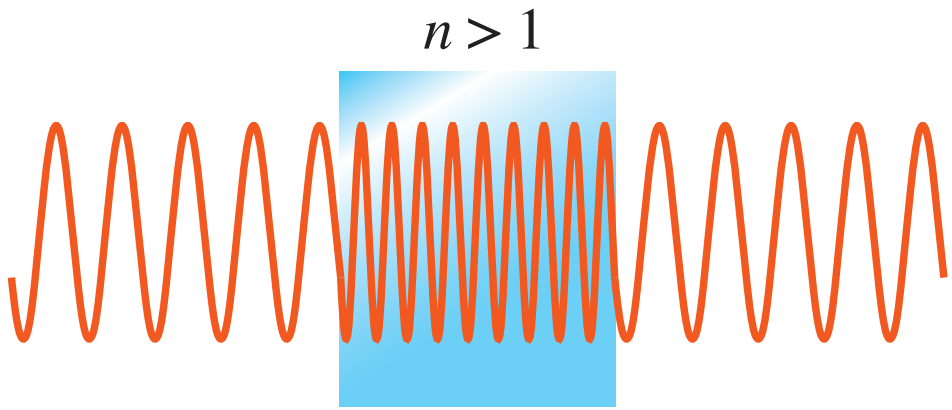
2 zero index

Q: If $n = 0$, which of the following is true?

1. the frequency goes to zero.
2. the phase velocity becomes infinite. ✔
3. both of the above.
4. neither of the above.

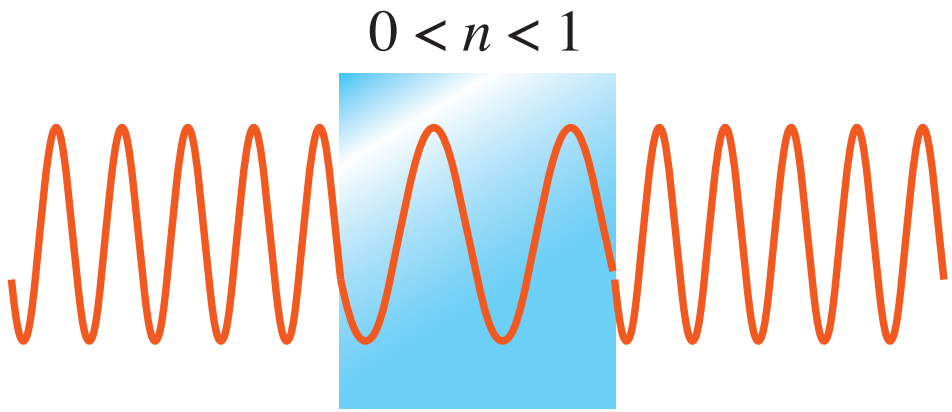
1 index

2 zero index



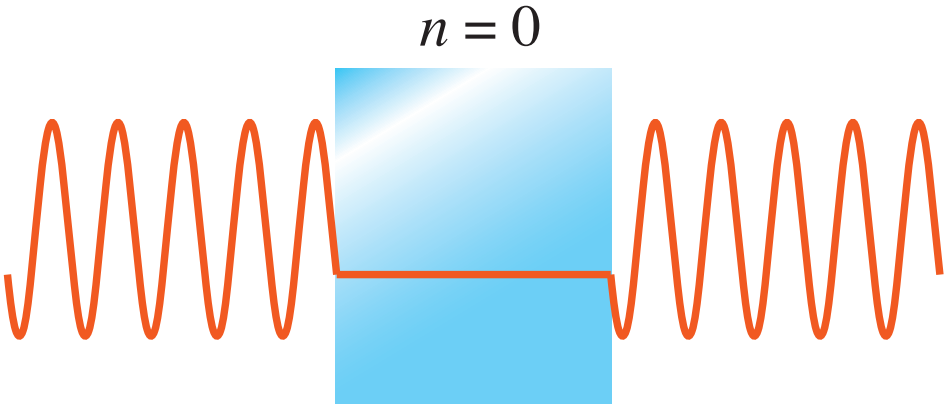
1 index

2 zero index



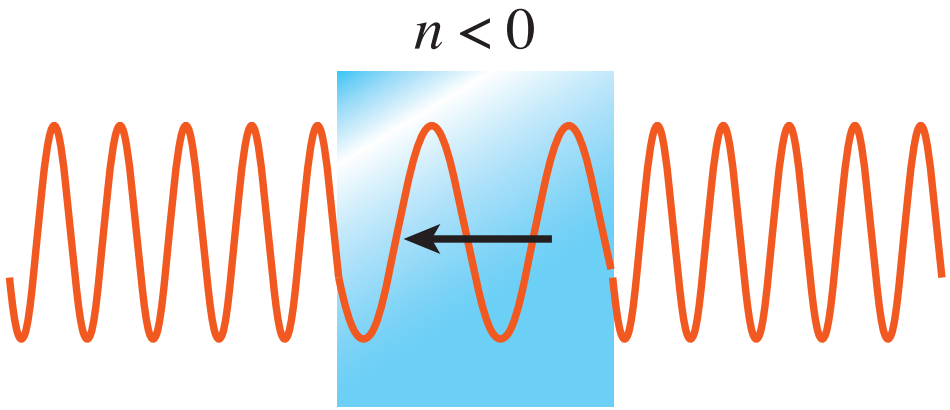
1 index

2 zero index



1 index

2 zero index



1 index

2 zero index

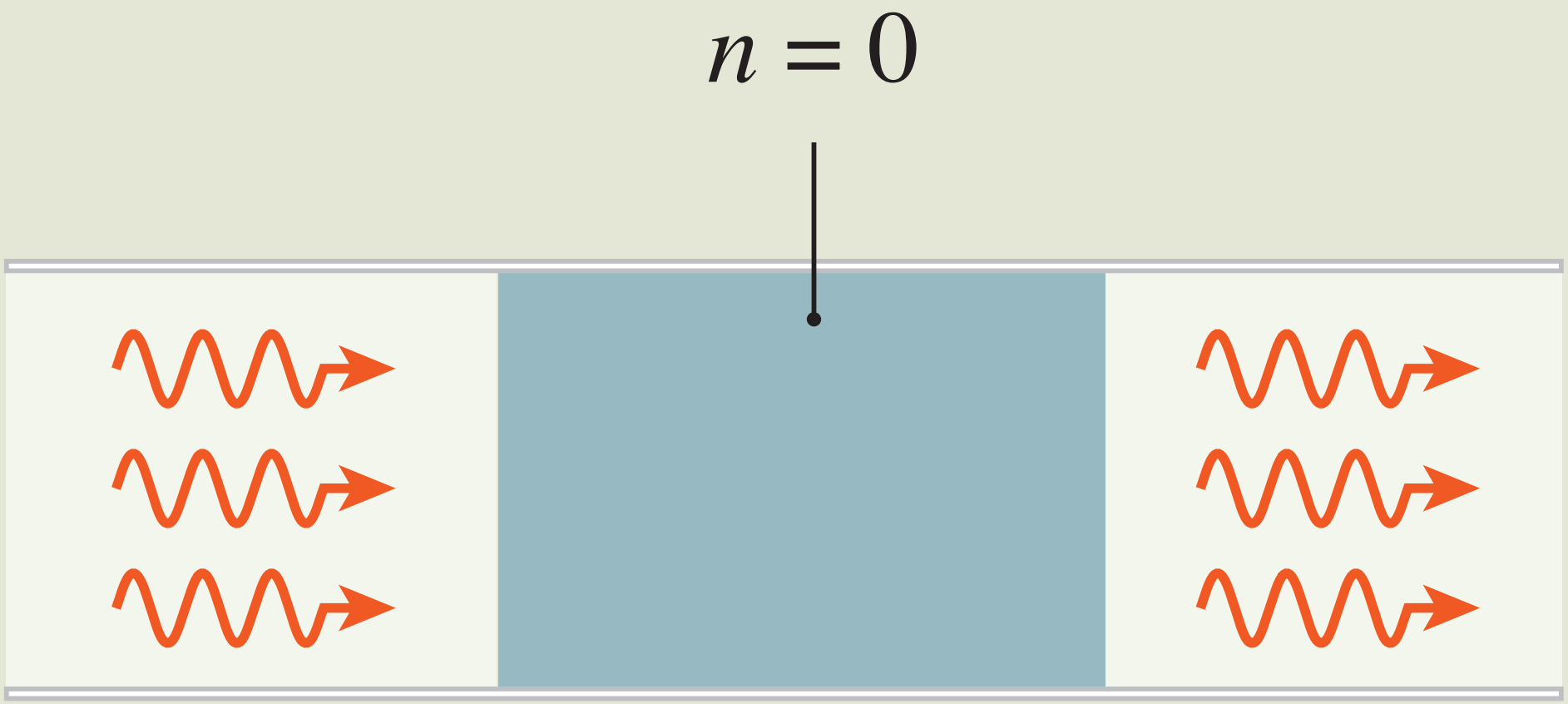
1 index

2 zero index

What can we do with uniform phase?

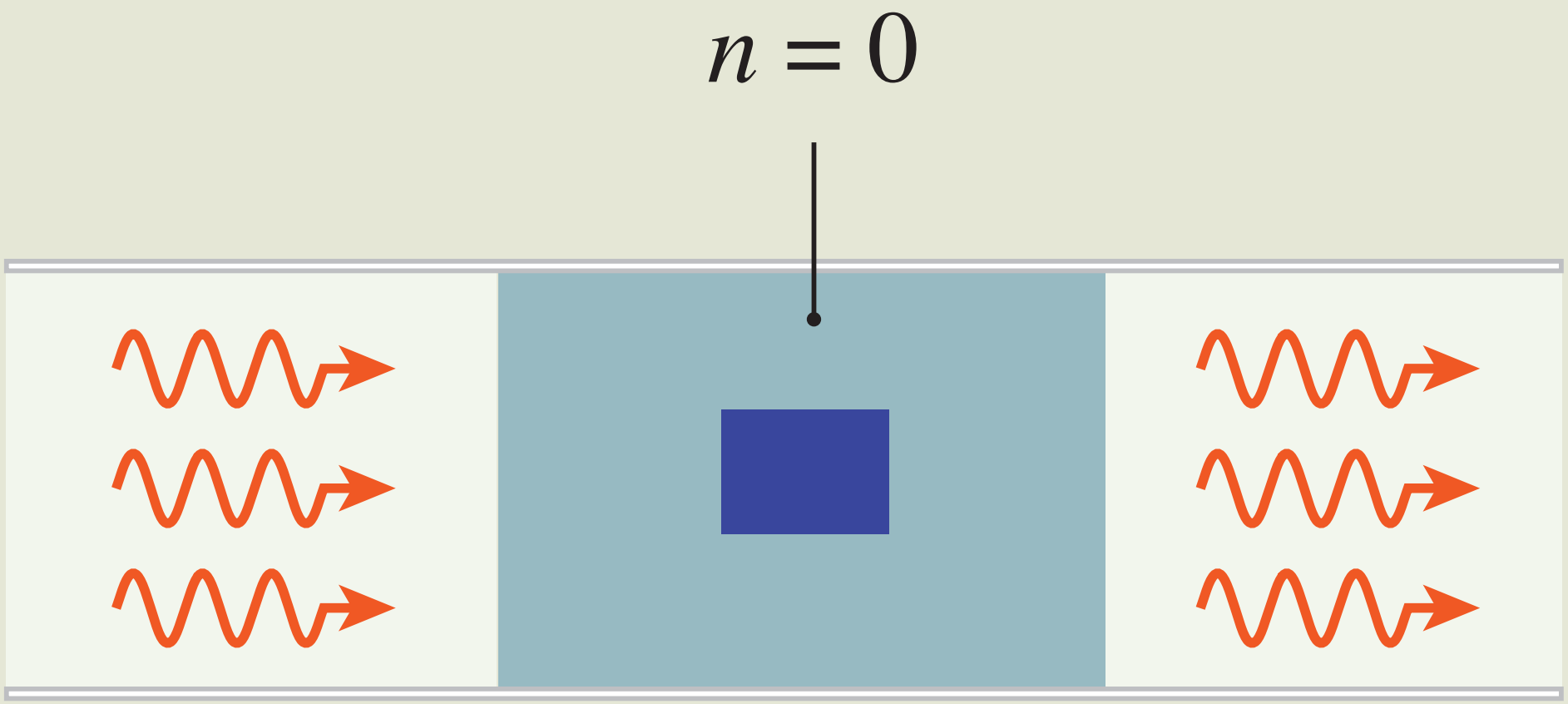
1 index

2 zero index



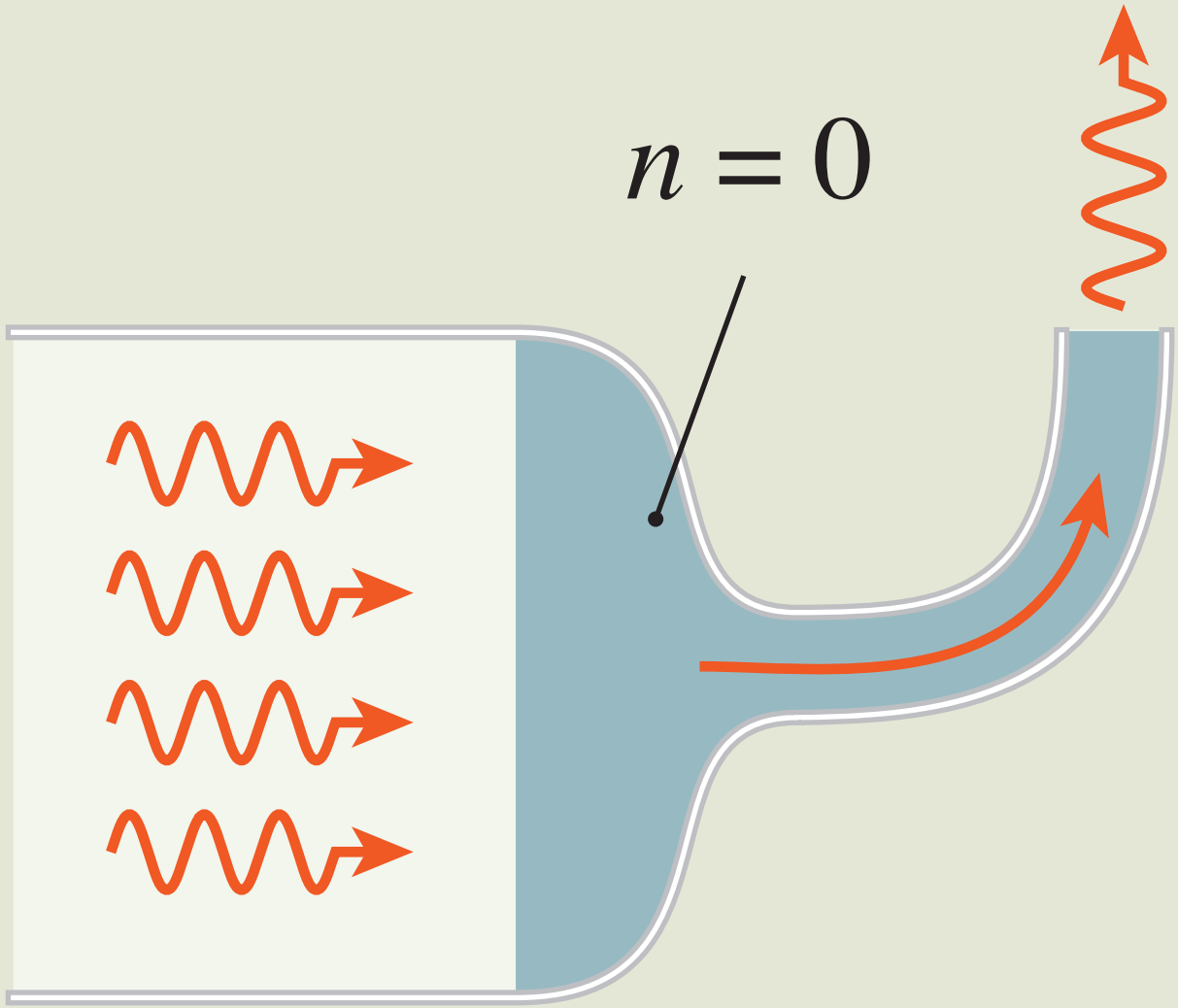
1 index

2 zero index



1 index

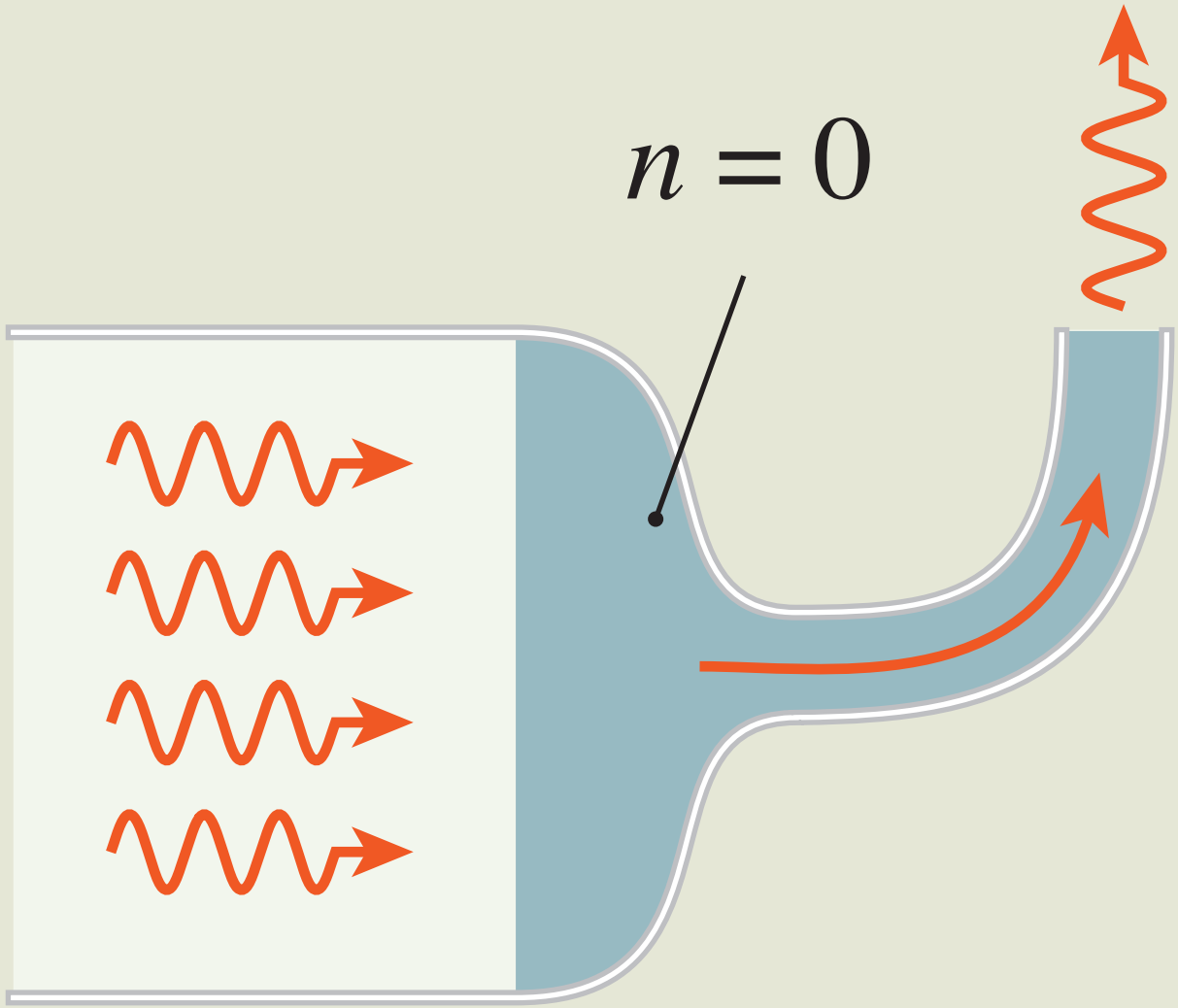
2 zero index



1 index

2 zero index

“tunneling with infinite decay length”



1 index

2 zero index

how?

$$n = \sqrt{\varepsilon\mu}$$

1 index

2 zero index

how?

$$n = \sqrt{\varepsilon\mu}$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

how?

$$n = \sqrt{\varepsilon\mu}$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

how?

$$\varepsilon \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

1 index

2 zero index

how?

$$\varepsilon \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to \infty$$

1 index 2 zero index

how?

$$\varepsilon \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1} \to 1$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to \infty$$

1 index

2 zero index

how?

$$\mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

1 index 2 zero index

how?

$$\mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to 0$$

1 index

2 zero index

how?

$$\mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1} \to -1$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to 0$$

1 index

2 zero index

how?

$$\varepsilon, \mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

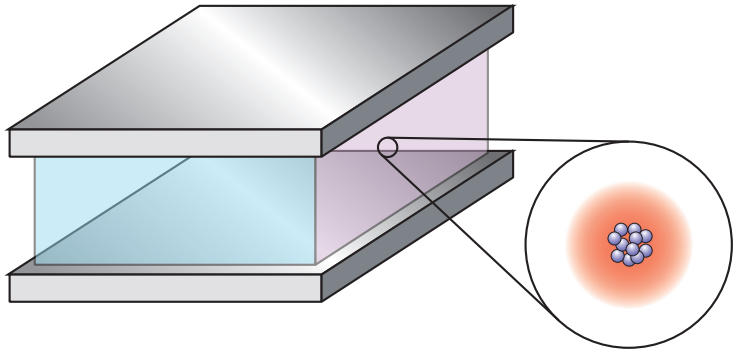
finite!

but $\mu \neq 1$ requires a magnetic response!

Engineering a magnetic response

Engineering a magnetic response

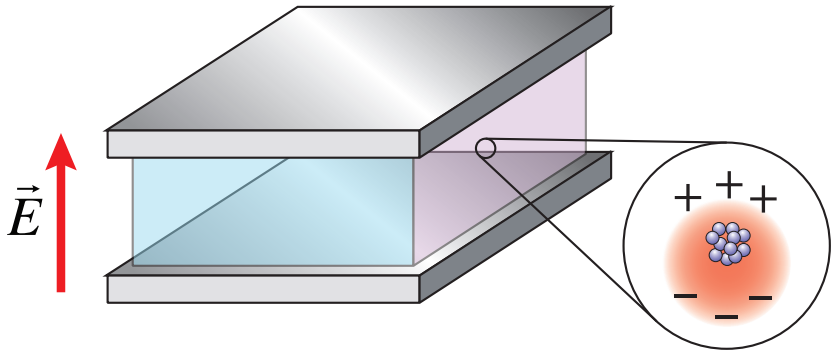
bulk material



properties derive from
constituent atoms

Engineering a magnetic response

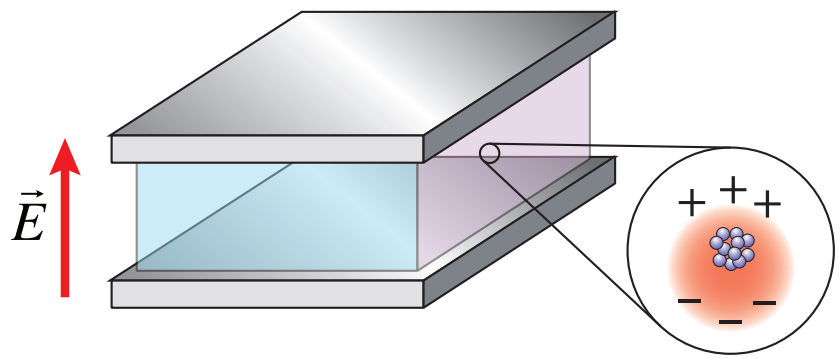
bulk material



properties derive from constituent atoms

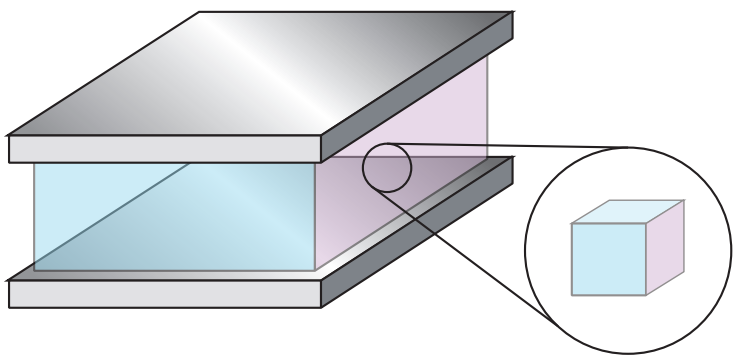
Engineering a magnetic response

bulk material



properties derive from constituent atoms

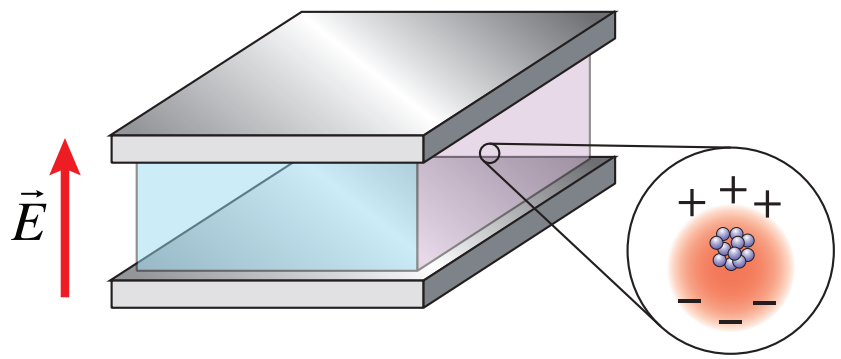
composite material



properties derive from constituent units

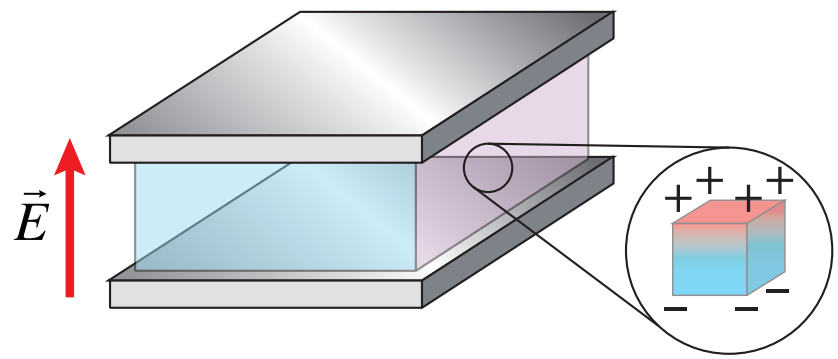
Engineering a magnetic response

bulk material



properties derive from constituent atoms

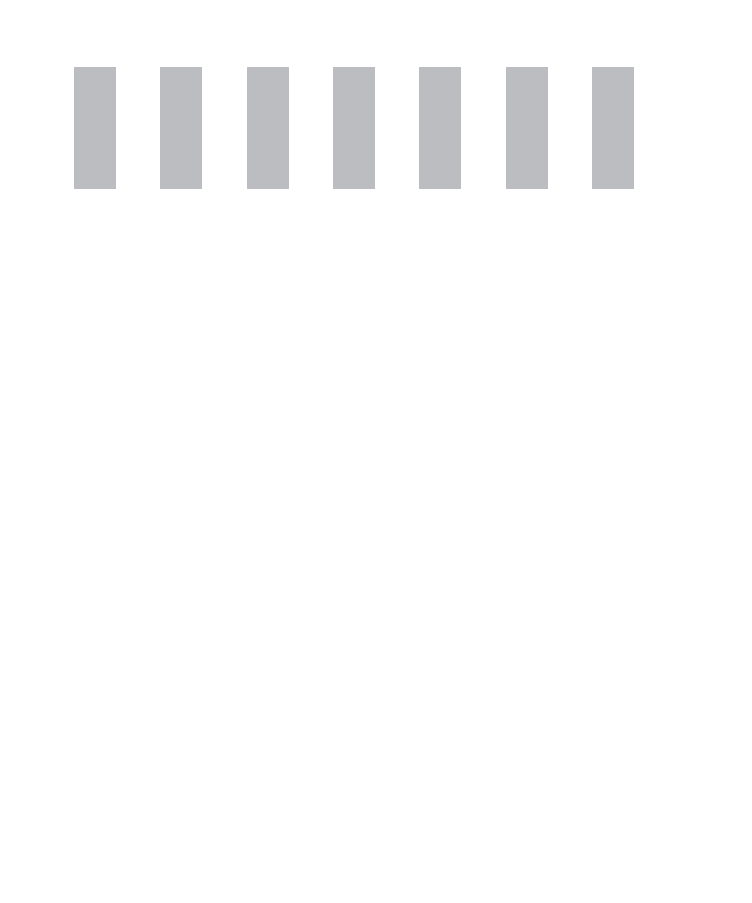
composite material



properties derive from constituent units

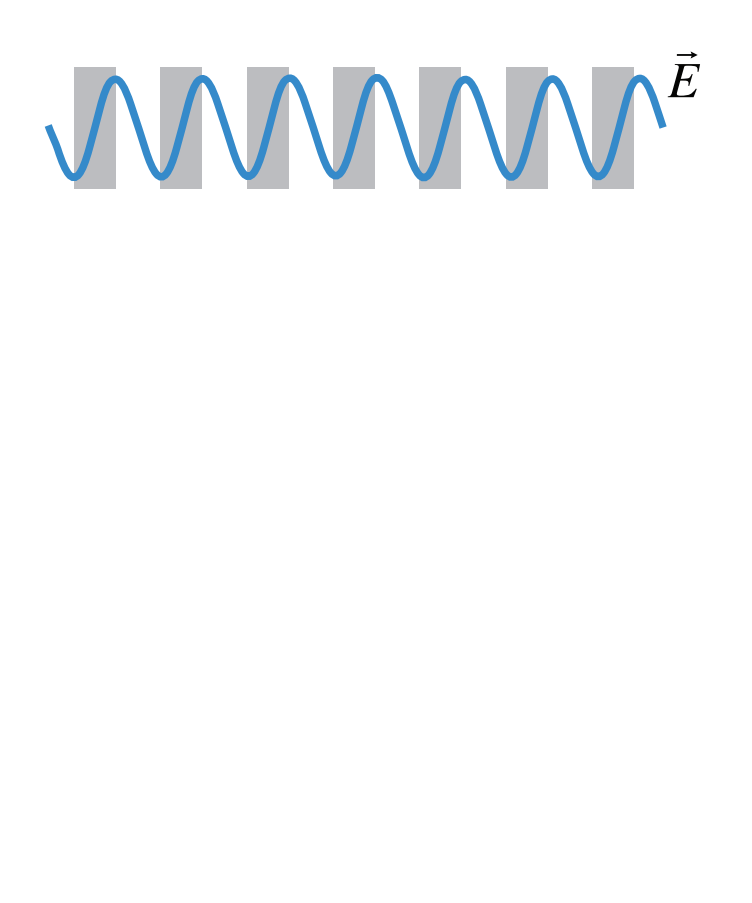
Engineering a magnetic response

use array of dielectric rods



Engineering a magnetic response

incident electromagnetic wave ($\lambda_{\text{eff}} \approx a$)

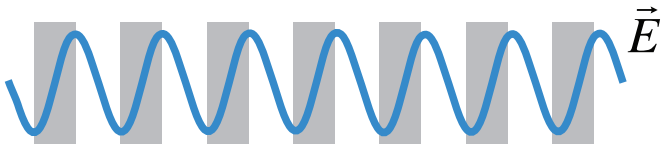


1 index

2 zero index

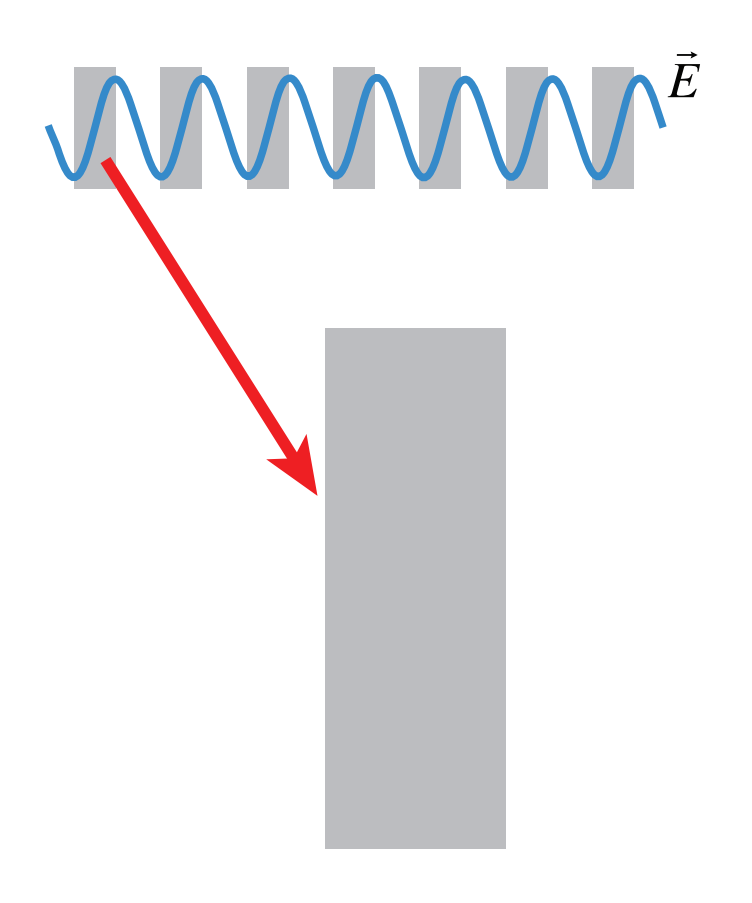
Engineering a magnetic response

produces an electric response…



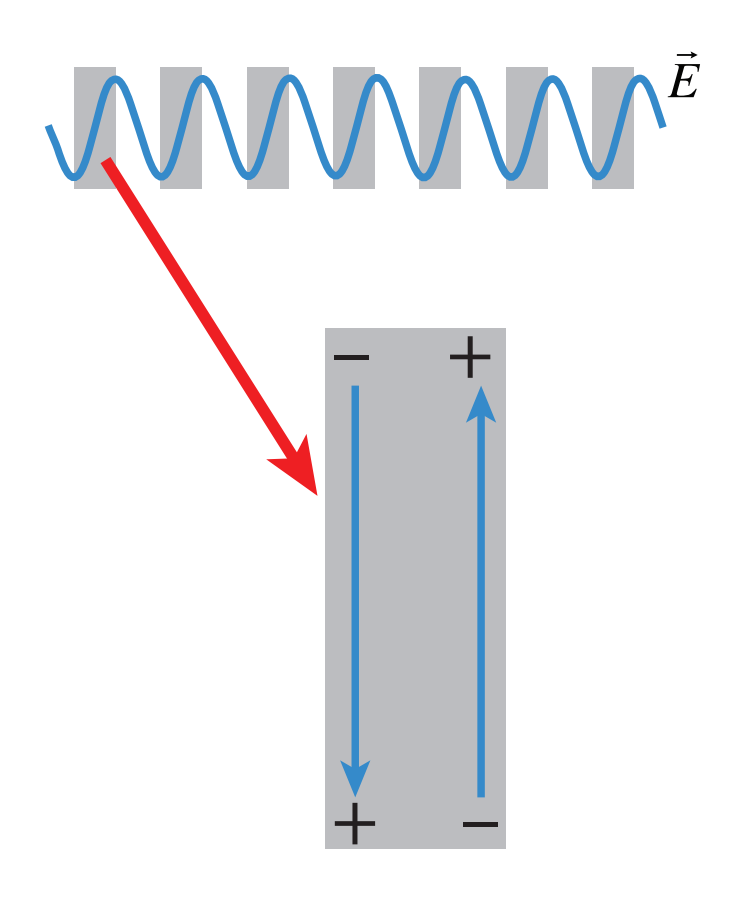
Engineering a magnetic response

… but different electric fields front and back…



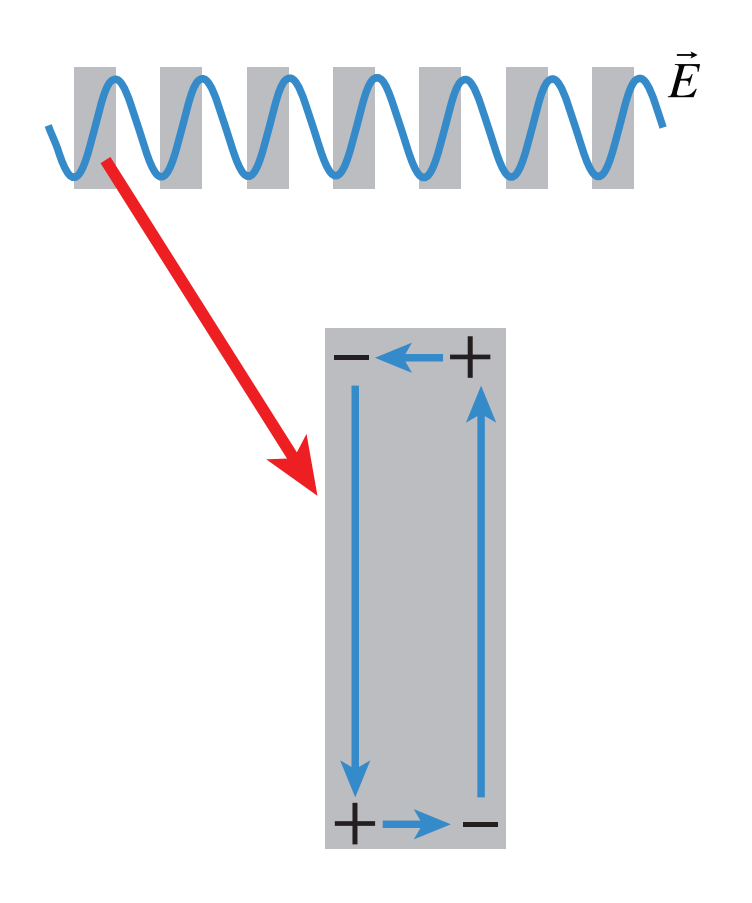
Engineering a magnetic response

…induce different polarizations on opposite sides…



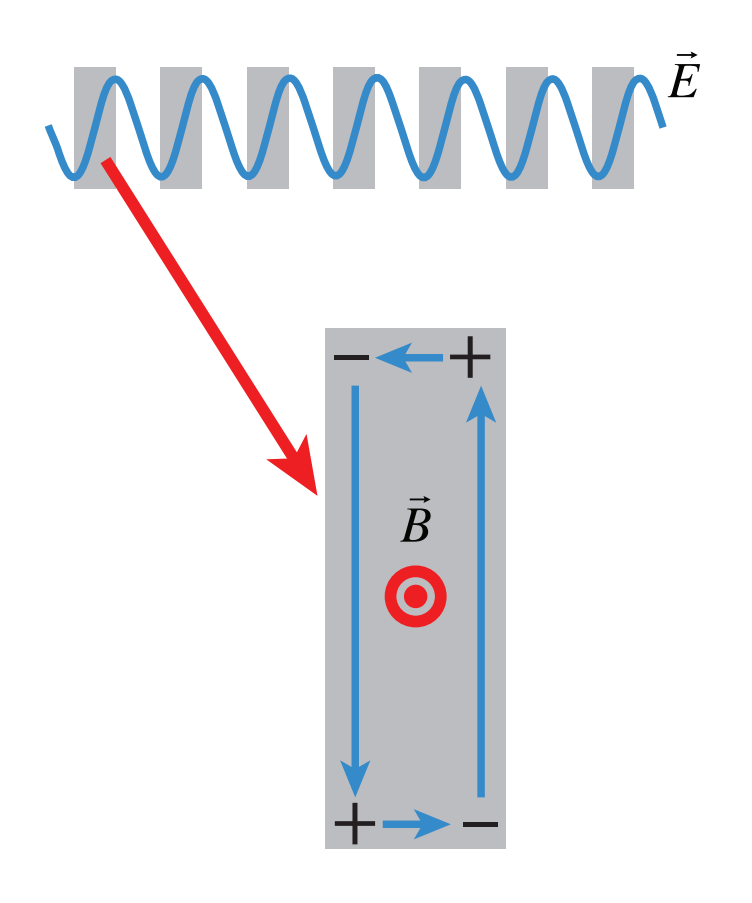
Engineering a magnetic response

…causing a current loop…



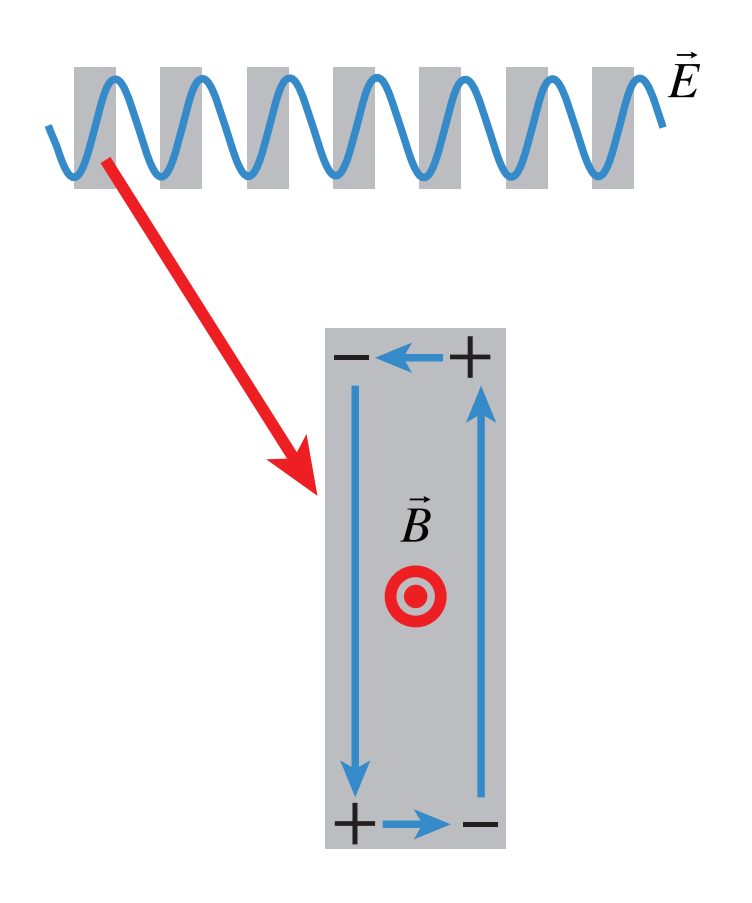
Engineering a magnetic response

…which, in turn, produces an induced magnetic field



Engineering a magnetic response

adjust design so electrical and magnetic resonances coincide



Engineering a magnetic response

adjustable parameters

Engineering a magnetic response

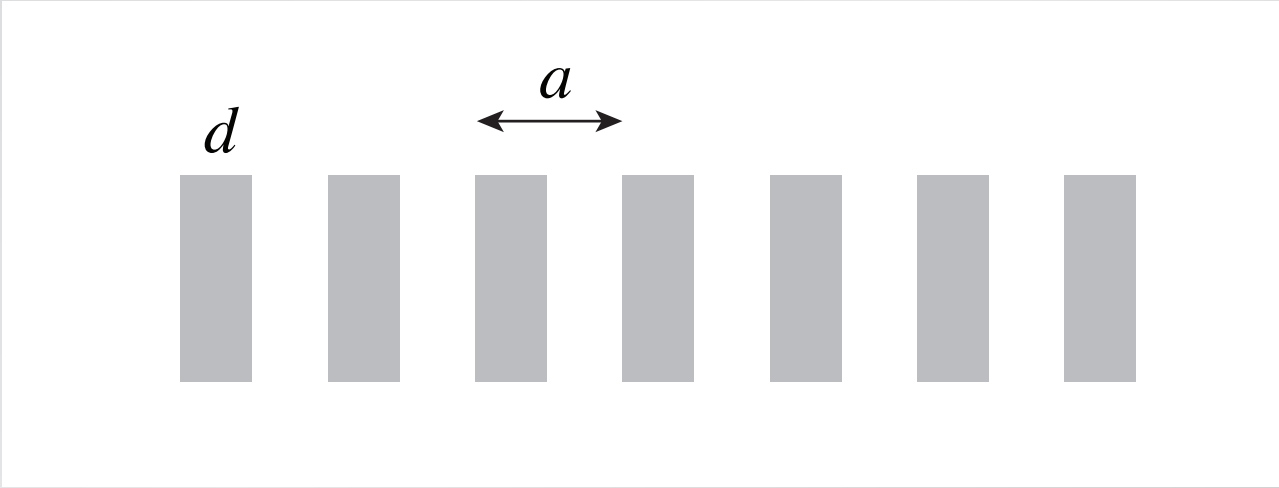
adjustable parameters

Engineering a magnetic response

adjustable parameters

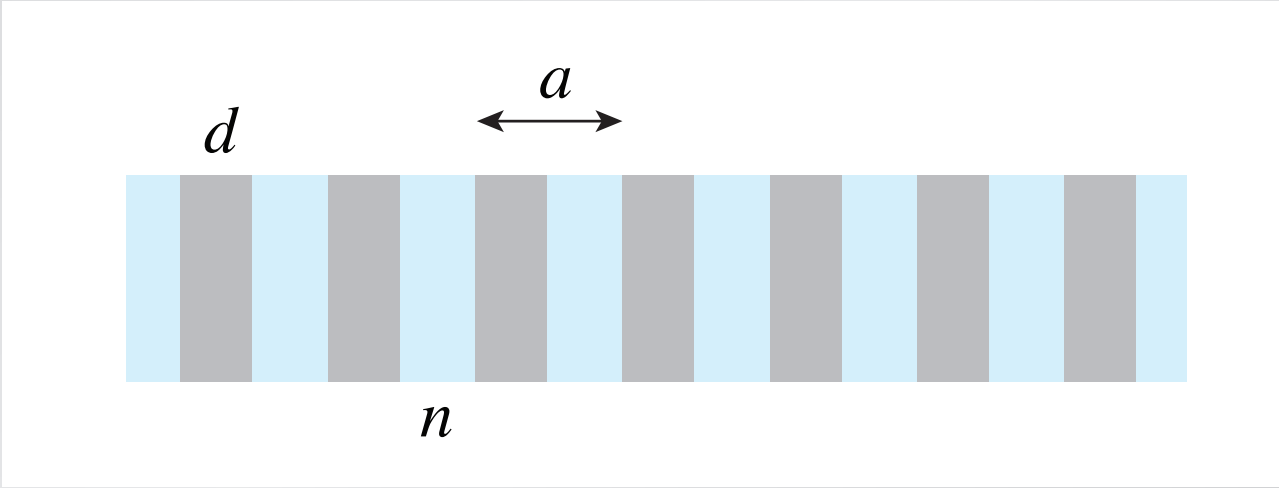
Engineering a magnetic response

adjustable parameters



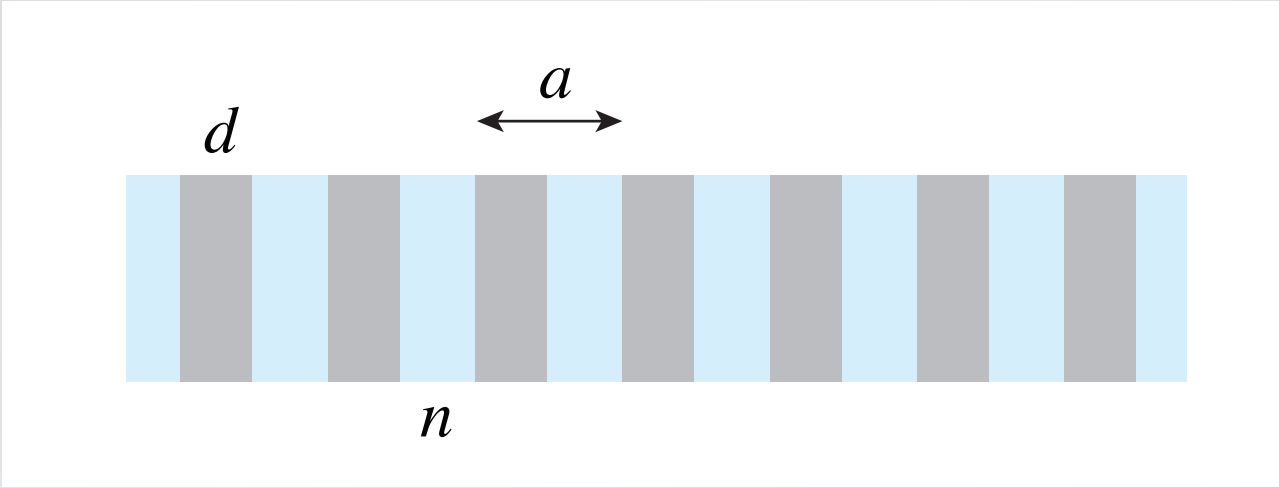
Engineering a magnetic response

adjustable parameters

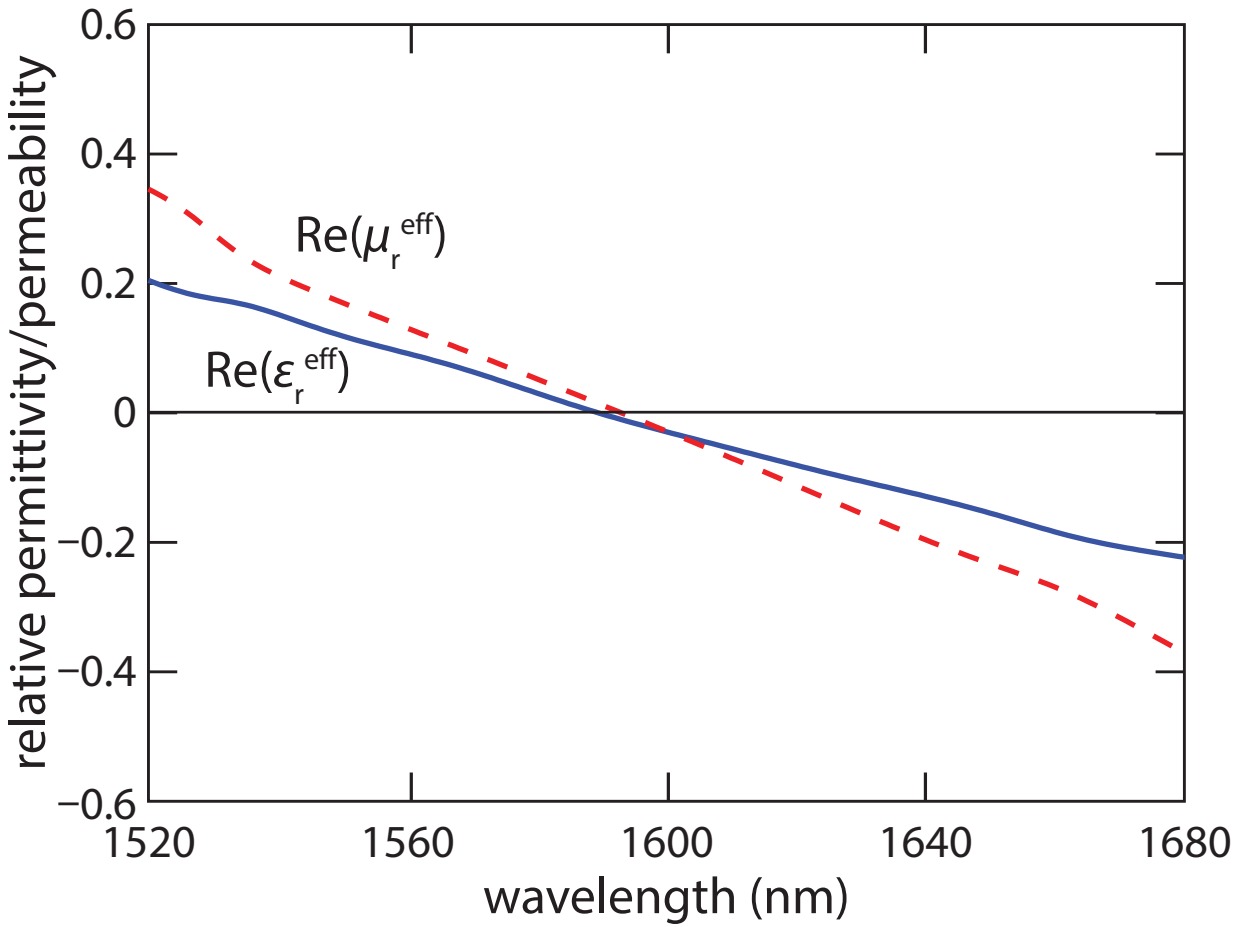


Engineering a magnetic response

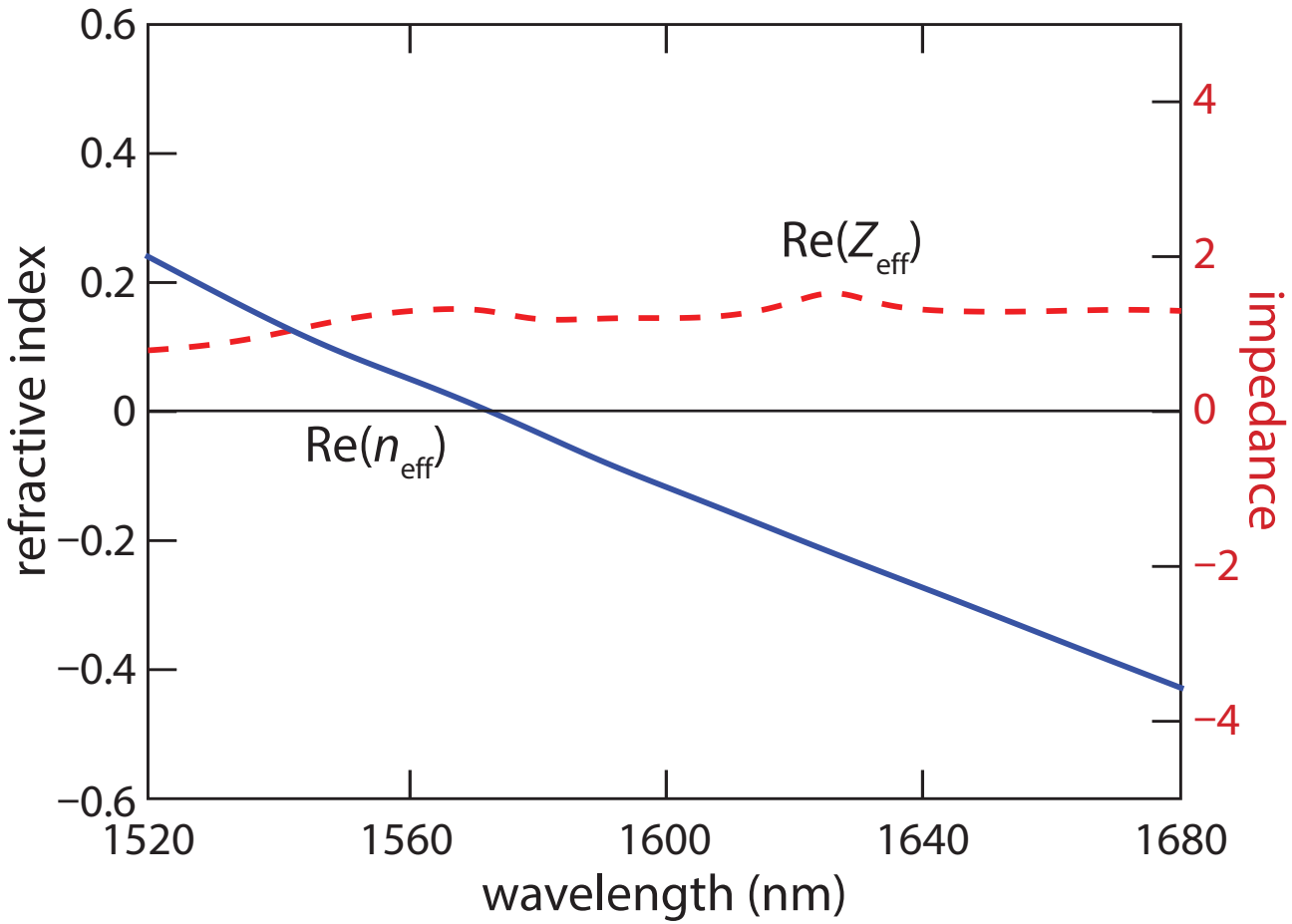
adjustable parameters



d = 422 nm, a = 690 nm, n = 1.57 (SU8)

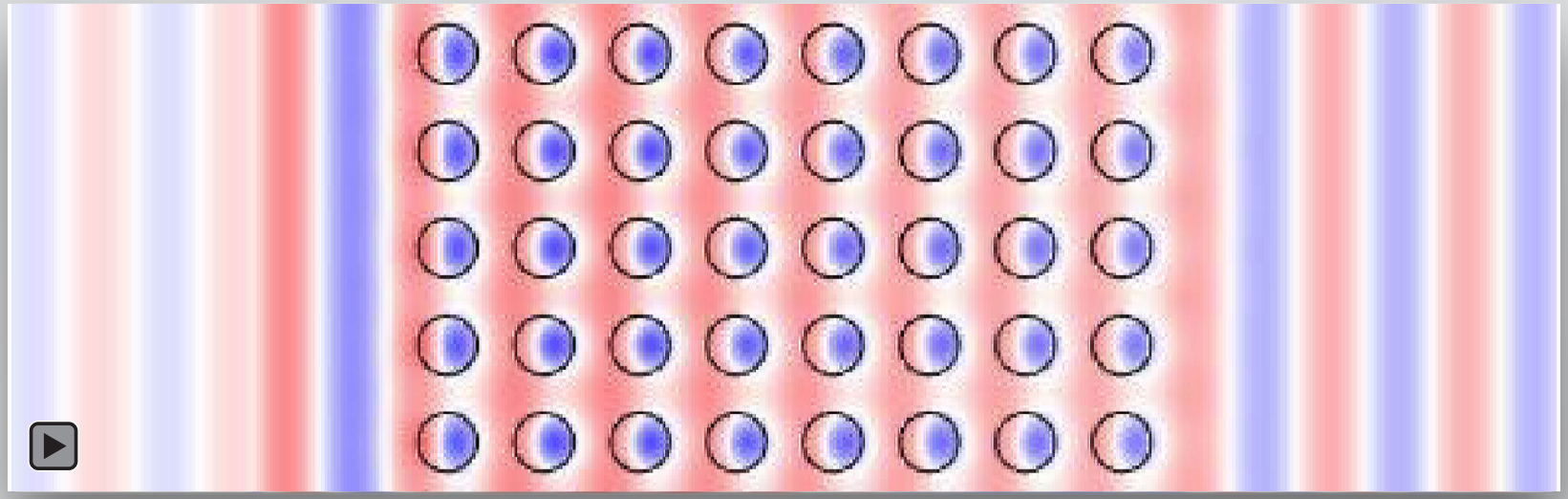


1 index 2 zero index 3 experiments

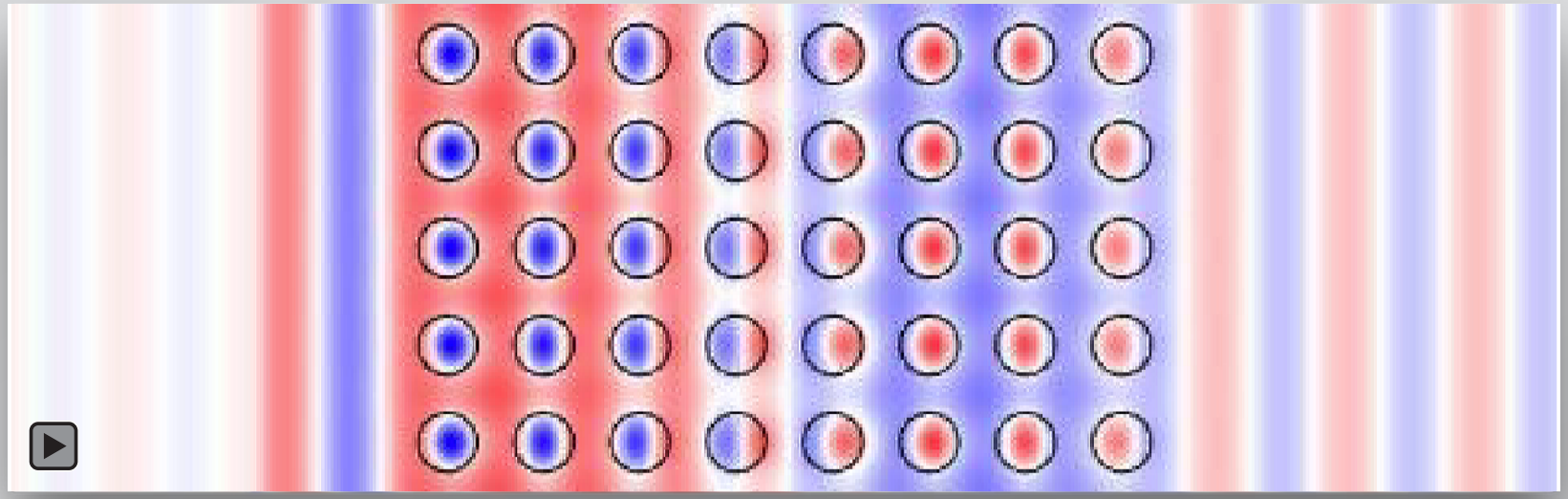


1 index 2 zero index 3 experiments

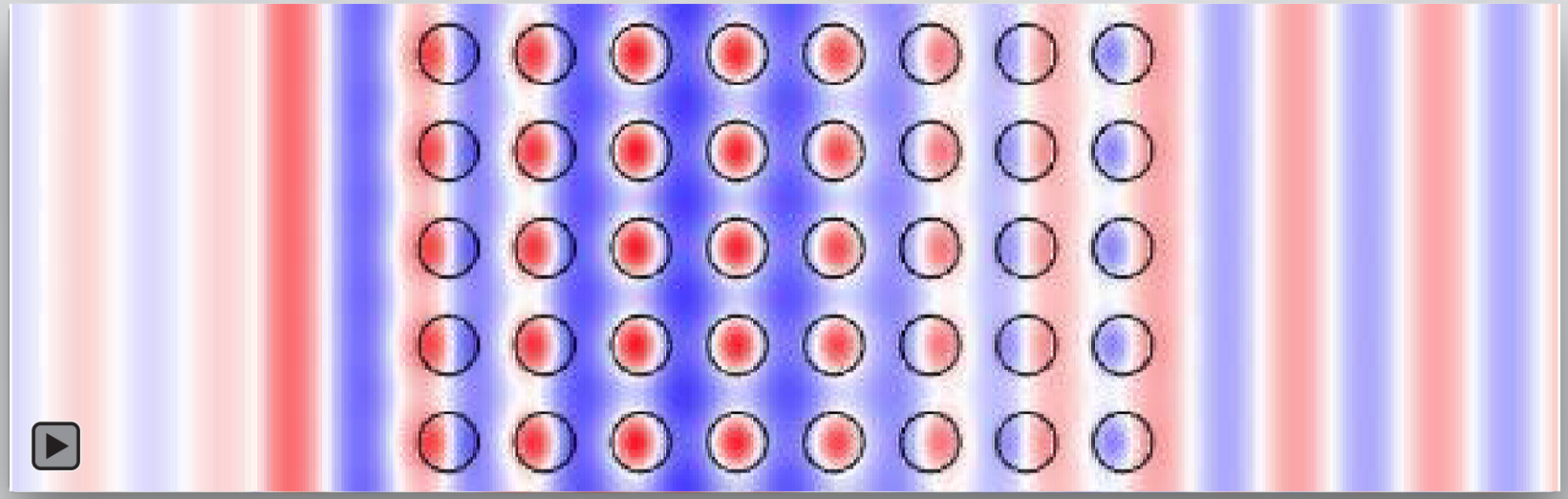
at design wavelength (1590 nm)



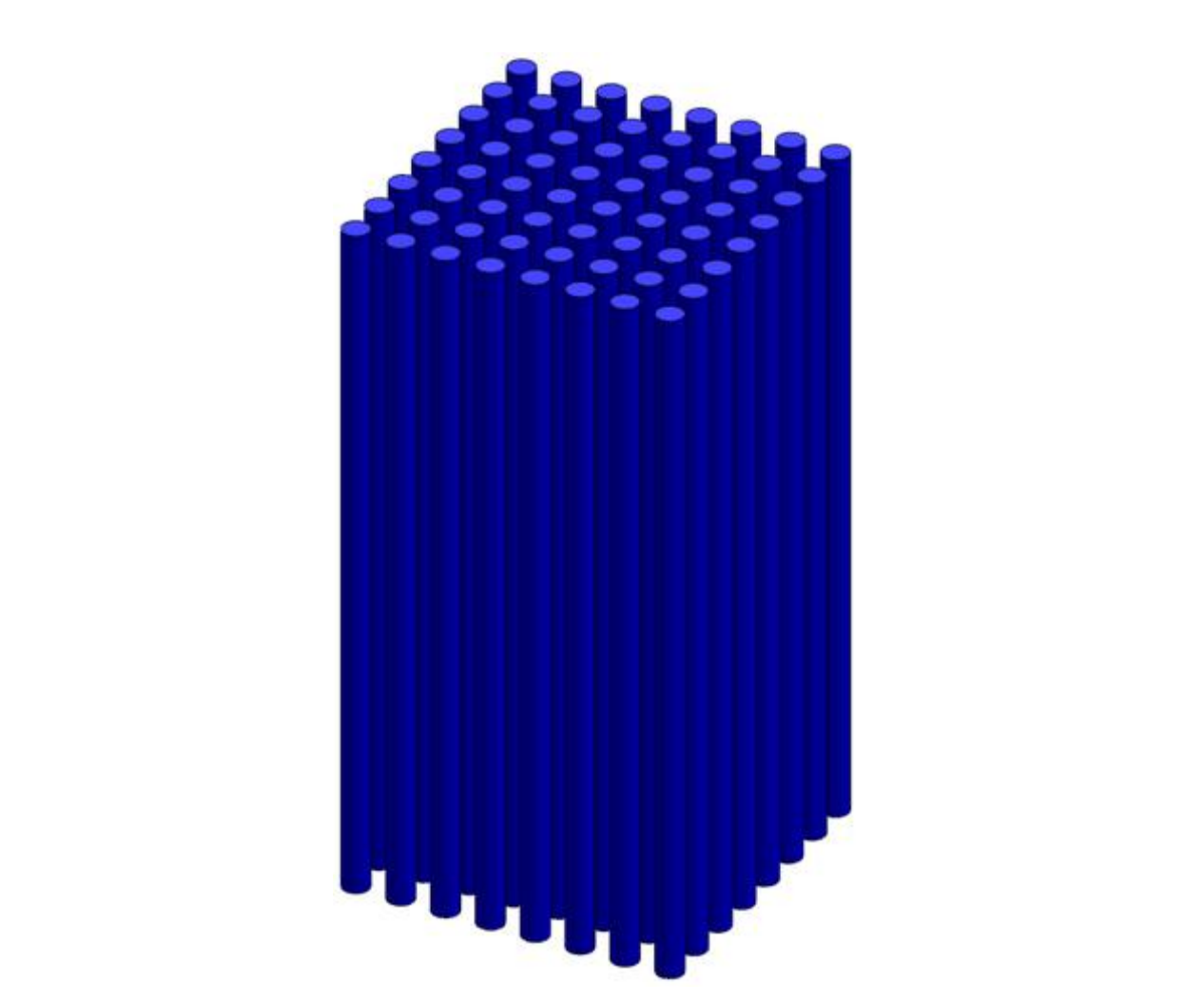
below design wavelength (1530 nm)



above design wavelength (1650 nm)

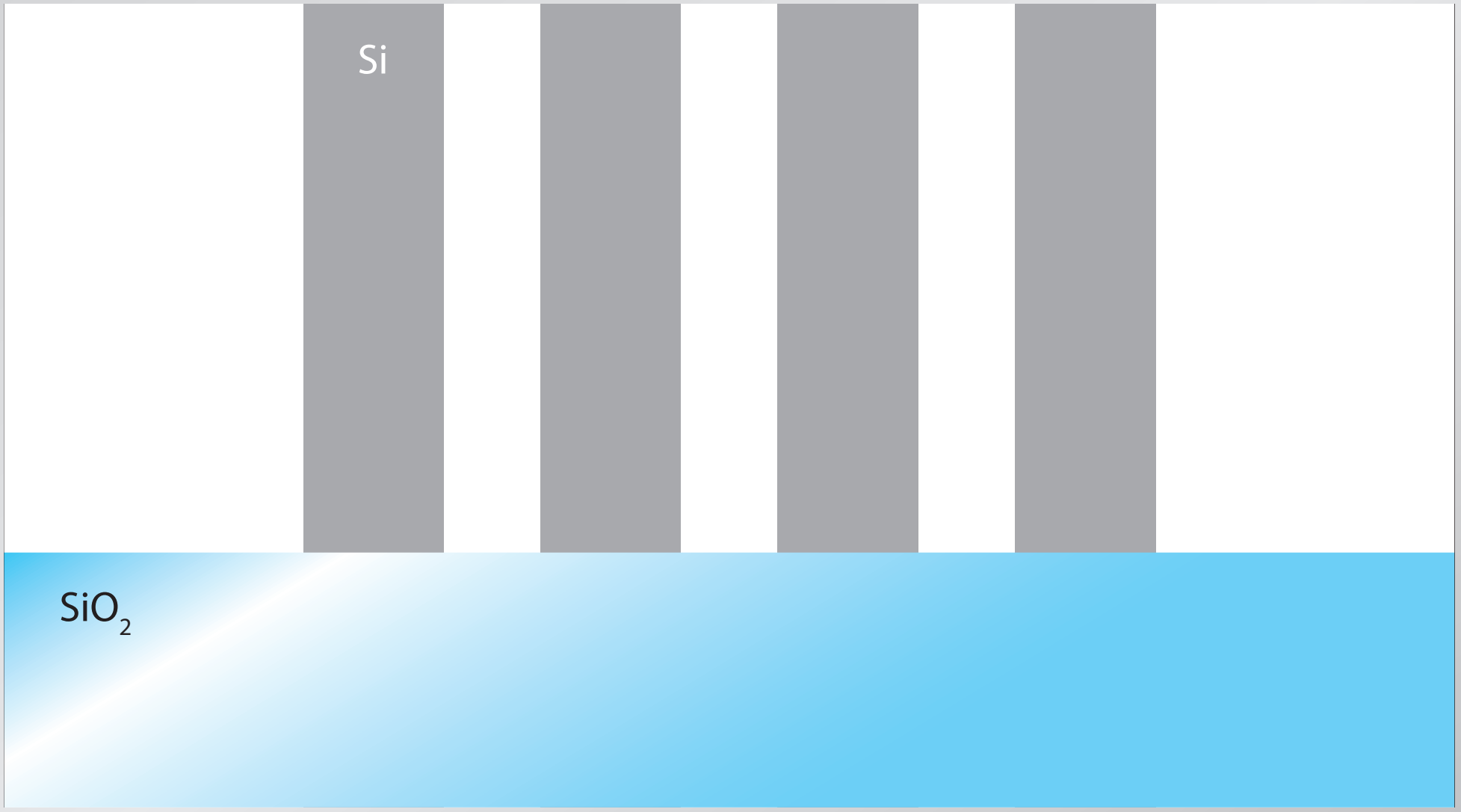


How to fabricate?



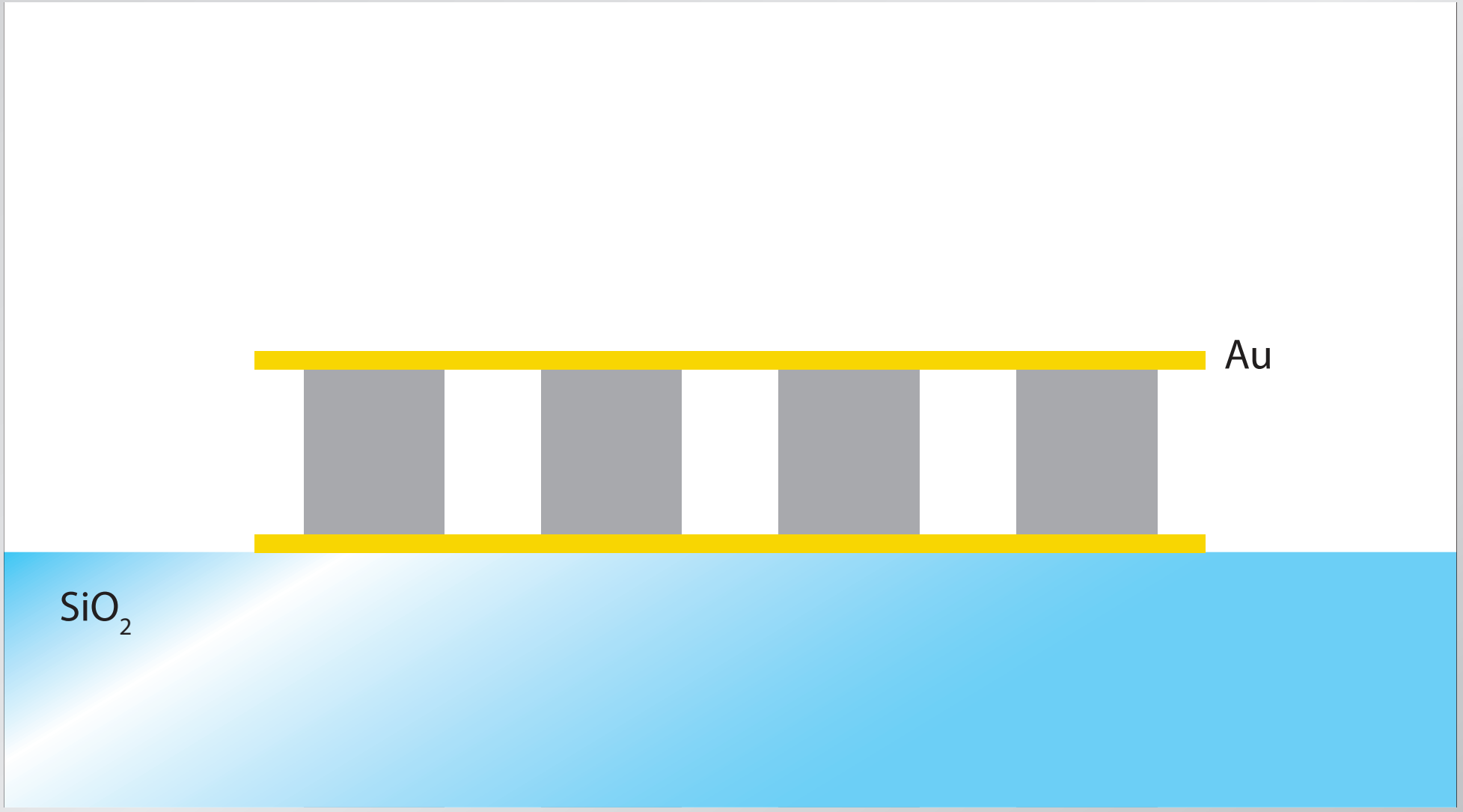
On-chip zero-index fabrication

On-chip zero-index fabrication

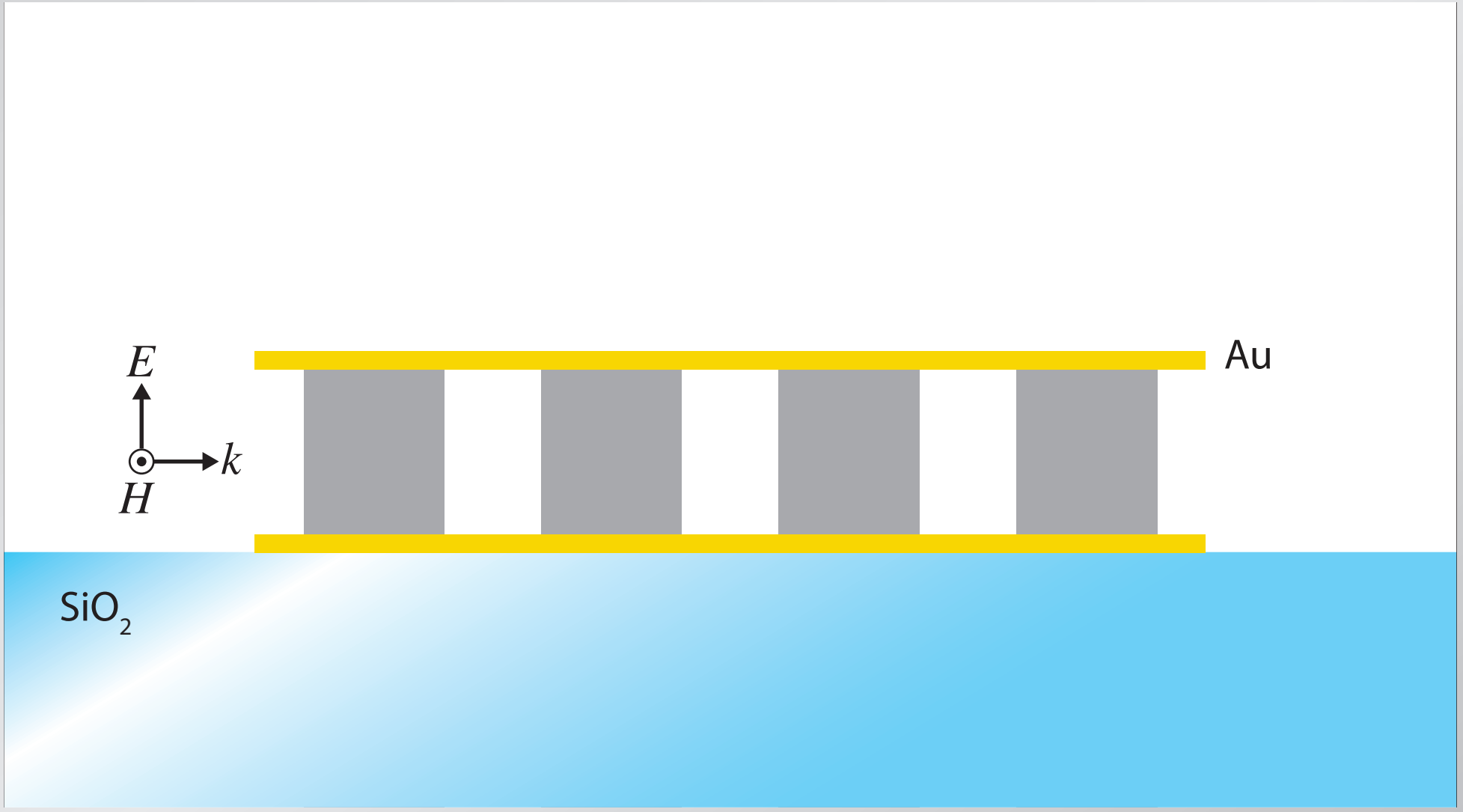


On-chip zero-index fabrication

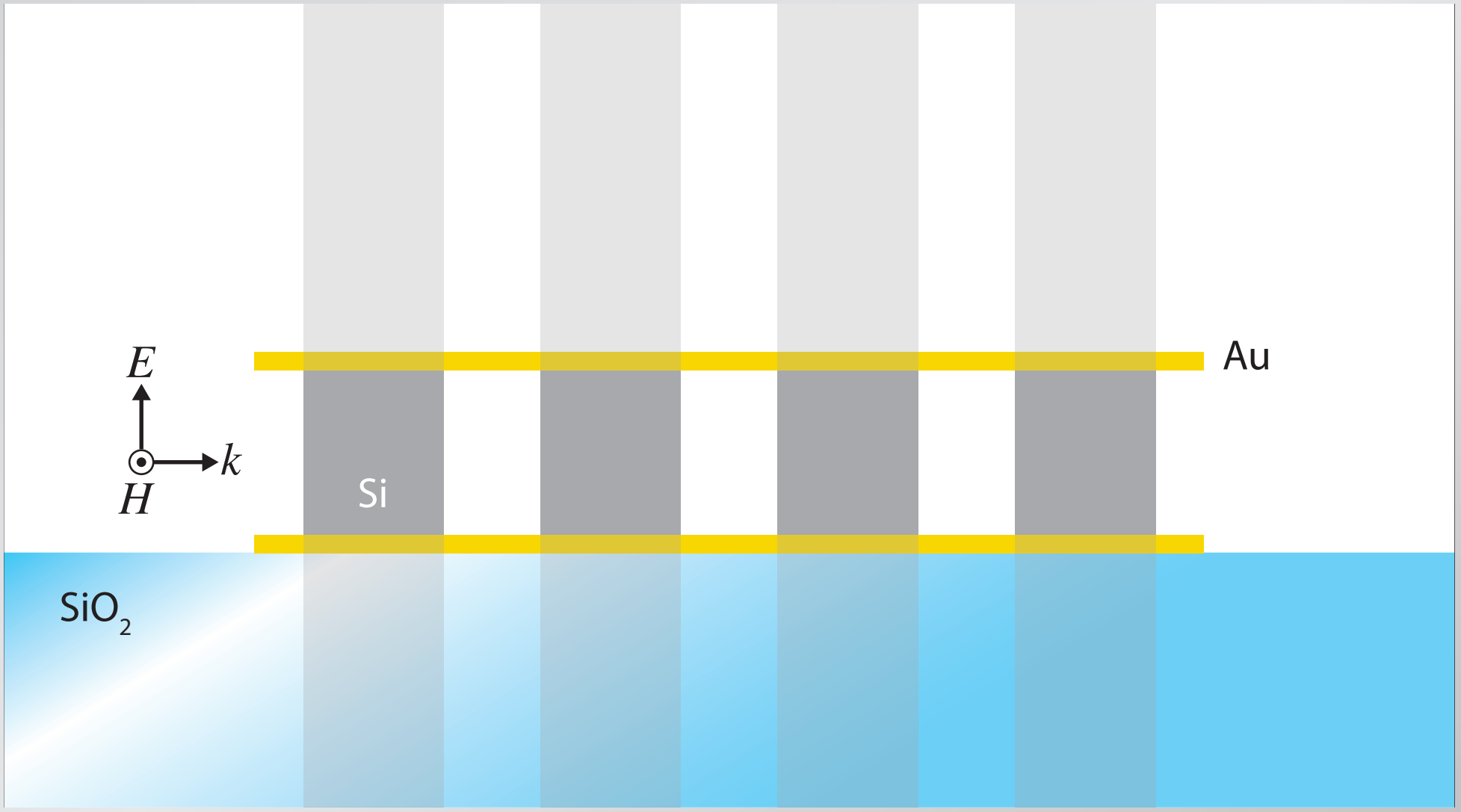
On-chip zero-index fabrication

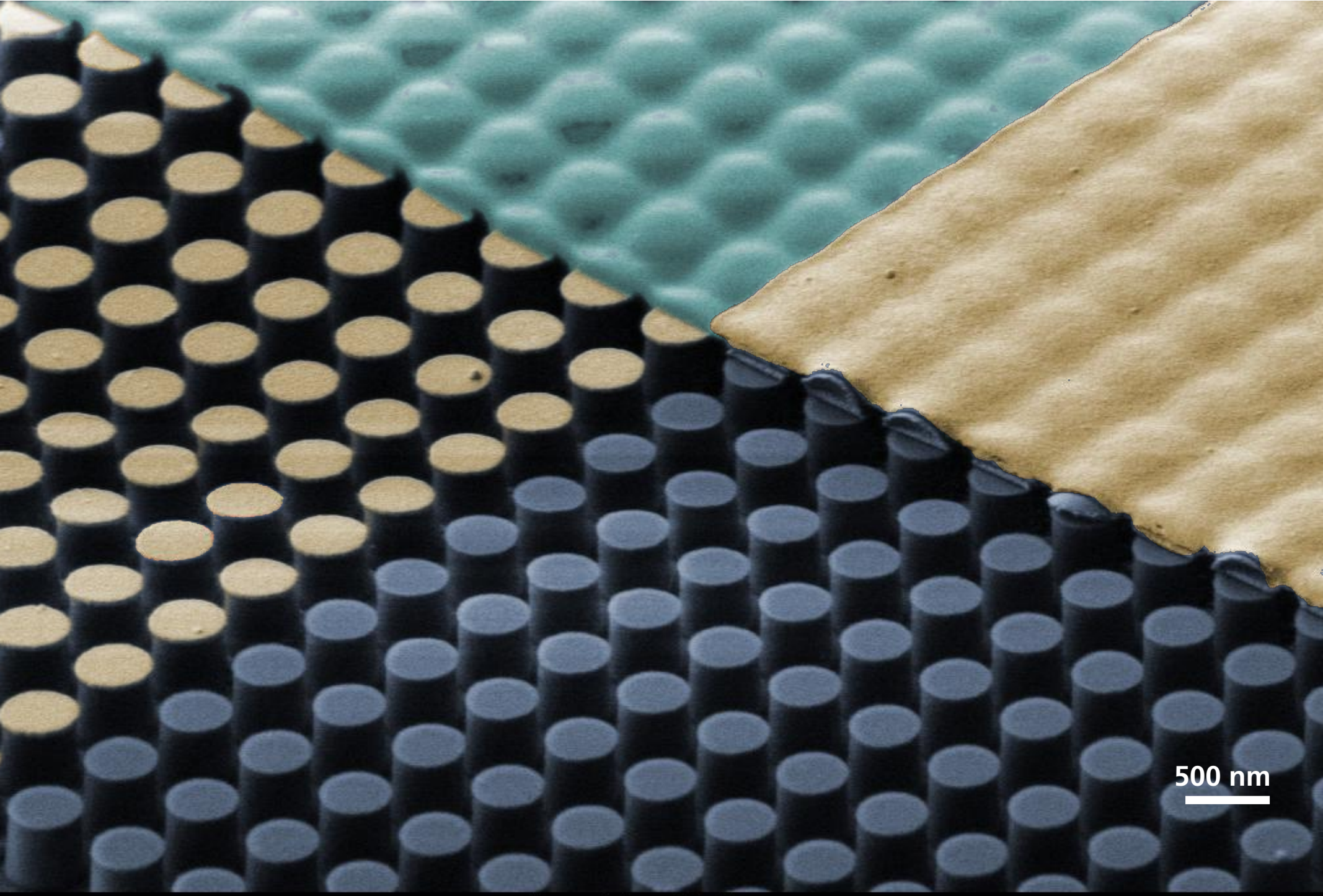


On-chip zero-index fabrication



On-chip zero-index fabrication

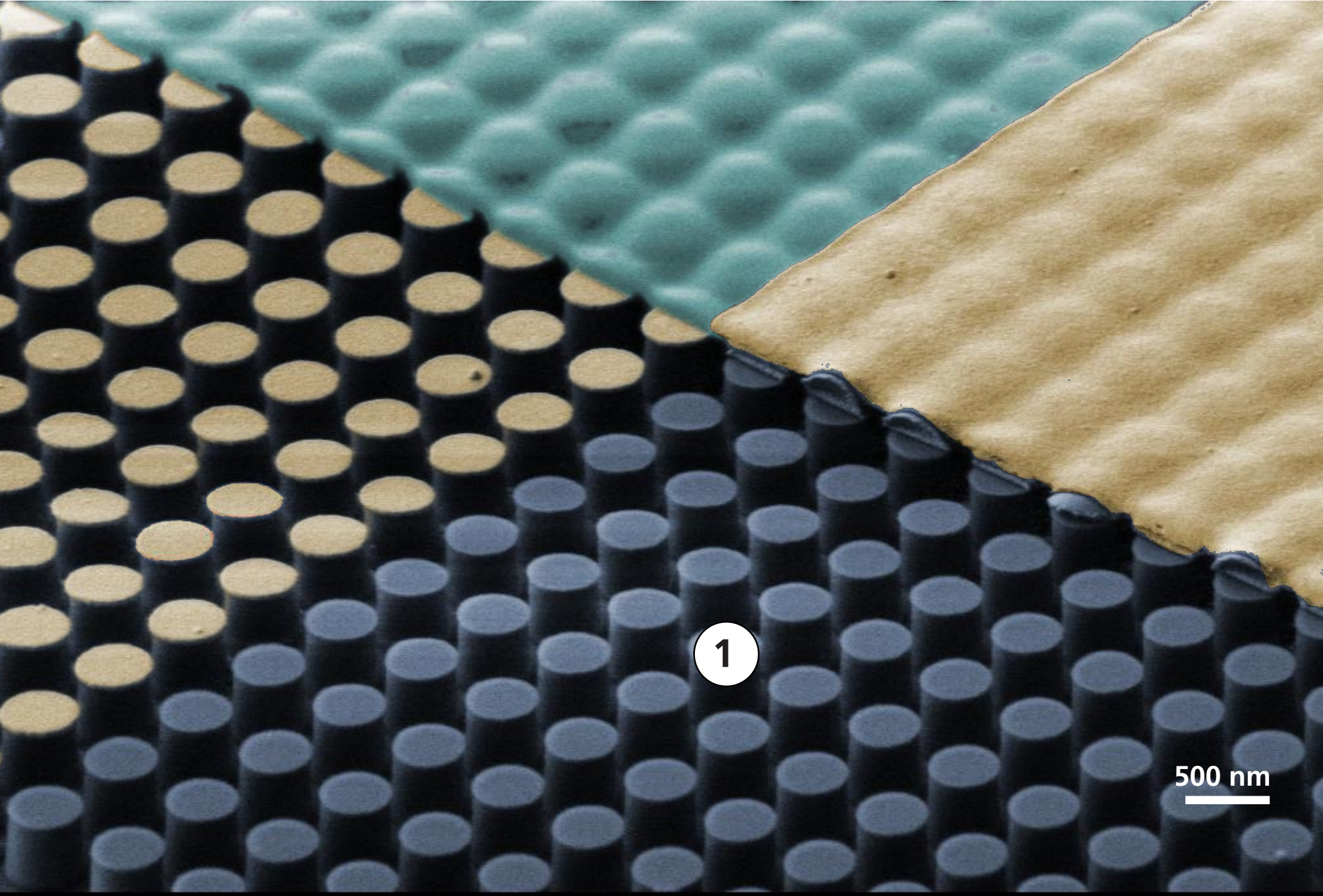




1 index

2 zero index

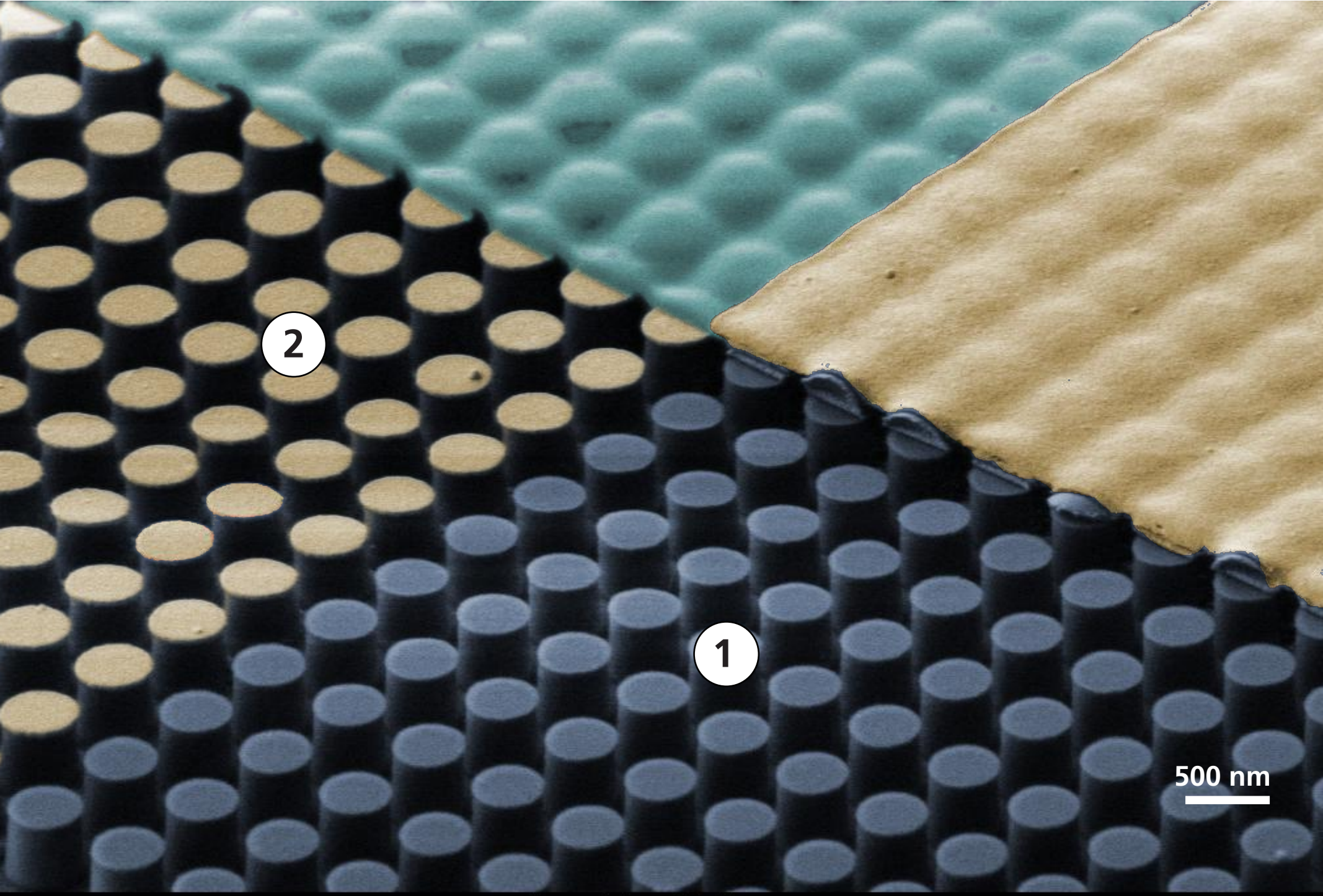
3 experiments



1 index

2 zero index

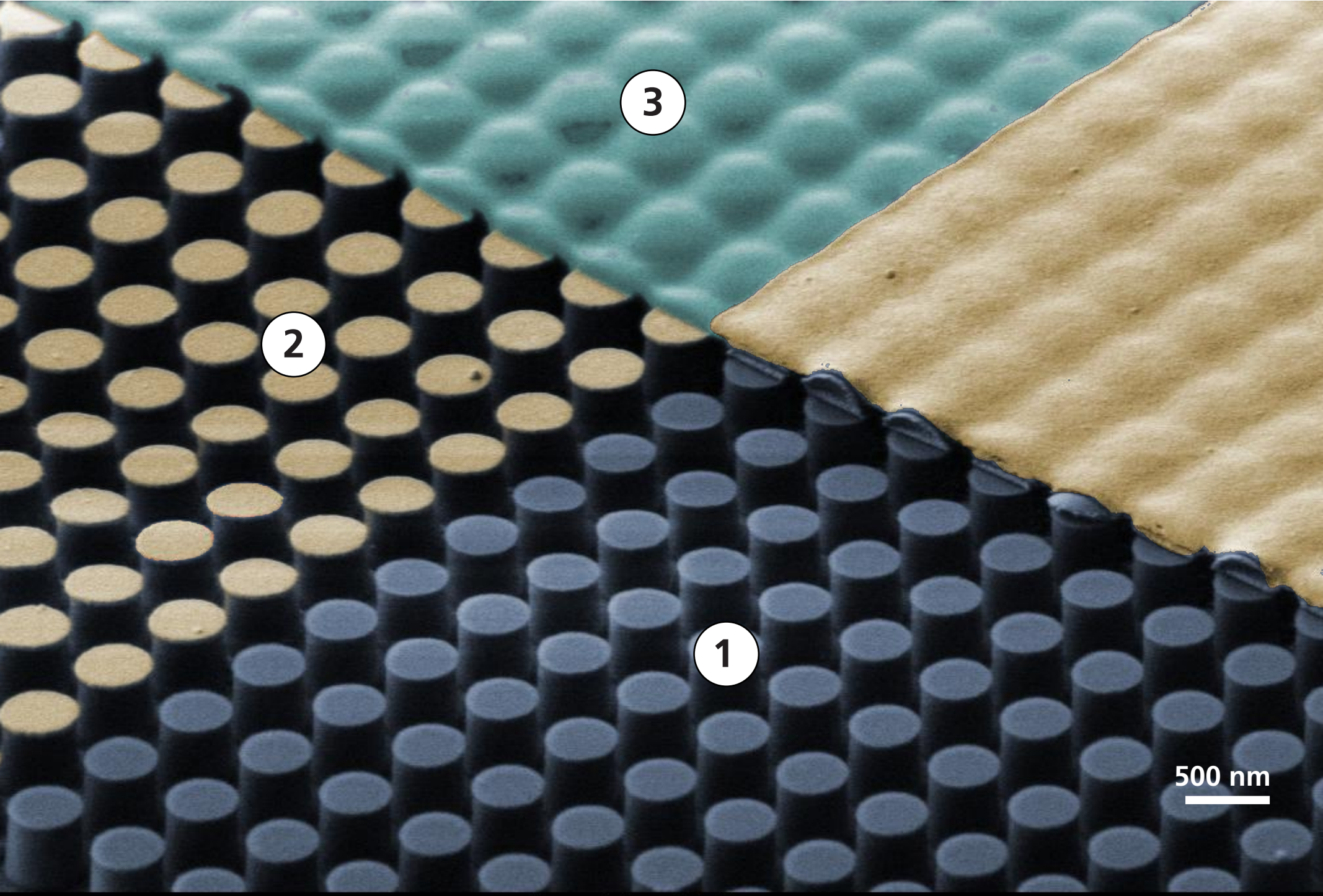
3 experiments



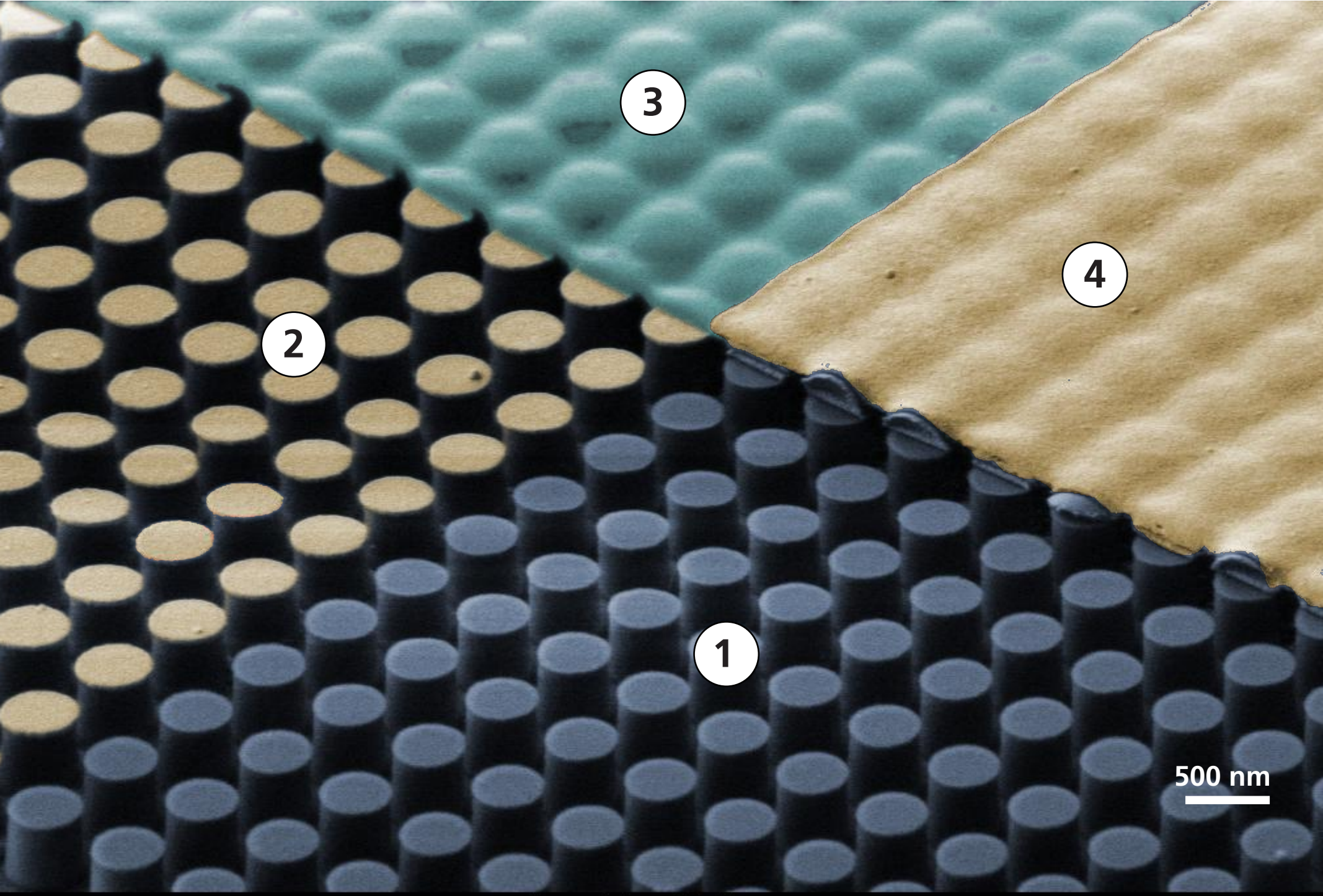
1 index

2 zero index

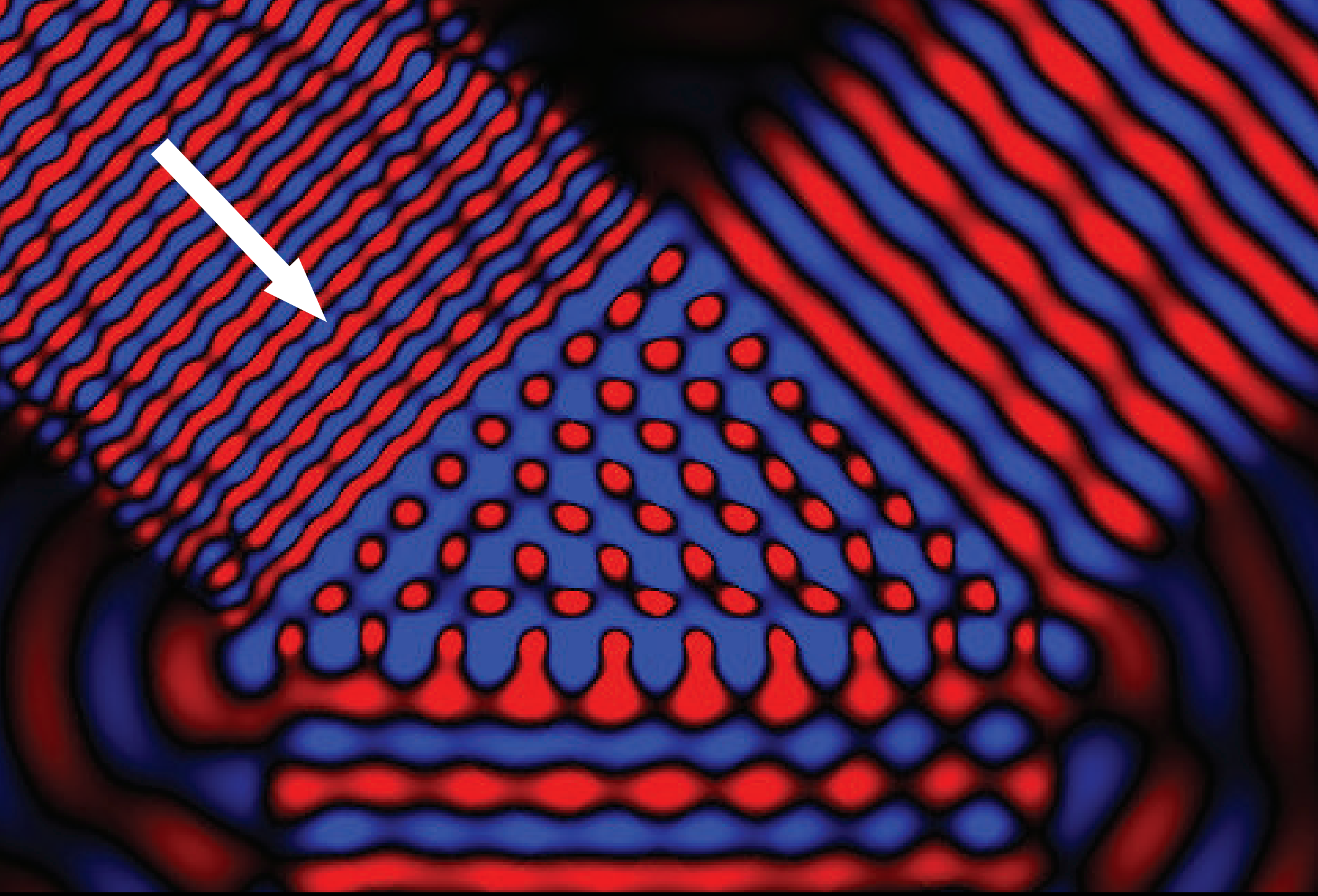
3 experiments



1 index 2 zero index 3 experiments



1 index 2 zero index 3 experiments



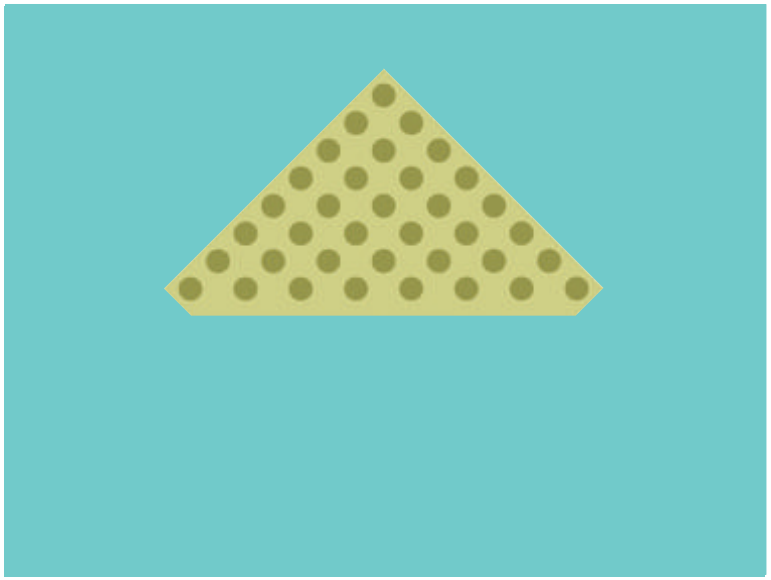
1 index

2 zero index

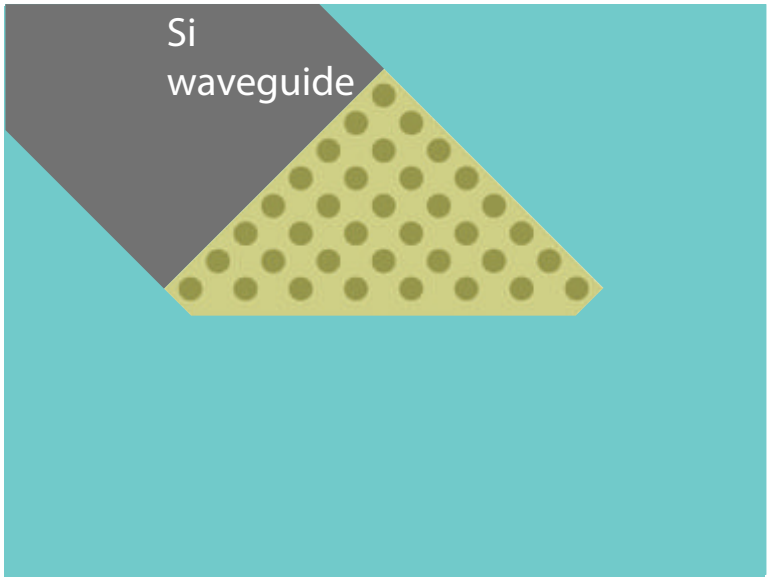
3 experiments

On-chip zero-index prism

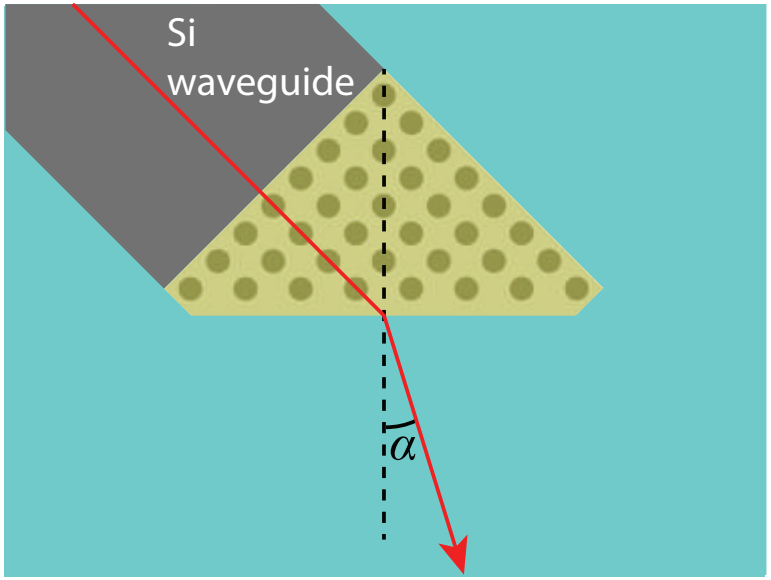
On-chip zero-index prism



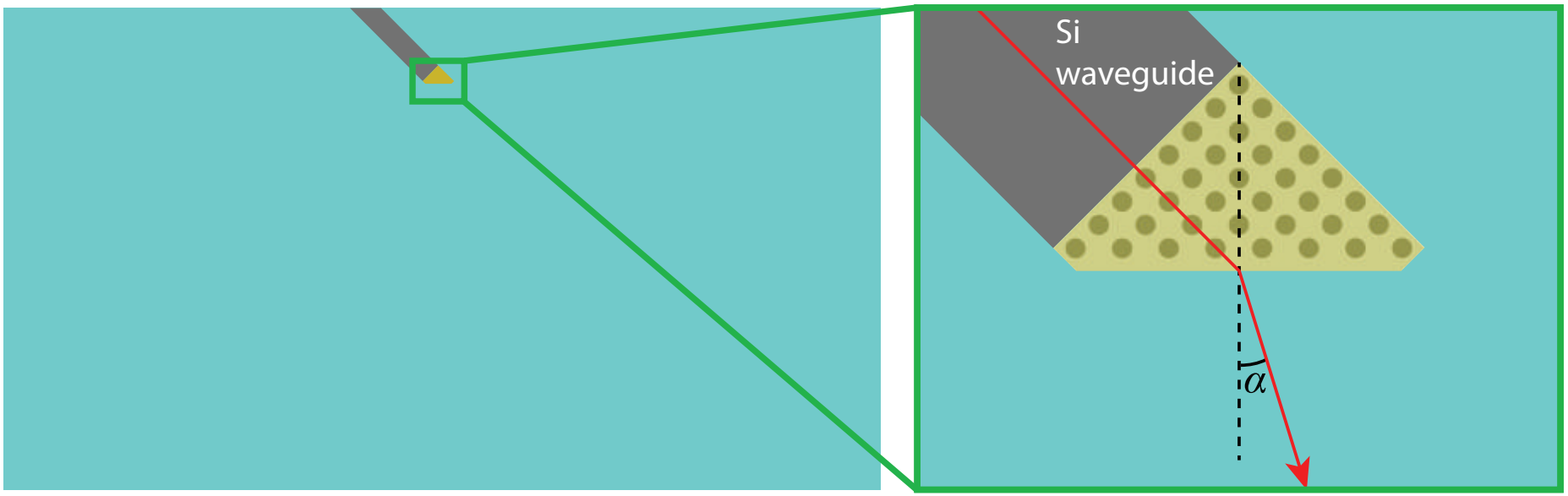
On-chip zero-index prism



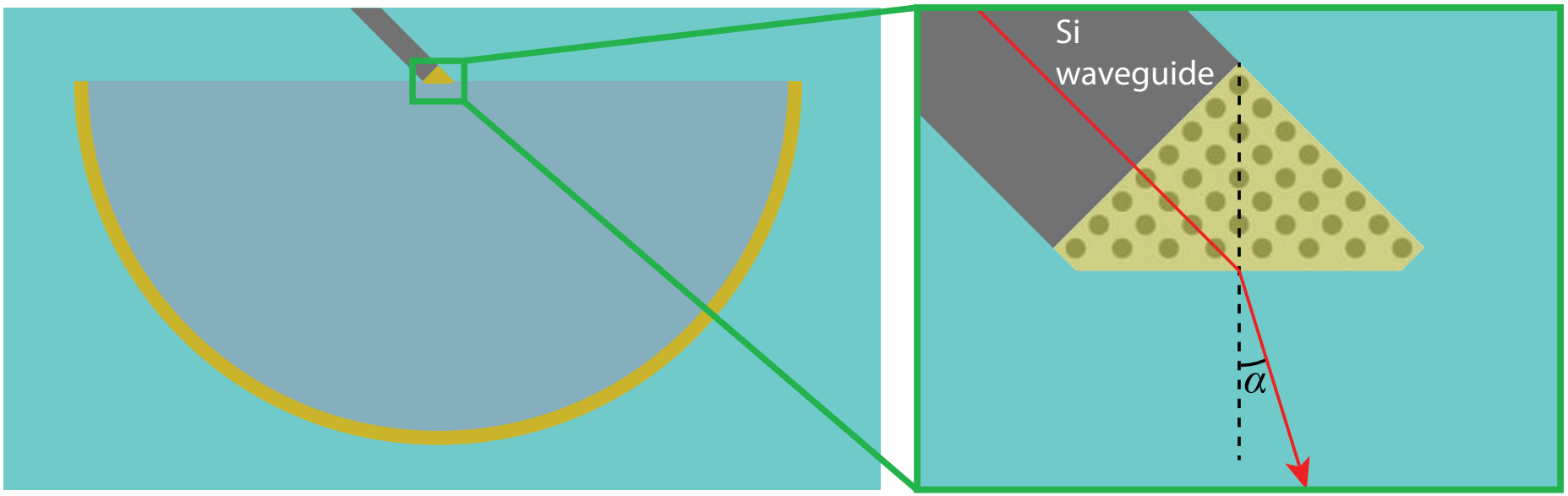
On-chip zero-index prism



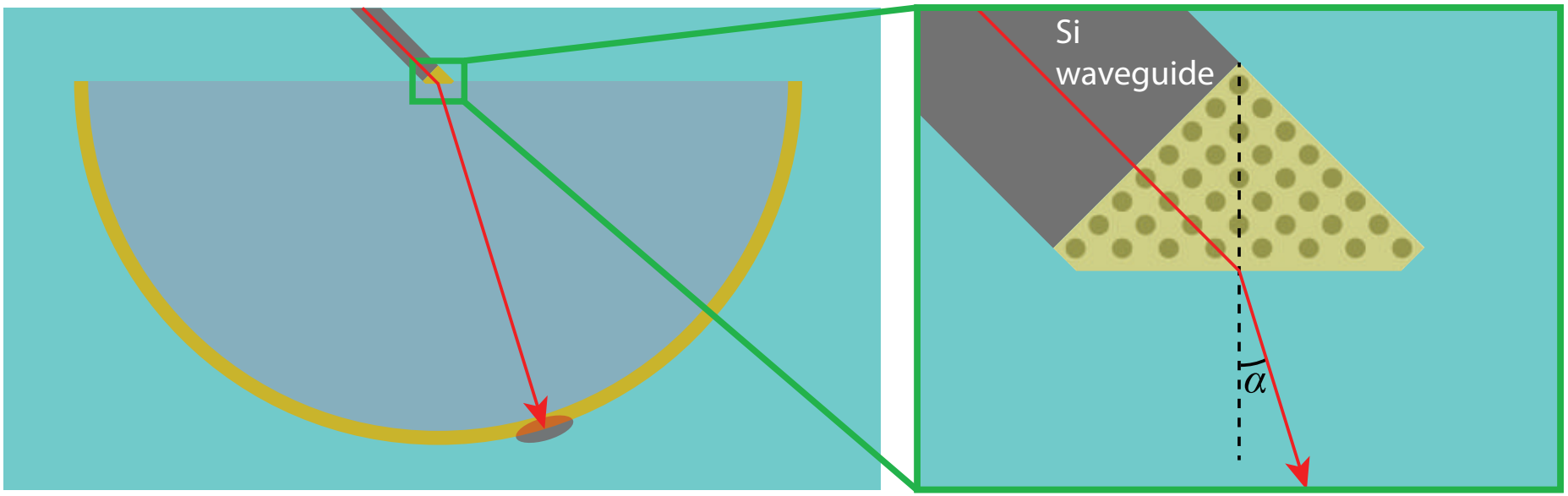
On-chip zero-index prism



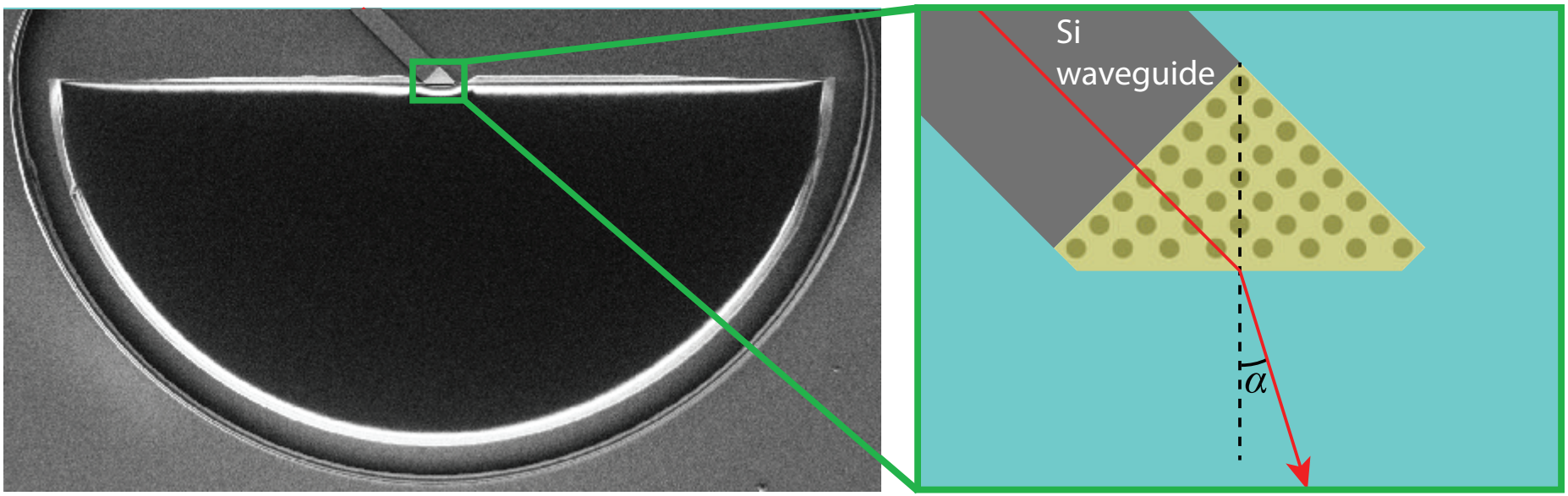
On-chip zero-index prism



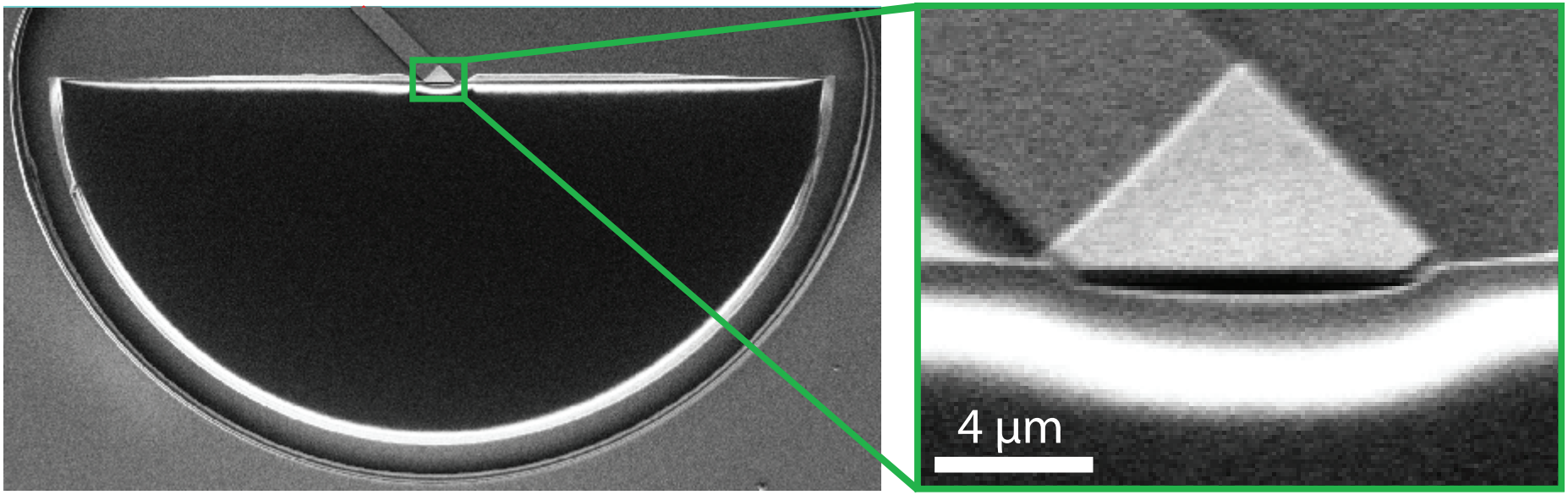
On-chip zero-index prism

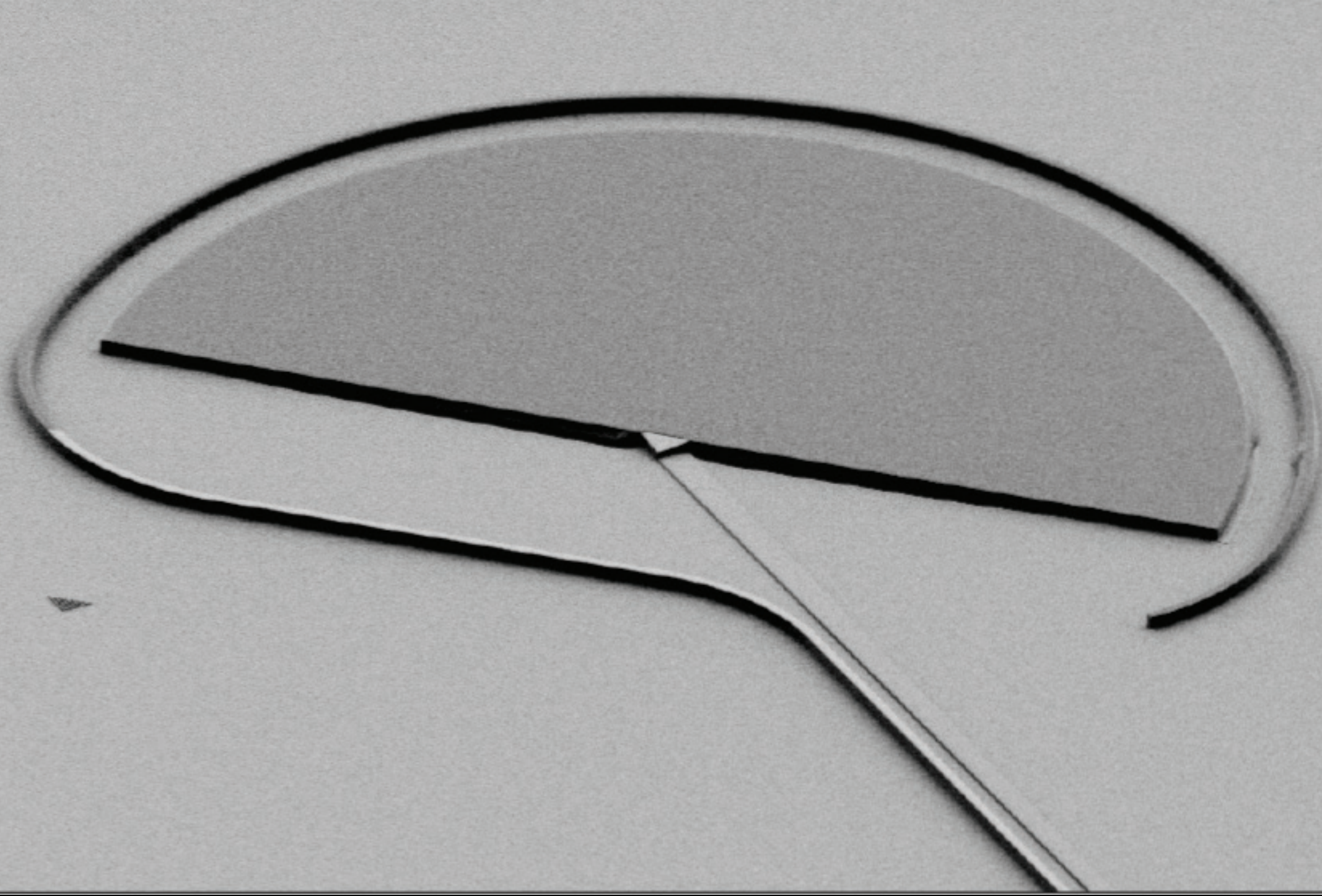


On-chip zero-index prism

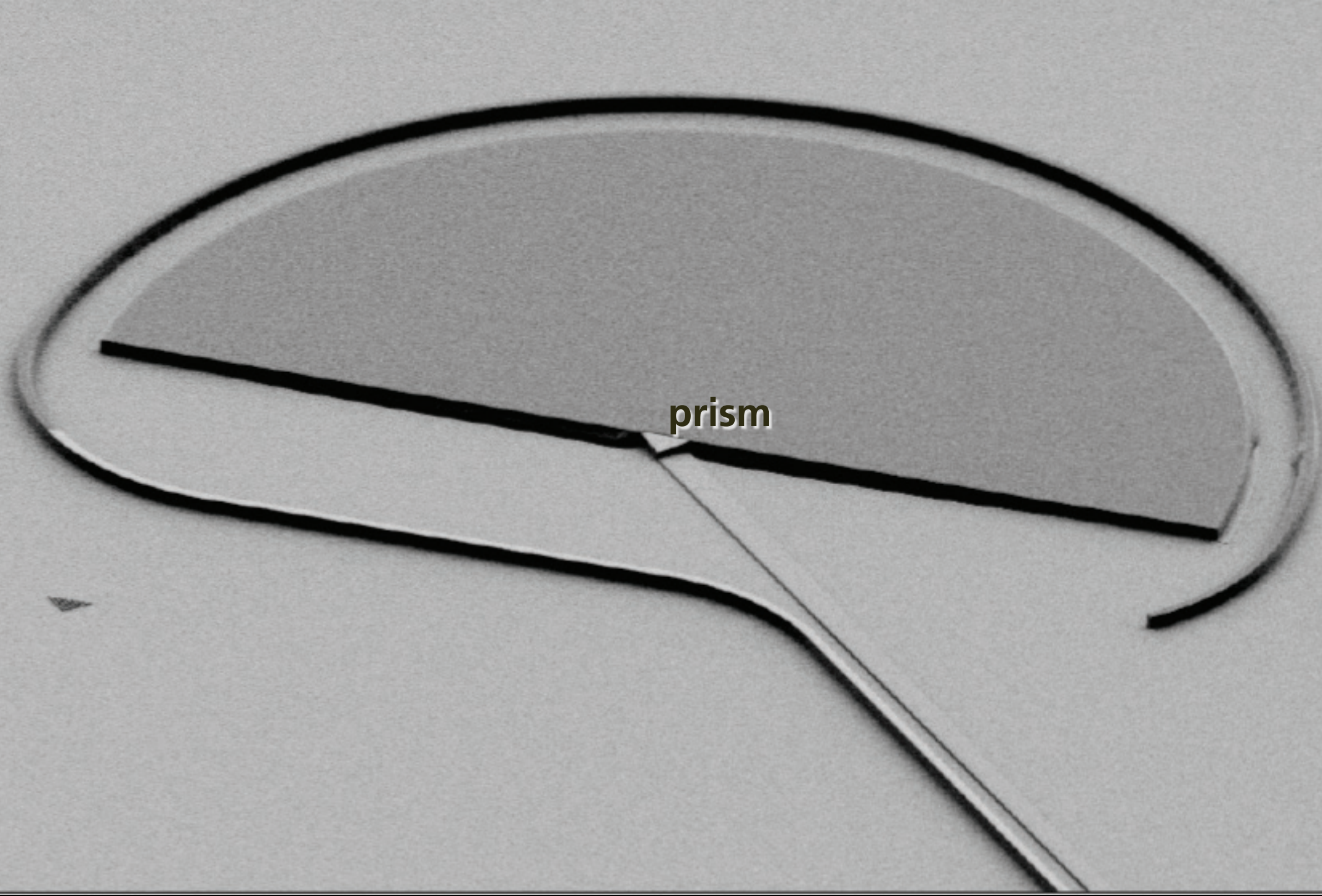


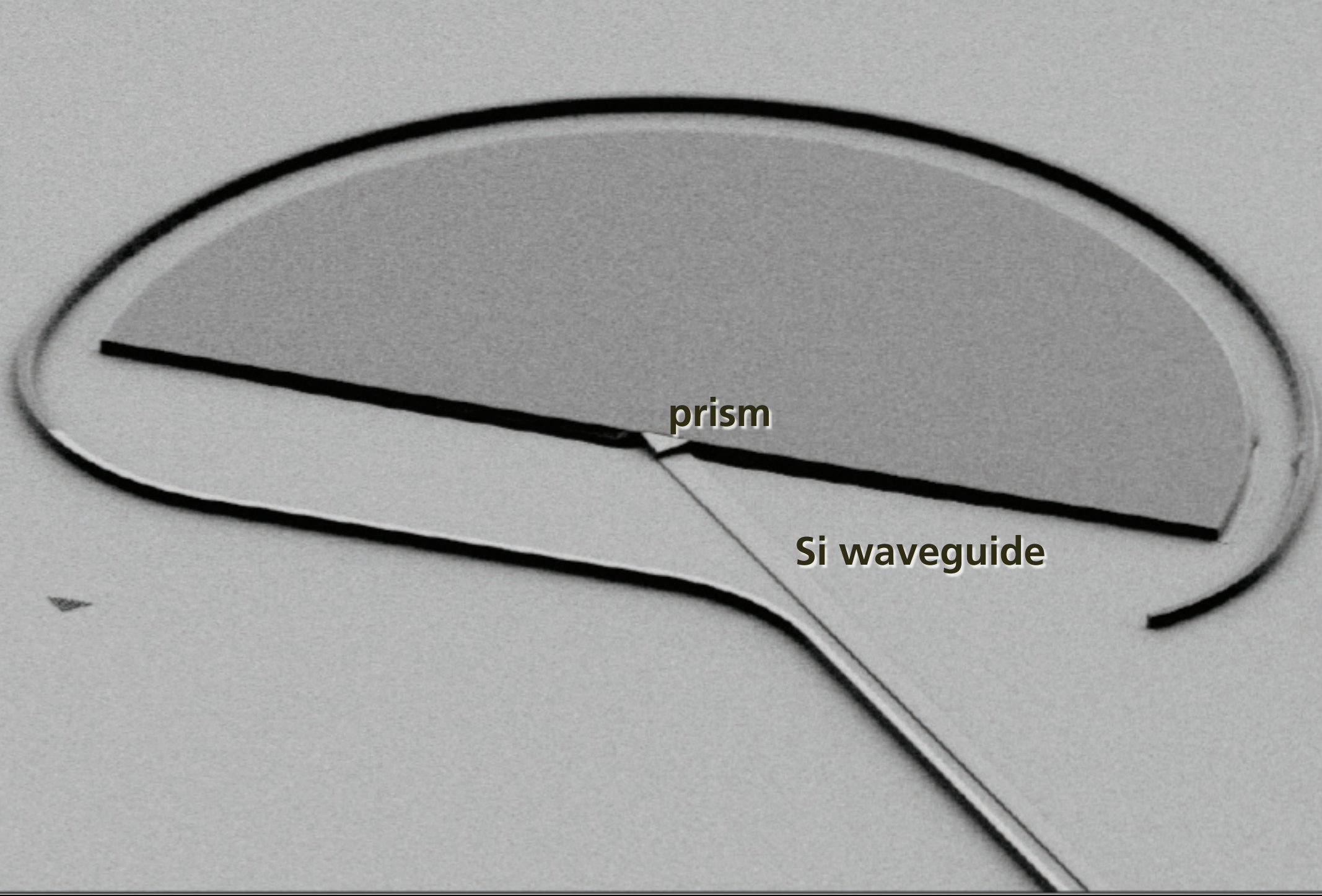
On-chip zero-index prism

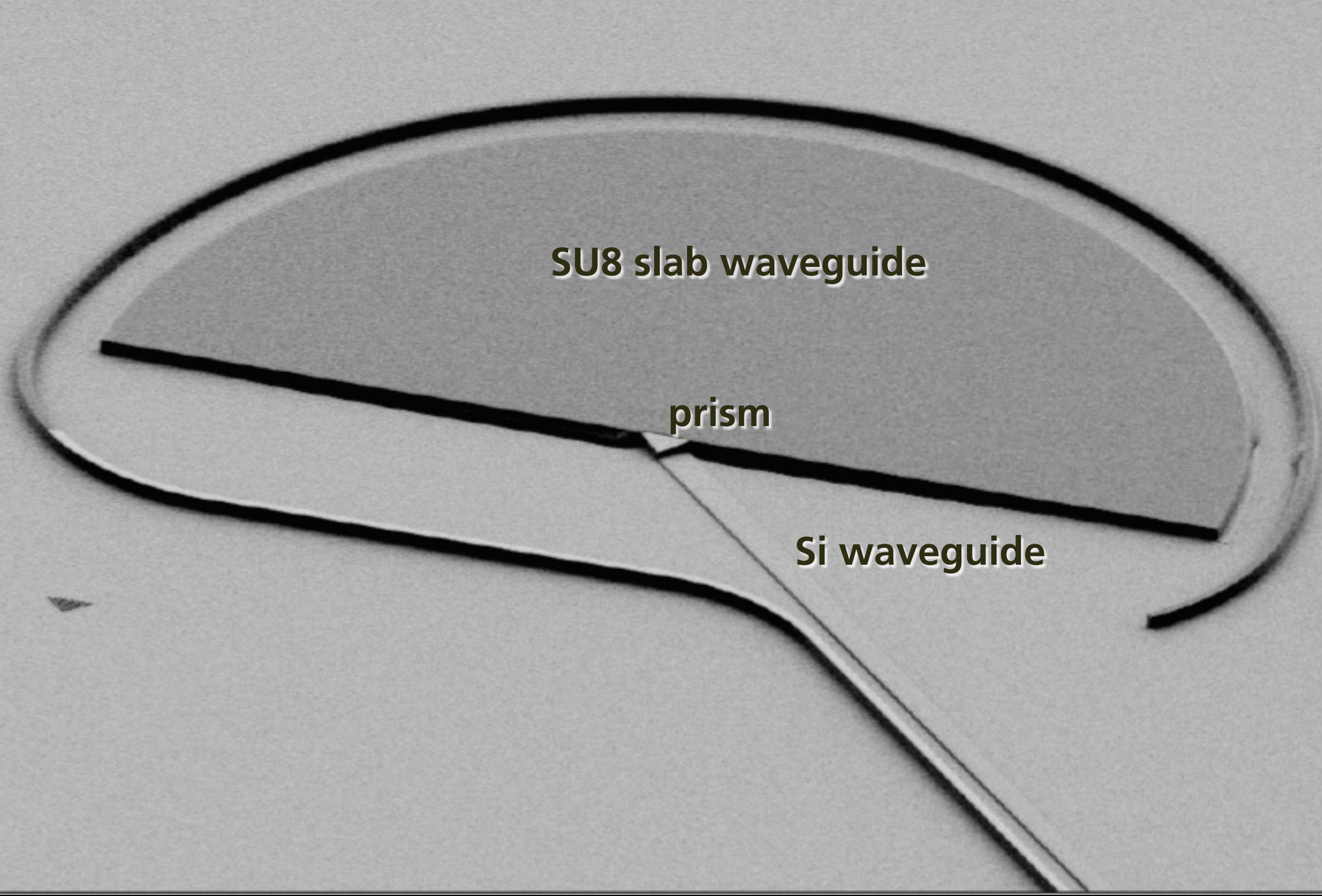


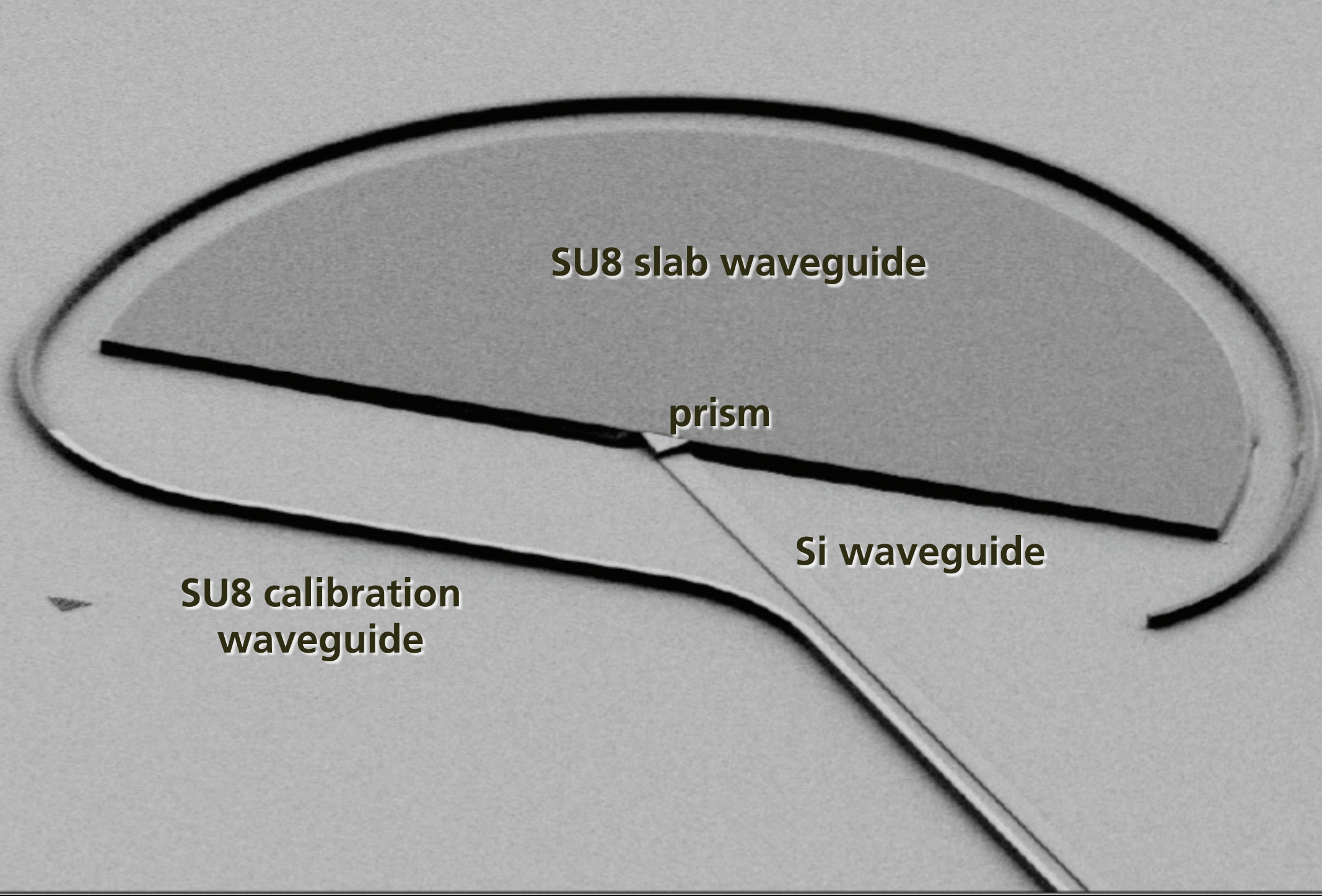


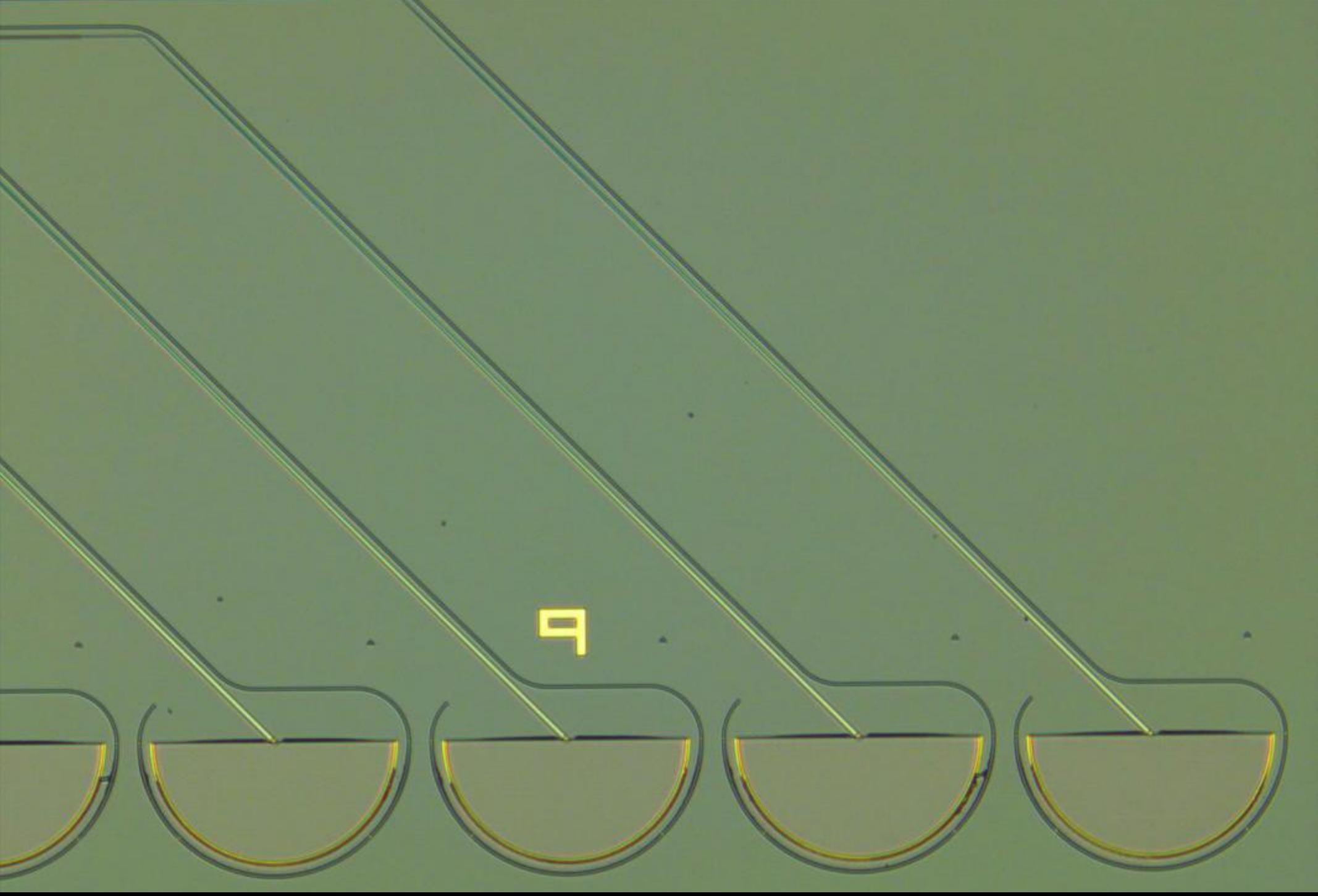
1 index 2 zero index 3 experiments


prism


prism
Si waveguide


SU8 slab waveguide
prism
Si waveguide


SU8 slab waveguide
prism
Si waveguide
SU8 calibration
waveguide

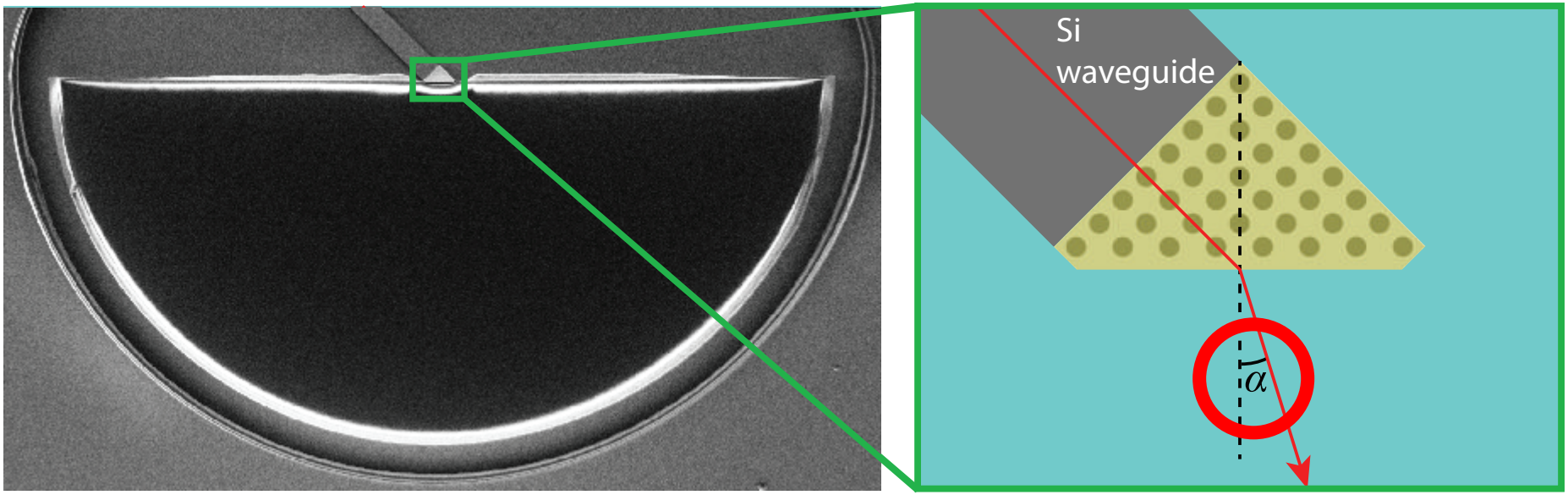


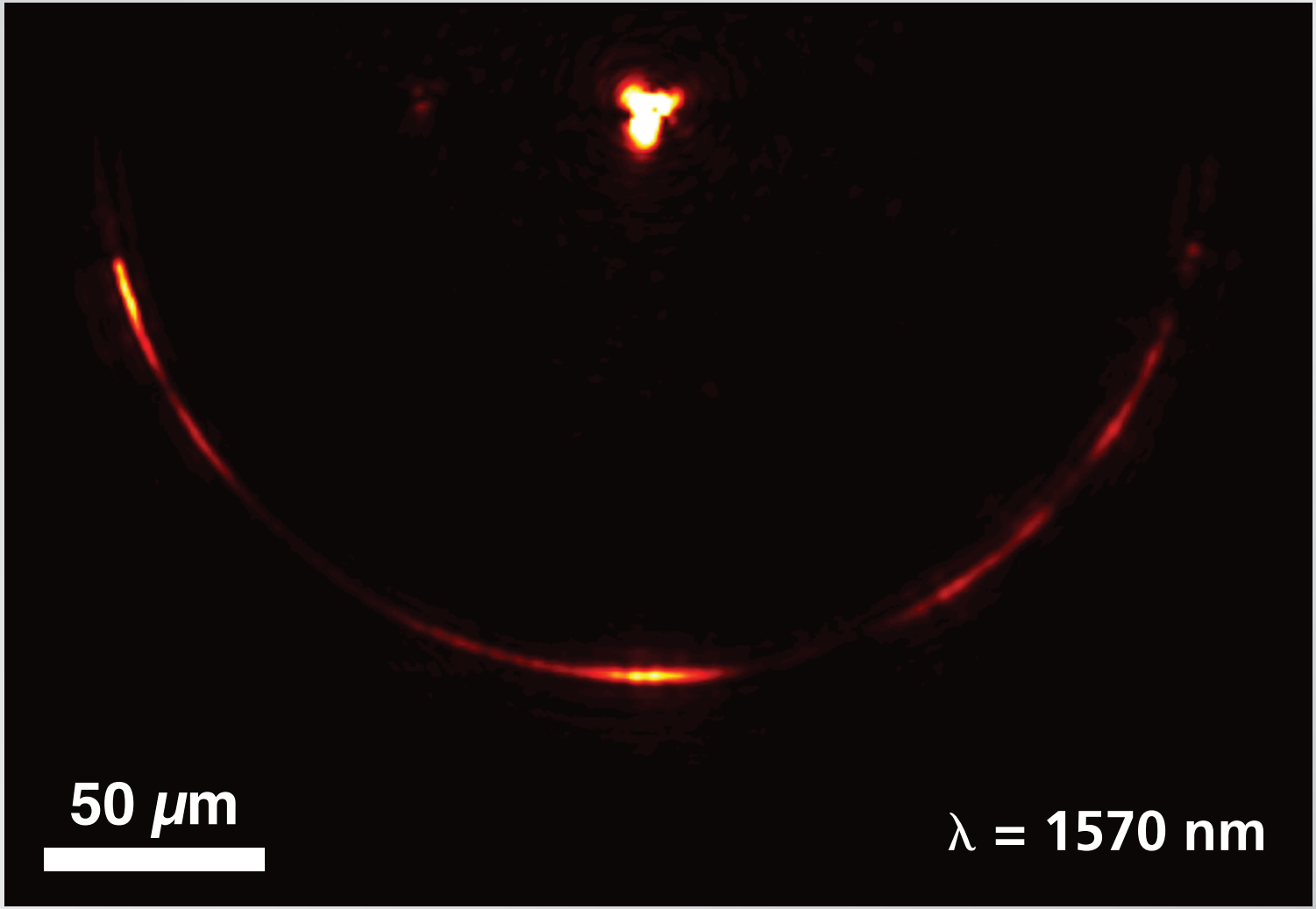
1 index

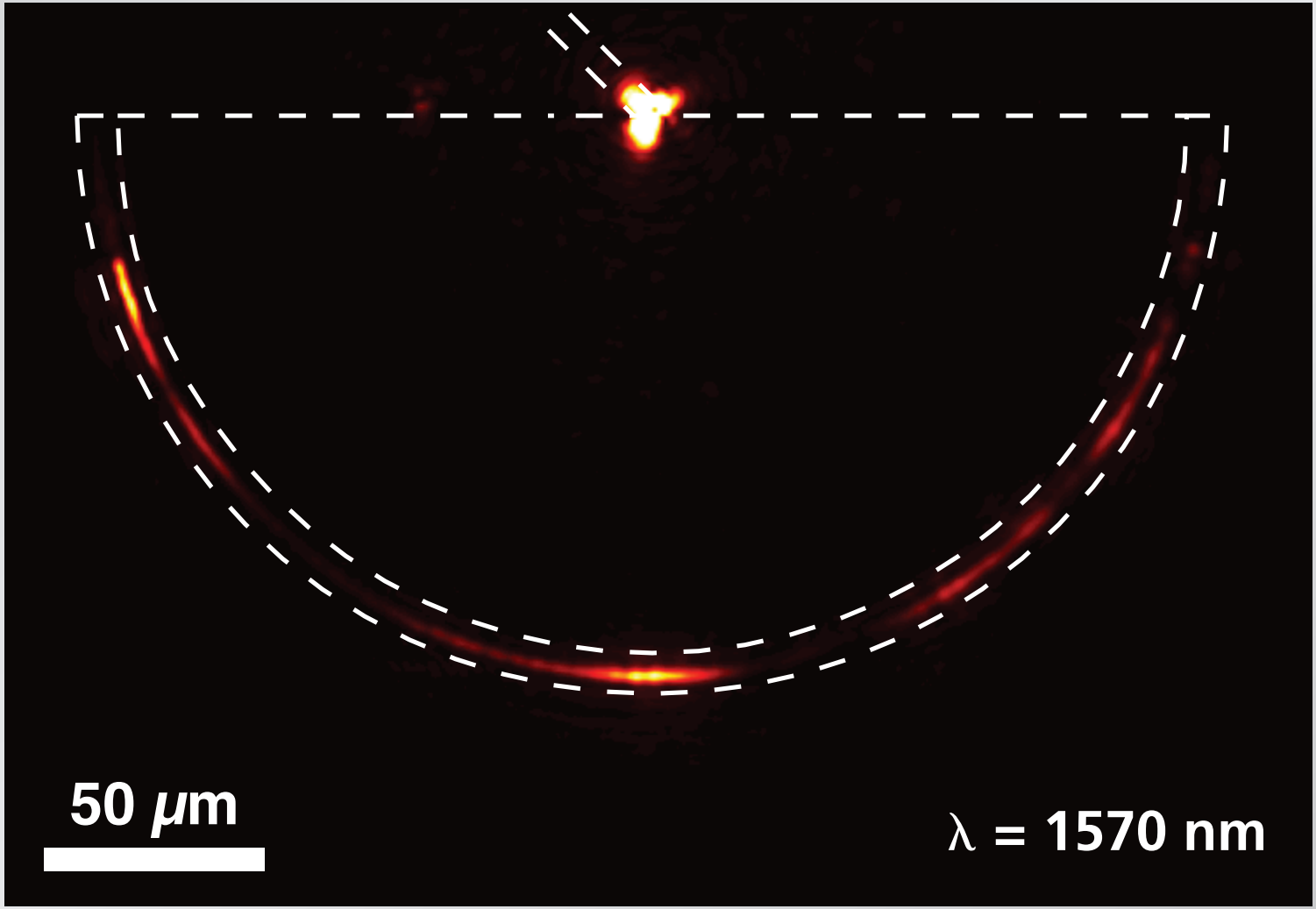
2 zero index

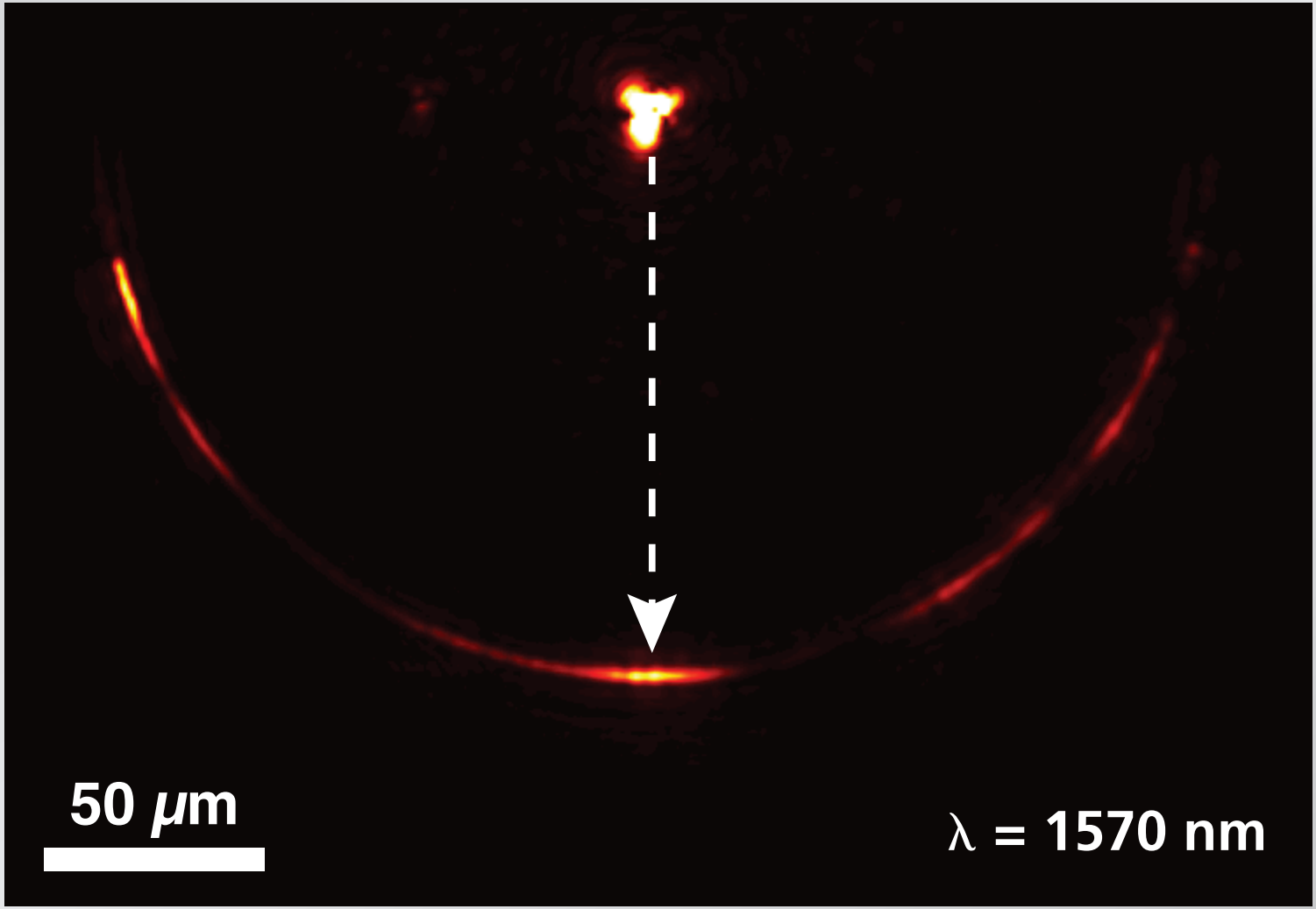
3 experiments

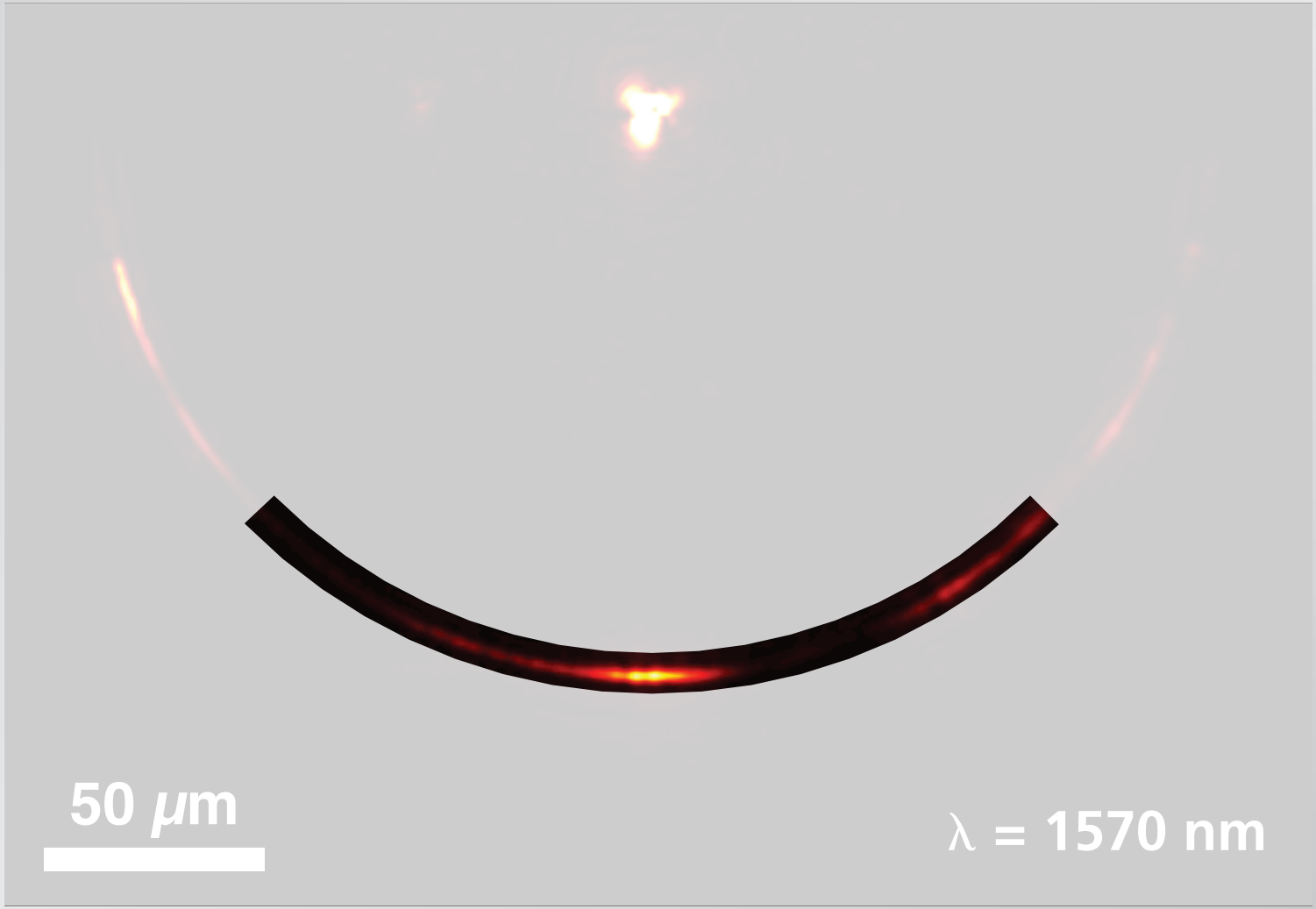
On-chip zero-index prism



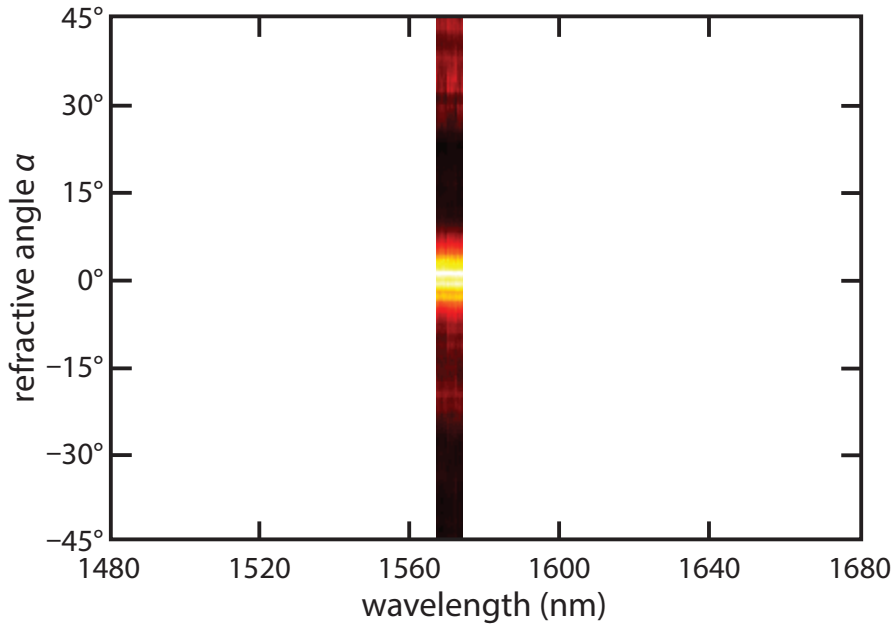

50 μm
λ = 1570 nm


50 μm
λ = 1570 nm

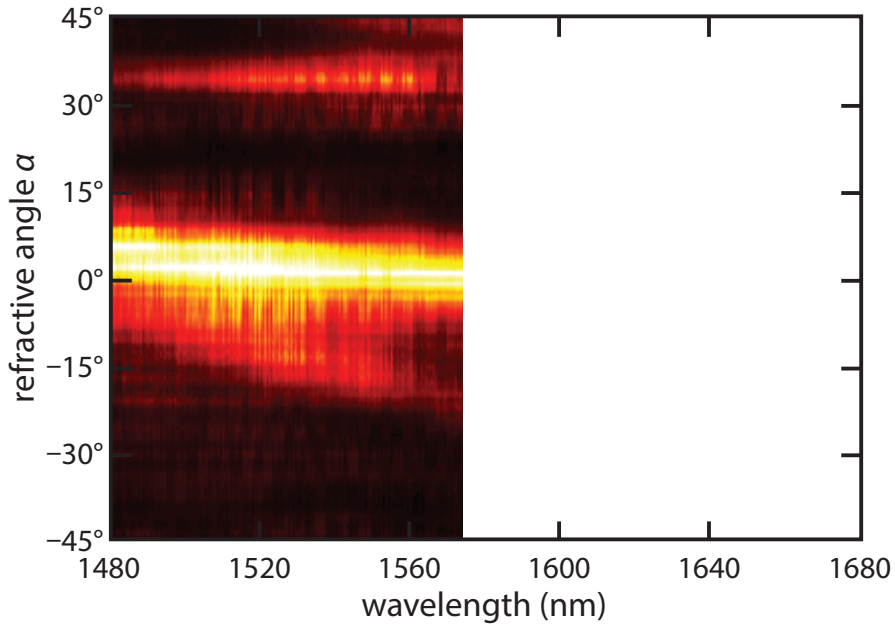

50 μm
λ = 1570 nm


50 μm
λ = 1570 nm

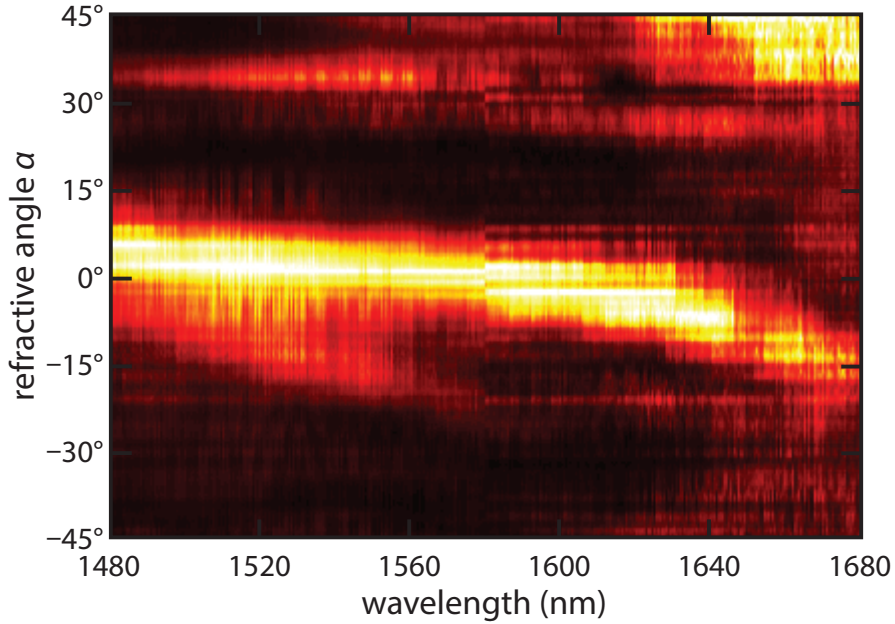
Wavelength dependence of refraction angle



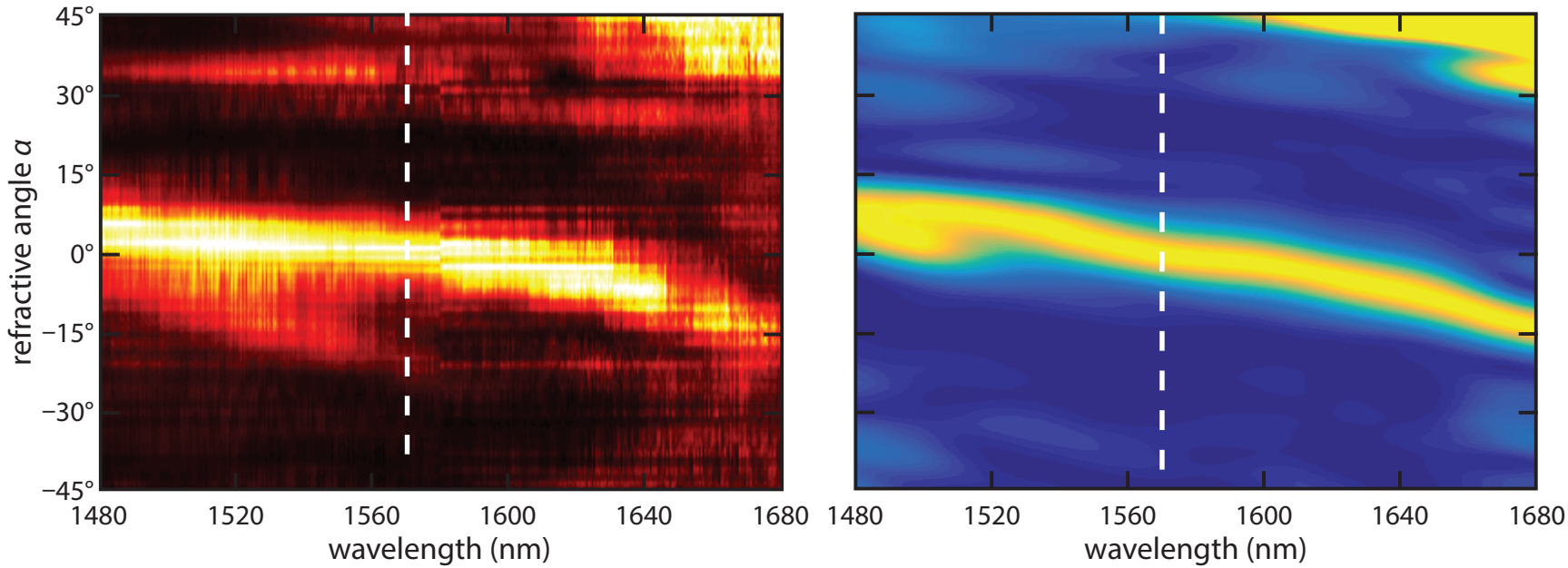
Wavelength dependence of refraction angle



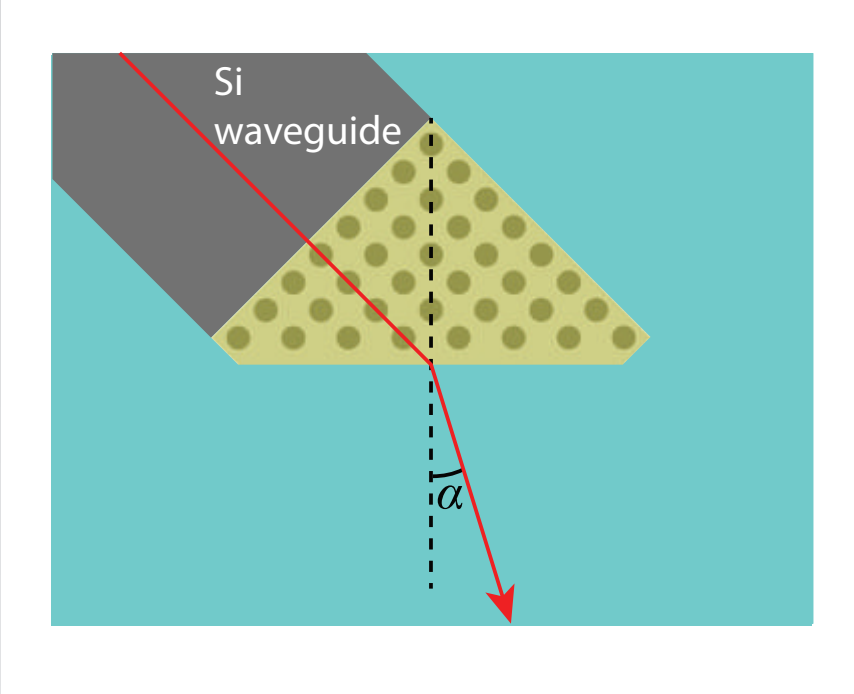
Wavelength dependence of refraction angle



Wavelength dependence of refraction angle

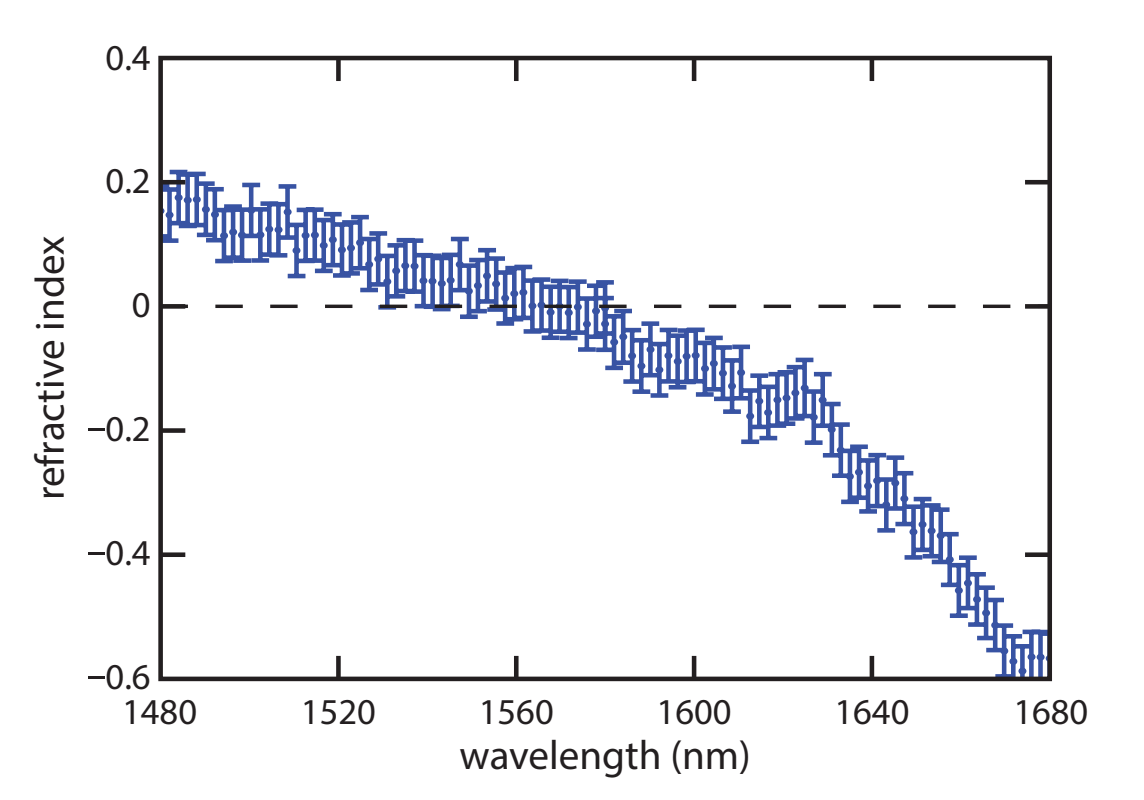


Wavelength dependence of index

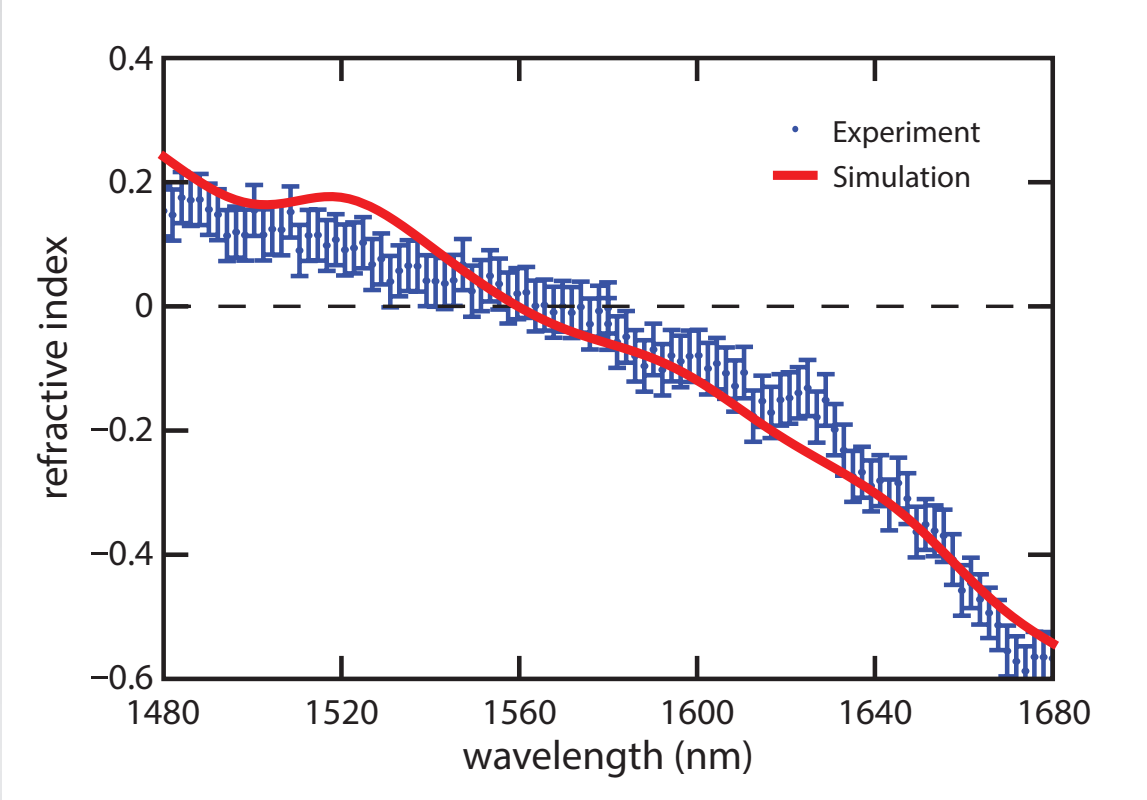


$$n_{\text{prism}} = n_{\text{slab}} \frac{\sin\alpha}{\sin 45^\circ}$$

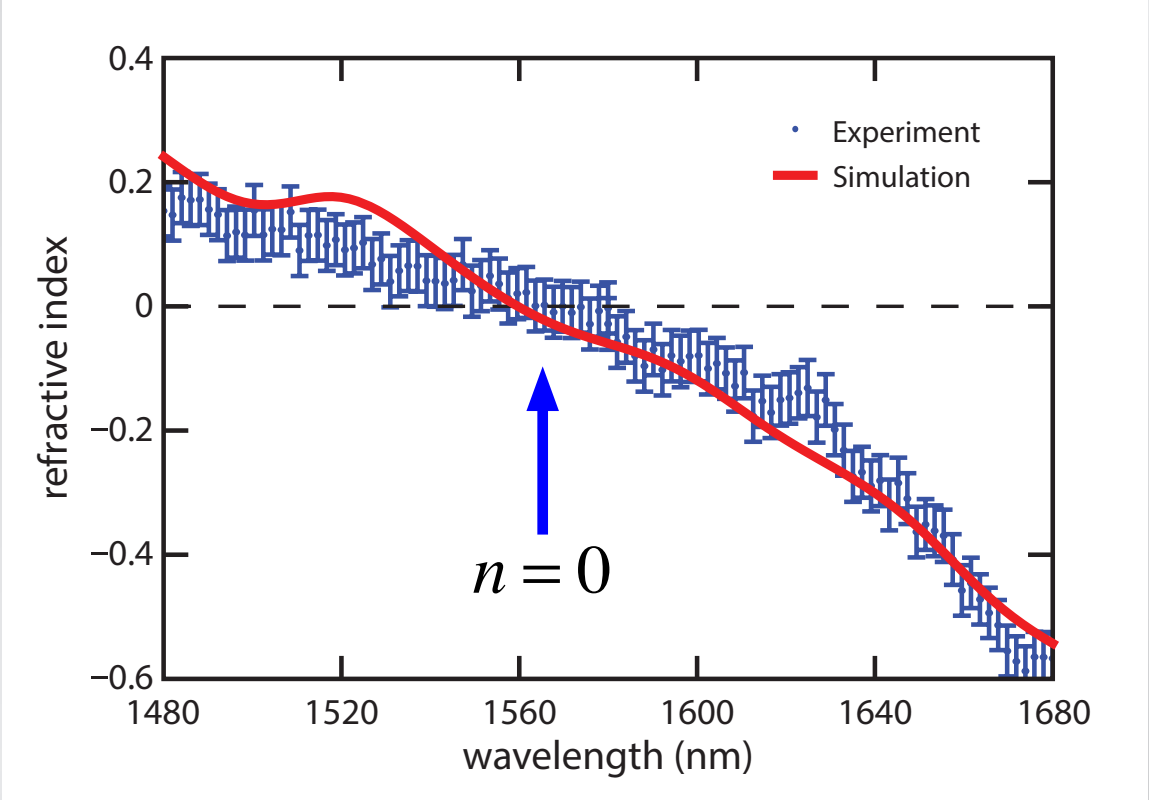
Wavelength dependence of index



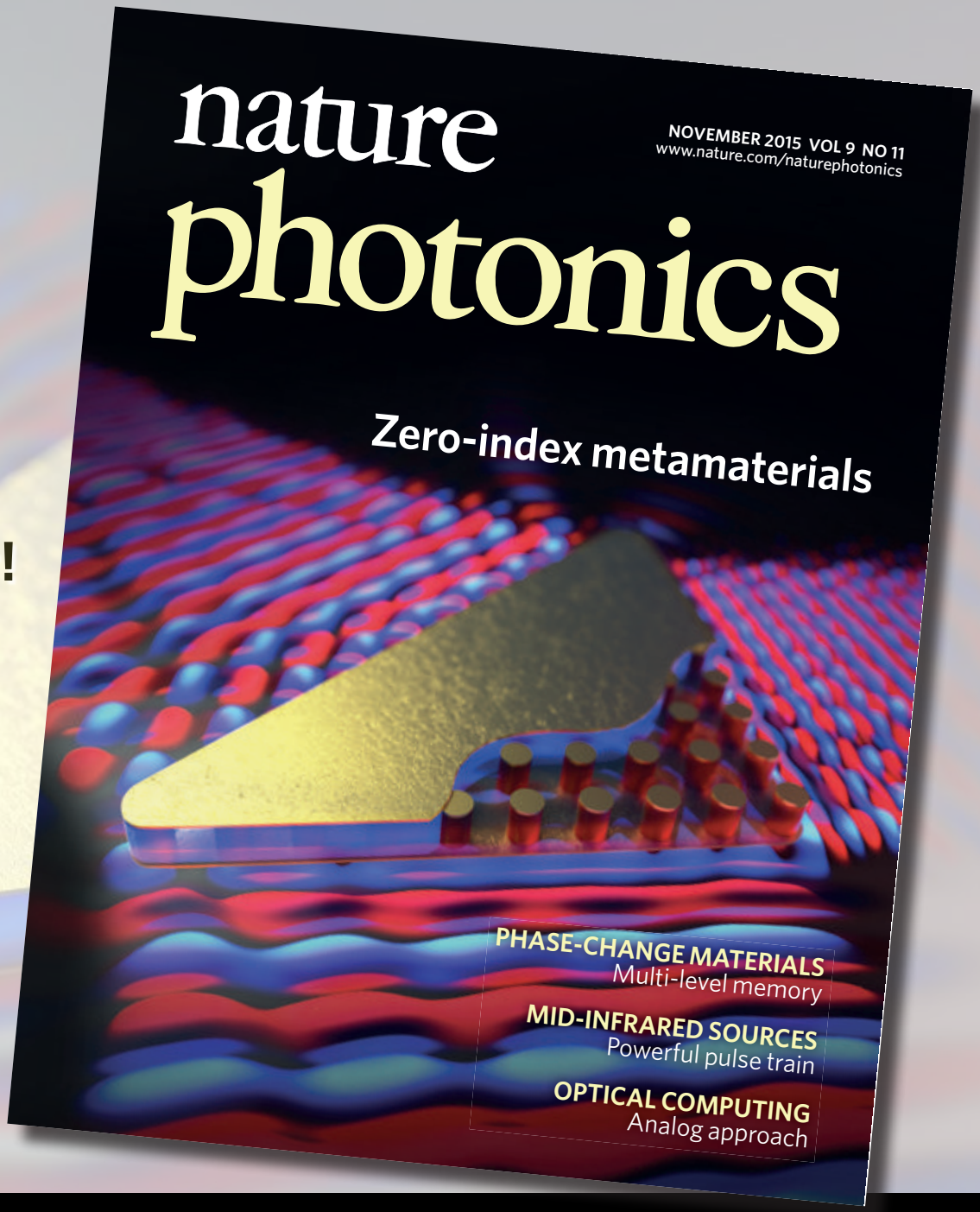
Wavelength dependence of index



Wavelength dependence of index



More info: download paper!

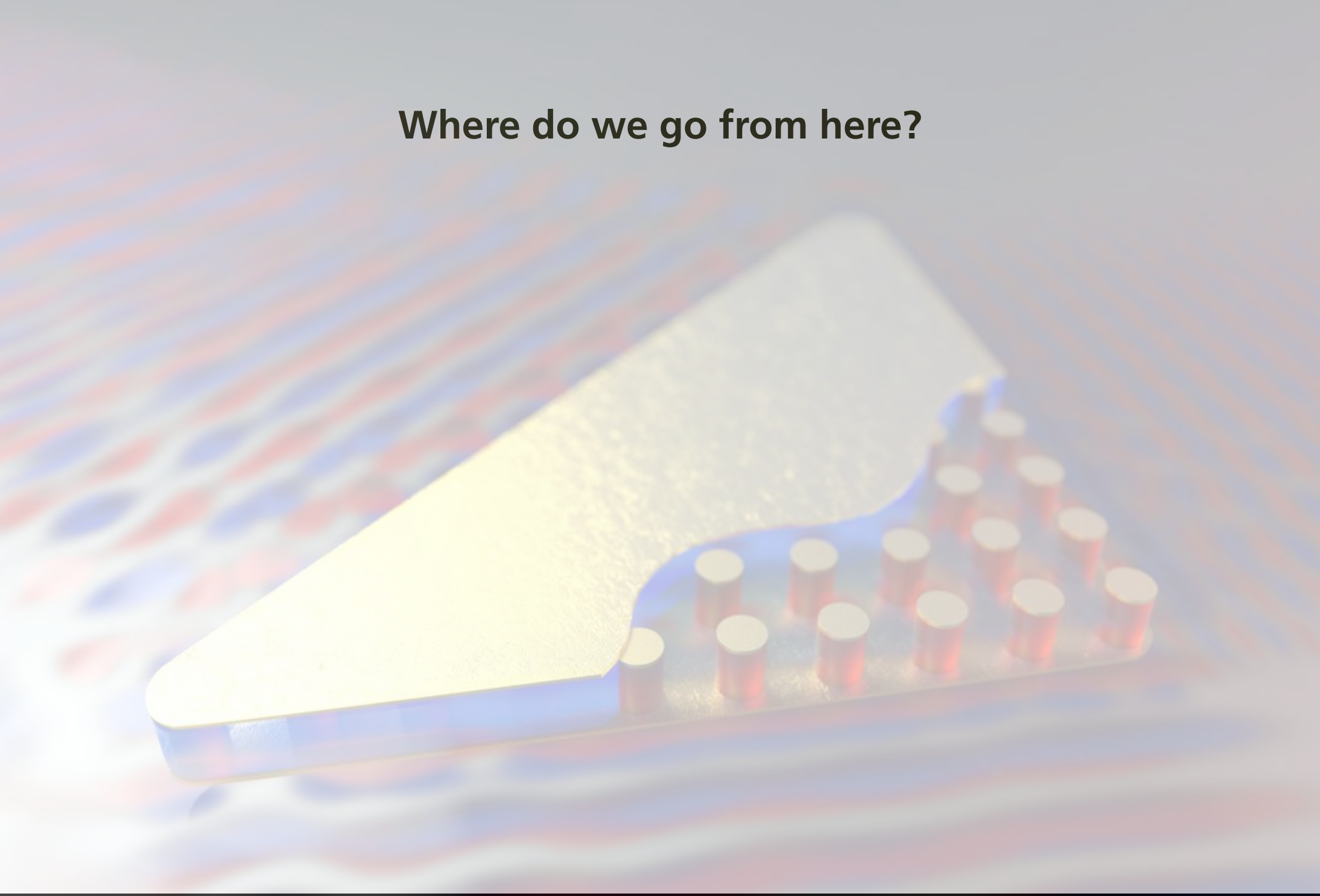


1 index

2 zero index

3 experiments

Where do we go from here?

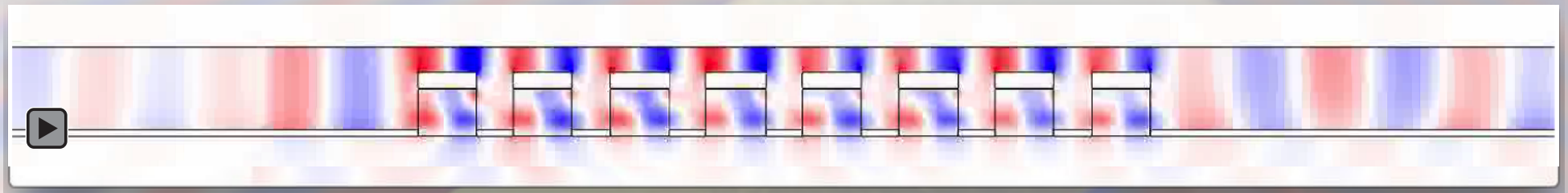


1 index

2 zero index

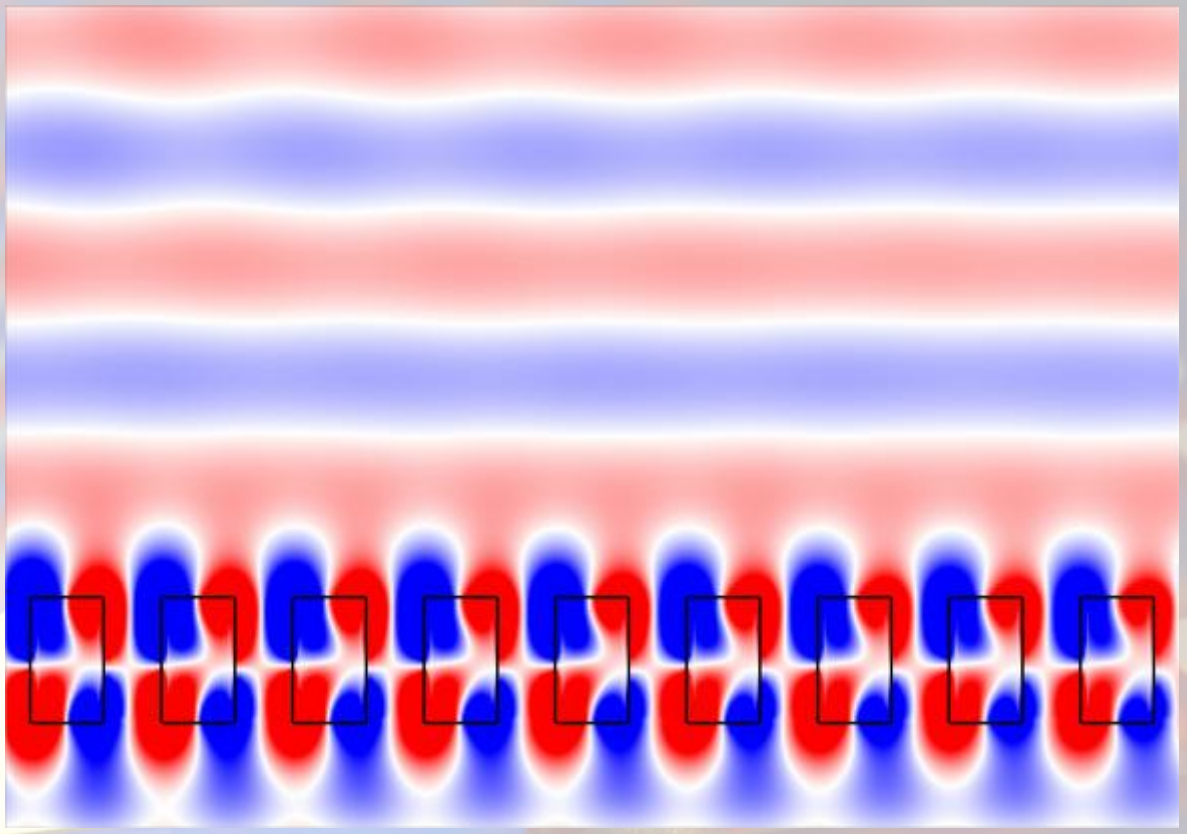
3 experiments

Where do we go from here?



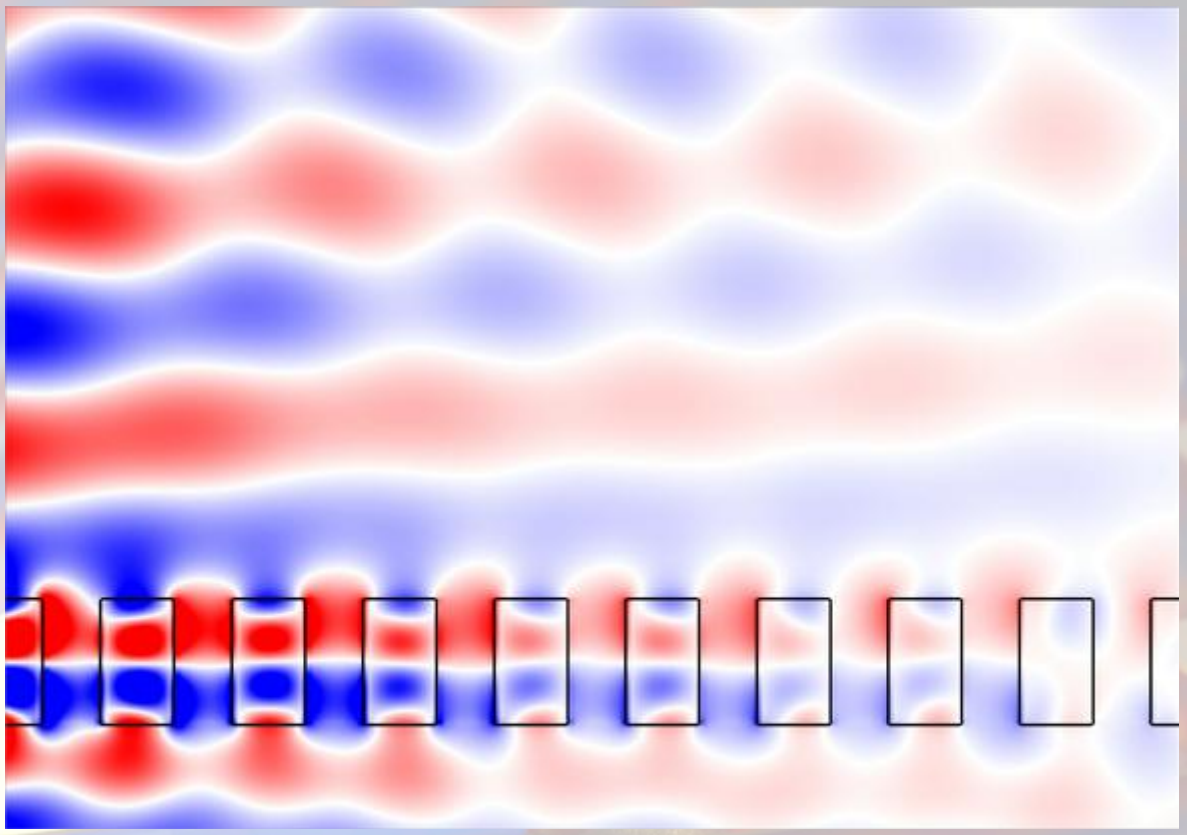
Need to eliminate losses in metal mirrors

Where do we go from here?



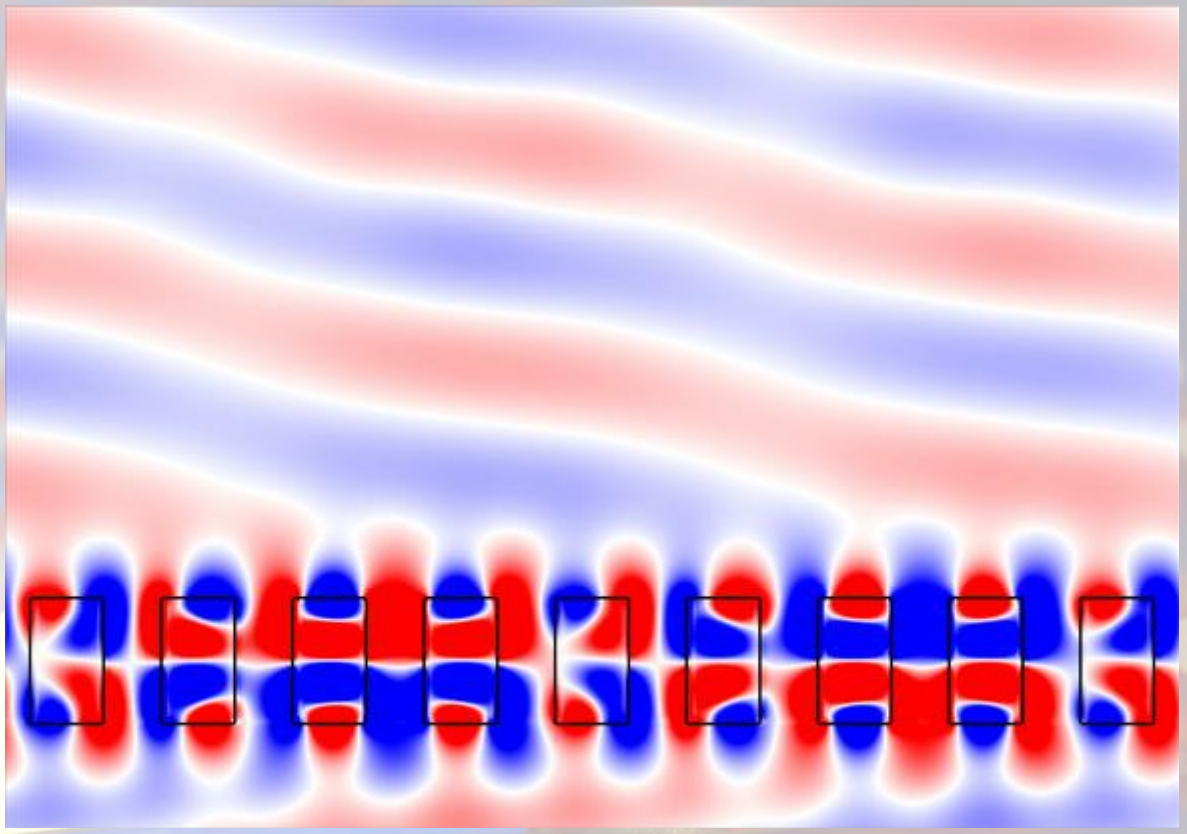
Removing mirrors causes radiative losses

Where do we go from here?



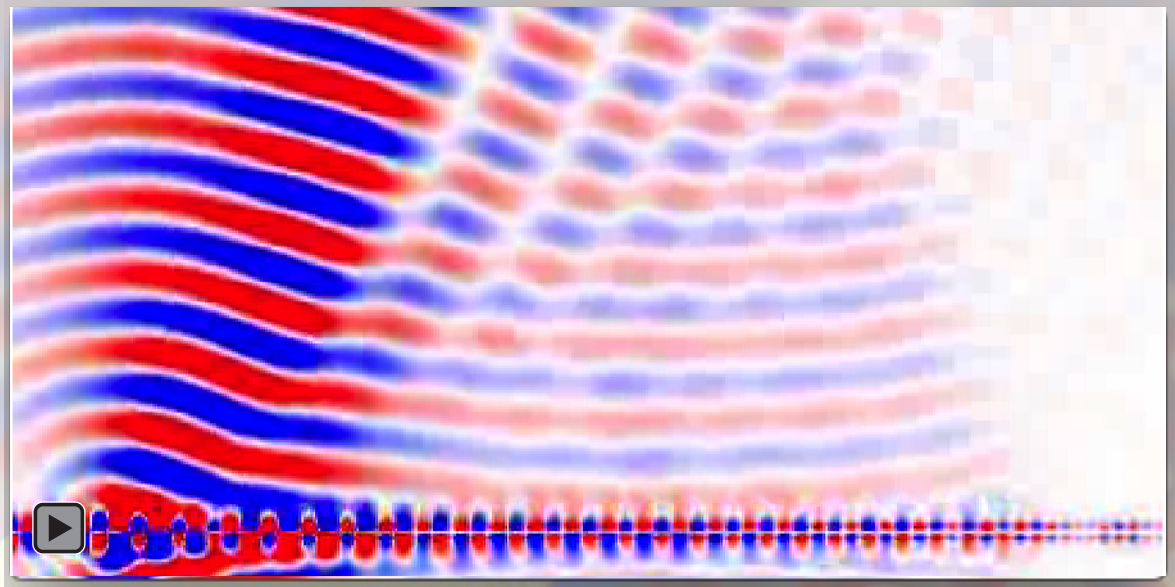
Radiative losses can be steered…

Where do we go from here?



Radiative losses can be steered…

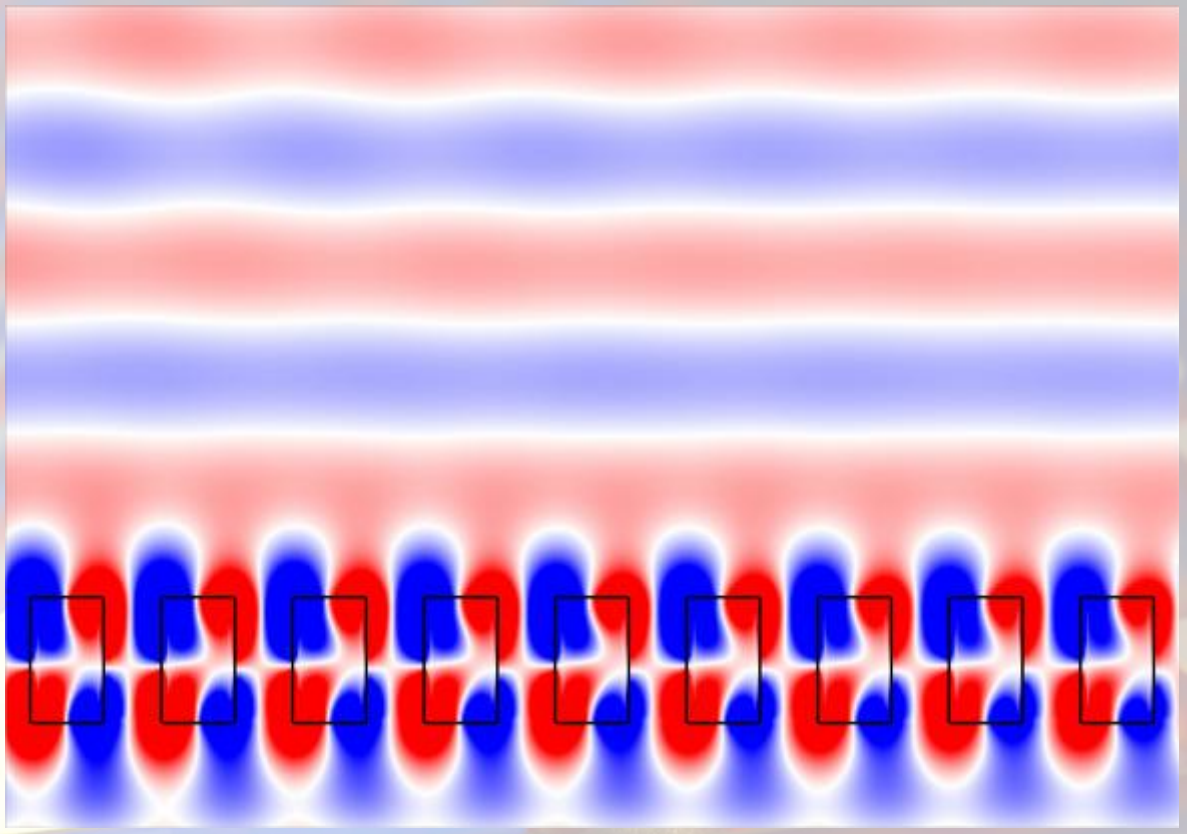
Where do we go from here?



…or arranged to cause focusing…

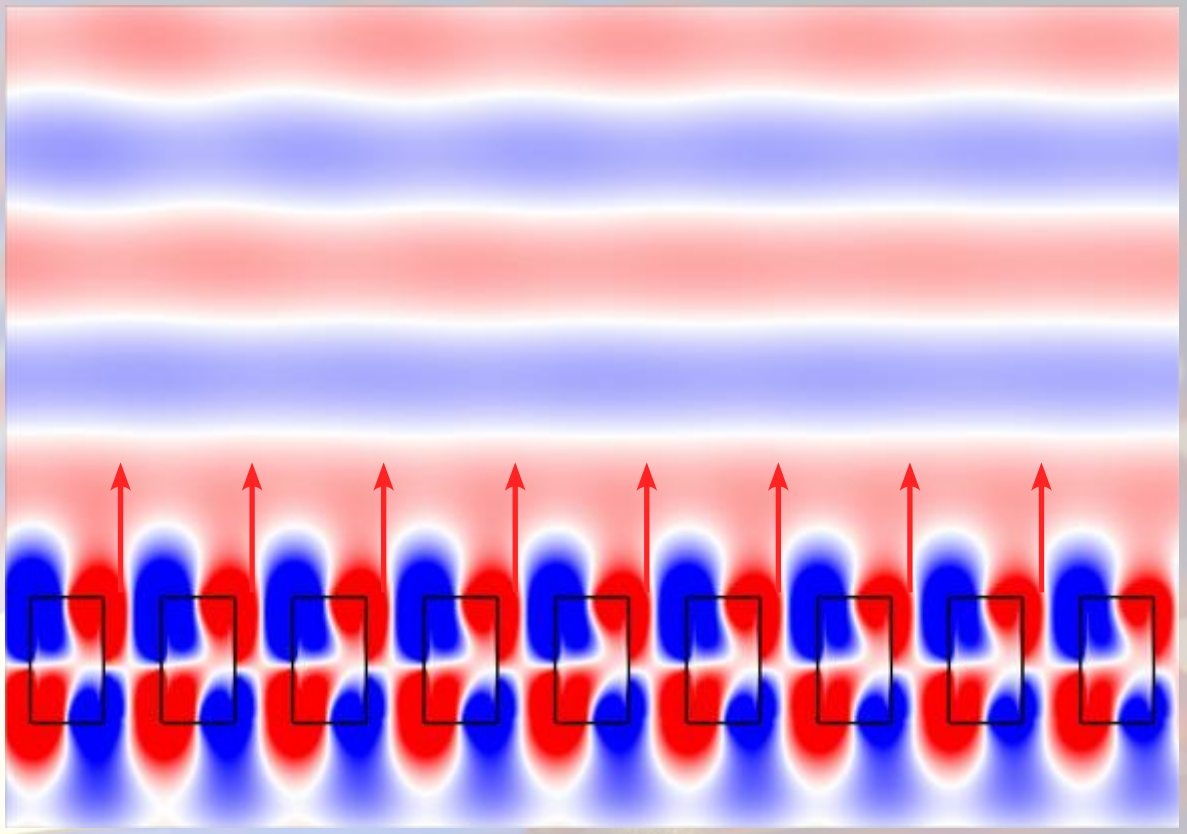
1 index 2 zero index 3 experiments

Where do we go from here?



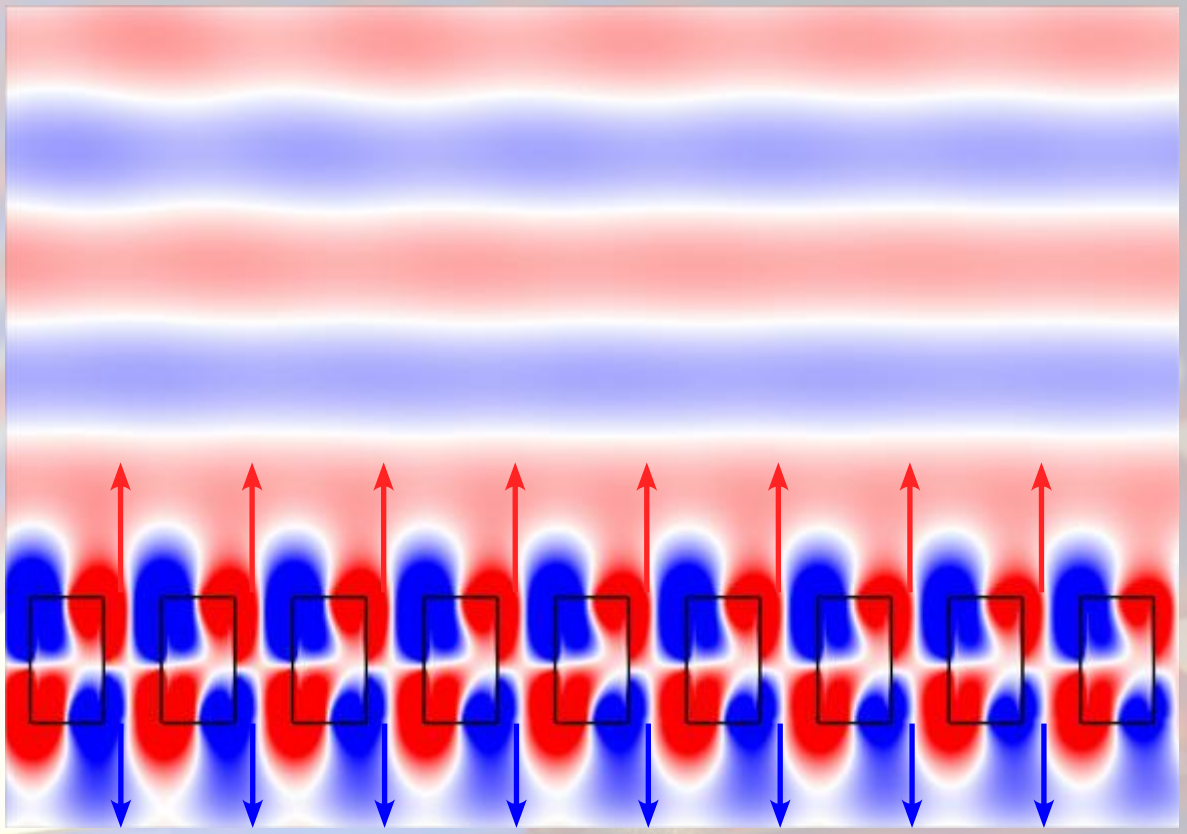
…or eliminated causing “bound in continuum” state

Where do we go from here?



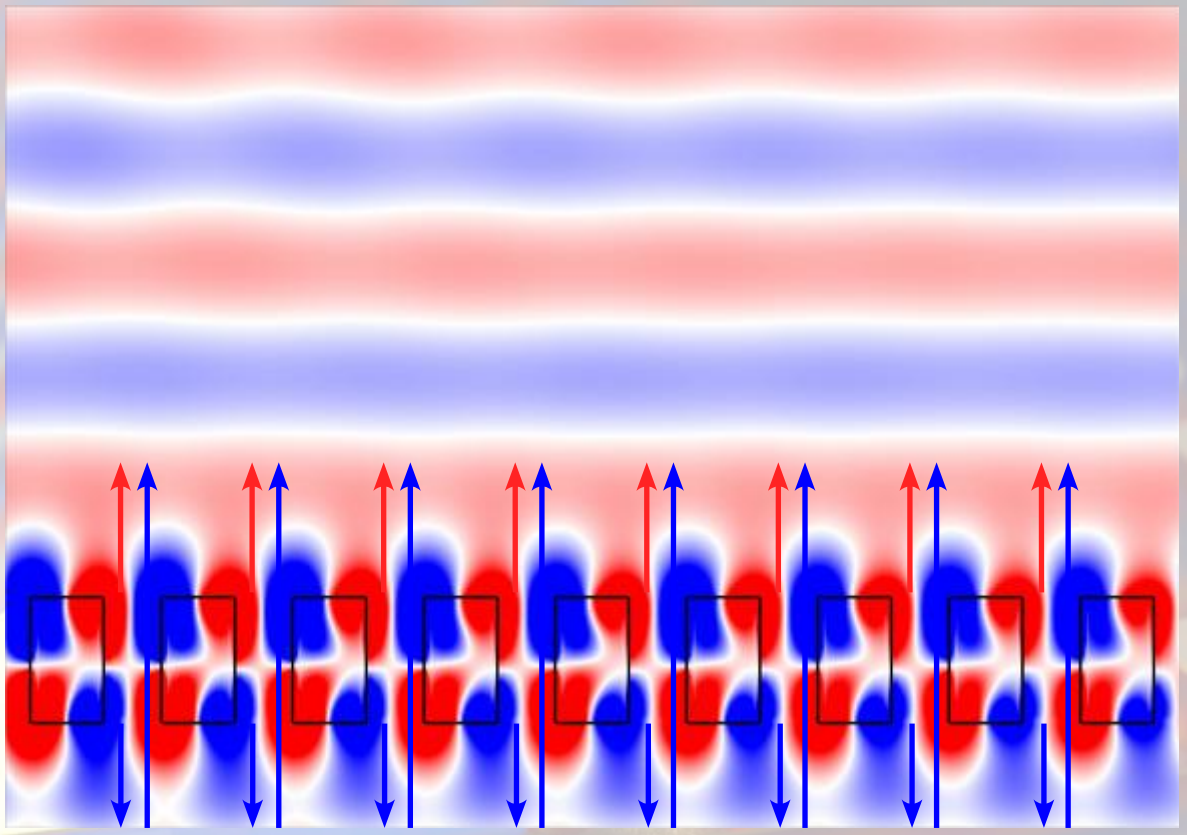
…or eliminated causing “bound in continuum” state

Where do we go from here?



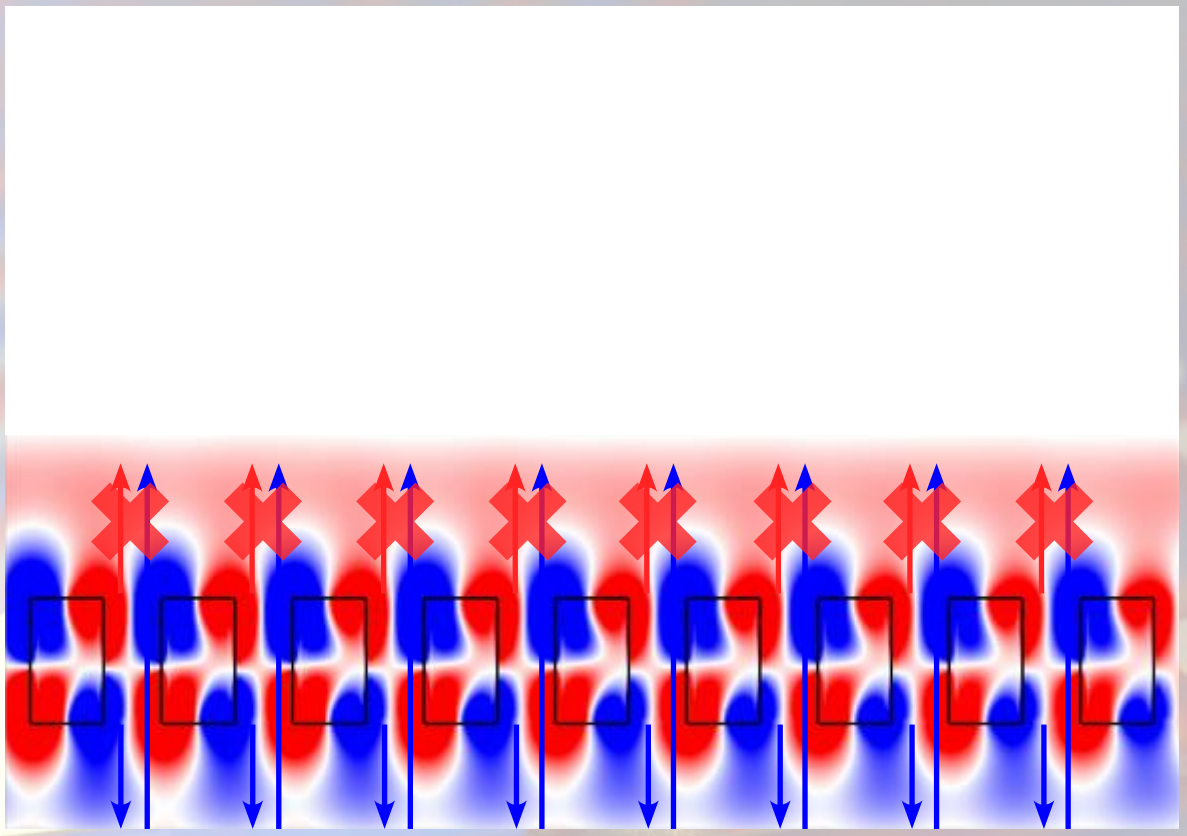
…or eliminated causing “bound in continuum” state

Where do we go from here?



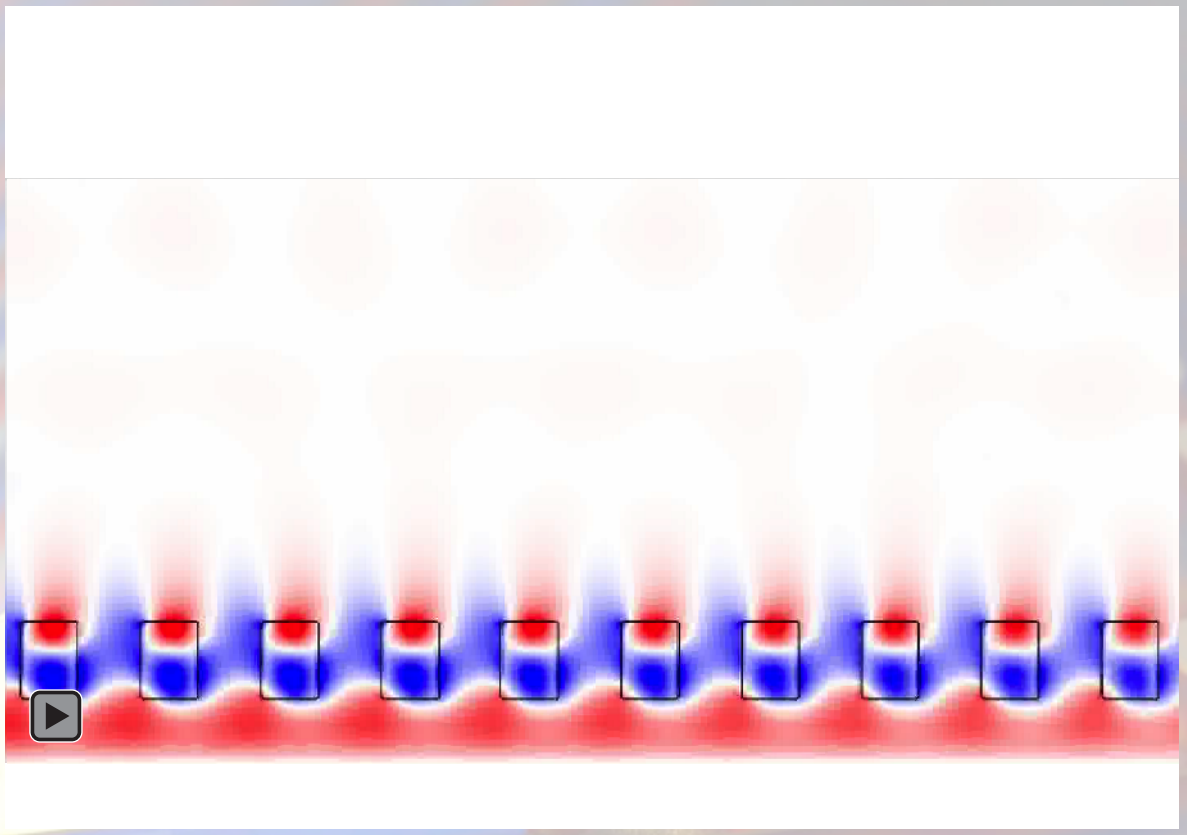
…or eliminated causing “bound in continuum” state

Where do we go from here?



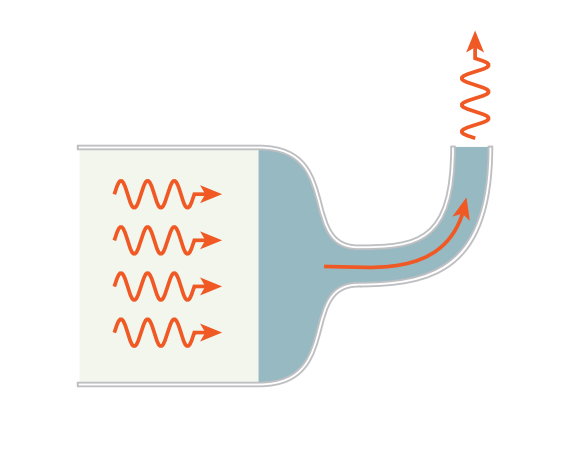
…or eliminated causing “bound in continuum” state

Where do we go from here?



…or eliminated causing “bound in continuum” state

Exciting applications ahead



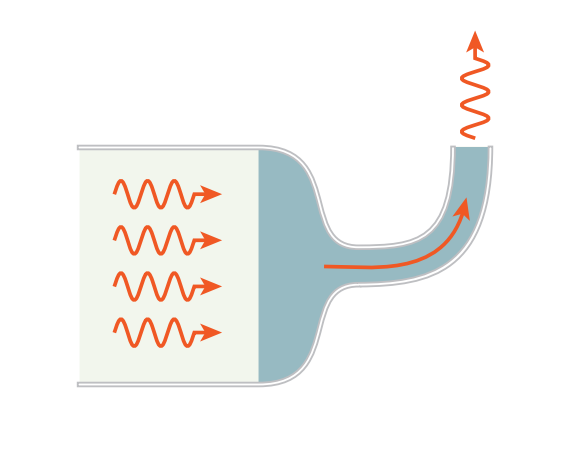
supercoupling

1 index

2 zero index

3 experiments

Exciting applications ahead



supercoupling

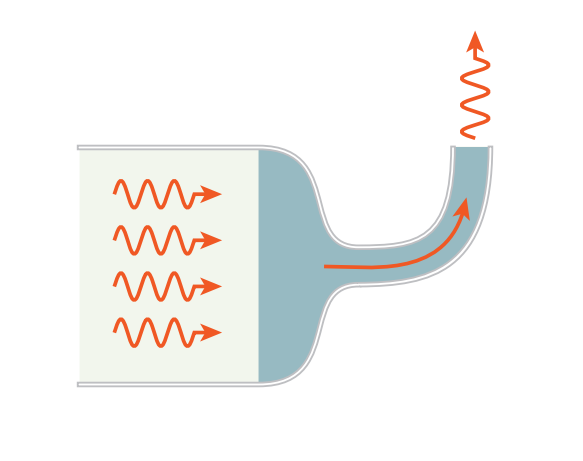
NLO

1 index

2 zero index

3 experiments

Exciting applications ahead



supercoupling

SHG $\omega' = 2\omega$

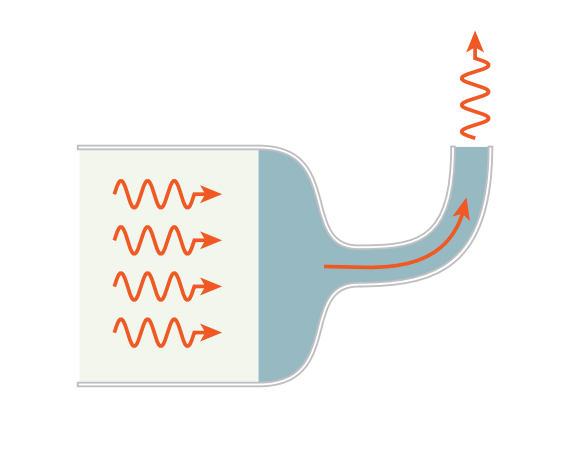
NLO

1 index

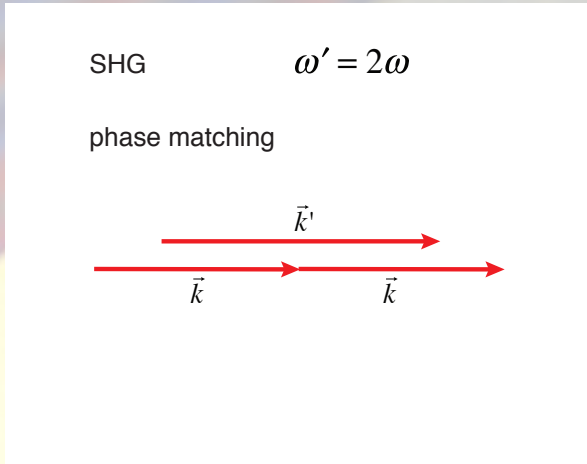
2 zero index

3 experiments

Exciting applications ahead



supercoupling



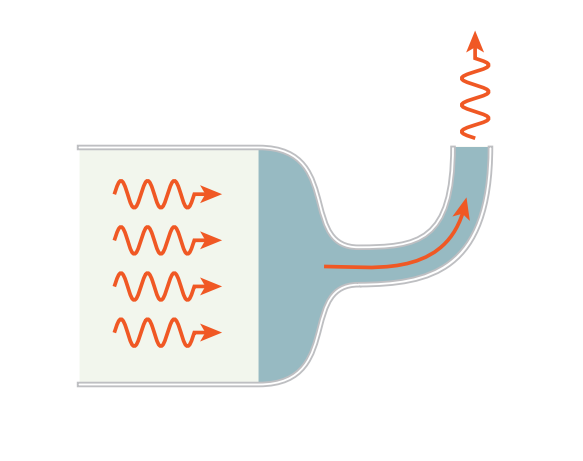
NLO

1 index

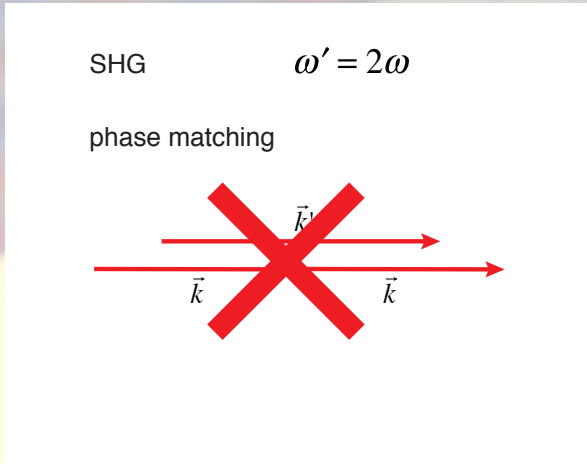
2 zero index

3 experiments

Exciting applications ahead



supercoupling



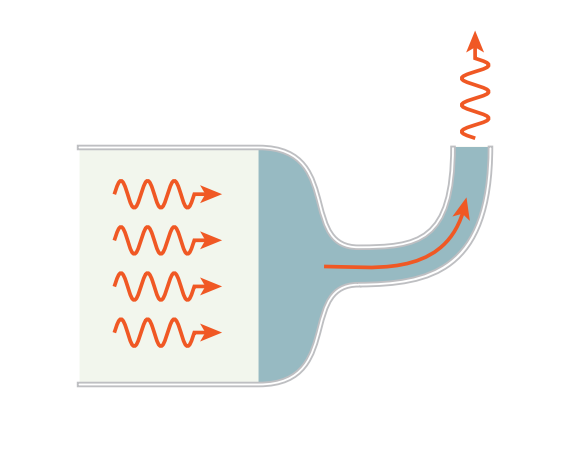
NLO

1 index

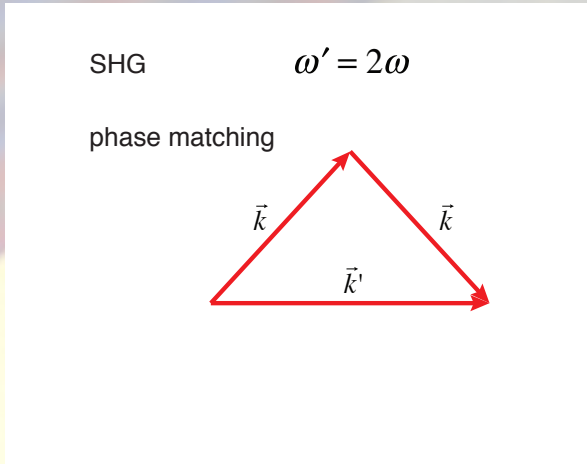
2 zero index

3 experiments

Exciting applications ahead



supercoupling



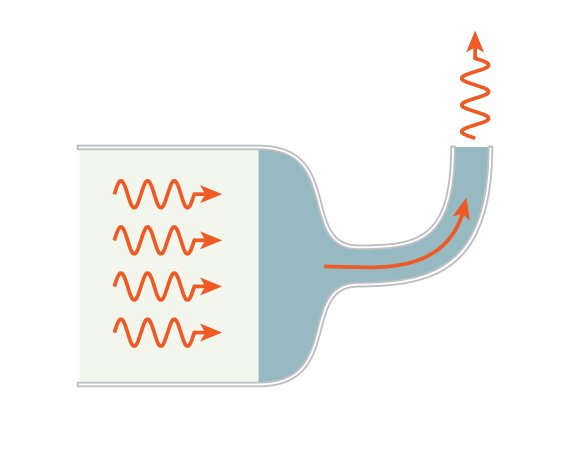
NLO

1 index

2 zero index

3 experiments

Exciting applications ahead



supercoupling

SHG $\omega' = 2\omega$

at zero index

$\vec{k} = 0$

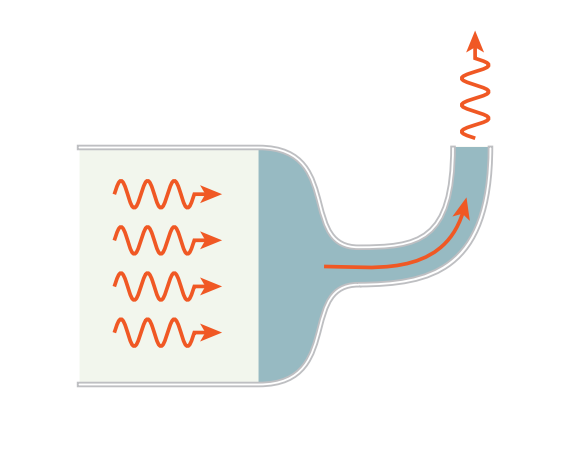
NLO

1 index

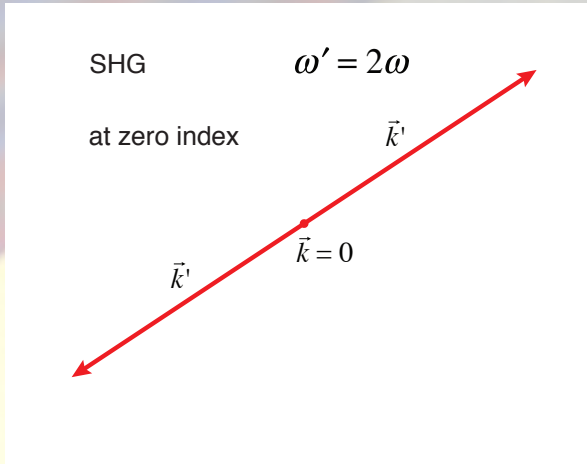
2 zero index

3 experiments

Exciting applications ahead



supercoupling



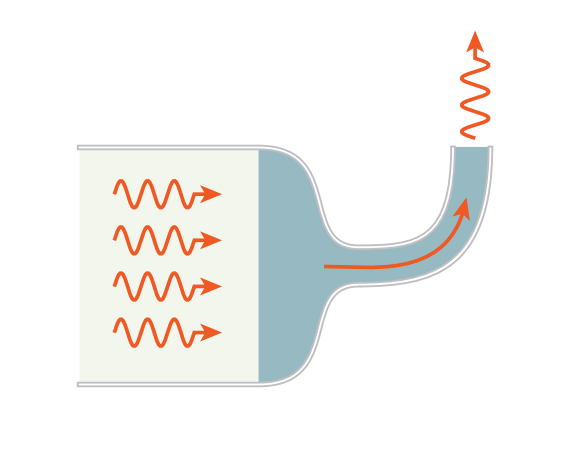
NLO

1 index

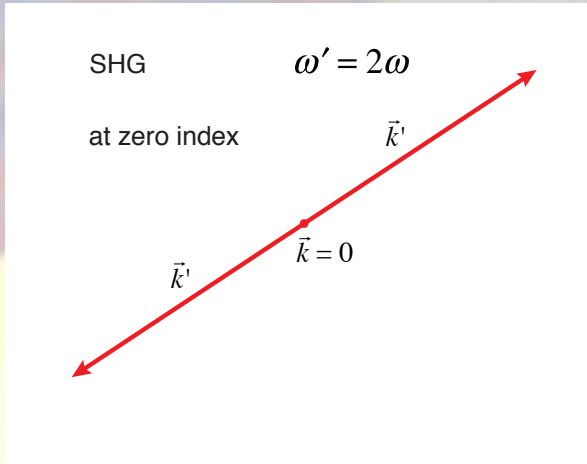
2 zero index

3 experiments

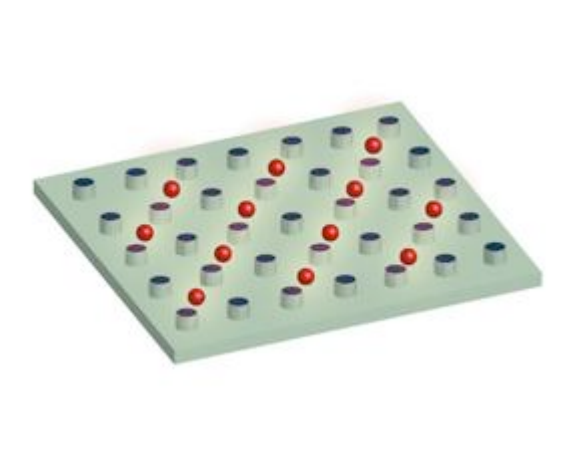
Exciting applications ahead



supercoupling



NLO



quantum optics

1 index

2 zero index

3 experiments

Yang Li, Shota Kita, Phil Muñoz, Orad Reshef,
Daryl Vulis, Mei Yin, Lysander Christakis, Zin Lin,
Cleaven Chia, Olivia Mello, Haoning Tang, Marko Lončar

National Science Foundation
Harvard Center for Nanoscale Systems

for a copy of this presentation:

http://ericmazur.com

Follow me! eric_mazur