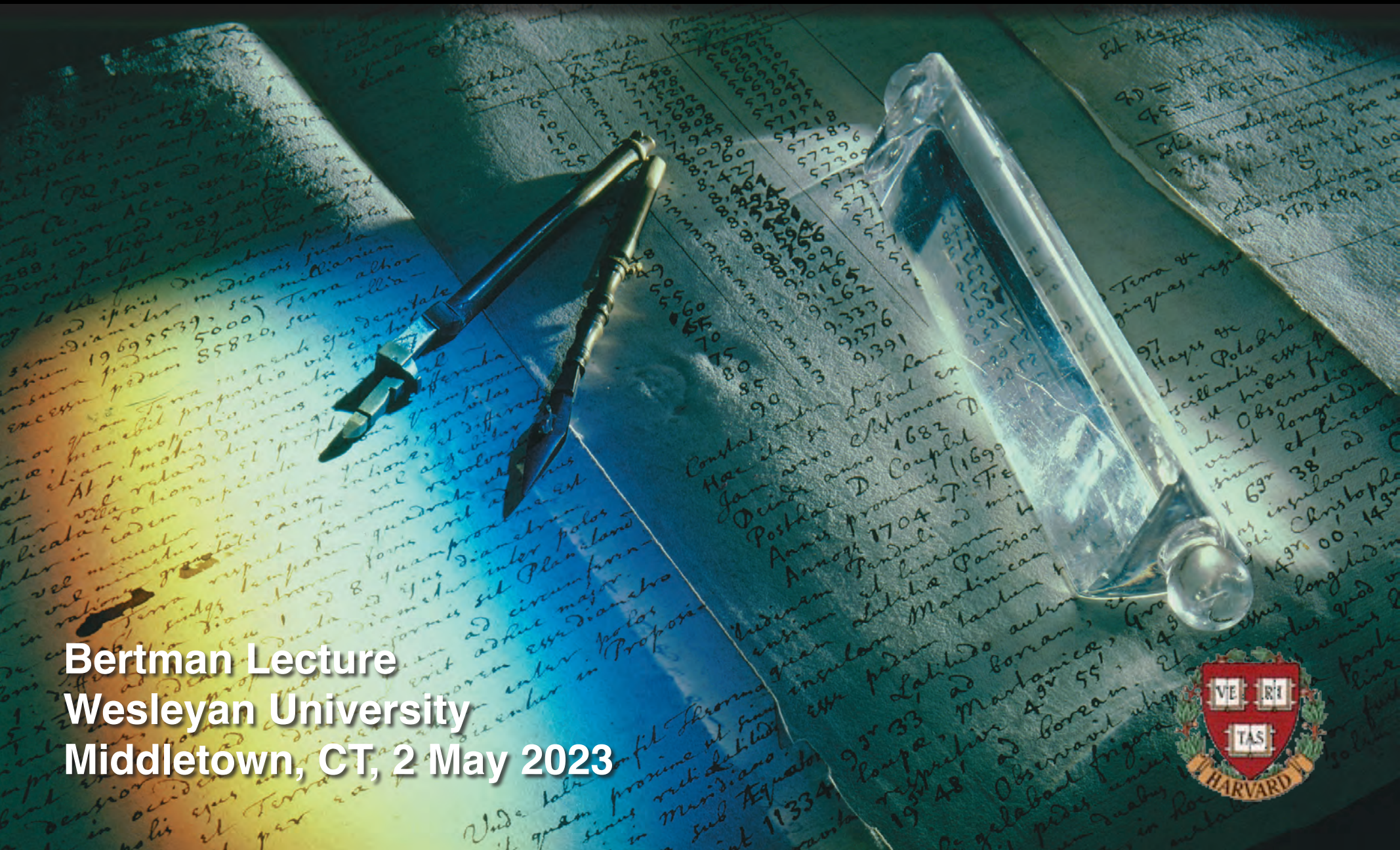
The surprising world where optical properties approach zero



Bertman Lecture
Wesleyan University
Middletown, CT, 2 May 2023

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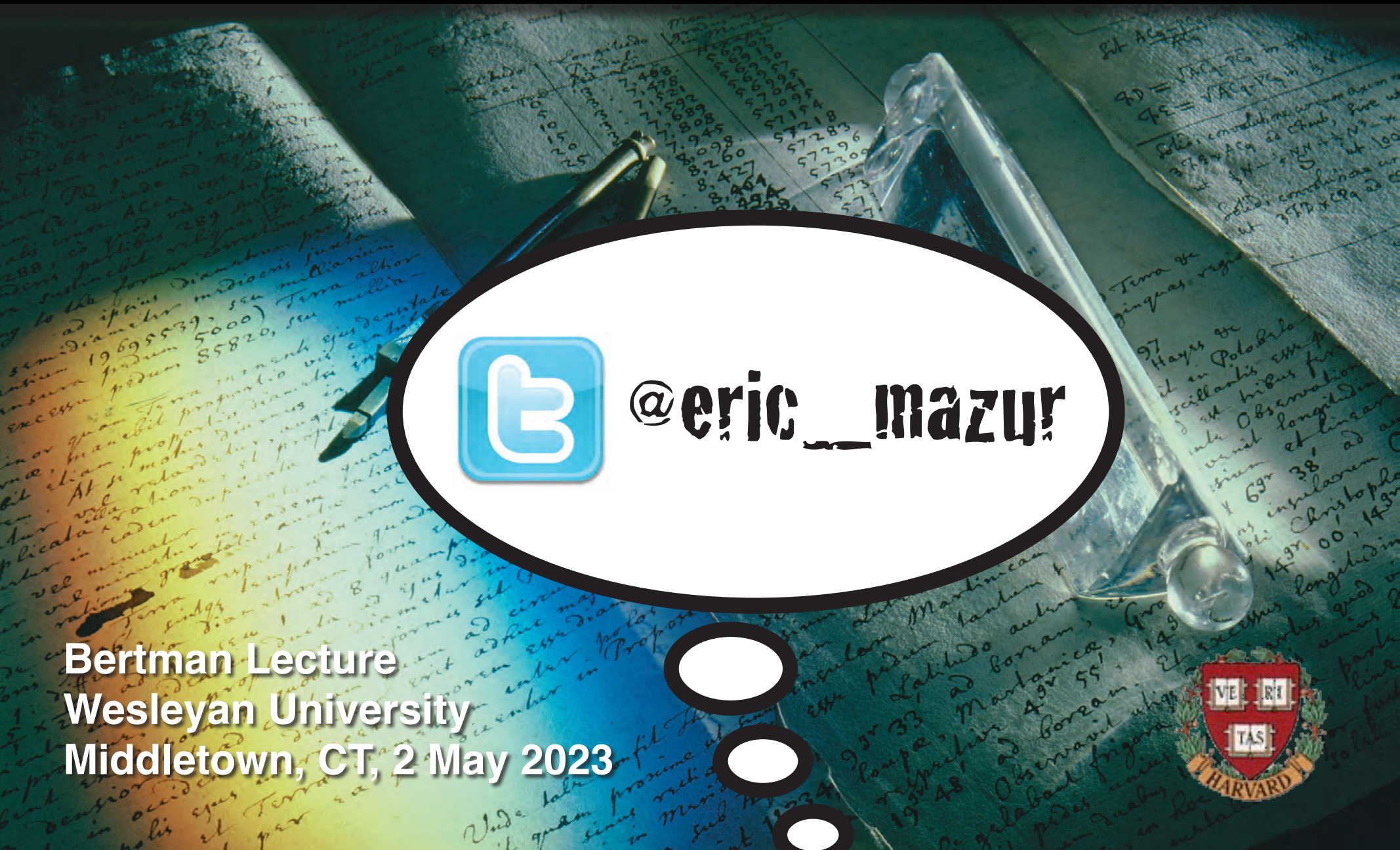

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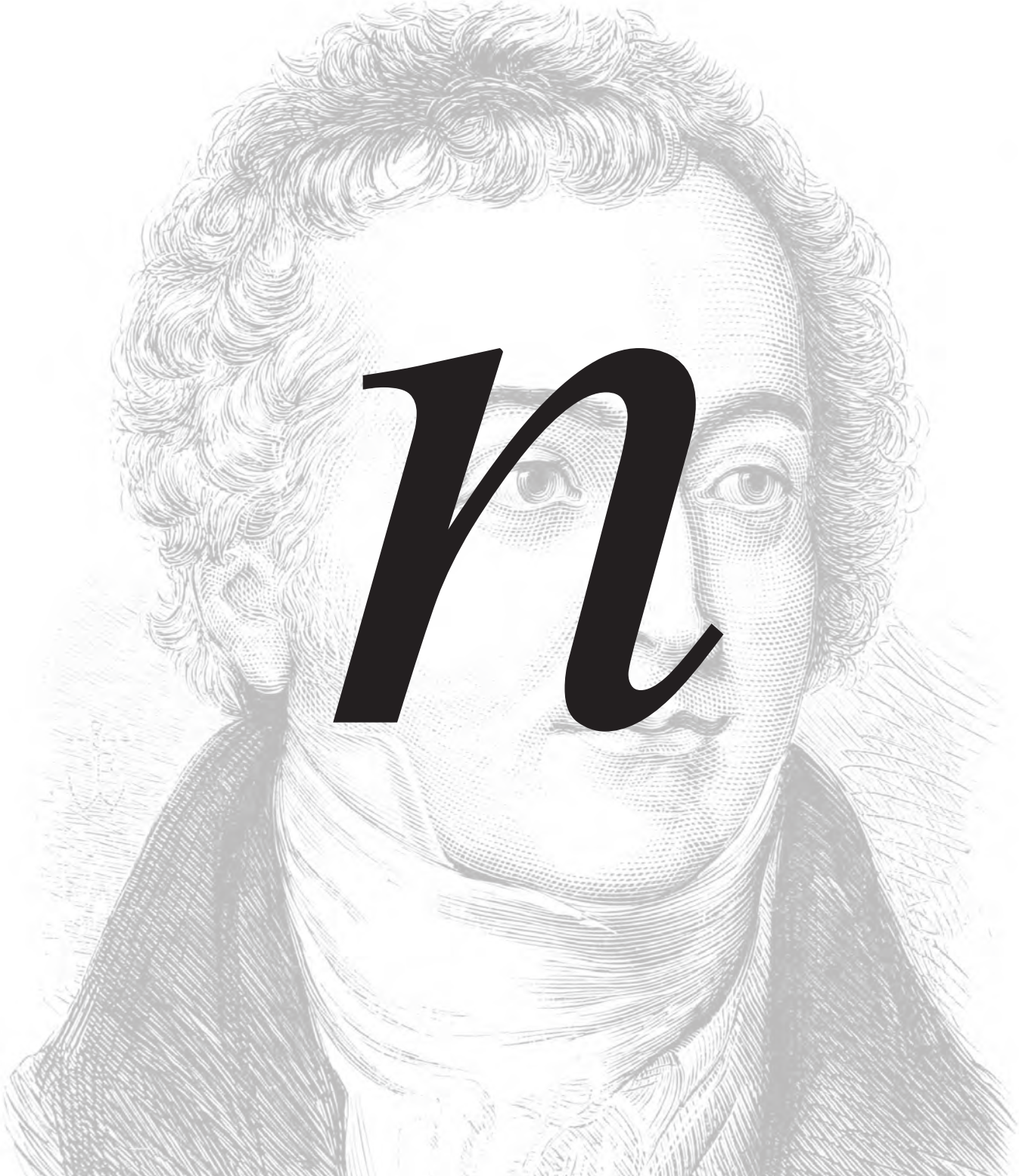
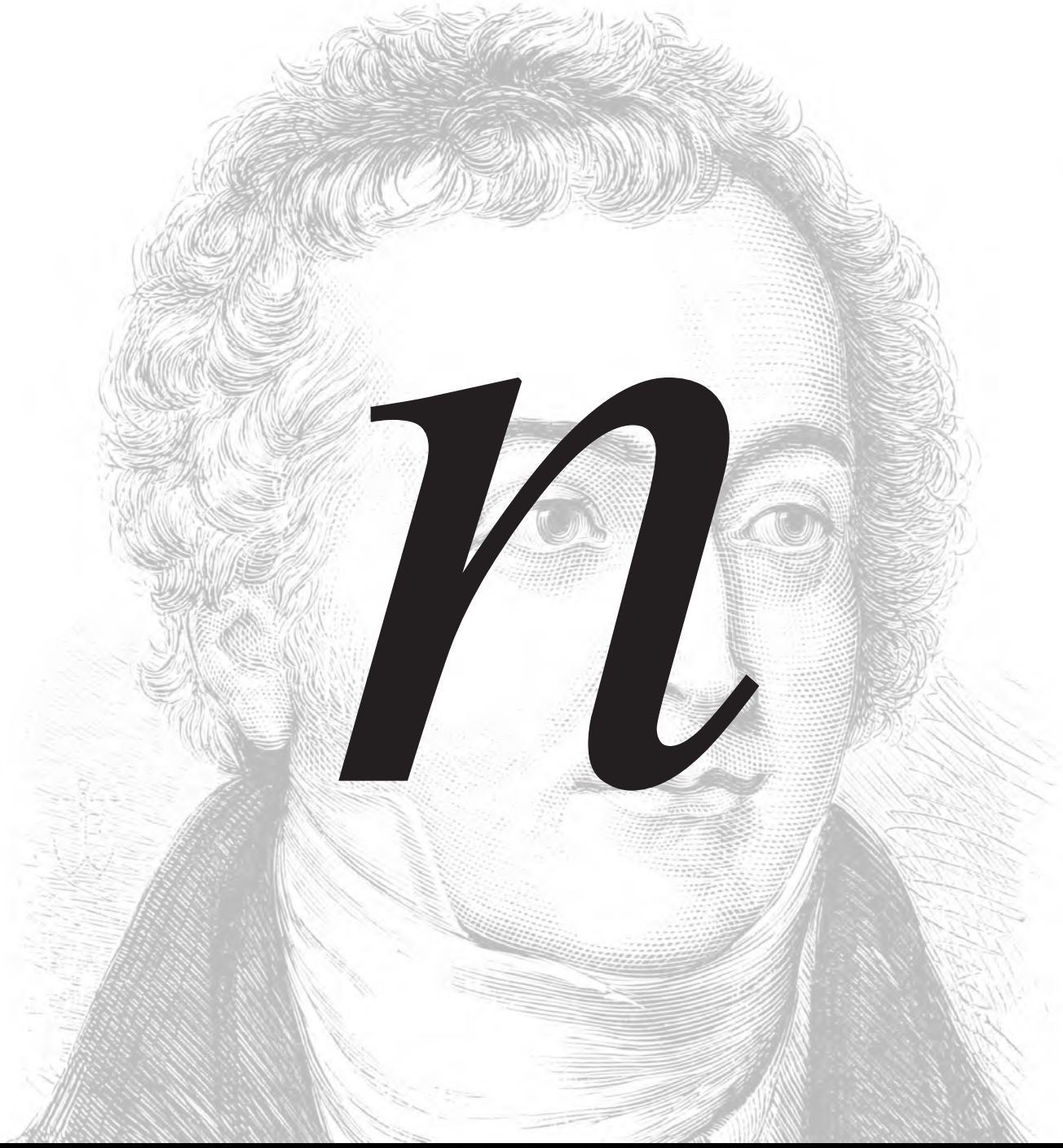


n



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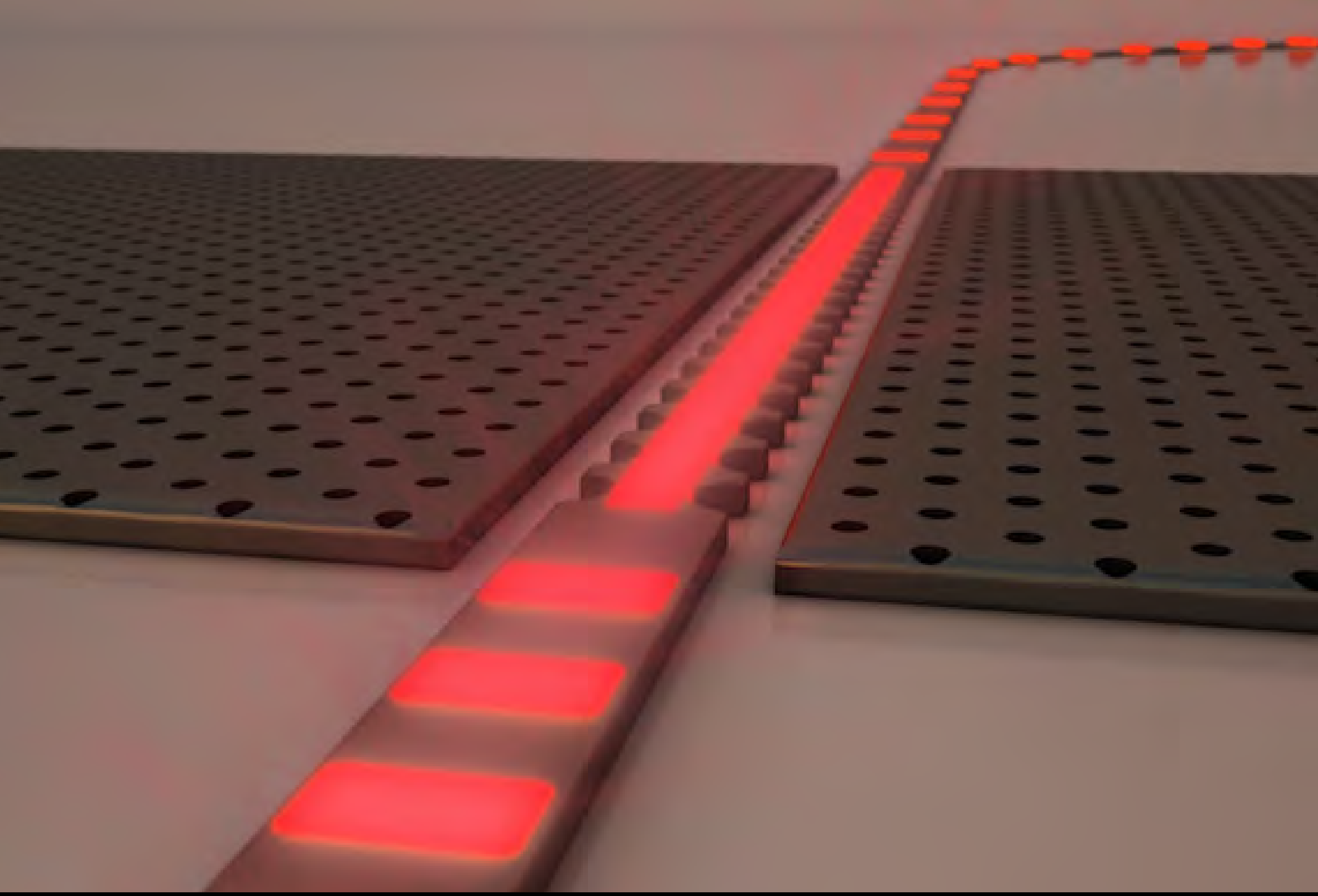

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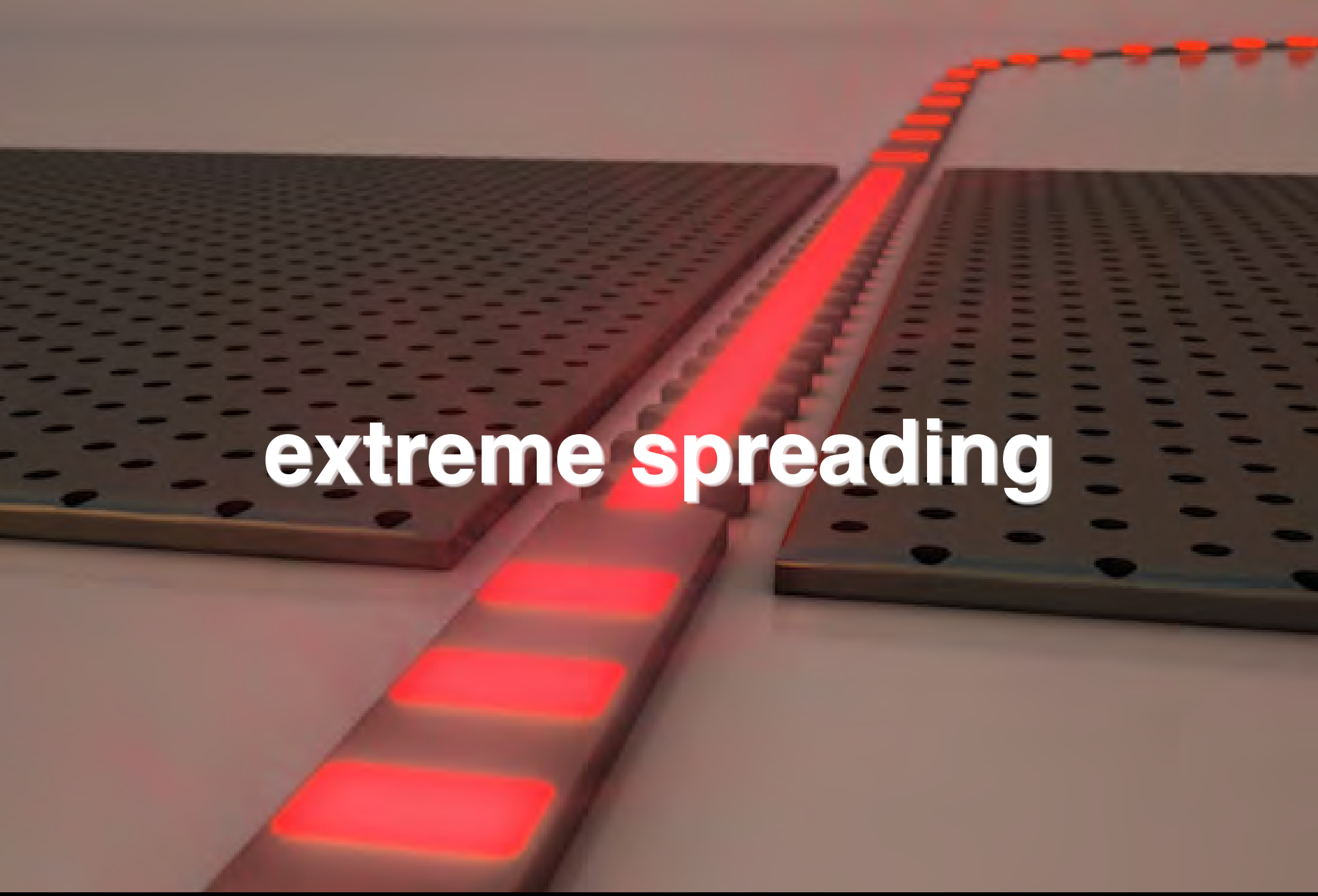

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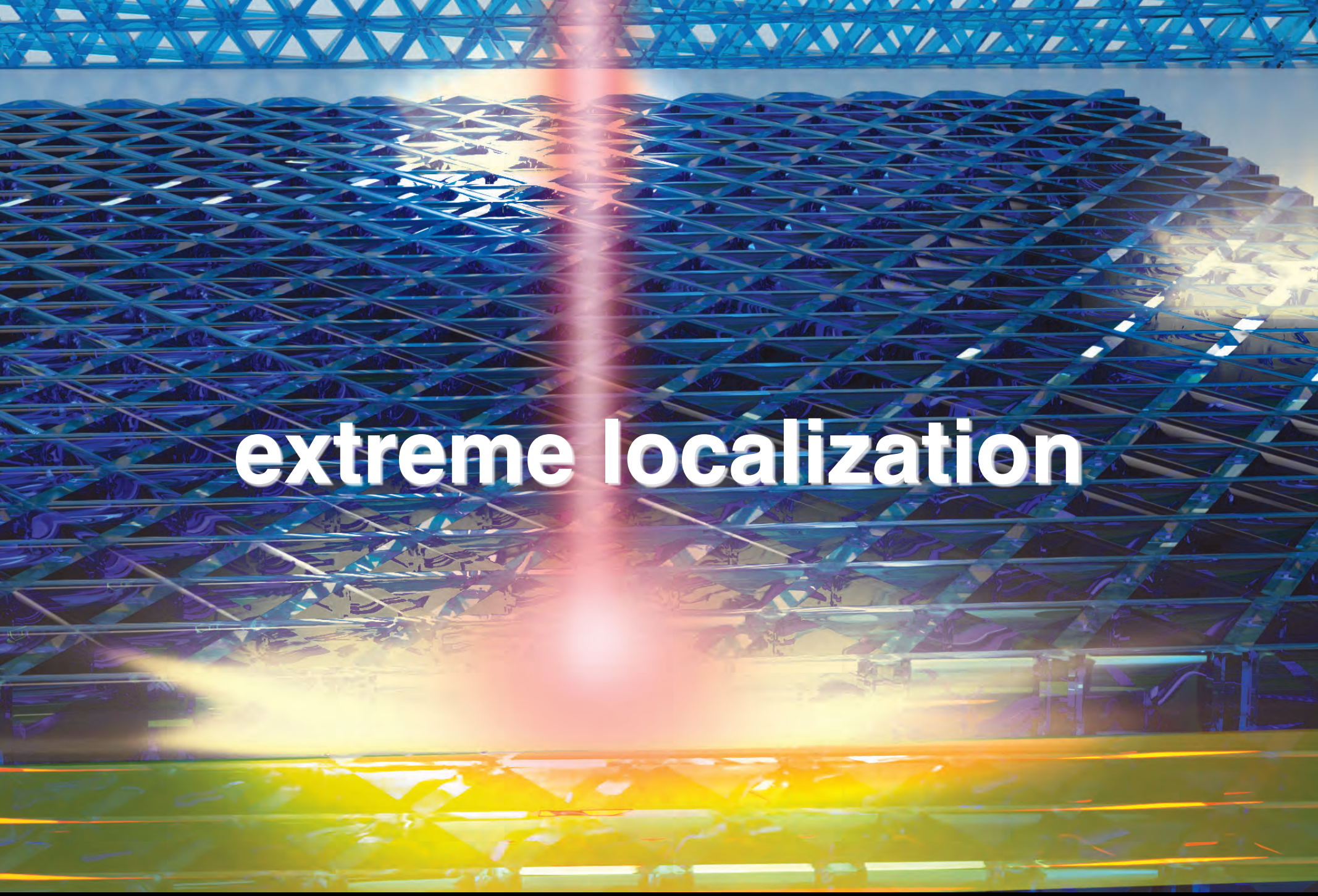

1 index

2 zero index

3 zero group velocity

Propagation of EM wave

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governed by wave equation

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In dispersive media $n = n\,(\omega)$.

Index of refraction

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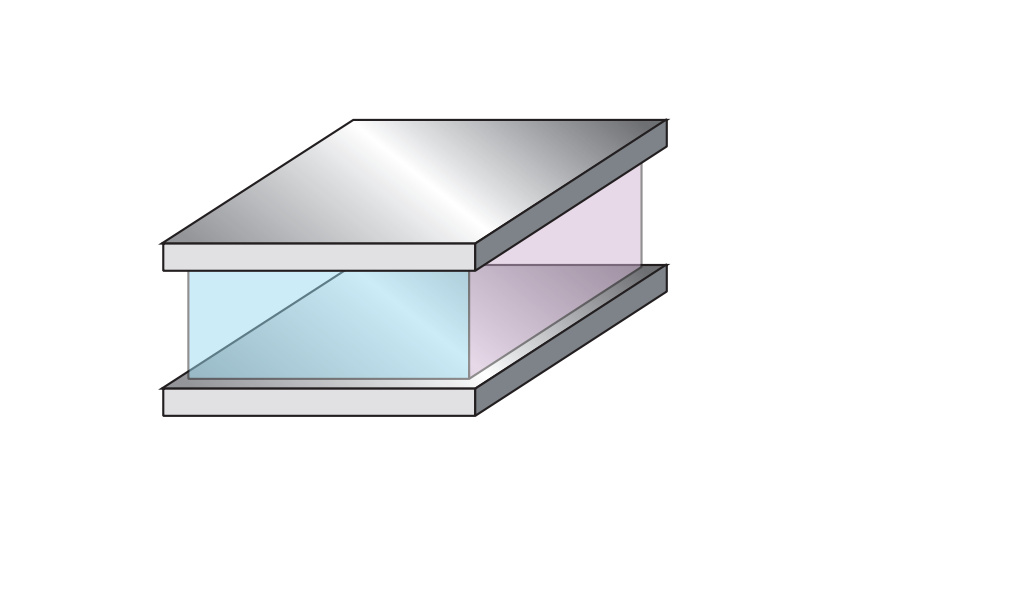

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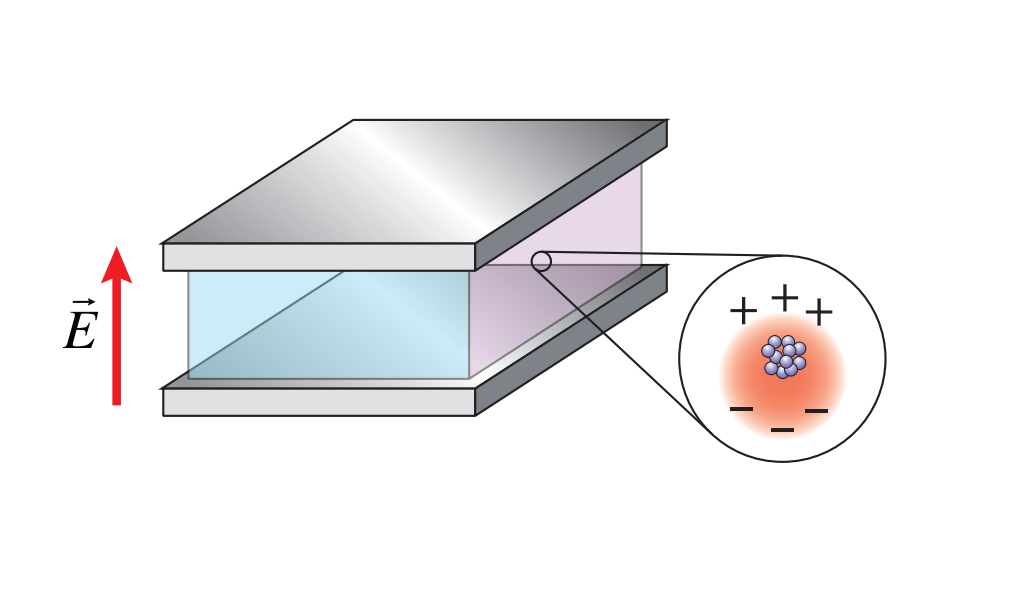

$\epsilon(\omega)$ measure of attenuation of electric field

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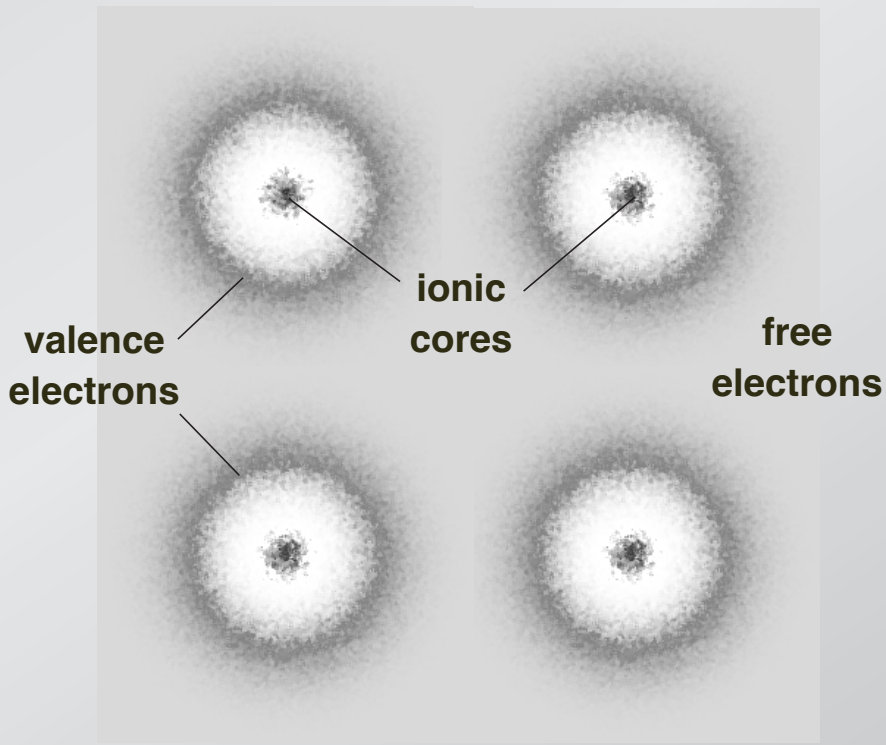
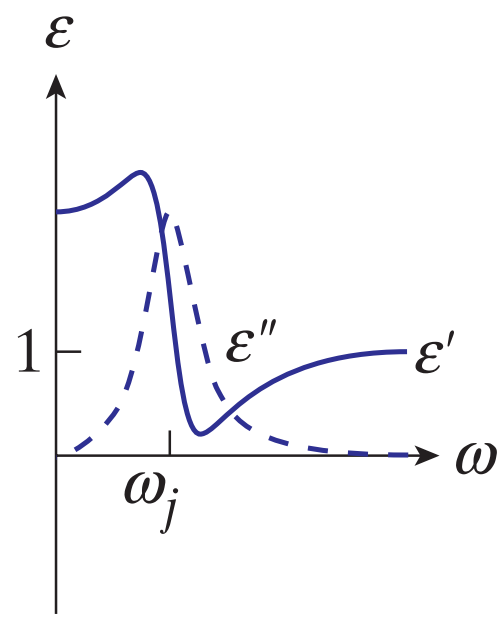


Dielectric constant

Lorentz oscillator



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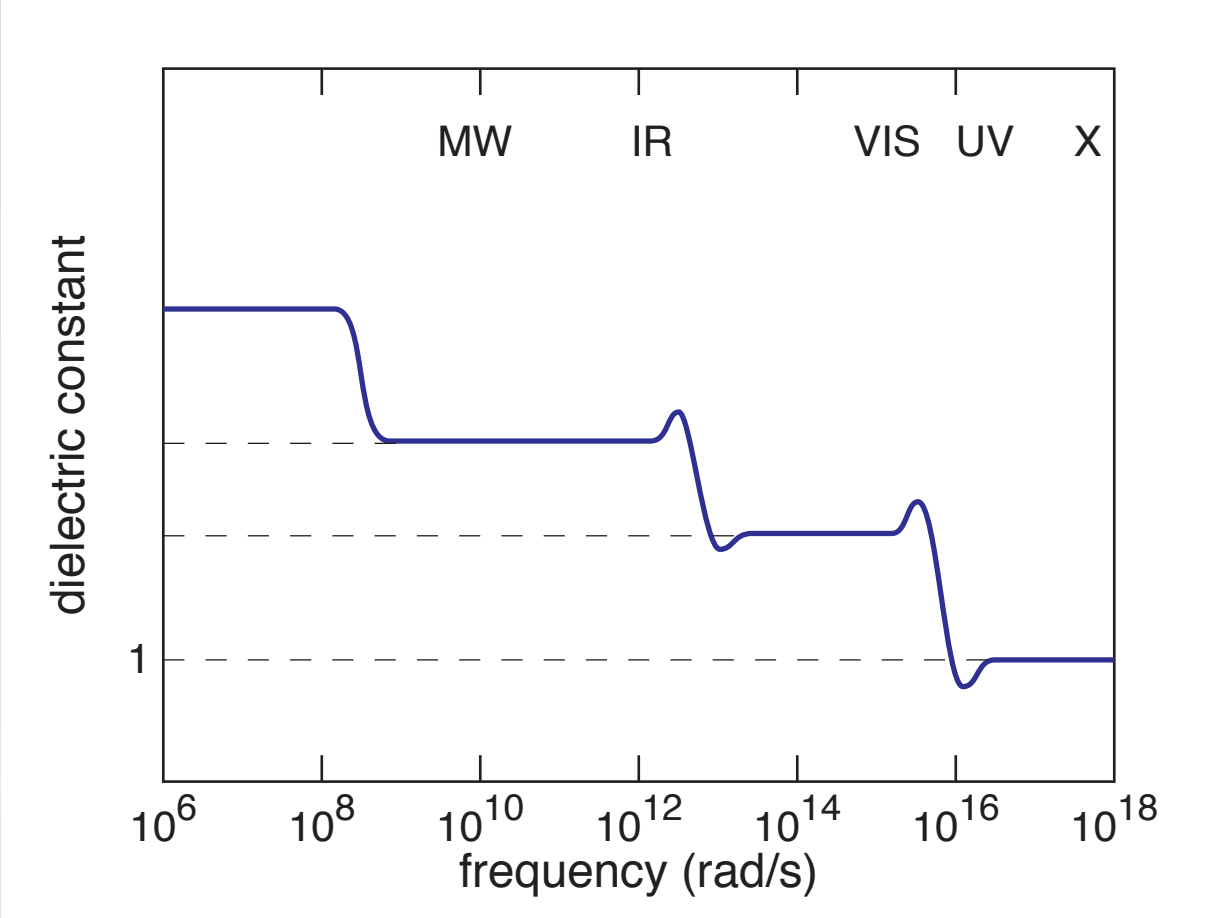

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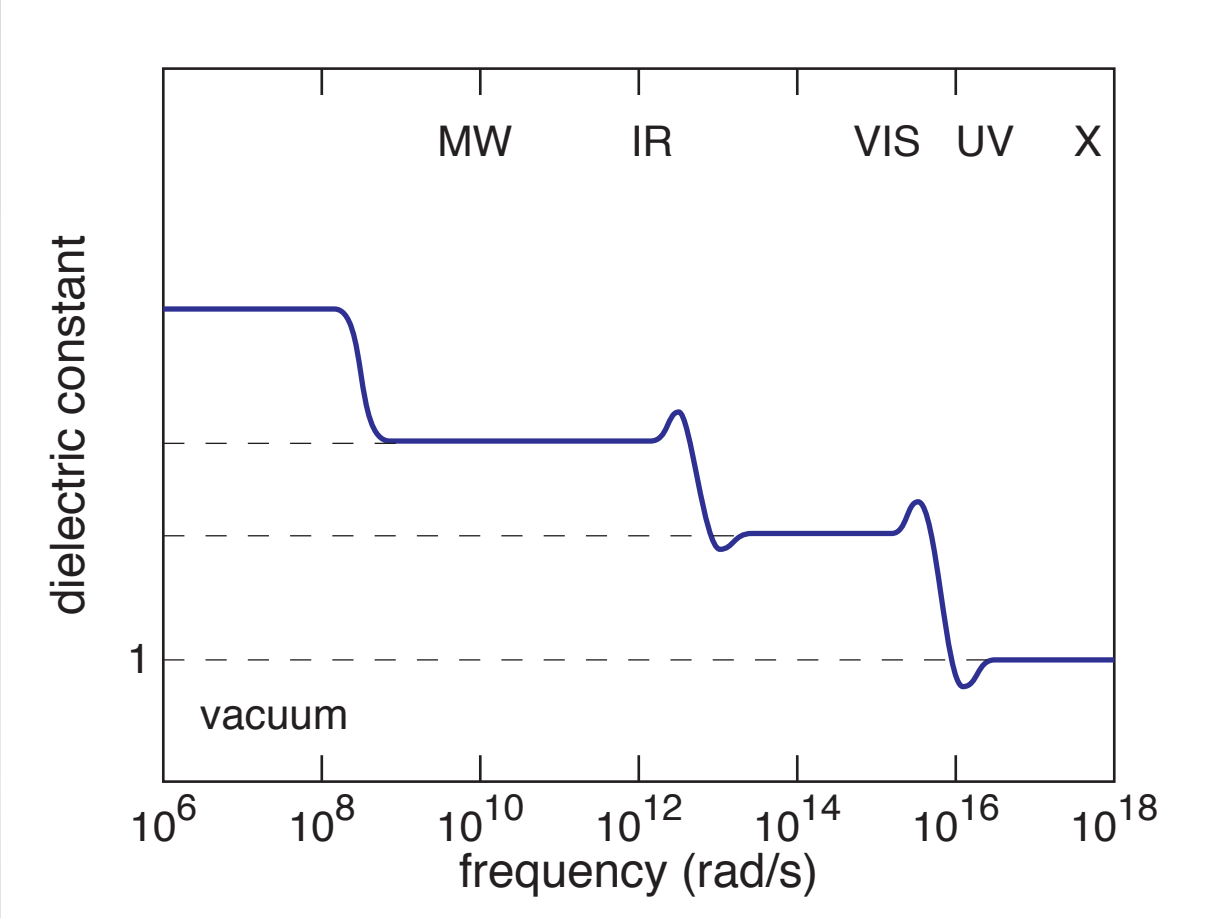

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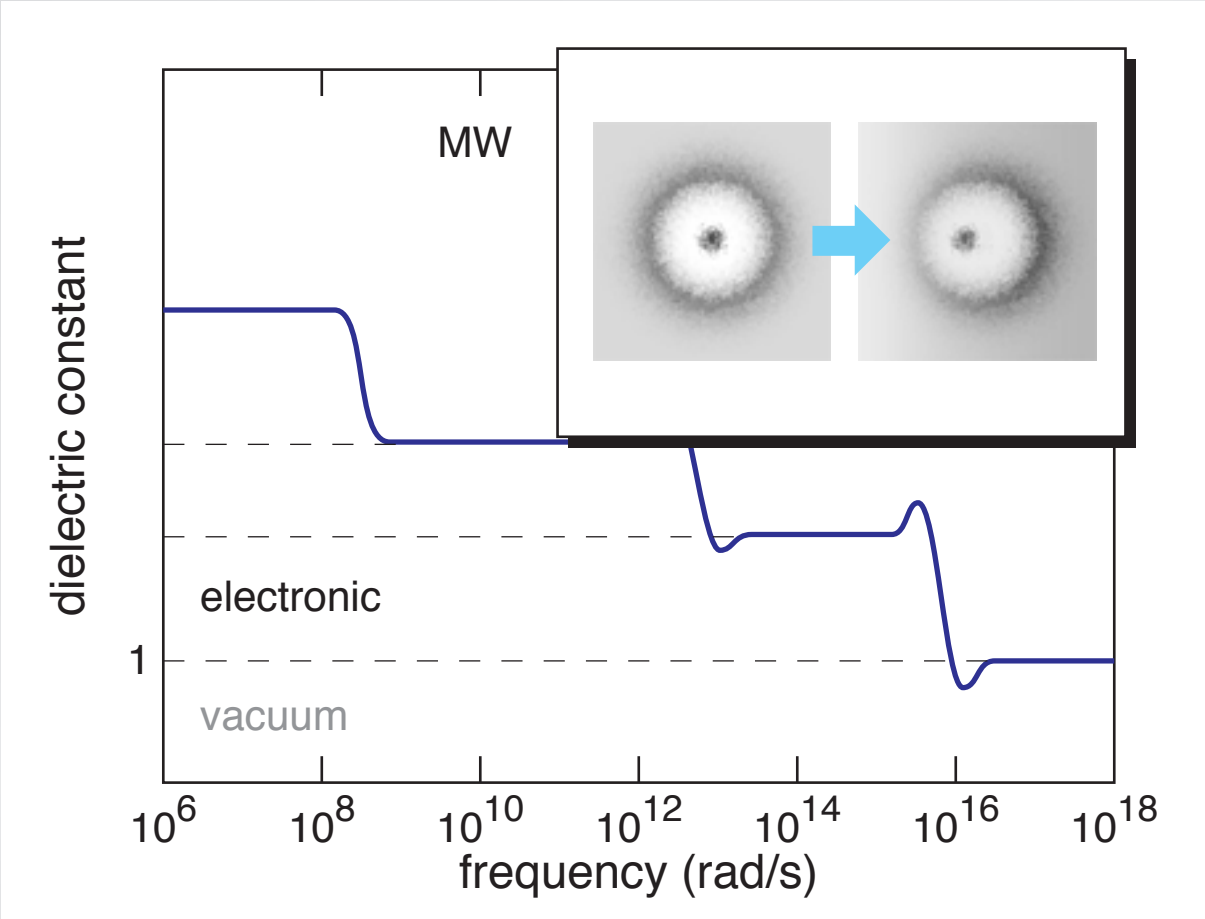

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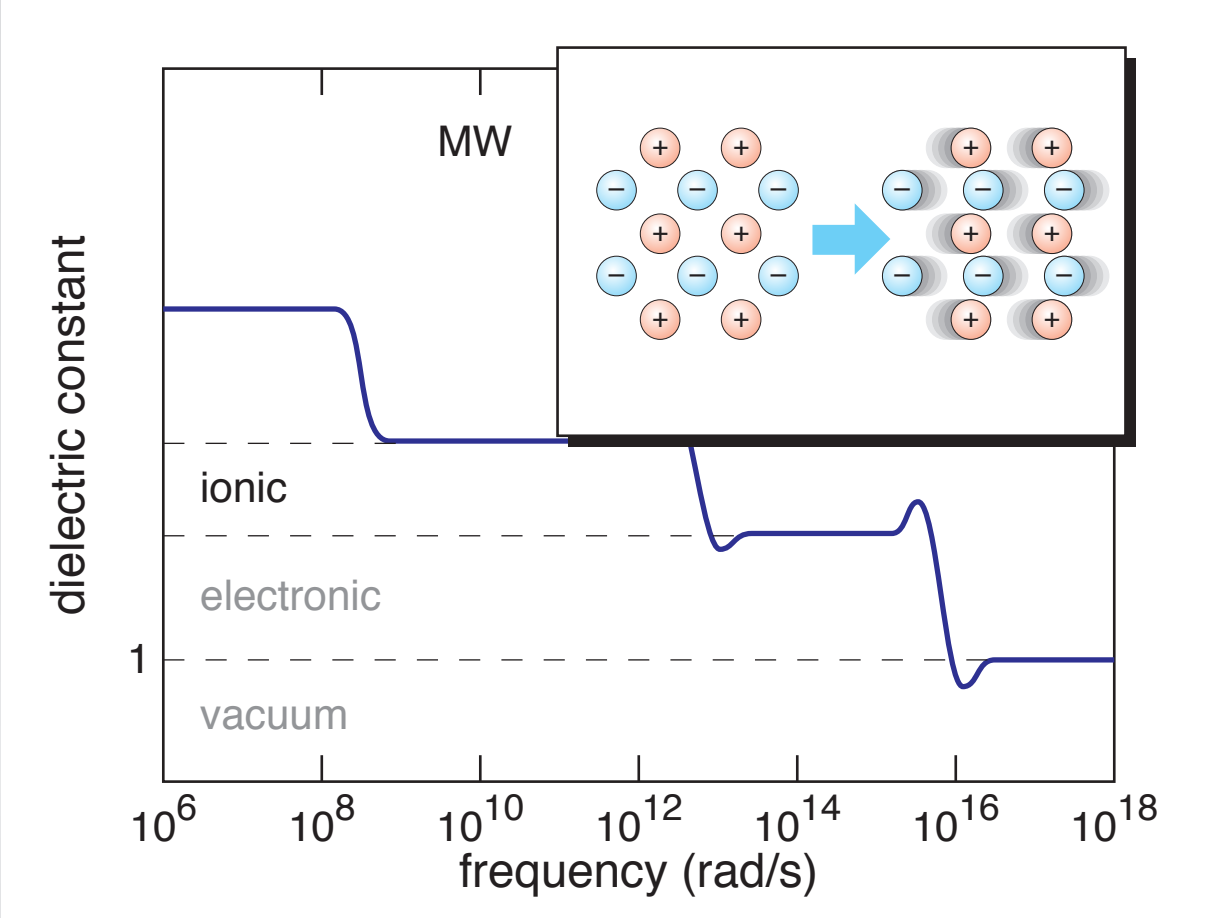

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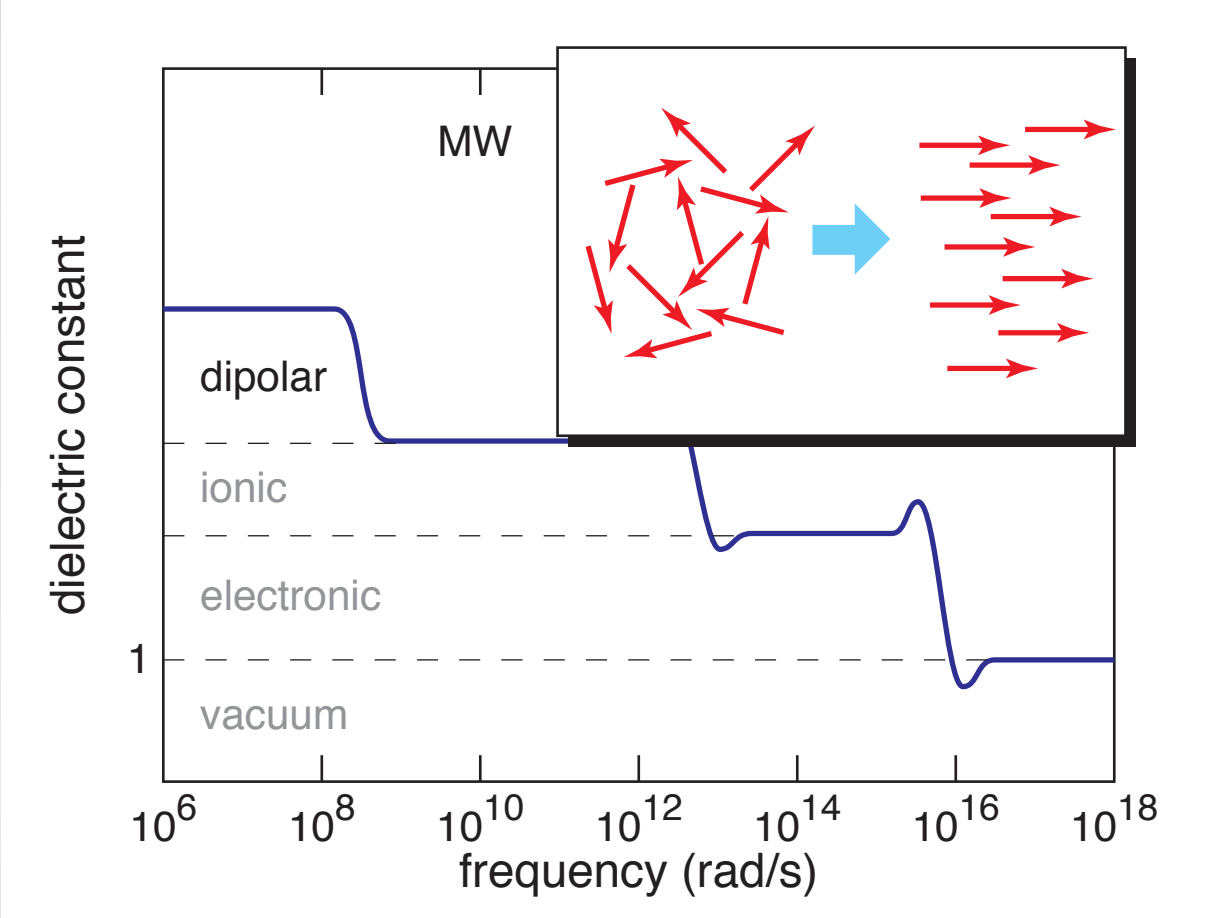

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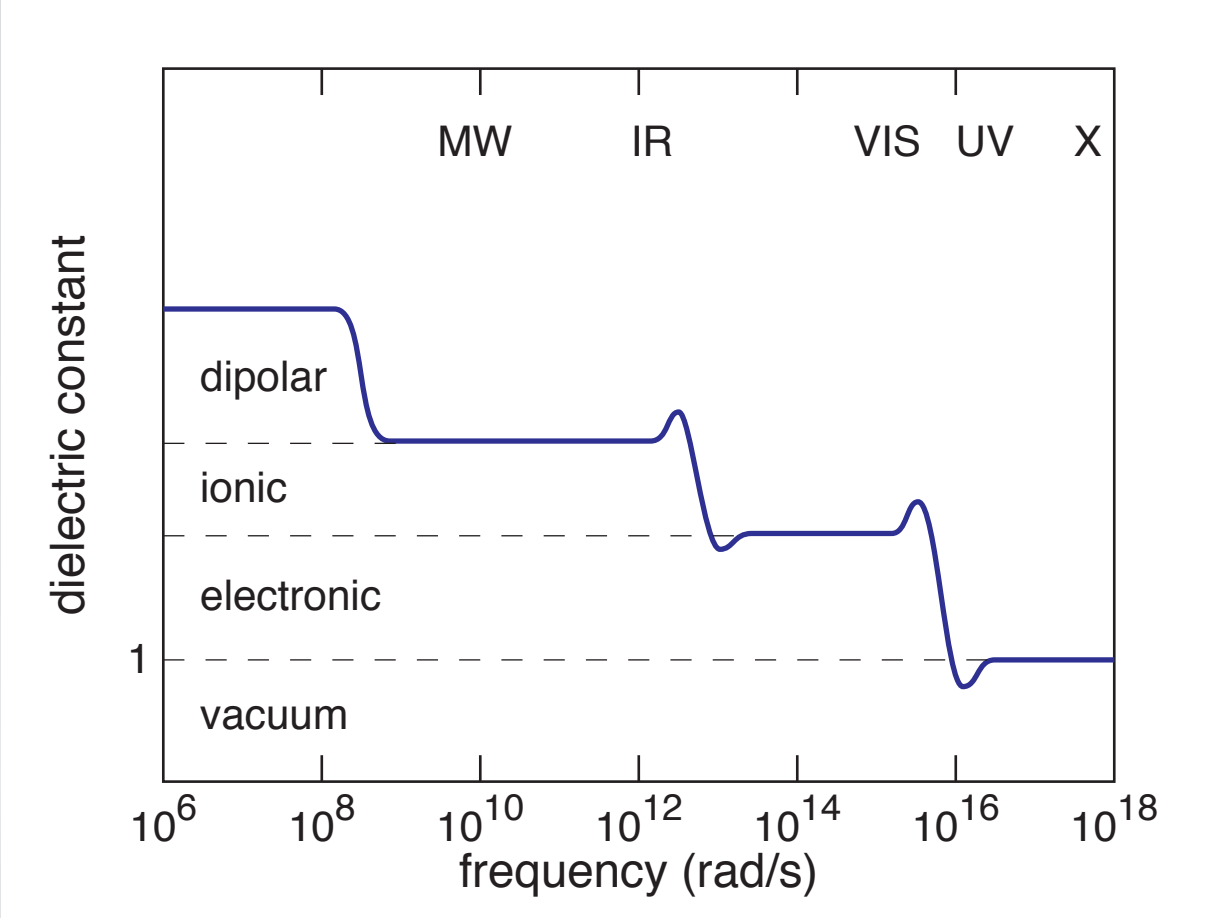
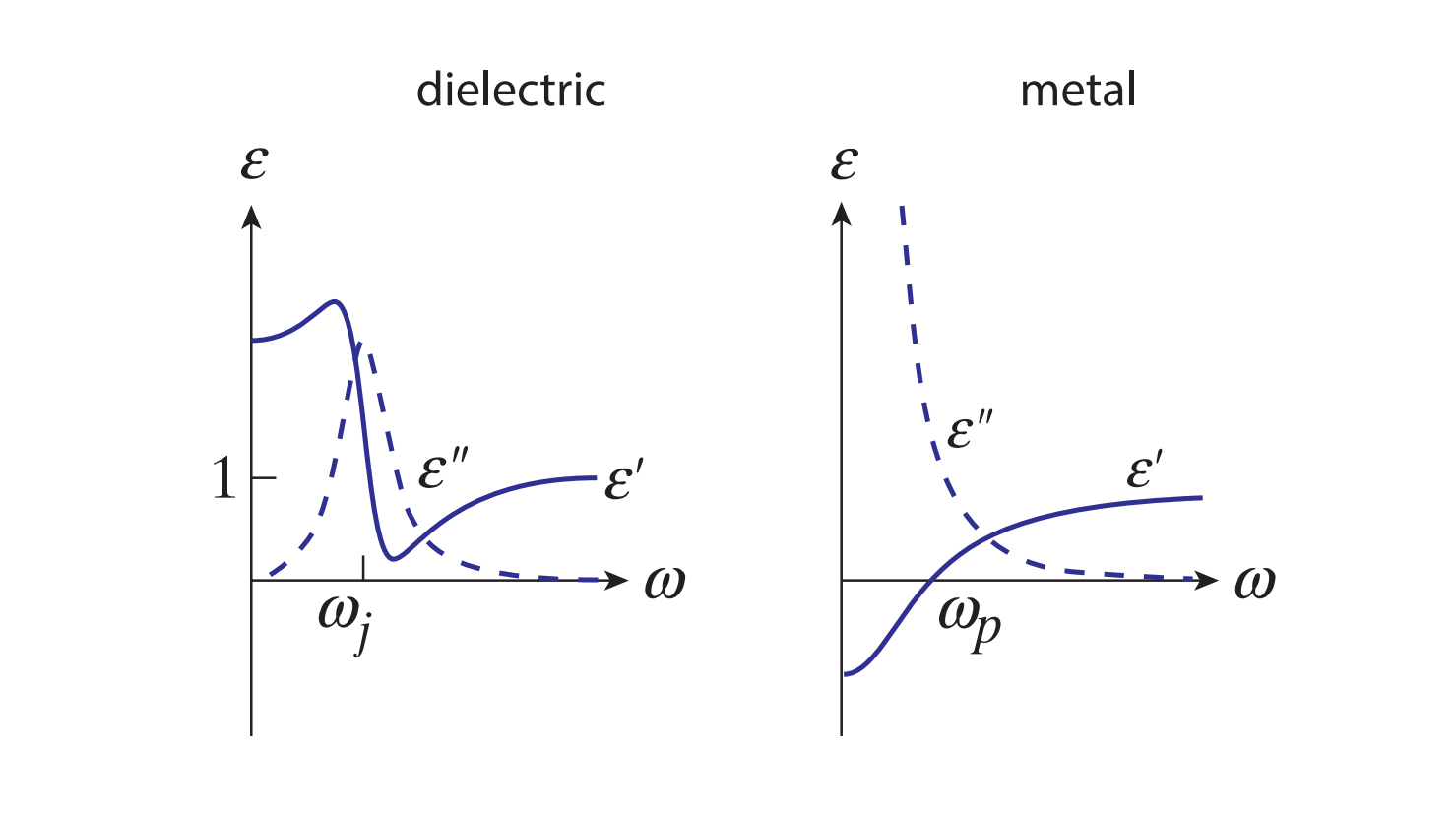
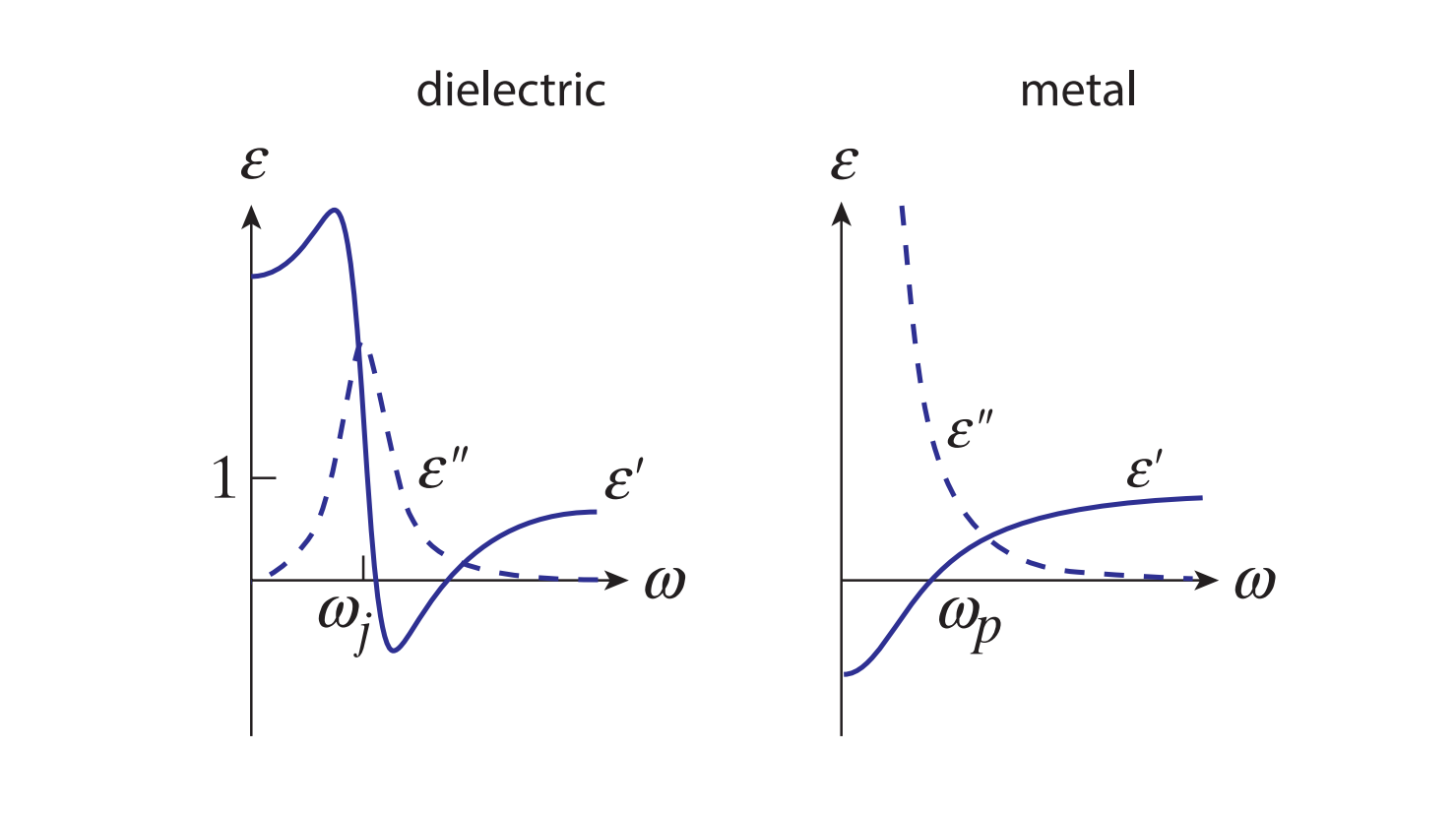


Lorentz and Drude models



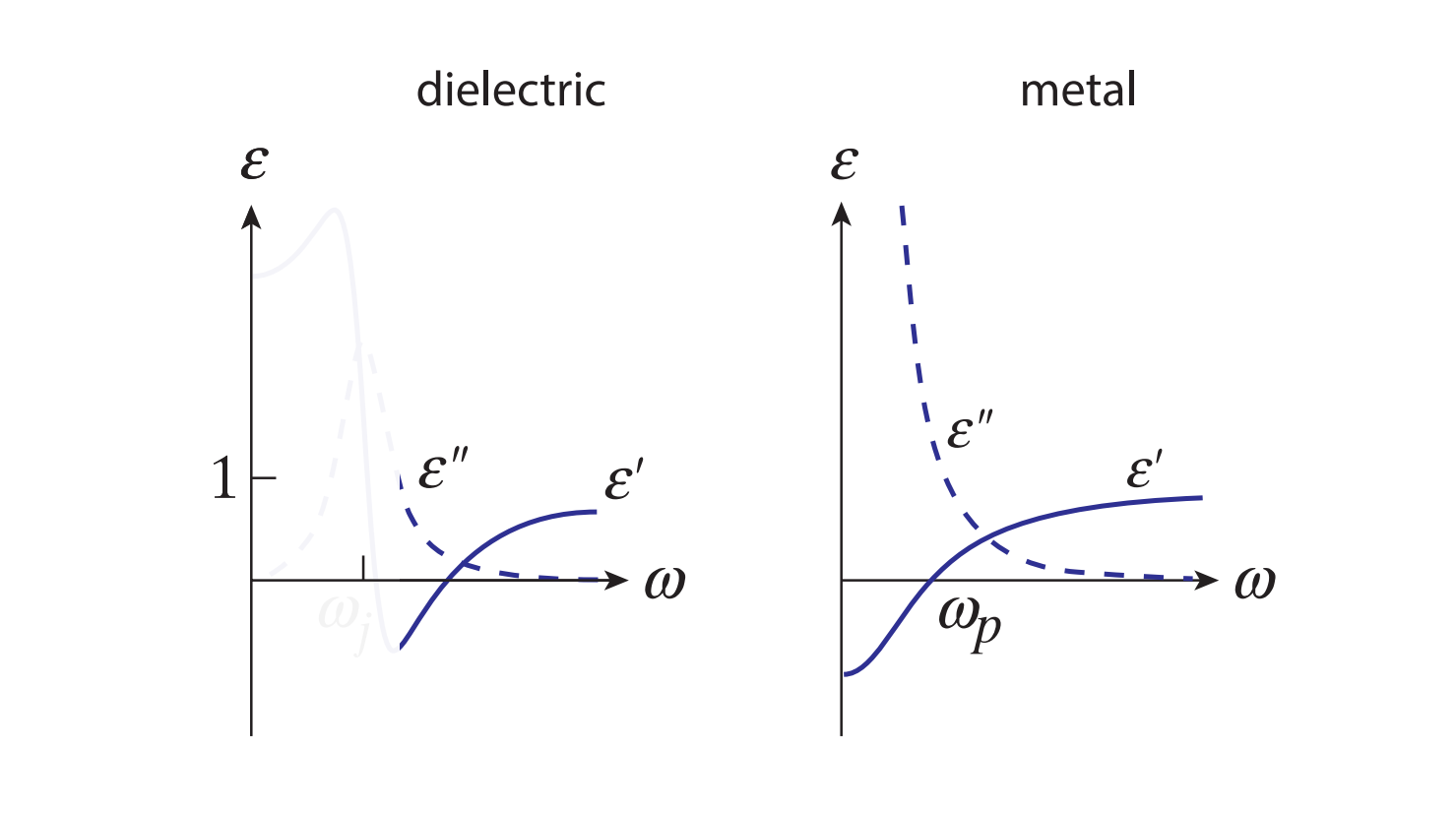
Lorentz and Drude models

for a strong (dielectric) resonance ε can become negative



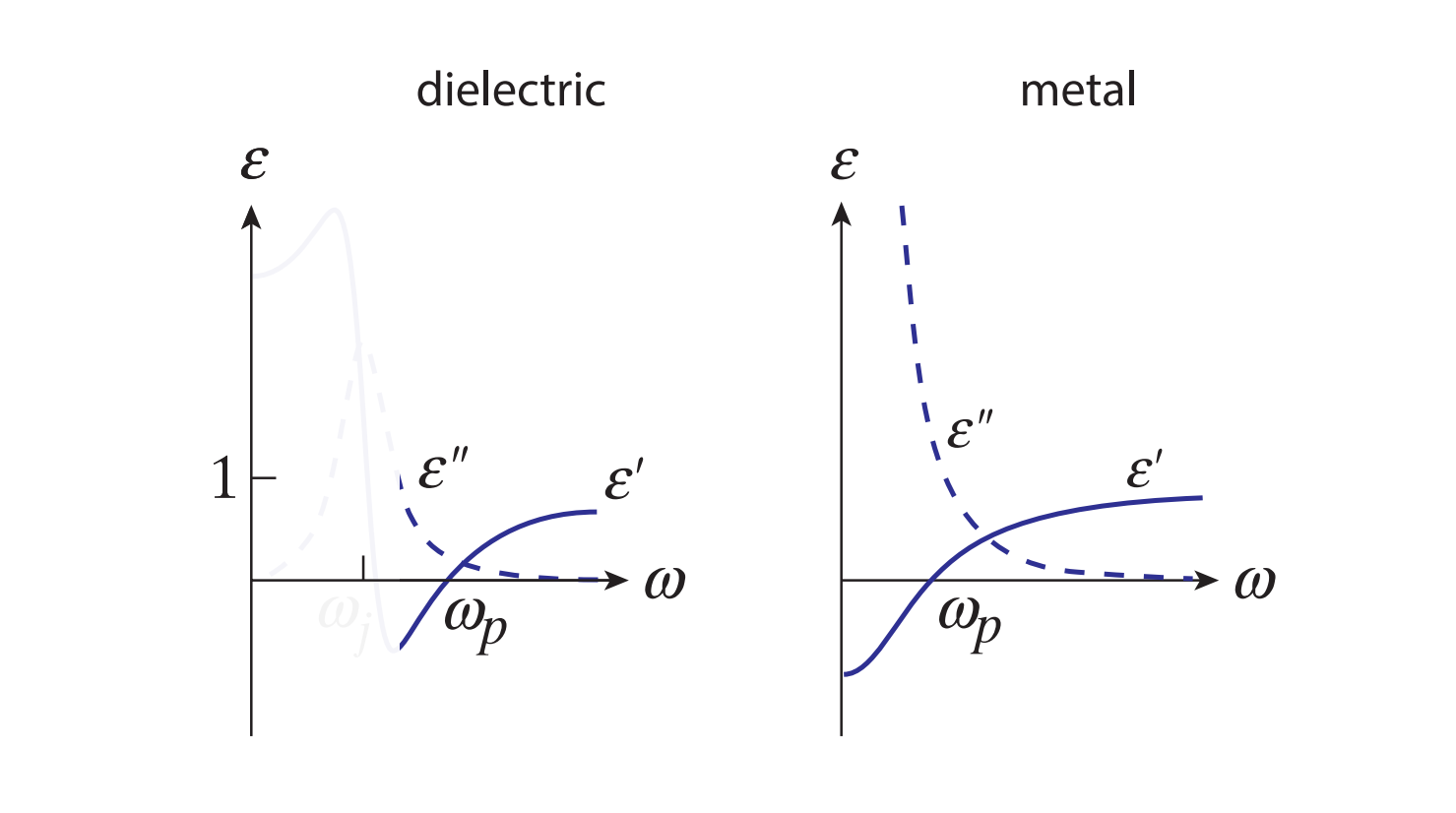
Lorentz and Drude models

valence electrons in dielectric then behave like a plasma



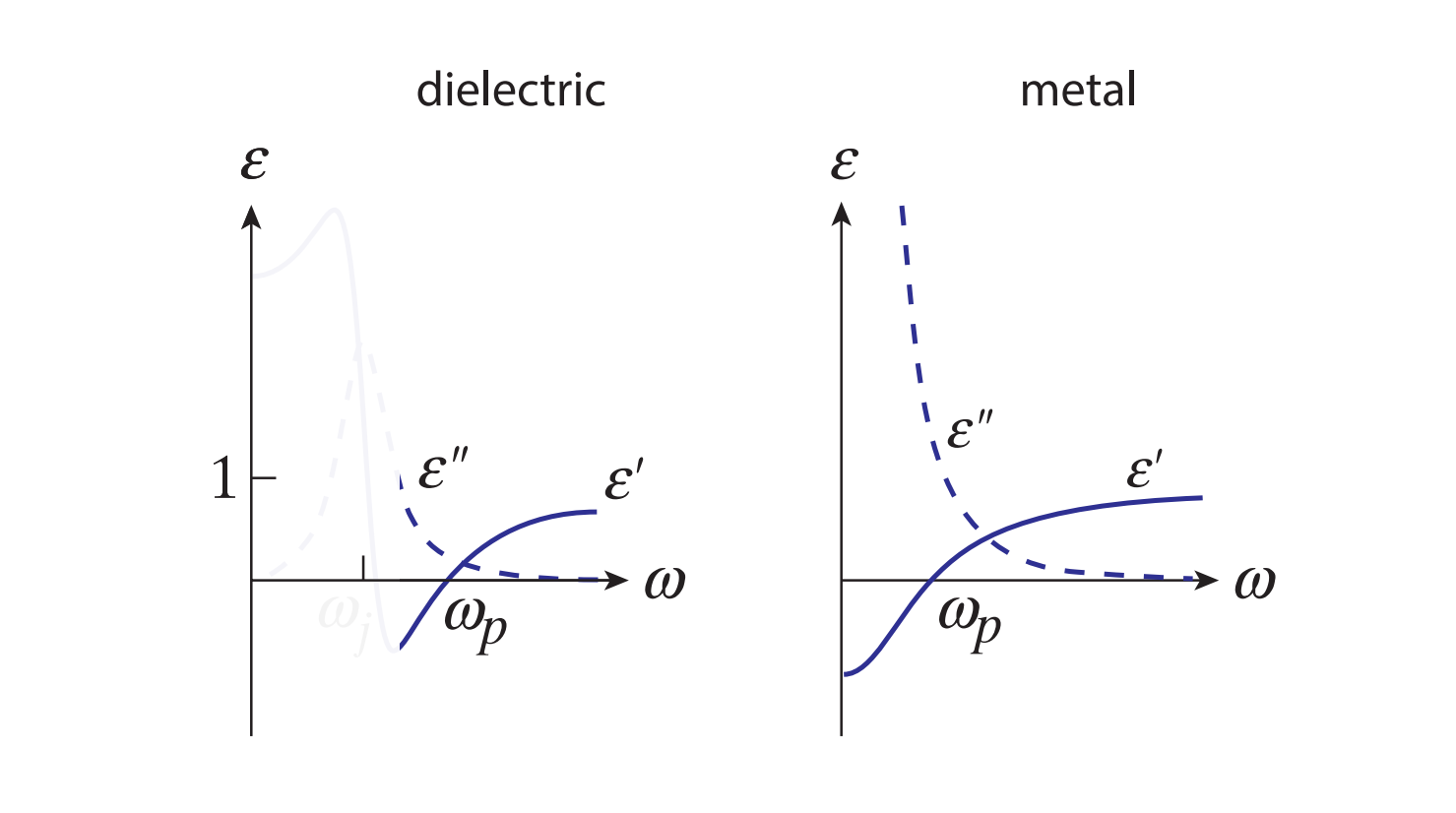
Lorentz and Drude models

with plasma frequency above the resonance



Lorentz and Drude models

(and far below the UV region)



Index also determined by magnetic response

$$n = \sqrt{\varepsilon\mu}$$

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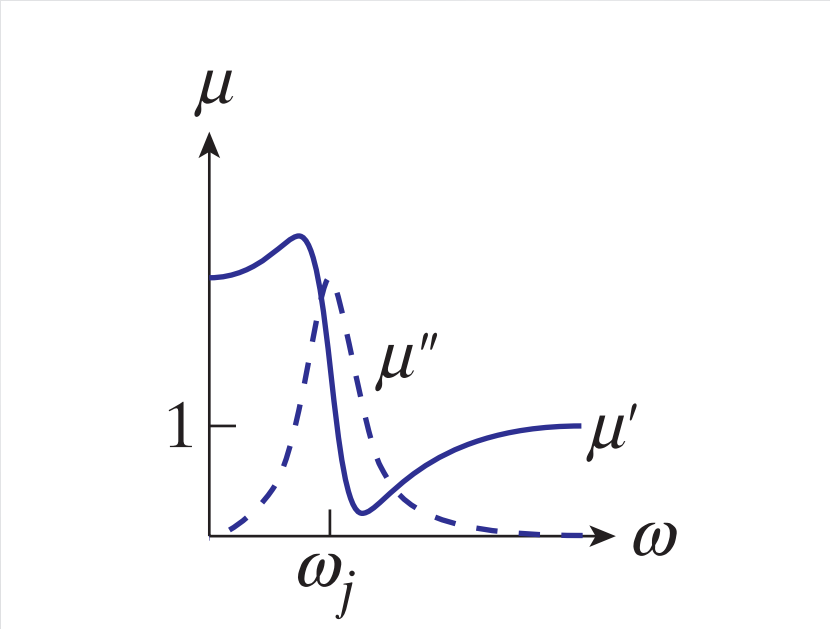
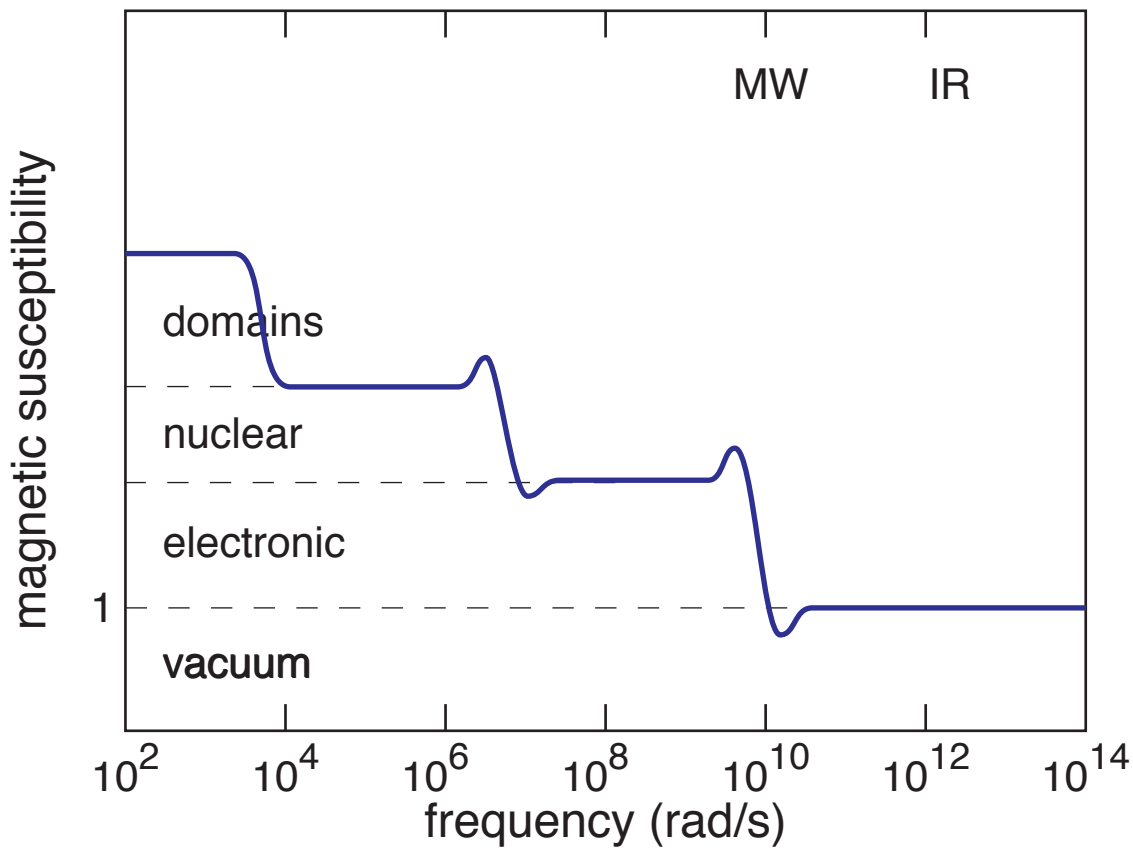
and magnetic response shows similar resonances

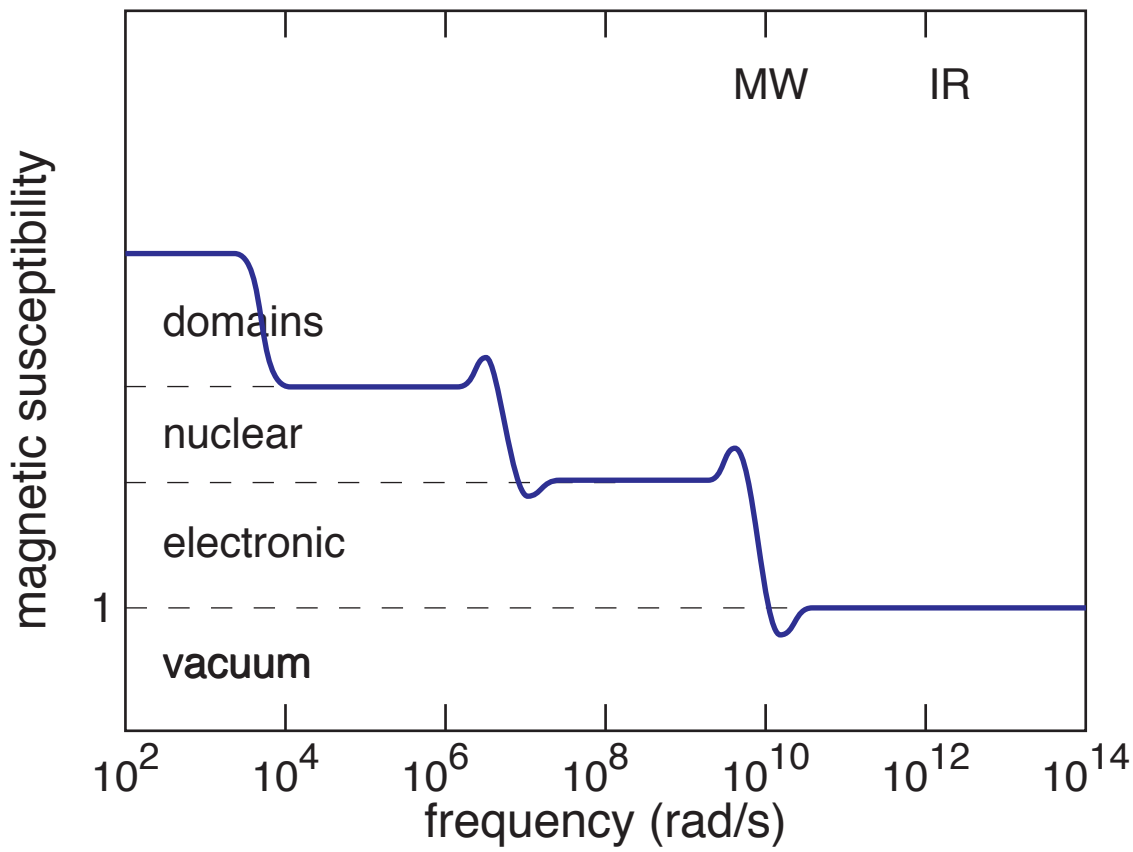


Magnetic response



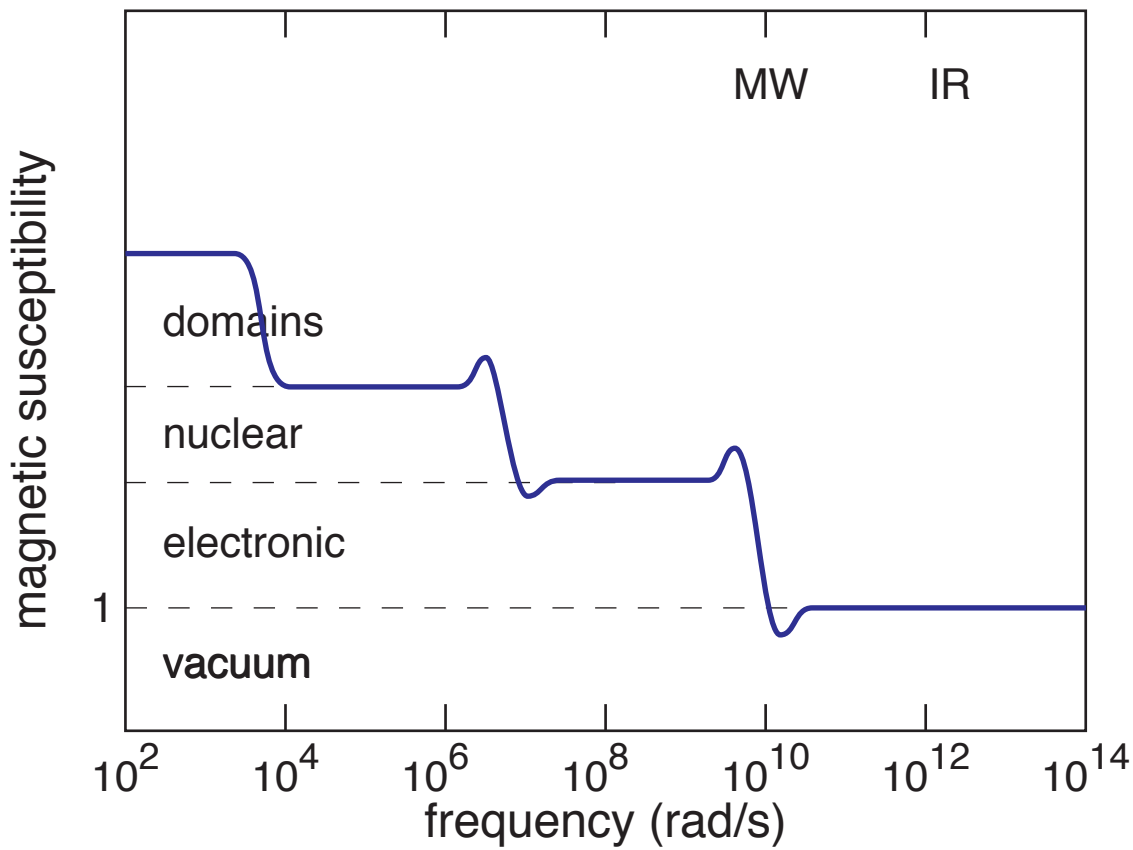
Magnetic response

but magnetic resonances occur below optical frequencies



Magnetic response

so, in optical regime, $\mu \approx 1$



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Both ε and μ are complex and their real parts can be negative.

What happens when $\mathrm{Re}\,\varepsilon$ and/or $\mathrm{Re}\,\mu$ is negative?

Write complex quantities as

$$\varepsilon = |\varepsilon| e^{i\theta} \qquad \mu = |\mu| e^{i\phi}$$

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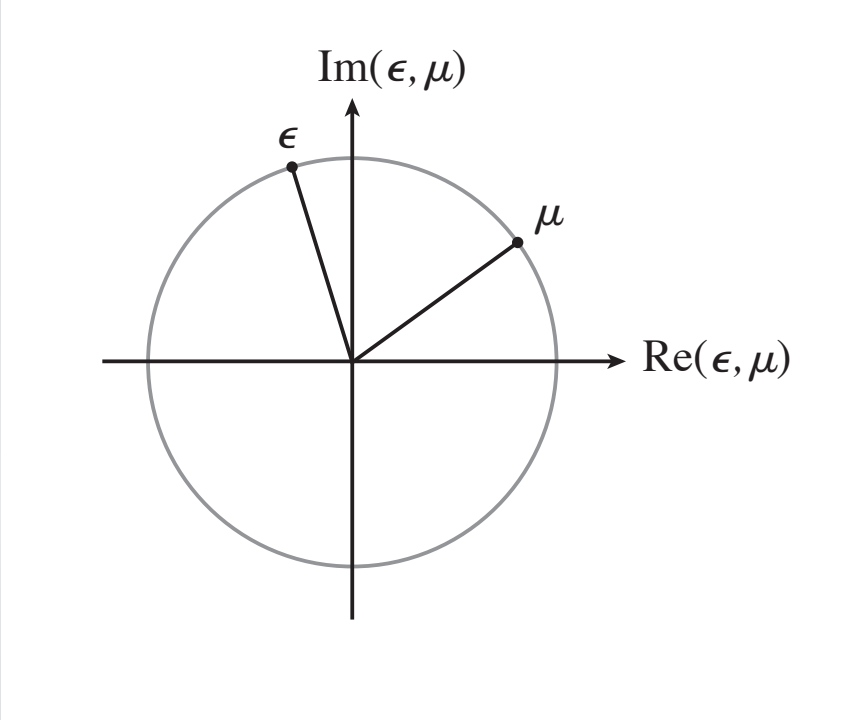

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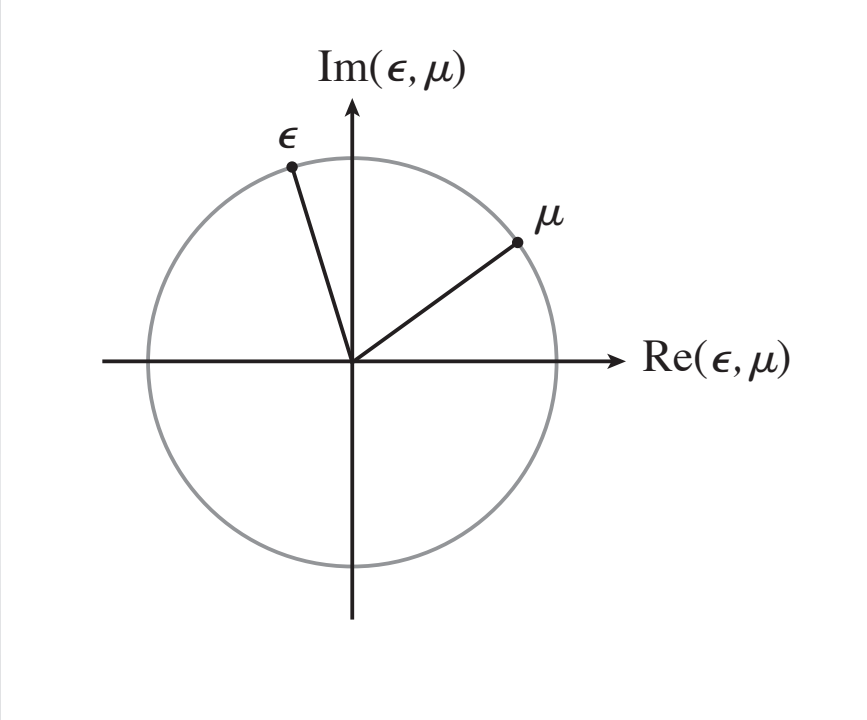

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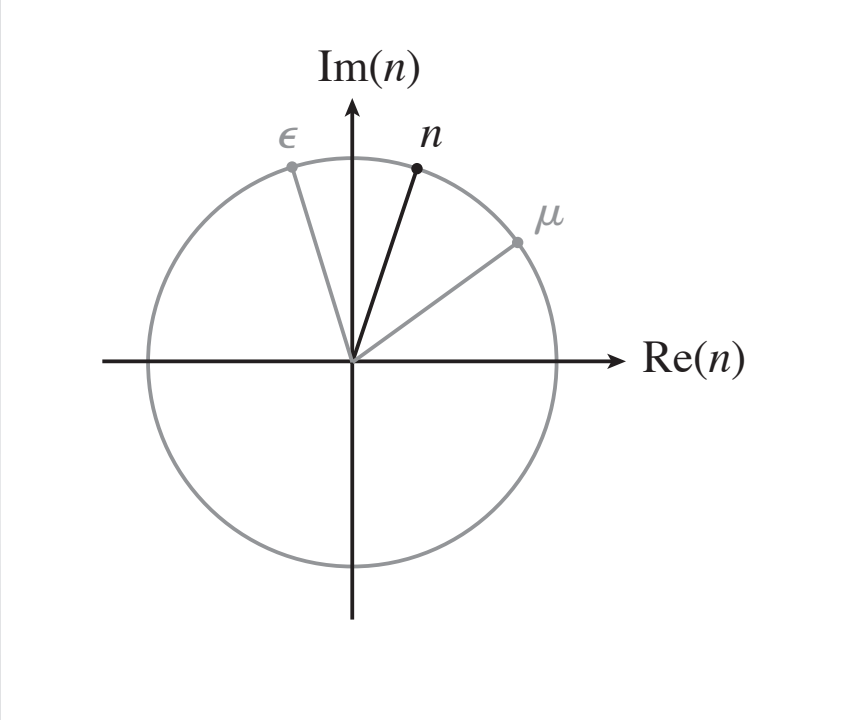

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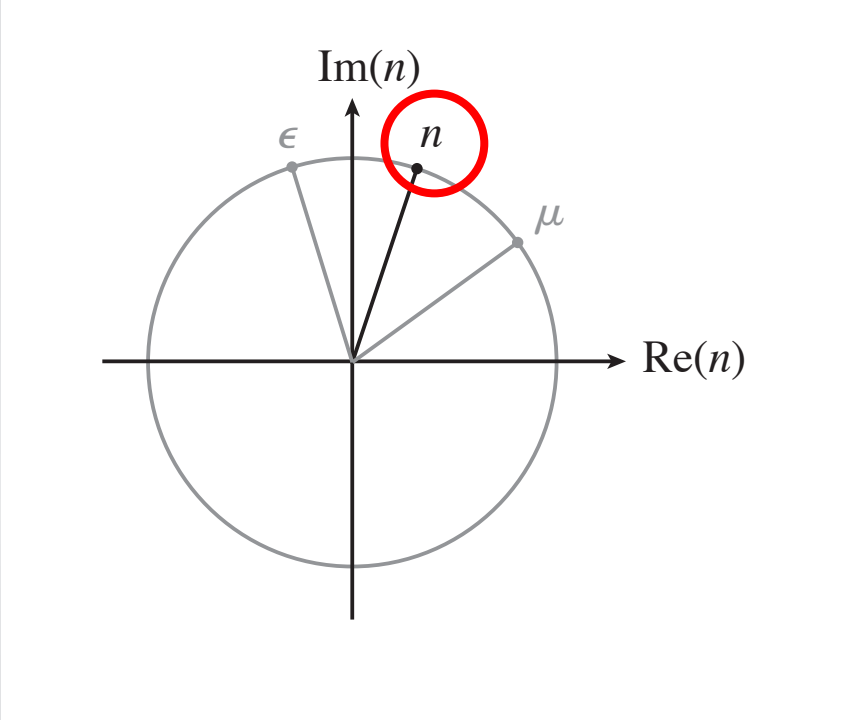
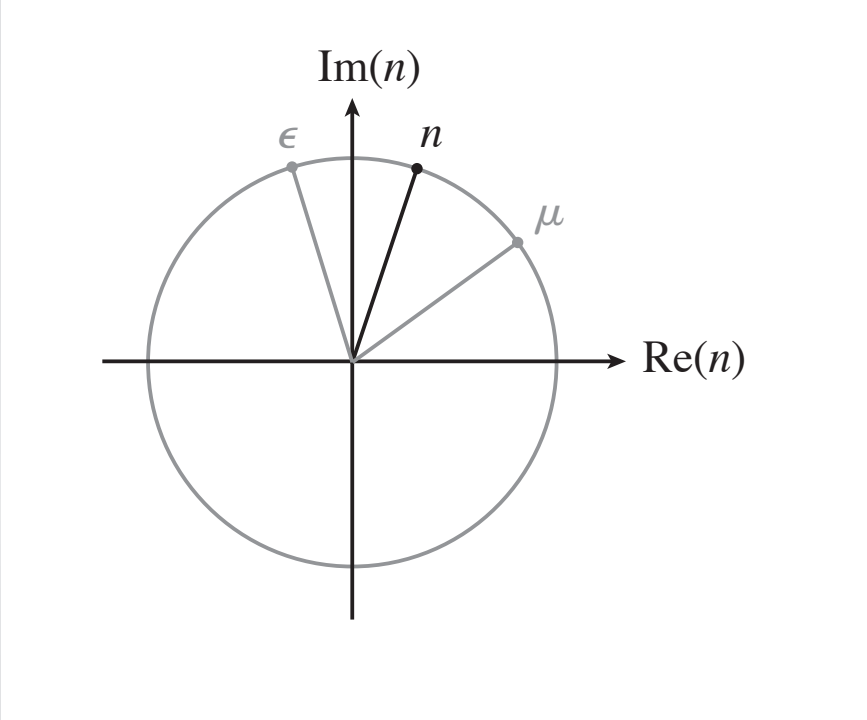


Q: Is this only possible value?

1. yes
2. no, there's one more
3. there are many more
4. it depends

There *is* another root…



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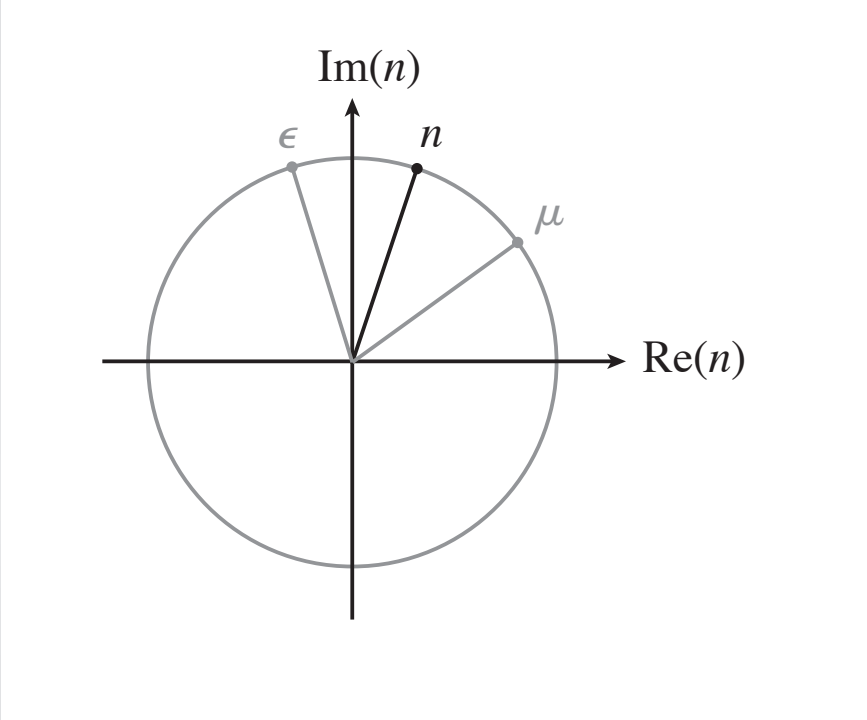
Can add 2π to exponent

$$e^{+i(\theta+\phi)} = e^{+i[\theta+\phi+2\pi]}$$



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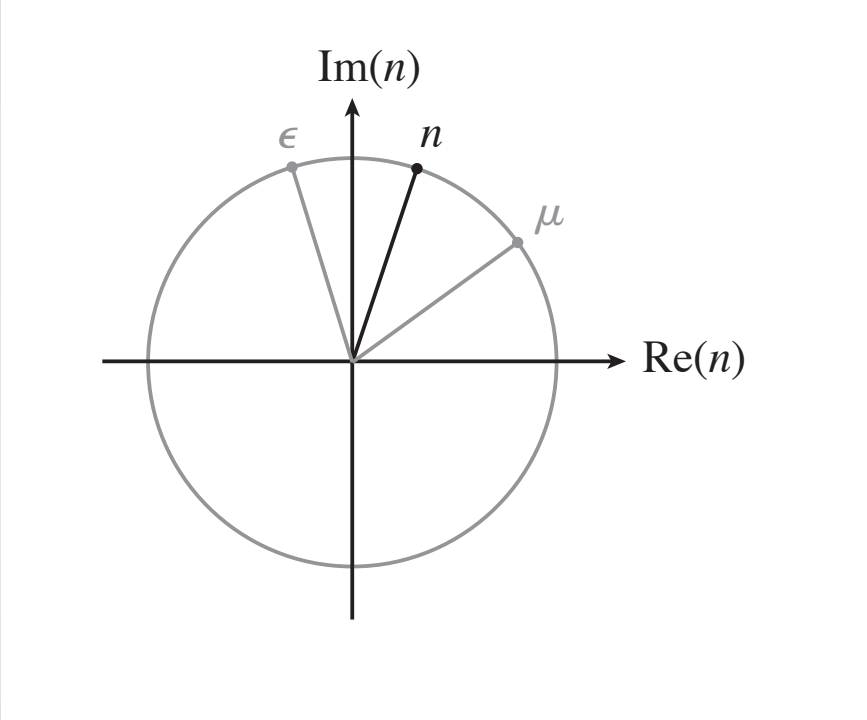

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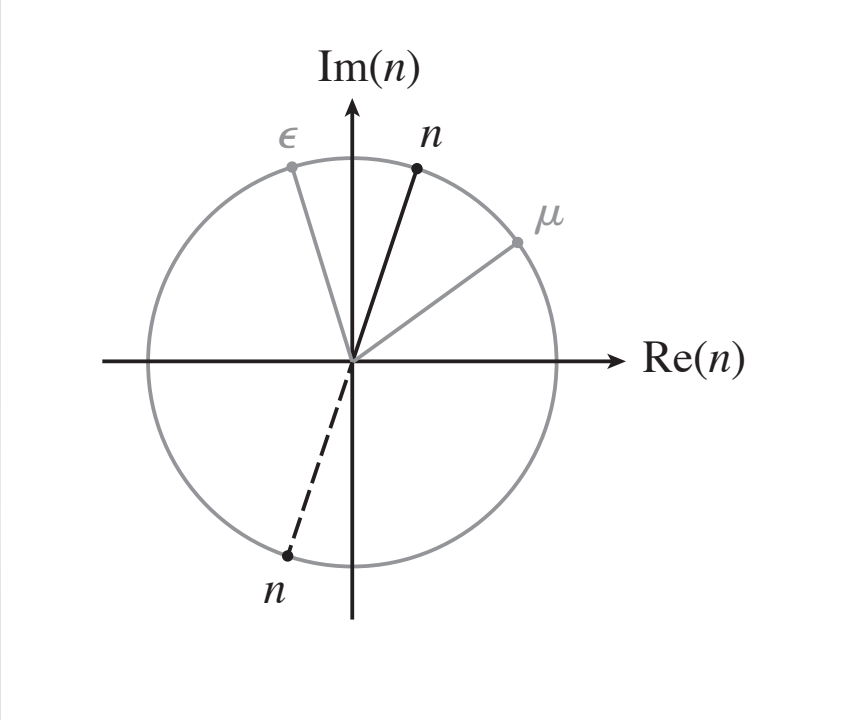

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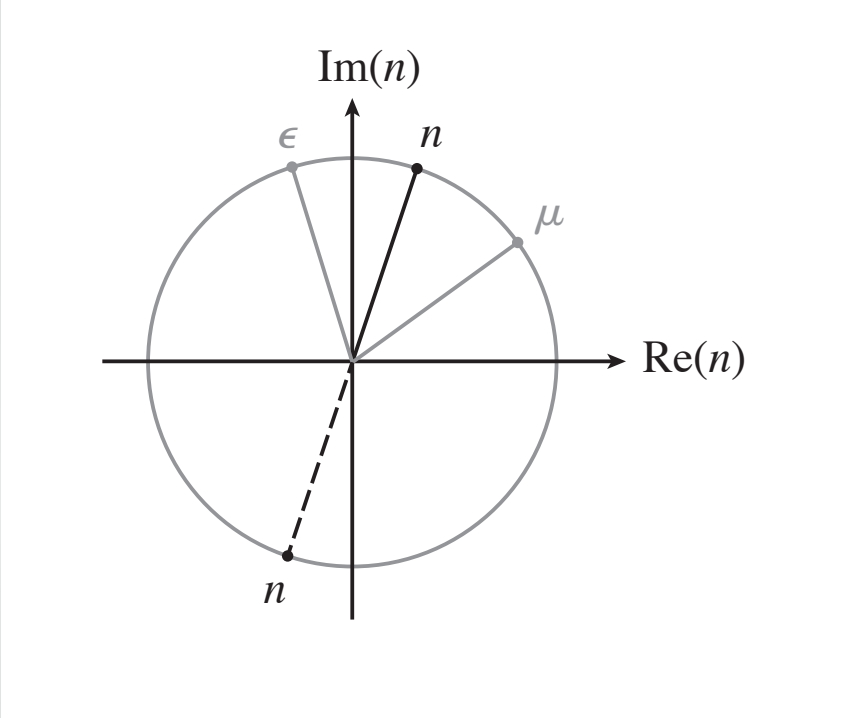

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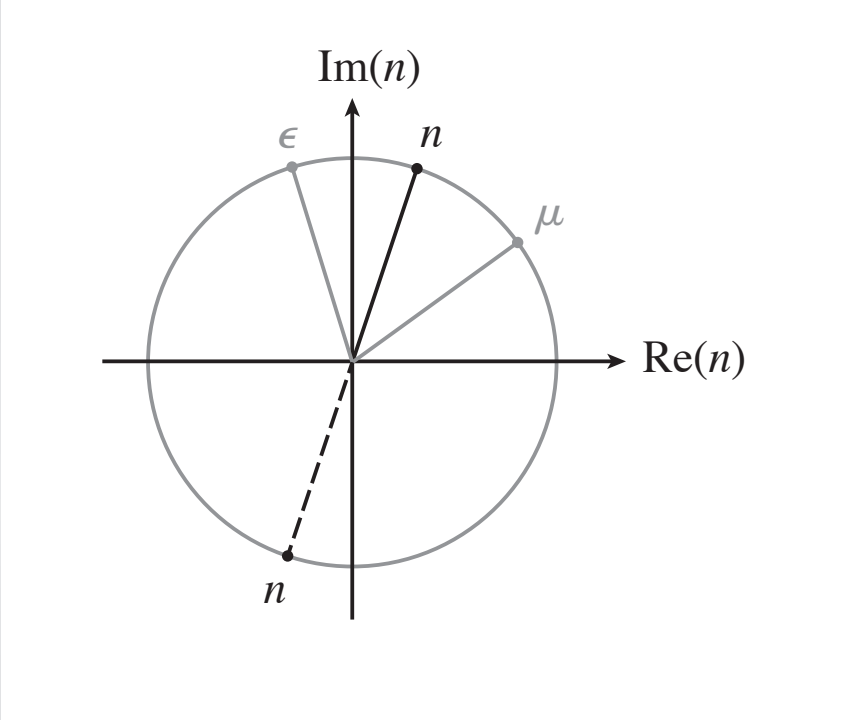

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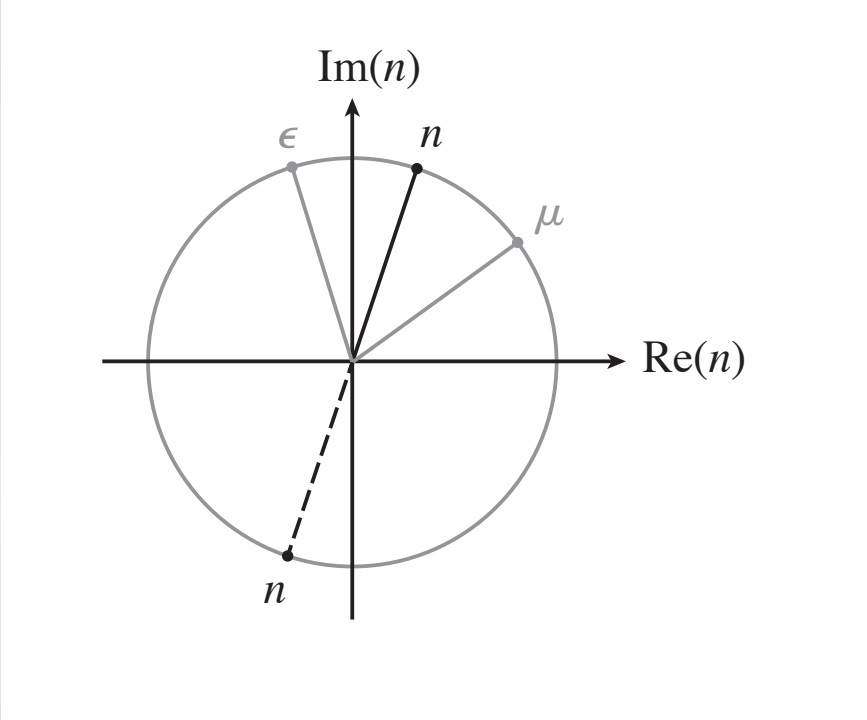

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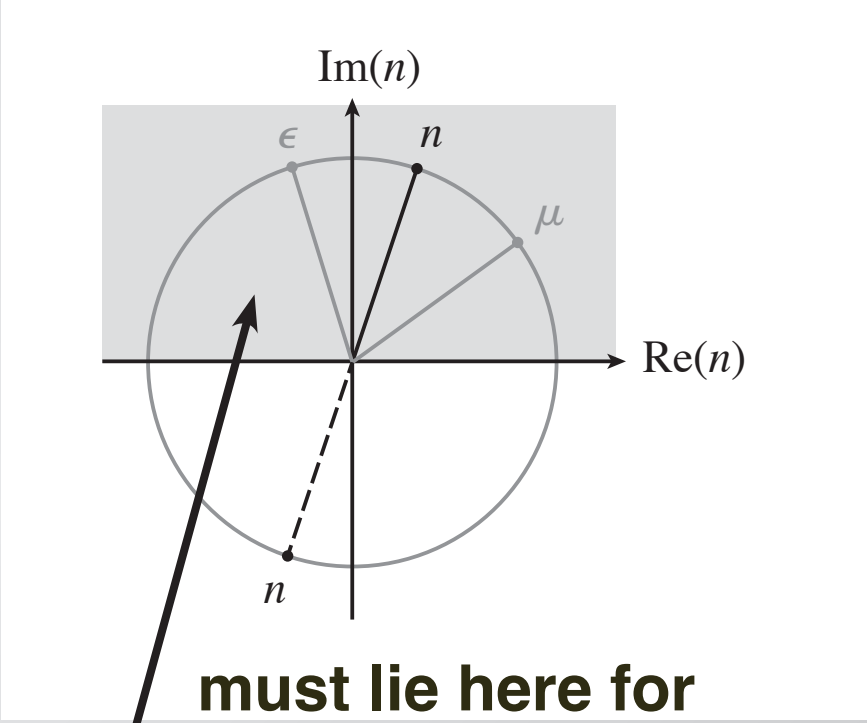

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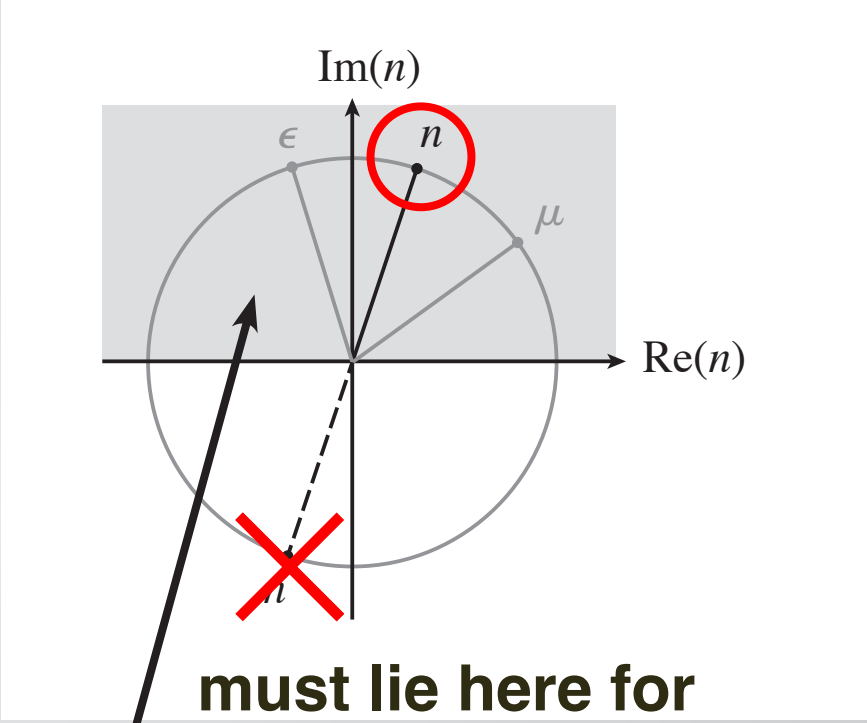

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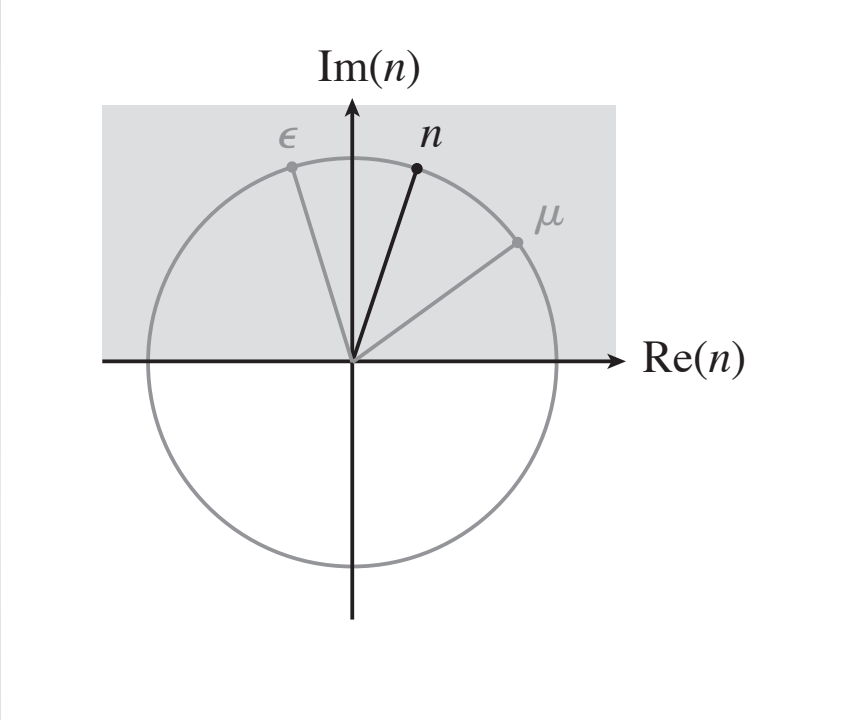

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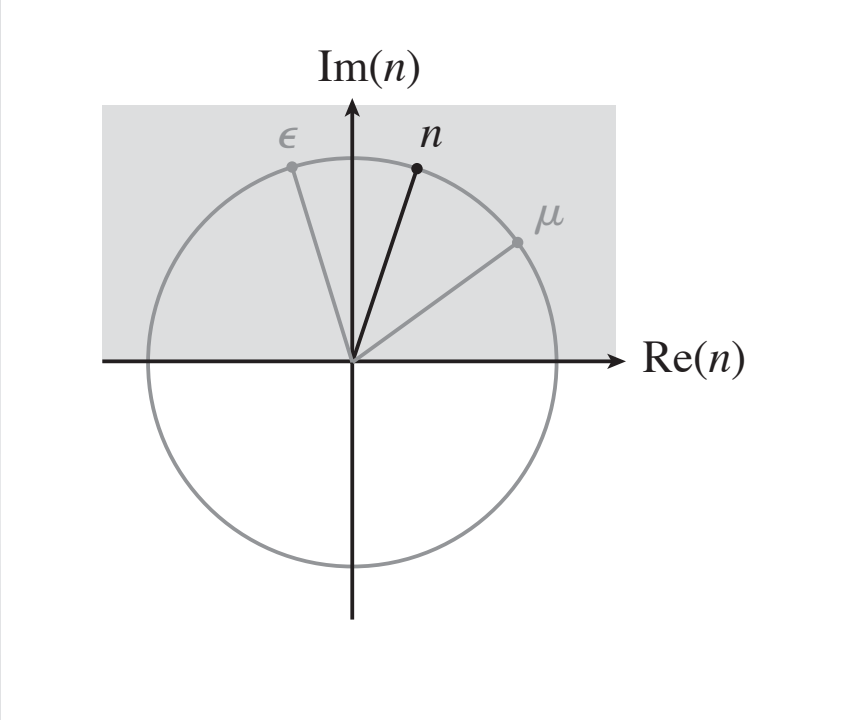

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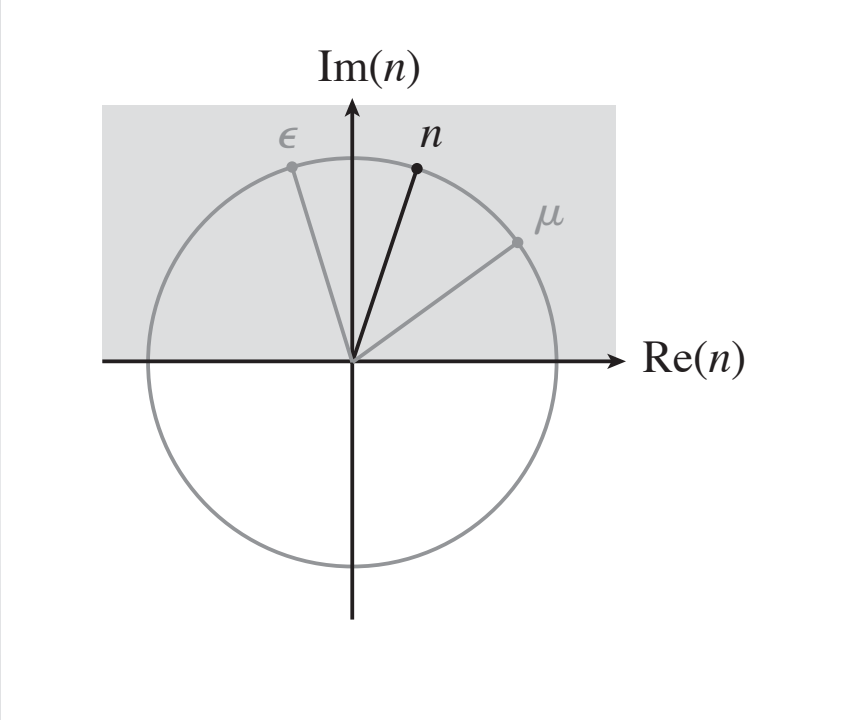

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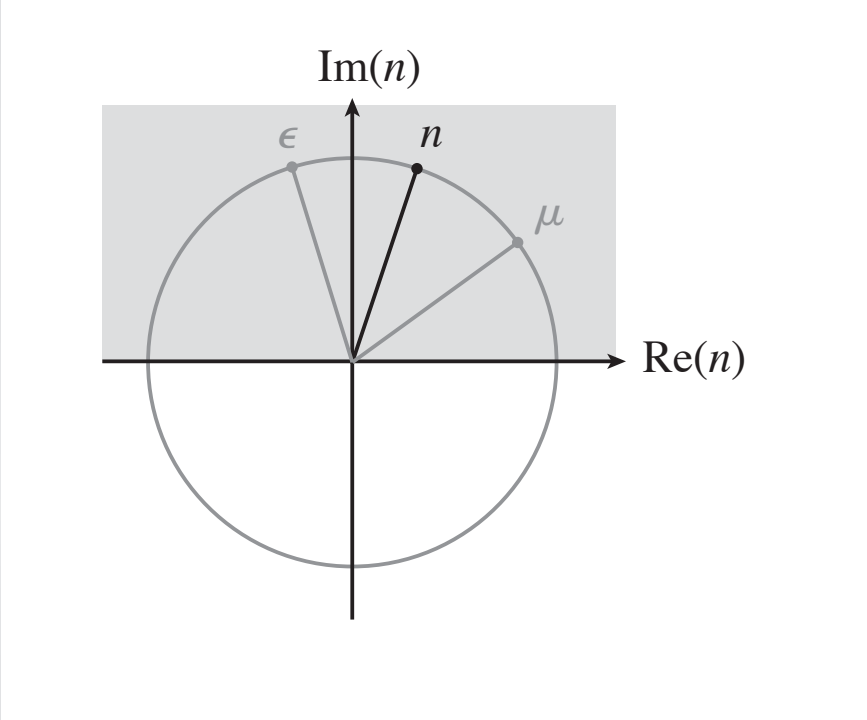
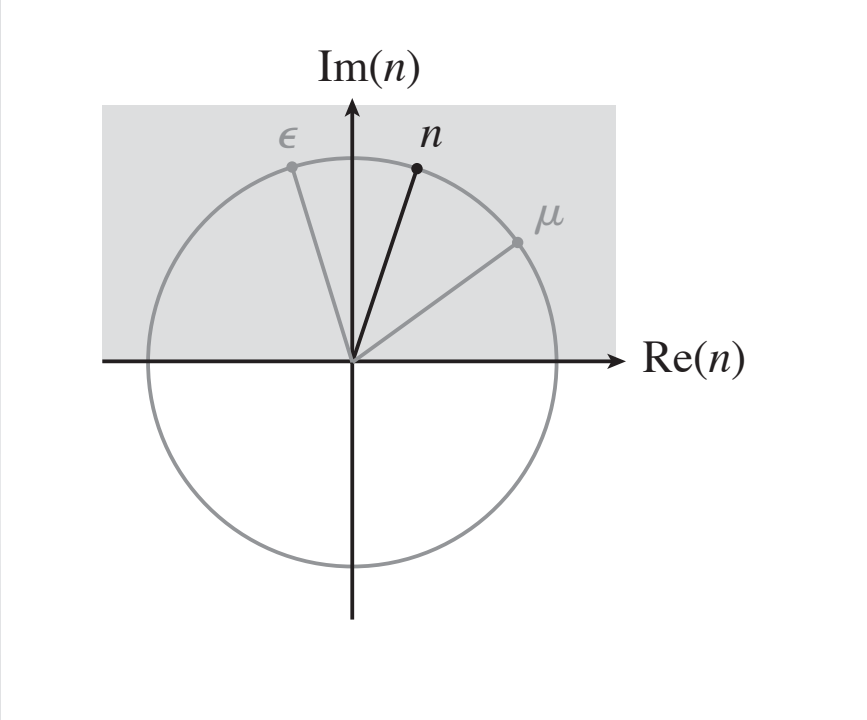


To find n (passive materials):

1. Draw line that bisects ε and μ

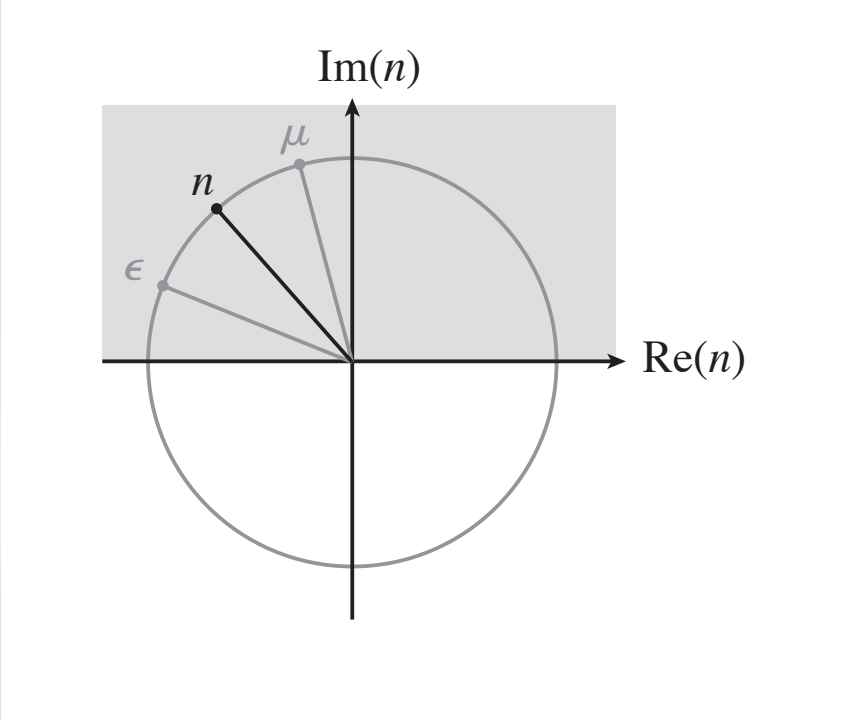
2. Choose upper branch



What happens when $\mathrm{Re}\varepsilon$ and/or $\mathrm{Re}\mu$ is negative?

For certain values of ε and μ

we can get a *negative* $\mathrm{Re}(n)$!

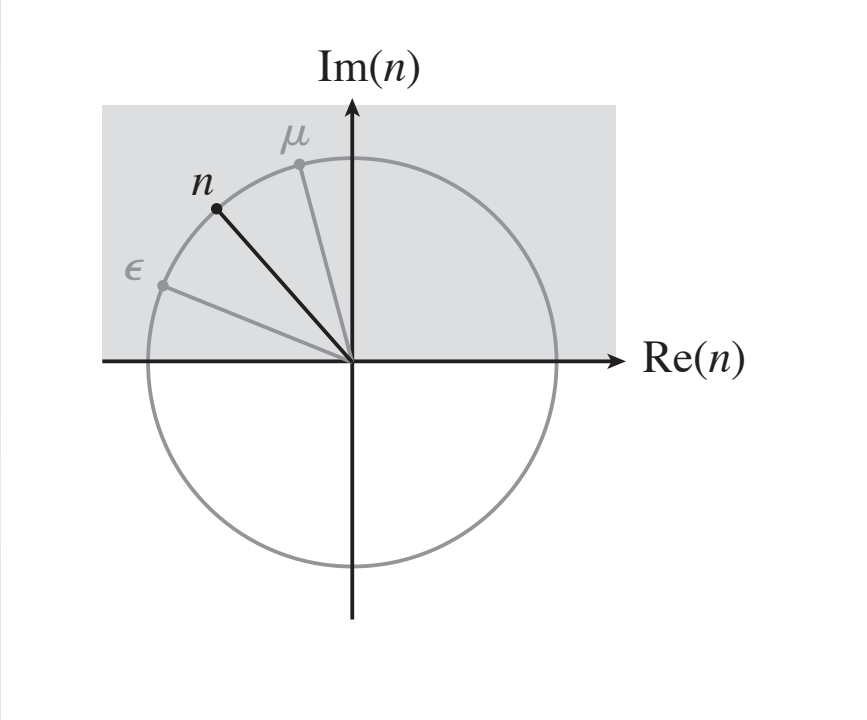


Q: Must both $\mathrm{Re}\varepsilon < 0$ and $\mathrm{Re}\mu < 0$

to get a negative $\mathrm{Re}(n)$?

1. yes

2. no



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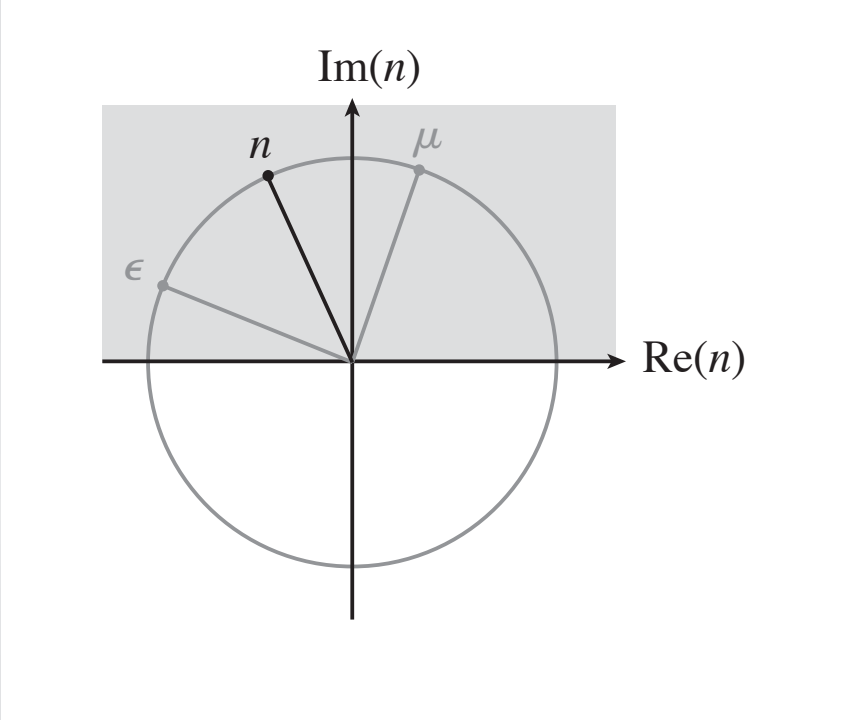
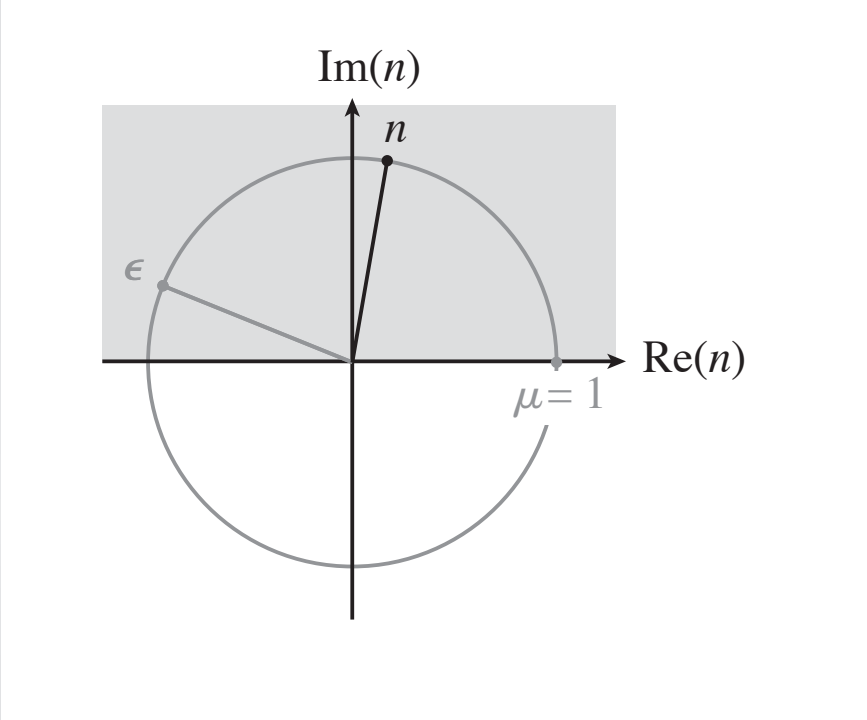


However, need magnetic response

to achieve $\mathrm{Re}(n) \leq 0$**!**



What happens when $\mathrm{Re}(n) < 0$?

What happens when $\mathrm{Re}(n) < 0$?

Remember

$$k = \frac{2\pi n}{\lambda_o} = \frac{2\pi(n' + in'')}{\lambda_o} = k' + ik''$$

$$E = E_o e^{i(kx - \omega t)} = E_o e^{-k''x} e^{i(k'x - \omega t)}$$

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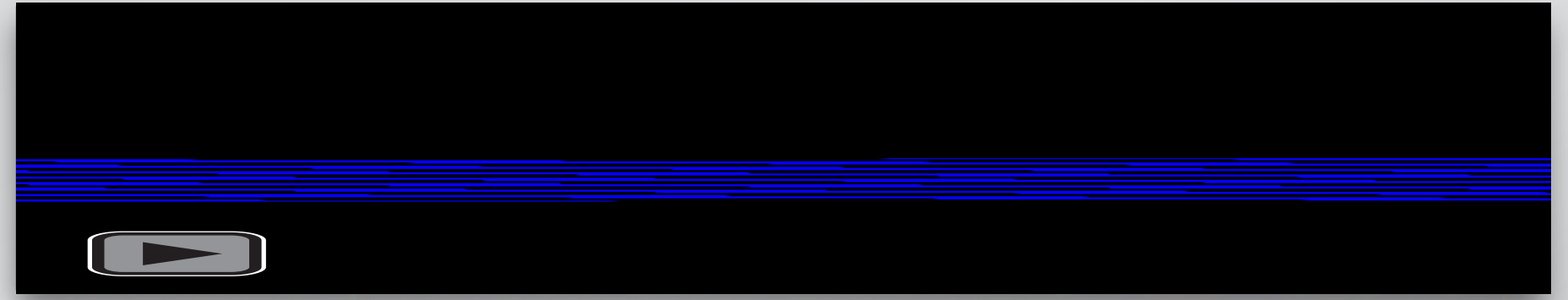
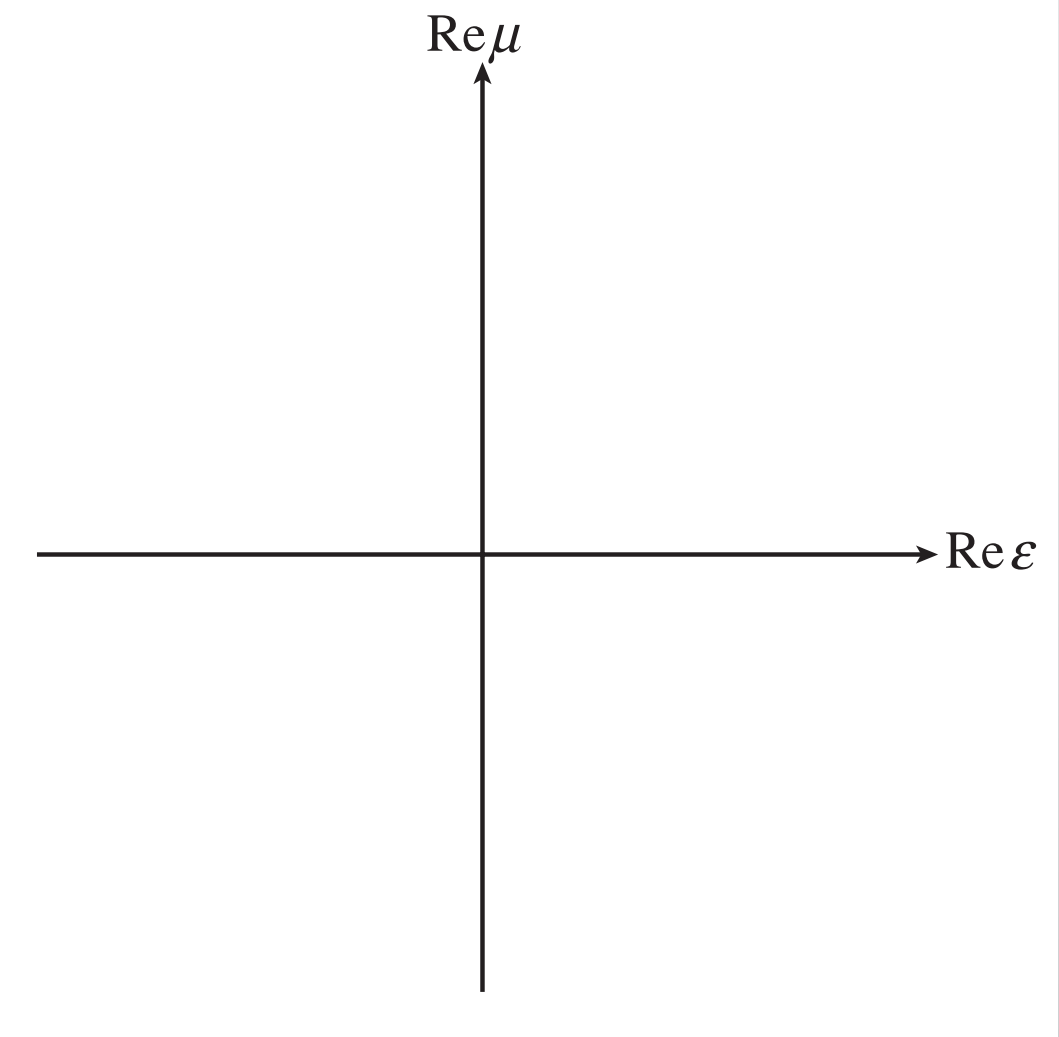
When $\mathrm{Re}(n) < 0,\ k' < 0,$ **and so phase velocity reversed!**

What happens when $\mathrm{Re}(n) < 0$?

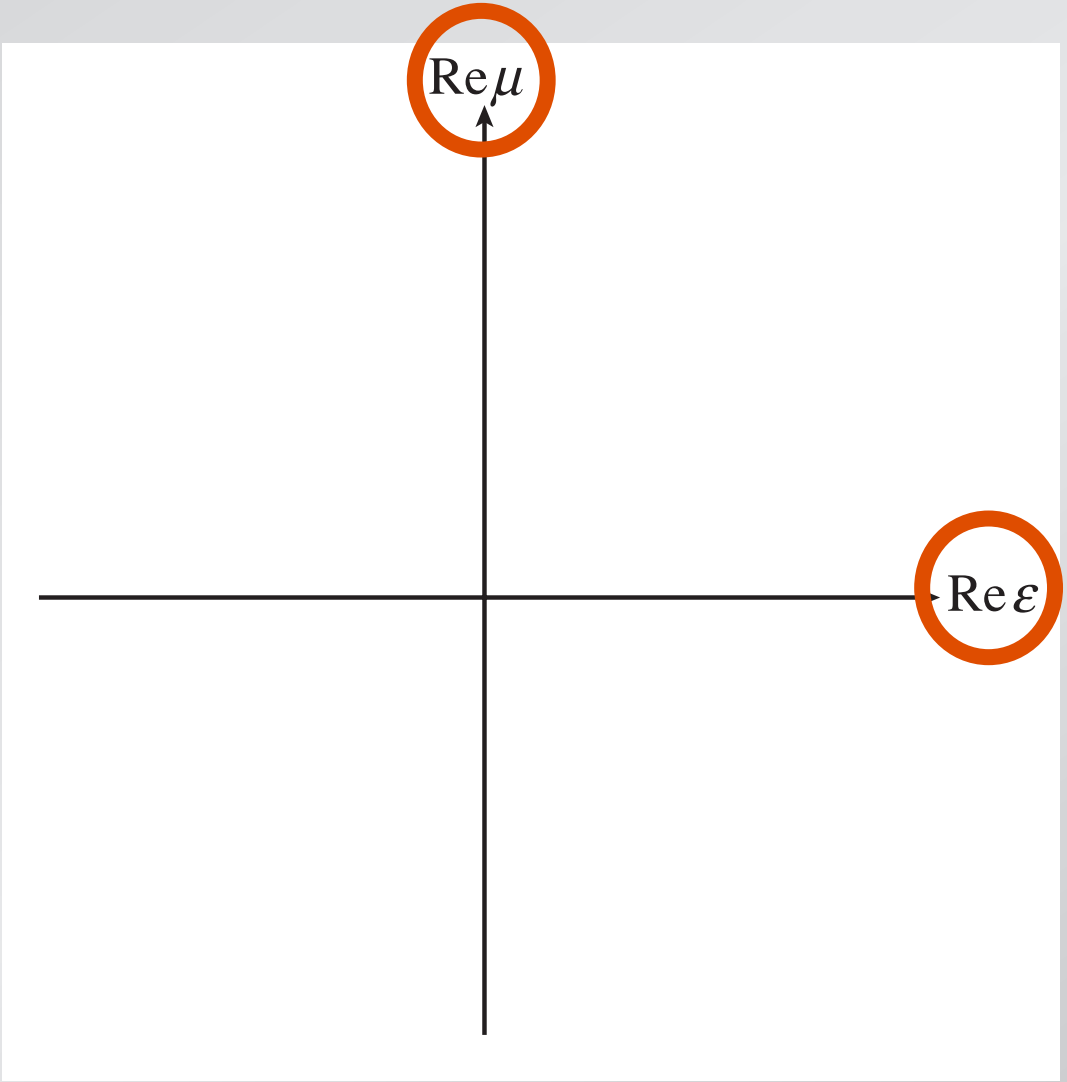


When $\mathrm{Re}(n) < 0,\ k' < 0,$ and so phase velocity reversed!

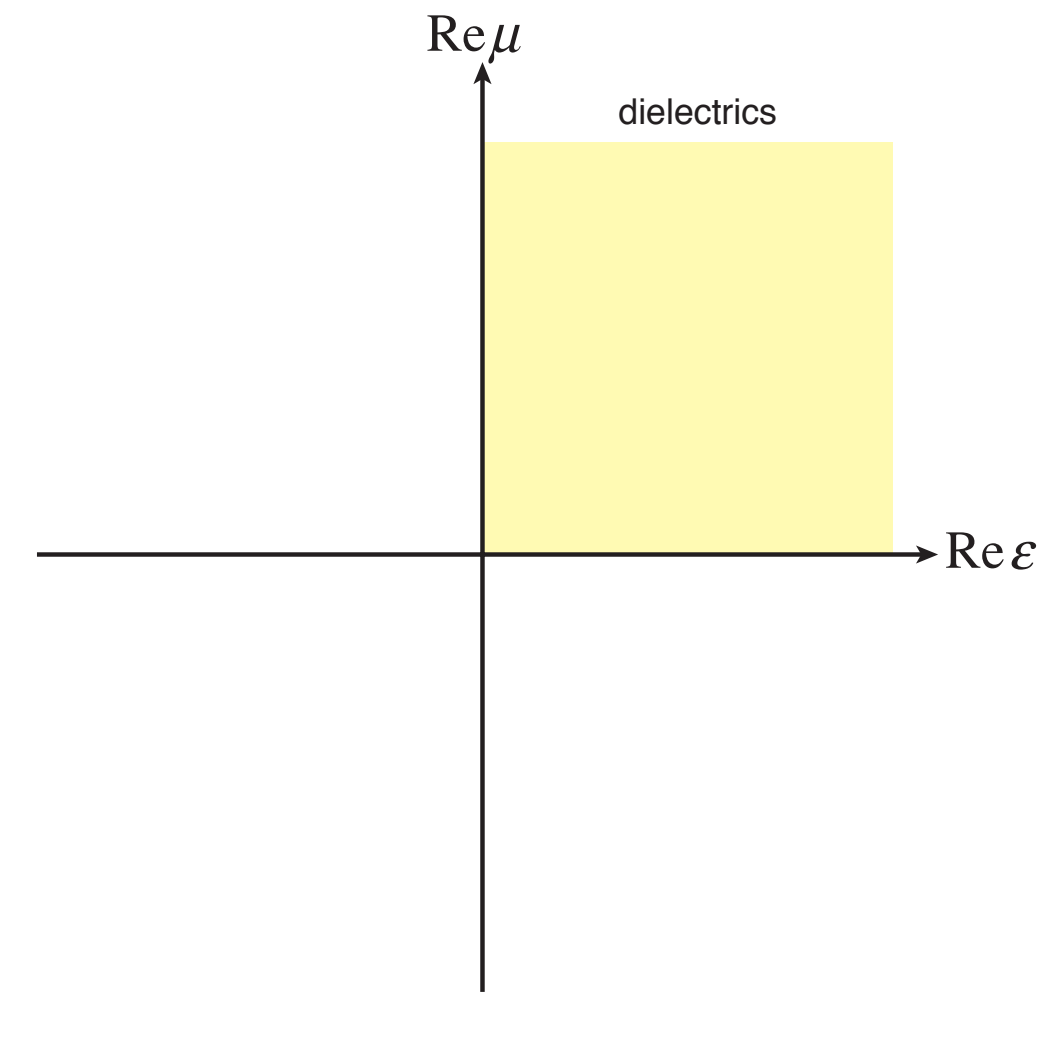
classification of (non-lossy) materials



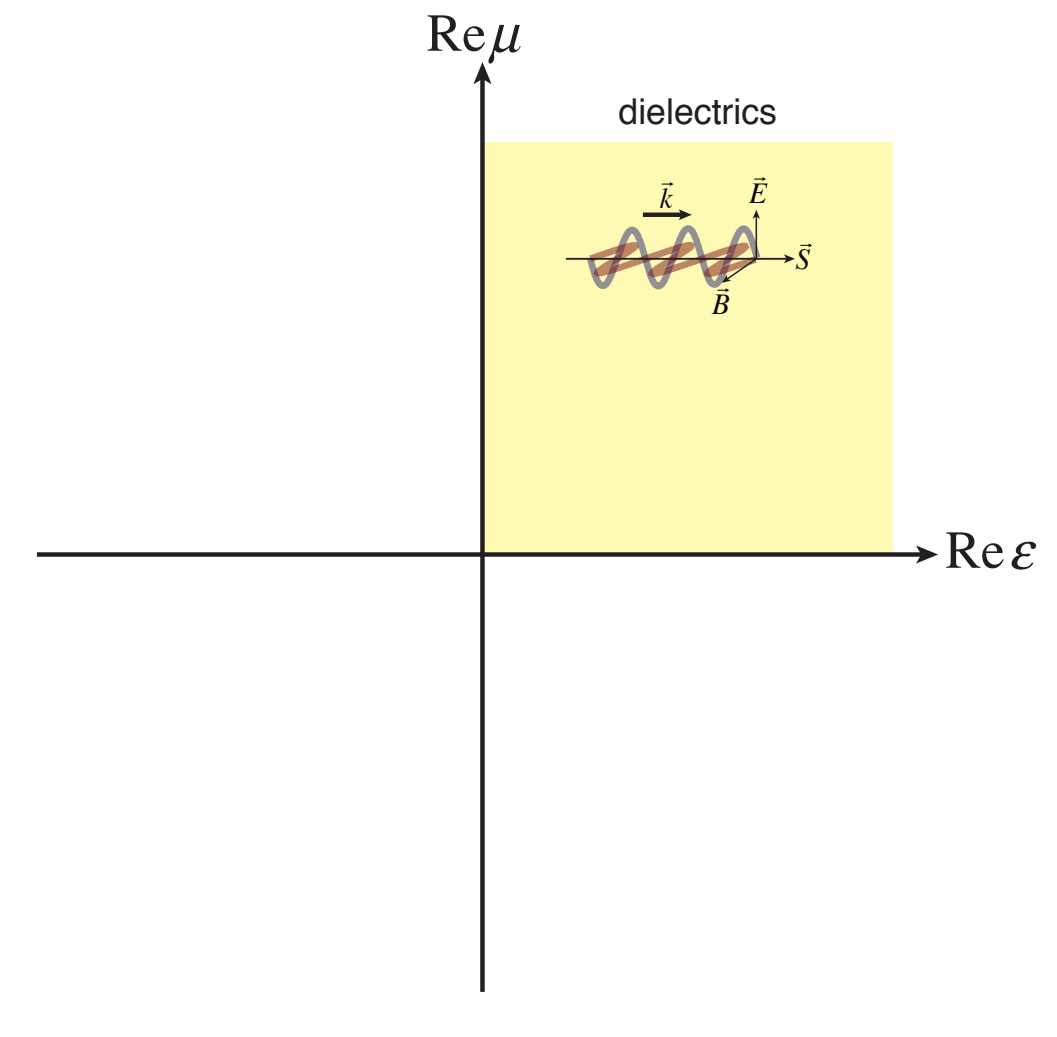
classification of (non-lossy) materials



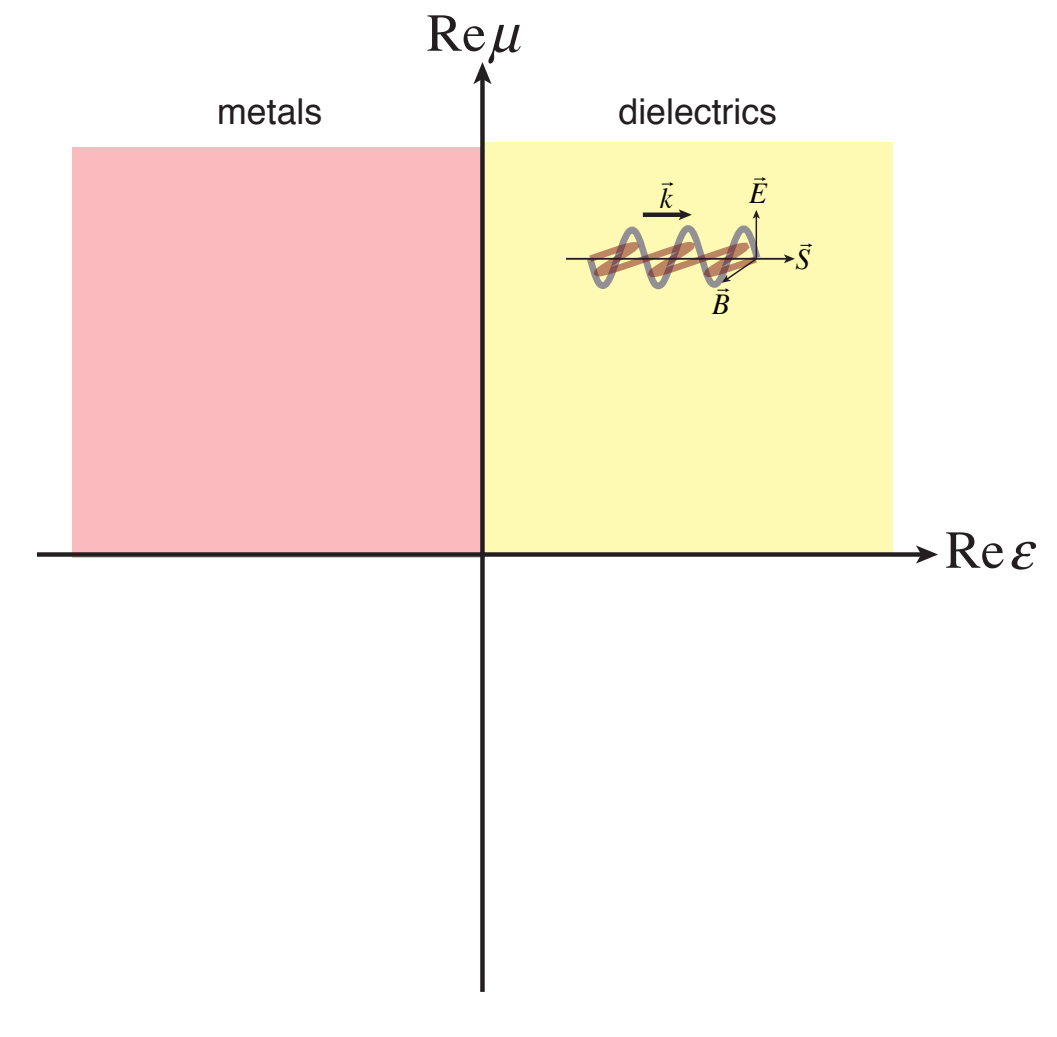
classification of (non-lossy) materials



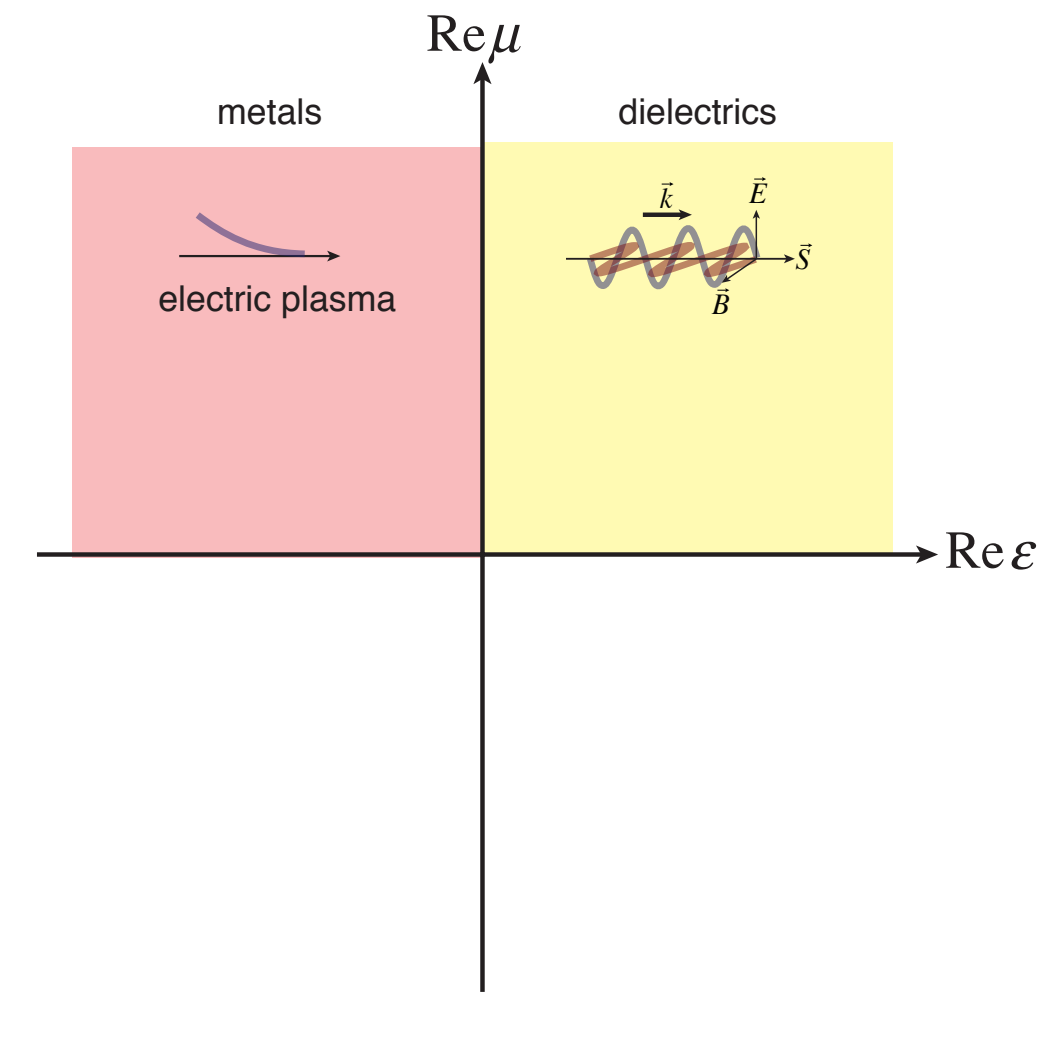
classification of (non-lossy) materials



classification of (non-lossy) materials



classification of (non-lossy) materials



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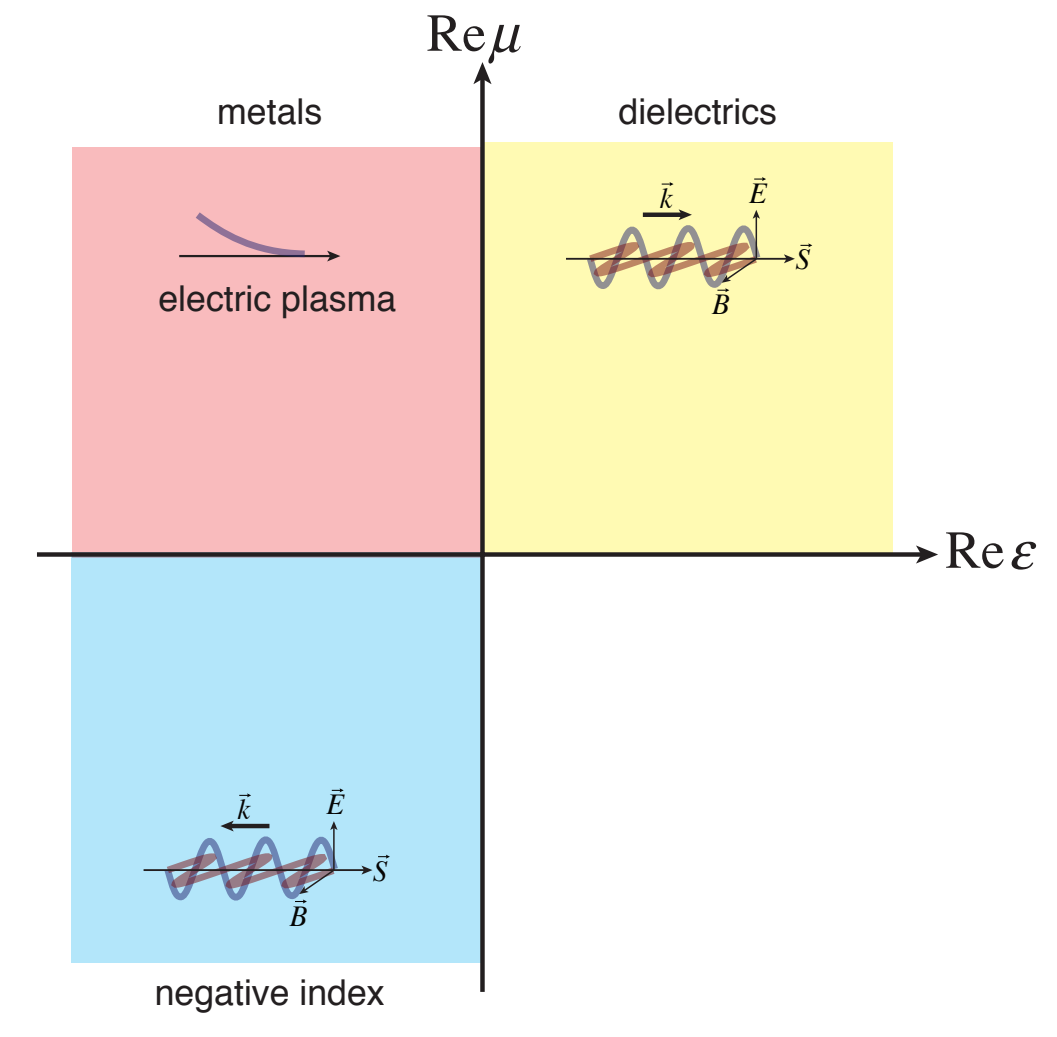

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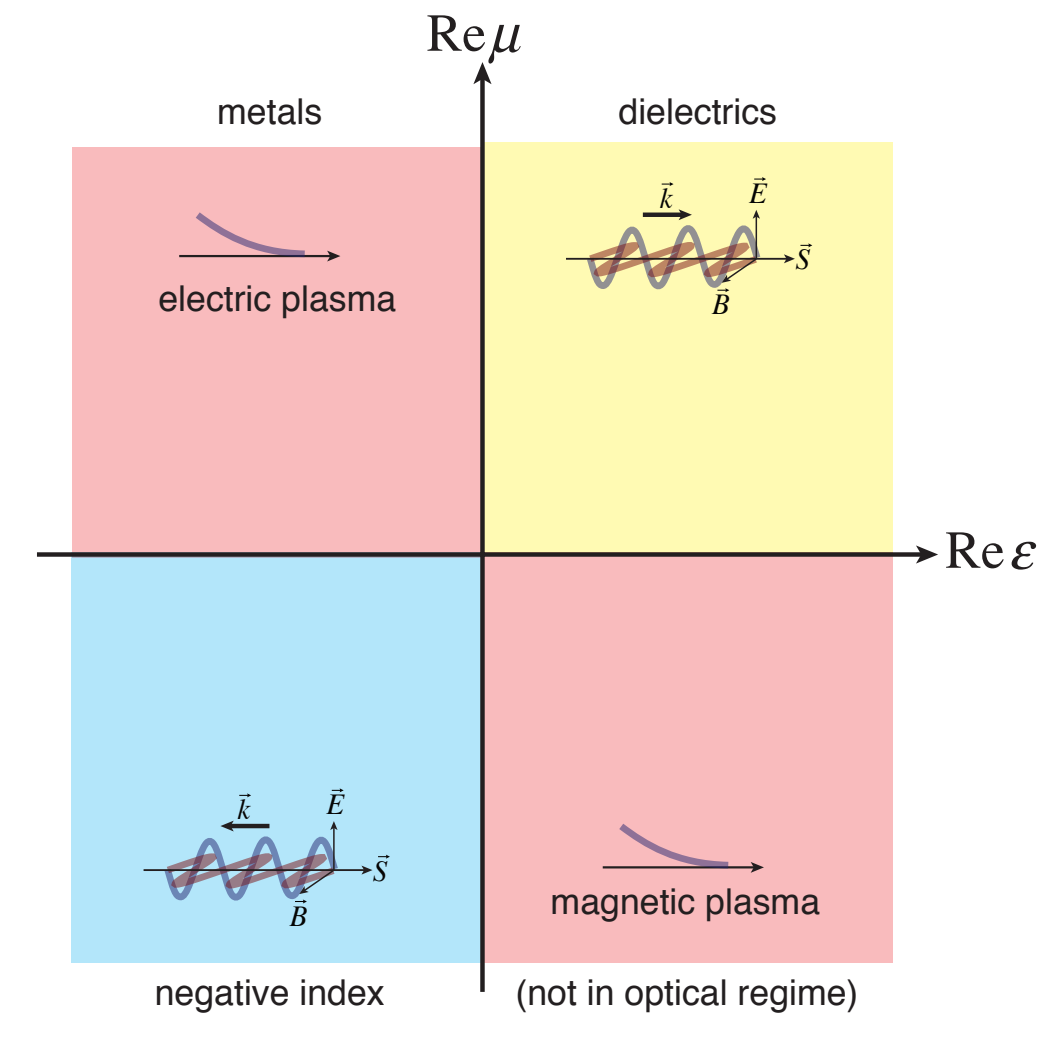

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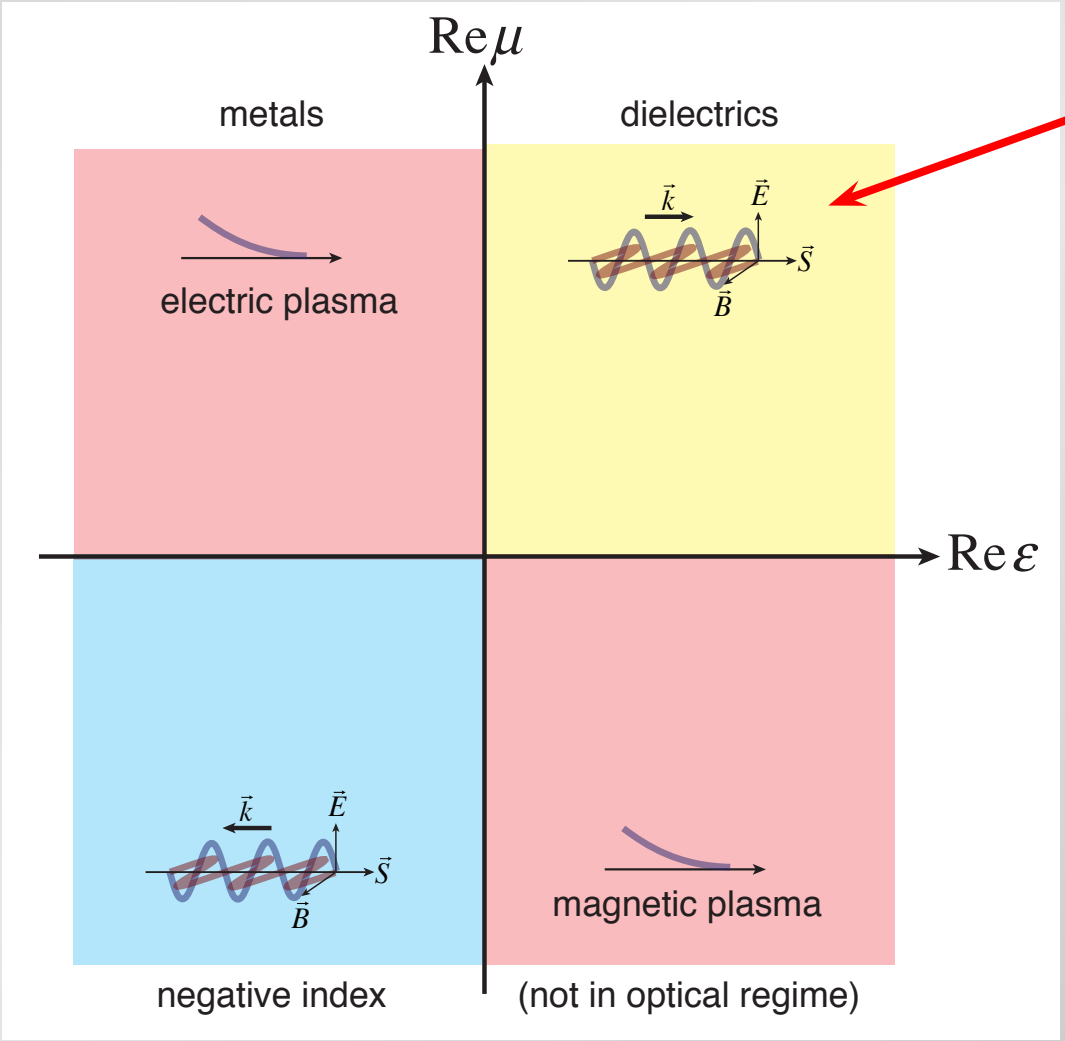
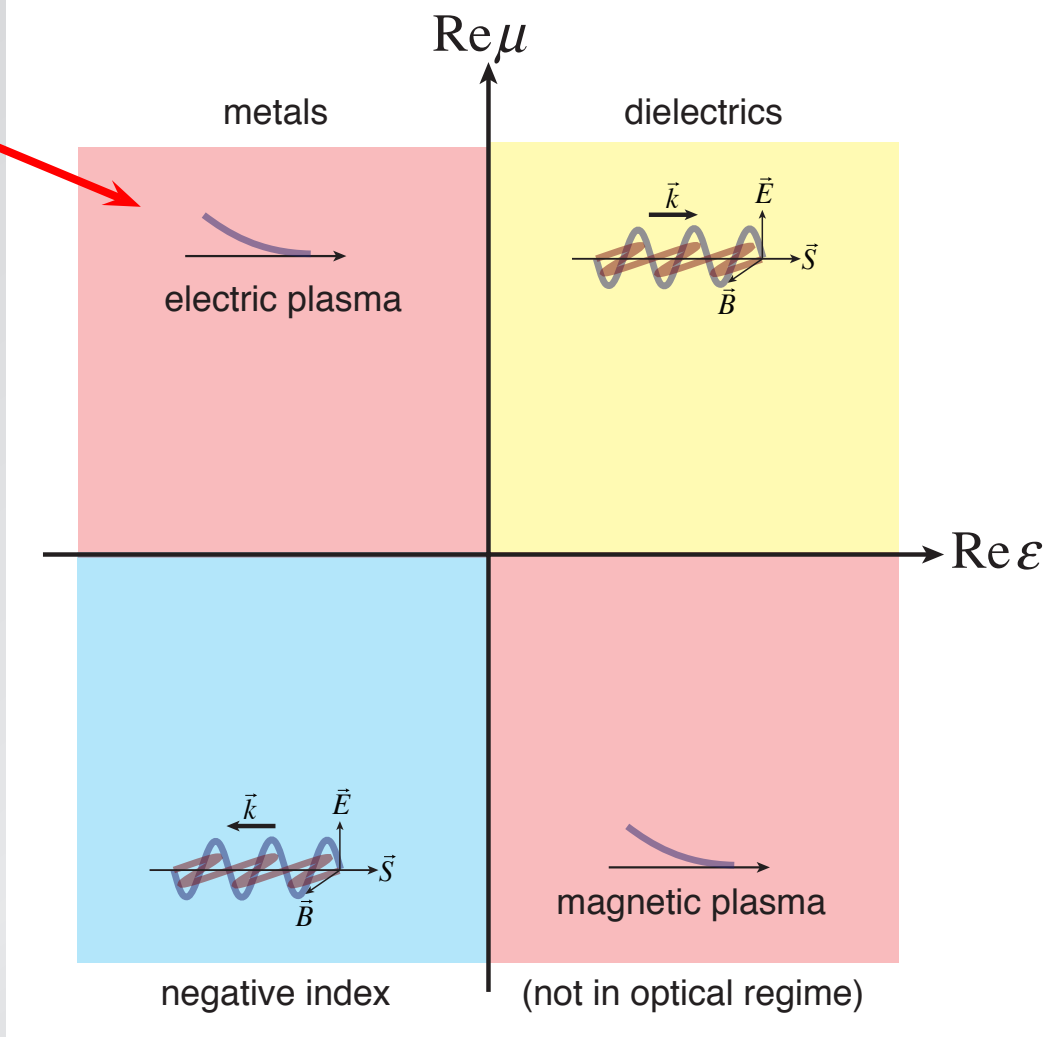


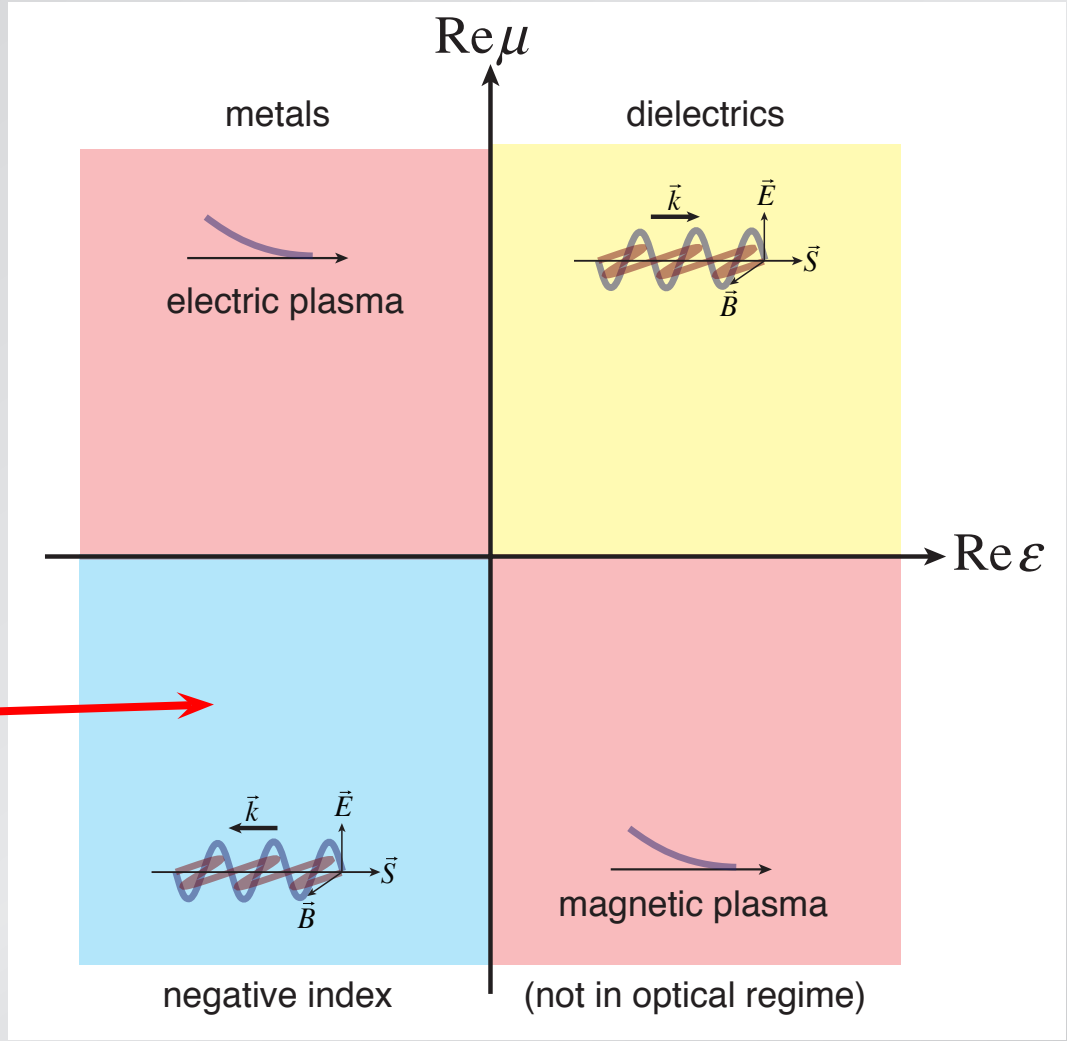
limited by diffraction

classification of (non-lossy) materials

no propagation

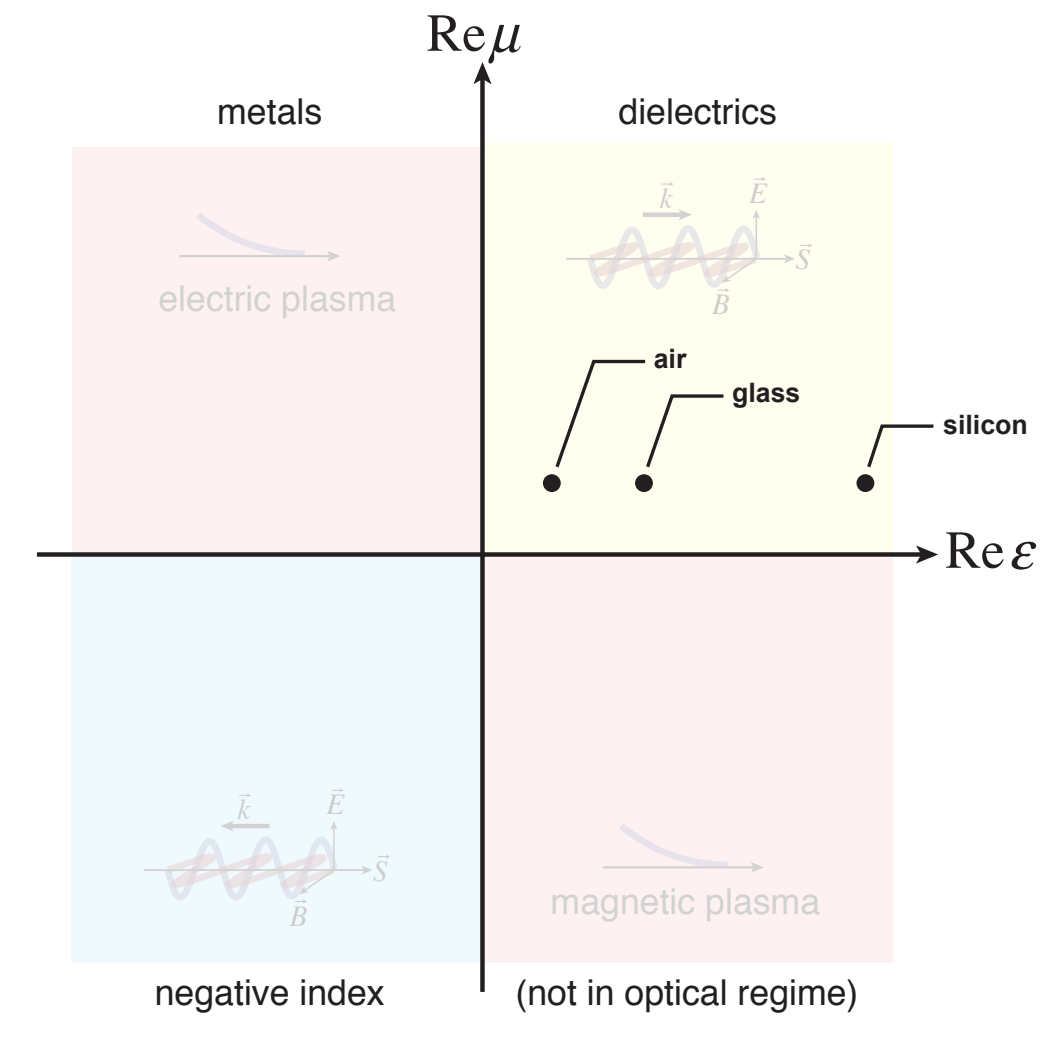


classification of (non-lossy) materials

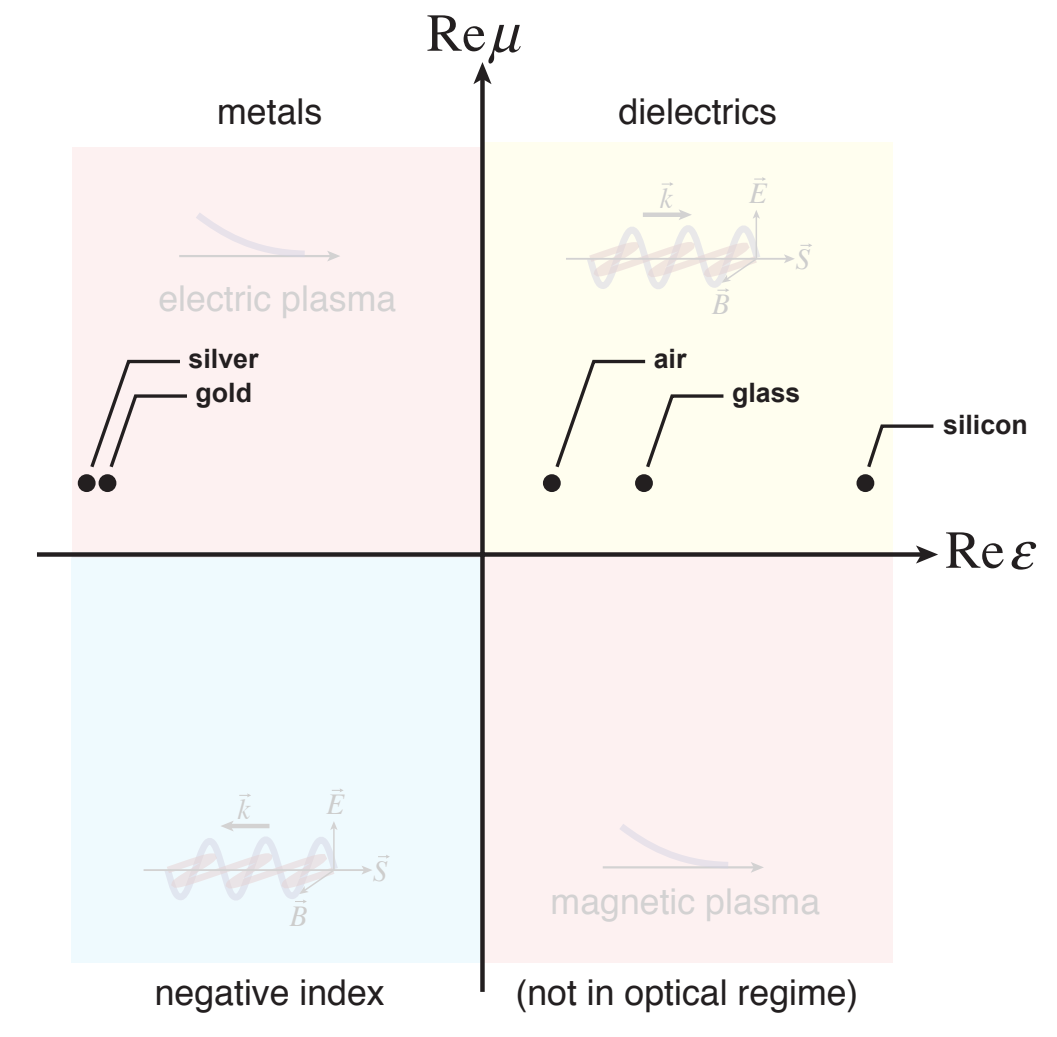


superlensing but…

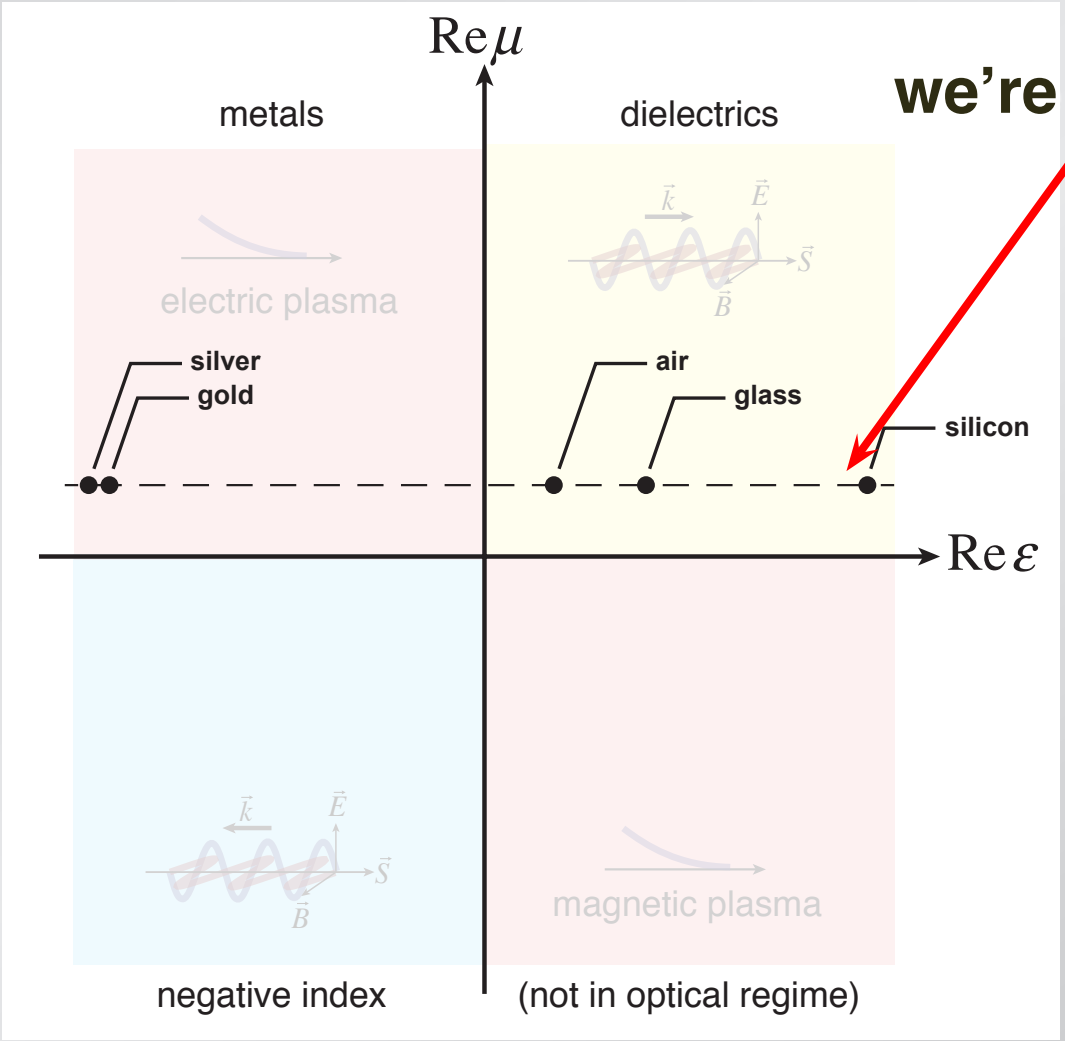
common materials very limited



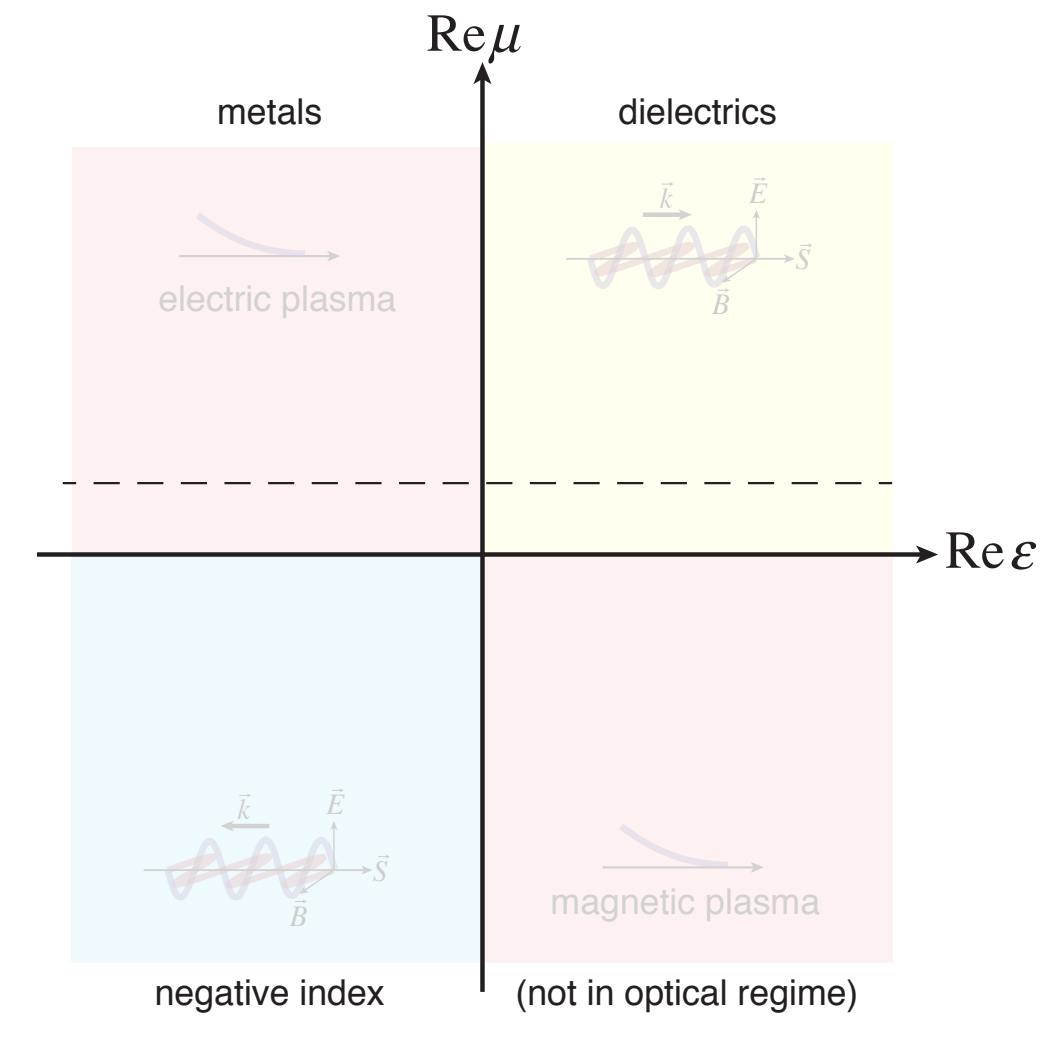
common materials very limited



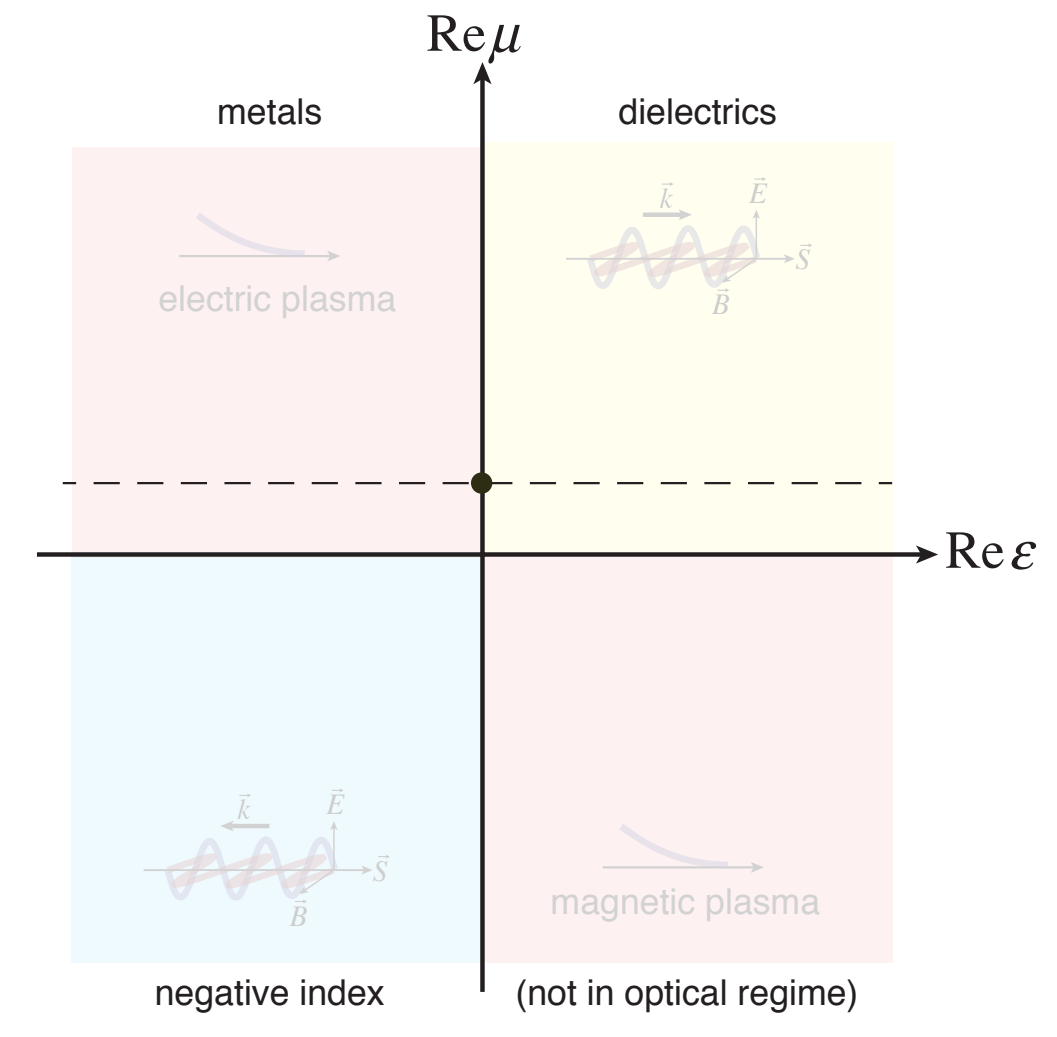
common materials very limited



What happens on the axes?



what if we let $\varepsilon = 0$?



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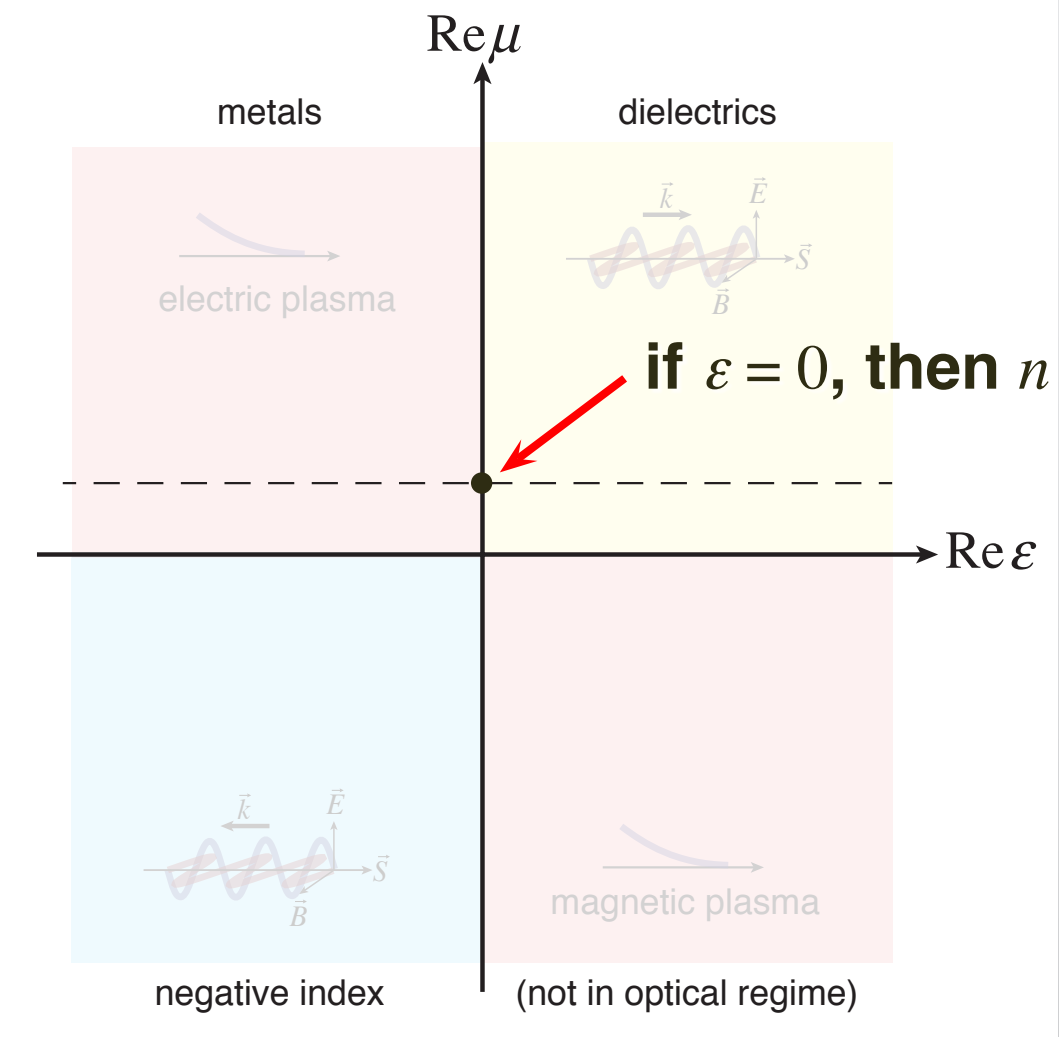

1 index

2 zero index

Q: If $n = 0$, which of the following is true?

1. the frequency goes to zero.
2. the phase velocity becomes infinite.
3. both of the above.
4. neither of the above.

1 index

2 zero index

wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

solution

$$\vec{E} = \vec{E}_o\, e^{i\,(kx - \omega t)}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}}\, c = \frac{1}{n}\, c$$

1 index 2 zero index

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1 index

2 zero index

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1 index 2 zero index

wave equation

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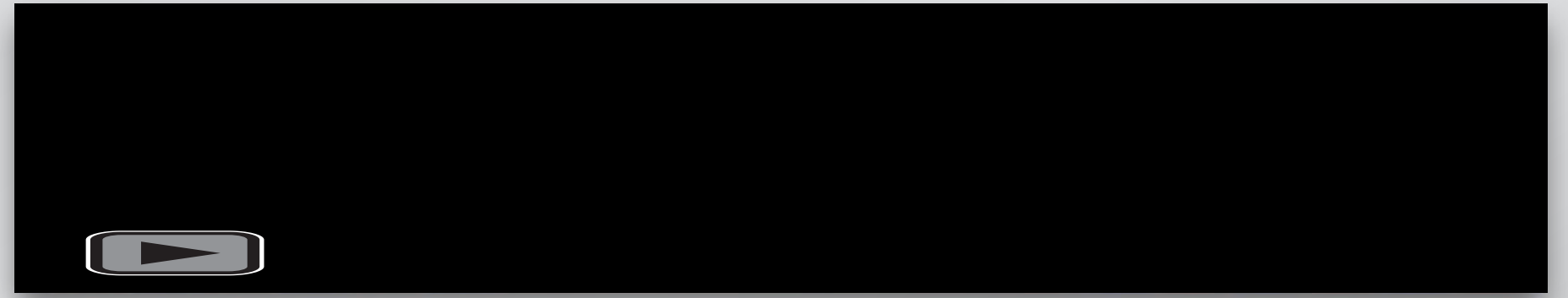
where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}} c = \frac{1}{n} c \longrightarrow \infty$$

1 index 2 zero index



1 index

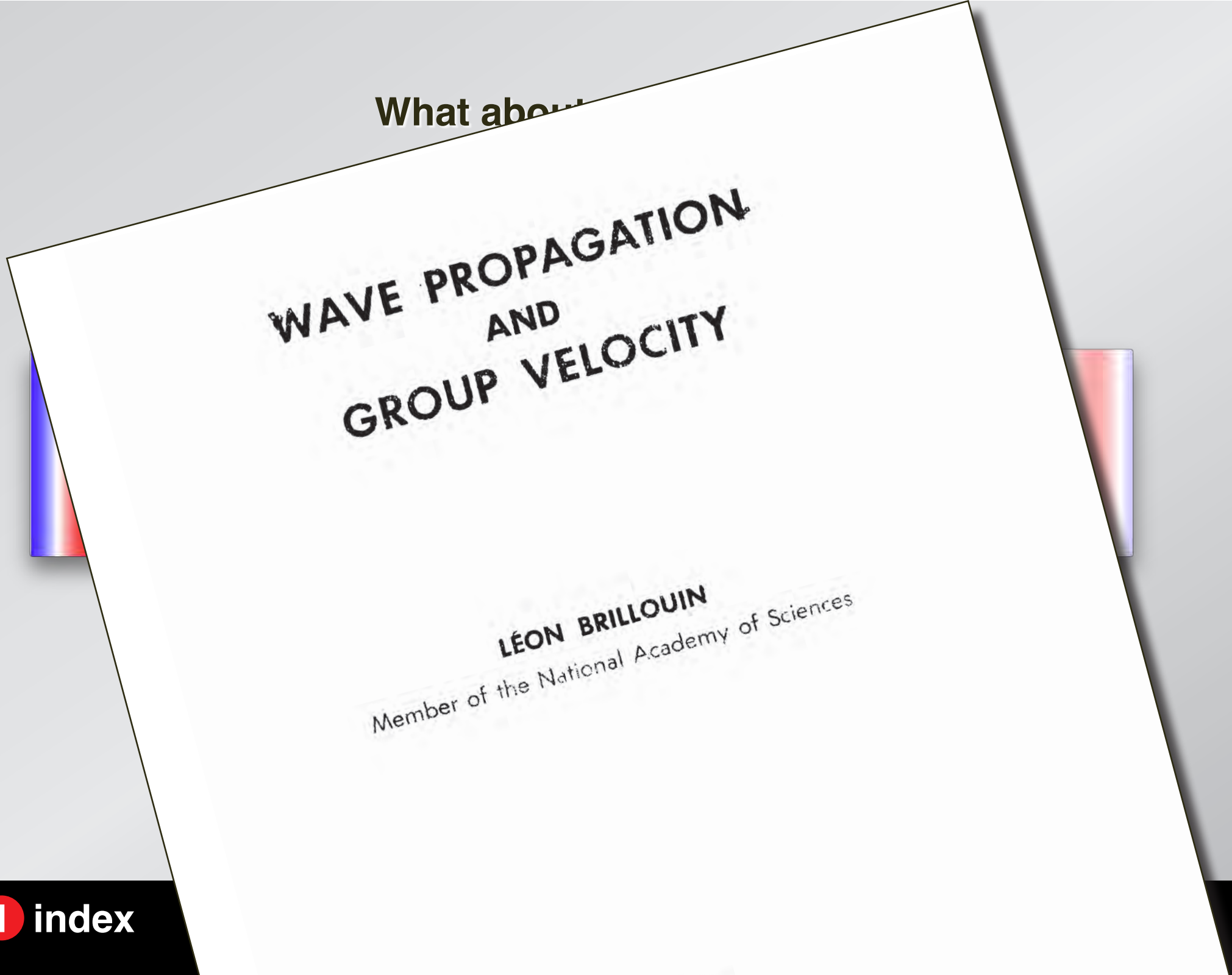
2 zero index

“Superluminal”?!

1 index

2 zero index

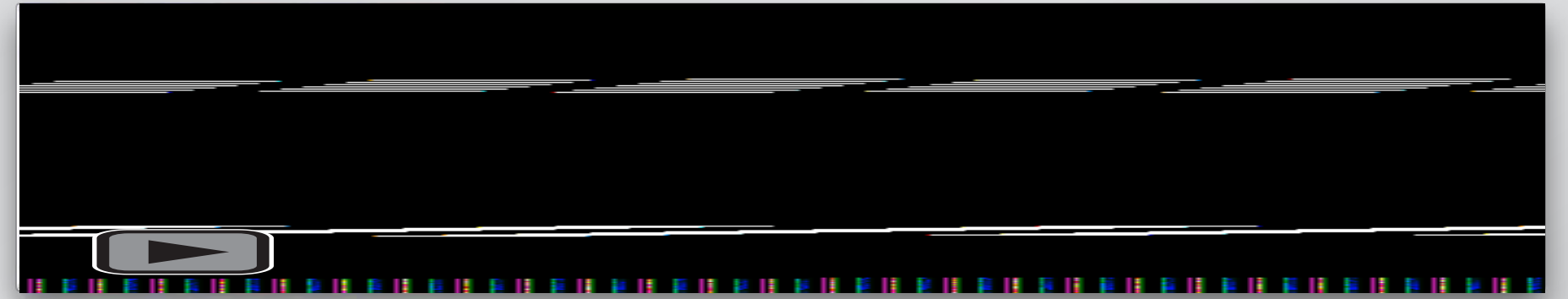
What abou



WAVE PROPAGATION
AND
GROUP VELOCITY

LÉON BRILLOUIN
Member of the National Academy of Sciences

What about causality?



1 index

2 zero index

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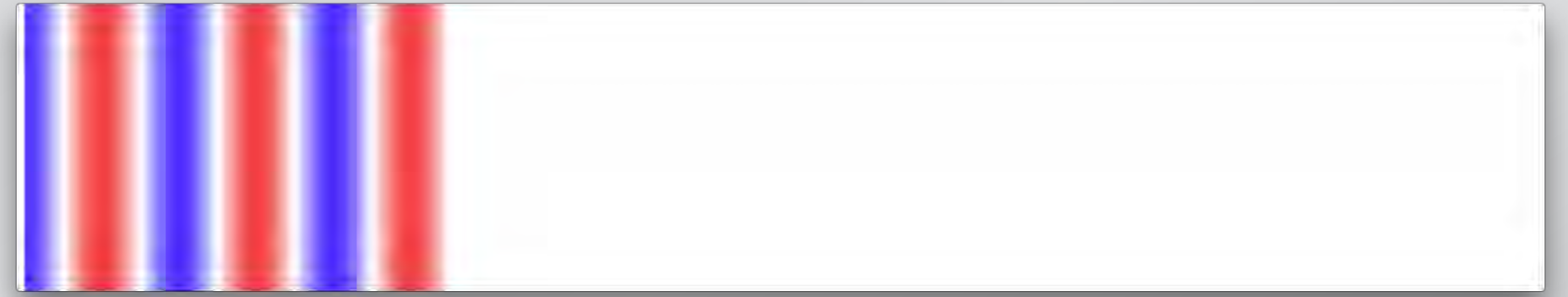

1 index

2 zero index

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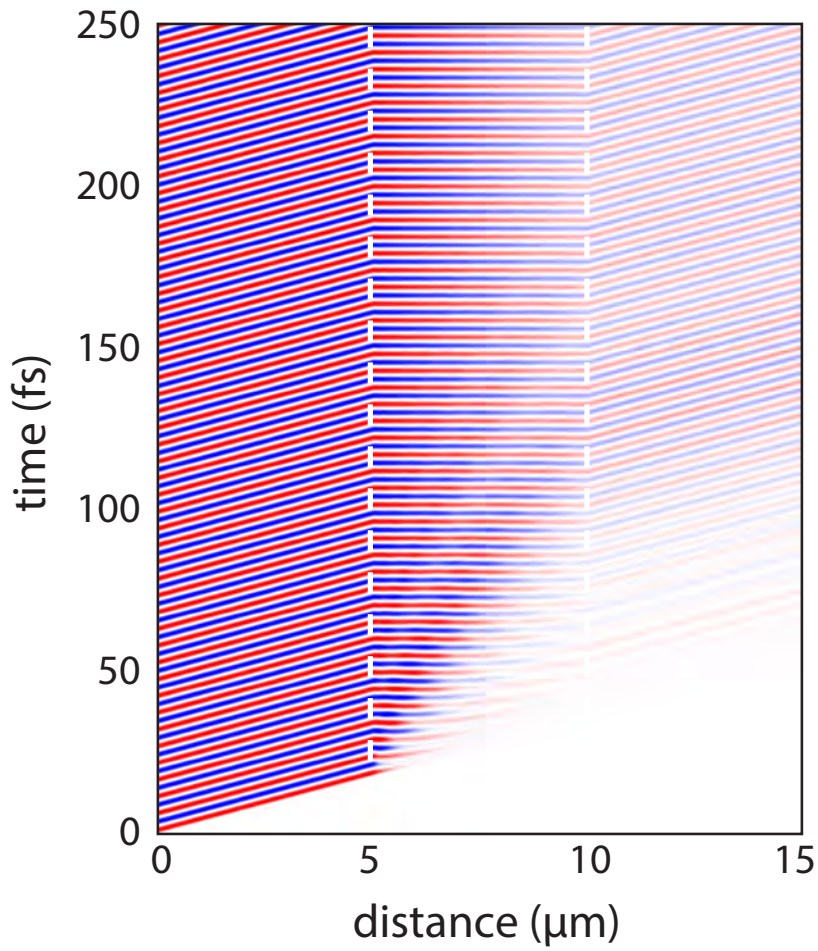
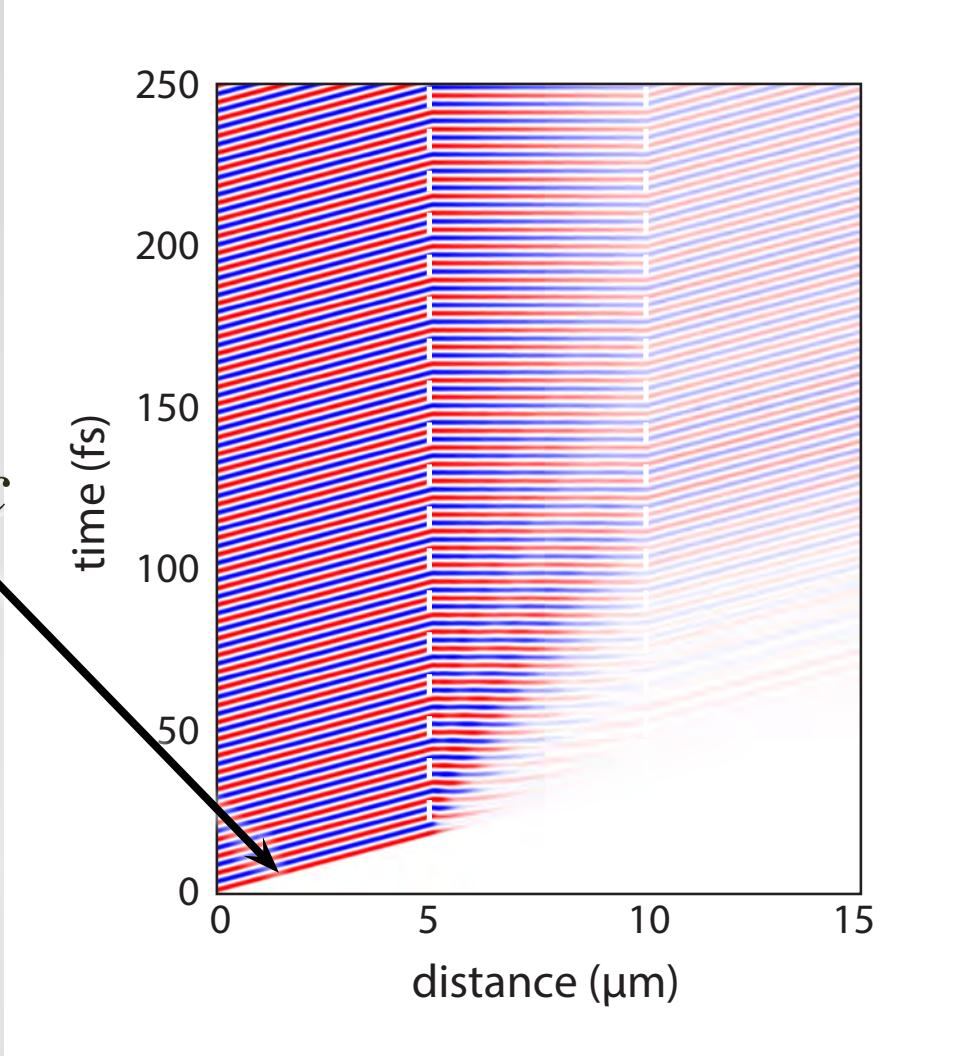


1 index

2 zero index

What about causality?

speed of light c



1 index

2 zero index

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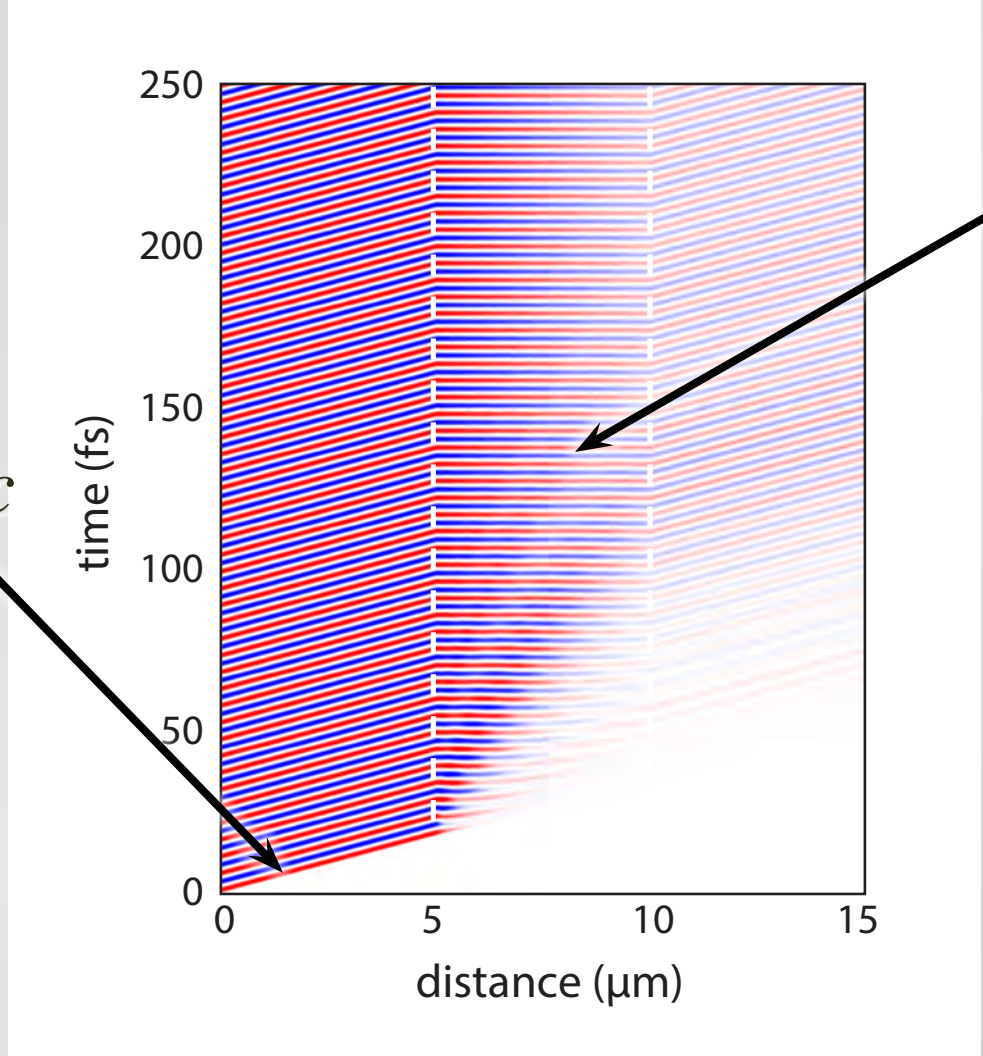


constant phase

speed of light c

1 index

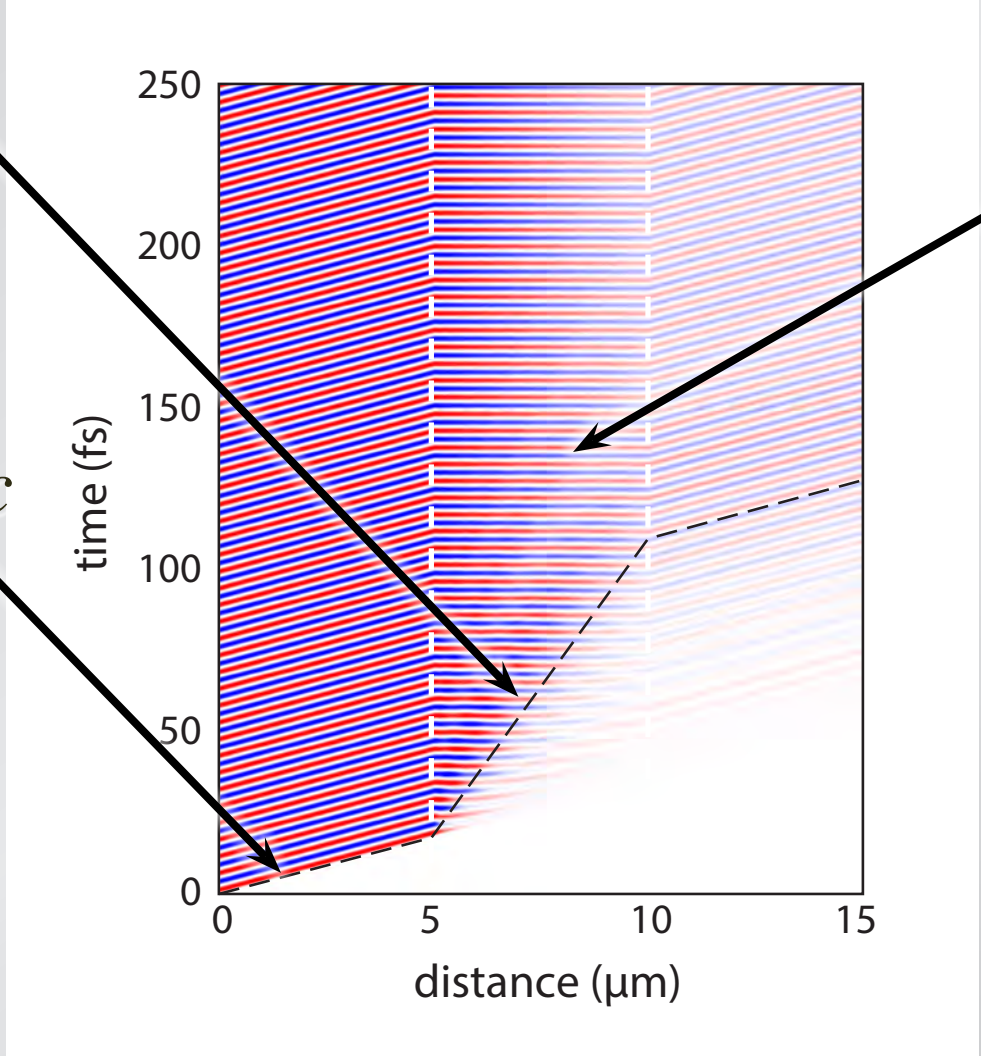
2 zero index

What about causality?

group velocity

$v_g < c$

speed of light c



constant phase

1 index

2 zero index

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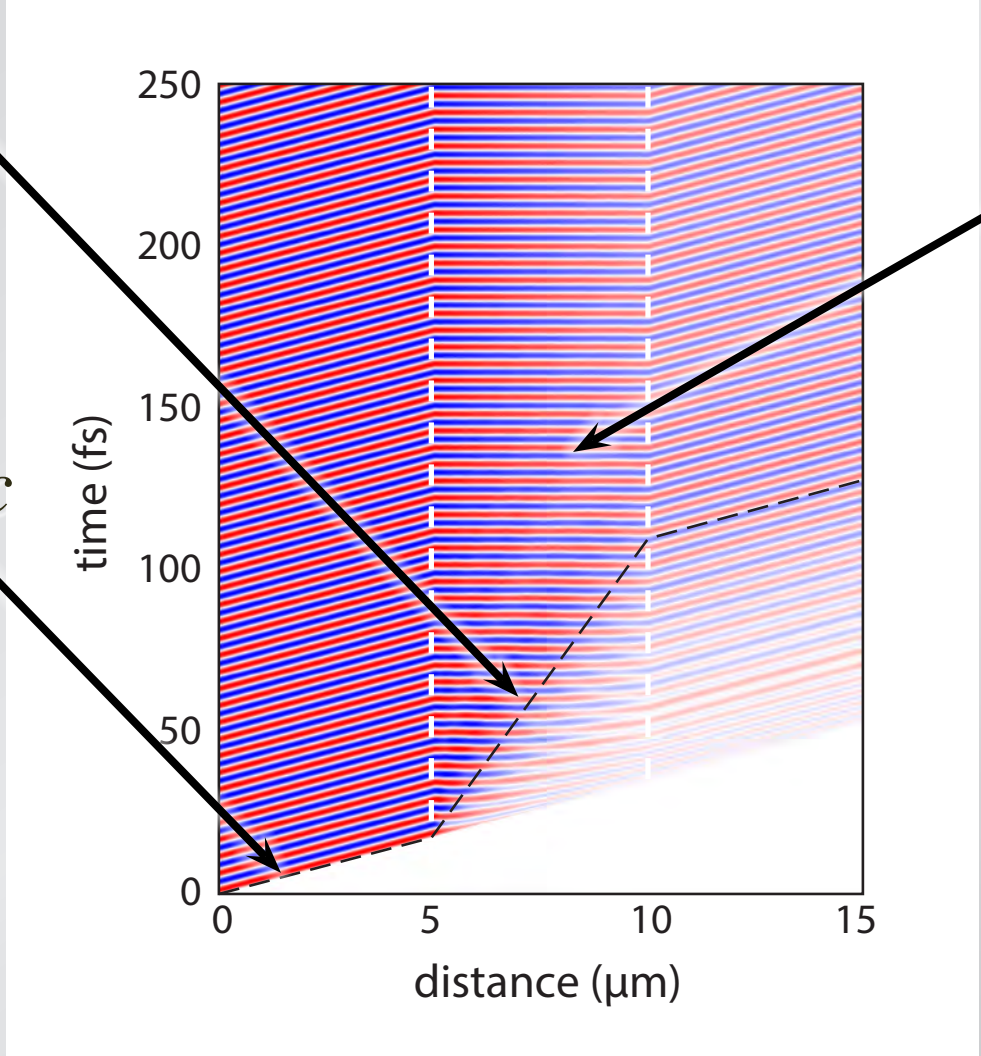
speed of light c



constant phase

1 index

2 zero index

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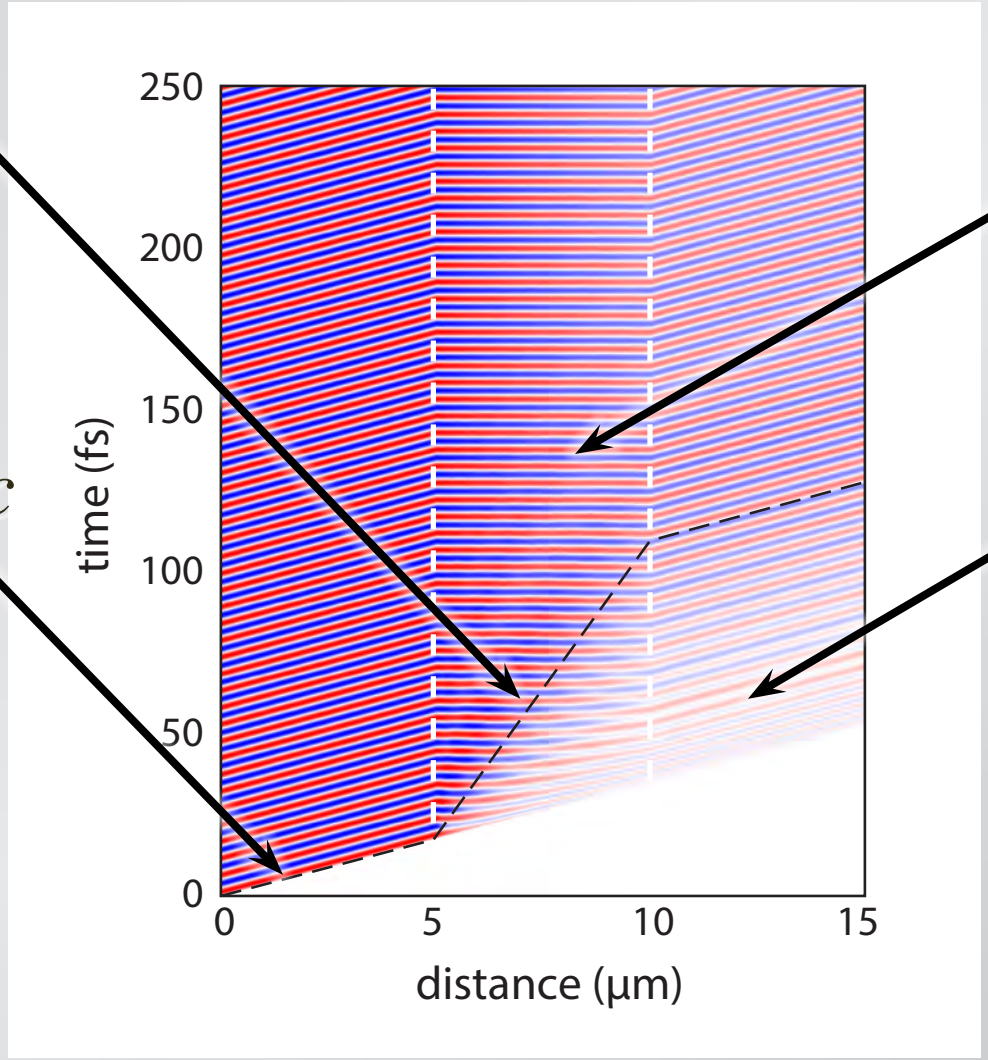
speed of light c



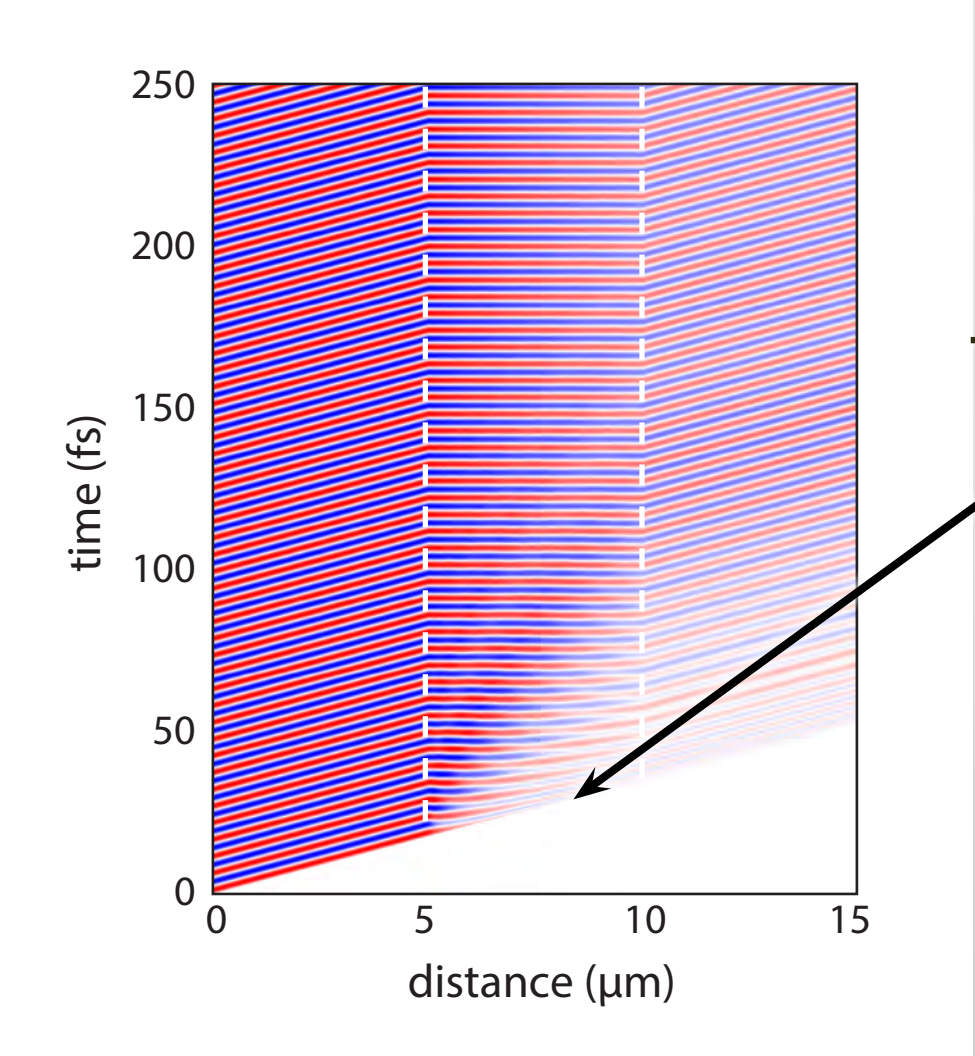
constant phase

high-frequency precursors

1 index

2 zero index

What about causality?

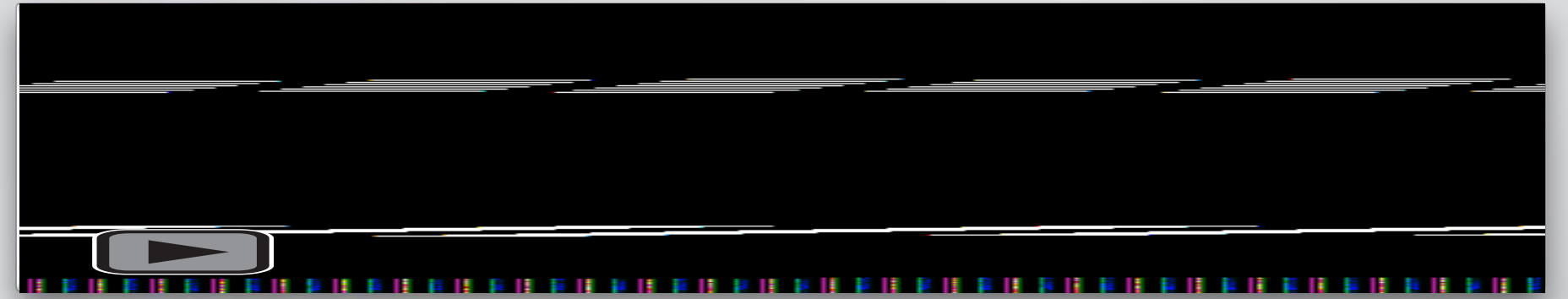


signal *always* travels at speed c!

1 index

2 zero index

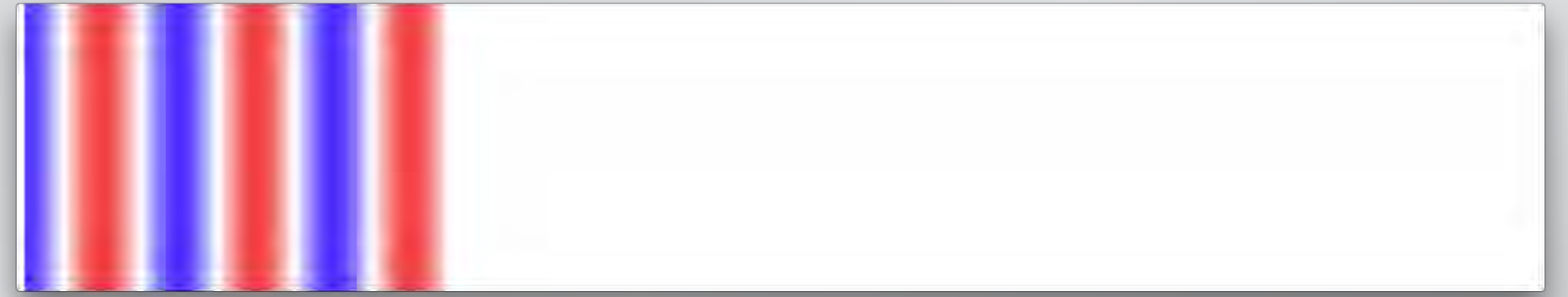
What about causality?



1 index

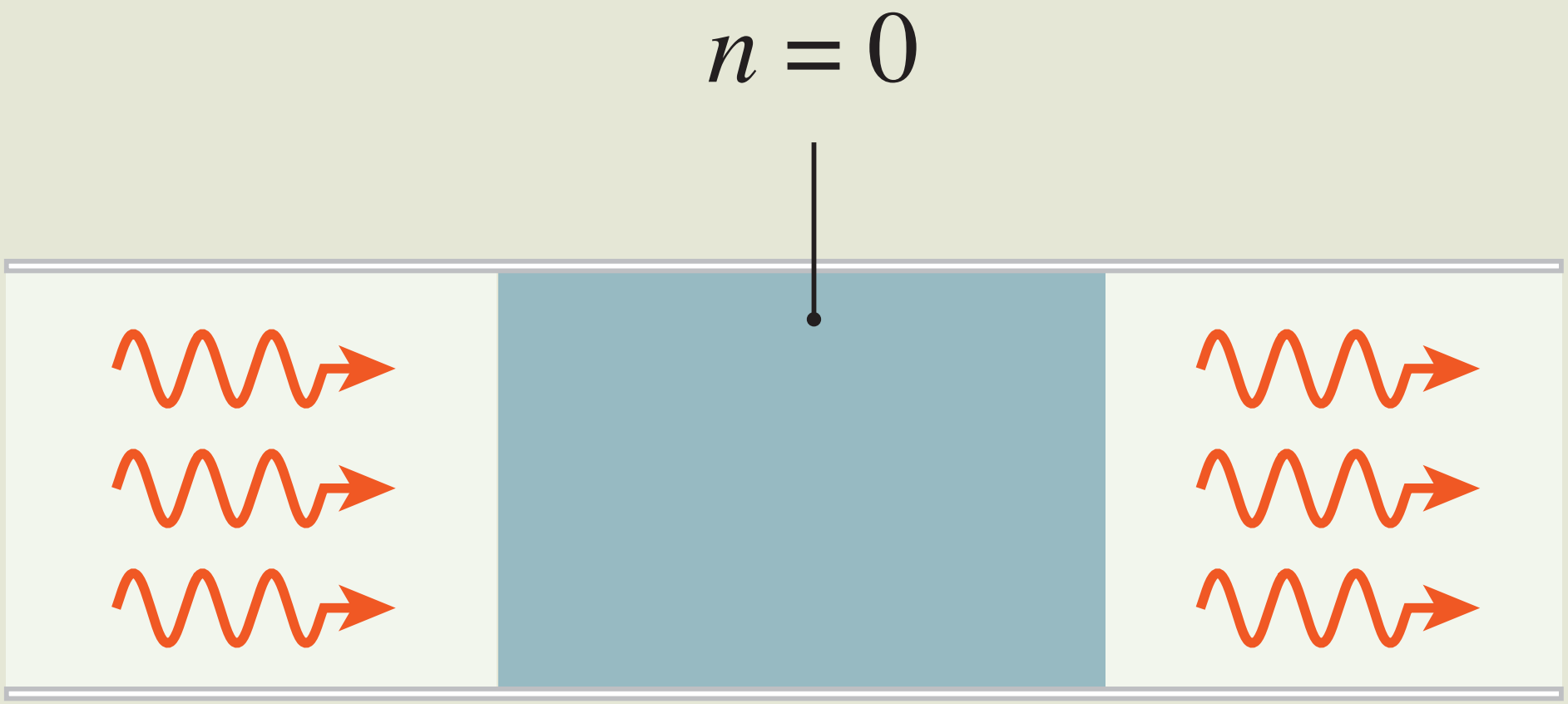
2 zero index

What about causality?



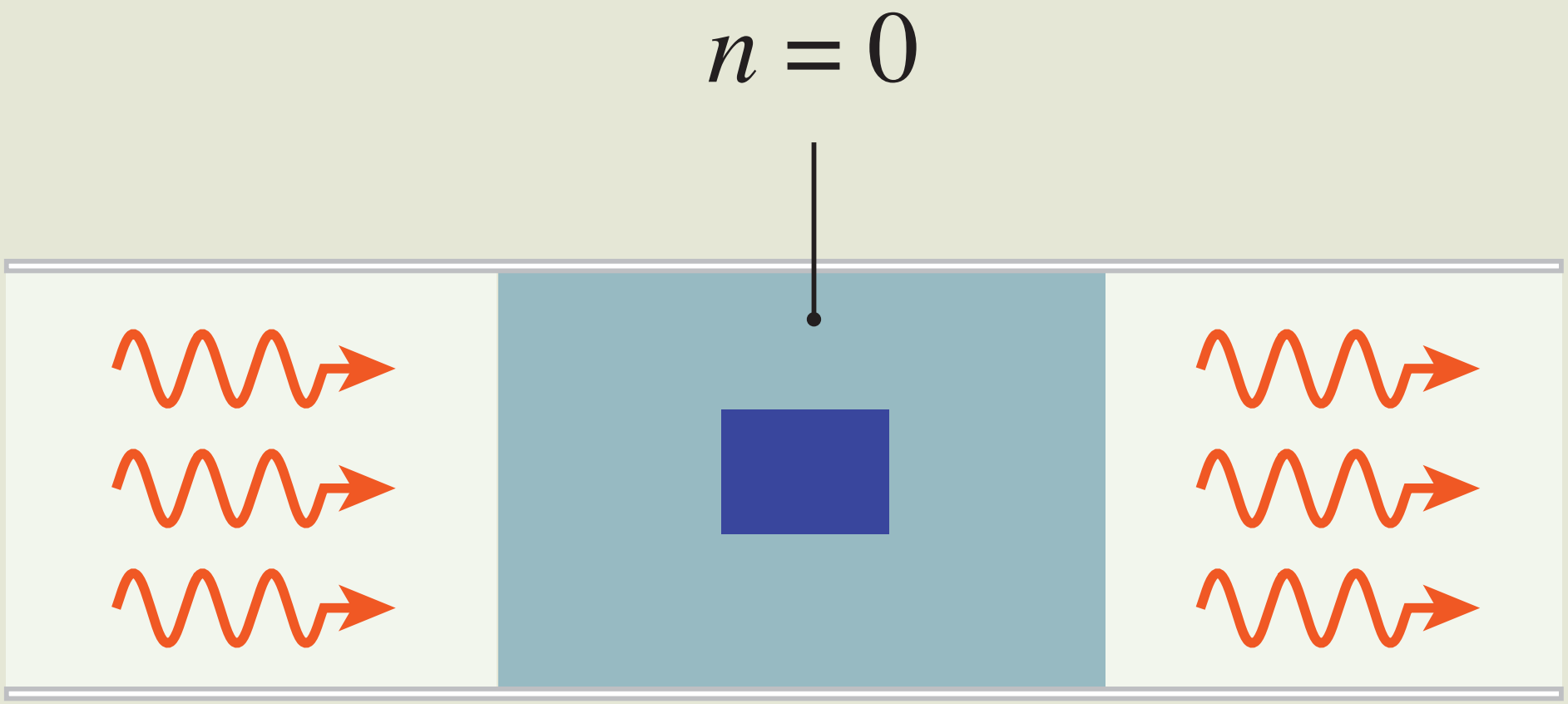
1 index

2 zero index



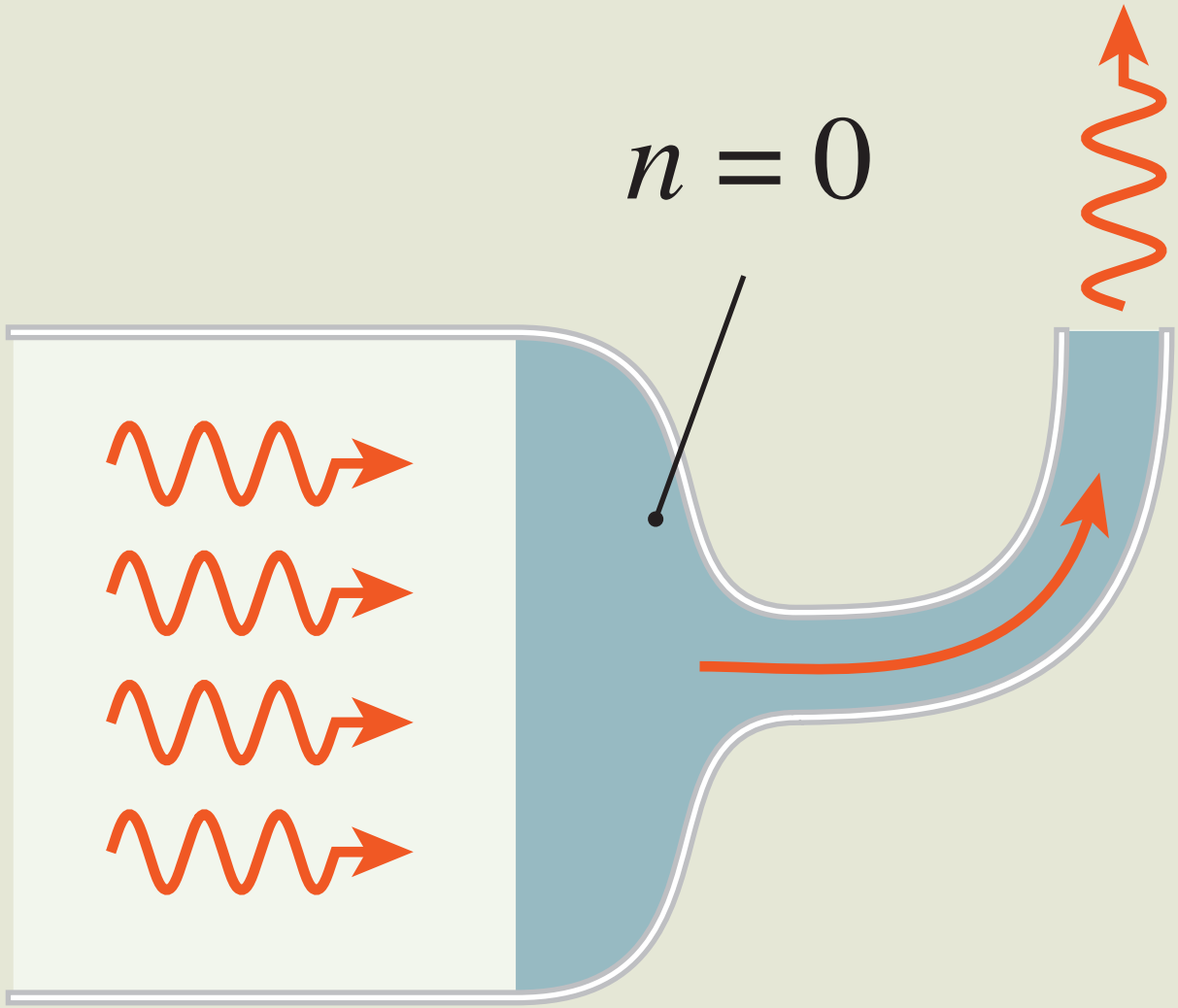
1 index

2 zero index



1 index

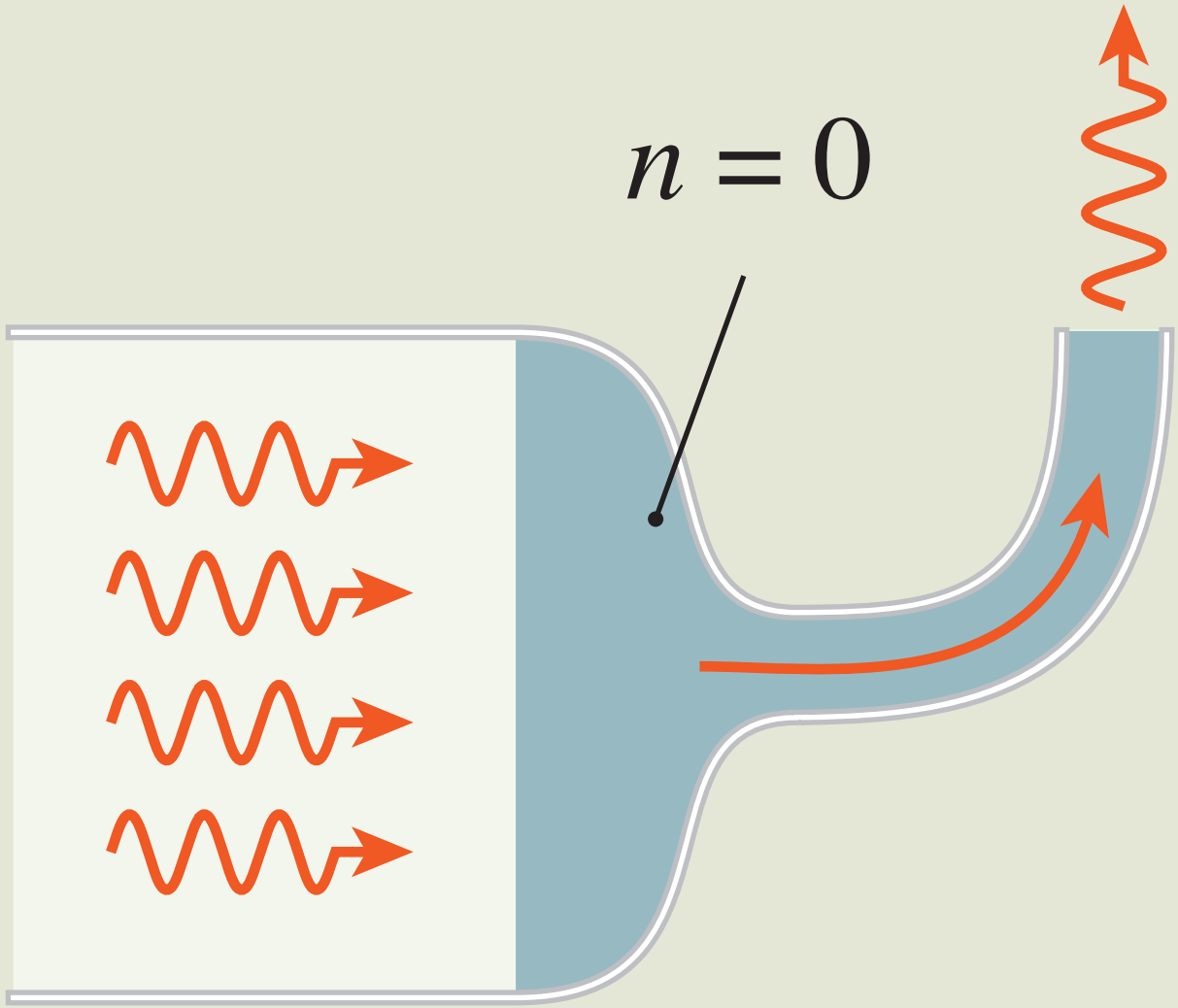
2 zero index



1 index

2 zero index

“tunneling with infinite decay length”



1 index

2 zero index

how?

$$n = \sqrt{\varepsilon\mu}$$

but ε and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

1 index

2 zero index

how?

$$n = \sqrt{\varepsilon\mu}$$

but ε and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

how?

$$\varepsilon, \mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ε and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

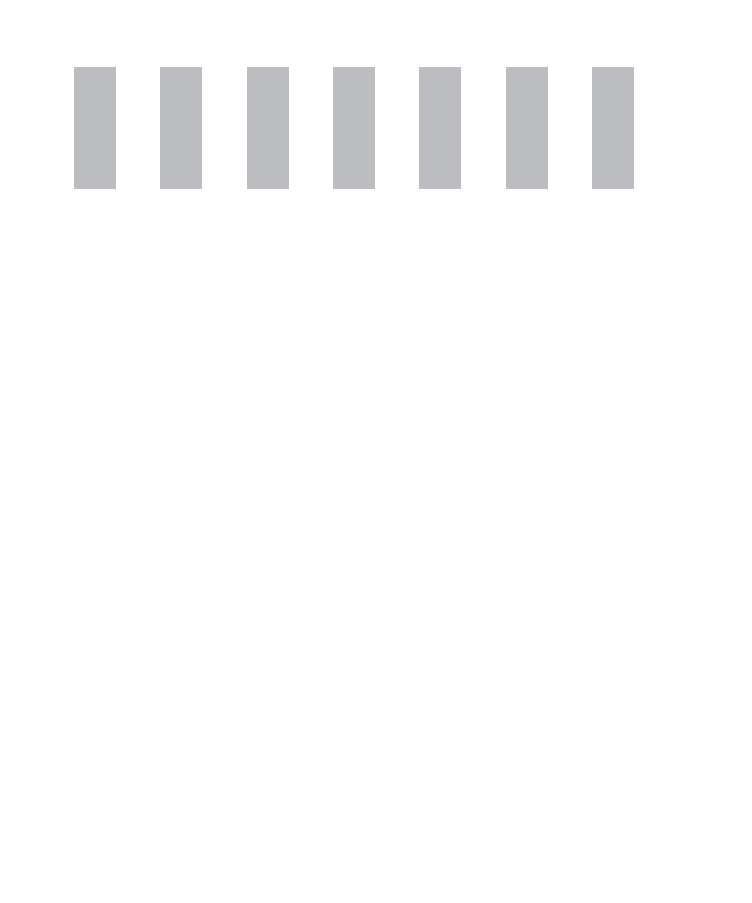
$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

finite!

1 index 2 zero index

Engineering a magnetic response

use array of dielectric rods

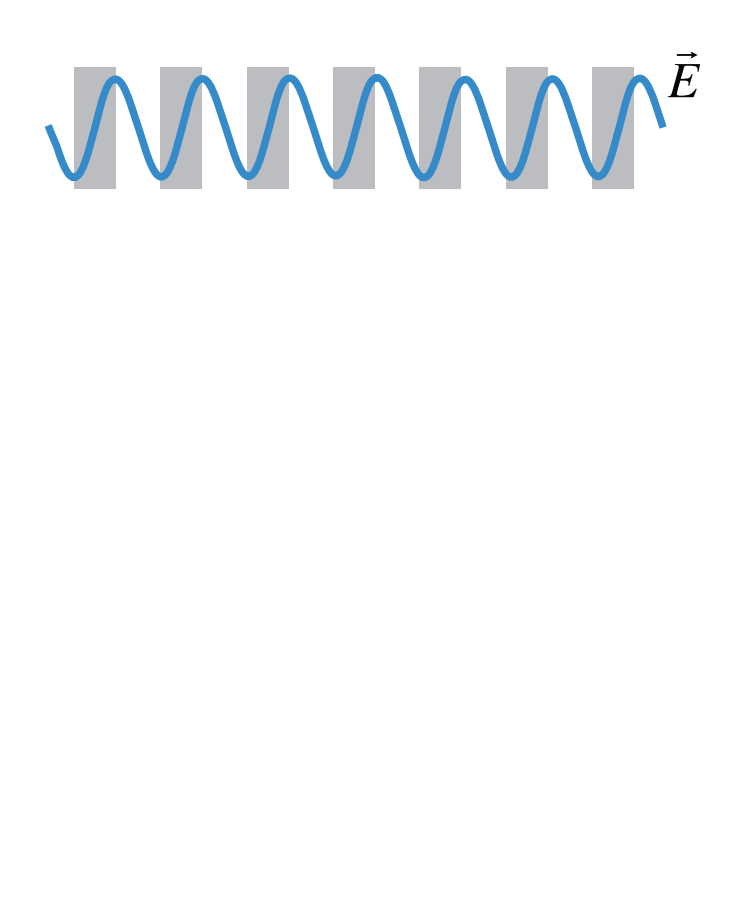


1 index

2 zero index

Engineering a magnetic response

incident electromagnetic wave ($\lambda_{\text{eff}} \approx a$)

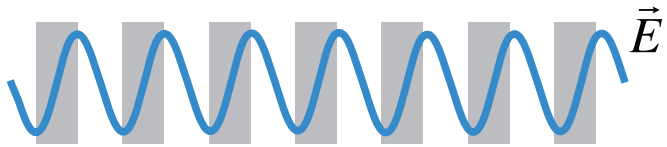


1 index

2 zero index

Engineering a magnetic response

produces an electric response…

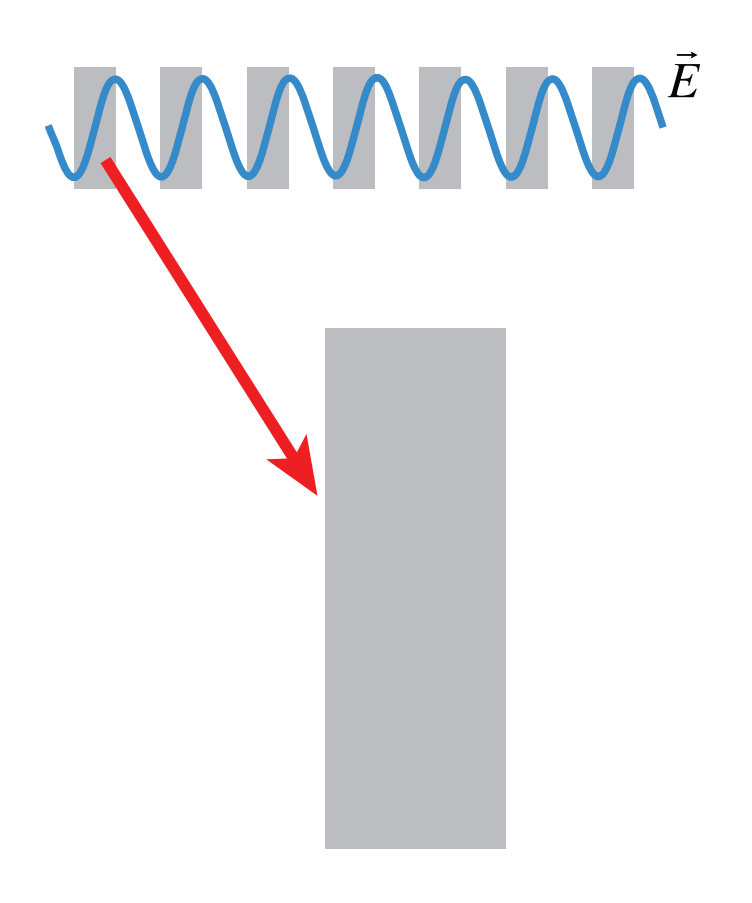


1 index

2 zero index

Engineering a magnetic response

… but different electric fields front and back…

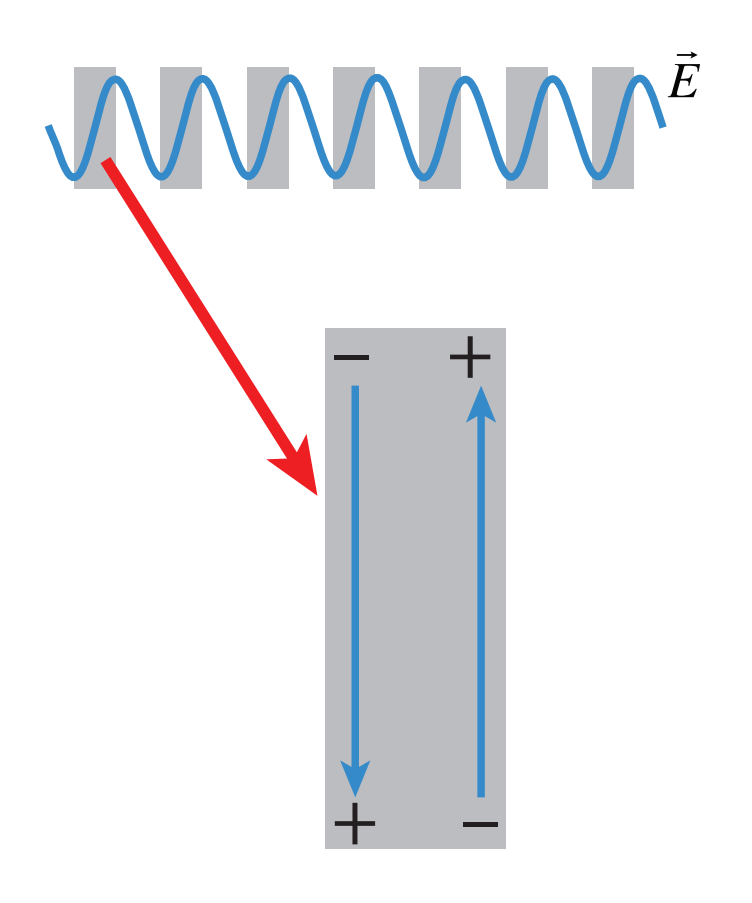


1 index

2 zero index

Engineering a magnetic response

…induce different polarizations on opposite sides…

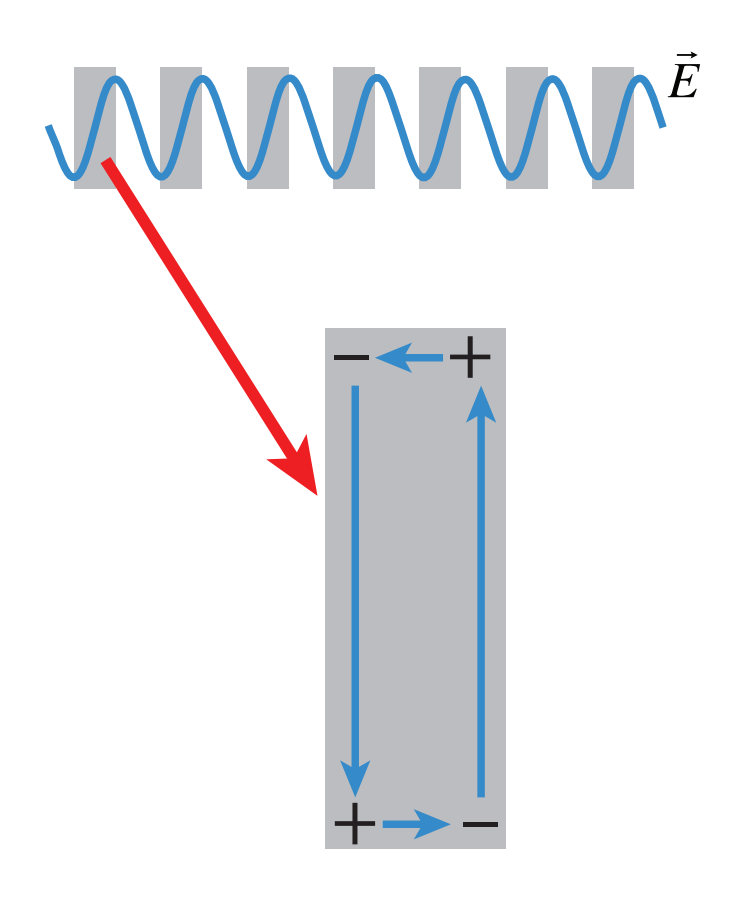


1 index

2 zero index

Engineering a magnetic response

…causing a current loop…

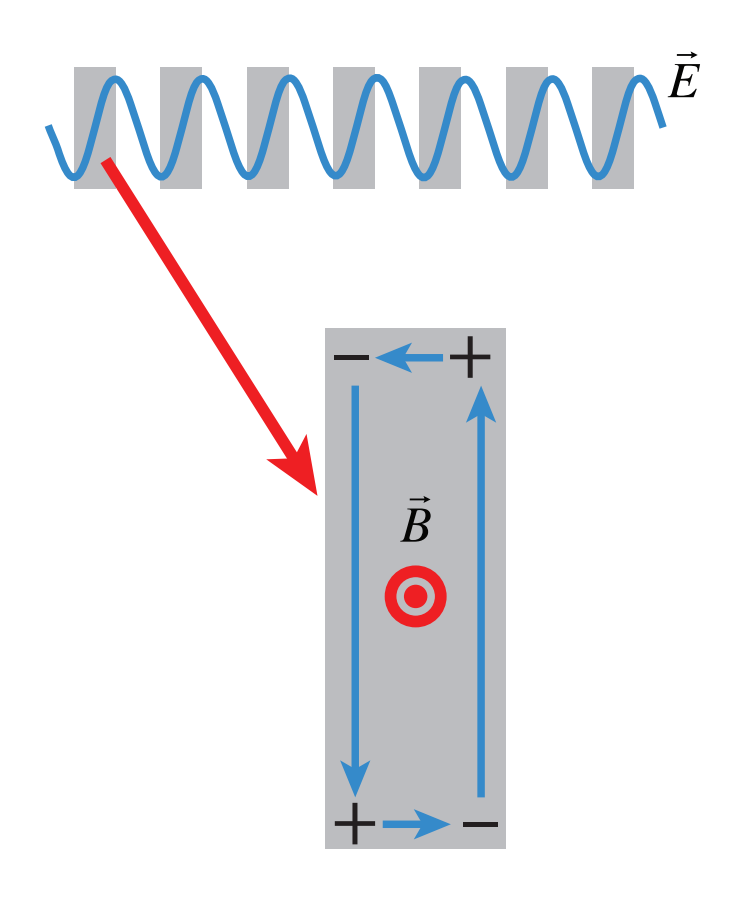


1 index

2 zero index

Engineering a magnetic response

…which, in turn, produces an induced magnetic field

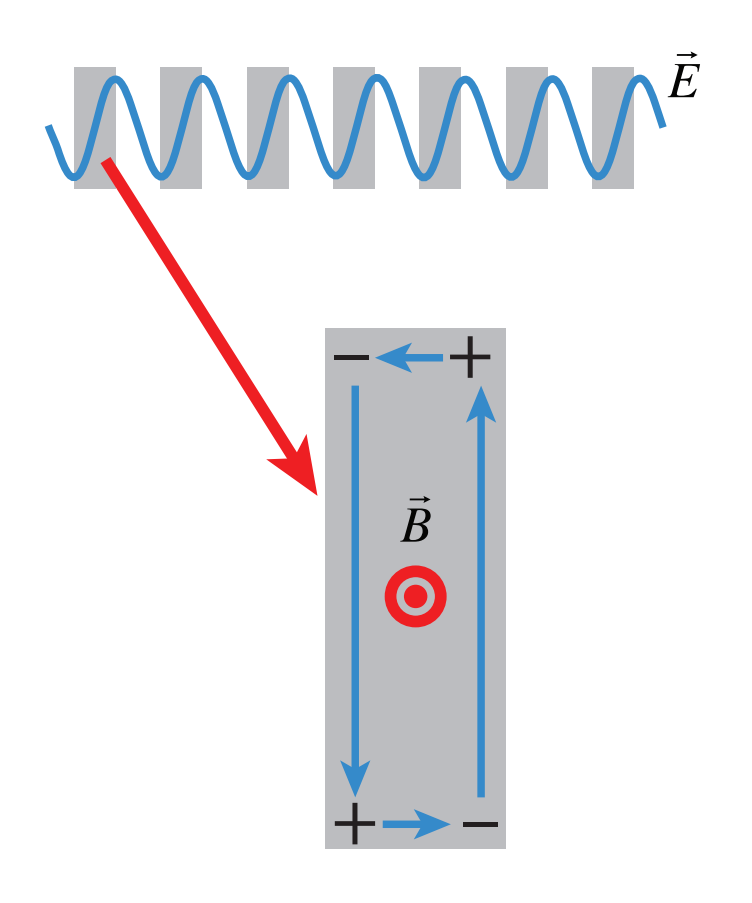


1 index

2 zero index

Engineering a magnetic response

adjust design so electrical and magnetic resonances coincide



1 index

2 zero index

Engineering a magnetic response

adjustable parameters

1 index

2 zero index

Engineering a magnetic response

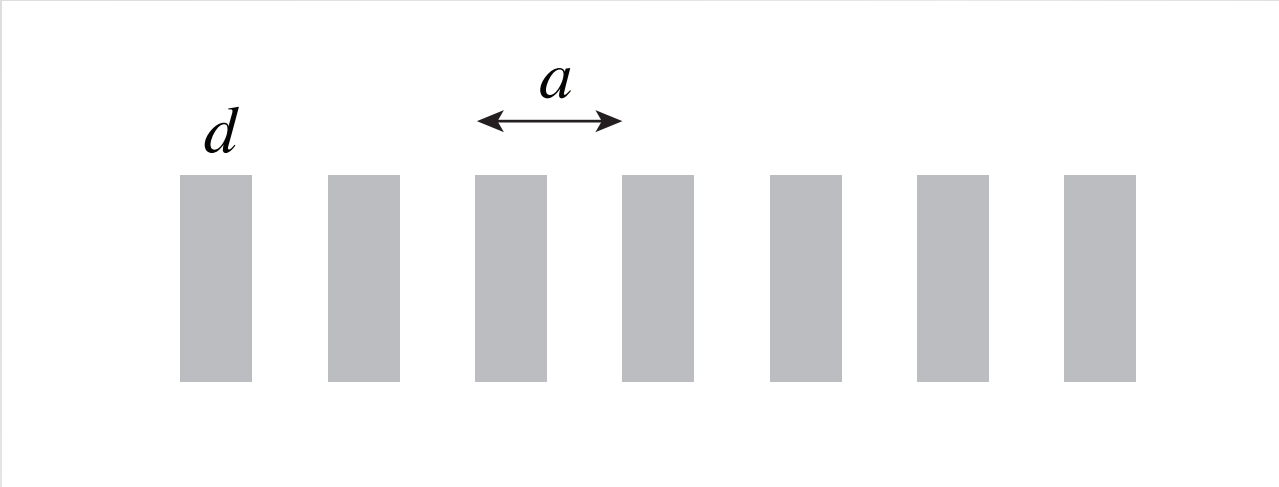
adjustable parameters

1 index

2 zero index

Engineering a magnetic response

adjustable parameters

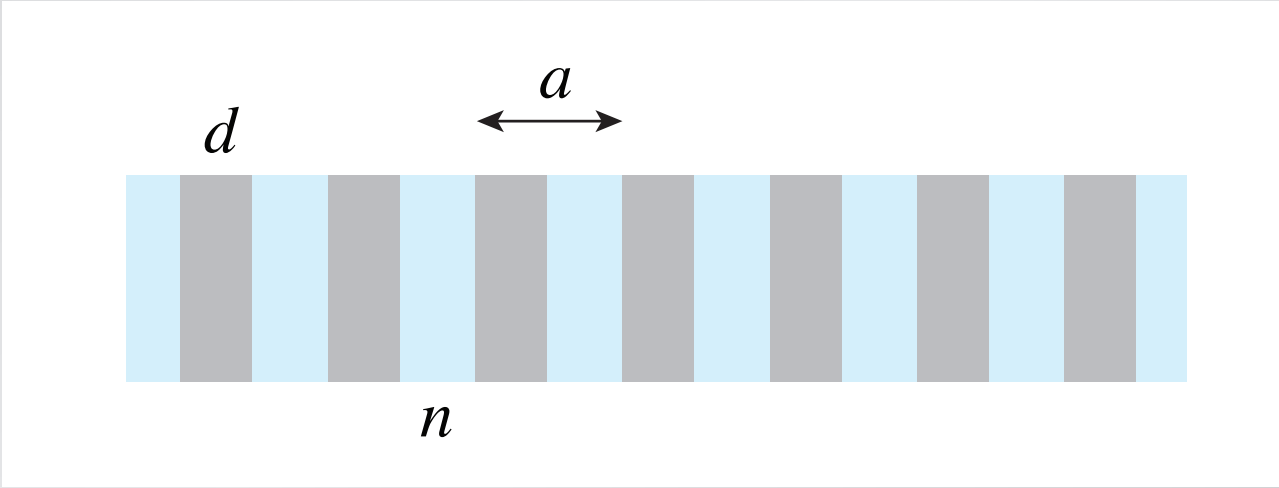


1 index

2 zero index

Engineering a magnetic response

adjustable parameters

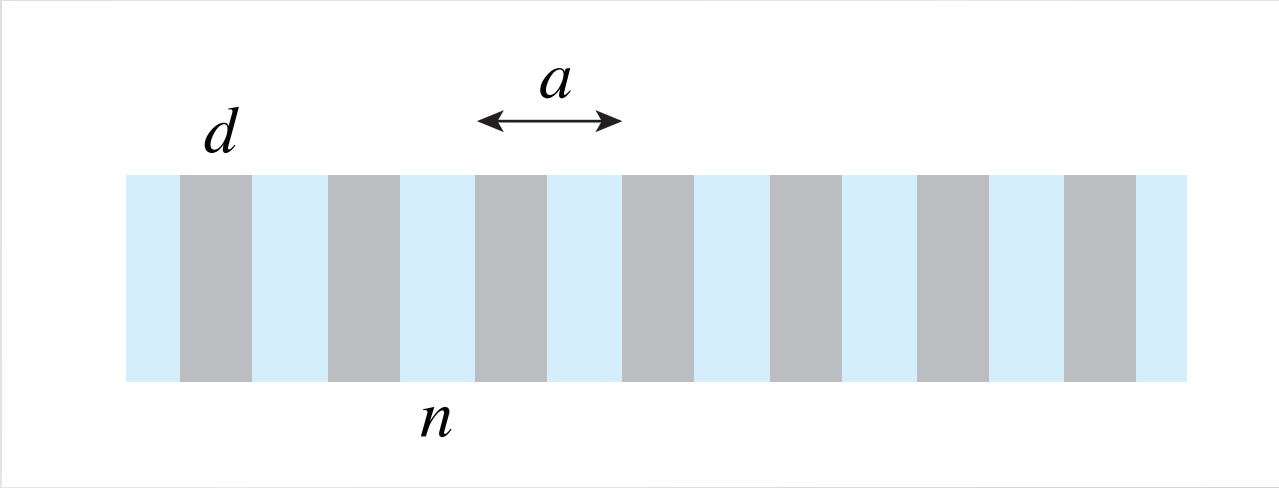


1 index

2 zero index

Engineering a magnetic response

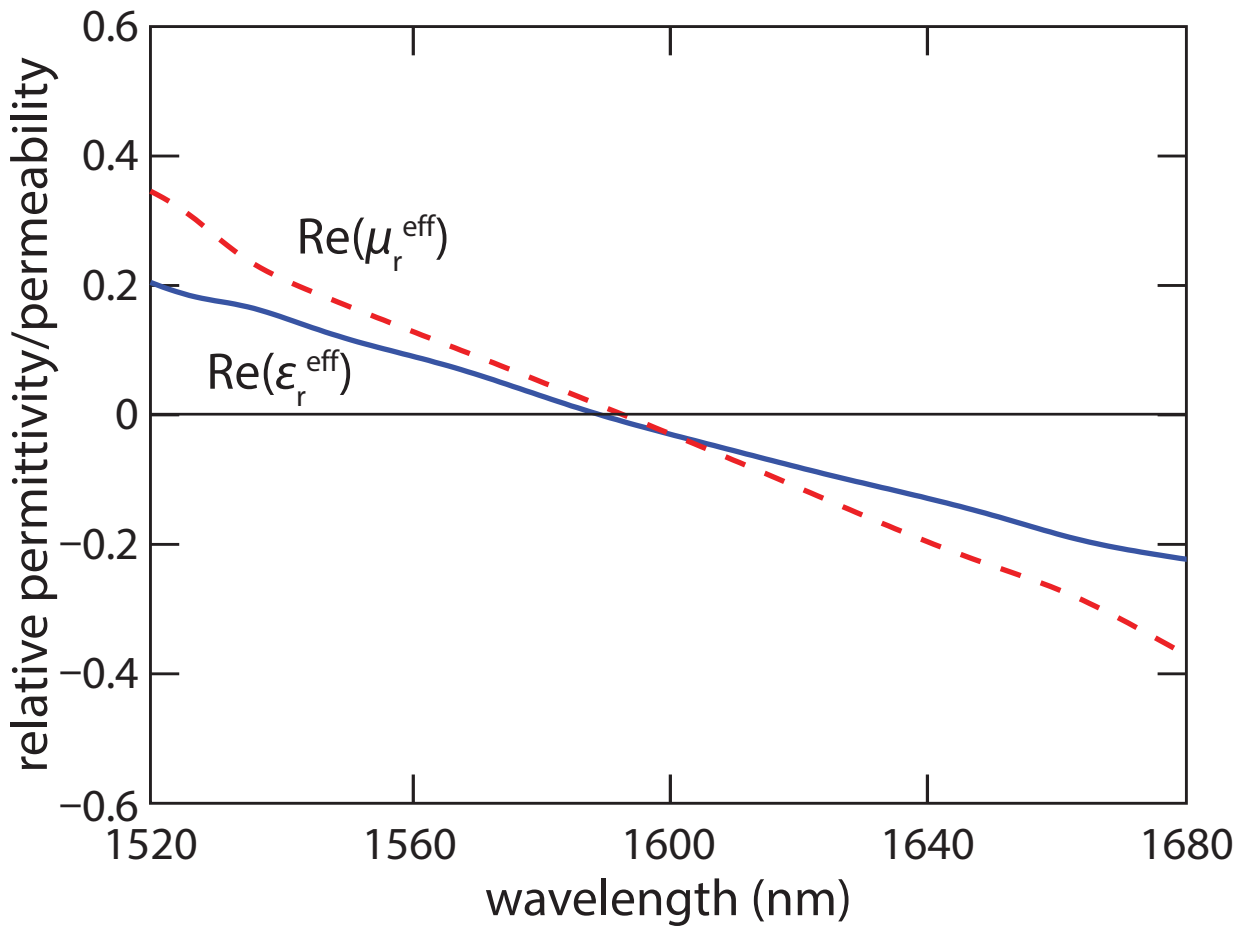
adjustable parameters



d = 422 nm, a = 690 nm, n = 1.57 (SU8)

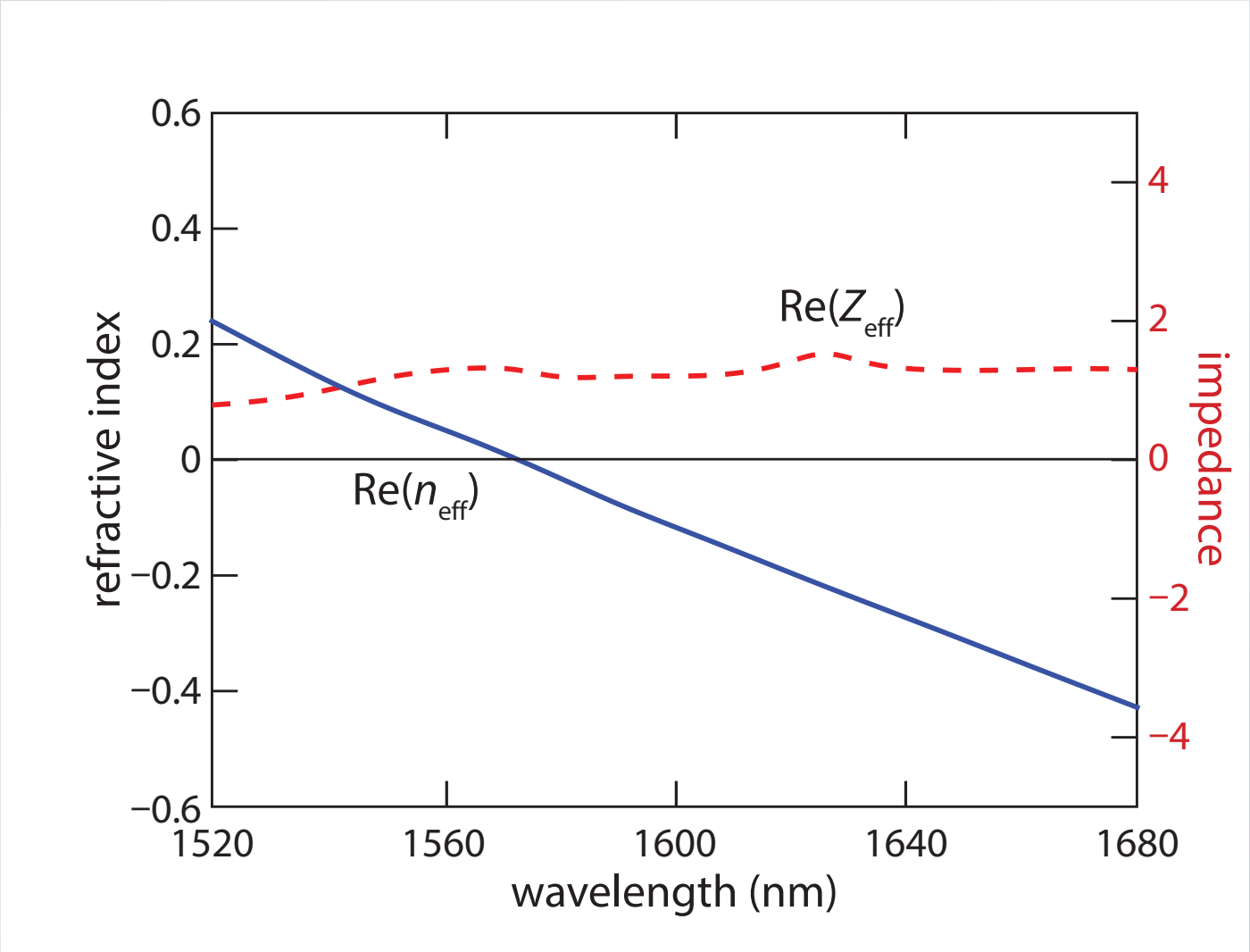
1 index

2 zero index



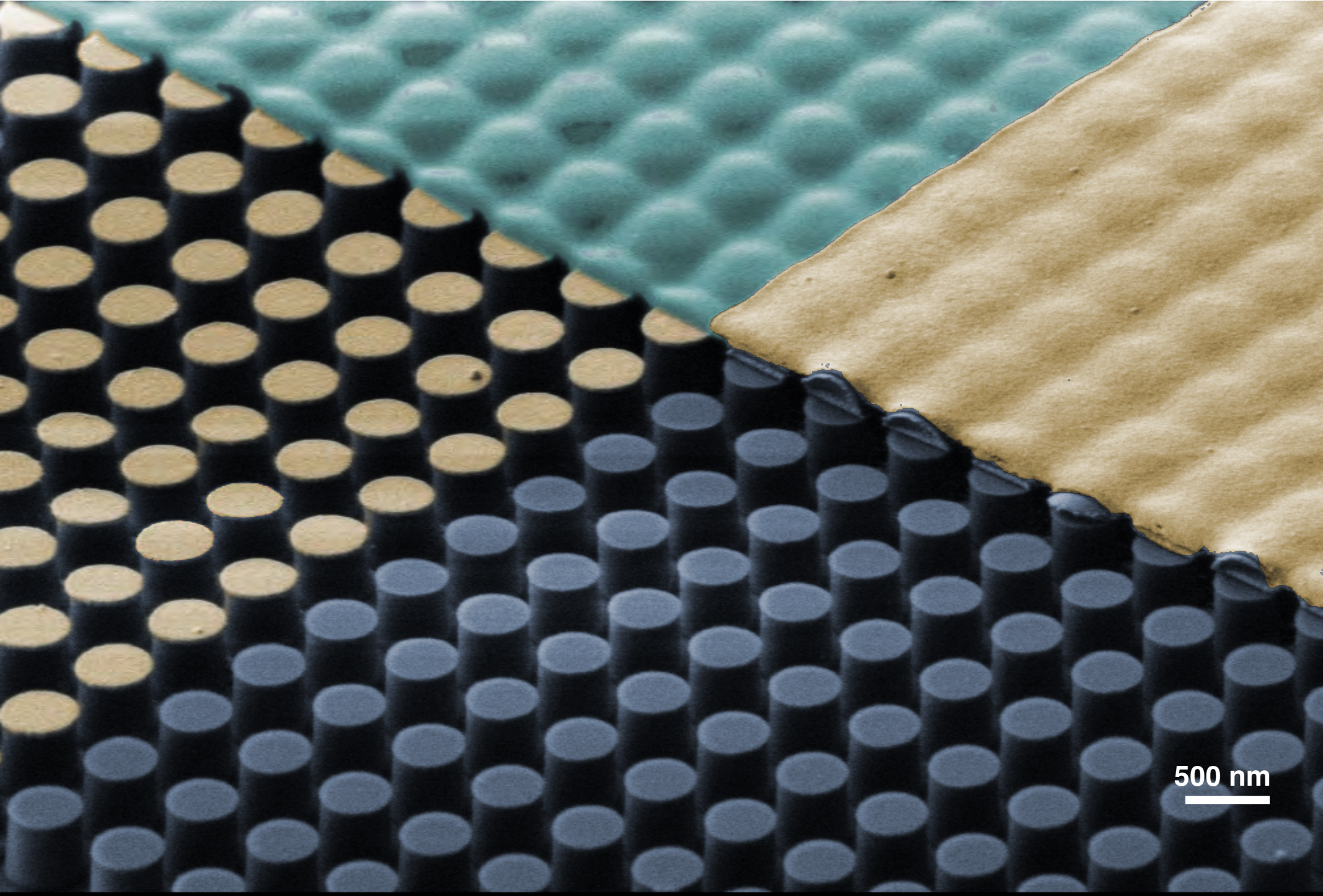
1 index

2 zero index



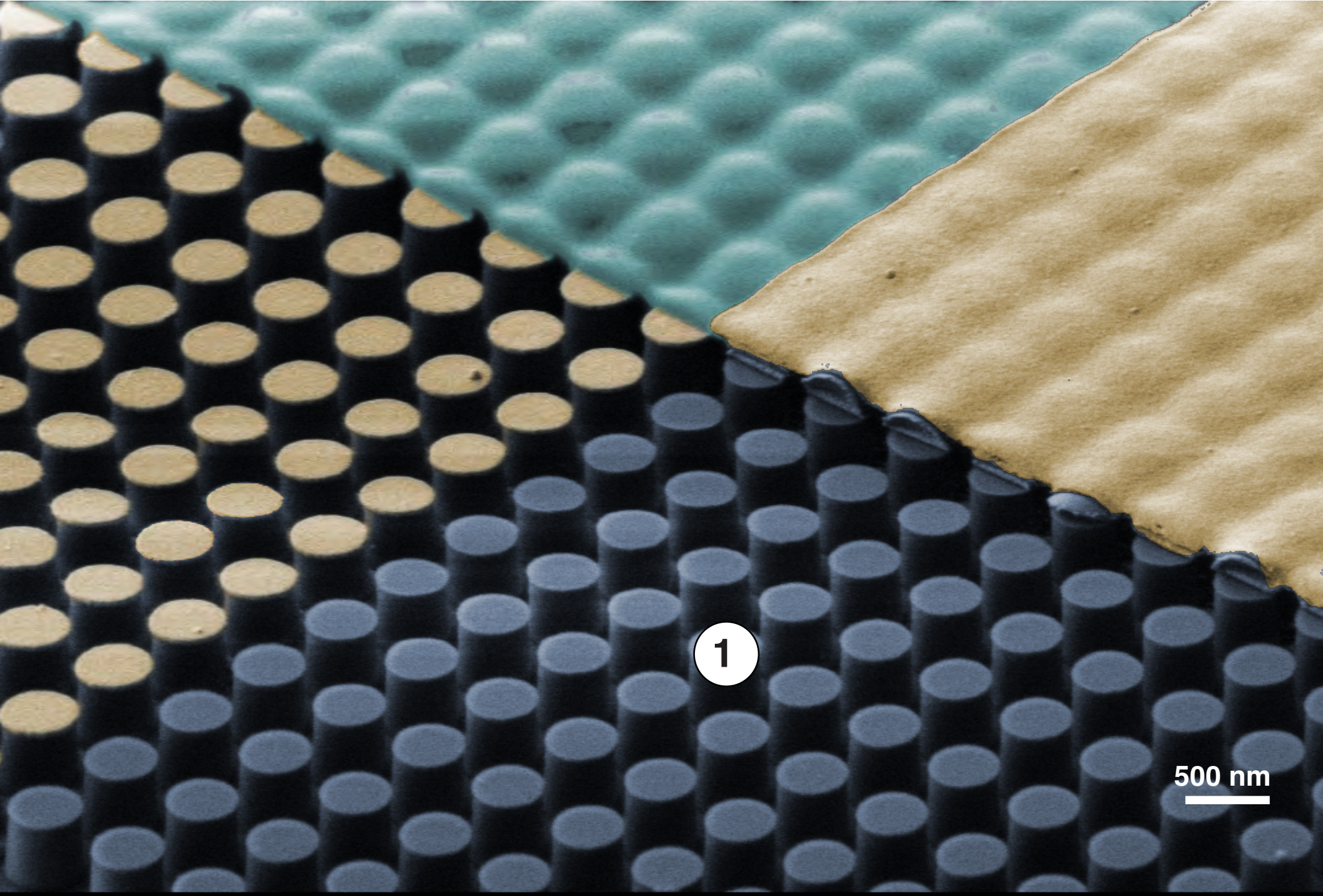
1 index

2 zero index



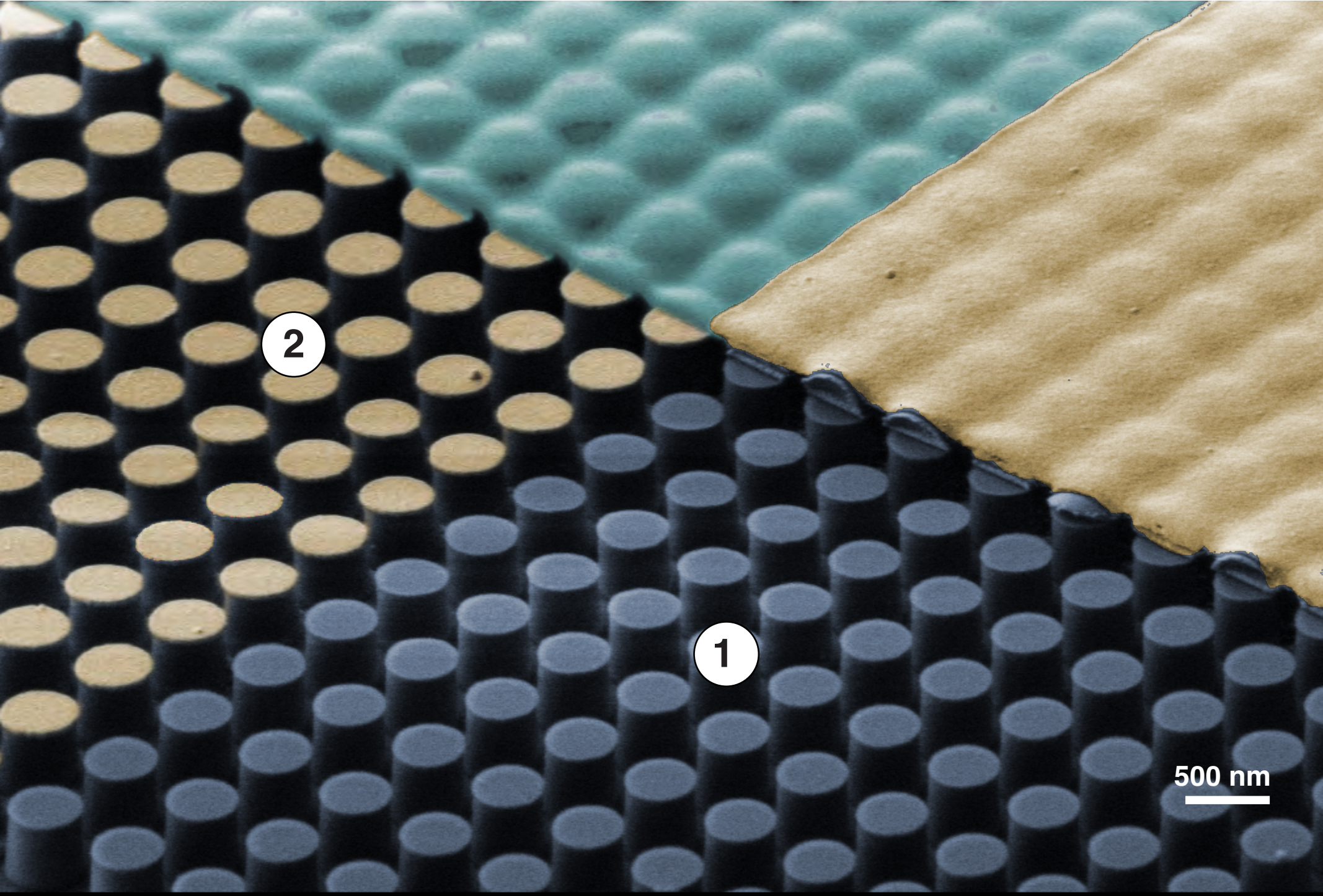
1 index

2 zero index



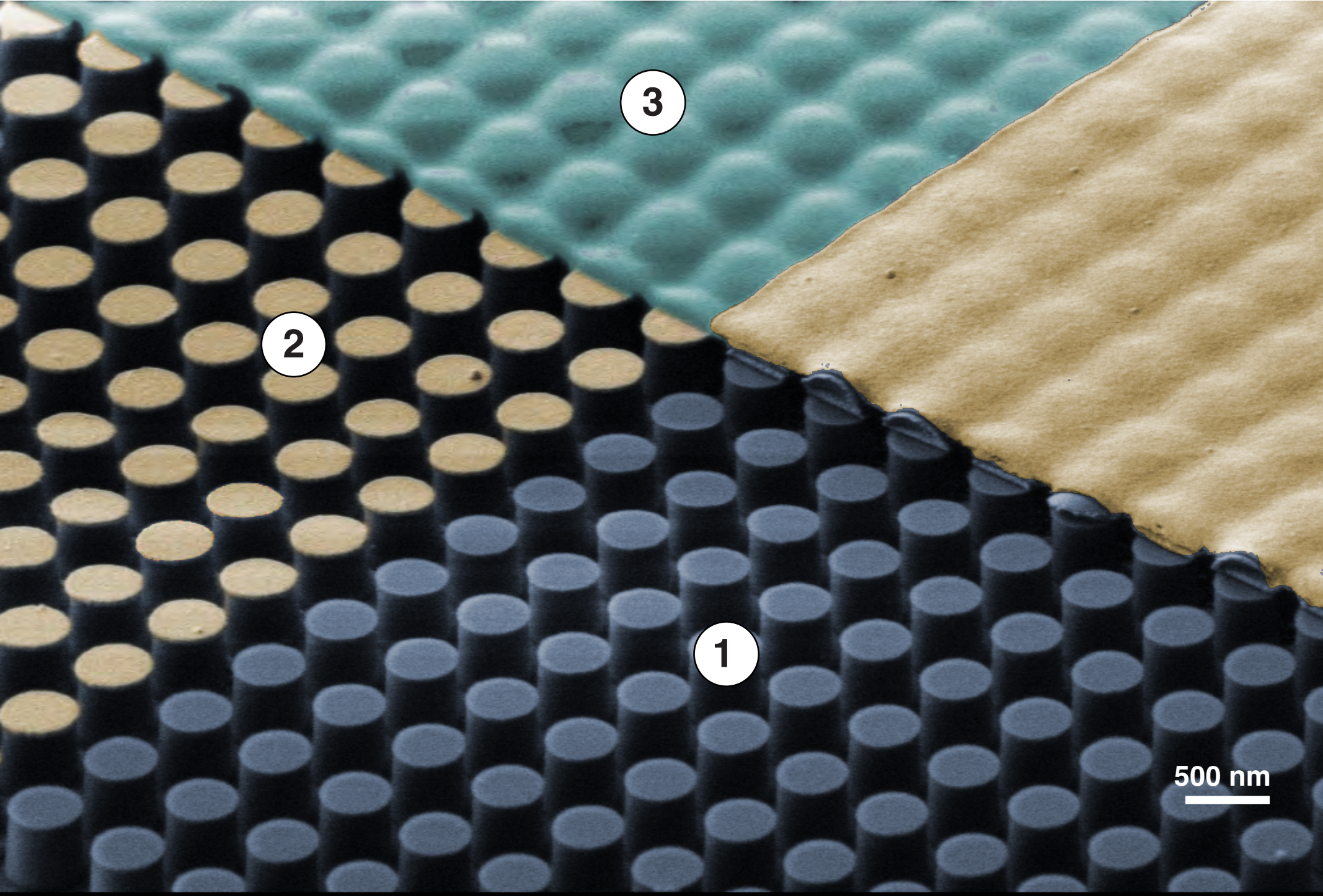
1 index

2 zero index



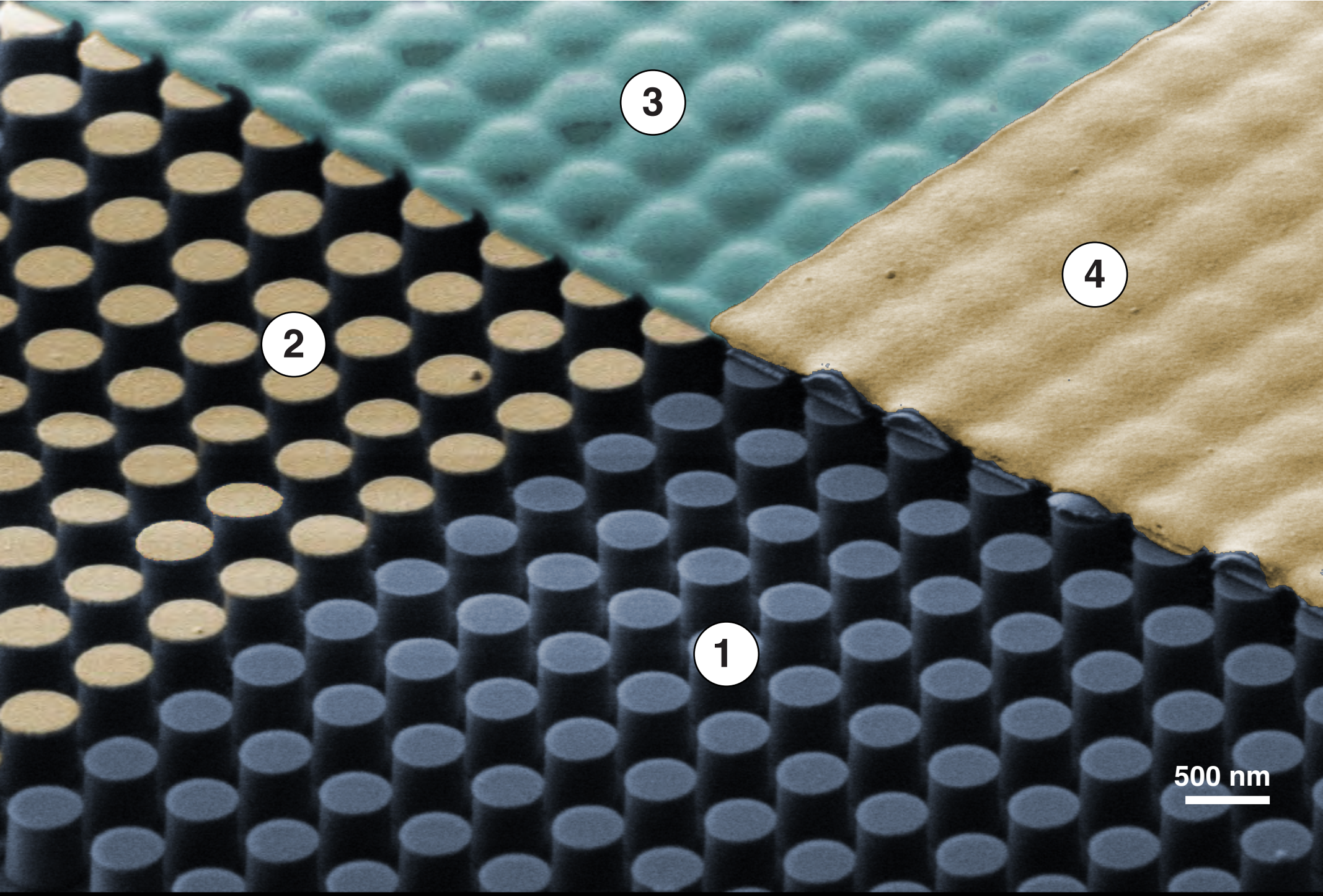
1 index

2 zero index



1 index

2 zero index



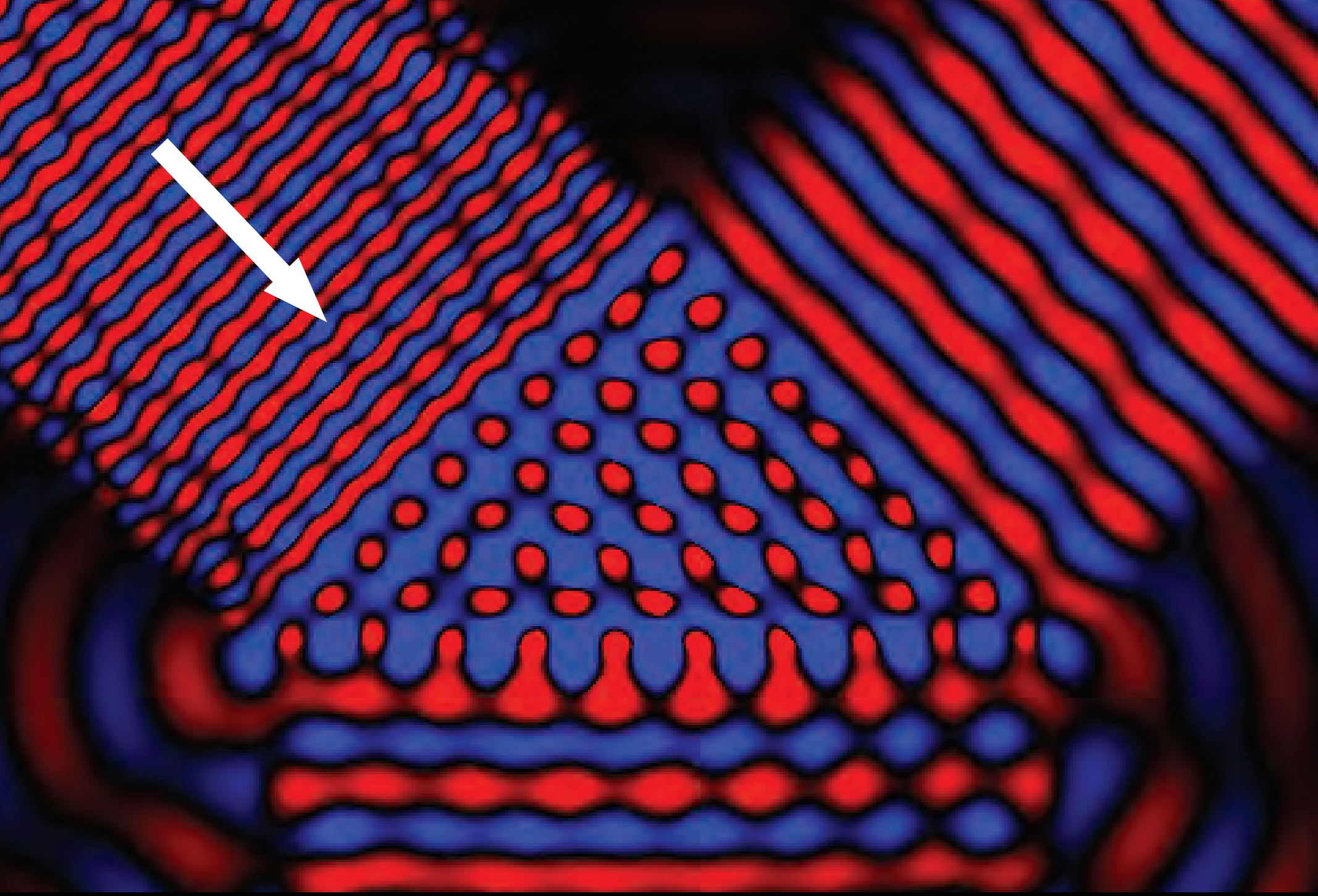
1 index

2 zero index

Can make this in any shape!

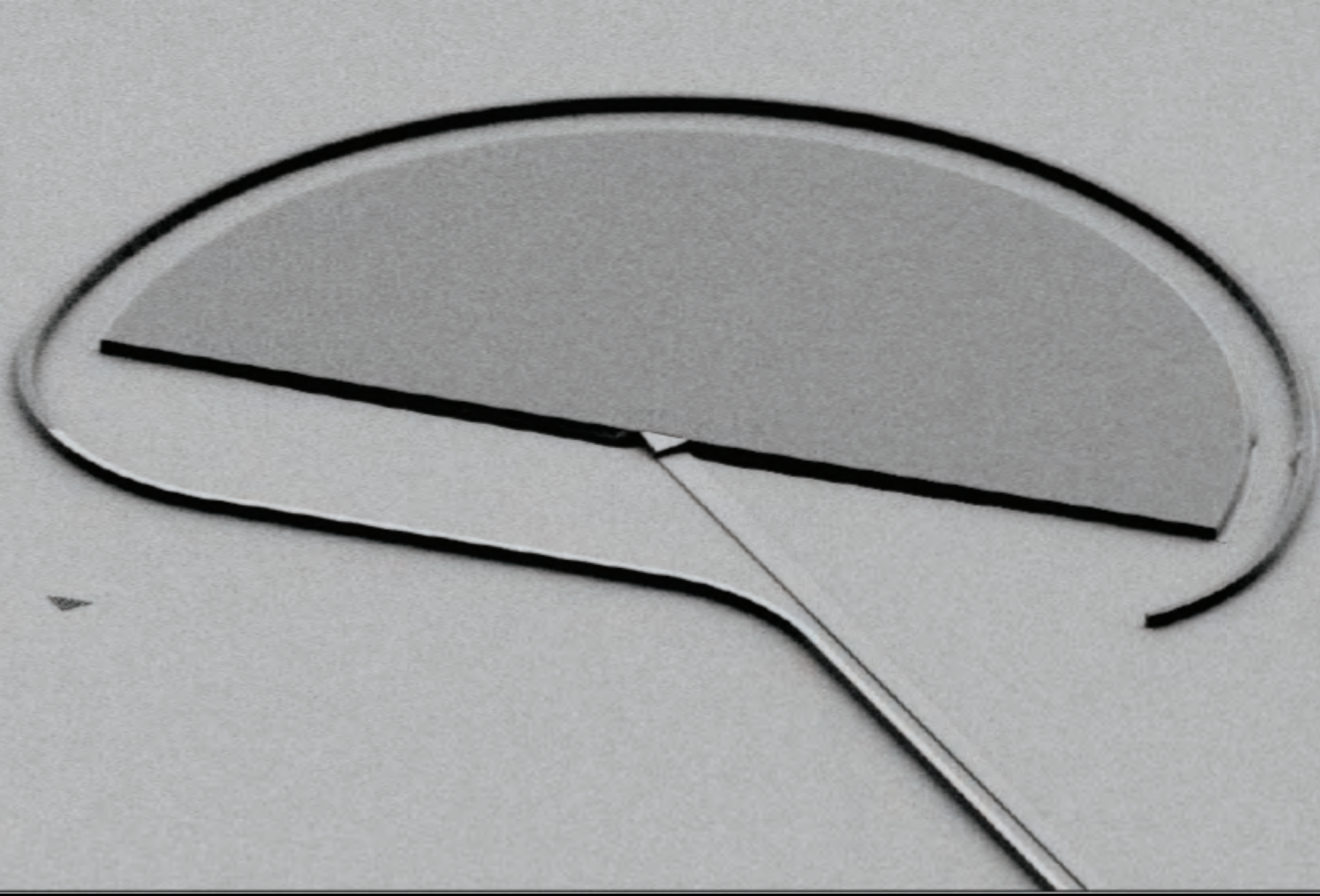
1 index

2 zero index



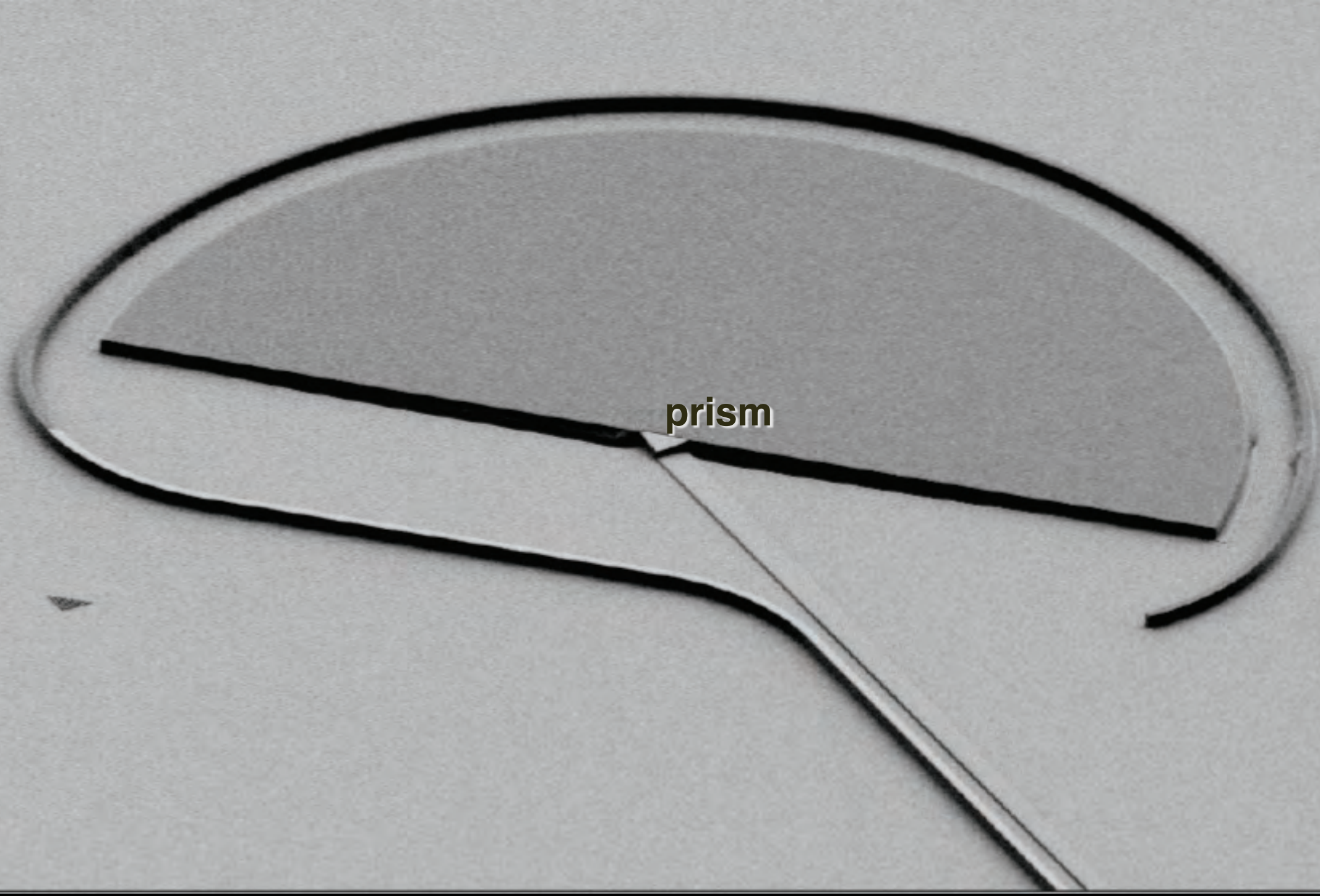
1 index

2 zero index



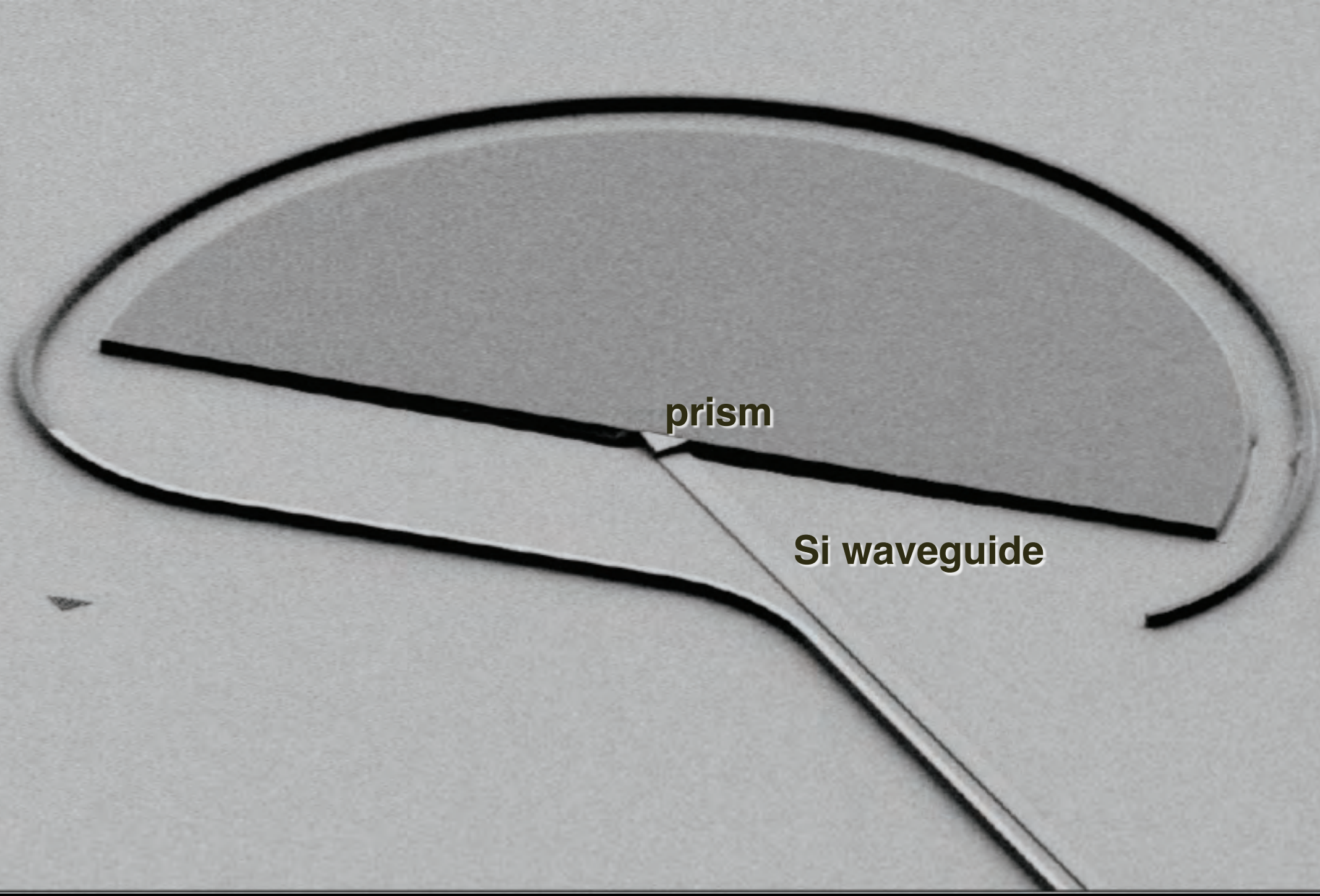
1 index

2 zero index



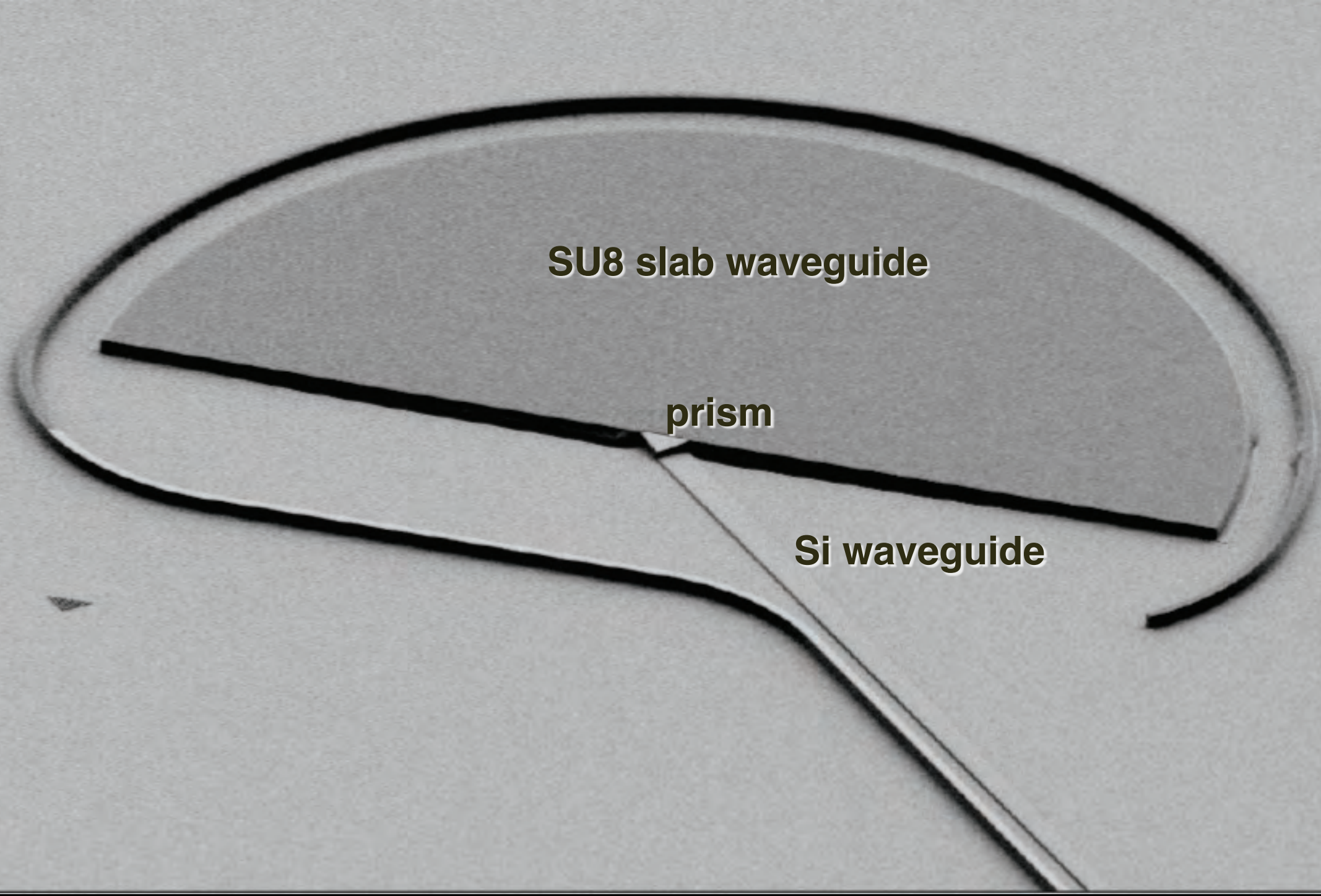
1 index

2 zero index



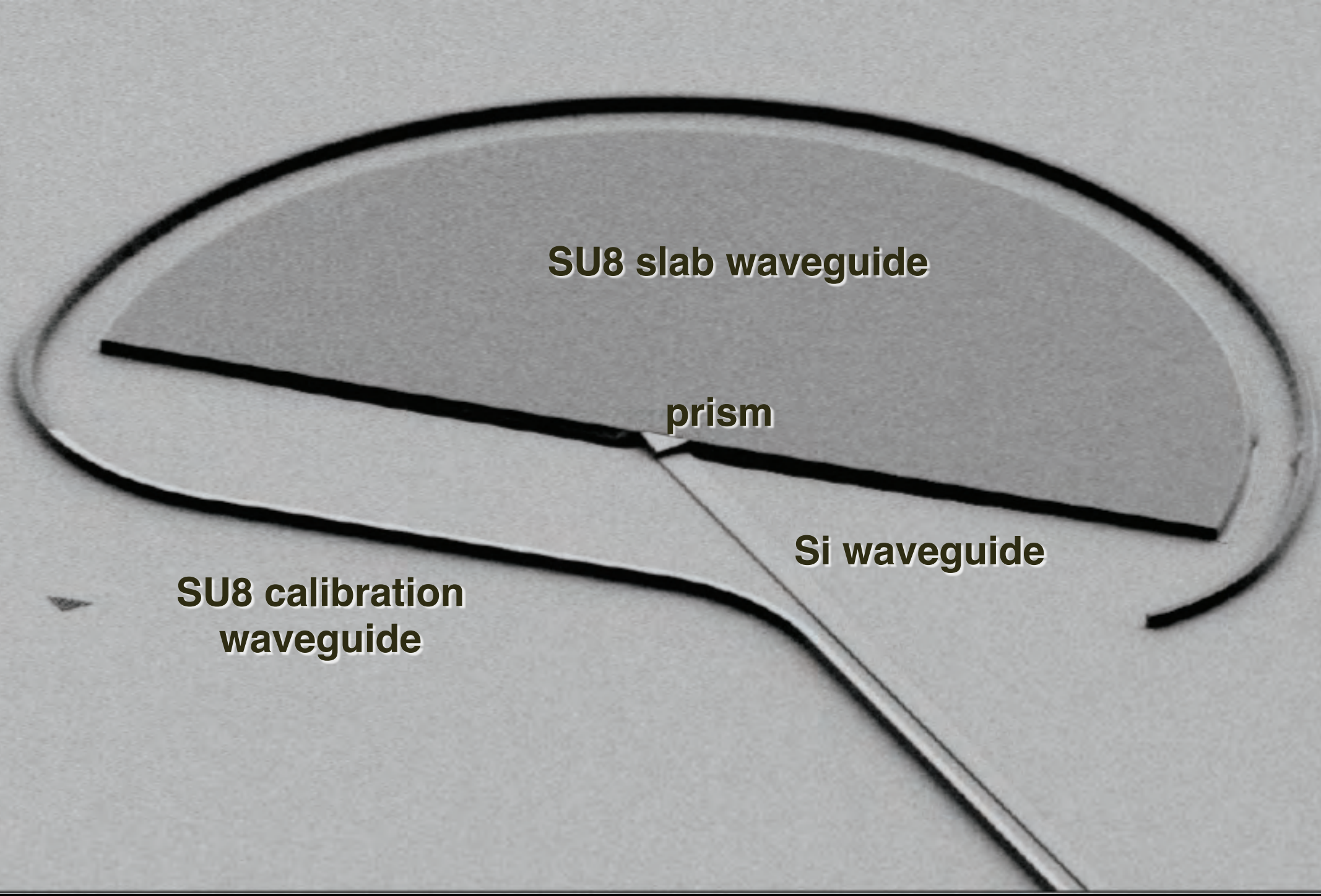
1 index

2 zero index



1 index

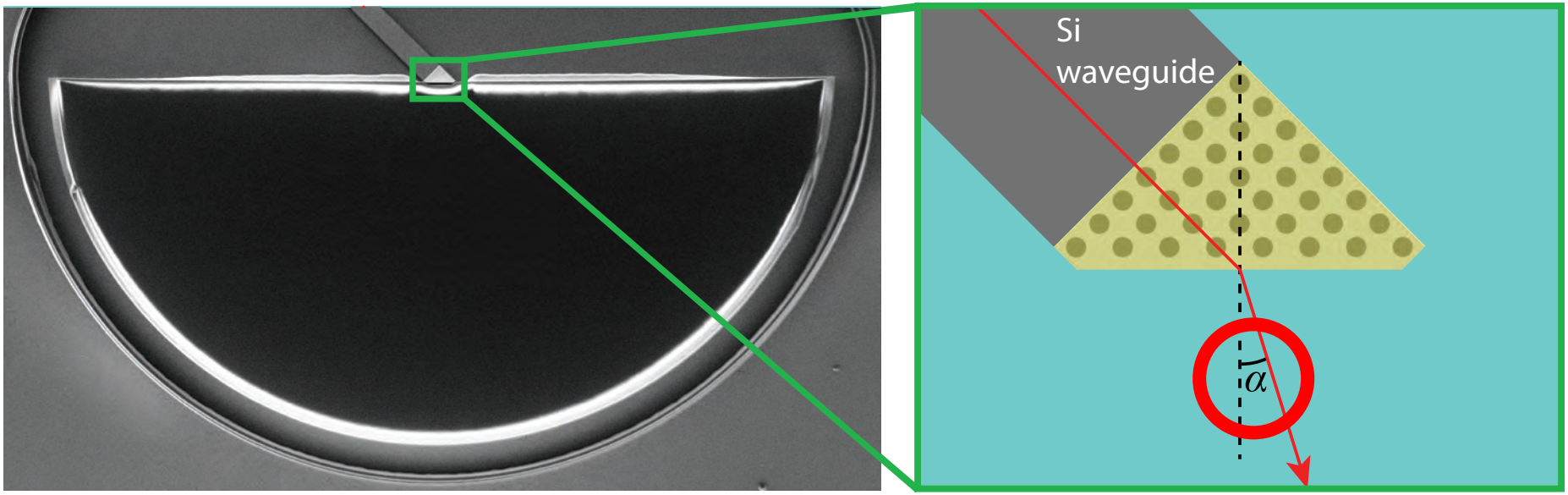
2 zero index



1 index

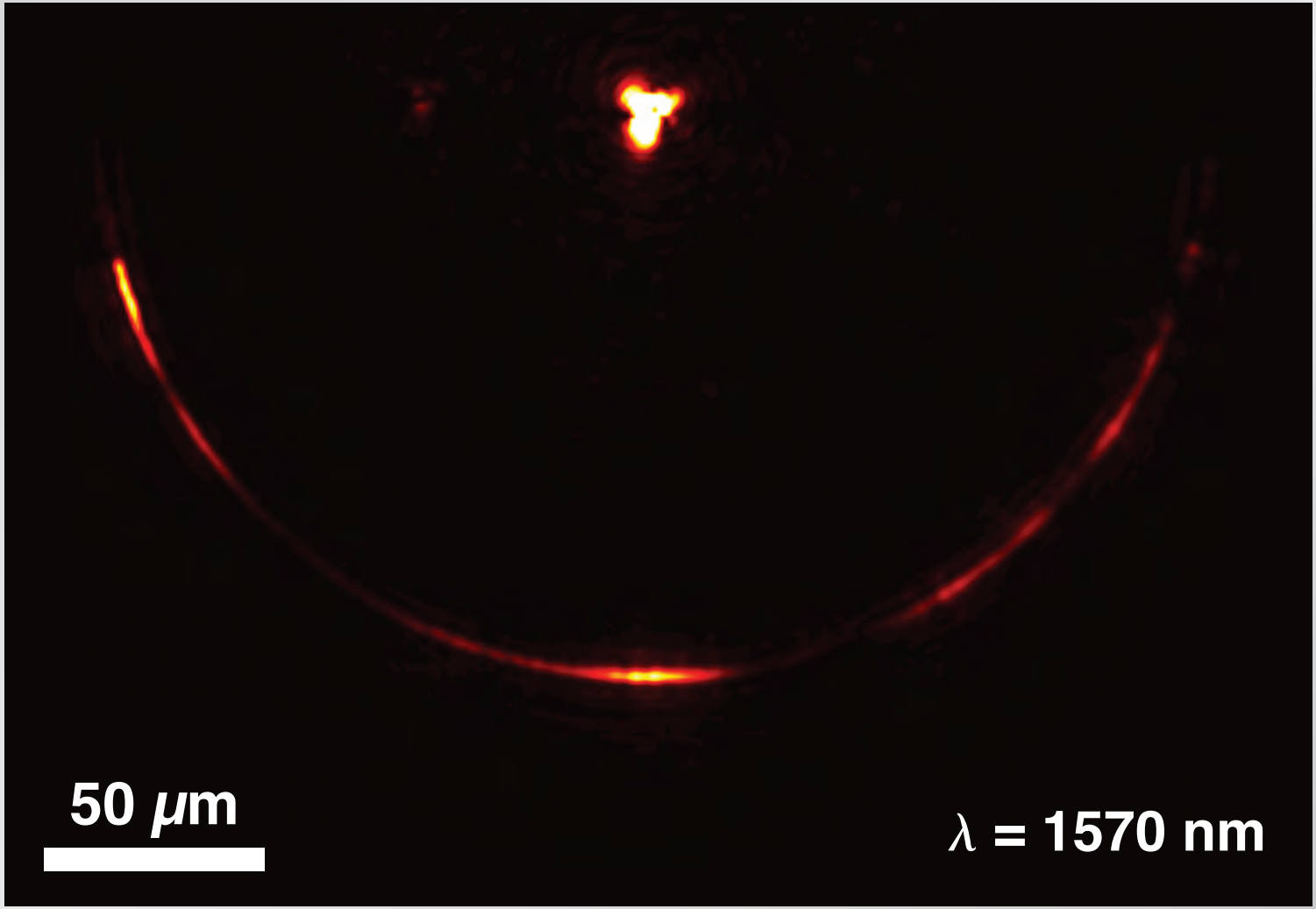
2 zero index

On-chip zero-index prism



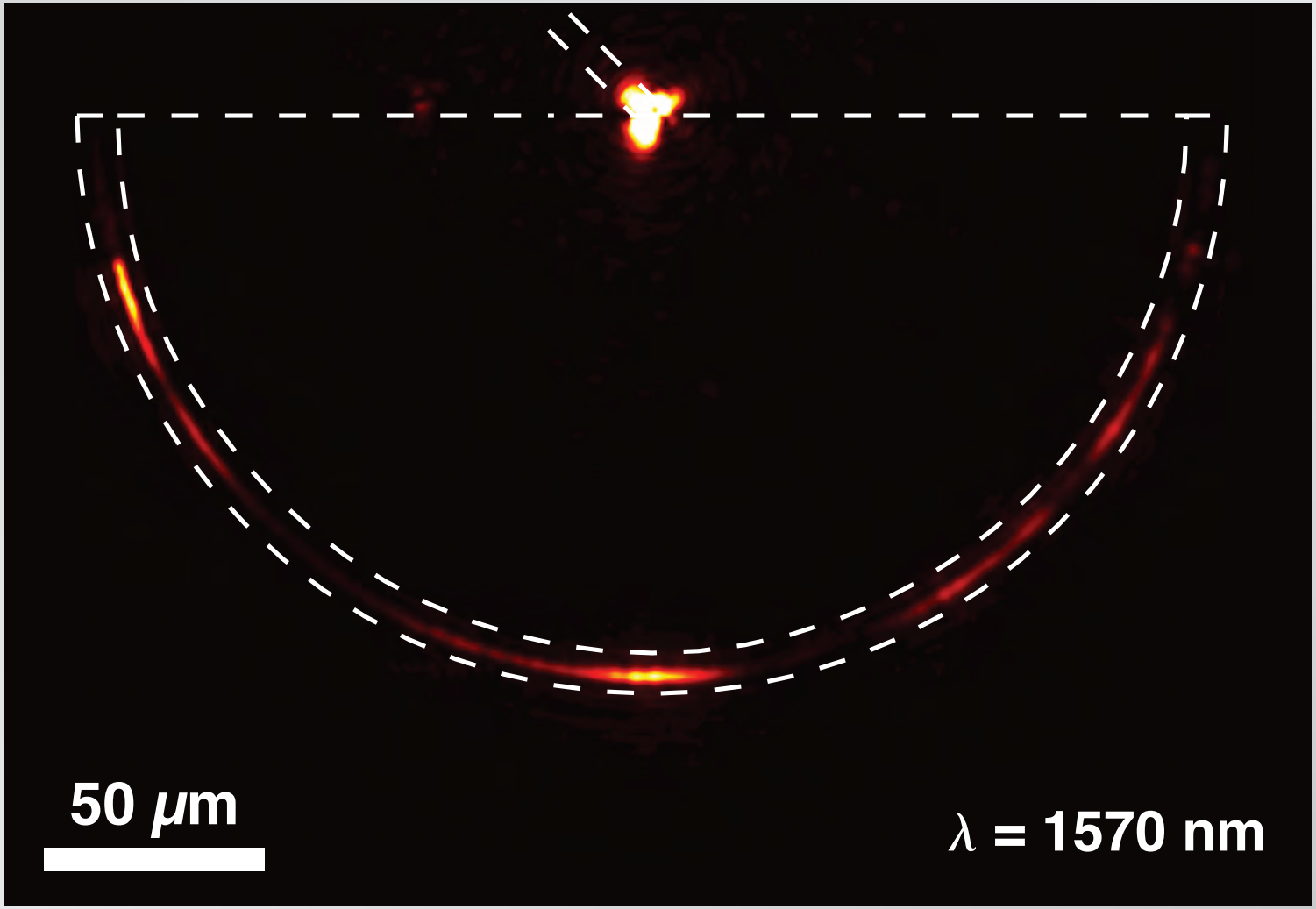
1 index

2 zero index



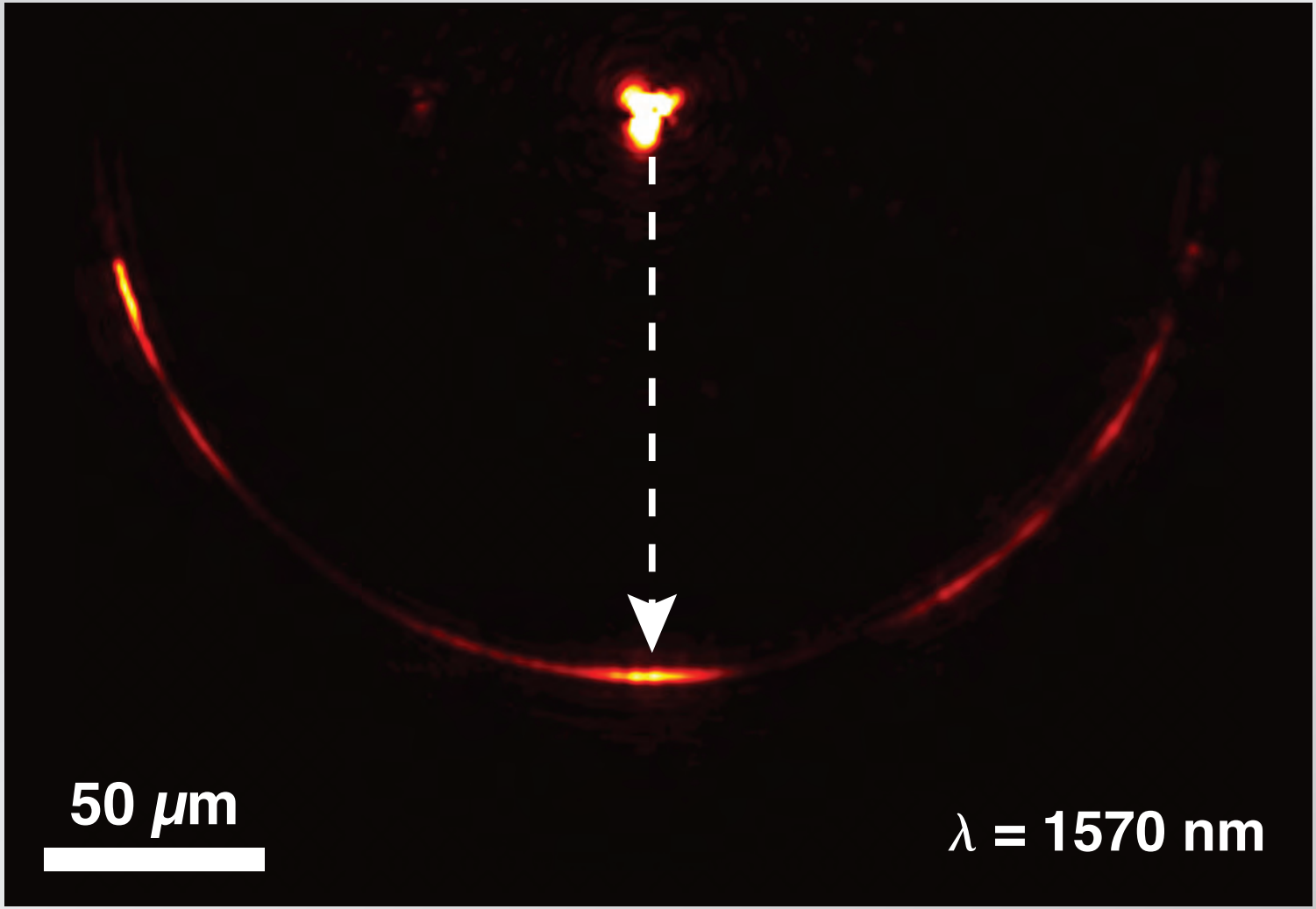
1 index

2 zero index



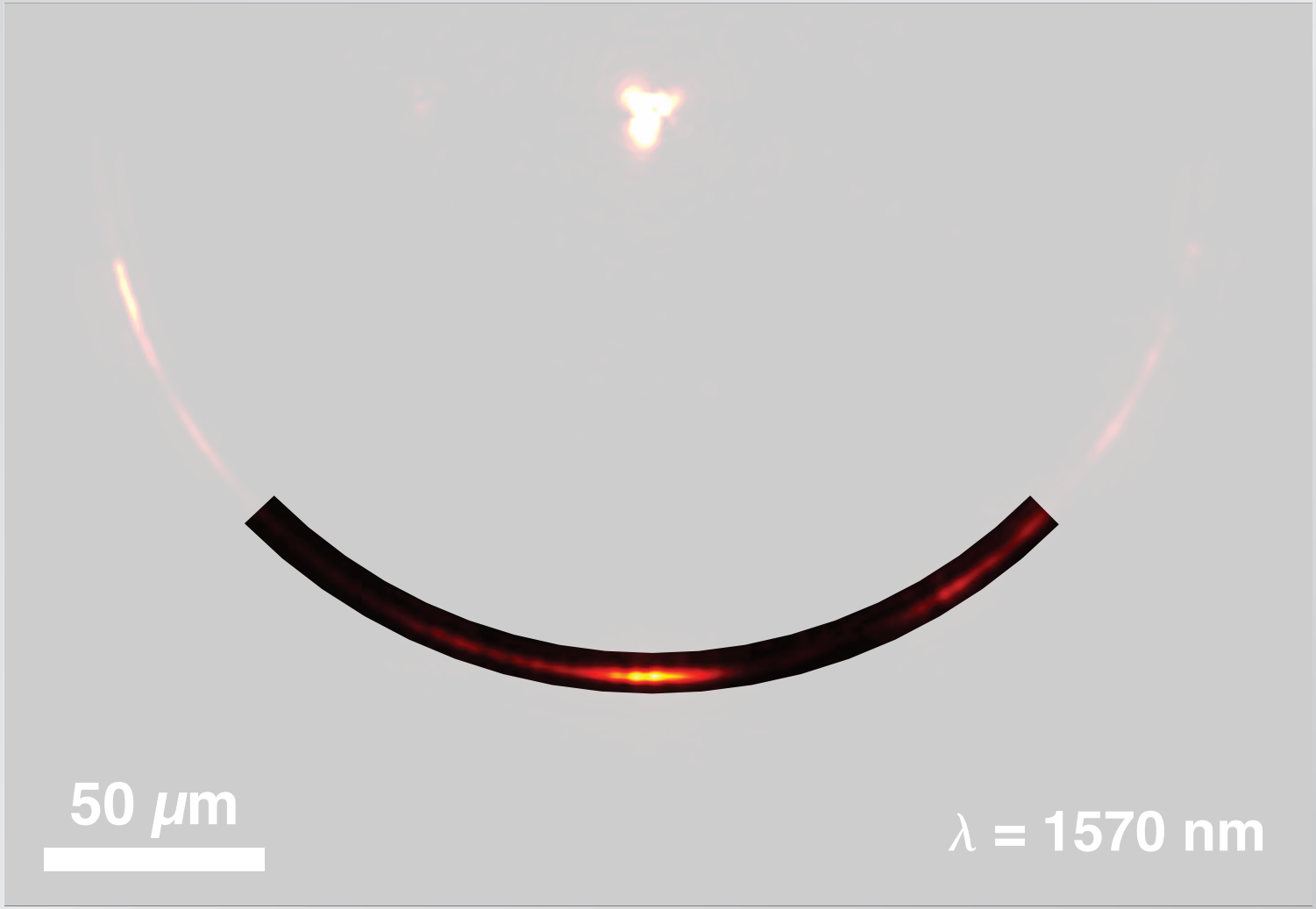
1 index

2 zero index



1 index

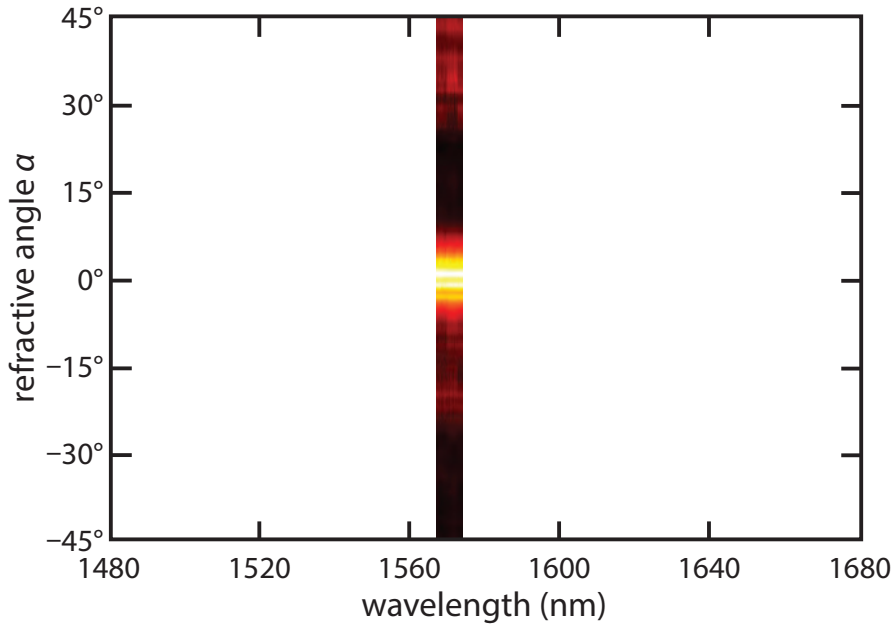
2 zero index



1 index

2 zero index

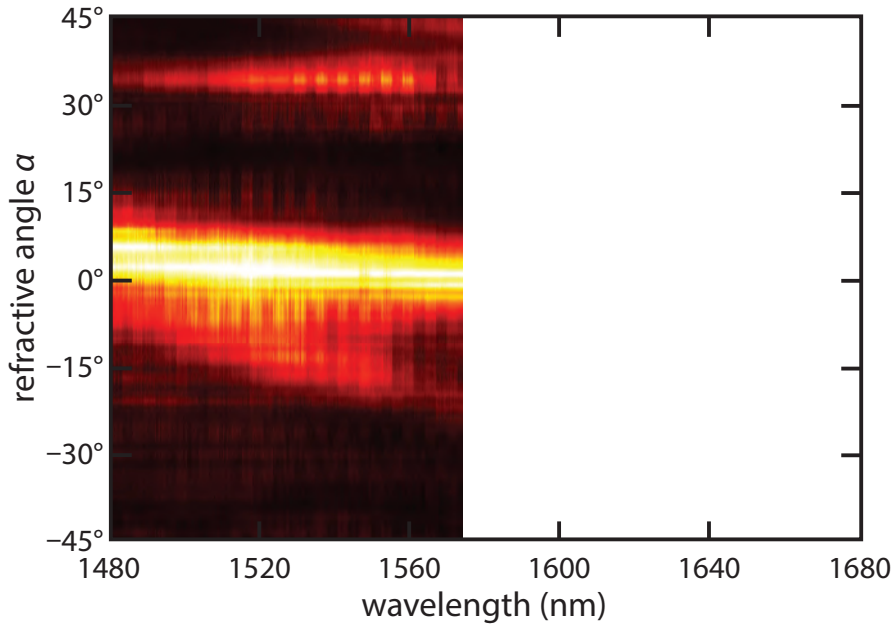
Wavelength dependence of refraction angle



1 index

2 zero index

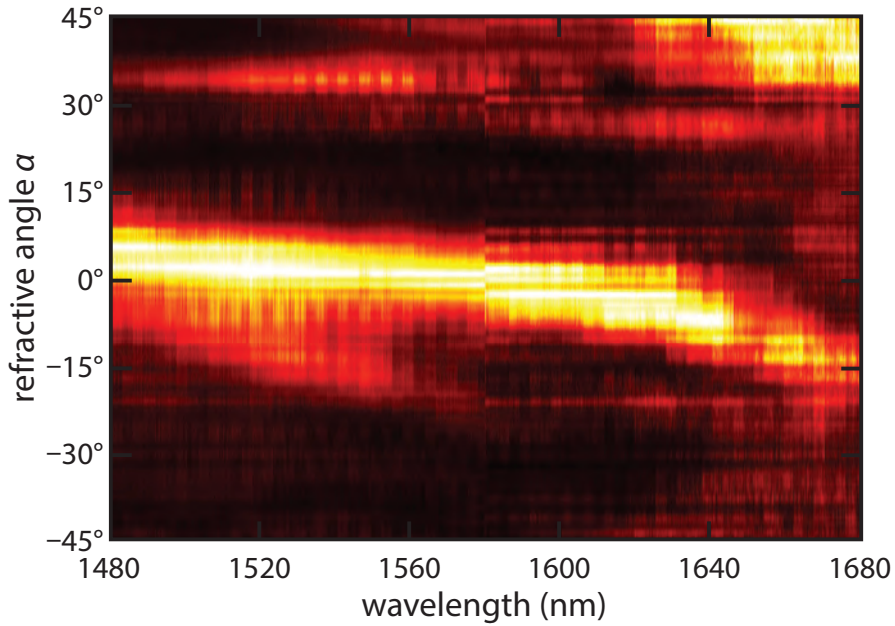
Wavelength dependence of refraction angle



1 index

2 zero index

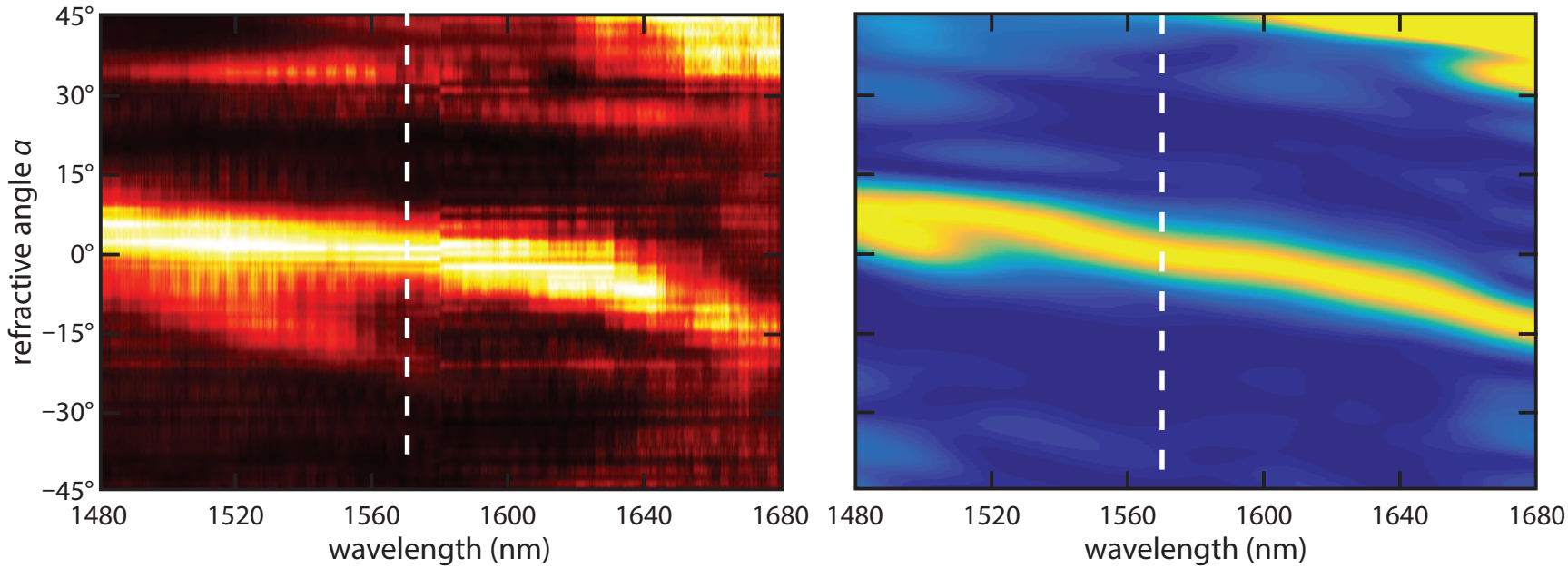
Wavelength dependence of refraction angle



1 index

2 zero index

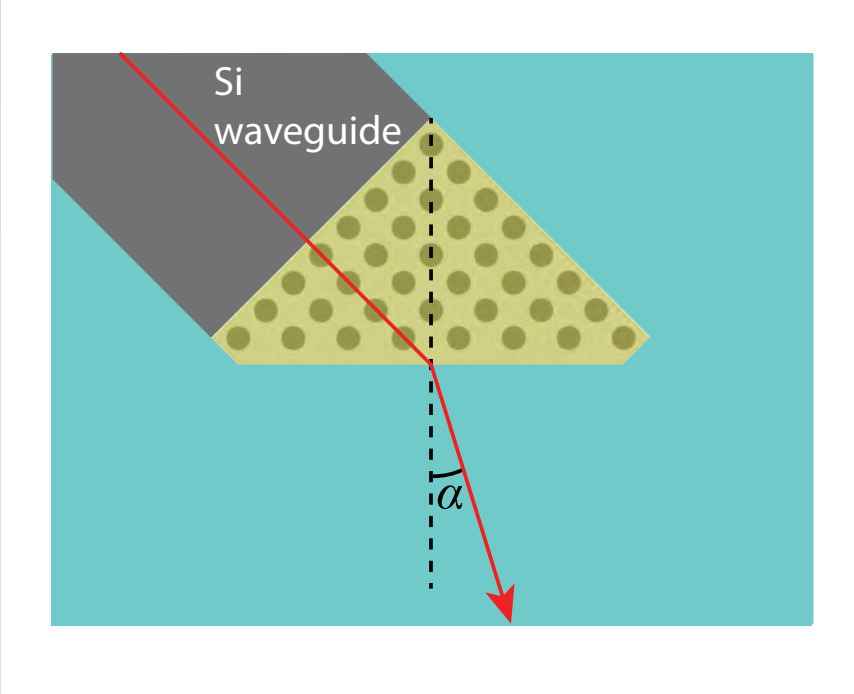
Wavelength dependence of refraction angle



1 index

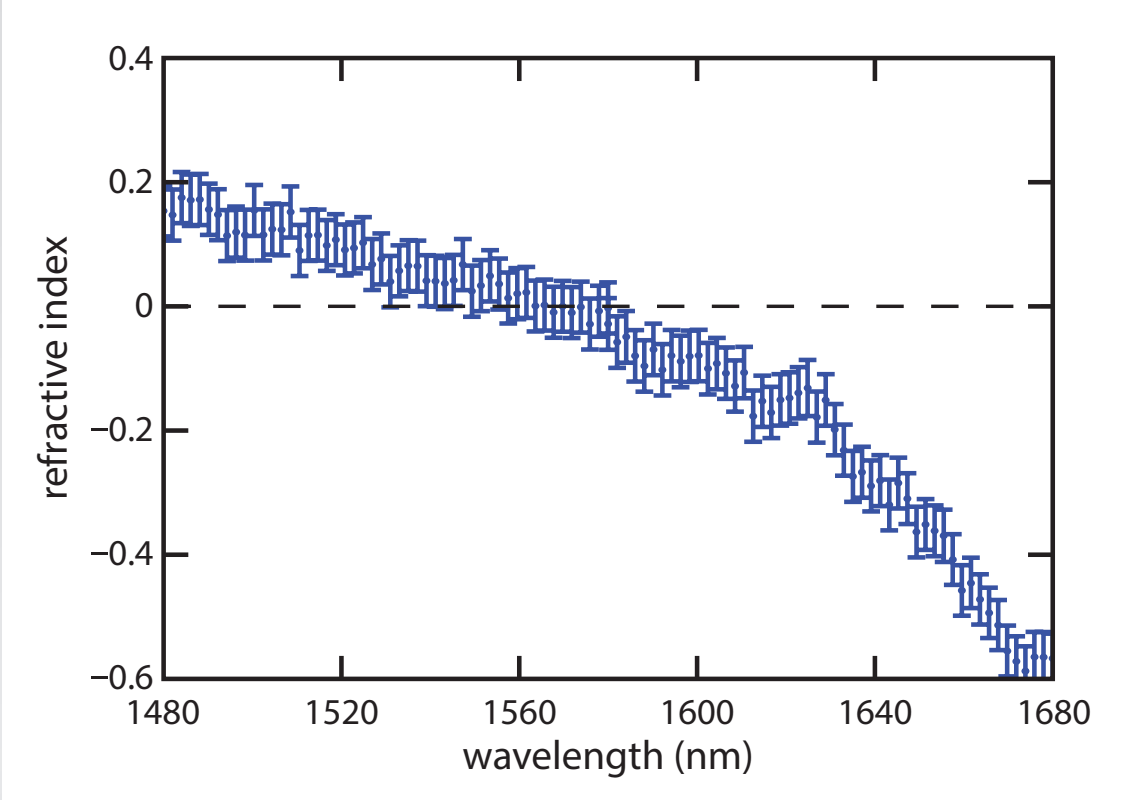
2 zero index

Wavelength dependence of index



$$n_{\text{prism}} = n_{\text{slab}} \frac{\sin\alpha}{\sin 45^\circ}$$

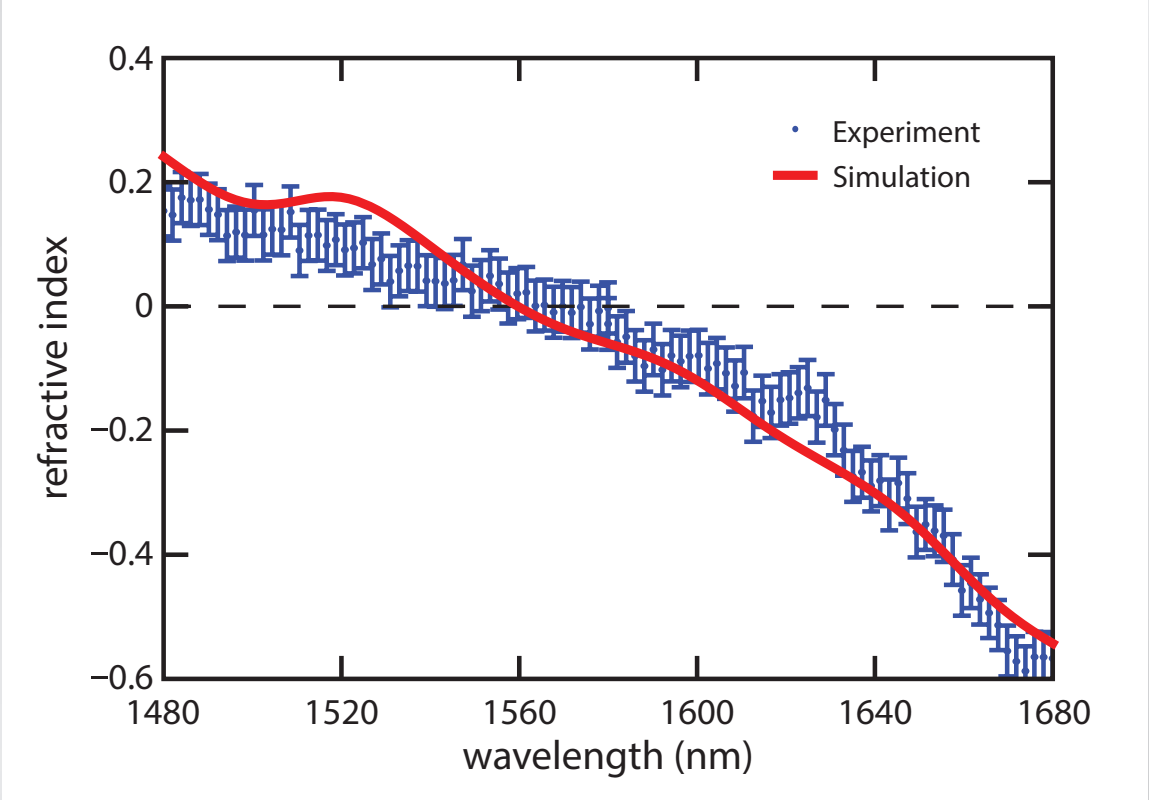
Wavelength dependence of index



1 index

2 zero index

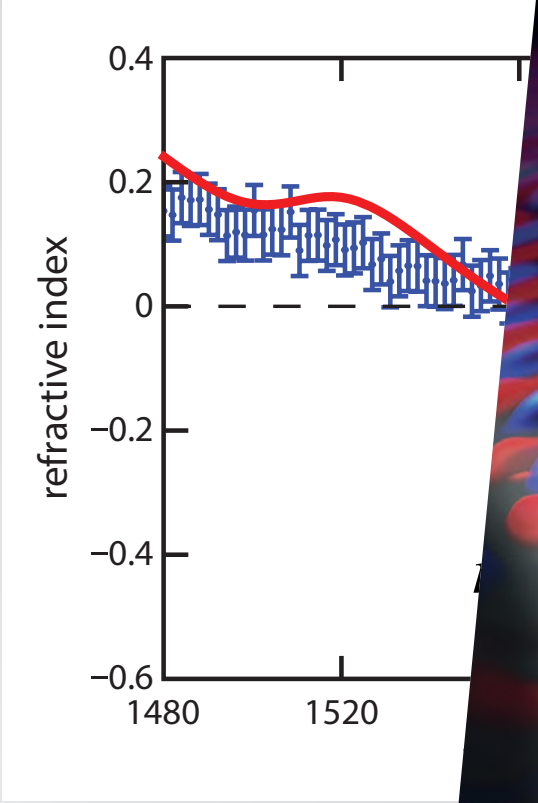
Wavelength dependence of index

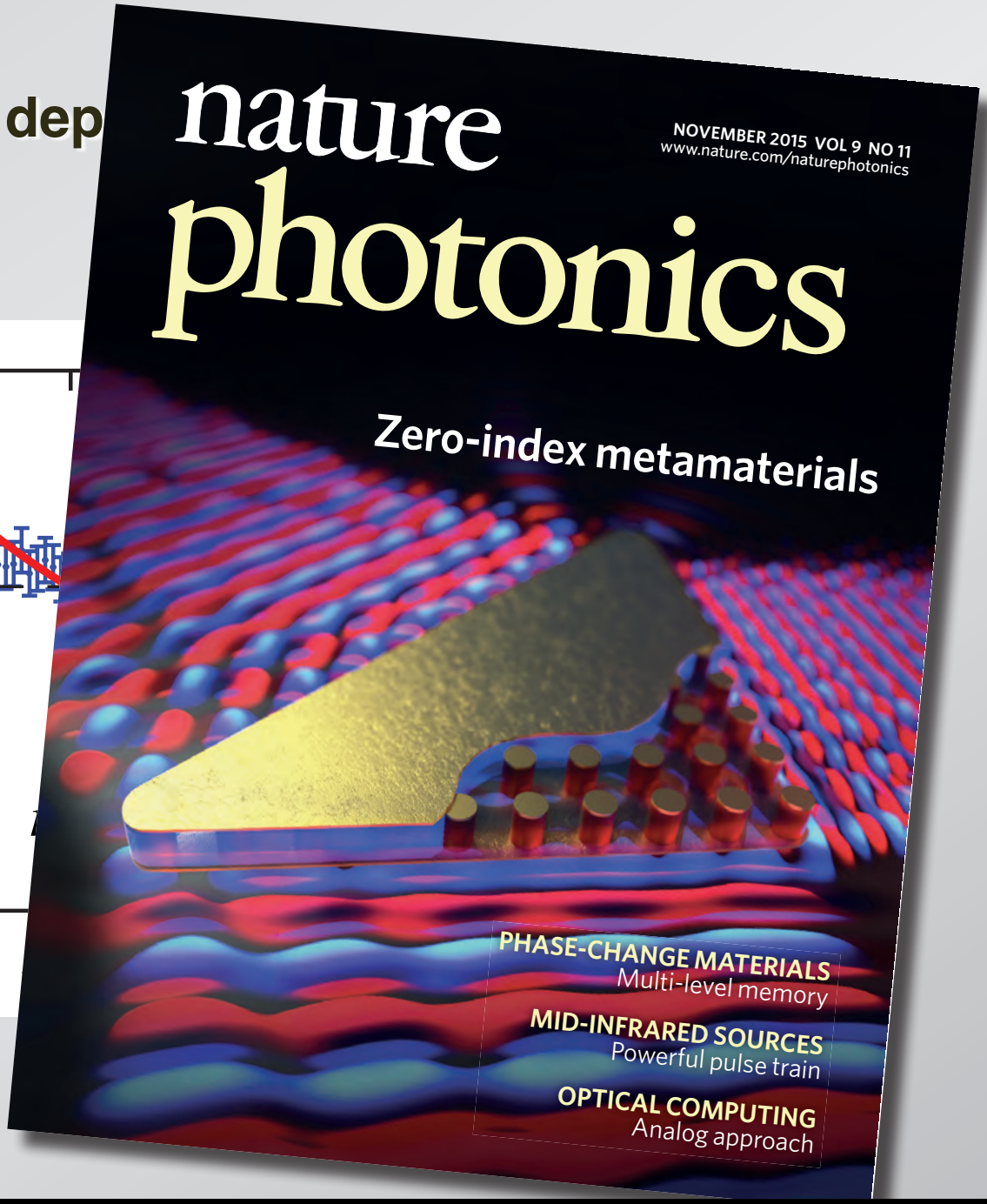


1 index

2 zero index

Wavelength dep



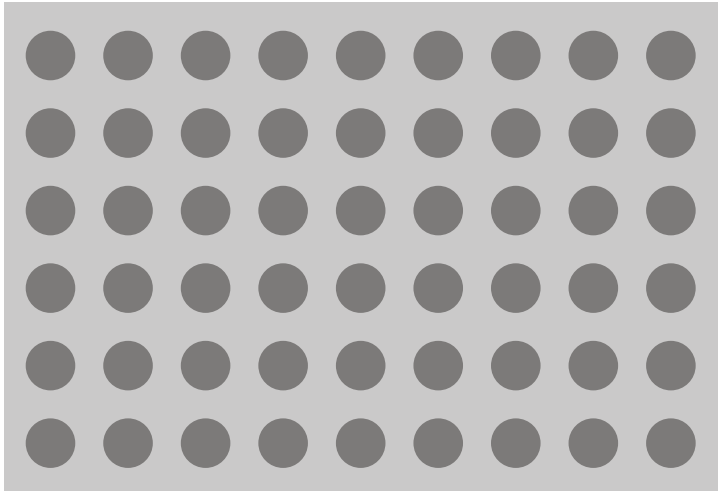


1 index

2 zero index

simplify fabrication

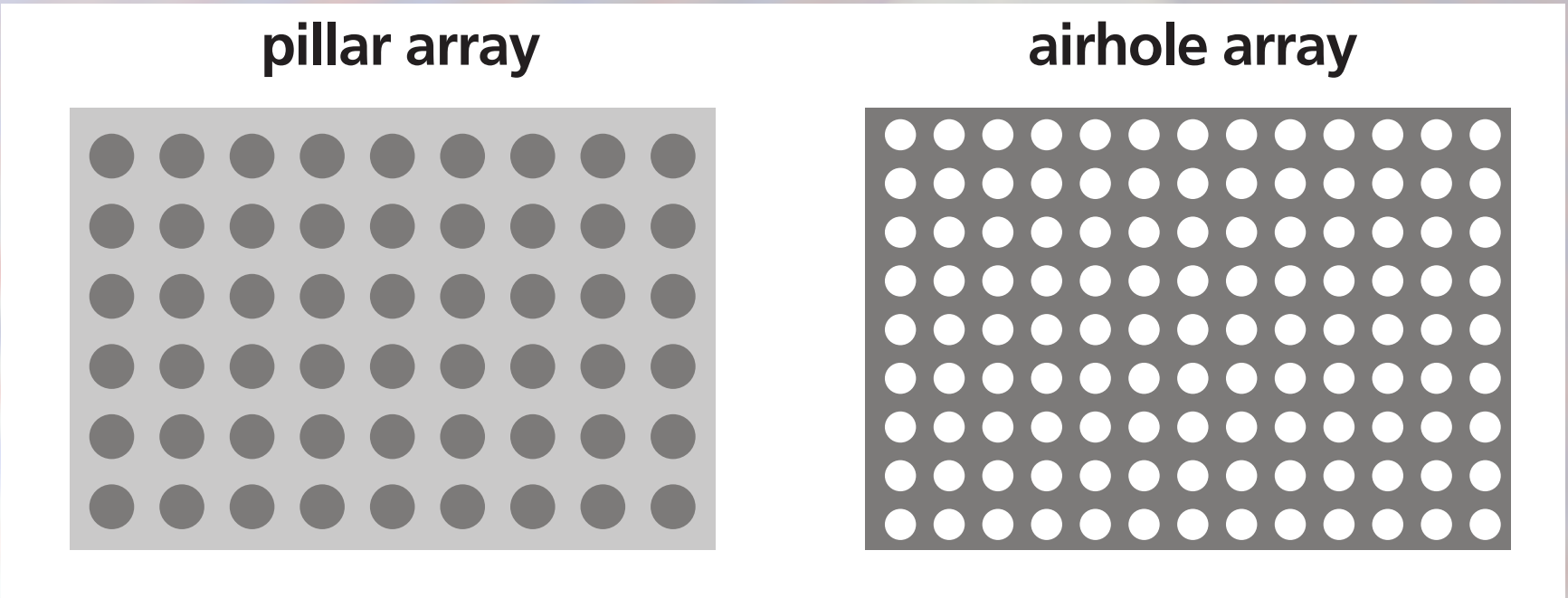
pillar array



1 index

2 zero index

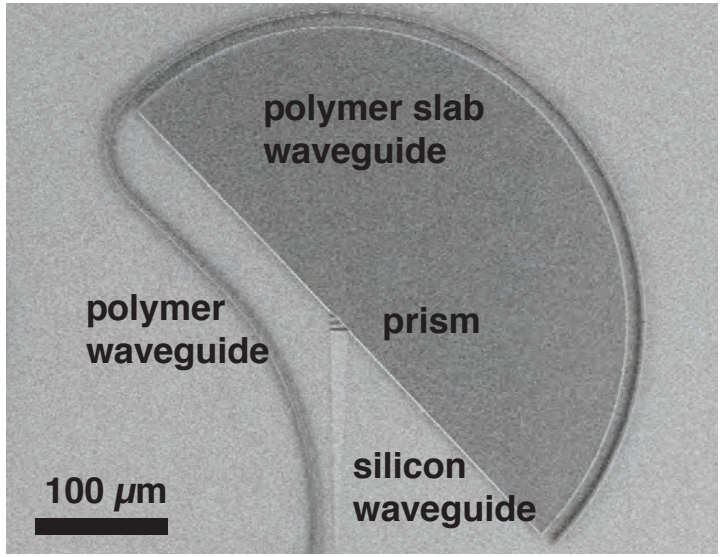
simplify fabrication



1 index

2 zero index

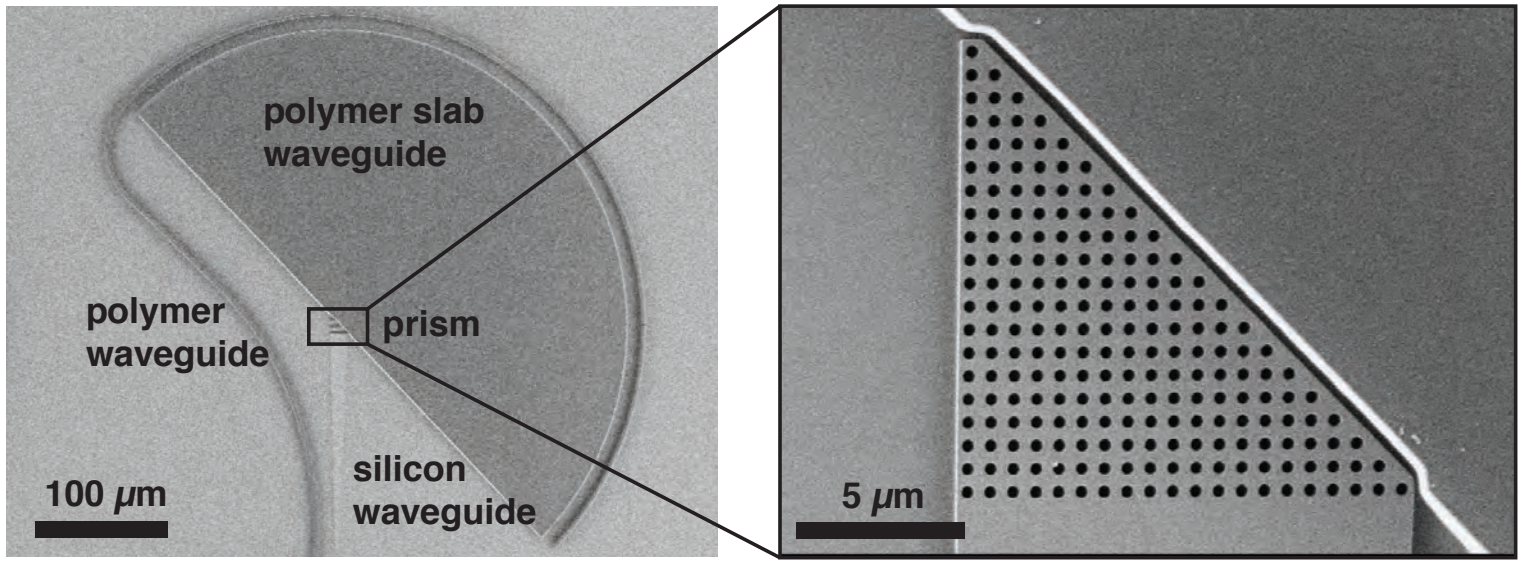
simplify fabrication



1 index

2 zero index

simplify fabrication

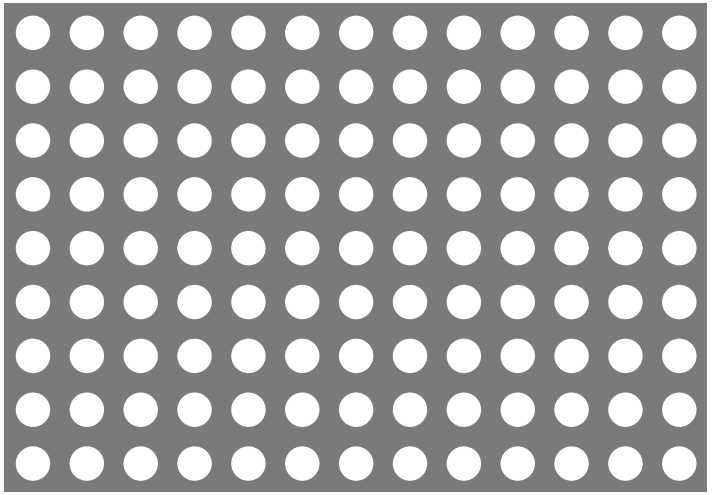


1 index

2 zero index

simplify further!

airhole array

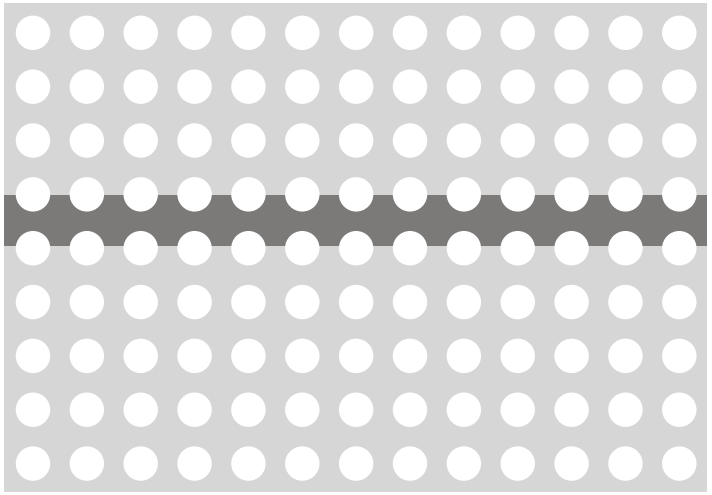


1 index

2 zero index

simplify further!

airhole array

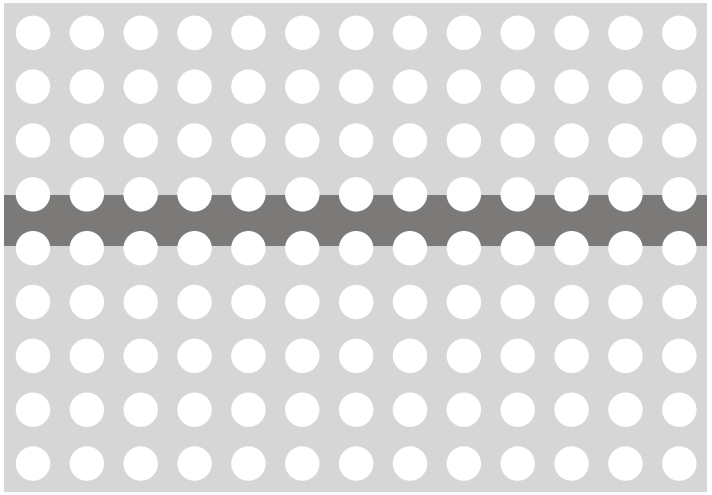


1 index

2 zero index

simplify further!

airhole array



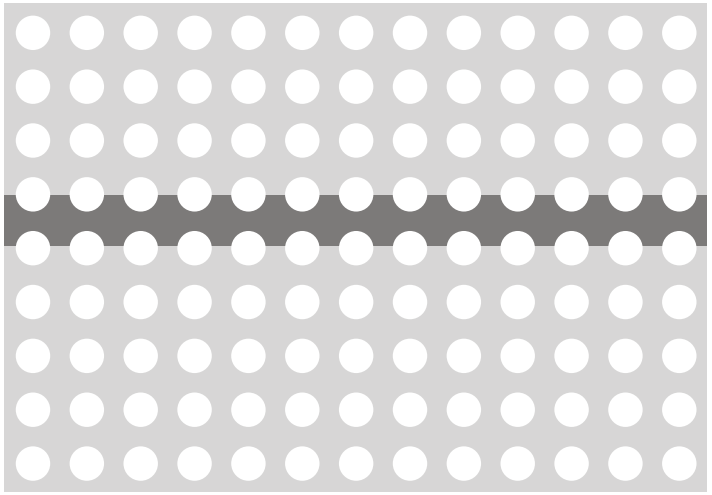
1D ZIM waveguide

1 index

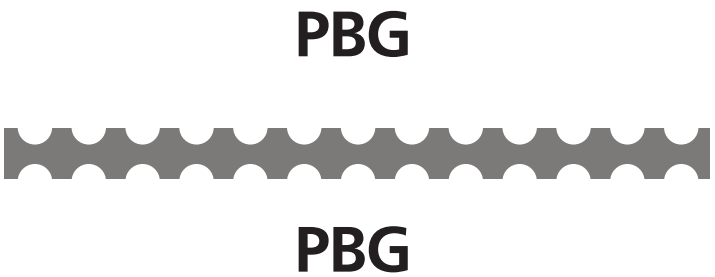
2 zero index

simplify further!

airhole array



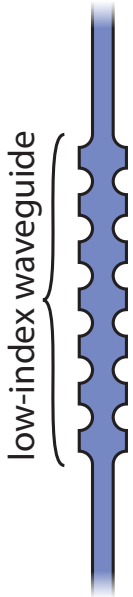
1D ZIM waveguide



1 index

2 zero index

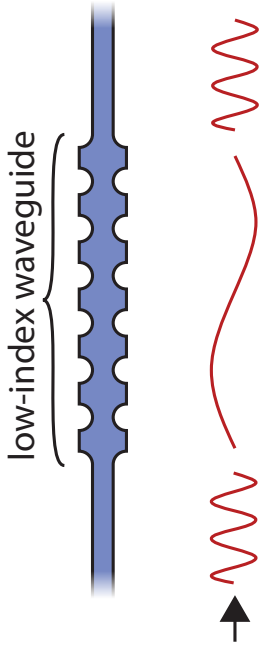
waveguiding



1 index

2 zero index

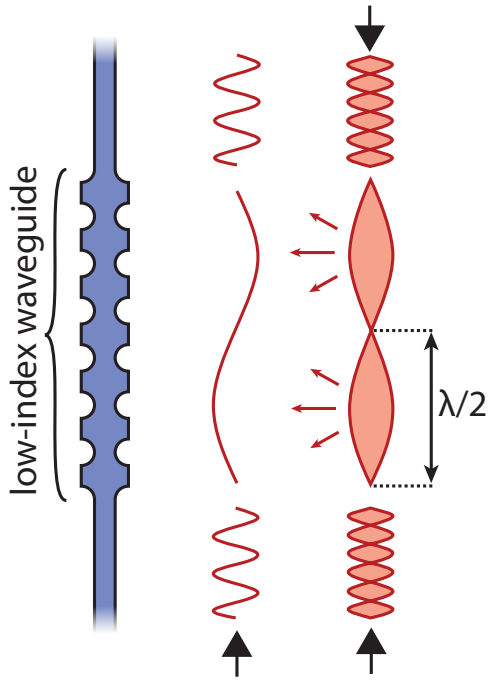
waveguiding



1 index

2 zero index

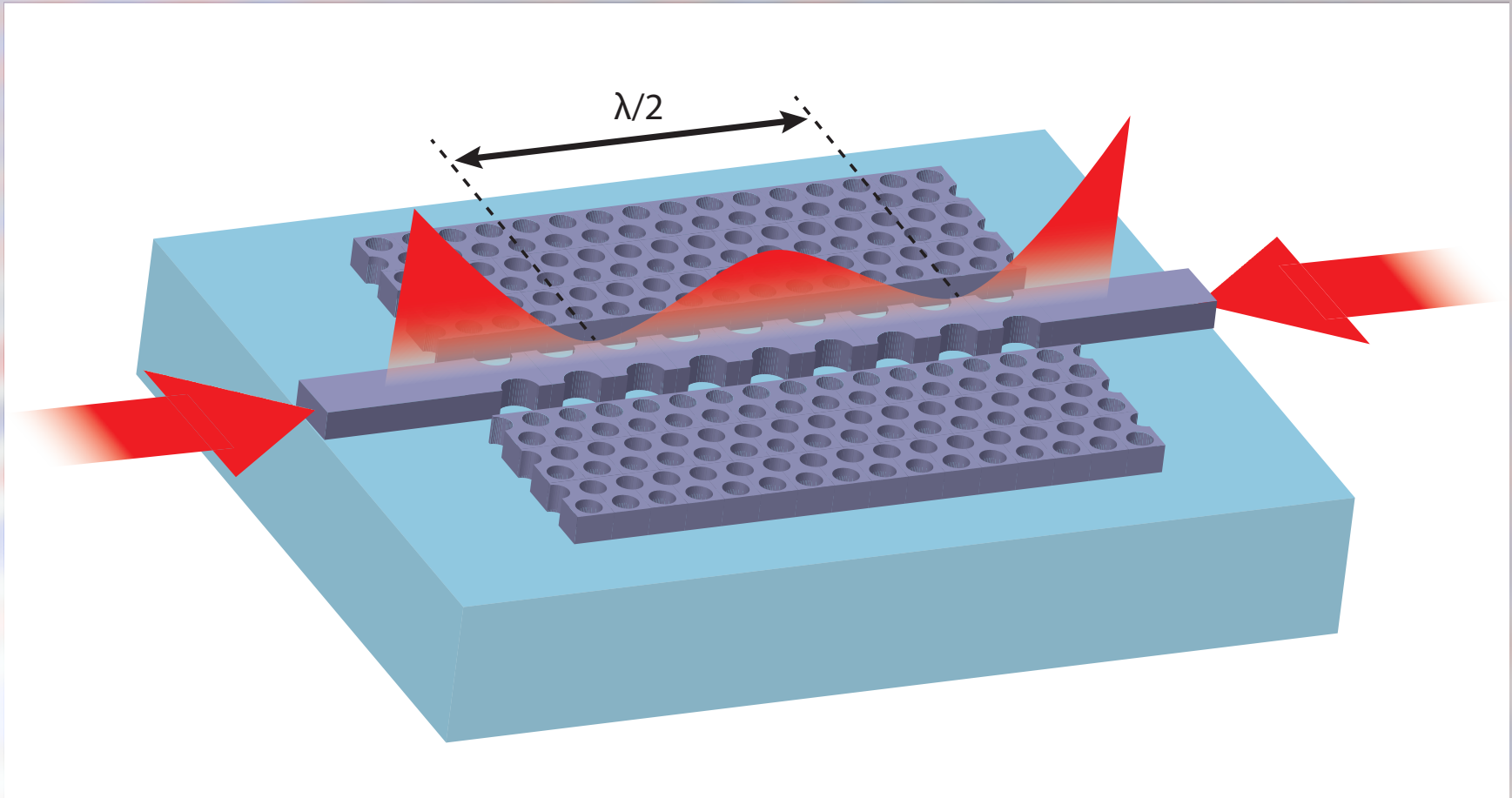
waveguiding



1 index

2 zero index

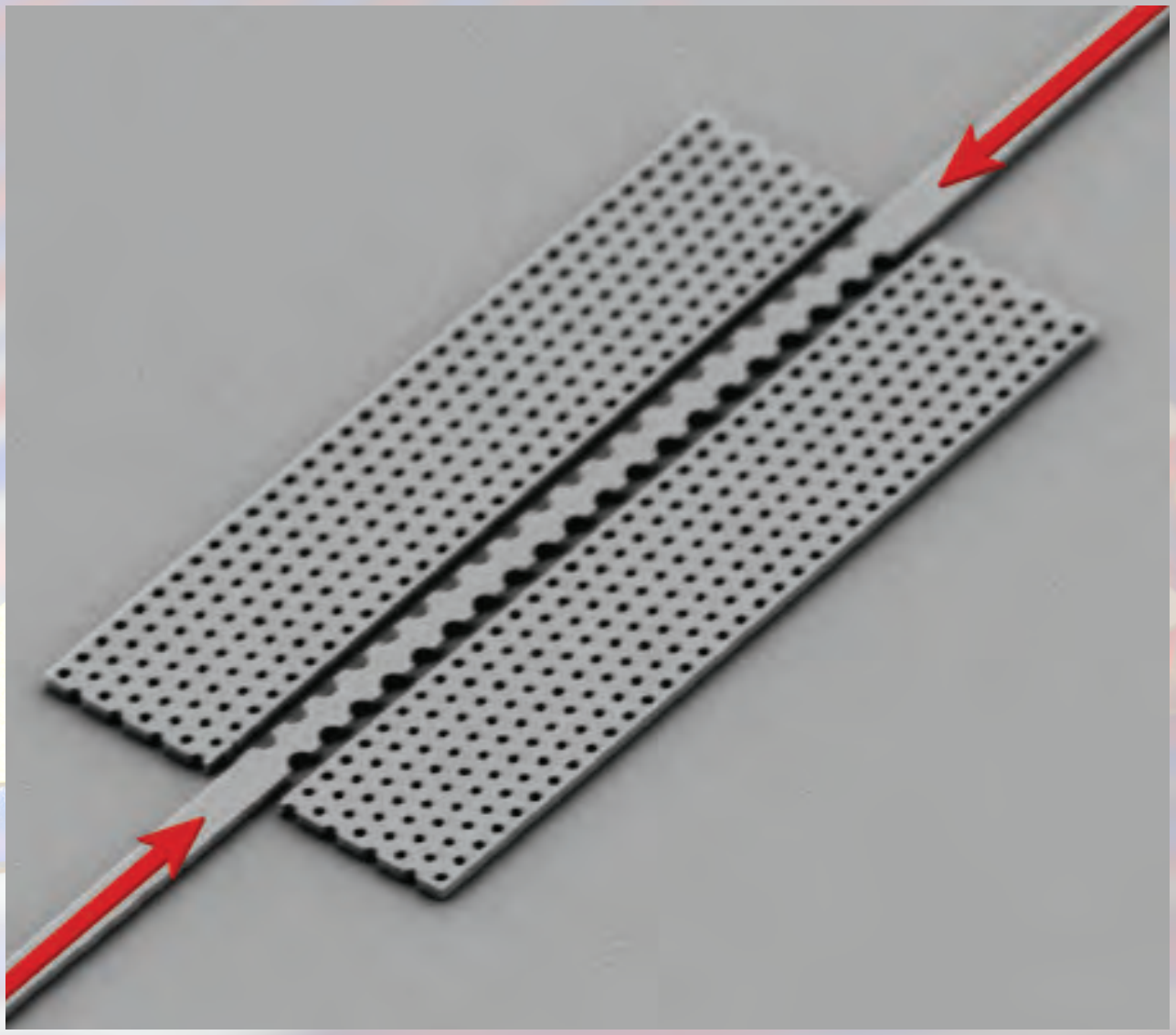
look at standing waves



1 index

2 zero index

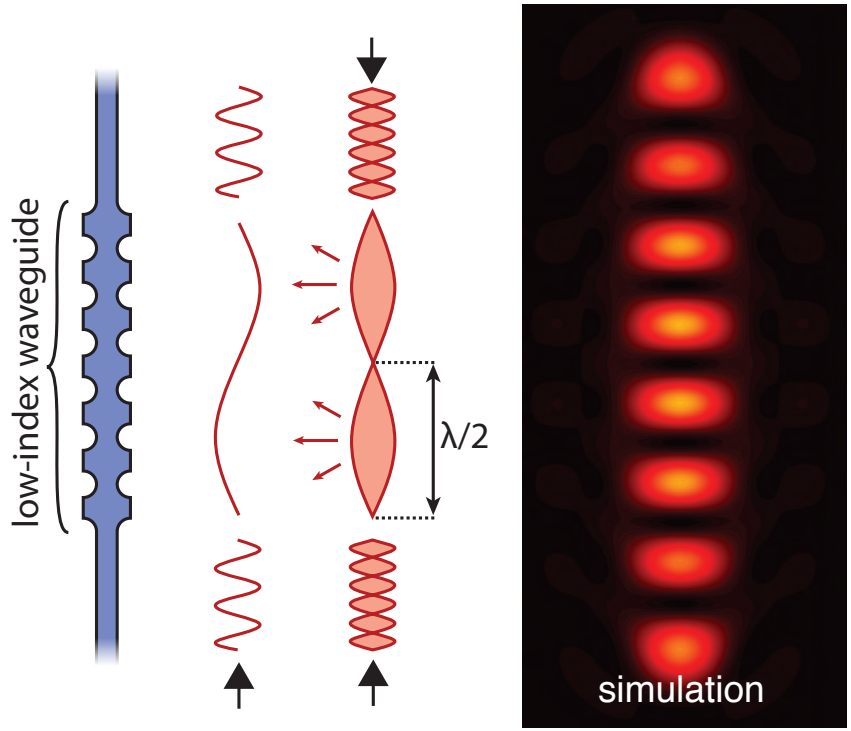
look at standing waves



1 index

2 zero index

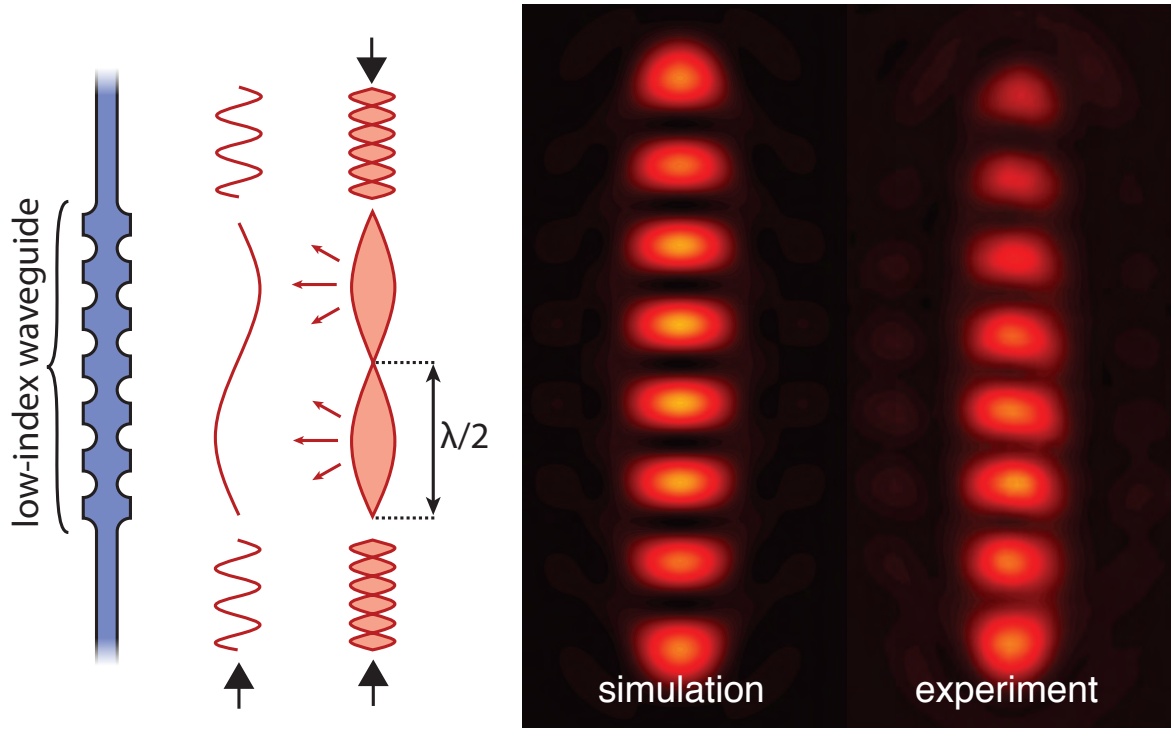
look at standing waves



1 index

2 zero index

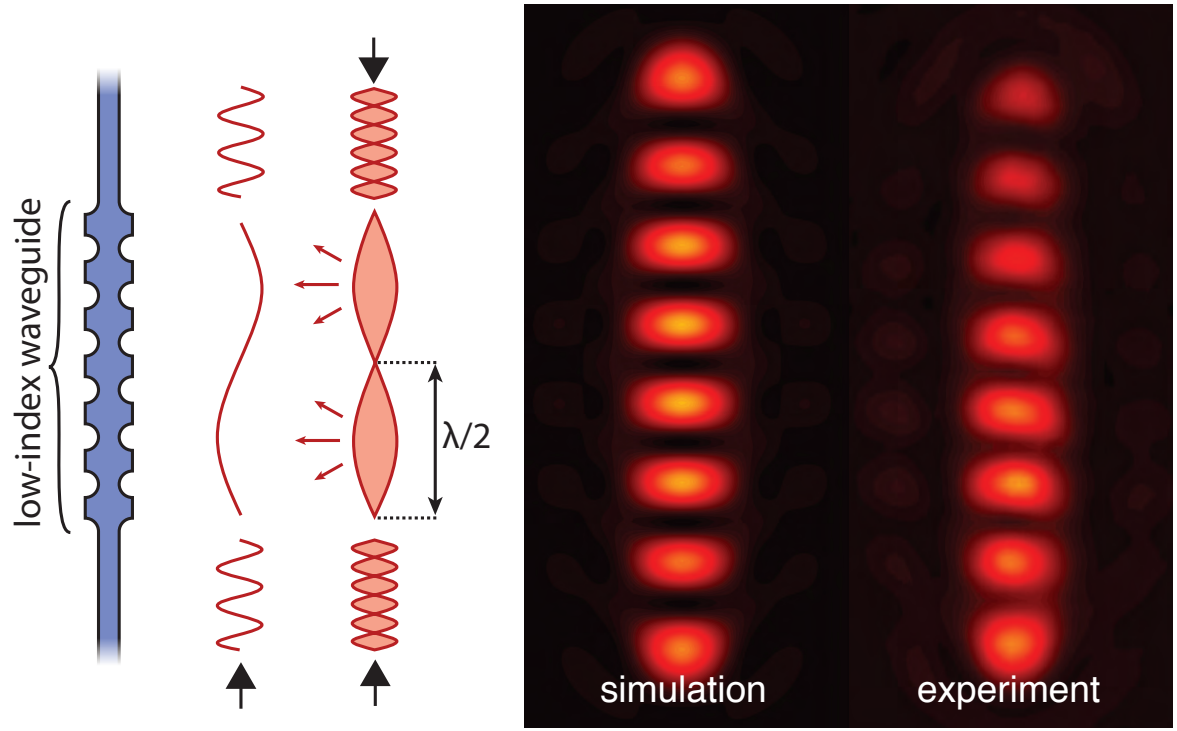
look at standing waves



1 index

2 zero index

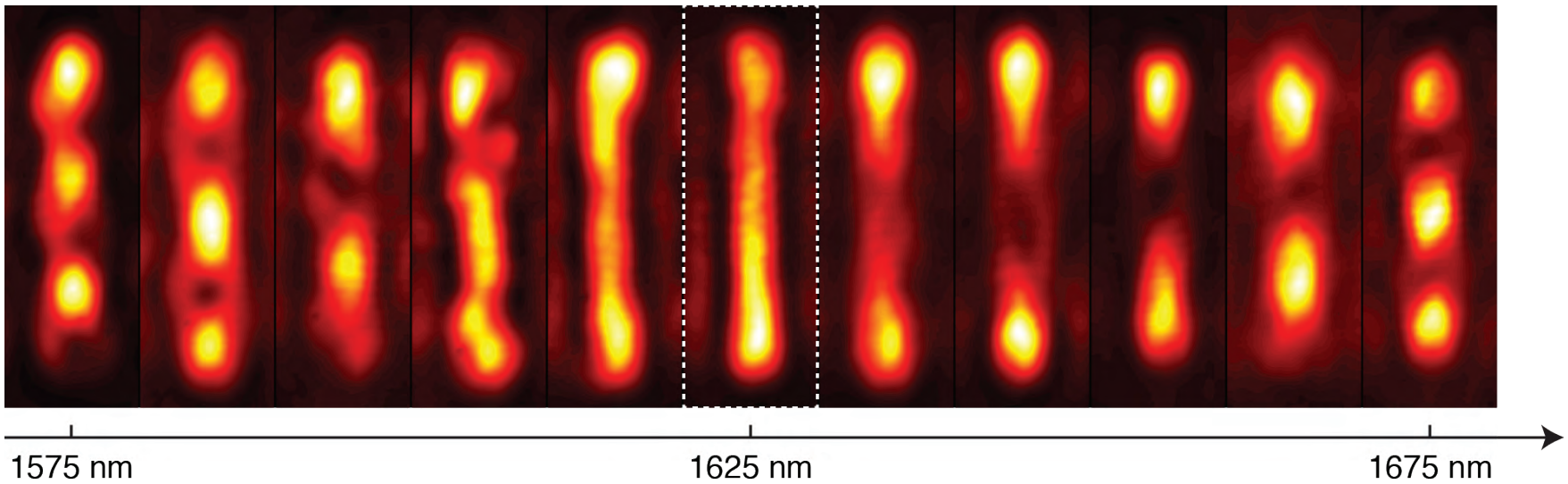
direct observation of effective wavelength!!



1 index

2 zero index

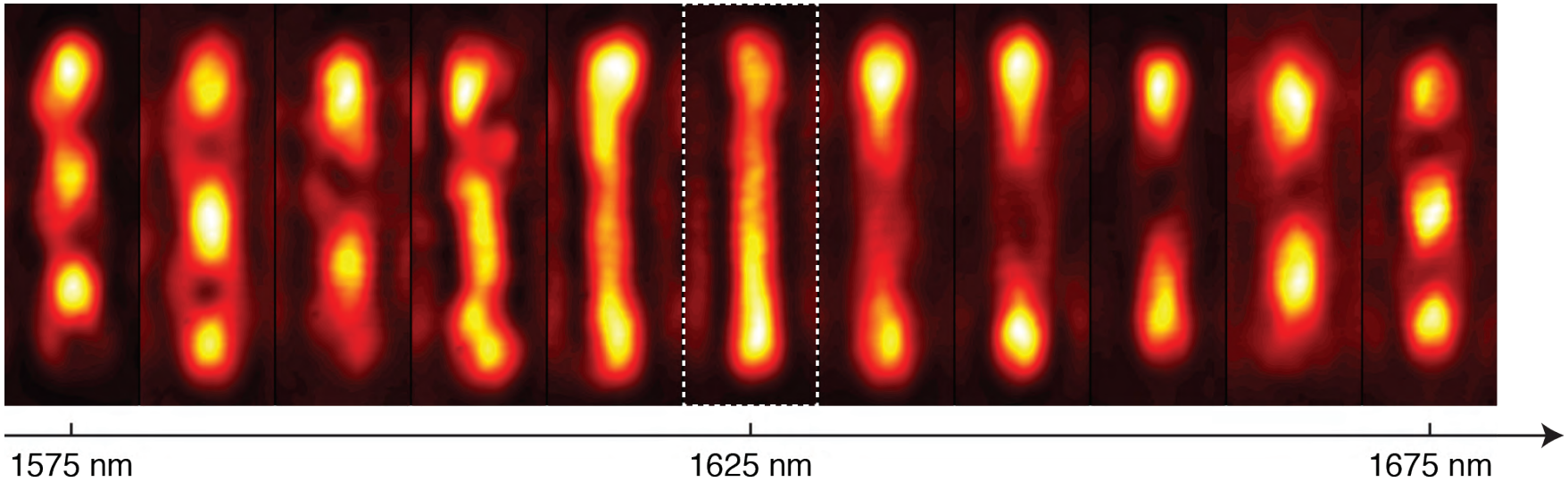
look at standing waves



1 index

2 zero index

look at standing waves

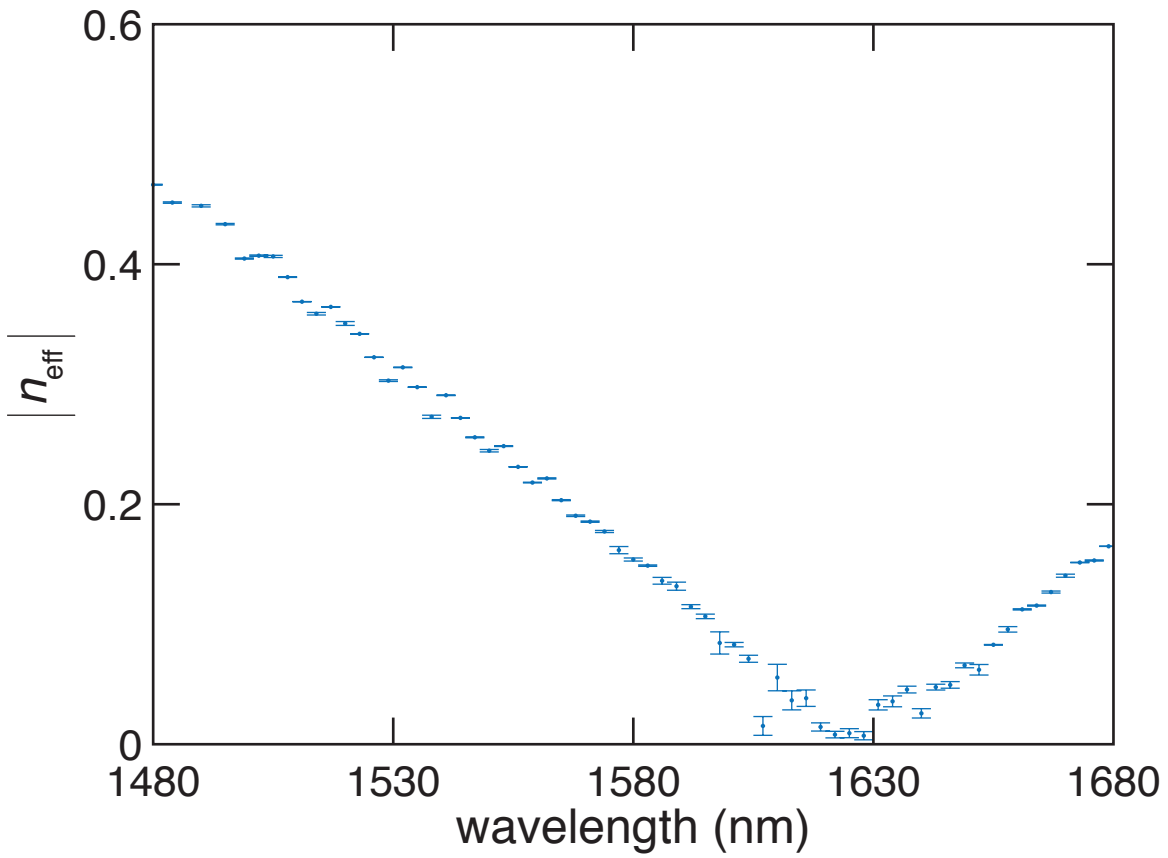


$$|n_{\text{eff}}| = \frac{\lambda_0}{\lambda_{\text{eff}}}$$

1 index

2 zero index

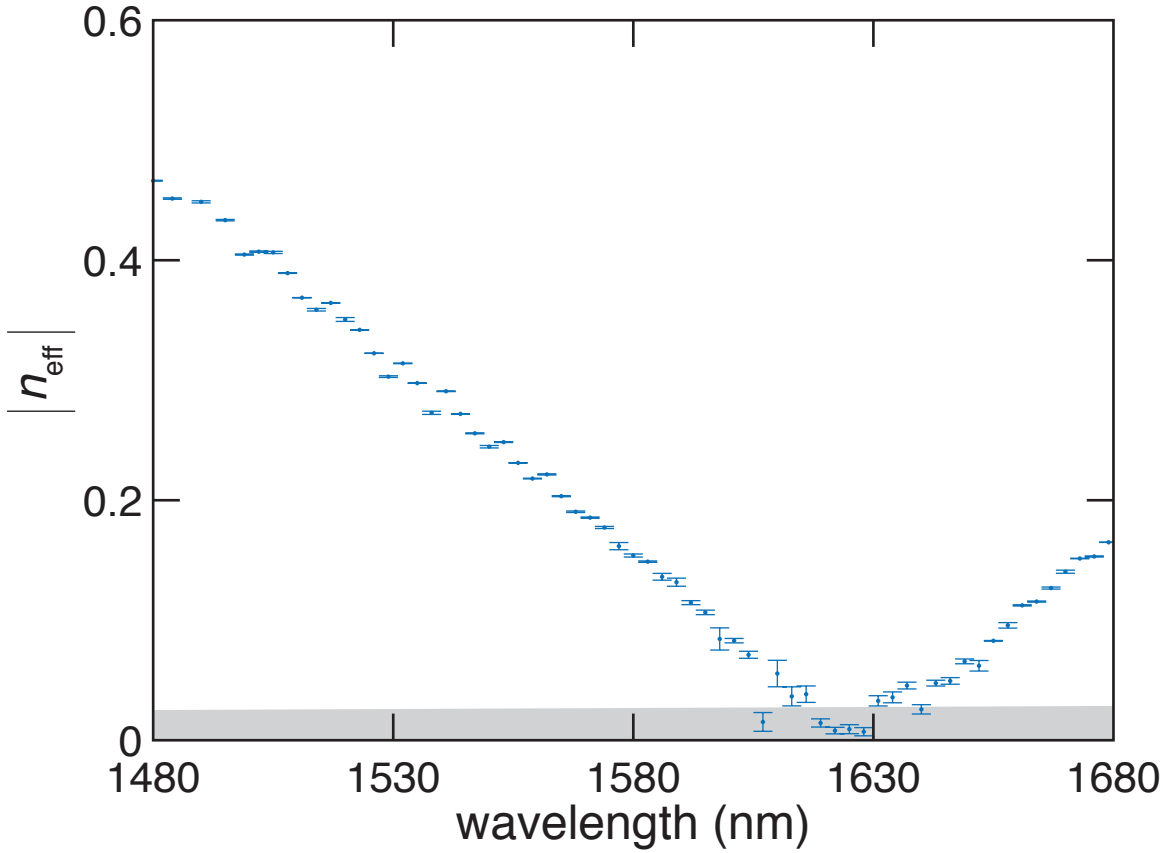
comparison of experiment and simulation



1 index

2 zero index

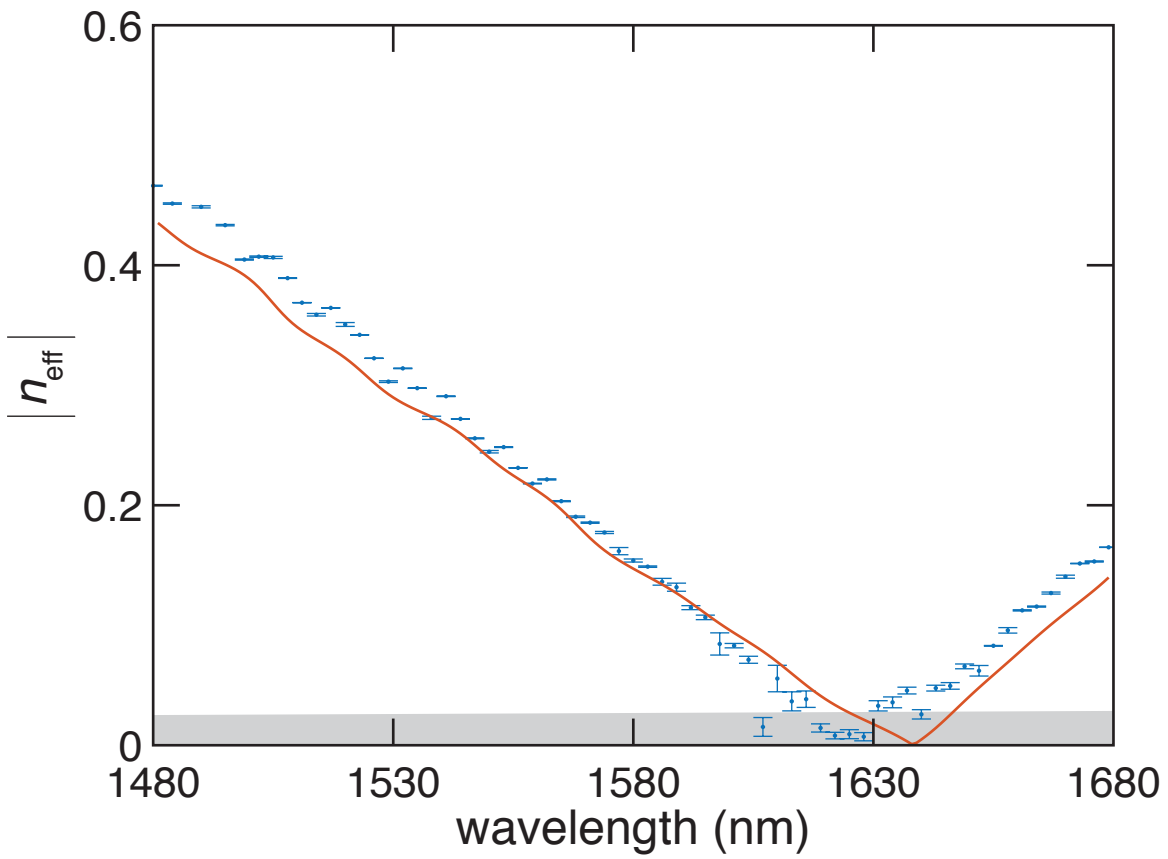
comparison of experiment and simulation



1 index

2 zero index

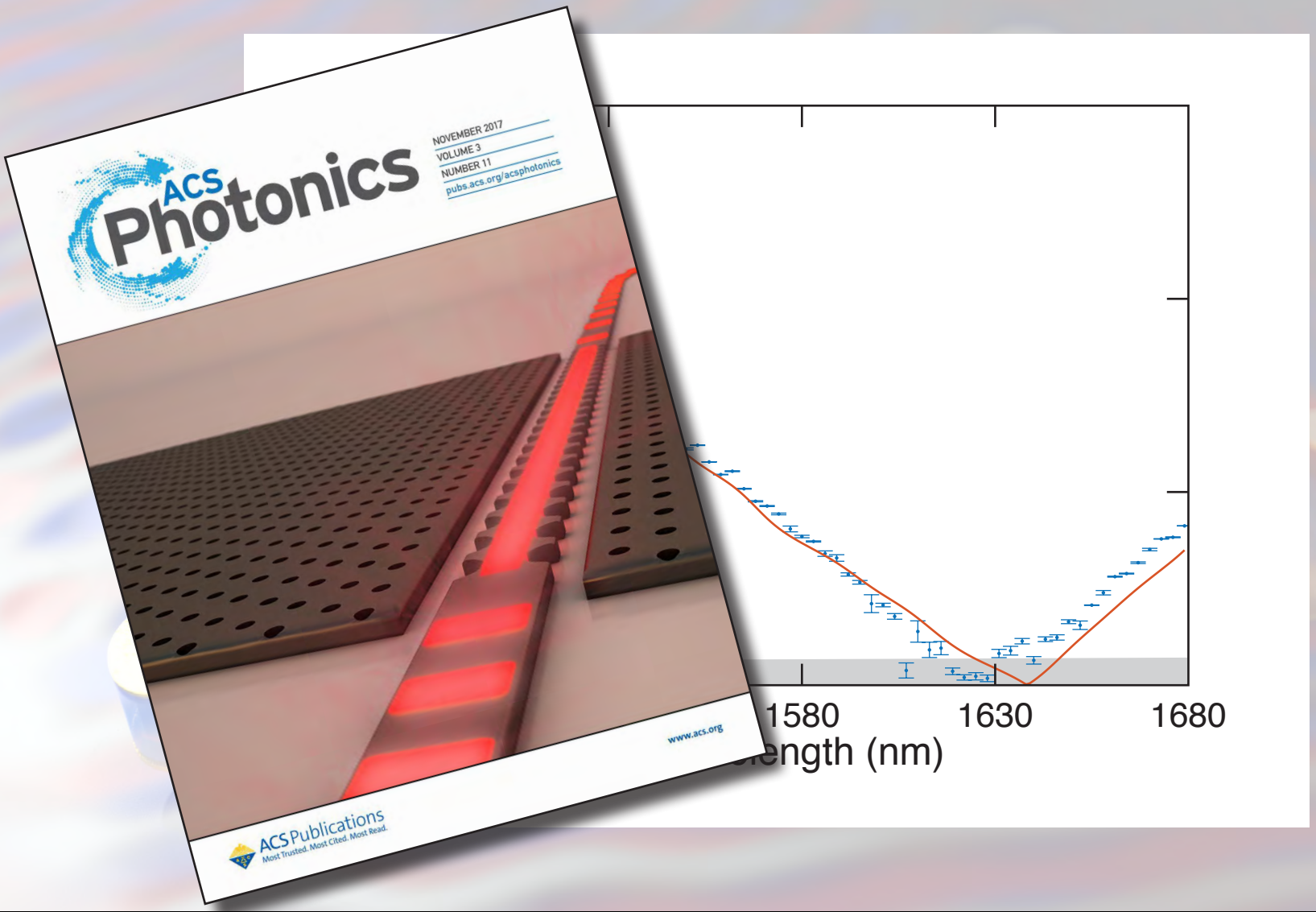
comparison of experiment and simulation



1 index

2 zero index

comparison of experiment and simulation



1 index

2 zero index

$$n = 0$$

1 index

2 zero index

(extreme) opportunities

relaxed phase matching constraints as $k \rightarrow 0$

1 index

2 zero index

(extreme) opportunities

PHYSICAL REVIEW LETTERS **128,** 203902 (2022)

Editors' Suggestion

Relaxed Phase-Matching Constraints in Zero-Index Waveguides

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The utility of all parametric nonlinear optical processes is hampered by phase-matching requirements. Quasi-phase-matching, birefringent phase matching, and higher-order-mode phase matching have all been developed to address this constraint, but the methods demonstrated to date suffer from the inconvenience of only being phase matched for a single, specific arrangement of beams, typically copropagating, resulting in cumbersome experimental configurations and large footprints for integrated devices. Here, we exper-

that these phase-matching requirements may be satisfied in a parametric nonlinear

of input and output beams when using low-index

independent phase

1 index

2 zero index

$$v_g = \frac{d\omega}{dk}$$

1 index

2 zero index

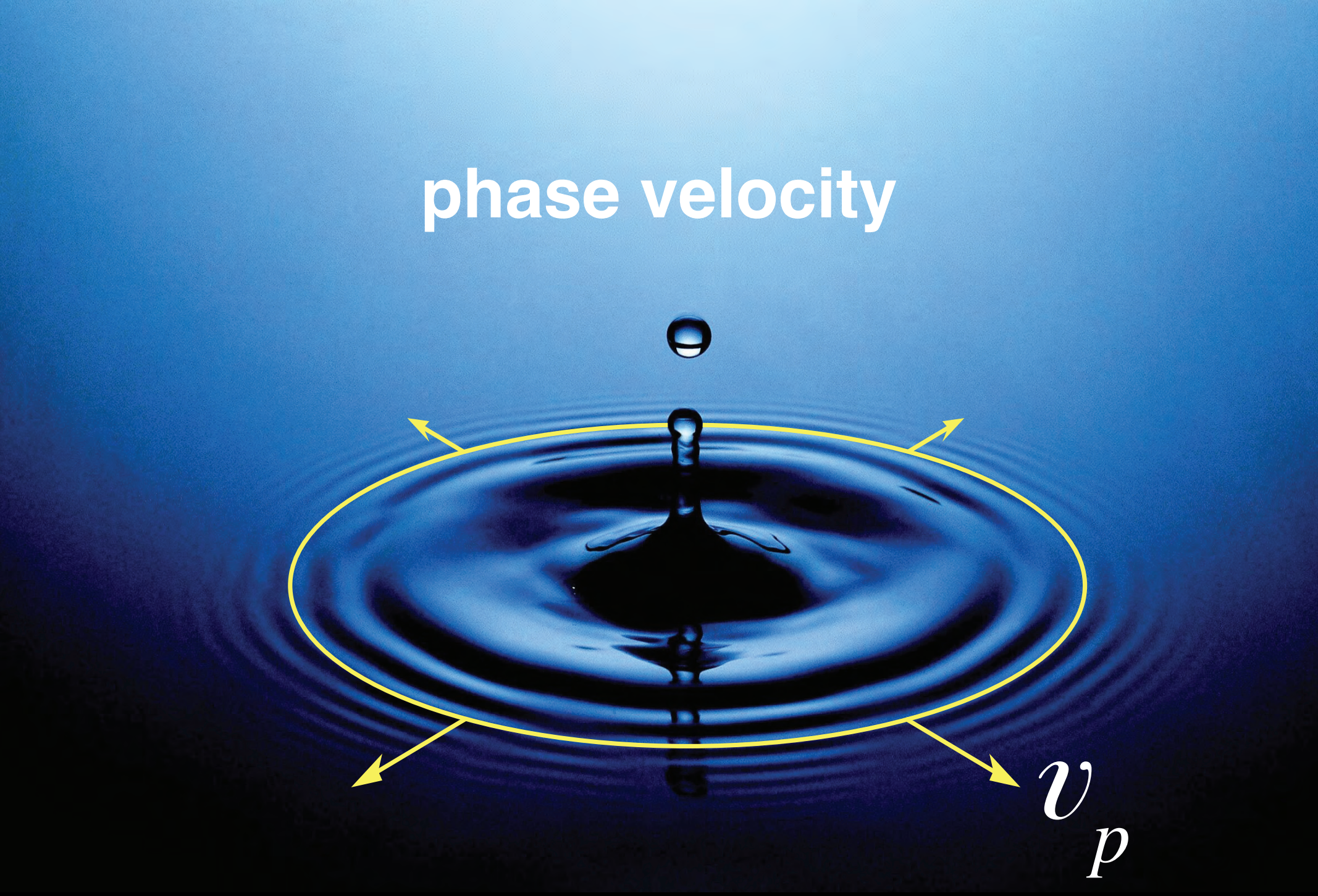
3 zero group velocity

1 index

2 zero index

3 zero group velocity

phase velocity

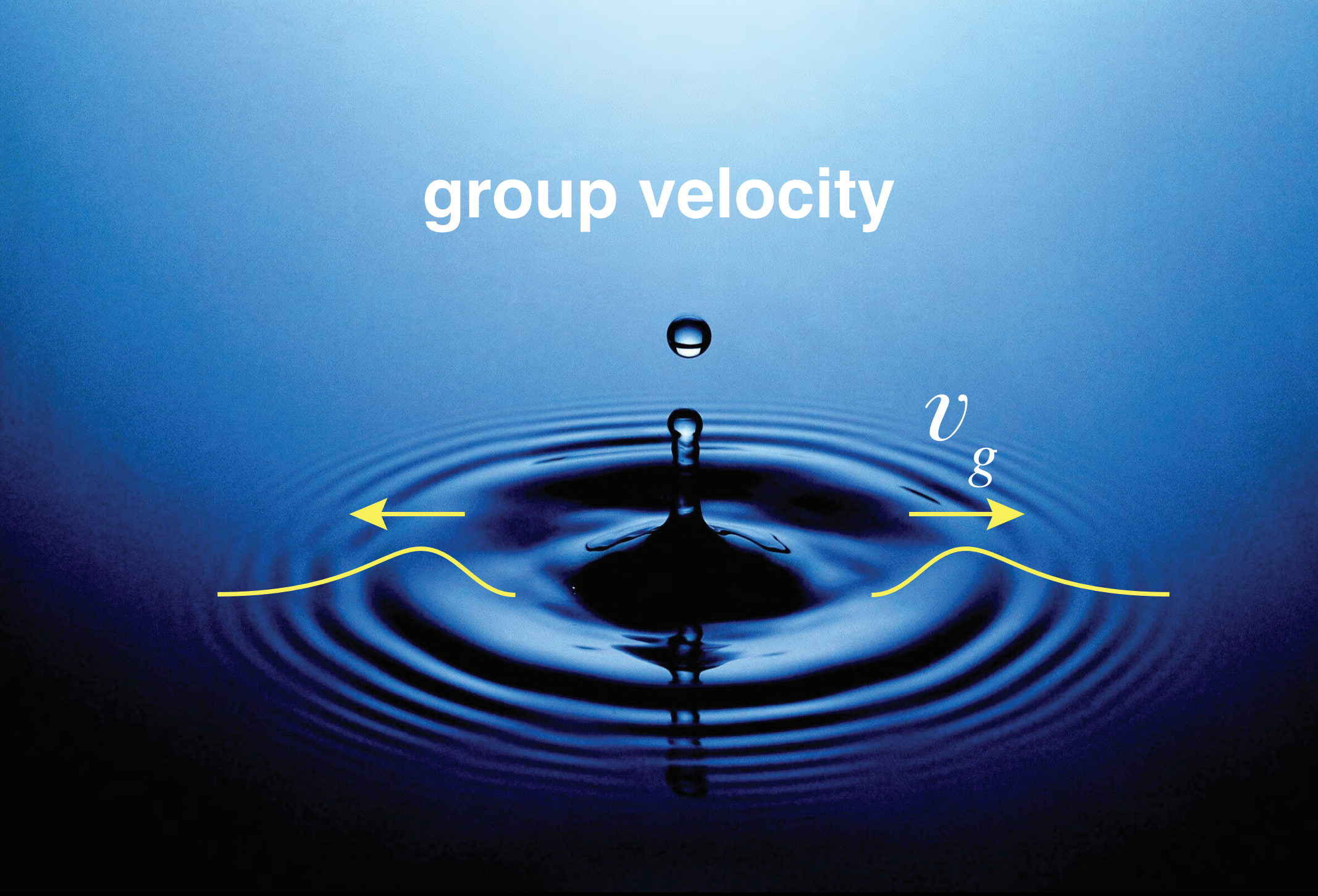


1 index

2 zero index

3 zero group velocity

group velocity

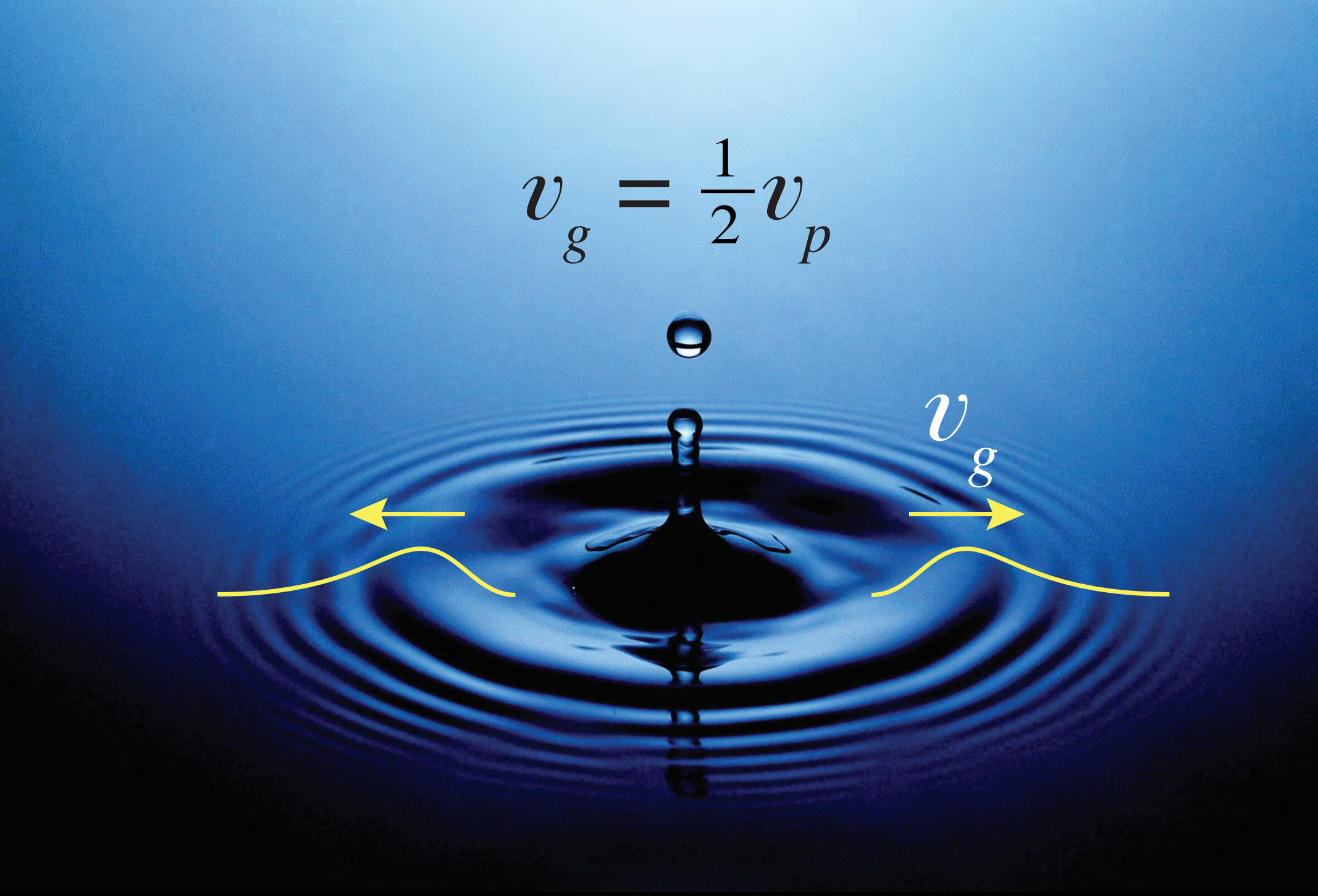


1 index

2 zero index

3 zero group velocity

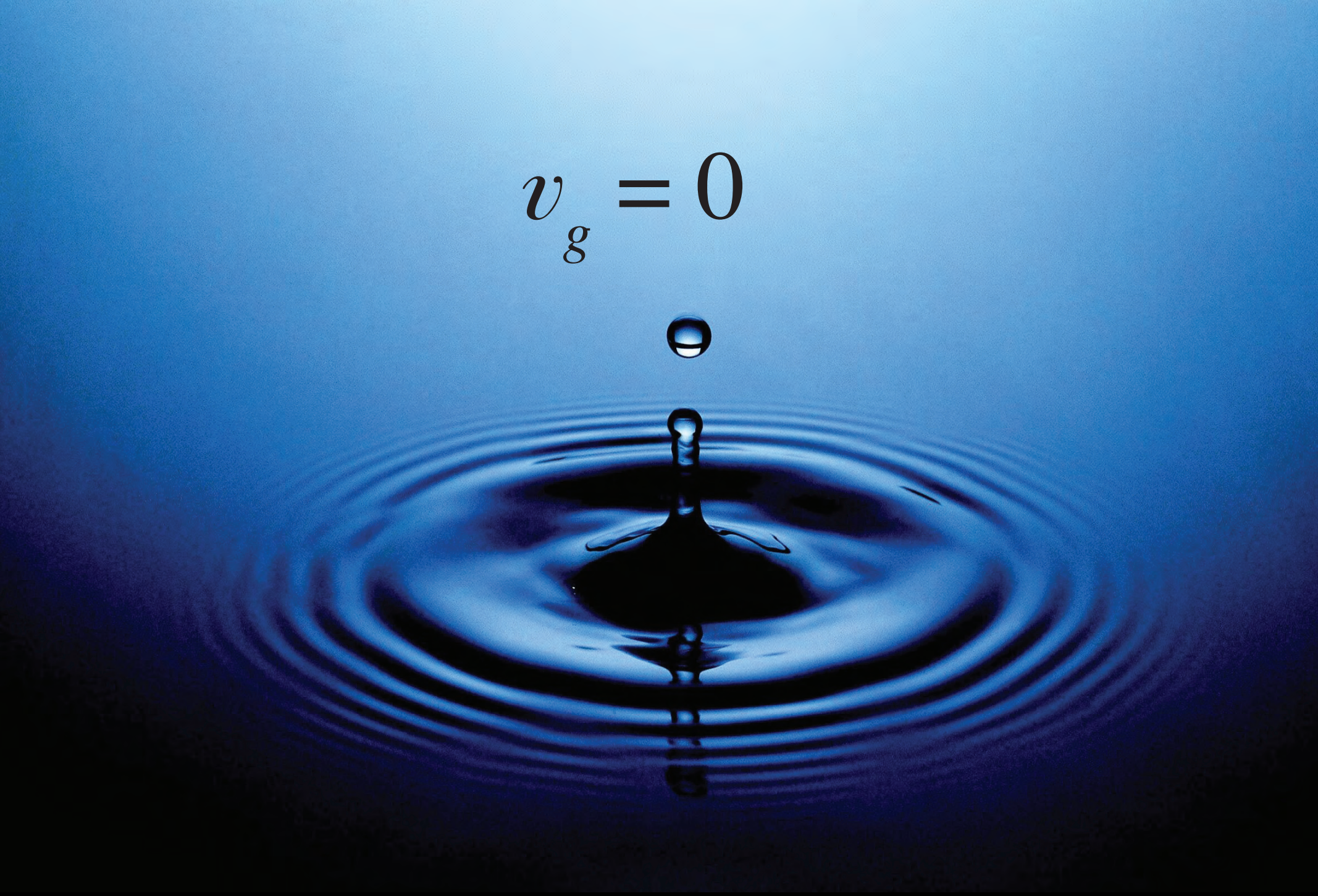
$$v_g = \frac{1}{2} v_p$$



1 index

2 zero index

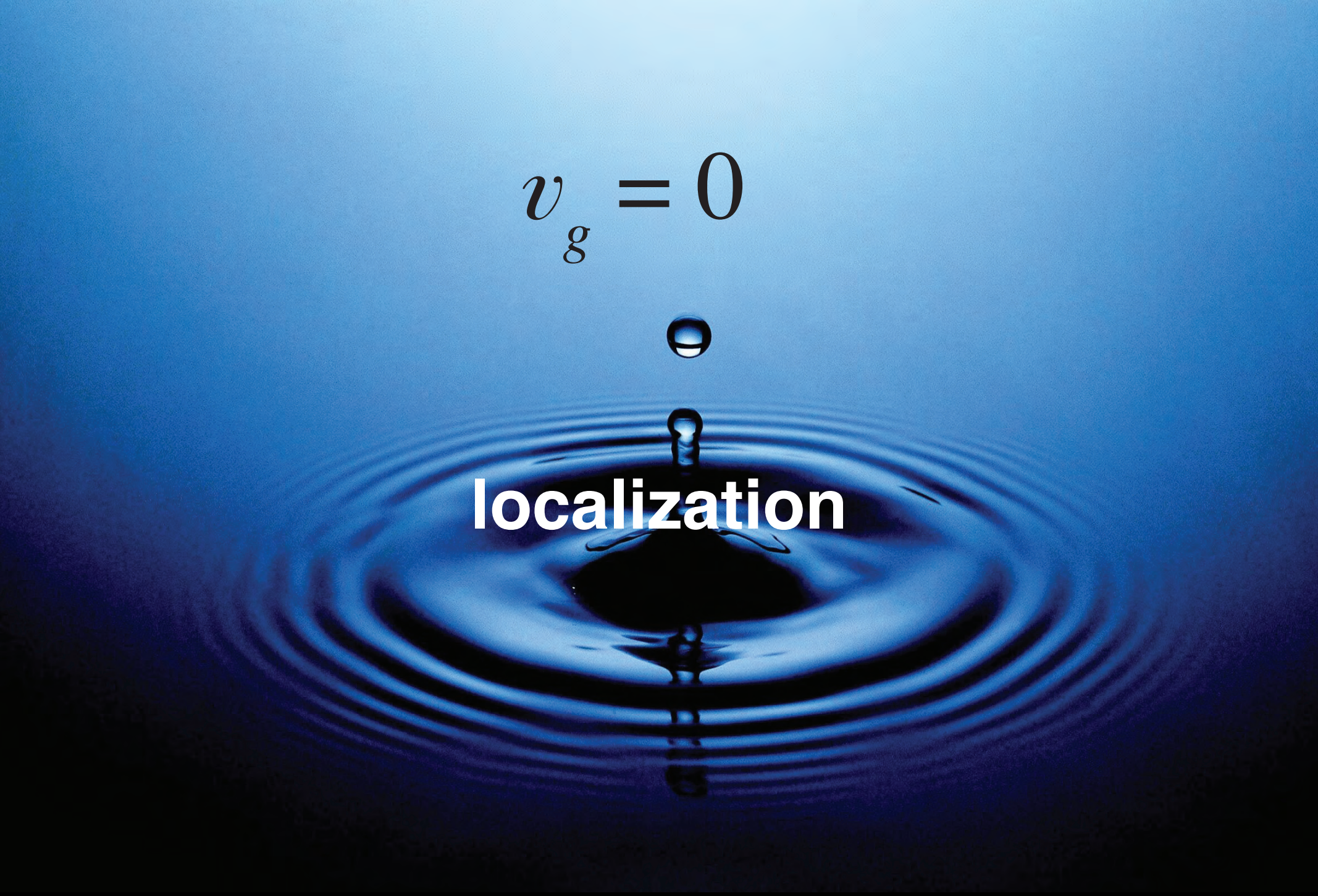
3 zero group velocity



1 index

2 zero index

3 zero group velocity



$$v_g = 0$$

localization

1 index

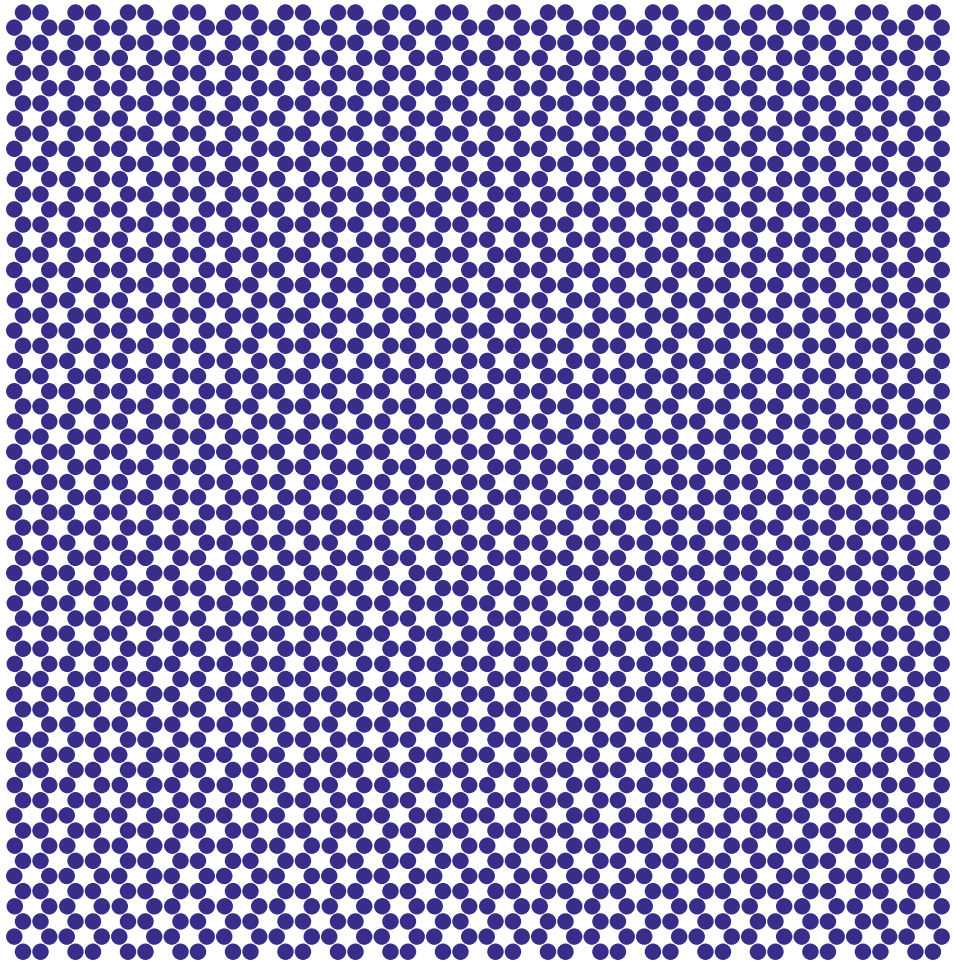
2 zero index

3 zero group velocity

1 index

2 zero index

3 zero group velocity

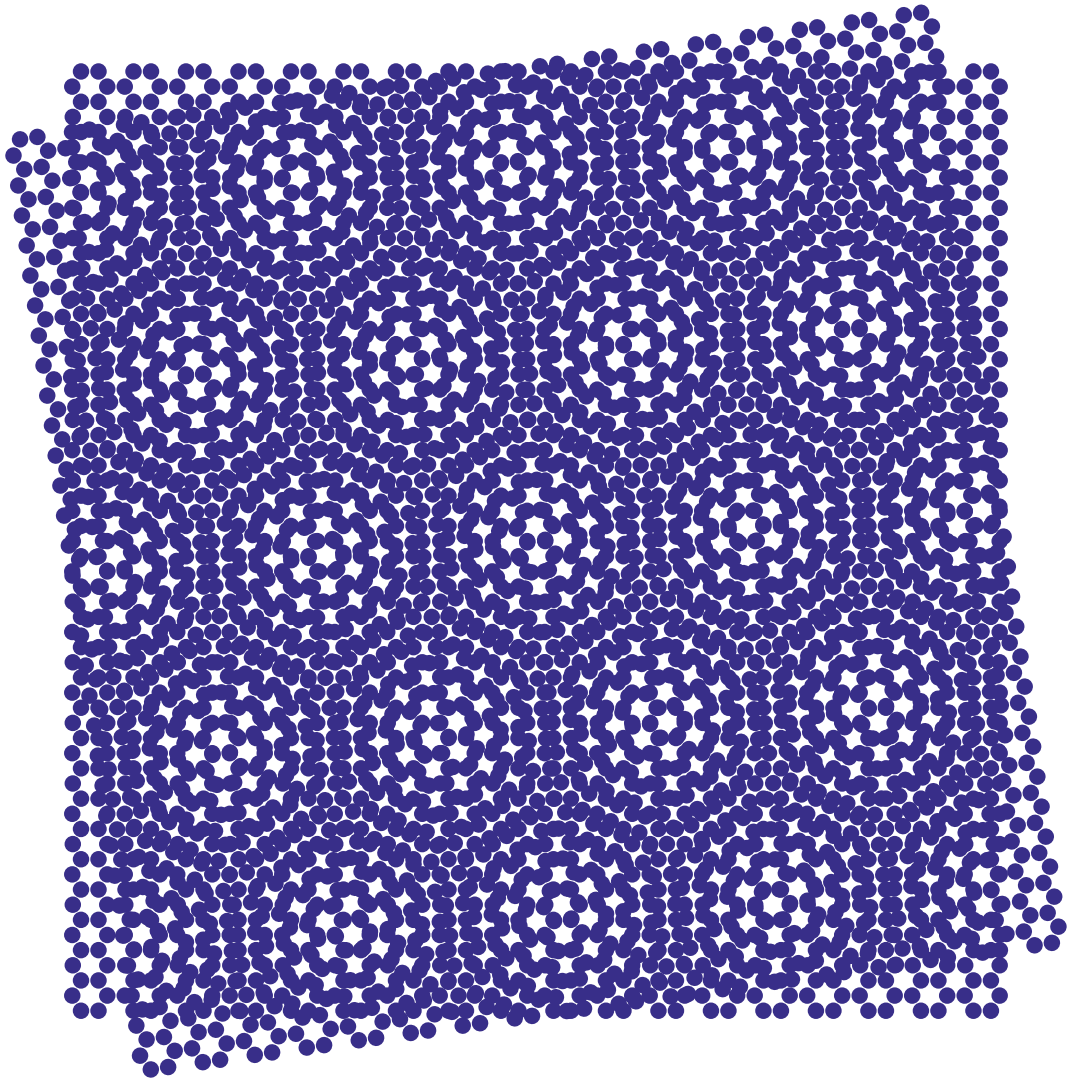


1 index

2 zero index

3 zero group velocity

8°

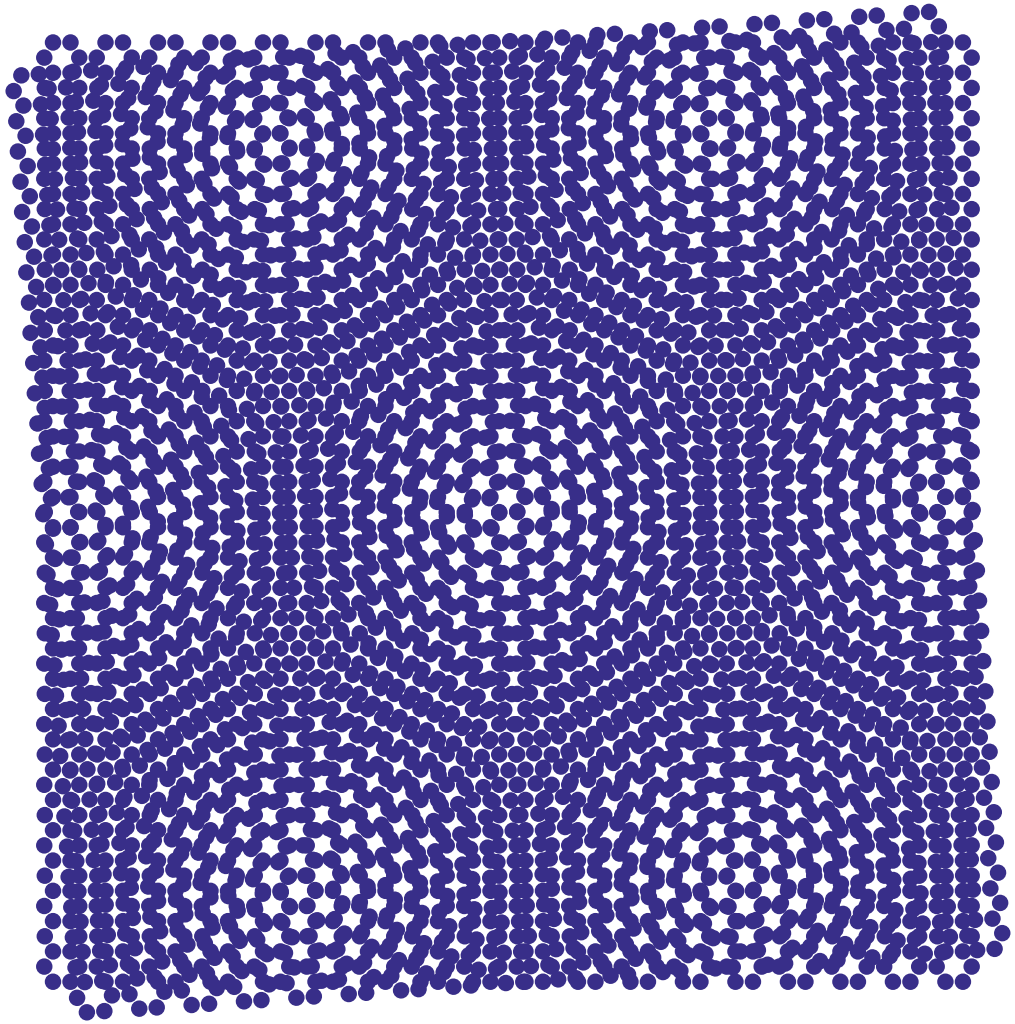


1 index

2 zero index

3 zero group velocity

4°

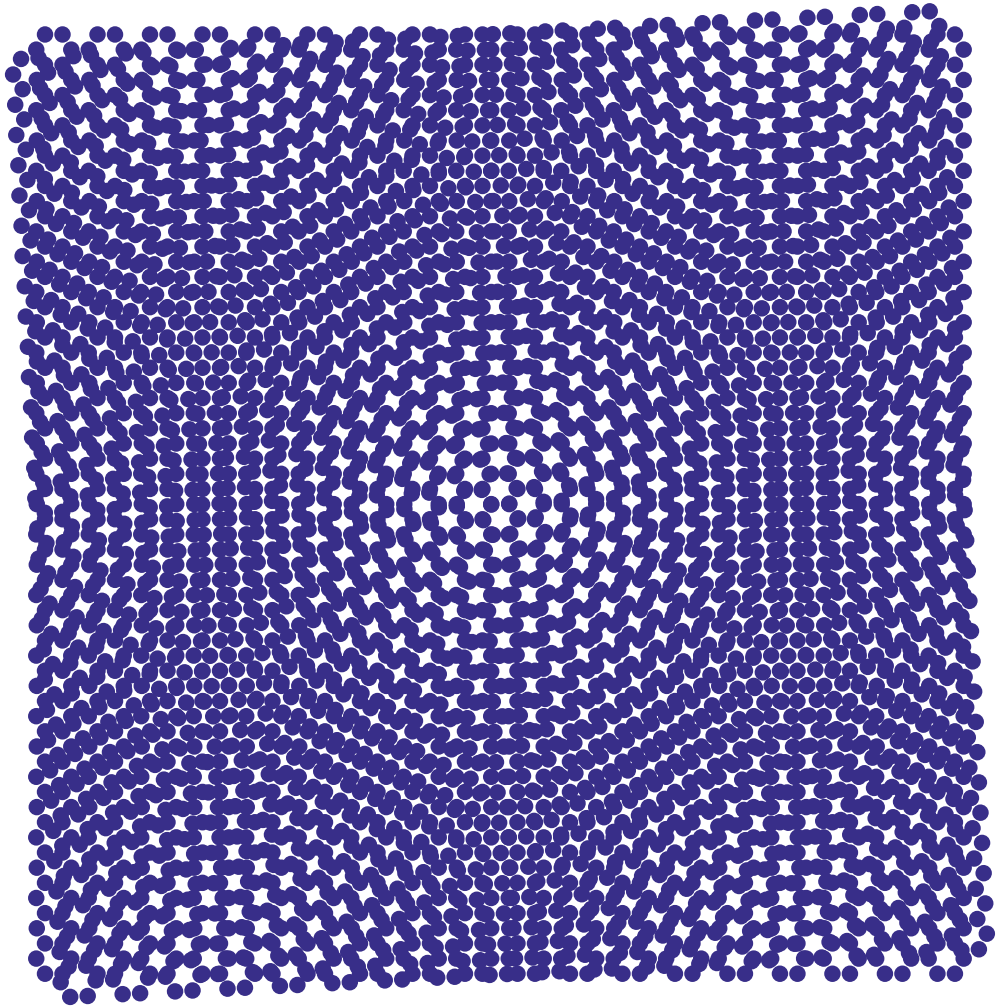


1 index

2 zero index

3 zero group velocity

3°

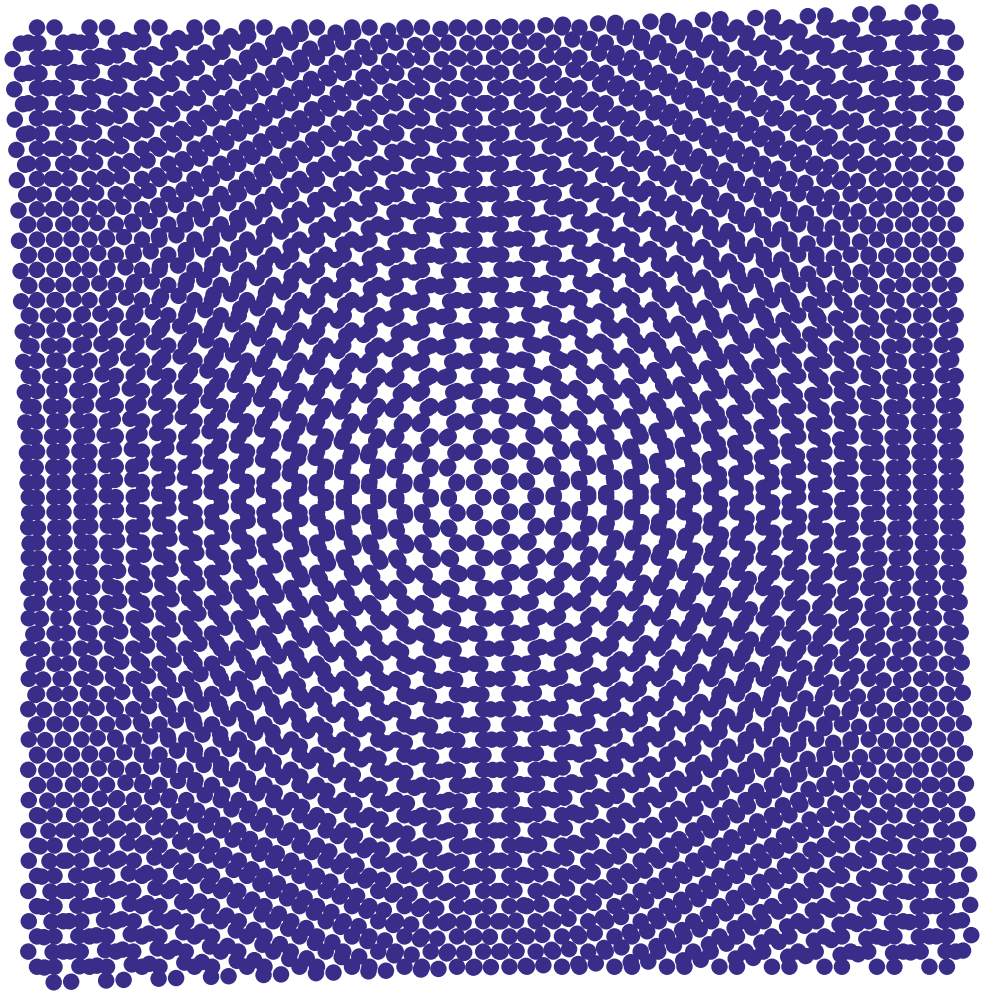


1 index

2 zero index

3 zero group velocity

2°

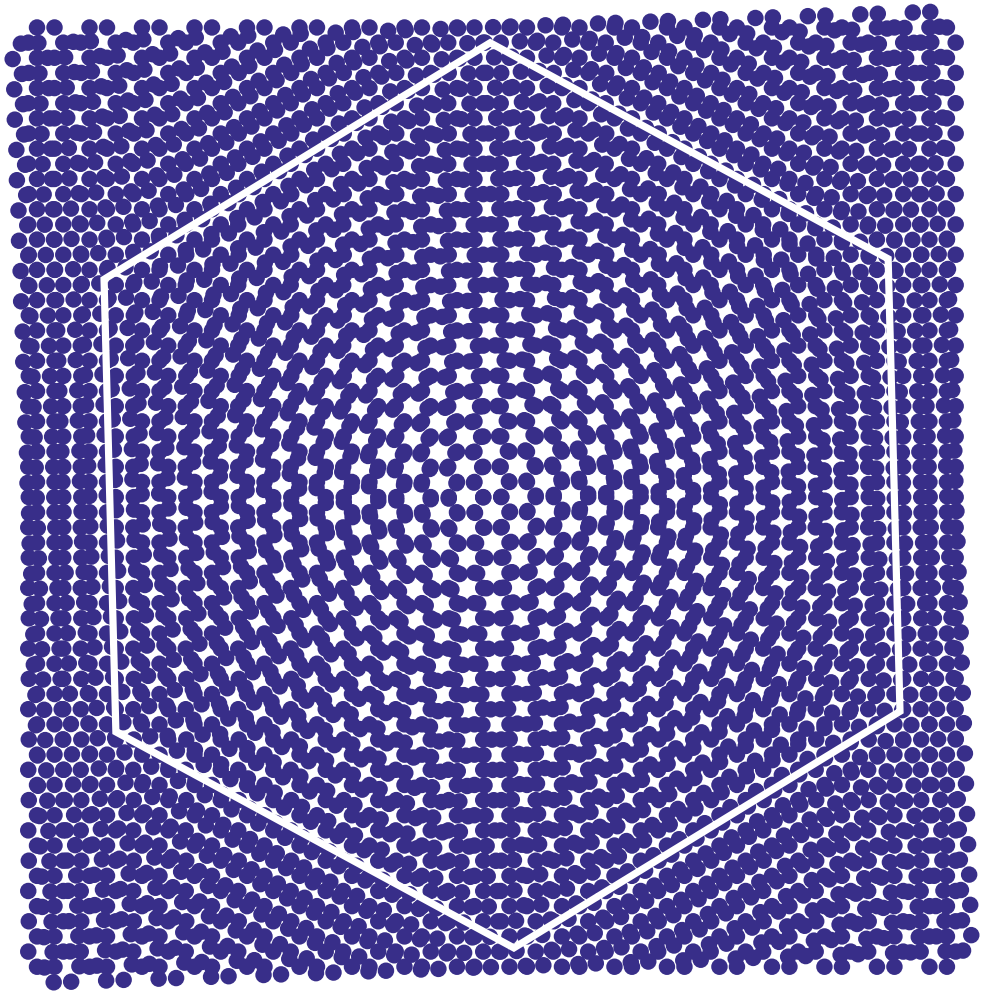


1 index

2 zero index

3 zero group velocity

2°

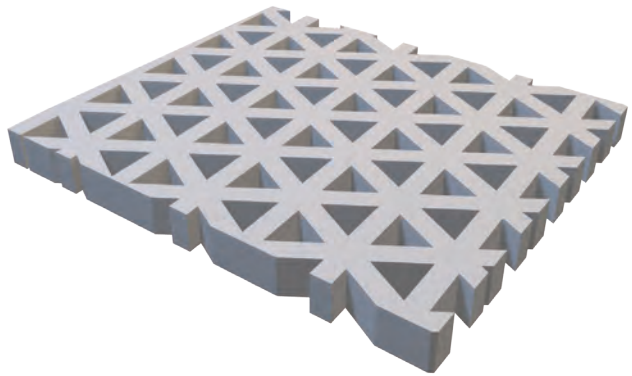


1 index

2 zero index

3 zero group velocity

twisted bilayer photonic crystals

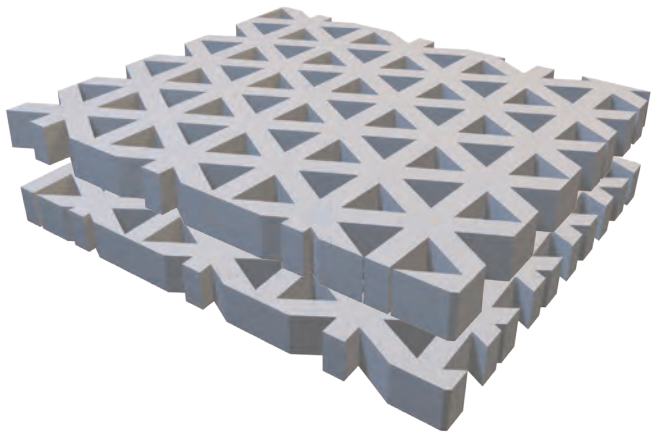


1 index

2 zero index

3 zero group velocity

twisted bilayer photonic crystals

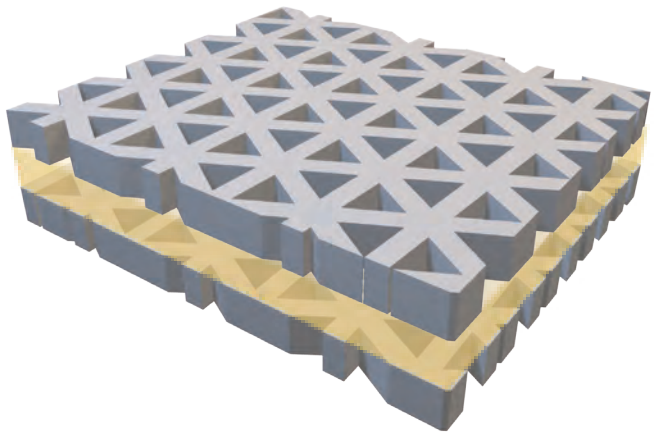


1 index

2 zero index

3 zero group velocity

twisted bilayer photonic crystals

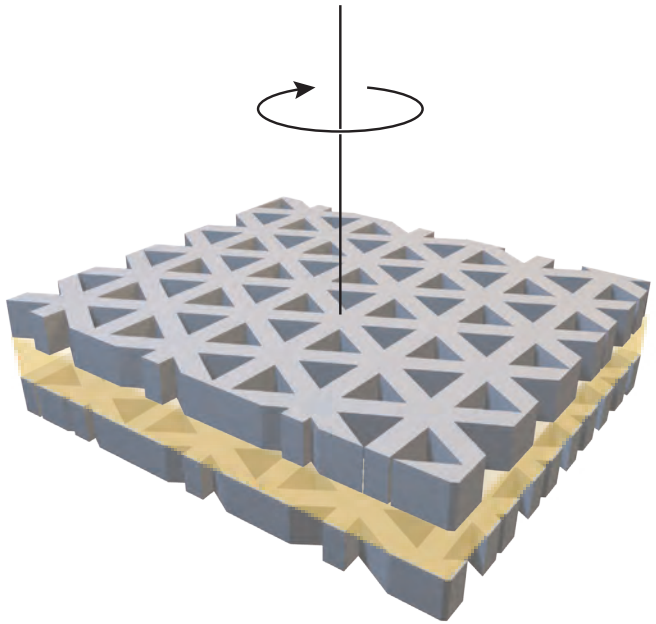


1 index

2 zero index

3 zero group velocity

twisted bilayer photonic crystals

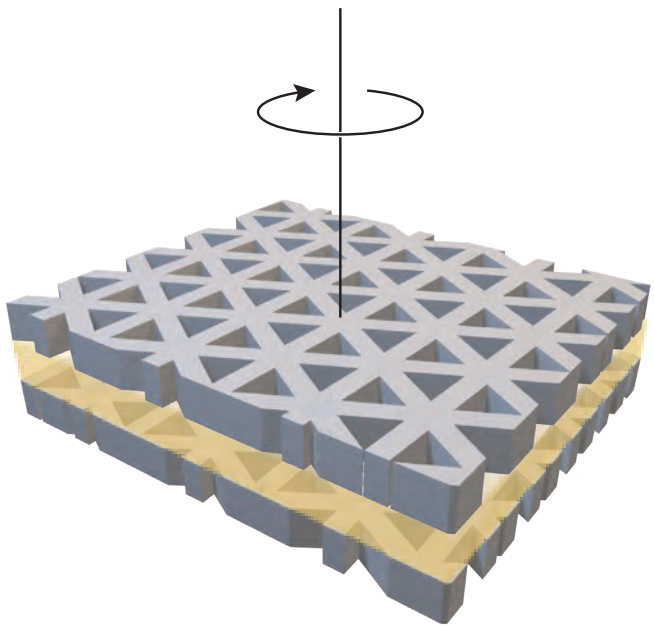


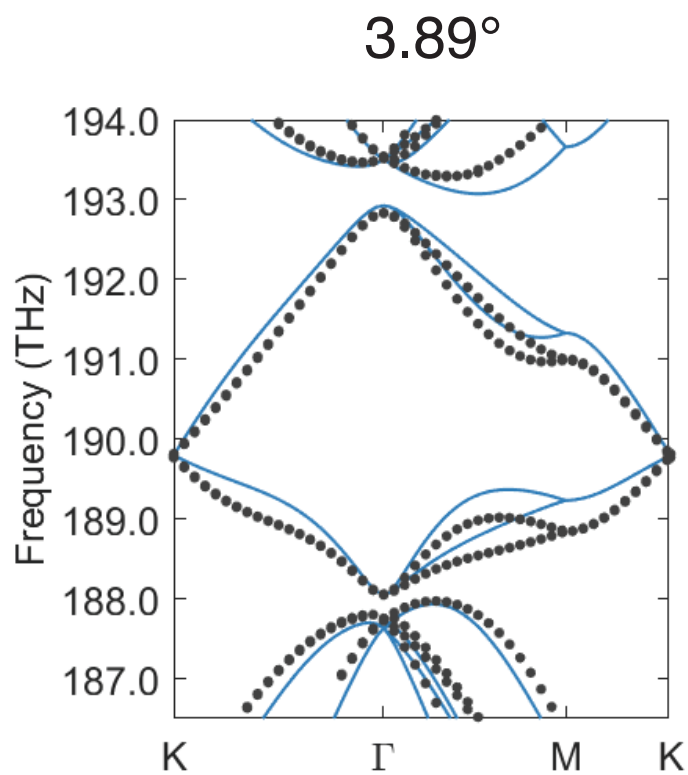
1 index

2 zero index

3 zero group velocity

twisted bilayer photonic crystals



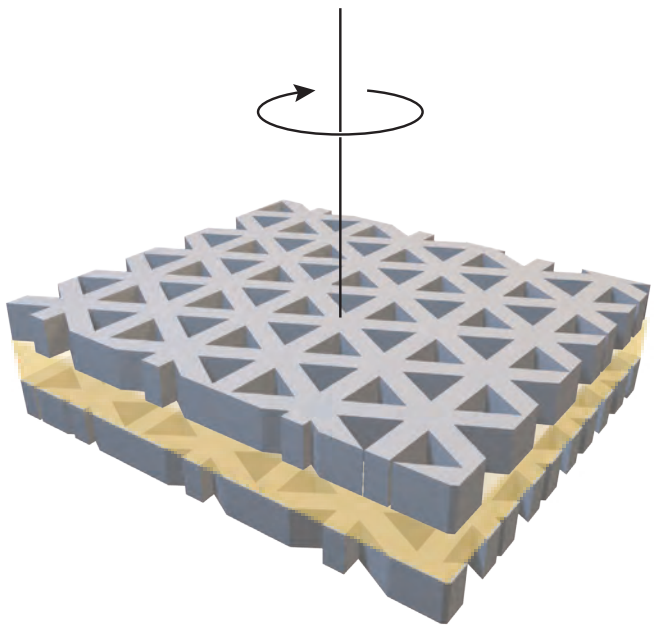


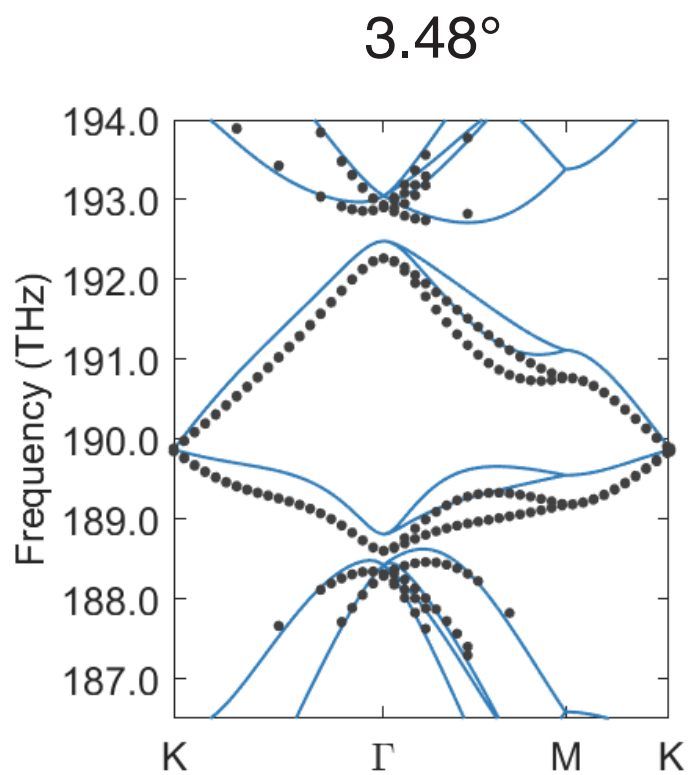
1 index

2 zero index

3 zero group velocity

twisted bilayer photonic crystals



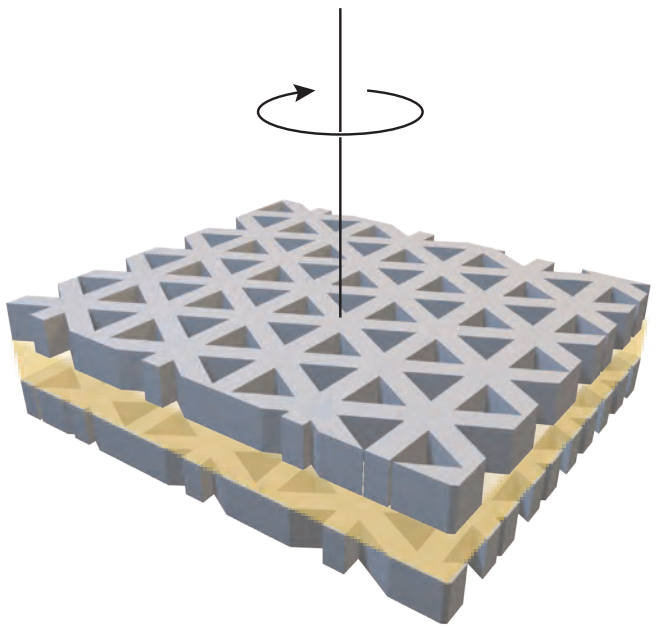


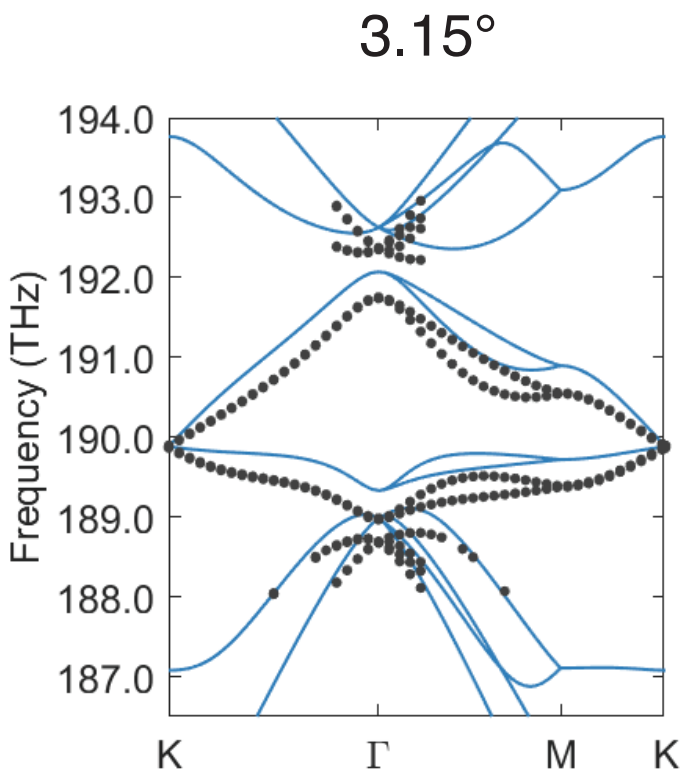
1 index

2 zero index

3 zero group velocity

twisted bilayer photonic crystals



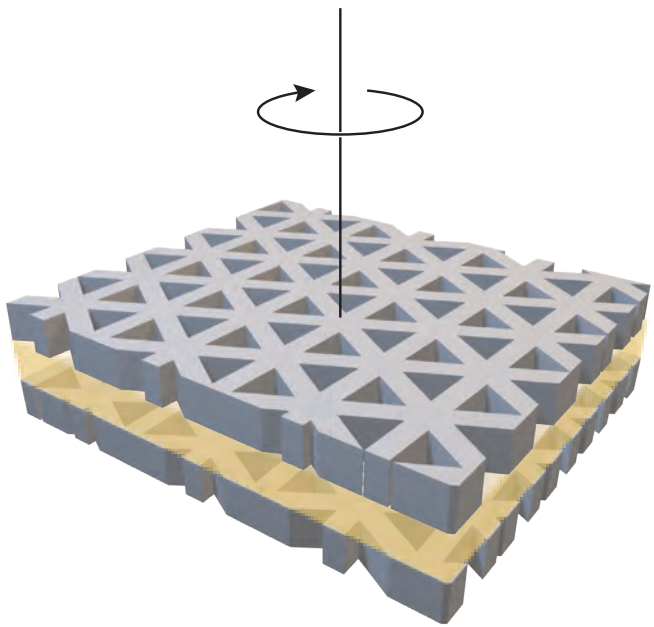


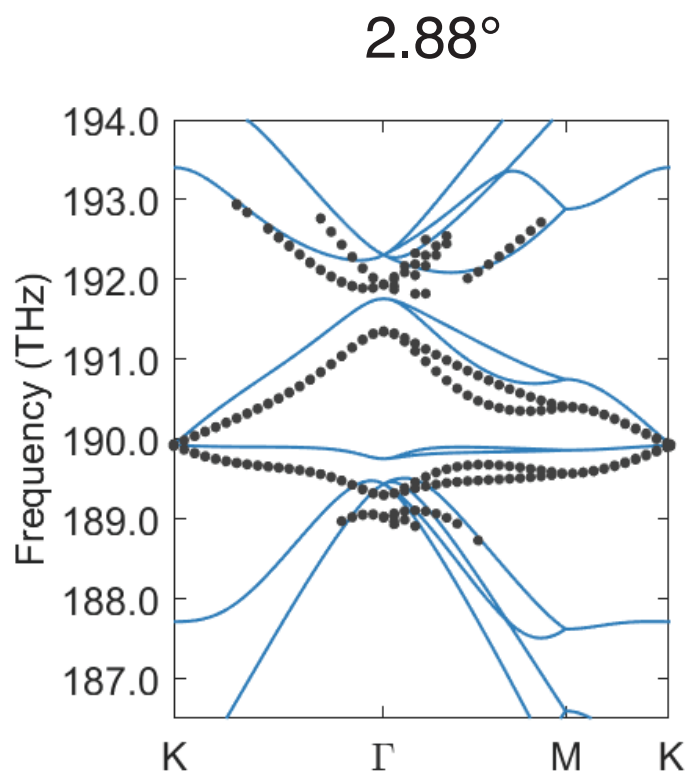
1 index

2 zero index

3 zero group velocity

twisted bilayer photonic crystals



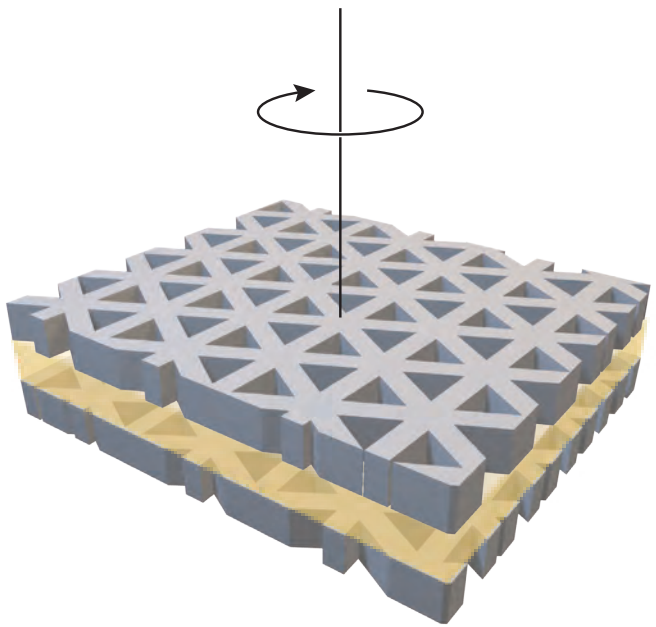


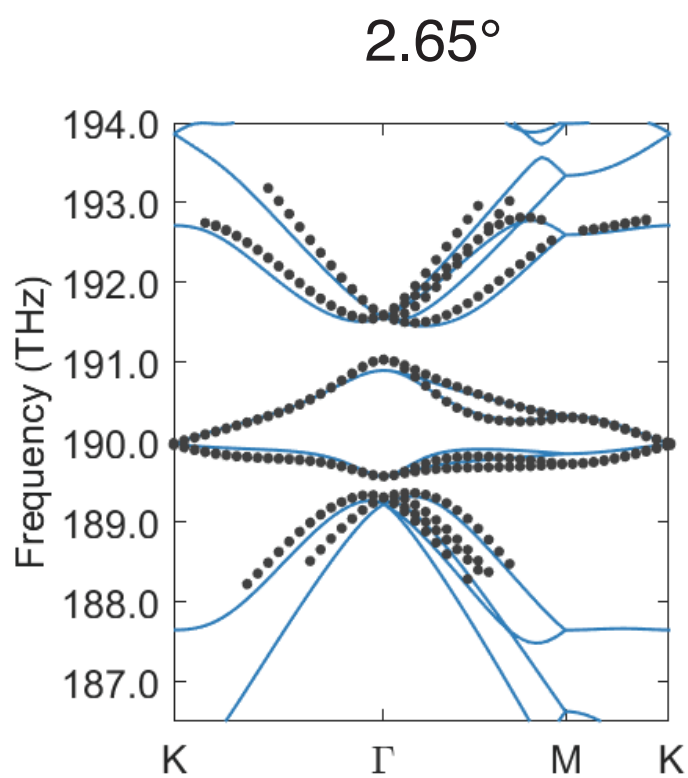
1 index

2 zero index

3 zero group velocity

twisted bilayer photonic crystals



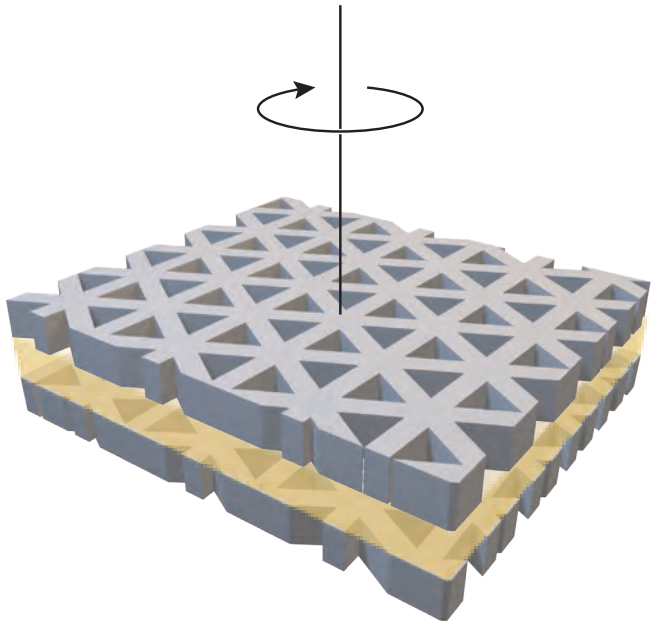


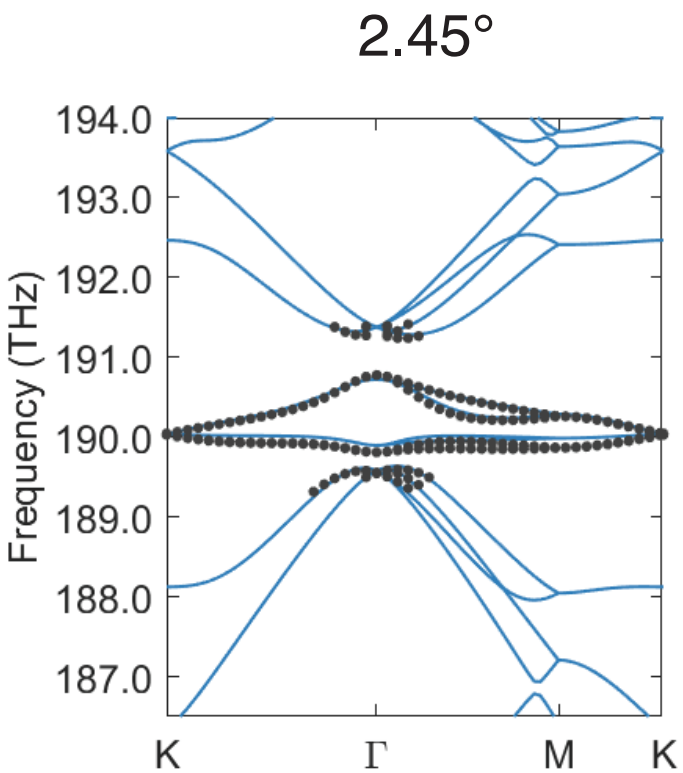
1 index

2 zero index

3 zero group velocity

twisted bilayer photonic crystals



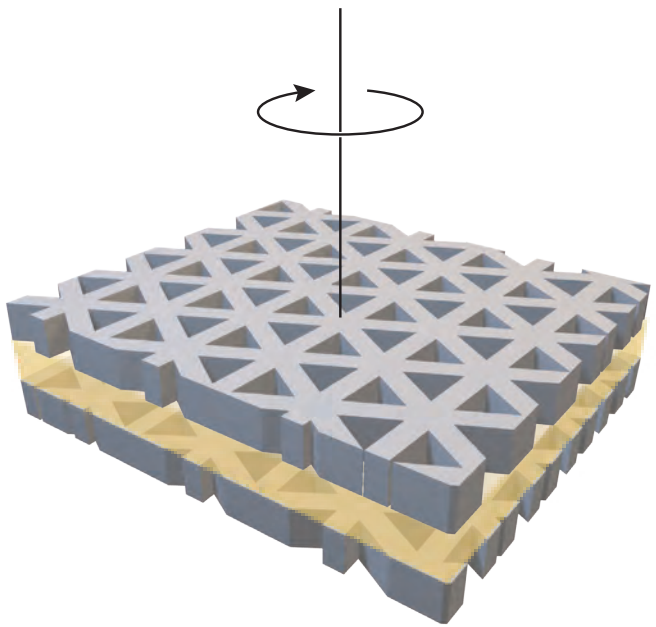


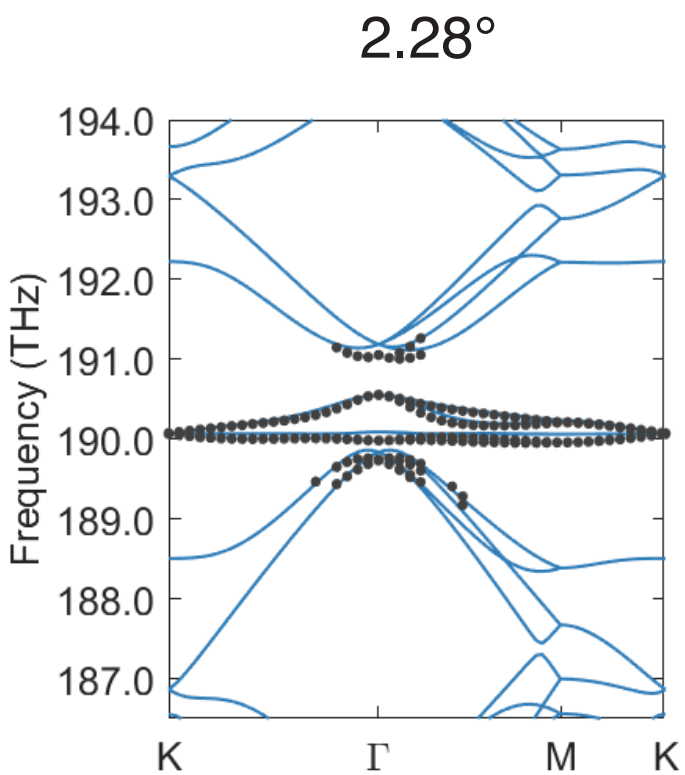
1 index

2 zero index

3 zero group velocity

twisted bilayer photonic crystals



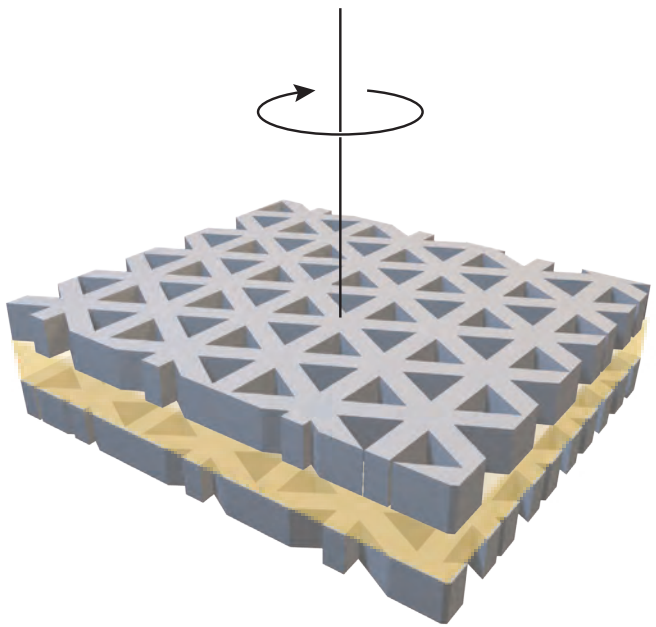


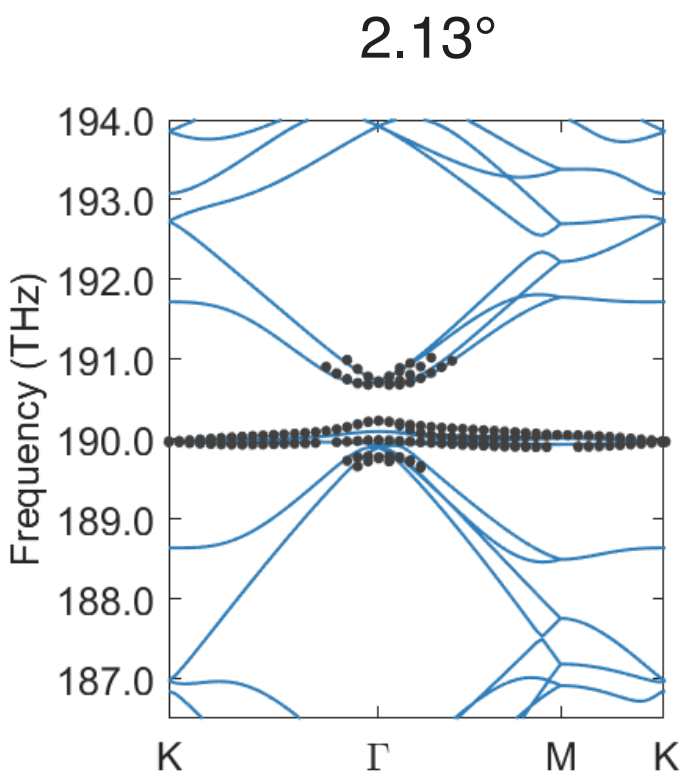
1 index

2 zero index

3 zero group velocity

twisted bilayer photonic crystals



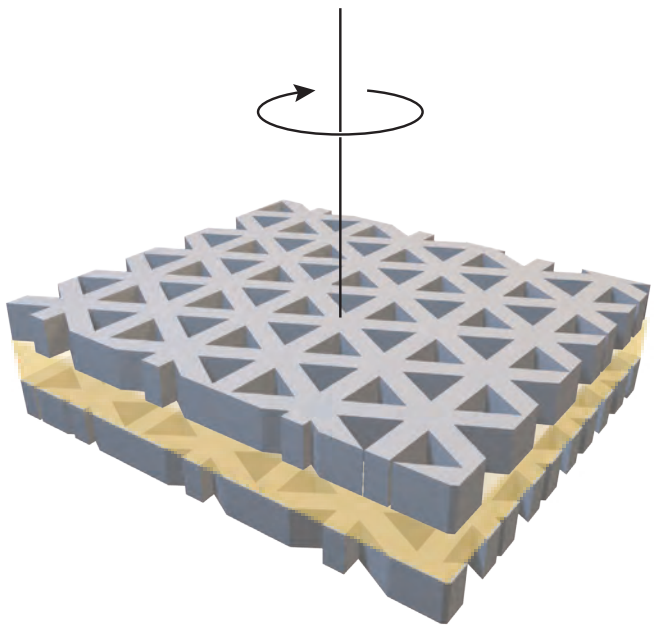


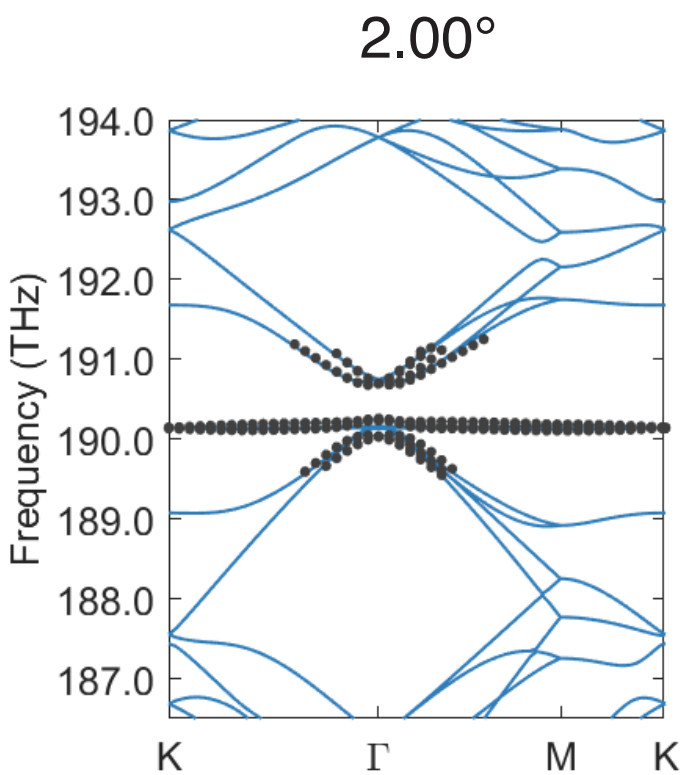
1 index

2 zero index

3 zero group velocity

twisted bilayer photonic crystals



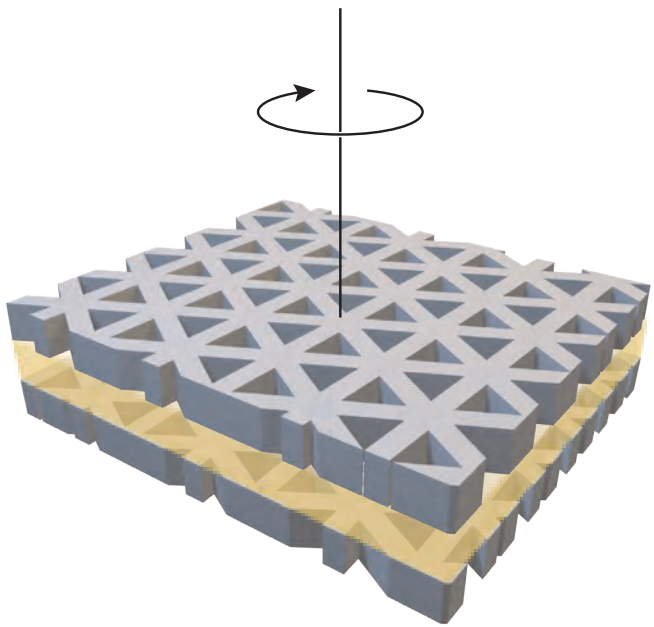


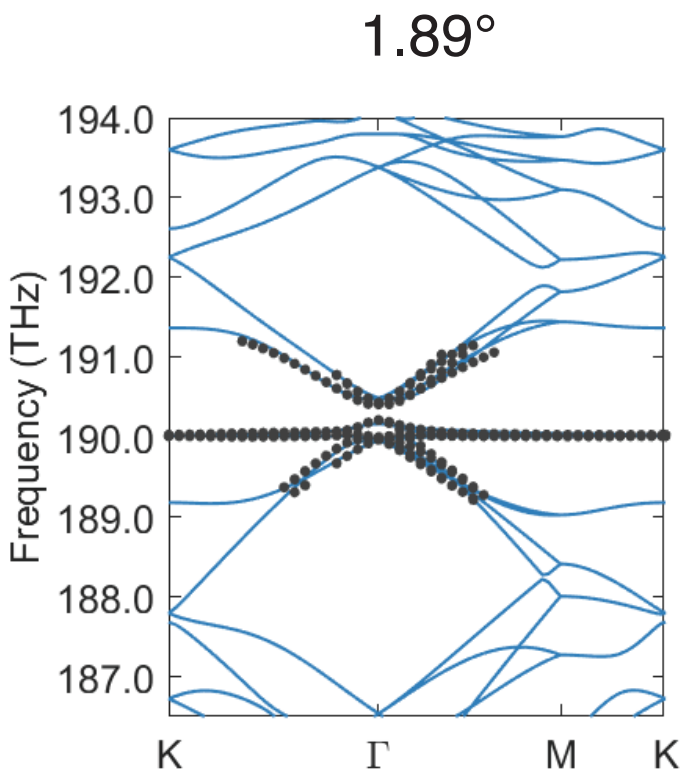
1 index

2 zero index

3 zero group velocity

twisted bilayer photonic crystals



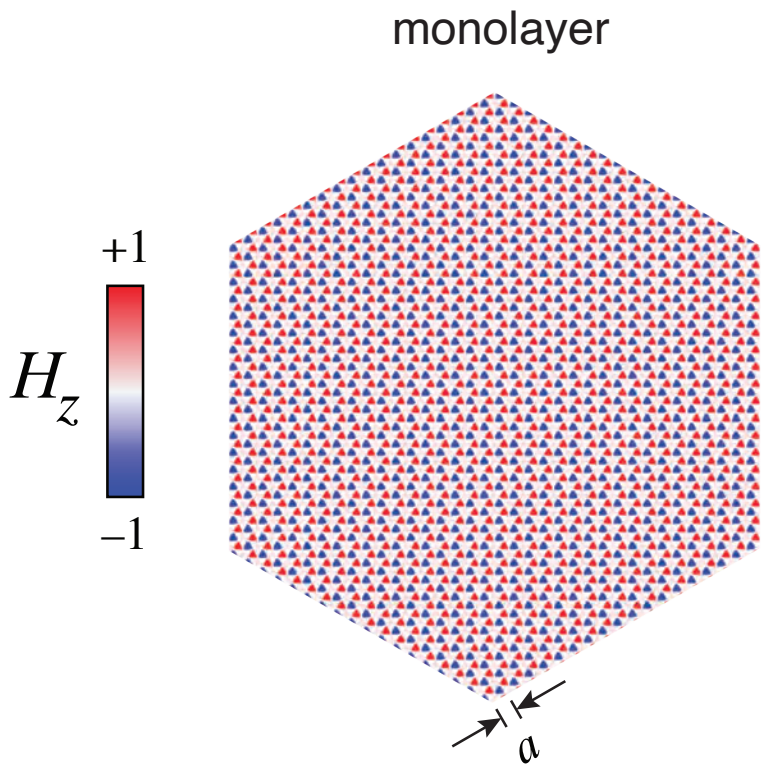


1 index

2 zero index

3 zero group velocity

twisted bilayer photonic crystals



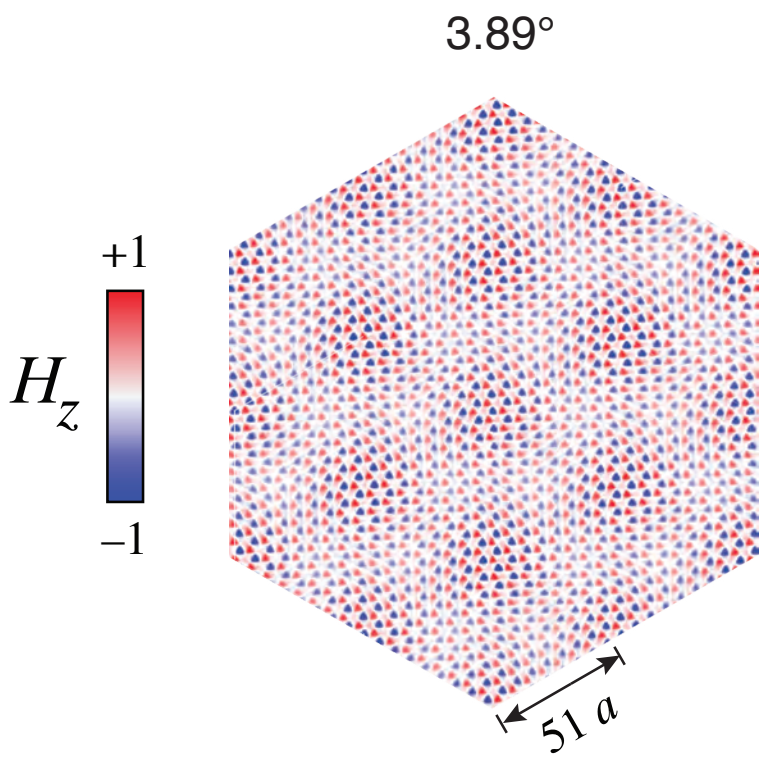
$$v_g = 0.23\ c$$

1 index

2 zero index

3 zero group velocity

twisted bilayer photonic crystals



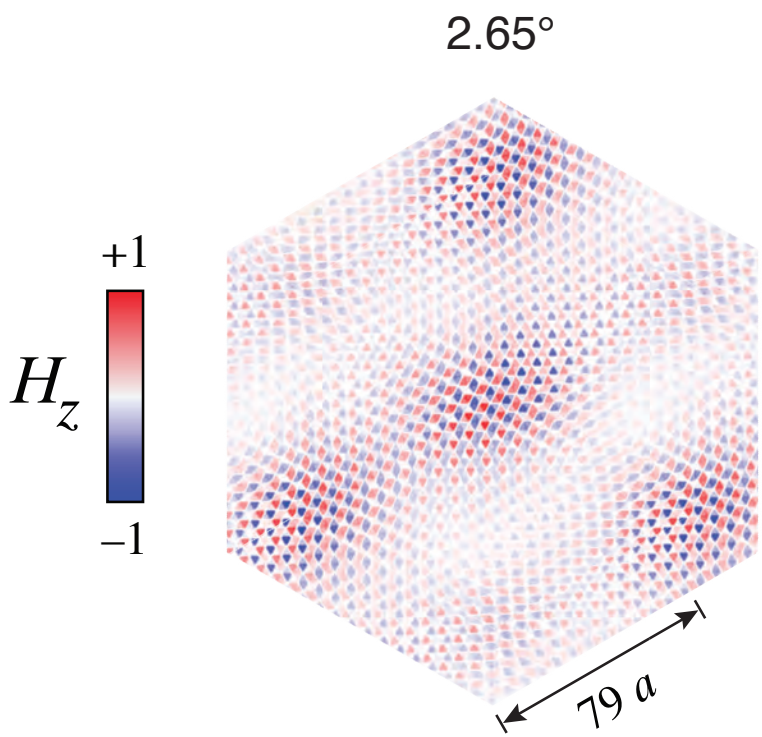
$$v_g = 0.26\,c$$

1 index

2 zero index

3 zero group velocity

twisted bilayer photonic crystals



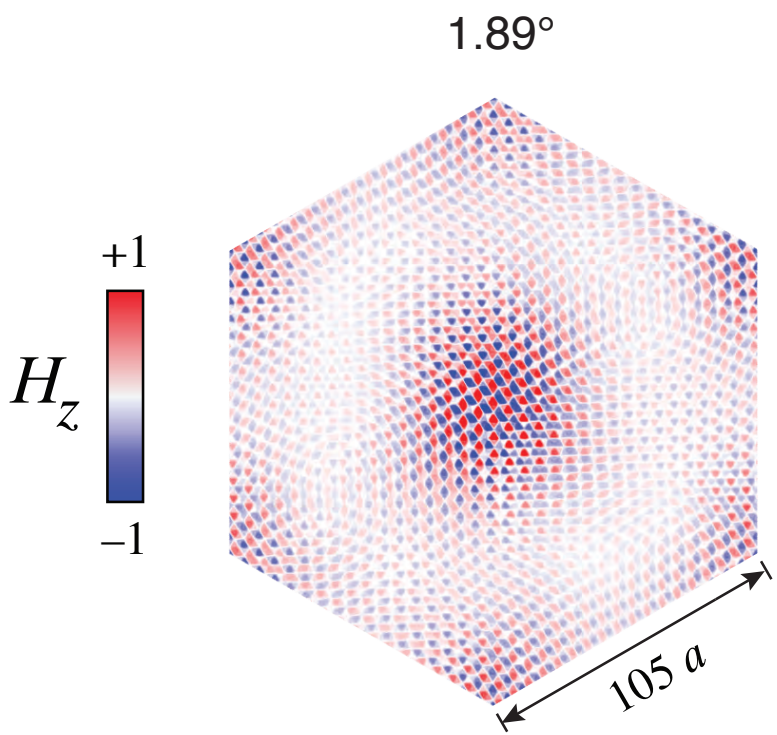
$$v_g = 0.04\ c$$

1 index

2 zero index

3 zero group velocity

twisted bilayer photonic crystals



$$v_g = 0!$$

1 index

2 zero index

3 zero group velocity

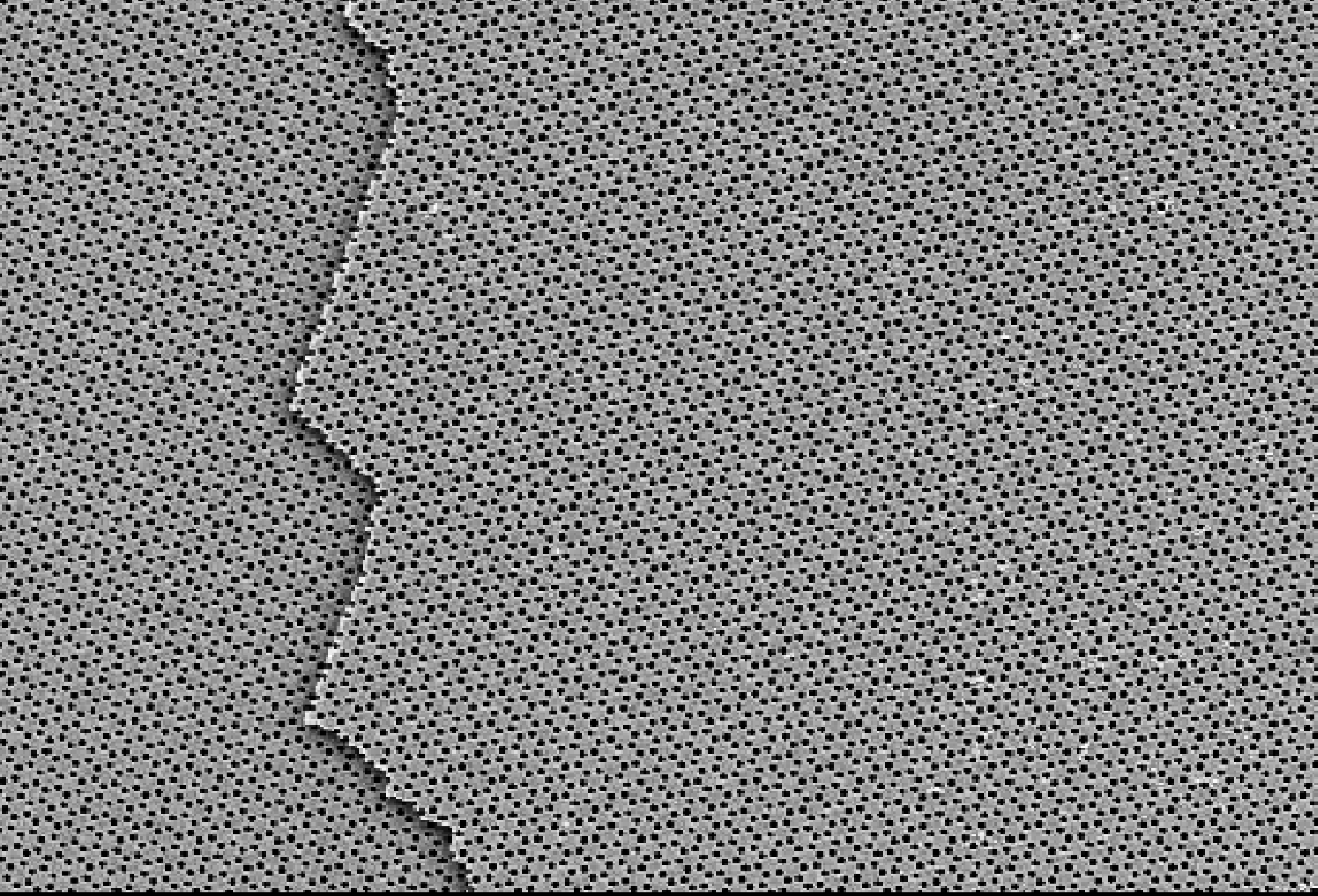
twisted bilayer photonic crystals

$$v_g = 0!$$

1 index

2 zero index

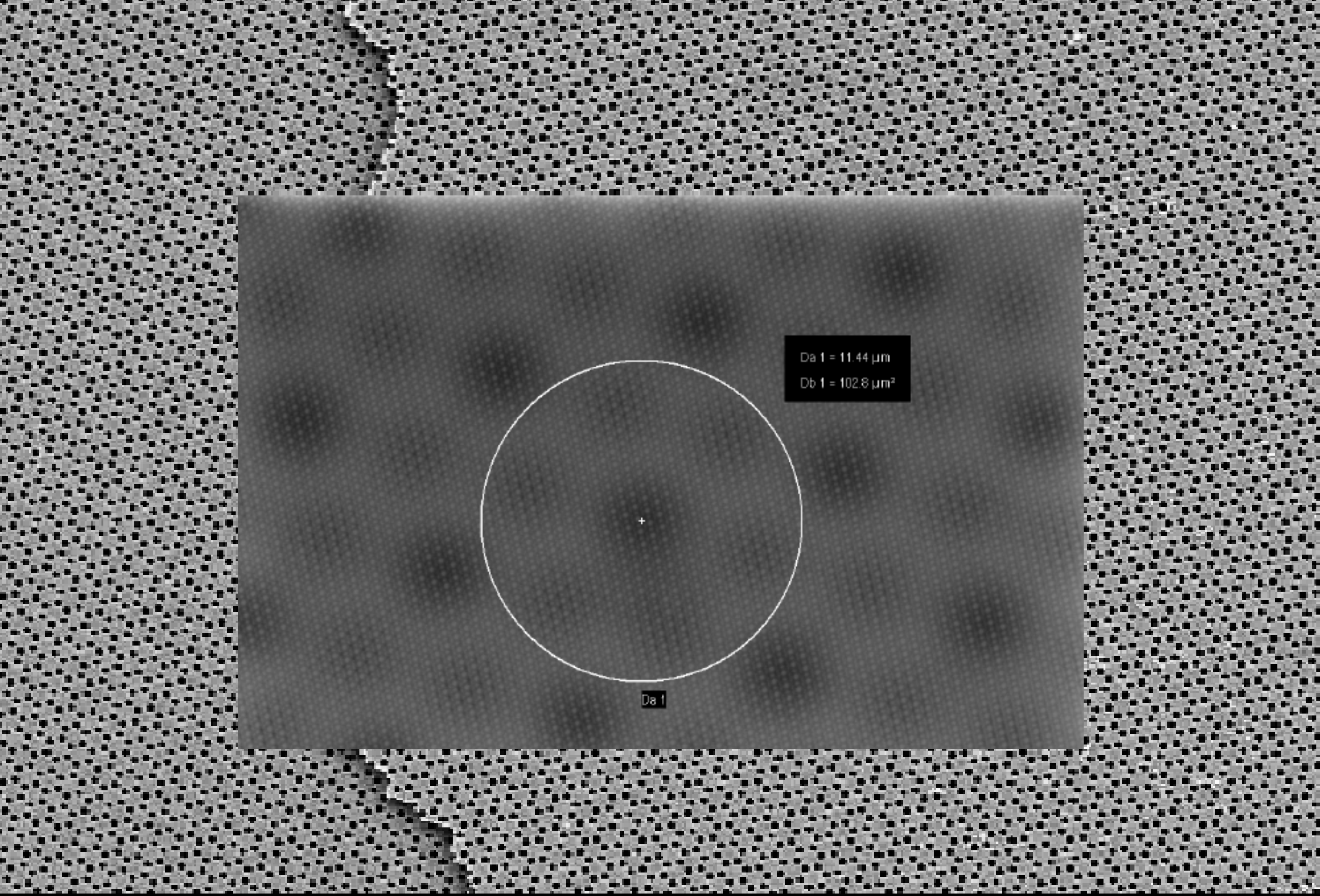
3 zero group velocity



1 index

2 zero index

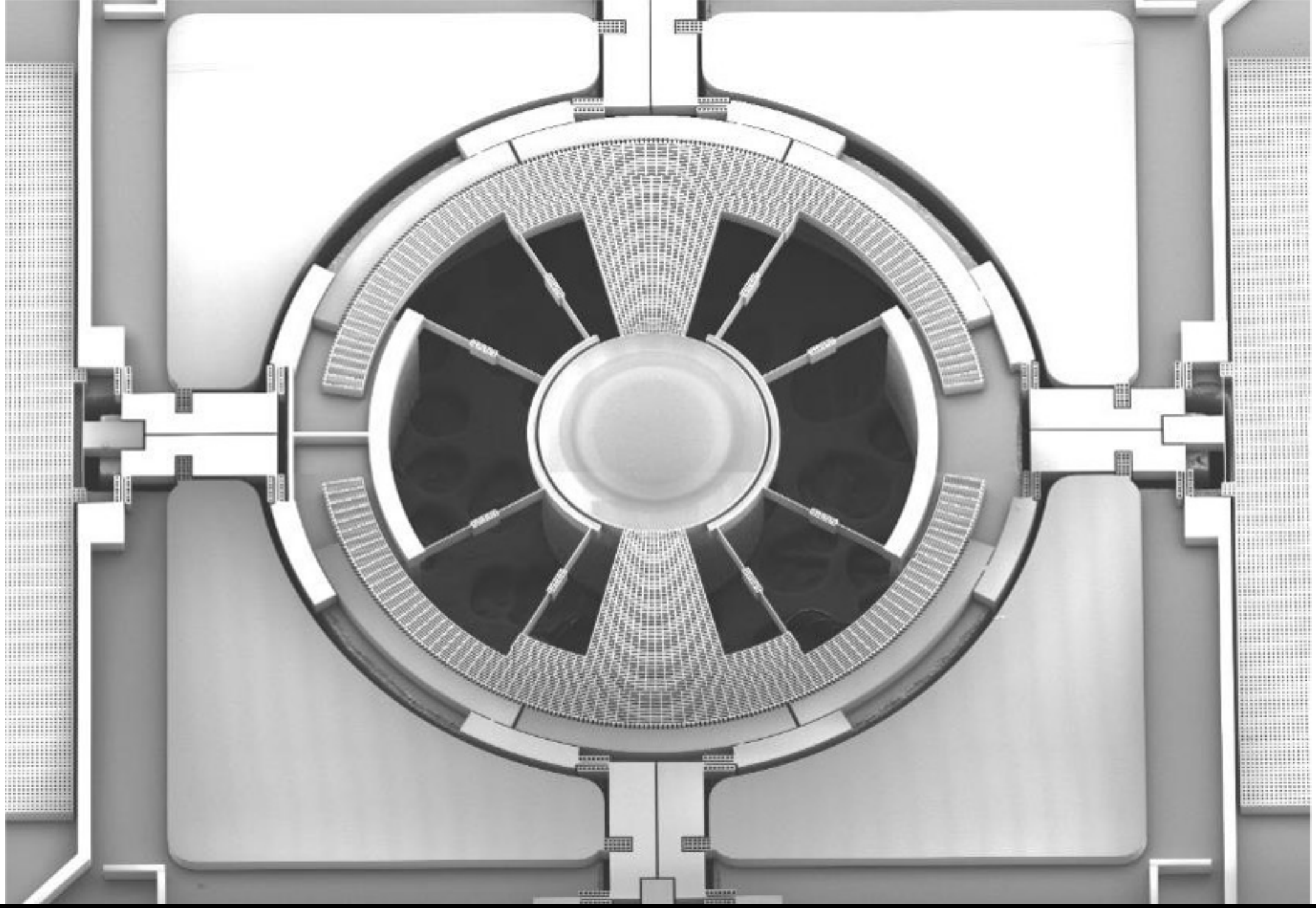
3 zero group velocity



1 index

2 zero index

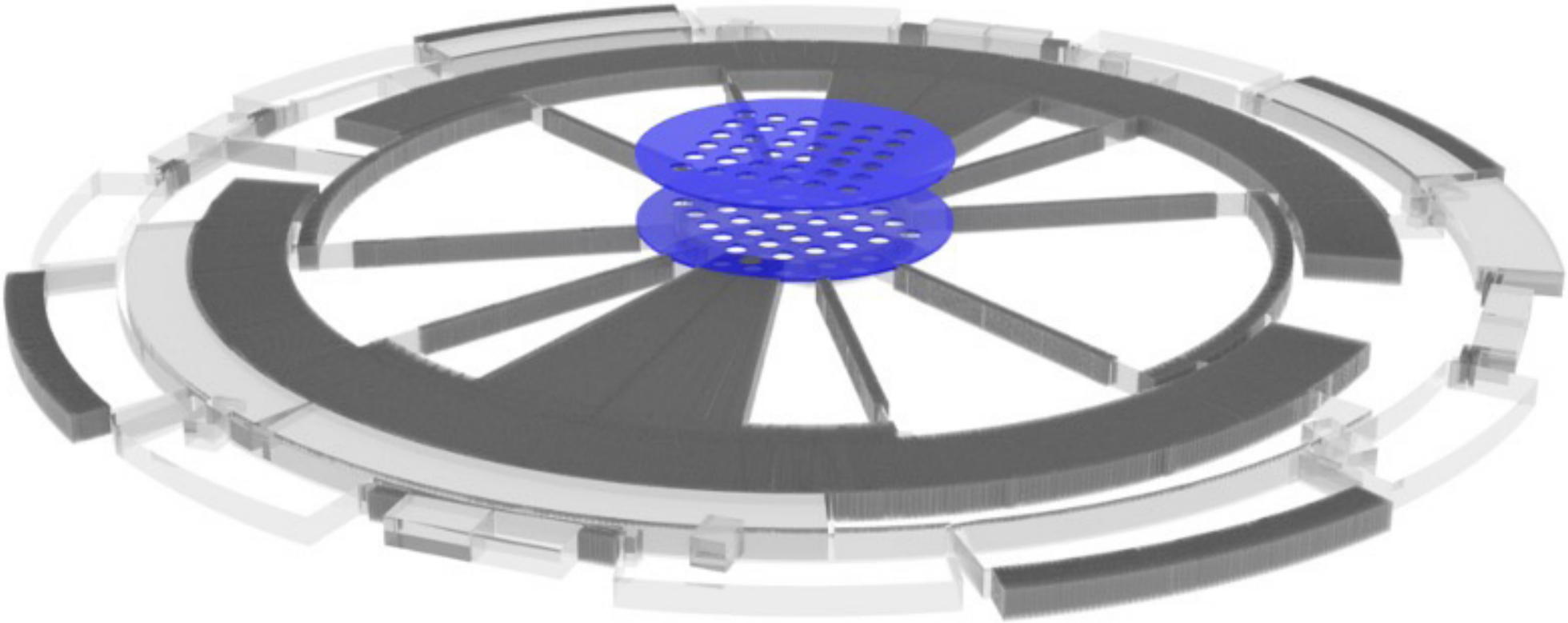
3 zero group velocity



1 index

2 zero index

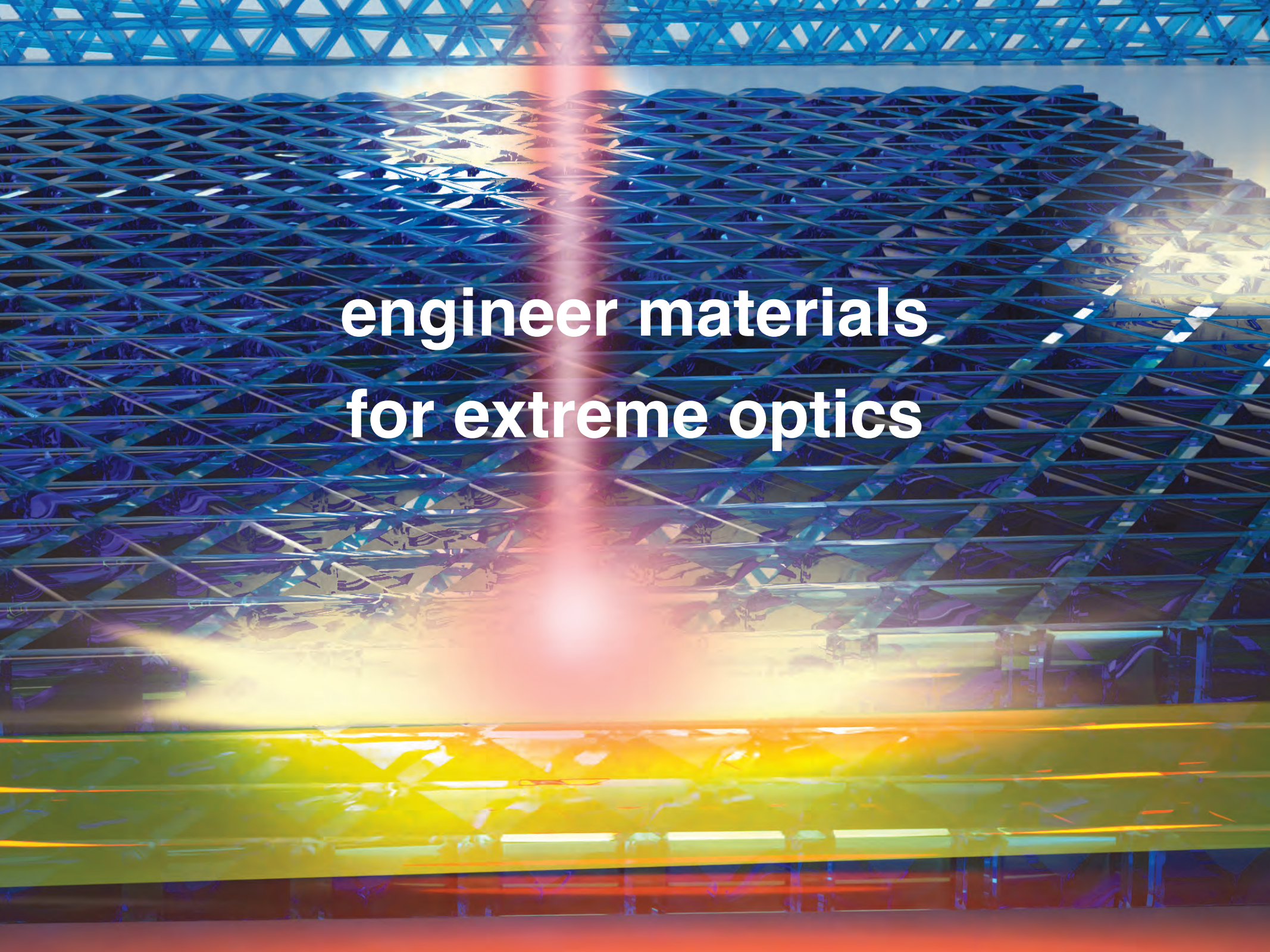
3 zero group velocity



1 index

2 zero index

3 zero group velocity


engineer materials
for extreme optics

Zero-Index Metamaterials:

Yang Li
Shota Kita
Phil Muñoz
Orad Reshef,
Daryl Vulis
Mei Yin
Lysander Christakis
Zin Lin, Cleaven Chia
Olivia Mello
Haoning Tang

Zero-Index Waveguide:

Orad Reshef
Justin Gagnon
Marko Loncar
Phil Muñoz
Daryl Vulis

Twisted Bilayer Photonic Crystals

Haoning Tang
Fan Du
Stephen Carr
Xueqi Ni
Huanyu Zhou
Vishantak Srikrishna
Chentong Li
Clayton DeVault
Michael Lobet

Profs. Bob Boyd , Nader Engheta, Alan Willner

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