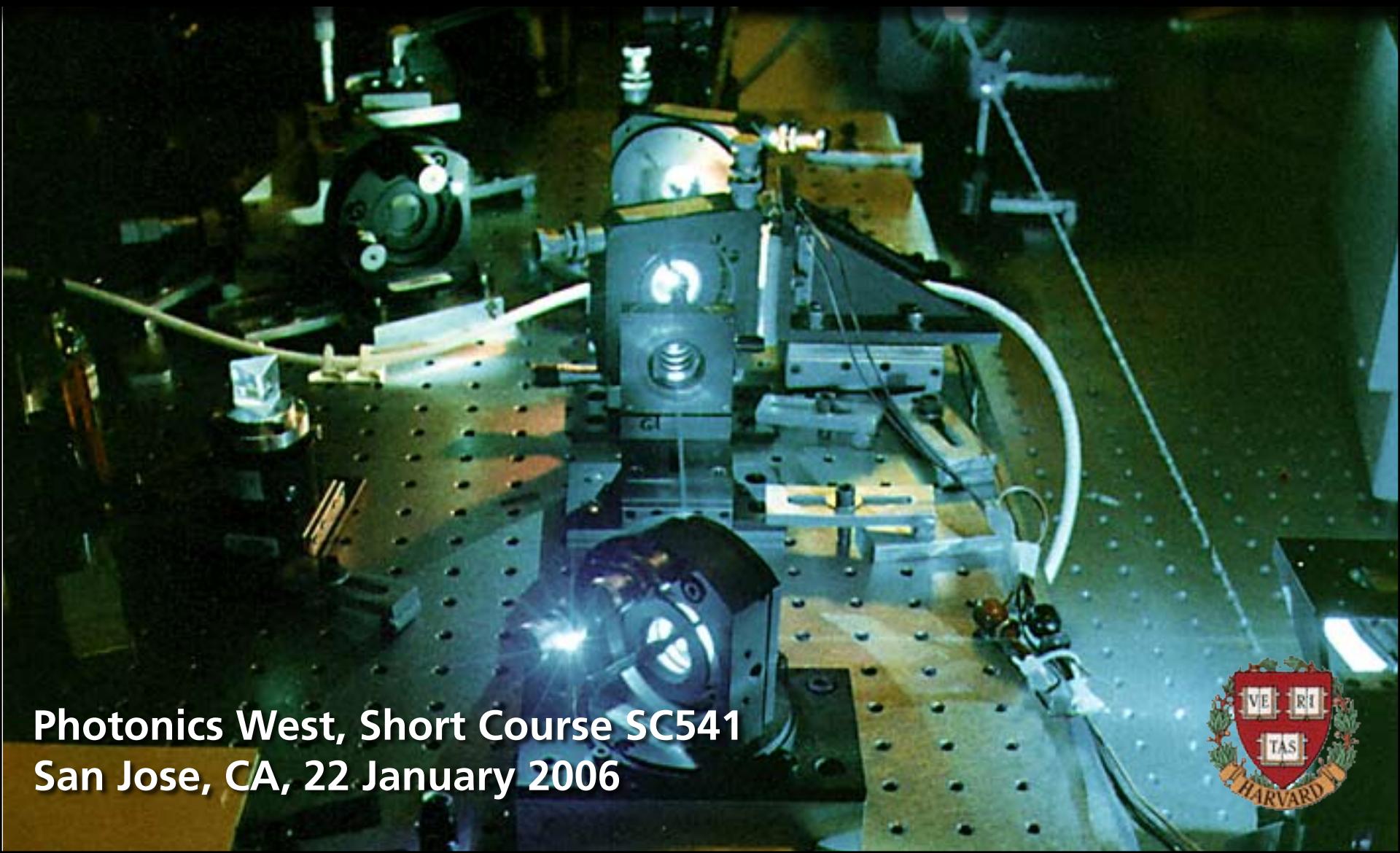


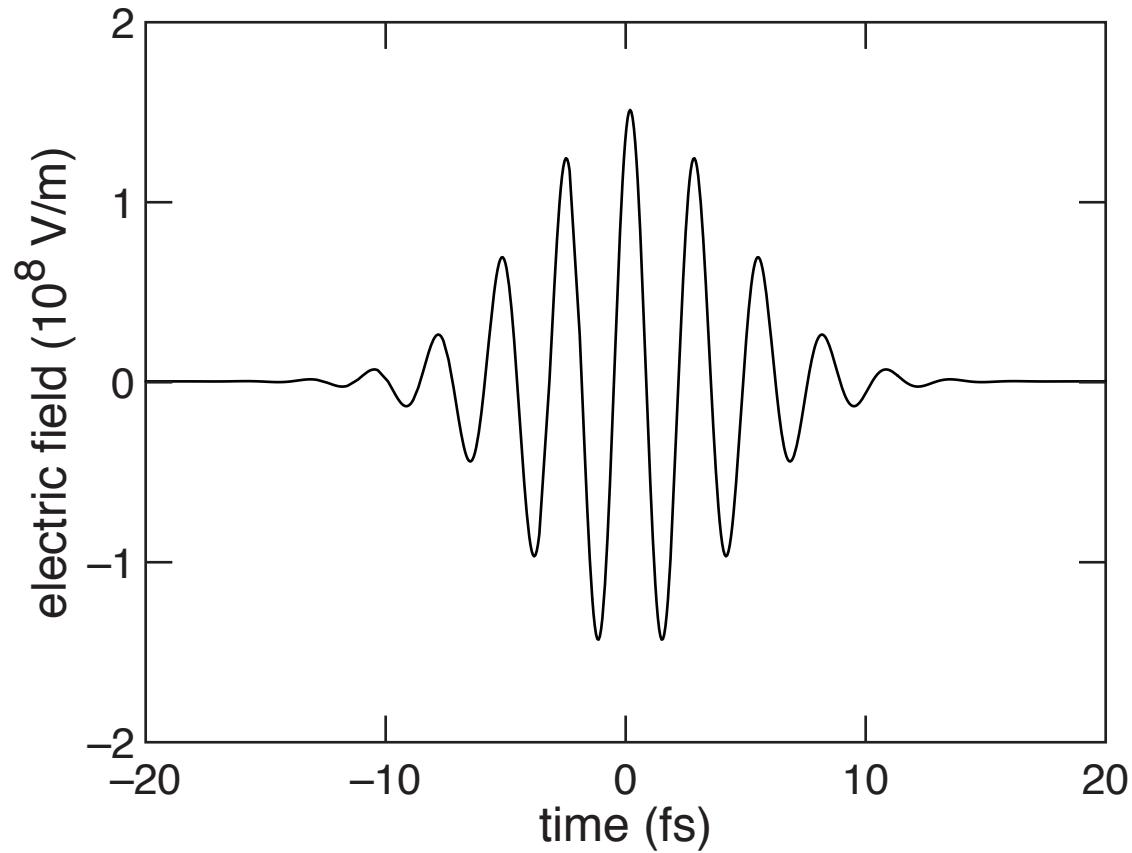
An introduction to femtosecond laser techniques



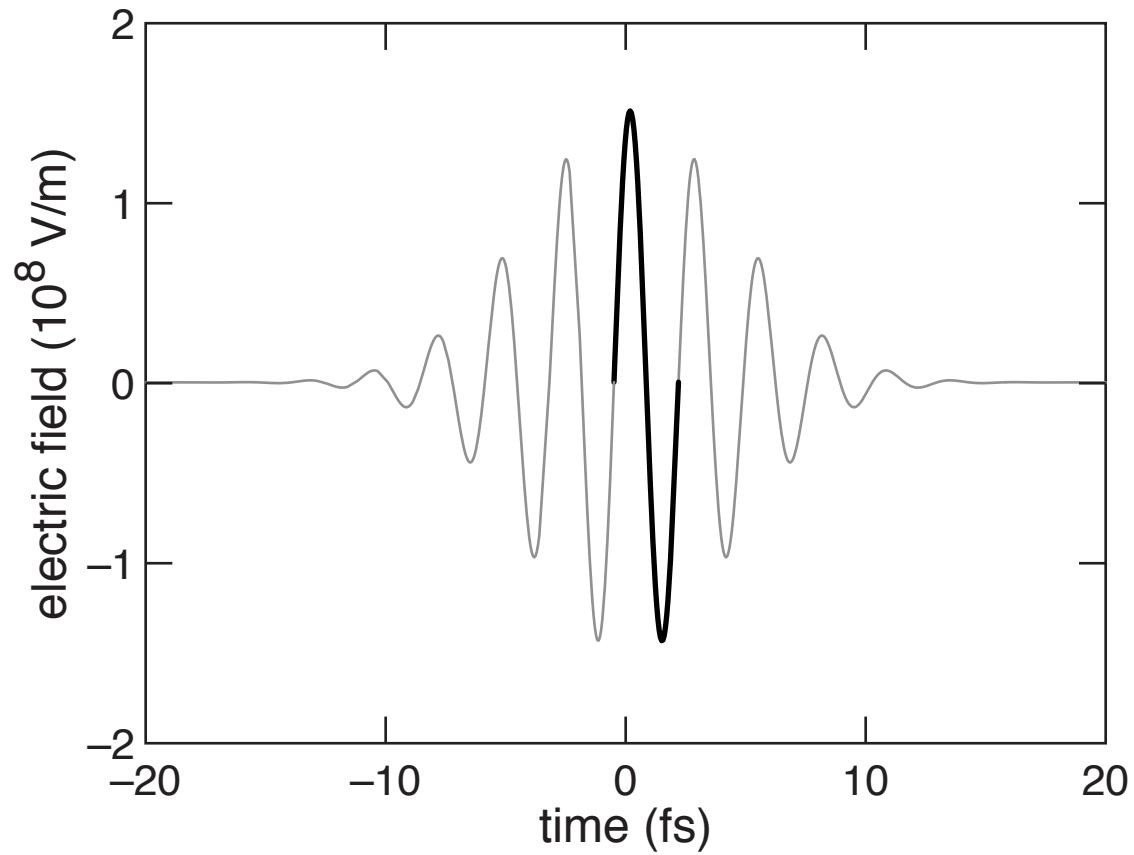
Photonics West, Short Course SC541
San Jose, CA, 22 January 2006



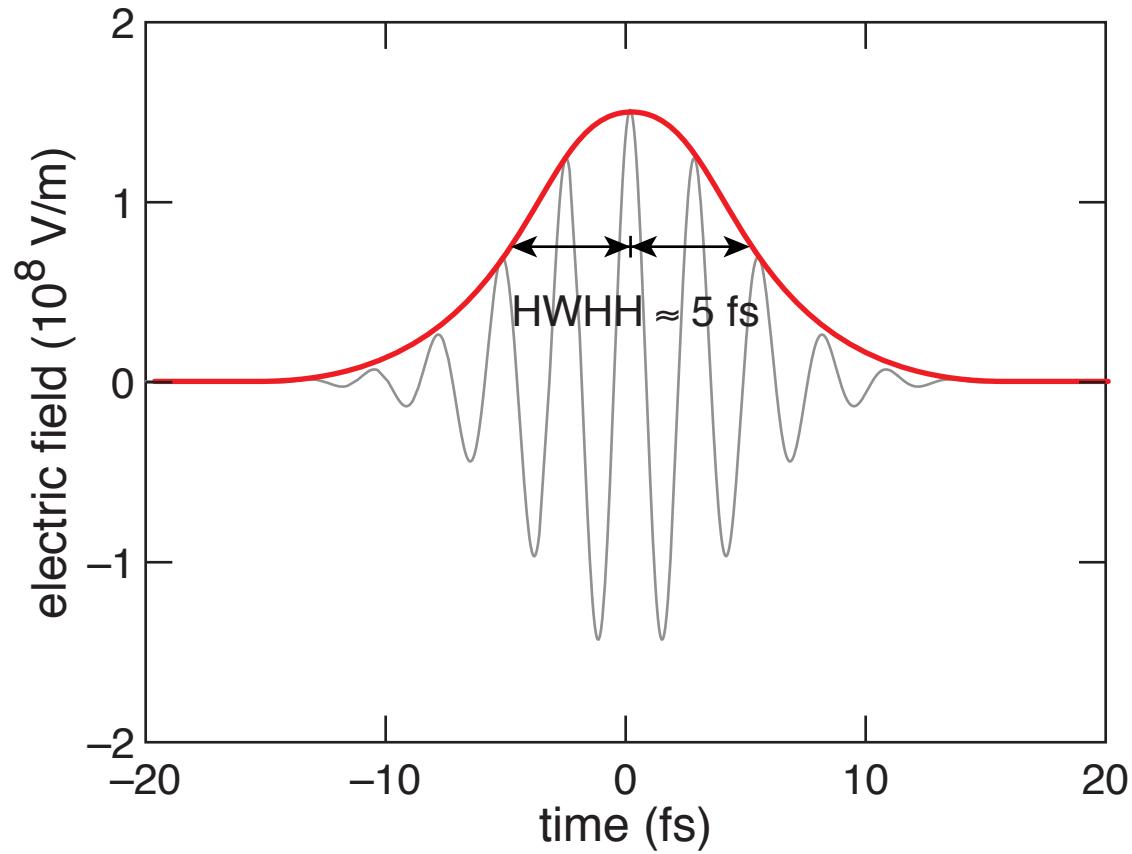
Introduction



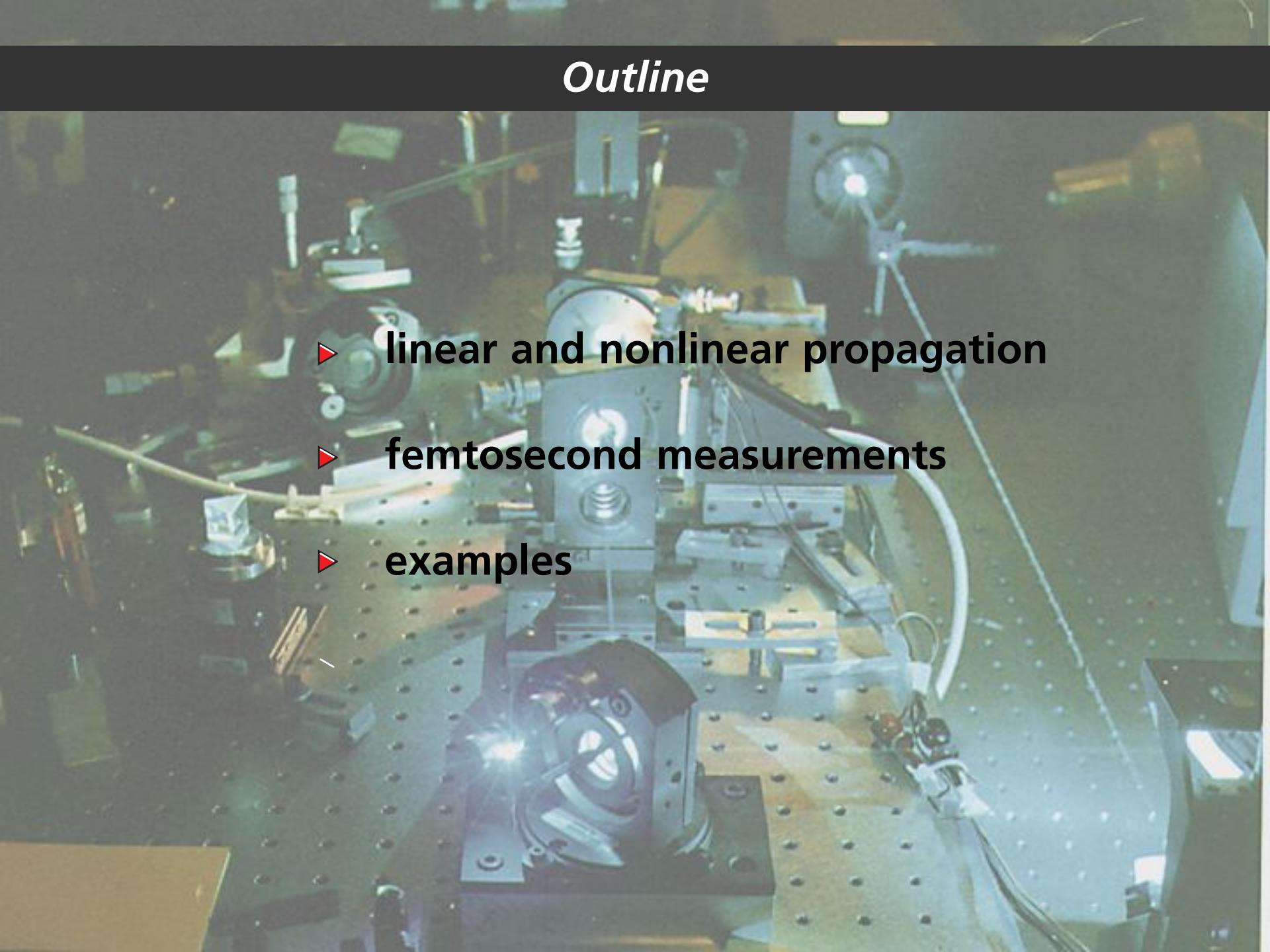
Introduction



Introduction



Outline

- 
- ▶ linear and nonlinear propagation
 - ▶ femtosecond measurements
 - ▶ examples

Propagation of EM waves through medium

Governed by wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

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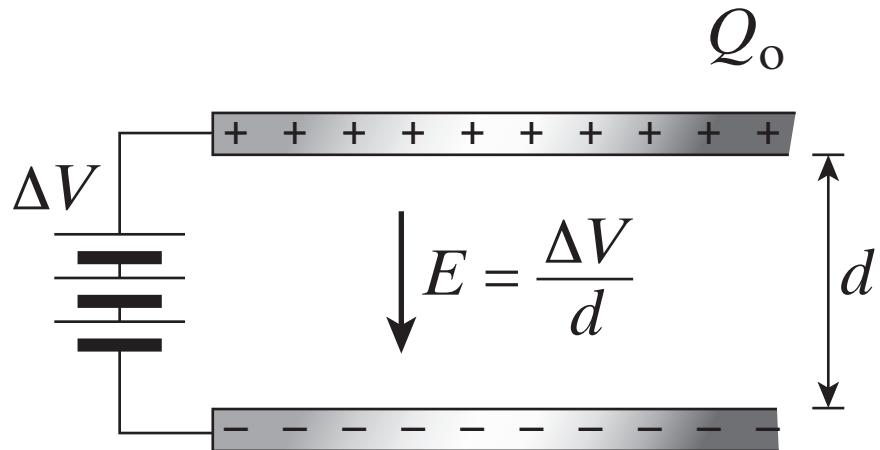
In non-ferromagnetic media $\mu \approx 1$, and so $n \approx \sqrt{\epsilon}$.

In dispersive media $n = n(\omega)$.

Propagation of EM waves through medium

Dielectric constant measures increase in capacitance

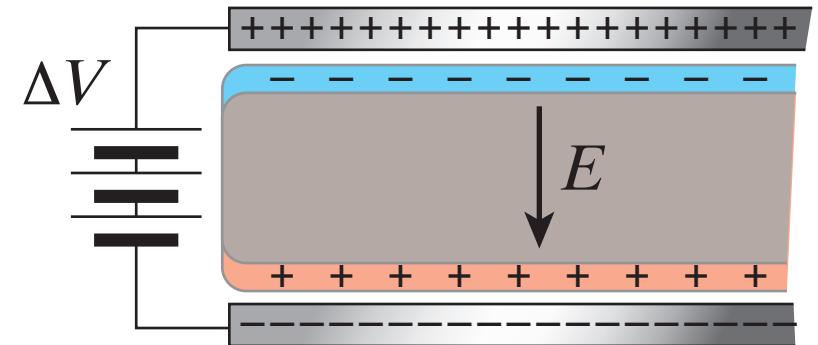
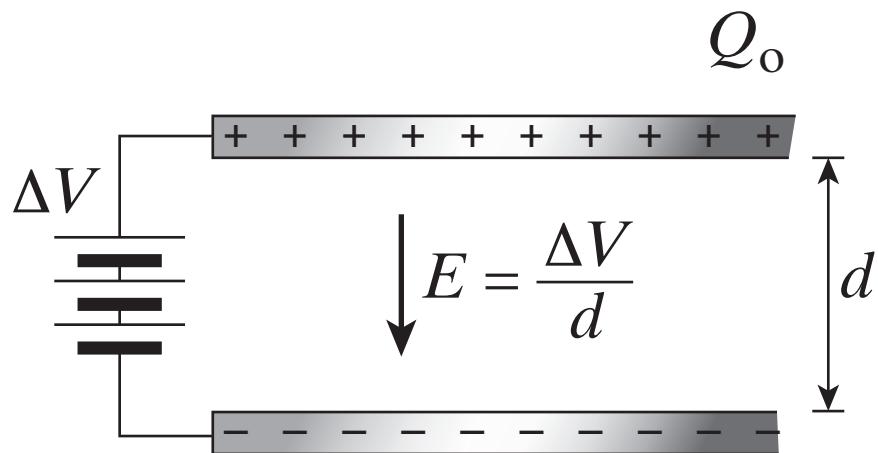
$$\epsilon = \frac{C_d}{C_o}$$



Propagation of EM waves through medium

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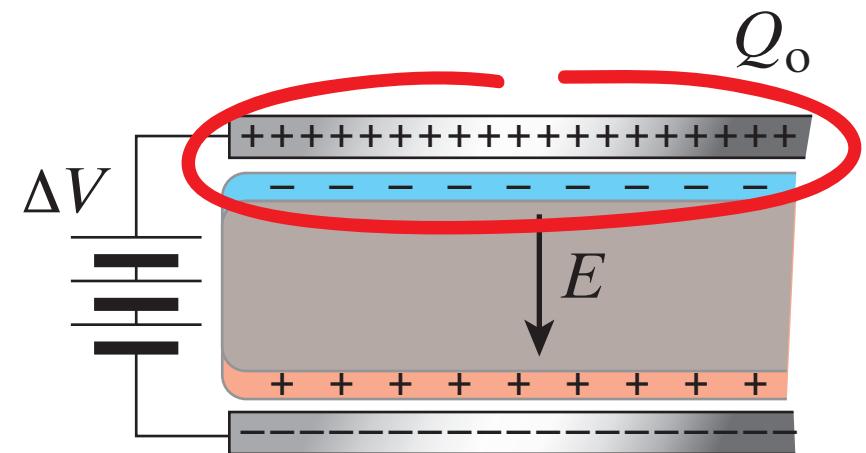
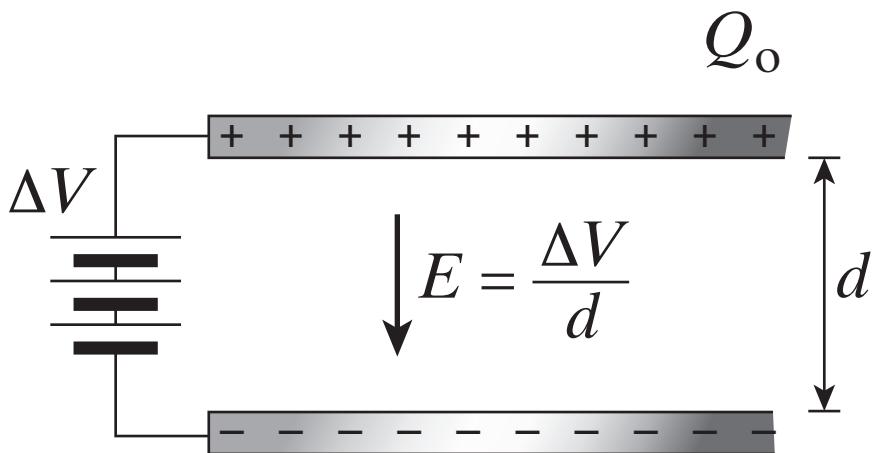
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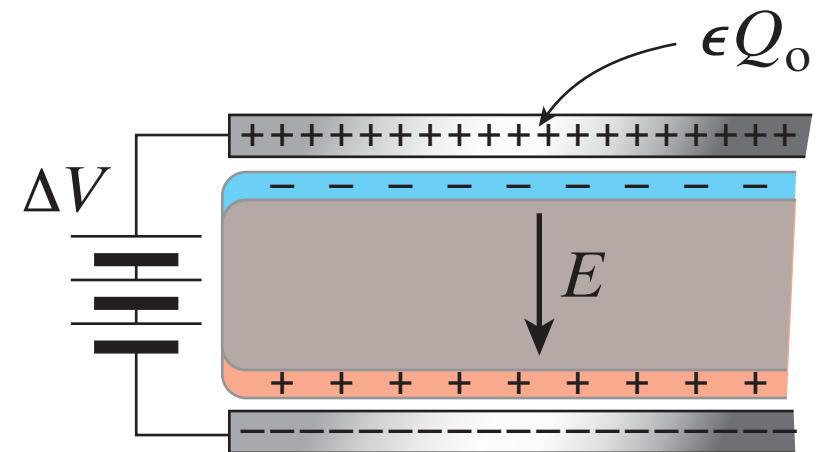
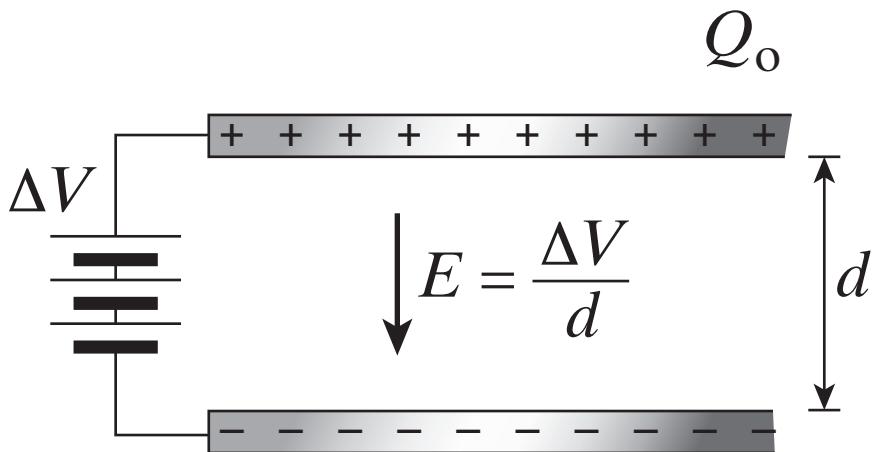
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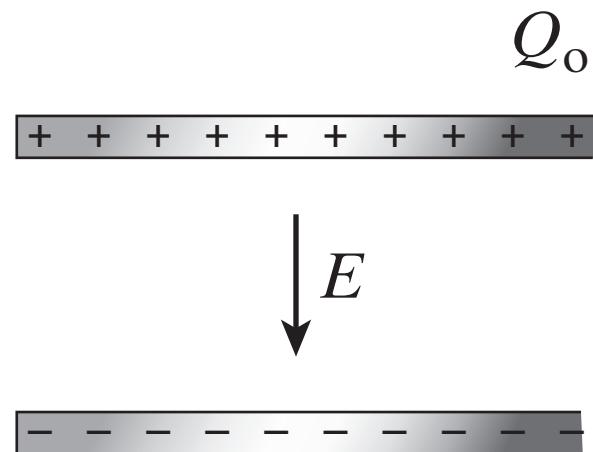
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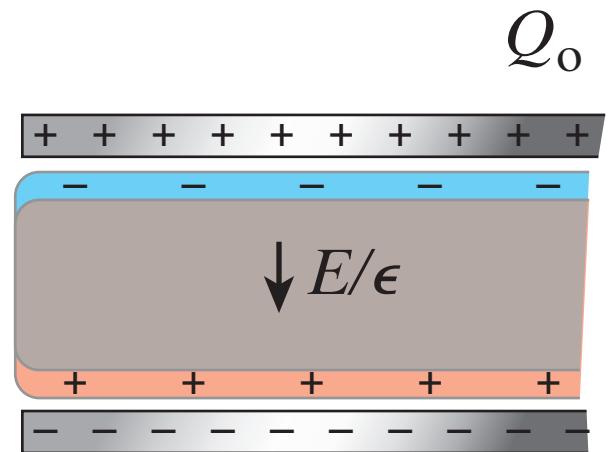
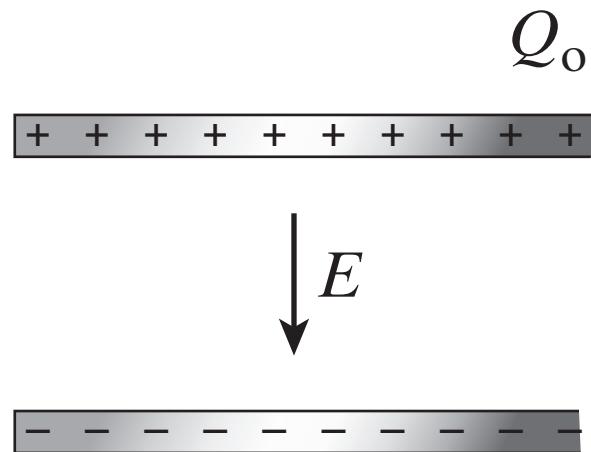
Propagation of EM waves through medium

Alternatively, ϵ is measure of the attenuation of the field



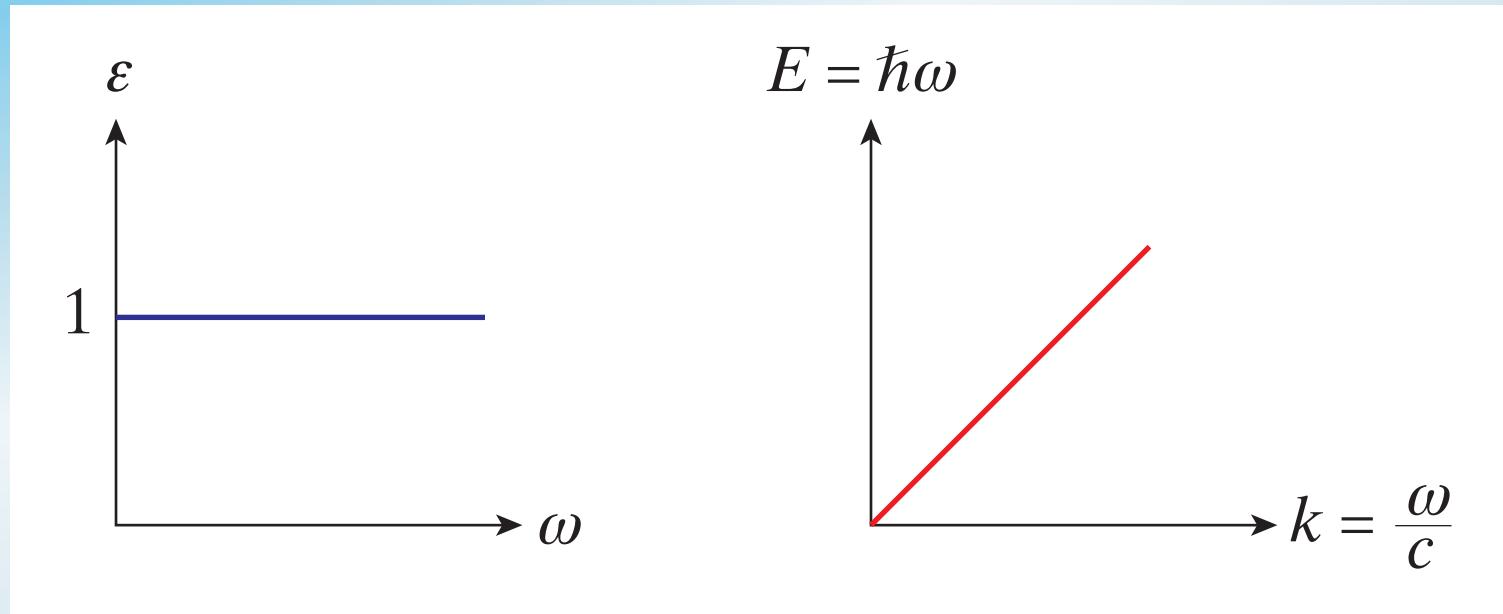
Propagation of EM waves through medium

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Propagation of EM waves through medium

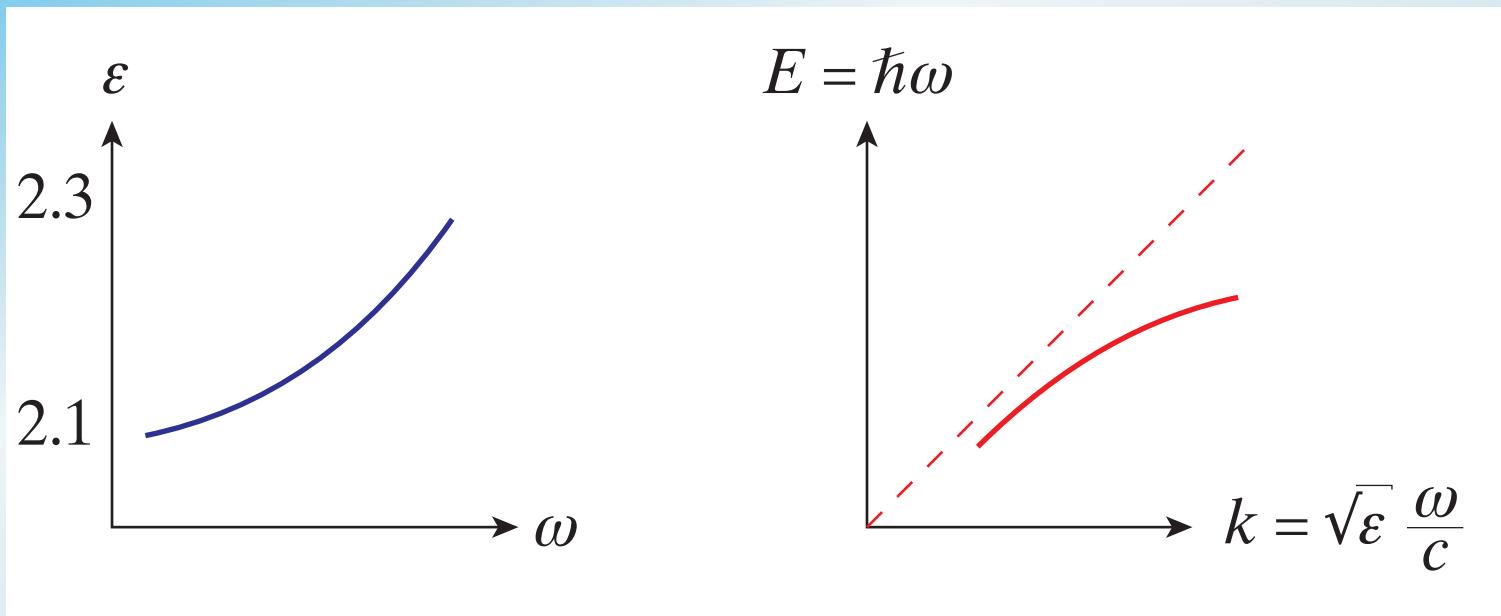
In vacuum: $f\lambda = \frac{\omega}{k} = c \quad \Rightarrow \quad \omega = c k$



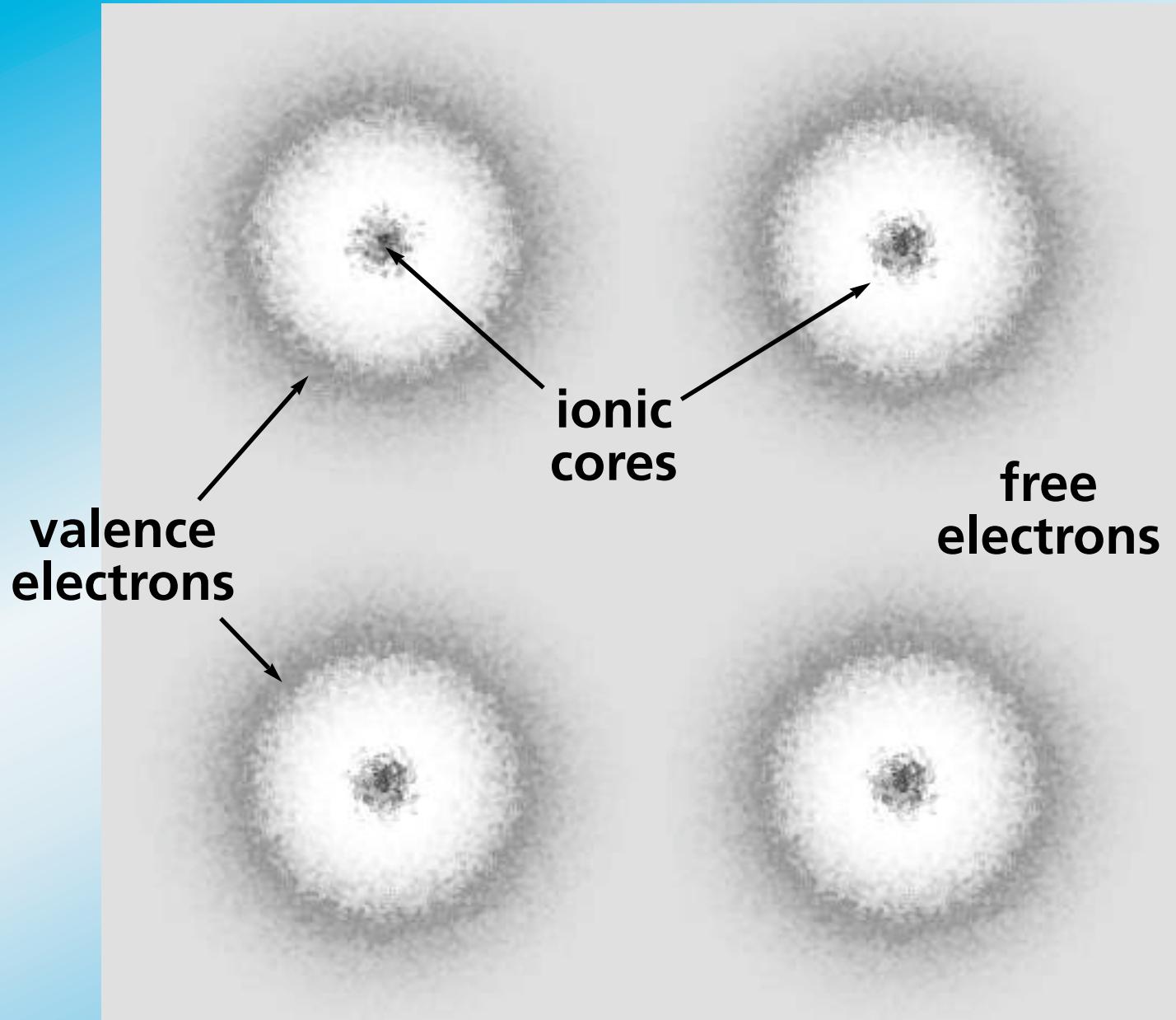
Propagation of EM waves through medium

In medium:

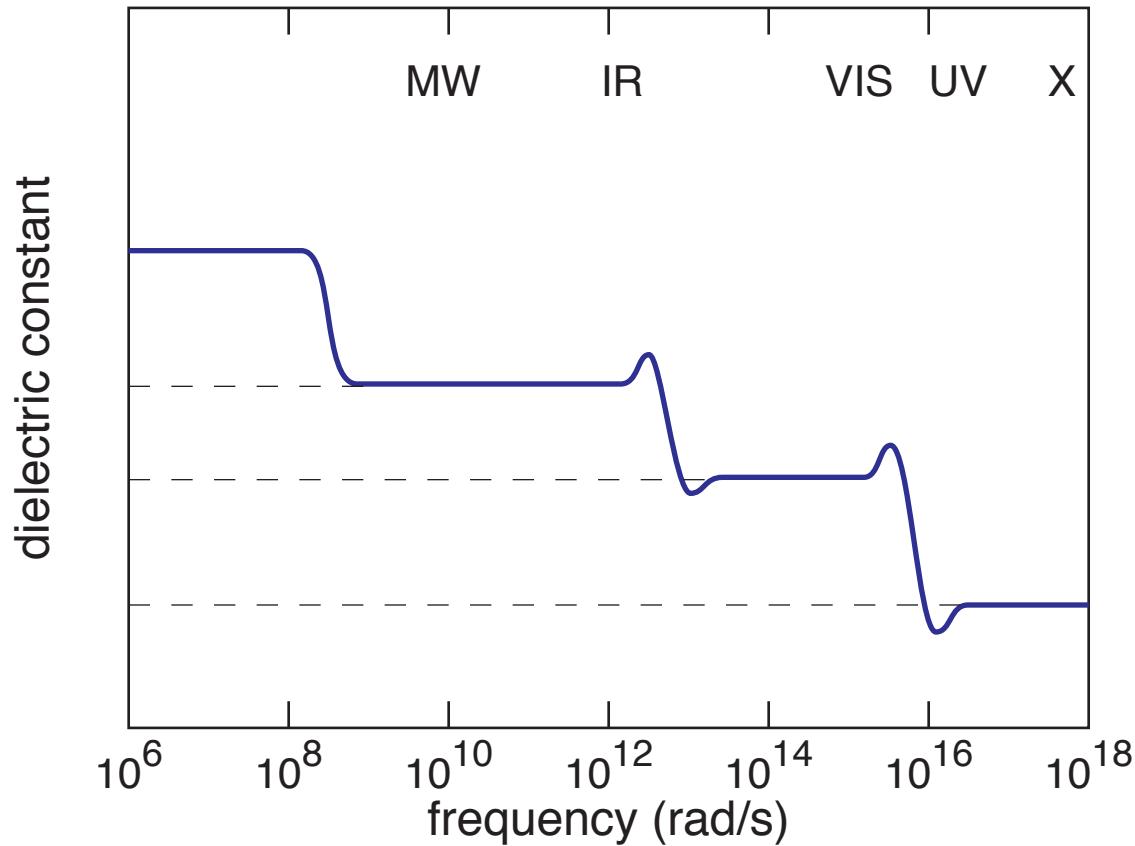
$$v = \frac{c}{\sqrt{\epsilon(\omega)}} = \frac{c}{n} \quad \Rightarrow \quad \omega = \frac{c}{\sqrt{\epsilon}} k$$



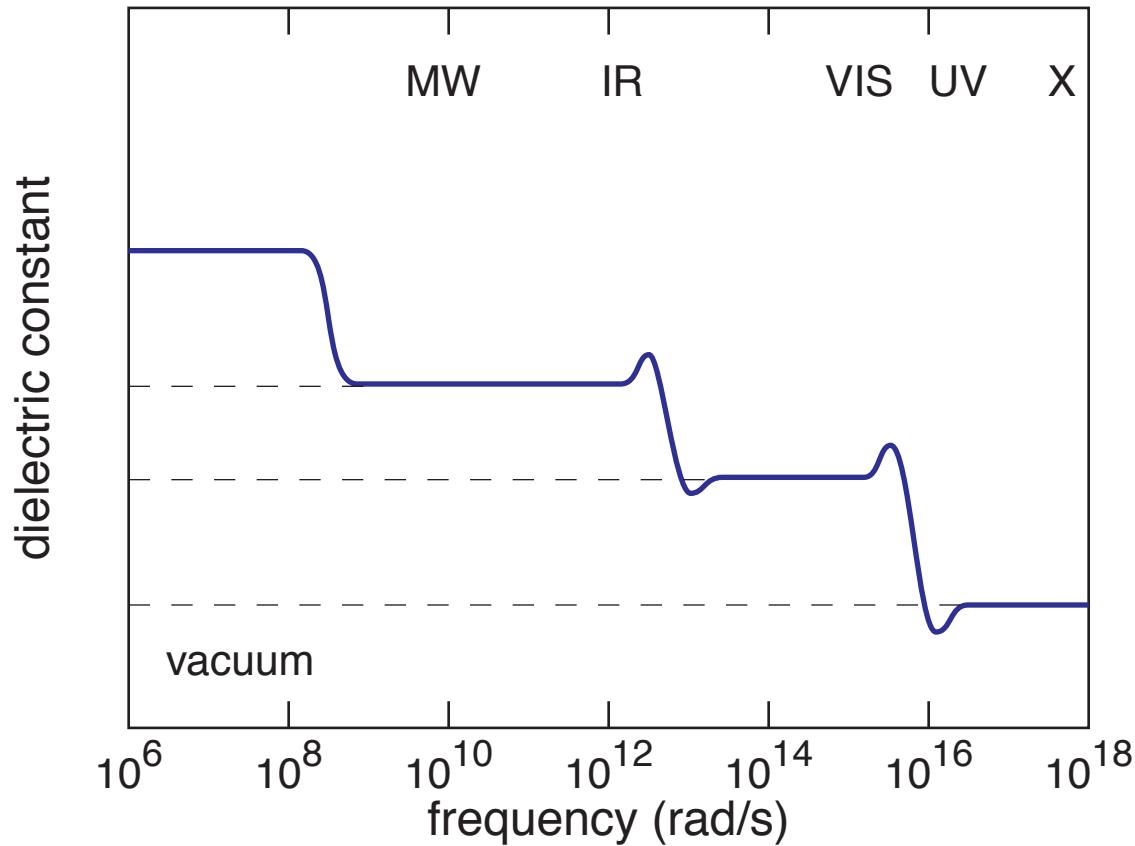
Which charges participate?



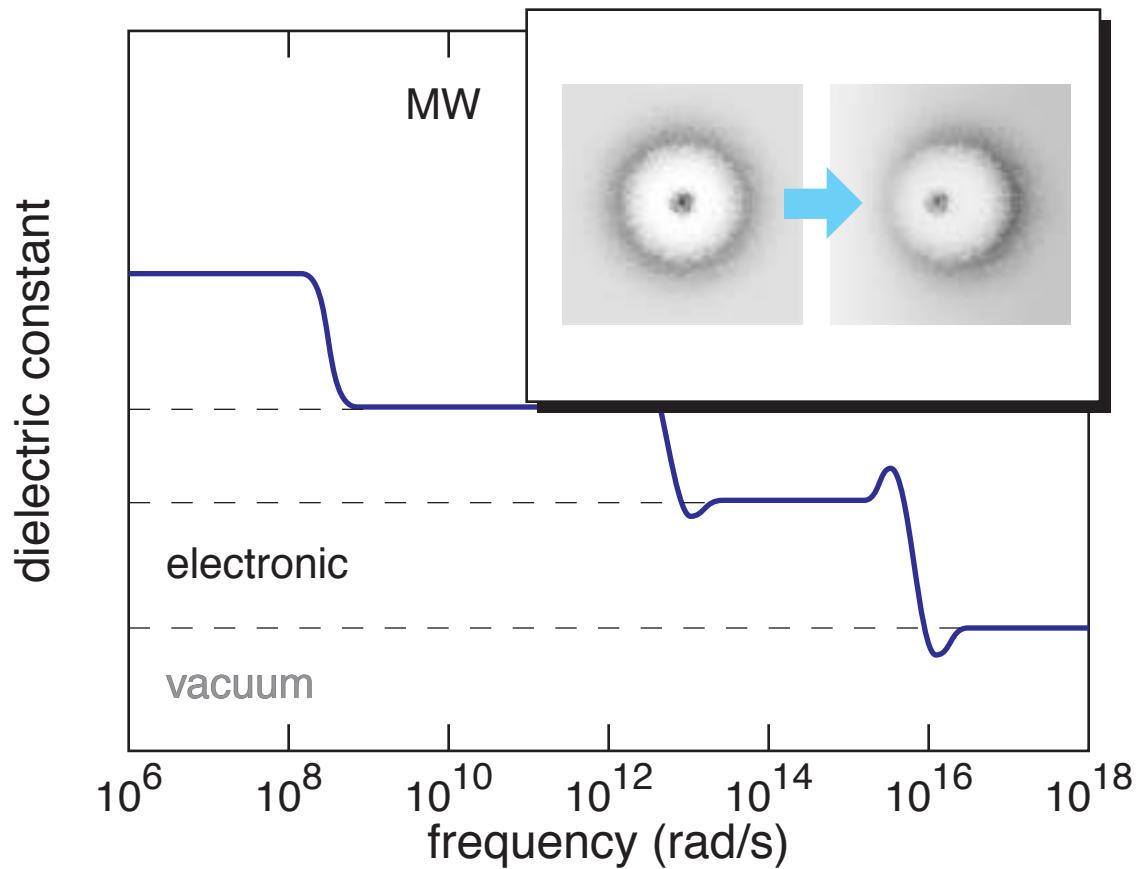
Dielectric function



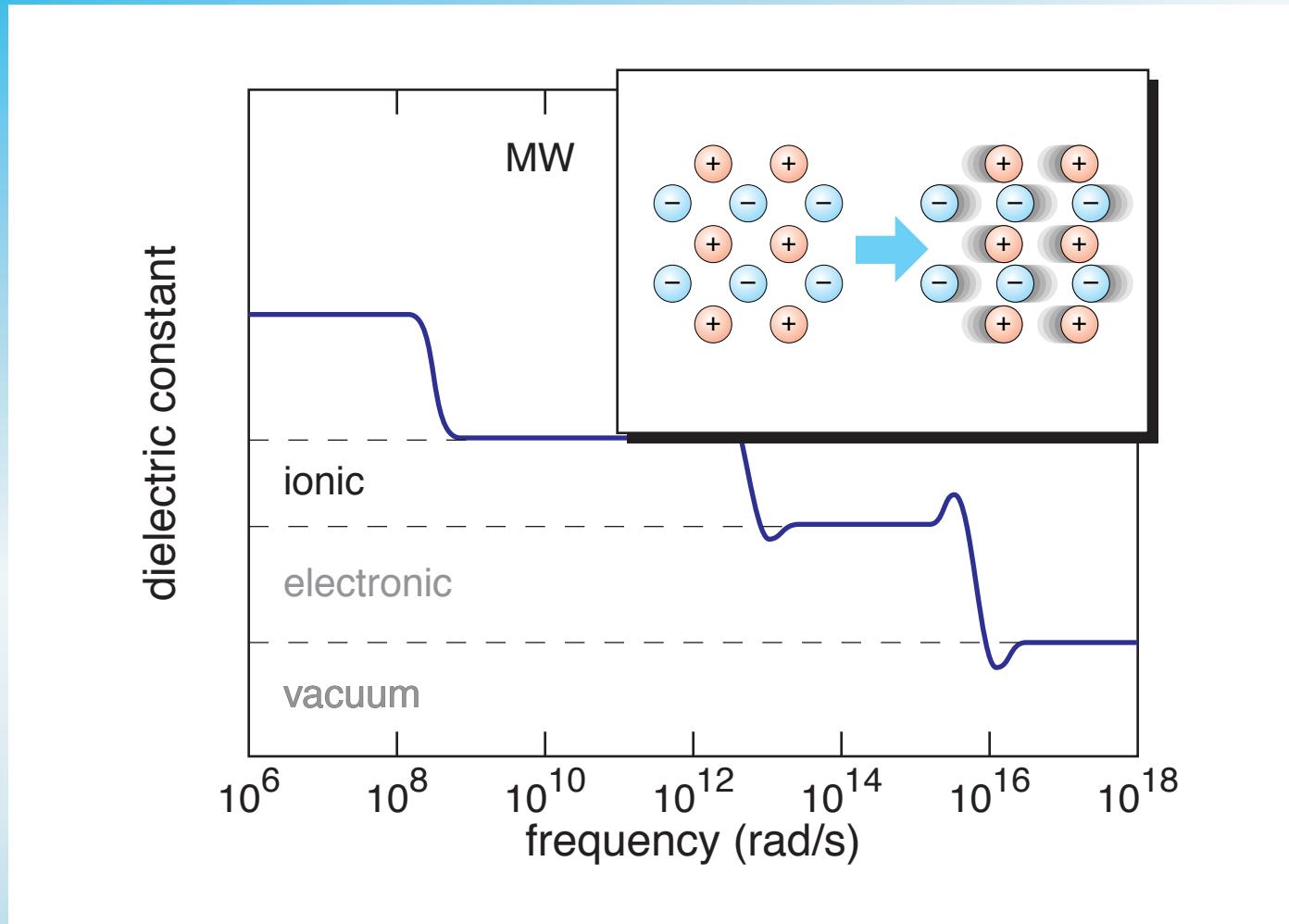
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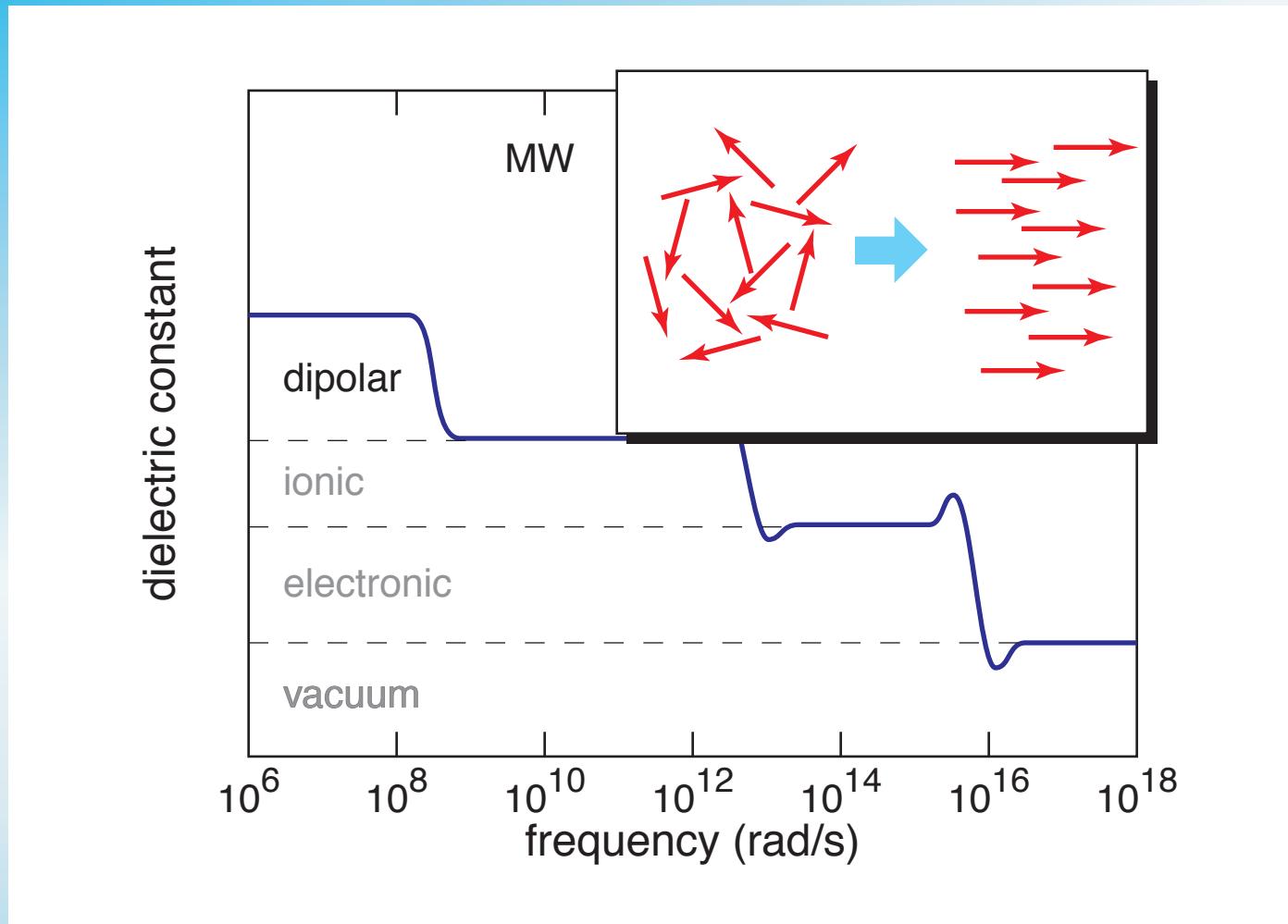
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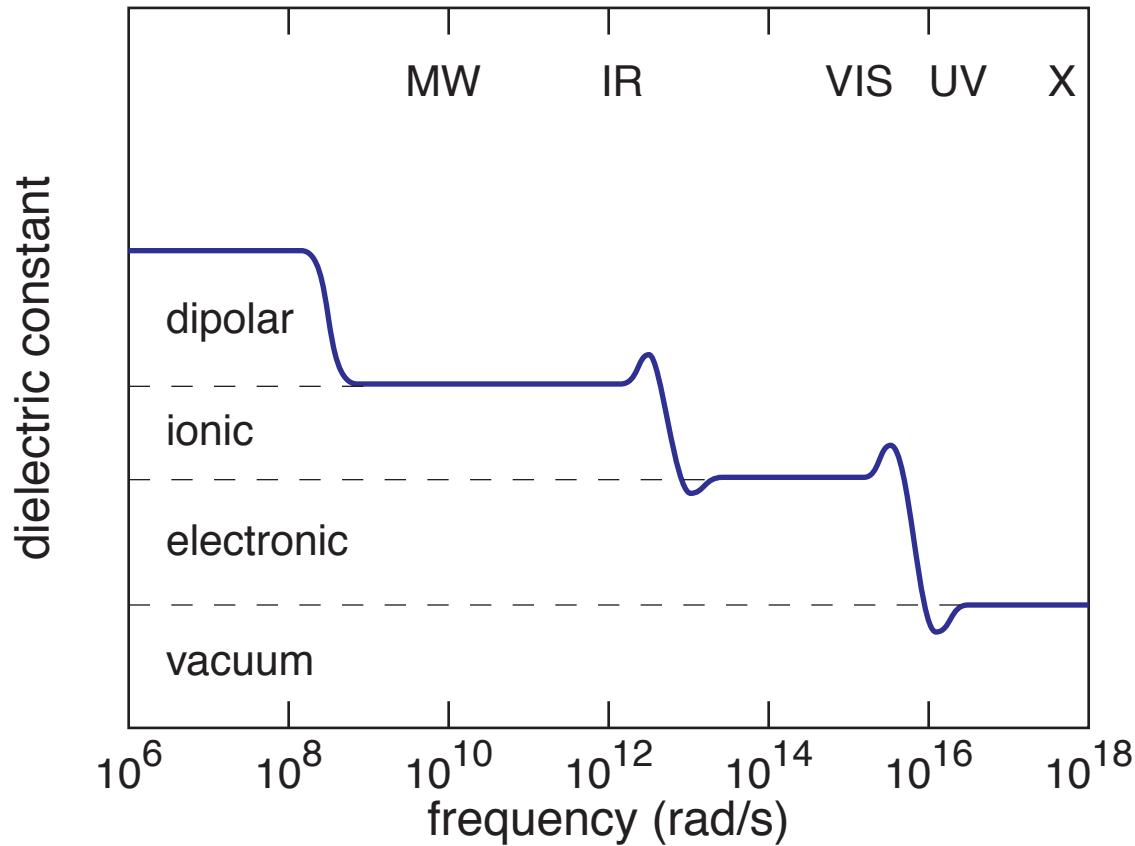
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Dielectric function



Bound electrons

Electron on a string:

$$F_{binding} = - m_e \omega_o^2 x$$

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$$m \frac{d^2x}{dt^2} = \sum F$$

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$$m \frac{d^2x}{dt^2} = \sum F$$

$$m \frac{d^2x}{dt^2} + m\gamma \frac{dx}{dt} + m\omega_o^2 x = - eE$$

Bound electrons

Steady state: electron oscillates at driving frequency

$$x(t) = x_o e^{-i\omega t} \quad x_o = - \frac{e}{m} \frac{1}{(\omega_o^2 - \omega^2) - i\gamma\omega} E_o$$

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Polarization

$$P(t) = \left(\frac{Ne^2}{m} \right) \sum_j \frac{f_j}{(\omega_j^2 - \omega^2) - i\gamma_j\omega} E(t) \equiv \epsilon_o \chi_e E(t)$$

Bound electrons

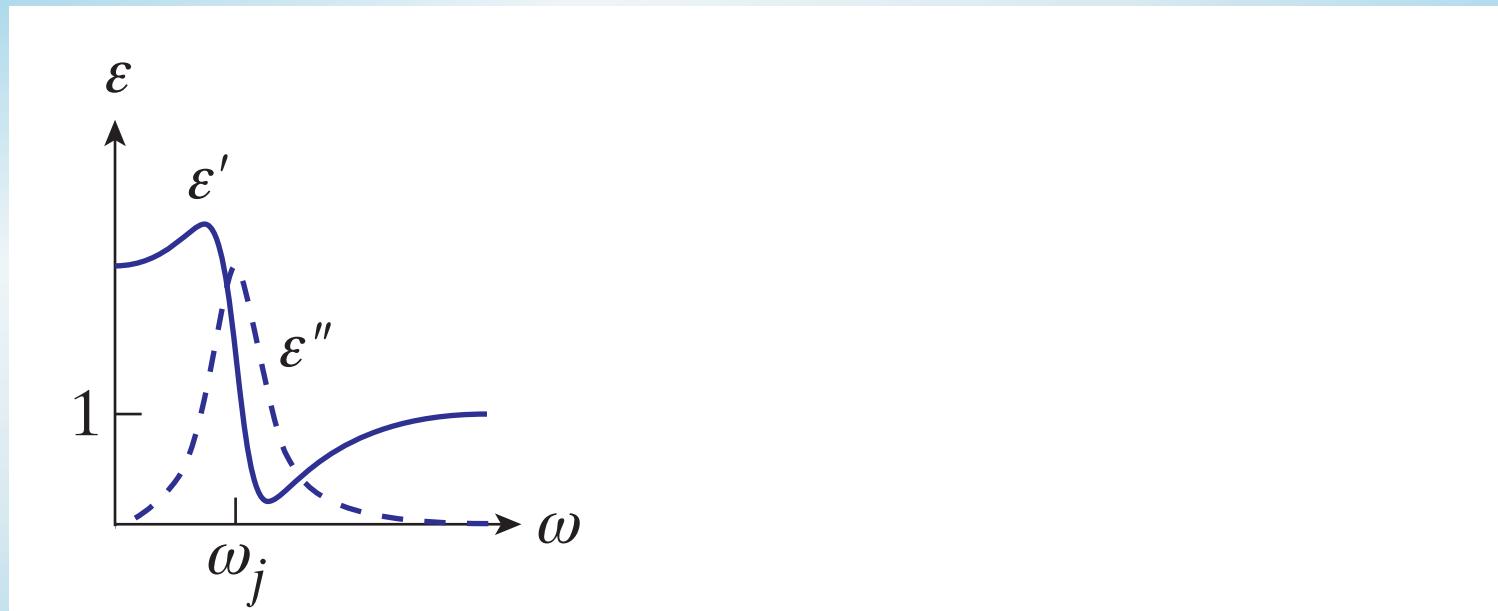
Dielectric function

$$\epsilon(\omega) \equiv 1 + \chi_e = 1 + \frac{Ne^2}{\epsilon_0 m} \sum_j \frac{f_j}{(\omega_j^2 - \omega^2) - i\gamma_j \omega}$$

Bound electrons

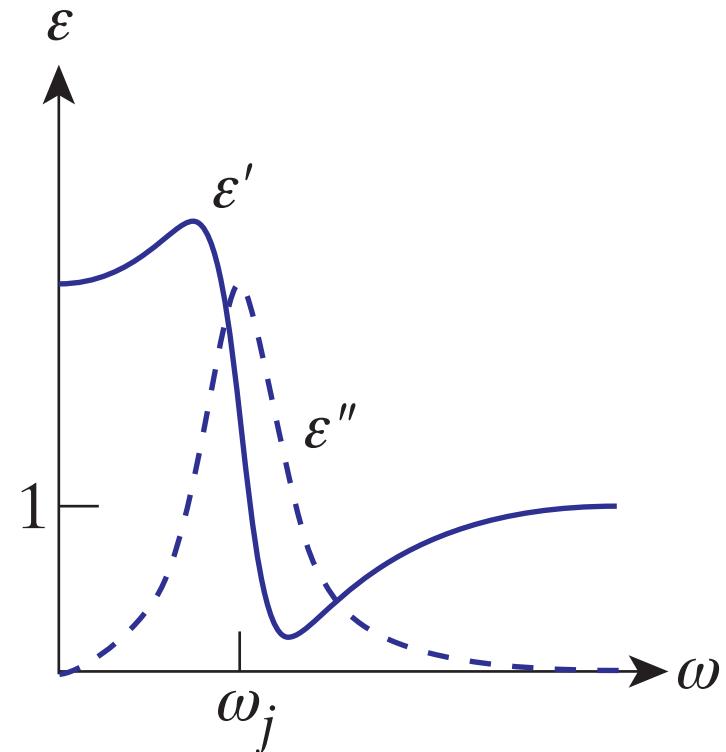
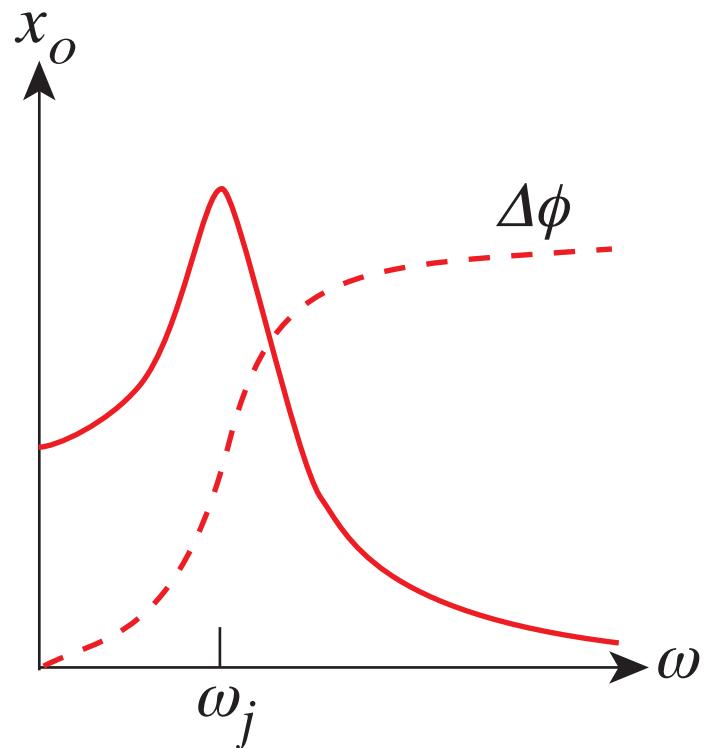
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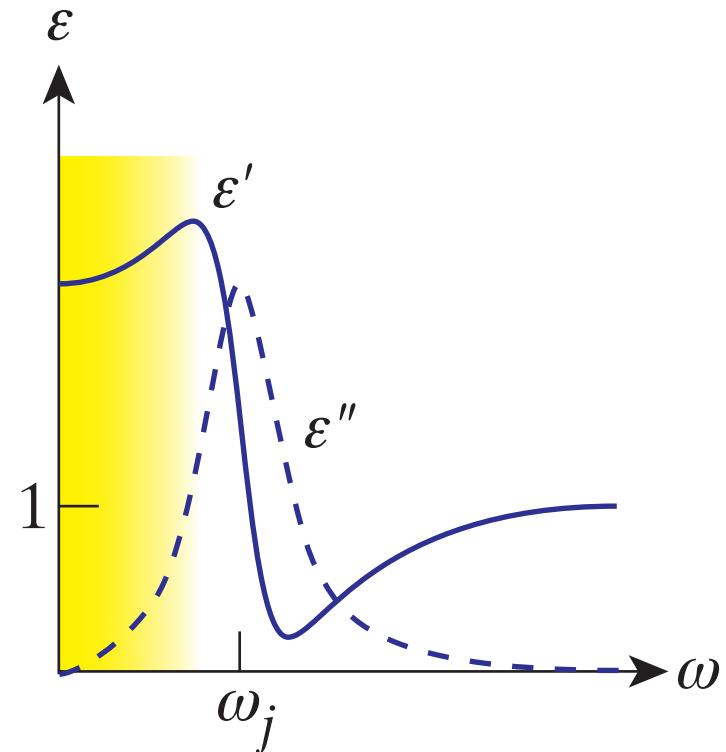
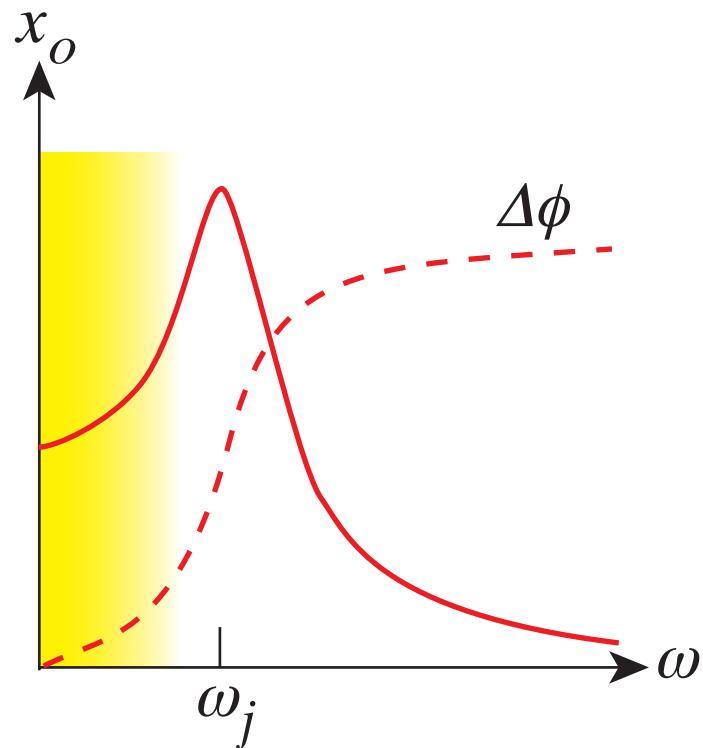
Bound electrons

amplitude of bound charge oscillation



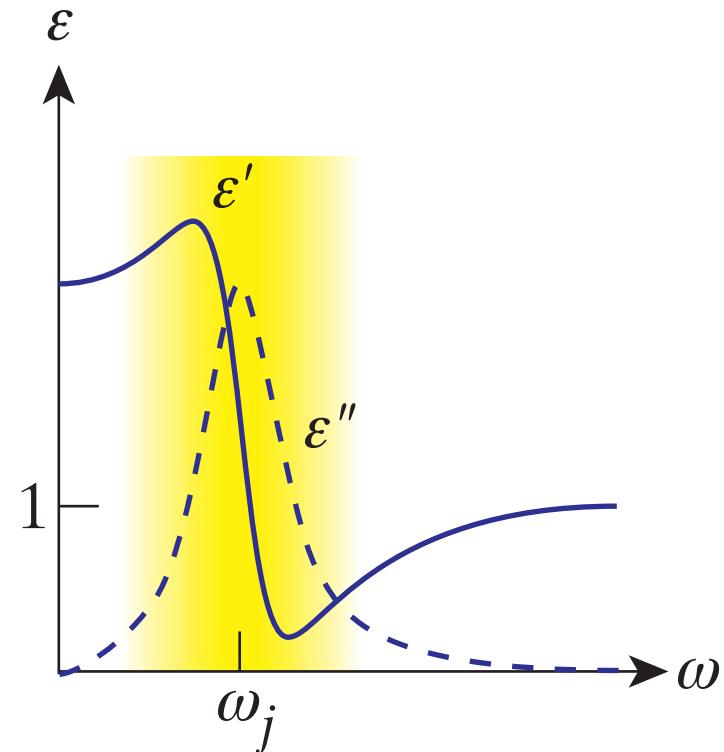
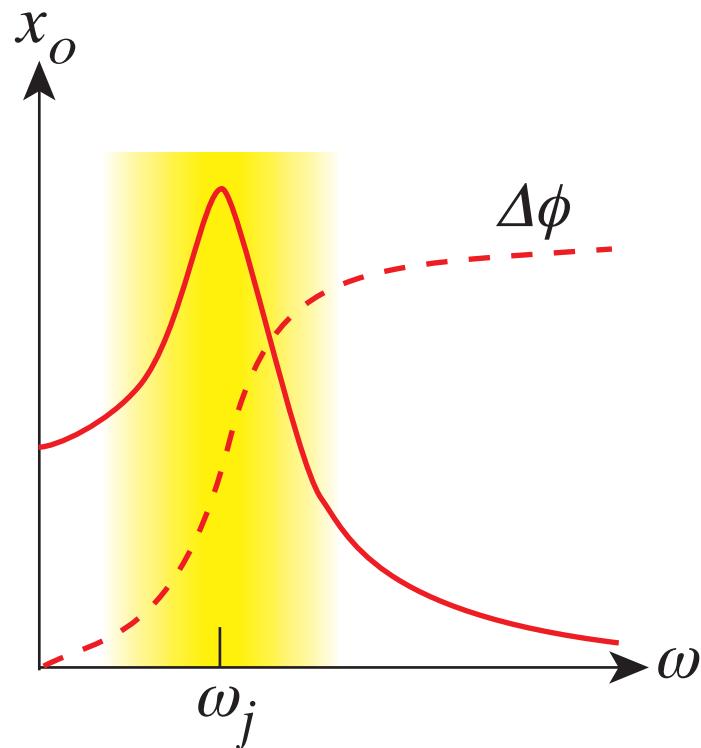
Bound electrons

Below resonance: bound charges keep up with driving field \Rightarrow field attenuated, wave propagates more slowly



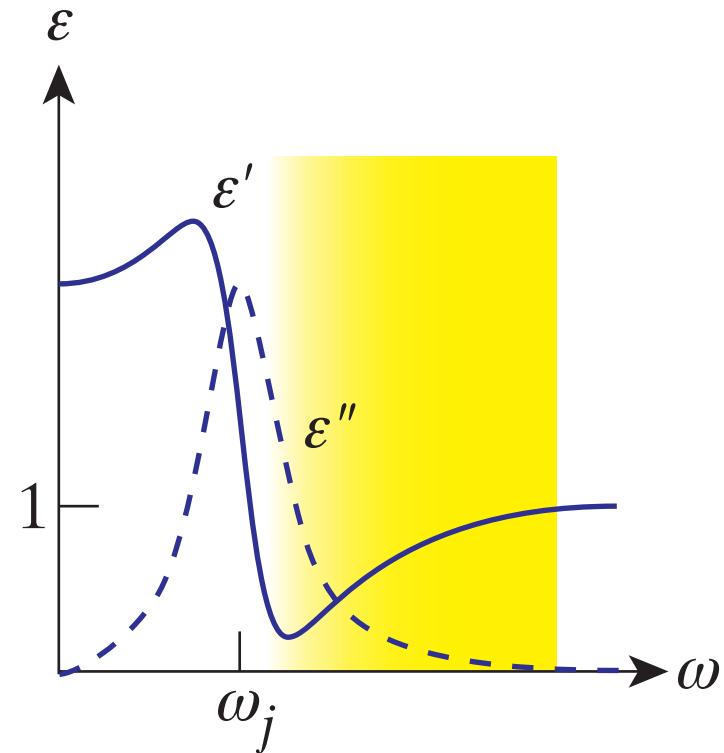
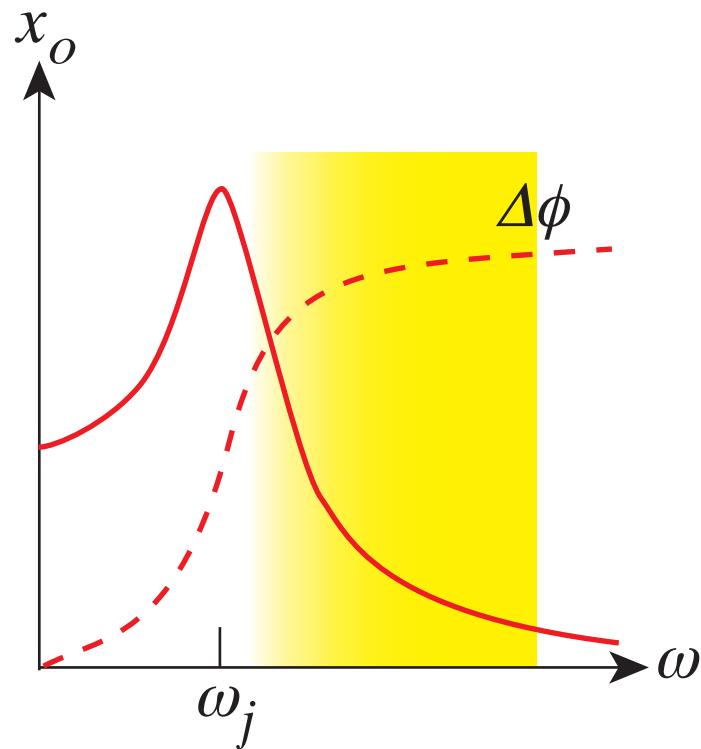
Bound electrons

At resonance: energy transfer from wave to bound charges \Rightarrow wave attenuates (absorption)



Bound electrons

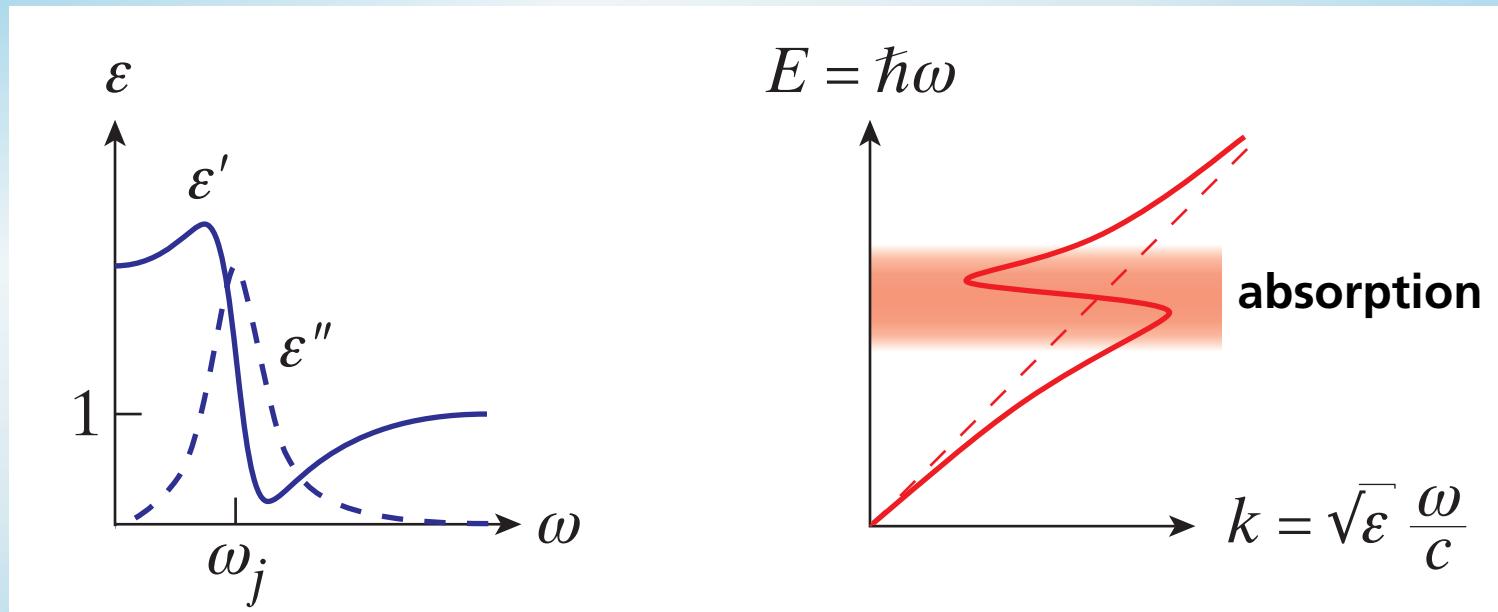
Above resonance: bound charges cannot keep up with driving field \Rightarrow dielectric like a vacuum



Bound electrons

Dielectric function

$$\epsilon(\omega) \equiv 1 + \chi_e = 1 + \frac{Ne^2}{\epsilon_0 m} \sum_j \frac{f_j}{(\omega_j^2 - \omega^2) - i\gamma_j \omega}$$



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Low frequency ($\omega \ll \gamma$) \Rightarrow current generated

$$J = -Ne \frac{dx}{dt} = \frac{Ne^2}{m} \frac{1}{\gamma - i\omega} E \approx \frac{Ne^2}{m\gamma} E \equiv \sigma E$$

Free electrons

$\omega \gg \gamma$: **σ complex** \Rightarrow **J out of phase with E**

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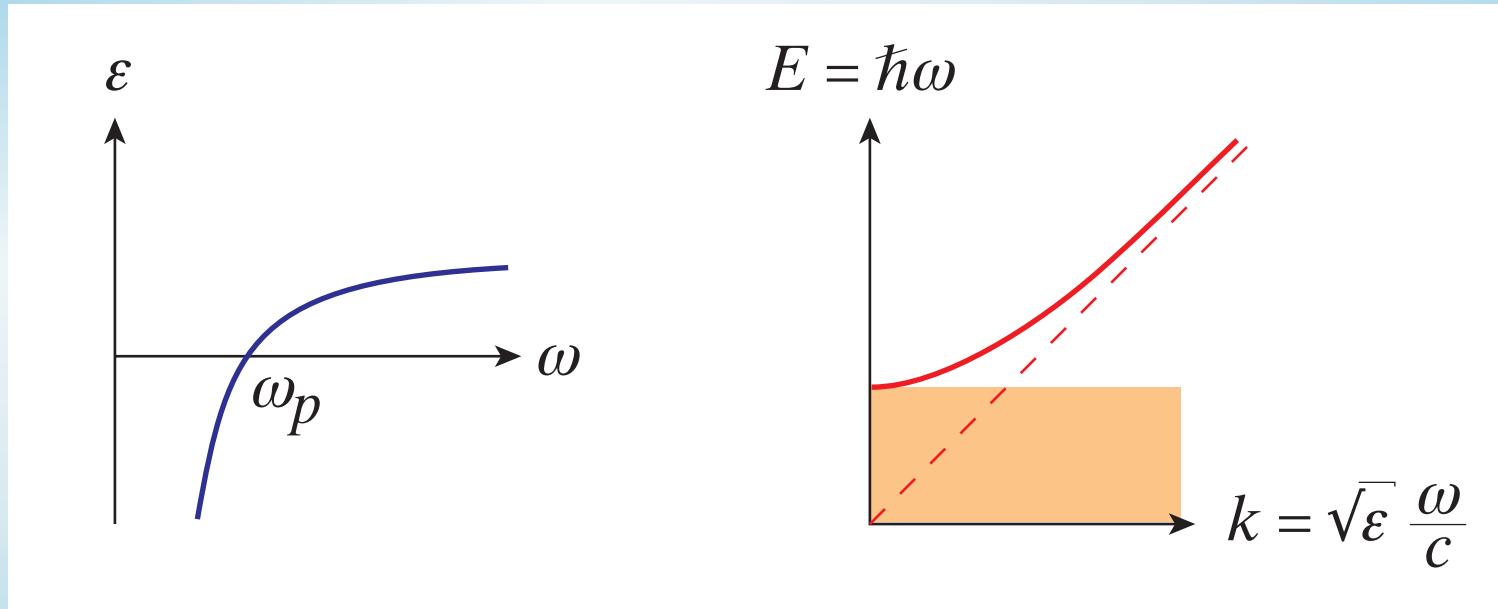
Dielectric function

$$\epsilon(\omega) \equiv 1 + \chi_e = 1 - \frac{Ne^2}{m\epsilon_0} \frac{1}{\omega^2 + i\gamma\omega} = \epsilon'(\omega) + i\epsilon''(\omega)$$

Plasma

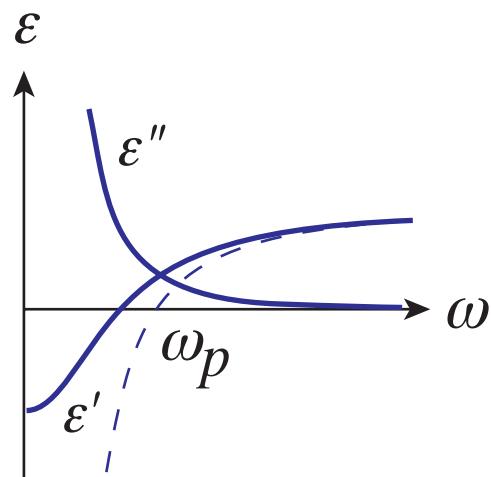
$$\gamma \approx 0 \quad \Rightarrow \quad \epsilon'' = 0$$

$$\epsilon'(\omega) = 1 - \frac{Ne^2}{m\epsilon_0} \frac{1}{\omega^2} \equiv 1 - \frac{\omega_p^2}{\omega^2}$$

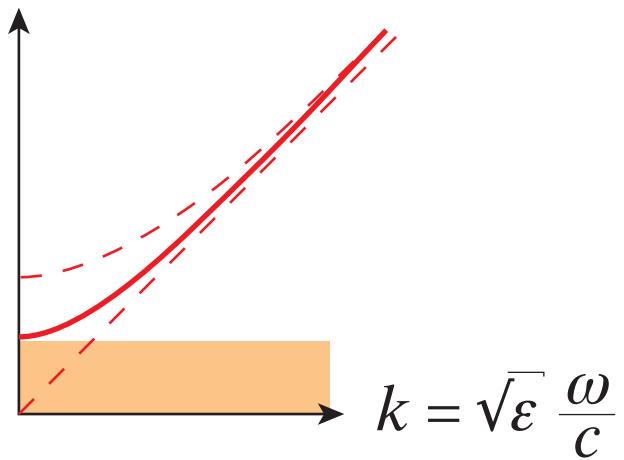


Add damping

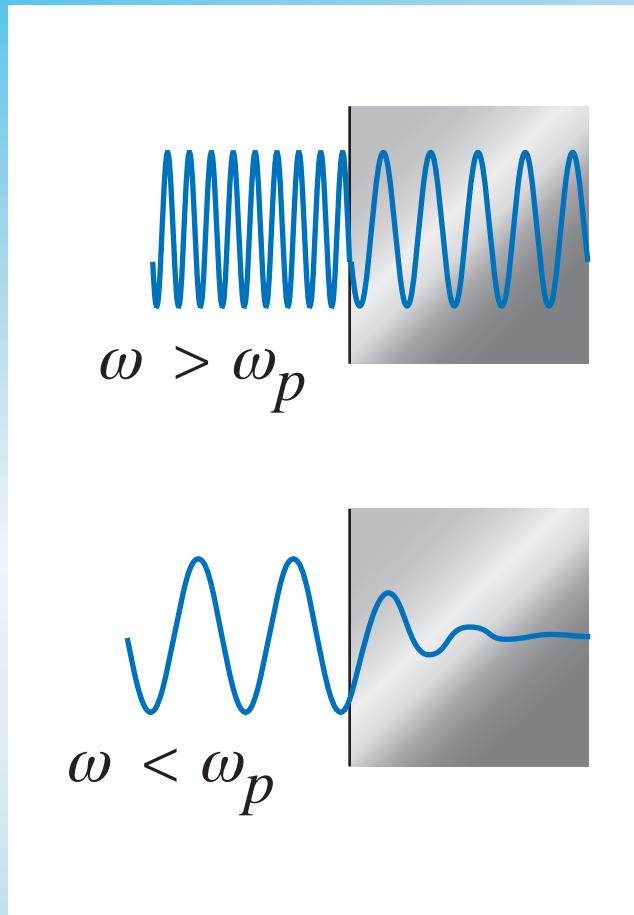
$$\gamma \lesssim \omega_p$$



$$E = \hbar\omega$$

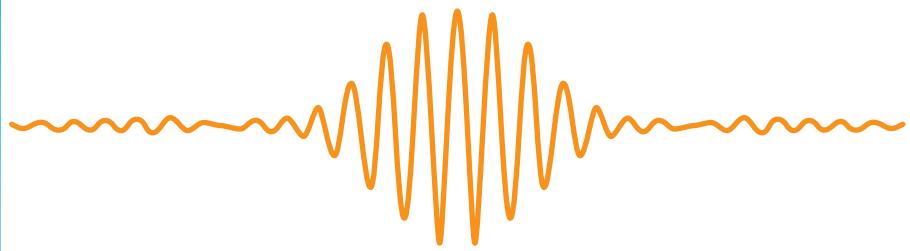


Plasma acts like a high-pass filter:

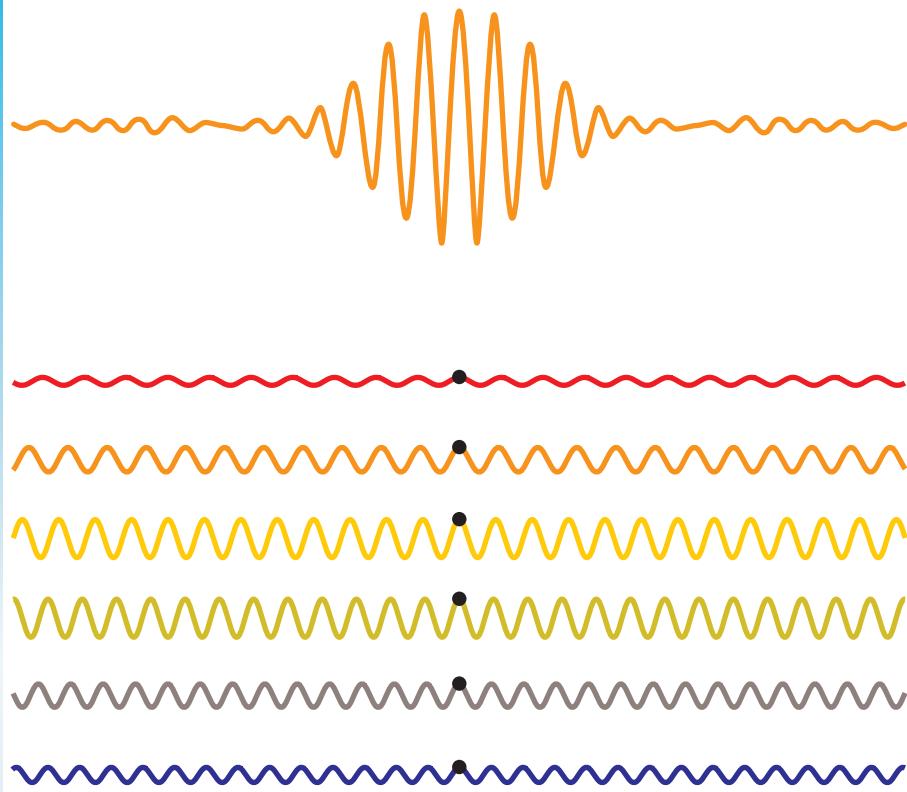


$\log N$ (cm $^{-3}$)	ω_p (rad s $^{-1}$)	λ_p
22	6×10^{15}	330 nm
18	6×10^{13}	33 μm
14	6×10^{11}	3.3 mm
10	6×10^9	0.33 m

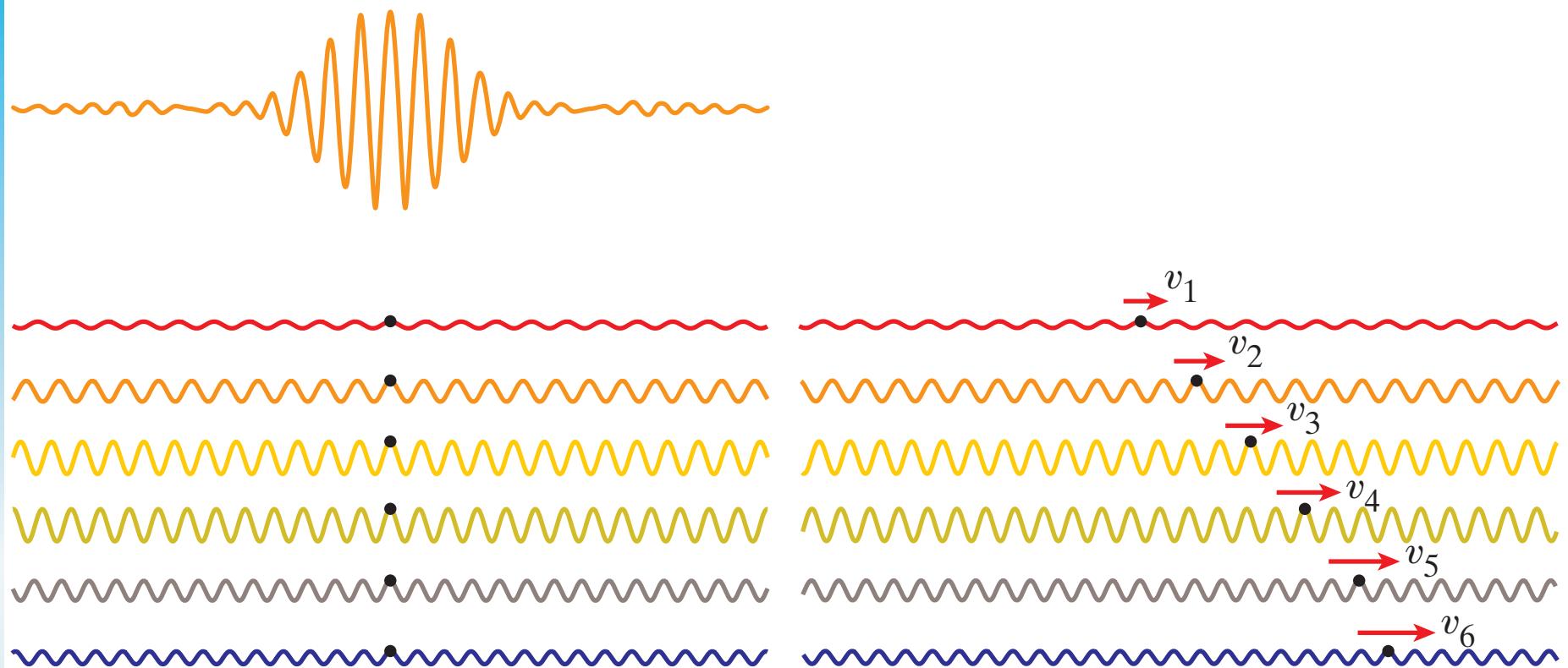
Pulse dispersion



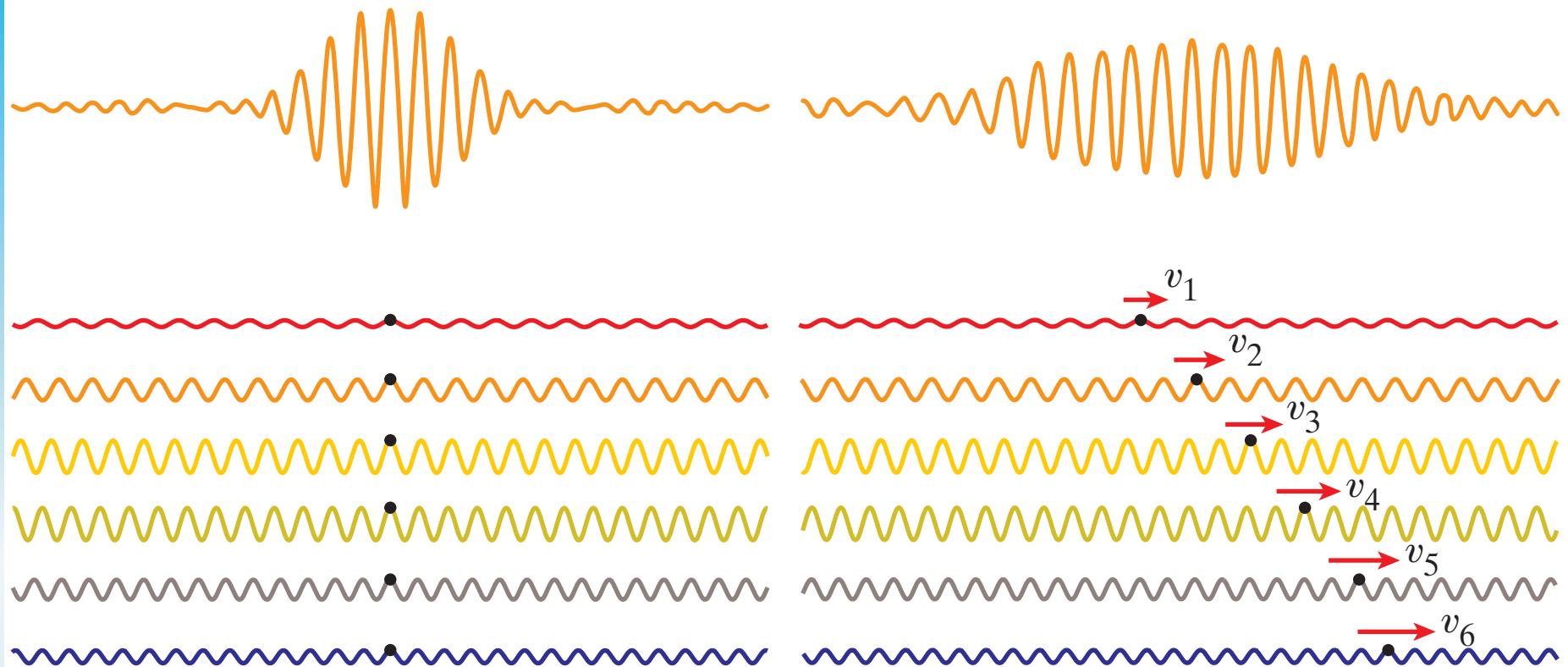
Pulse dispersion



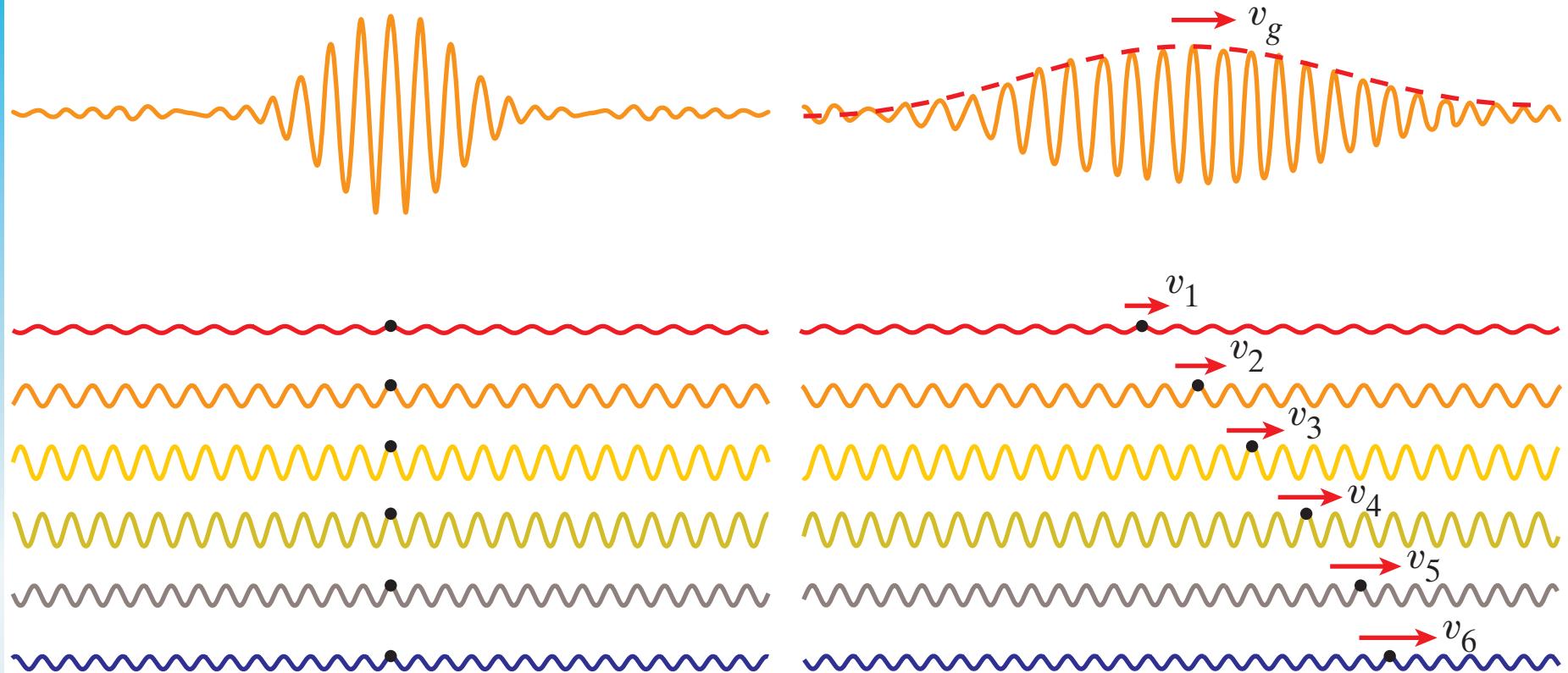
Pulse dispersion



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Pulse dispersion



Pulse dispersion

Consider two propagating waves:

$$y_1 = A \sin 2\pi(k_1 x - f_1 t) \quad \text{and} \quad y_2 = A \sin 2\pi(k_2 x - f_2 t)$$

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propagating at speeds

$$v_1 = \frac{f_1}{k_1} = f_1 \lambda_1 \quad \text{and} \quad v_2 = \frac{f_2}{k_2} = f_2 \lambda_2.$$

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Superposition:

$$y = A[\sin 2\pi(k_1 x - f_1 t) + \sin 2\pi(k_2 x - f_2 t)]$$

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$$y = A[\sin 2\pi(k_1 x - f_1 t) + \sin 2\pi(k_2 x - f_2 t)]$$

$$\sin \alpha + \sin \beta = 2\cos\left(\frac{\alpha - \beta}{2}\right)\sin\left(\frac{\alpha + \beta}{2}\right)$$

Pulse dispersion

$$y = 2A \cos\pi[(k_1 - k_2)x - (f_1 - f_2)t] \sin 2\pi\left[\frac{k_1 + k_2}{2}x - \frac{f_1 + f_2}{2}t\right]$$

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$$y = 2A \cos \pi(x\Delta k - t\Delta f) \sin 2\pi(kx - ft)$$

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traveling sine wave, with amplitude modulation

Pulse dispersion

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Pulse dispersion

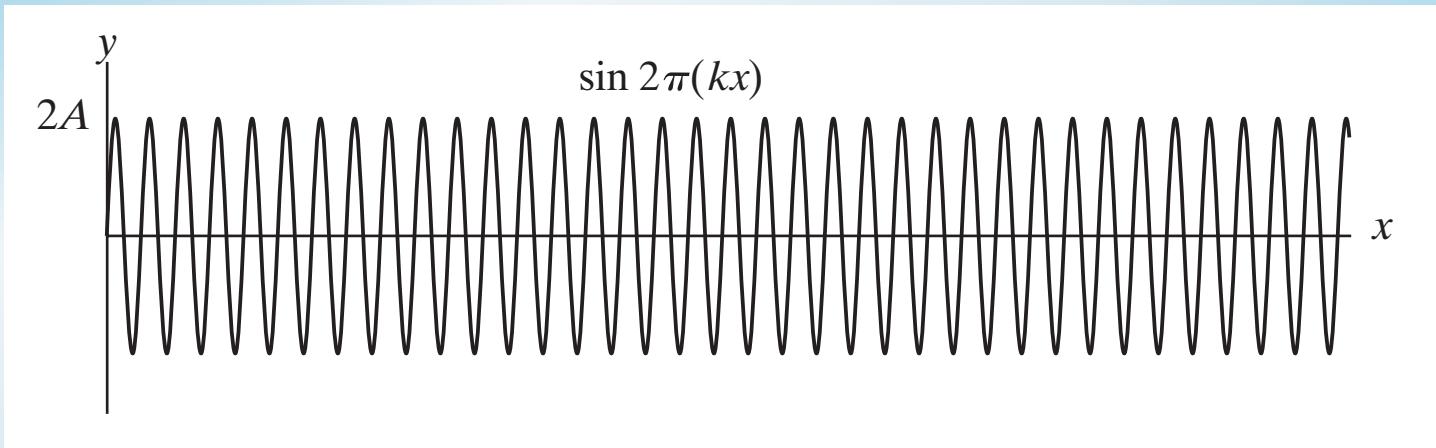
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at $t = 0$: $y = 2A \cos \pi(x\Delta k) \sin 2\pi(kx)$

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$$y = 2A \cos \pi(x\Delta k - t\Delta f) \sin 2\pi(kx - ft)$$

at $t = 0$: $y = 2A \cos \pi(x\Delta k) \sin 2\pi(kx)$



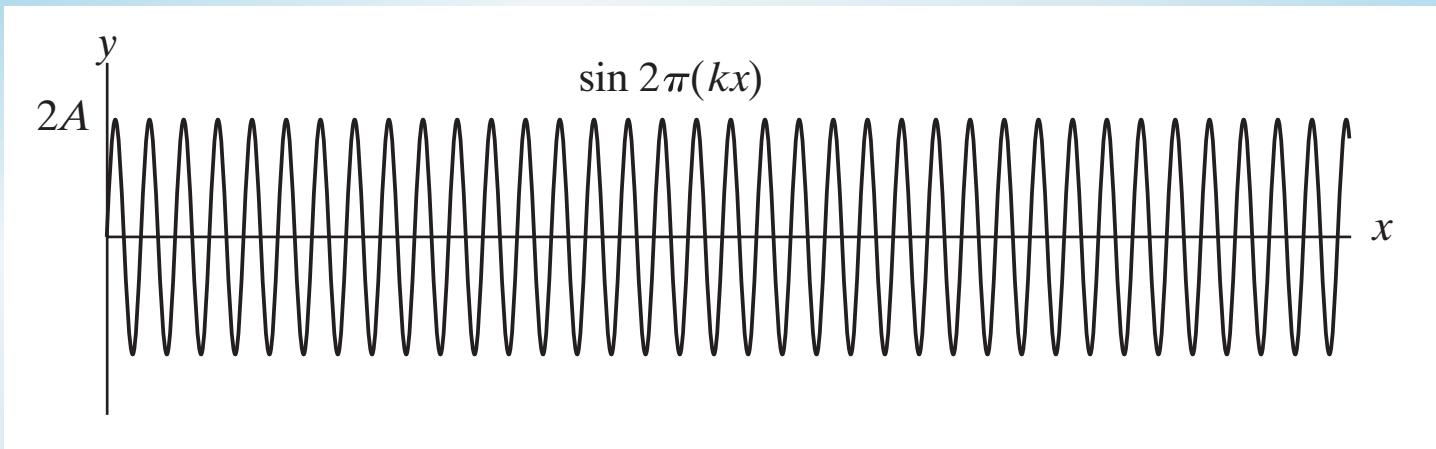
Pulse dispersion

$$y = 2A \cos \pi(x\Delta k - t\Delta f) \sin 2\pi(kx - ft)$$

at $t = 0$:

$$y = 2A \cos \pi(x\Delta k) \sin 2\pi(kx)$$

carrier



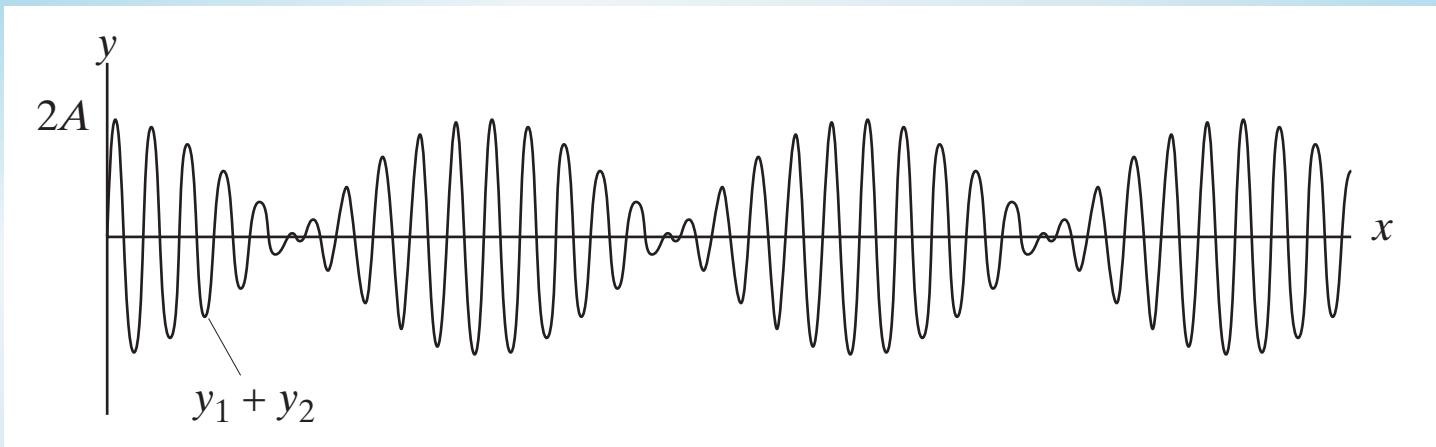
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Pulse dispersion

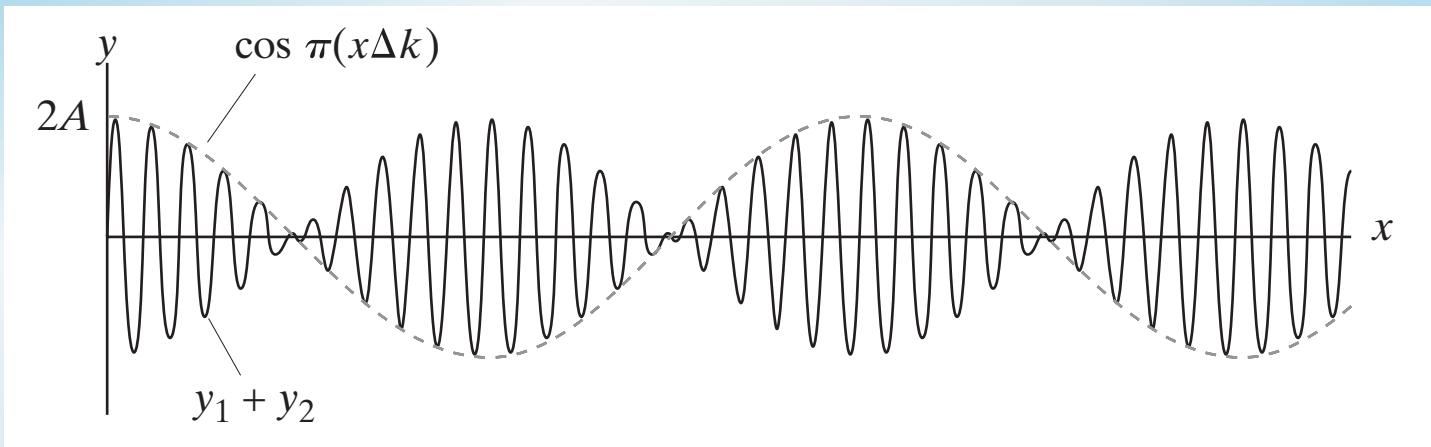
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envelope

carrier



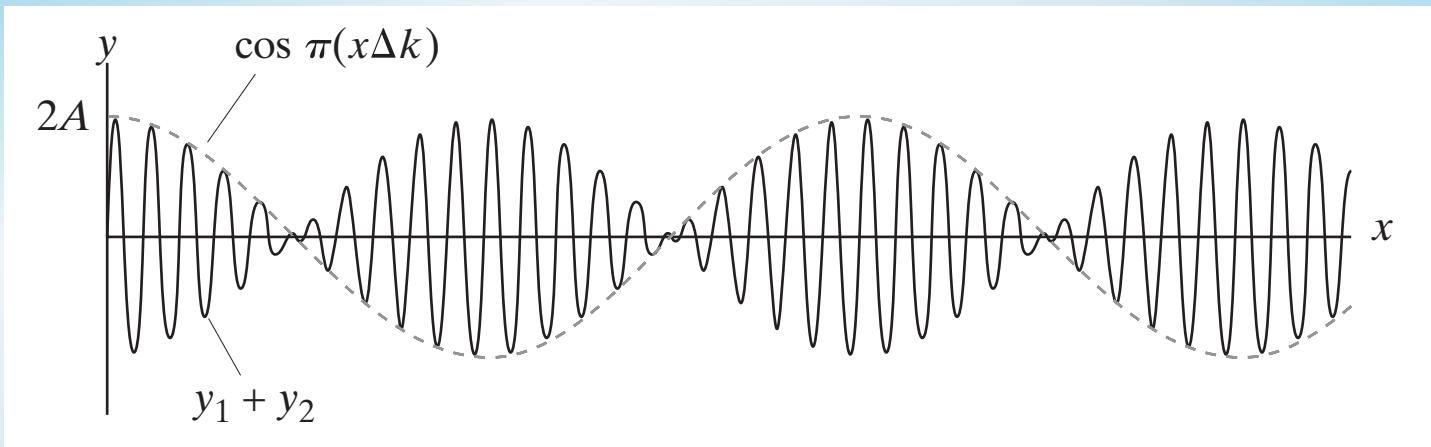
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$$y = 2A \cos \pi(x\Delta k) \sin 2\pi(kx)$$

envelope carrier



both carrier and envelope travel!

Pulse dispersion

$$y = 2A \cos \pi(x\Delta k - t\Delta f) \sin 2\pi(kx - ft)$$

speed of carrier

$$v_p = \frac{f}{k} = f\lambda$$

Pulse dispersion

$$y = 2A \cos \pi(x\Delta k - t\Delta f) \sin 2\pi(kx - ft)$$

speed of carrier

$$v_p = \frac{f}{k} = f\lambda$$

speed of envelope

$$v_g = \frac{\Delta f}{\Delta k} = \frac{df}{dk}$$

Pulse dispersion

$$y = 2A \cos \pi(x\Delta k - t\Delta f) \sin 2\pi(kx - ft)$$

speed of carrier ('phase velocity'):

$$v_p = \frac{f}{k} = f\lambda$$

speed of envelope ('group velocity'):

$$v_g = \frac{\Delta f}{\Delta k} = \frac{df}{dk}$$

Pulse dispersion

For each wave, determine the wavevector k , the frequency f , the wavelength λ , the propagation speed v :

$$a\left(\frac{x}{b} - t\right)$$

$$k_1 = \frac{8}{2\pi} \quad \text{and} \quad k_2 = \frac{7.2}{2\pi(0.95)} < k_1$$

$$2\pi(kx - ft)$$

$$f_1 = \frac{8}{2\pi} \quad \text{and} \quad f_2 = \frac{7.2}{2\pi}$$

$$k = \frac{a}{2\pi b} \quad \lambda = \frac{1}{k} = \frac{2\pi b}{a}$$

$$\lambda_1 = \frac{2\pi}{8} \quad \textcircled{B} \quad \text{and} \quad \lambda_2 = \frac{2\pi(0.95)}{7.2} > \lambda_1$$

$$f = \frac{a}{2\pi} \quad v = b$$

$$v_1 = 1.0 \quad \text{and} \quad v_2 = 0.95$$

Does the red get ahead of blue or the other way around? Why?

Pulse dispersion

What is the phase velocity of the superposition of y_1 and y_2 ?

$$v_p = \frac{\langle \omega \rangle}{\langle k \rangle} = \frac{7.6/2\pi}{7.8/2\pi} = 0.98$$

What is the group velocity of the superposition of y_1 and y_2 ?

$$v_g = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{8 - 7.2}{8 - 7.5 / 0.95} = \frac{0.8}{0.1} = 8$$

Do the crests of the carrier wave travel forward or backward through the envelope? Does your prediction agree with your observation?

Pulse dispersion

$$y = 2A \cos \pi(x\Delta k - t\Delta f) \sin 2\pi(kx - ft)$$

how does this expression change if no dispersion?

Pulse dispersion

$$y = 2A \cos \pi(x\Delta k - t\Delta f) \sin 2\pi(kx - ft)$$

if no dispersion:

$$v_p = \frac{f_1}{k_1} = \frac{f_2}{k_2}$$

Pulse dispersion

$$y = 2A \cos \pi(x\Delta k - t\Delta f) \sin 2\pi(kx - ft)$$

if no dispersion:

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group velocity:

$$v_g = \frac{\Delta f}{\Delta k} = \frac{f_1 - f_2}{k_1 - k_2} = \frac{f_1/k_1 - f_2/k_1}{1 - k_2/k_1} = \frac{v_p - f_2/k_1}{1 - k_2/k_1}$$

Pulse dispersion

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$$v_g = \frac{v_p - f_2/k_2(k_2/k_1)}{1 - k_2/k_1} = v_p \frac{1 - k_2/k_1}{1 - k_2/k_1} = v_p$$

Pulse dispersion

$$y = 2A \cos \pi(x\Delta k - t\Delta f) \sin 2\pi(kx - ft)$$

if no dispersion:

$$v_p = \frac{f_1}{k_1} = \frac{f_2}{k_2}$$

group and phase velocities are the same:

$$v_g = v_p$$

and so the envelope and carrier wave travel together

Pulse dispersion

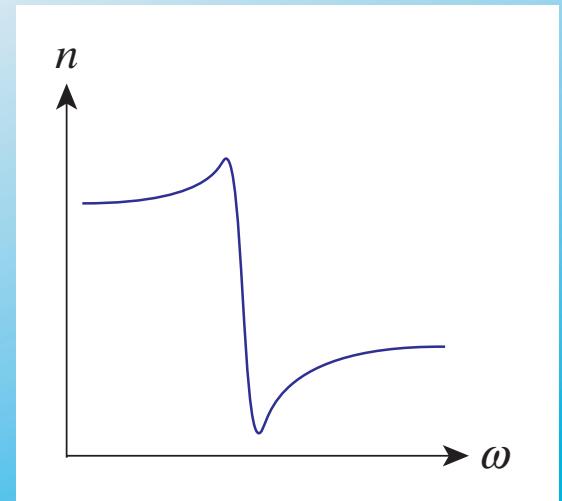
$$y = 2A \cos \pi(x\Delta k - t\Delta f) \sin 2\pi(kx - ft)$$

types of dispersion:

$$\frac{dn}{d\omega} > 0 \text{ (**normal dispersion**)} \quad v_g < v_p$$

$$\frac{dn}{d\omega} = 0 \text{ (**no dispersion**)} \quad v_g = v_p$$

$$\frac{dn}{d\omega} < 0 \text{ (**anomalous dispersion**)} \quad v_g > v_p$$



Pulse dispersion

consider a traveling Gaussian pulse:

$$y(t) = \exp\left[-\frac{(x-v_g t)^2}{2\sigma_t^2}\right] \sin 2\pi(kx - ft)$$

Pulse dispersion

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Pulse dispersion

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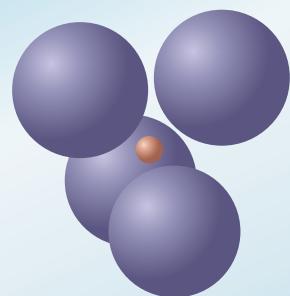
$$v_p = \frac{f}{k}$$

Gaussian envelope travels at group velocity v_g .

Nonlinear optics

Linear response

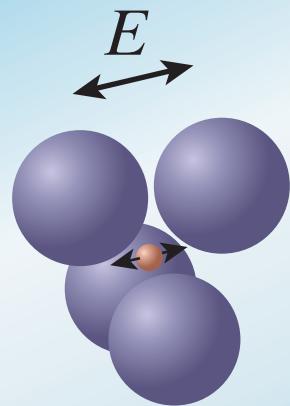
$$P(t) = \epsilon_0 \chi_e E(t)$$



Nonlinear optics

Linear response

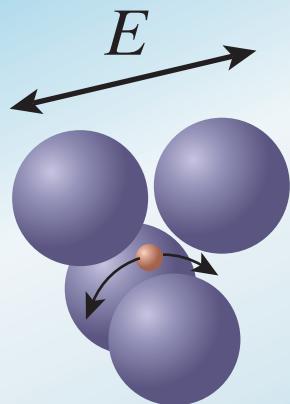
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Nonlinear optics

Linear response

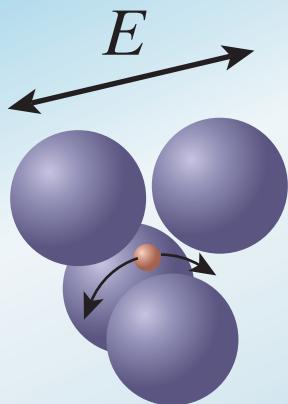
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Nonlinear optics

Linear response

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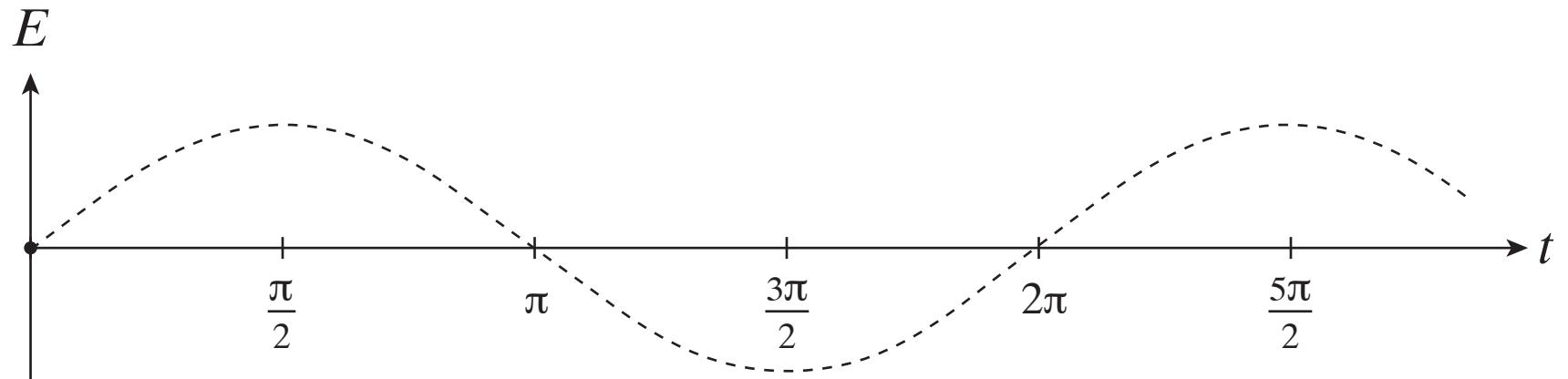


Nonlinear polarization:

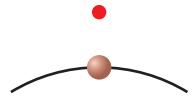
$$P = \chi^{(1)}E + \chi^{(2)}E^2 + \chi^{(3)}E^3 + \dots$$

$$P = P^{(1)} + P^{(2)} + P^{(3)} + \dots$$

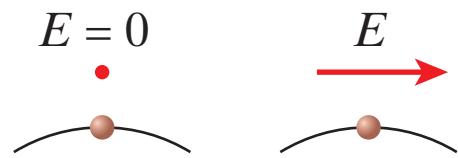
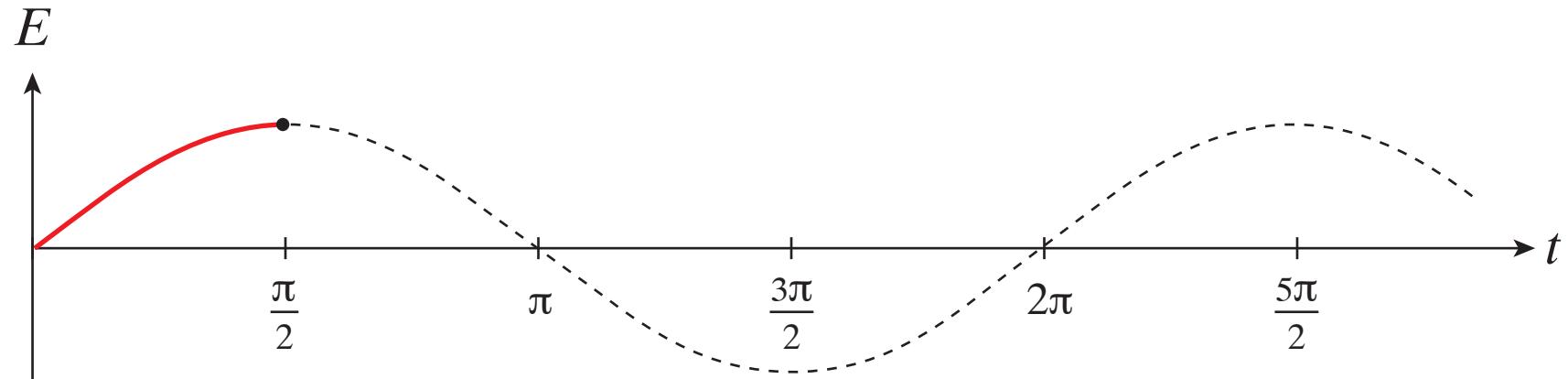
Nonlinear optics



$$E = 0$$

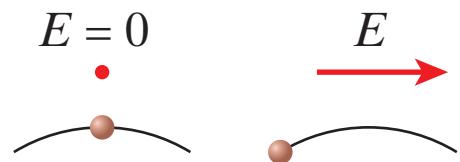
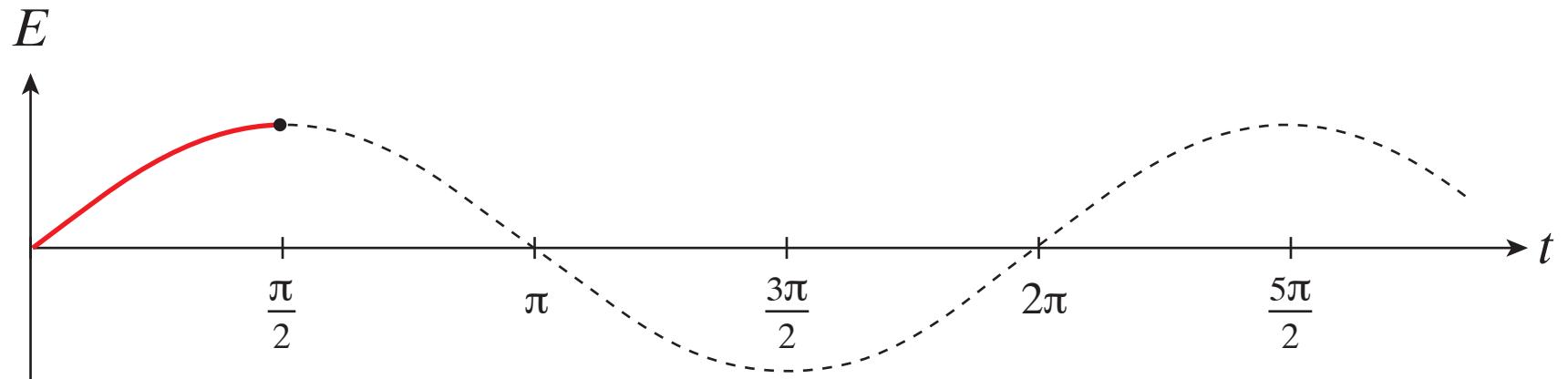


Nonlinear optics

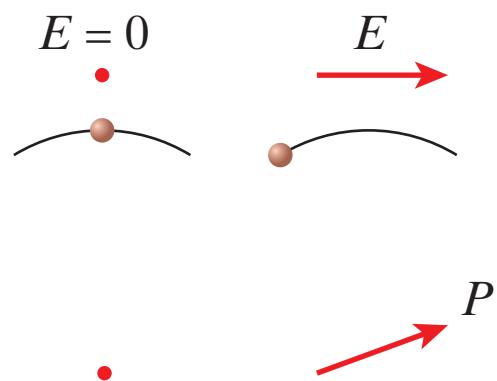
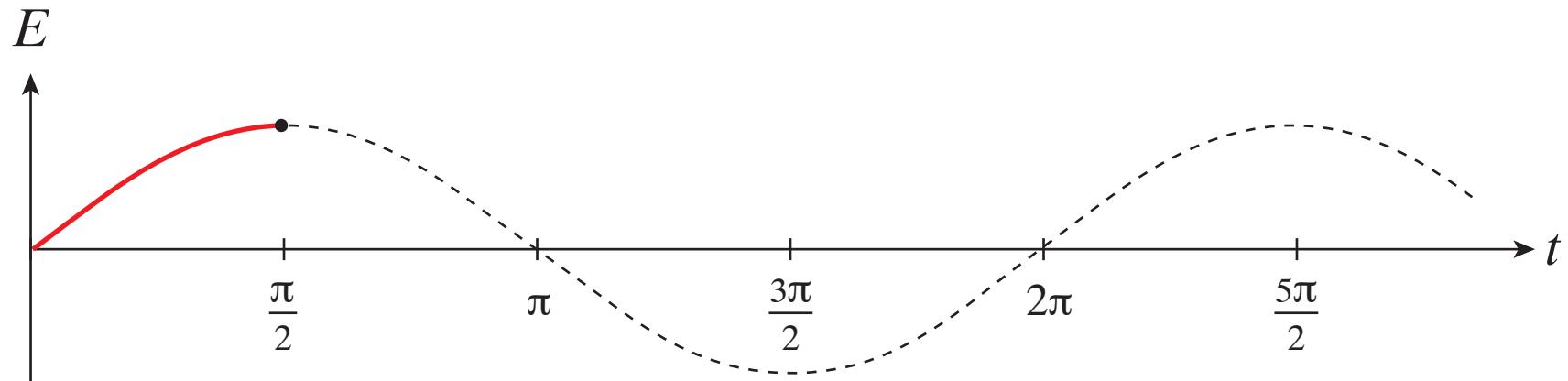


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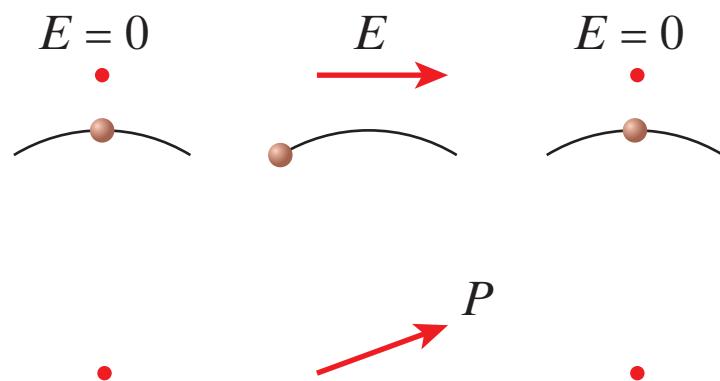
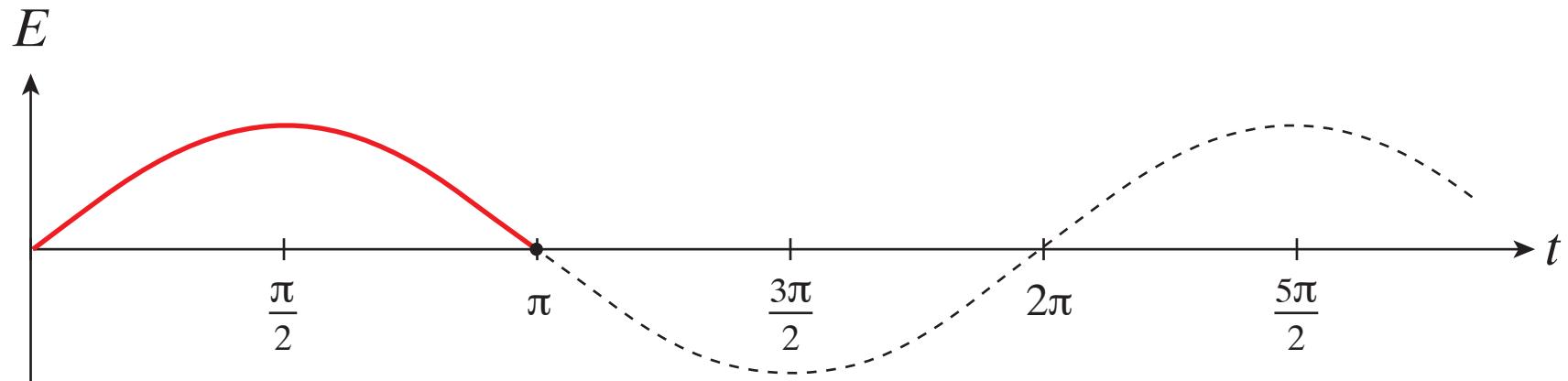
Nonlinear optics



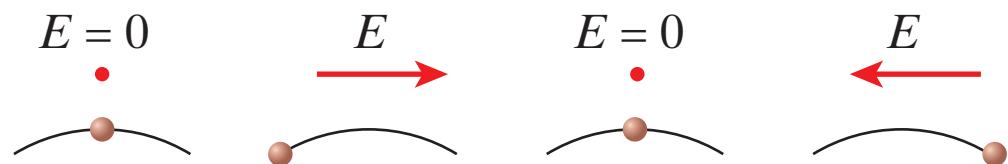
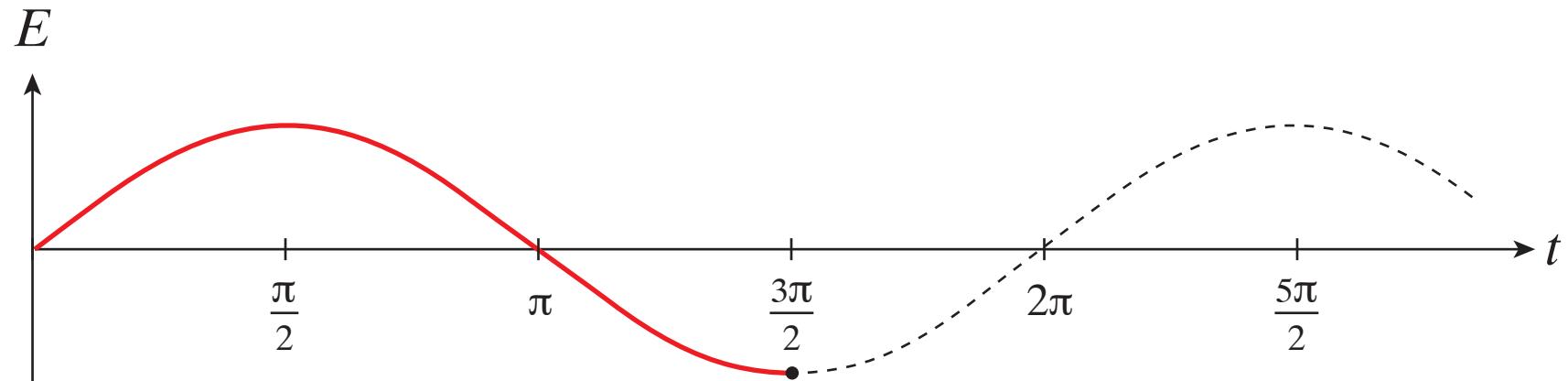
Nonlinear optics



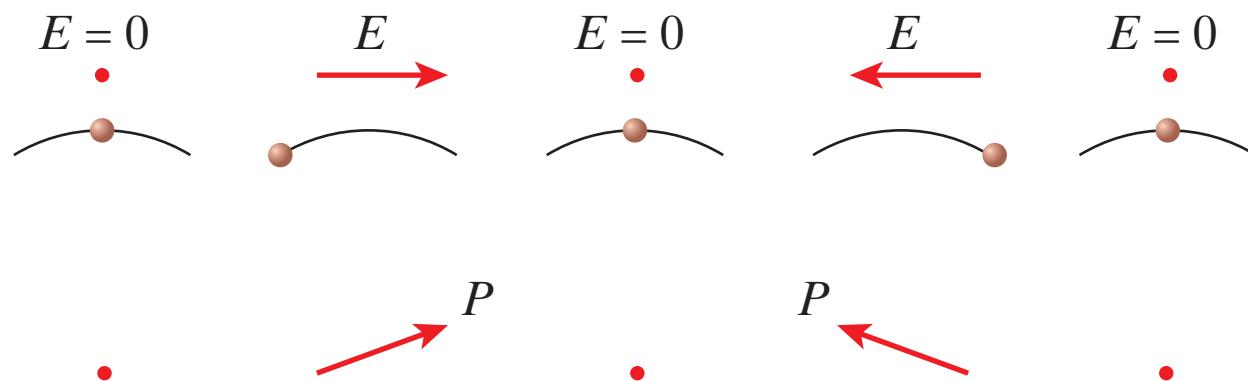
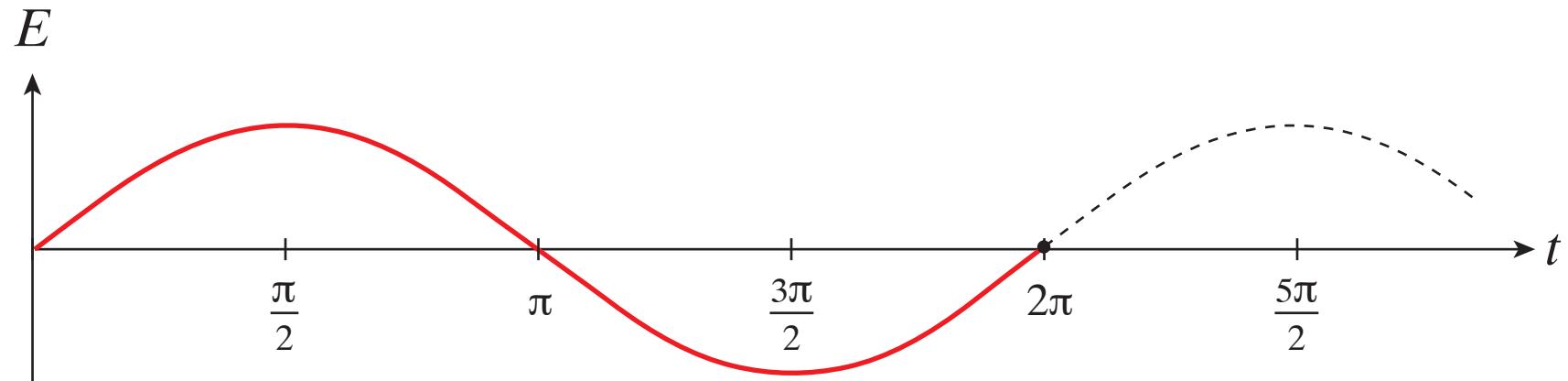
Nonlinear optics



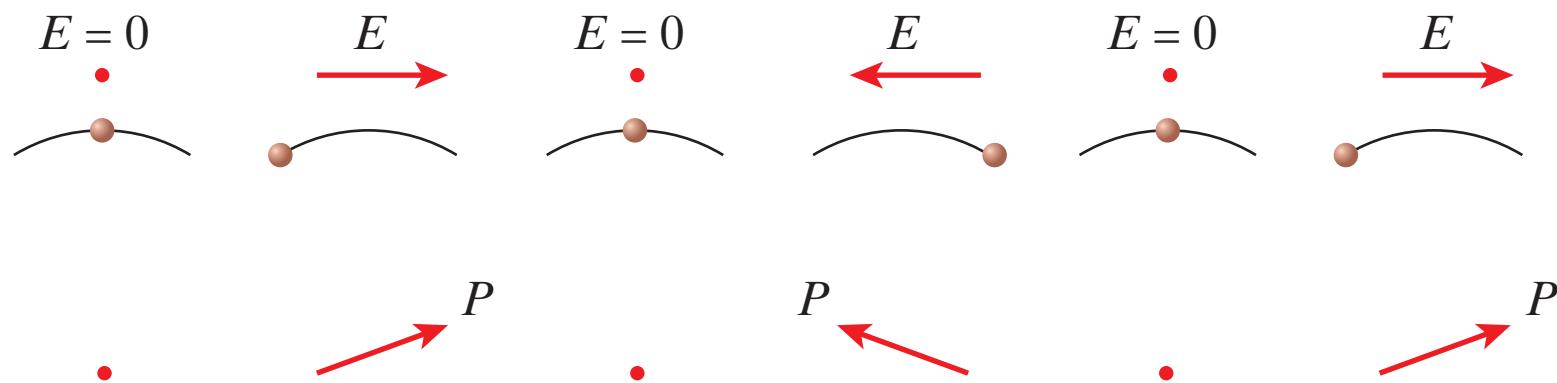
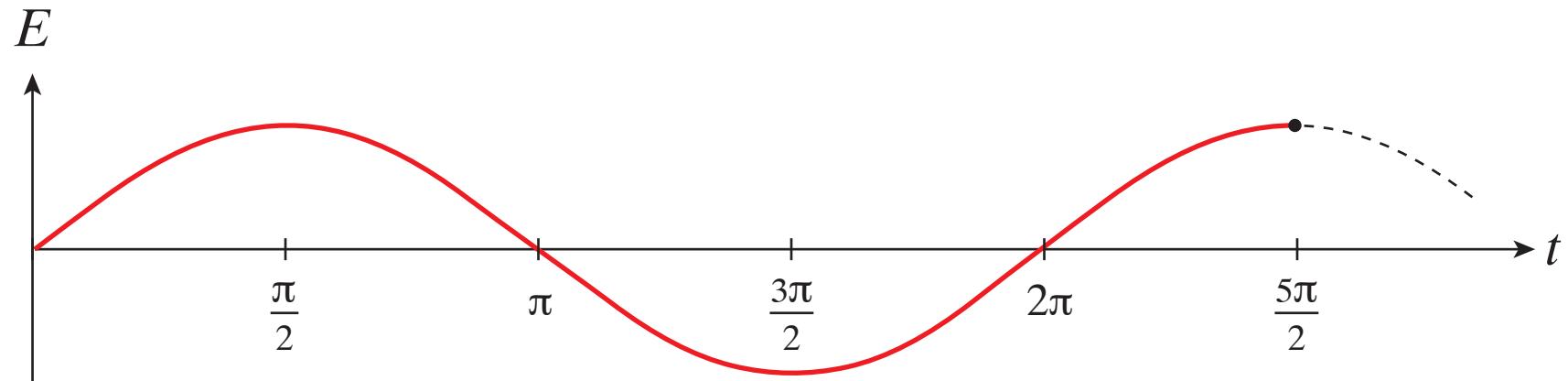
Nonlinear optics



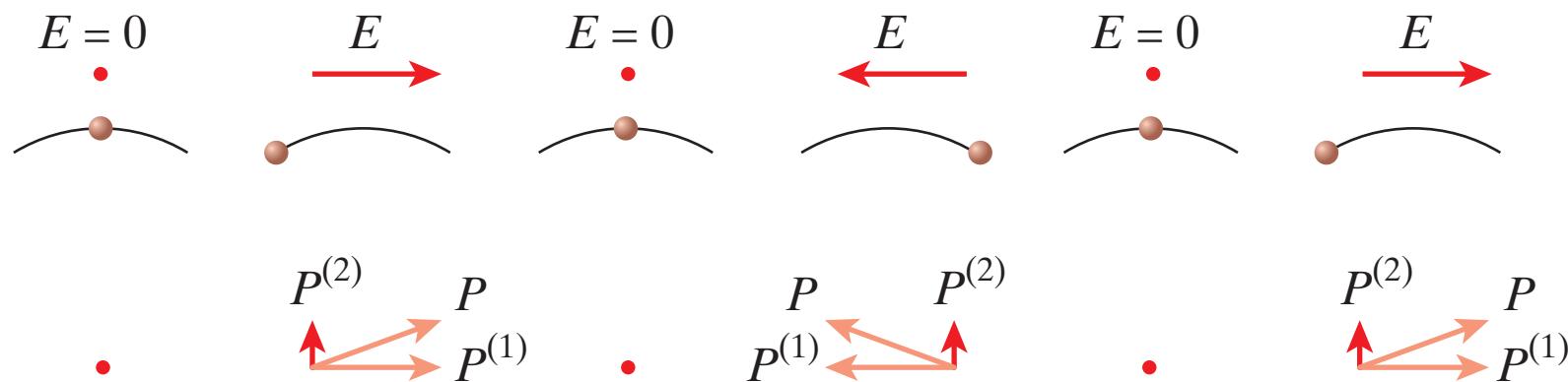
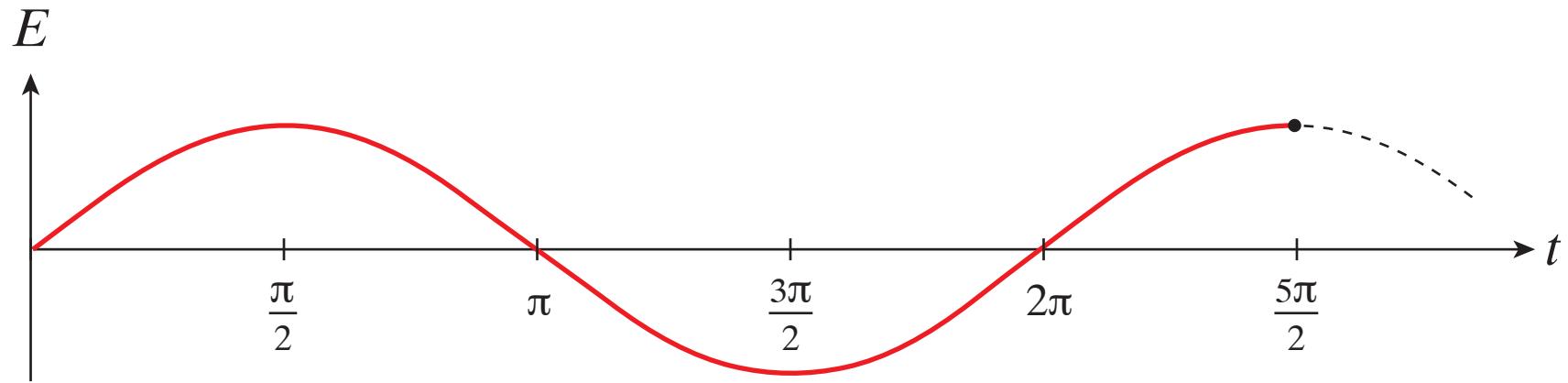
Nonlinear optics



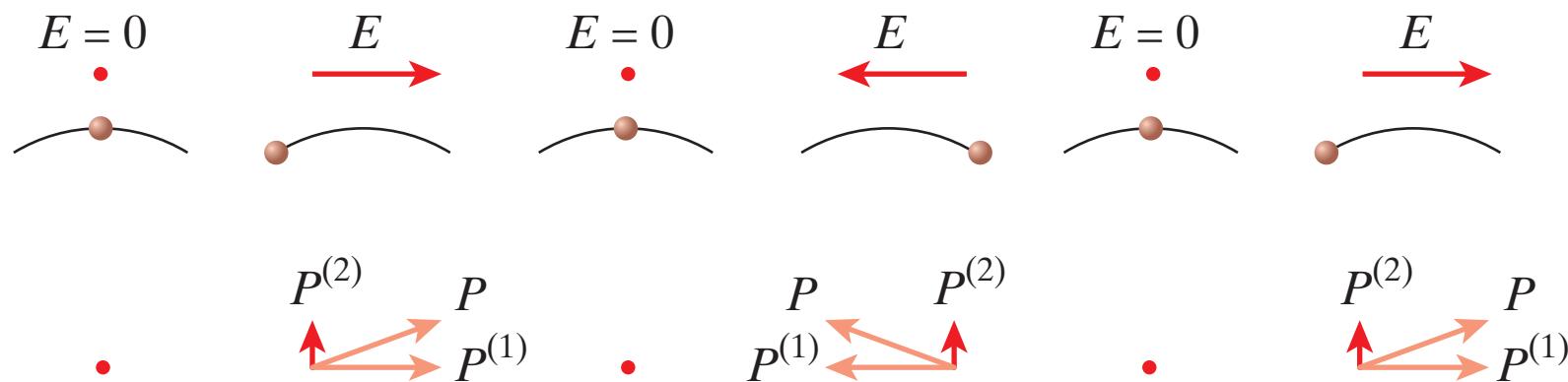
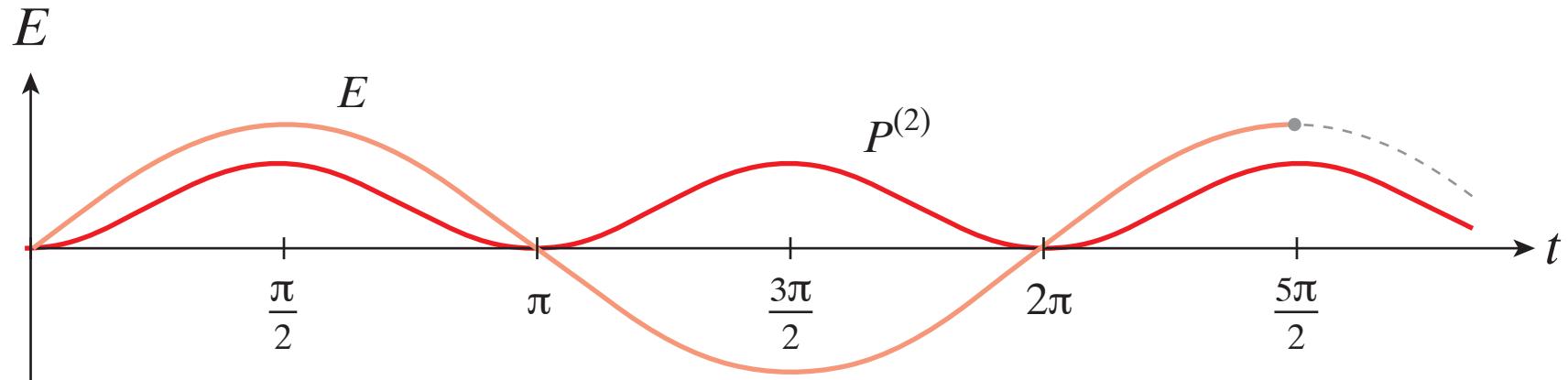
Nonlinear optics



Nonlinear optics



Nonlinear optics



Nonlinear optics

In medium with inversion symmetry

$$\vec{P}^{(2)} = \vec{\chi}^{(2)} : \vec{E} \vec{E} \quad \Rightarrow \quad -\vec{P}^{(2)} = \vec{\chi}^{(2)} : (-\vec{E})(-\vec{E})$$

Nonlinear optics

In medium with inversion symmetry

$$\vec{P}^{(2)} = \vec{\chi}^{(2)} : \vec{E} \vec{E} \quad \Rightarrow \quad -\vec{P}^{(2)} = \vec{\chi}^{(2)} : (-\vec{E})(-\vec{E})$$

and so

$$\chi^{(2)} = -\chi^{(2)} = 0$$

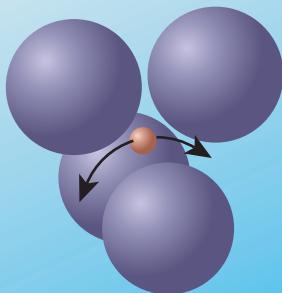
Nonlinear optics

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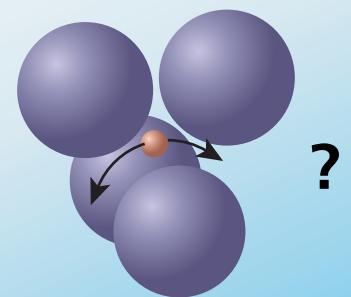
$$\chi^{(2)} = -\chi^{(2)} = 0$$



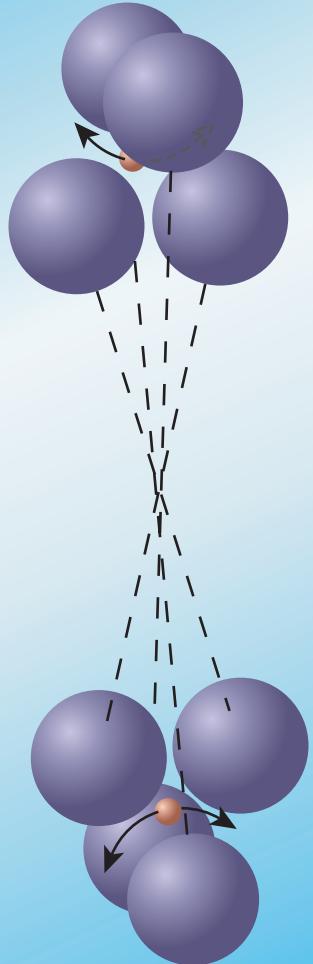
... but ...

Nonlinear optics

How to reconcile $\chi^{(2)} = -\chi^{(2)} = 0$ with



Nonlinear optics



Nonlinear optics

Nonlinear polarization:

$$P = \chi^{(1)}E + \chi^{(3)}E^3 + \dots$$

Nonlinear optics

Nonlinear polarization:

$$P = \chi^{(1)}E + \chi^{(3)}E^3 + \dots$$

Third order polarization

$$P^{(3)}(t) = \chi^{(3)}E(t)E^*(t)E(t) = \chi^{(3)}I(t)E(t)$$

Nonlinear optics

Nonlinear polarization:

$$P = \chi^{(1)}E + \chi^{(3)}E^3 + \dots$$

Third order polarization

$$P^{(3)}(t) = \chi^{(3)}E(t)E^*(t)E(t) = \chi^{(3)}I(t)E(t)$$

and so $P = P^{(1)} + P^{(3)} = (\chi^{(1)} + \chi^{(3)}I)E \equiv \chi_{eff}E$

Nonlinear polarization:

$$P = \chi^{(1)}E + \chi^{(3)}E^3 + \dots$$

Third order polarization

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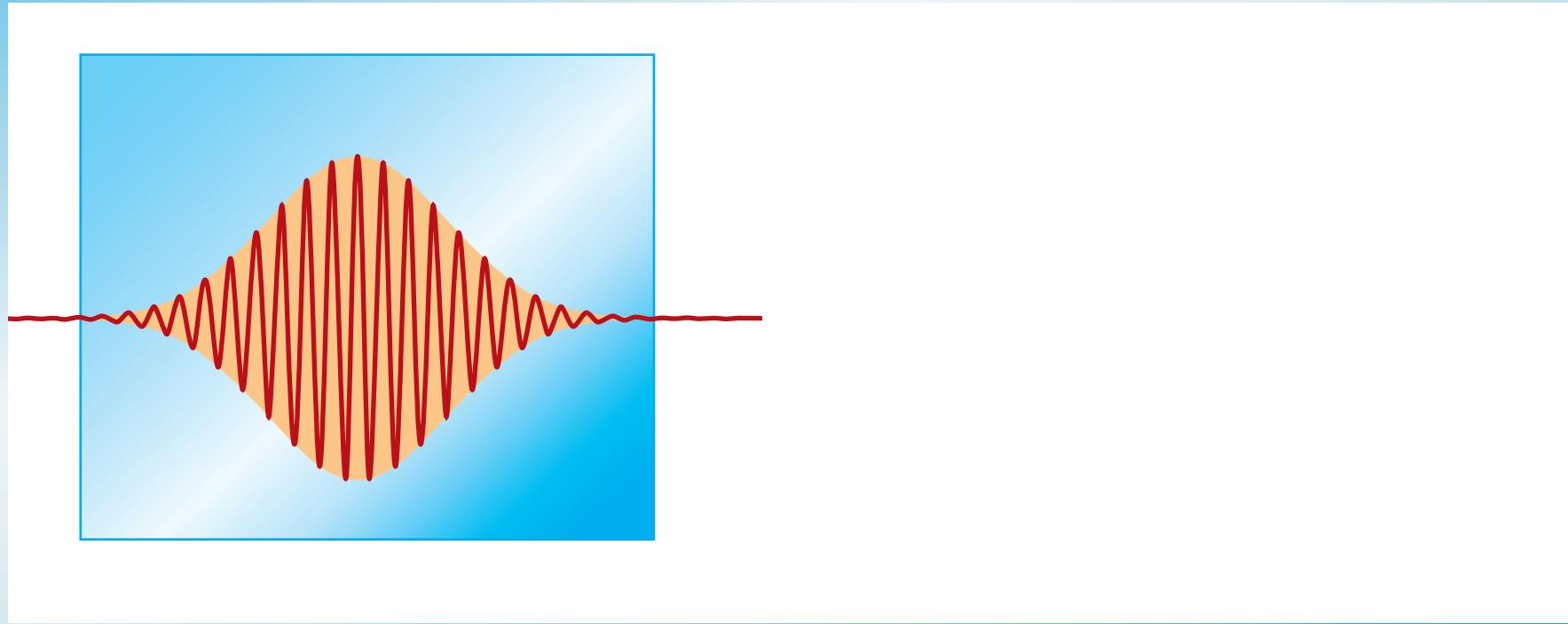
and so $P = P^{(1)} + P^{(3)} = (\chi^{(1)} + \chi^{(3)}I)E \equiv \chi_{eff}E$

$$n = \sqrt{\epsilon} = \sqrt{1 + \chi_{eff}} \approx \sqrt{1 + \chi^{(1)}} + \frac{1}{2} \frac{\chi^{(3)}I}{\sqrt{1 + \chi^{(1)}}} = n_o + n_2 I$$

Nonlinear optics

Intensity dependent index of refraction:

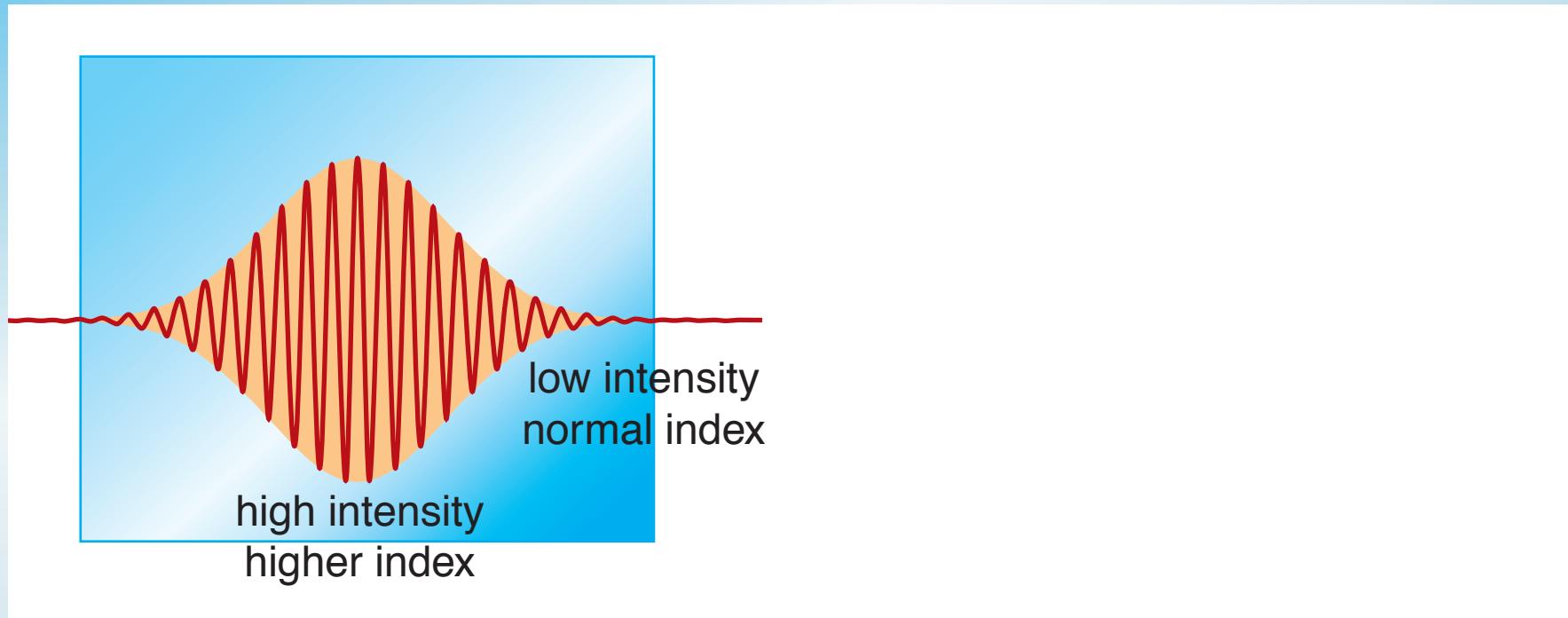
$$n = n_o + n_2 I$$



Nonlinear optics

Intensity dependent index of refraction:

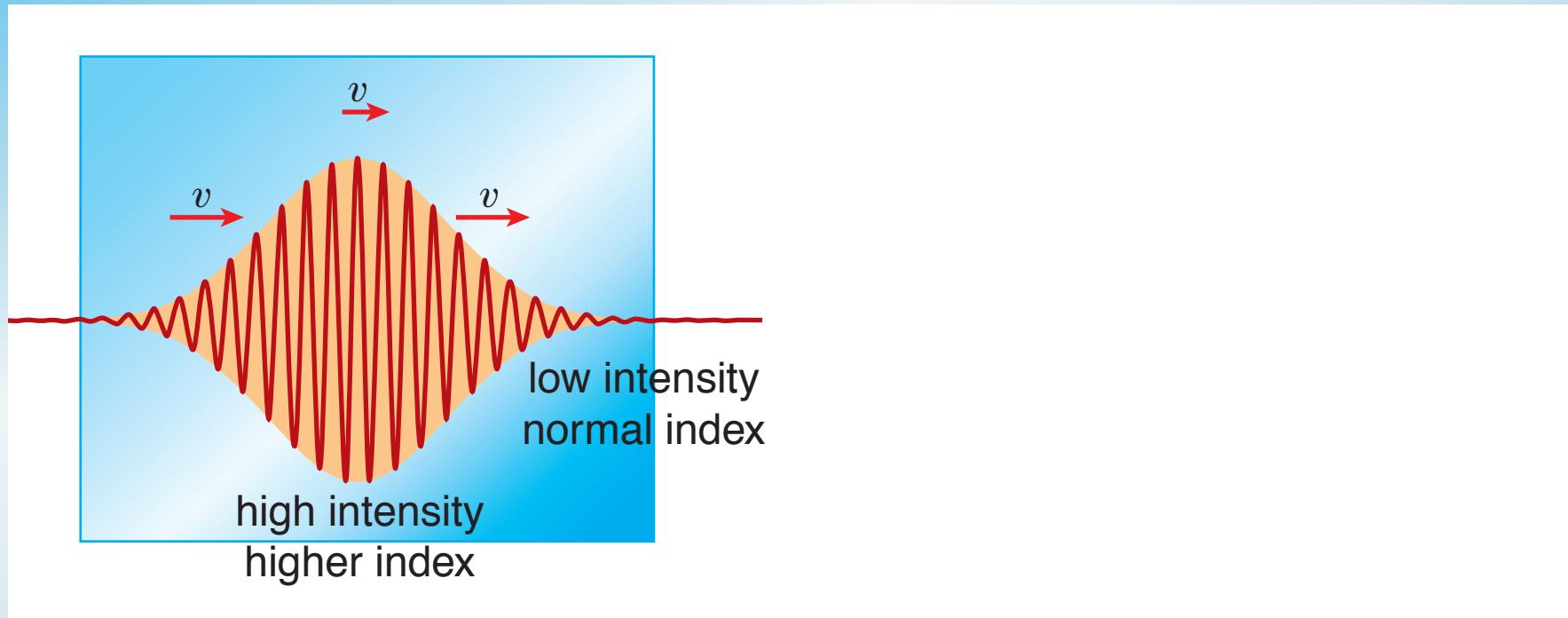
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Nonlinear optics

Intensity dependent index of refraction:

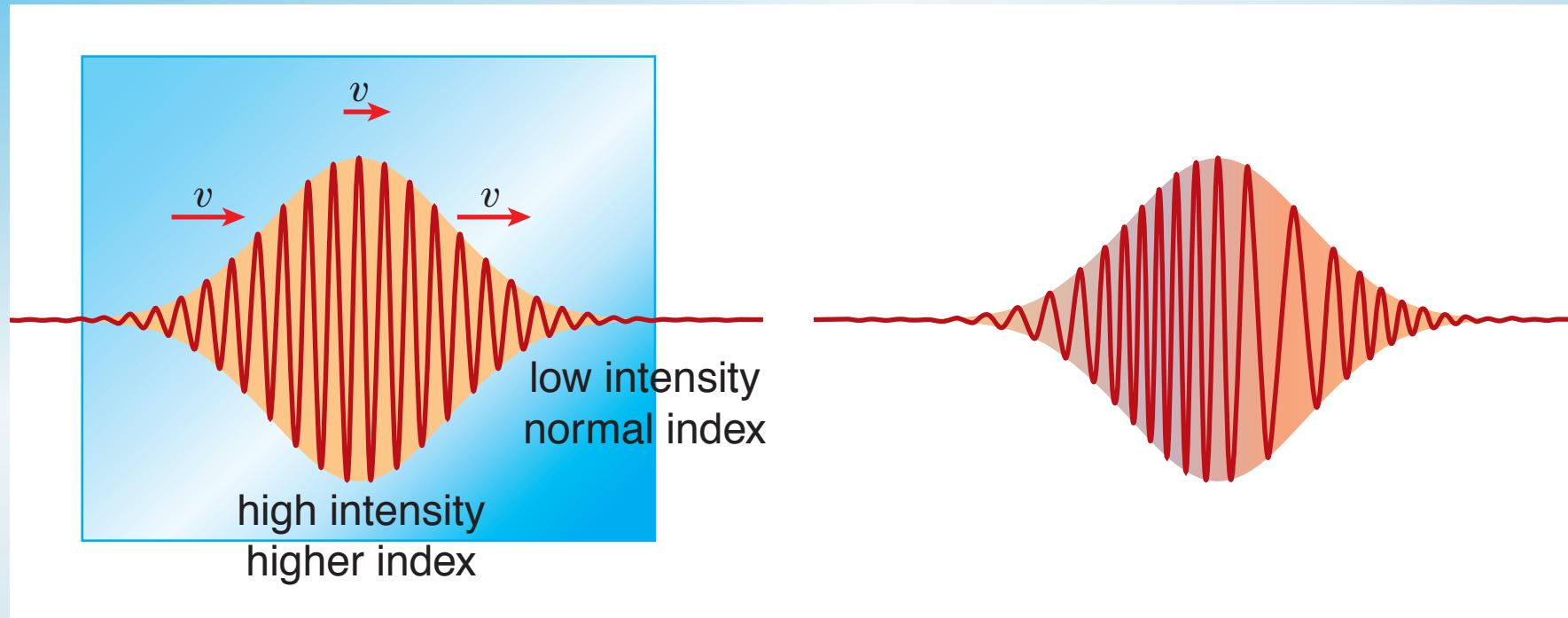
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Nonlinear optics

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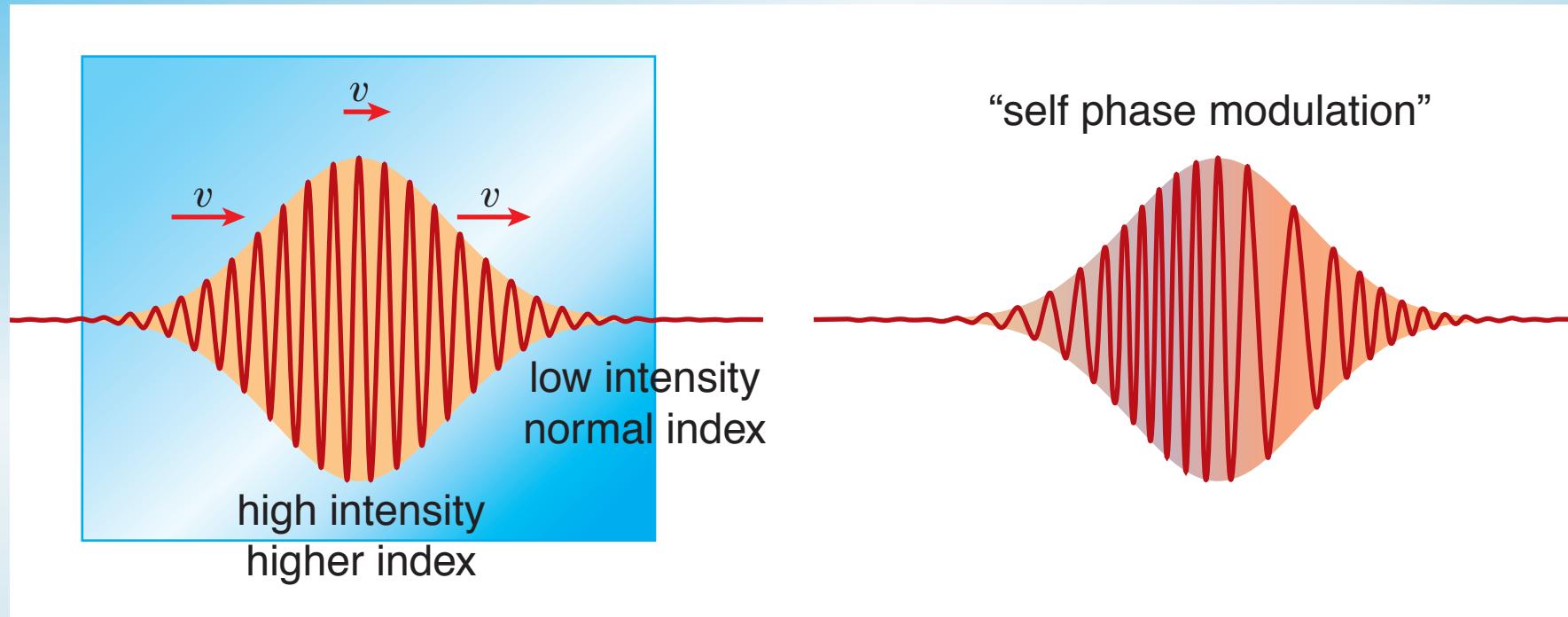
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Nonlinear optics

Intensity dependent index of refraction:

$$n = n_o + n_2 I$$



Nonlinear optics

Phase:

$$\frac{\phi}{2\pi} = \frac{nL}{\lambda}$$

Nonlinear optics

Phase:

$$\frac{\phi}{2\pi} = \frac{nL}{\lambda}$$

$$\phi = \frac{2\pi}{\lambda} L(n_o + n_2 I)$$

Nonlinear optics

Phase:

$$\frac{\phi}{2\pi} = \frac{nL}{\lambda}$$

$$\phi = \frac{2\pi}{\lambda} L(n_o + n_2 I)$$

Frequency change:

$$\Delta\omega = -\frac{d\phi}{dt} = \frac{-2\pi}{\lambda} L n_2 \frac{dI}{dt}$$

Nonlinear optics

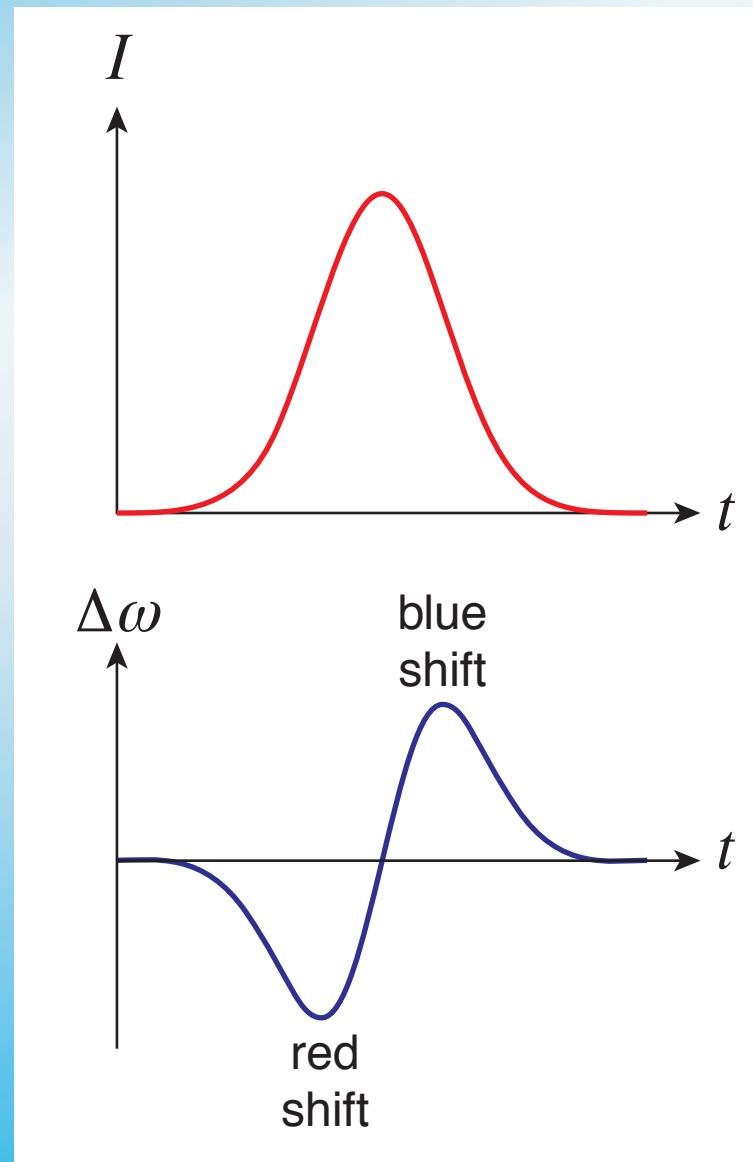
Phase:

$$\frac{\phi}{2\pi} = \frac{nL}{\lambda}$$

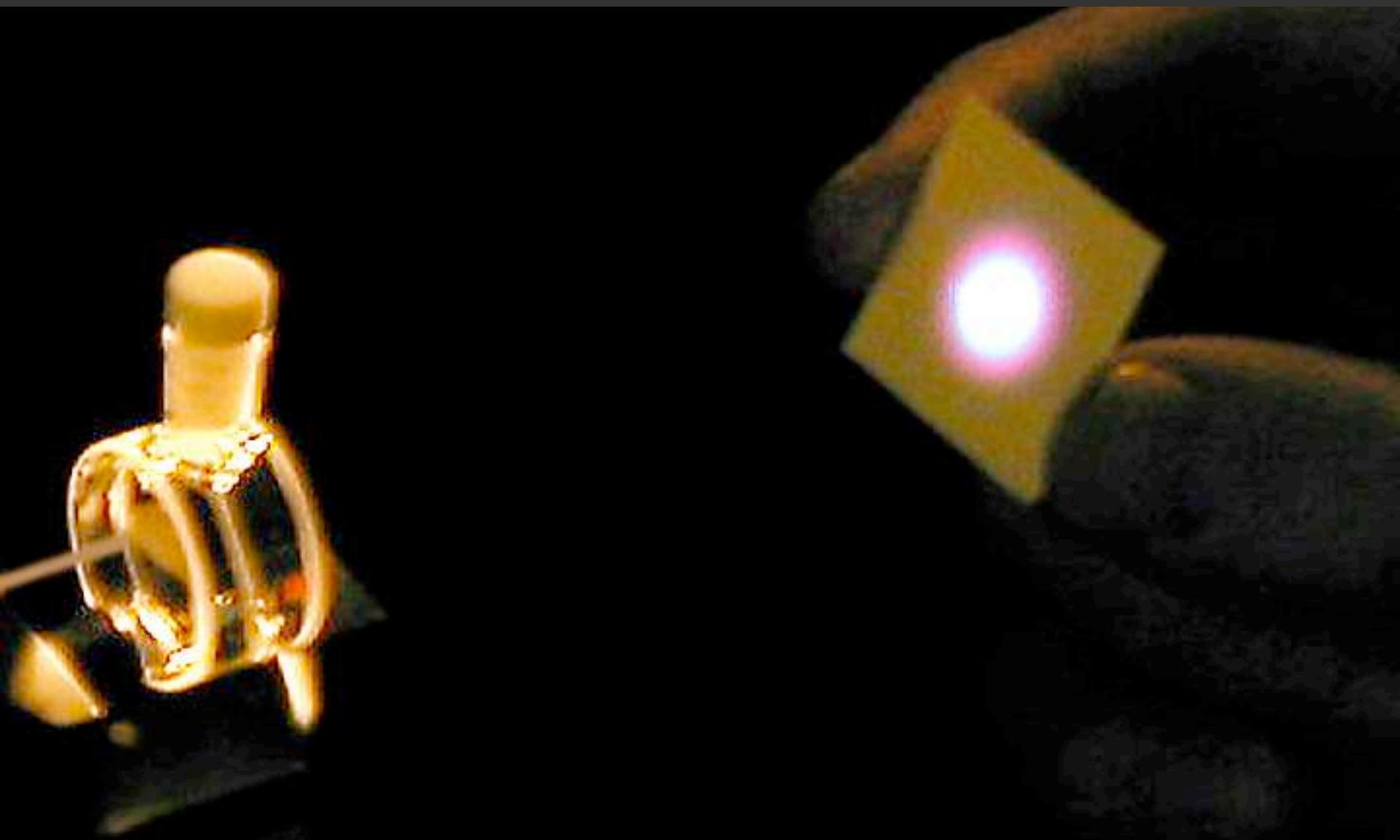
$$\phi = \frac{2\pi}{\lambda} L(n_o + n_2 I)$$

Frequency change:

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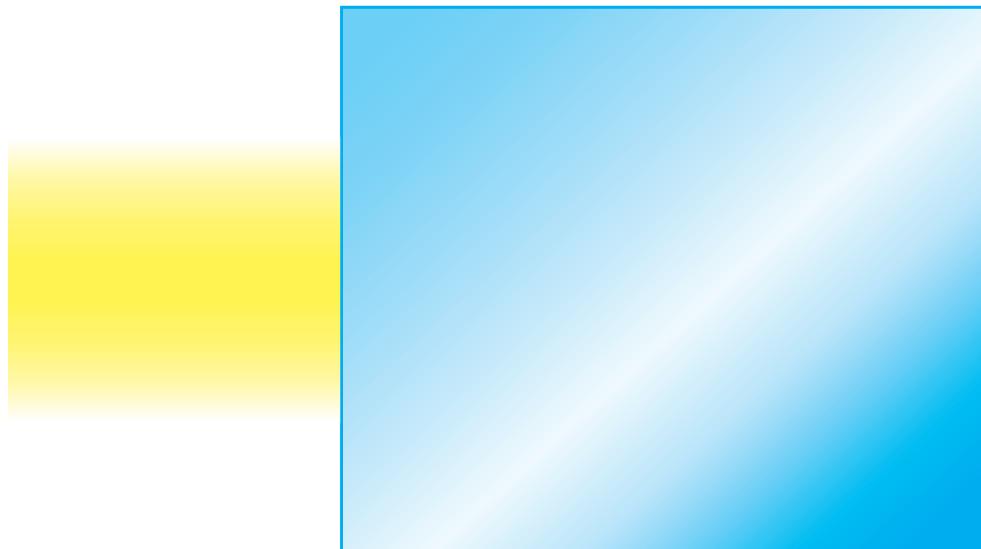


Nonlinear optics



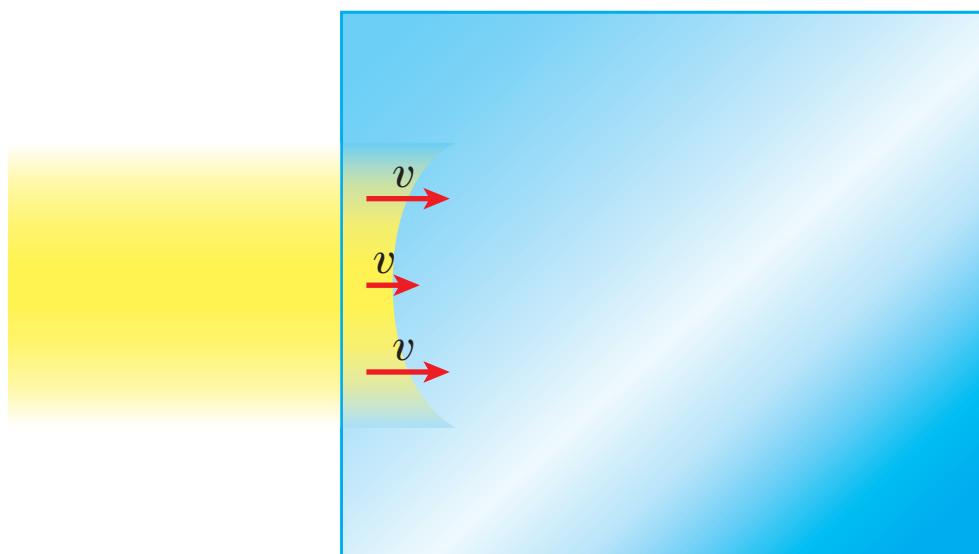
Nonlinear optics

Spatial intensity profile...



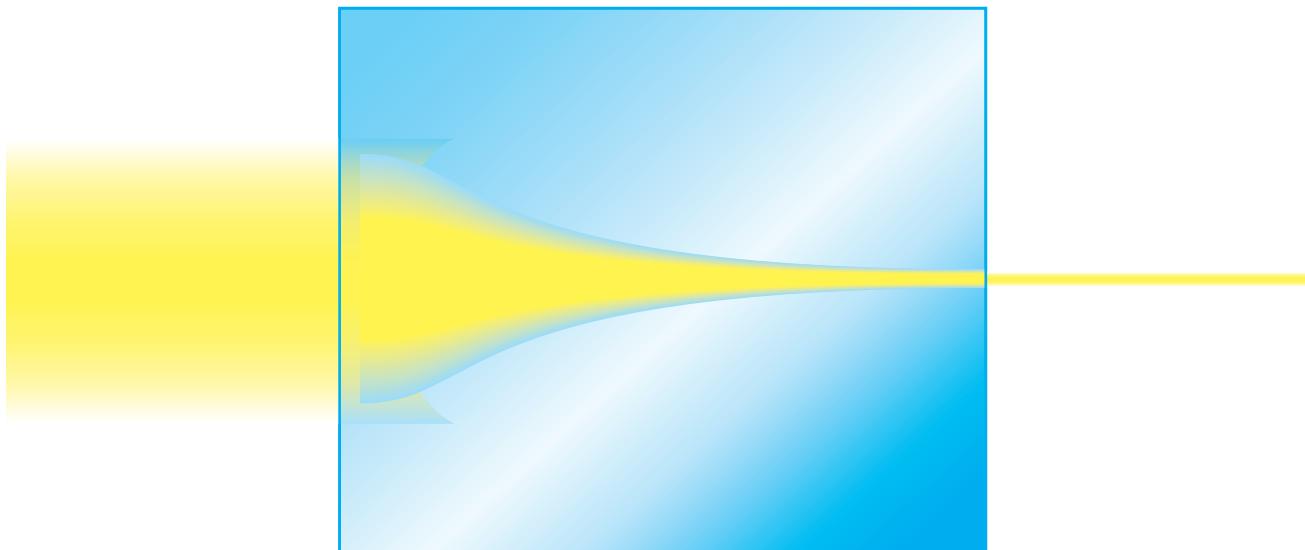
Nonlinear optics

Spatial intensity profile...

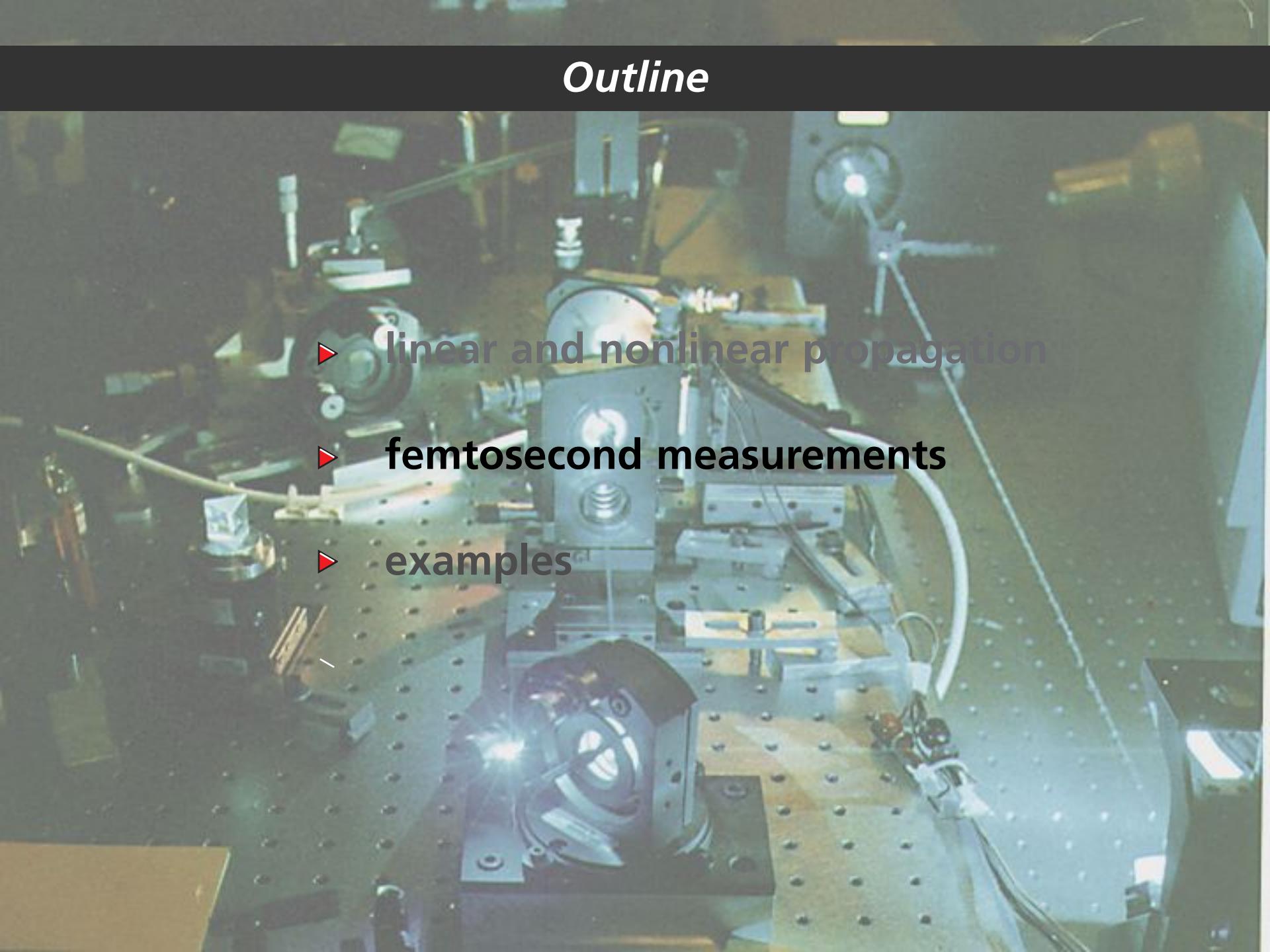


Nonlinear optics

...causes self-focusing

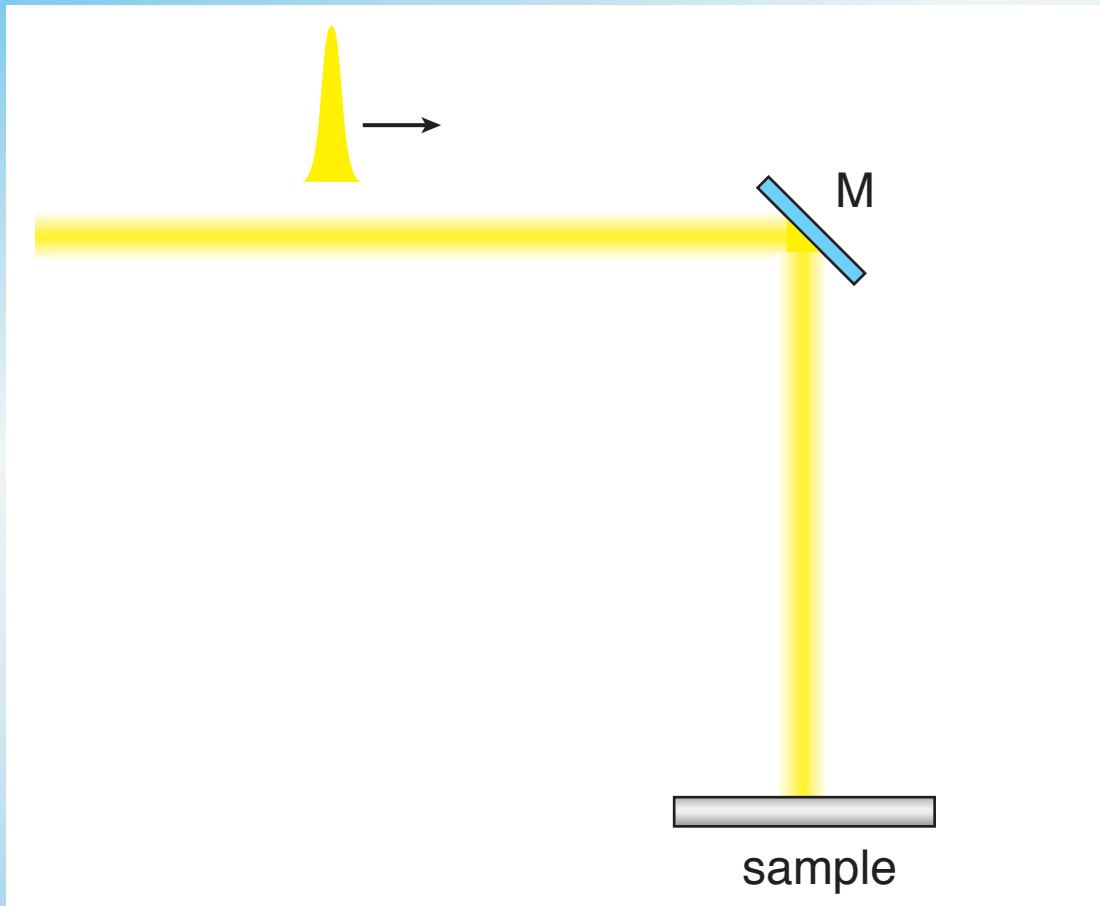


Outline

- 
- ▶ linear and nonlinear propagation
 - ▶ femtosecond measurements
 - ▶ examples

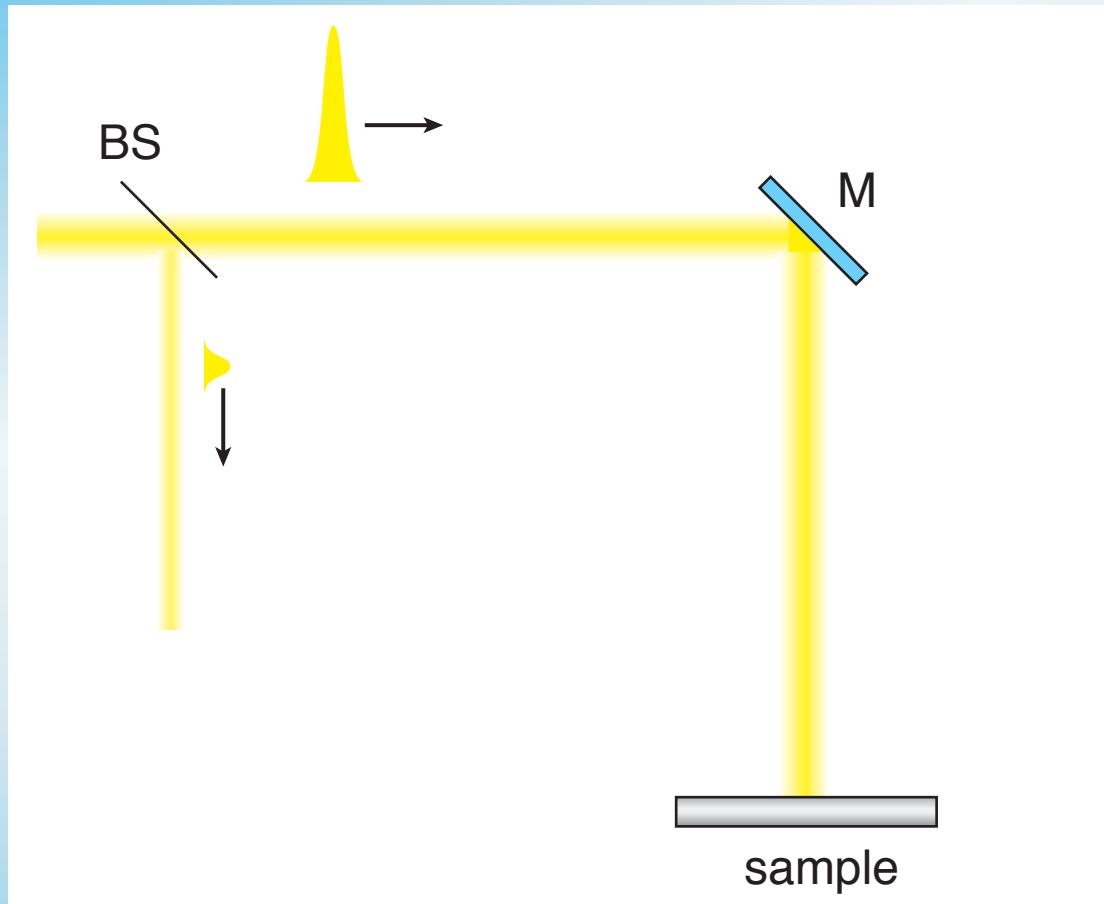
Introduction

How to measure on the femtosecond time scale?



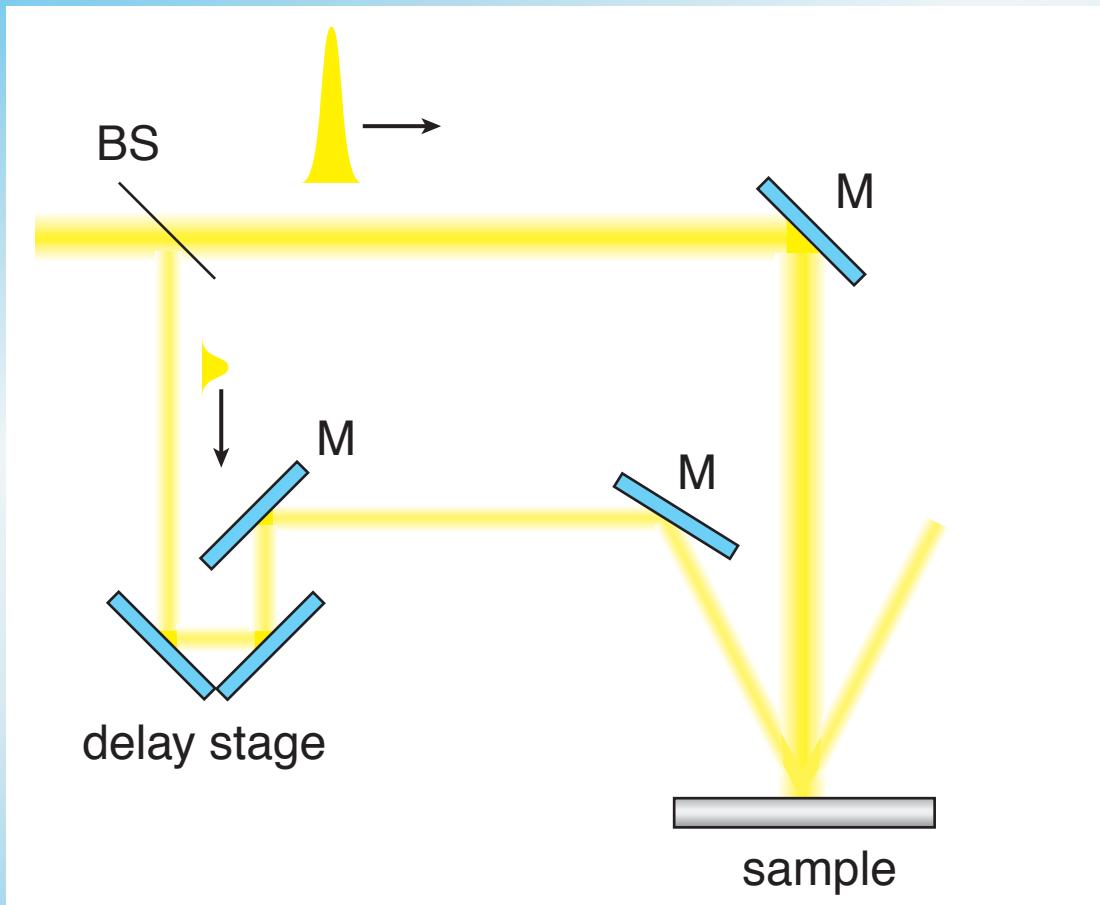
Introduction

Use pump-probe technique



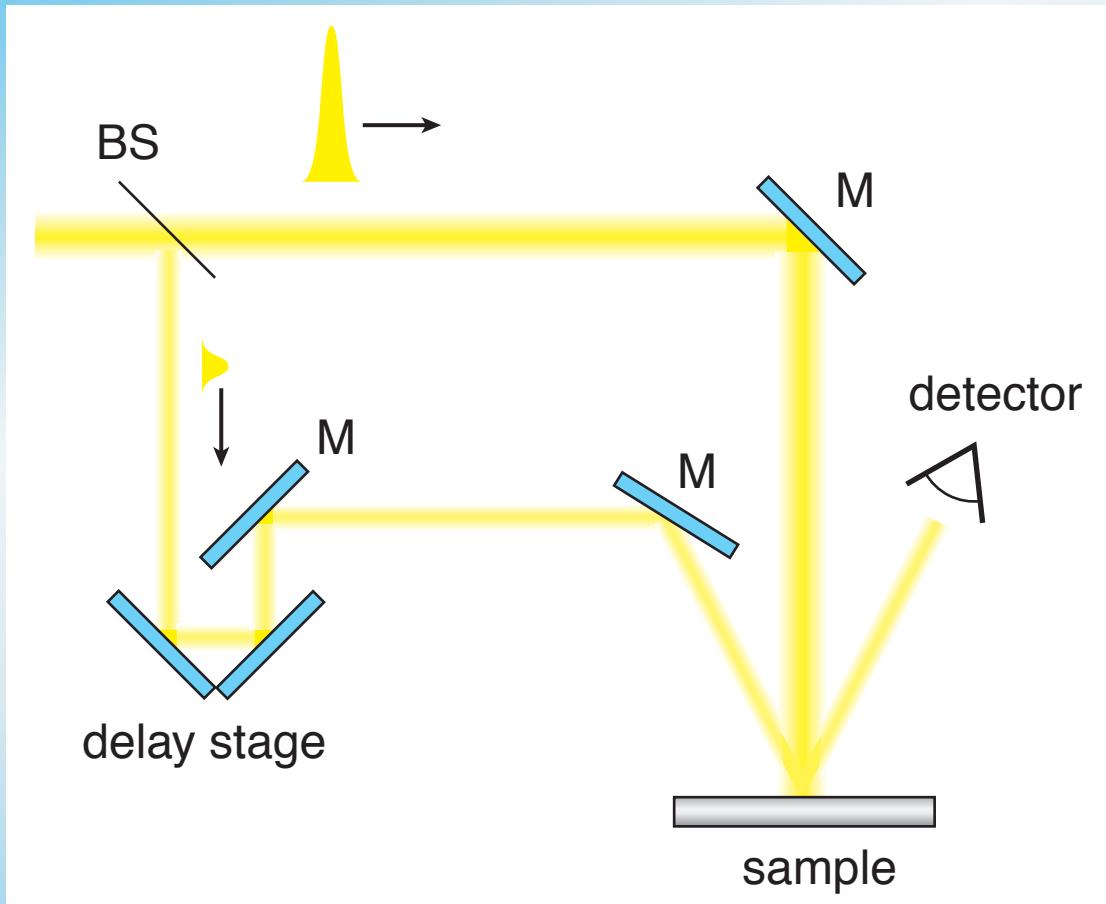
Introduction

Use pump-probe technique



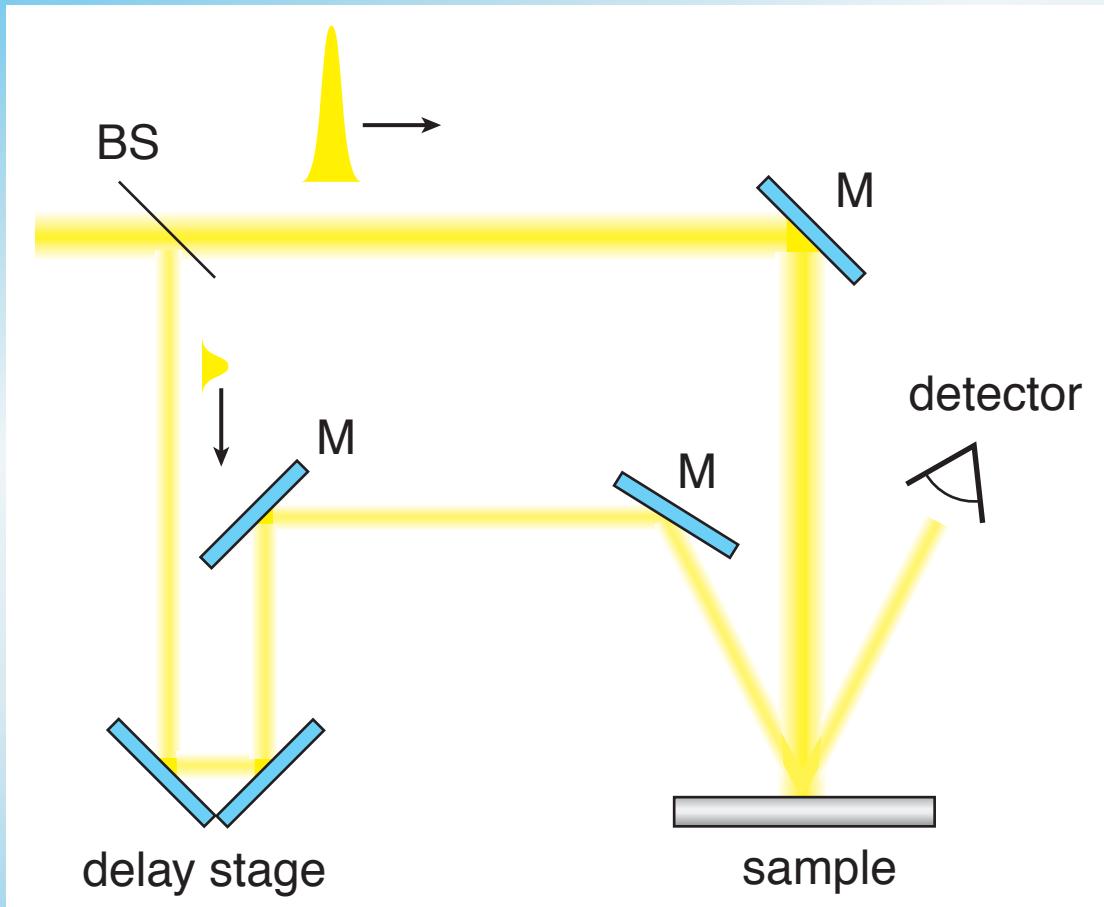
Introduction

Use pump-probe technique



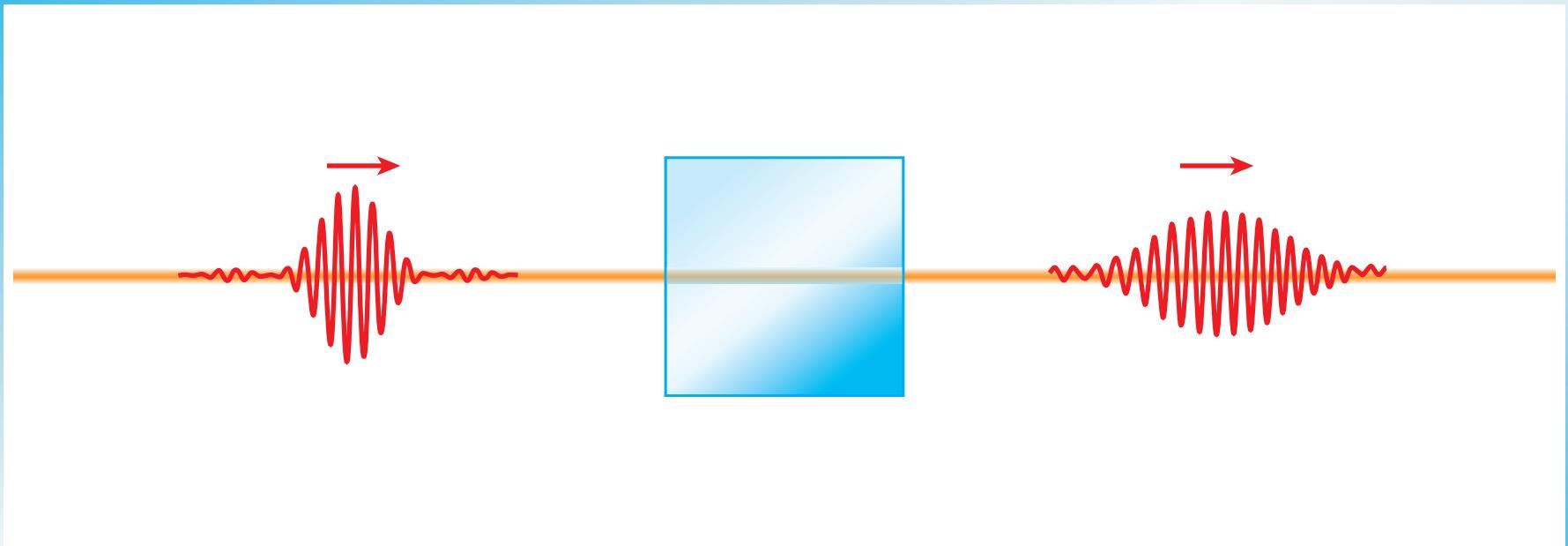
Introduction

Vary delay to get time resolution



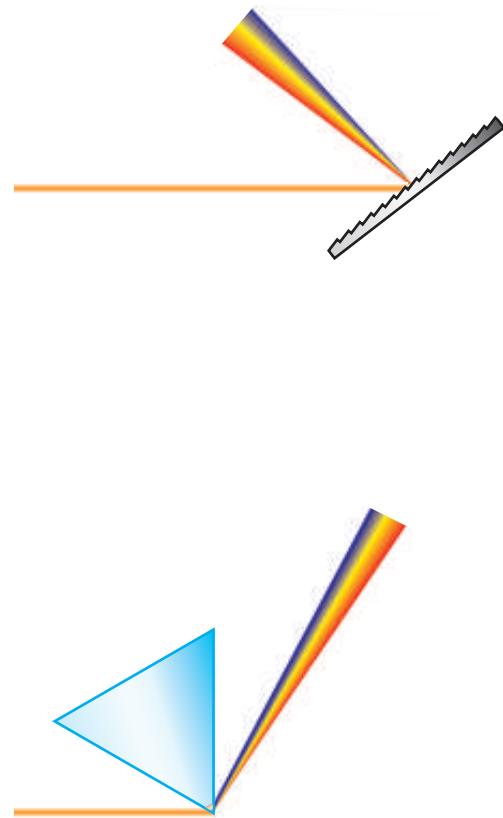
Dispersion compensation

Dispersion stretches the pulse

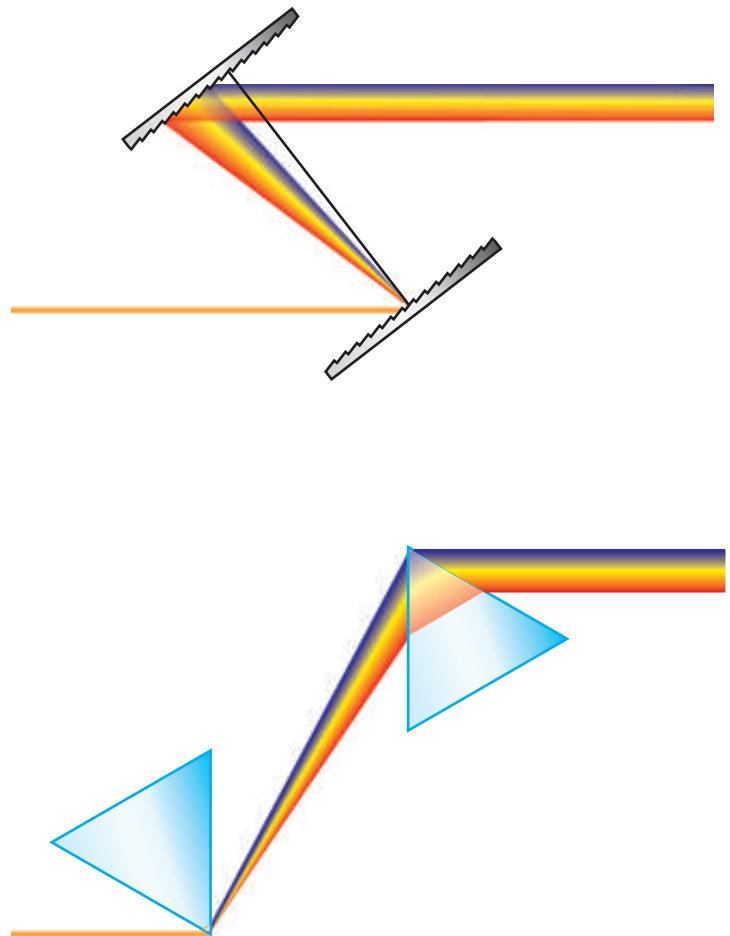


Compensate by rearranging spectral components!

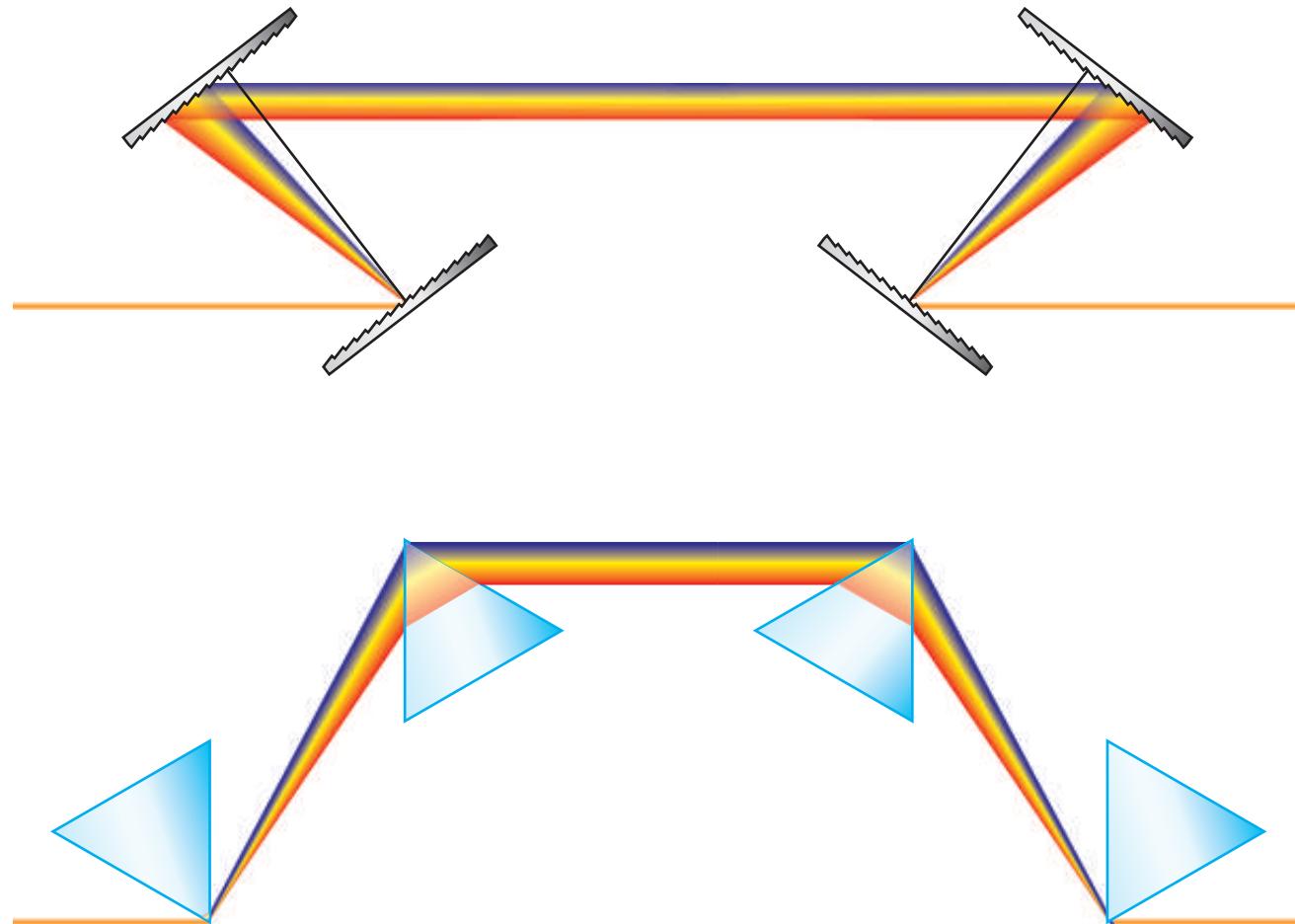
Dispersion compensation



Dispersion compensation



Dispersion compensation

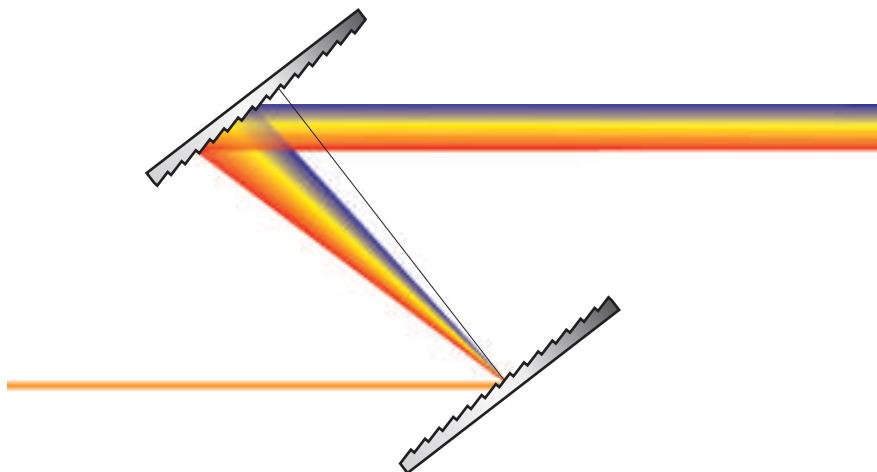


Dispersion compensation

How do these arrangements work?

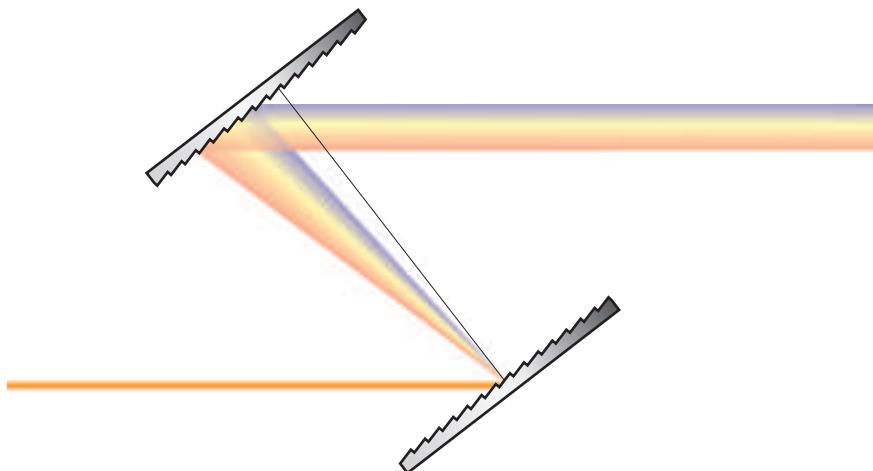
Dispersion compensation

Does path length difference compensate?



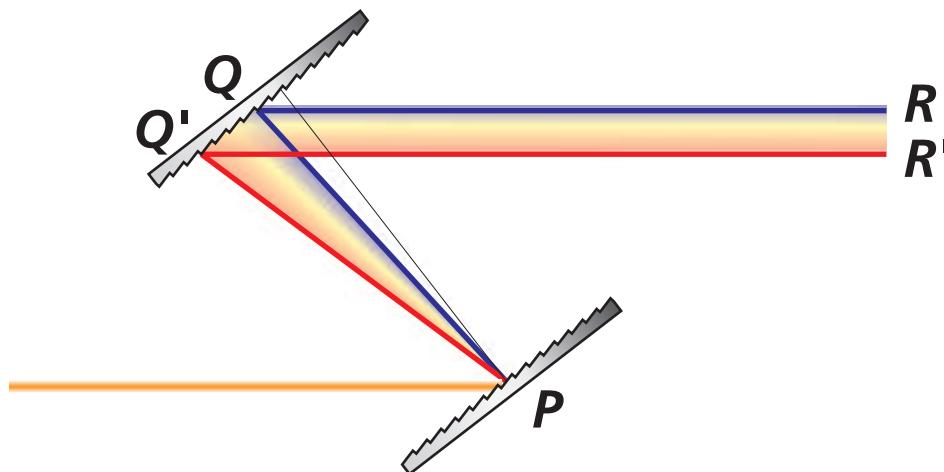
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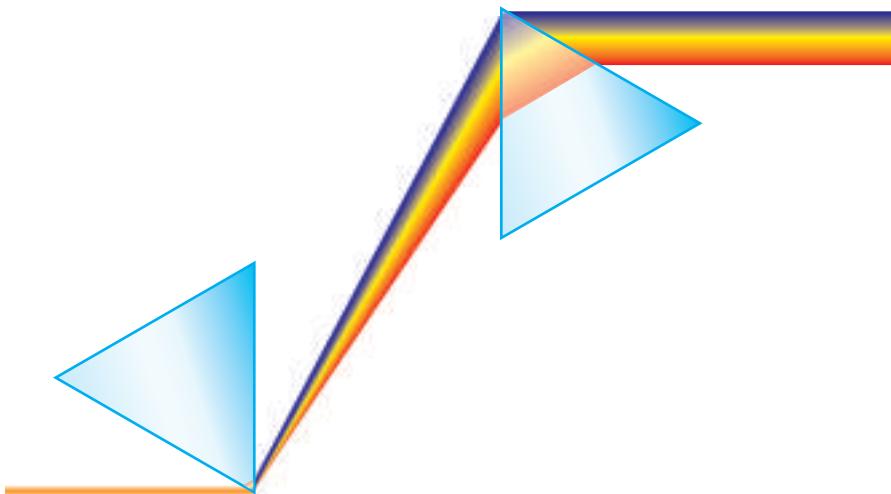
Does path length difference compensate?



Grating gives low frequency longer path length...

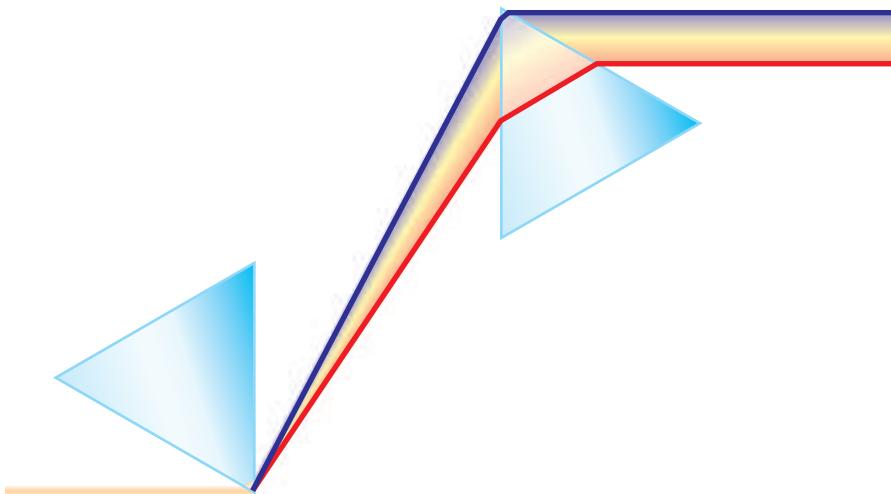
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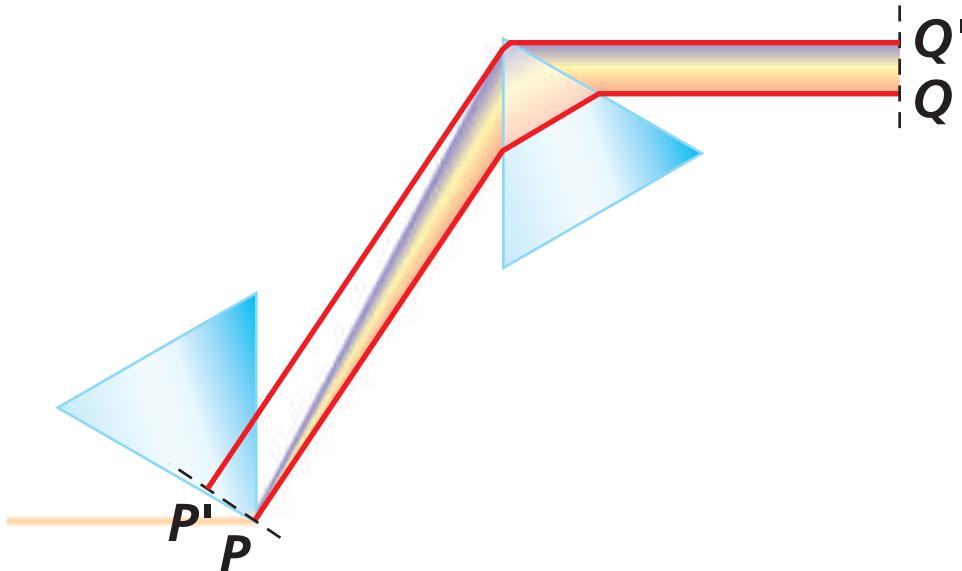
Dispersion compensation

Does path length difference compensate?



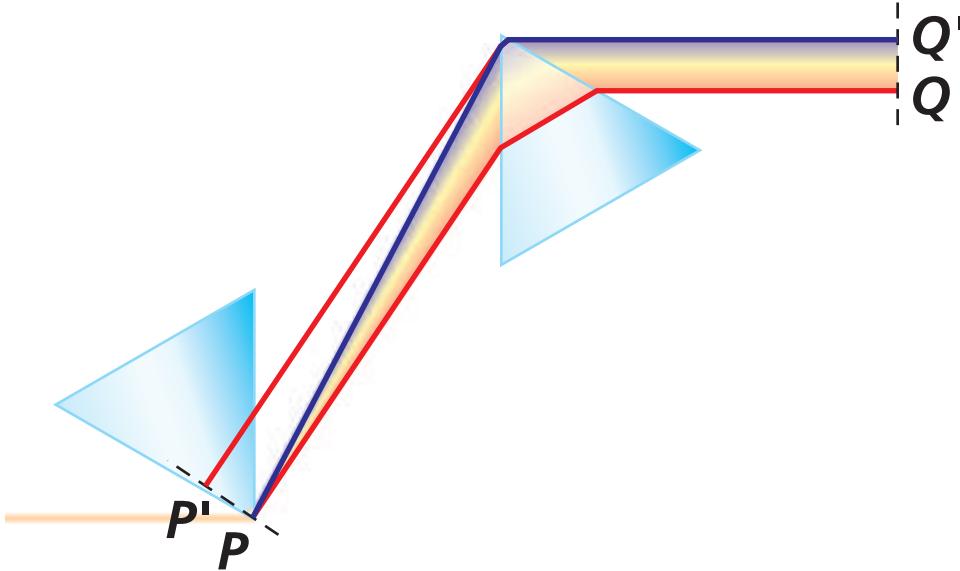
Dispersion compensation

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Dispersion compensation

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...so prism gives low frequency *shorter* path length...

Dispersion compensation

consider traveling Gaussian pulse again:

$$y(t) = \exp\left[-\frac{(x-v_g t)^2}{2\sigma_t^2}\right] \sin 2\pi(kx - ft)$$

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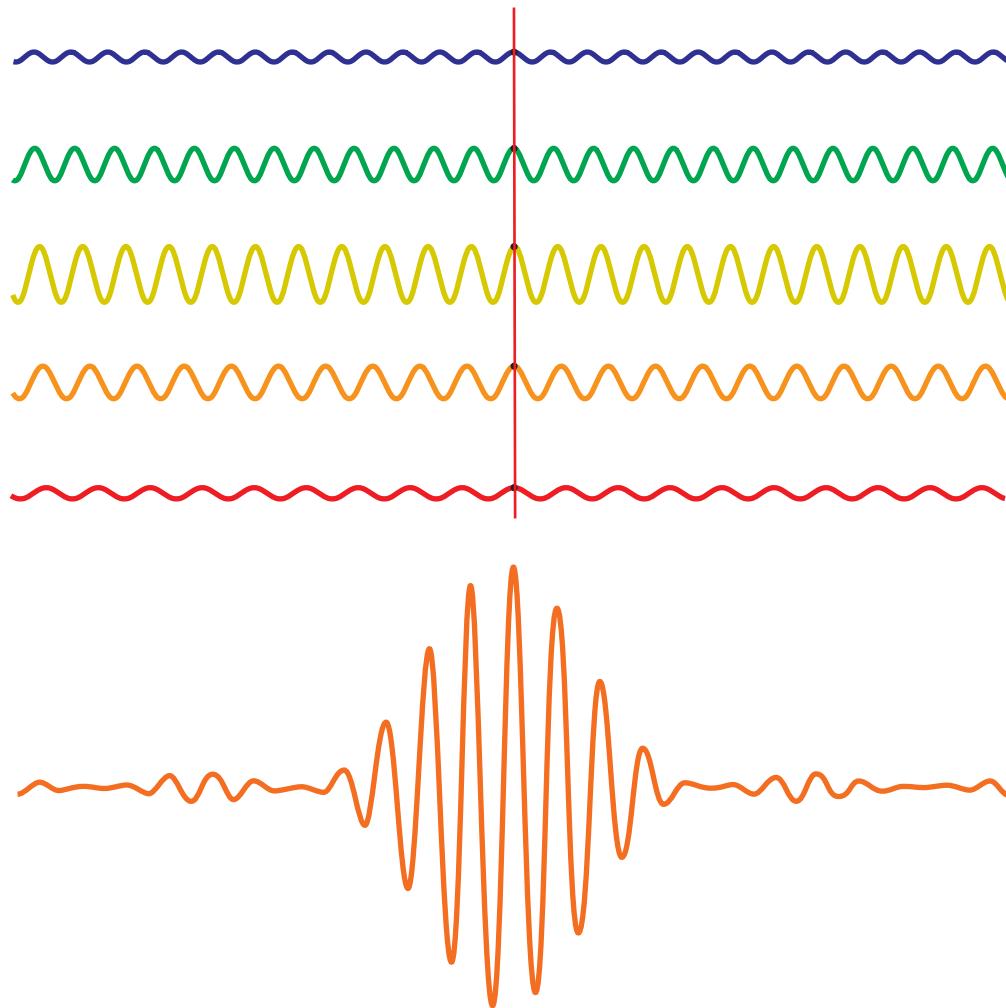
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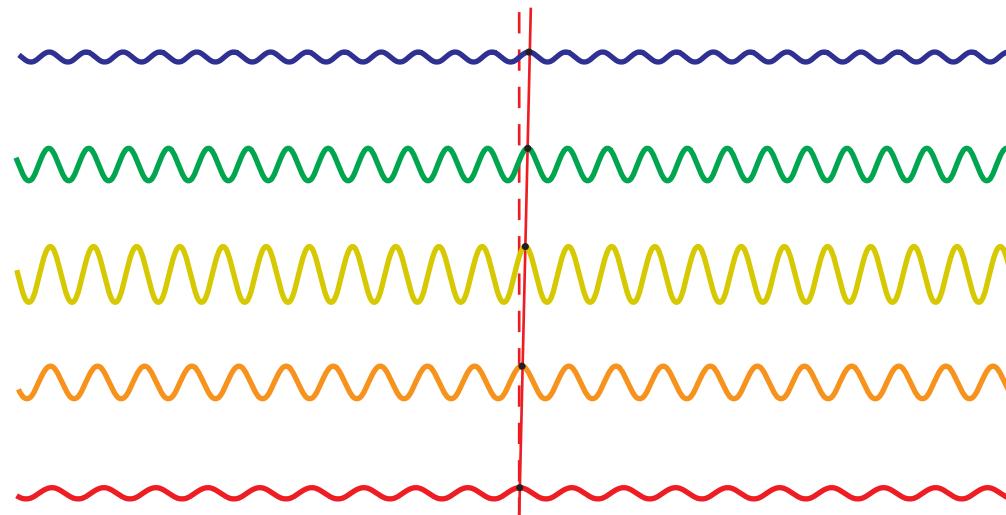
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...but Gaussian shape of pulse is constant!

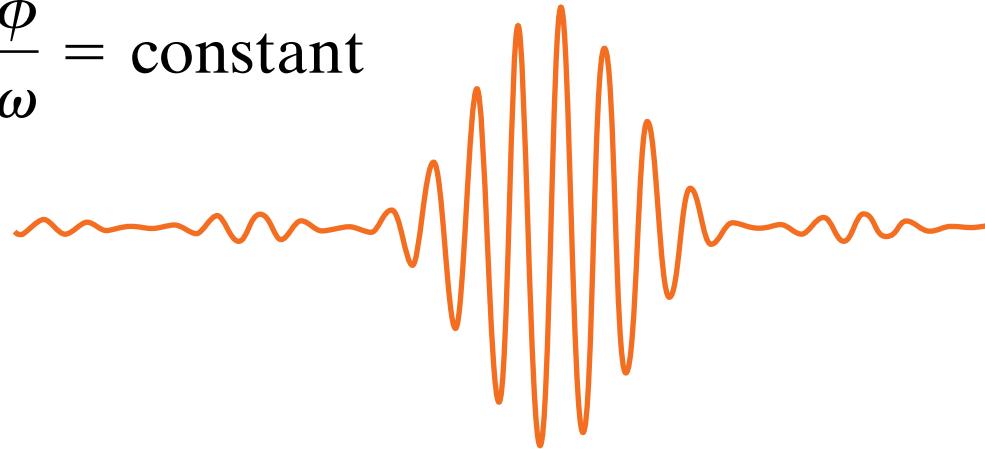
Dispersion compensation



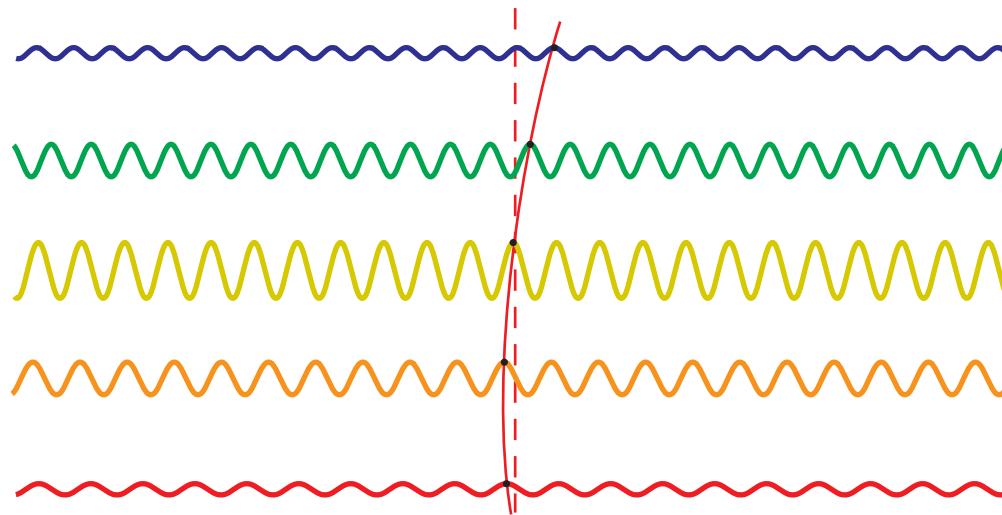
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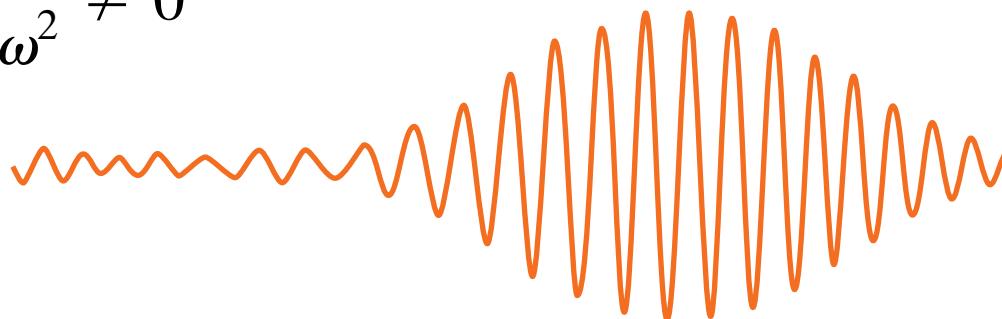
$$\frac{d\phi}{d\omega} = \text{constant}$$



Dispersion compensation



$$\frac{d^2\phi}{d\omega^2} \neq 0$$



Dispersion compensation

Write dispersion equation as Taylor series:

$$f(k) = f_o + \left(\frac{df}{dk} \right)_{k=k_o} (k - k_o) + \frac{1}{2} \left(\frac{d^2f}{dk^2} \right)_{k=k_o} (k - k_o)^2$$

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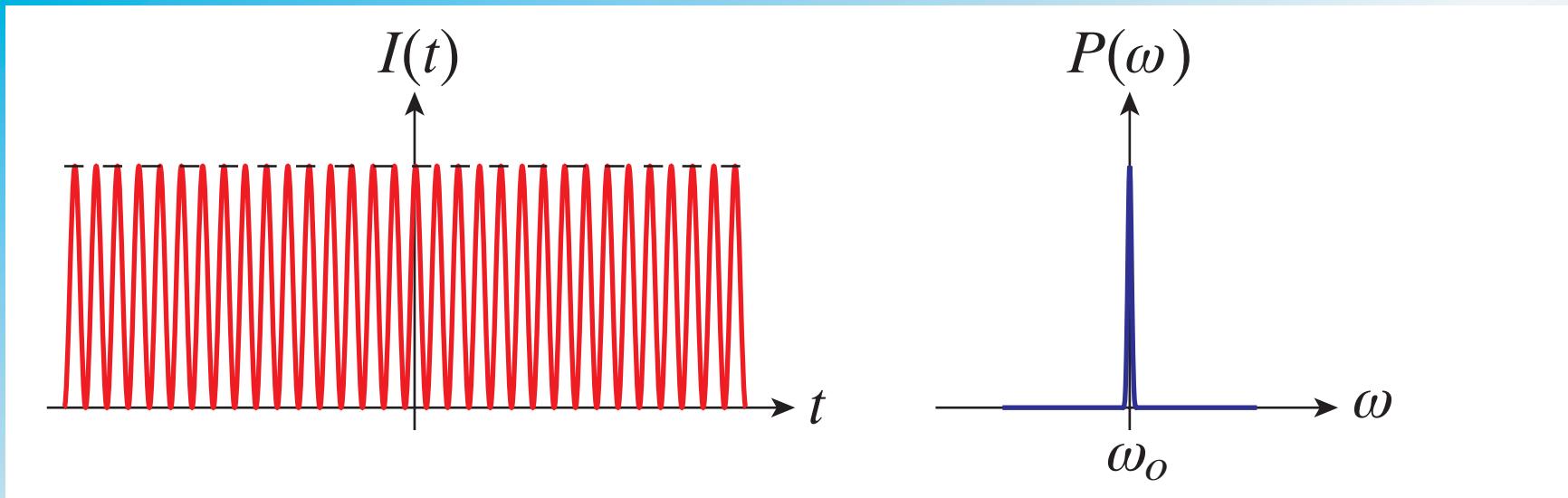
if $w = 0$, then group velocity and pulse shape constant!

Dispersion compensation

So not path length but $\frac{d^2\phi}{d\omega^2}$ matters!

	$\frac{dl_{eff}}{d\omega}$	$\frac{d^2\phi}{d\omega^2}$
dispersion	+	+
gratings	-	-
prisms	+	-

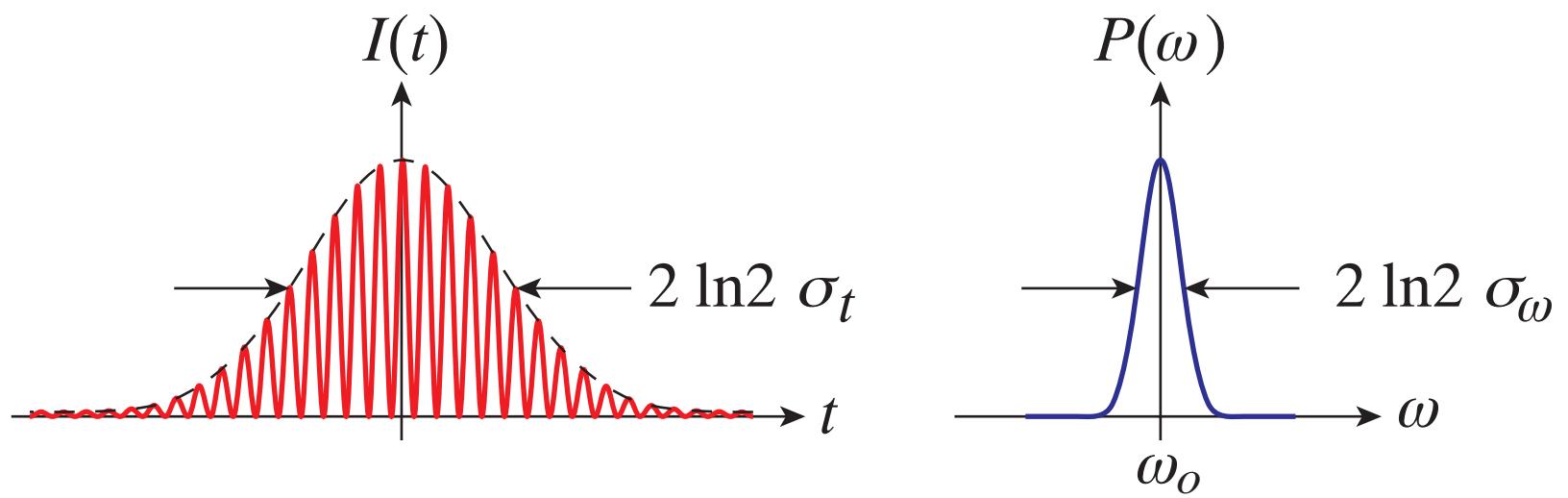
Representation of pulses



Spectrum of sinusoidal intensity is a delta function

$$I(t) = \cos^2(\omega_o t) \quad \Rightarrow \quad P(\omega) = \delta(\omega - \omega_o)$$

Representation of pulses



Modulate amplitude

$$I(t) = \exp\left[-\frac{t^2}{\sigma_t^2}\right] \cos^2(\omega_o t)$$

Representation of pulses

Fourier relations:

$$E(t) = \exp\left[-\frac{t^2}{2\sigma_t^2} - i\omega_o t\right]$$

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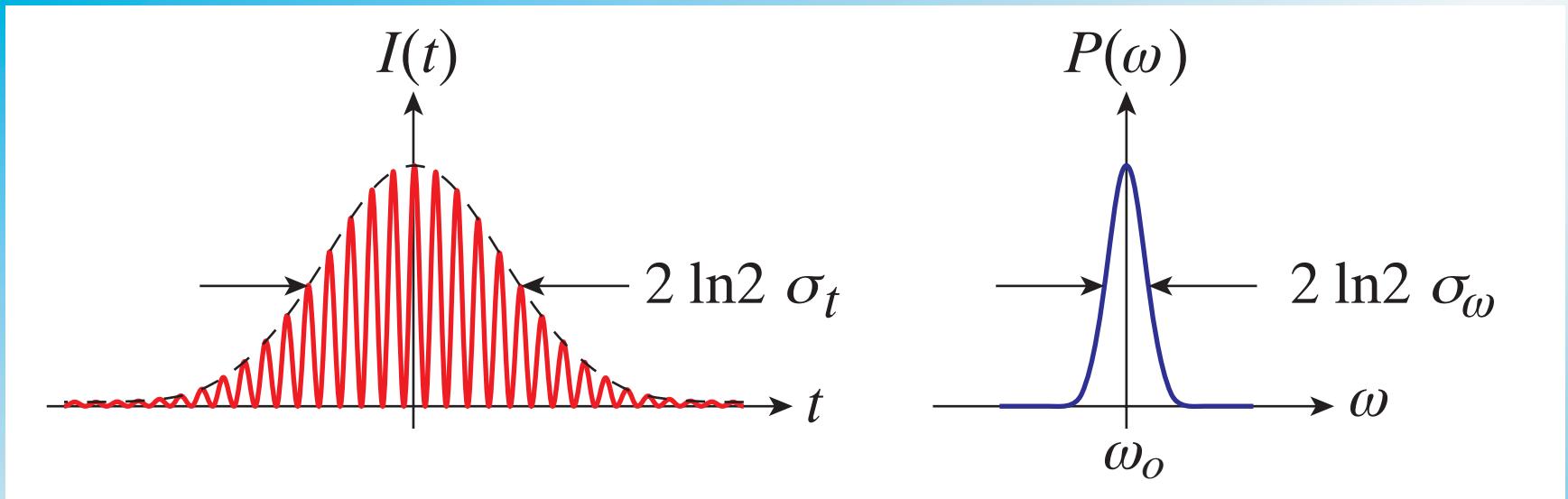
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$$= \sigma_t \exp\left[-\frac{\sigma_t^2(\omega - \omega_o)^2}{2}\right] \equiv \sigma_t \exp\left[-\frac{(\omega - \omega_o)^2}{2\sigma_\omega^2}\right]$$

Representation of pulses

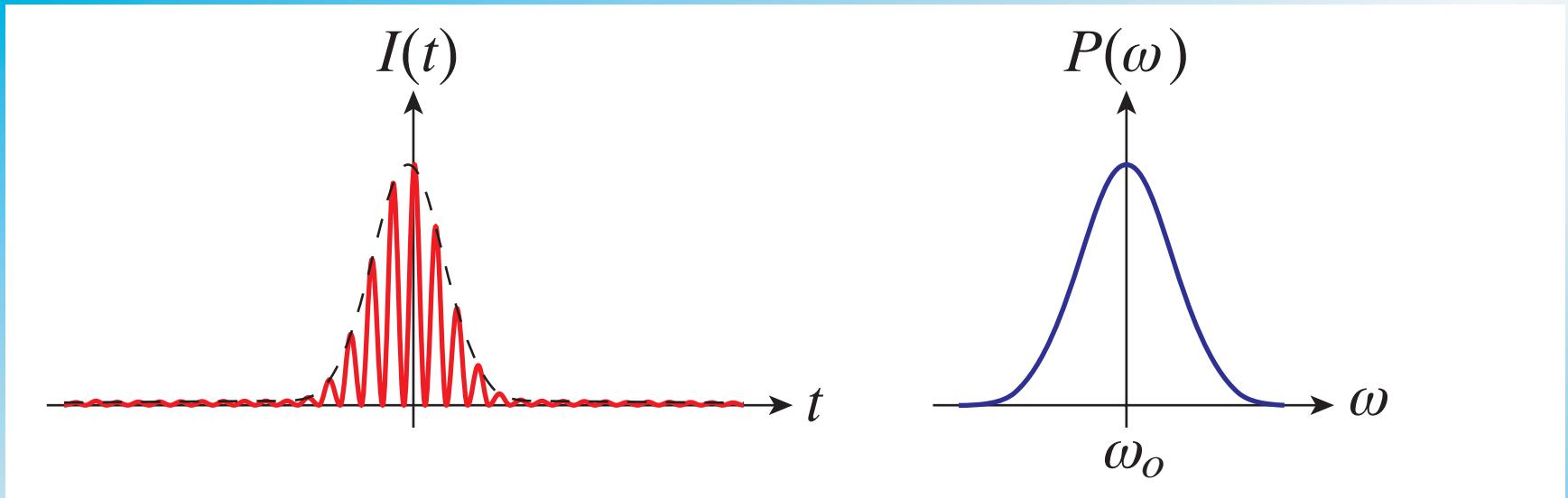


Pulse duration-bandwidth product: $\sigma_t \sigma_\omega = 1$

$$I(t) = [\operatorname{Re} E(t)]^2 \propto \exp\left[-\frac{t^2}{\sigma_t^2}\right] \cos^2(\omega_o t)$$

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Joint time-frequency representation

Wigner representation:

$$\begin{aligned} W(t, \omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} E\left(\omega + \frac{\omega'}{2}\right) E^*\left(\omega - \frac{\omega'}{2}\right) e^{-i\omega't} d\omega' = \\ &= \int_{-\infty}^{\infty} E\left(t + \frac{t'}{2}\right) E^*\left(t - \frac{t'}{2}\right) e^{-i\omega t'} dt' \end{aligned}$$

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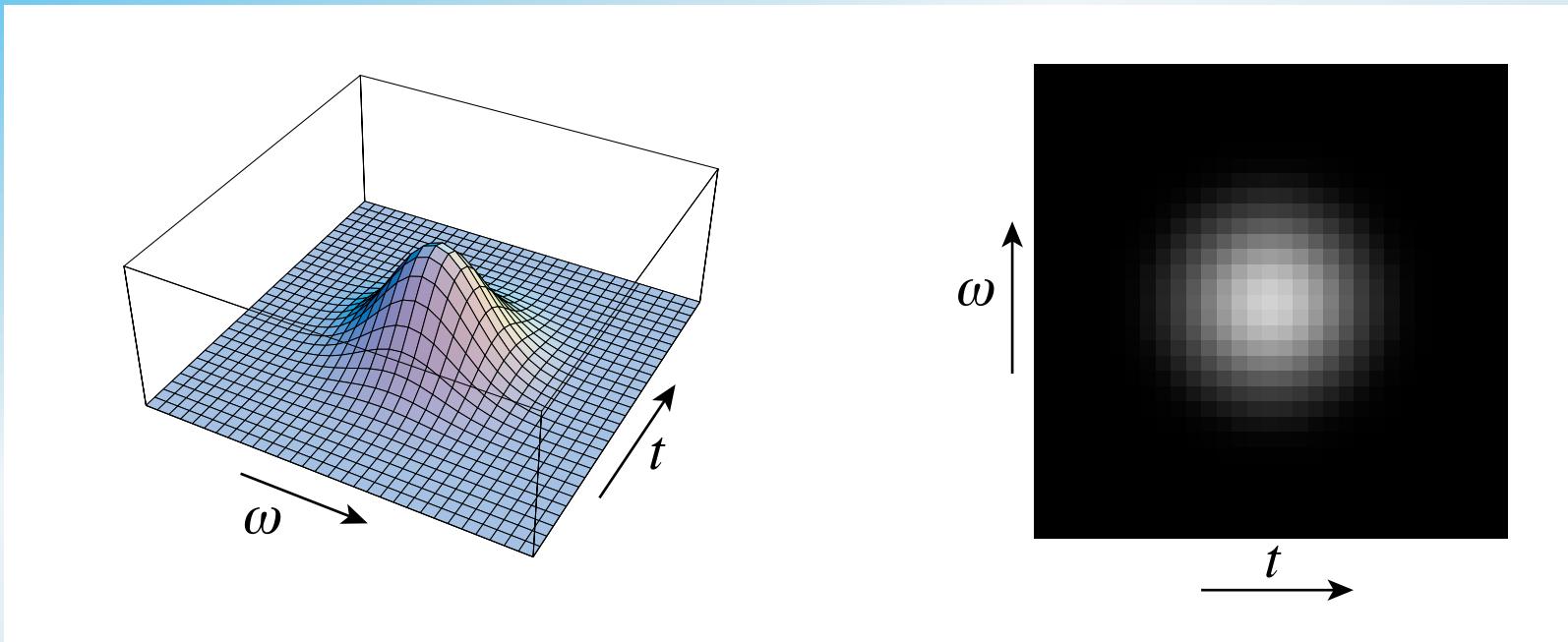
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Joint time-frequency representation

Energy:

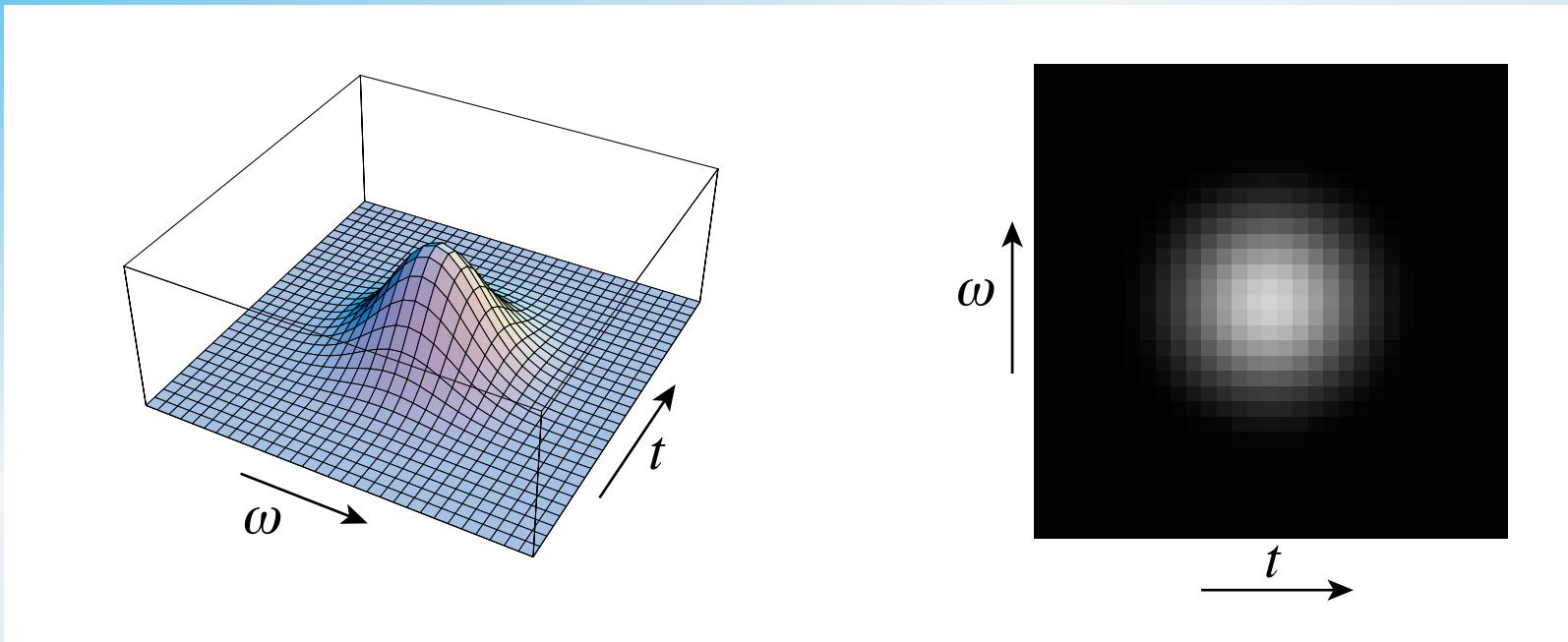
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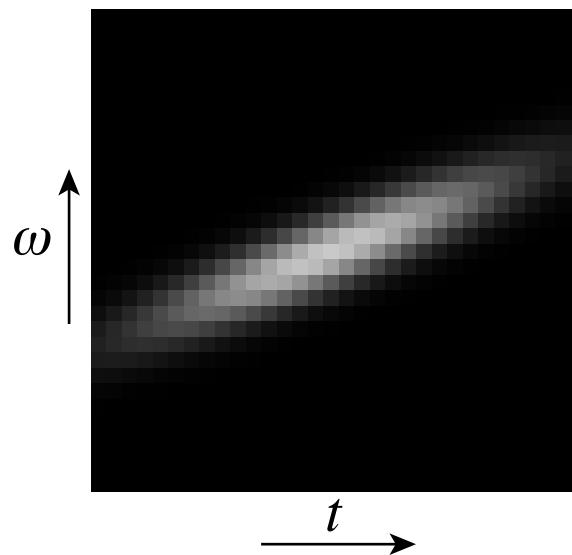
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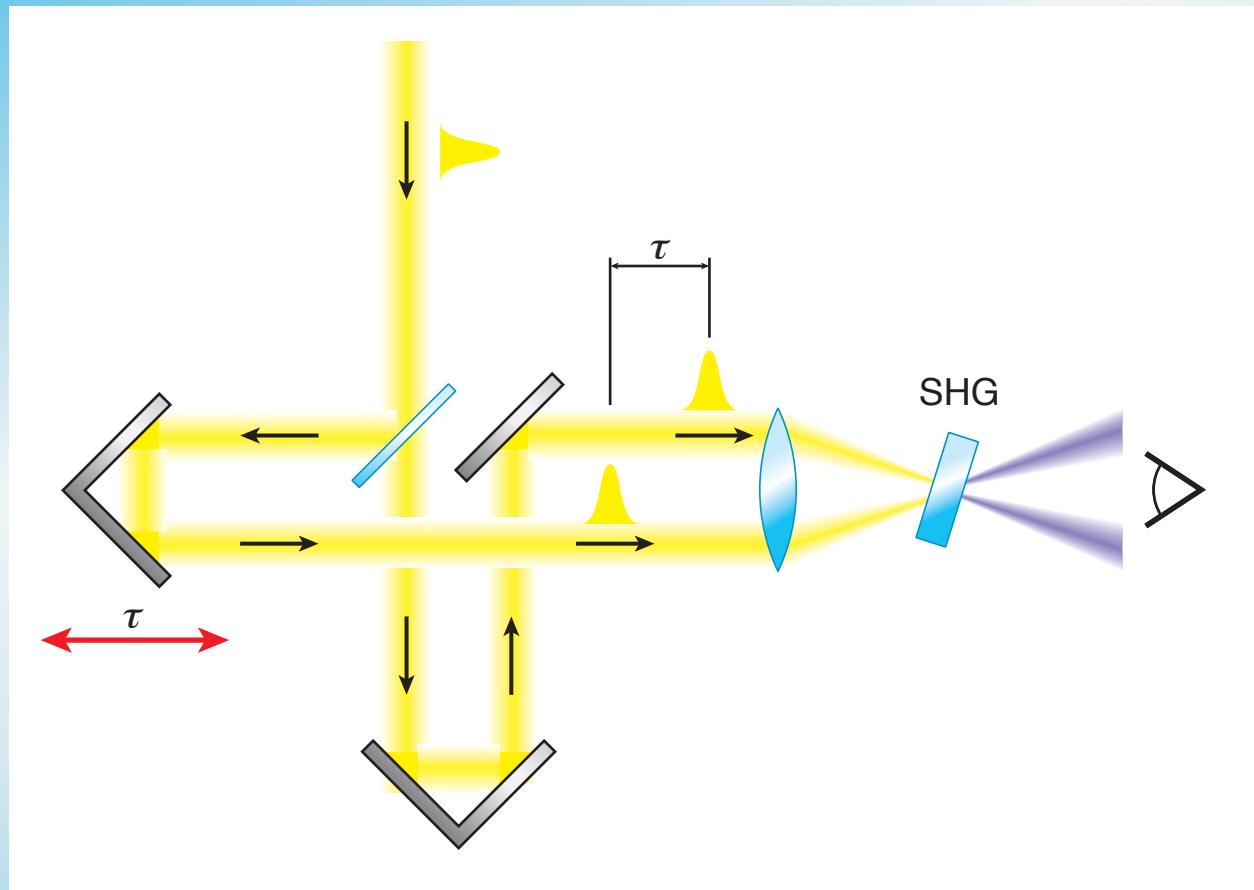
chirped pulse



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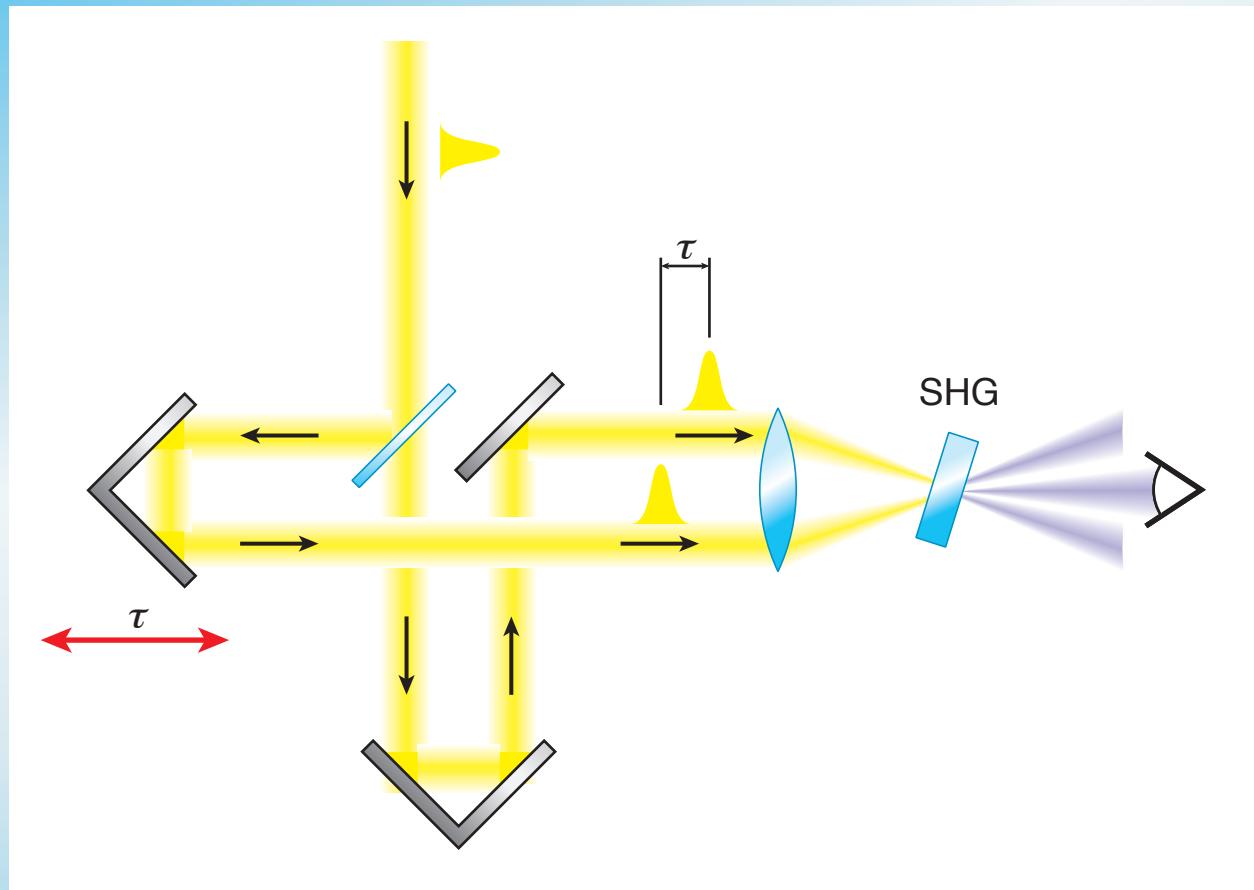
Temporal characterization

Use pulse to measure itself...



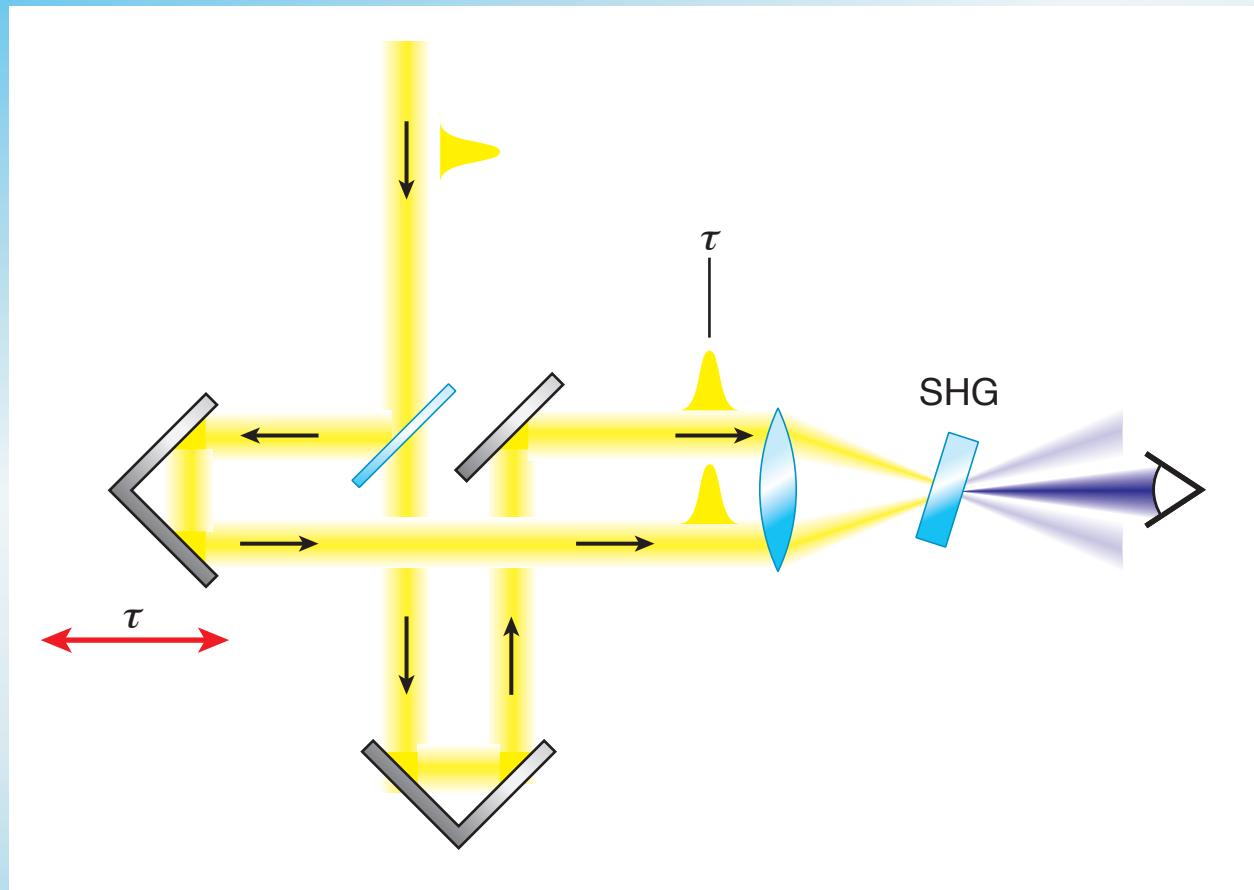
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Temporal characterization

Electric field at SHG crystal

$$E_{tot}(t,\tau) = \frac{1}{\sqrt{2}}E_1(t) + \frac{1}{\sqrt{2}}E_2(t + \tau)$$

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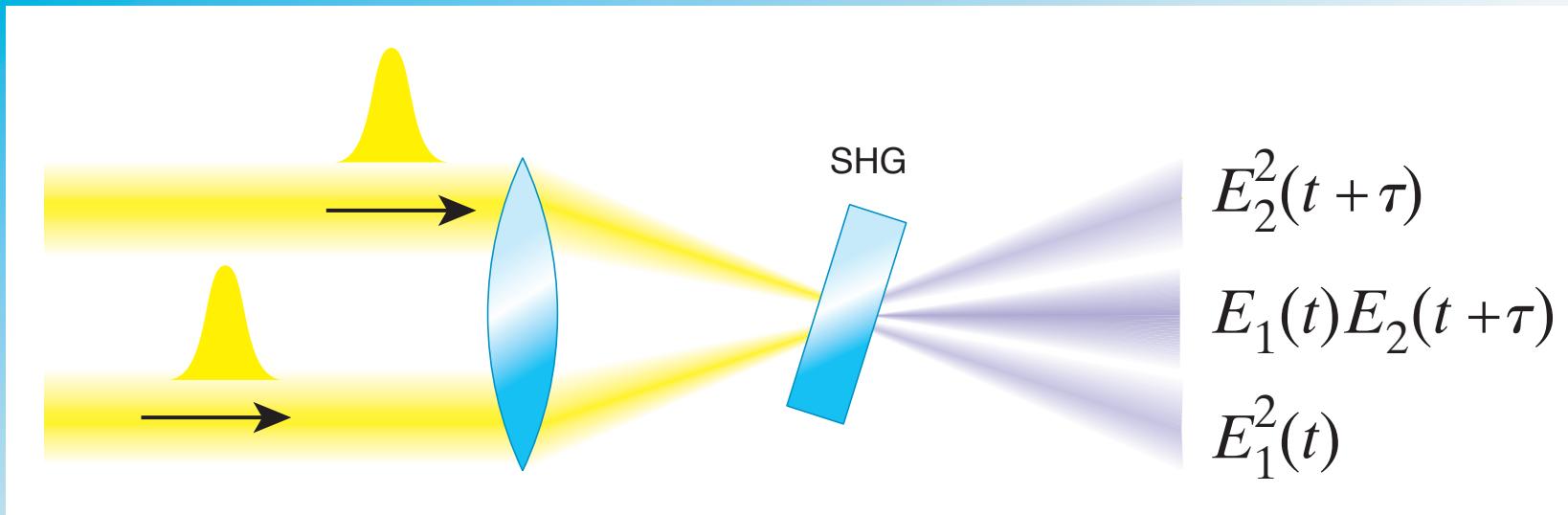
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Second harmonic intensity

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Temporal characterization



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detector selects middle term

Temporal characterization

Integrated detector signal yields intensity autocorrelation

$$A(\tau) = \int I_{2\omega}(t, \tau) dt \propto \int |\chi^{(2)}|^2 4 |E_1(t)|^2 |E_2(t + \tau)|^2 dt$$

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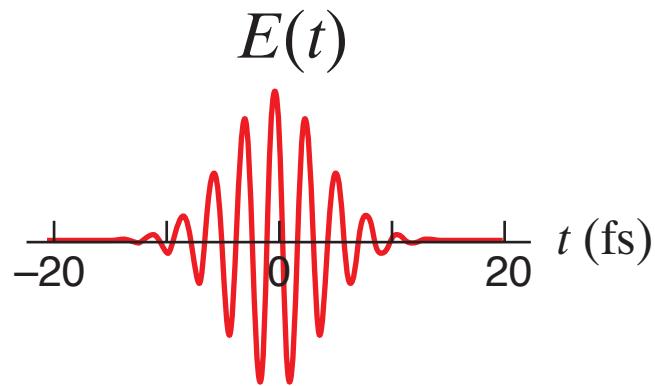
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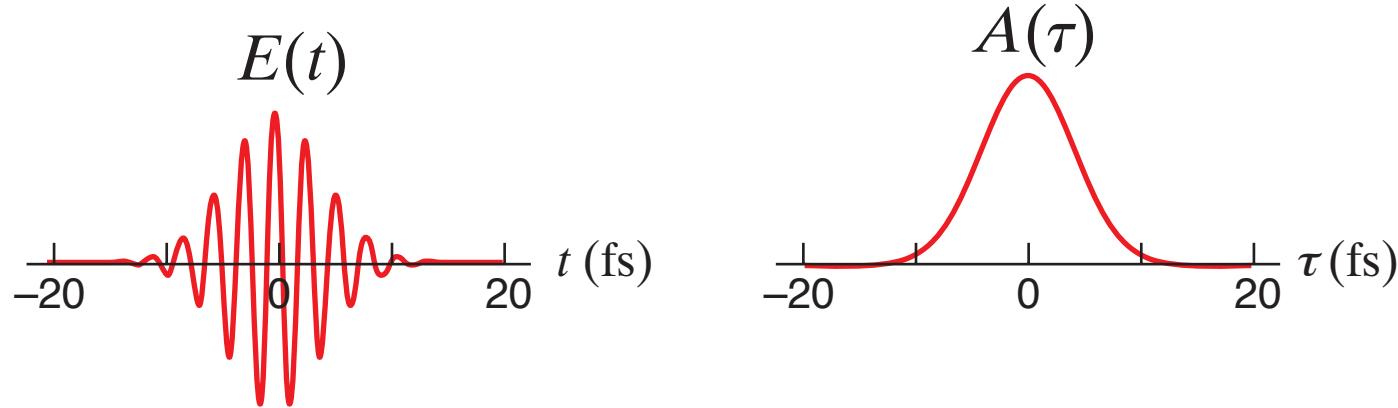


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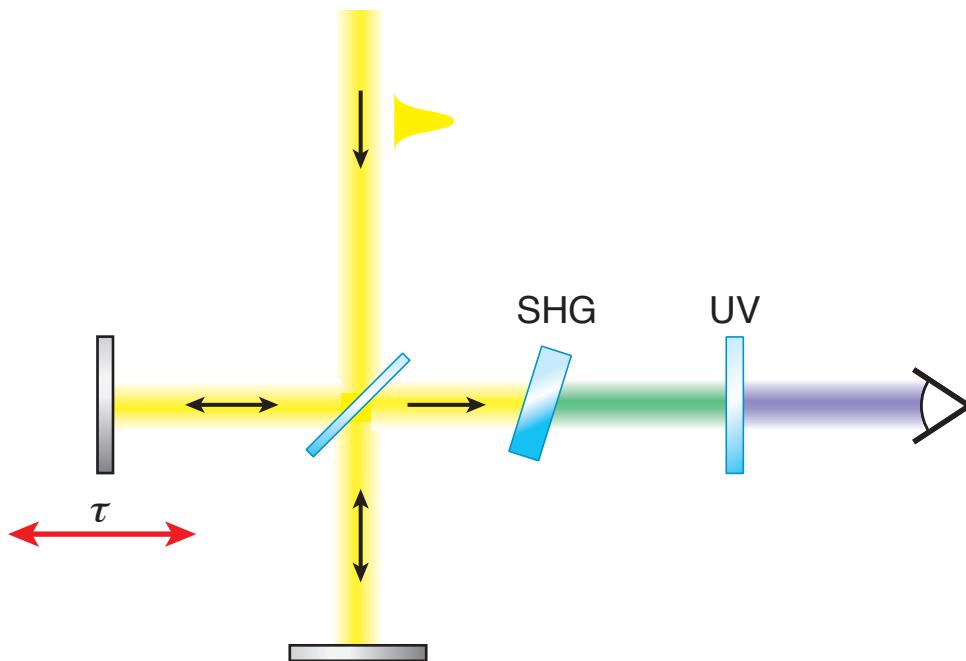
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Temporal characterization

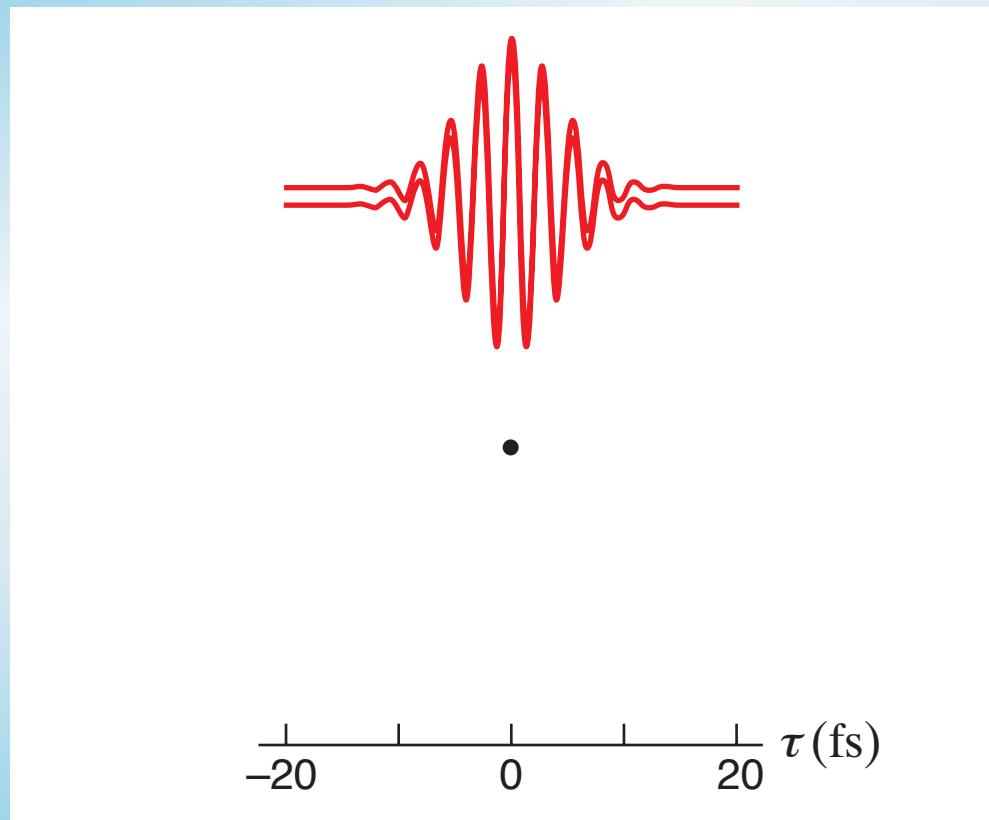
Alternative colinear geometry



Temporal characterization

All terms now contribute:

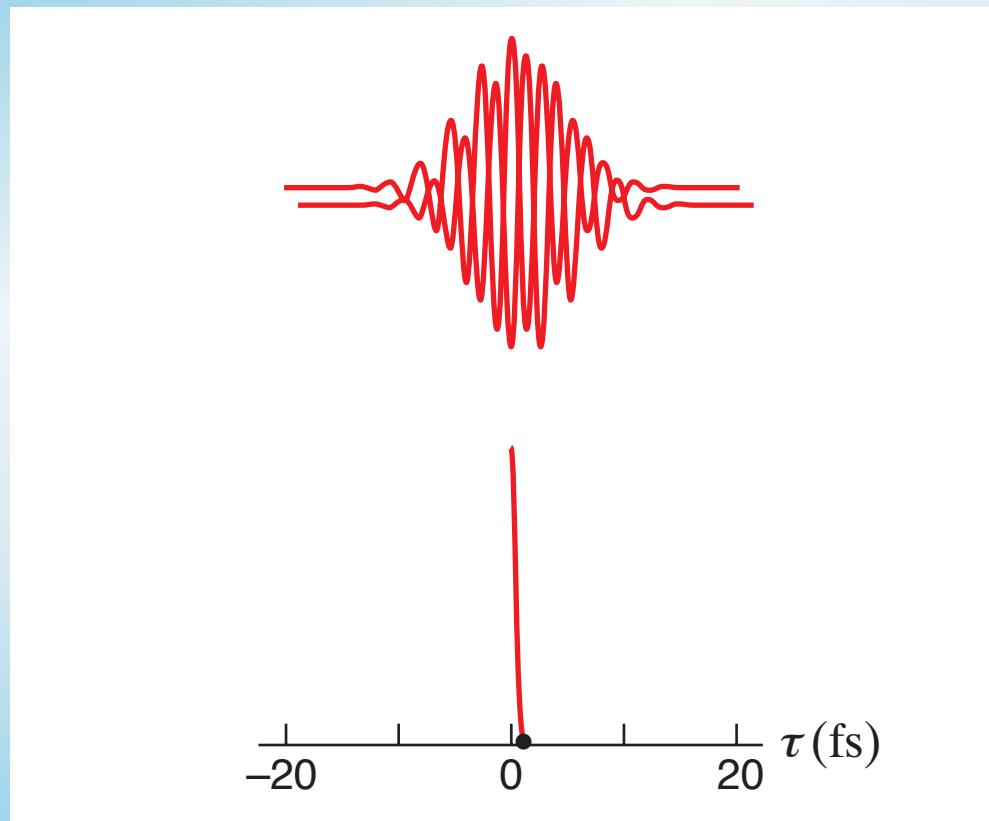
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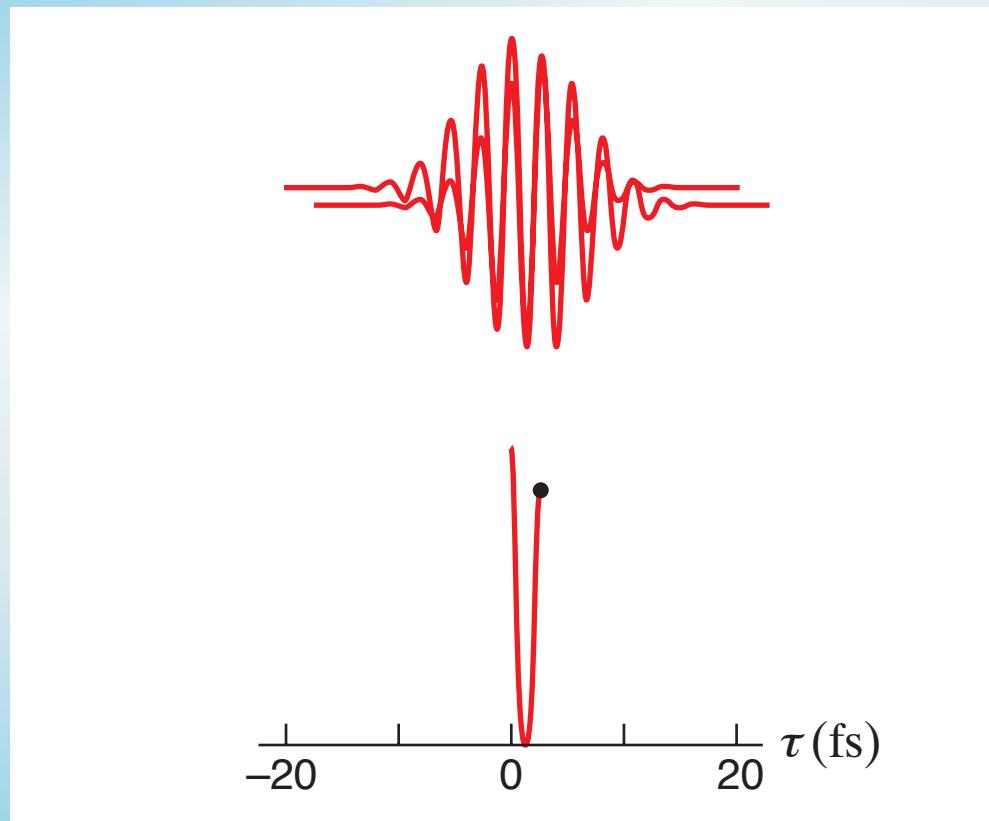
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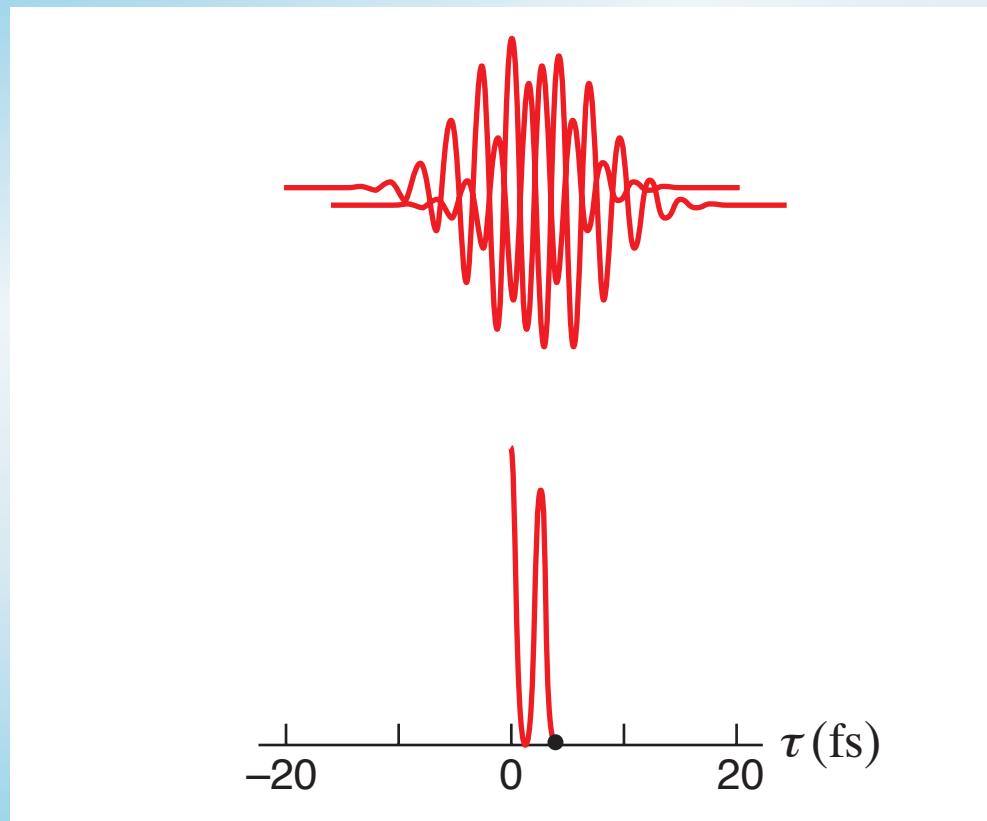
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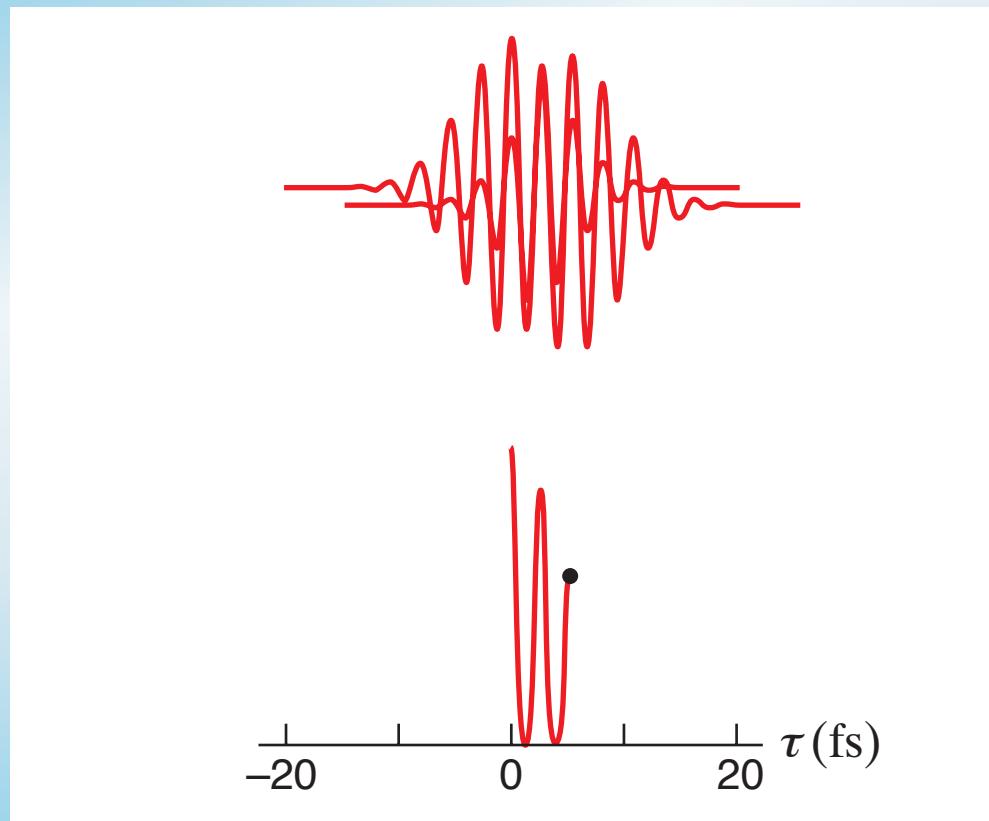
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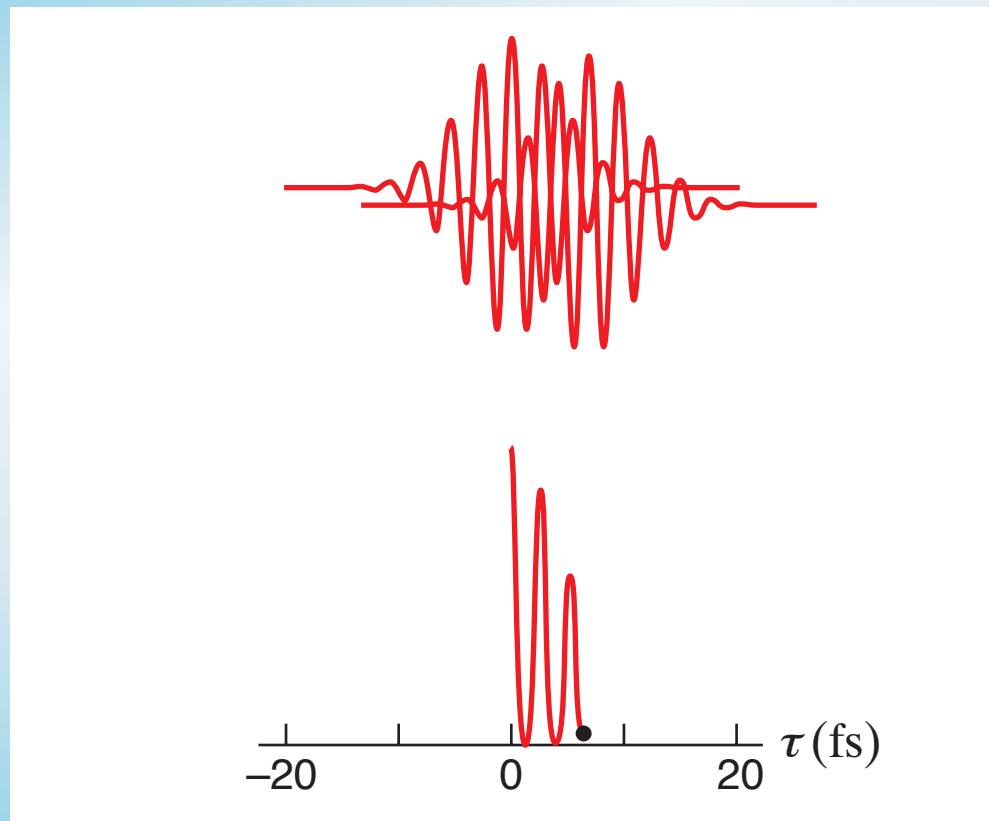
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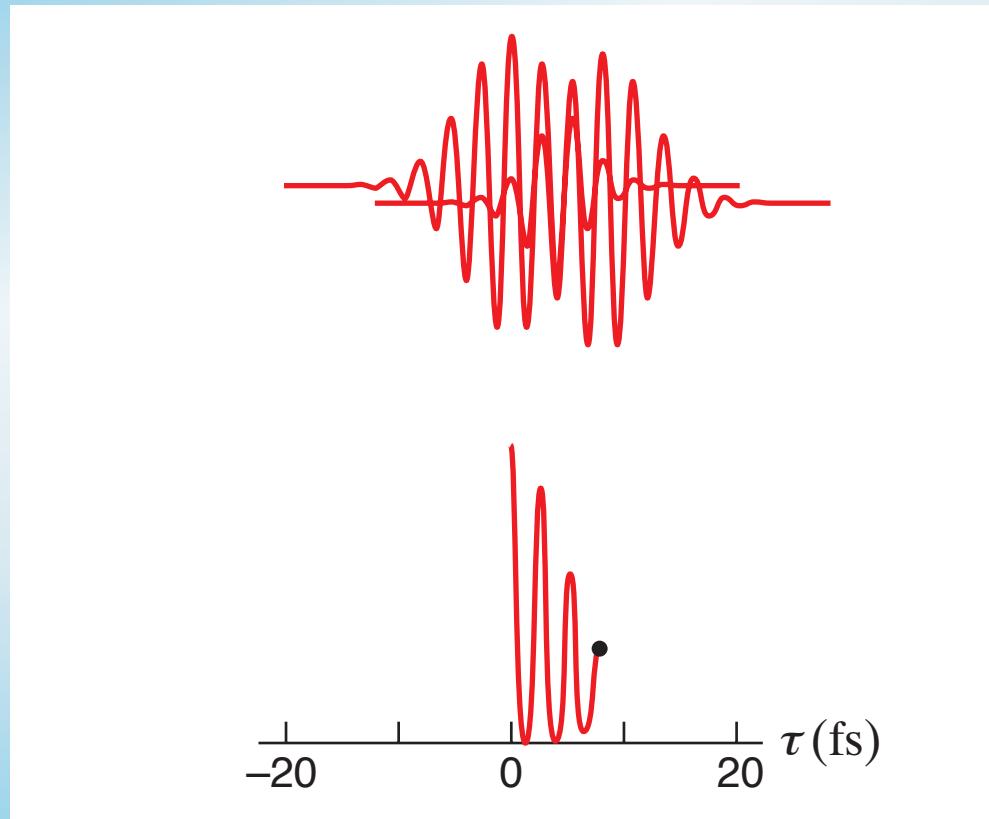
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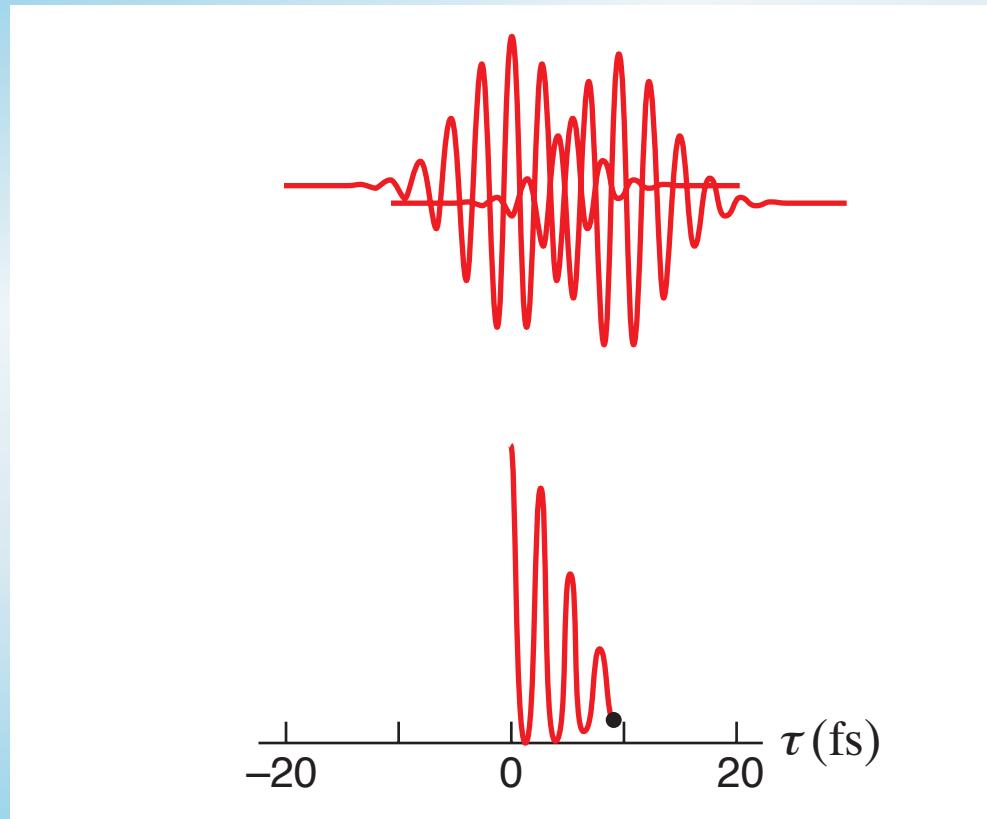
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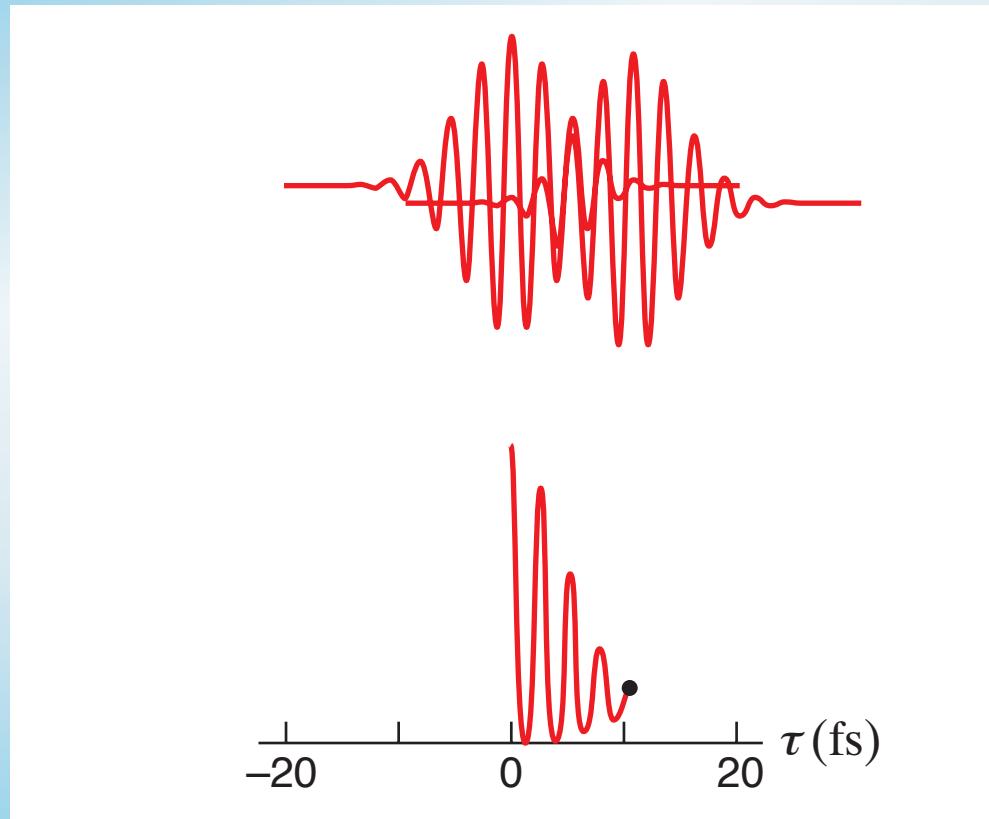
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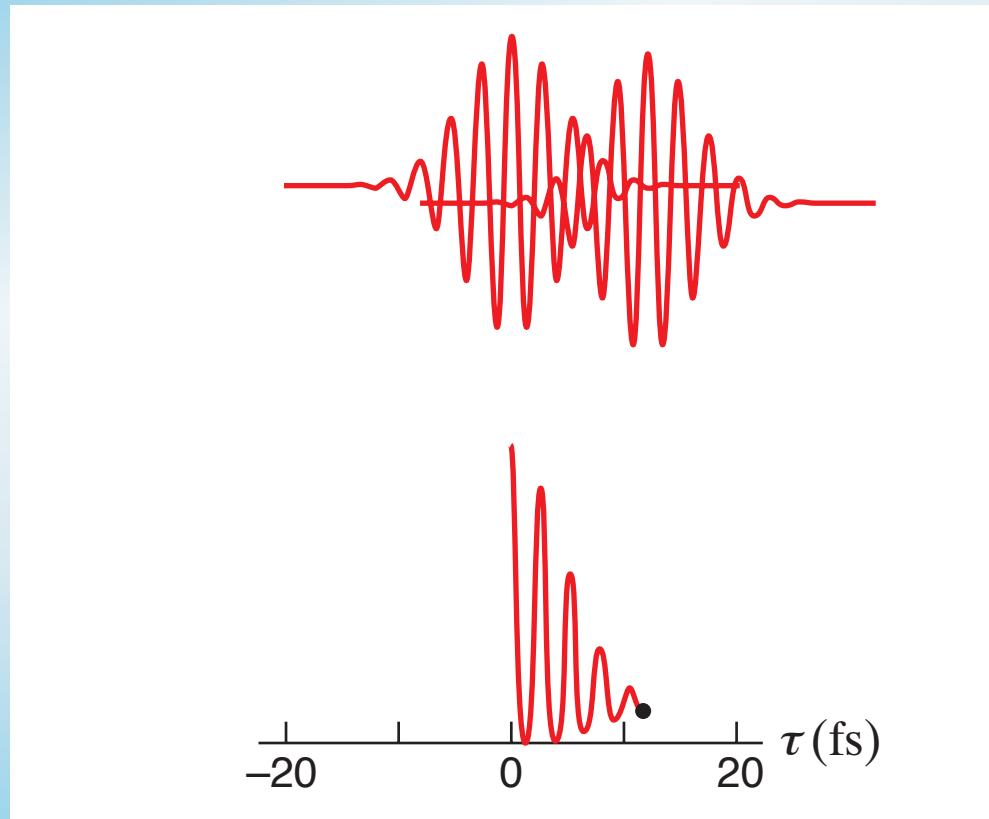
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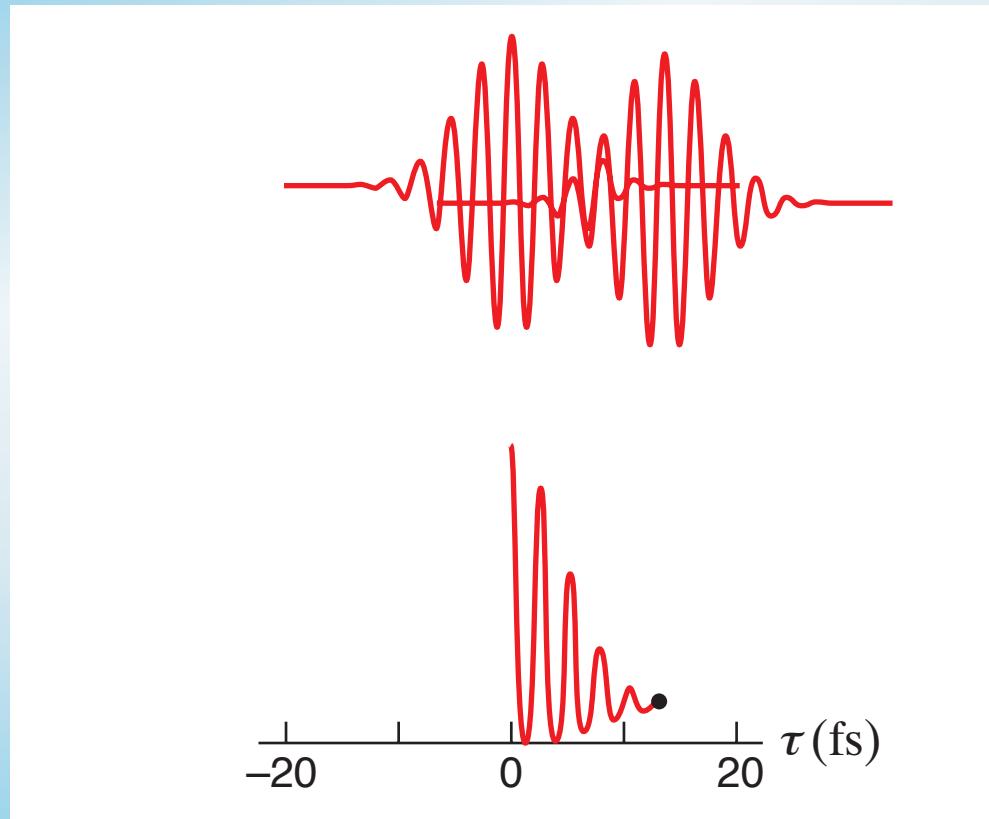
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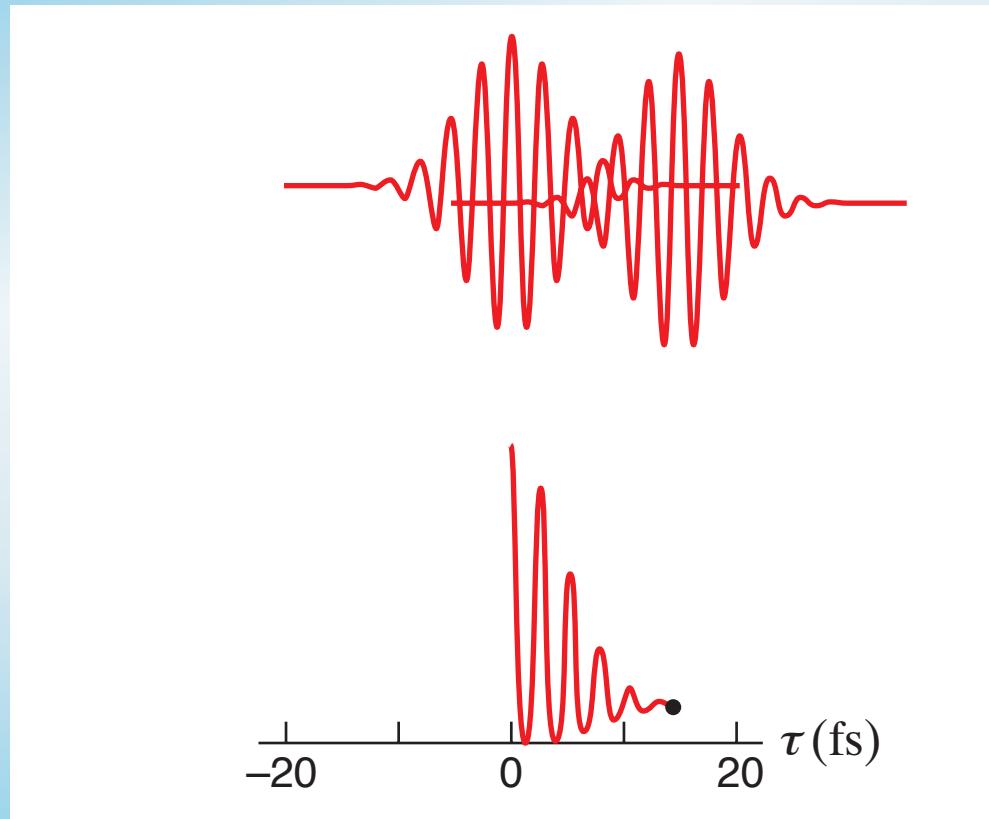
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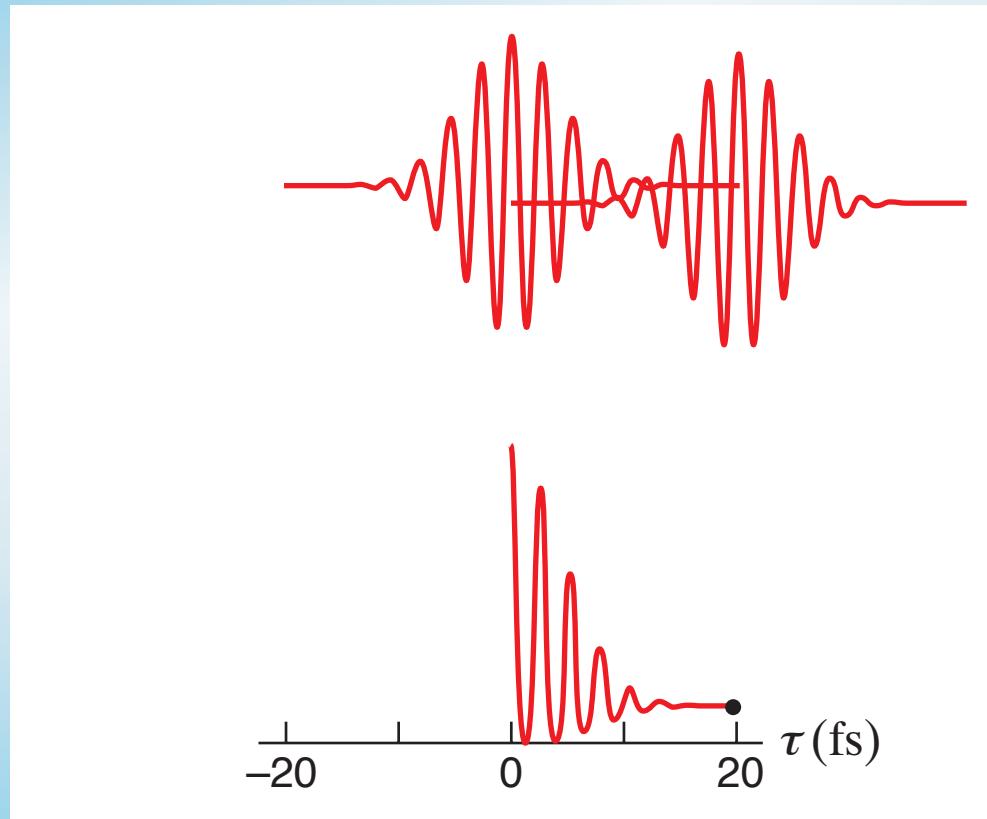
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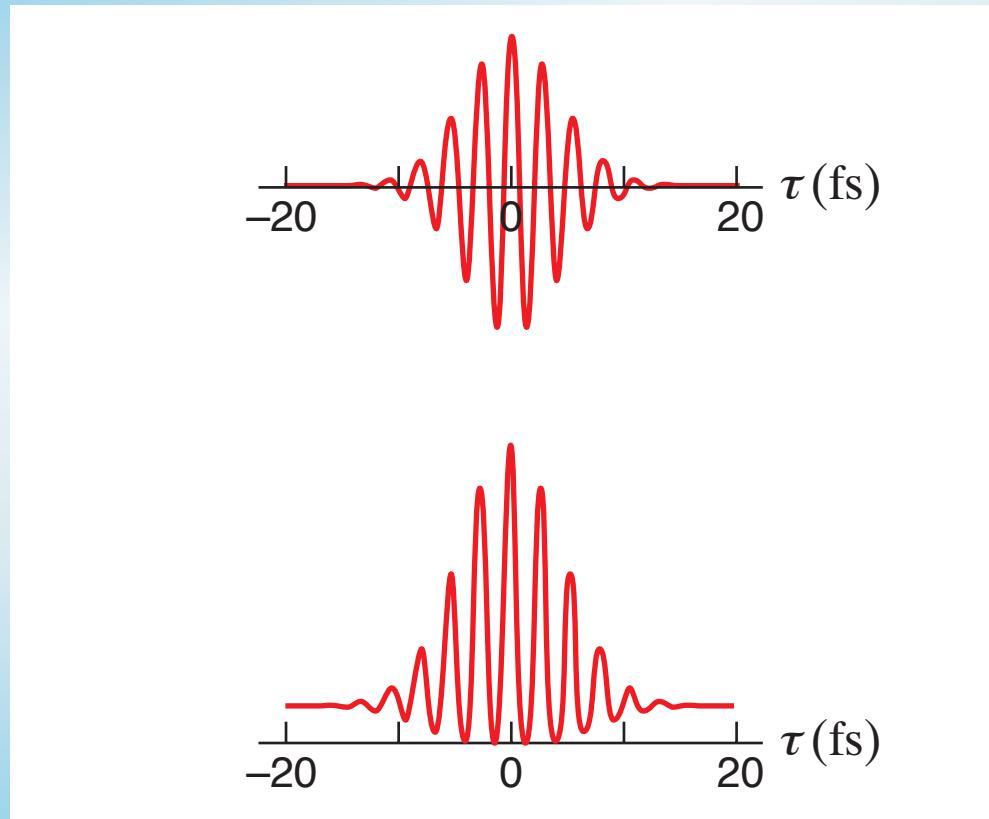
$$I_{2\omega}(t, \tau) \propto |\chi^{(2)}|^2 |E_{tot}^2|^2 = |\chi^{(2)}|^2 |E_1^2(t) + 2E_1(t)E_2(t+\tau) + E_2^2(t+\tau)|^2$$



Temporal characterization

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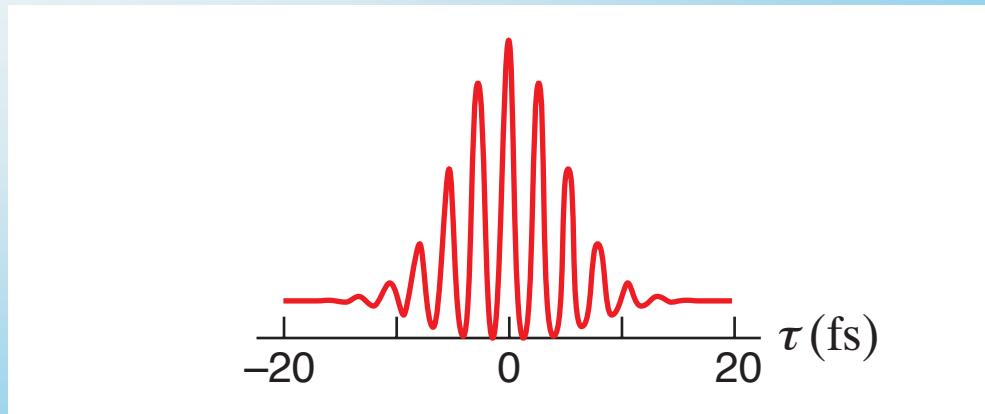
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at $\tau = 0$:

$$I_{2\omega}(t, \tau) \propto 16E^4(t)$$



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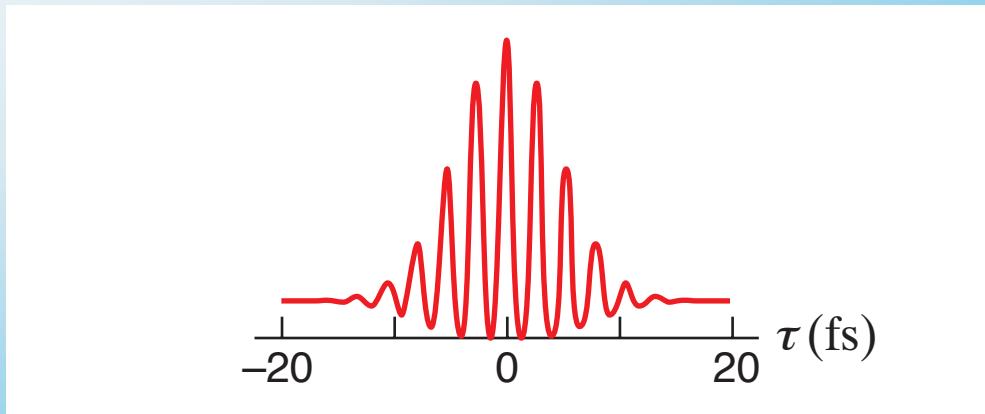
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at $\tau = 0$:

$$I_{2\omega}(t, \tau) \propto 16E^4(t)$$

as $\tau \rightarrow \pm\infty$:

$$I_{2\omega}(t, \tau) \propto 2E^4(t)$$



Temporal characterization

All terms now contribute:

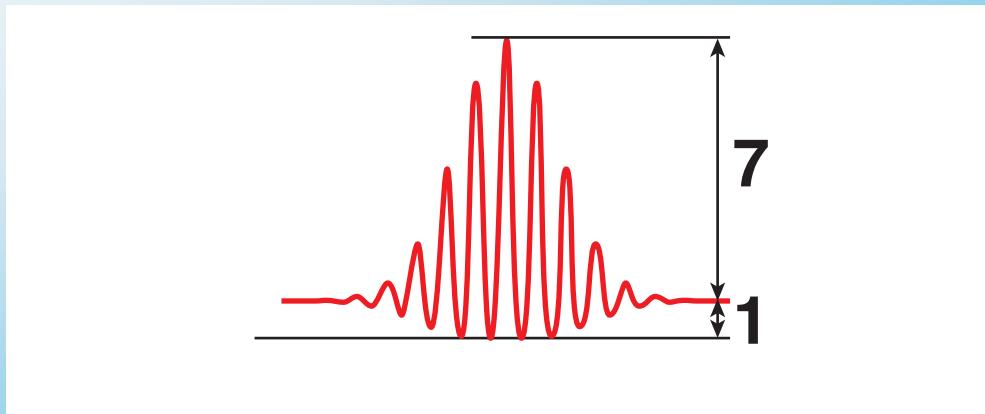
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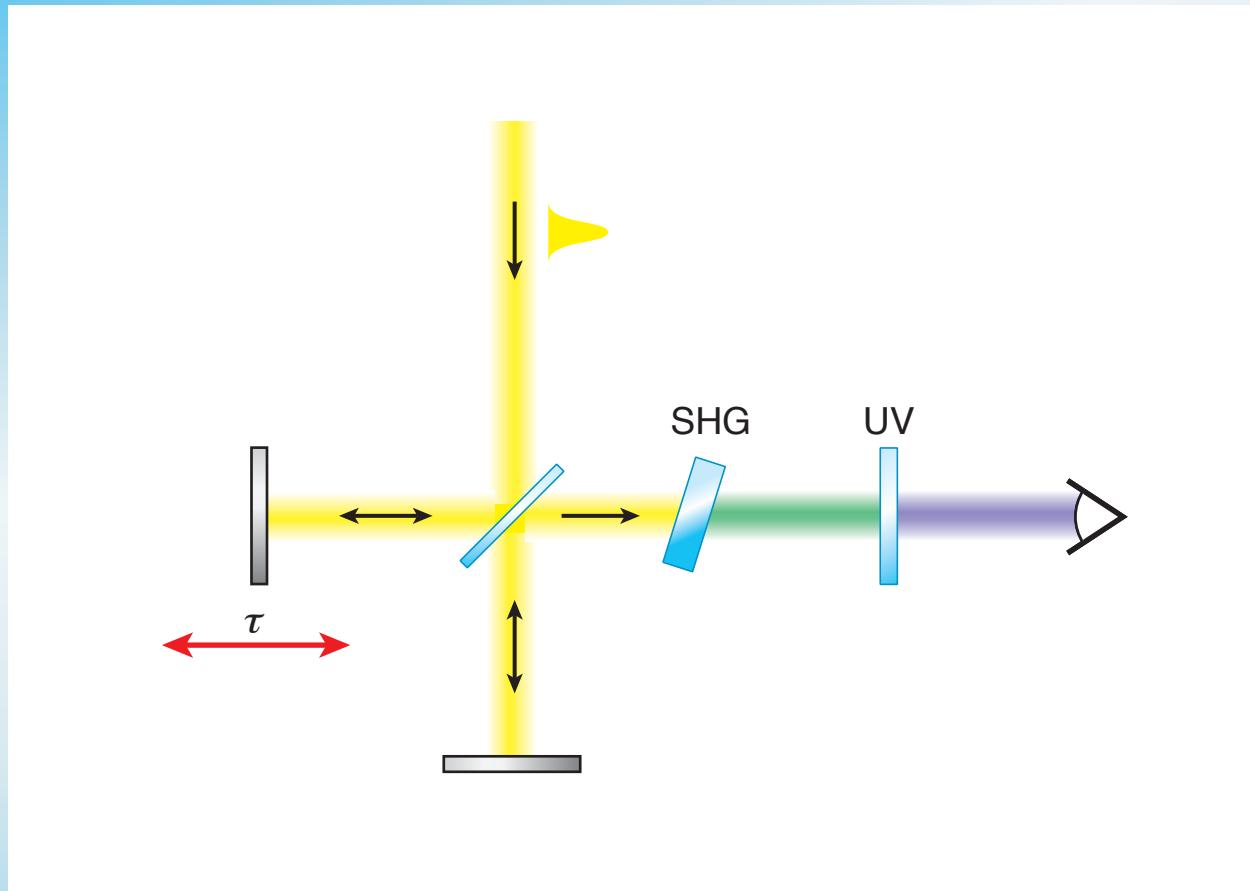
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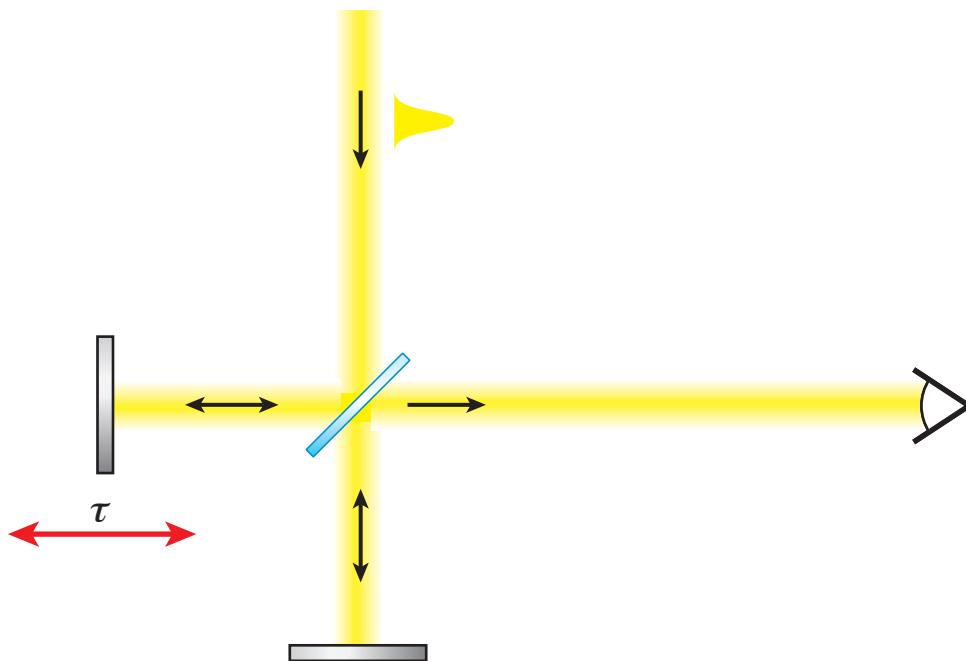
Temporal characterization

Do we really need the second-harmonic crystal...?



Temporal characterization

Would this work?



Temporal characterization

Intensity at detector

$$I_\omega(t, \tau) \propto |E_1(t) + E_2(t + \tau)|^2$$

Temporal characterization

Intensity at detector

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Detected signal

$$S_\omega(\tau) = \int I_\omega(t, \tau) dt$$

Temporal characterization

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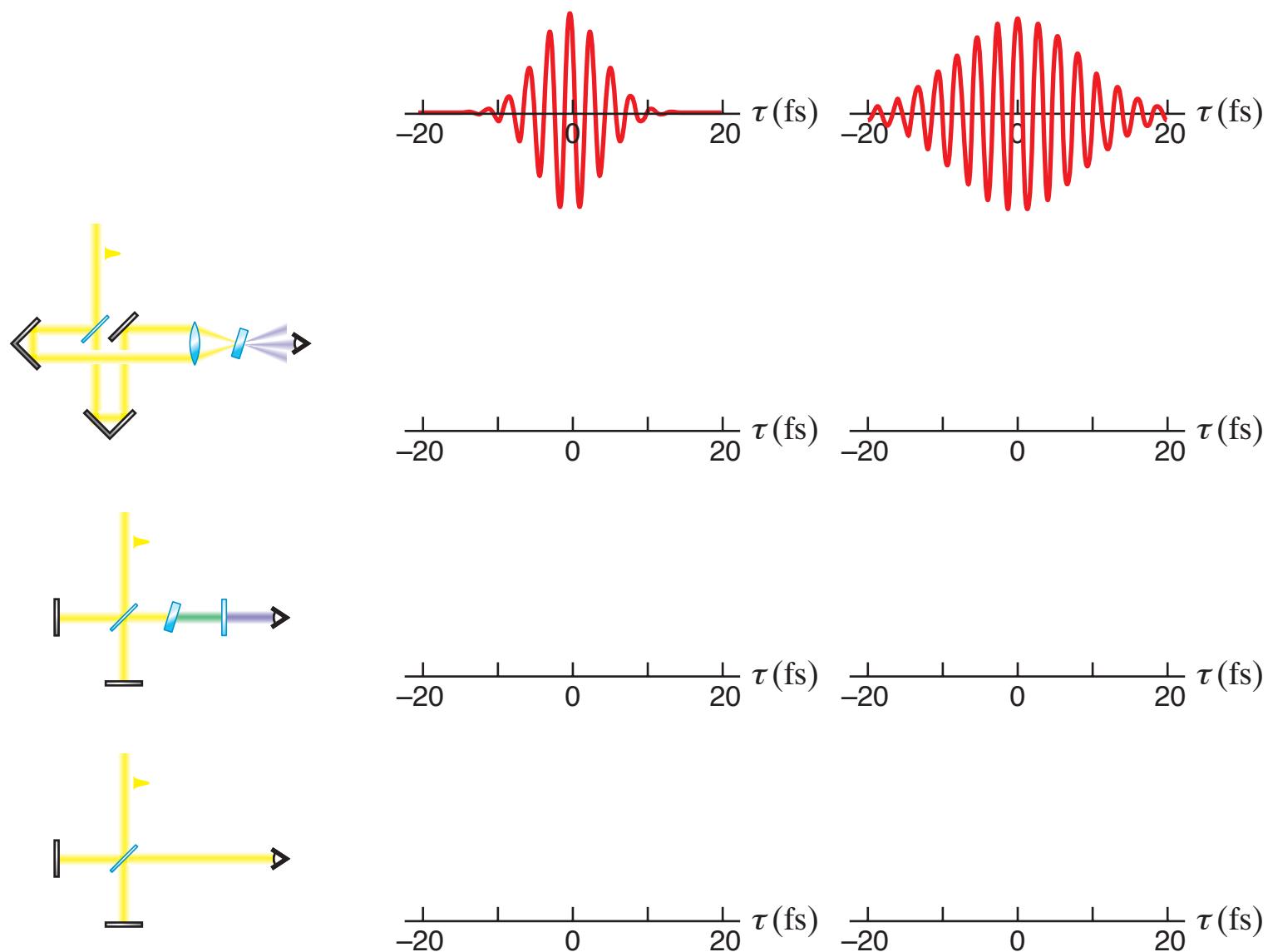
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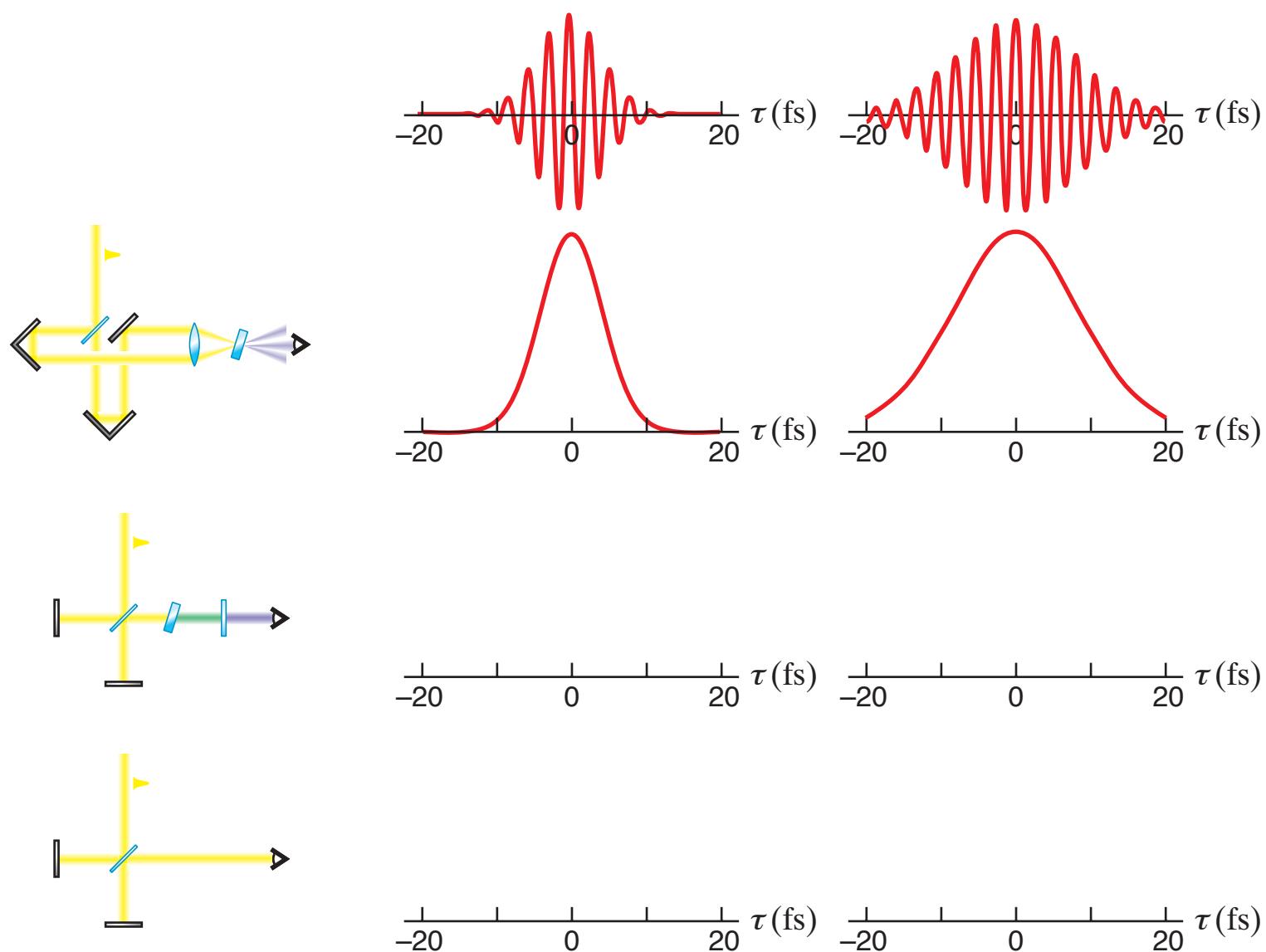
so

$$S_\omega(\tau) \propto \int \{|E_1(t)|^2 + |E_2(t + \tau)|^2 + E_1(t)E_2^*(t + \tau) + E_1^*(t)E_2(t + \tau)\} dt$$

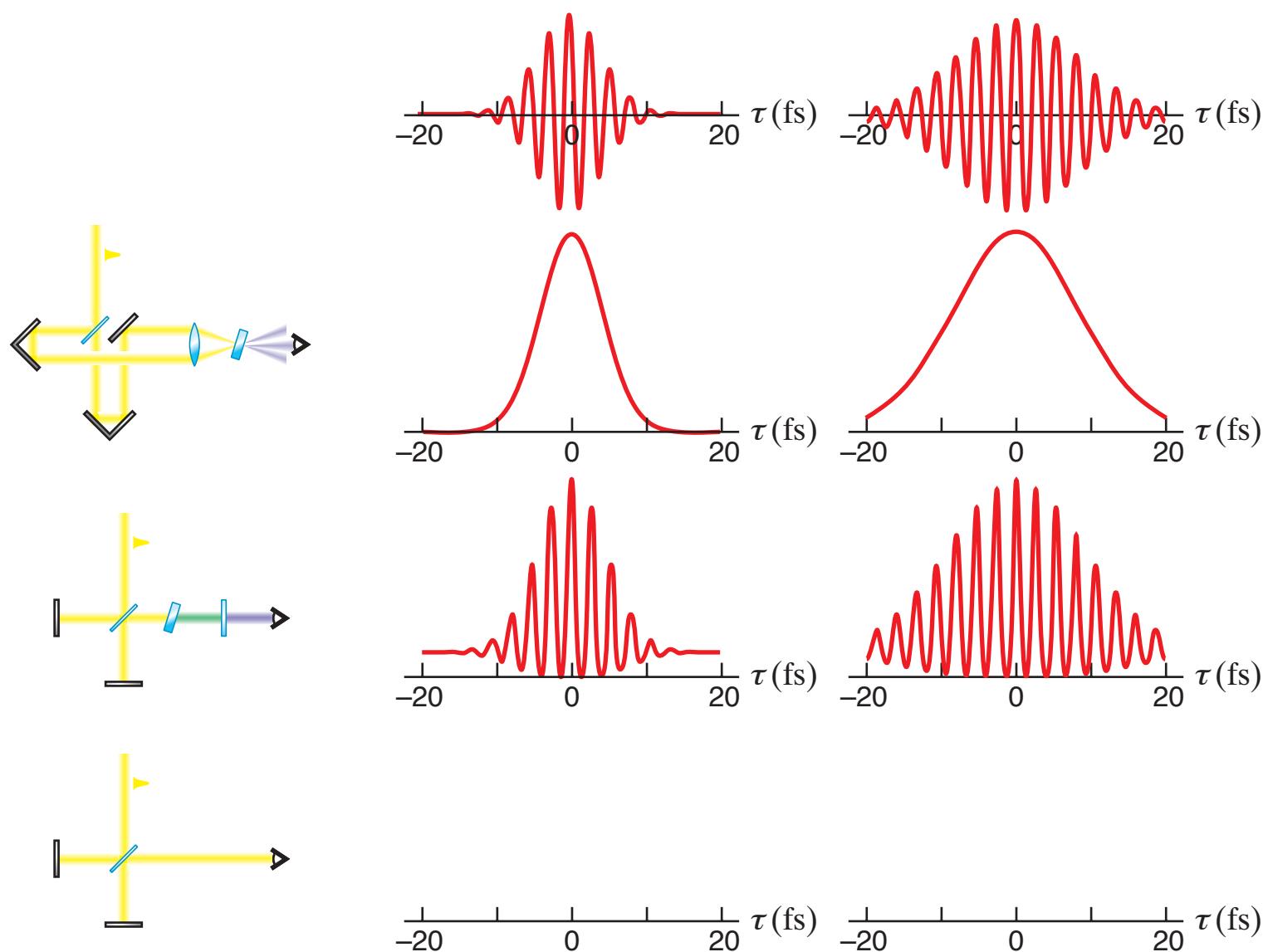
Temporal characterization



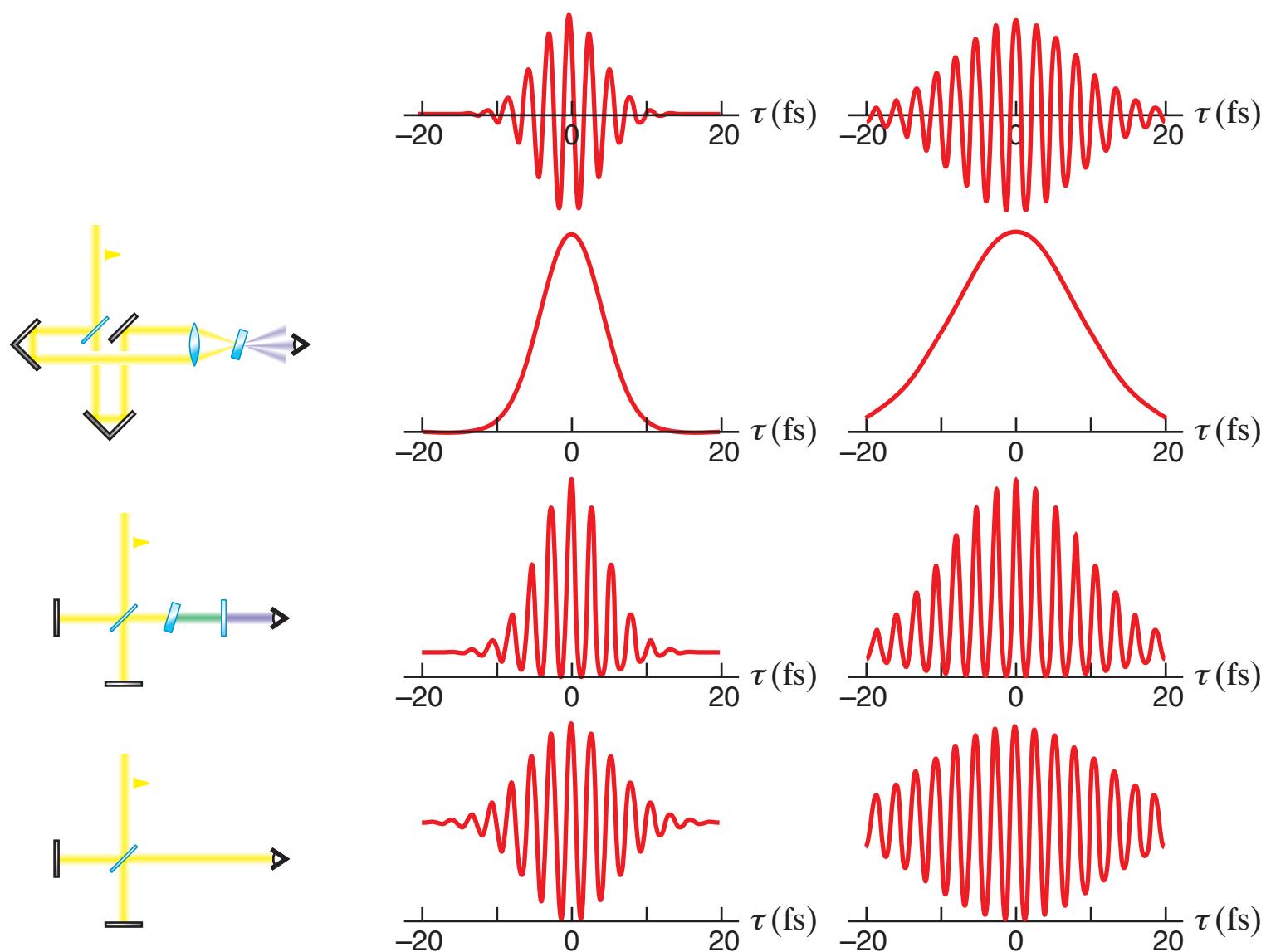
Temporal characterization



Temporal characterization



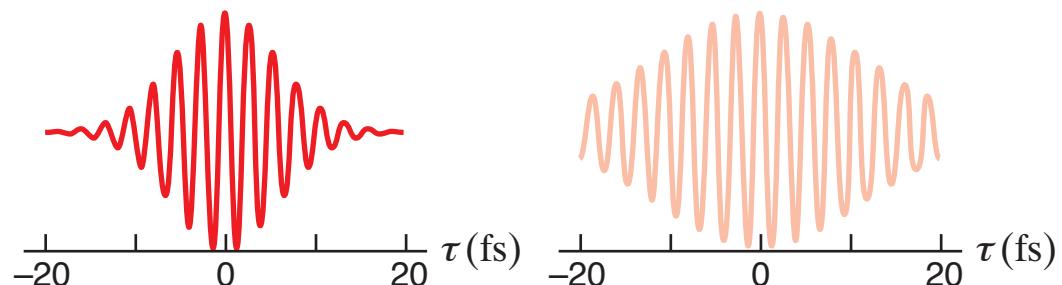
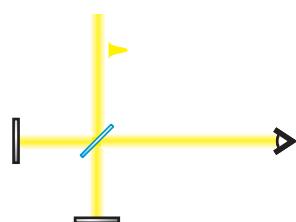
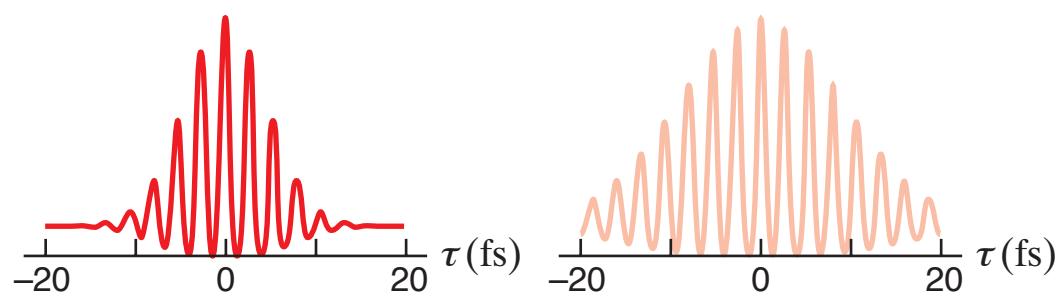
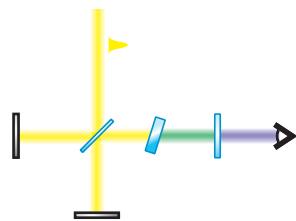
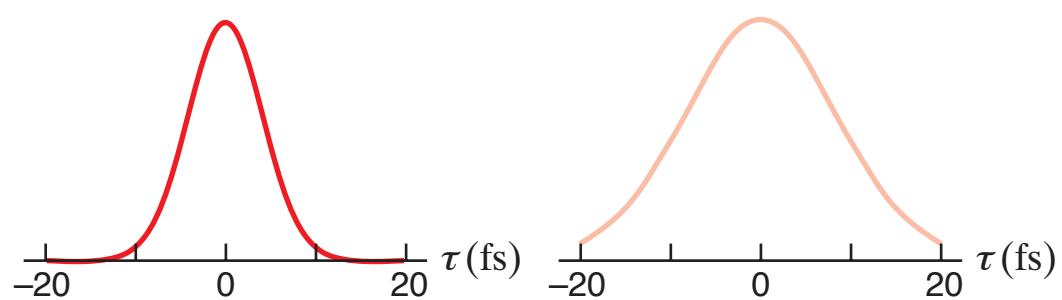
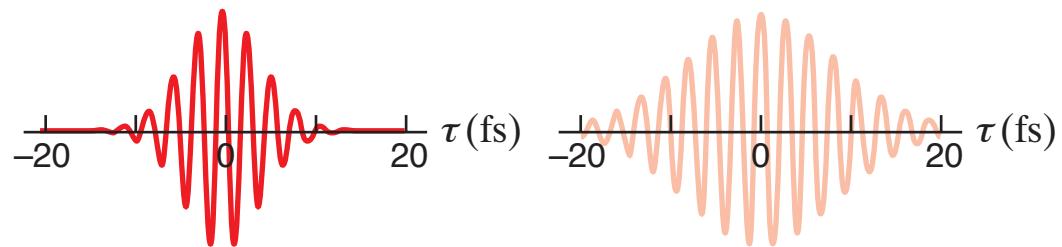
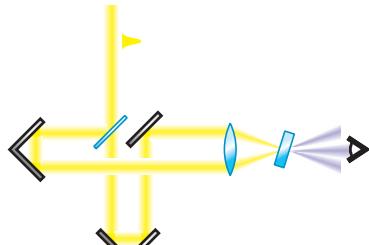
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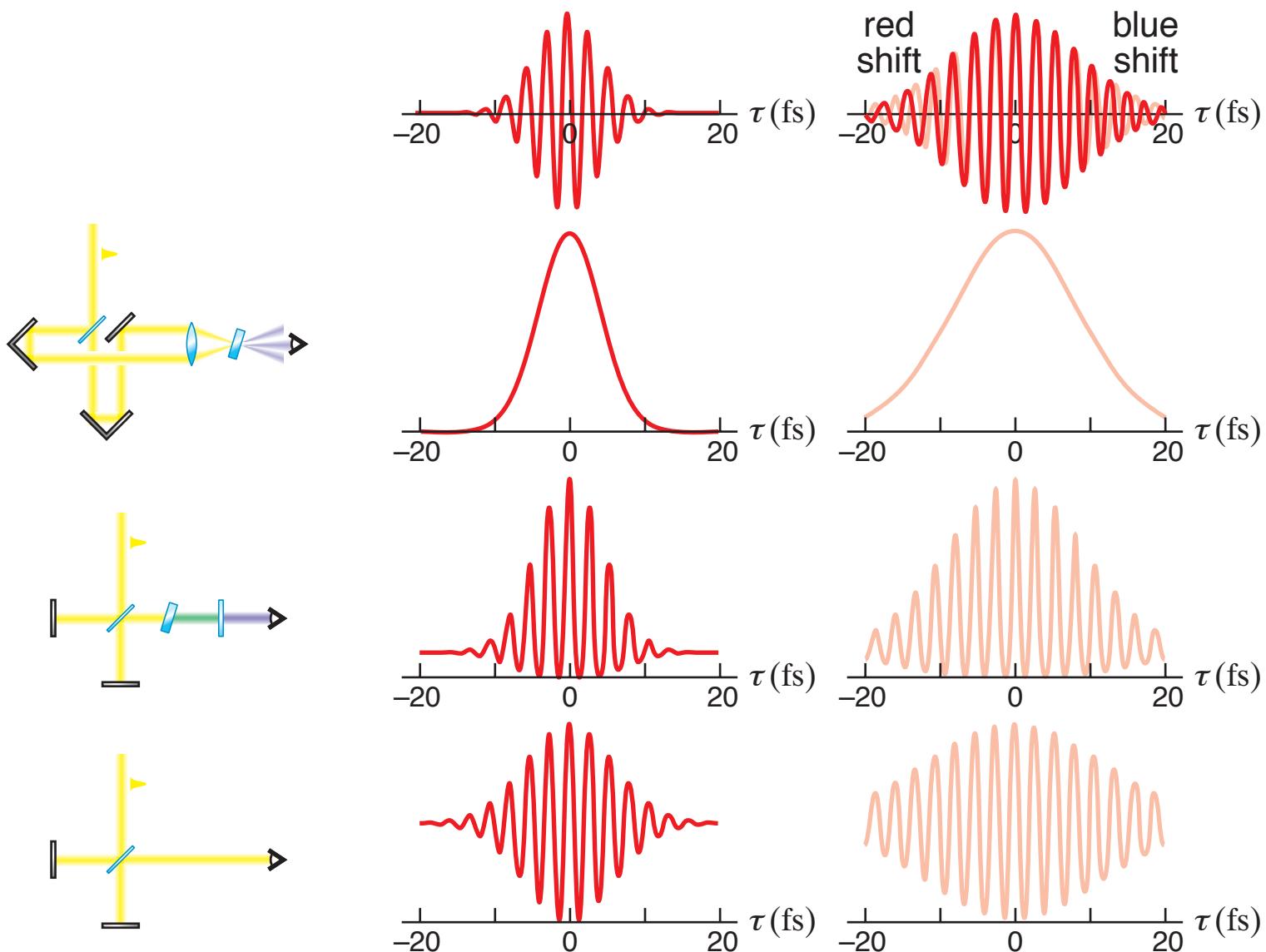
Temporal characterization

But what about dispersion?

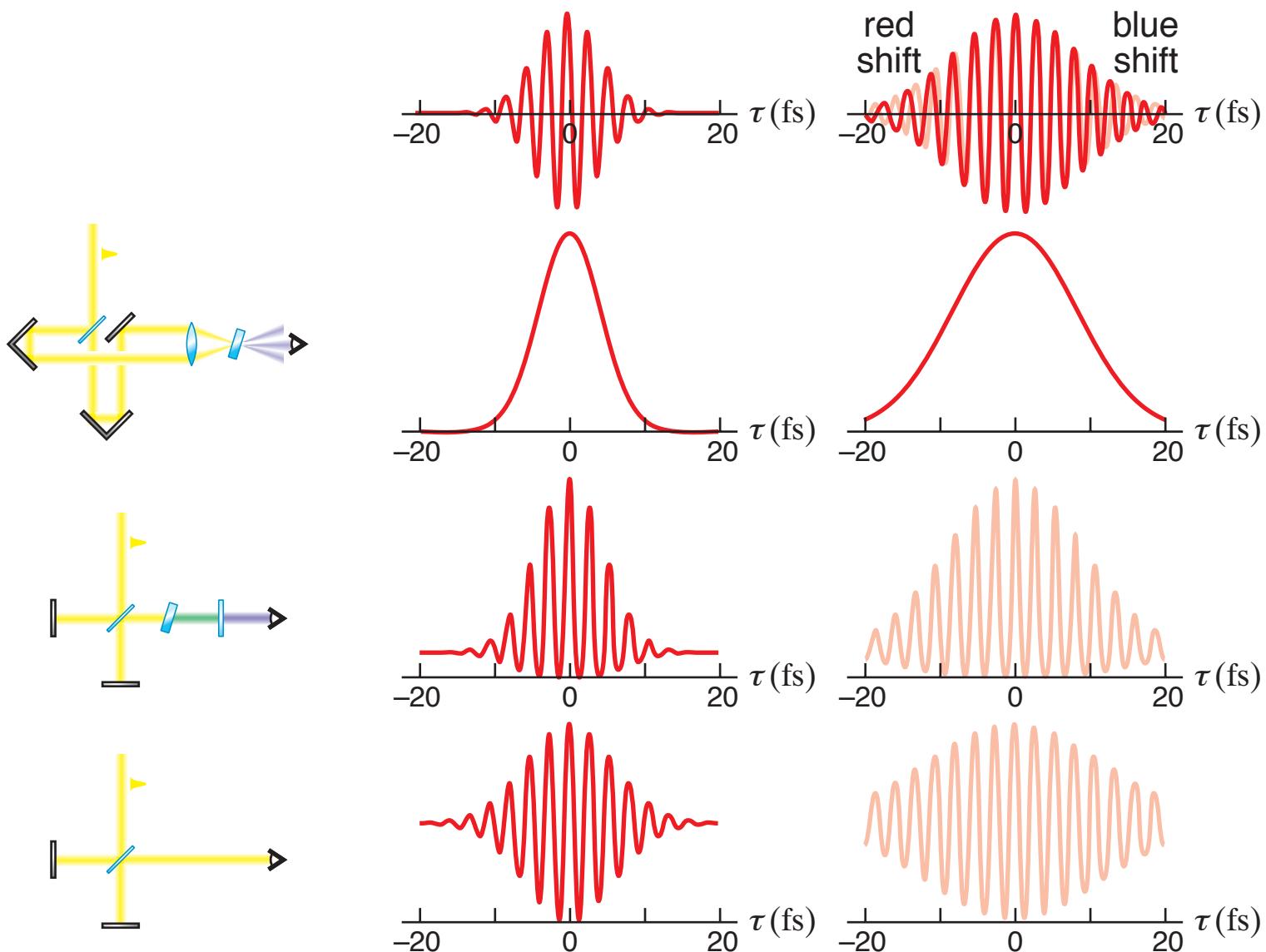
Temporal characterization



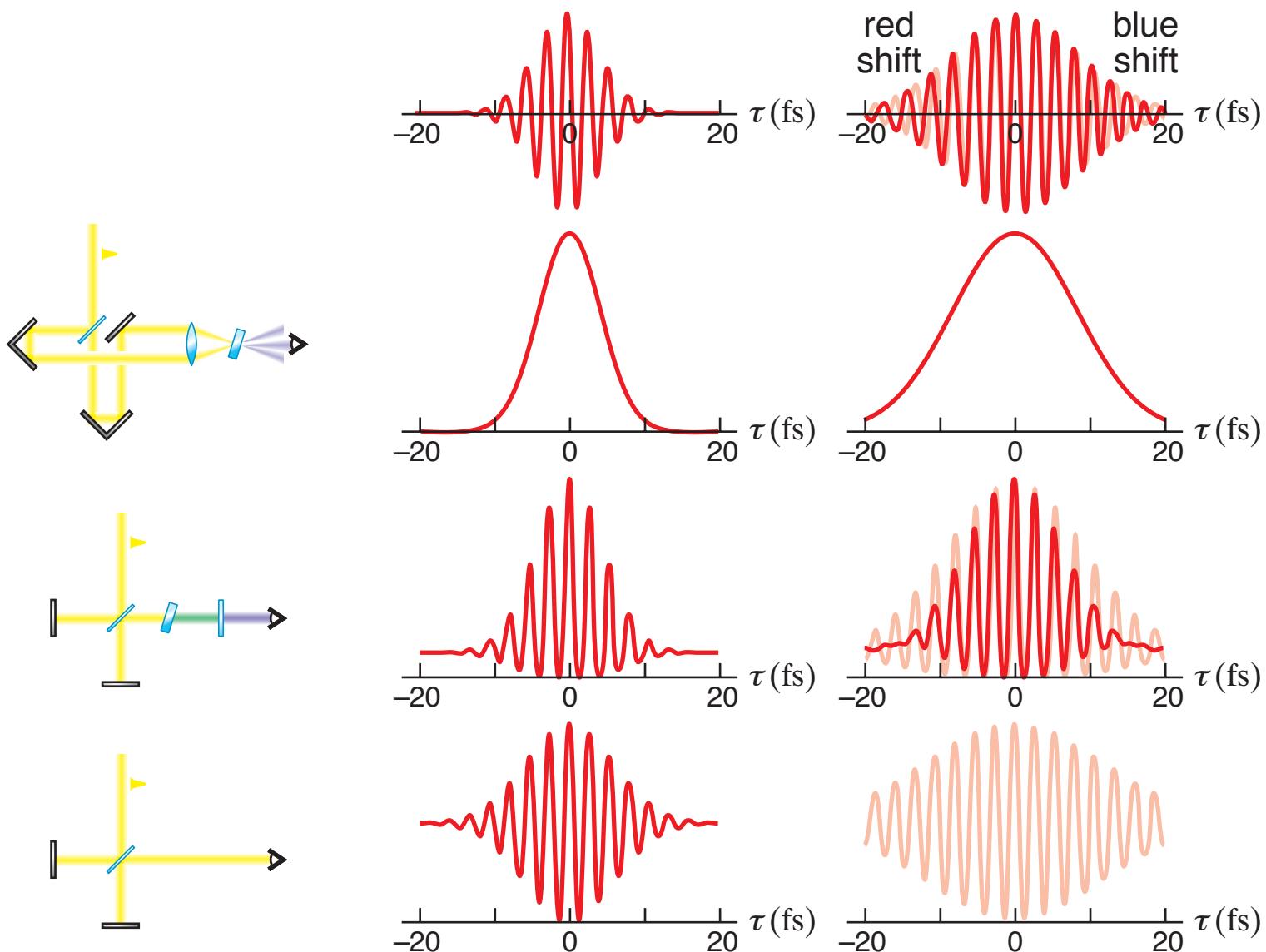
Temporal characterization



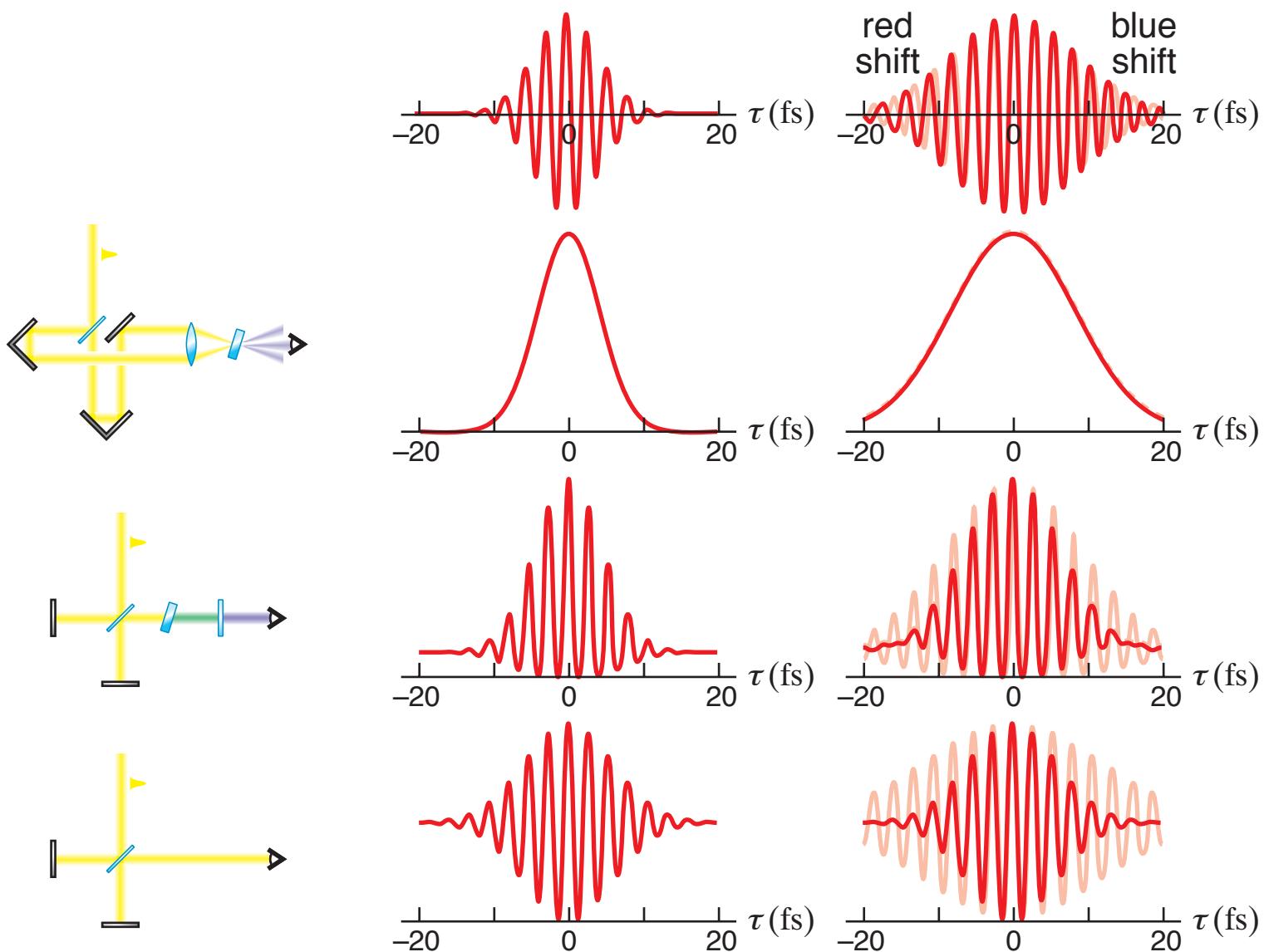
Temporal characterization



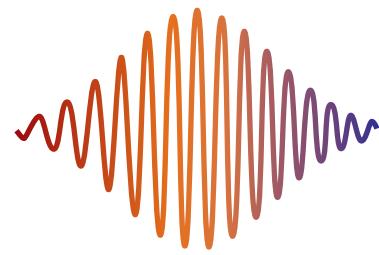
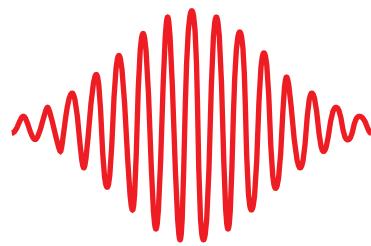
Temporal characterization



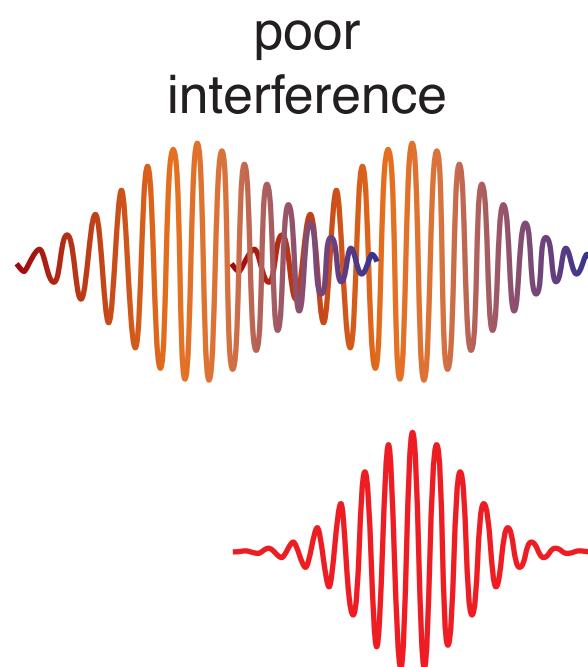
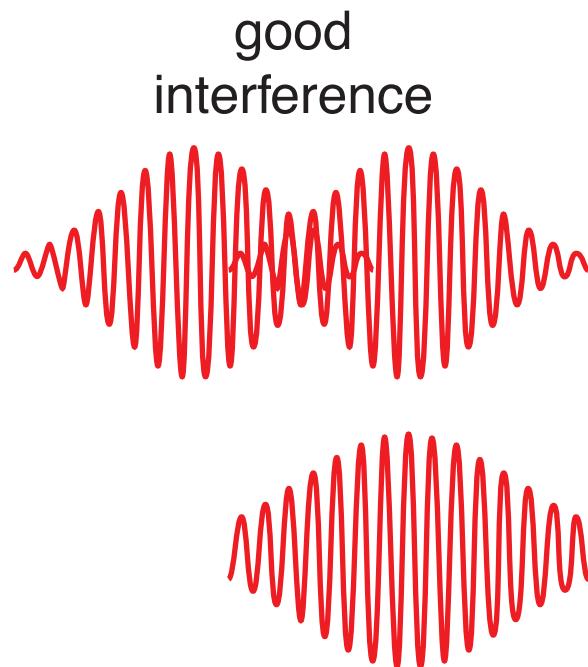
Temporal characterization



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Temporal characterization

Let $E_{disp}(\omega) = E_{orig}(\omega)e^{-i\phi(\omega)}$.

Temporal characterization

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$$f_1(t) \otimes f_2(t) \equiv \int f_1(t+\tau) {f_2}^*(\tau) d\tau = \mathcal{F}^{-1}\{f_1(\omega) {f_2}^*(\omega)\}$$

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Interference term in linear autocorrelation:

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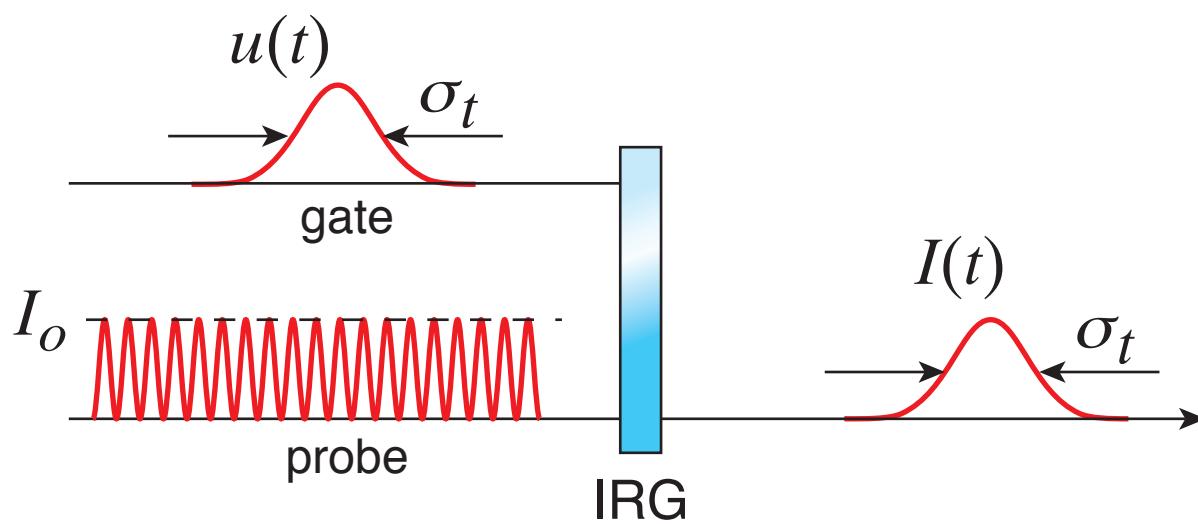
Interference term in linear autocorrelation:

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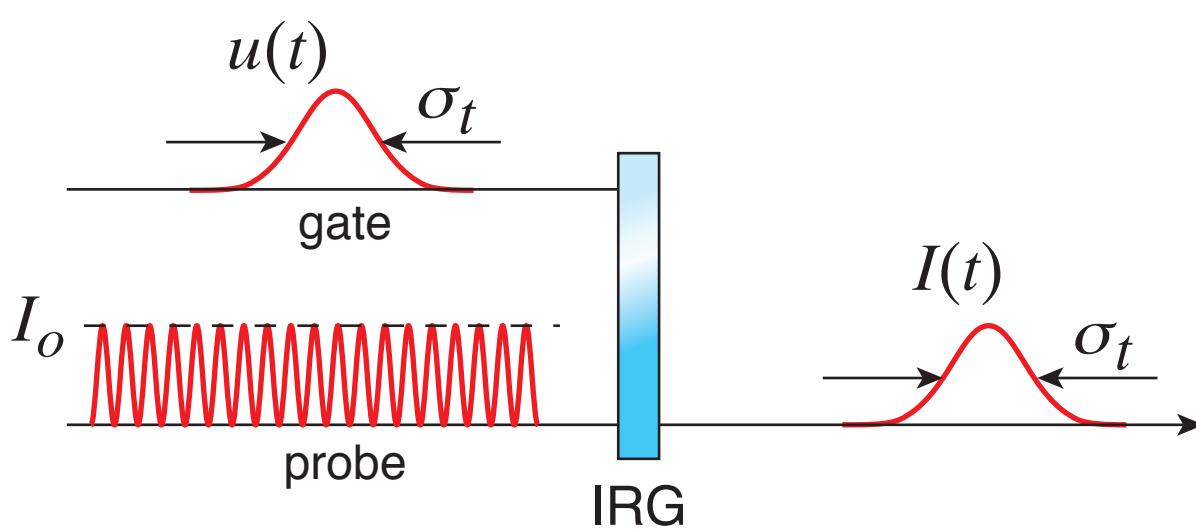
Joint time-frequency measurements



IRG (“instantaneous response gate”): device whose transmittance of a weak probe pulse is proportional to the intensity envelope of the pump (“gate”)

$$T(t) = u(t)$$

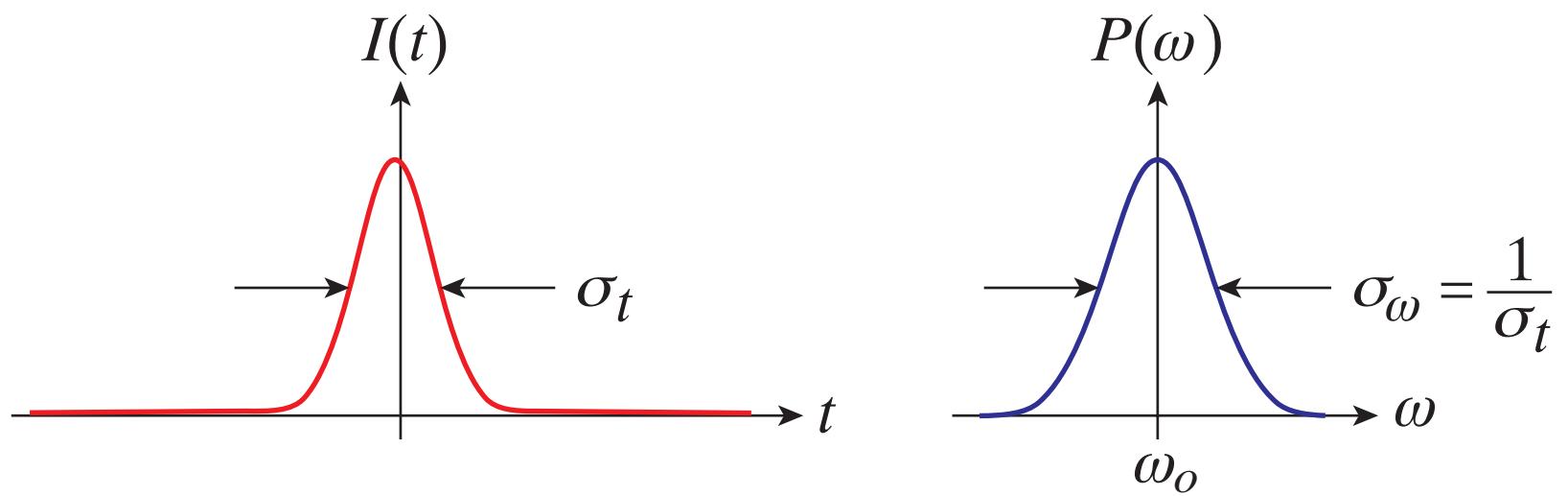
Joint time-frequency measurements



Transmitted intensity

$$I(t) = I_o T(t) = I_o u(t) = I_o \exp\left[-\frac{t^2}{\sigma_t^2}\right]$$

Joint time-frequency measurements

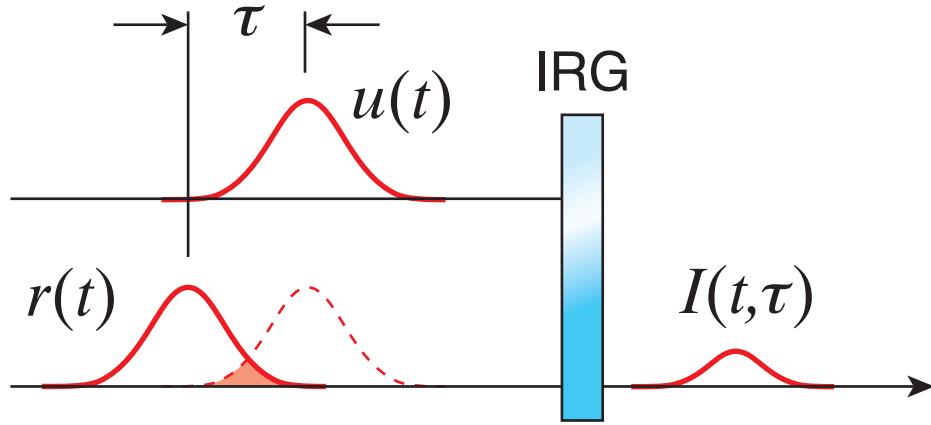


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$$\sigma_t \sigma_\omega = 1$$

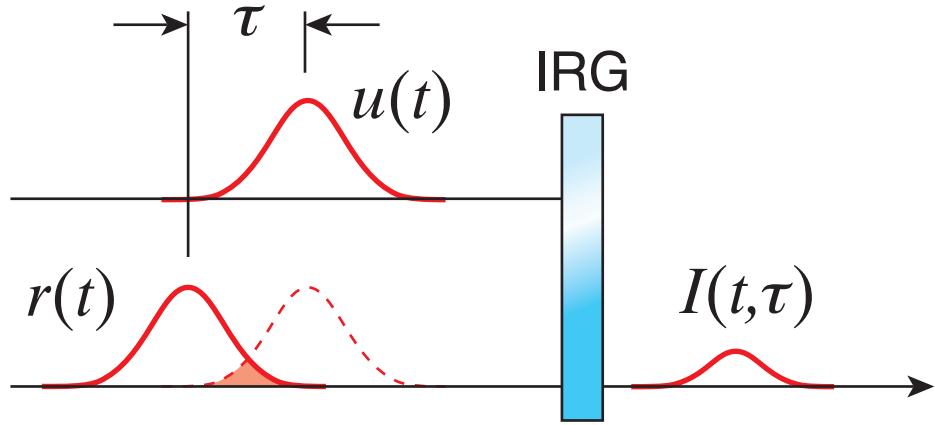
Joint time-frequency measurements



Transmitted intensity

$$\begin{aligned} I(t, \tau) &= u(t)r(t+\tau) = \exp\left[-\frac{t^2}{\sigma^2}\right] \exp\left[-\left(\frac{t+\tau}{\sigma}\right)^2\right] = \\ &= \exp\left[-\frac{2t^2+2t\tau+\tau^2}{\sigma^2}\right] = \exp\left[-\frac{2t^2+2t\tau+\tau^2/2}{\sigma^2}\right] \exp\left[-\frac{\tau^2}{2\sigma^2}\right] = \end{aligned}$$

Joint time-frequency measurements

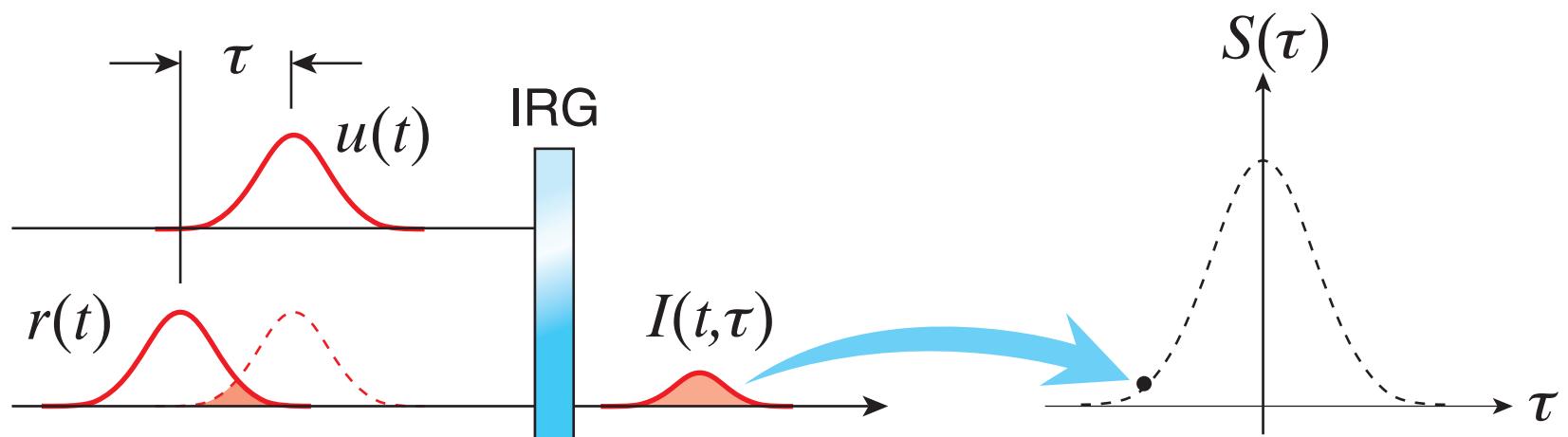


Transmitted intensity

$$I(t, \tau) = \exp\left[-\frac{\tau^2}{2\sigma^2}\right] \exp\left[-\left(\frac{t + \tau/2}{\sigma/\sqrt{2}}\right)^2\right]$$

so $I(t, \tau)$ narrowed by $\sqrt{2}$

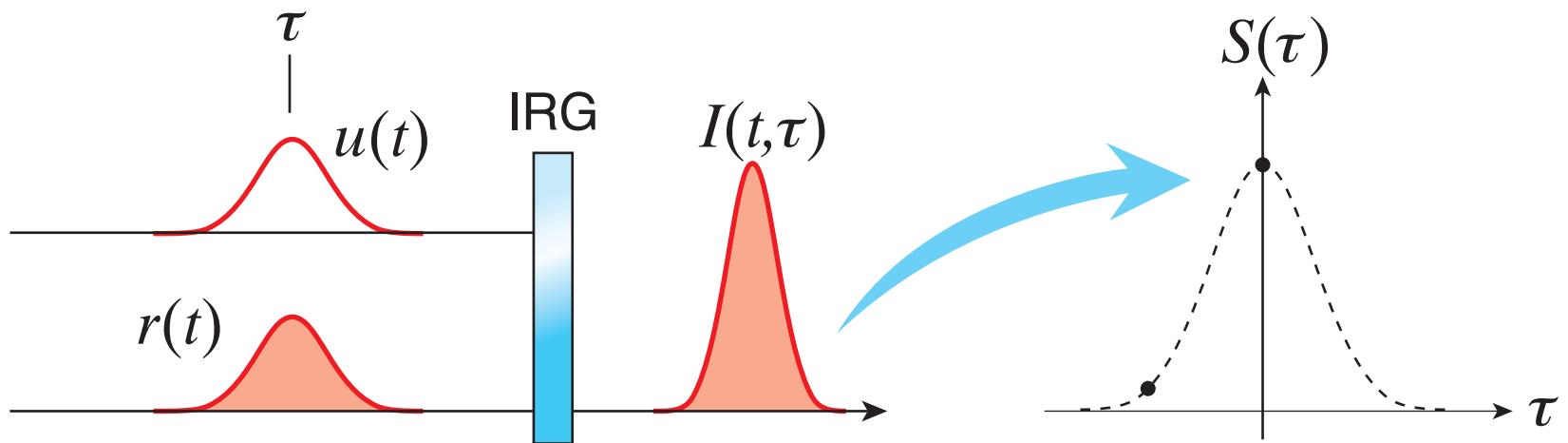
Joint time-frequency measurements



...but detector integrates $I(t, \tau)$:

$$S(\tau) = \int_{-\infty}^{\infty} I(t, \tau) dt = \int_{-\infty}^{\infty} \exp\left[-\frac{\tau^2}{2\sigma^2}\right] \exp\left[-2\left(\frac{t + \tau/2}{\sigma}\right)^2\right] dt$$

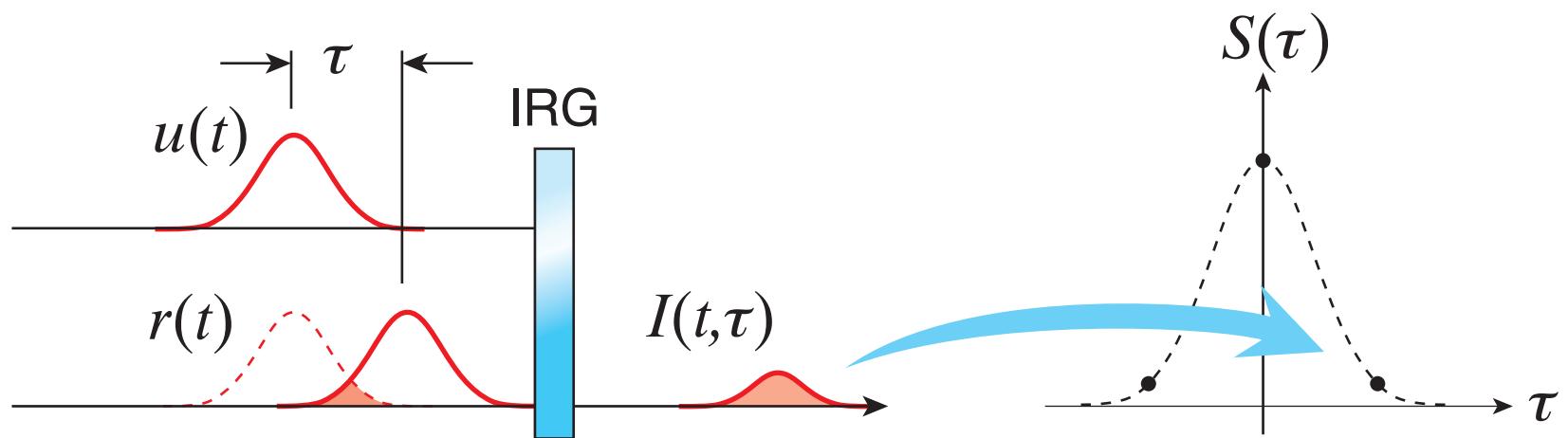
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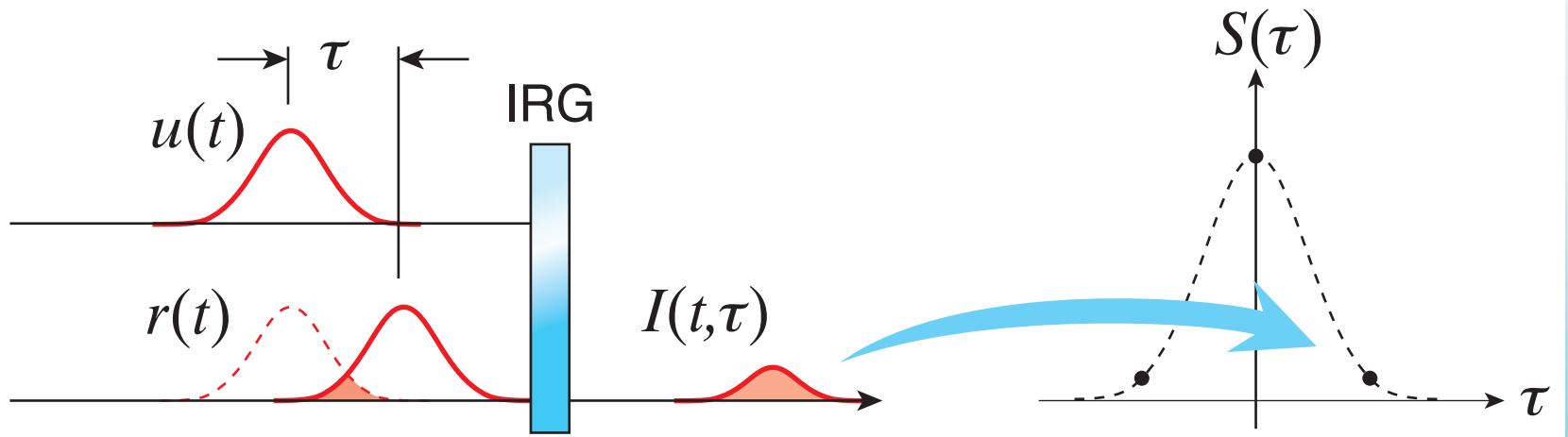
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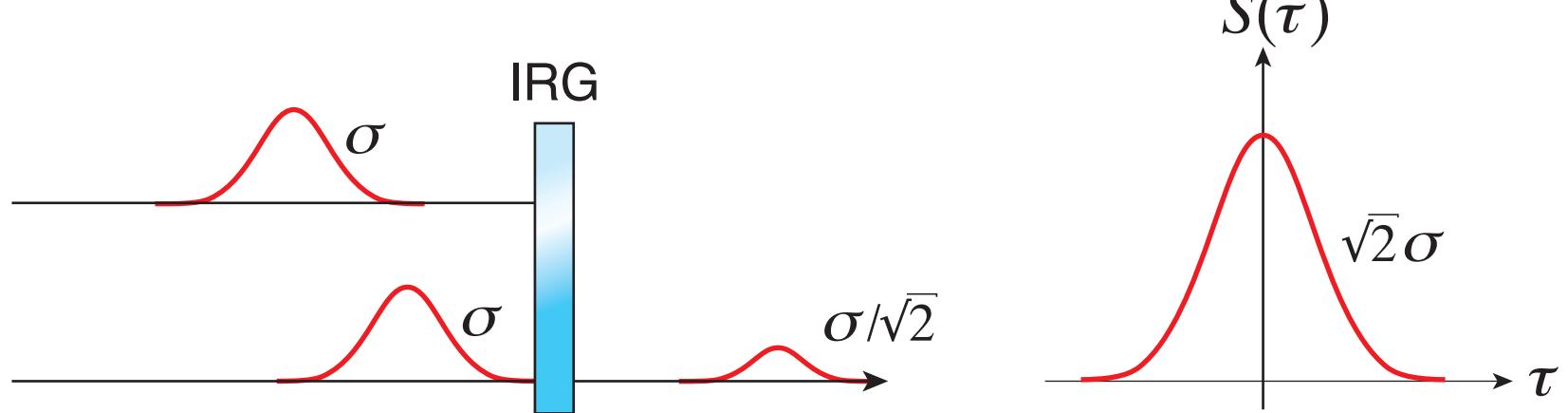
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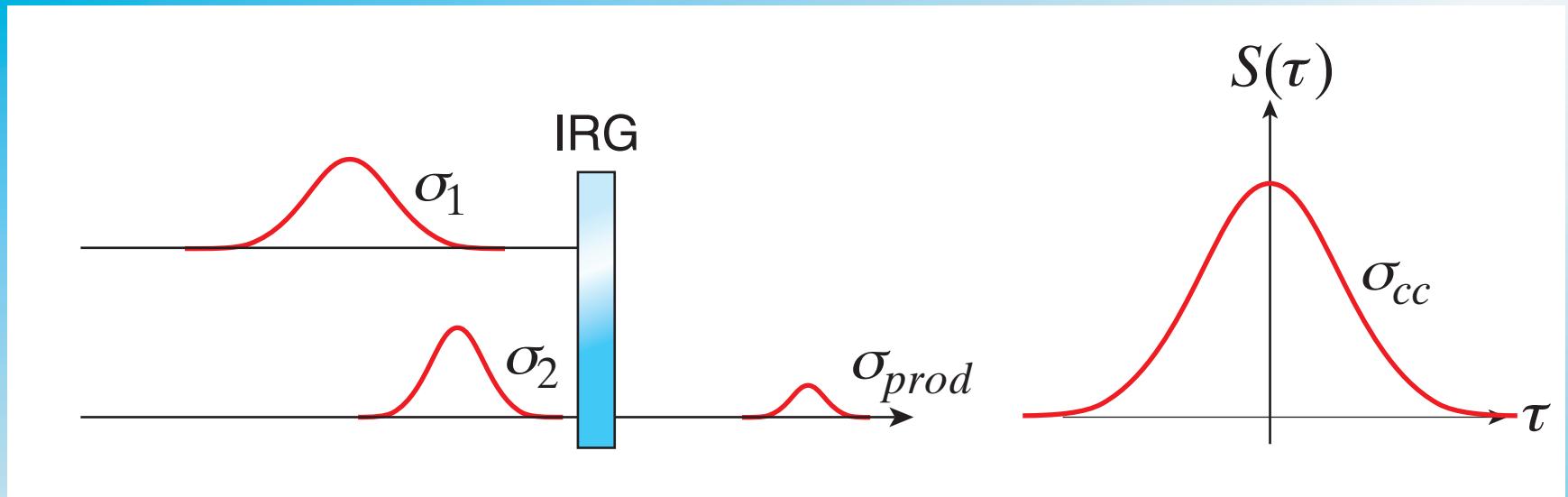
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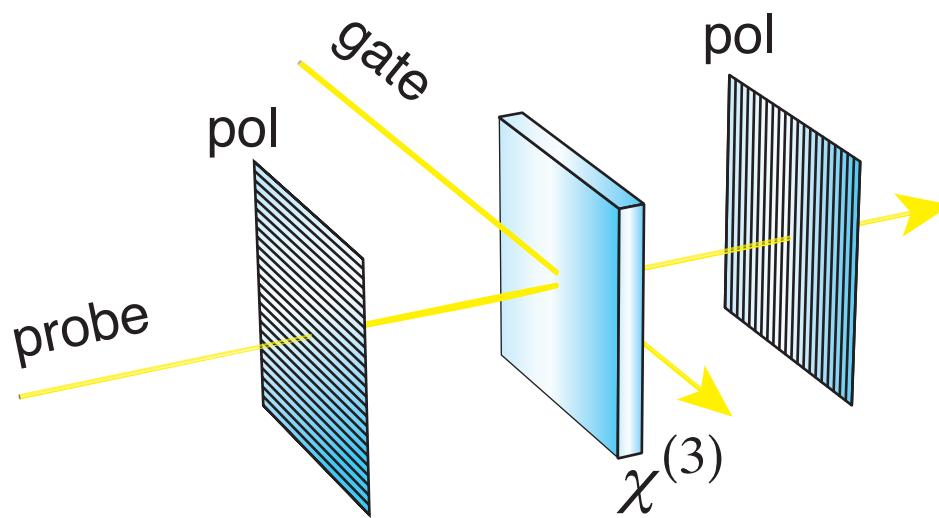


If gate and probe unequal:

$$\sigma_{prod}^2 = \frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 + \sigma_2^2} \quad (\text{narrower than both})$$

$$\sigma_{cc}^2 = \sigma_1^2 + \sigma_2^2 \quad (\text{wider than both})$$

Joint time-frequency measurements

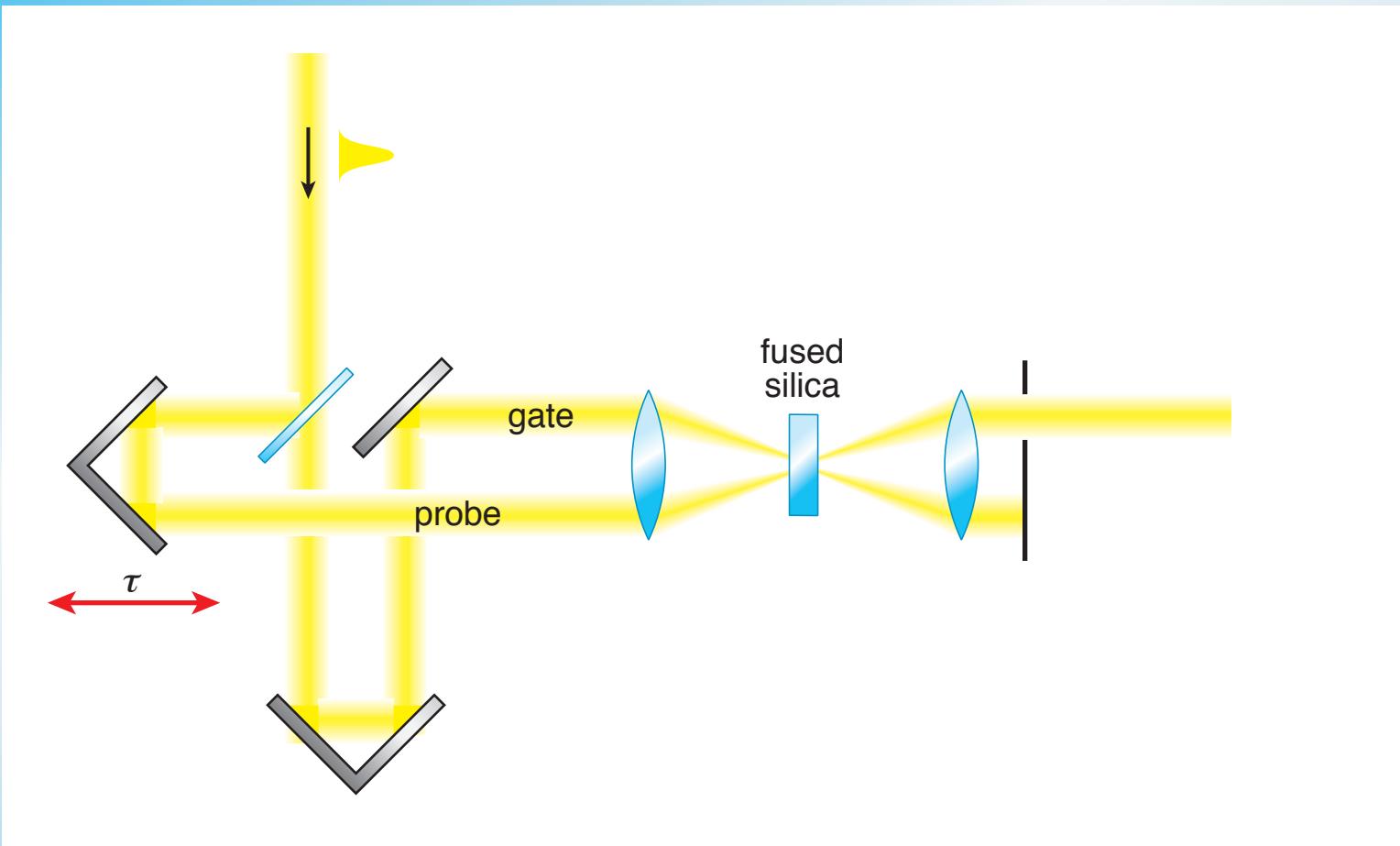


Transmitted field:

$$E_{trans}(t, \tau) \propto \chi^{(3)} E_{probe}(t) |E_{gate}(t + \tau)|^2$$

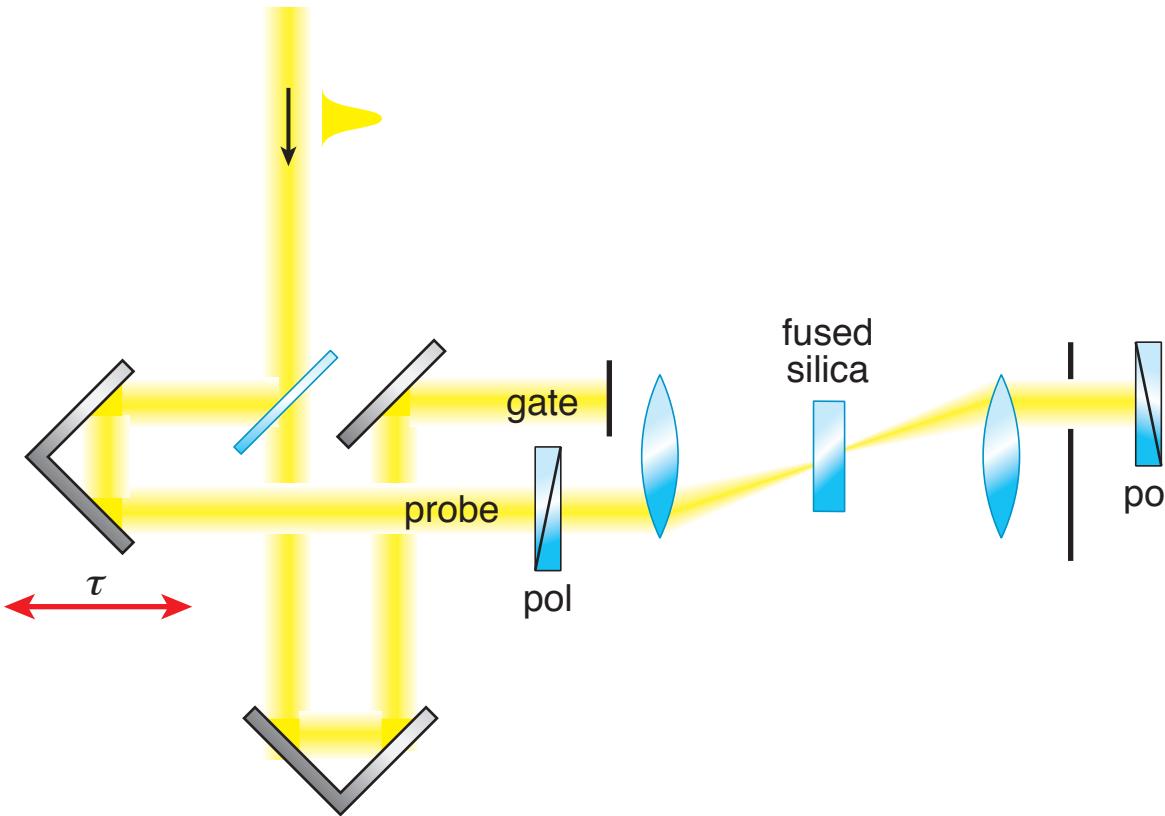
Joint time-frequency measurements

FROG: frequency-resolved optical gating



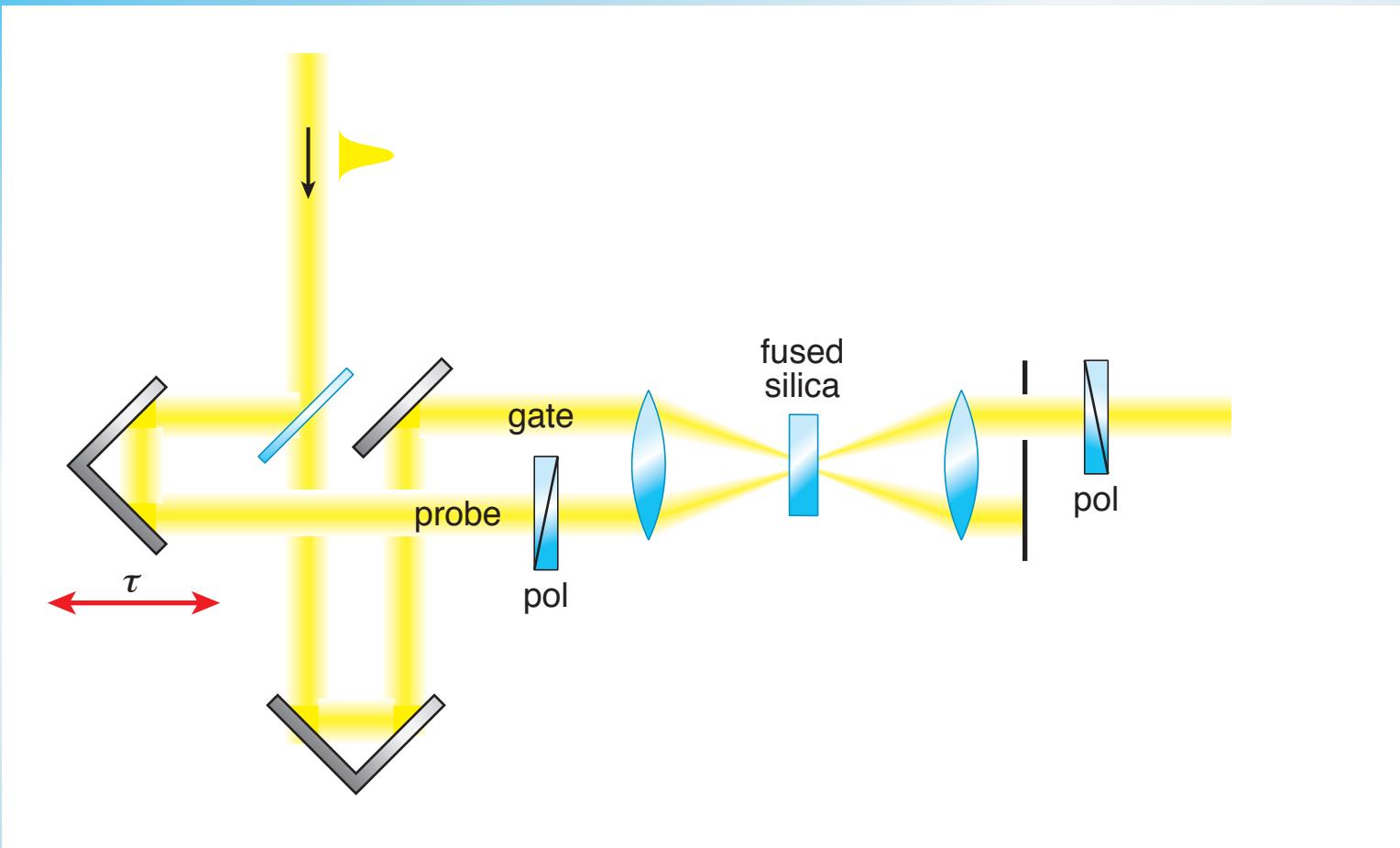
Joint time-frequency measurements

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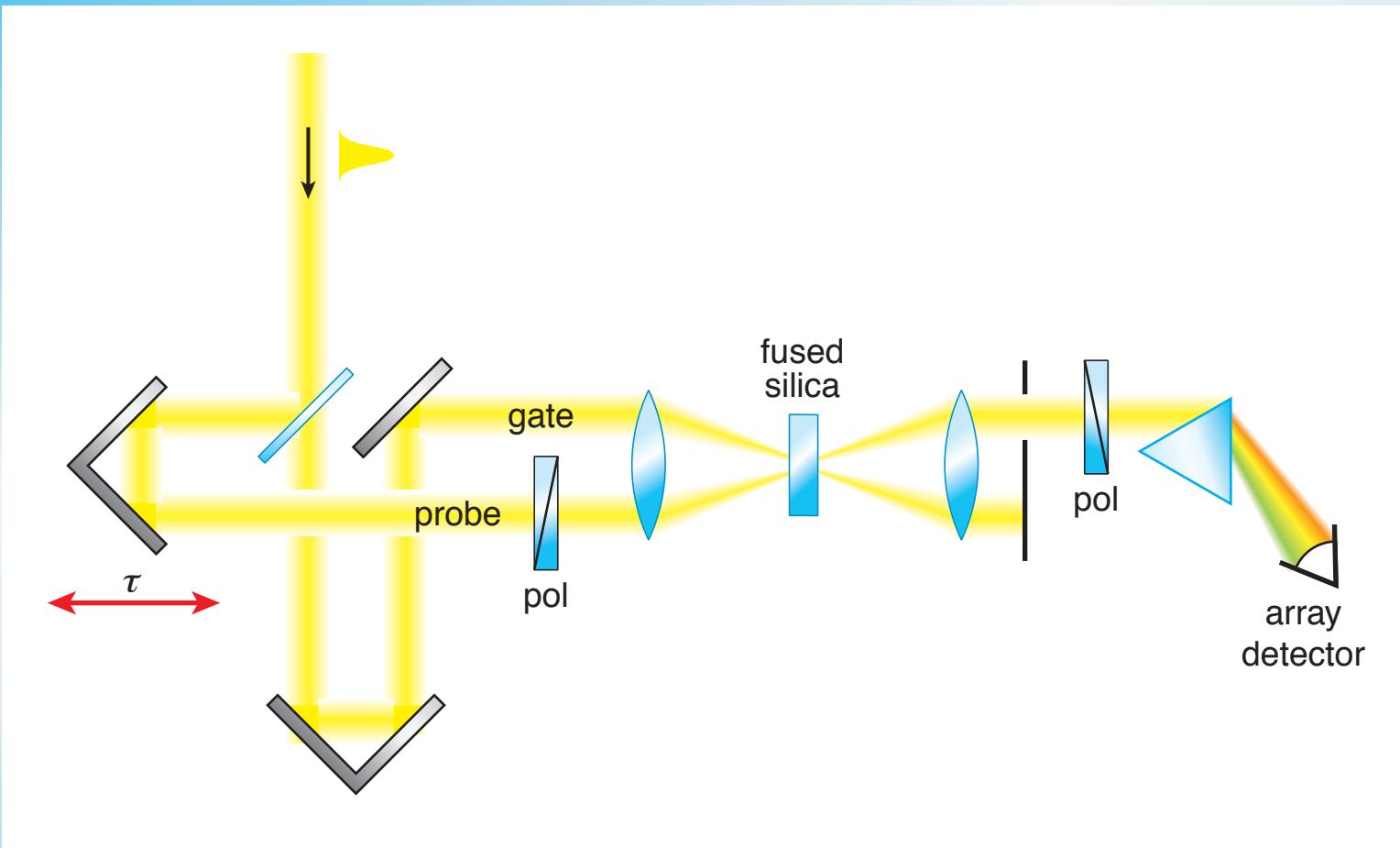
Joint time-frequency measurements

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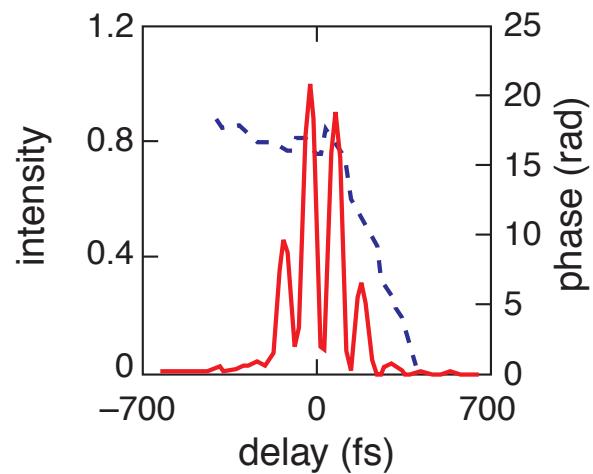
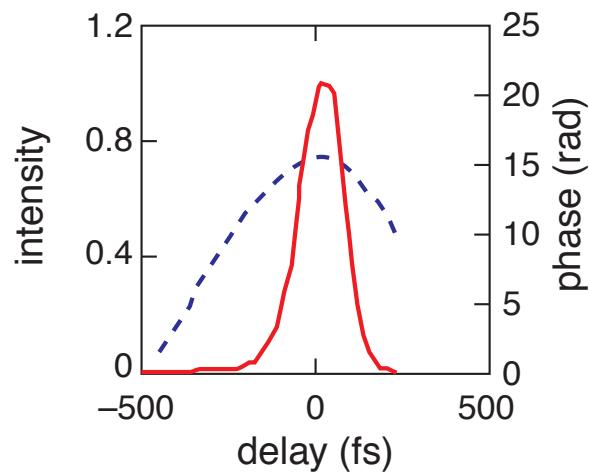
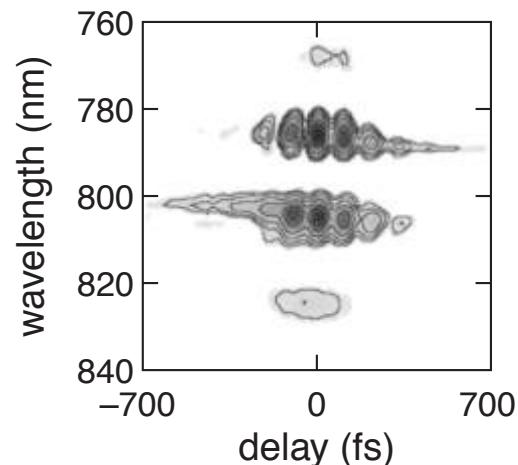
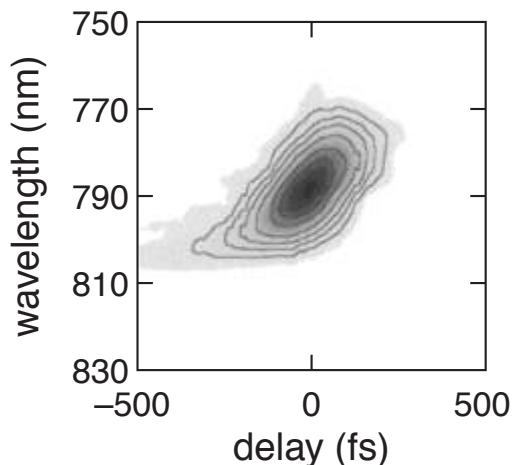


Joint time-frequency measurements

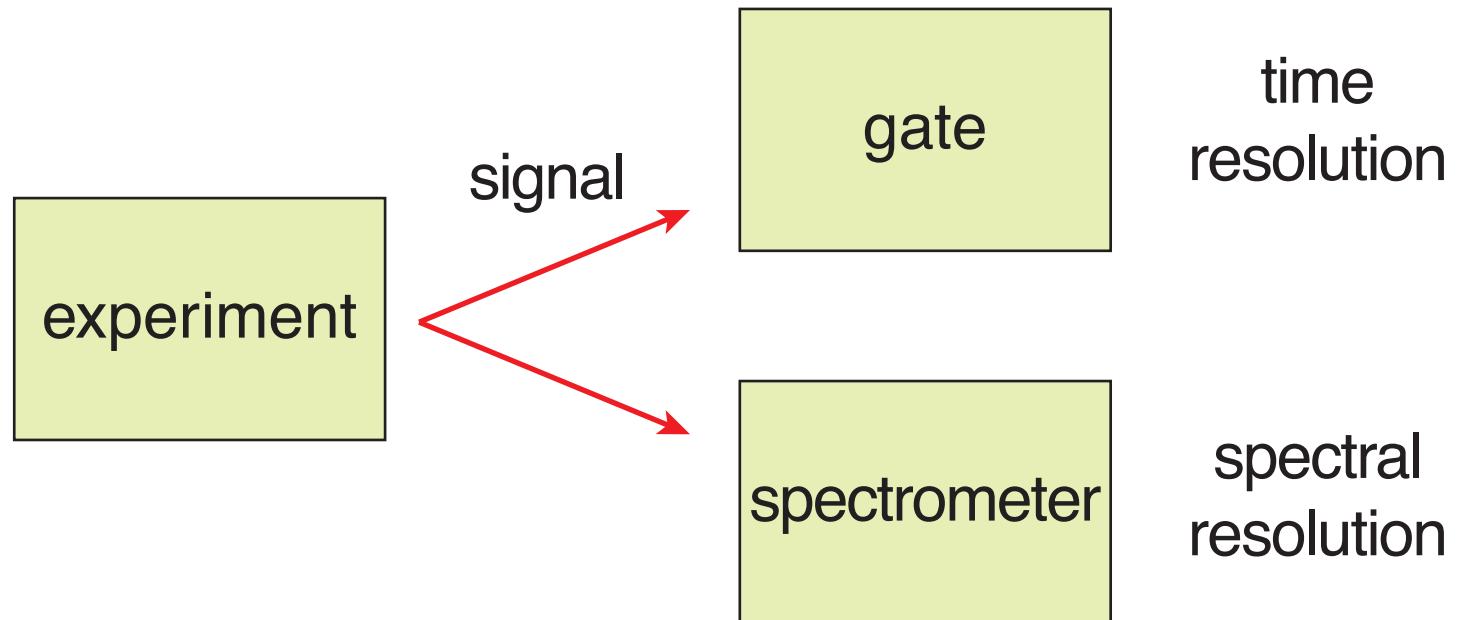
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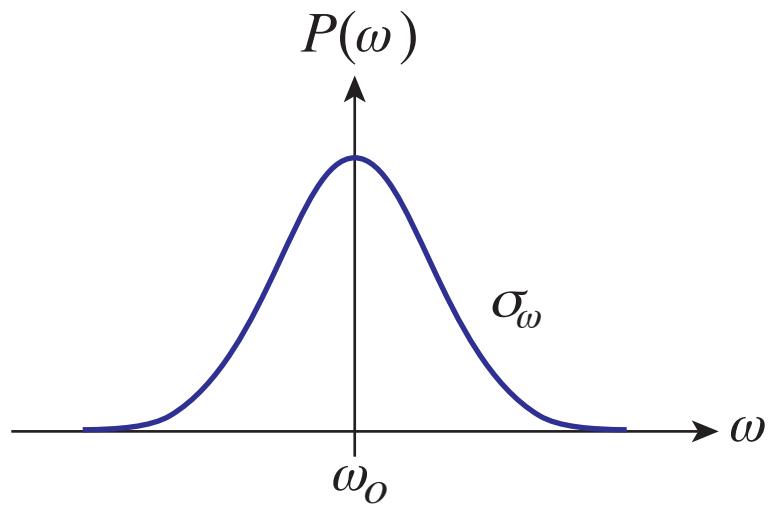
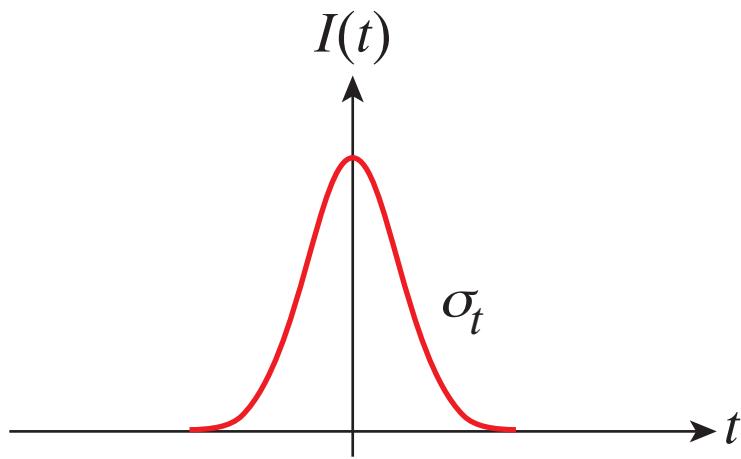
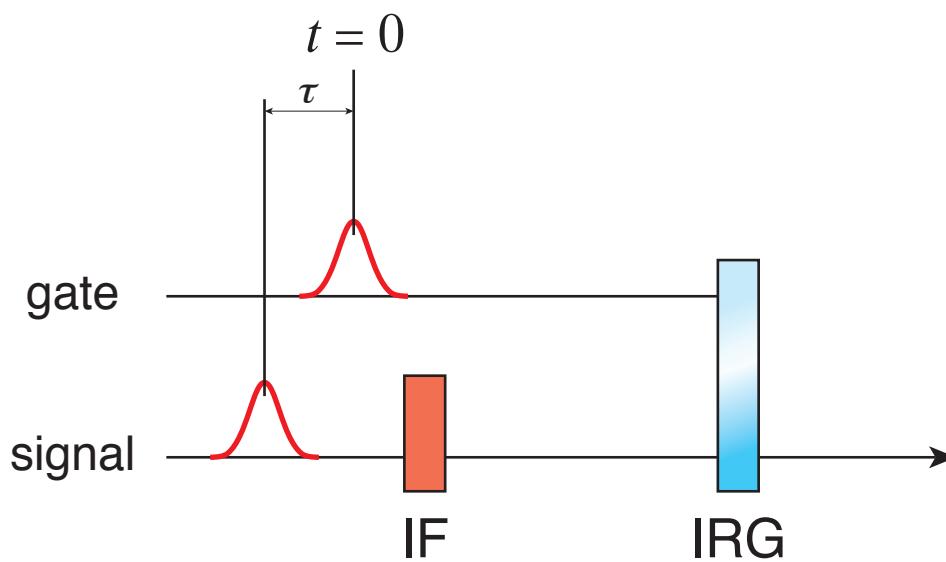
Joint time-frequency measurements



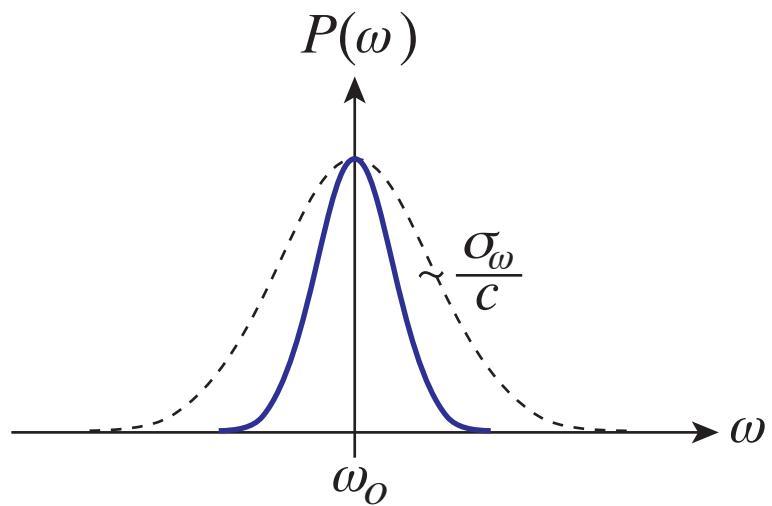
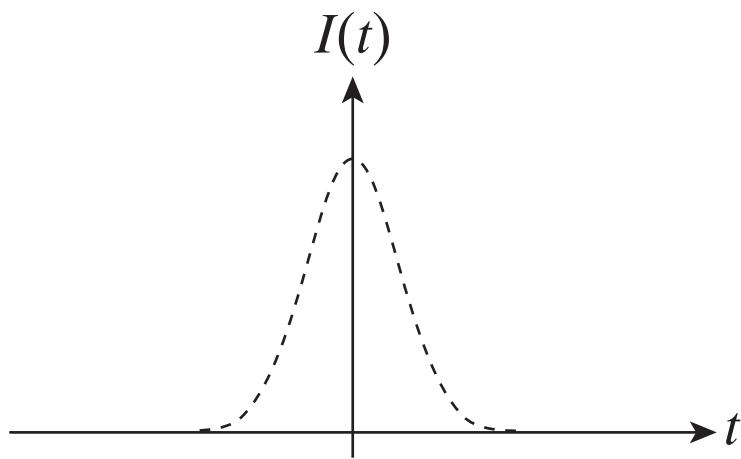
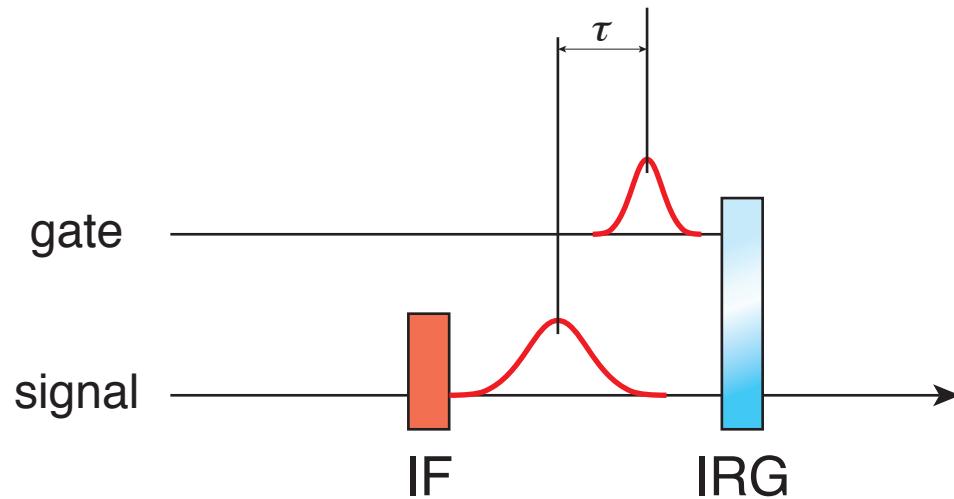
Joint time-frequency measurements

What are the resolution limits?

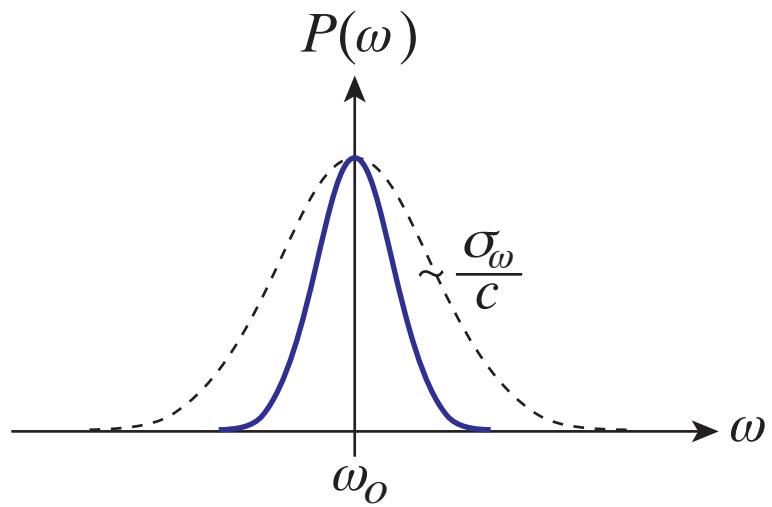
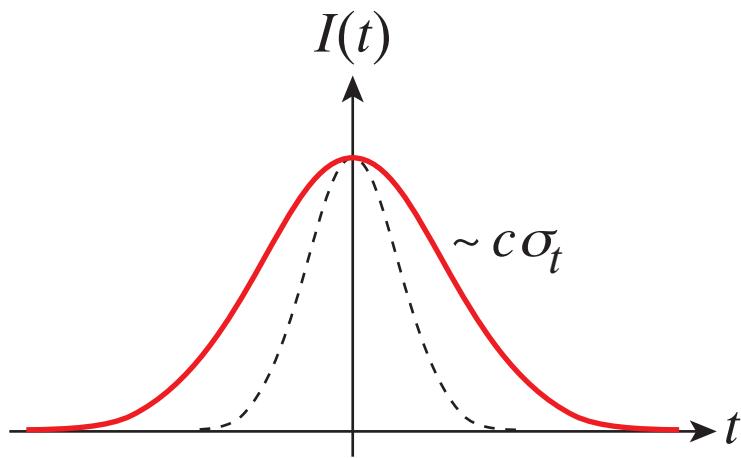
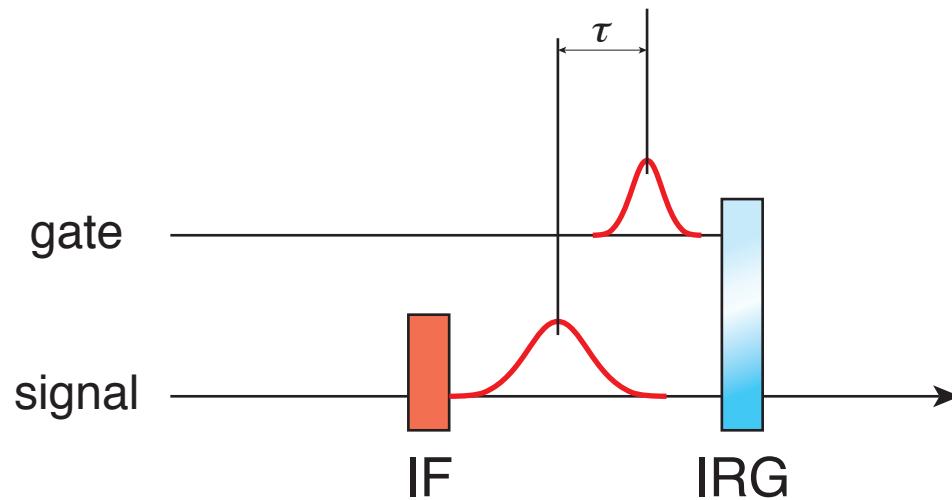
Experiment 1



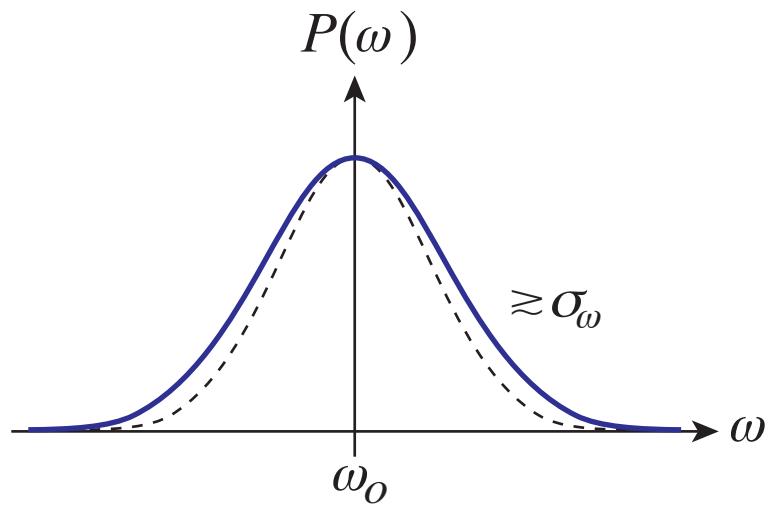
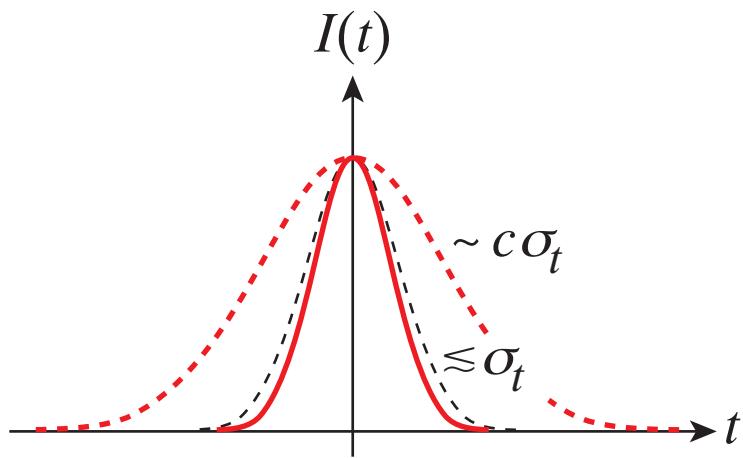
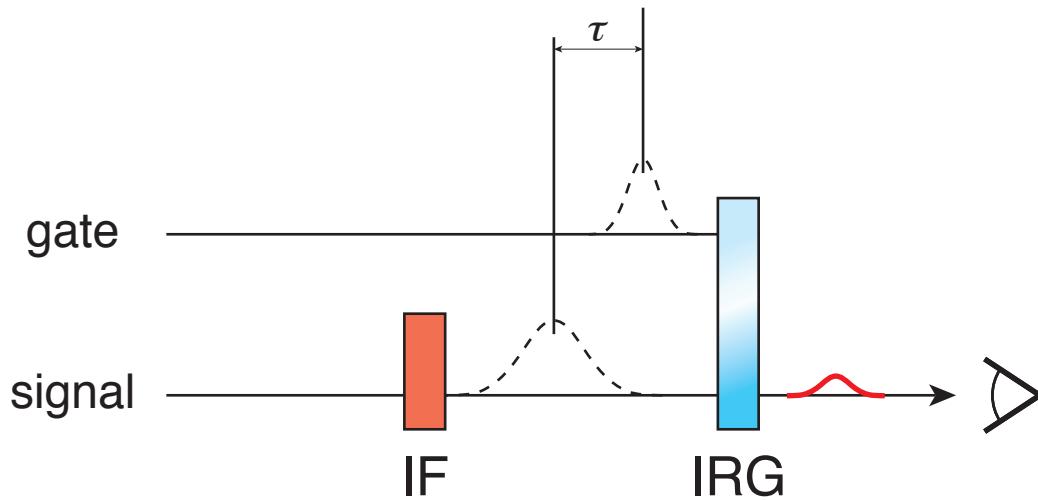
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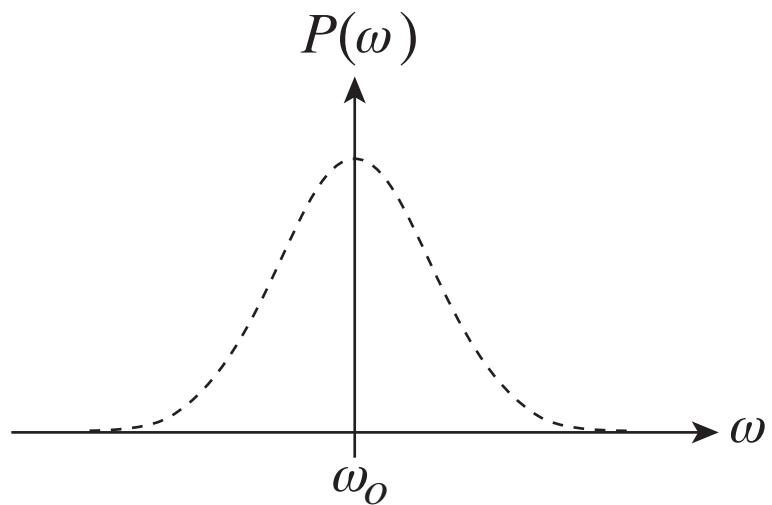
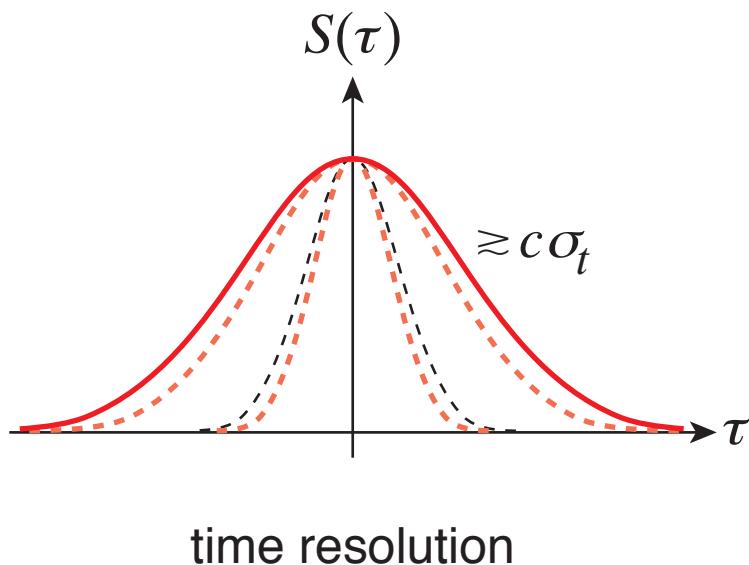
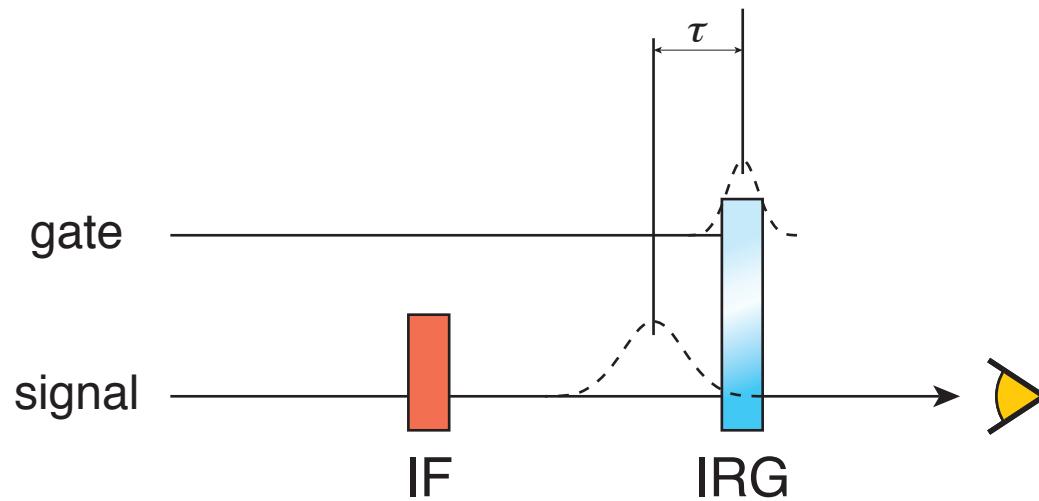
Experiment 1



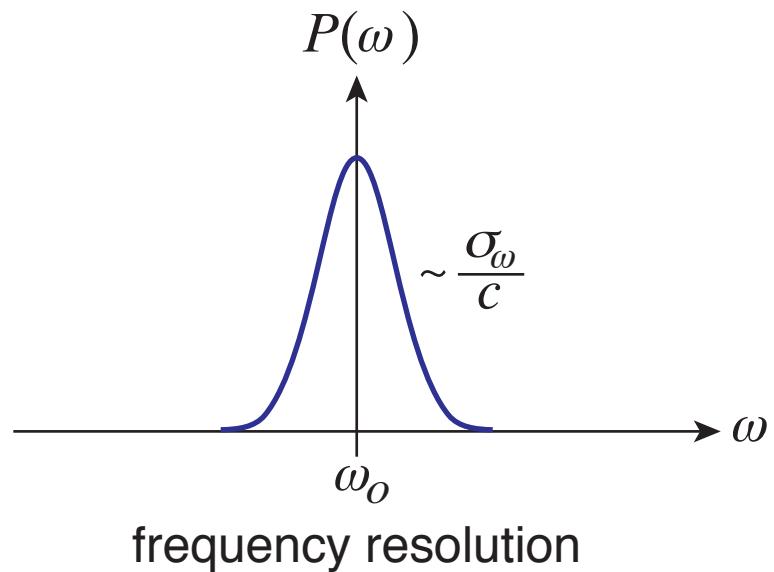
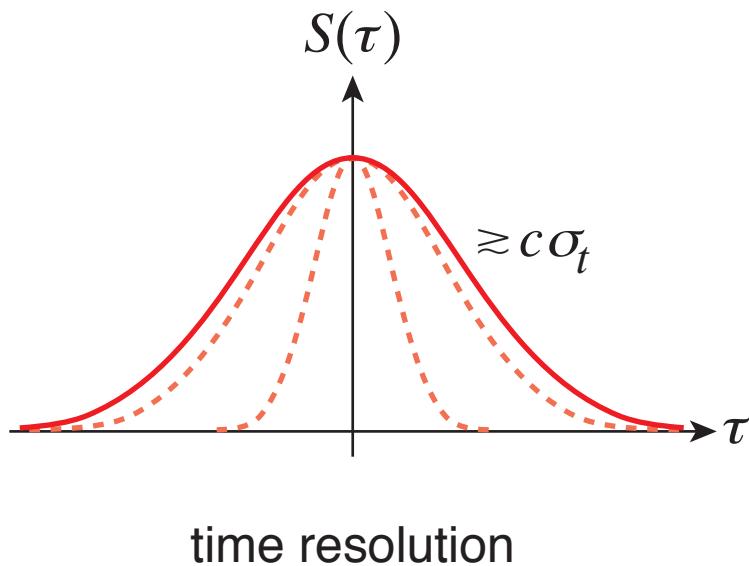
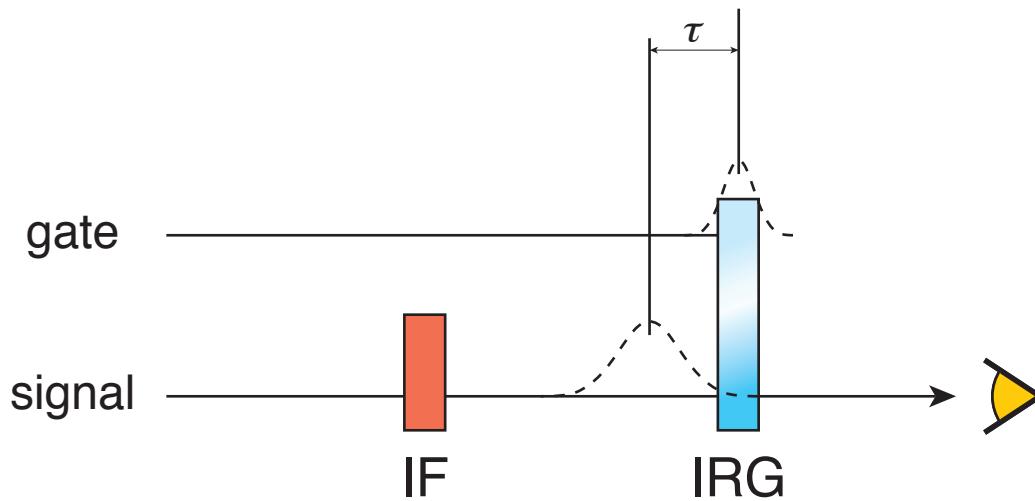
Experiment 1



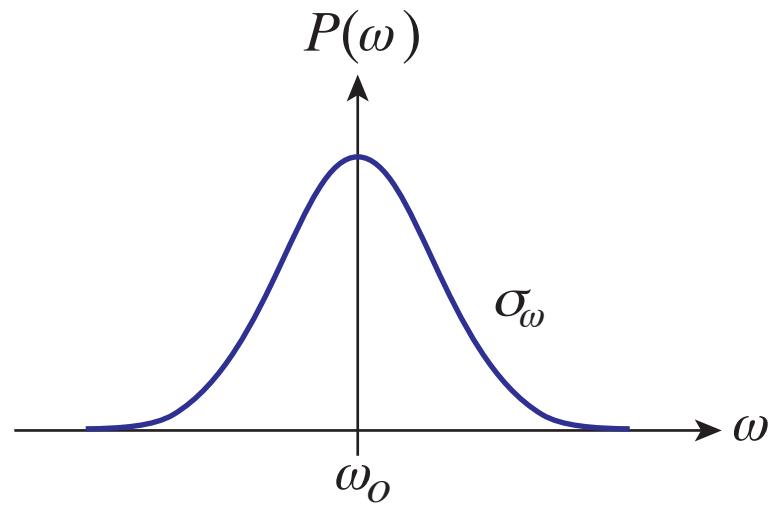
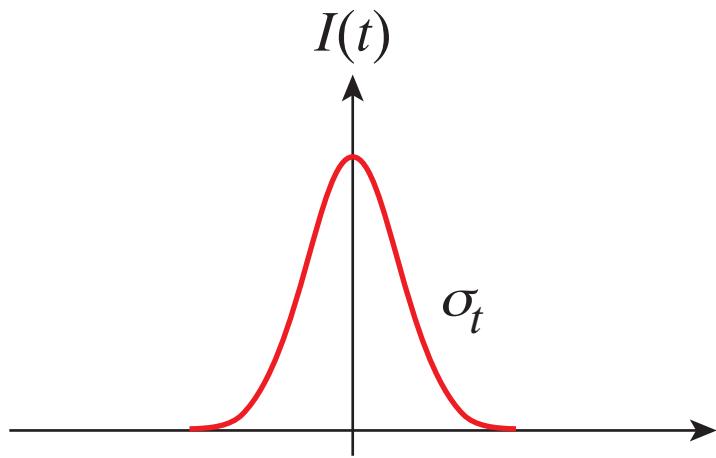
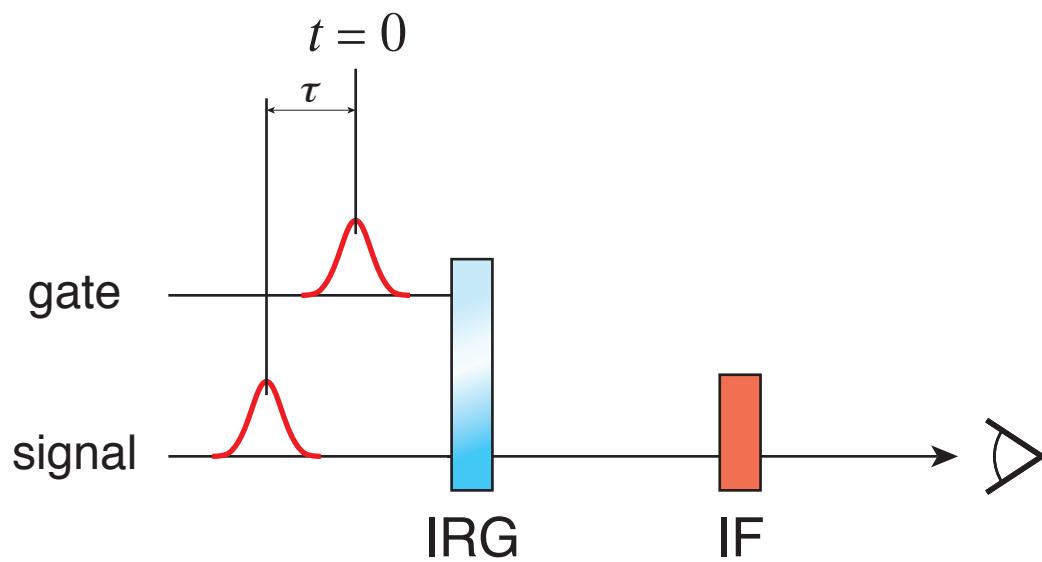
Experiment 1



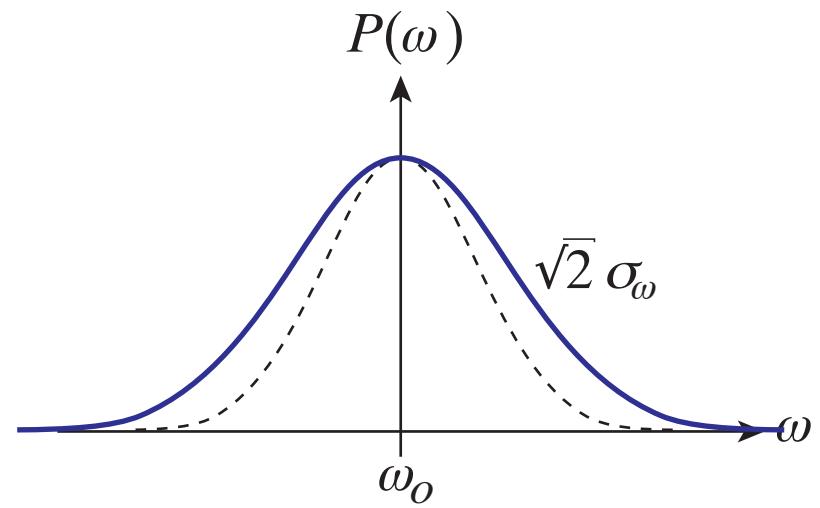
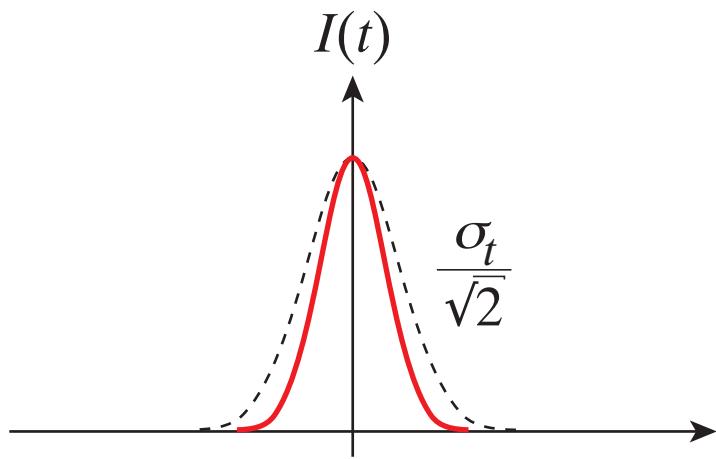
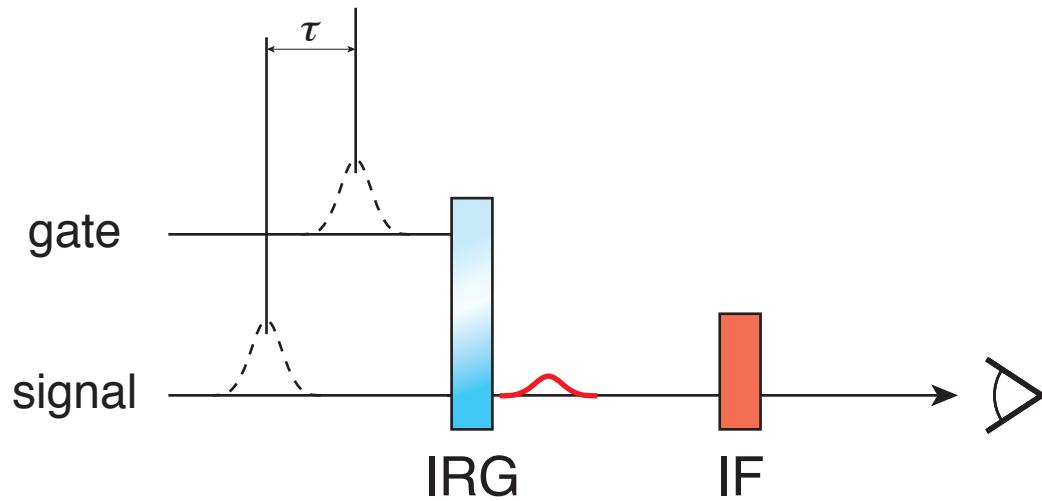
Experiment 1



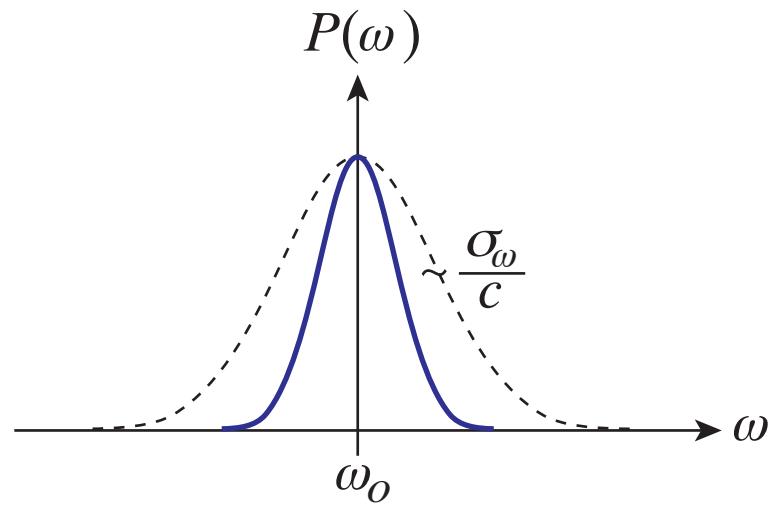
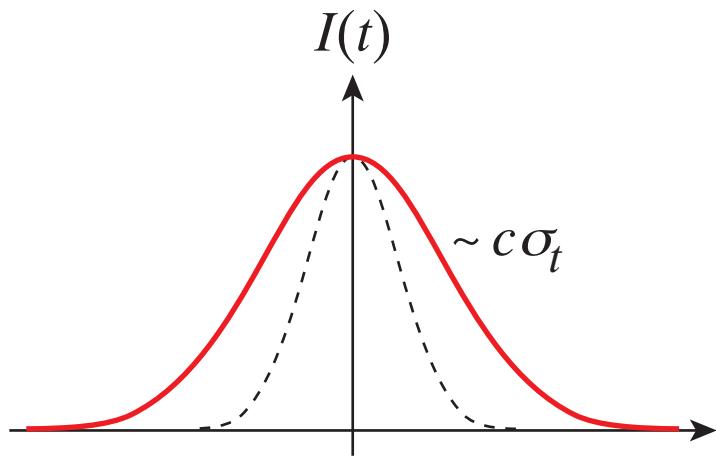
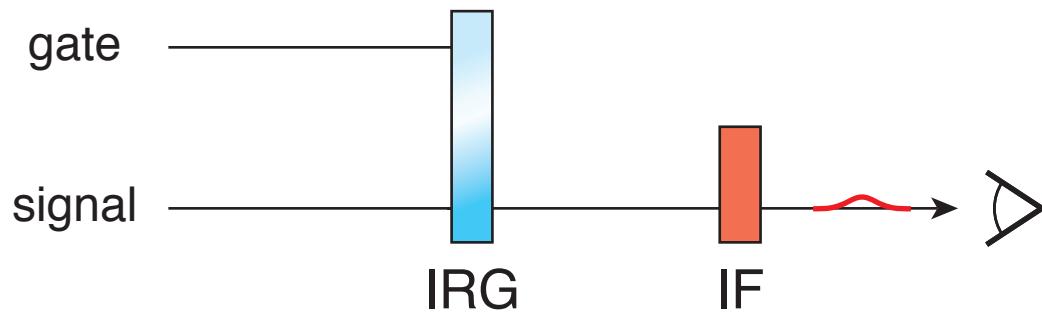
Experiment 2



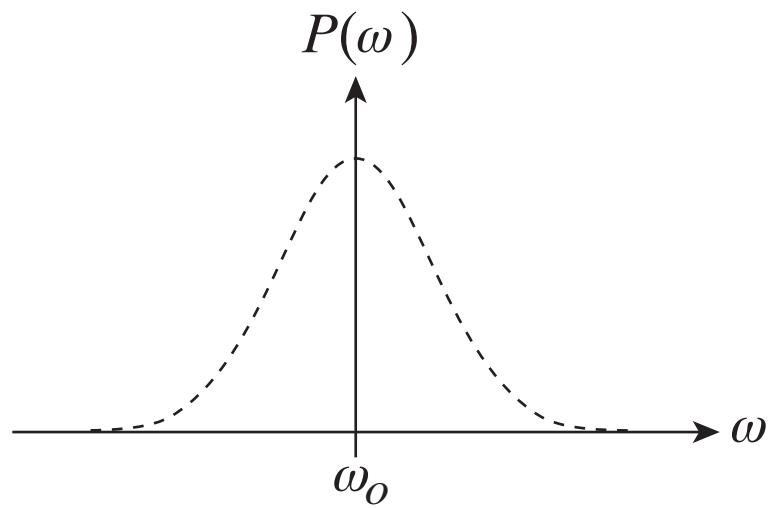
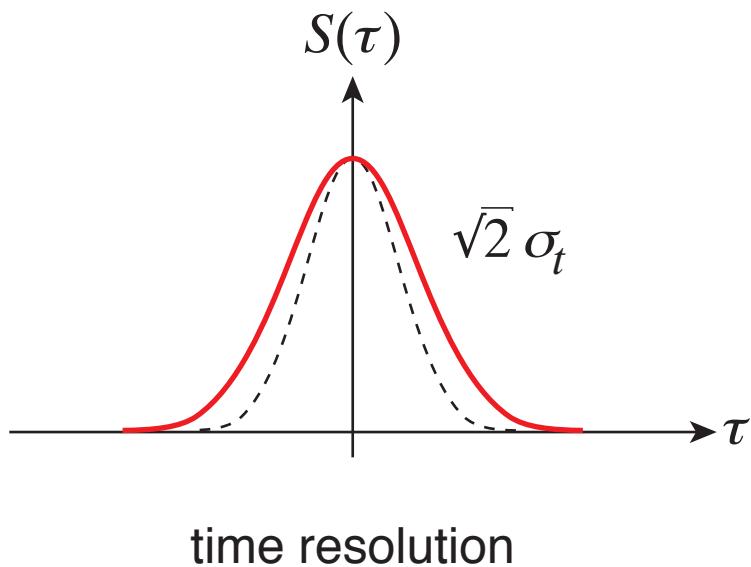
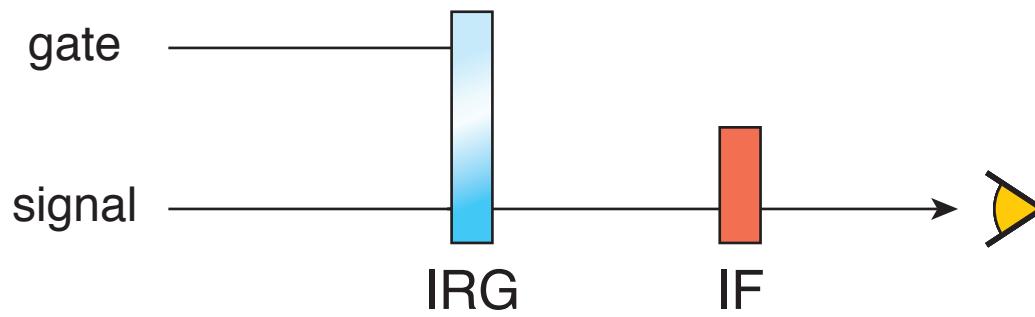
Experiment 2



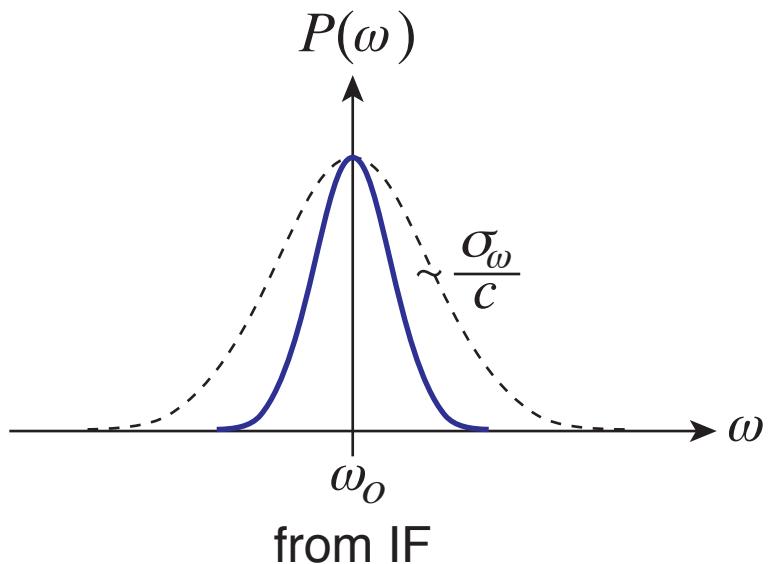
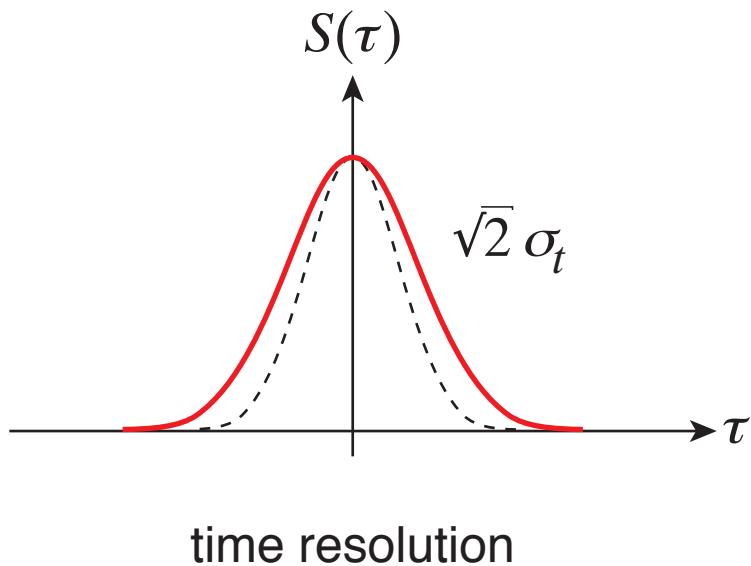
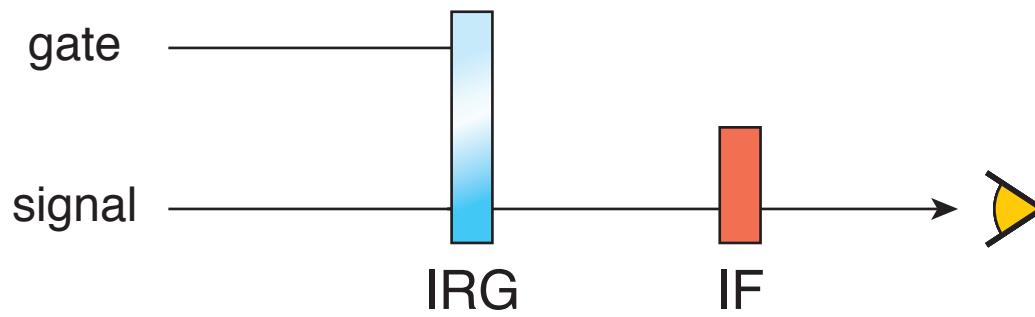
Experiment 2



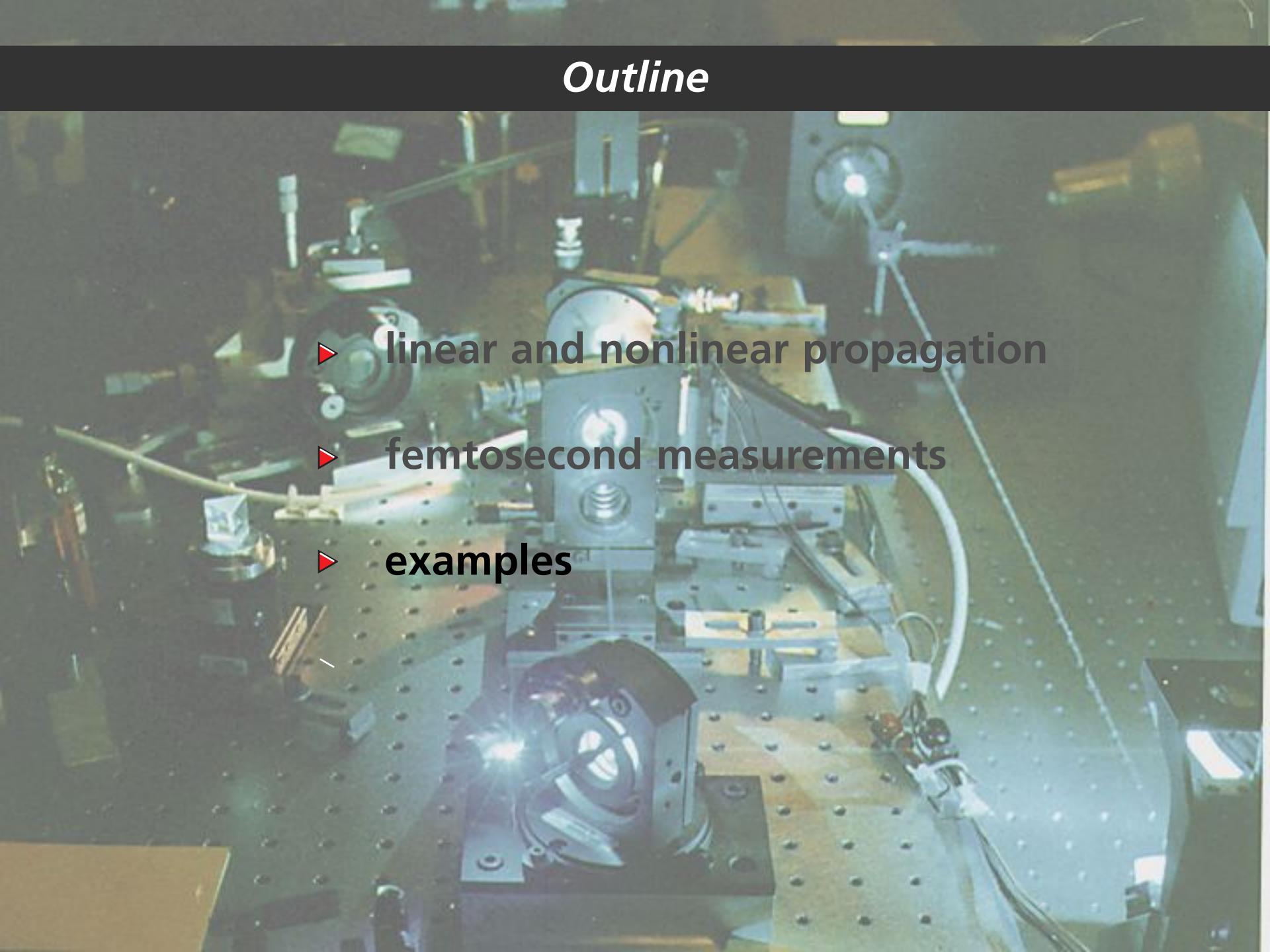
Experiment 2



Experiment 2



Outline

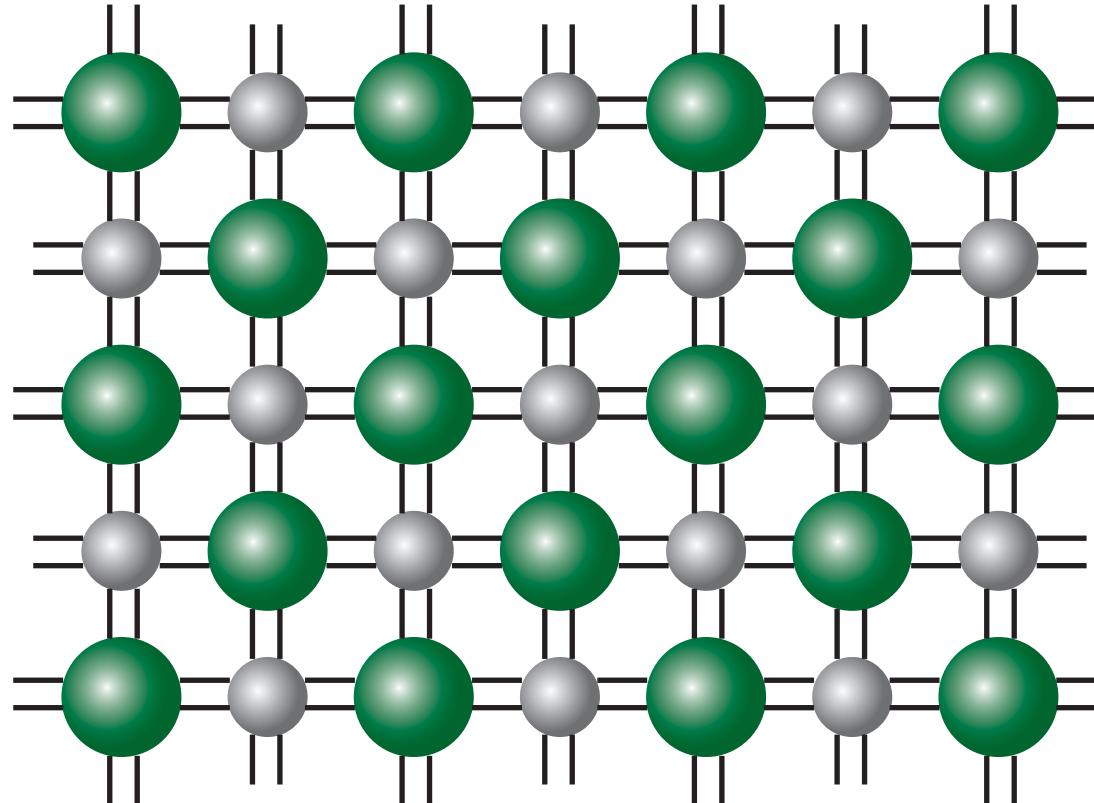
- 
- ▶ linear and nonlinear propagation
 - ▶ femtosecond measurements
 - ▶ examples

Introduction

short laser pulses can drive structural transitions

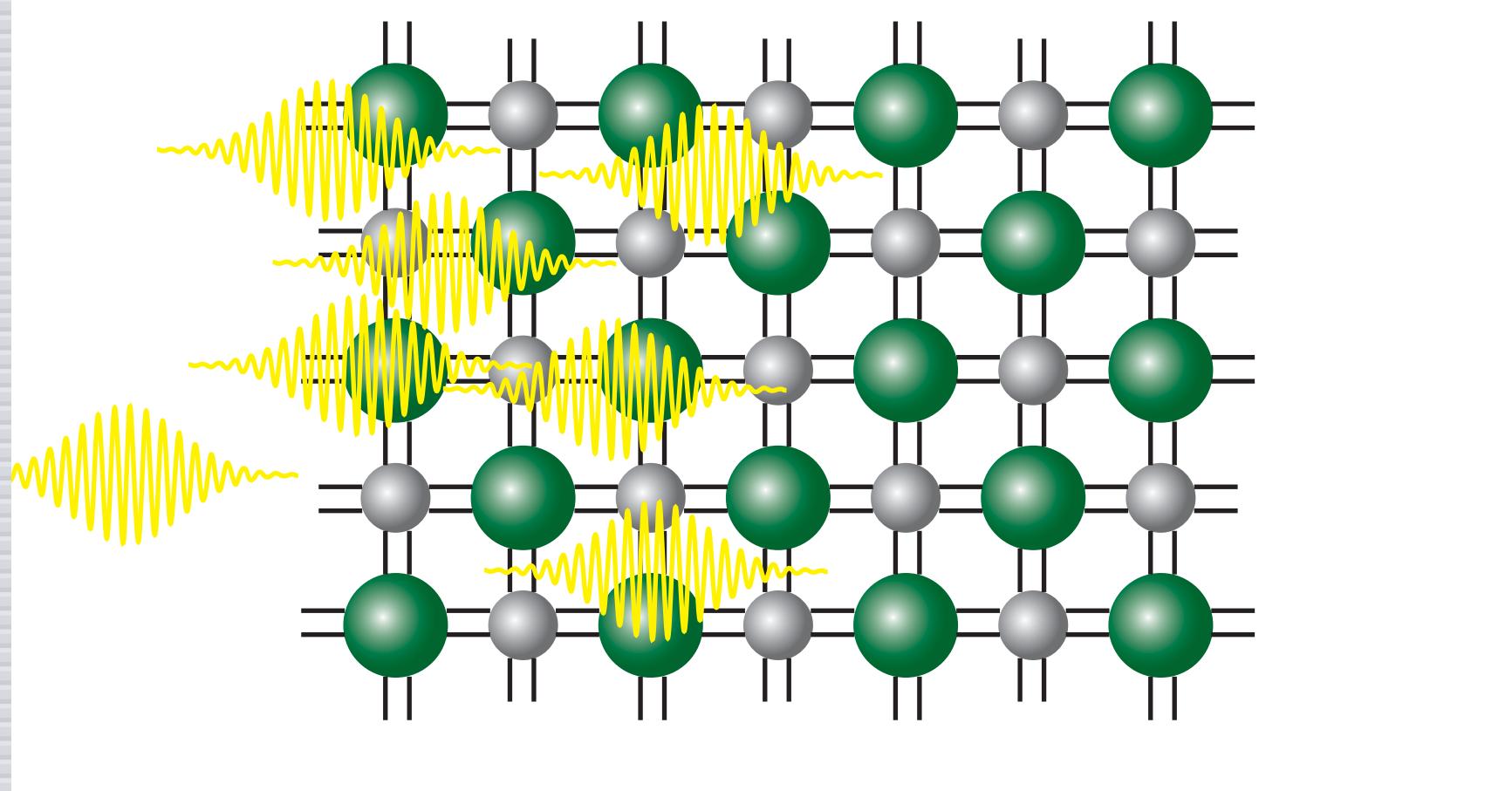
Introduction

how do femtosecond laser pulses alter a solid?



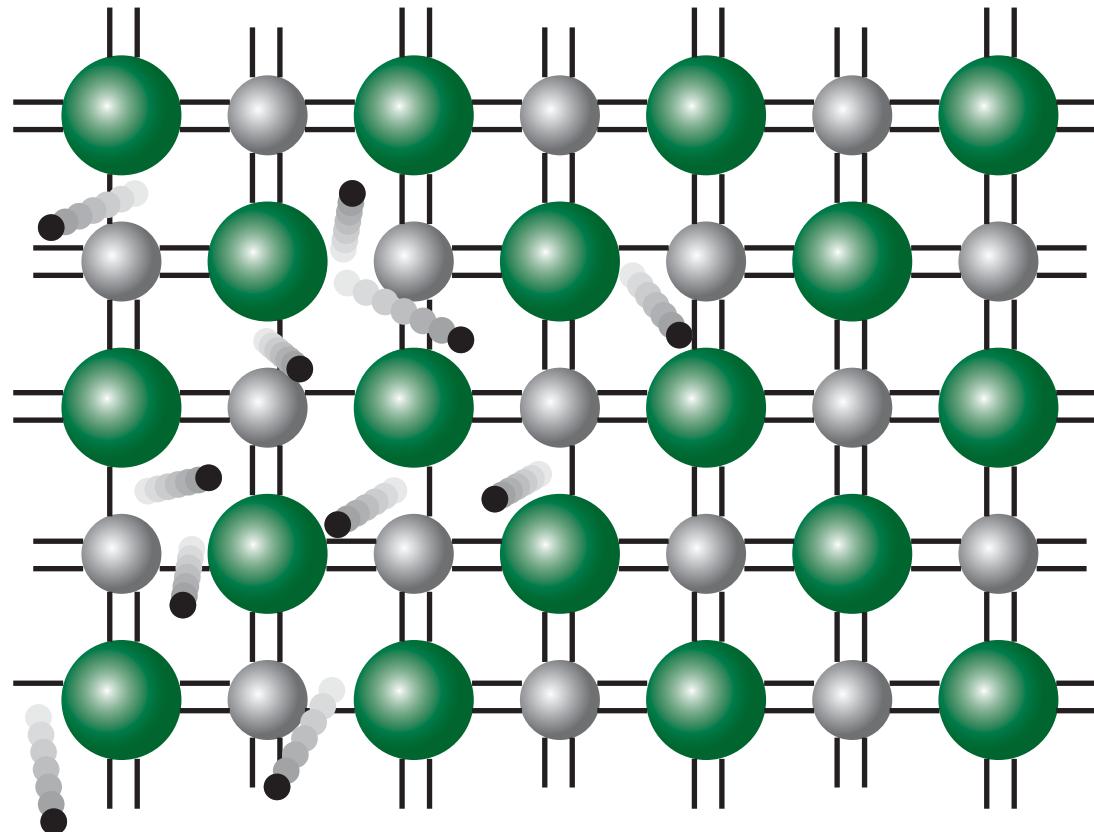
Introduction

photons excite valence electrons...



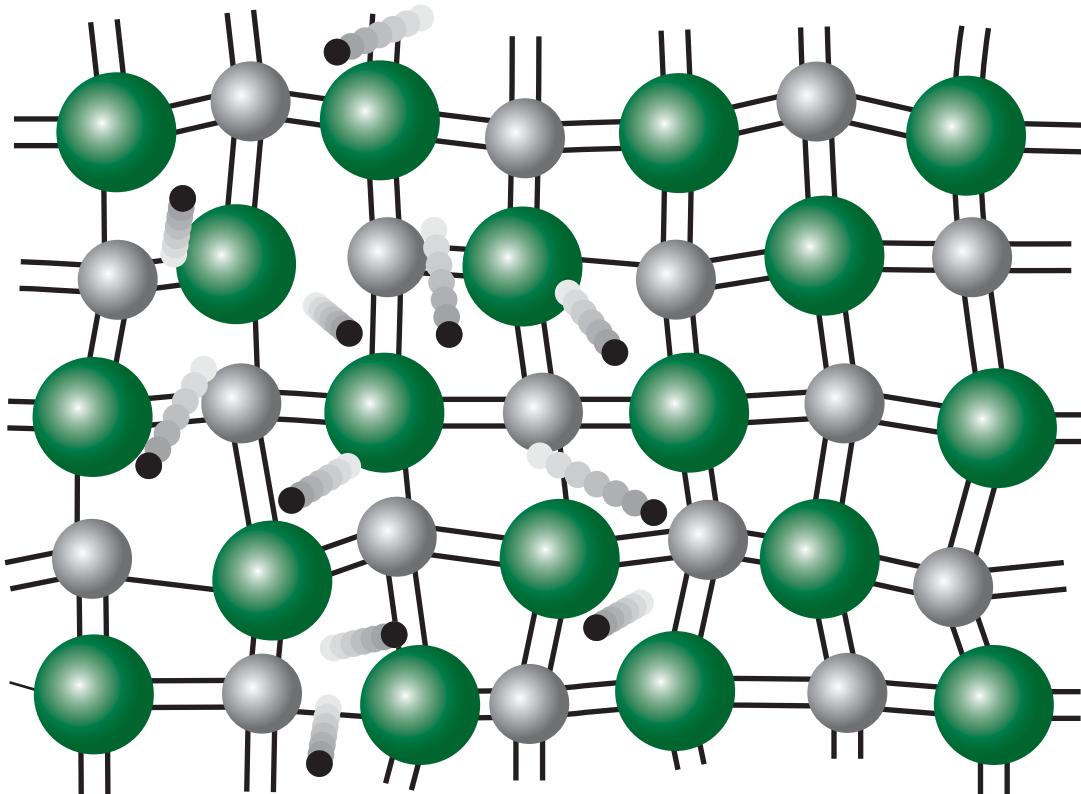
Introduction

... and create free electrons...



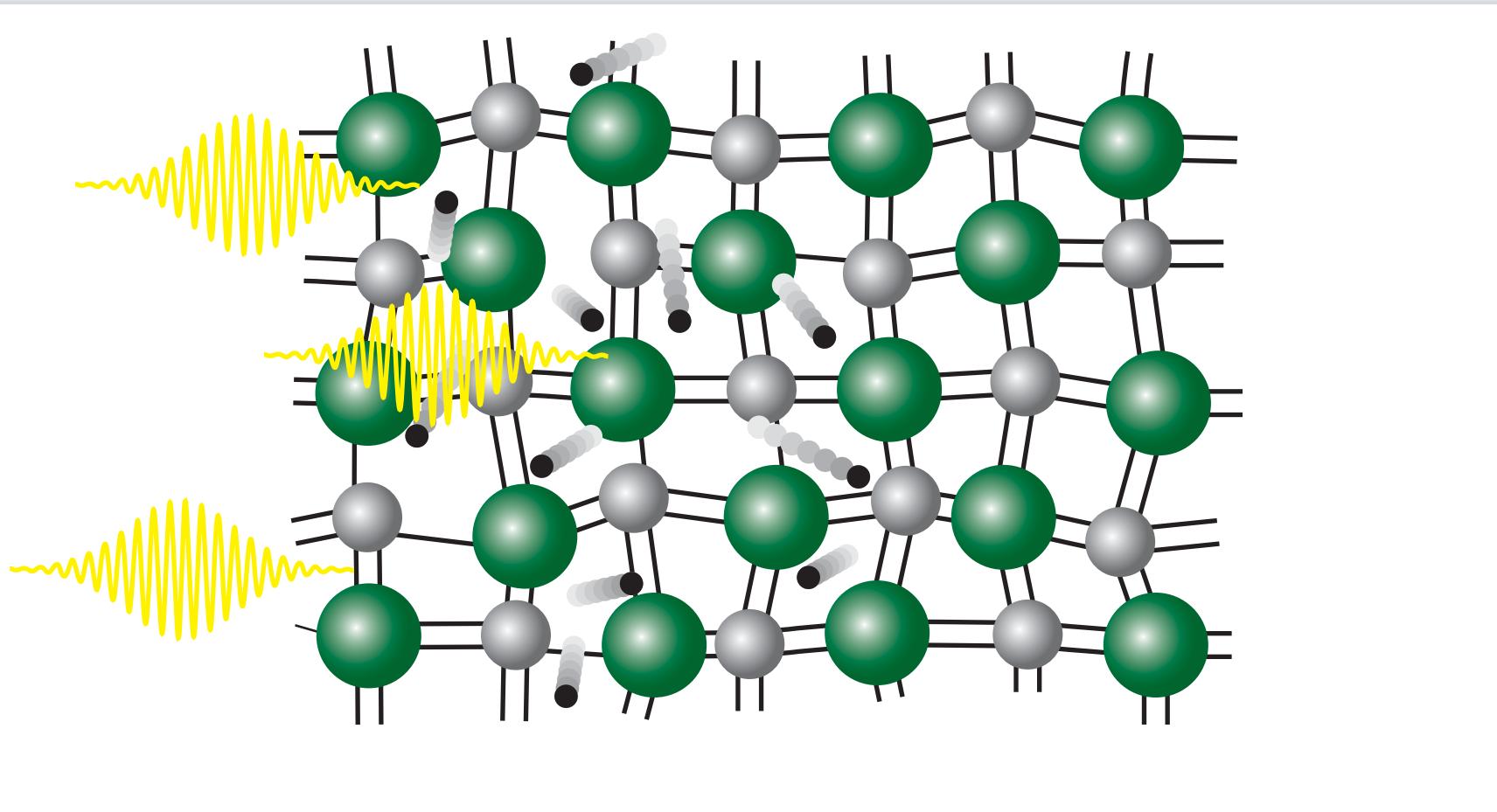
Introduction

... causing electronic and structural changes...



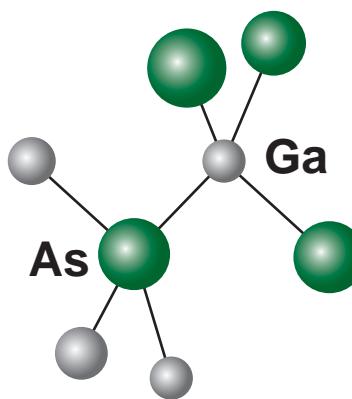
Introduction

... which we detect with a second laser pulse

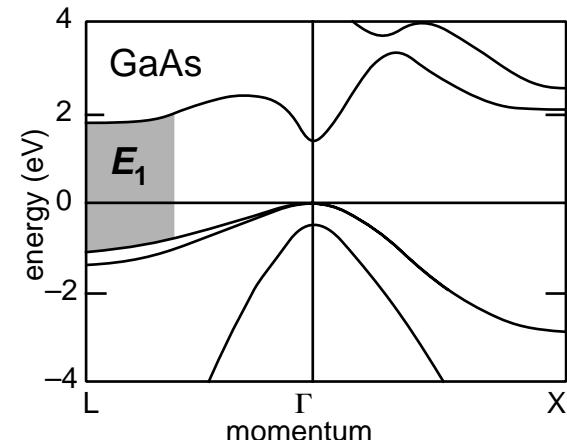


Introduction

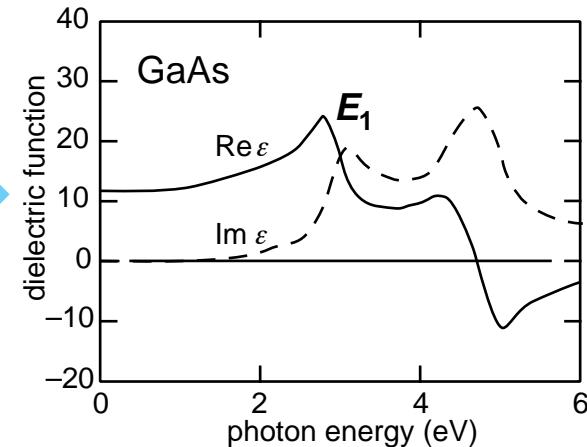
structure



band structure

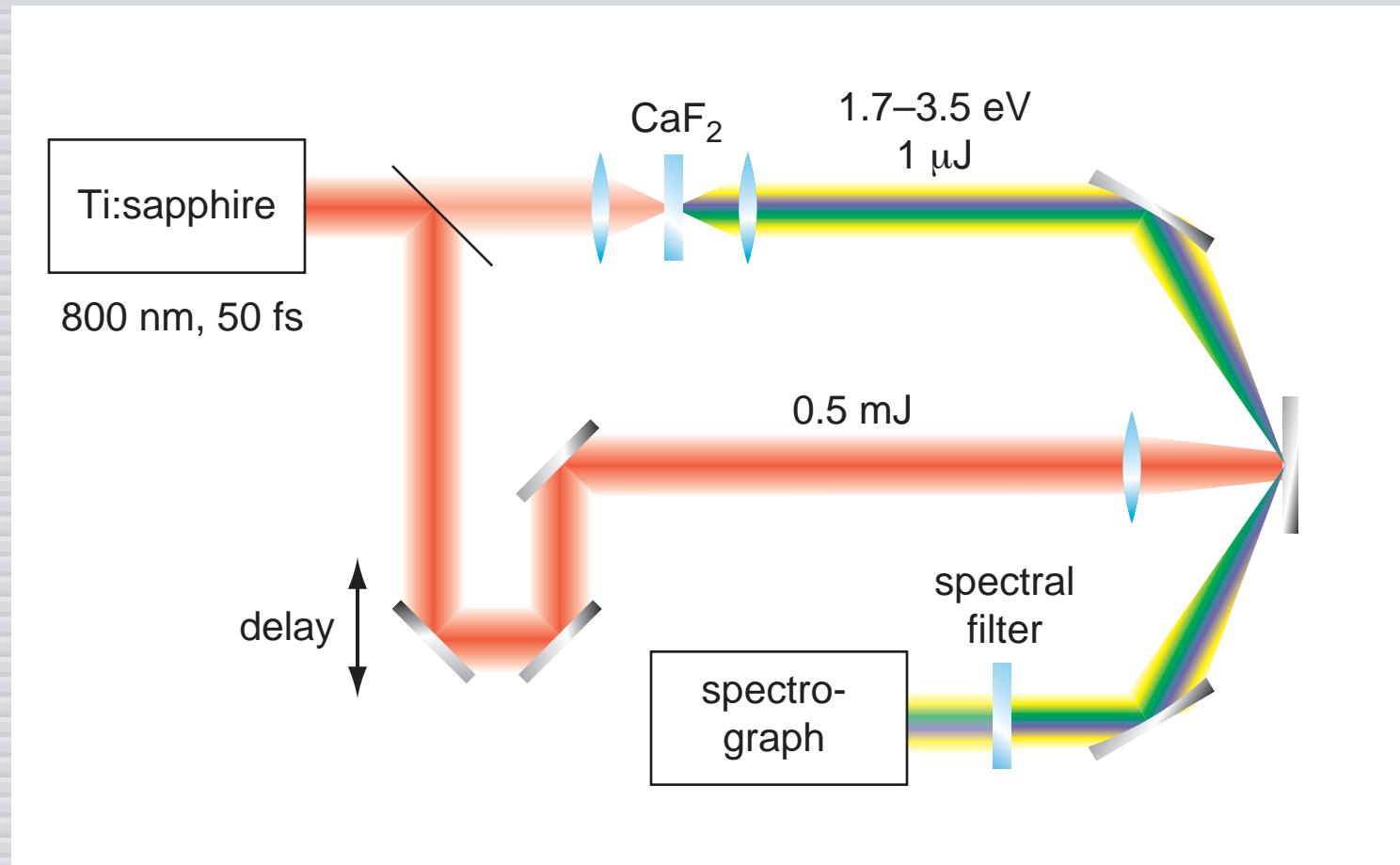


dielectric function



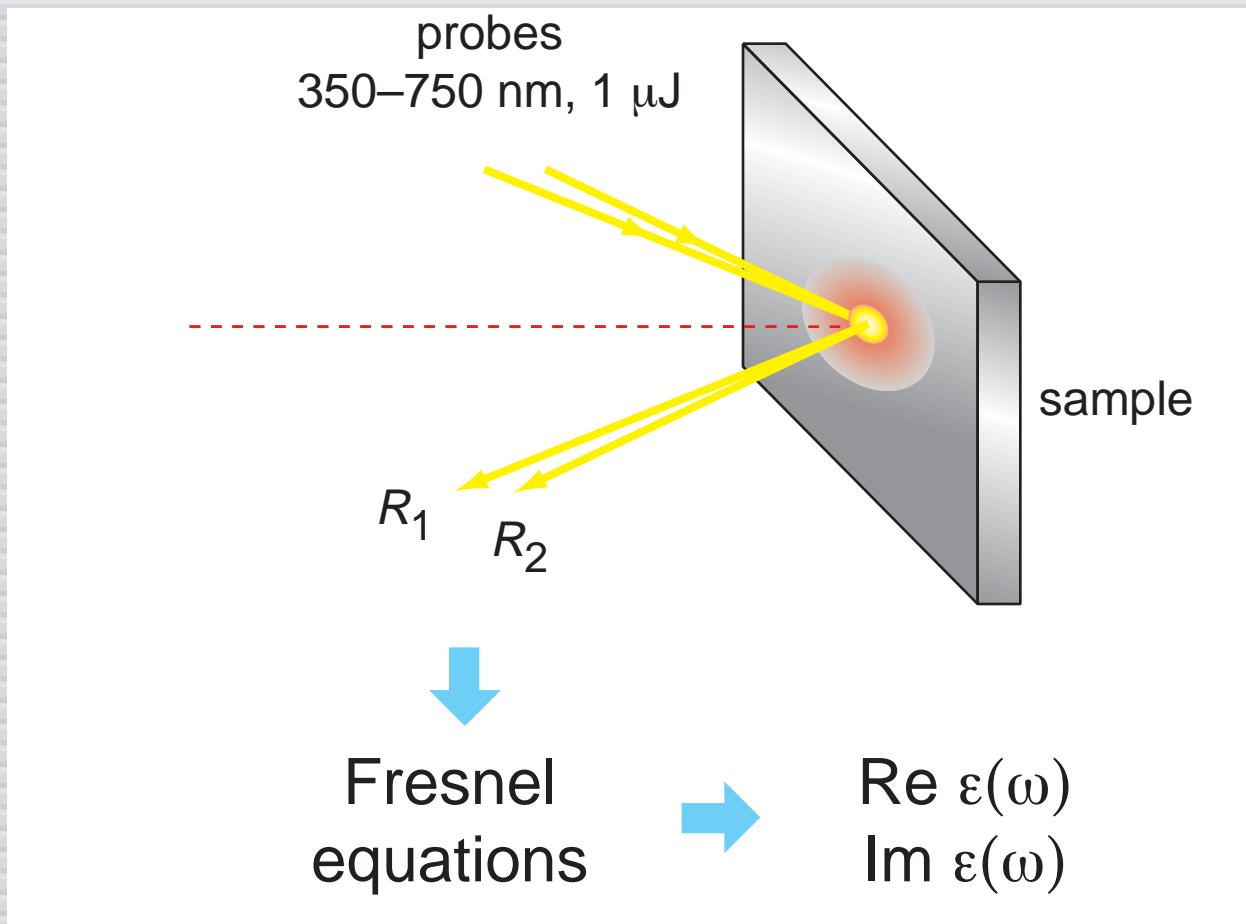
Technique

broadband time-resolved ellipsometry



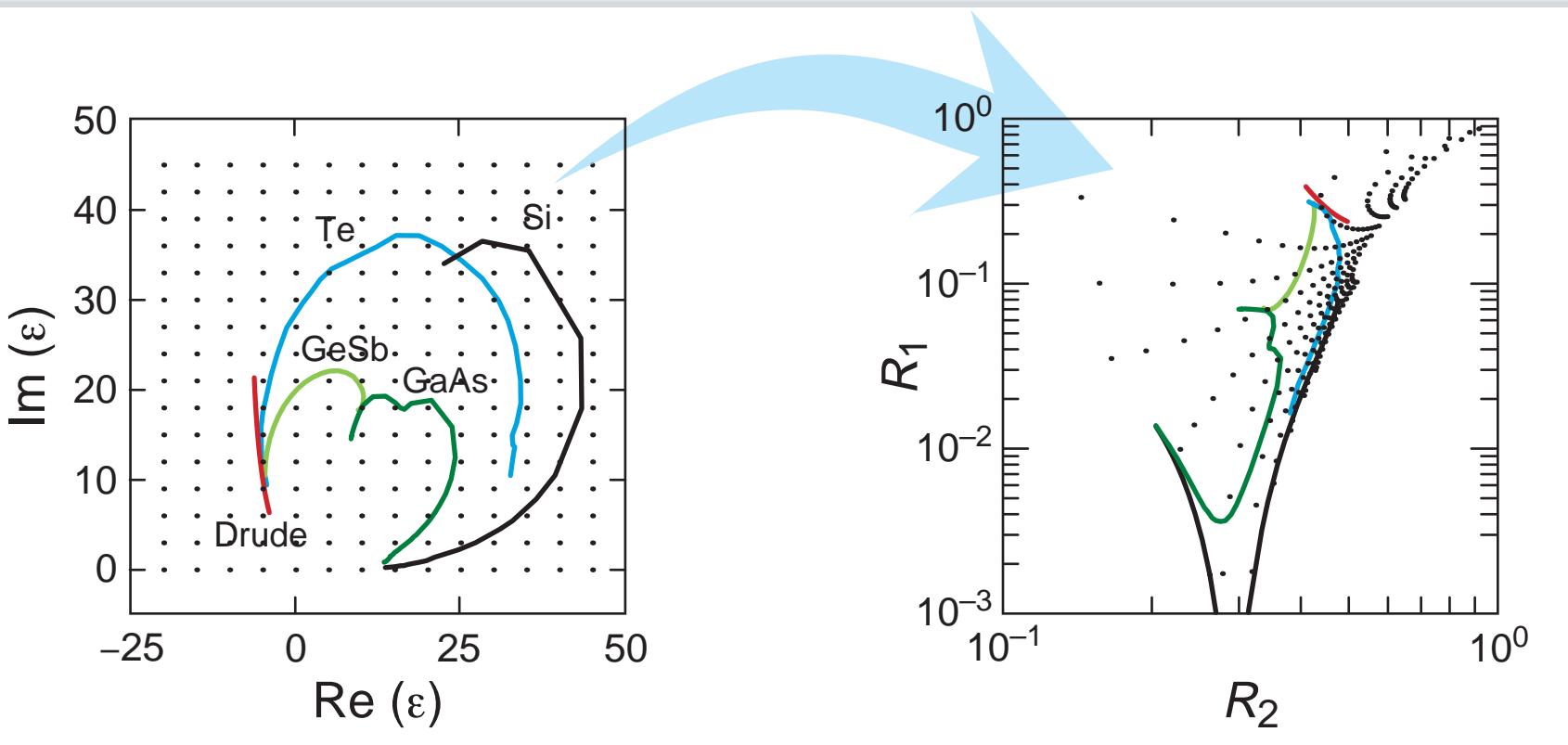
Technique

broadband time-resolved ellipsometry



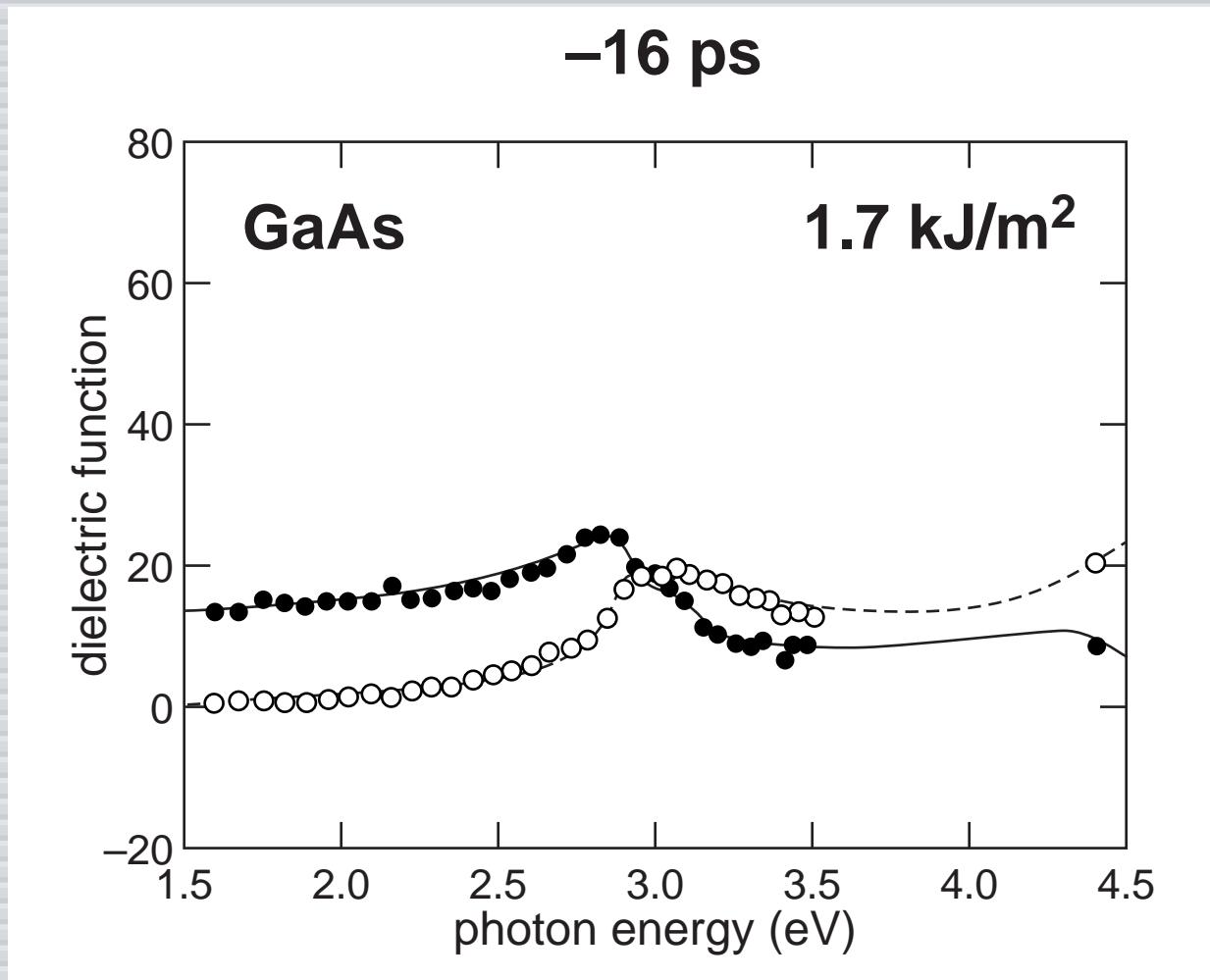
Technique

choice of angles

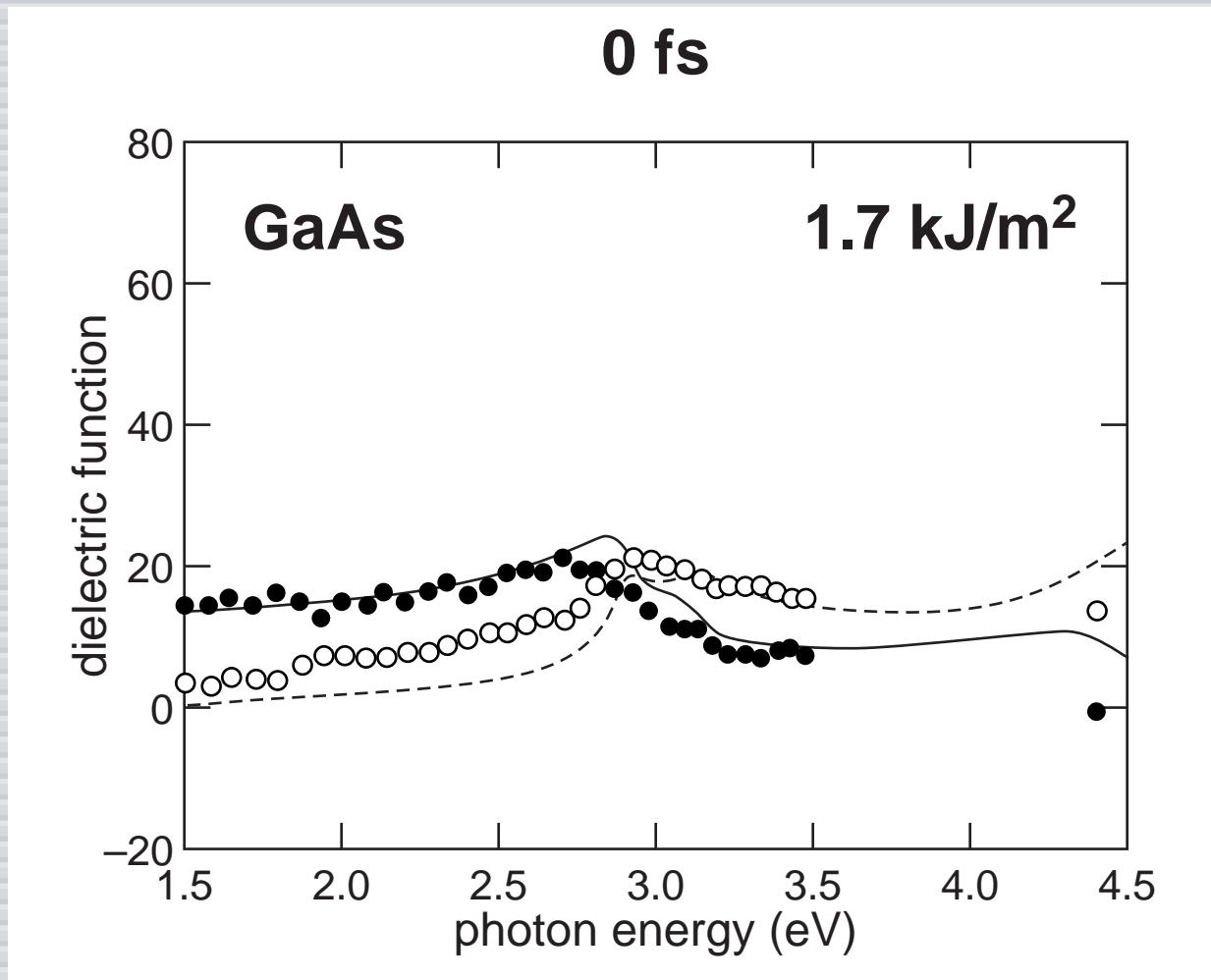


$$R_1 = 78^\circ \text{ } p\text{-pol}, R_2 = 45^\circ \text{ } p\text{-pol}$$

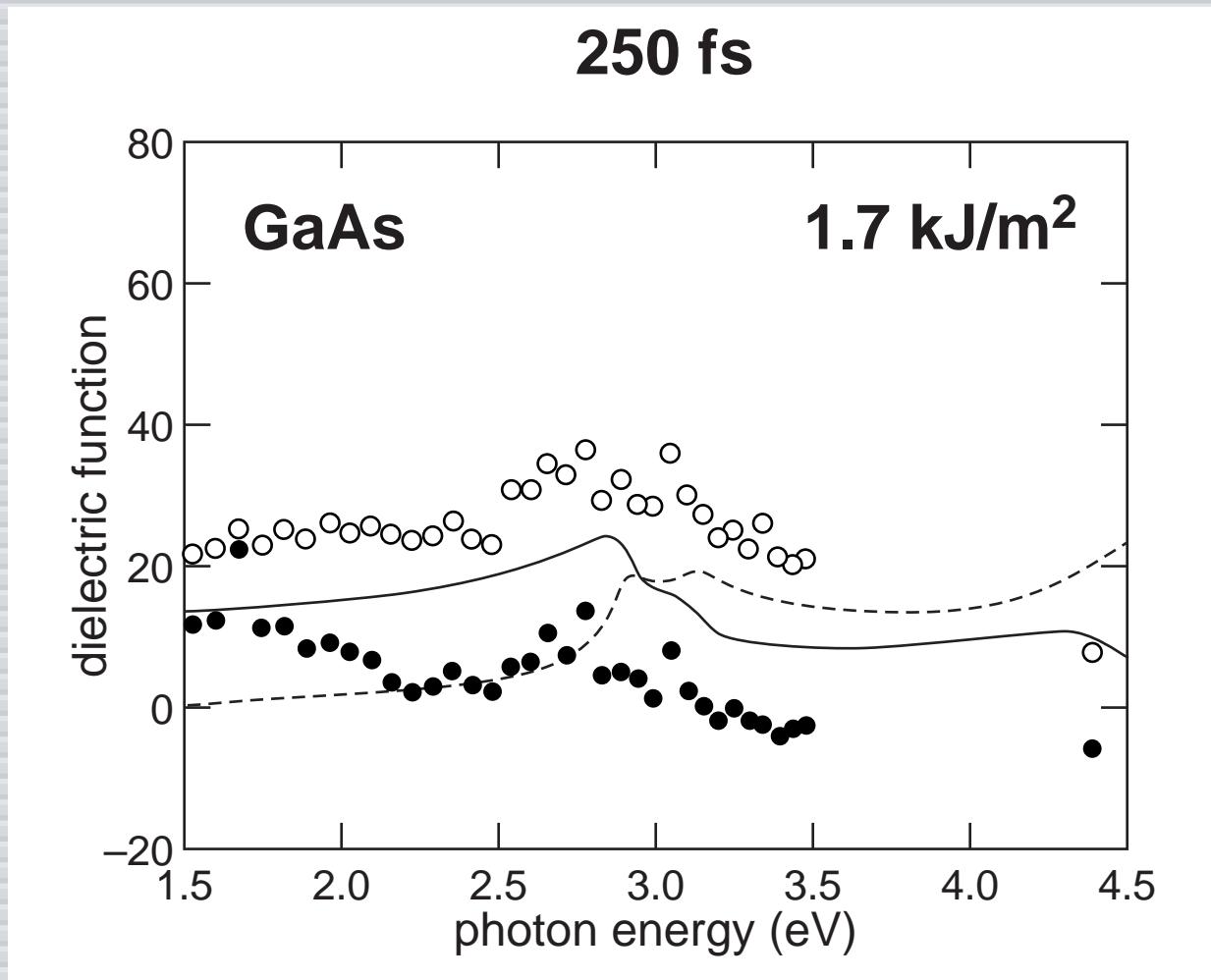
Technique



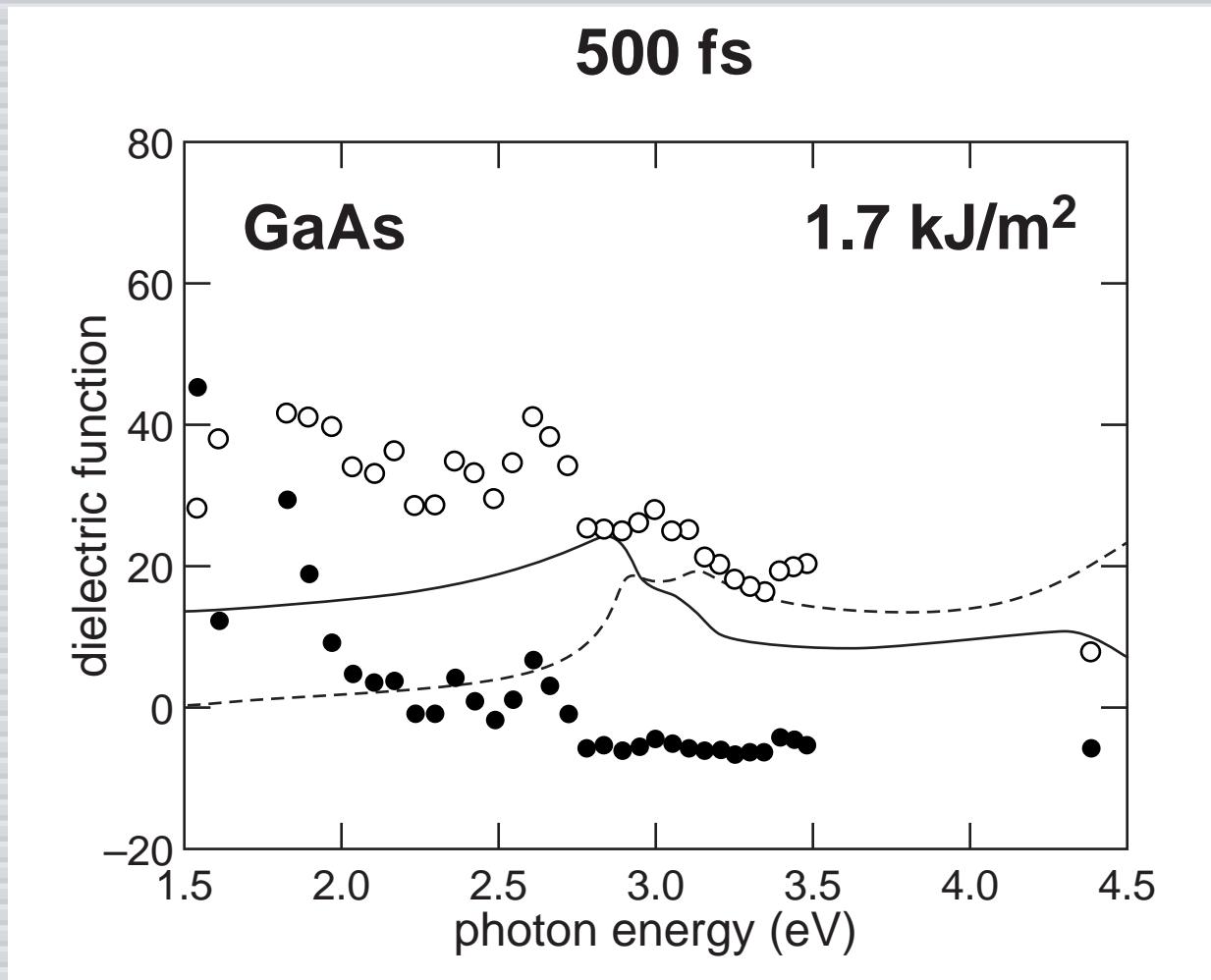
Technique



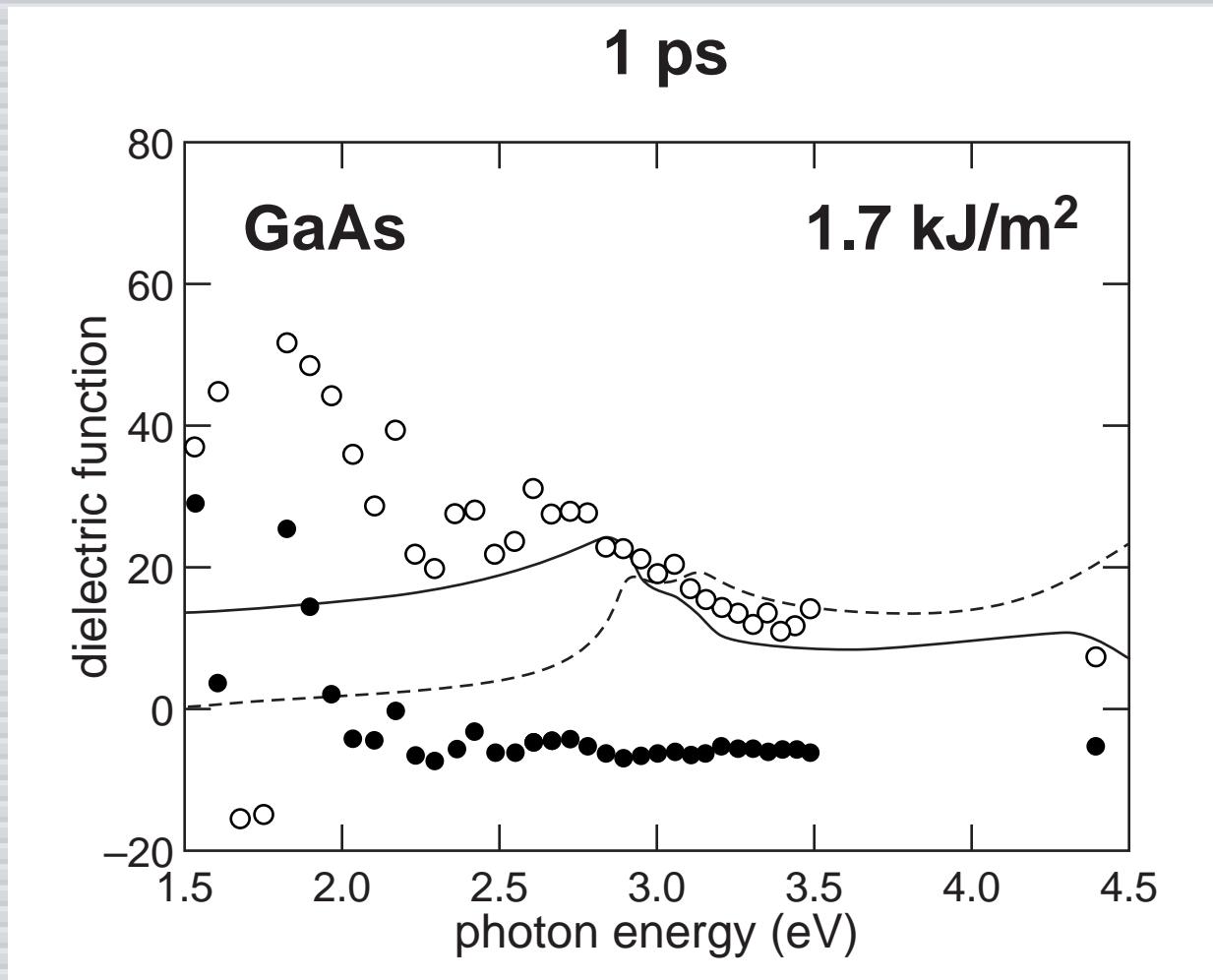
Technique



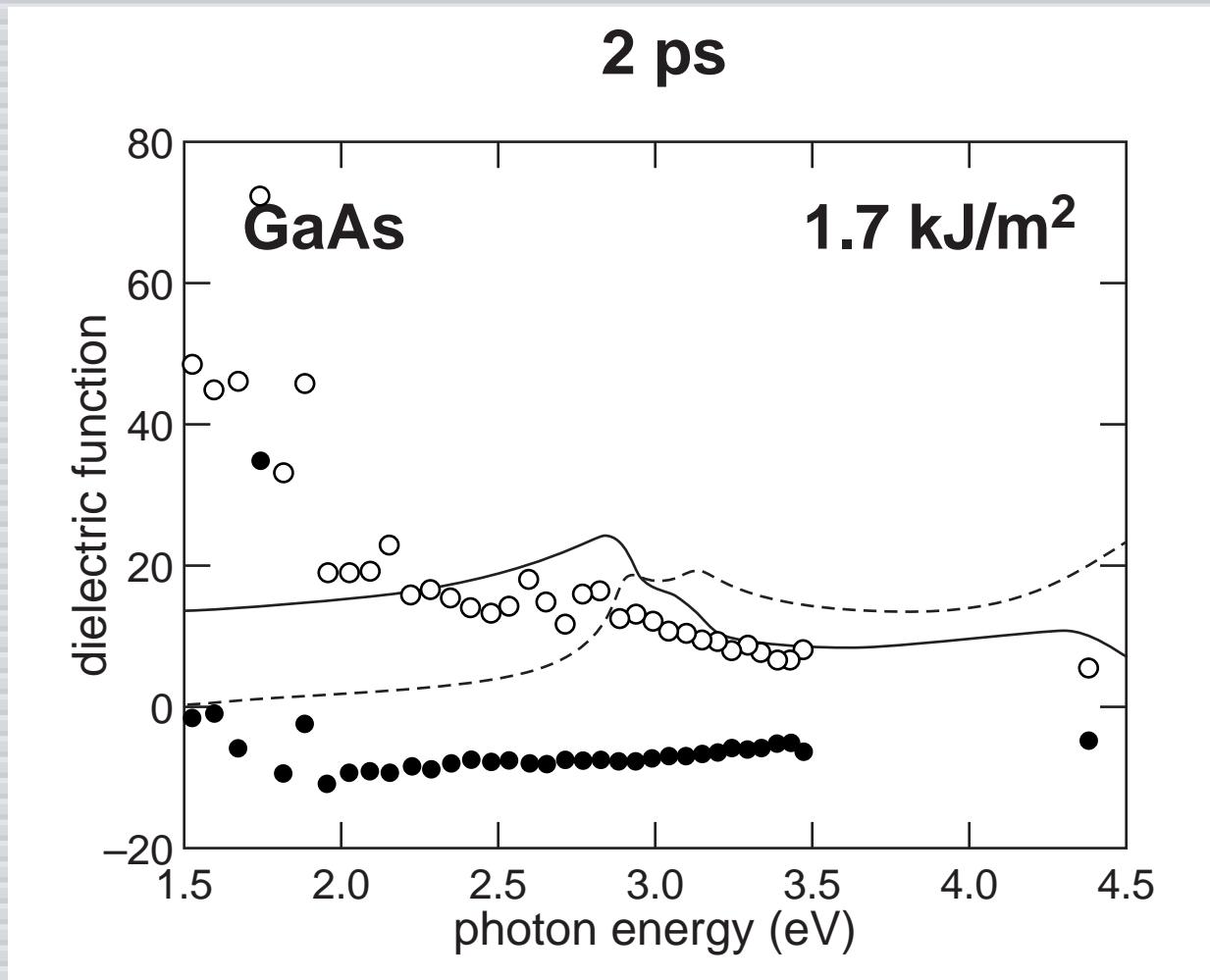
Technique



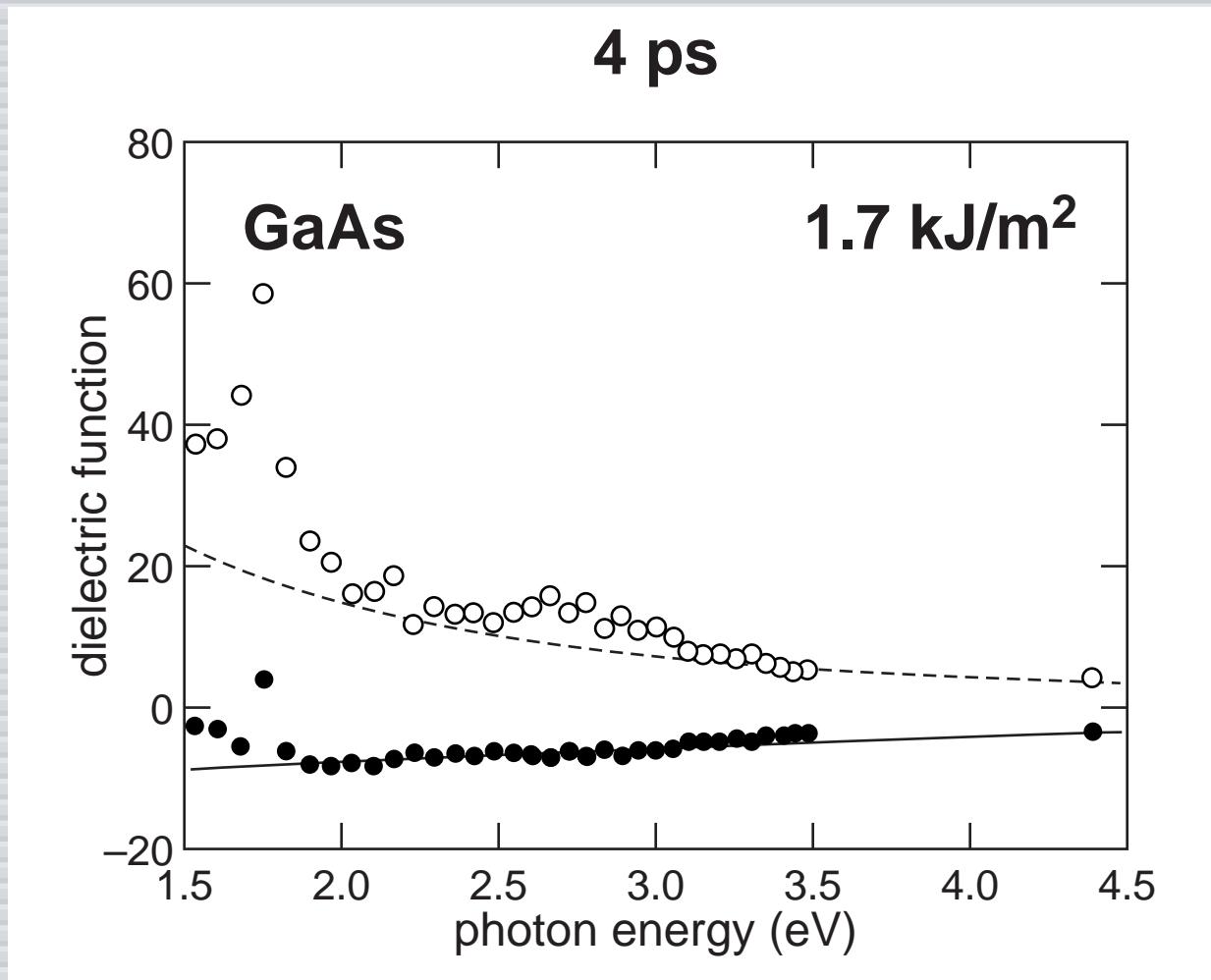
Technique



Technique

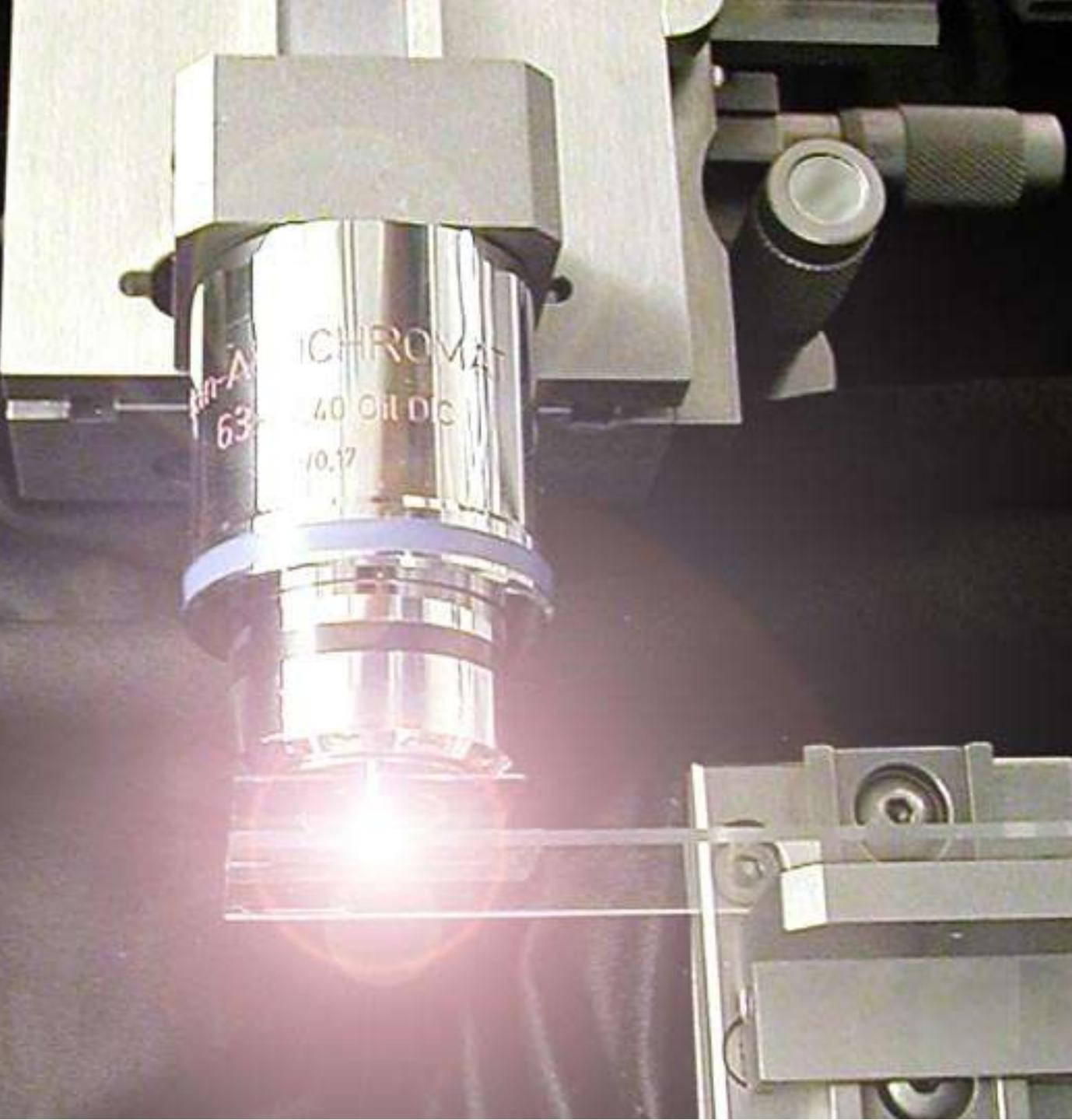


Technique

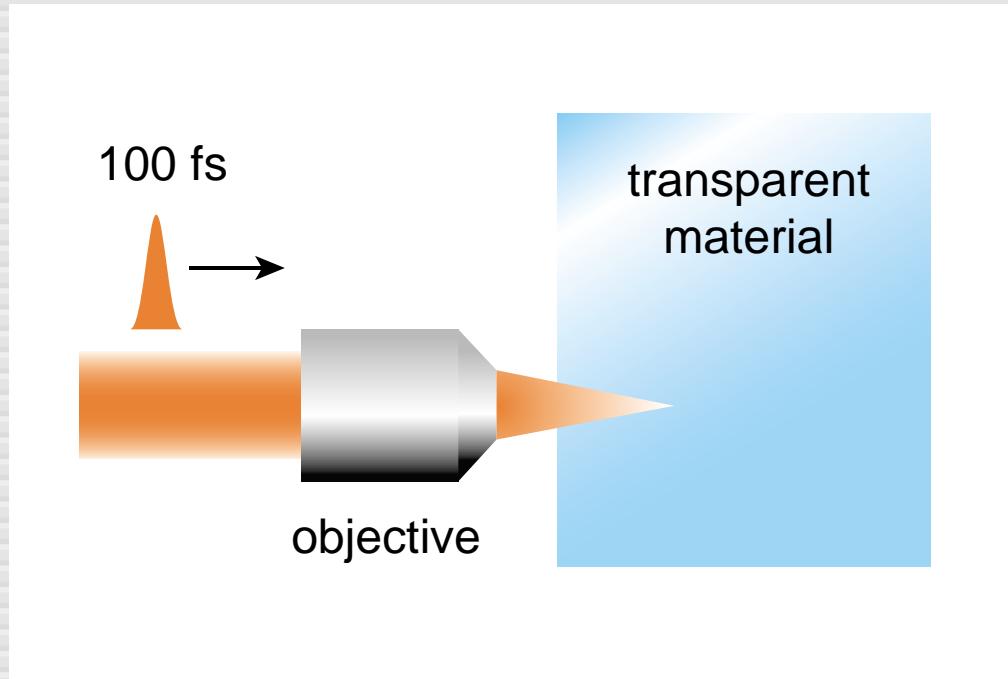


Technique

- ▶ **direct observation of semiconductor-to-metal transition**
- ▶ **order-disorder transition**
- ▶ **transition structural, not electronic**

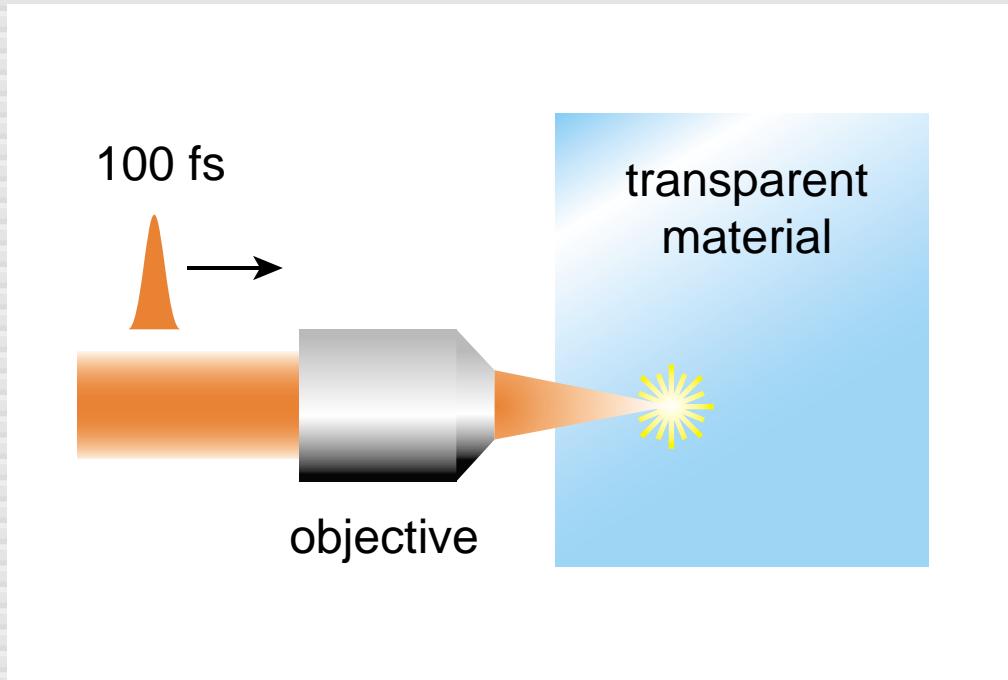


Processing with fs pulses



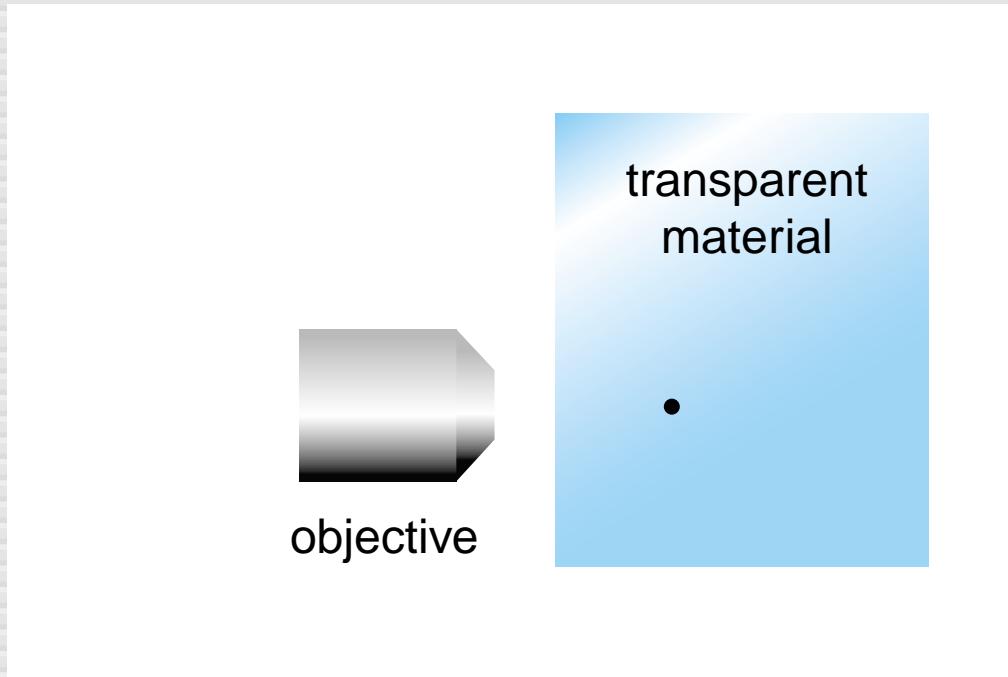
high intensity at focus...

Processing with fs pulses



... causes nonlinear ionization...

Processing with fs pulses



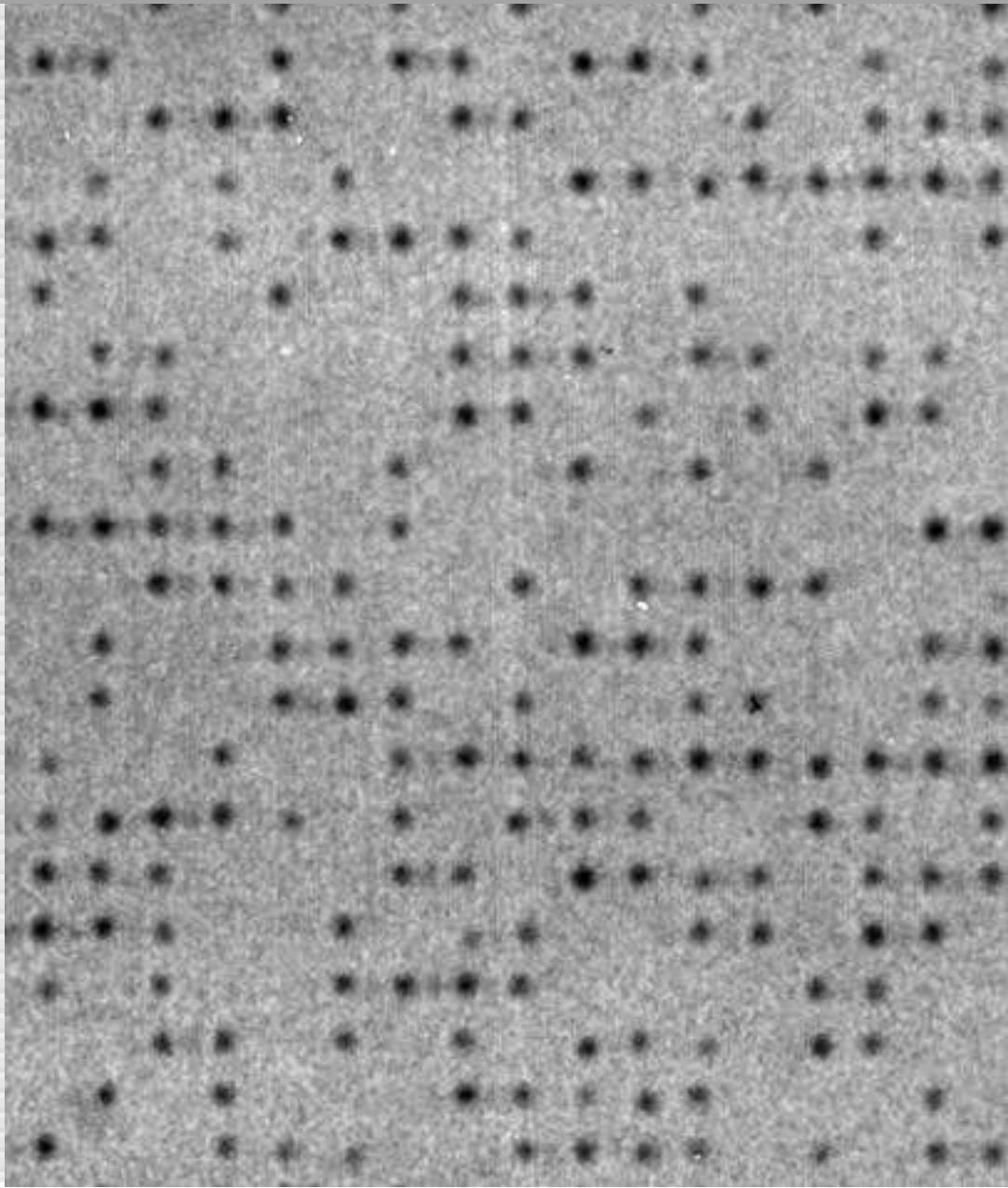
and 'microexplosion' causes microscopic damage

Processing with fs pulses

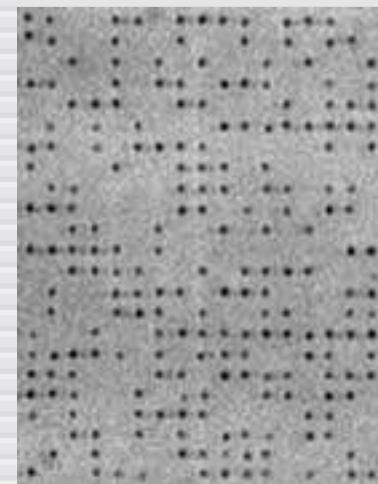
2 x 2 μm array

fused silica, 0.65 NA

0.5 μJ , 100 fs, 800 nm



Processing with fs pulses



**200 ps
9 μJ**

**100 fs
0.5 μJ**

Processing with fs pulses

100 nm

5 x 5 μm array

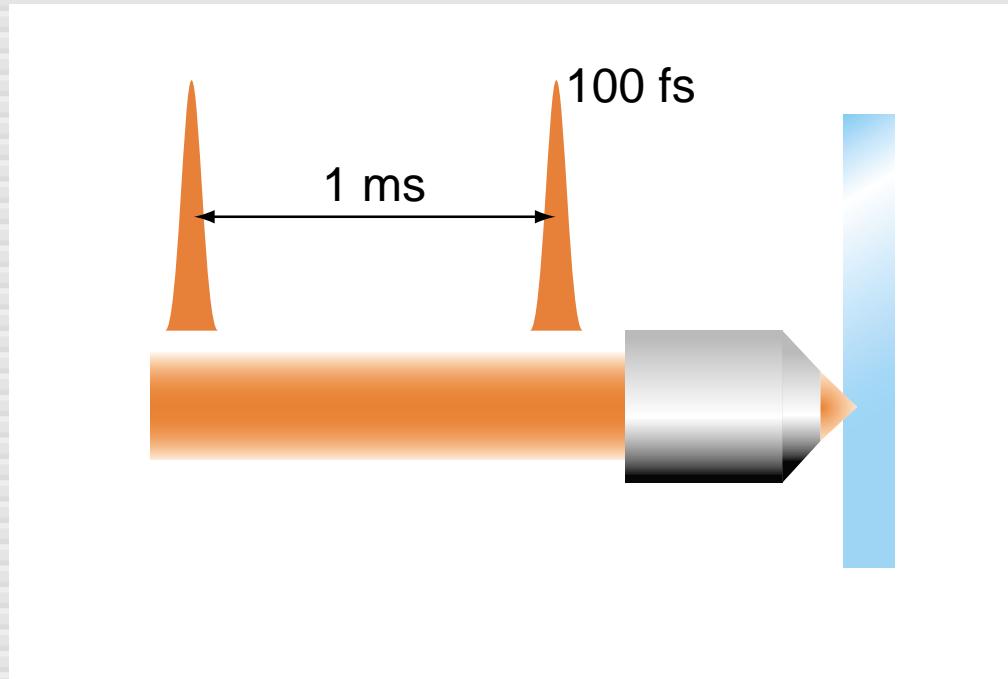
fused silica, 0.65 NA

0.5 μJ , 100 fs, 800 nm

Opt. Lett. 21, 2023 (1996)

Low-energy processing

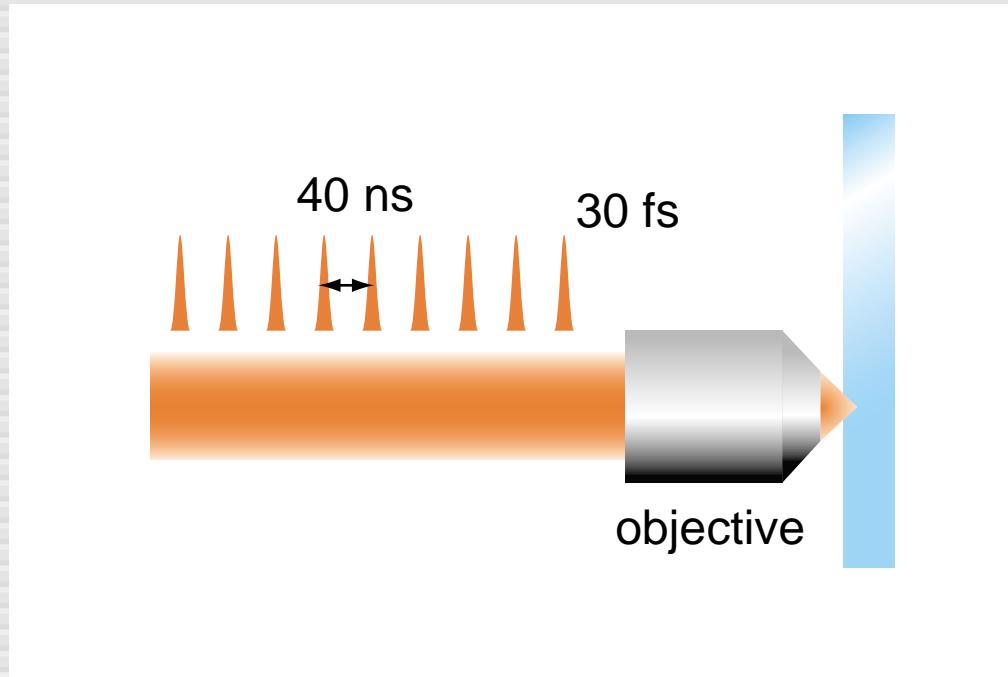
amplified laser



heat-diffusion time: $\tau_{diff} \approx 1 \mu s$

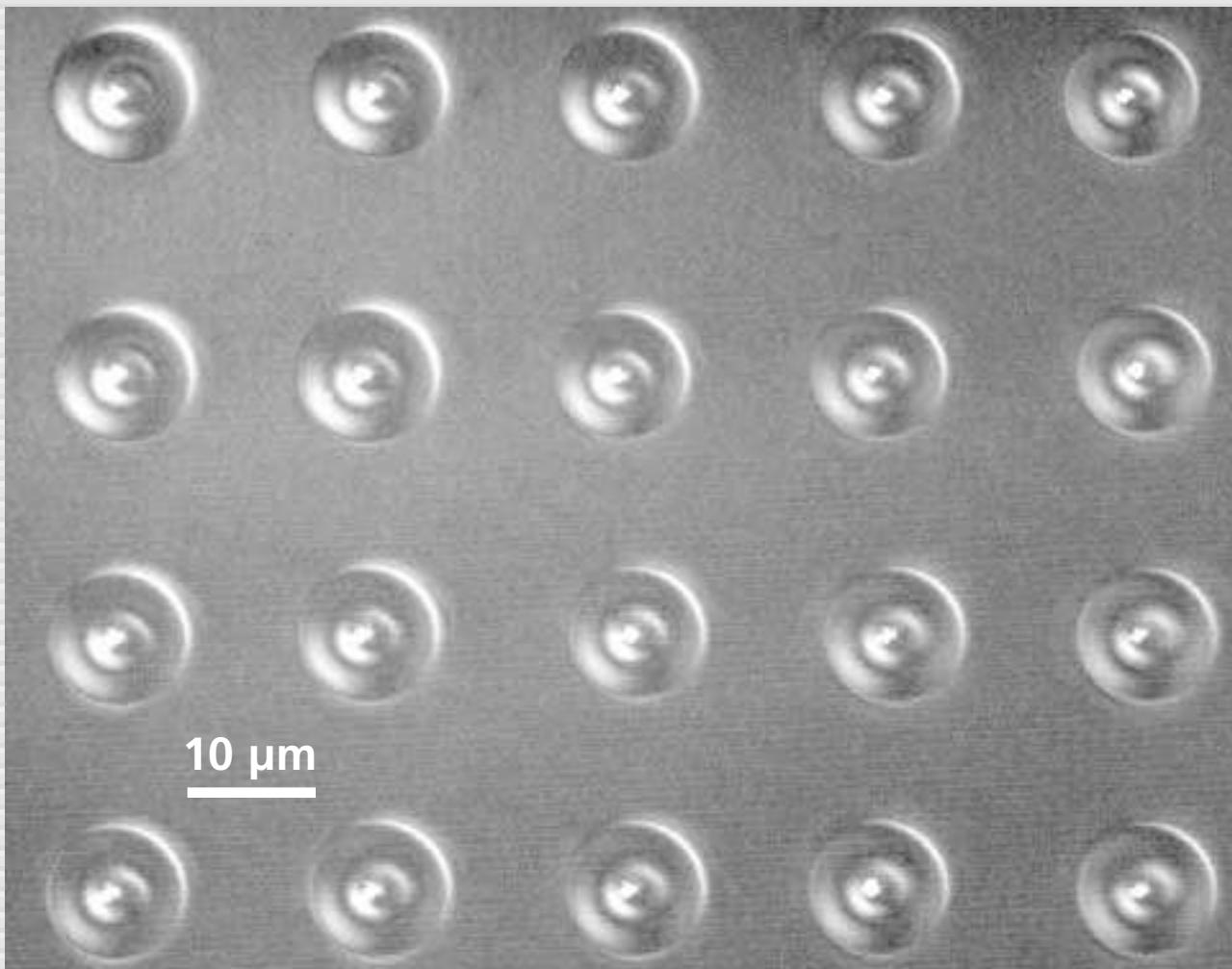
Low-energy processing

long-cavity Ti:sapphire oscillator

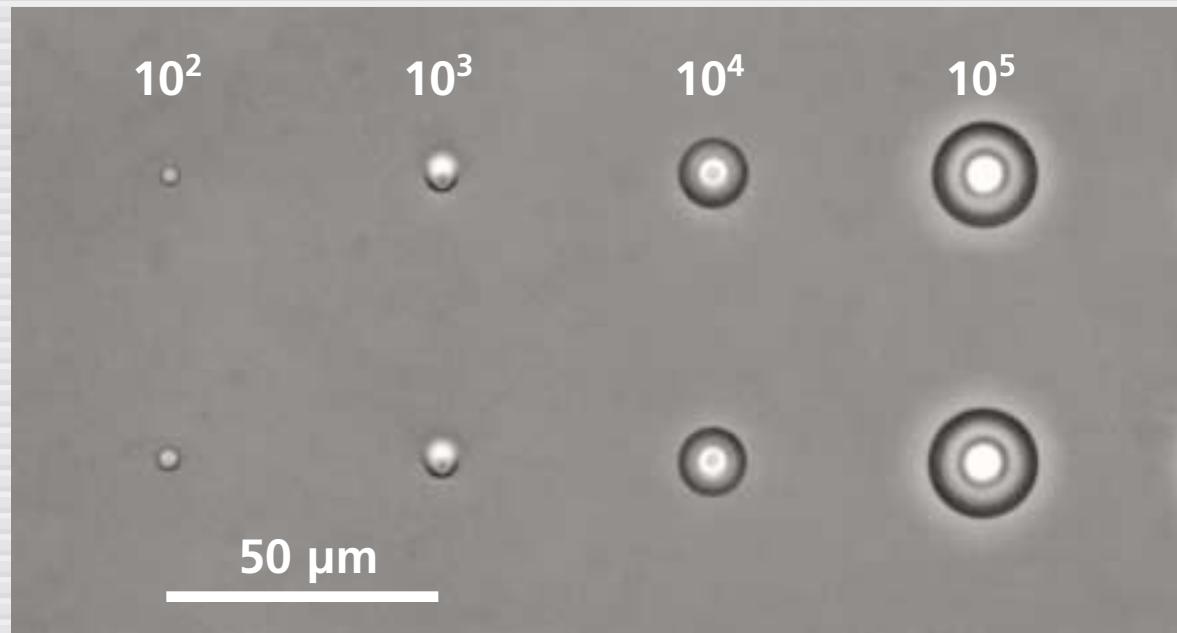


heat-diffusion time: $\tau_{diff} \approx 1 \mu s$

Low-energy processing

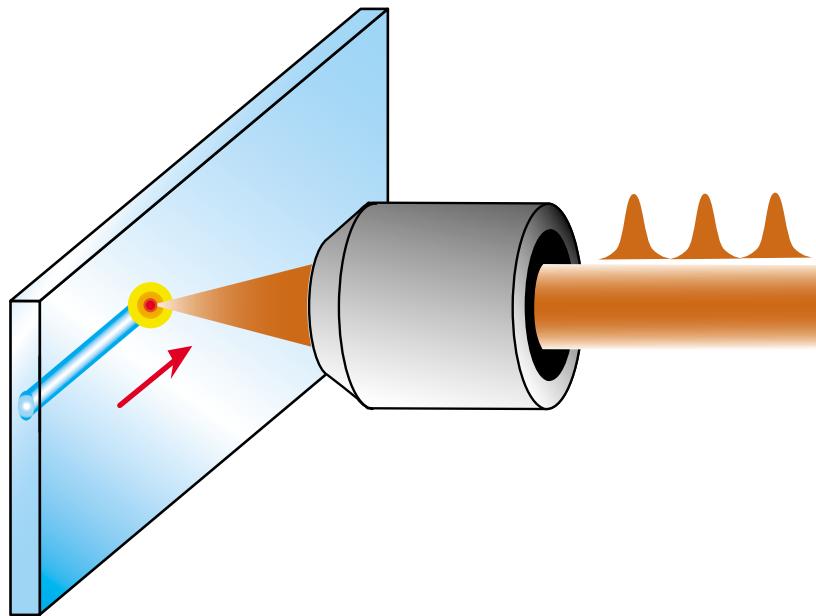


Low-energy processing



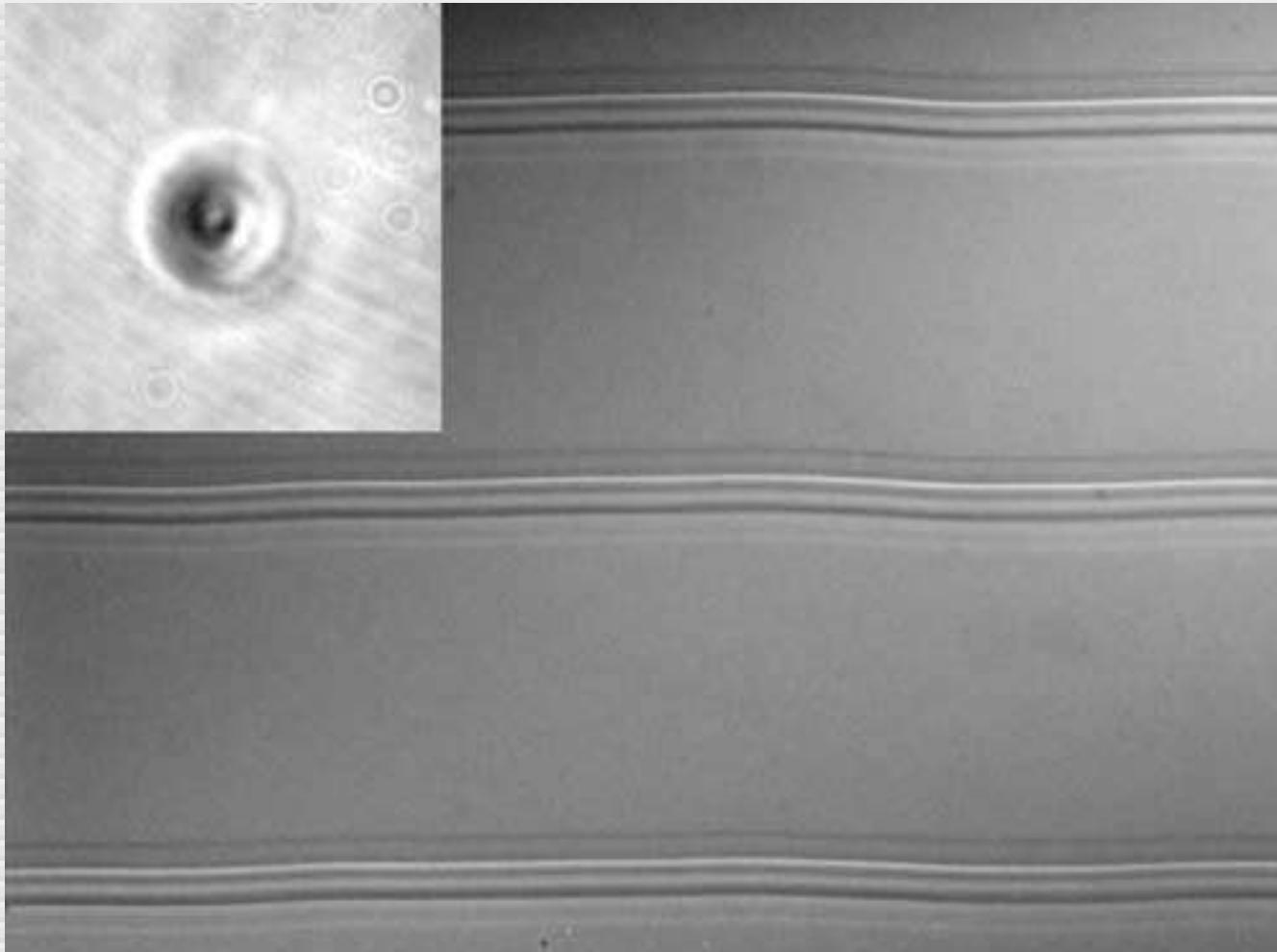
Low-energy processing

waveguide machining



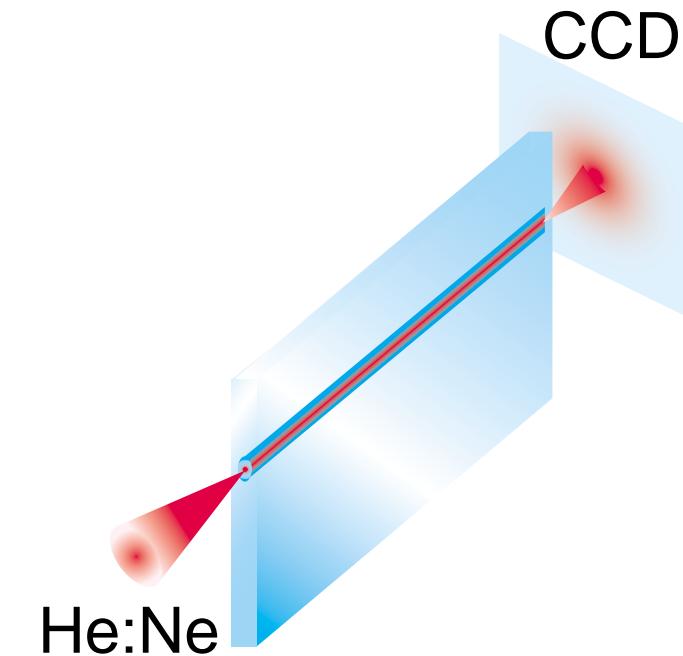
Low-energy processing

waveguide machining



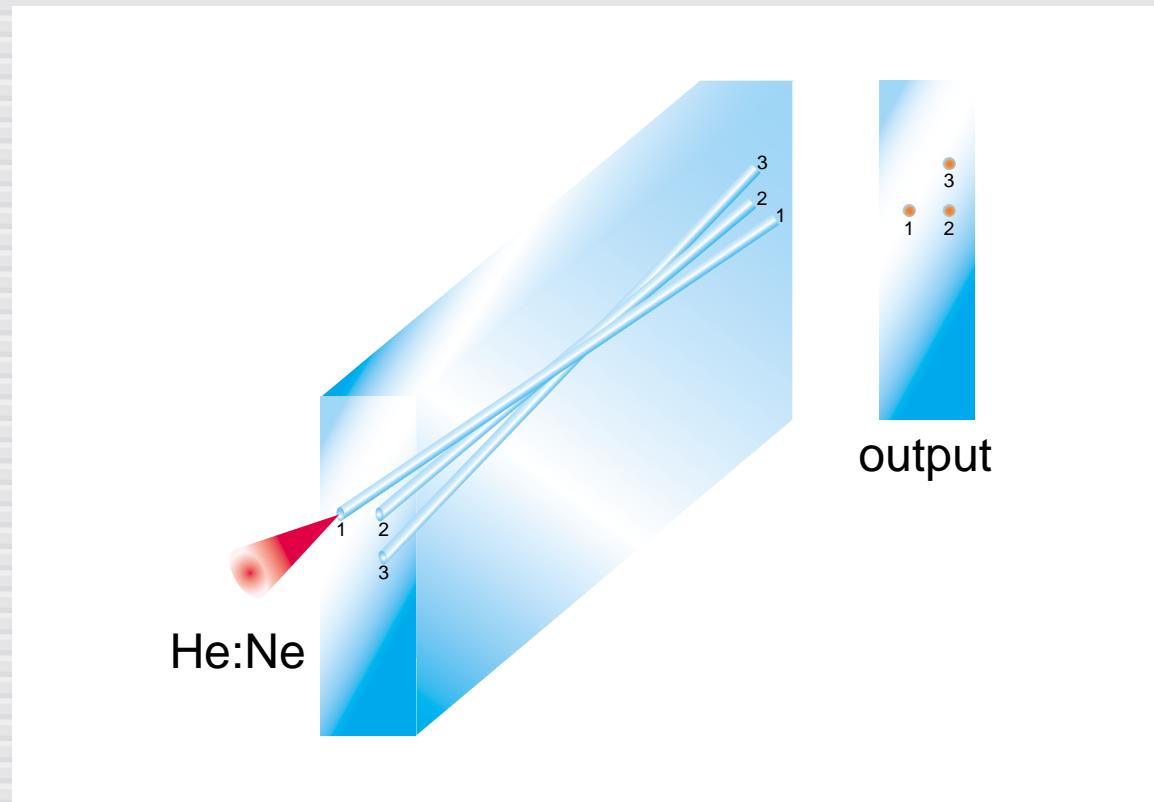
Low-energy processing

waveguide mode analysis



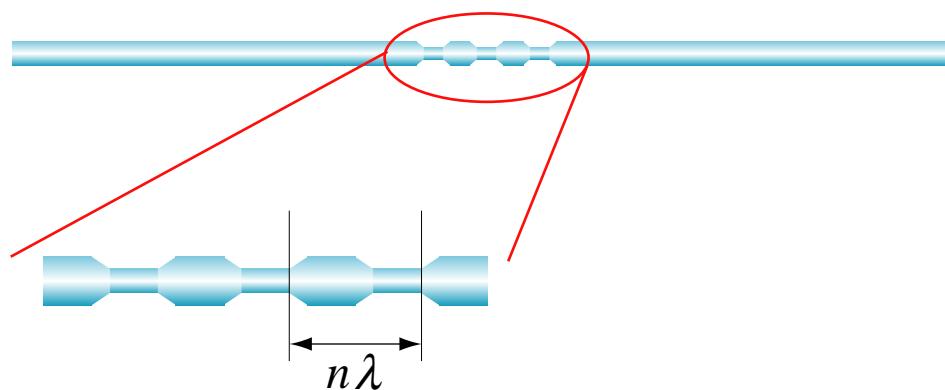
Low-energy processing

3D wave splitter



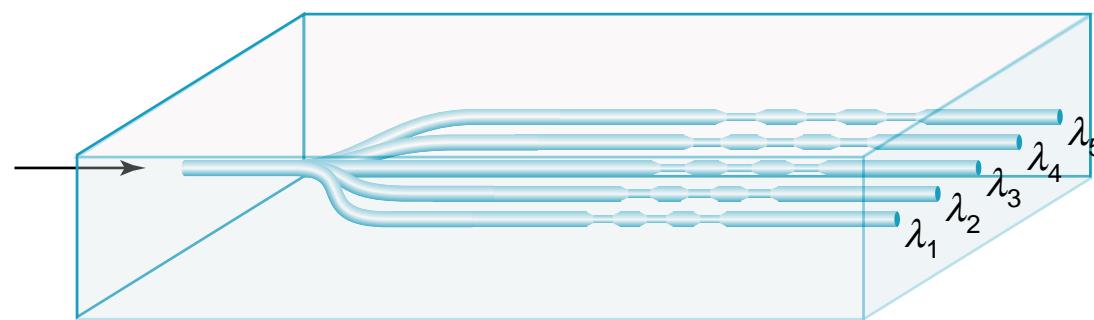
Low-energy processing

Bragg grating



Low-energy processing

Bragg grating



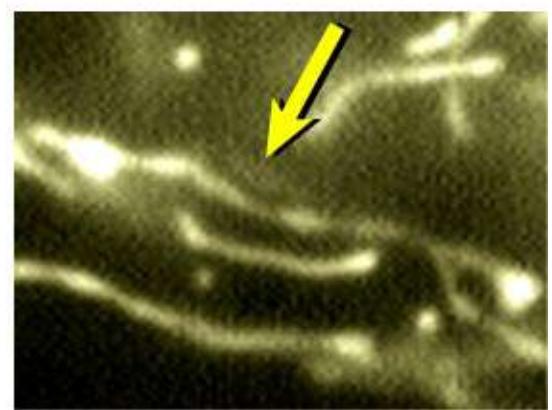
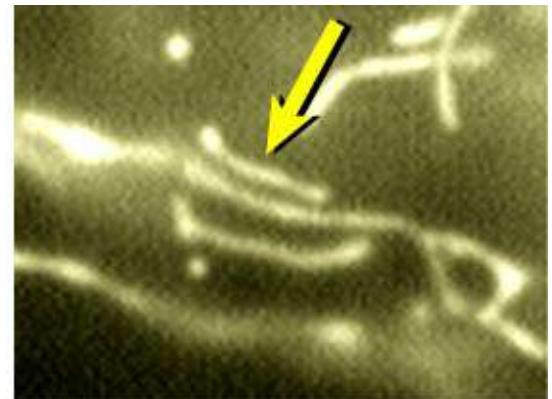
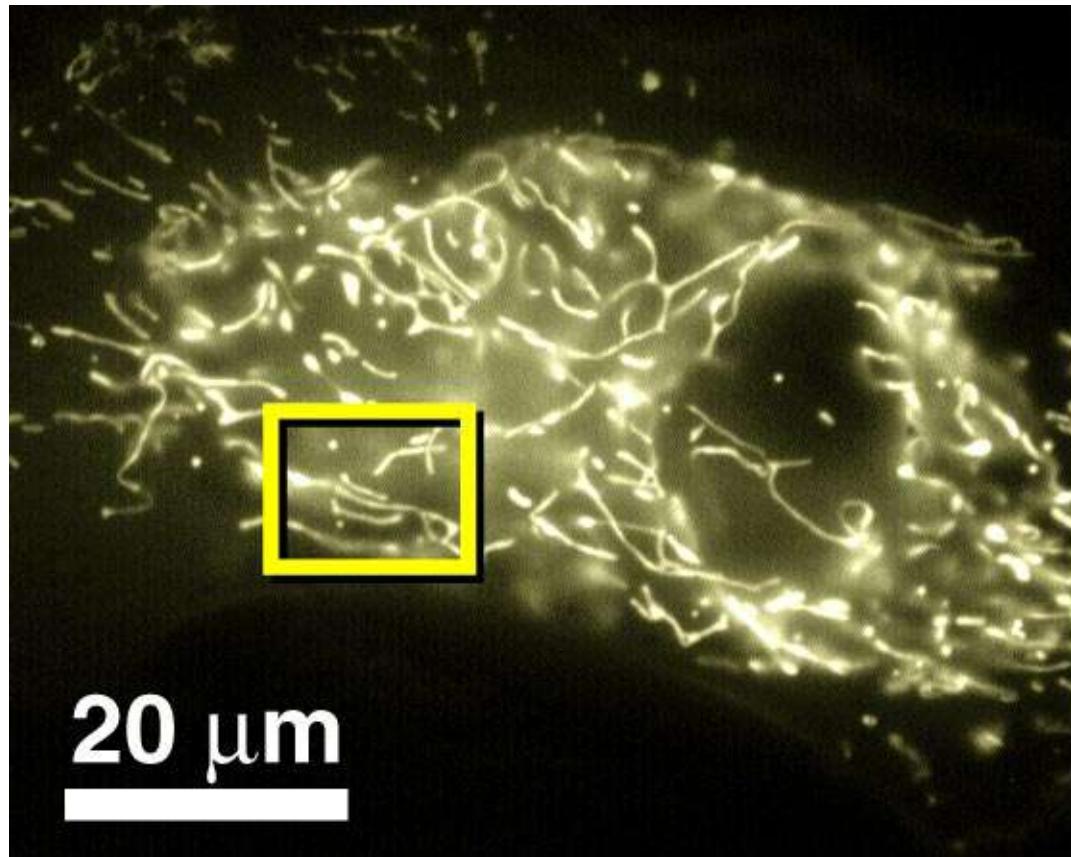
Low-energy processing

monolithic amplifier

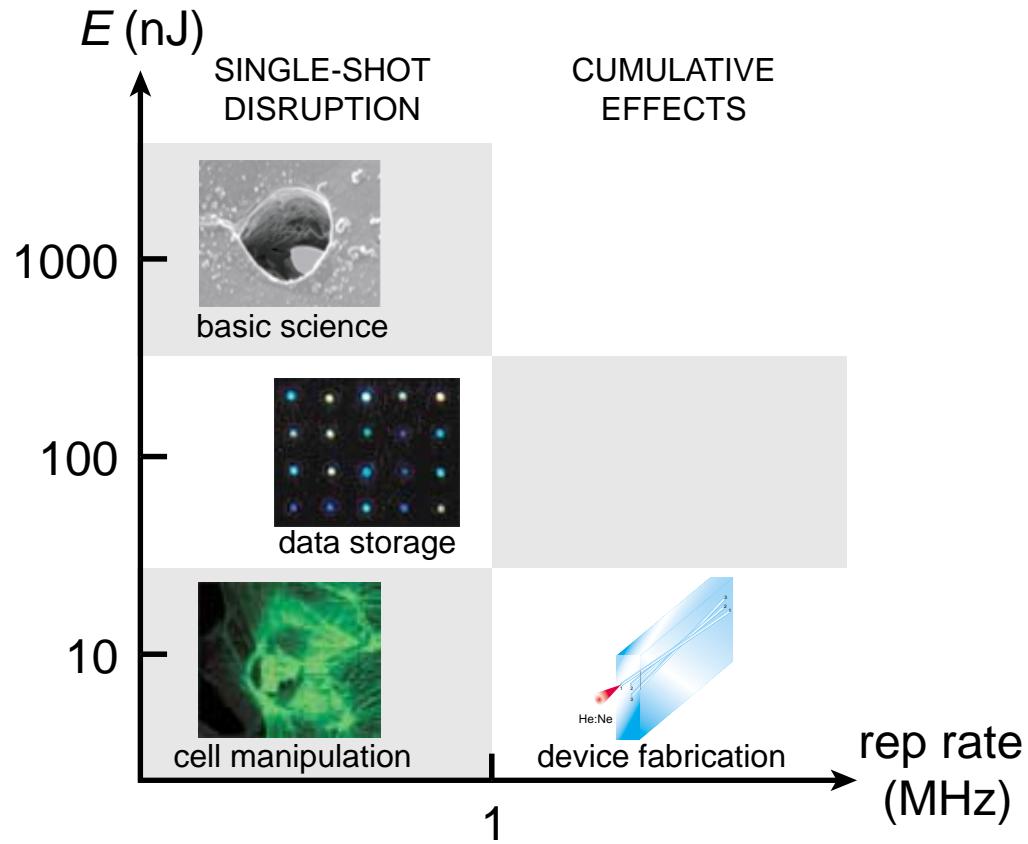


laser active glass

Low-energy processing



Summary



Conclusion

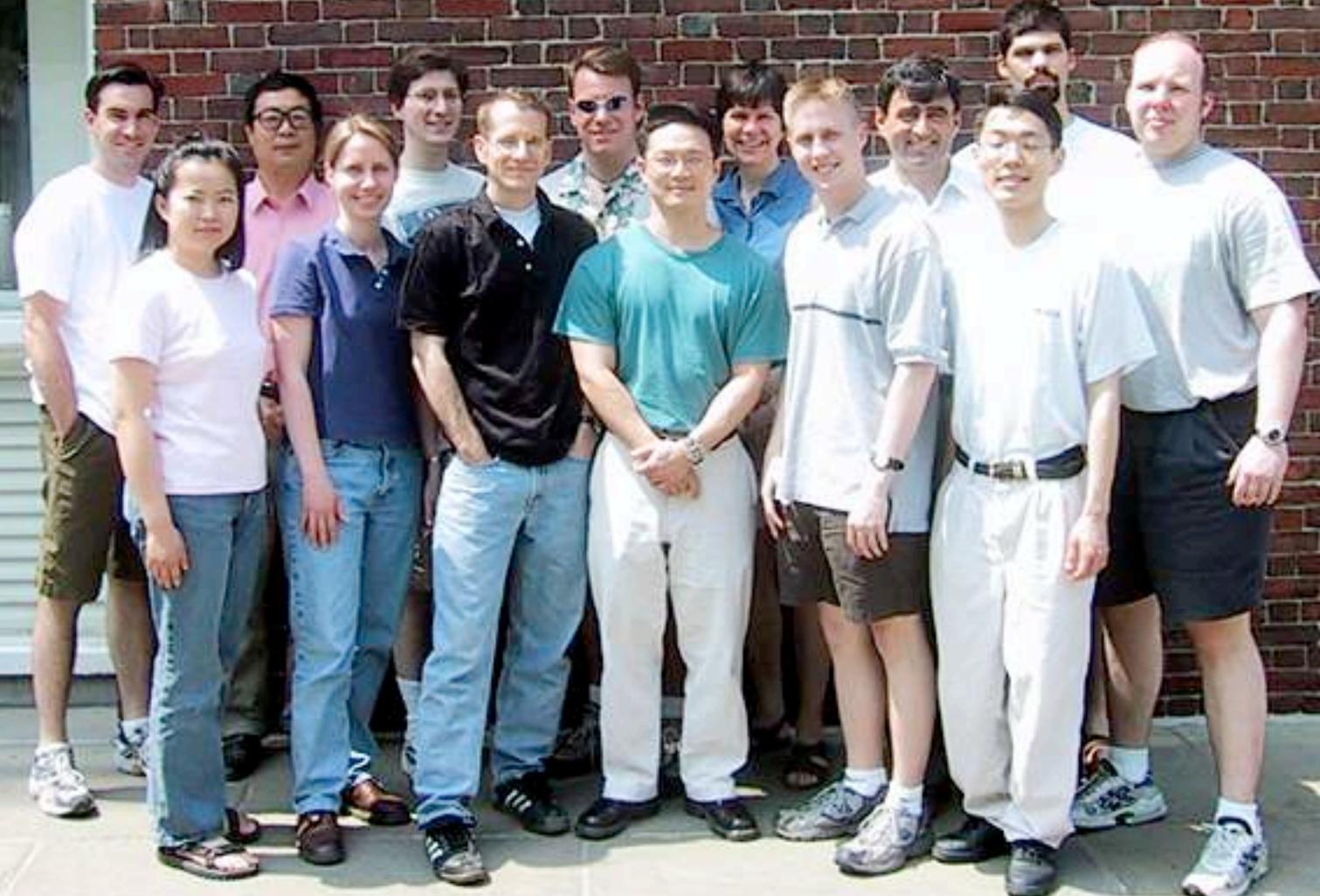
- ▶ **wiring optoelectronics circuits of the future**
- ▶ **manipulating the machinery of life**

Summary

Femtosecond laser pulses offer:

- ▶ **Unprecedented view into dynamics**
- ▶ **Extreme conditions with little energy**
- ▶ **New opportunities for research and processing**

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LABORATORY OF
APPLIED SCIENCE





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Prof. H. Ehrenreich
Prof. T. Kaxiras**

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