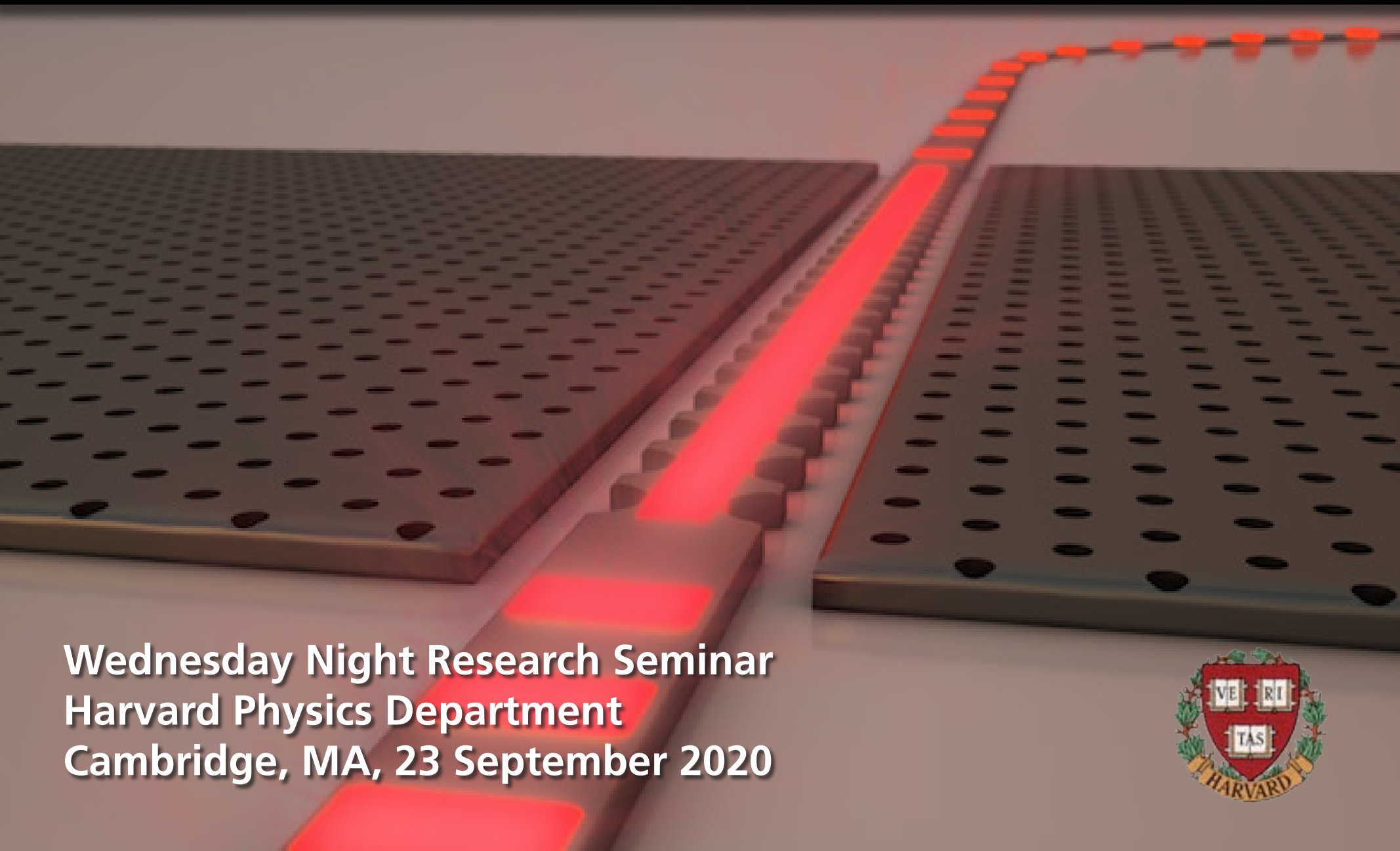


# The surprising world of zero-index metamaterials



Wednesday Night Research Seminar  
Harvard Physics Department  
Cambridge, MA, 23 September 2020



# The surprising world of zero-index metamaterials



**@eric\_mazur**

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A 3D perspective rendering of a photonic crystal waveguide. Two parallel gray plates with a grid of circular holes are shown. A red light path is depicted as a series of glowing rectangular segments within a central channel between the plates. The path starts from the bottom left, moves towards the right, and then turns 90 degrees upwards at the top right. The text "Enabling nanophotonics" is centered over the middle of the structure.

**Enabling nanophotonics**

A 3D perspective illustration of a photonic crystal slab waveguide. Two parallel slabs of material with a periodic array of circular holes are shown. A central waveguide channel is formed by a layer of material with a different periodicity. A red light beam is shown propagating through this channel, which is highlighted with a glowing red effect. The text "Enabling nanophotonics" is overlaid in the center of the image.

# Enabling nanophotonics

A 3D perspective rendering of a photonic crystal slab waveguide. The structure consists of two parallel slabs of material with a periodic array of circular holes. A central waveguide channel is formed by a row of larger holes. A red light path is shown entering from the bottom left, traveling through the waveguide, and exiting at the top right. The text "Enabling nanophotonics" is overlaid in the center.

# Enabling nanophotonics

**1** zero index

**2** fabrication

A 3D perspective rendering of a photonic crystal slab waveguide. The structure consists of two parallel slabs of material with a periodic array of circular holes. A central waveguide channel is formed by a row of larger holes. A red light path is shown entering from the bottom left, traveling through the waveguide, and exiting at the top right. The text "Enabling nanophotonics" is centered over the waveguide.

# Enabling nanophotonics

**1** zero index

**2** fabrication

**3** results

wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

solution

$$\vec{E} = \vec{E}_0 e^{i(kx - \omega t)}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}} c = \frac{1}{n} c$$

wave equation

$$\nabla^2 \vec{E} - \frac{\mu \epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

solution

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wave equation

$$\nabla^2 \vec{E} - \frac{\mu \epsilon \omega^2}{c^2} \vec{E} = 0$$

solution

$$\vec{E} = \vec{E}_0 e^{i(kx - \omega t)} \longrightarrow \vec{E} = \vec{E}_0 e^{-i\omega t}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}} c = \frac{1}{n} c$$

wave equation

$$\nabla^2 \vec{E} - \frac{\mu \epsilon \omega^2}{c^2} \vec{E} = 0$$

solution

$$\vec{E} = \vec{E}_0 e^{i(kx - \omega t)} \longrightarrow \vec{E} = \vec{E}_0 e^{-i\omega t}$$

where

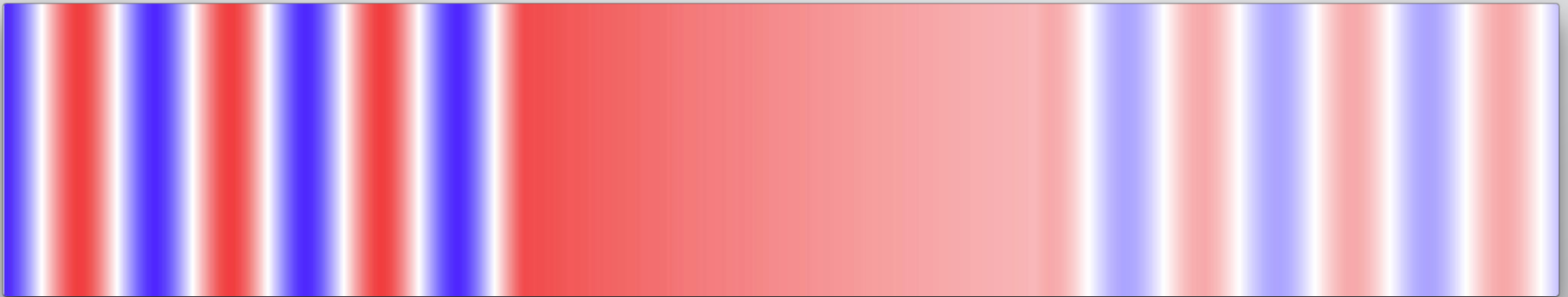
$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}} c = \frac{1}{n} c \longrightarrow \infty$$



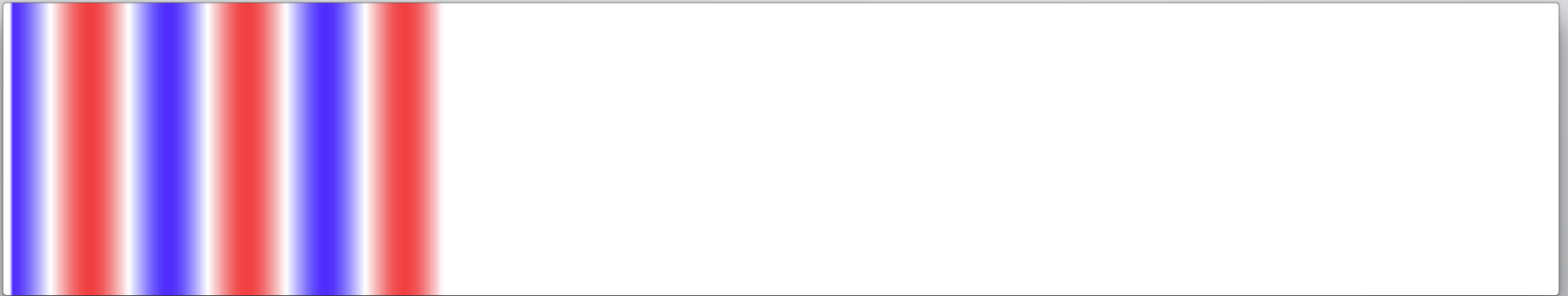
**1** zero index

**“Superluminal”?!**

# What about causality?

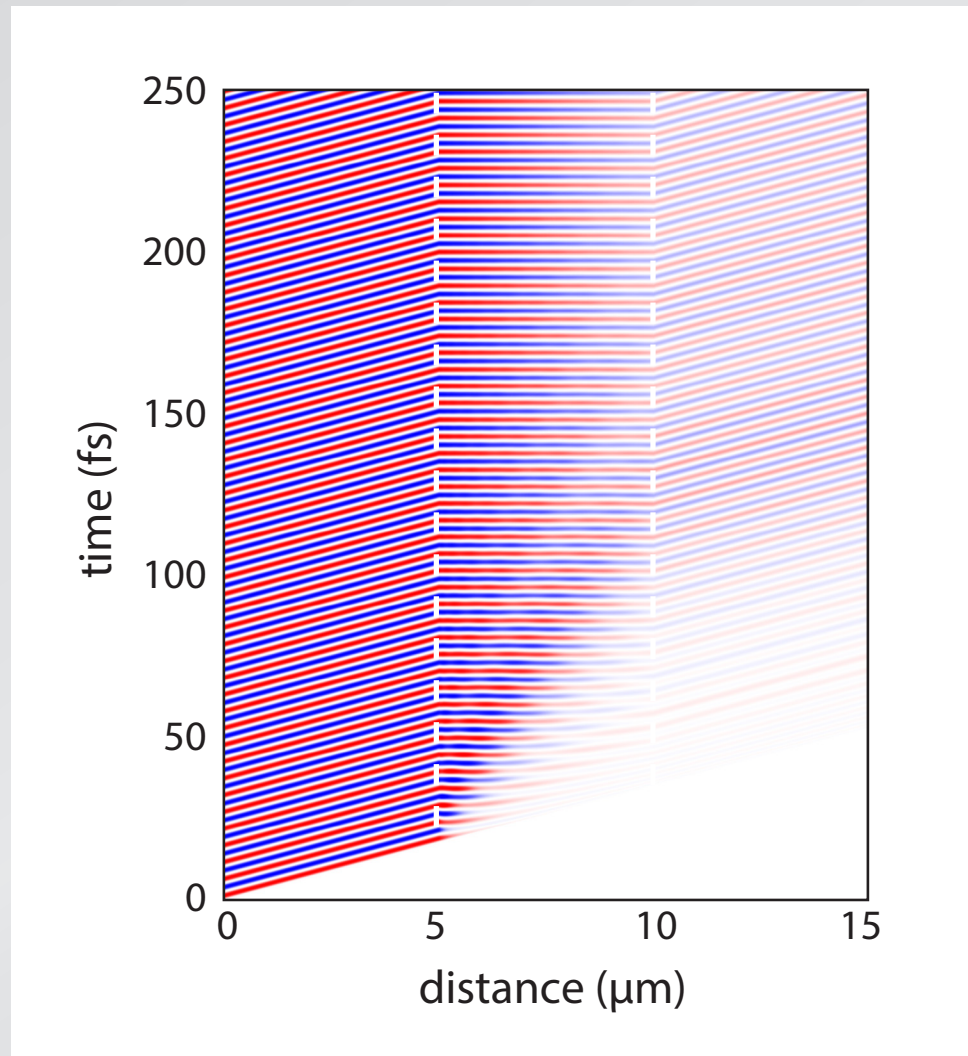


# What about causality?



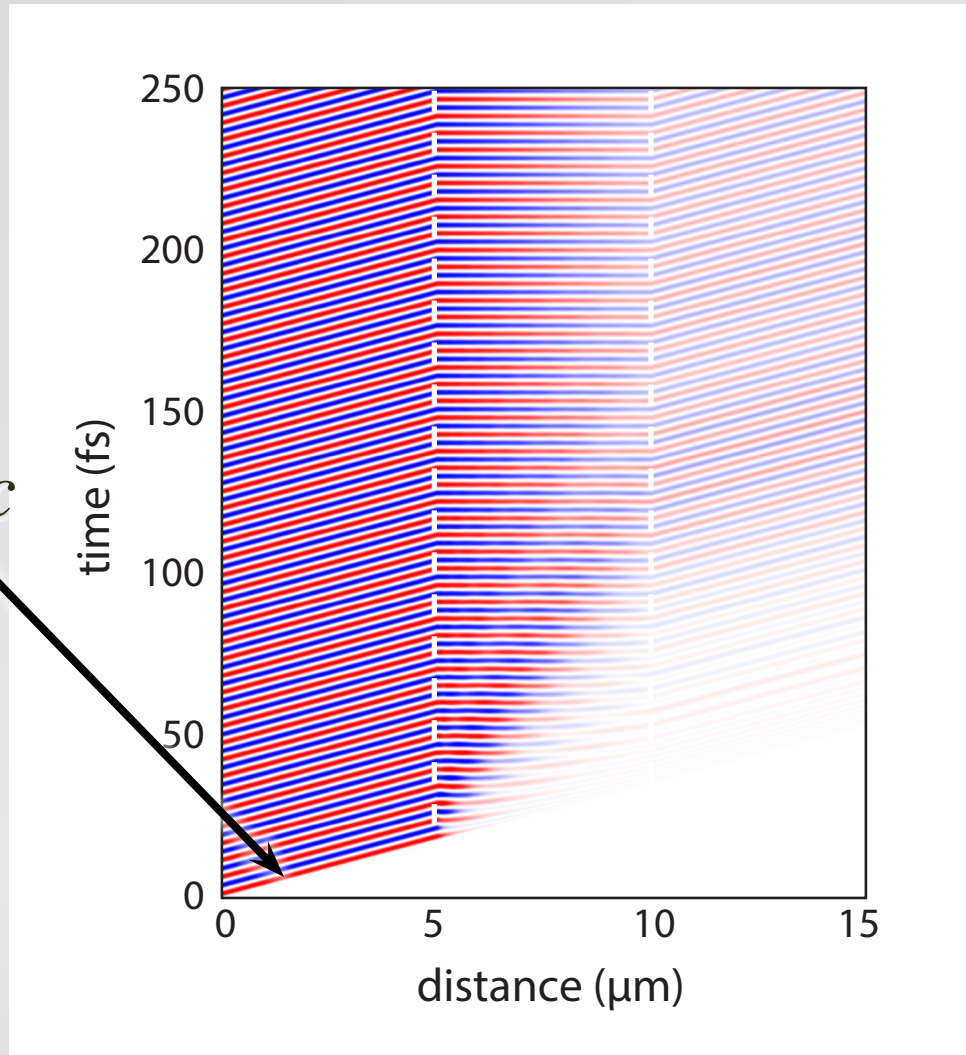
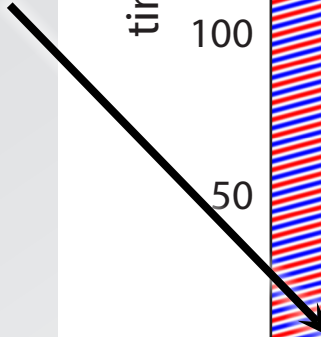


# What about causality?

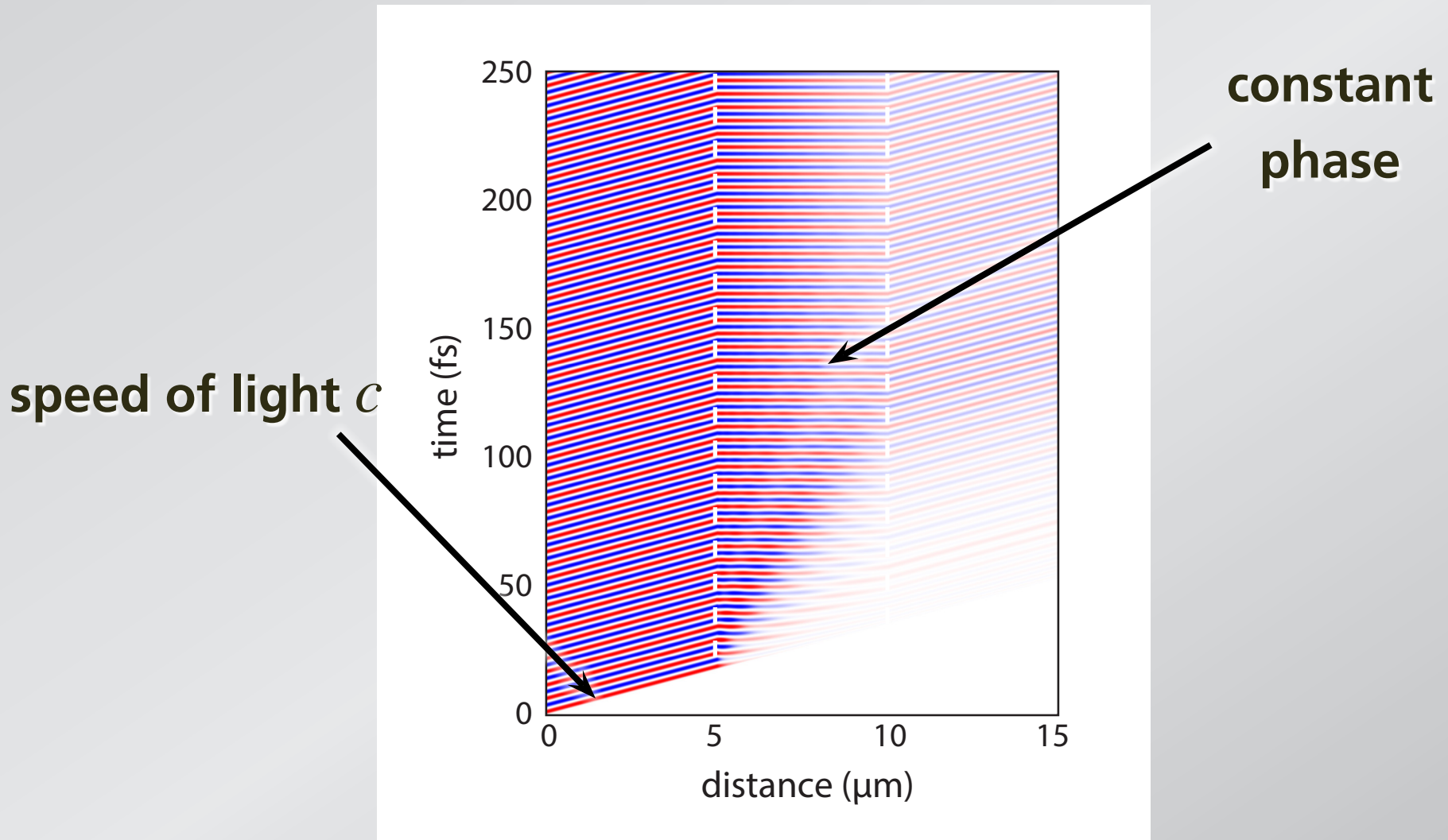


# What about causality?

speed of light  $c$



# What about causality?

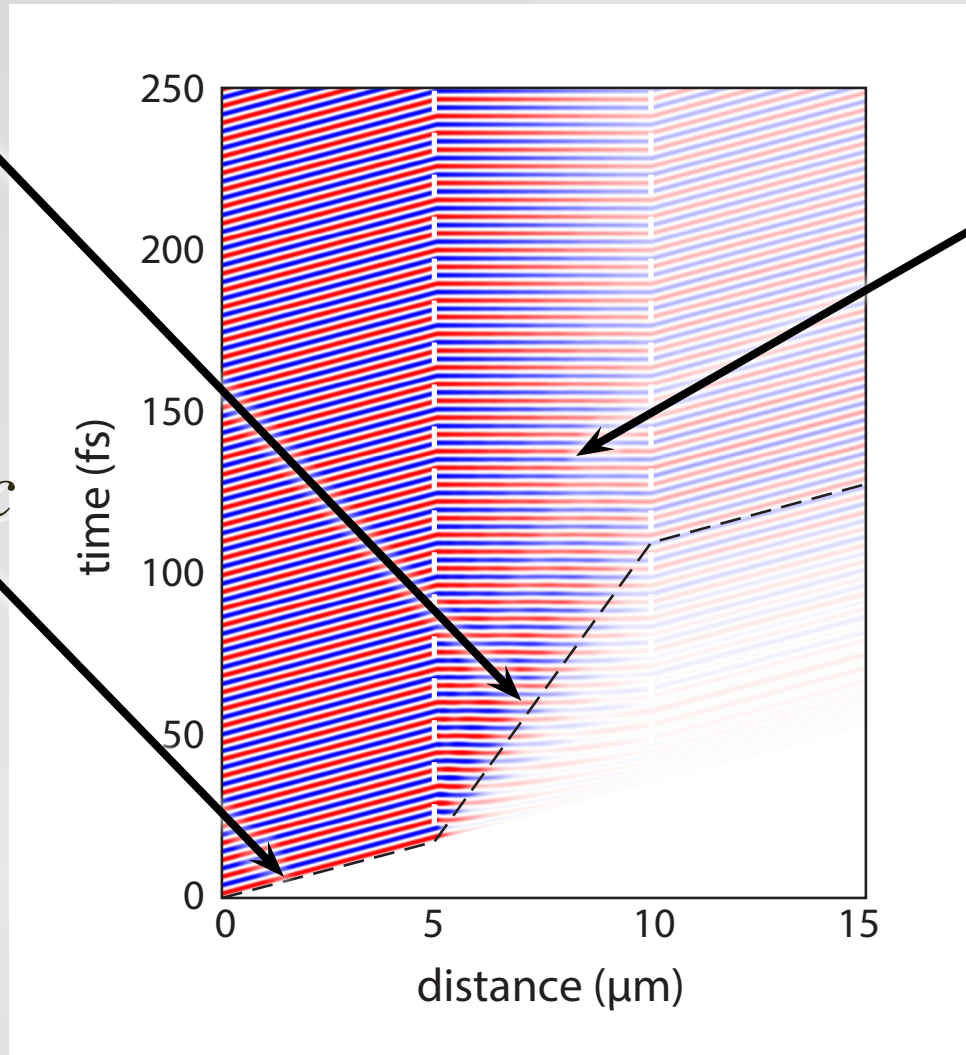


# What about causality?

group velocity

$$v_g < c$$

speed of light  $c$



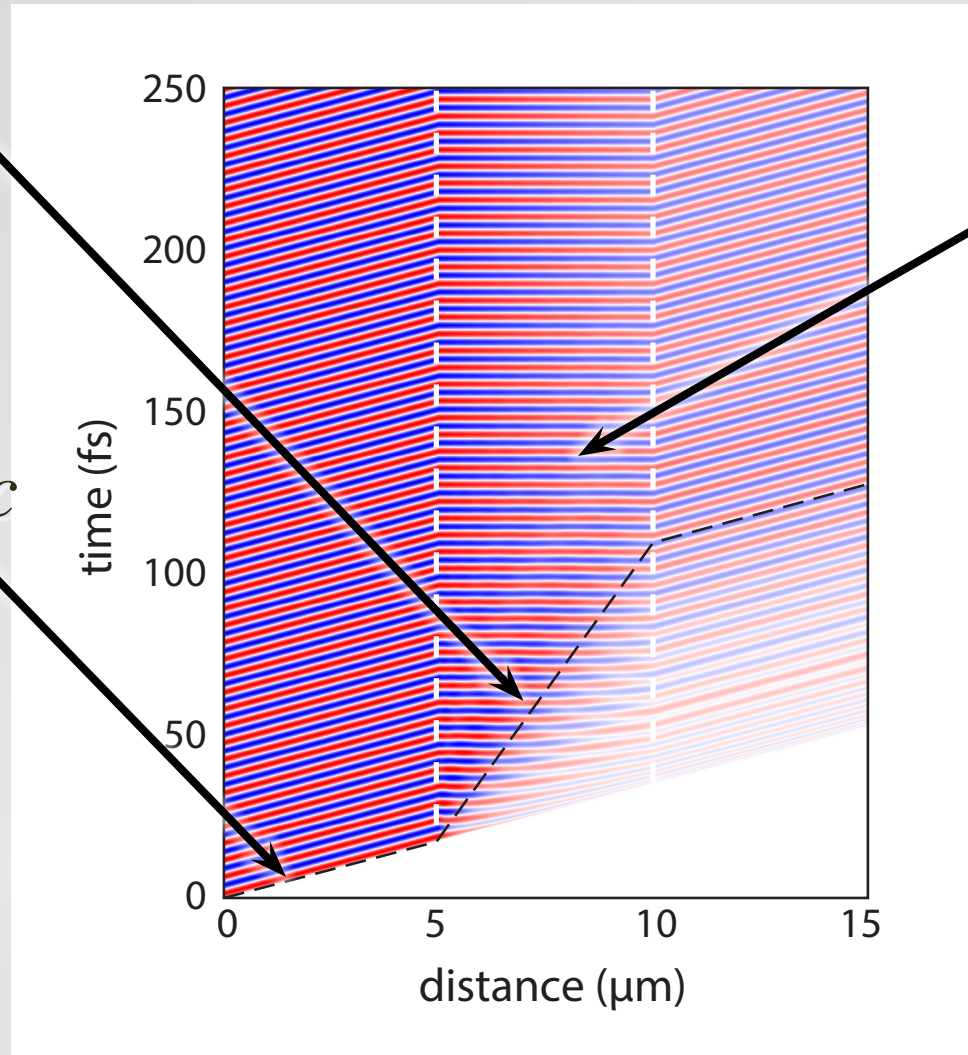
constant phase

# What about causality?

group velocity

$$v_g < c$$

speed of light  $c$



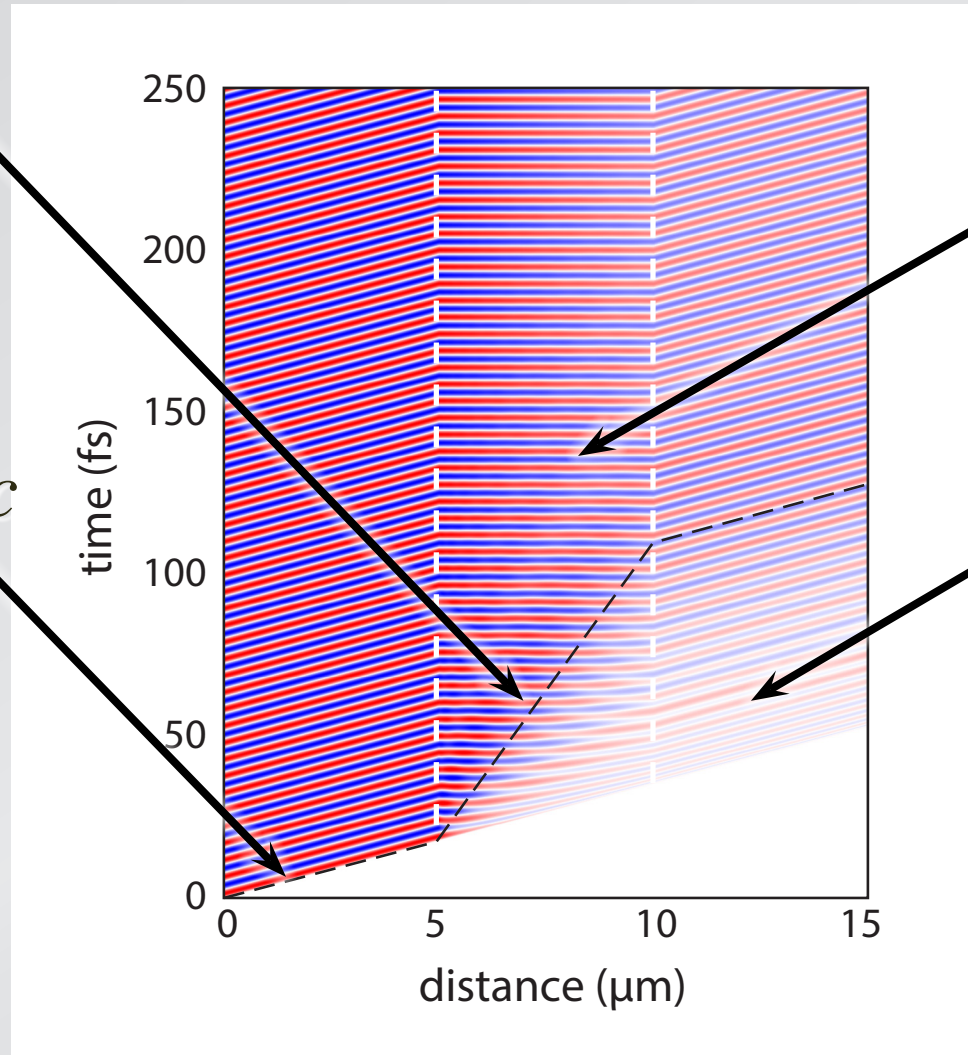
constant phase

# What about causality?

group velocity

$$v_g < c$$

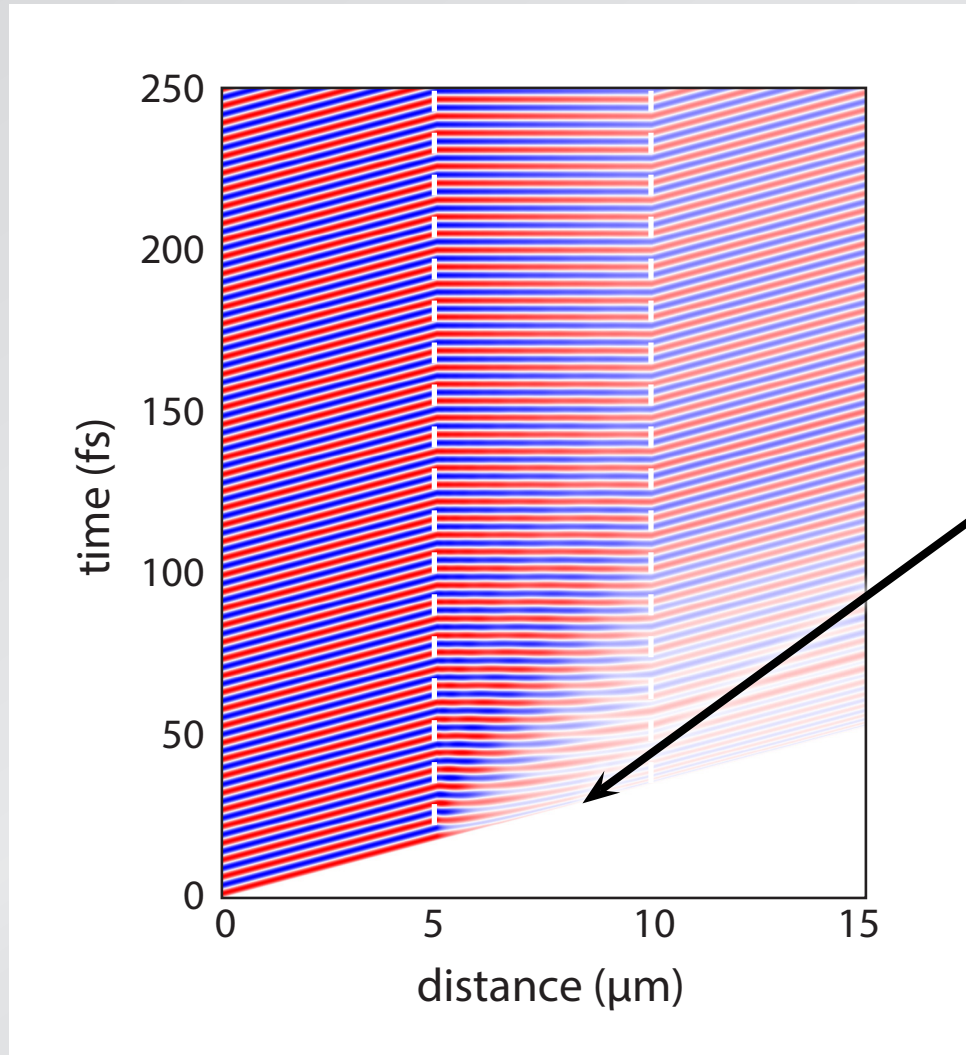
speed of light  $c$



constant phase

high-frequency precursors

# What about causality?



signal *always*  
travels at speed  $c$ !



how?

$$n = \sqrt{\epsilon\mu}$$

but  $\epsilon$  and  $\mu$  also determine reflectivity

$$R = \frac{Z - 1}{Z + 1}$$

how?

$$n = \sqrt{\epsilon\mu}$$

but  $\epsilon$  and  $\mu$  also determine reflectivity

$$R = \frac{Z - 1}{Z + 1}$$

where

$$Z = \sqrt{\frac{\mu}{\epsilon}}$$

how?

$$\epsilon, \mu \rightarrow 0$$

$$n = \sqrt{\epsilon\mu} \rightarrow 0$$

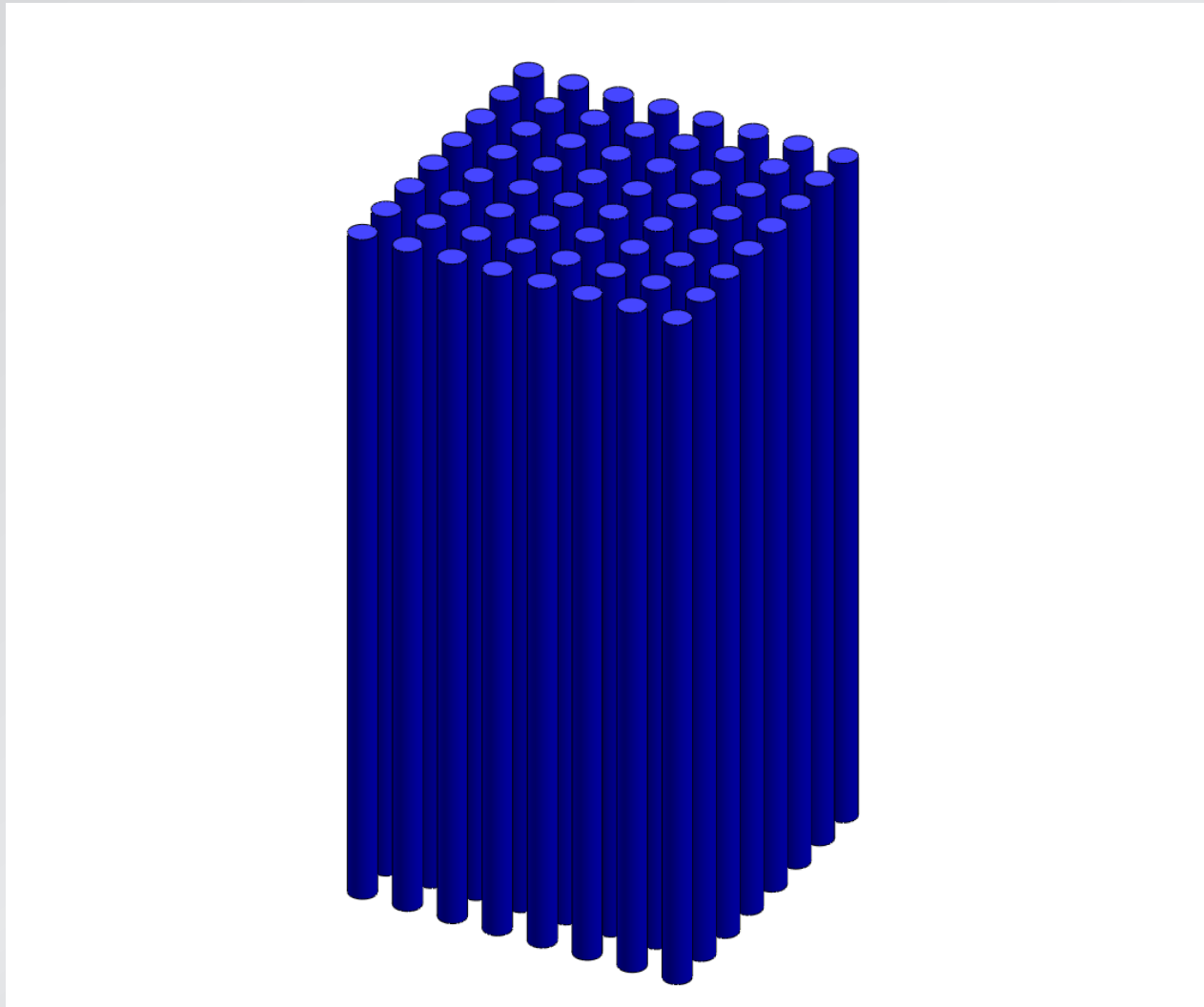
but  $\epsilon$  and  $\mu$  also determine reflectivity

$$R = \frac{Z - 1}{Z + 1}$$

where

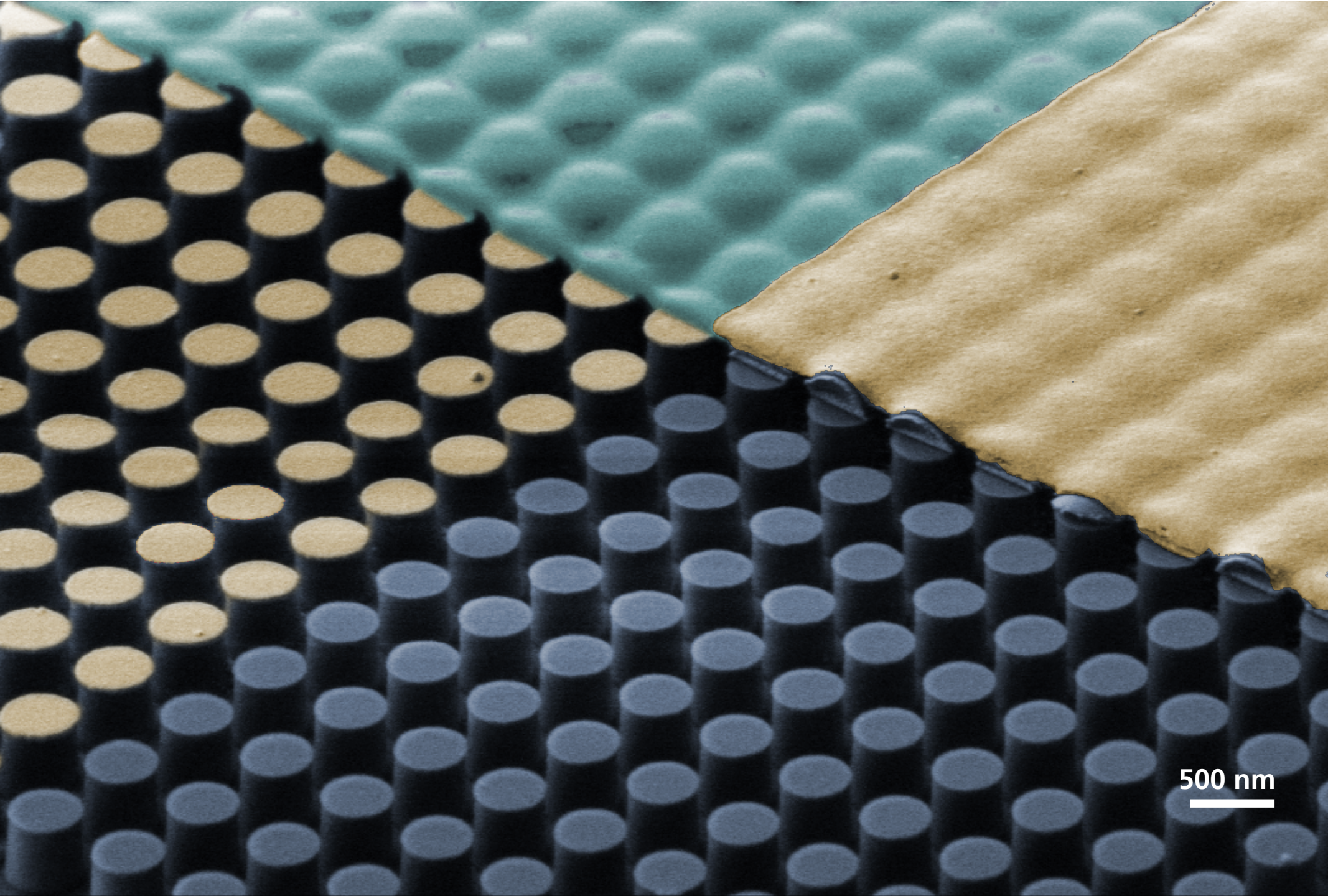
$$Z = \sqrt{\frac{\mu}{\epsilon}} \quad \text{finite!}$$

# How to fabricate?



**1** zero index

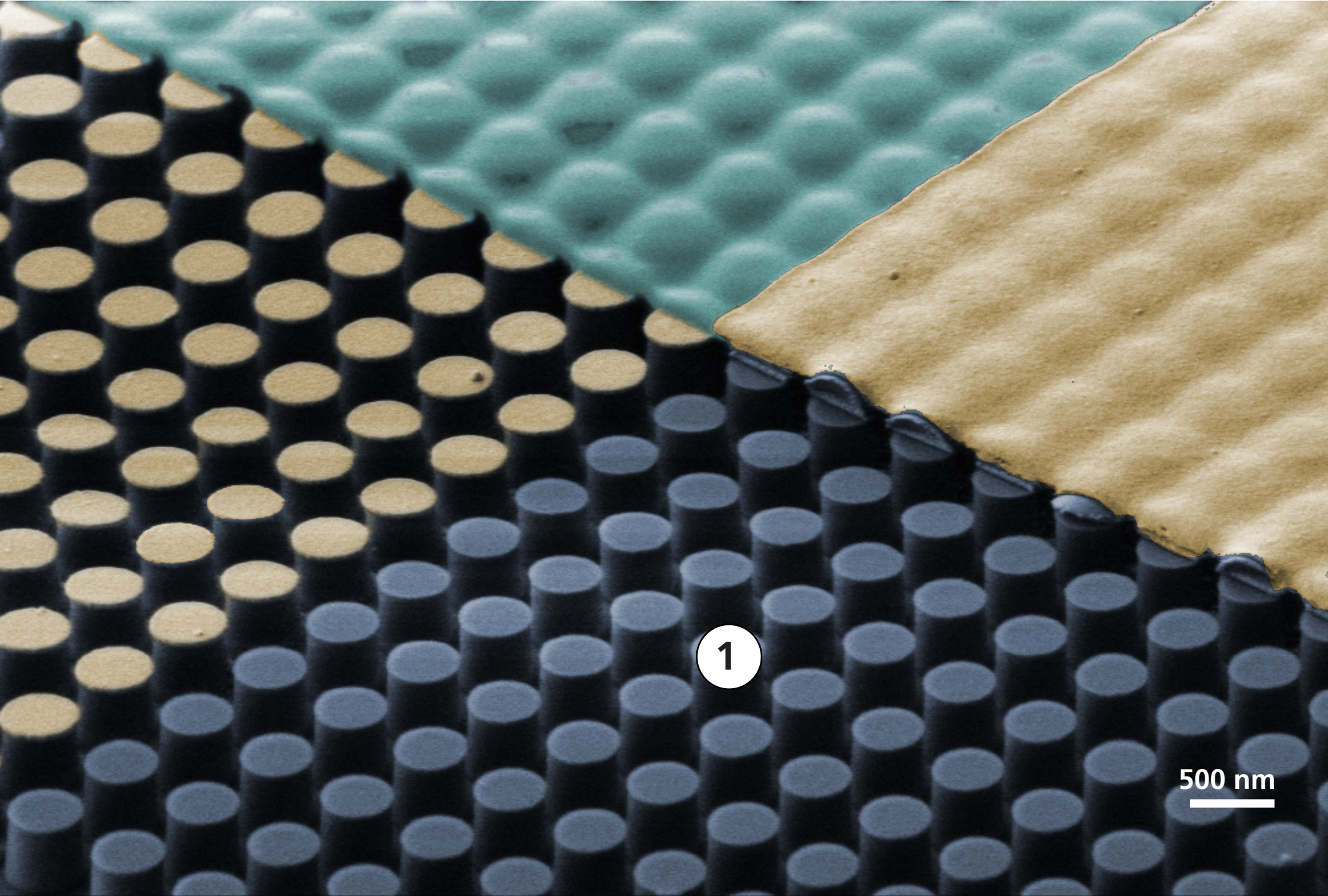
**2** fabrication



500 nm

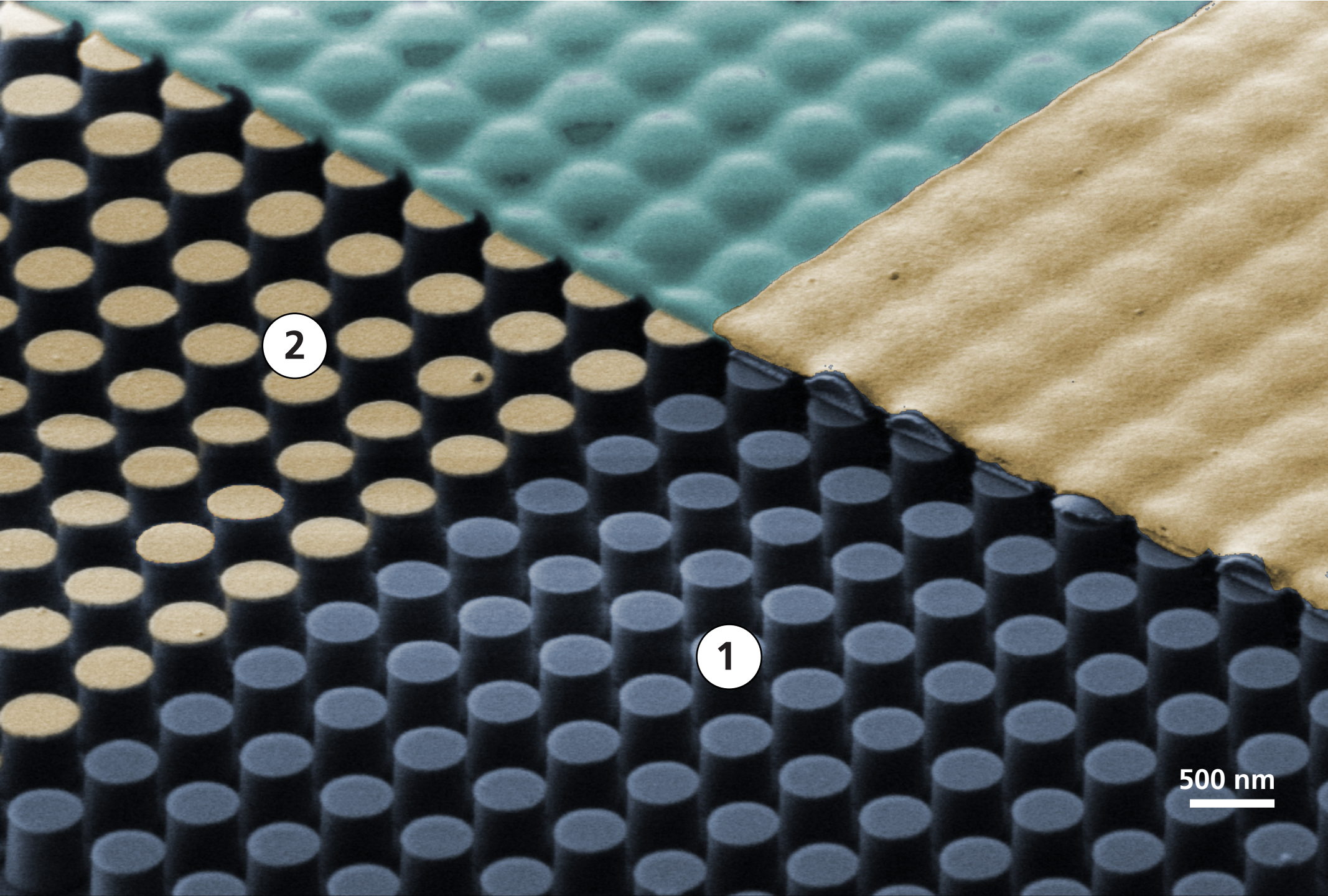
**1** zero index

**2** fabrication



**1** zero index

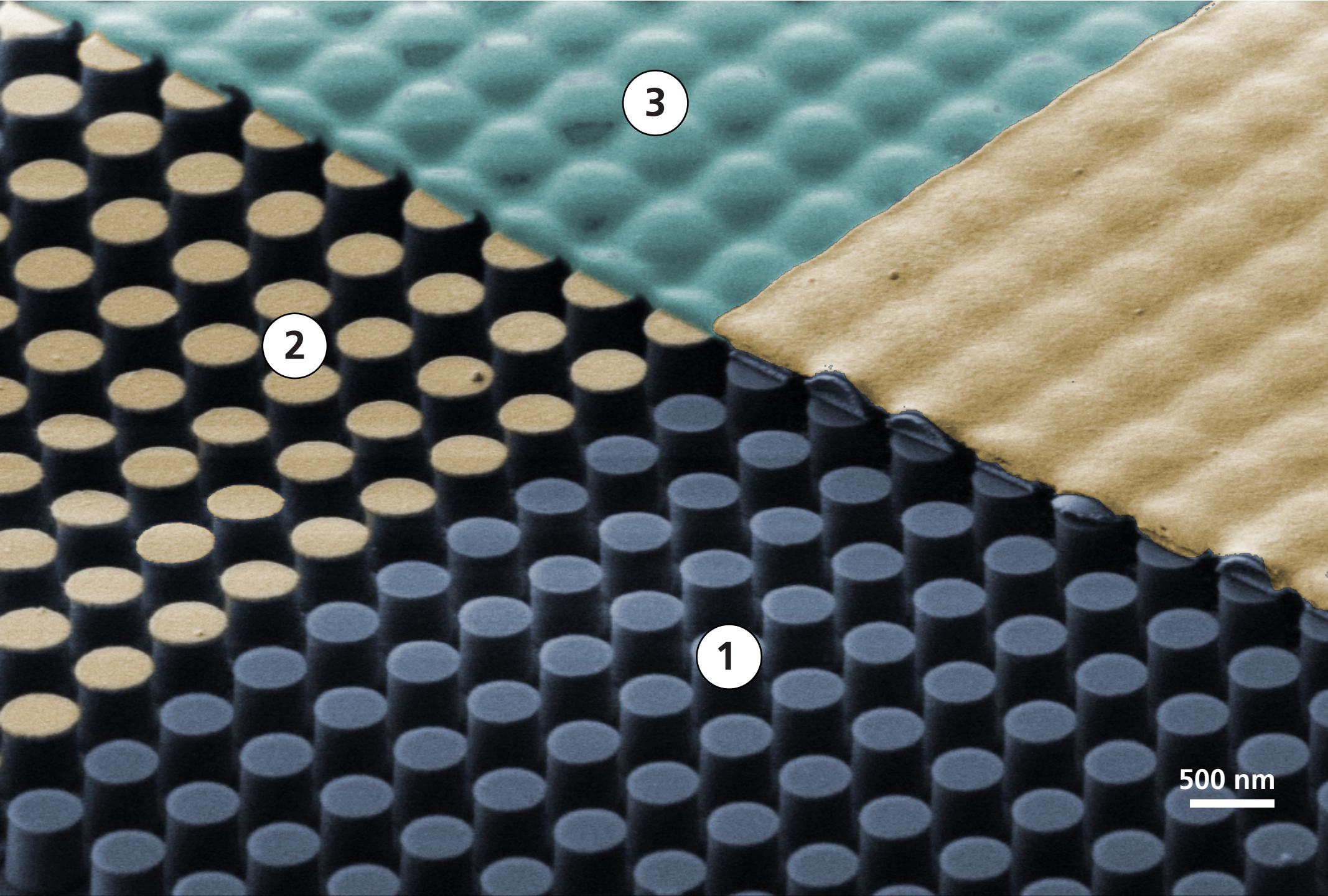
**2** fabrication



500 nm

**1** zero index

**2** fabrication



3

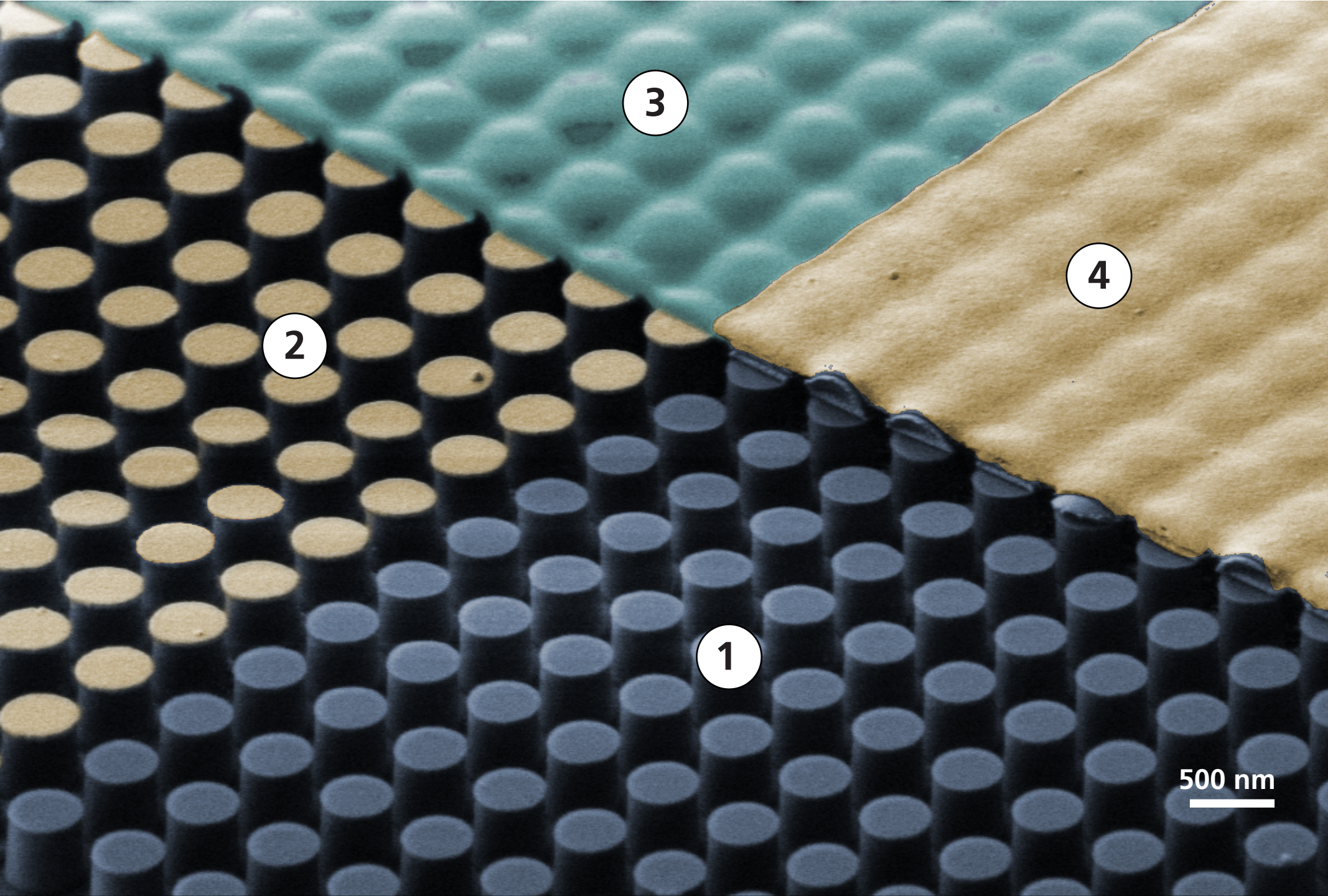
2

1

500 nm

1 zero index

2 fabrication



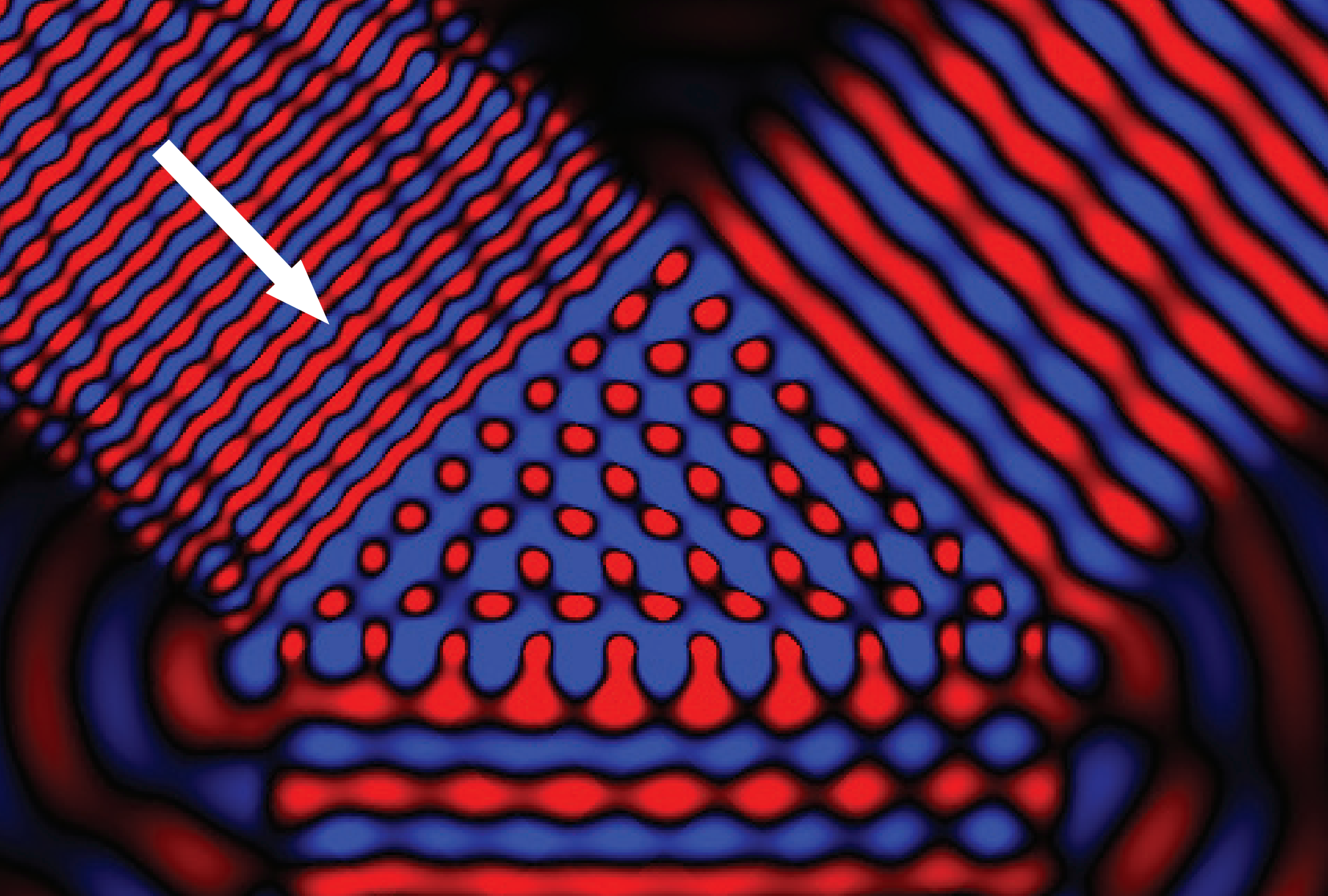
**1** zero index

**2** fabrication

**Can make this in any shape!**

**1 zero index**

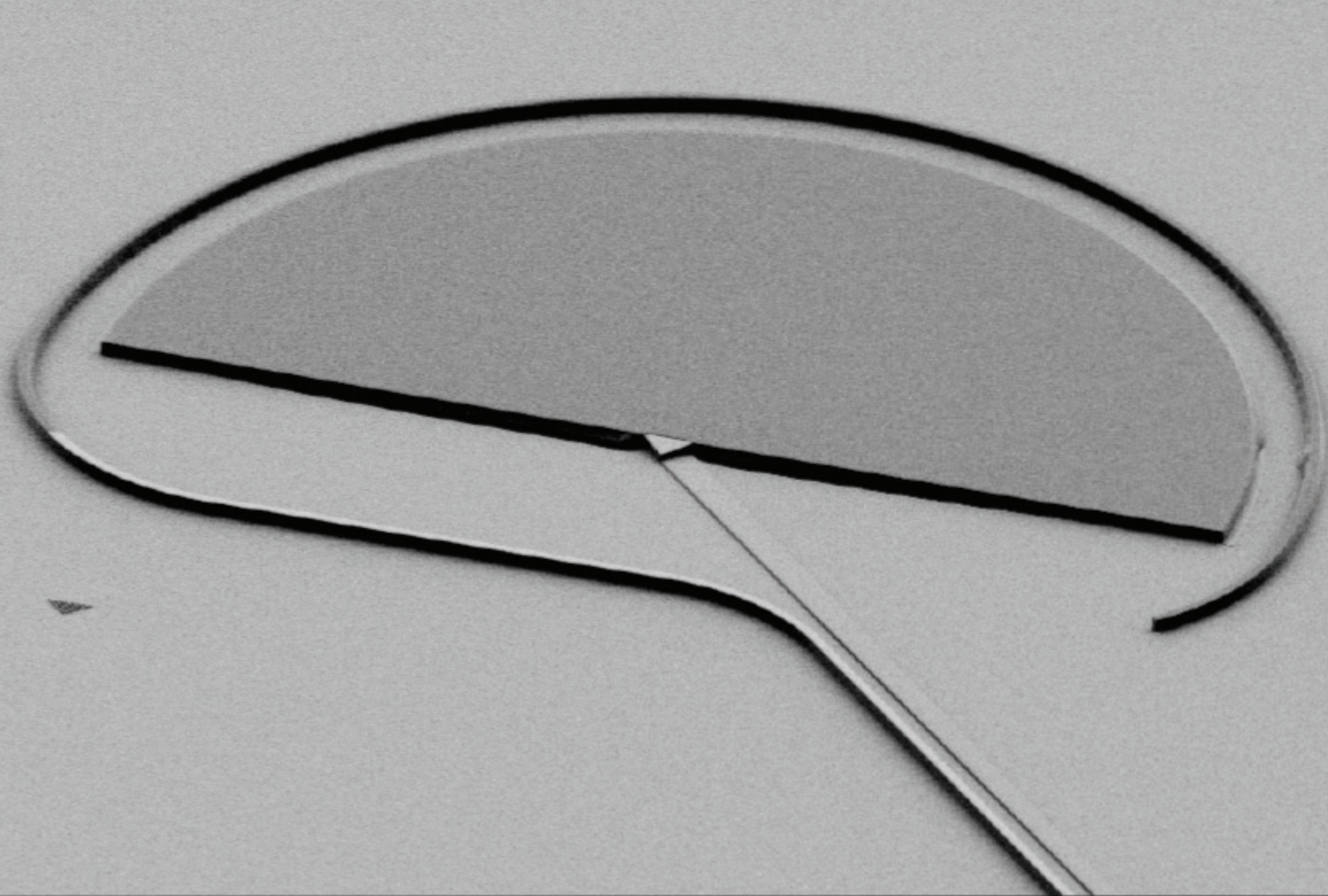
**2 fabrication**



**1** zero index

**2** fabrication

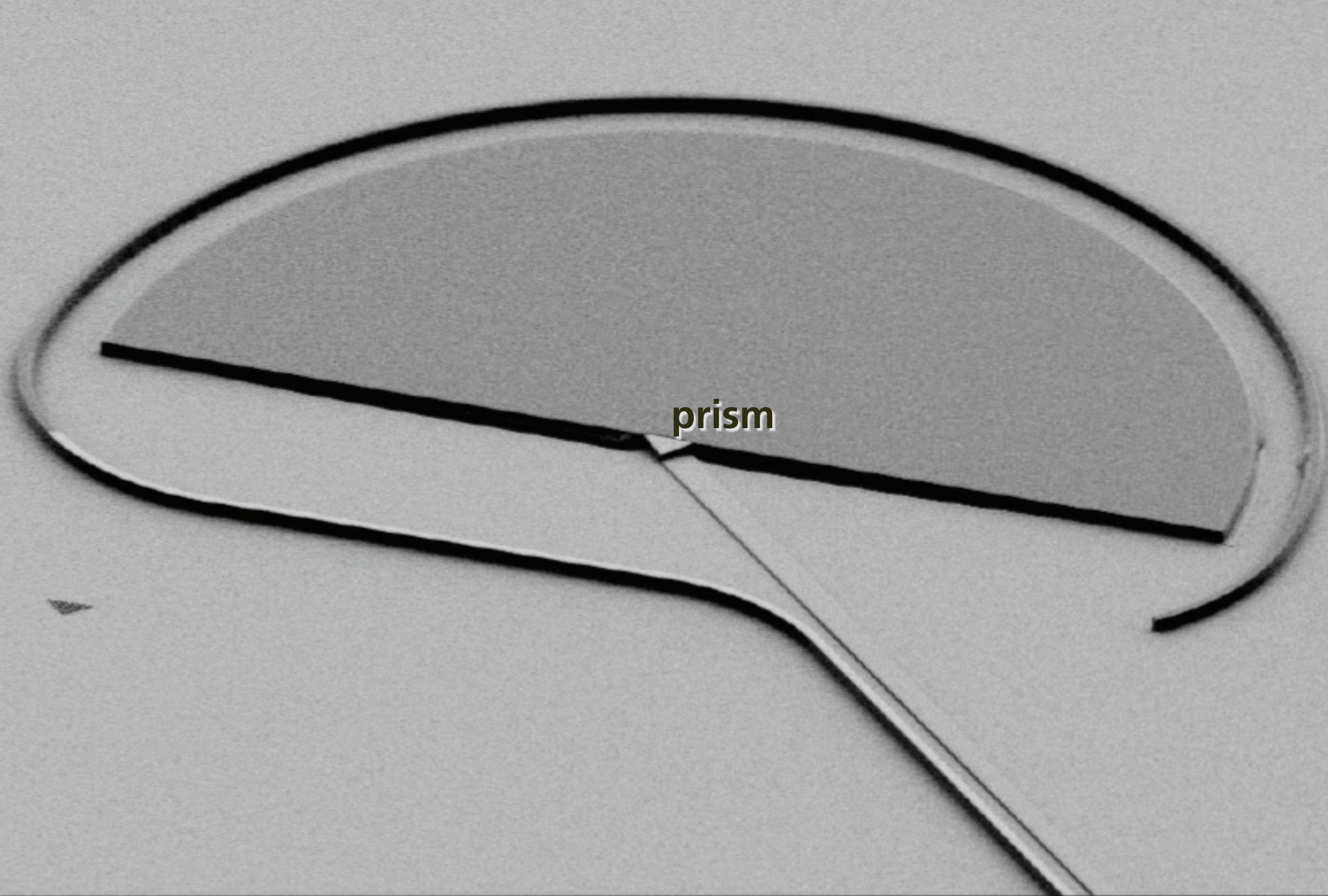
**3** results



**1** zero index

**2** fabrication

**3** results

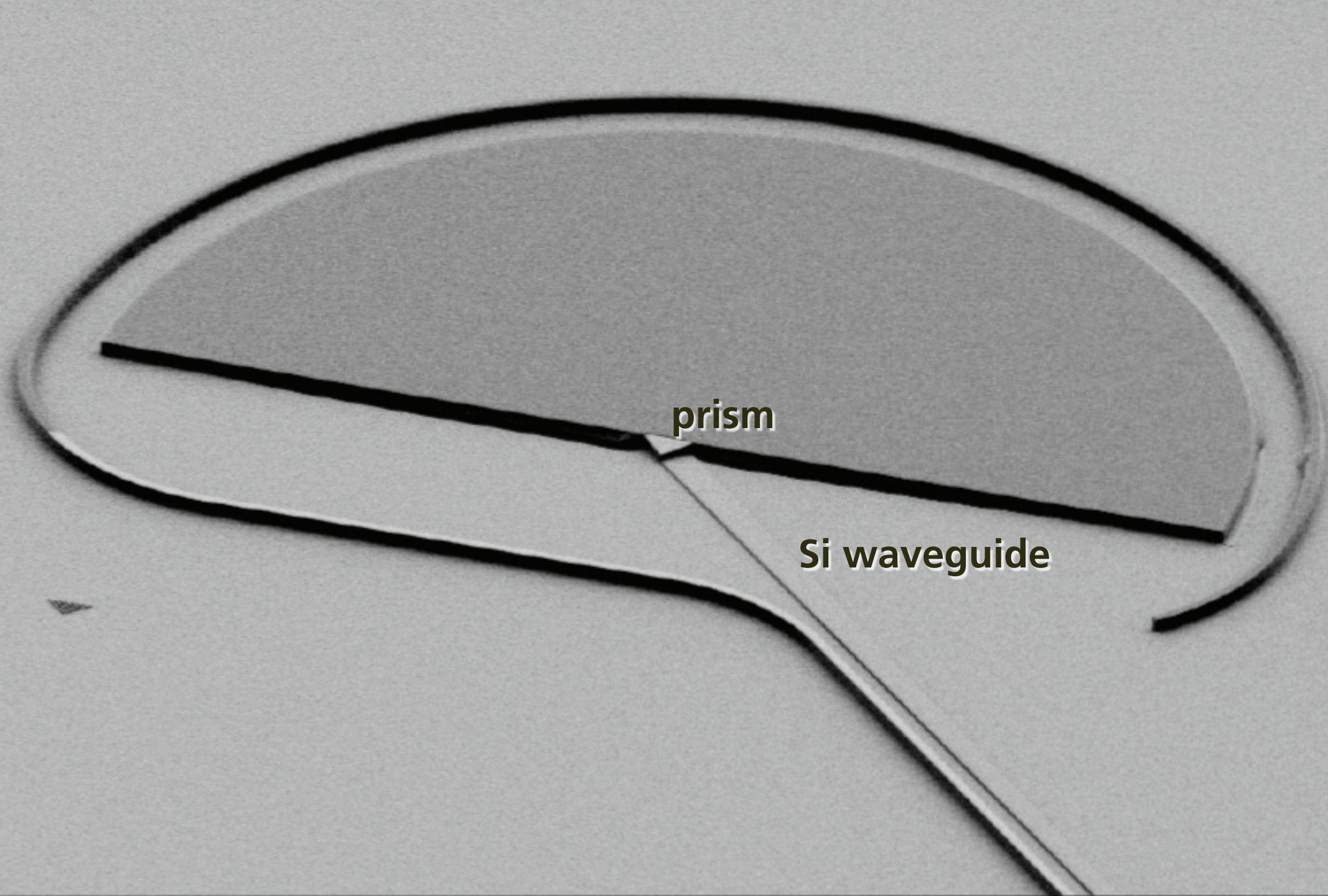


prism

1 zero index

2 fabrication

3 results



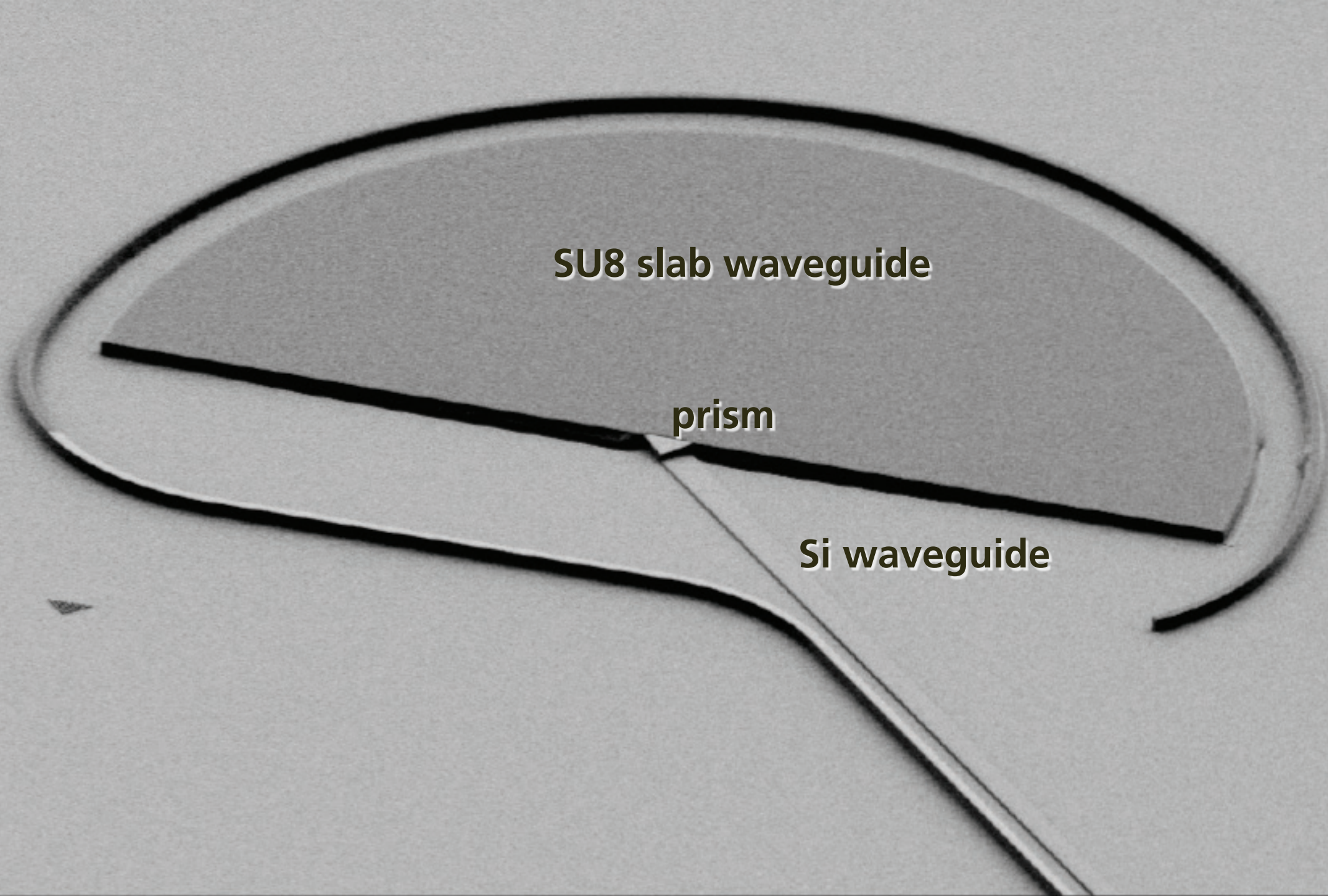
prism

Si waveguide

1 zero index

2 fabrication

3 results



SU8 slab waveguide

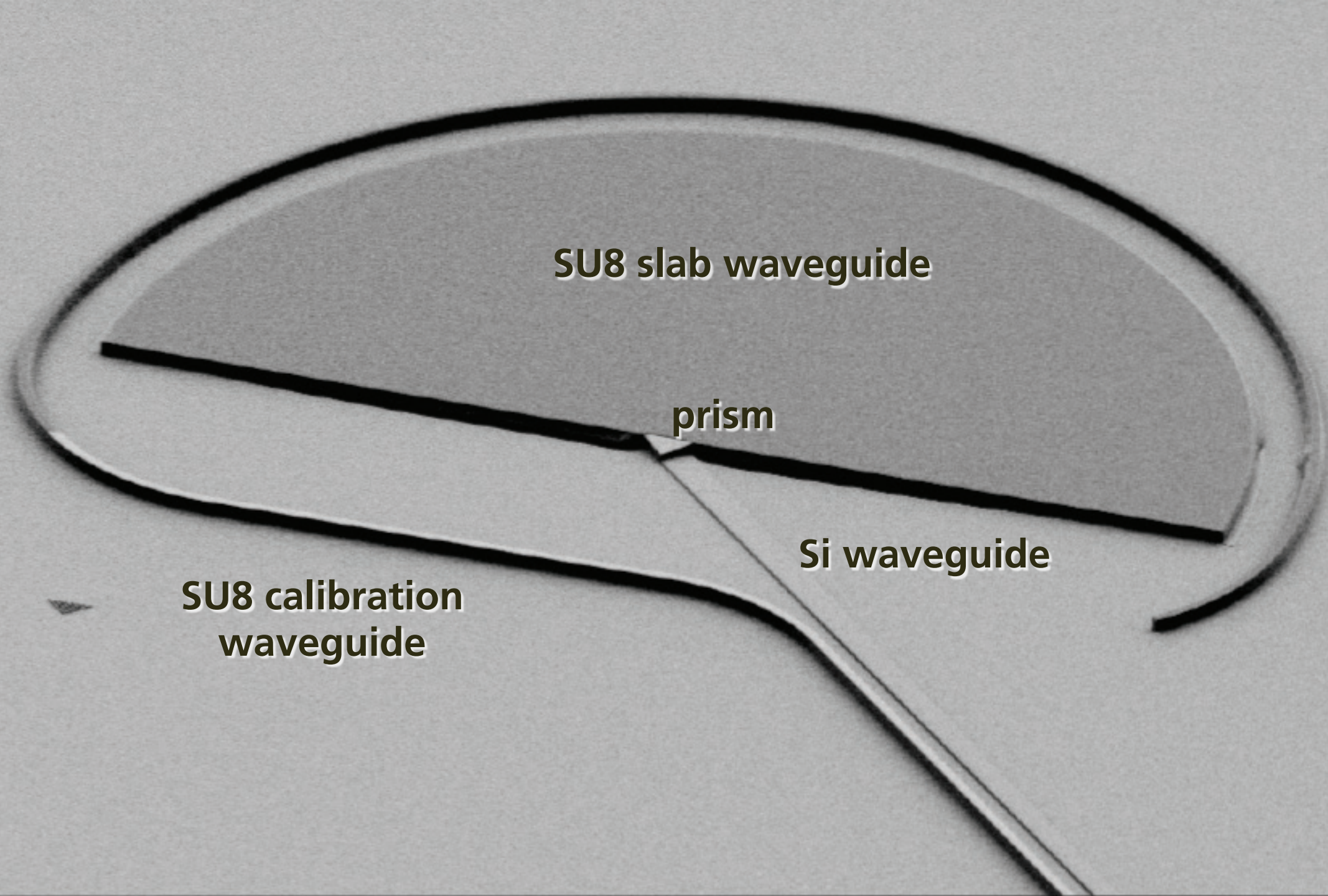
prism

Si waveguide

1 zero index

2 fabrication

3 results



SU8 slab waveguide

prism

Si waveguide

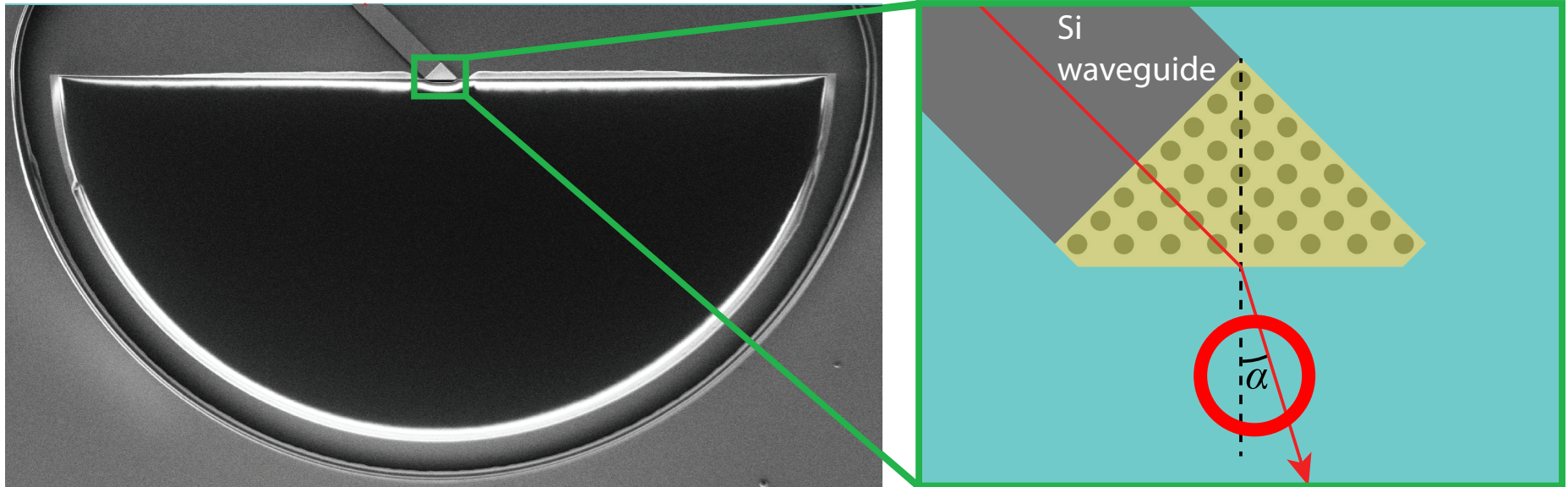
SU8 calibration  
waveguide

1 zero index

2 fabrication

3 results

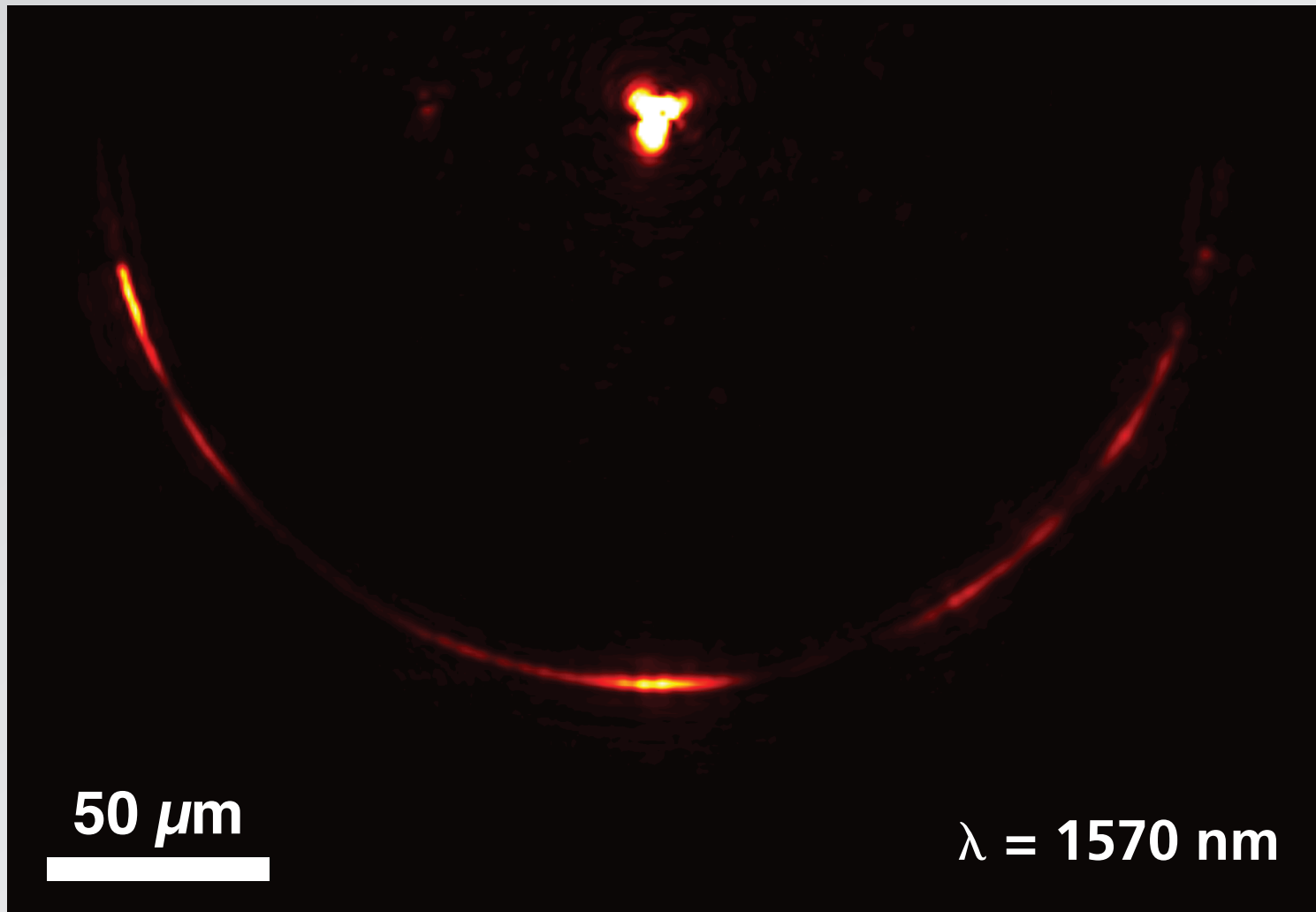
# On-chip zero-index prism



1 zero index

2 fabrication

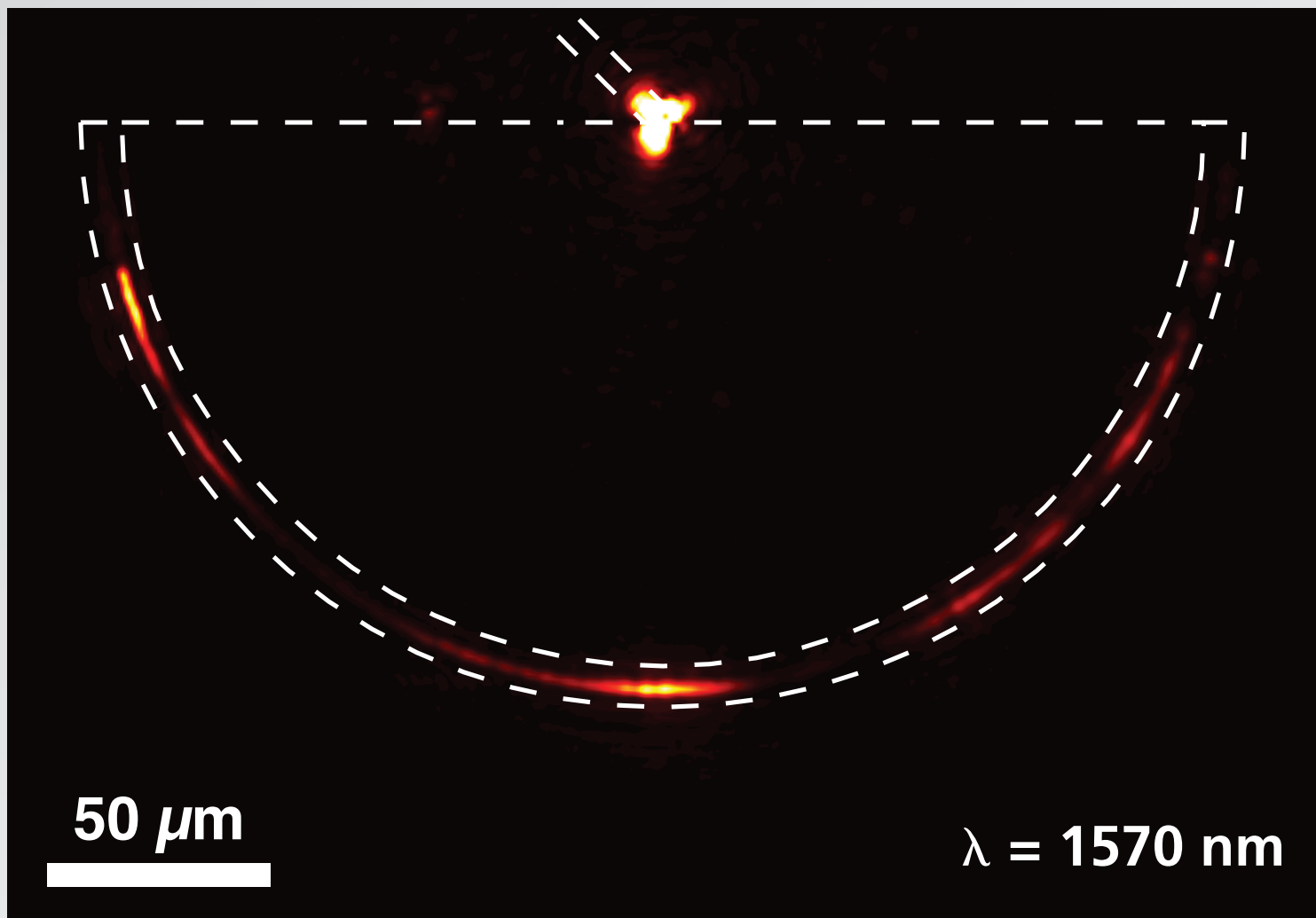
3 results



1 zero index

2 fabrication

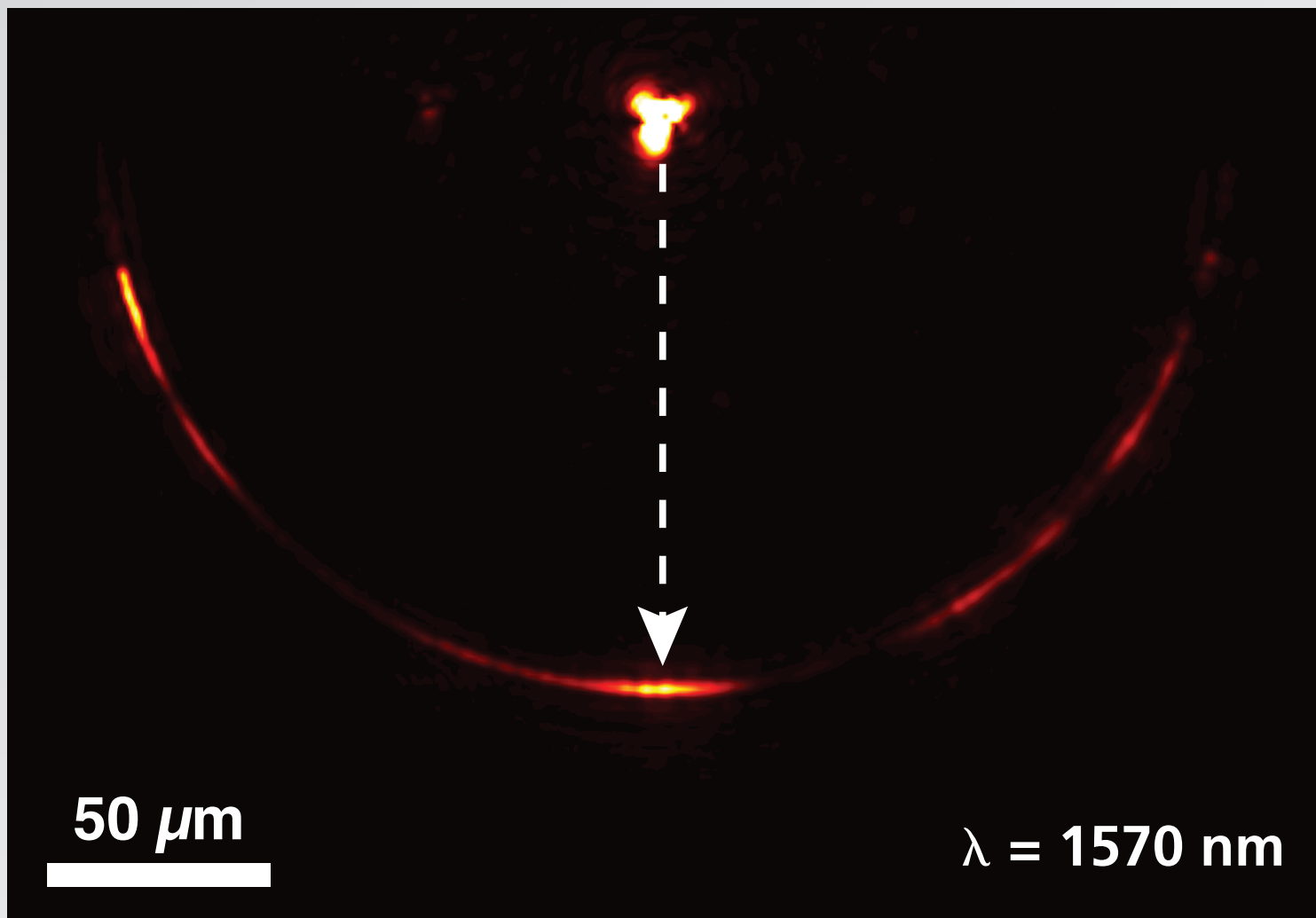
3 results



1 zero index

2 fabrication

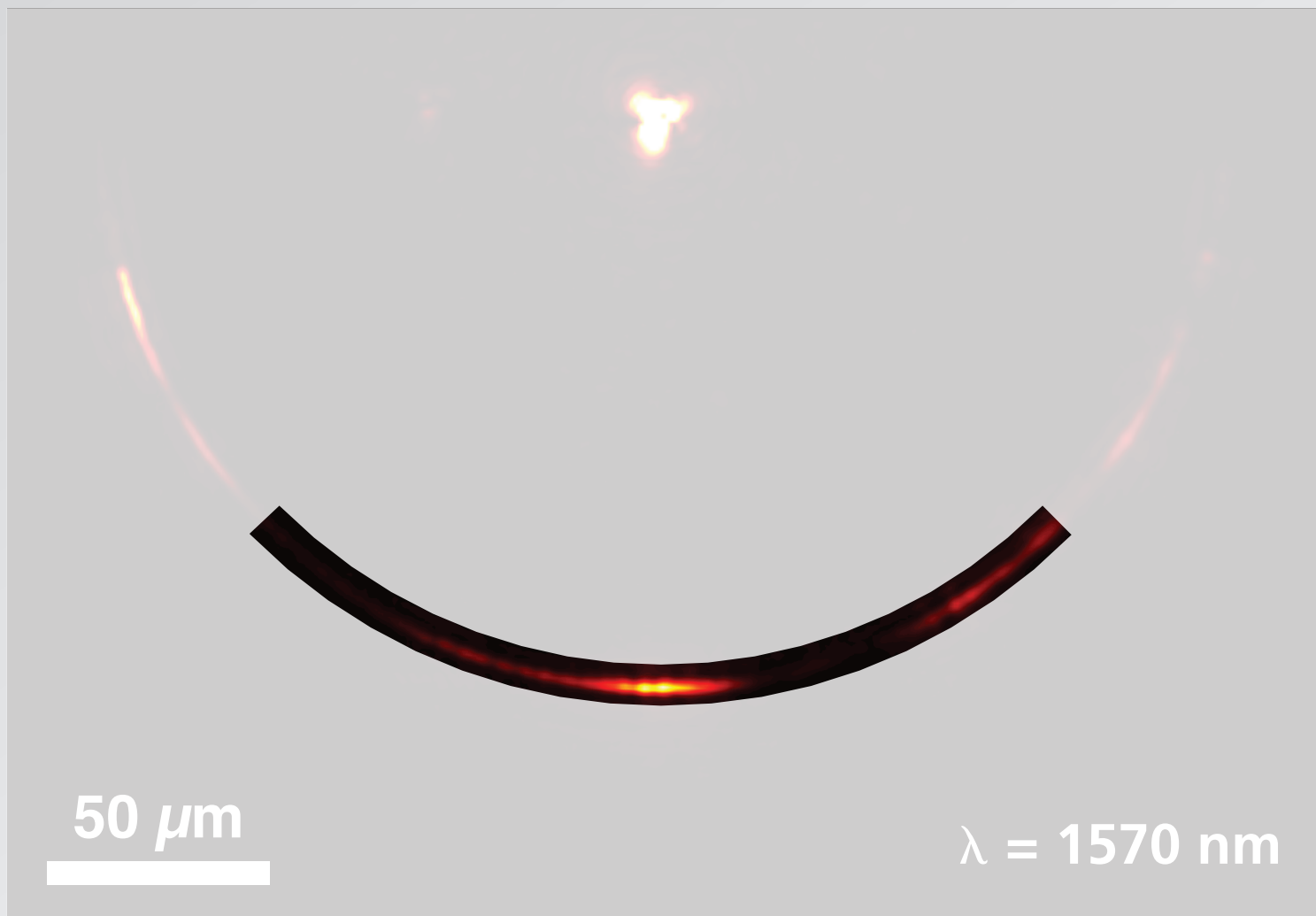
3 results



1 zero index

2 fabrication

3 results

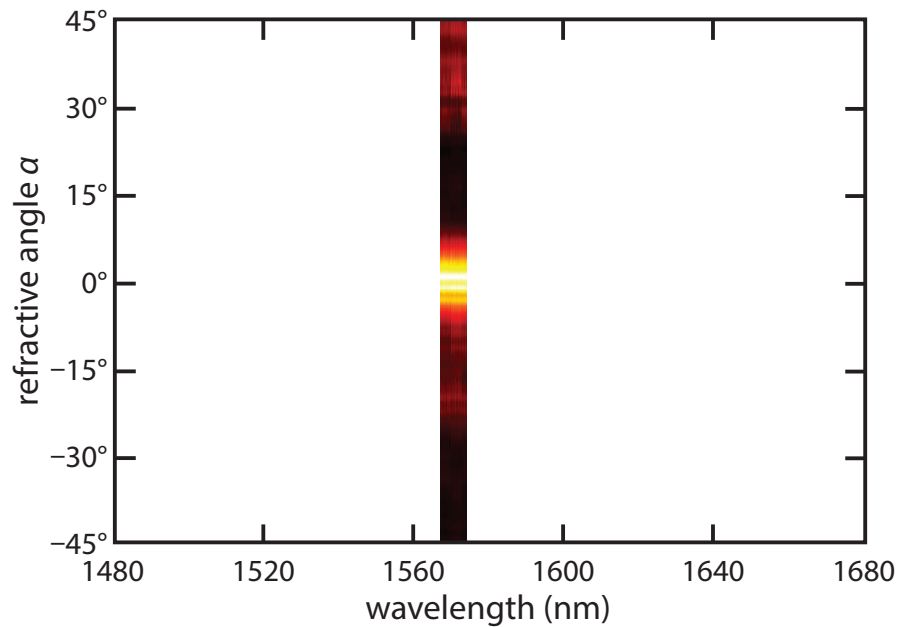


1 zero index

2 fabrication

3 results

# Wavelength dependence of refraction angle

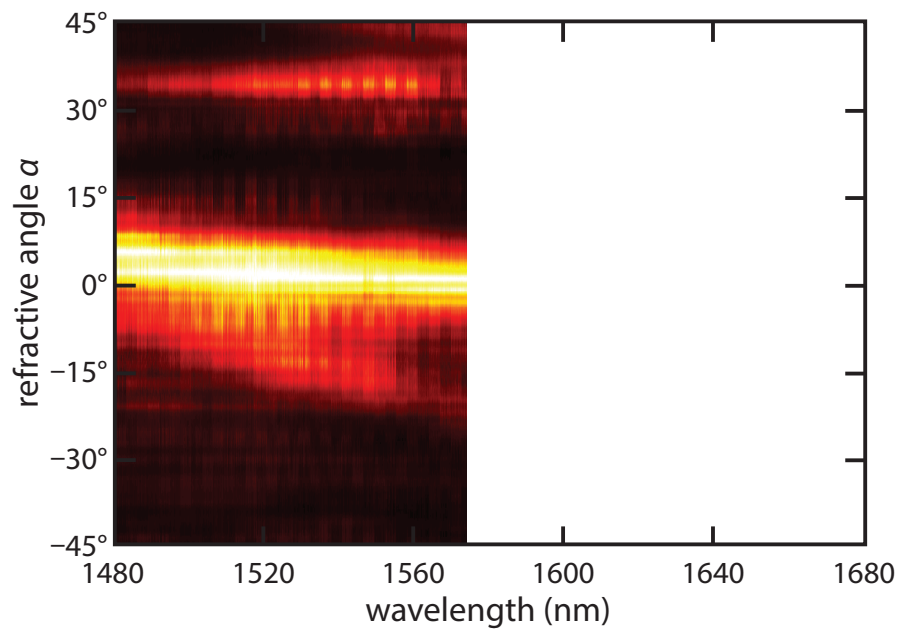


1 zero index

2 fabrication

3 results

# Wavelength dependence of refraction angle

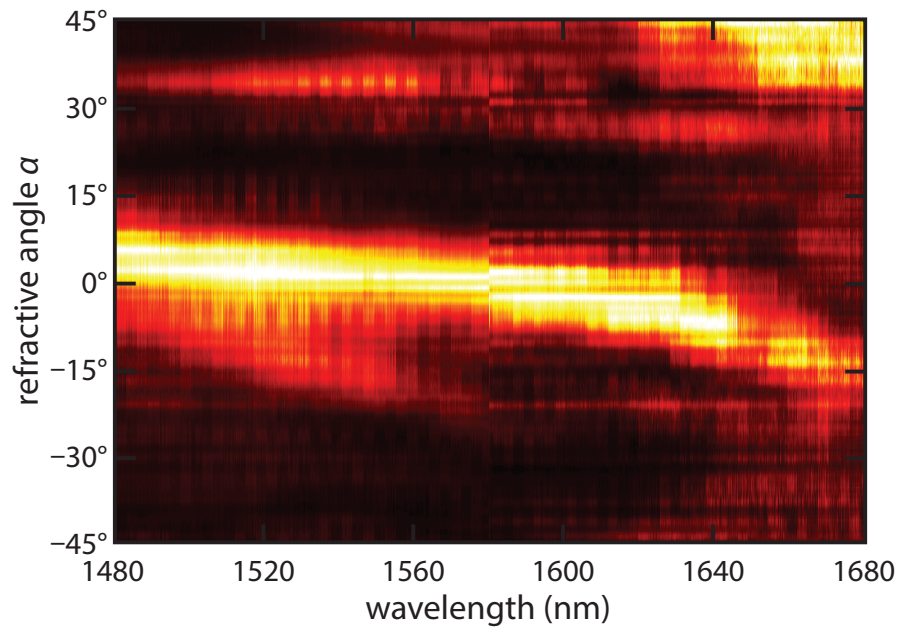


1 zero index

2 fabrication

3 results

# Wavelength dependence of refraction angle

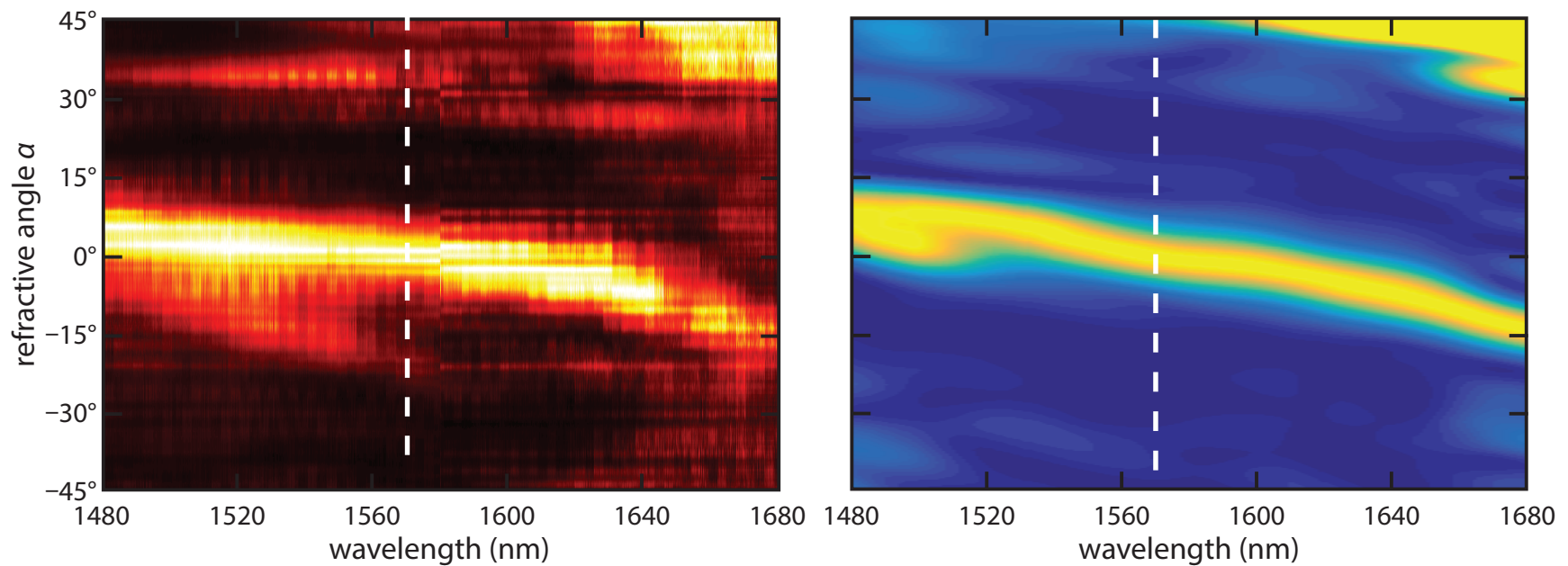


1 zero index

2 fabrication

3 results

# Wavelength dependence of refraction angle

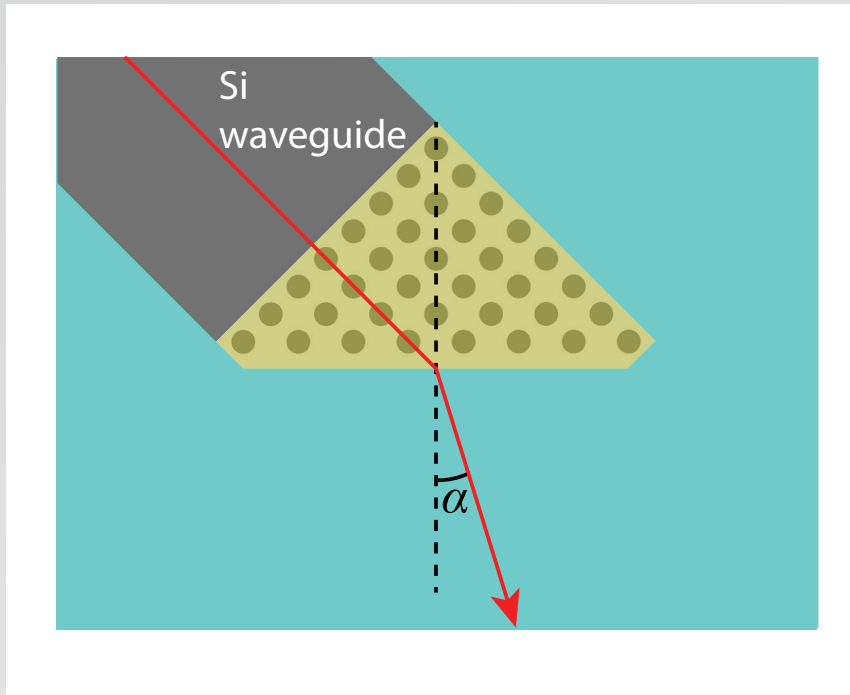


1 zero index

2 fabrication

3 results

# Wavelength dependence of index



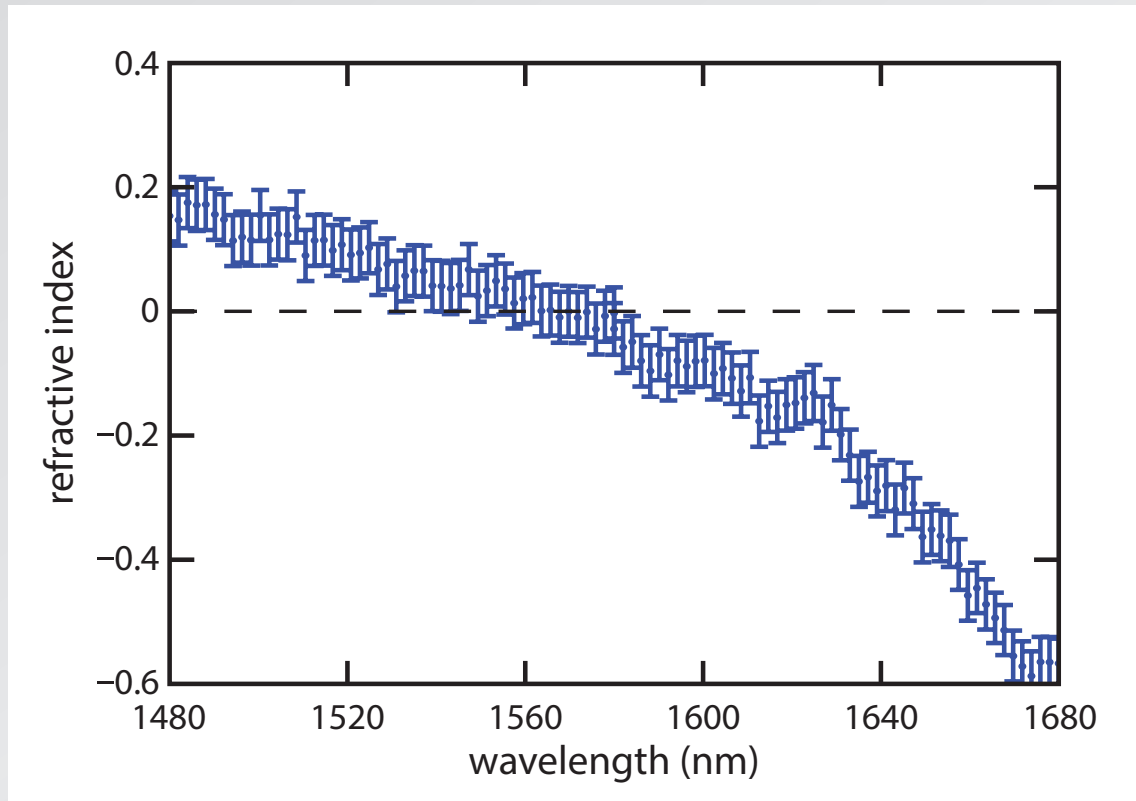
$$n_{\text{prism}} = n_{\text{slab}} \frac{\sin \alpha}{\sin 45^\circ}$$

1 zero index

2 fabrication

3 results

# Wavelength dependence of index

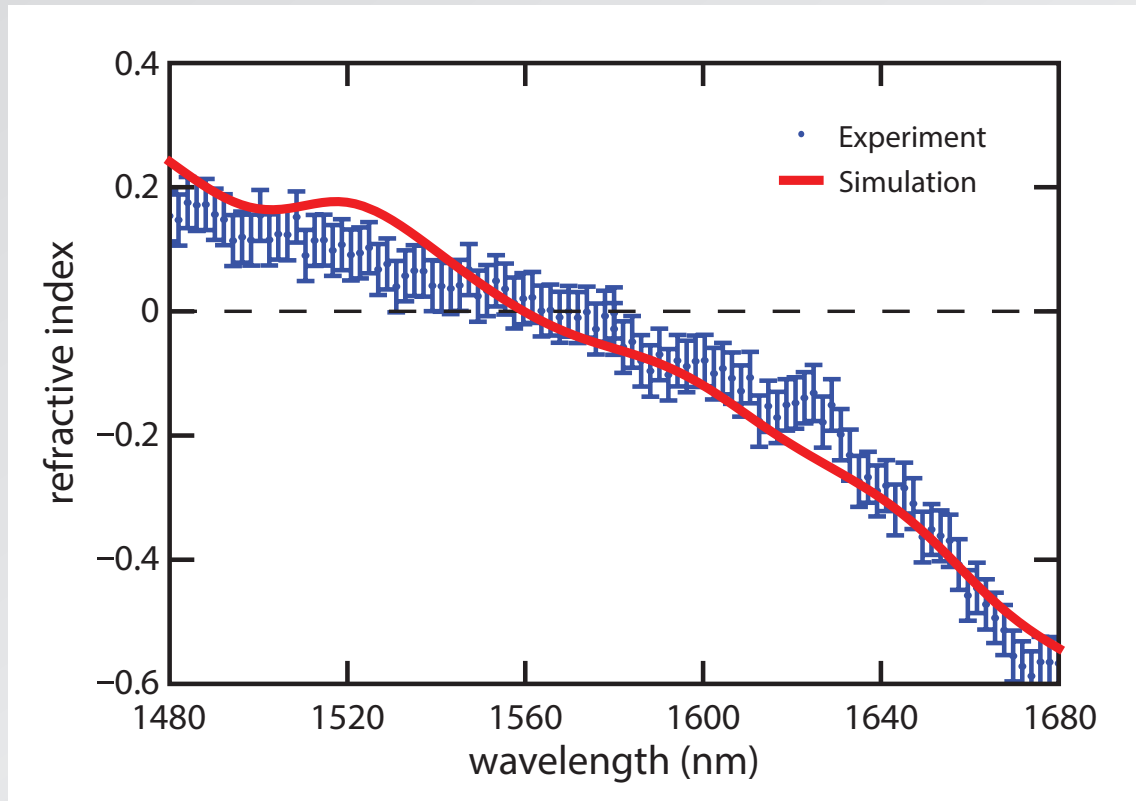


1 zero index

2 fabrication

3 results

# Wavelength dependence of index

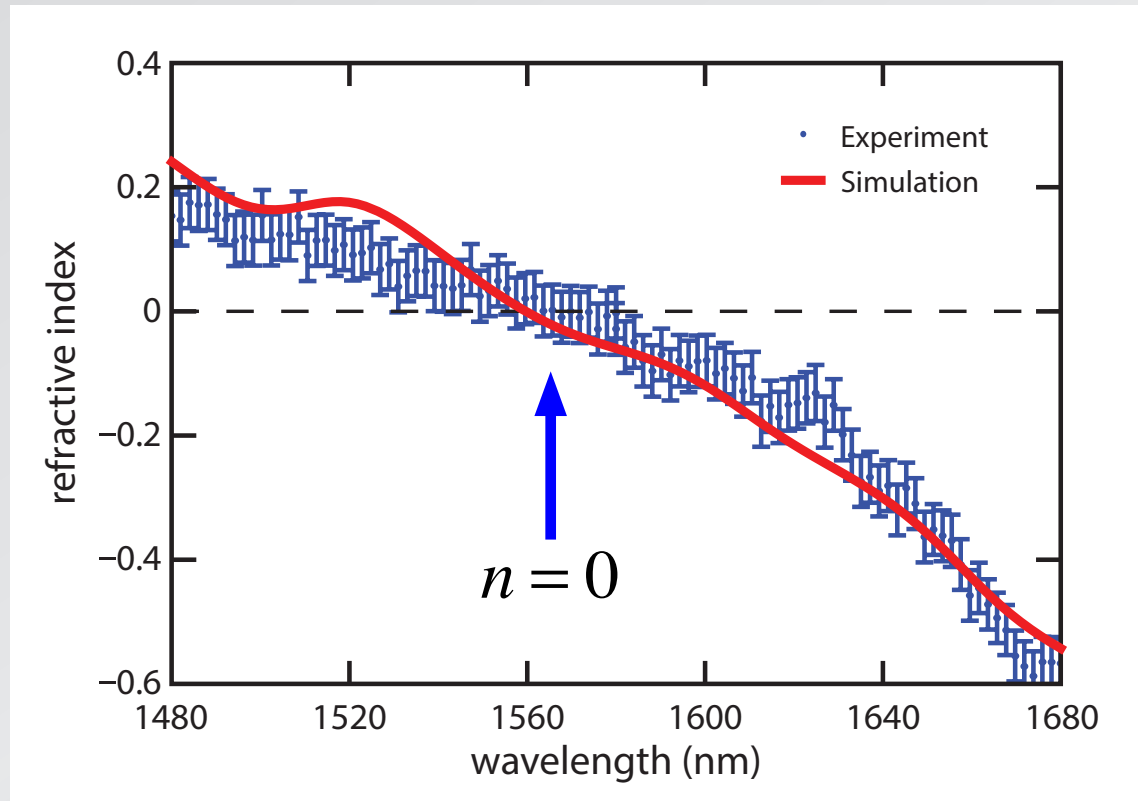


1 zero index

2 fabrication

3 results

# Wavelength dependence of index

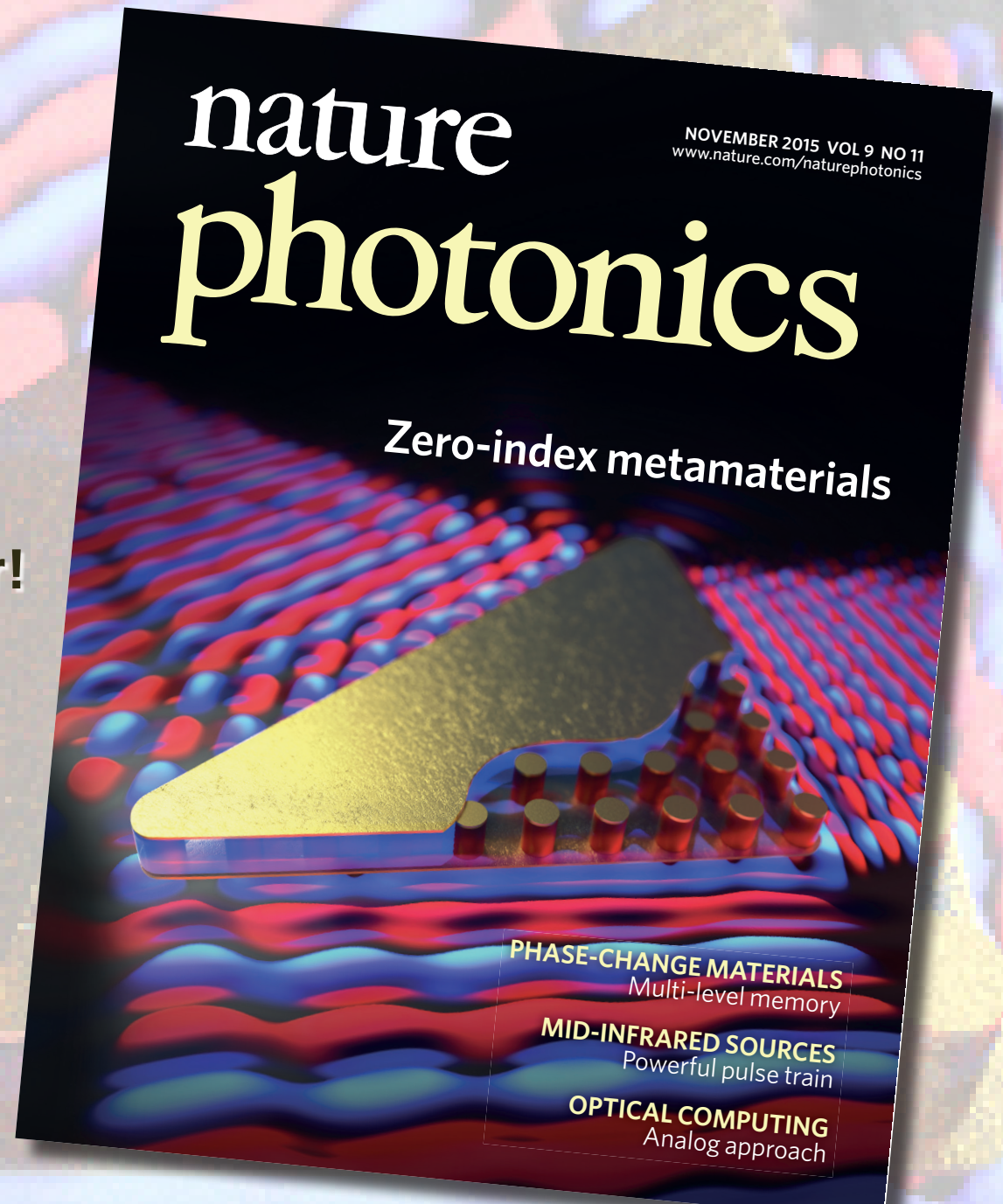


1 zero index

2 fabrication

3 results

More info: download paper!



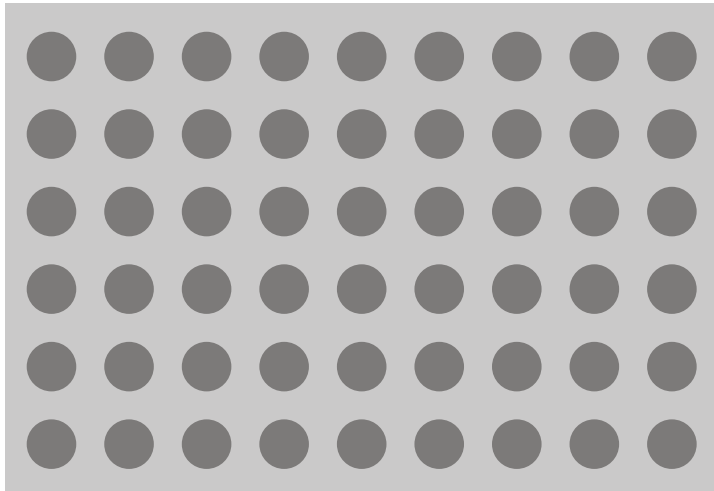
1 zero index

2 fabrication

3 results

# simplify fabrication

**pillar array**



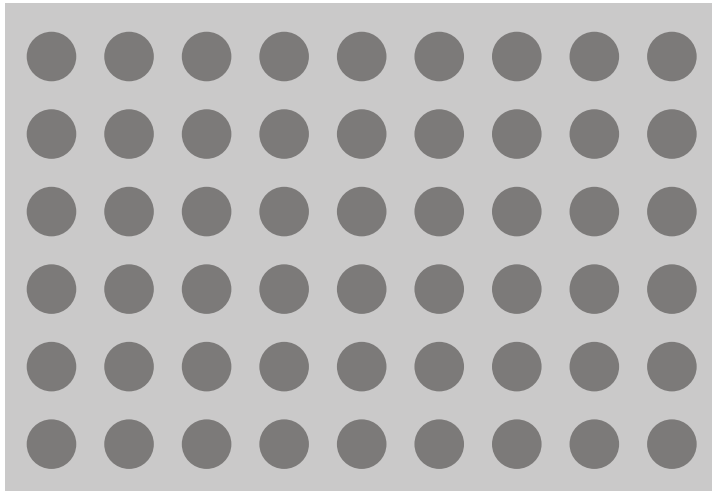
**1** zero index

**2** fabrication

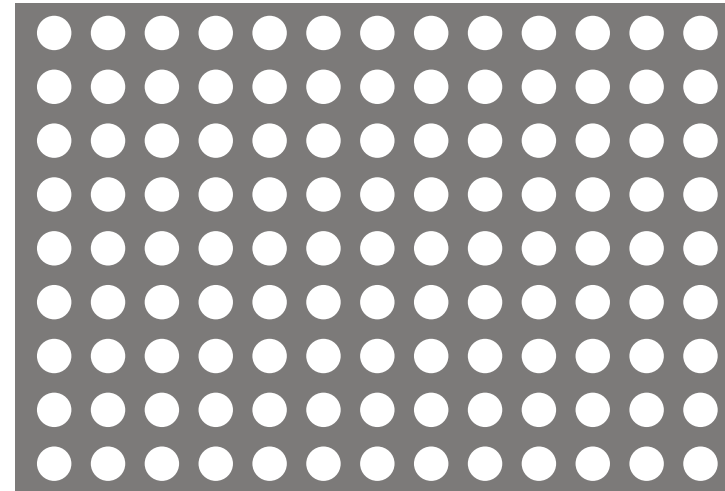
**3** results

## simplify fabrication

**pillar array**



**airhole array**

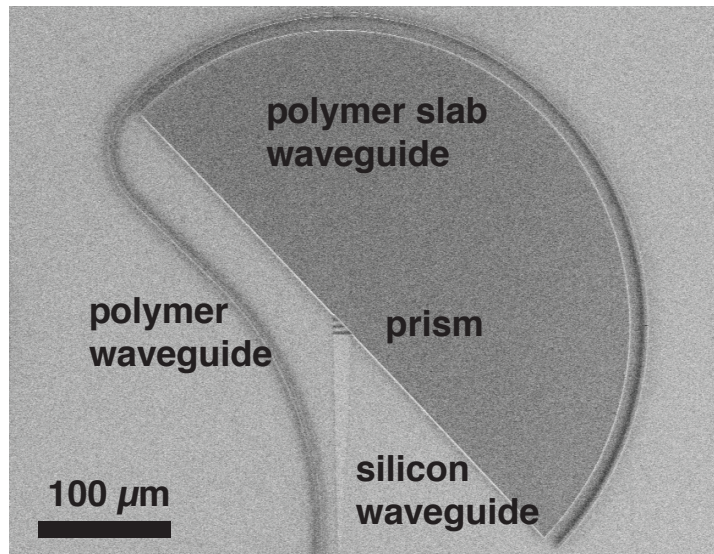


**1** zero index

**2** fabrication

**3** results

## simplify fabrication

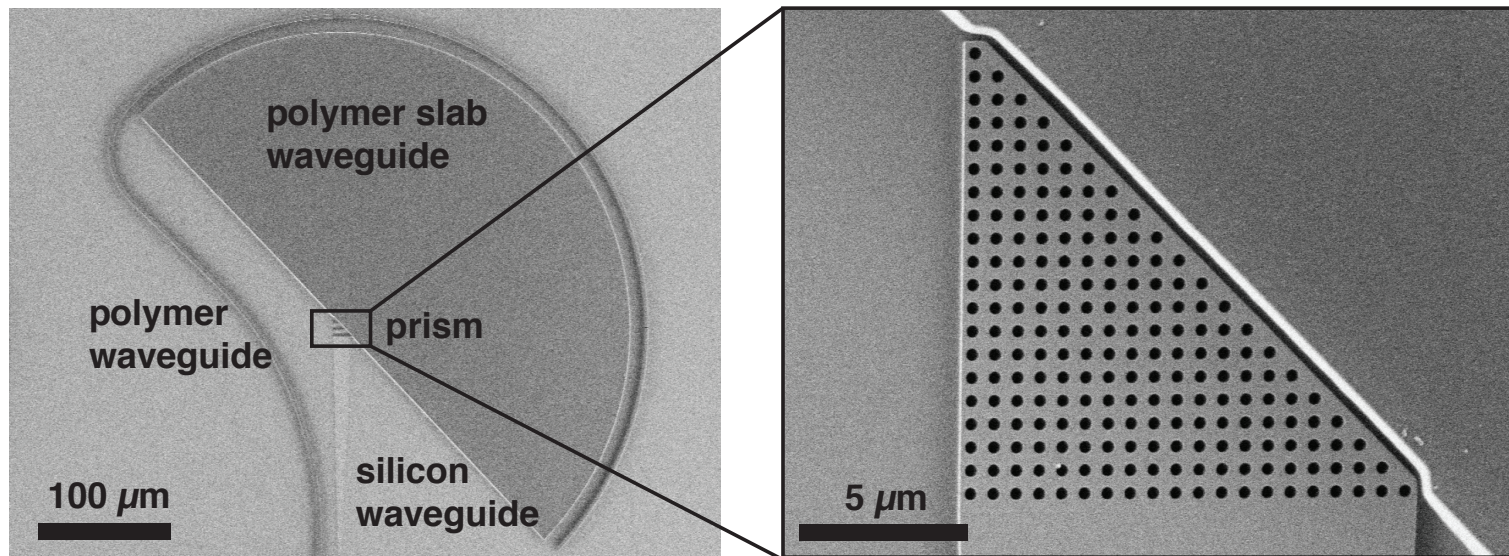


1 zero index

2 fabrication

3 results

## simplify fabrication



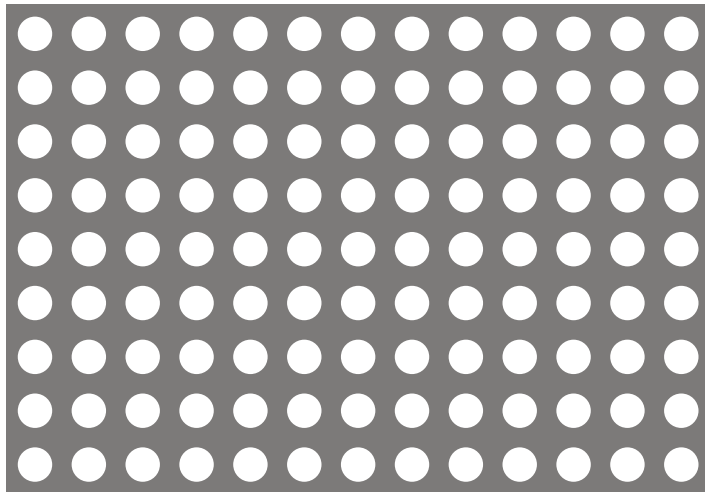
1 zero index

2 fabrication

3 results

**simplify further!**

**airhole array**



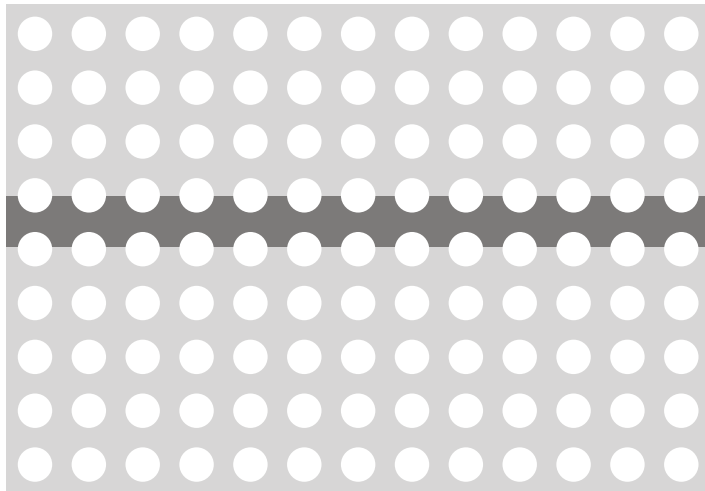
**1 zero index**

**2 fabrication**

**3 results**

**simplify further!**

**airhole array**



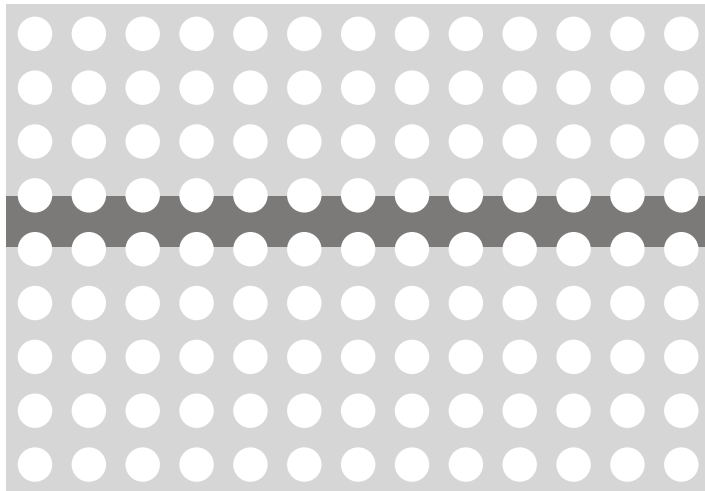
**1** zero index

**2** fabrication

**3** results

simplify further!

airhole array

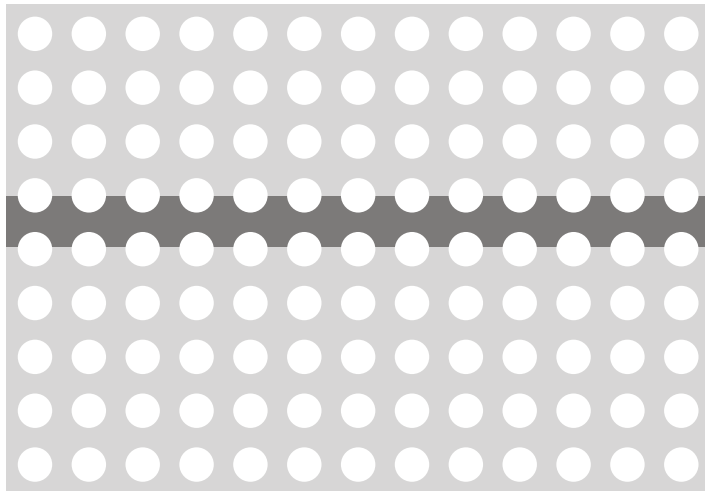


1D ZIM waveguide

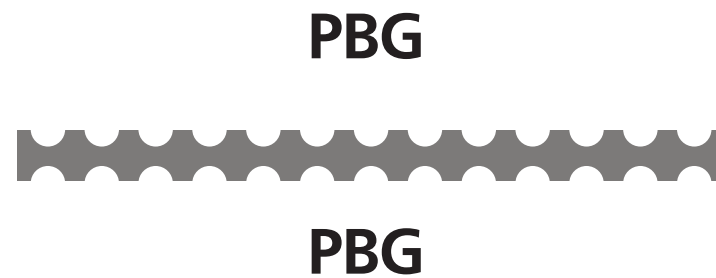


simplify further!

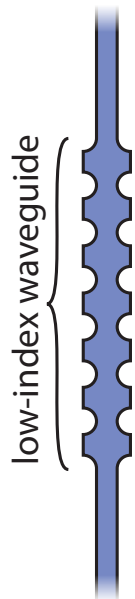
airhole array



1D ZIM waveguide



# waveguiding

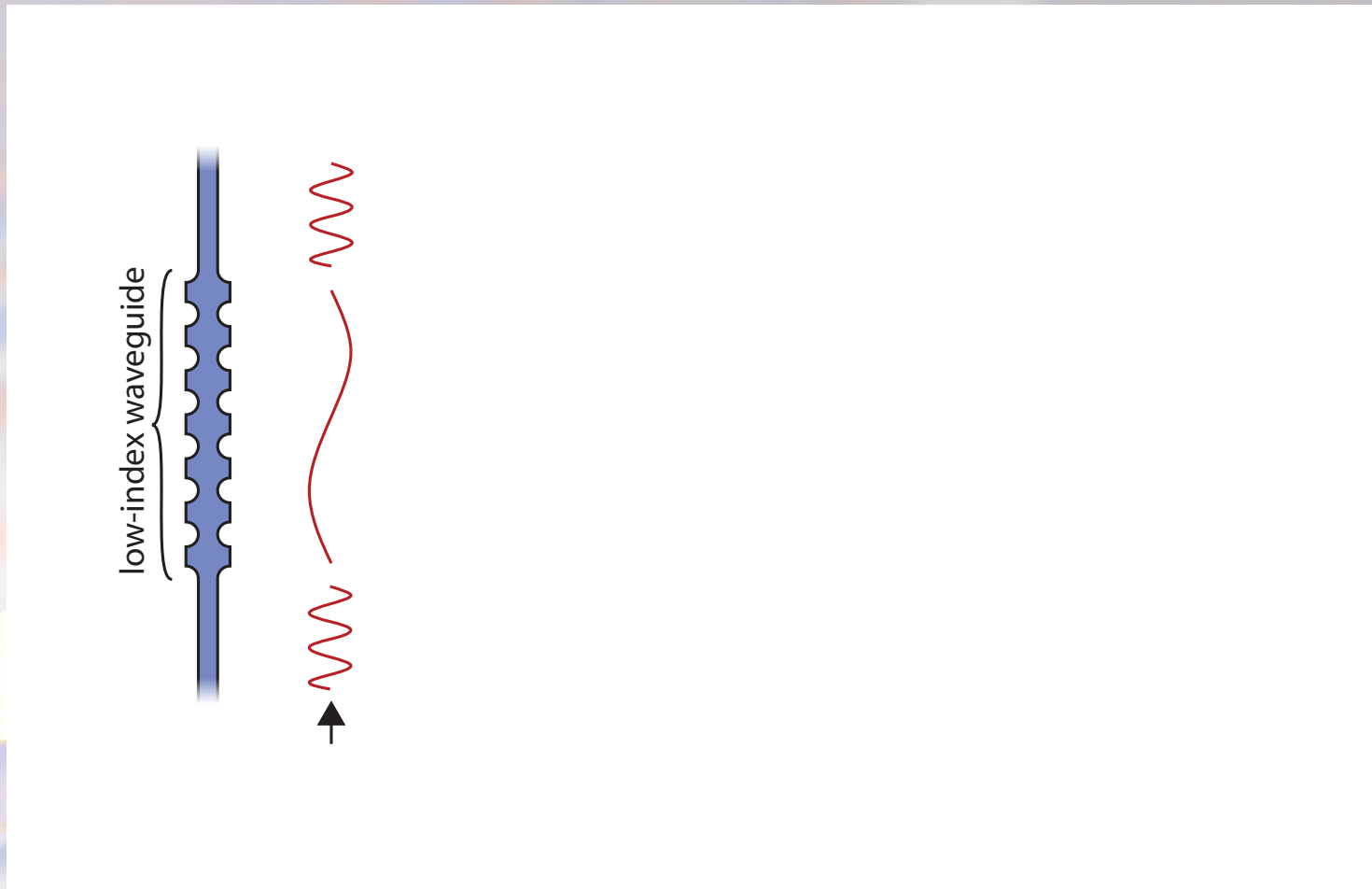


1 zero index

2 fabrication

3 results

# waveguiding

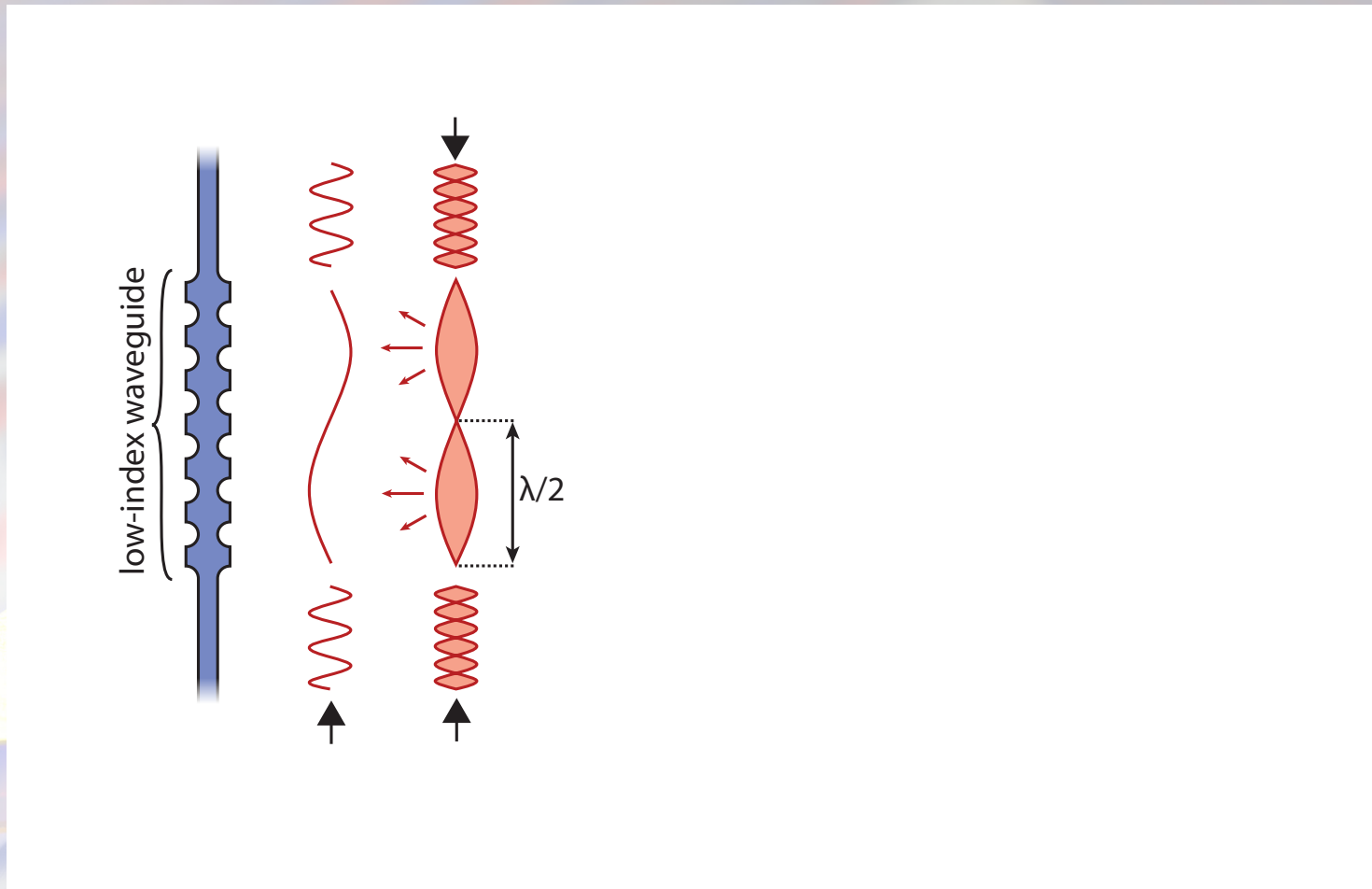


1 zero index

2 fabrication

3 results

# waveguiding

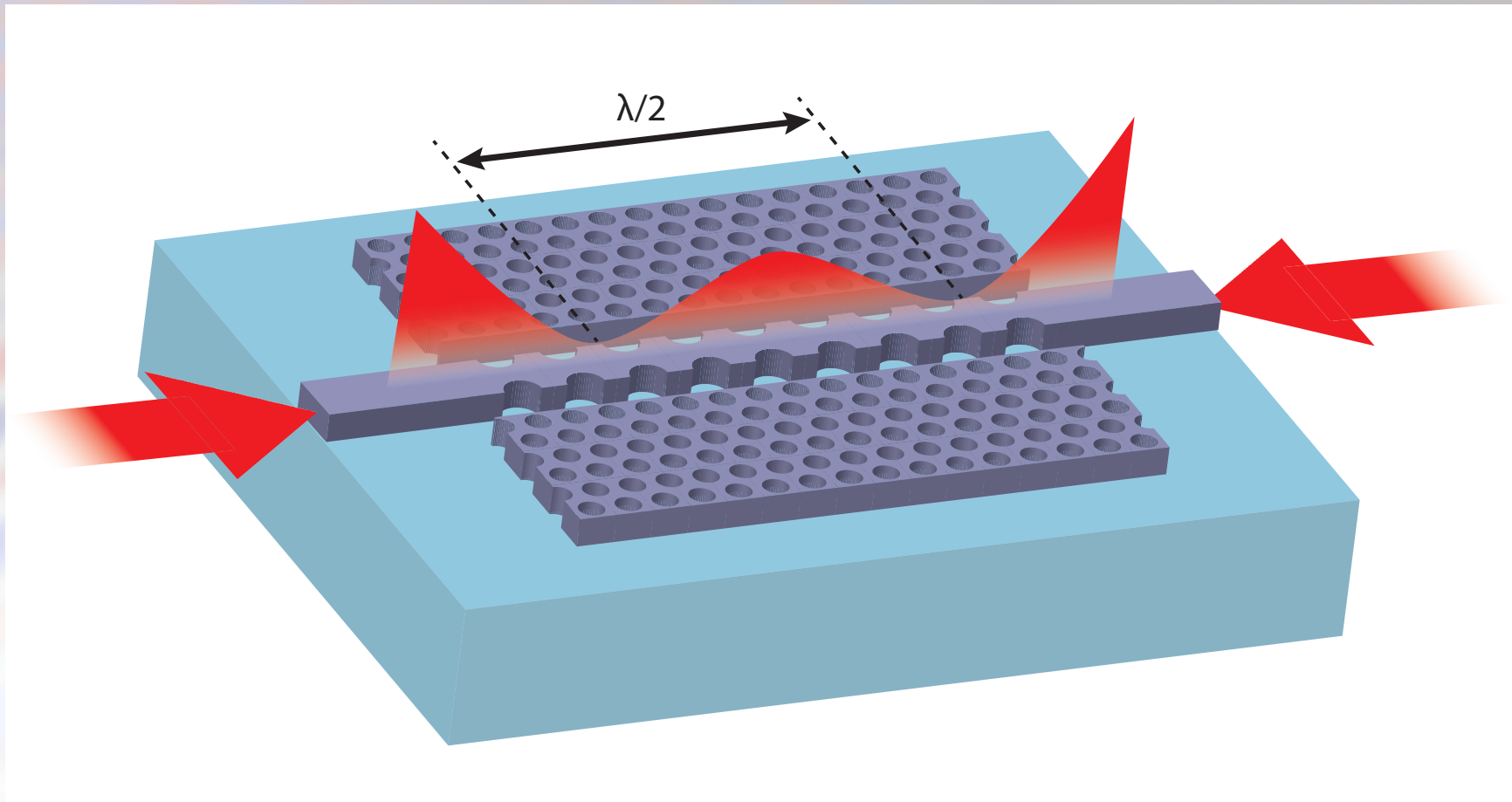


1 zero index

2 fabrication

3 results

look at standing waves

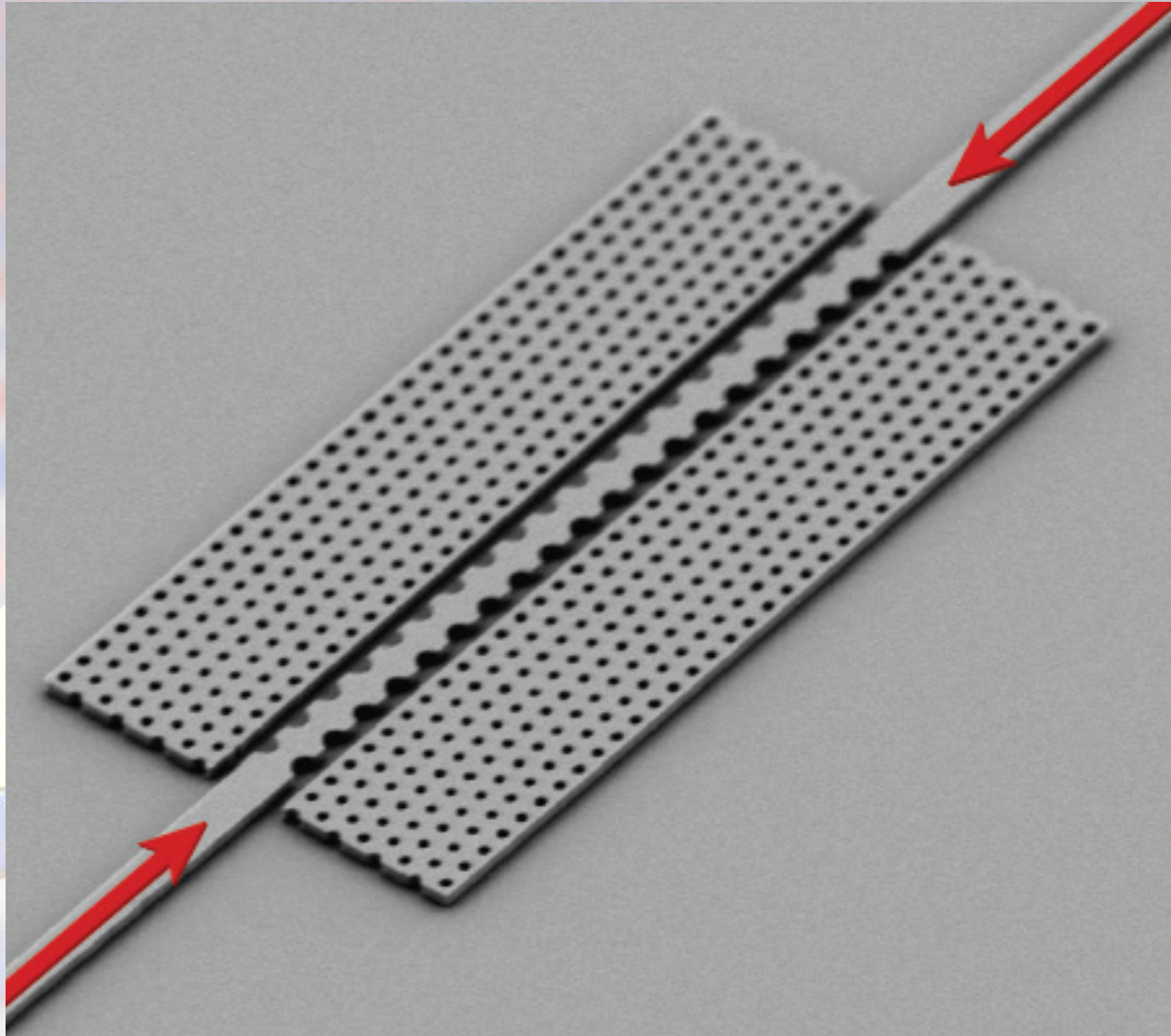


1 zero index

2 fabrication

3 results

look at standing waves

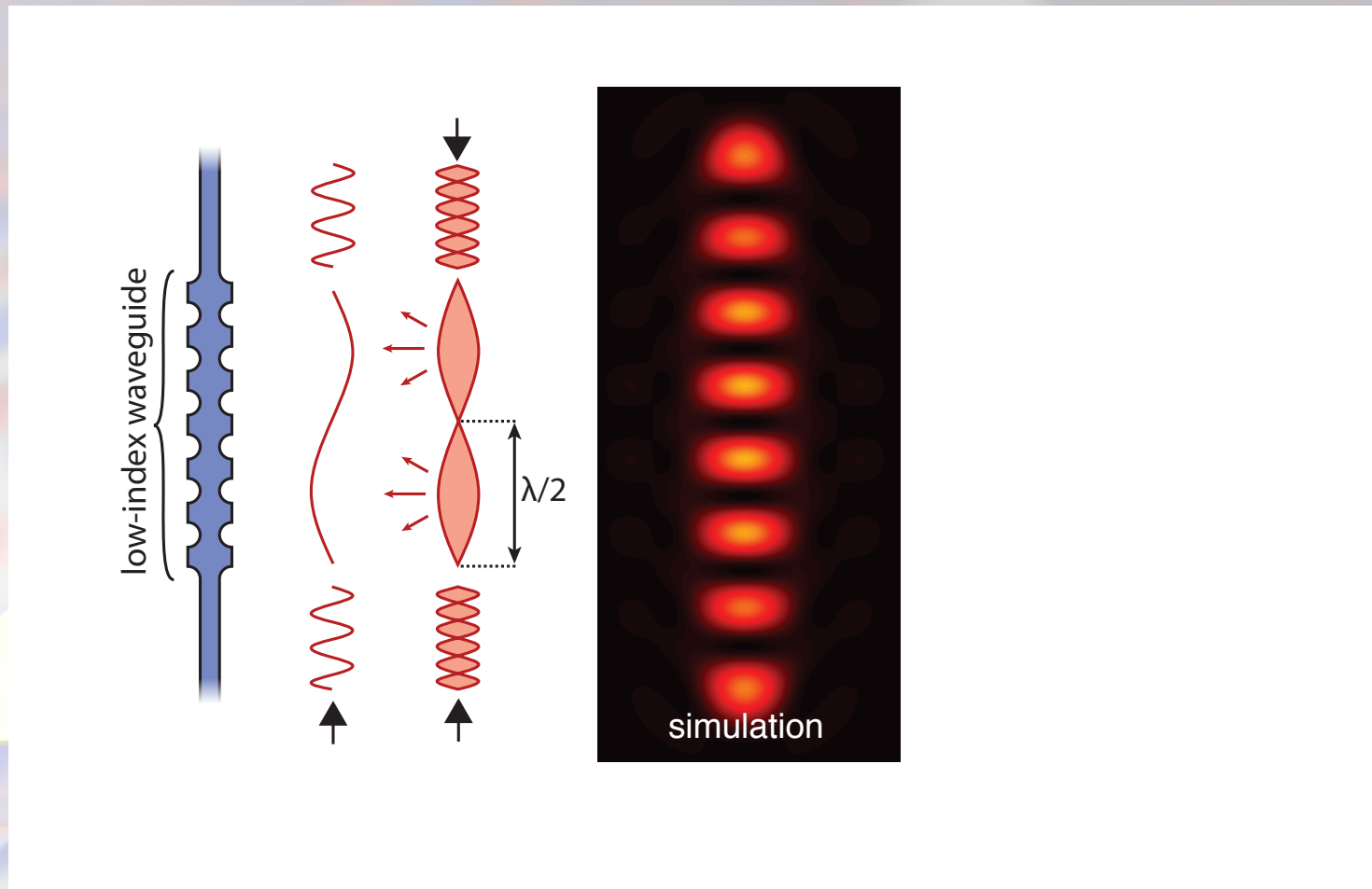


1 zero index

2 fabrication

3 results

# look at standing waves

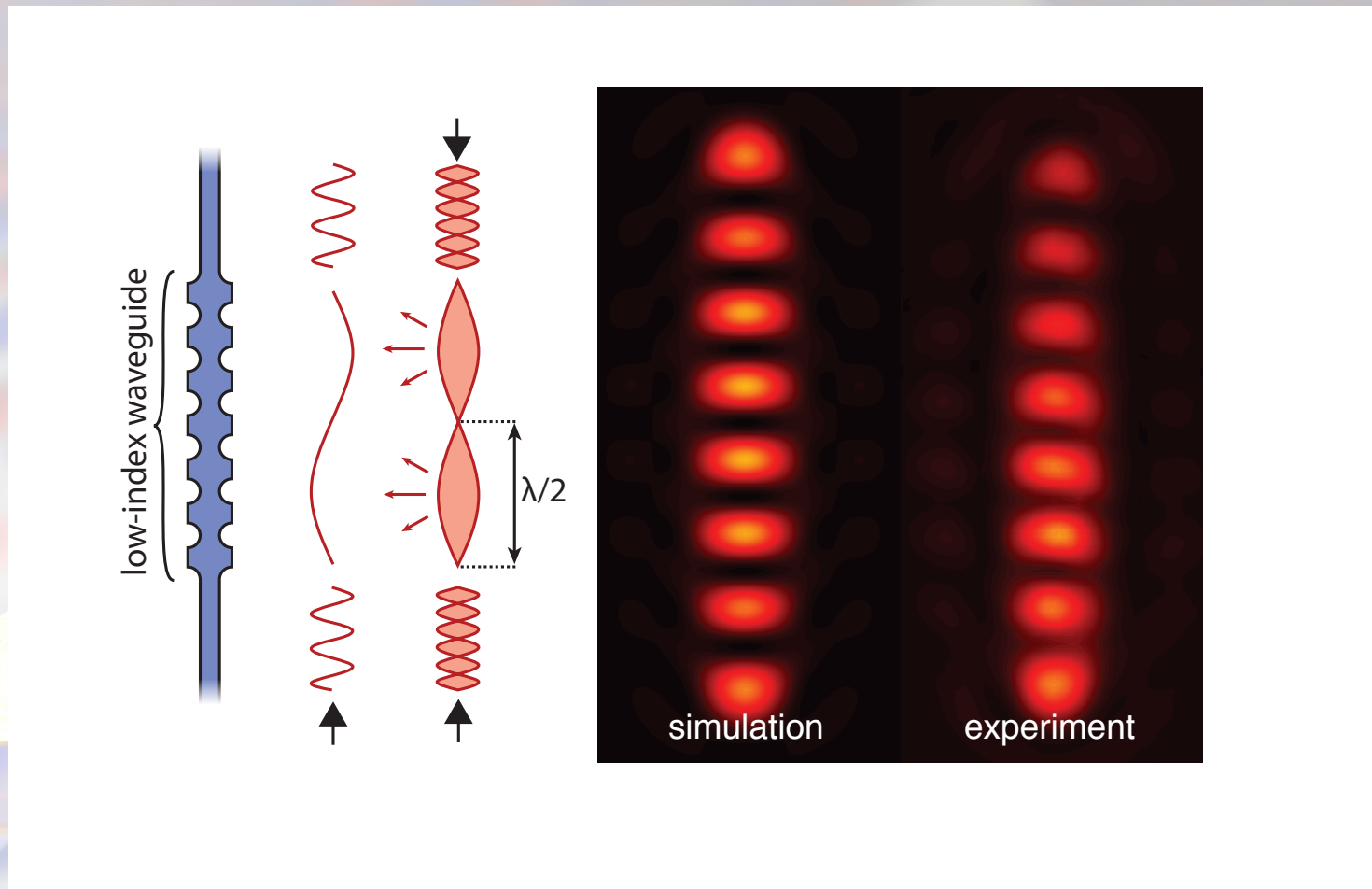


1 zero index

2 fabrication

3 results

## look at standing waves

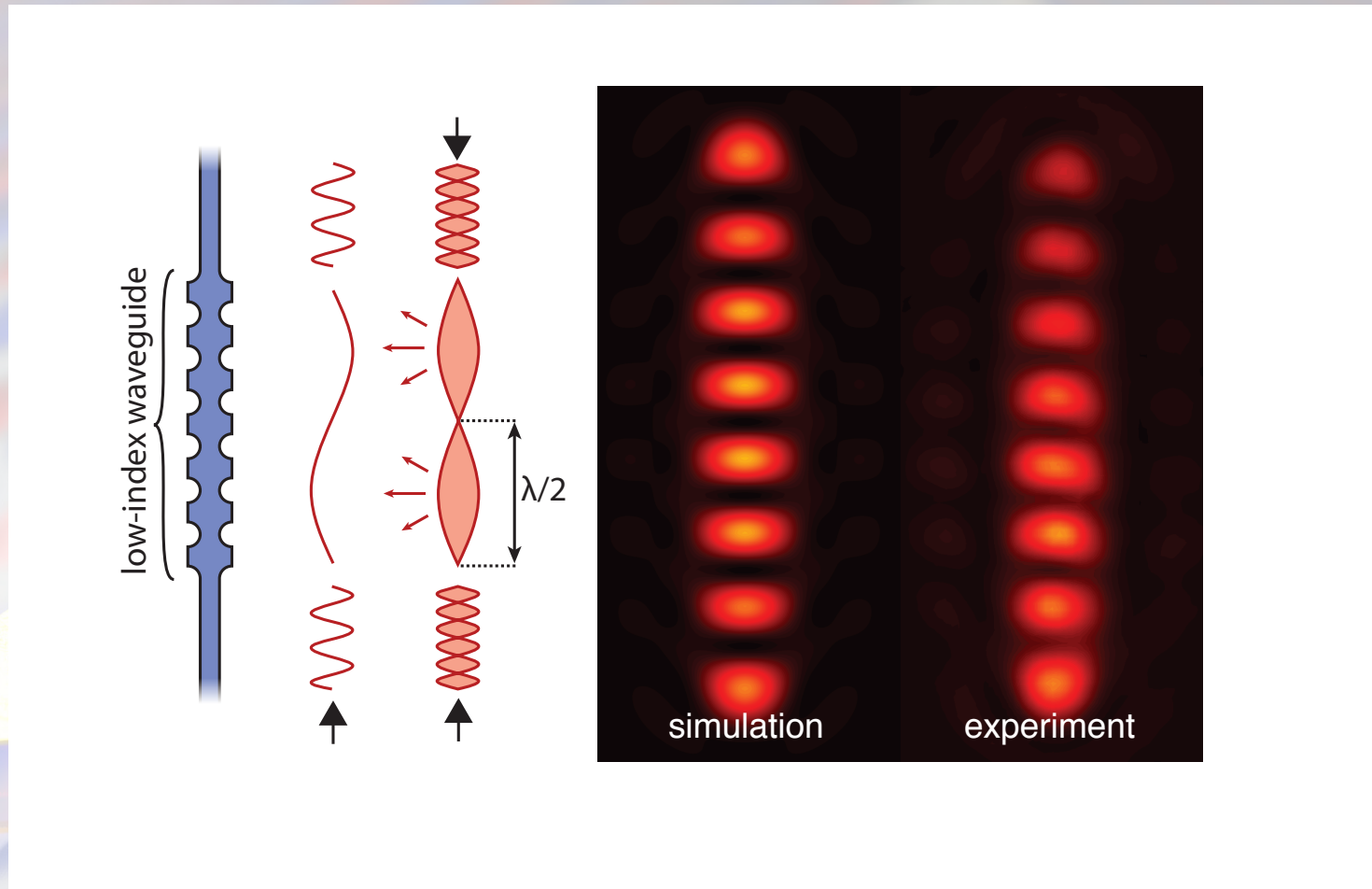


1 zero index

2 fabrication

3 results

# *direct observation of effective wavelength!!*

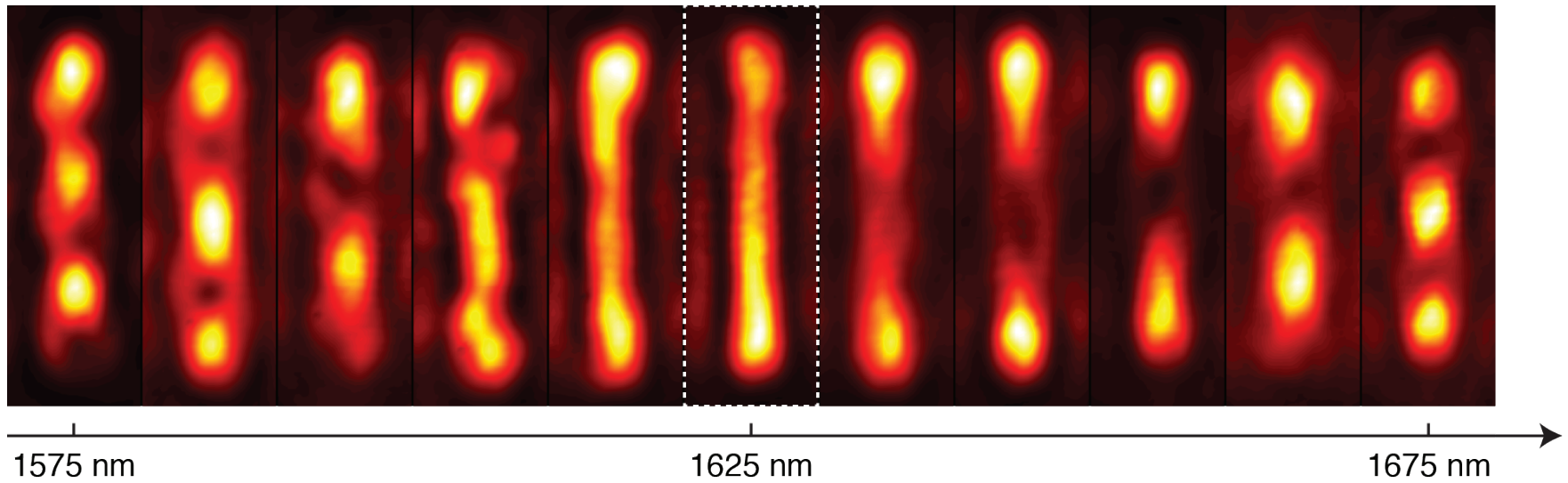


1 zero index

2 fabrication

3 results

look at standing waves

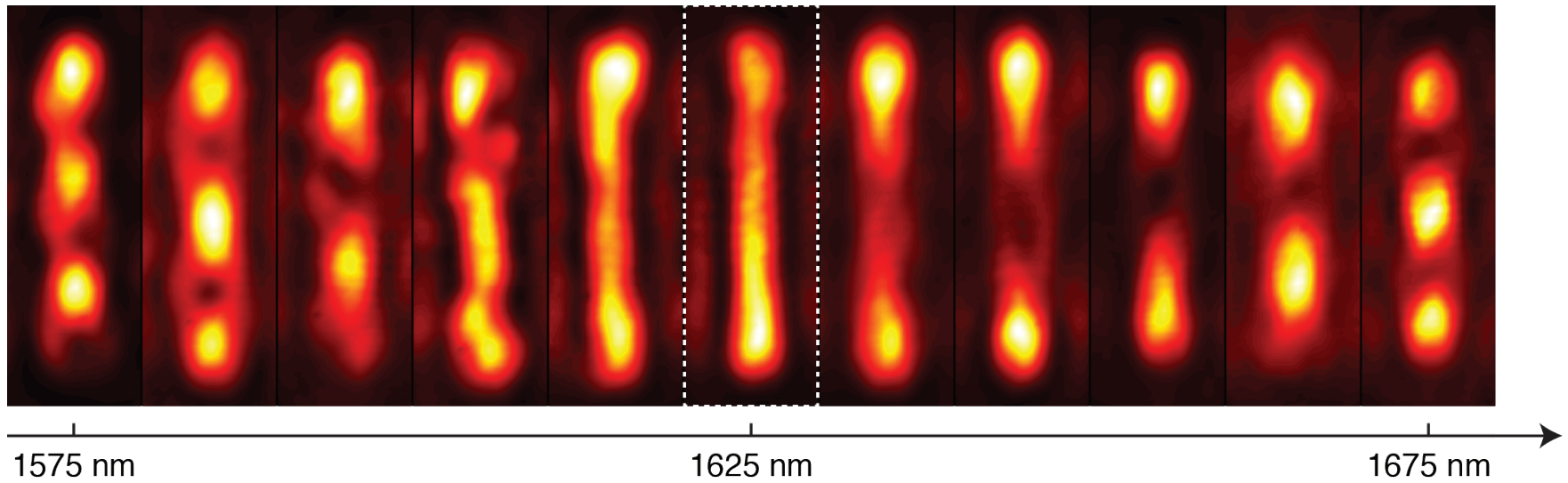


1 zero index

2 fabrication

3 results

look at standing waves



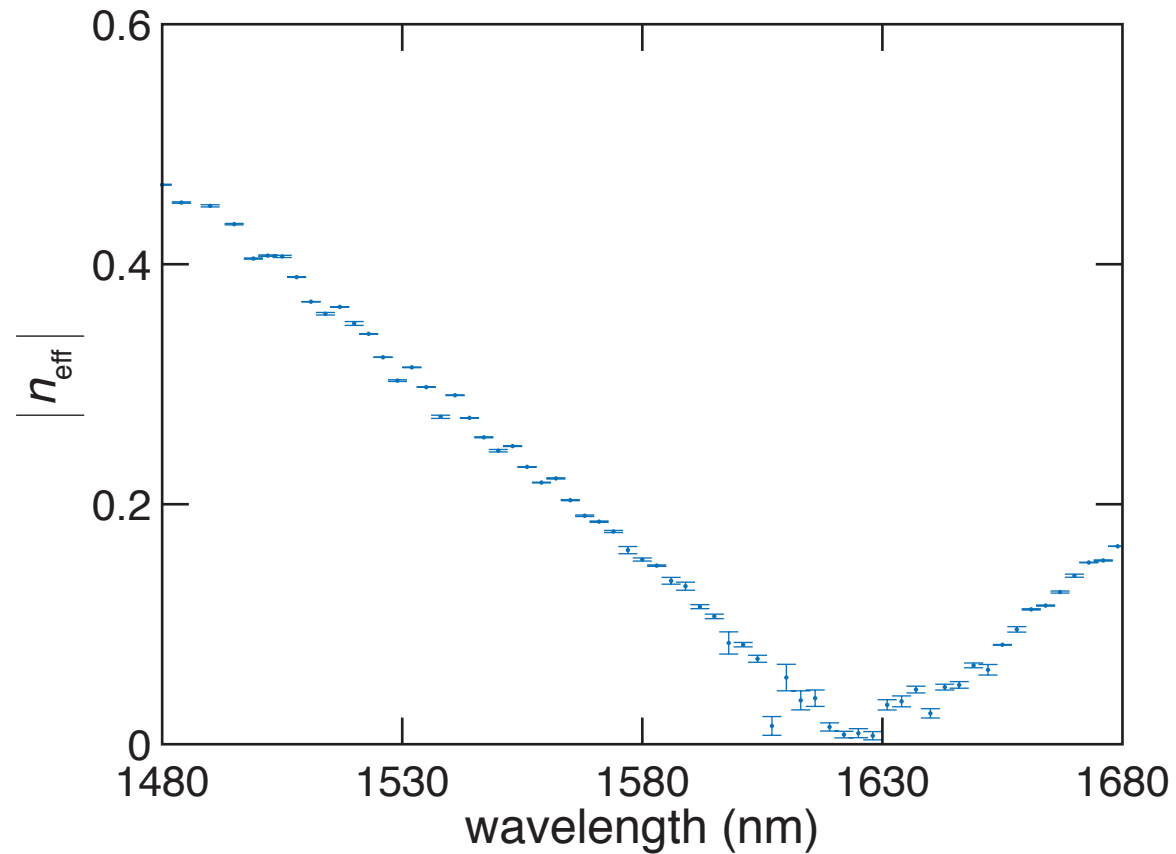
$$|n_{\text{eff}}| = \frac{\lambda_0}{\lambda_{\text{eff}}}$$

1 zero index

2 fabrication

3 results

## comparison of experiment and simulation

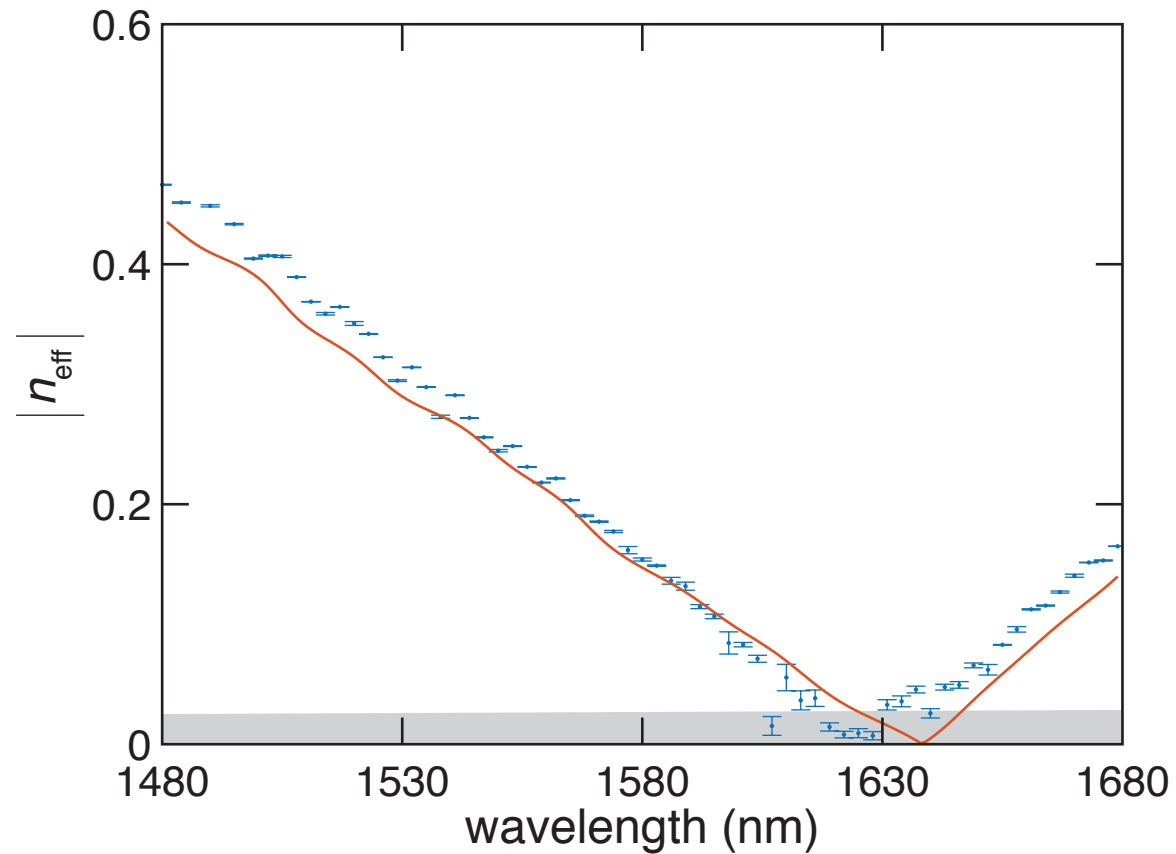


1 zero index

2 fabrication

3 results

## comparison of experiment and simulation

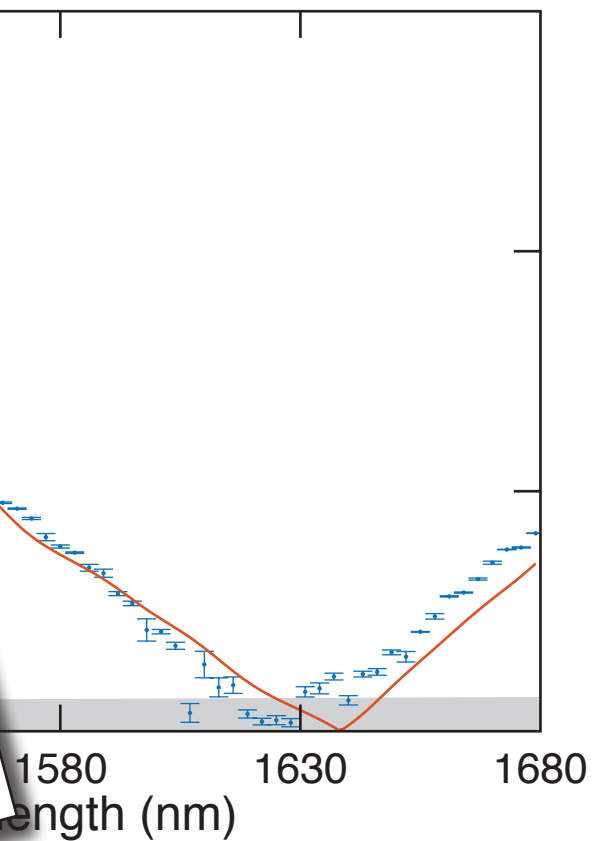
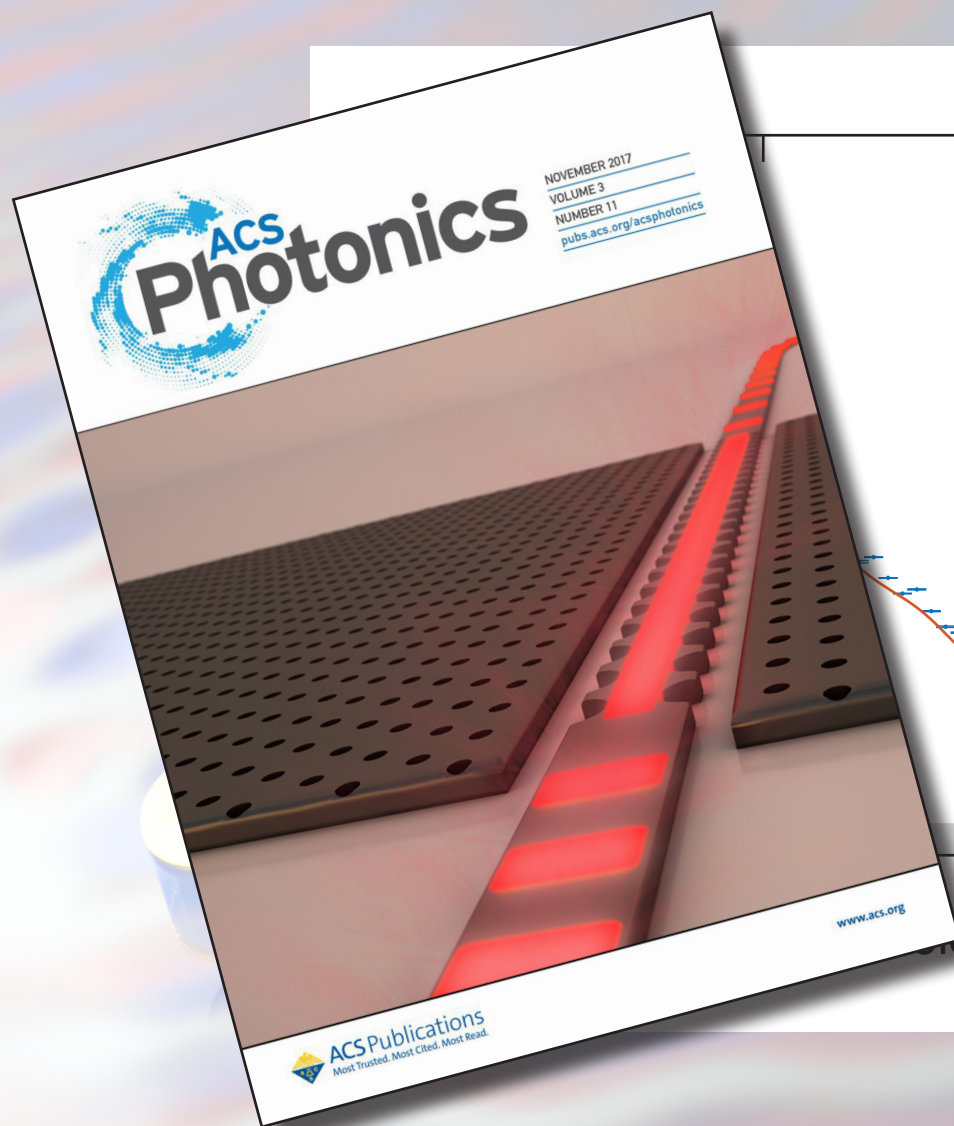


1 zero index

2 fabrication

3 results

# comparison of experiment and simulation



1 zero index

2 fabrication

3 results

Exciting applications ahead

$$n = 0$$

1 zero index

2 fabrication

3 results

## Exciting applications ahead

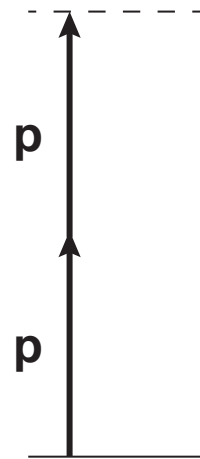
$$n \approx n_0 + \frac{\chi_R^{(3)}}{2n_0} |E(\omega)|^2$$

1 zero index

2 fabrication

3 results

# four-wave mixing

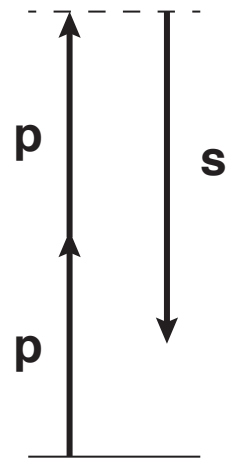


1 zero index

2 fabrication

3 results

# four-wave mixing

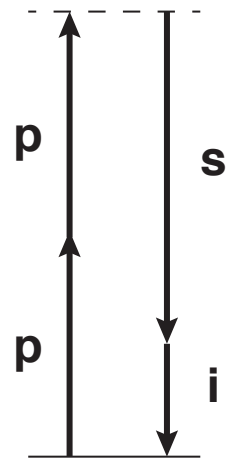


1 zero index

2 fabrication

3 results

# four-wave mixing



1 zero index

2 fabrication

3 results

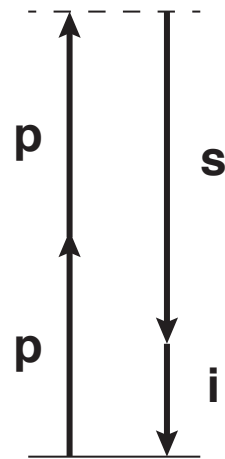
# four-wave mixing

energy  
conservation

$$\Delta\omega = 2\omega_p - \omega_s - \omega_i = 0$$

momentum  
conservation

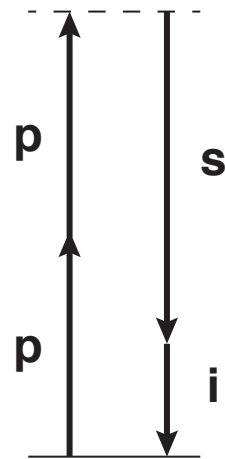
$$\Delta k = 2\vec{k}_p - \vec{k}_s - \vec{k}_i = 0$$



# four-wave mixing

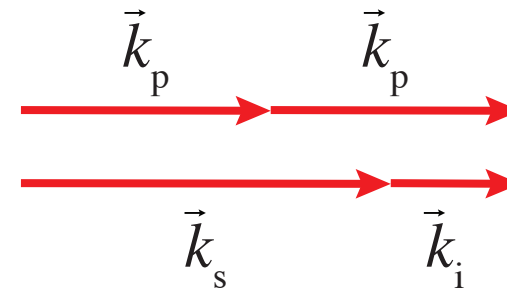
energy  
conservation

$$\Delta\omega = 2\omega_p - \omega_s - \omega_i = 0$$



momentum  
conservation

$$\Delta k = 2\vec{k}_p - \vec{k}_s - \vec{k}_i = 0$$



phase matching

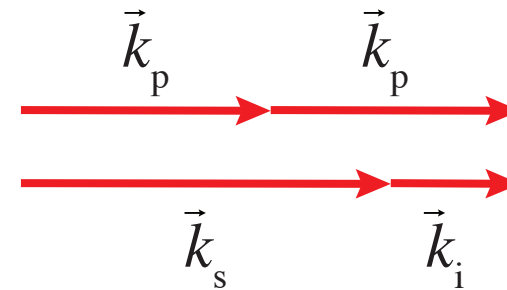
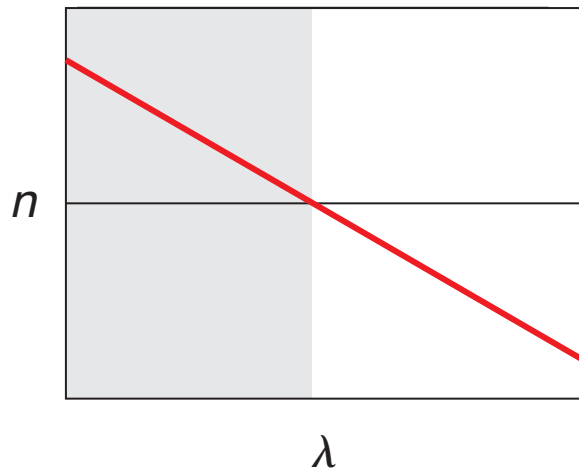
# four-wave mixing

energy  
conservation

$$\Delta\omega = 2\omega_p - \omega_s - \omega_i = 0$$

momentum  
conservation

$$\Delta k = 2\vec{k}_p - \vec{k}_s - \vec{k}_i = 0$$



phase matching

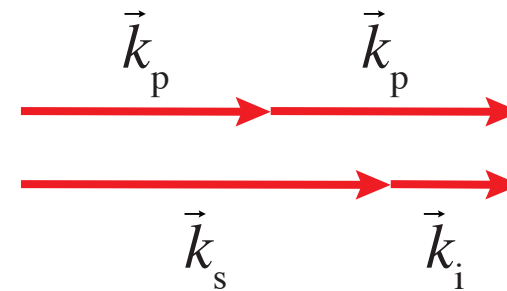
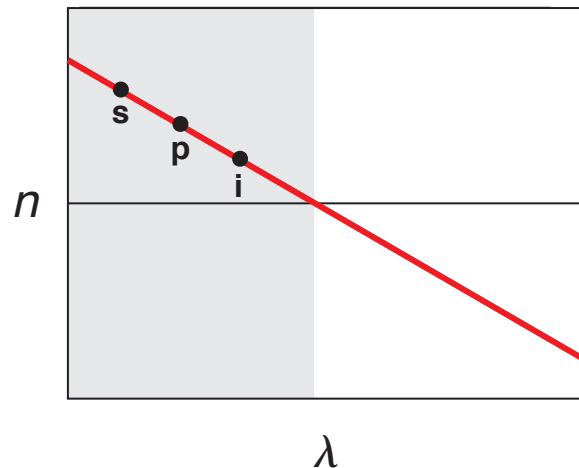
# four-wave mixing

energy  
conservation

$$\Delta\omega = 2\omega_p - \omega_s - \omega_i = 0$$

momentum  
conservation

$$\Delta k = 2\vec{k}_p - \vec{k}_s - \vec{k}_i = 0$$



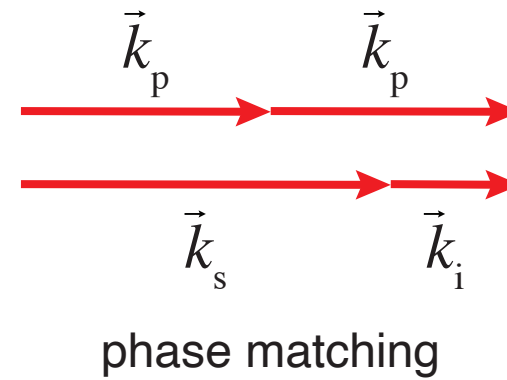
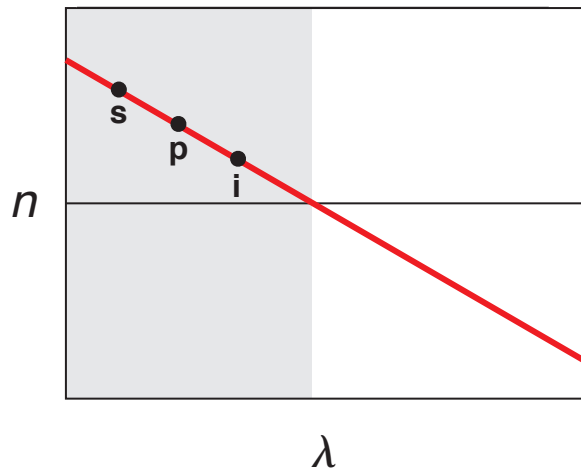
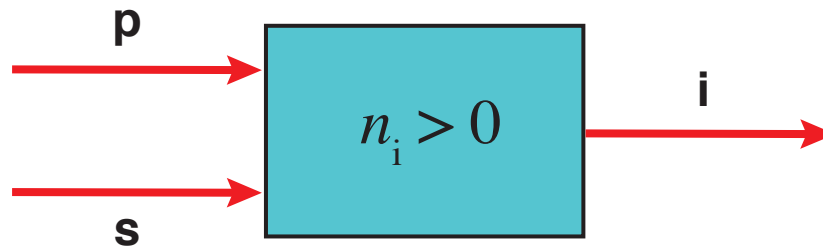
phase matching

1 zero index

2 fabrication

3 results

# four-wave mixing

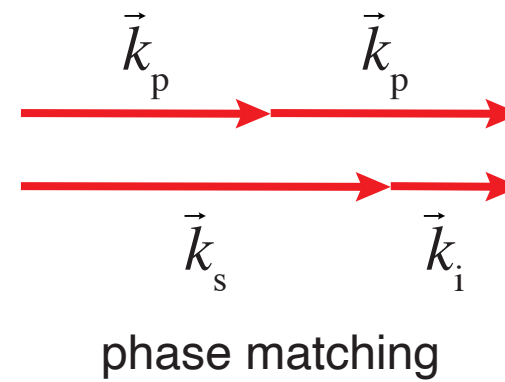
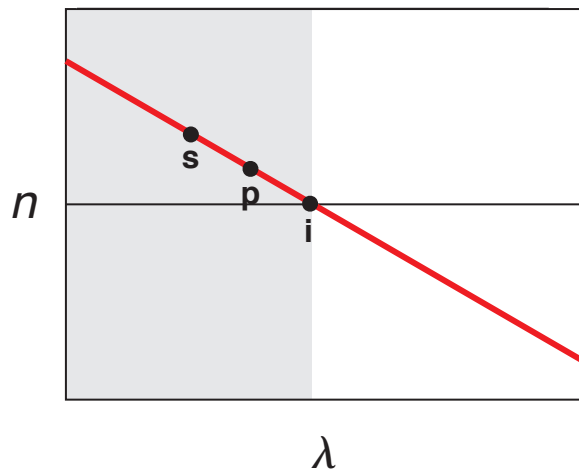
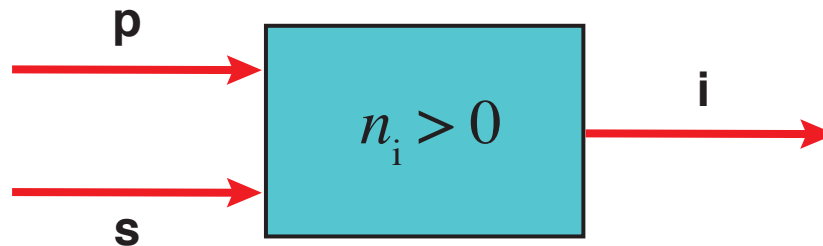


1 zero index

2 fabrication

3 results

# four-wave mixing

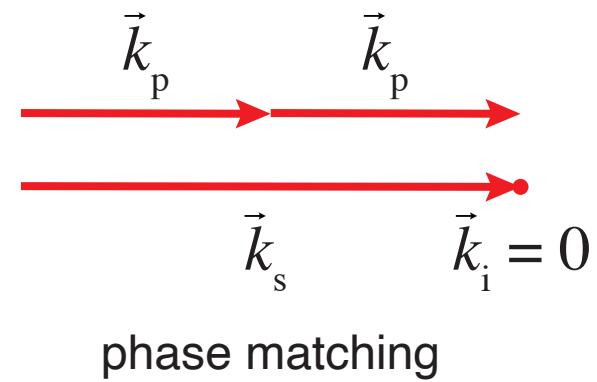
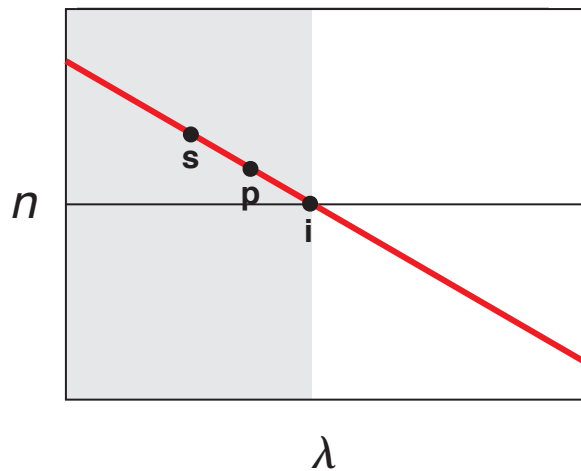
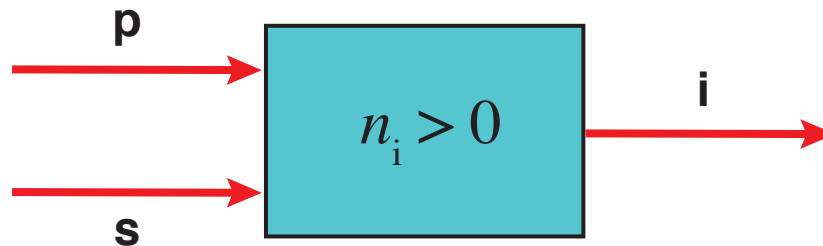


1 zero index

2 fabrication

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# four-wave mixing

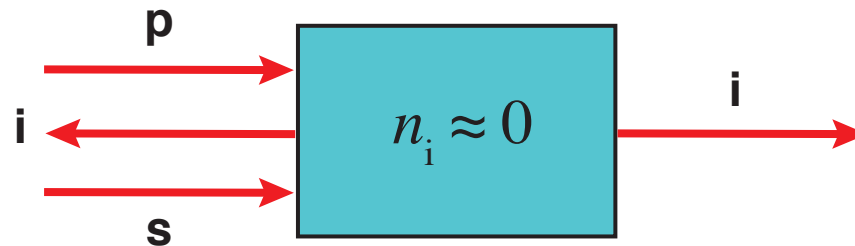


1 zero index

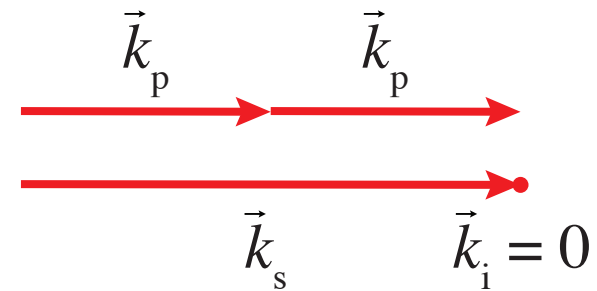
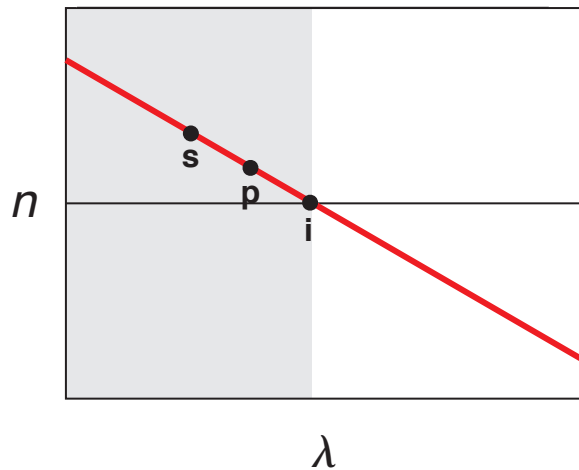
2 fabrication

3 results

# four-wave mixing



forward and backward



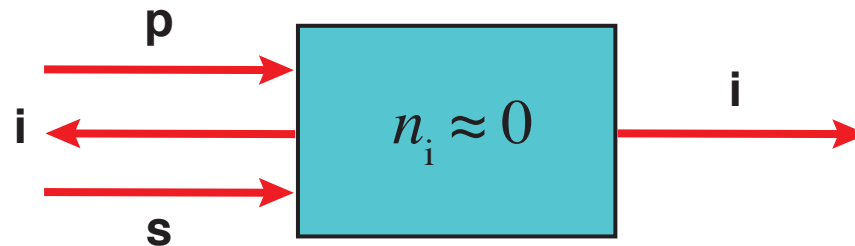
phase matching

1 zero index

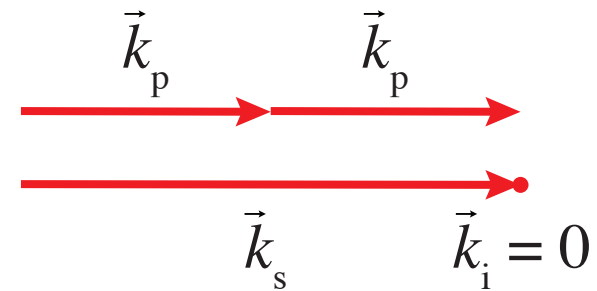
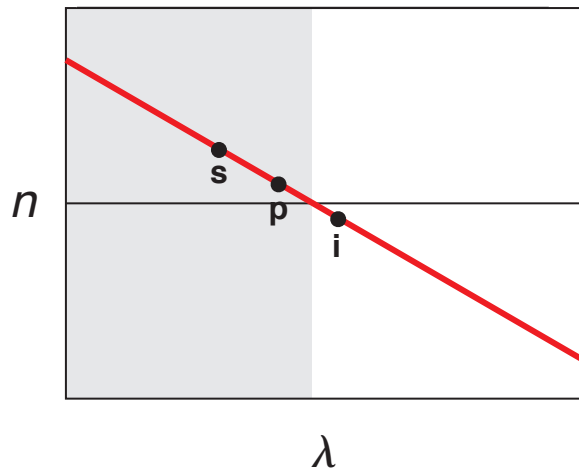
2 fabrication

3 results

# four-wave mixing



forward and backward



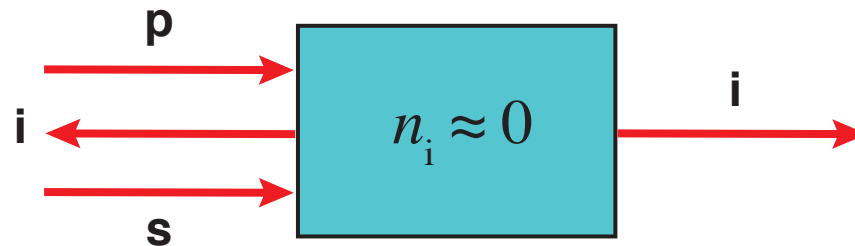
phase matching

1 zero index

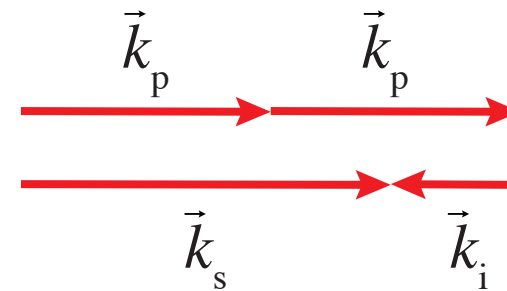
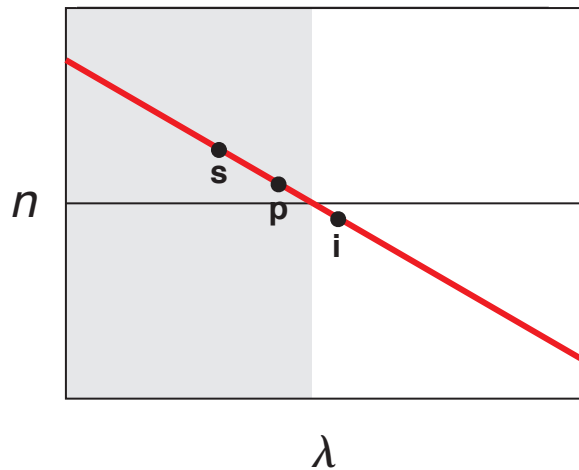
2 fabrication

3 results

# four-wave mixing



forward and backward



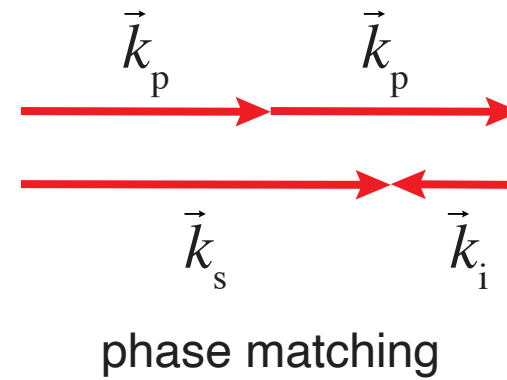
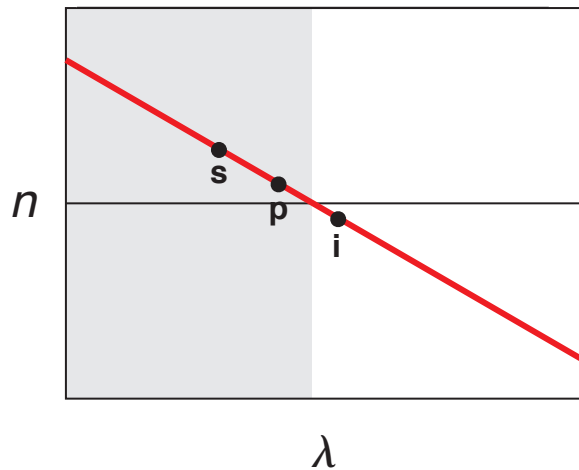
phase matching

1 zero index

2 fabrication

3 results

# four-wave mixing

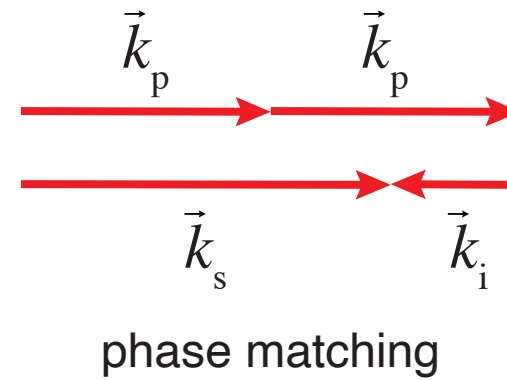
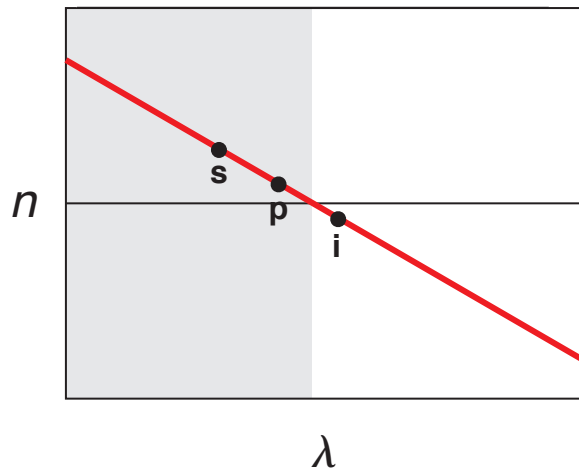
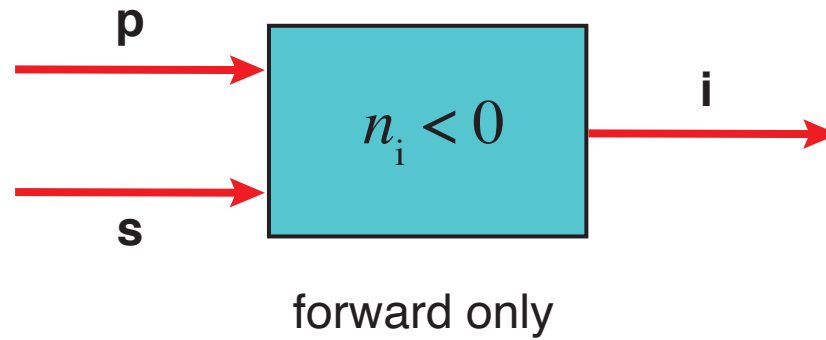


1 zero index

2 fabrication

3 results

# four-wave mixing

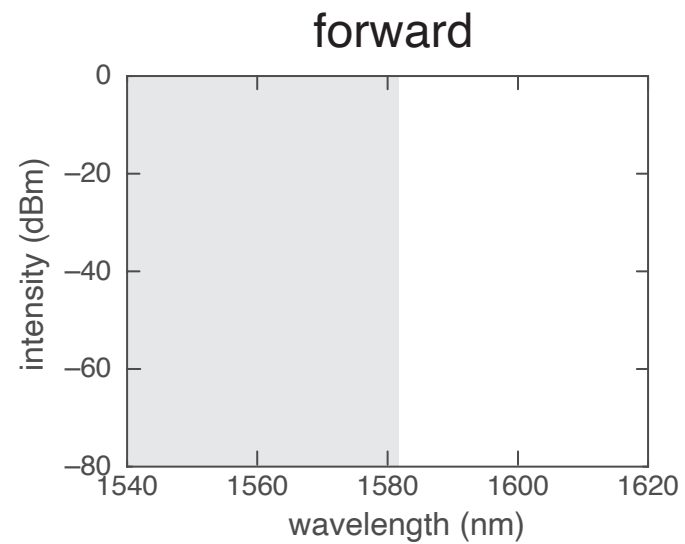
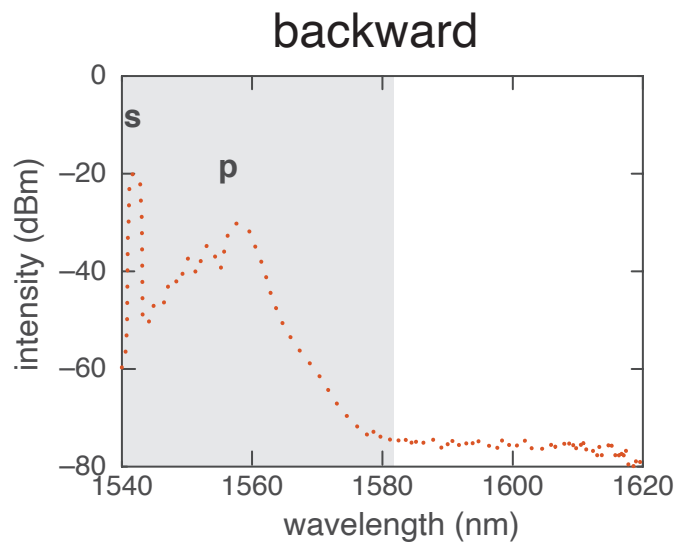
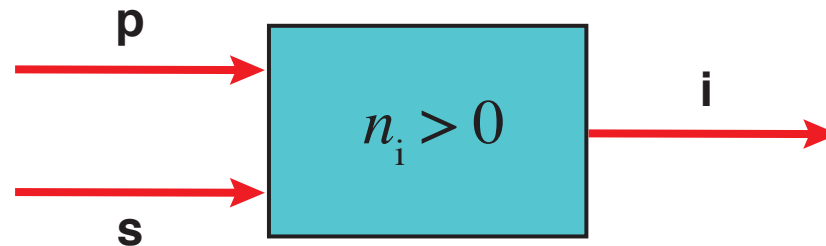


1 zero index

2 fabrication

3 results

# four-wave mixing

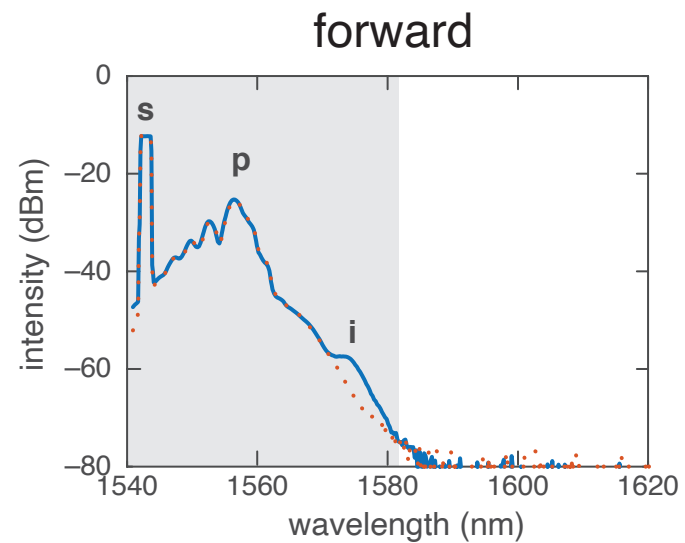
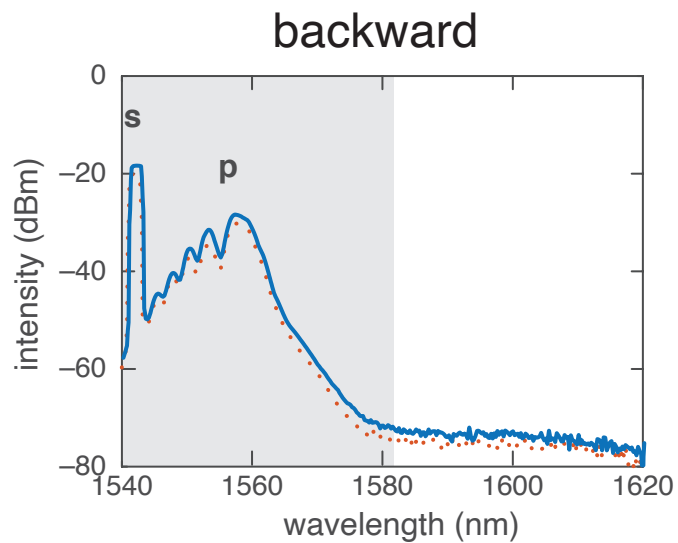
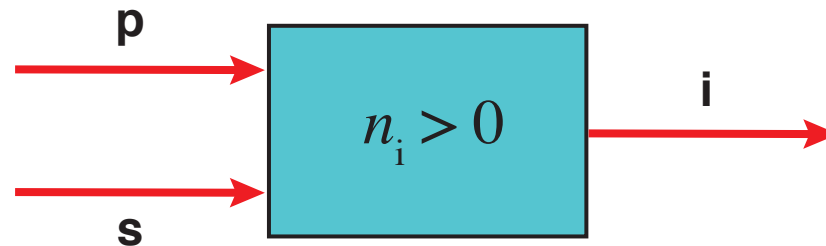


1 zero index

2 fabrication

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# four-wave mixing

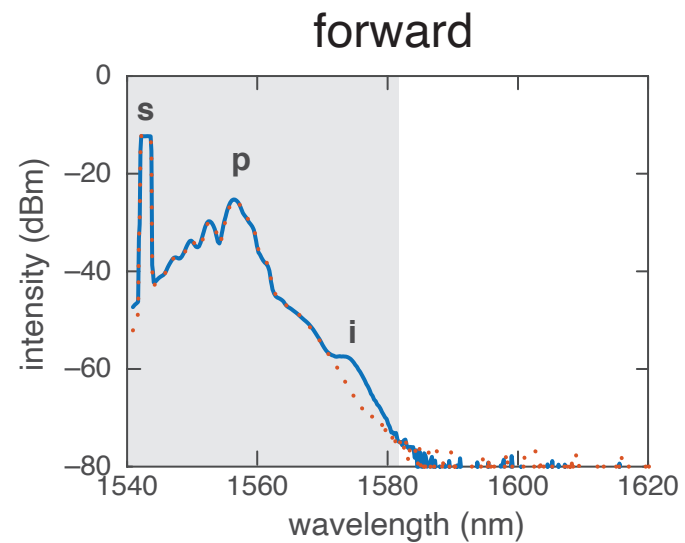
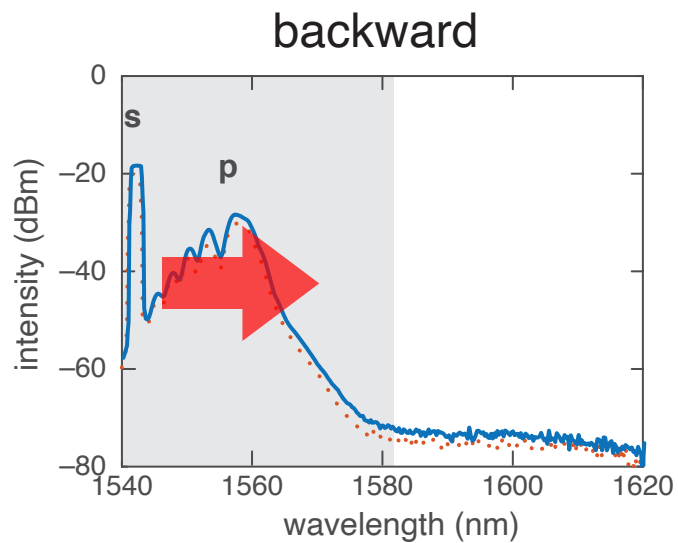
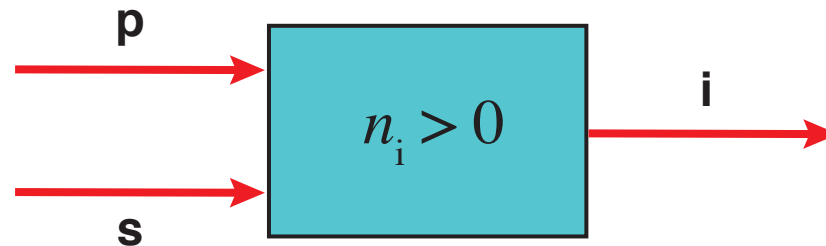


1 zero index

2 fabrication

3 results

# four-wave mixing

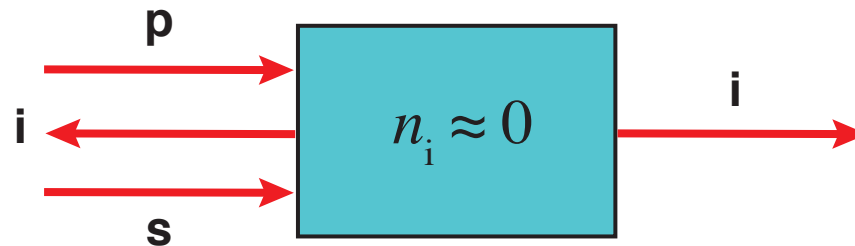


1 zero index

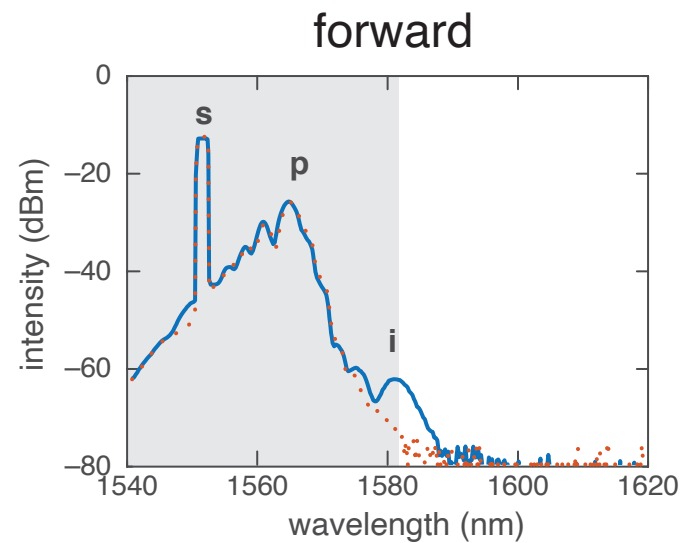
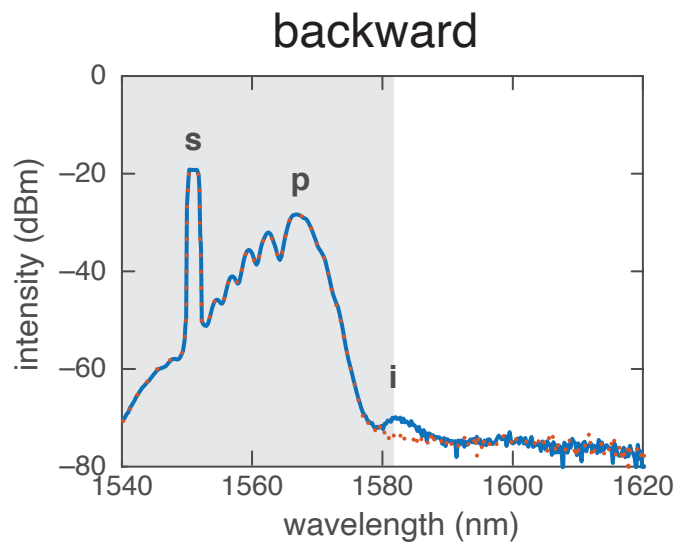
2 fabrication

3 results

# four-wave mixing



forward and backward

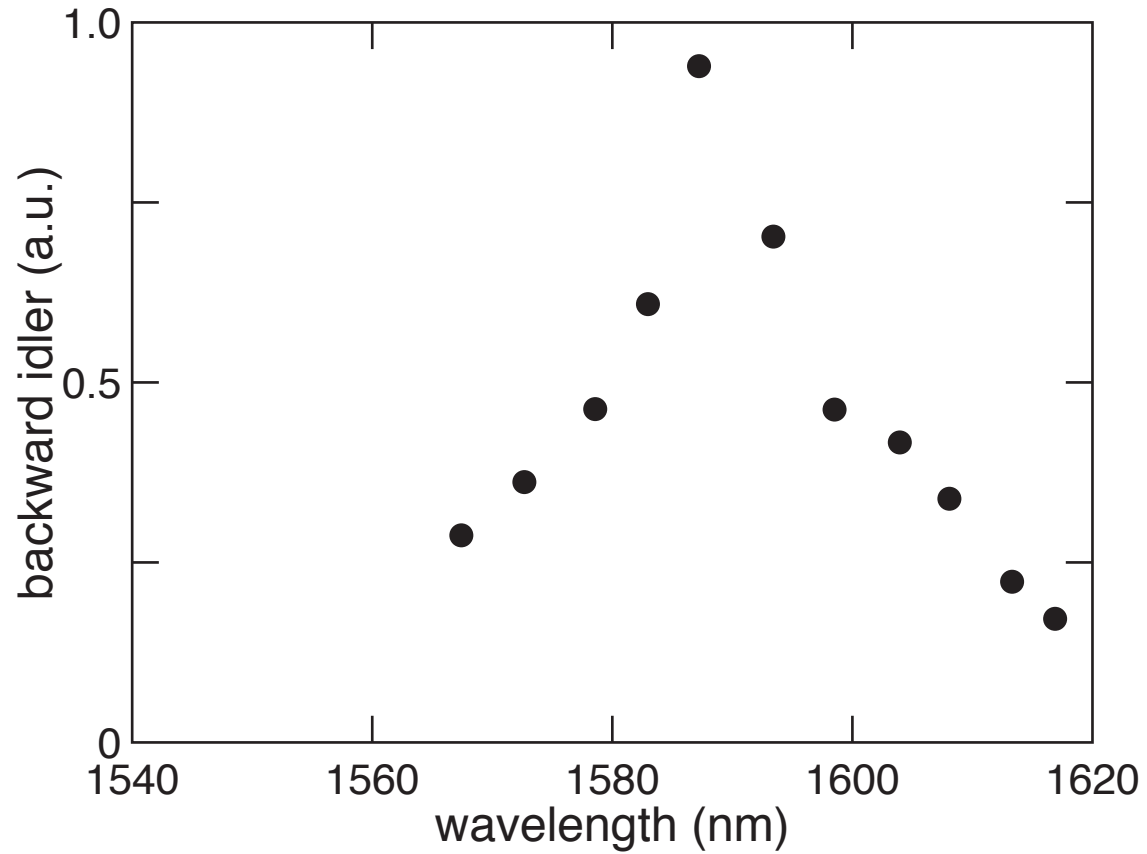


1 zero index

2 fabrication

3 results

## backward idler intensity



1 zero index

2 fabrication

3 results

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Daryl Vulis, Mei Yin, Lysander Christakis, Zin Lin, Cleaven Chia,  
Olivia Mello, Haoning Tang, Justin Gagnon, Marko Loñcar**

**Profs. Bob Boyd , Nader Engheta, Alan Willner**

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**1 zero index**

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