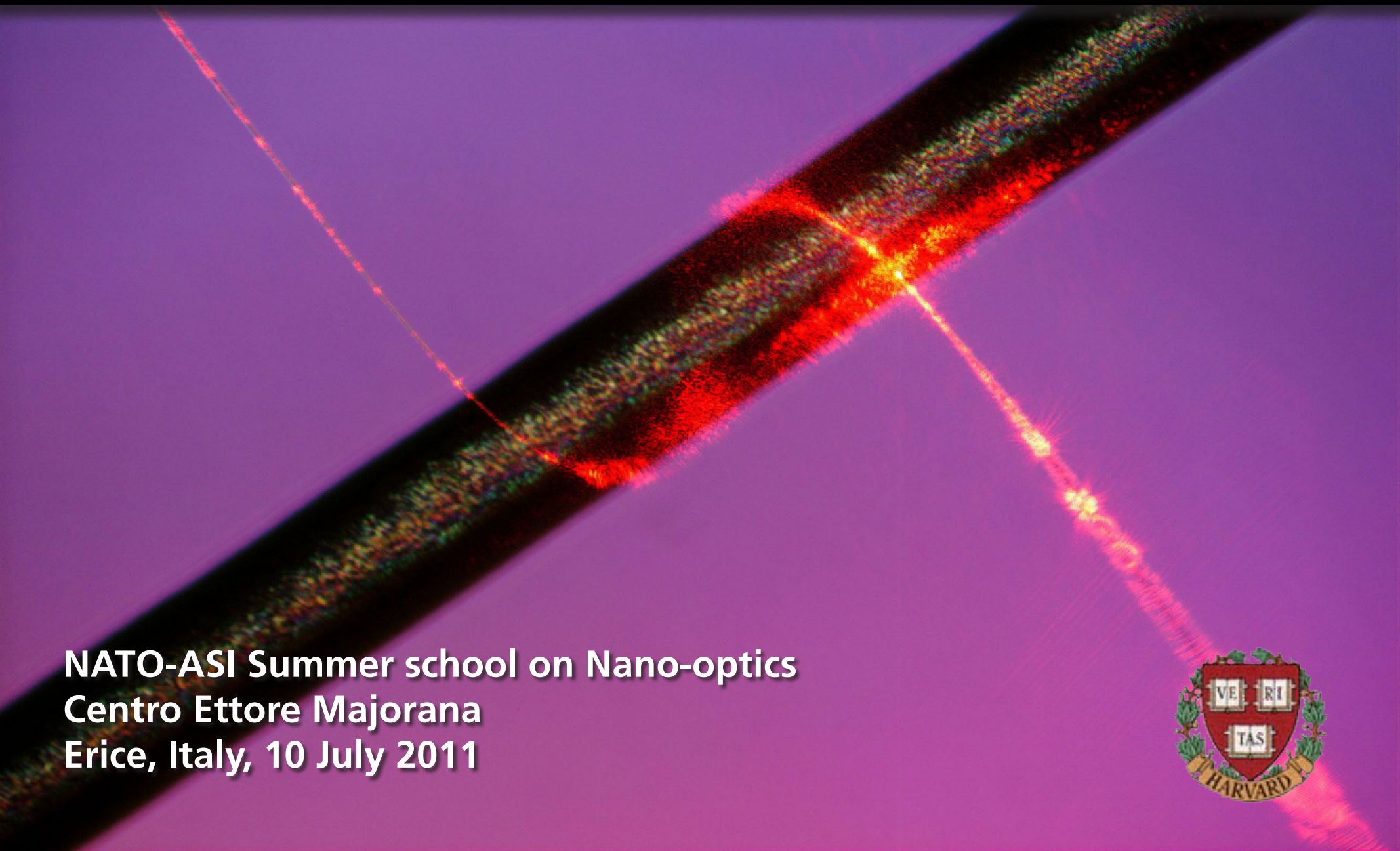
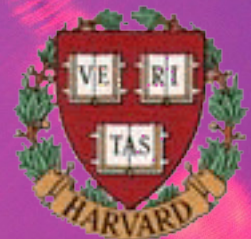
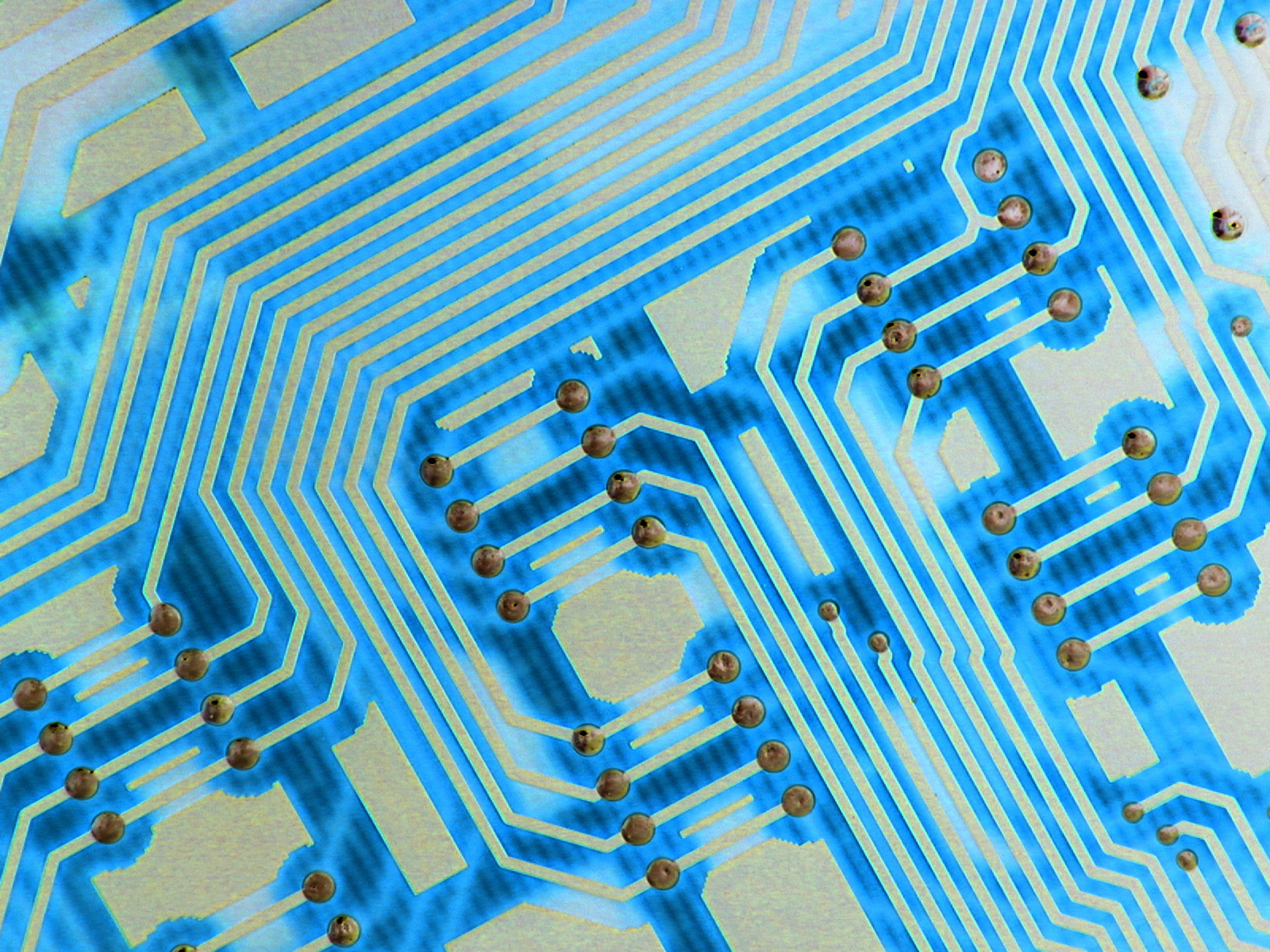


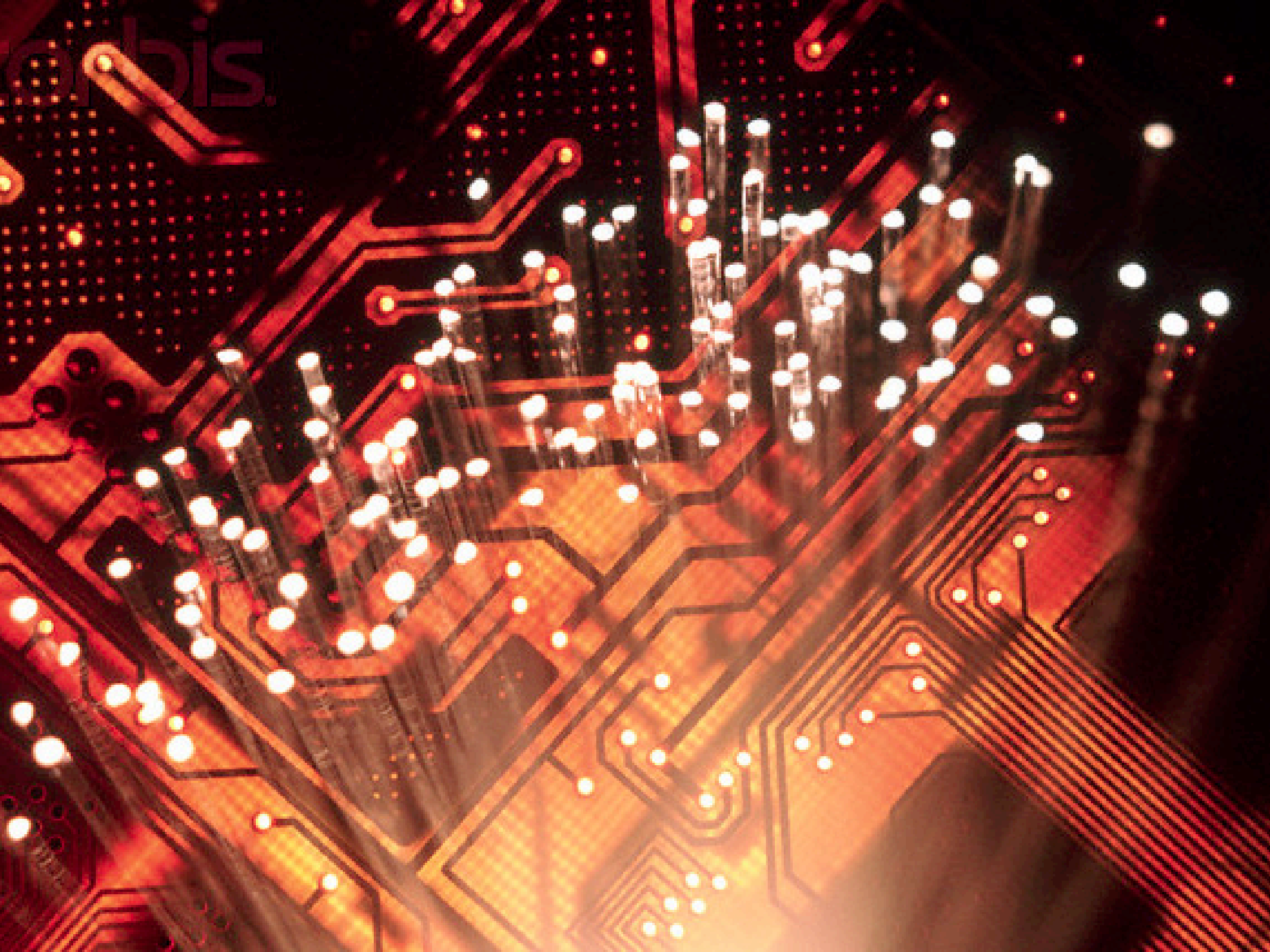
Nanophotonics: Linear and nonlinear optics at the nanoscale



NATO-ASI Summer school on Nano-optics
Centro Ettore Majorana
Erice, Italy, 10 July 2011







obis.

Outline

- propagation of pulses
- nonlinear optics
- nanoscale optics
- nonlinear optics at the nanoscale

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- nonlinear optics
- nanoscale optics
- nonlinear optics at the nanoscale

Propagation of EM wave through medium

Governed by wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

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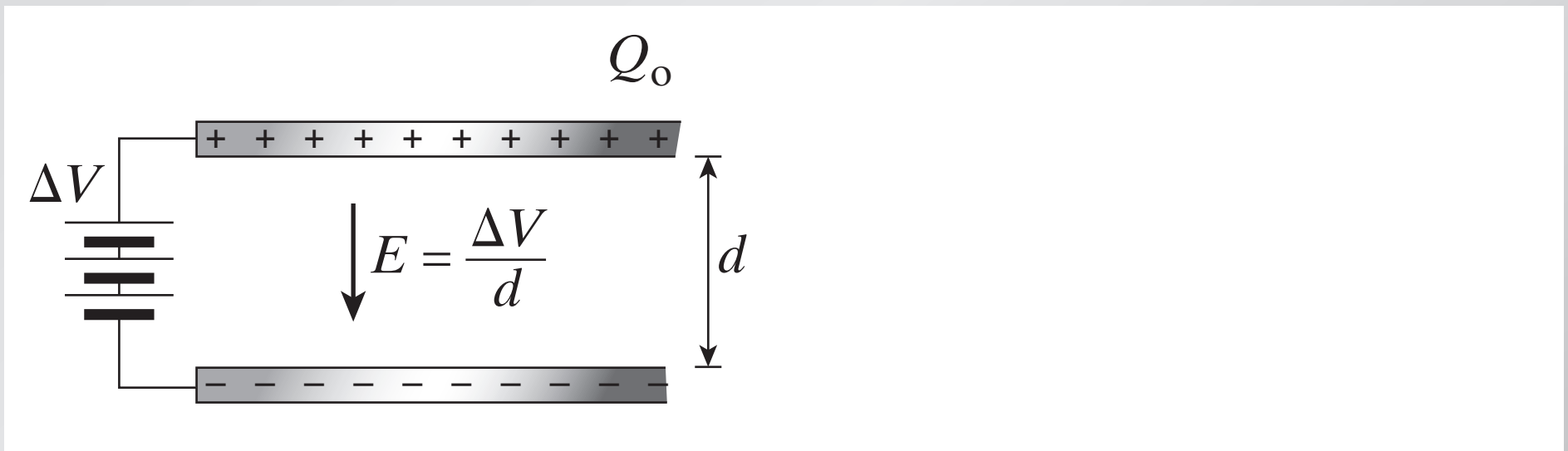
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In dispersive media $n = n(\omega)$.

Propagation of EM wave through medium

Dielectric constant measures increase in capacitance

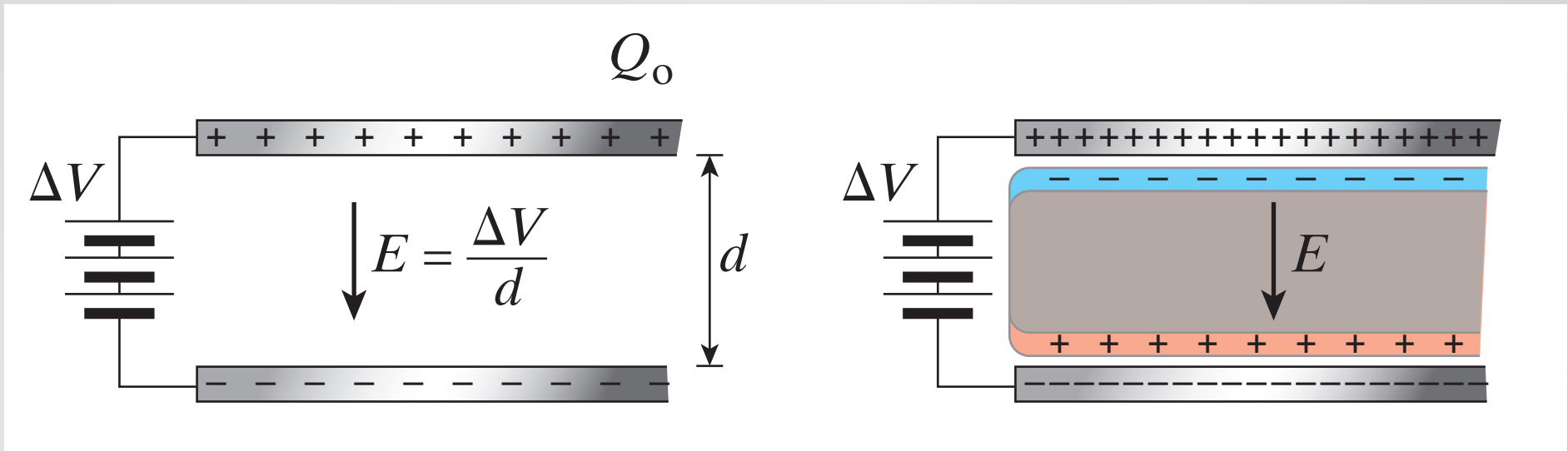
$$\epsilon = \frac{C_d}{C_o}$$



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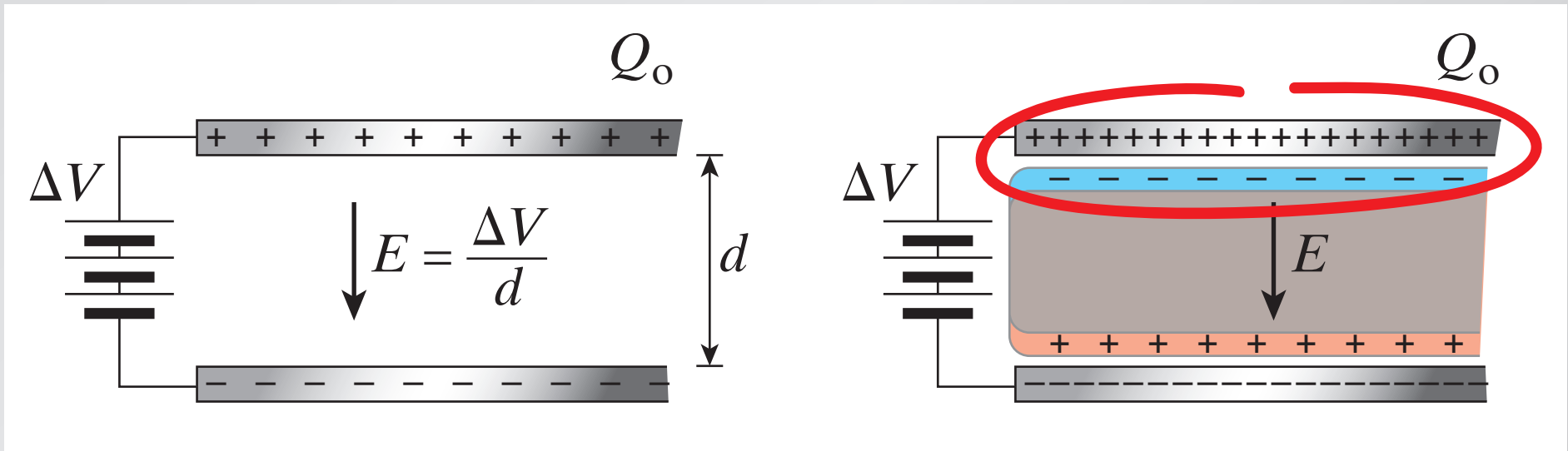
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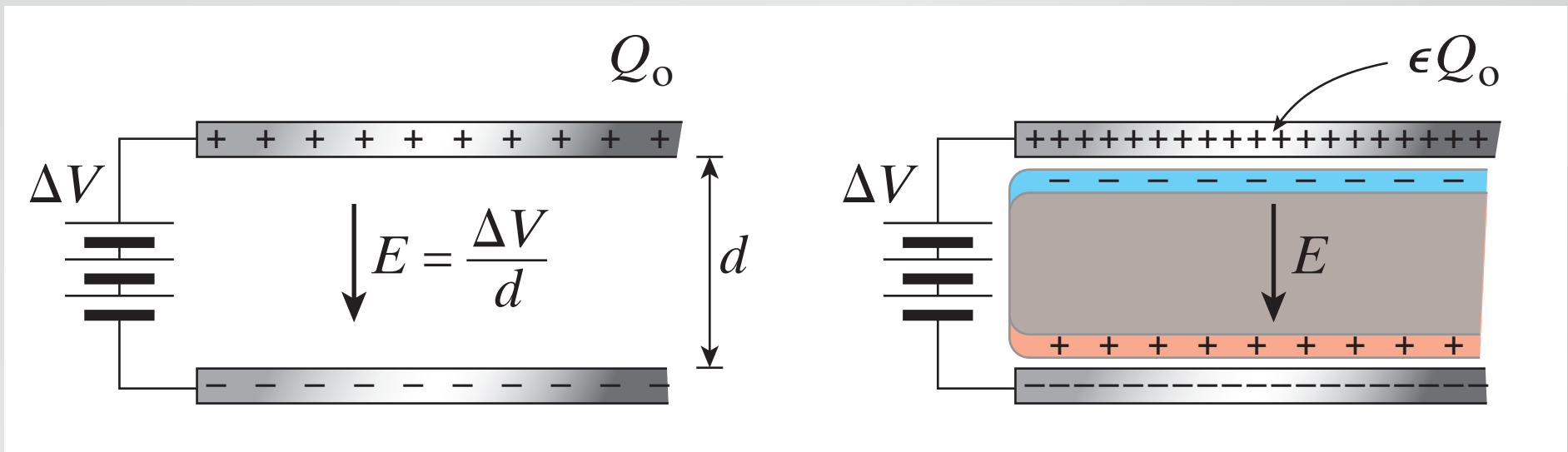
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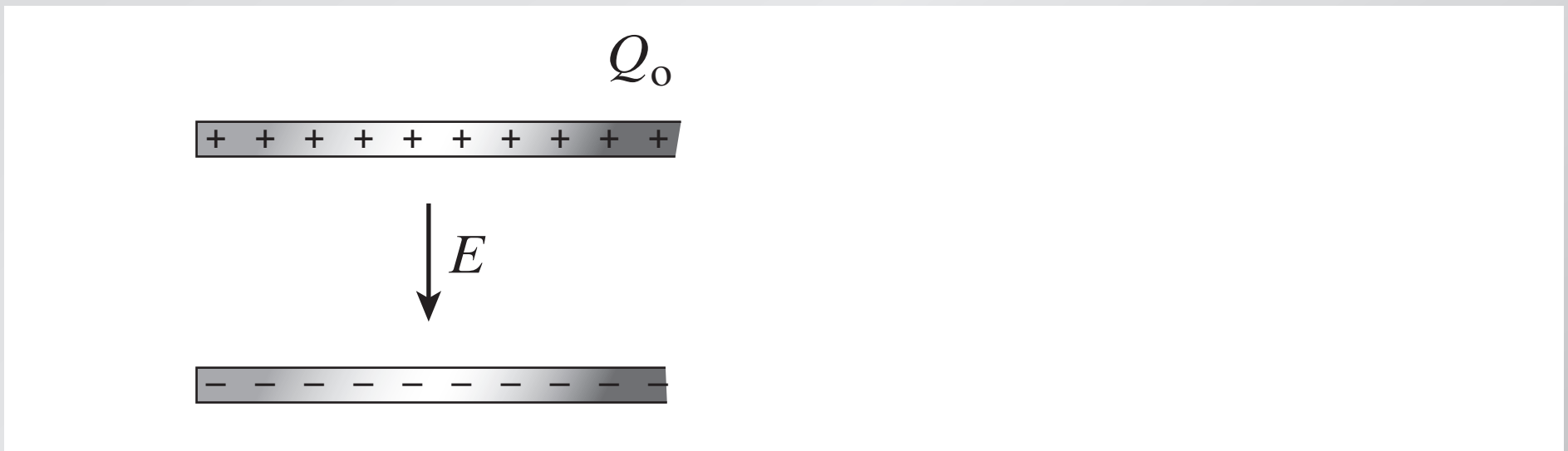
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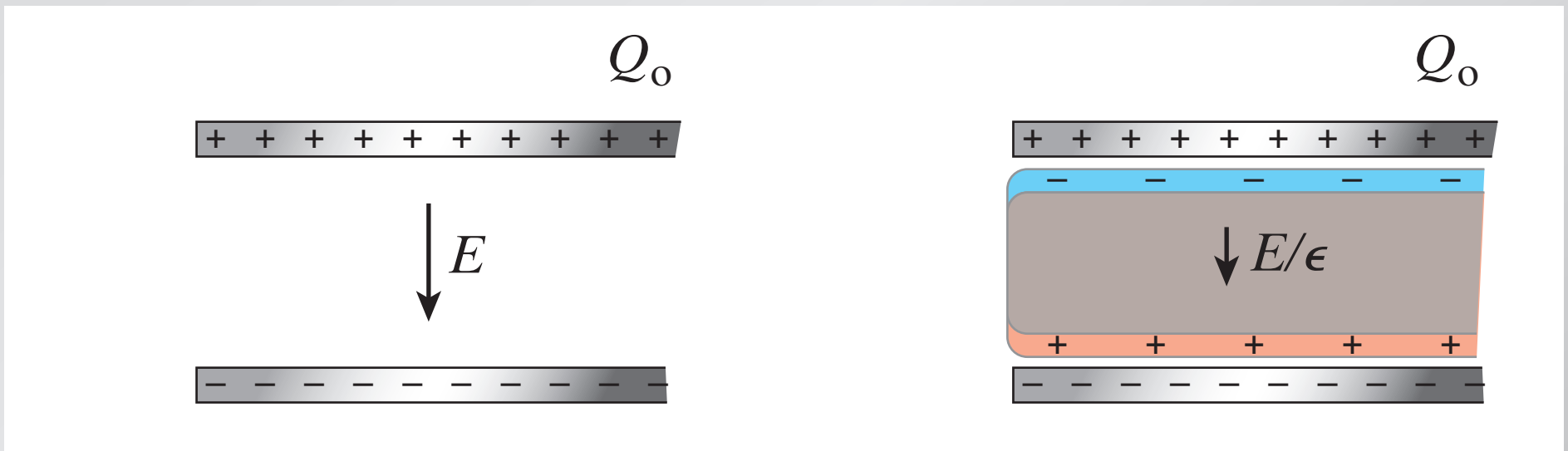
Propagation of EM wave through medium

Alternatively ϵ is measure of attenuation of electric field



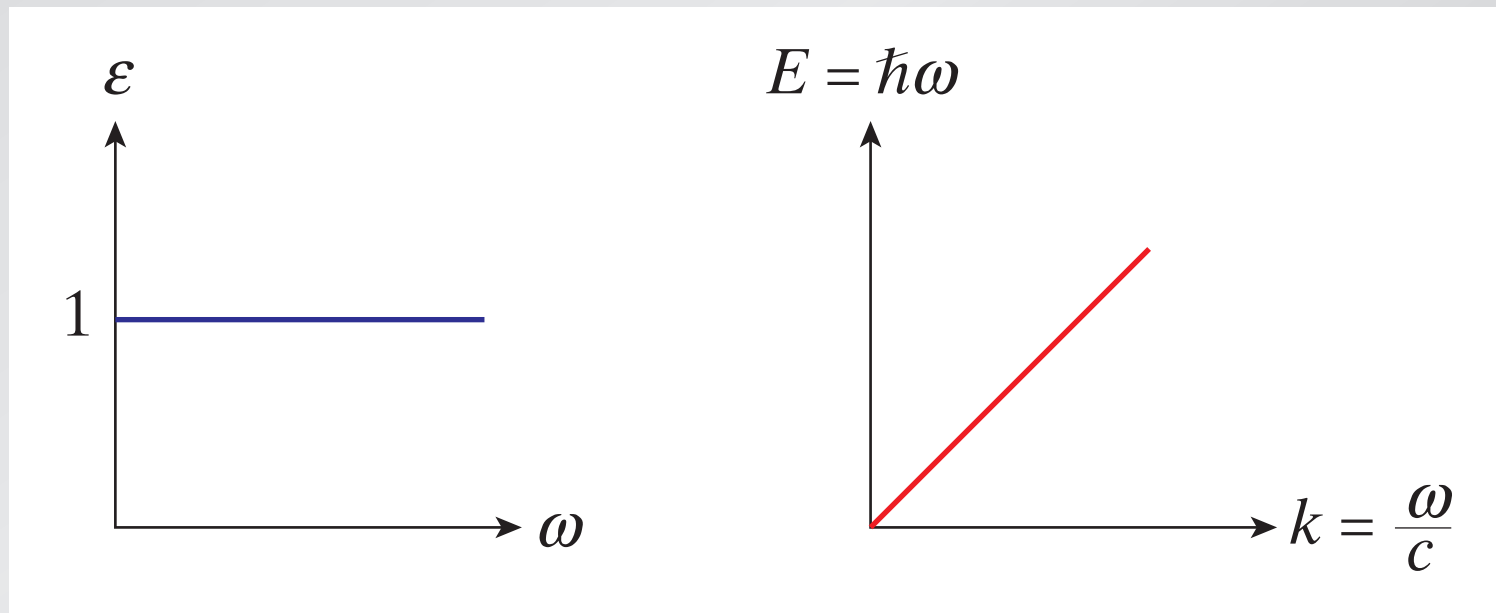
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Propagation of EM wave through medium

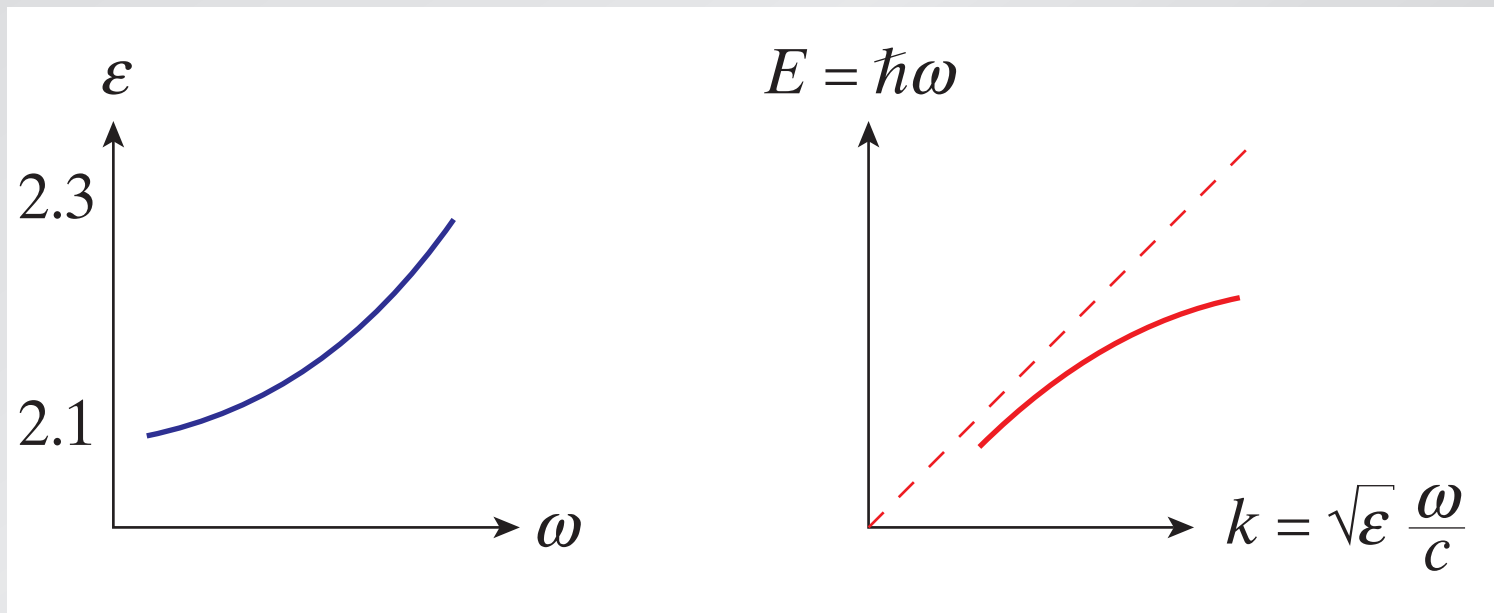
In vacuum: $f\lambda = \frac{\omega}{k} = c \quad \Rightarrow \quad \omega = c k$



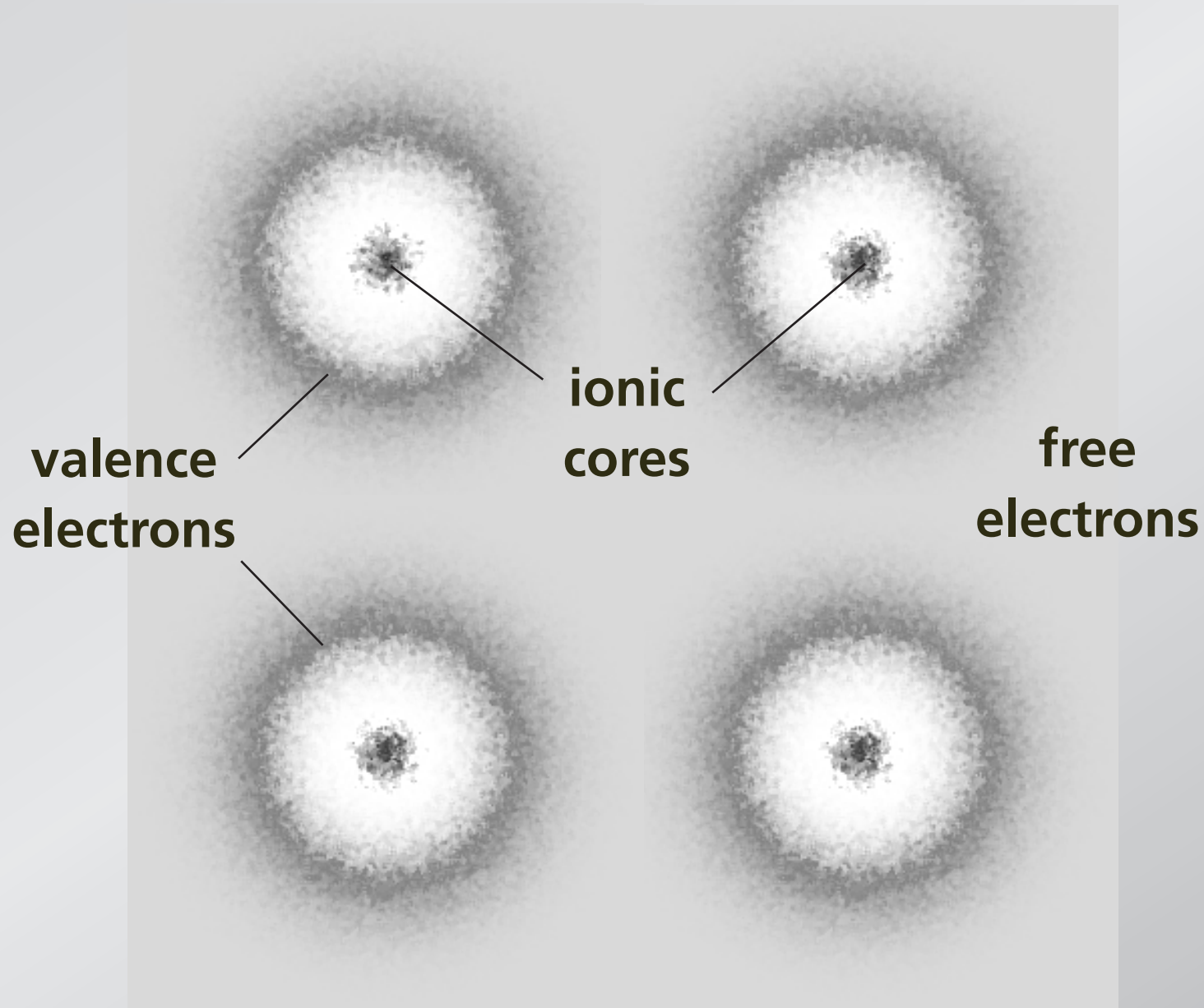
Propagation of EM wave through medium

In medium:

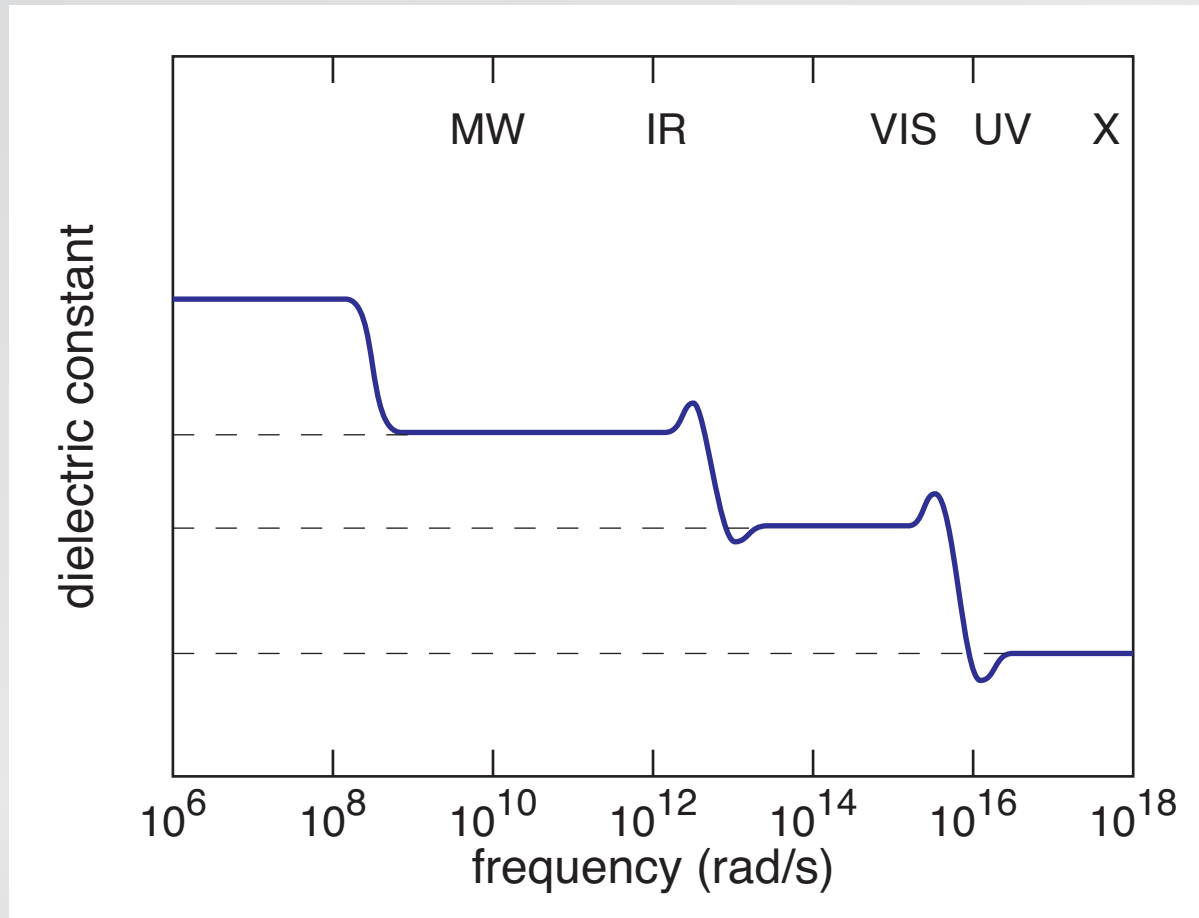
$$v = \frac{c}{\sqrt{\epsilon(\omega)}} = \frac{c}{n} \quad \Rightarrow \quad \omega = \frac{c}{\sqrt{\epsilon}} k$$



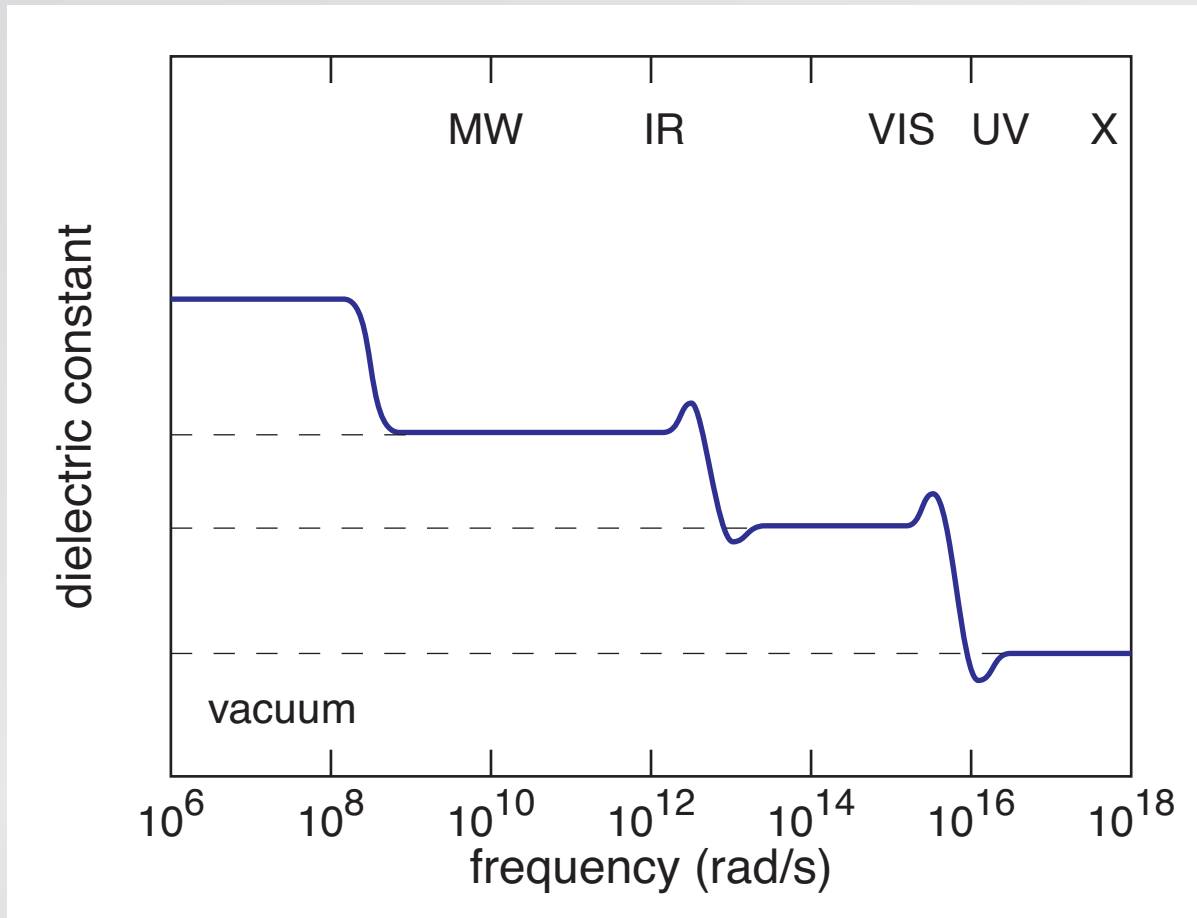
Which charges participate?



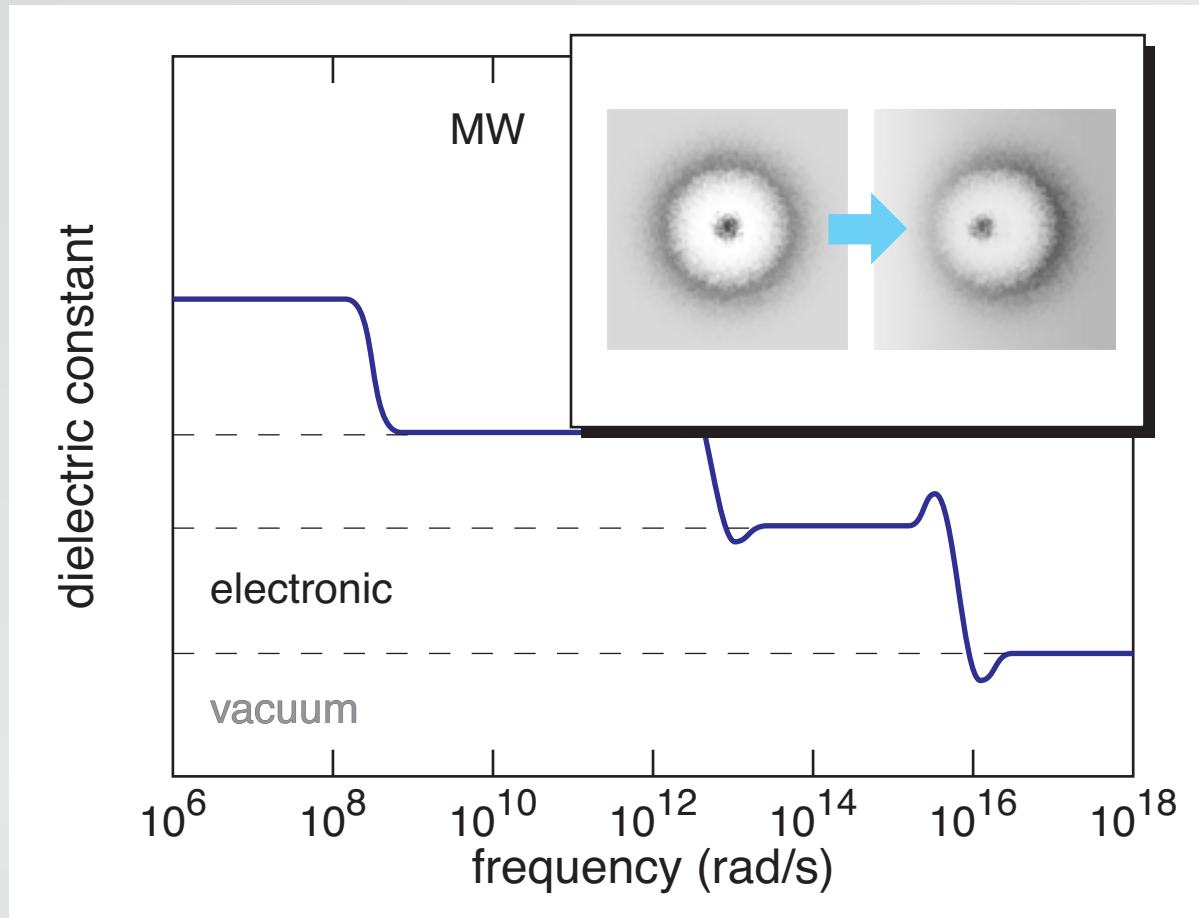
Dielectric function



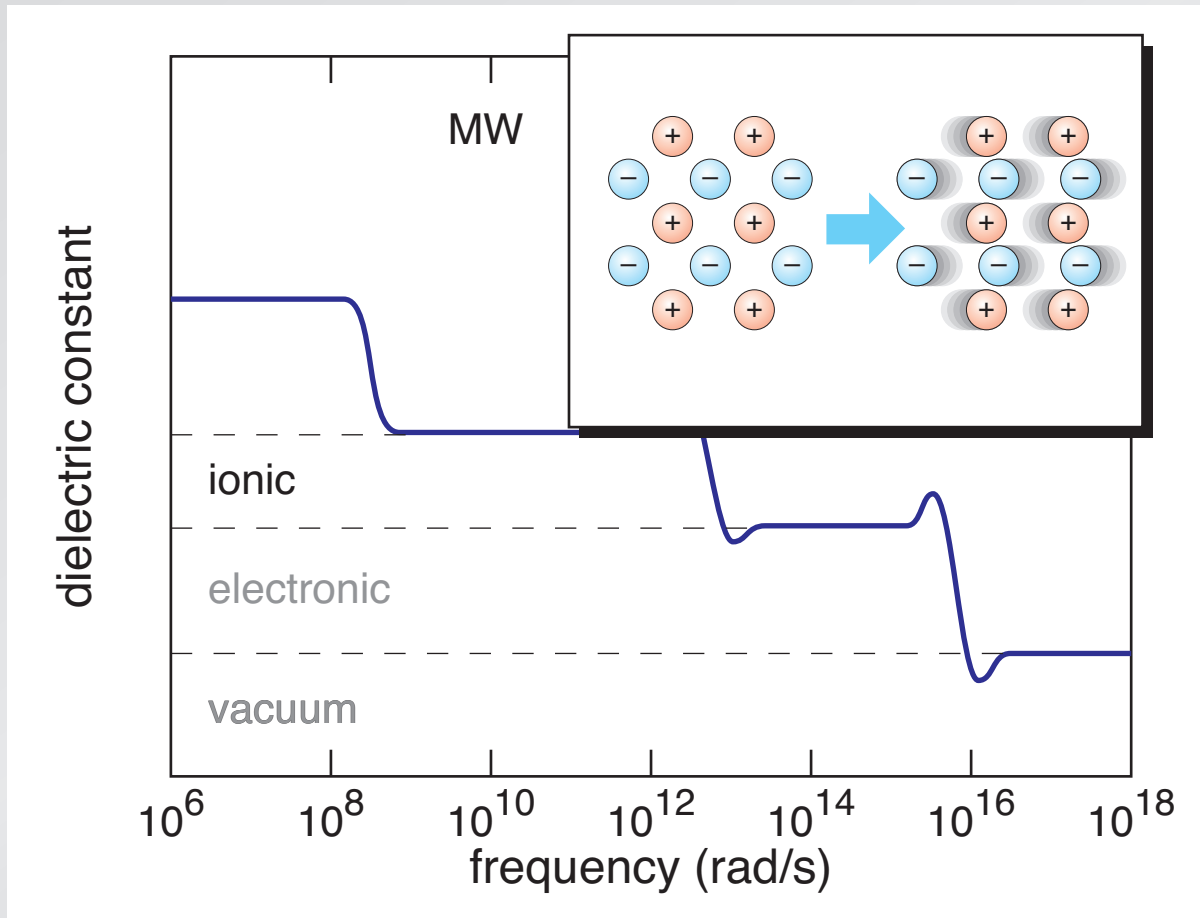
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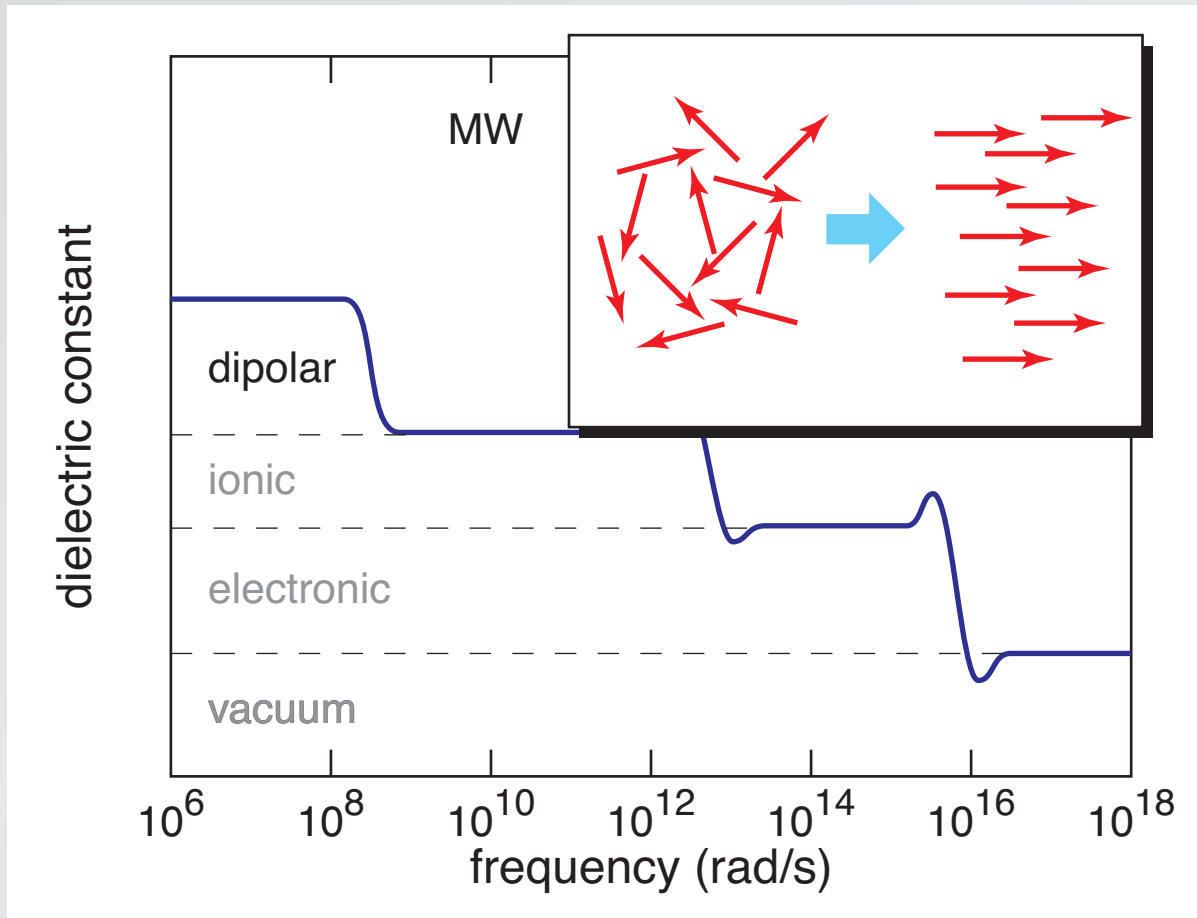
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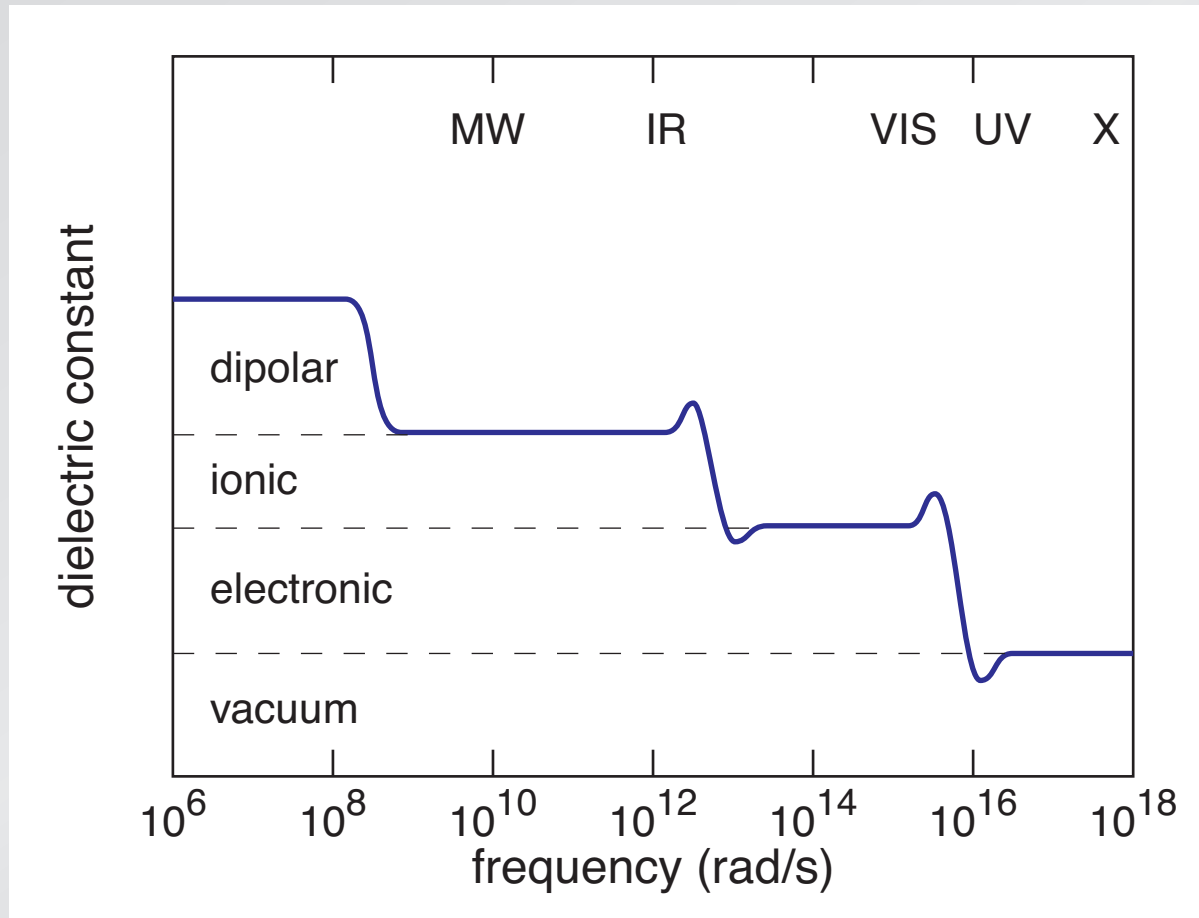
Dielectric function



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Bound electrons

Electron on a string: $F_{binding} = -m_e \omega_o^2 x$

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Steady state: electron oscillates at driving frequency

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$$P(t) = \left(\frac{Ne^2}{m} \right) \sum_j \frac{f_j}{(\omega_j^2 - \omega^2) - i\gamma_j\omega} E(t) \equiv \epsilon_o \chi_e E(t)$$

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$$\epsilon(\omega) \equiv 1 + \chi_e = 1 + \frac{Ne^2}{\epsilon_0 m} \sum_j \frac{f_j}{(\omega_j^2 - \omega^2) - i\gamma_j \omega}$$

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Q: For a single resonance, is the value of $\epsilon(\omega)$ at high frequency

1. larger than,

2. the same as, or

3. smaller than the value at low frequency?

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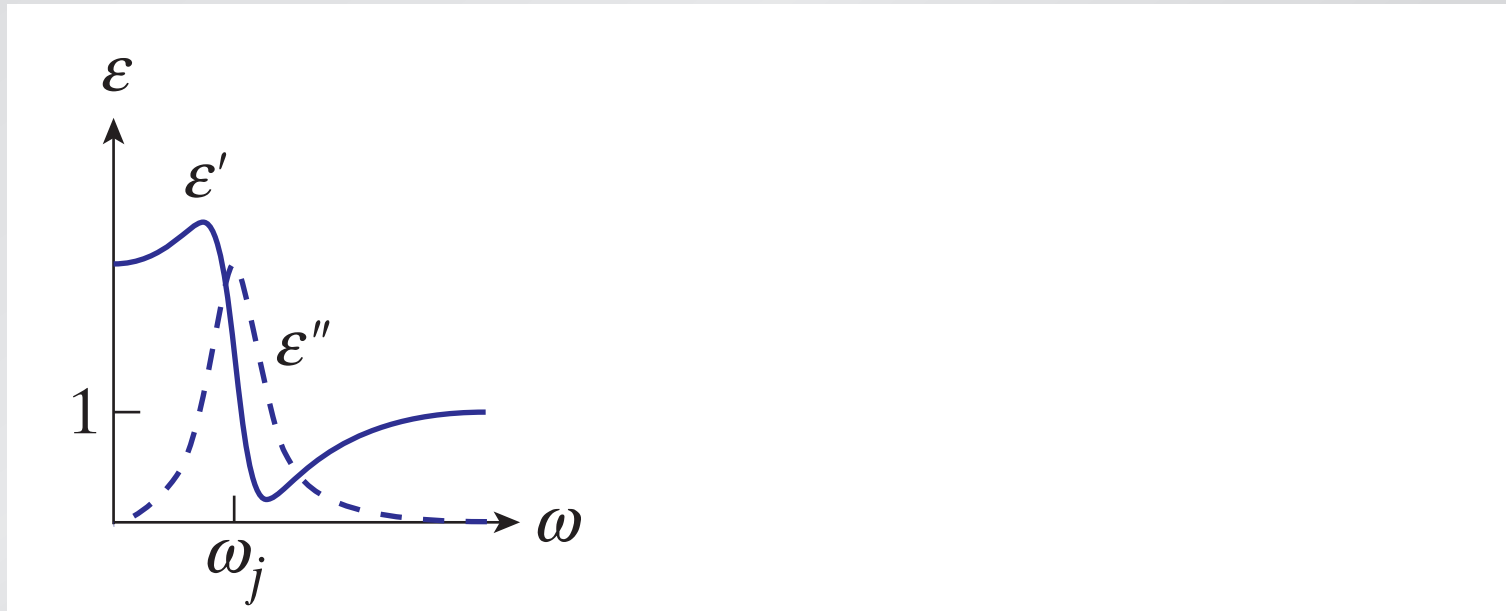
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Bound electrons

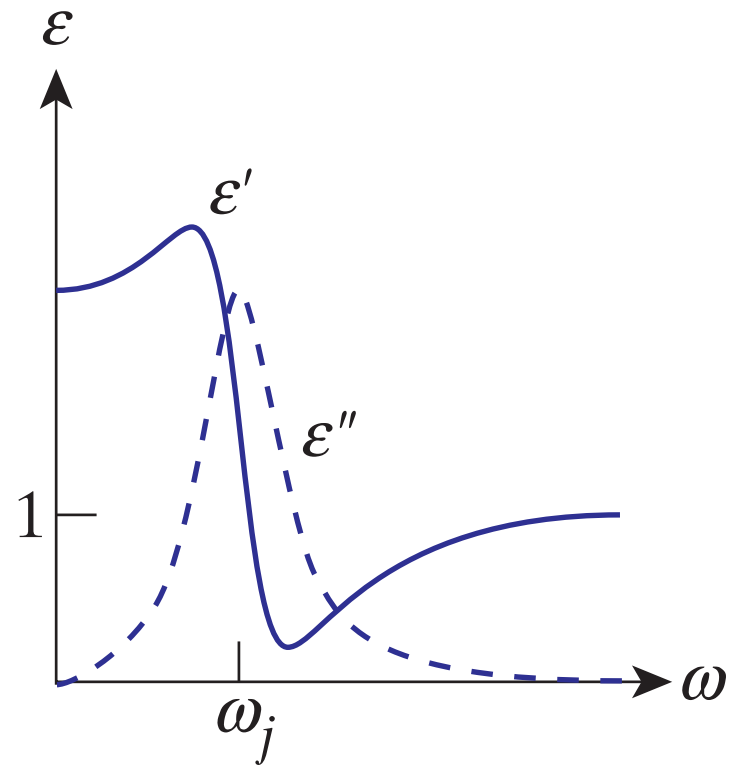
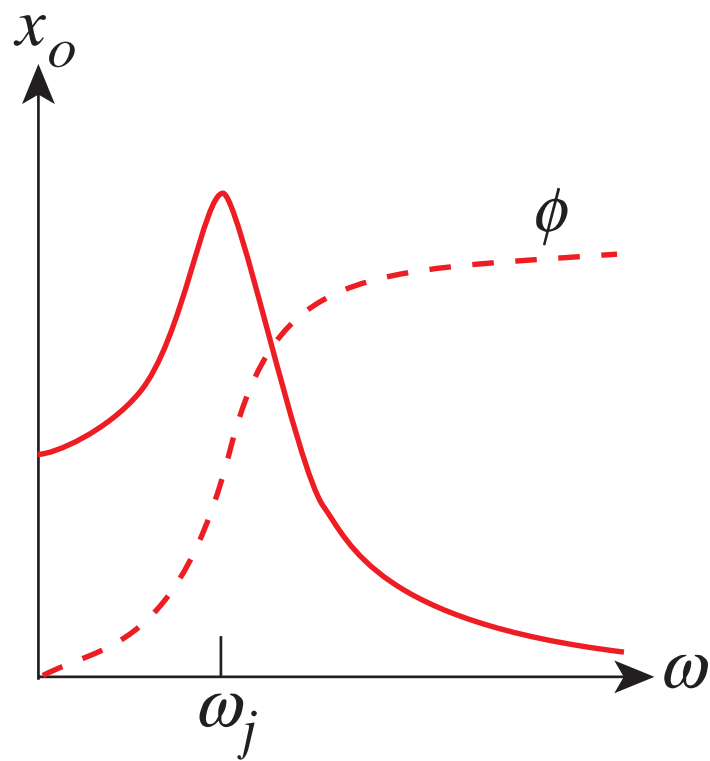
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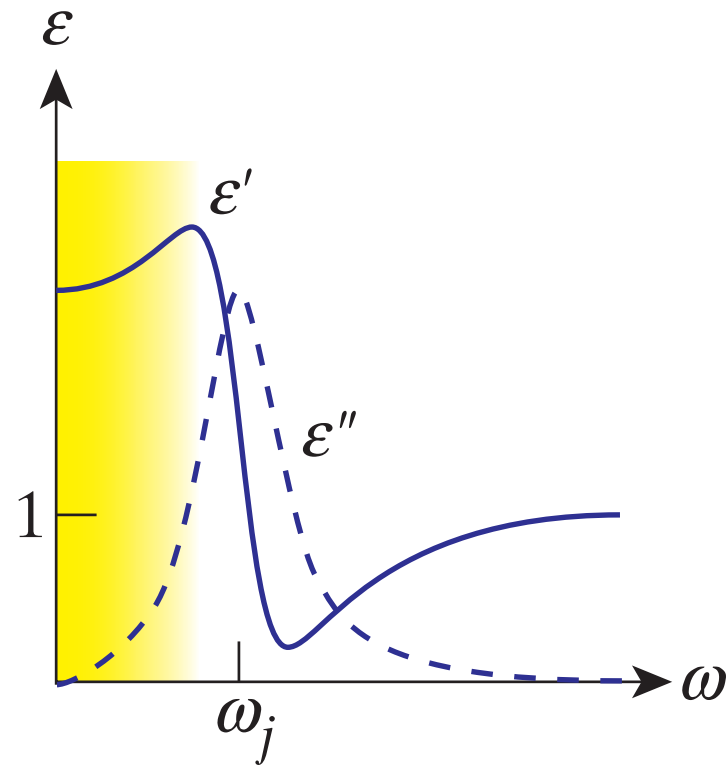
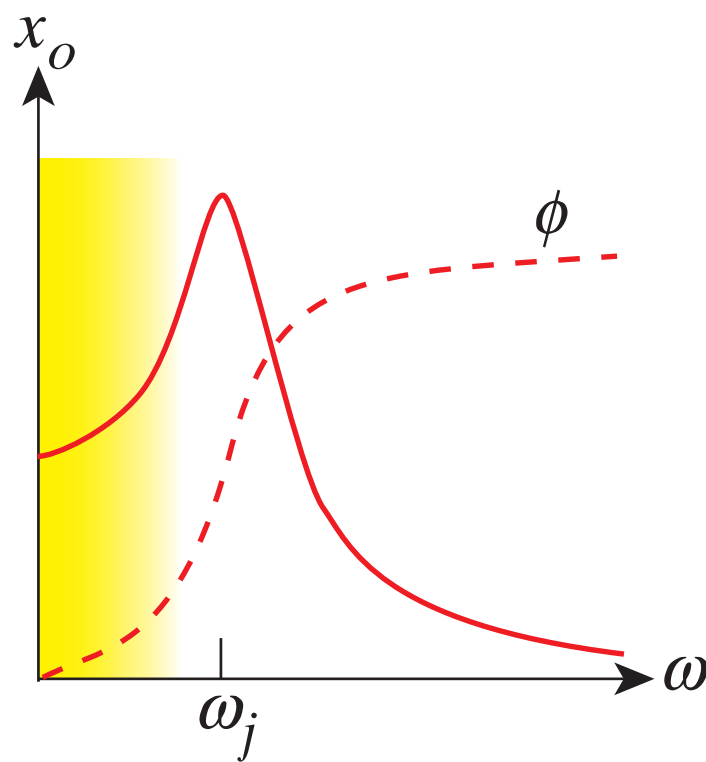
Bound electrons

Amplitude of bound charge oscillation



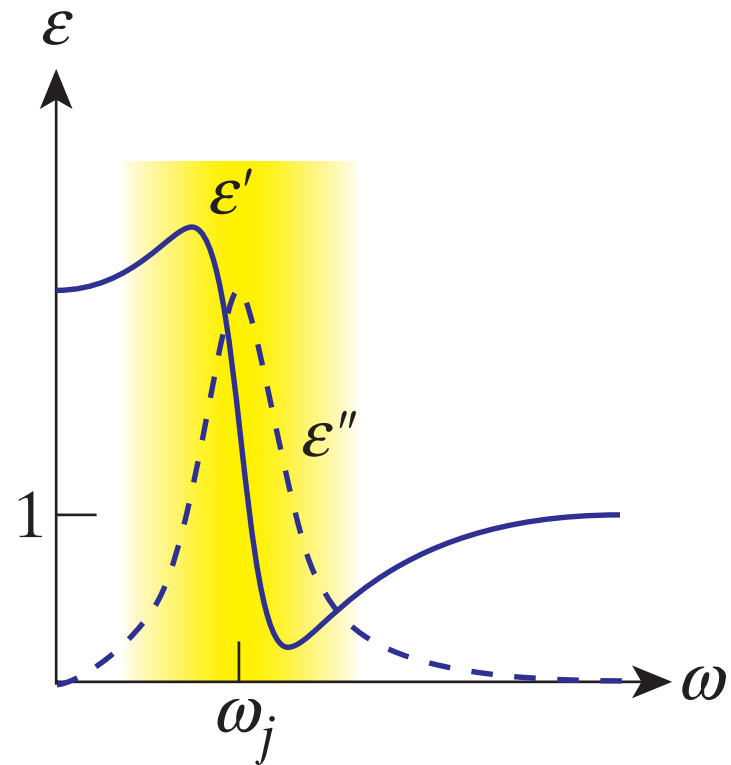
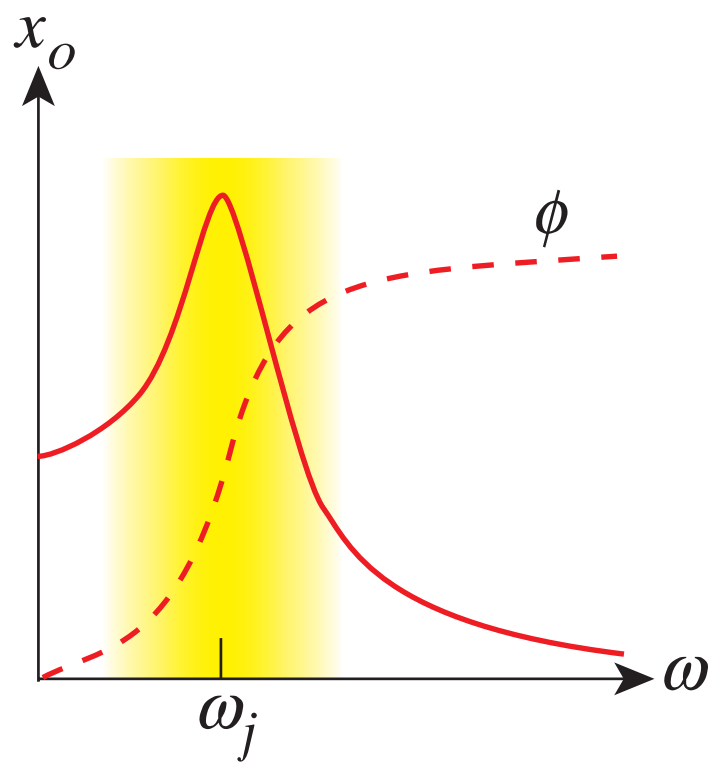
Bound electrons

Below resonance: bound charges keep up with driving field \Rightarrow field attenuated, wave propagates more slowly



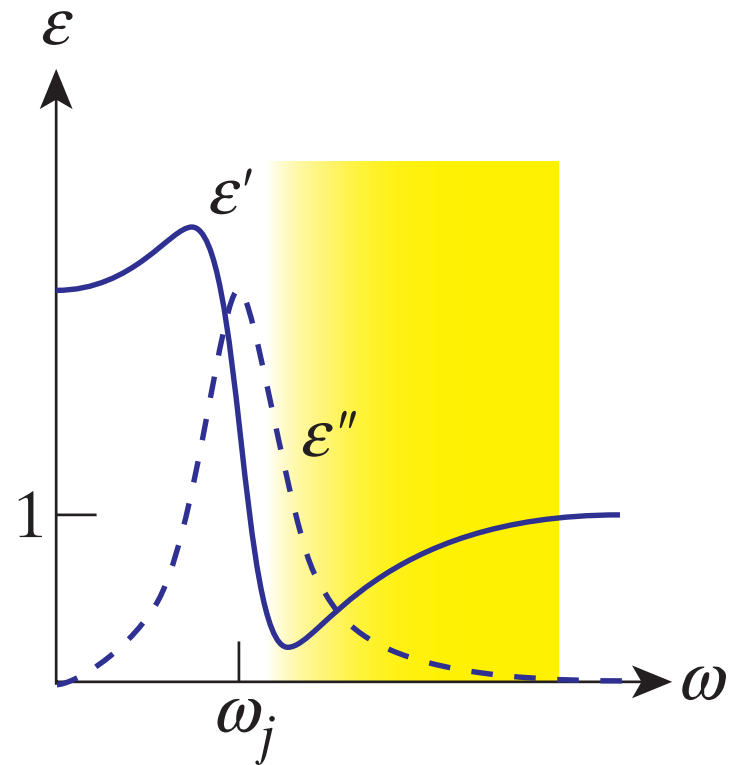
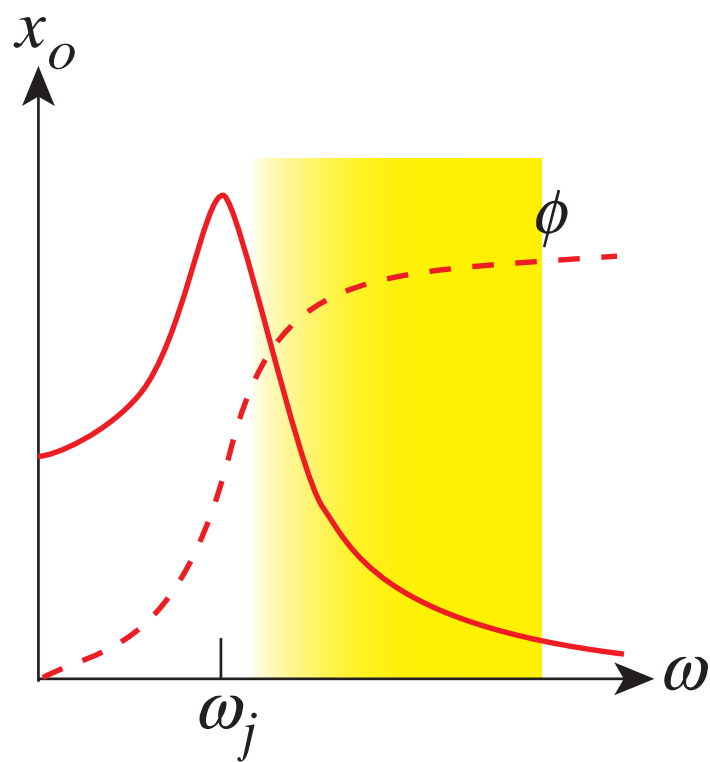
Bound electrons

At resonance: energy transfer from wave to bound charges \Rightarrow wave attenuates (absorption)



Bound electrons

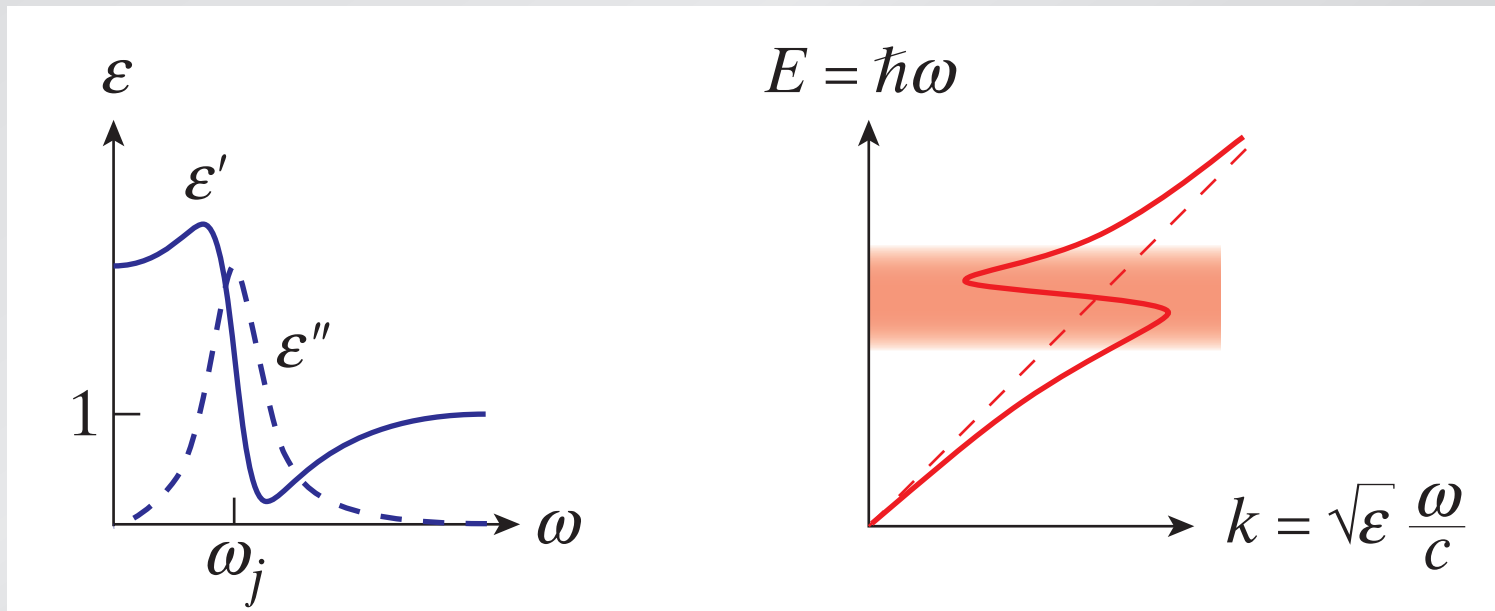
Above resonance: bound charges cannot keep up with driving field \Rightarrow dielectric like a vacuum



Bound electrons

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$$\epsilon(\omega) \equiv 1 + \chi_e = 1 + \frac{Ne^2}{\epsilon_0 m} \sum_j \frac{f_j}{(\omega_j^2 - \omega^2) - i\gamma_j \omega}$$



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Low frequency ($\omega \ll \gamma$) \Rightarrow current generated

$$J = -Ne \frac{dx}{dt} = \frac{Ne^2}{m} \frac{1}{\gamma - i\omega} E \approx \frac{Ne^2}{m\gamma} E \equiv \sigma E$$

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$\omega \gg \gamma$: σ complex $\Rightarrow J$ out of phase with E

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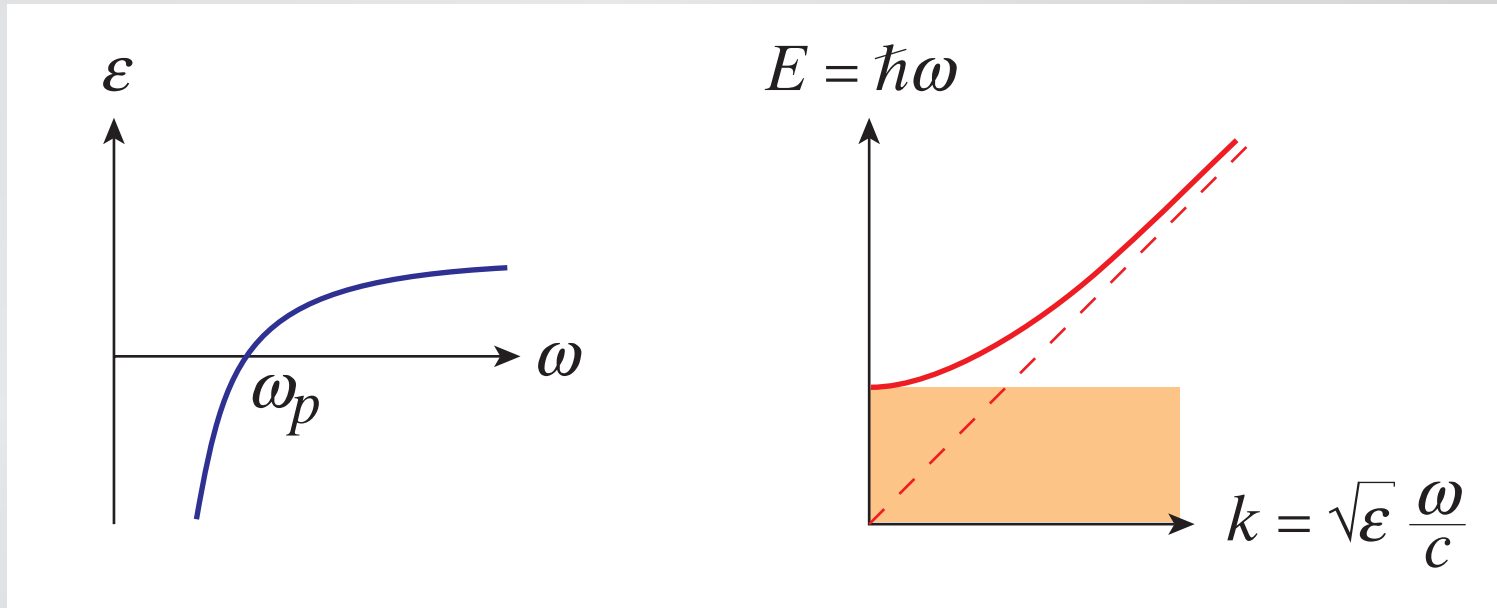
$$\epsilon'(\omega) = 1 - \frac{Ne^2}{m\epsilon_0} \frac{1}{\omega^2} \equiv 1 - \frac{\omega_p^2}{\omega^2}$$

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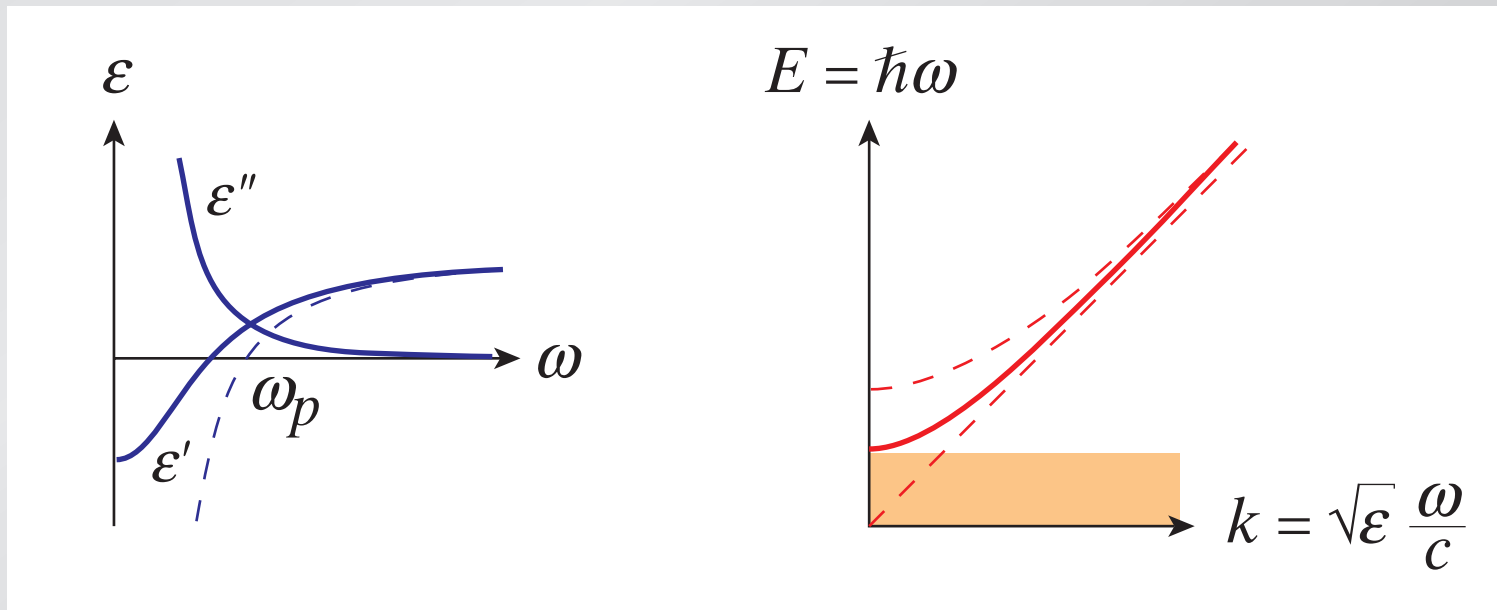


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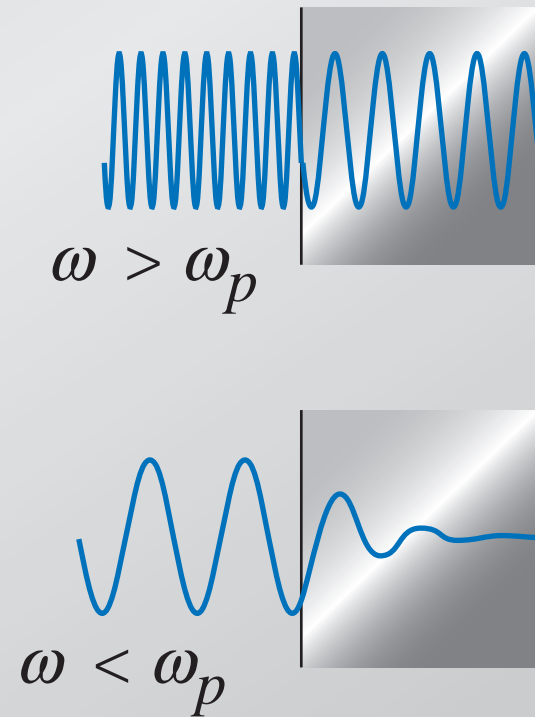
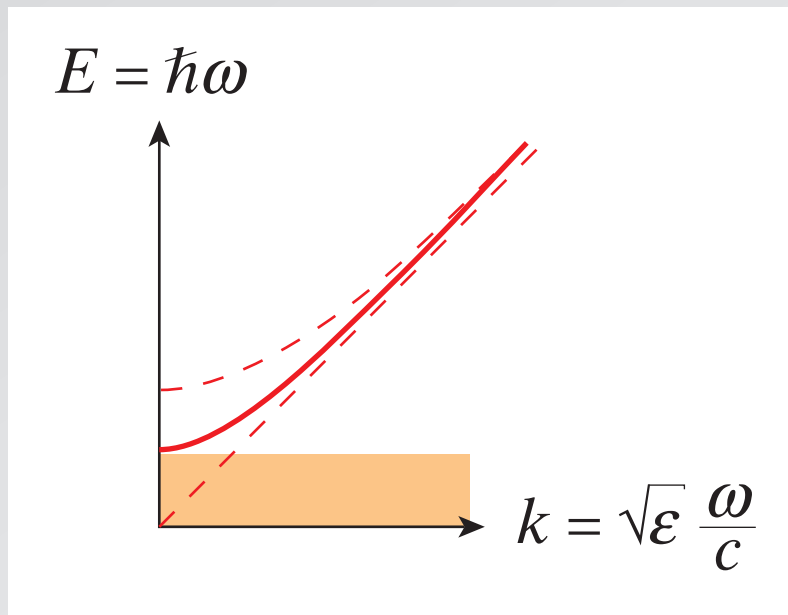
Add damping:

$$\gamma \lesssim \omega_p$$



Free electrons

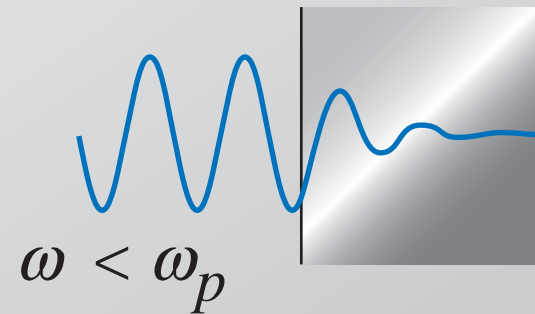
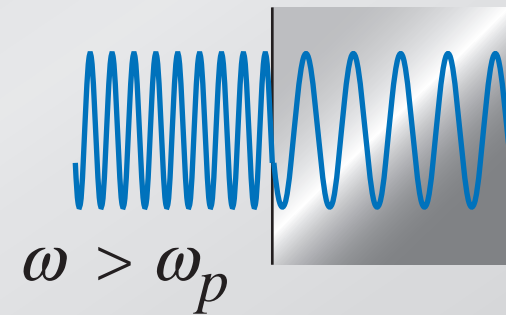
Plasma acts like a high-pass filter



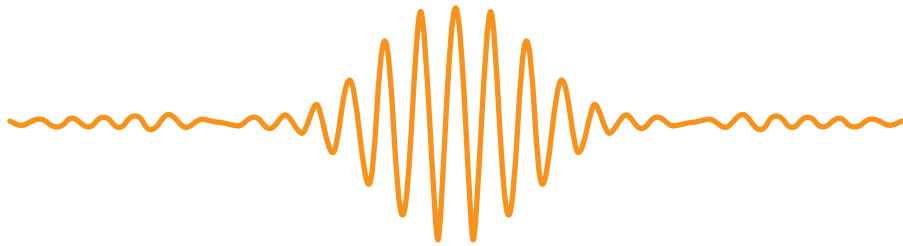
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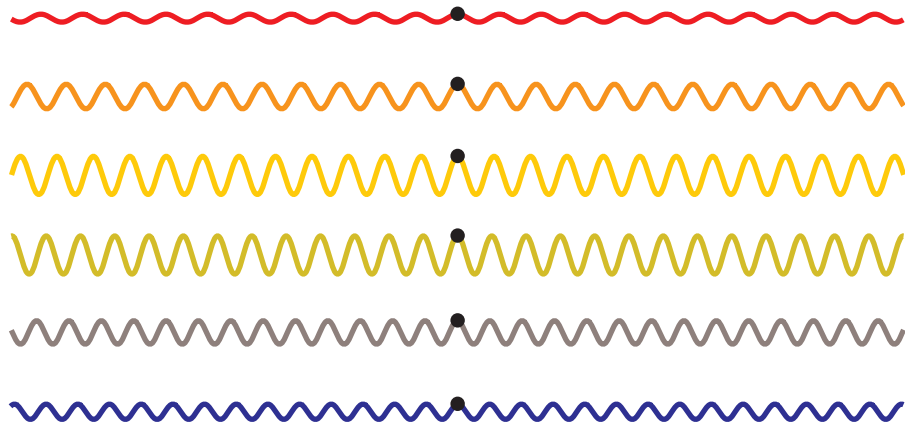
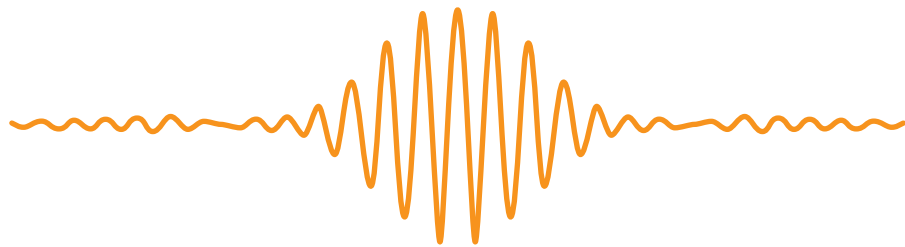
$\log N$ (cm^{-3})	ω_p (rad s^{-1})	λ_p
22	6×10^{15}	330 nm
18	6×10^{13}	33 μm
14	6×10^{11}	3.3 mm
10	6×10^9	0.33 m



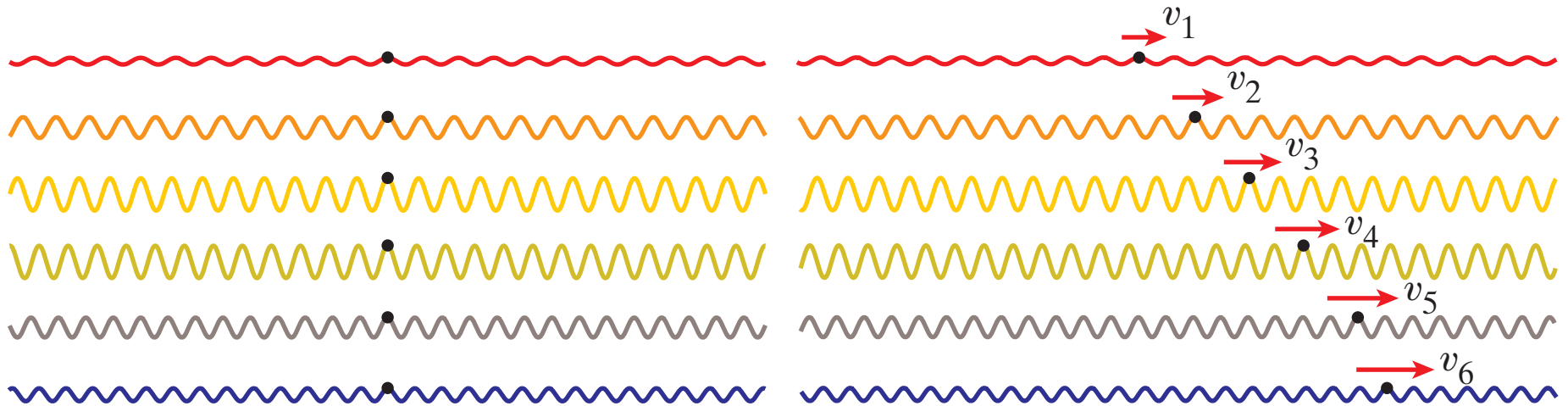
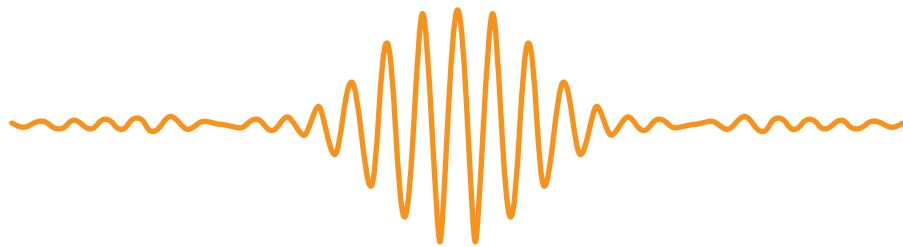
Pulse dispersion



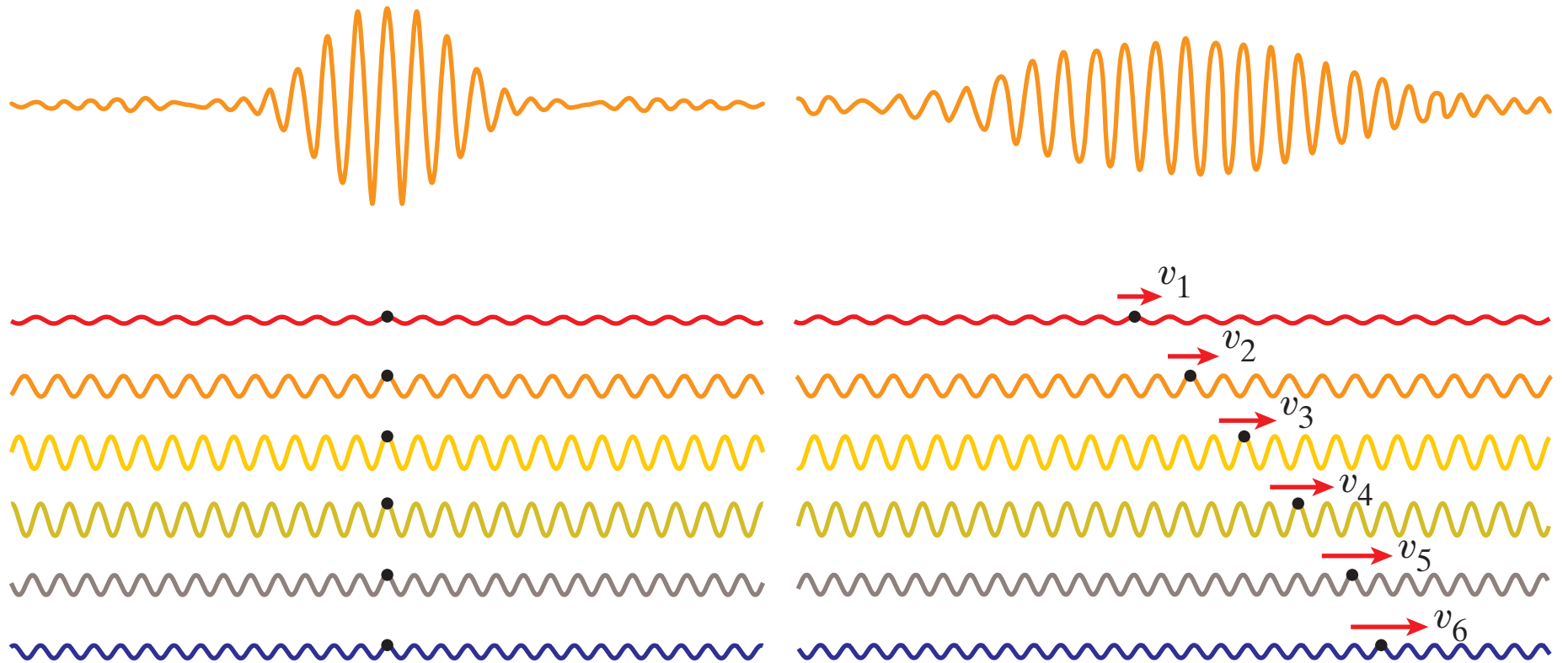
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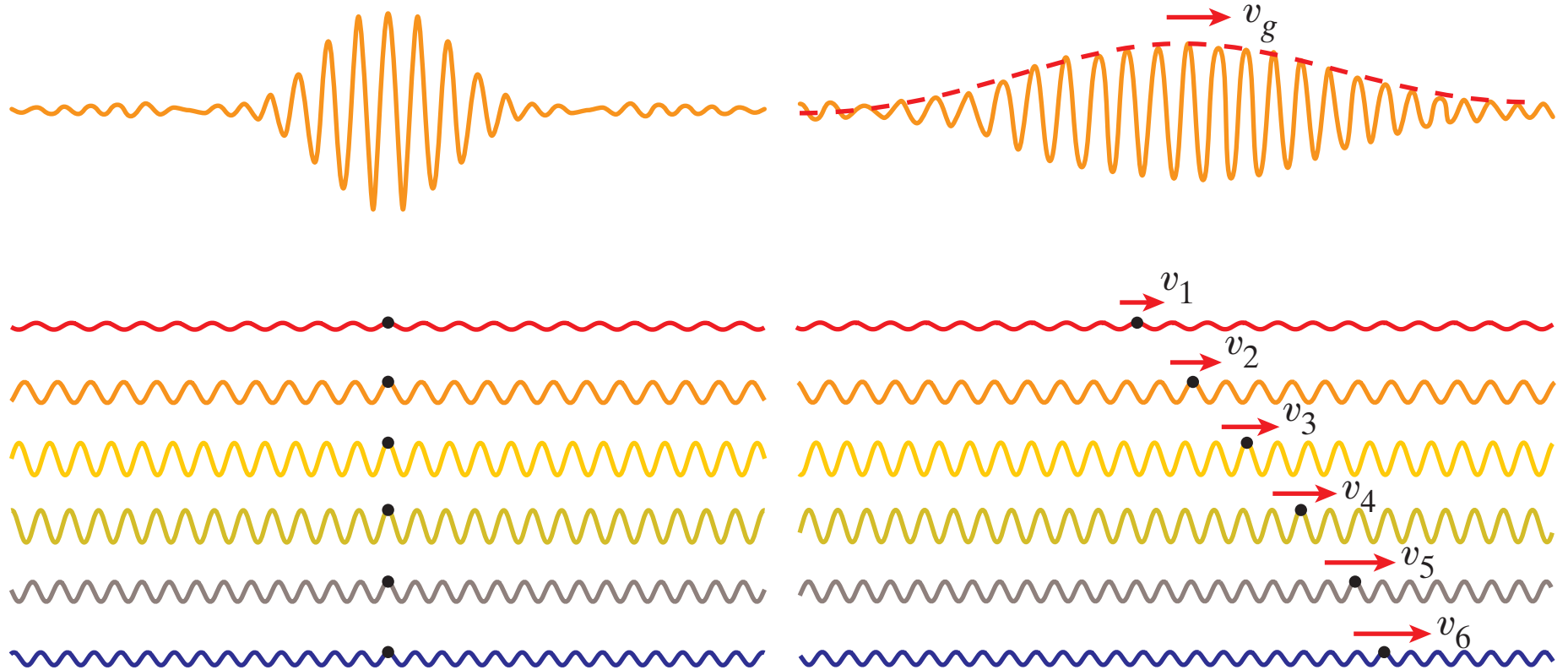
Pulse dispersion



Pulse dispersion



Pulse dispersion



Pulse dispersion

Consider two propagating waves:

$$y_1 = A \sin(k_1x - \omega_1t) \quad \text{and} \quad y_2 = A \sin(k_2x - \omega_2t)$$

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propagating at speeds

$$v_1 = \frac{\omega_1}{k_1} = f_1 \lambda_1 \quad \text{and} \quad v_2 = \frac{\omega_2}{k_2} = f_2 \lambda_2.$$

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Superposition:

$$y = A[\sin (k_1 x - \omega_1 t) + \sin (k_2 x - \omega_2 t)]$$

Pulse dispersion

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$$\sin \alpha + \sin \beta = 2 \cos \left(\frac{\alpha - \beta}{2} \right) \sin \left(\frac{\alpha + \beta}{2} \right)$$

Pulse dispersion

$$y = 2A \cos \frac{1}{2}[(k_1 - k_2)x - (\omega_1 - \omega_2)t] \sin \left[\frac{k_1 + k_2}{2}x - \frac{\omega_1 + \omega_2}{2}t \right]$$

Pulse dispersion

$$y = 2A \cos \frac{1}{2}[(k_1 - k_2)x - (\omega_1 - \omega_2)t] \sin \left[\frac{k_1 + k_2}{2}x - \frac{\omega_1 + \omega_2}{2}t \right]$$

Let: $k_1 - k_2 \equiv \Delta k$ **and** $\omega_1 - \omega_2 \equiv \Delta \omega$

Pulse dispersion

$$y = 2A \cos \frac{1}{2}[(k_1 - k_2)x - (\omega_1 - \omega_2)t] \sin \left[\frac{k_1 + k_2}{2}x - \frac{\omega_1 + \omega_2}{2}t \right]$$

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$$\frac{k_1 + k_2}{2} \equiv k \quad \text{and} \quad \frac{\omega_1 + \omega_2}{2} \equiv \omega$$

Pulse dispersion

$$y = 2A \cos \frac{1}{2}[(k_1 - k_2)x - (\omega_1 - \omega_2)t] \sin \left[\frac{k_1 + k_2}{2}x - \frac{\omega_1 + \omega_2}{2}t \right]$$

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and so:

$$y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta \omega) \sin (kx - \omega t)$$

Pulse dispersion

$$y = 2A \cos \frac{1}{2}[(k_1 - k_2)x - (\omega_1 - \omega_2)t] \sin \left[\frac{k_1 + k_2}{2}x - \frac{\omega_1 + \omega_2}{2}t \right]$$

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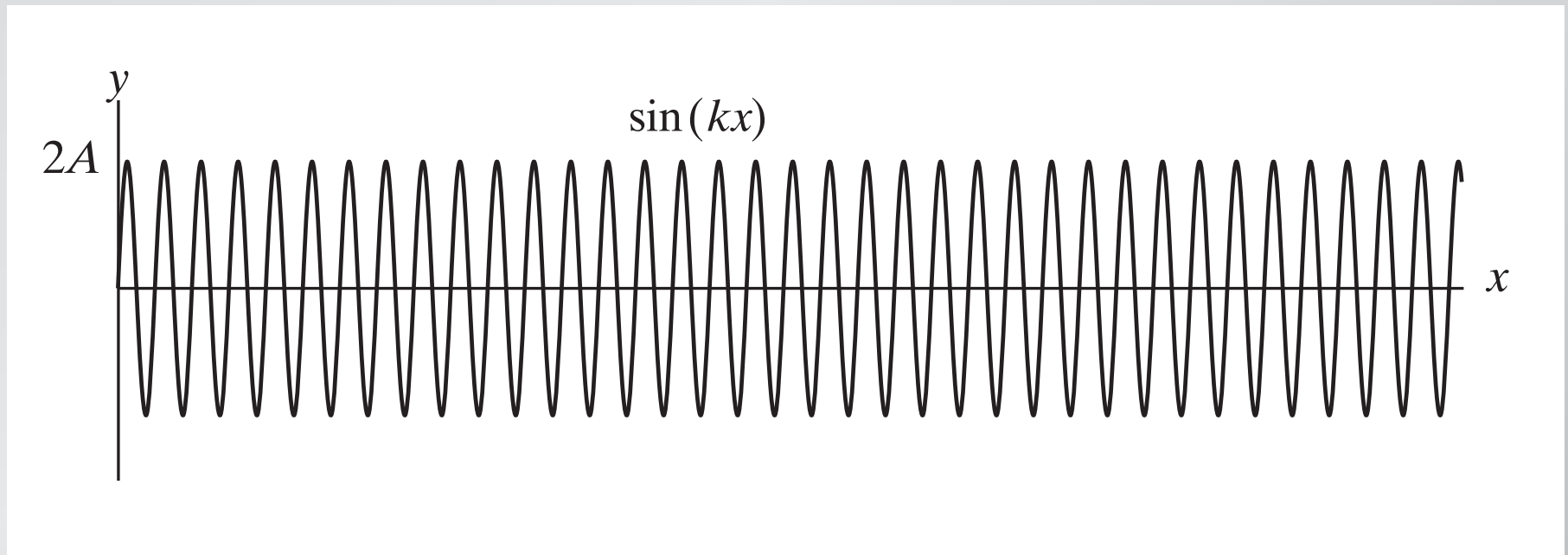
$$y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta \omega) \sin (kx - \omega t)$$

traveling sine wave, with amplitude modulation.

Pulse dispersion

$$y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta\omega) \sin (kx - \omega t)$$

At $t = 0$: $y = 2A \cos \frac{1}{2}(x\Delta k) \sin (kx)$



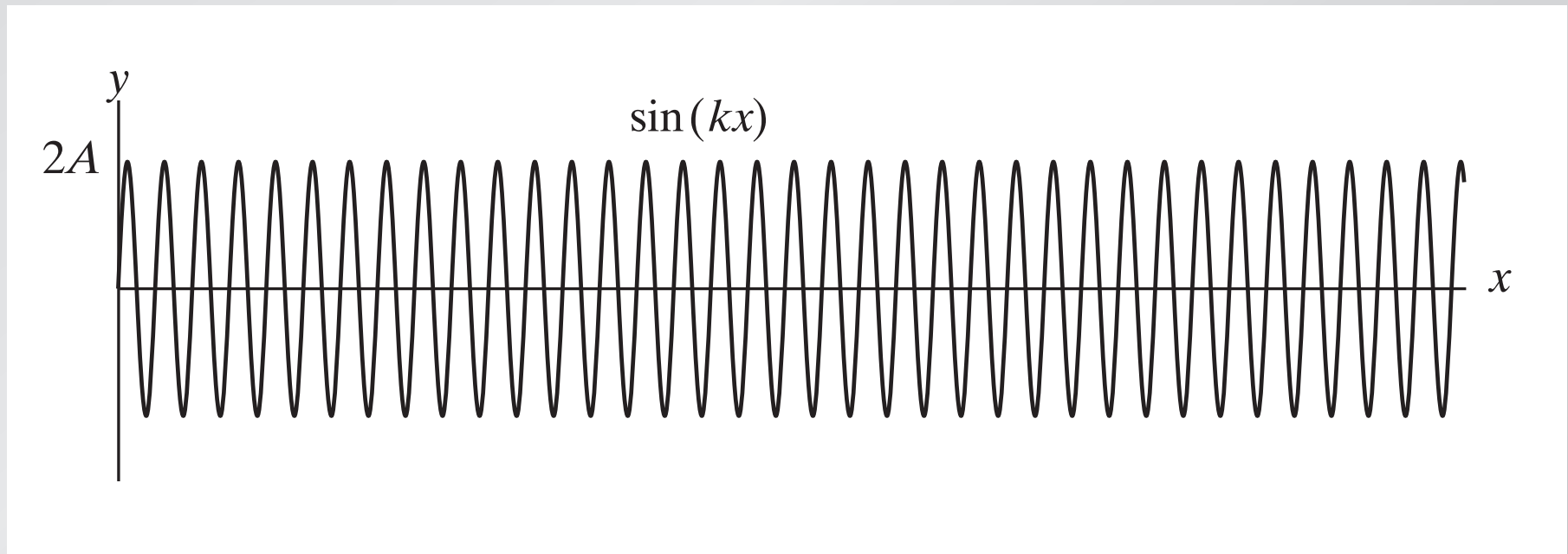
Pulse dispersion

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At $t = 0$:

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carrier



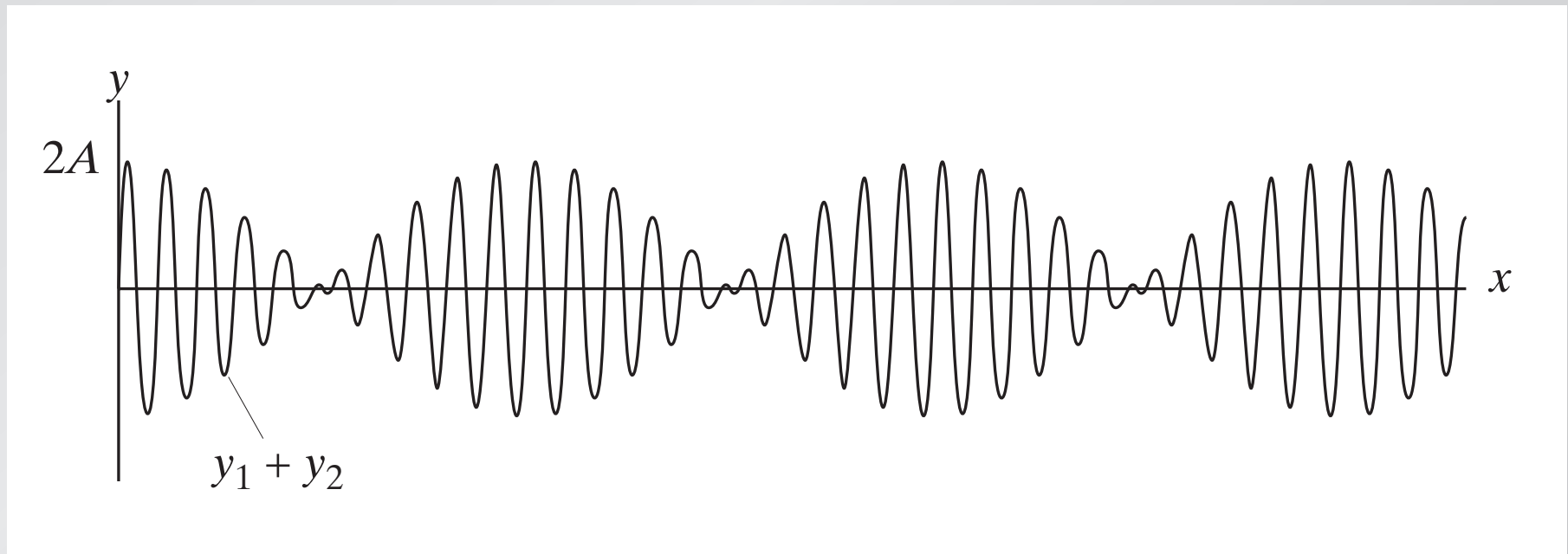
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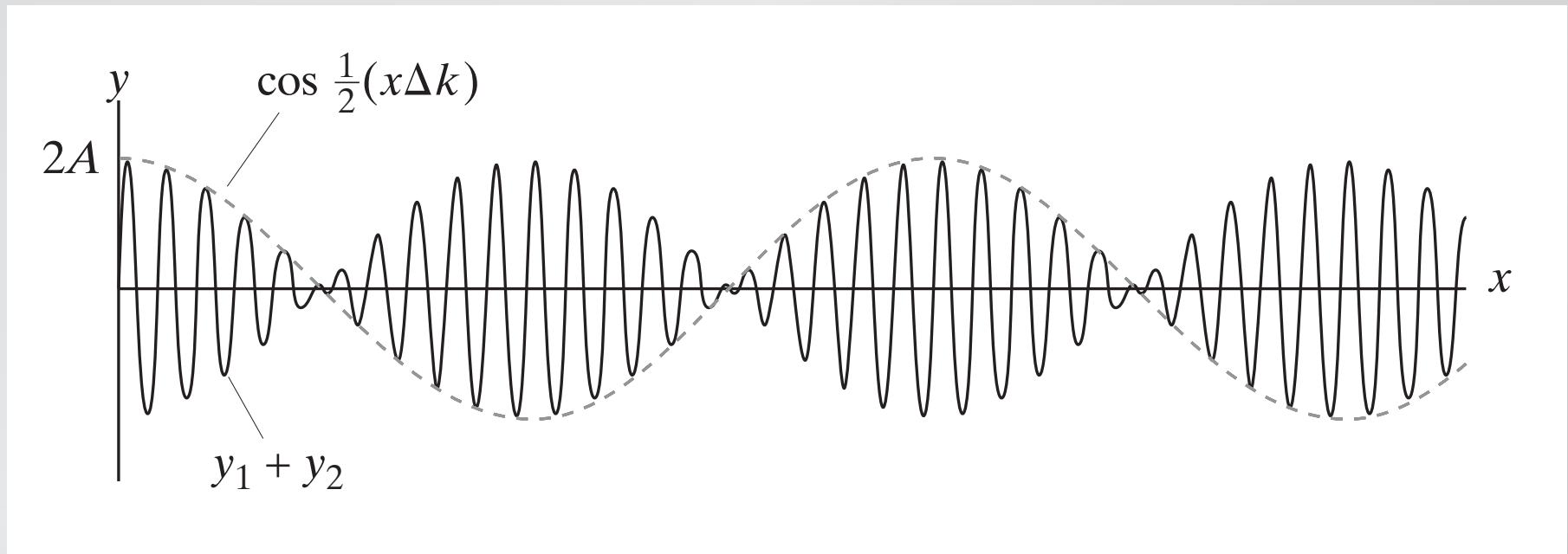
Pulse dispersion

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At $t = 0$:

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envelope **carrier**



Pulse dispersion

$$y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta \omega) \sin (kx - \omega t)$$

Pulse dispersion

$$y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta\omega) \sin (kx - \omega t)$$

speed of carrier

$$v_p = \frac{\omega}{k} = f\lambda$$

Pulse dispersion

$$y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta\omega) \sin (kx - \omega t)$$

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$$v_p = \frac{\omega}{k} = f\lambda$$

speed of envelope

$$v_g = \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk}$$

Pulse dispersion

$$y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta\omega) \sin (kx - \omega t)$$

speed of carrier ('phase velocity'):

$$v_p = \frac{\omega}{k} = f\lambda$$

speed of envelope ('group velocity'):

$$v_g = \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk}$$

Pulse dispersion

let's practice a bit!

(please complete worksheet)

Pulse dispersion

For each wave, determine the wavevector k , the frequency ω , and the propagation speed v :

$$\begin{array}{ll} k_1 = 8.0 & \text{and } k_2 = \frac{7.2}{0.95} = 7.6 < k_1 \\ \omega_1 = 8.0 & \text{and } \omega_2 = 7.2 \\ v_1 = \frac{\omega_1}{k_1} = 1. & \text{and } v_2 = \frac{\omega_2}{k_2} = \frac{7.2}{2.57} = 0.95 \end{array}$$

Does the red get ahead of the blue or the other way around? Why?

Is the dispersion in the medium through which these waves propagate normal or anomalous? Why?

Pulse dispersion

What is the phase velocity of the superposition of y_1 and y_2 ?

$$v_p = \frac{(\omega_1 + \omega_2)/2}{(k_1 + k_2)/2} = \frac{7.6}{7.8} = 0.98$$

What is the group velocity of the superposition of y_1 and y_2 ?

$$v_g = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{0.8}{0.4} = 2$$

Do the crests of the carrier wave travel forward or backward through the envelope? Does your prediction agree with your observation?

Pulse dispersion

$$y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta\omega) \sin (kx - \omega t)$$

If no dispersion

$$v_p = \frac{\omega_1}{k_1} = \frac{\omega_2}{k_2}$$

Pulse dispersion

$$y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta\omega) \sin(kx - \omega t)$$

If no dispersion

$$v_p = \frac{\omega_1}{k_1} = \frac{\omega_2}{k_2}$$

group velocity:

$$v_g = \frac{\Delta\omega}{\Delta k} = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{\omega_1/k_1 - \omega_2/k_1}{1 - k_2/k_1} = \frac{v_p - \omega_2/k_1}{1 - k_2/k_1}$$

Pulse dispersion

$$y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta\omega) \sin(kx - \omega t)$$

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$$v_g = \frac{v_p - \omega_2/k_2(k_2/k_1)}{1 - k_2/k_1} = v_p \frac{1 - k_2/k_1}{1 - k_2/k_1} = v_p$$

Pulse dispersion

$$y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta\omega) \sin (kx - \omega t)$$

If no dispersion

$$v_p = \frac{\omega_1}{k_1} = \frac{\omega_2}{k_2}$$

group and phase velocities are the same:

$$v_g = v_p$$

and so the envelope and carrier travel together

Pulse dispersion

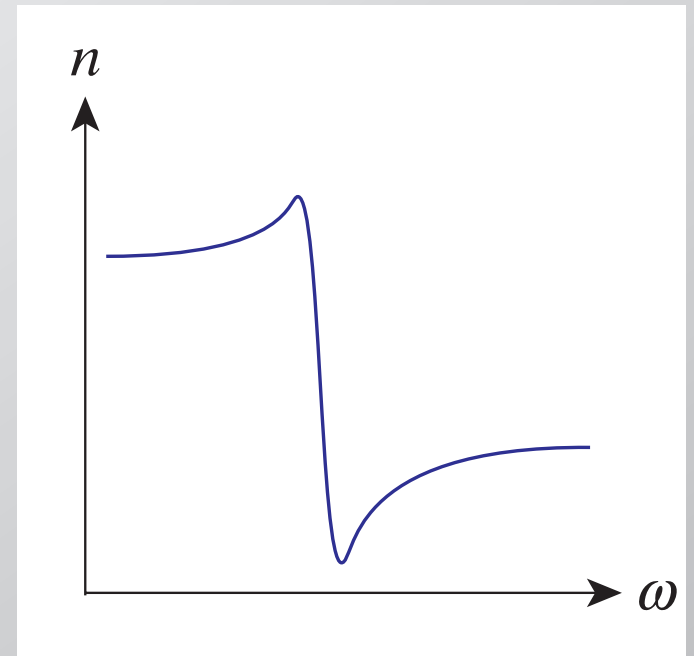
$$y = 2A \cos \frac{1}{2}(x\Delta k - t\Delta\omega) \sin (kx - \omega t)$$

Types of dispersion:

$\frac{dn}{d\omega} > 0$ **normal dispersion**

$\frac{dn}{d\omega} = 0$ **no dispersion**

$\frac{dn}{d\omega} < 0$ **anomalous dispersion**



Pulse dispersion

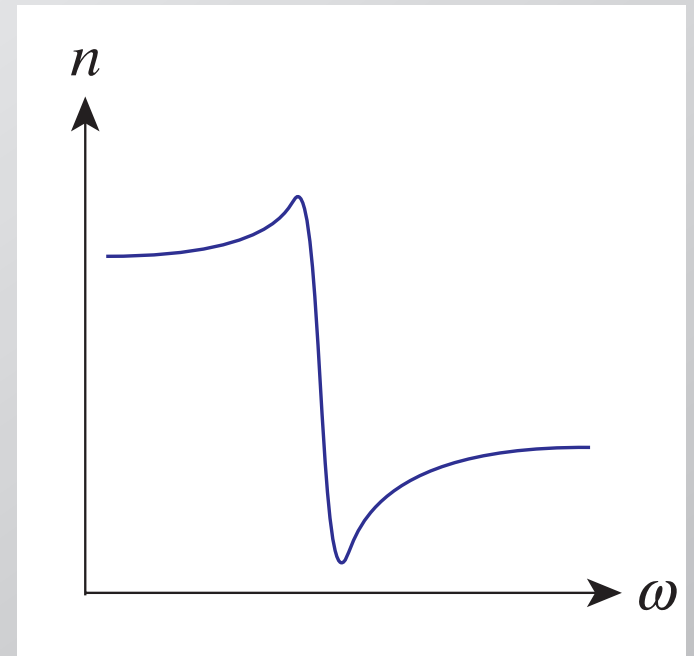
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Types of dispersion:

$\frac{dn}{d\omega} > 0$ **normal dispersion** $v_g < v_p$

$\frac{dn}{d\omega} = 0$ **no dispersion** $v_g = v_p$

$\frac{dn}{d\omega} < 0$ **anomalous dispersion** $v_g > v_p$



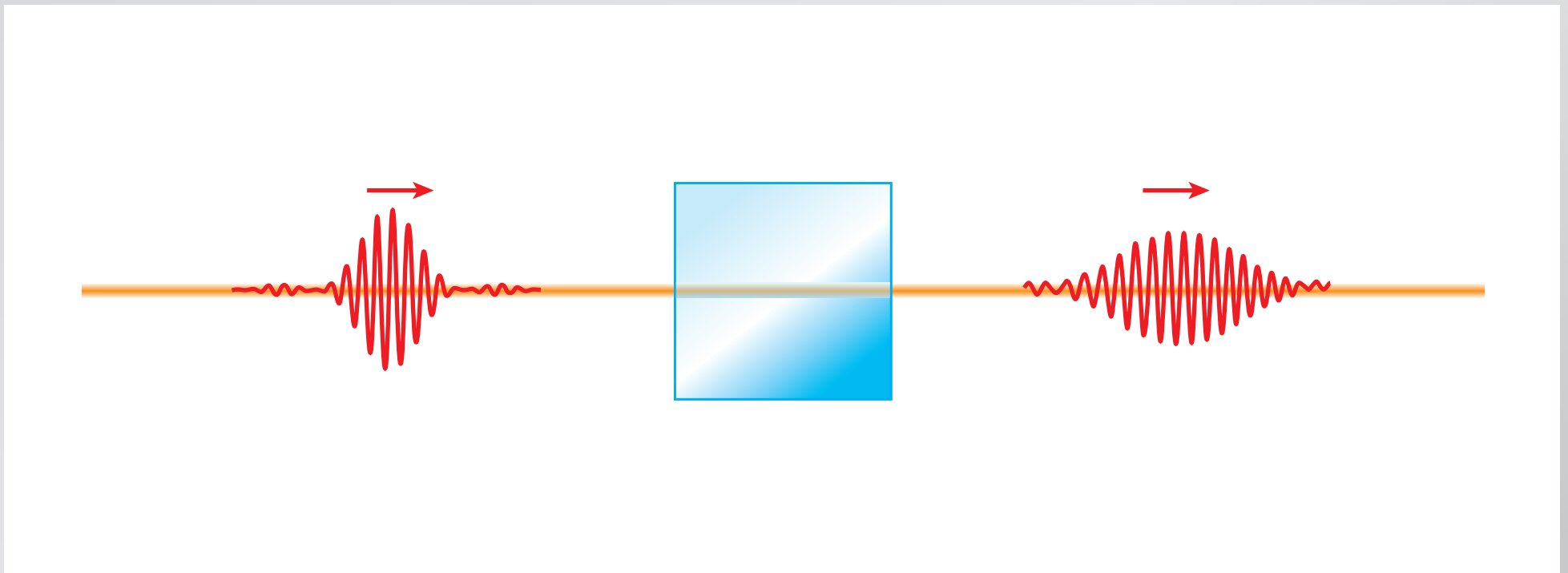
Pulse dispersion

medium causes pulse to stretch



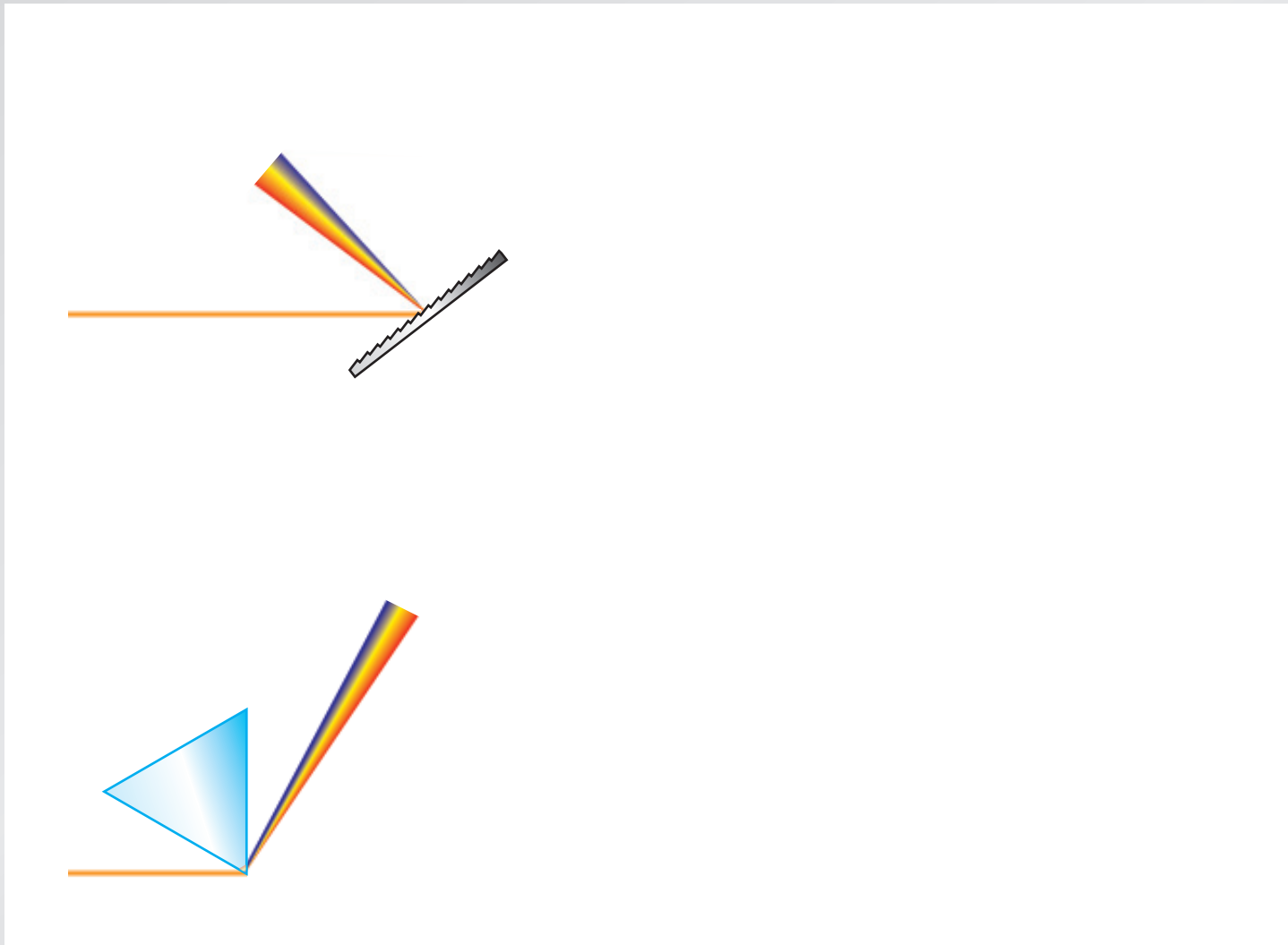
Pulse dispersion

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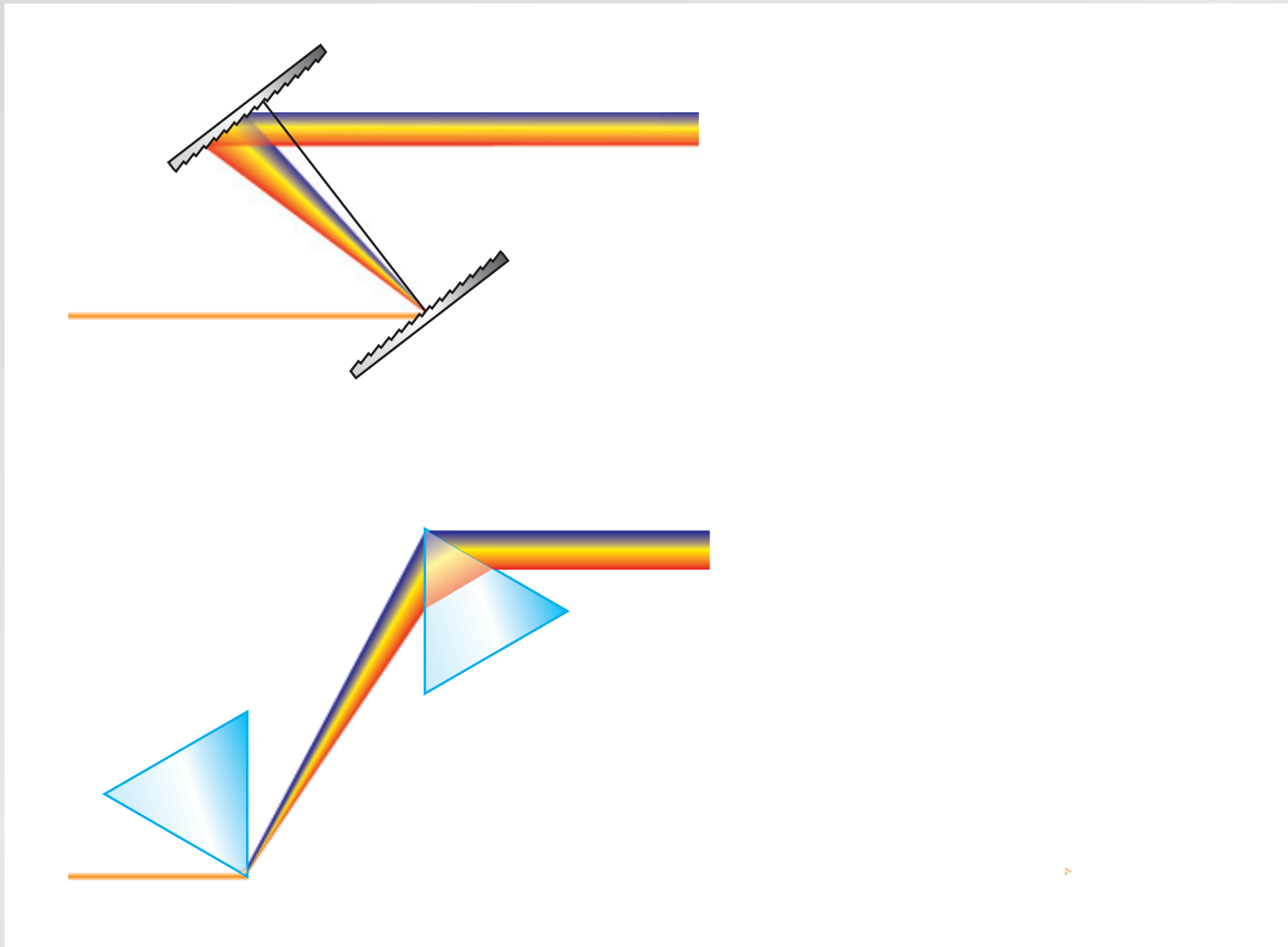


compensate by rearranging spectral components!

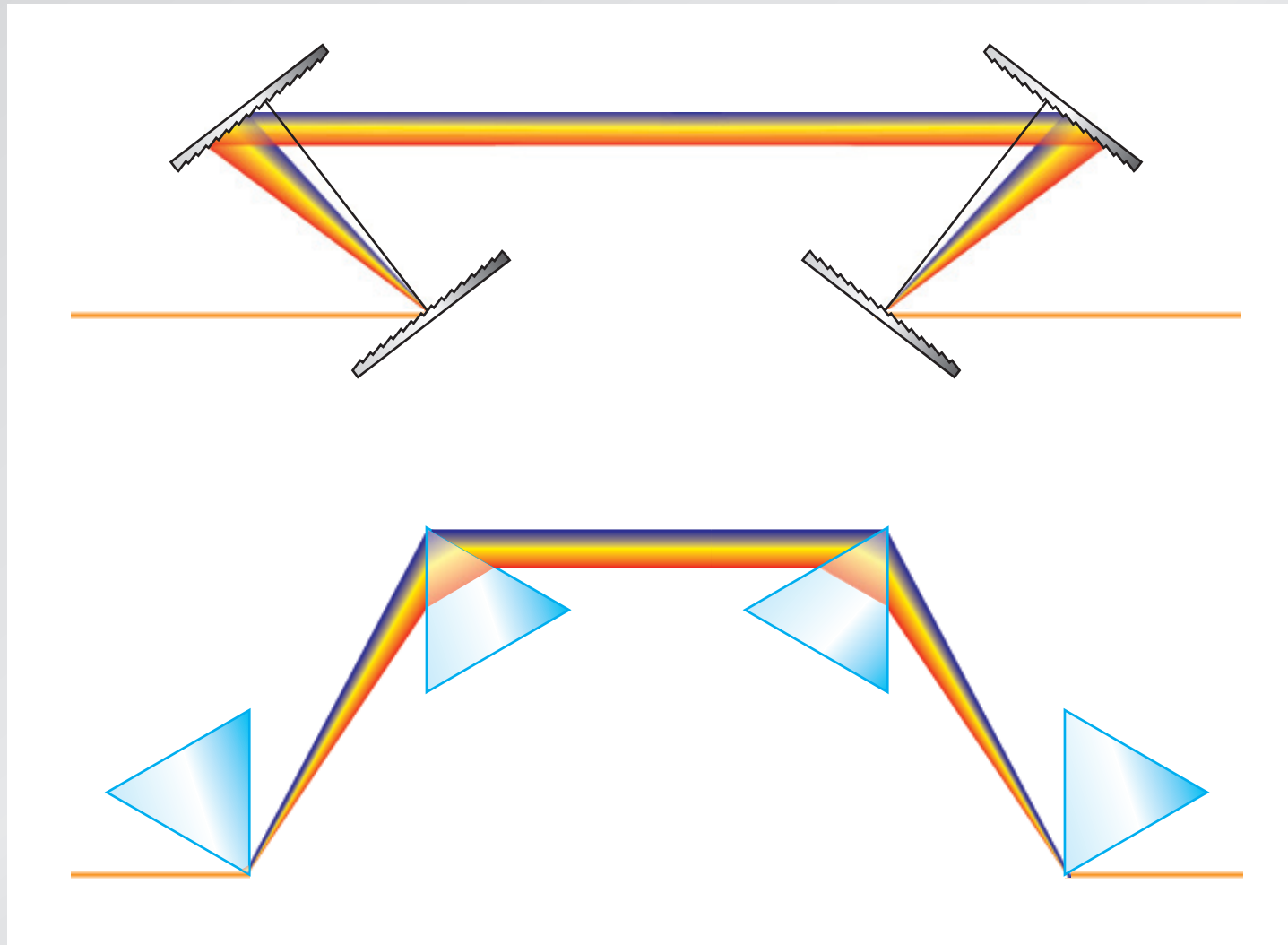
Pulse dispersion compensation



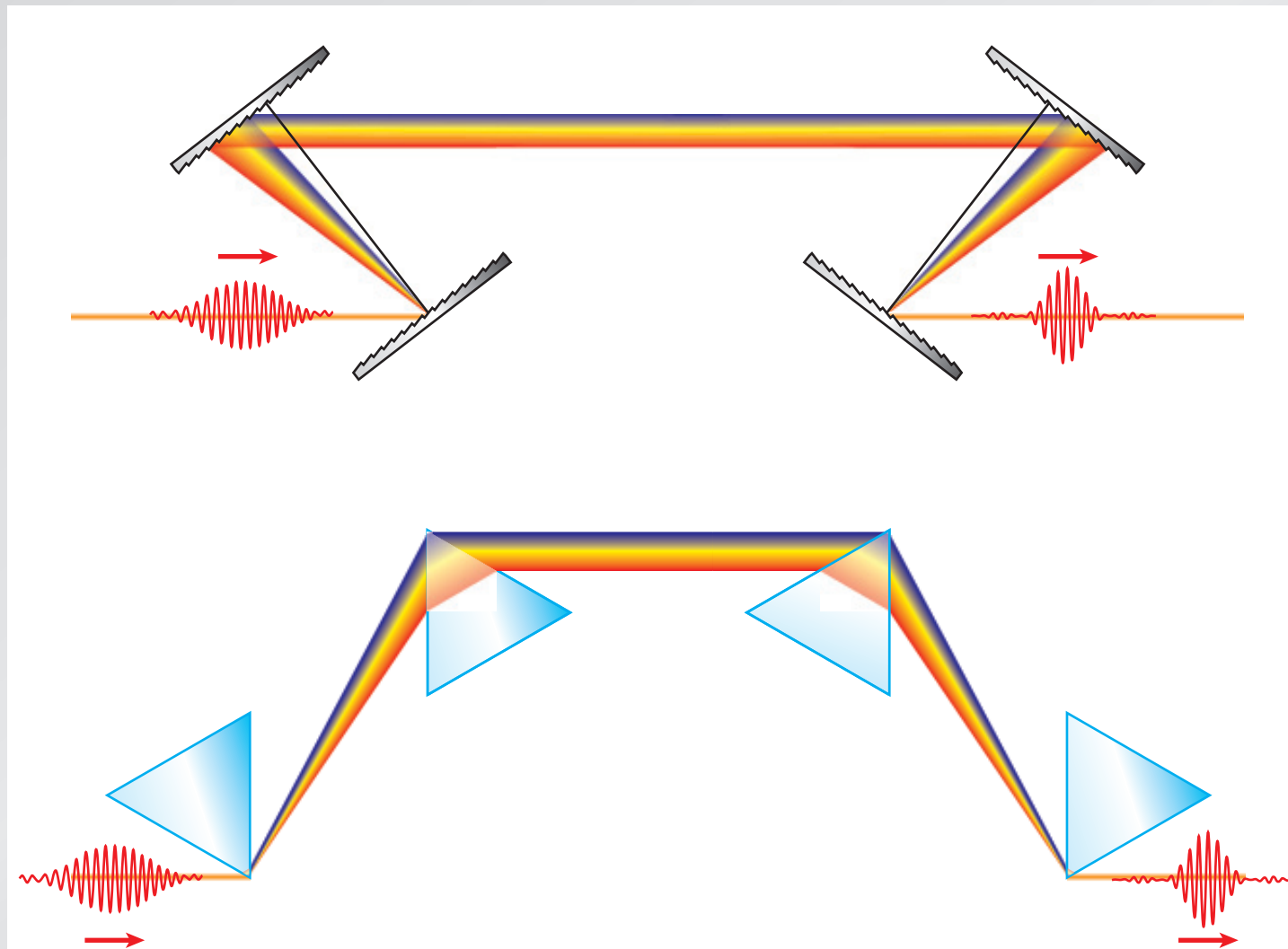
Pulse dispersion compensation



Pulse dispersion compensation



Pulse dispersion compensation



Pulse dispersion compensation

How do these arrangements work?

Pulse dispersion compensation

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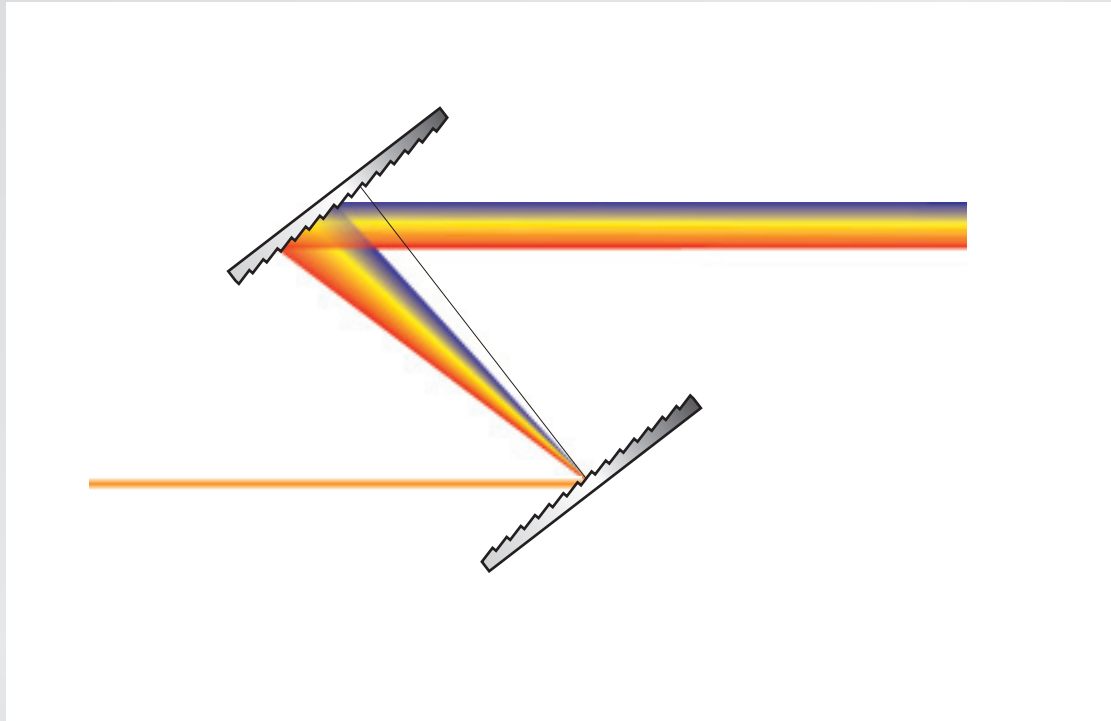
(please complete worksheet)

Pulse dispersion compensation

Does path length difference compensate?

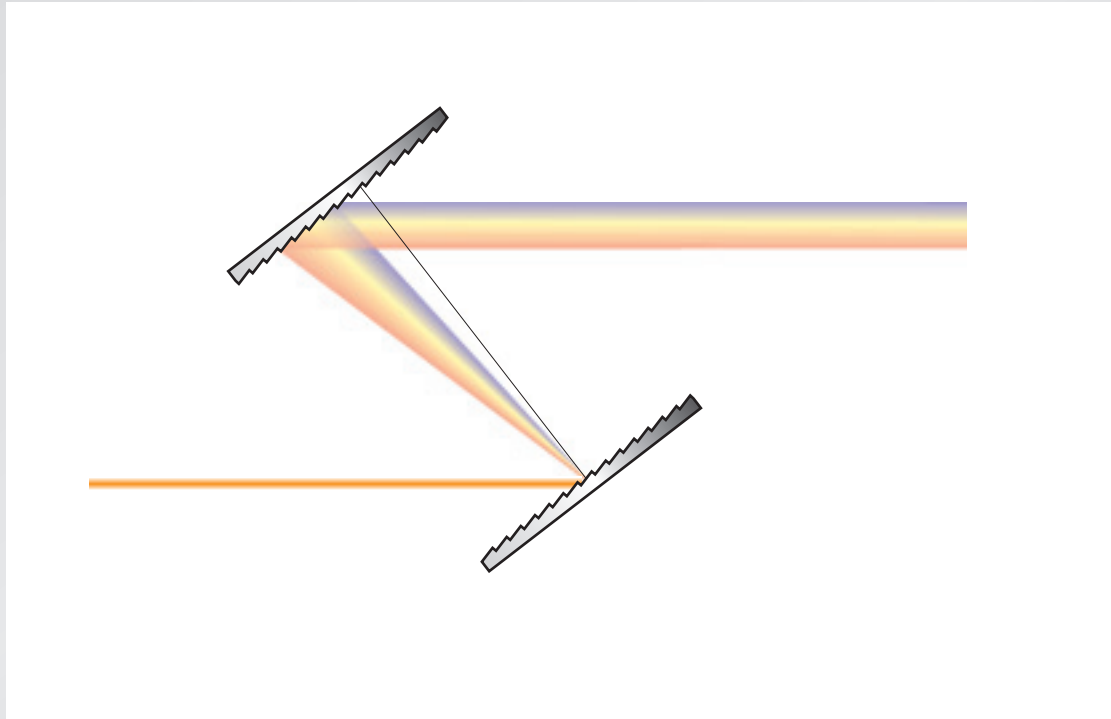
Pulse dispersion compensation

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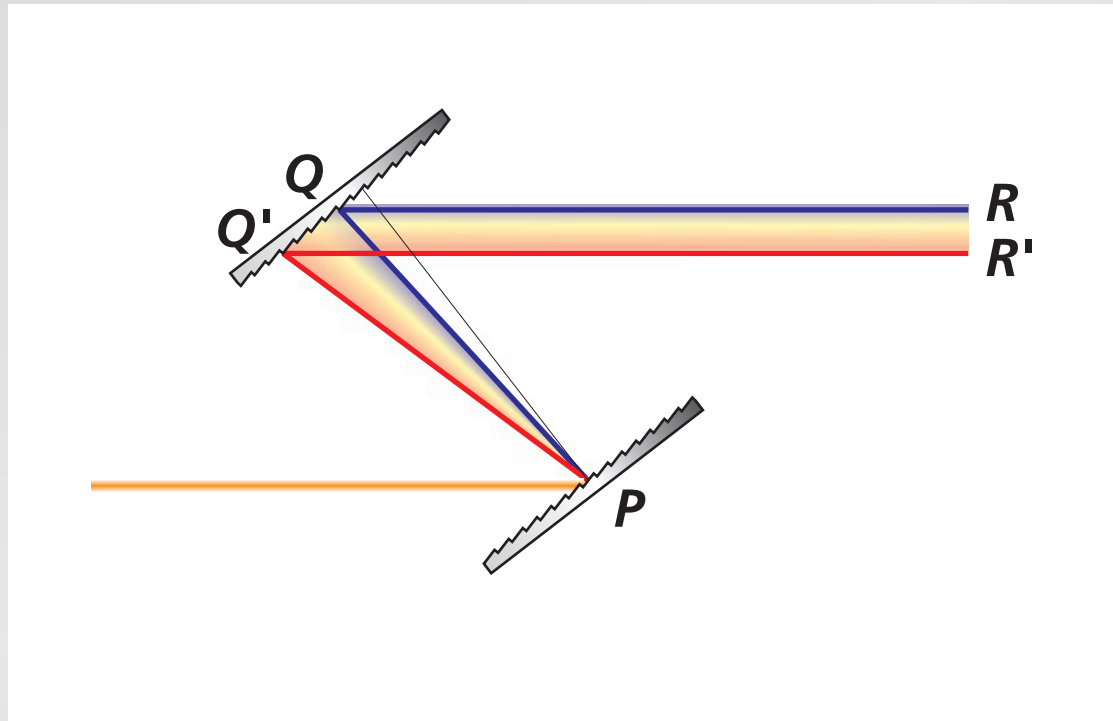
Pulse dispersion compensation

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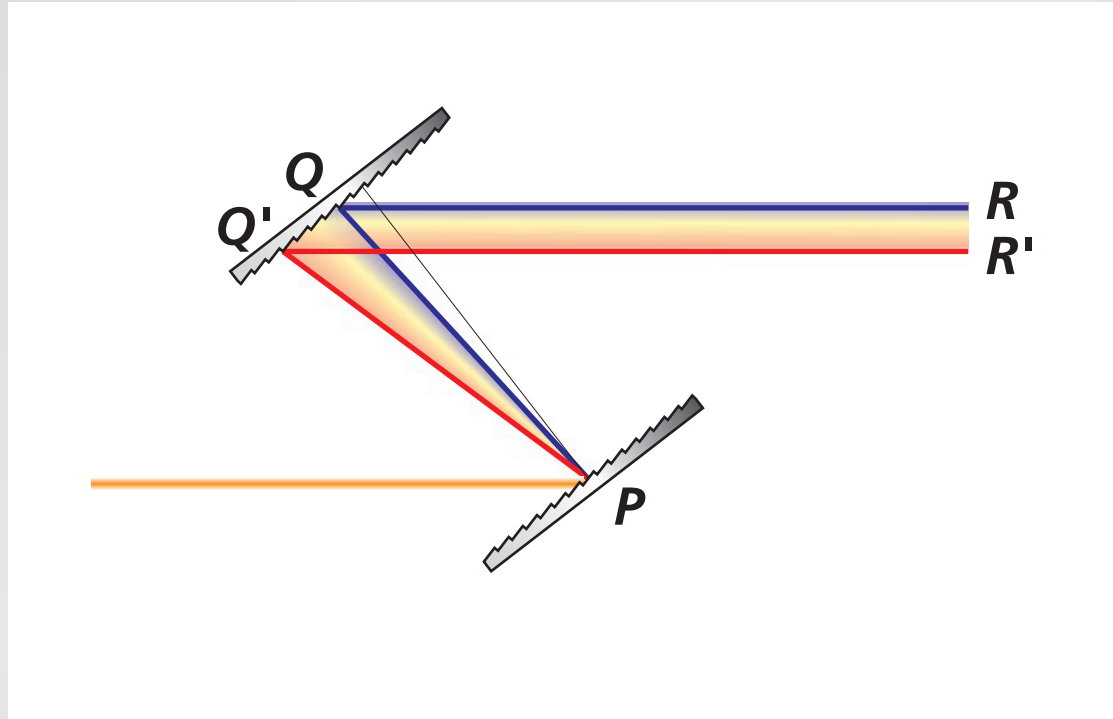
Pulse dispersion compensation

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Pulse dispersion compensation

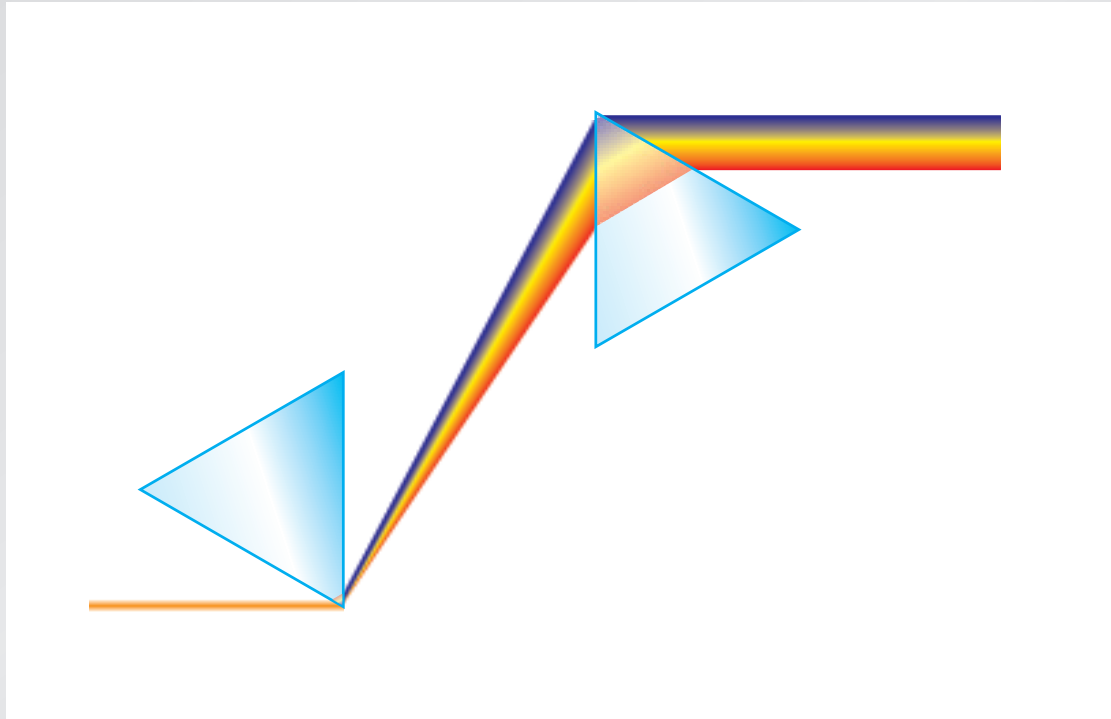
Does path length difference compensate?



grating gives low frequency longer path length!

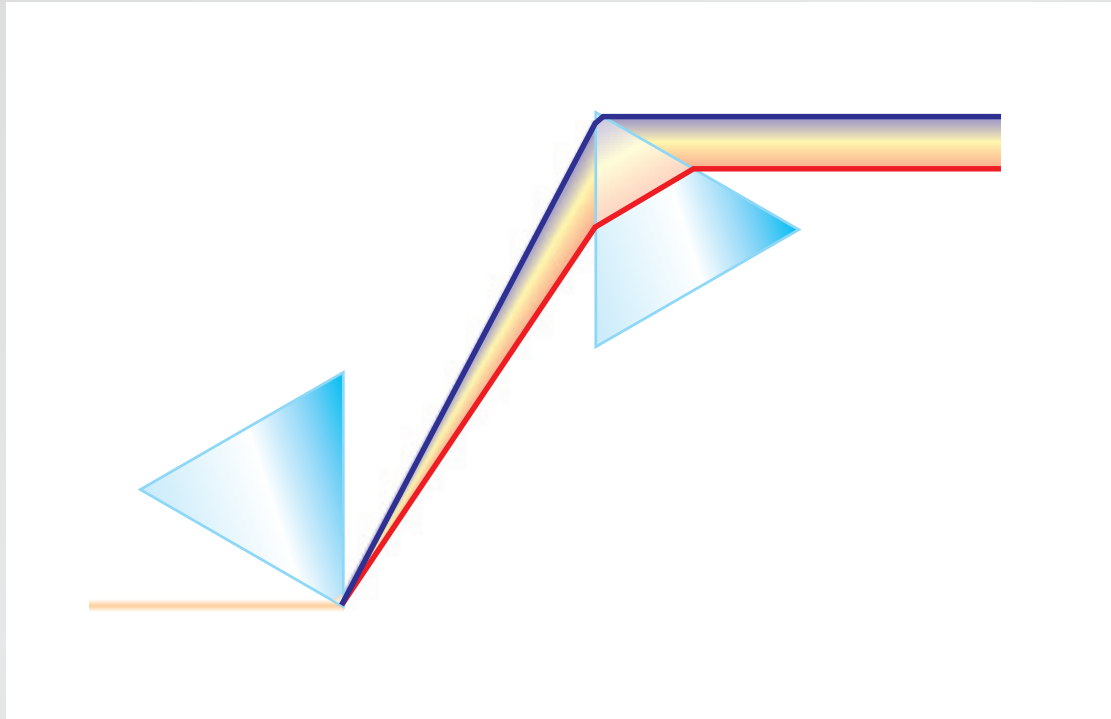
Pulse dispersion compensation

Does path length difference compensate?



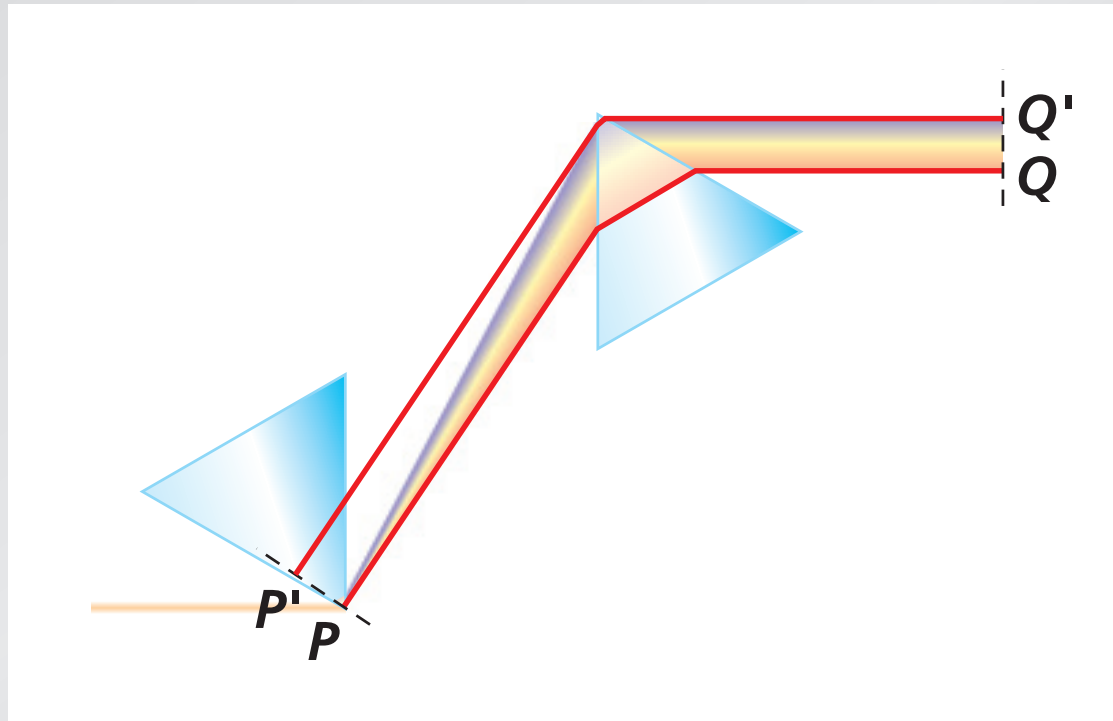
Pulse dispersion compensation

Does path length difference compensate?



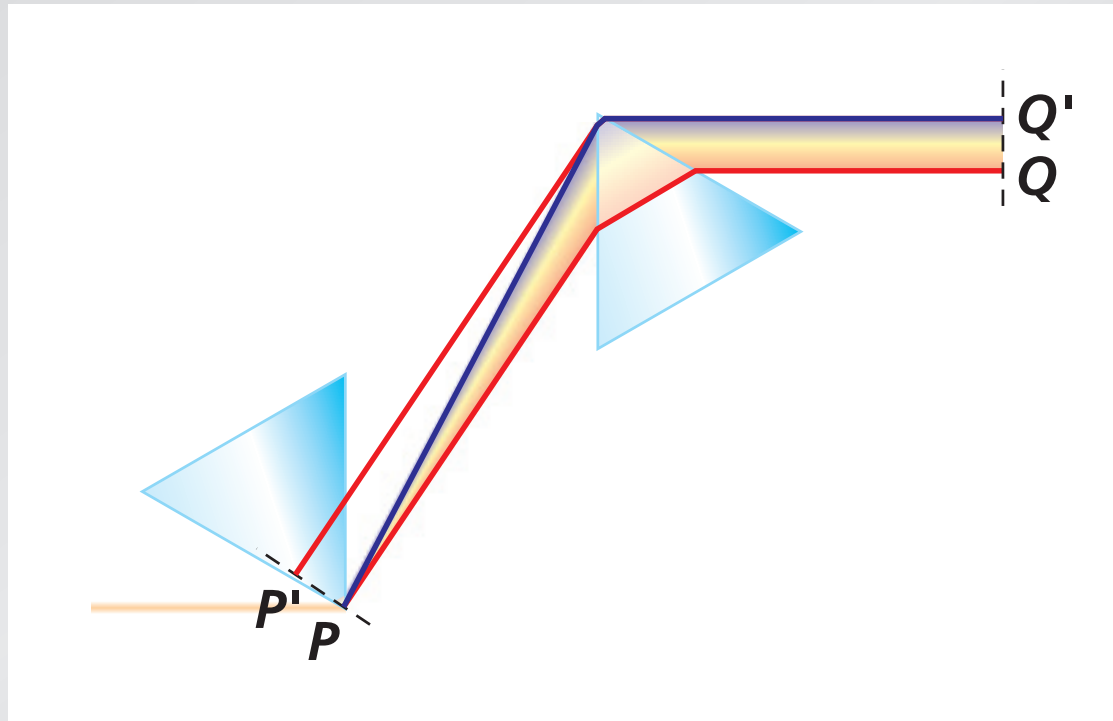
Pulse dispersion compensation

Does path length difference compensate?



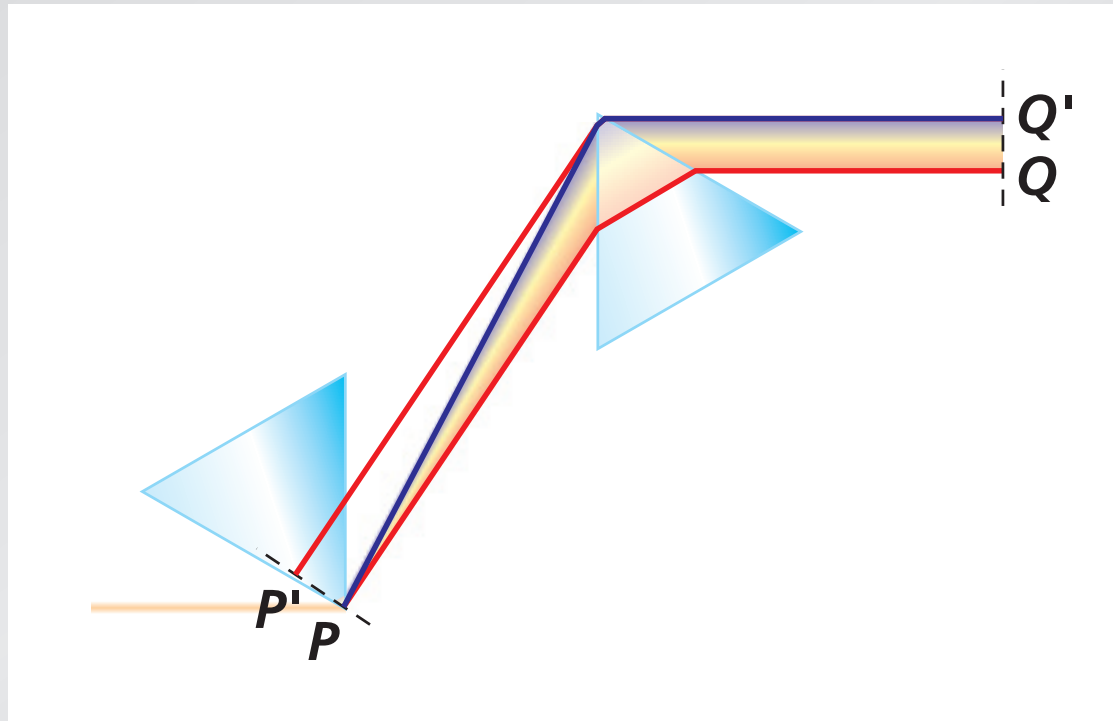
Pulse dispersion compensation

Does path length difference compensate?



Pulse dispersion compensation

Does path length difference compensate?



...so prism gives low frequency *shorter* path length!

Pulse dispersion compensation

consider traveling Gaussian pulse again:

$$y(t) = \exp\left[-\frac{(x - v_g t)^2}{2\sigma_t^2}\right] \sin(kx - \omega t)$$

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- 1. Yes, it is dispersive**
- 2. No, it is not dispersive (pulse shape is constant)**
- 3. Cannot tell**

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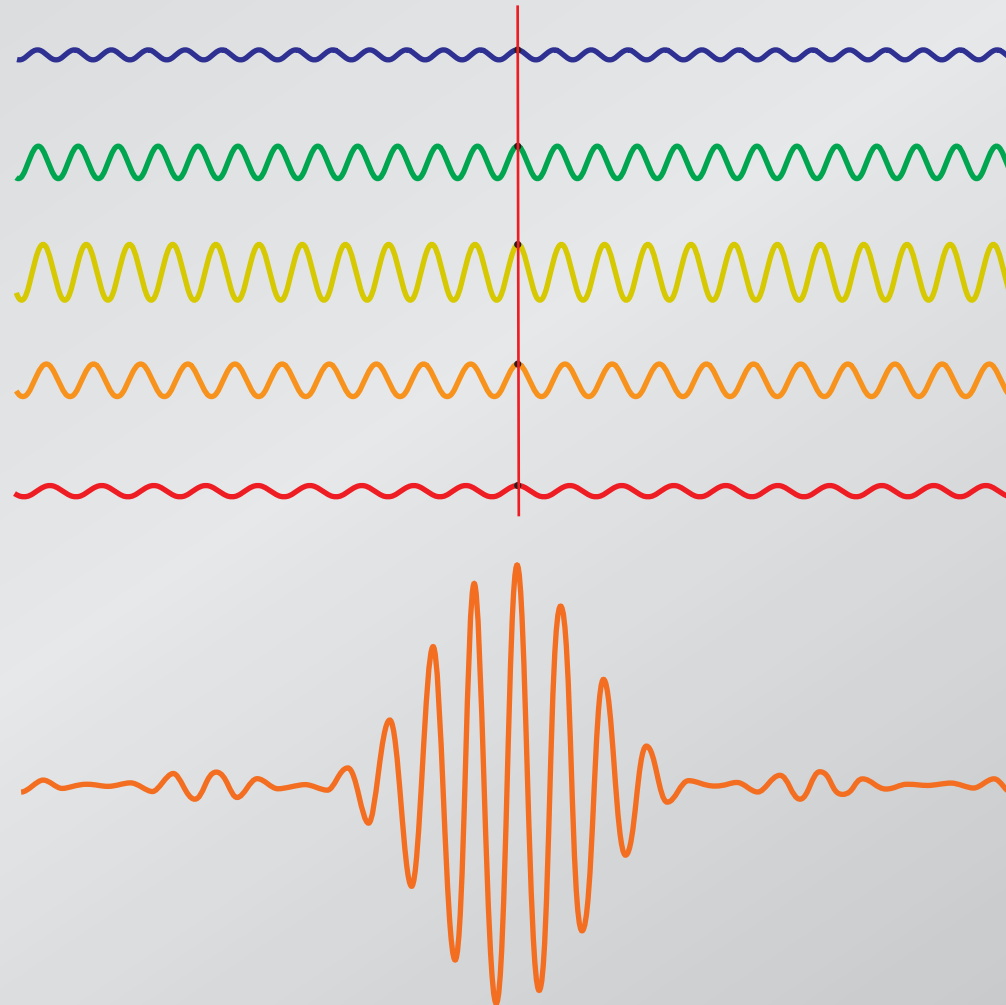
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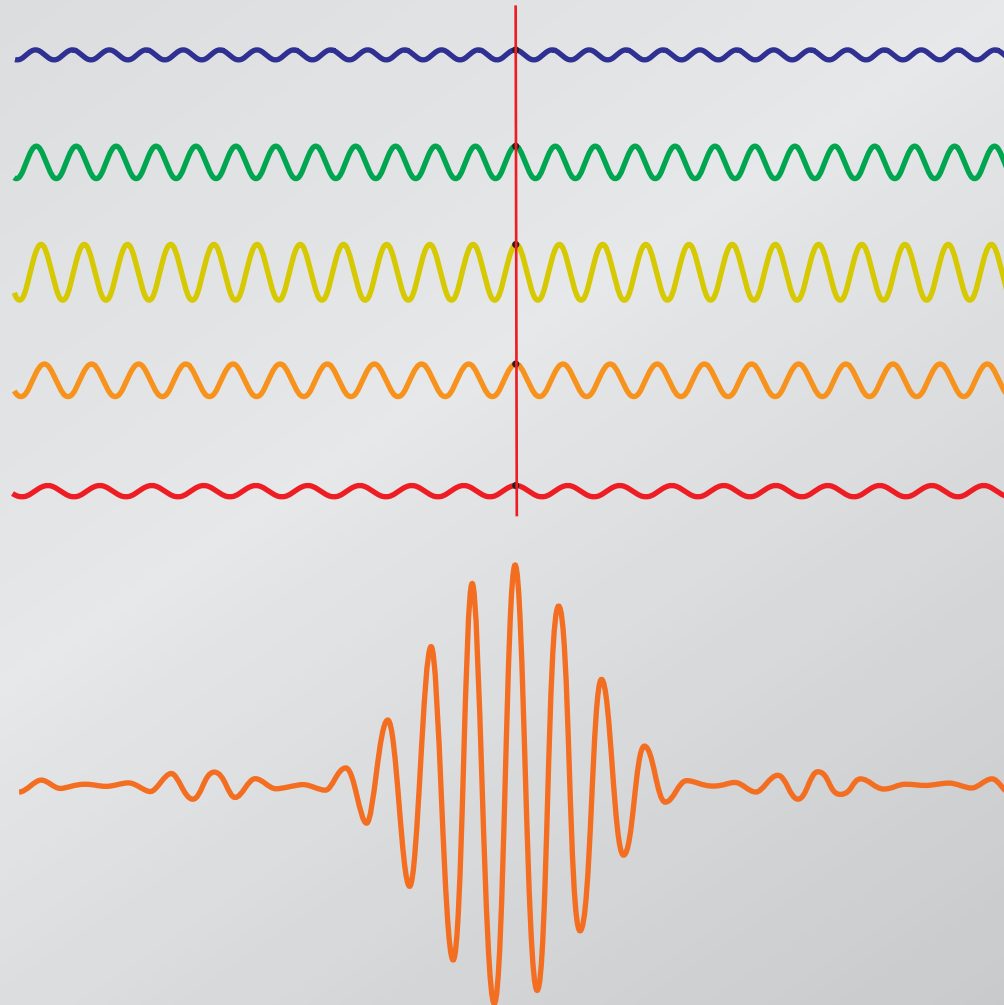
...but Gaussian shape of pulse is constant!

Pulse dispersion compensation



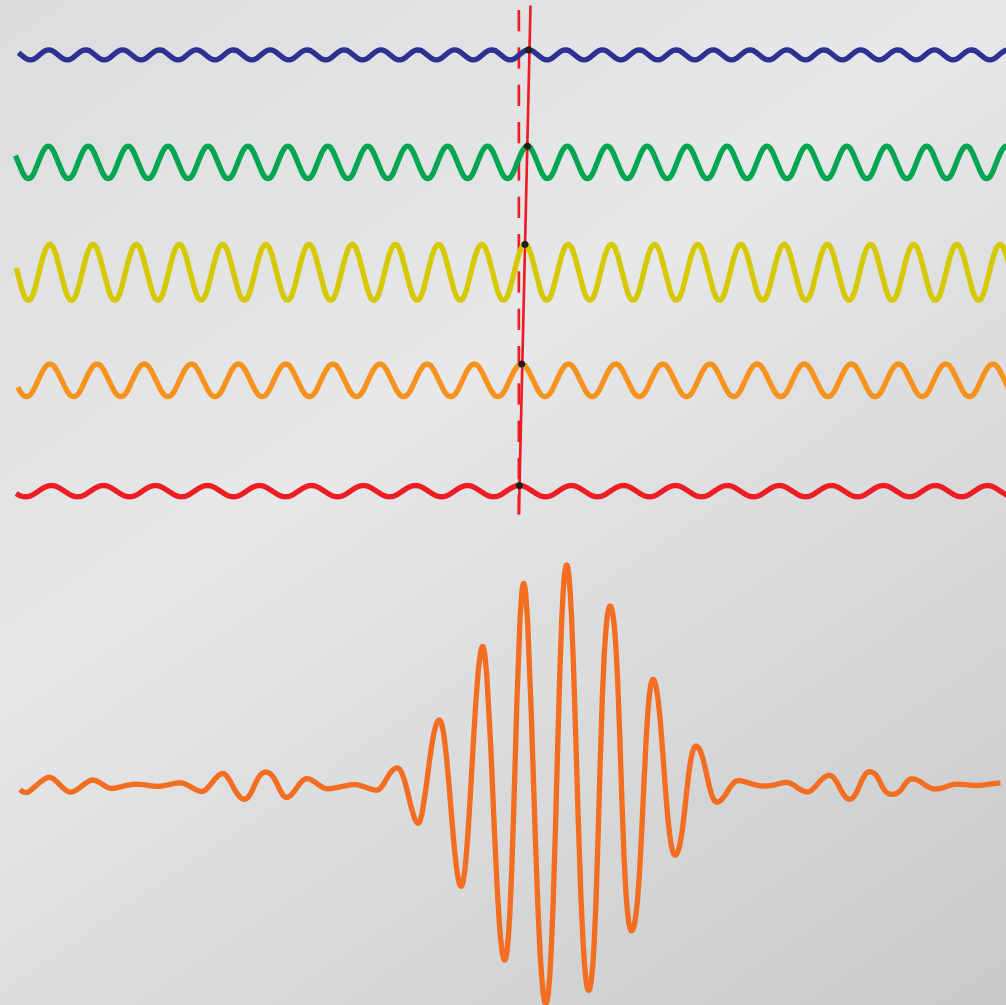
Pulse dispersion compensation

linear dispersion



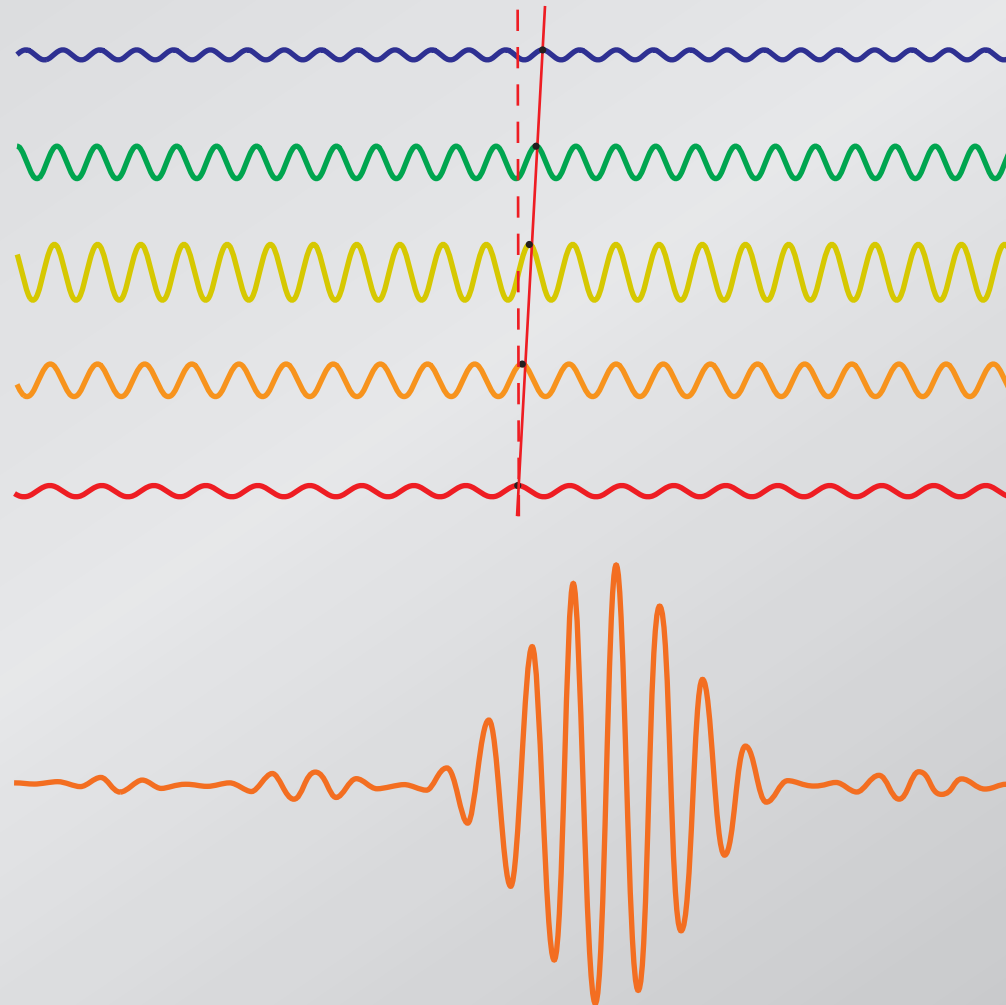
Pulse dispersion compensation

linear dispersion



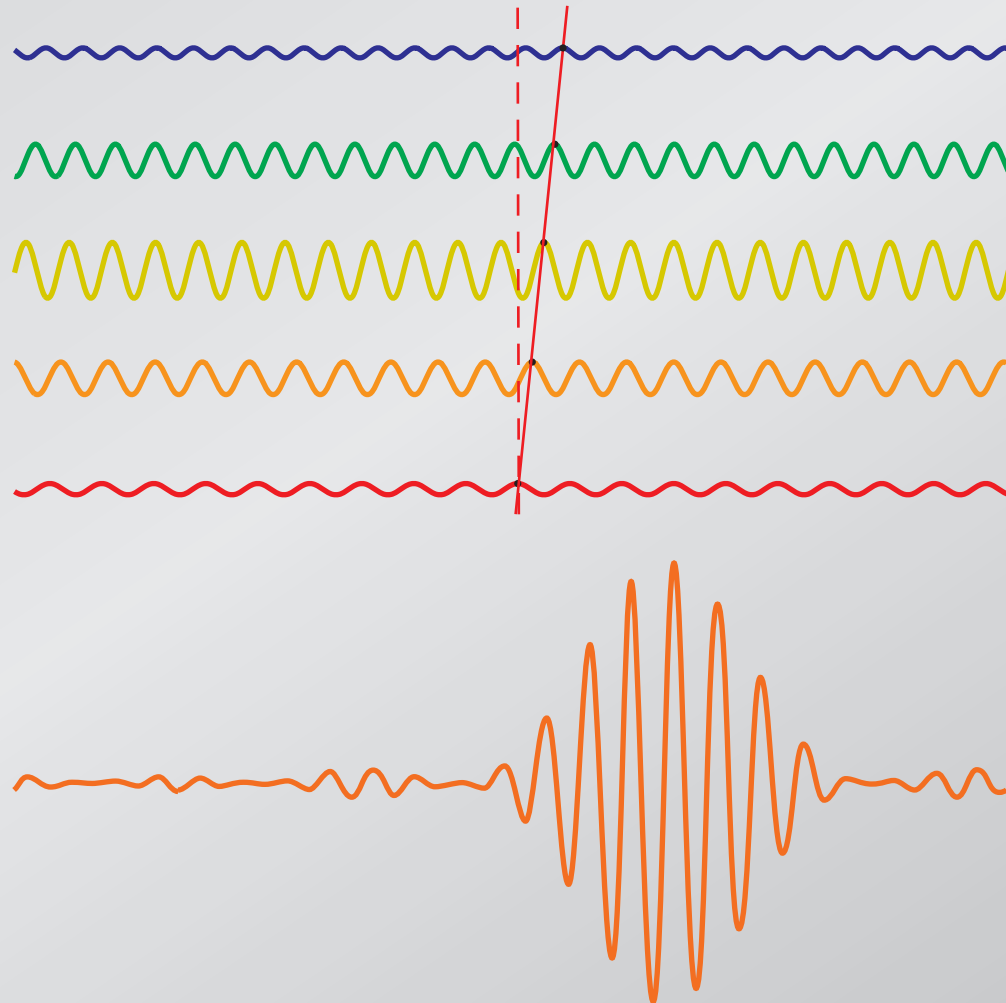
Pulse dispersion compensation

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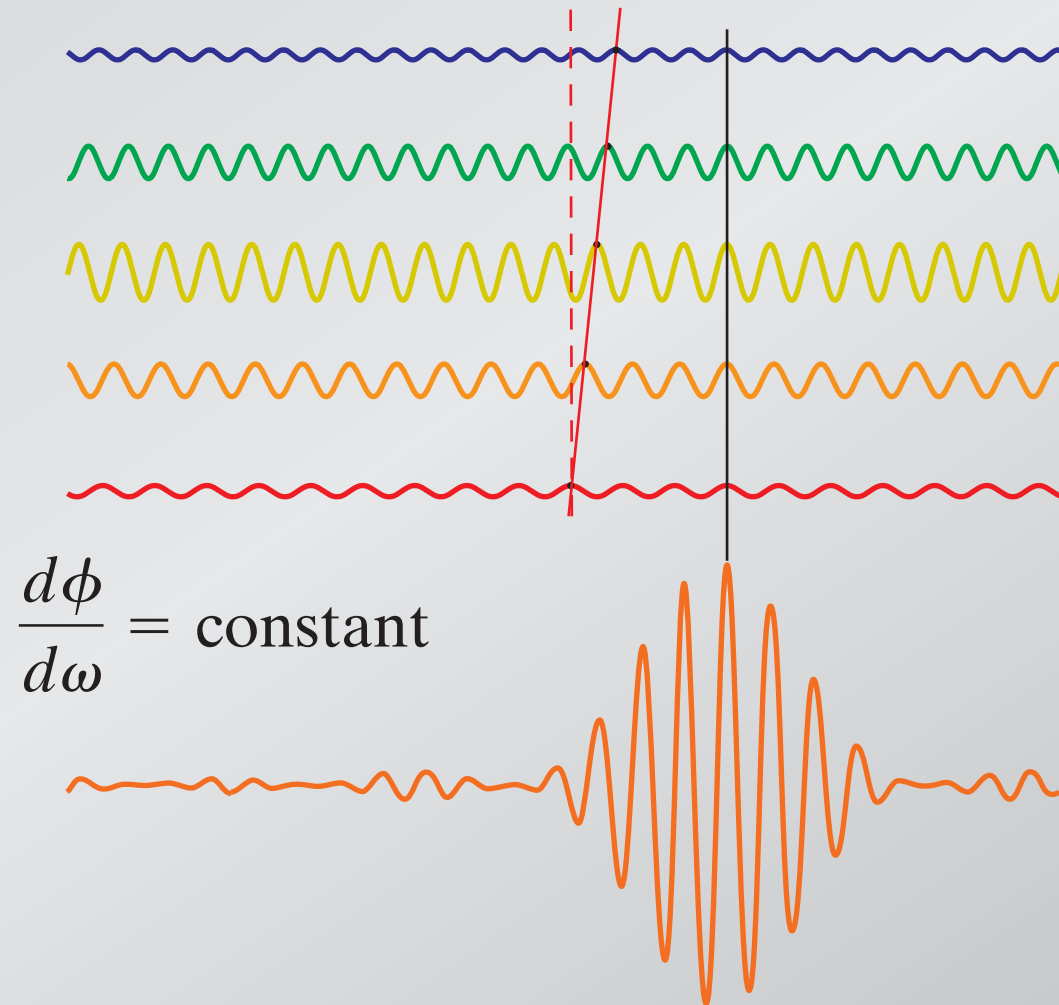
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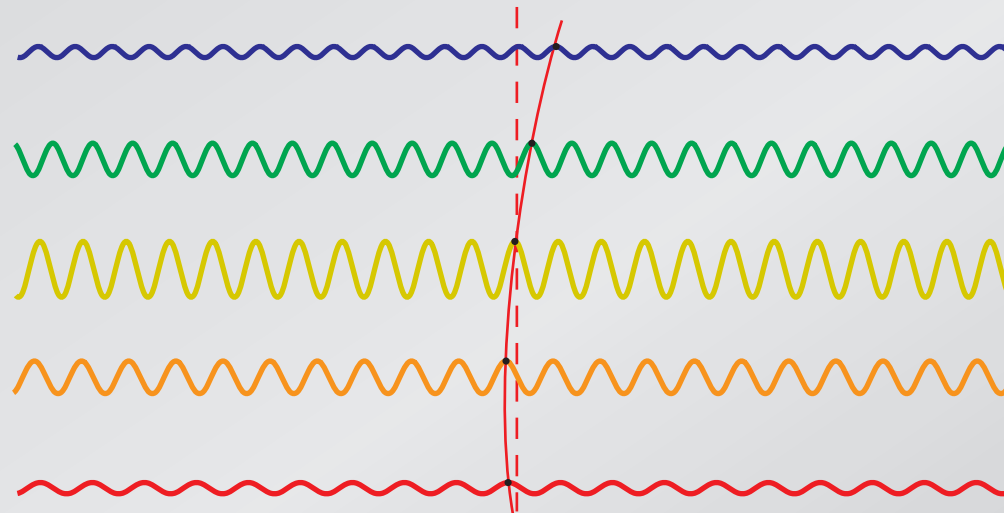
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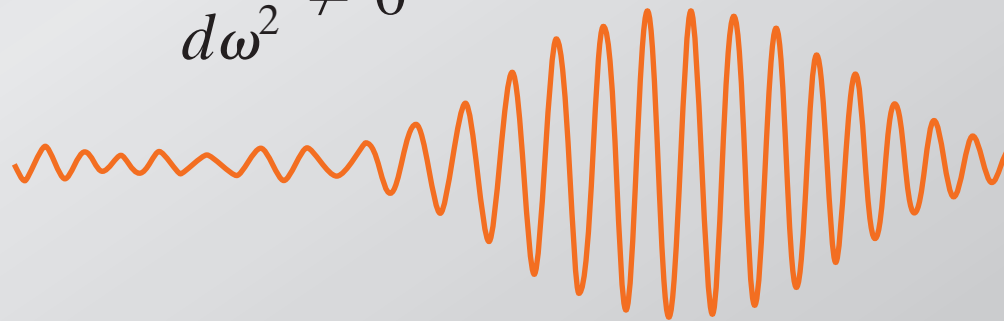


Pulse dispersion compensation

only *nonlinear* dispersion changes pulse shape!



$$\frac{d^2\phi}{d\omega^2} \neq 0$$



Pulse dispersion

Write dispersion as Taylor series:

Pulse dispersion

Write dispersion as Taylor series:

$$\omega(k) = \omega_o + \left(\frac{d\omega}{dk} \right)_{k=k_o} (k - k_o) + \frac{1}{2} \left(\frac{d^2\omega}{dk^2} \right)_{k=k_o} (k - k_o)^2$$

Pulse dispersion

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let

$$u \equiv \left(\frac{d\omega}{dk} \right)_{k=k_o} \quad \text{and} \quad w \equiv \left(\frac{d^2\omega}{dk^2} \right)_{k=k_o} .$$

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group velocity: $v_g = \frac{d\omega}{dk} = u + wk$

if $w = 0$, then group velocity and pulse shape constant!

Pulse dispersion compensation

So not path length but $\frac{d^2\phi}{d\omega^2}$ matters!

Pulse dispersion compensation

So not path length but $\frac{d^2\phi}{d\omega^2}$ matters!

$$\frac{dl_{eff}}{d\omega}$$

$$\frac{d^2\phi}{d\omega^2}$$

dispersion

+

+

Pulse dispersion compensation

So not path length but $\frac{d^2\phi}{d\omega^2}$ matters!

	$\frac{dl_{eff}}{d\omega}$	$\frac{d^2\phi}{d\omega^2}$
dispersion	+	+
gratings	-	-

Pulse dispersion compensation

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gratings	-	-
prisms	+	-

Outline

- propagation of pulses
- **nonlinear optics**
- nanoscale optics
- nonlinear optics at the nanoscale

Nonlinear optics

Linear optics:

$$\vec{P} = \chi \vec{E}$$

Nonlinear optics

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Nonlinear polarization:

$$P = \chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \dots$$

Nonlinear optics

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and so:

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Nonlinear optics

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and so:

$$P = P^{(1)} + P^{(2)} + P^{(3)} + \dots$$

$$P^{(2)} \approx P^{(1)} \text{ when } E = E_{at} \approx \frac{e}{a}, \text{ and so } \chi^{(n)} \approx \frac{\chi^{(1)}}{E_{at}^{n-1}}.$$

Nonlinear optics

Nonlinear polarization can drive new field:

$$\nabla^2 \vec{E} + \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{4\pi}{c^2} \frac{\partial^2 \vec{P}}{\partial t^2}$$

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But even terms disappear in media with inversion symmetry!

$$\vec{P}^{(2)} = \chi^{(2)} : \vec{E}\vec{E}$$

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Invert all vectors:

$$-\vec{P}^{(2)} = \chi^{(2)} : (-\vec{E})(-\vec{E})$$

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$$\vec{P}^{(2)} = \chi^{(2)} : \vec{E} \vec{E}$$

Invert all vectors:

$$-\vec{P}^{(2)} = \chi^{(2)} : (-\vec{E})(-\vec{E})$$

and so $\chi^{(2)} = -\chi^{(2)} = 0$.

Nonlinear optics

Consider oscillating electric field:

$$E(t) = E e^{i\omega t} + \text{c.c.}$$

Nonlinear optics

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$$P^{(2)}(t) = \chi^{(2)} E^2(t) = \frac{1}{2}\chi^{(2)} E E^* + \frac{1}{4}[\chi^{(2)} E^2 e^{-2\omega t} + \text{c.c.}]$$

Nonlinear optics

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Nonlinear optics

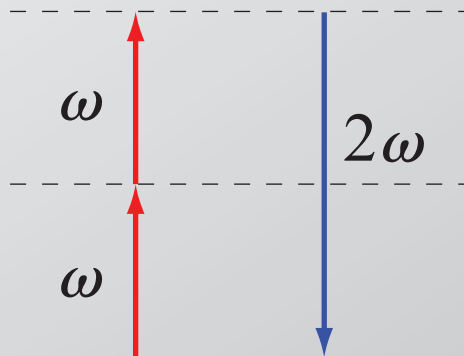
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$$E(t) = E e^{i\omega t} + \text{c.c.}$$

Second-order polarization:

$$P^{(2)}(t) = \chi^{(2)} E^2(t) = \frac{1}{2} \chi^{(2)} E E^* + \frac{1}{4} [\chi^{(2)} E^2 e^{-2\omega t} + \text{c.c.}]$$

Physical interpretation:



Nonlinear optics

Can also cause frequency mixing!

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$$E(t) = E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t}$$

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$$2\omega_1 \text{ (SHG)}, 2\omega_2 \text{ (SHG)}, \omega_1 + \omega_2 \text{ (SFG)}, \omega_1 - \omega_2 \text{ (DFG)}$$

Nonlinear optics

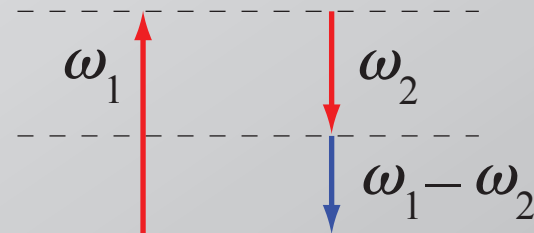
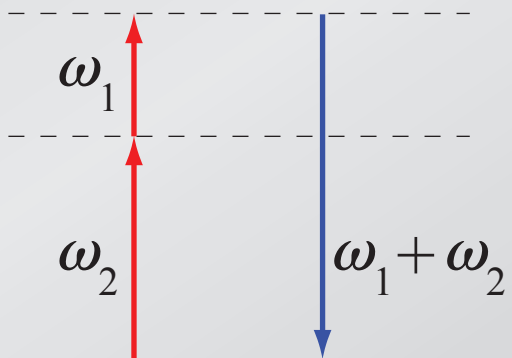
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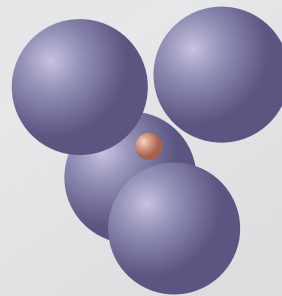
Physical interpretation:



Nonlinear optics

Linear response:

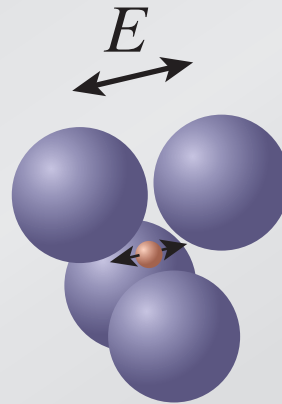
$$\vec{P} = \chi \vec{E}$$



Nonlinear optics

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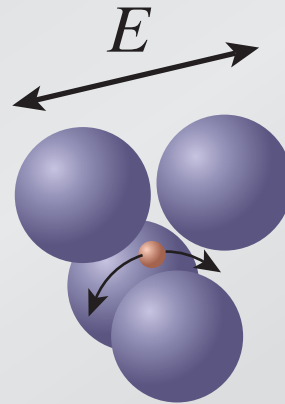
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Nonlinear optics

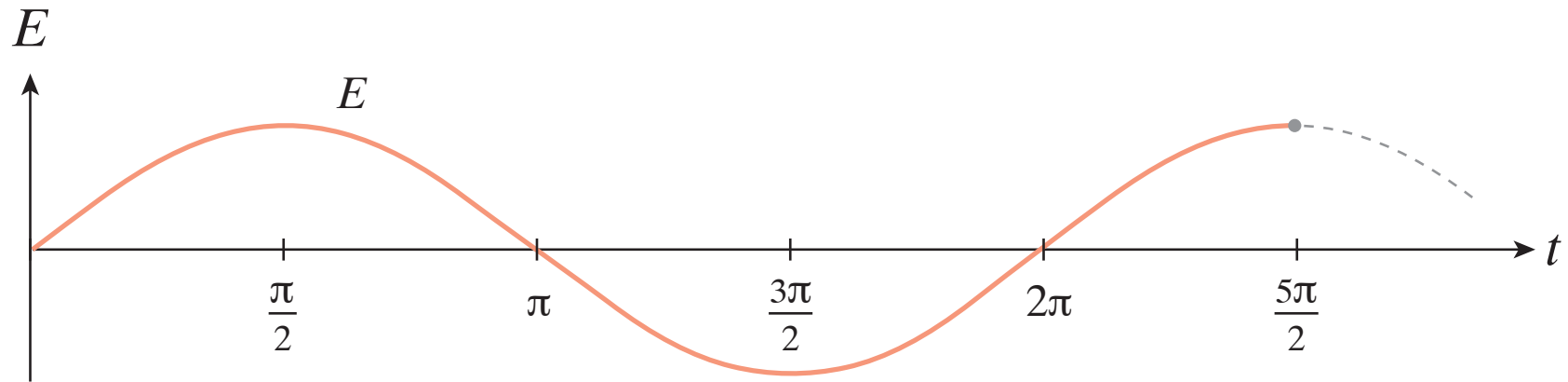
Nonlinear response:

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Nonlinear optics

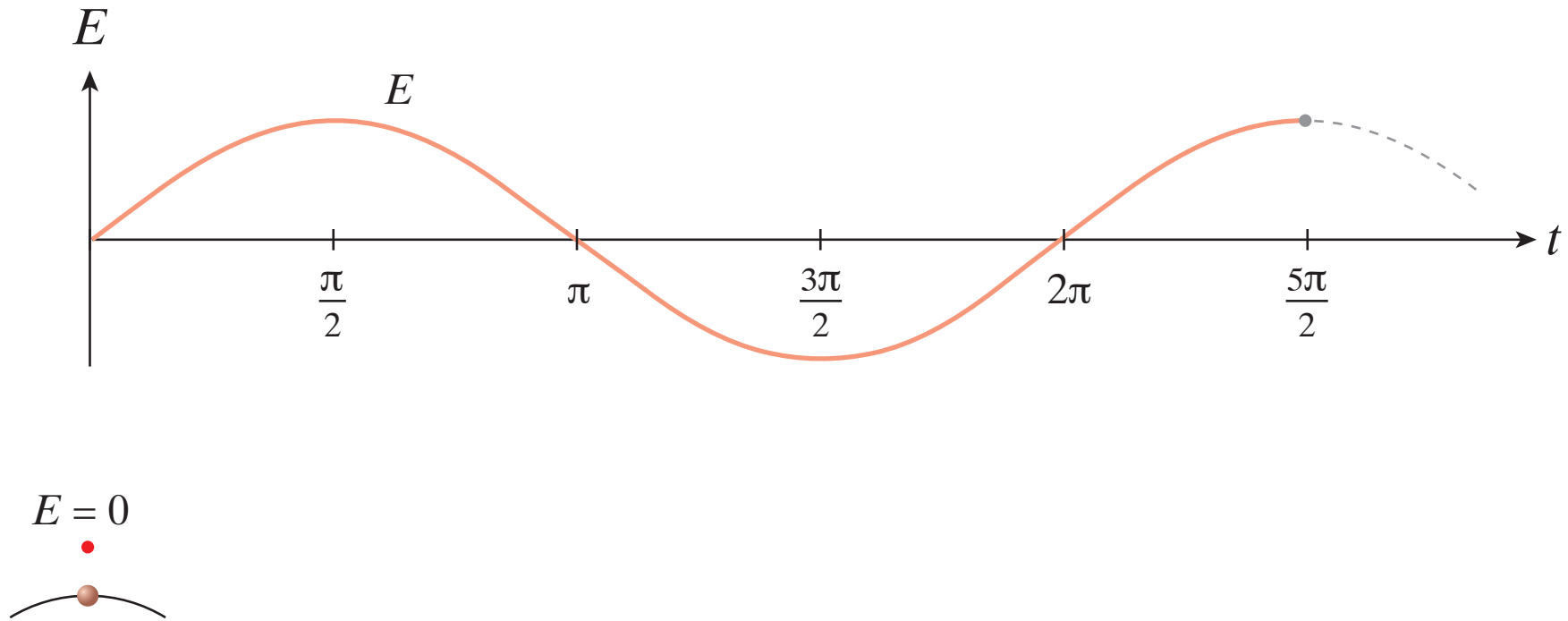
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Nonlinear optics

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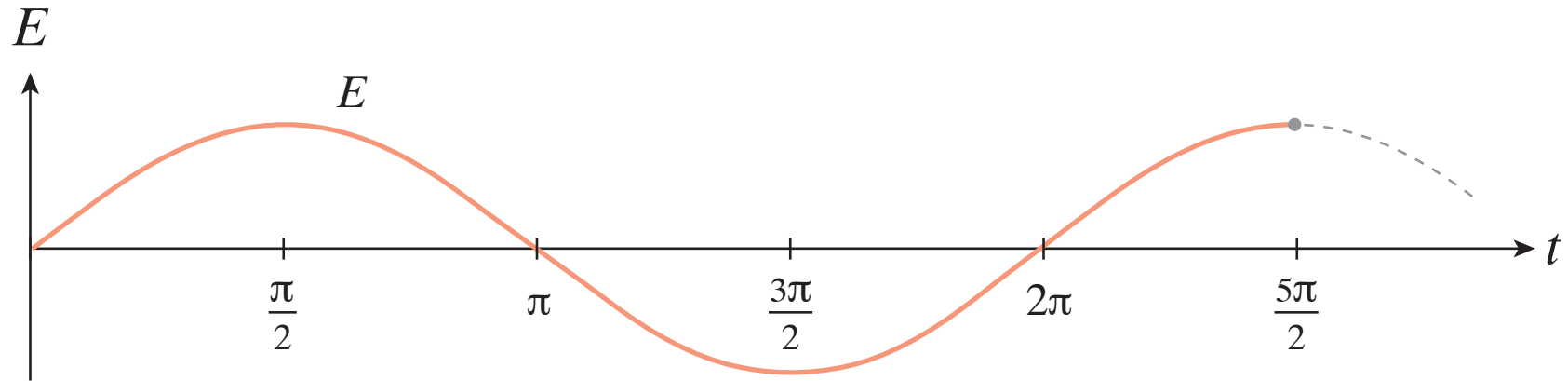
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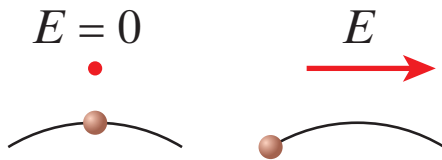
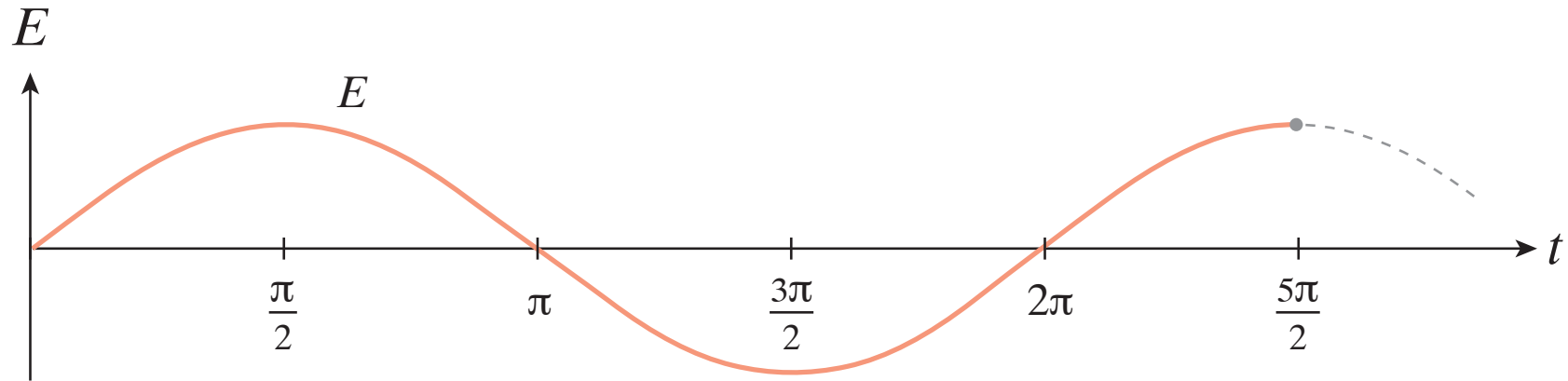
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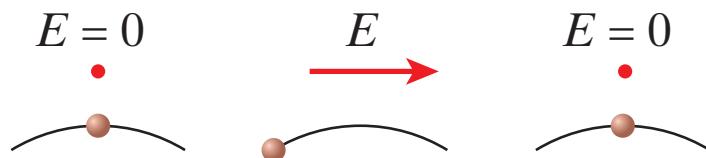
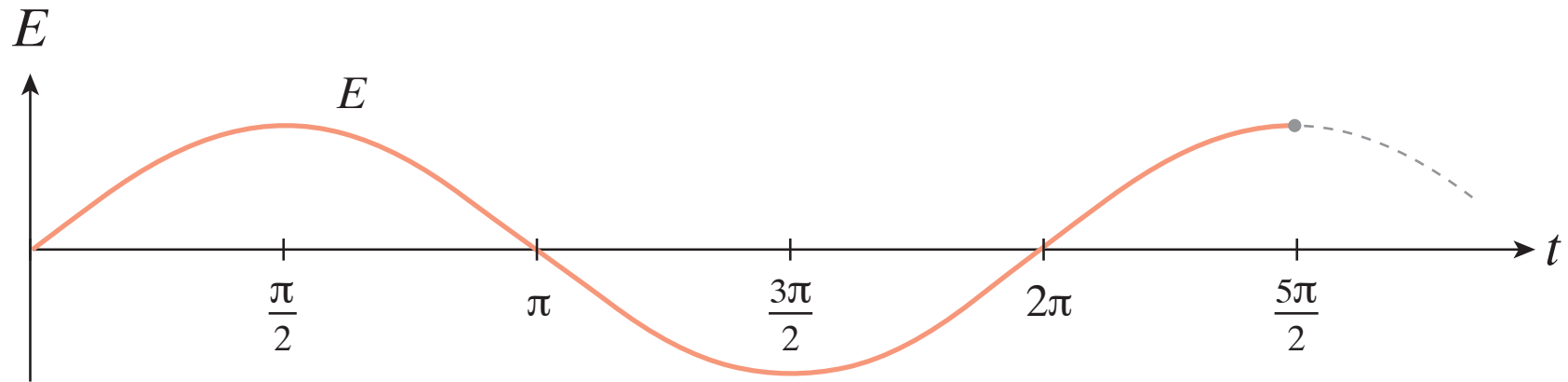
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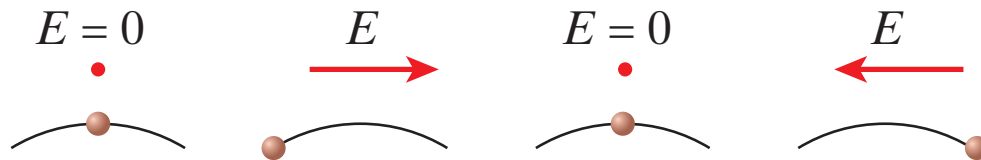
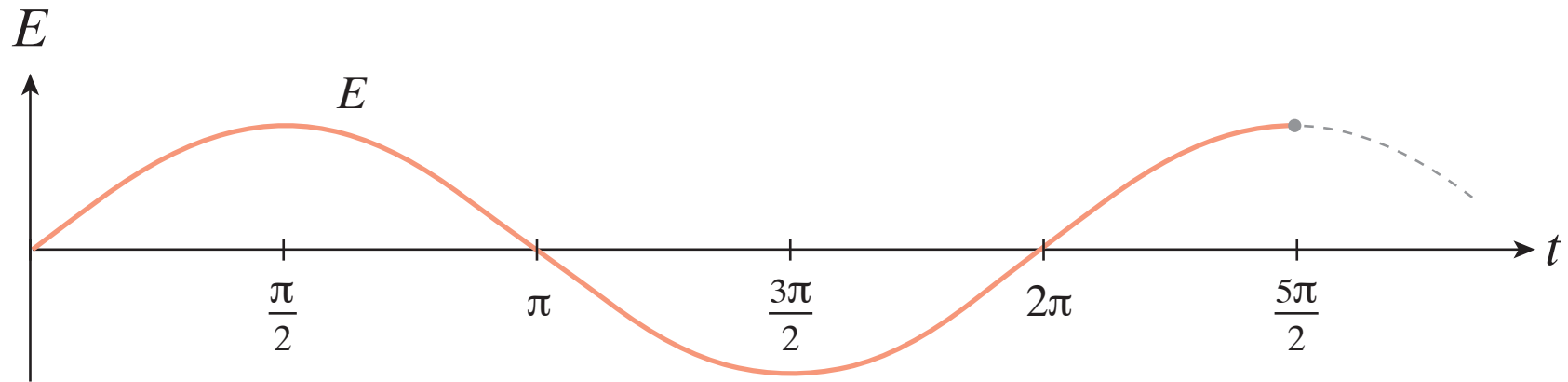
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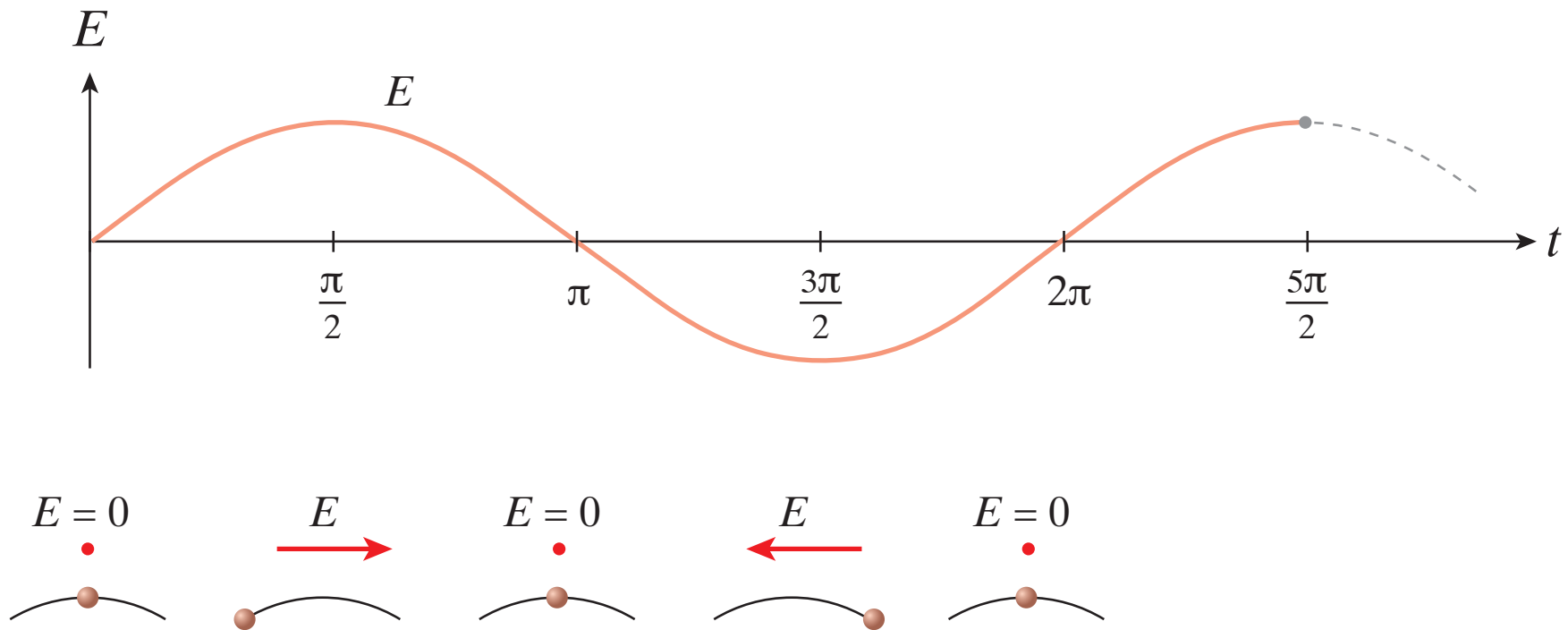
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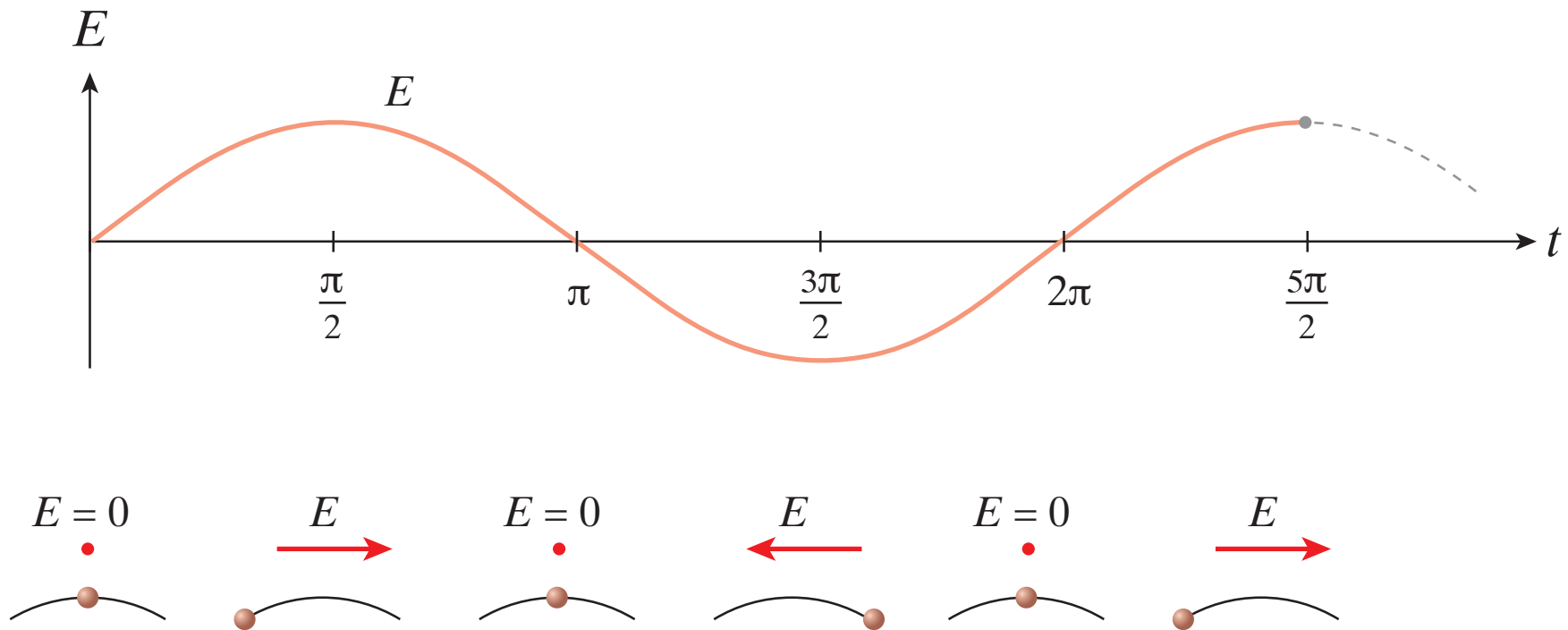
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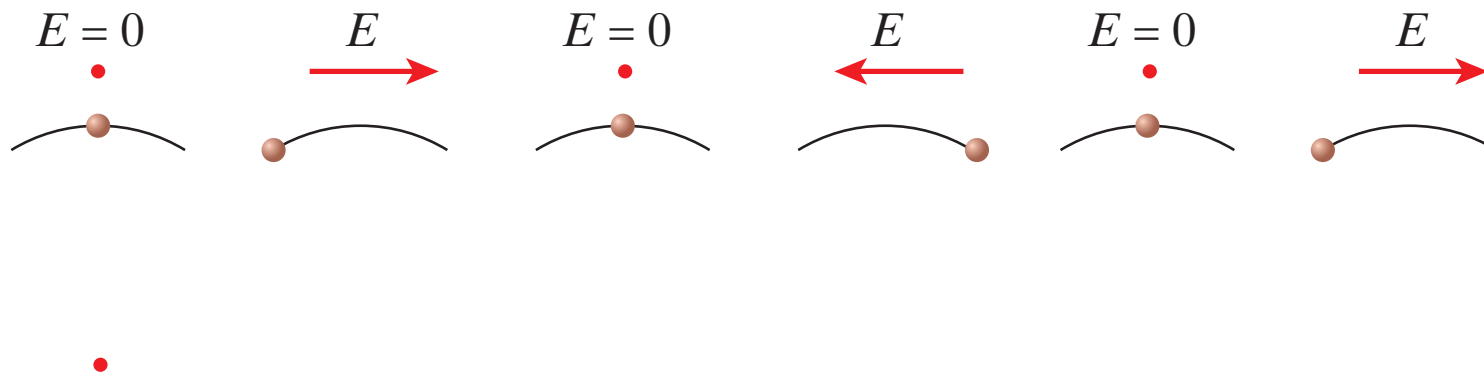
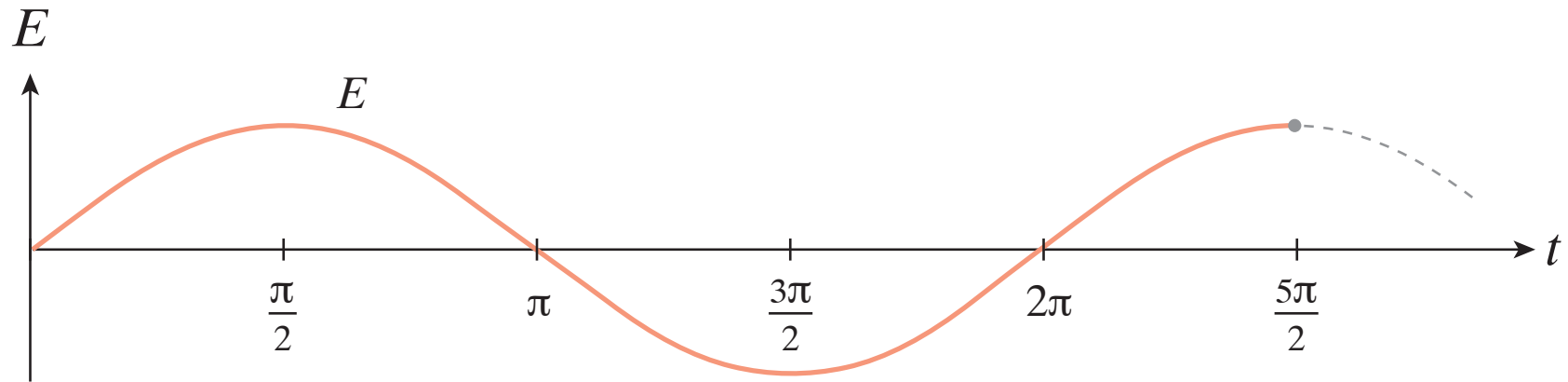
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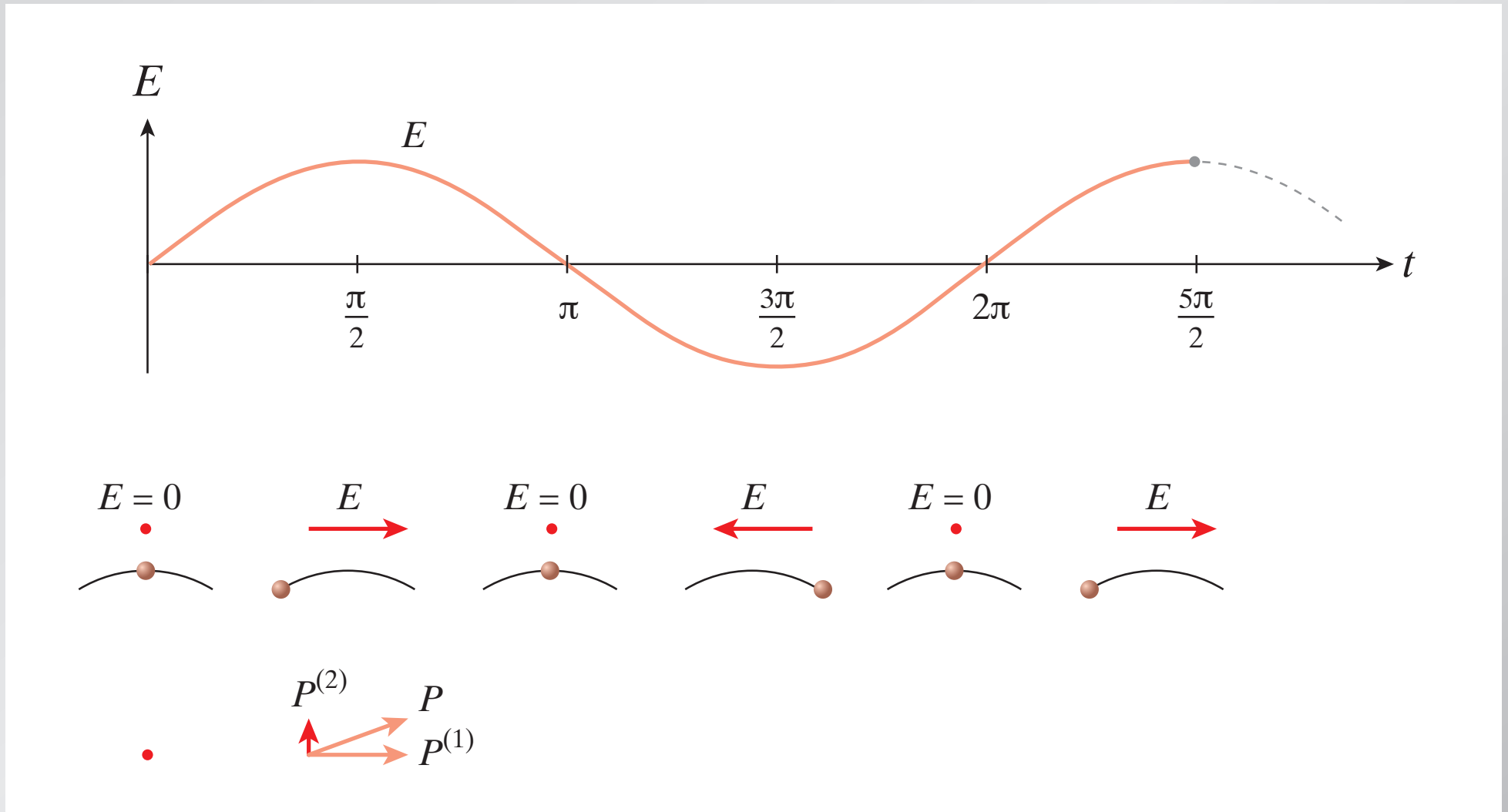
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Nonlinear optics

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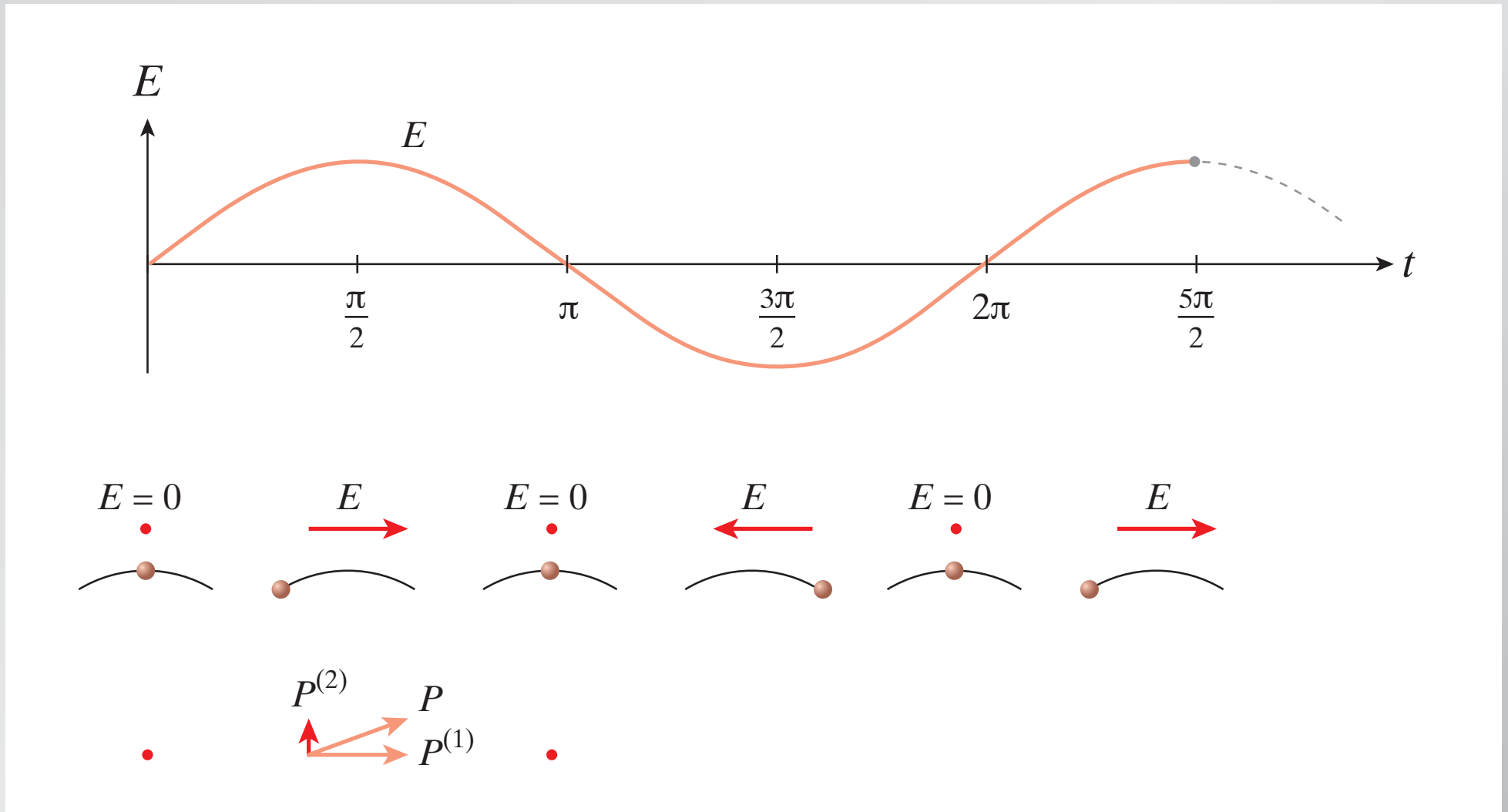
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Nonlinear optics

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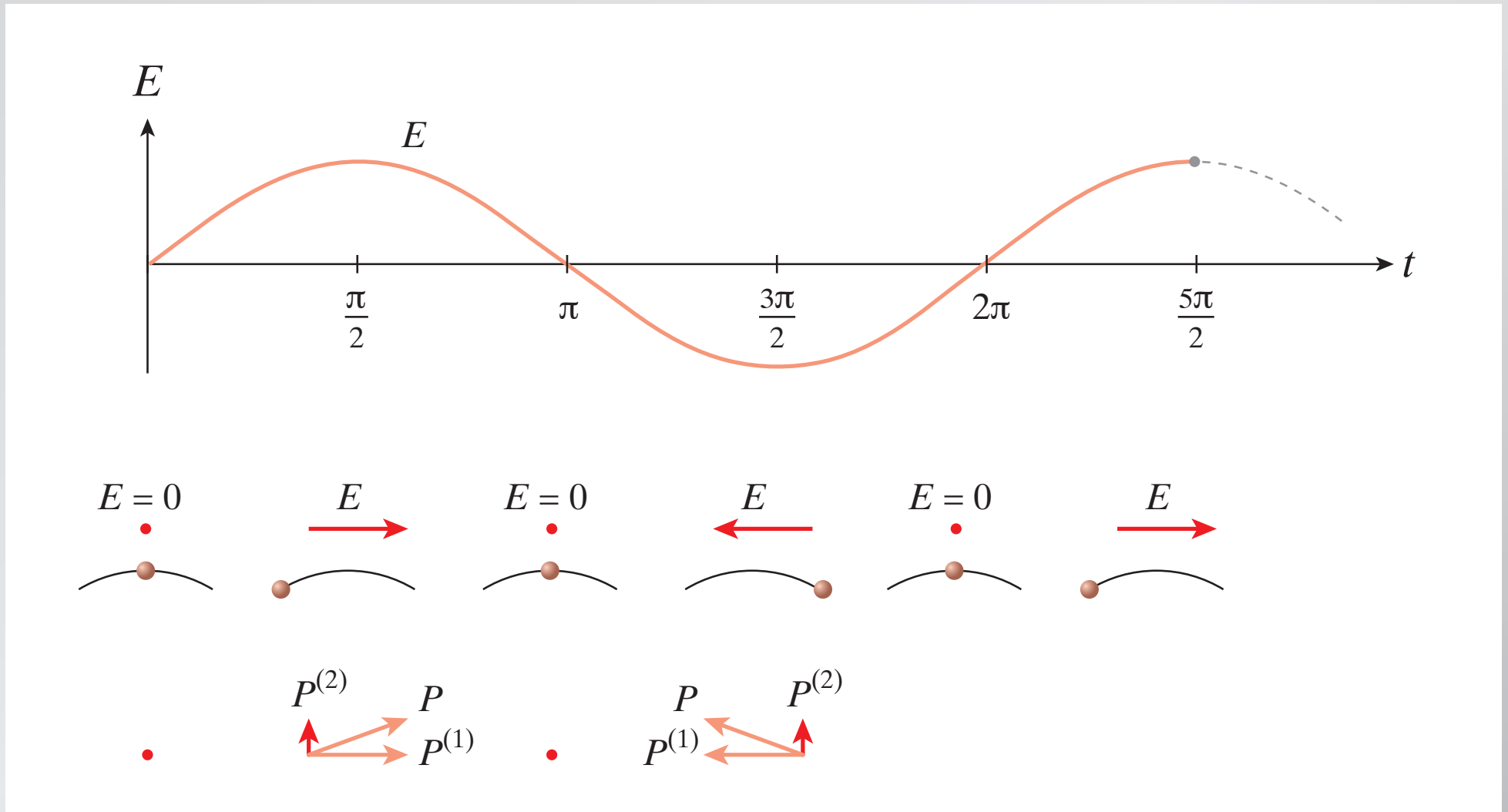
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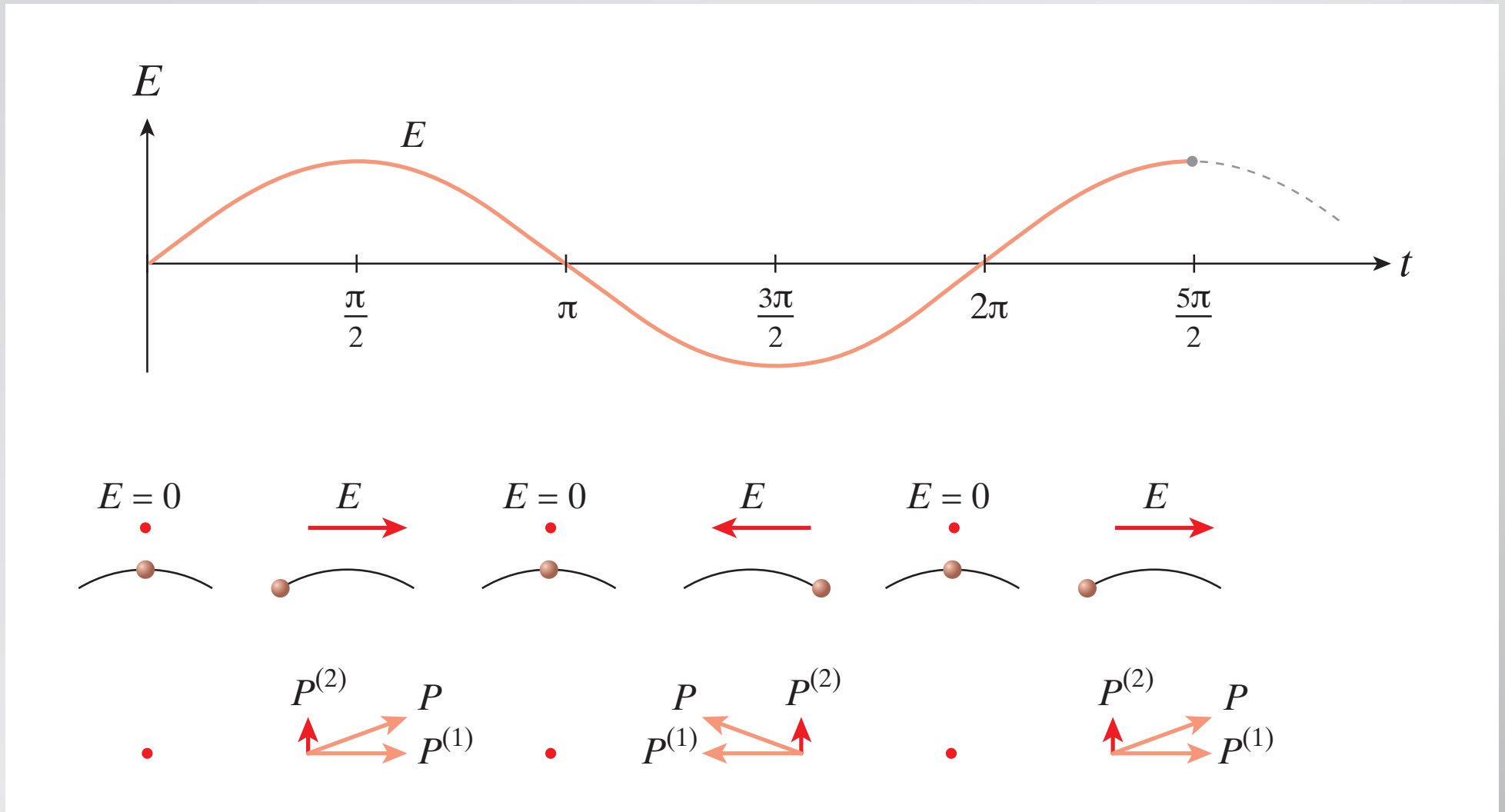
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Nonlinear optics

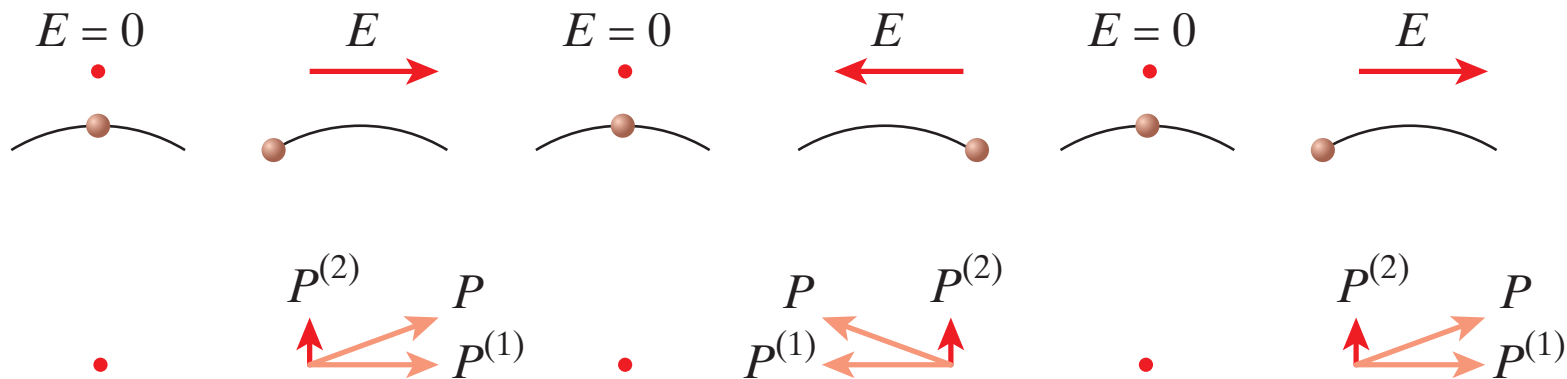
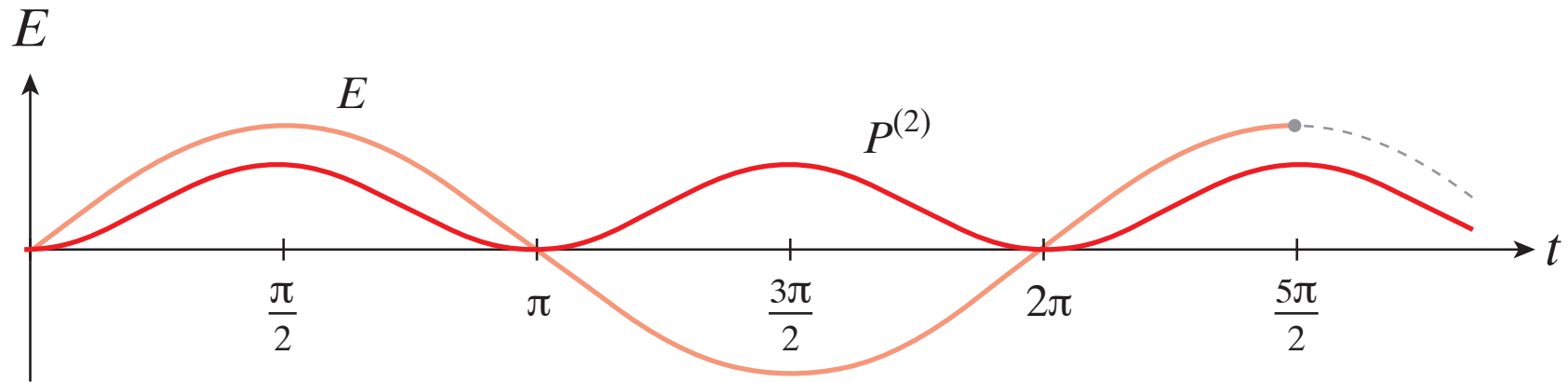
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Nonlinear optics

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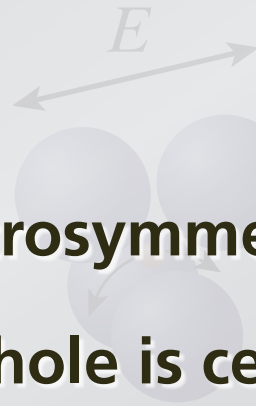


Nonlinear optics

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Q: Silicon atoms are arranged in this way. Does bulk silicon generate second harmonic?



- 1. Yes, silicon is not centrosymmetric (as the unit cell shows)**
- 2. No, the crystal as a whole is centrosymmetric**
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- 4. Other**

Nonlinear optics

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Nonlinear optics

Third-order polarization: $P^{(3)}(t) = \chi^{(3)} E^3(t)$

Nonlinear optics

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3 frequencies, 3 terms + c.c.: complicated! In general

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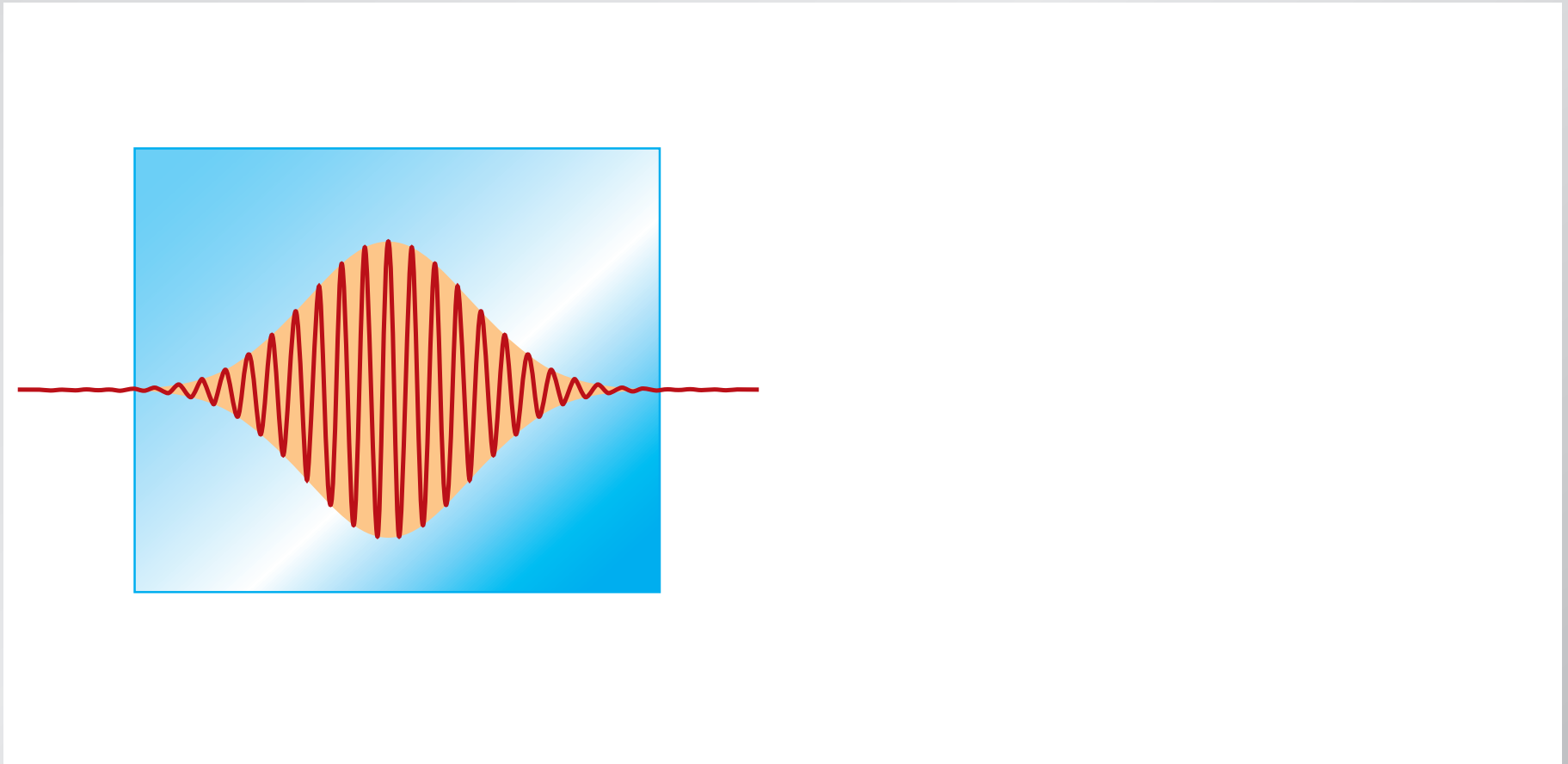
and so $P = P^{(1)} + P^{(3)} = (\chi^{(1)} + \chi^{(3)}I)E \equiv \chi_{eff}E$

$$n = \sqrt{\epsilon} = \sqrt{1 + \chi_{eff}} \approx \sqrt{1 + \chi^{(1)}} + \frac{1}{2} \frac{\chi^{(3)}I}{\sqrt{1 + \chi^{(1)}}} = n_o + n_2I$$

Nonlinear optics

Intensity-dependent index of refraction:

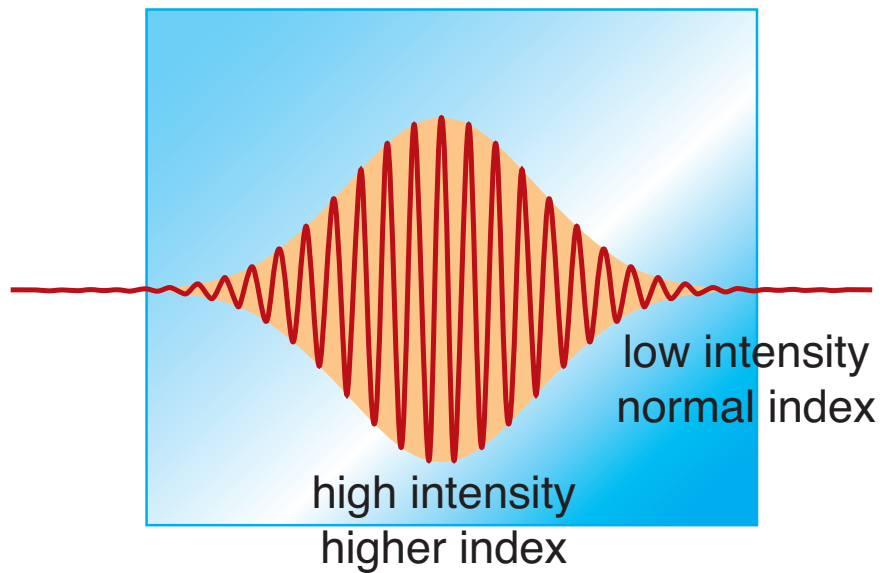
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Nonlinear optics

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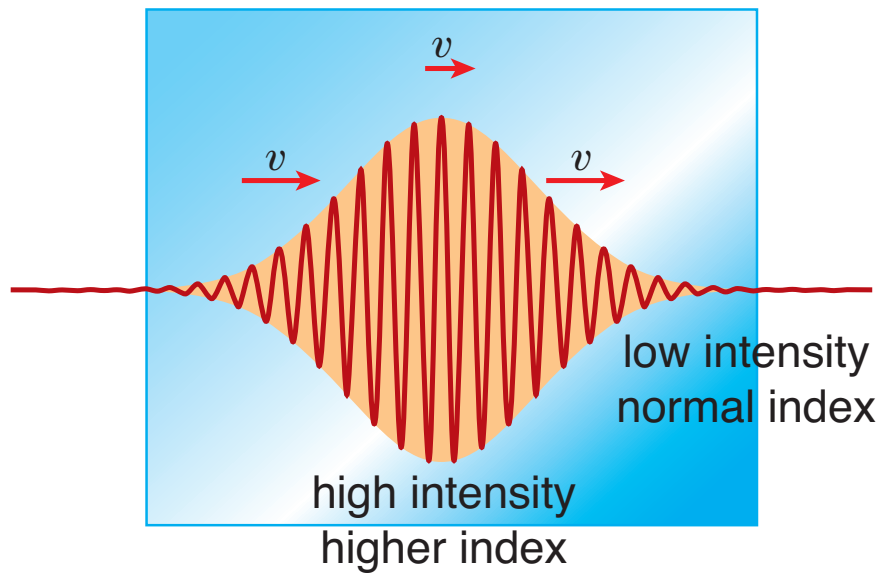
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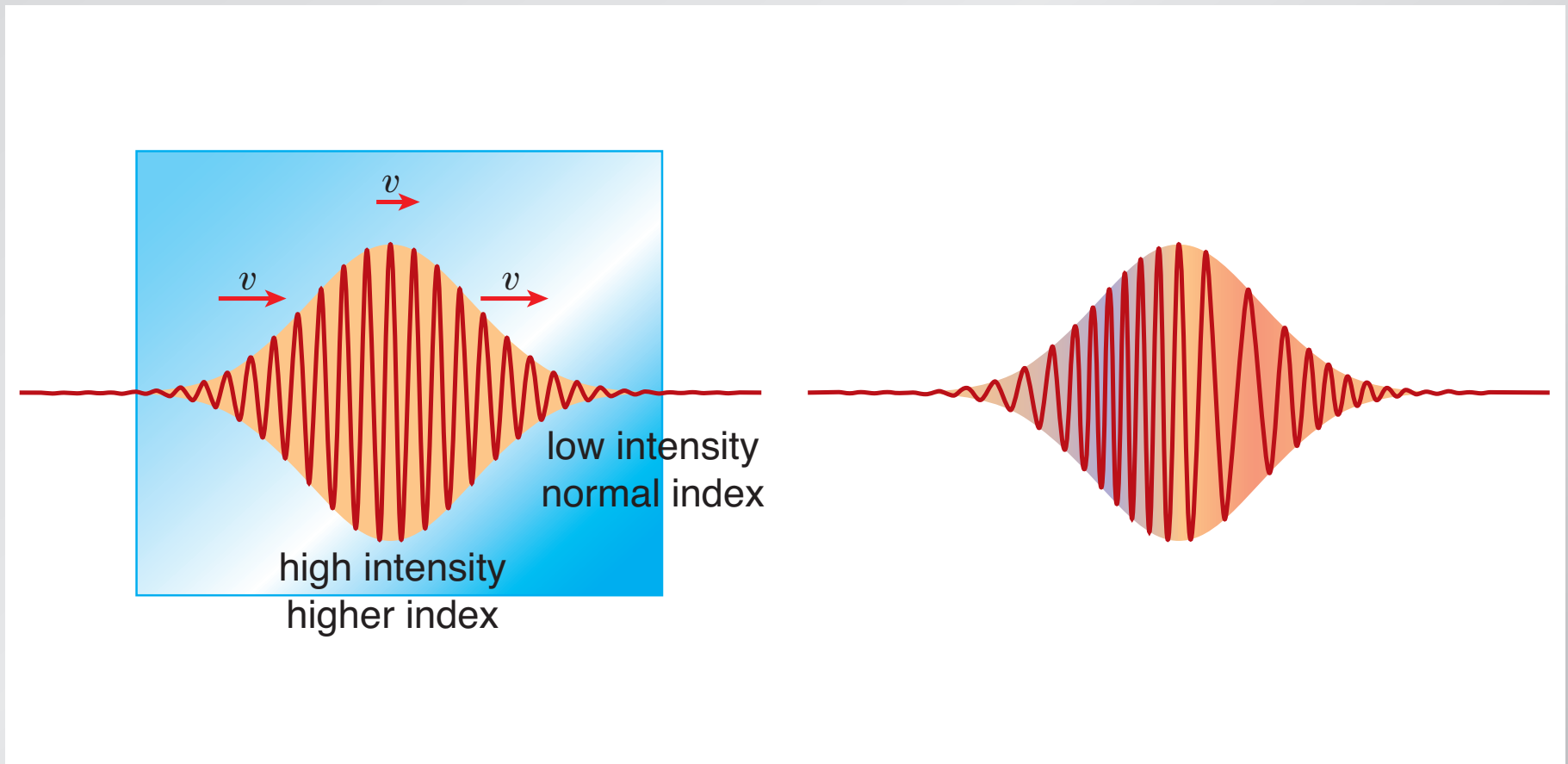
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Nonlinear optics

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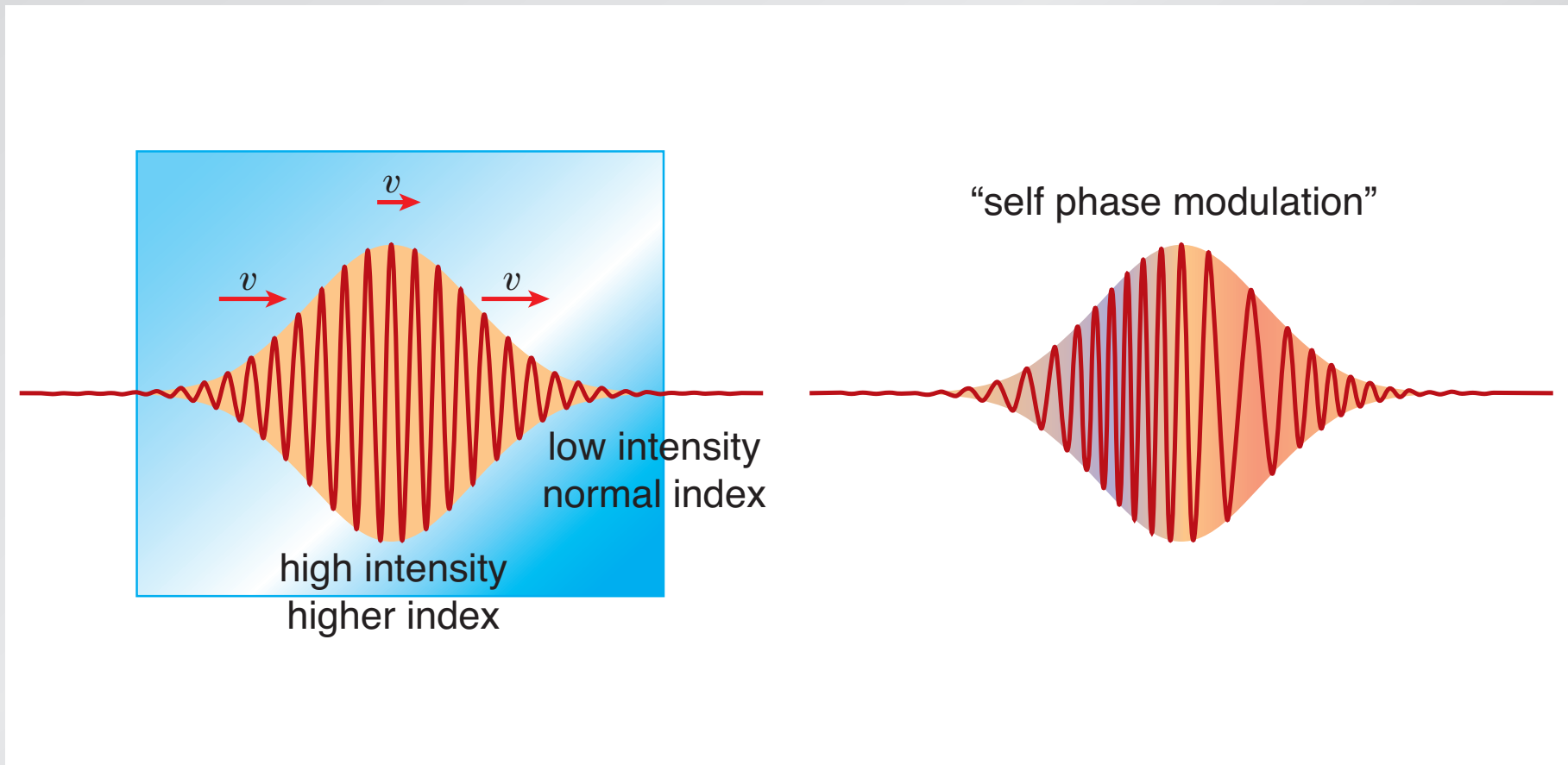
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Nonlinear optics

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Nonlinear optics

Phase:

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Nonlinear optics

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$$\phi = \frac{2\pi}{\lambda} L(n_o + n_2 I)$$

Nonlinear optics

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Frequency change: $\Delta\omega = -\frac{d\phi}{dt}$

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Nonlinear optics

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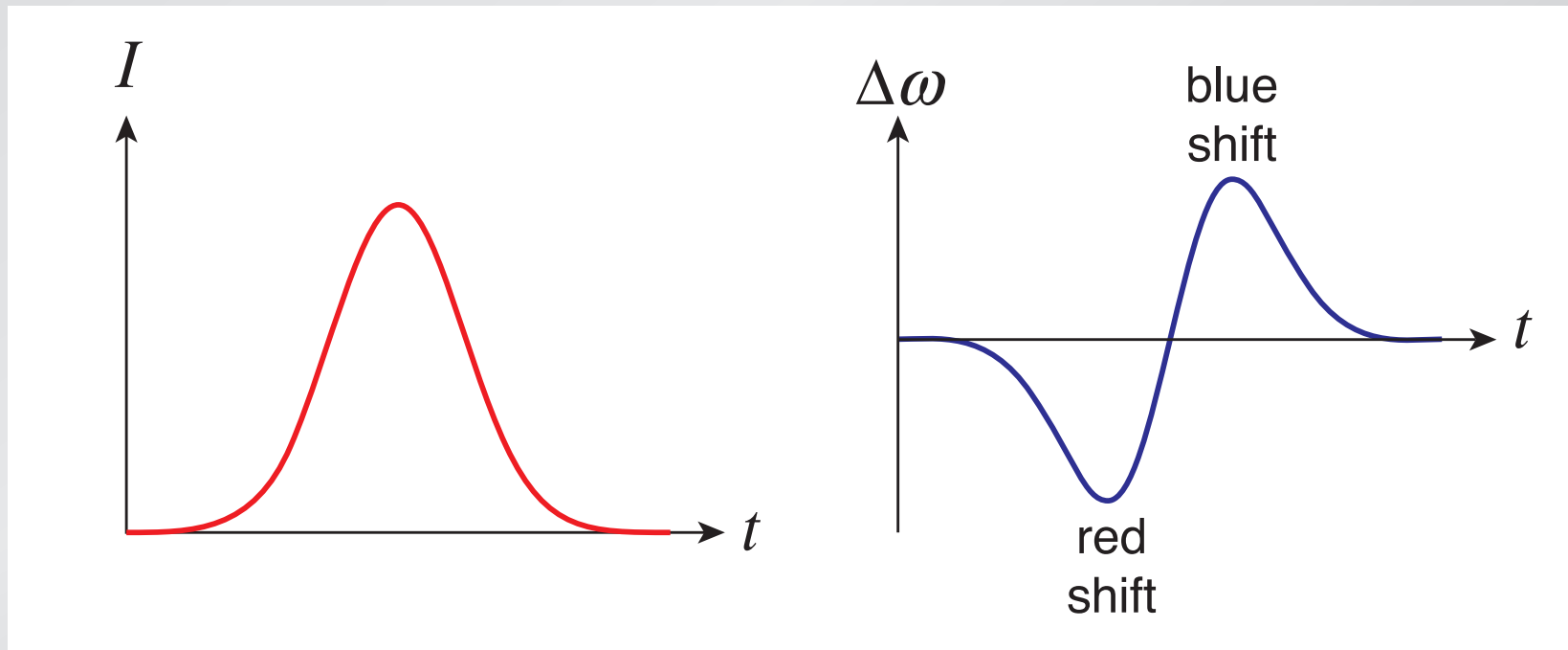
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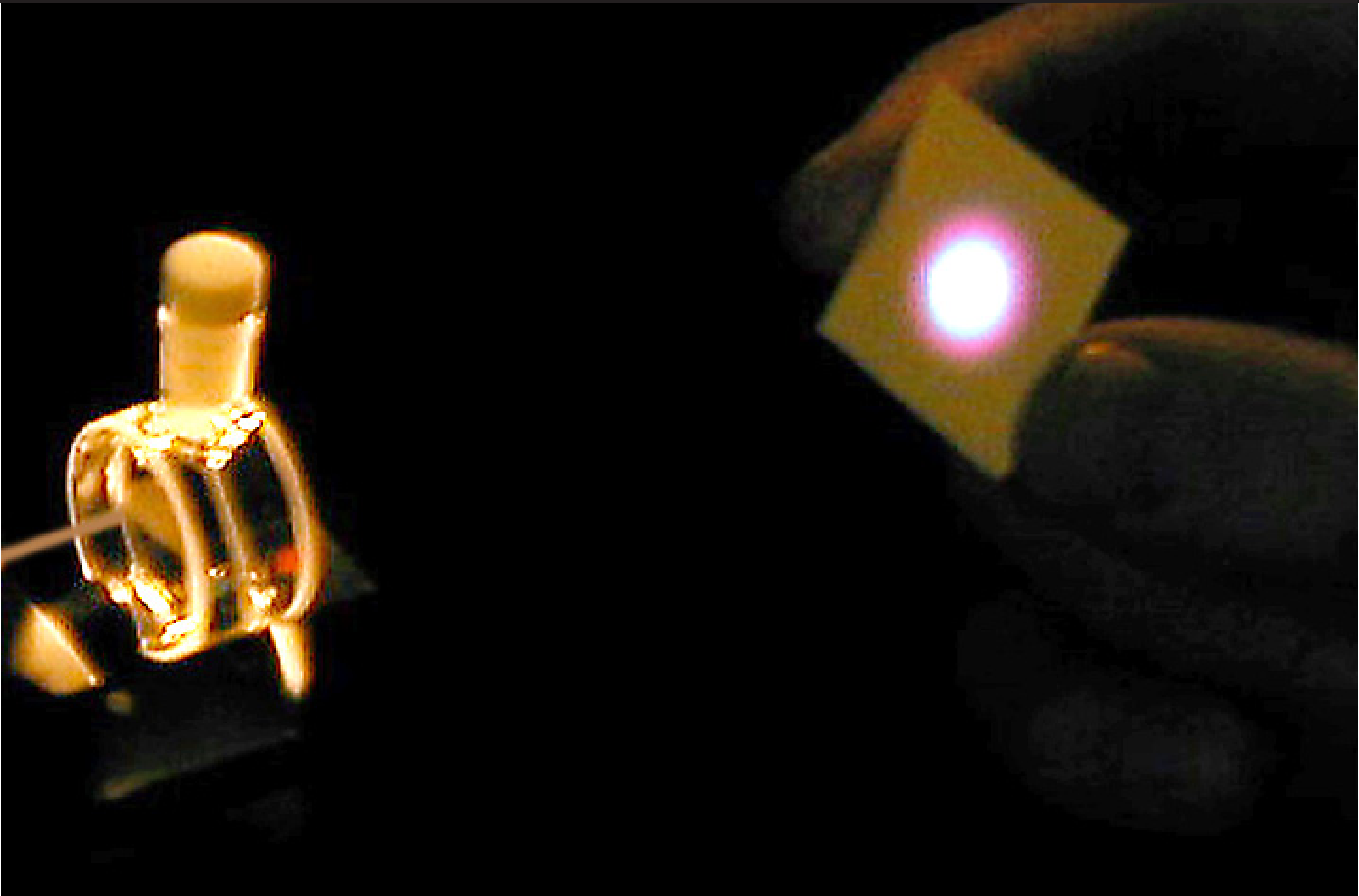
Nonlinear optics

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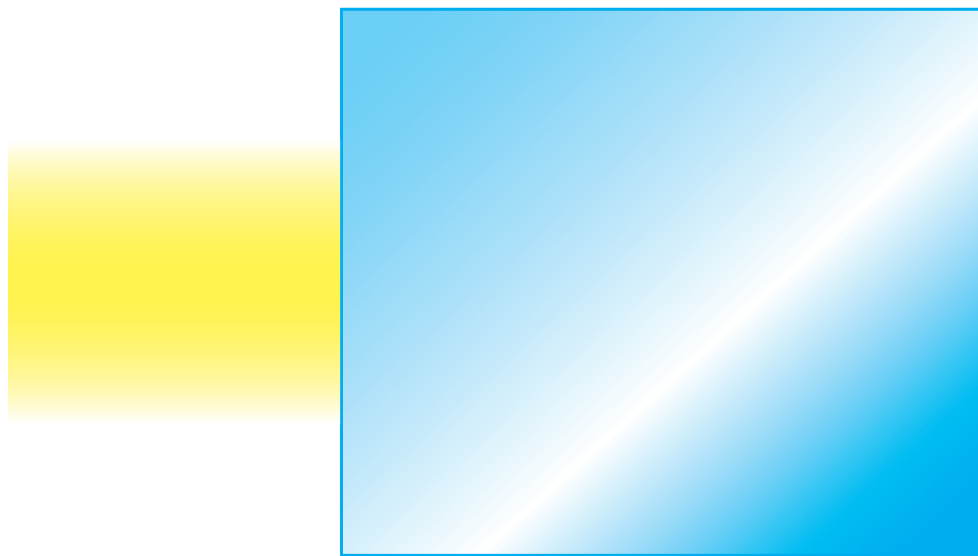
Nonlinear optics



Nonlinear optics

Intensity-dependent index of refraction:

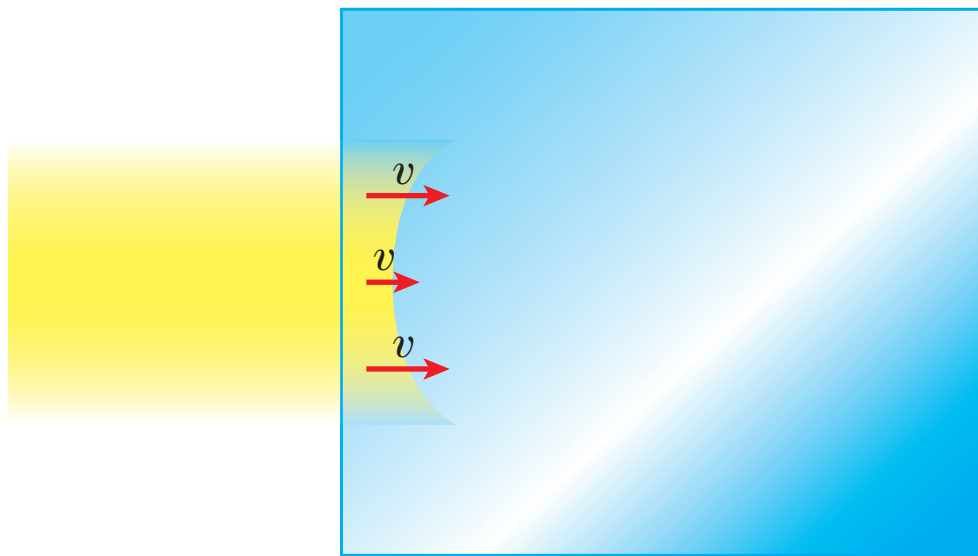
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Nonlinear optics

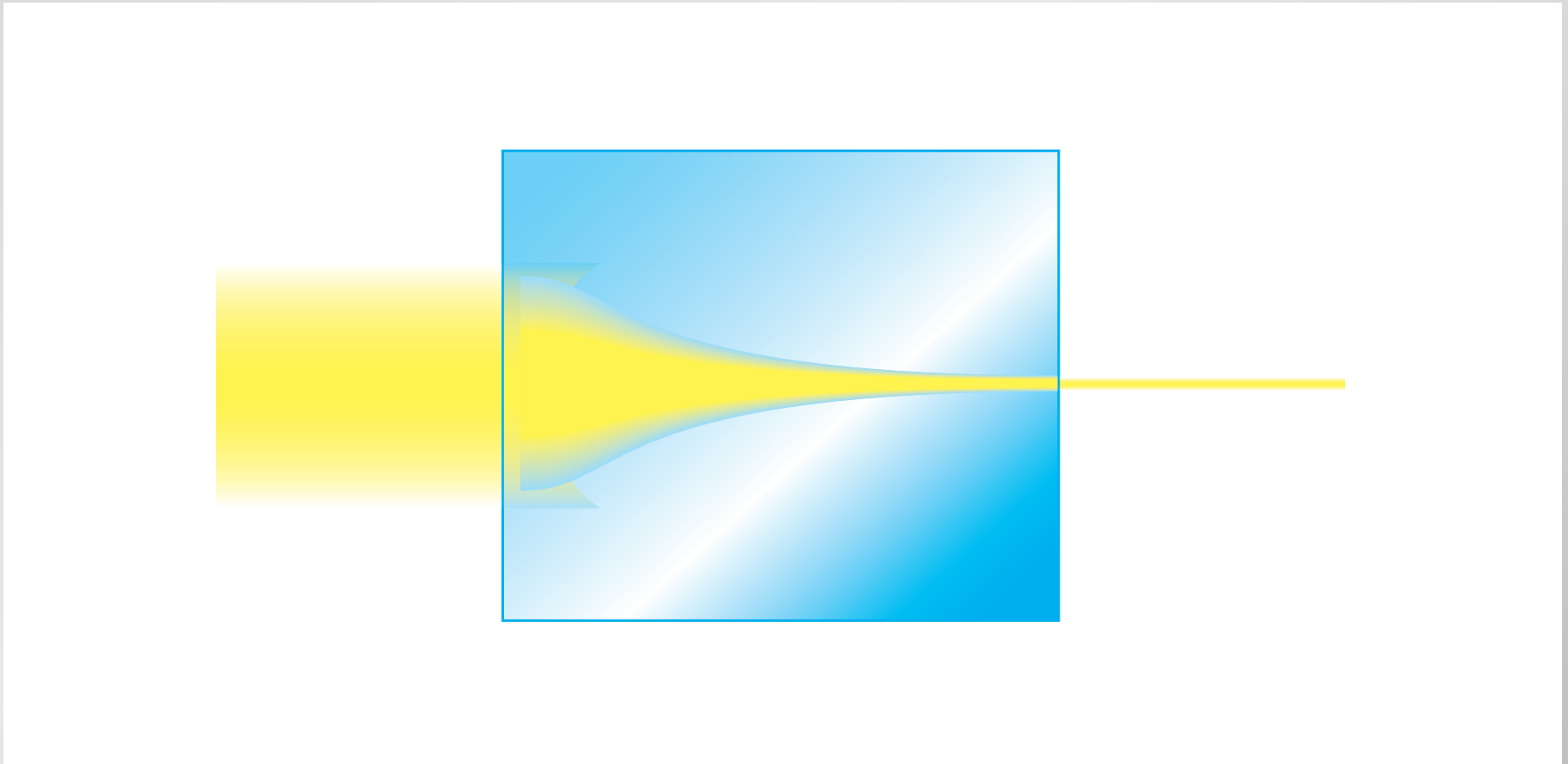
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Nonlinear optics

self-focusing



Nonlinear optics

but susceptibility is complex!

susceptibility	real part	imaginary part
linear	refraction	absorption
nonlinear	SHG, SFG, DFG, THG,...	multiphoton absorption

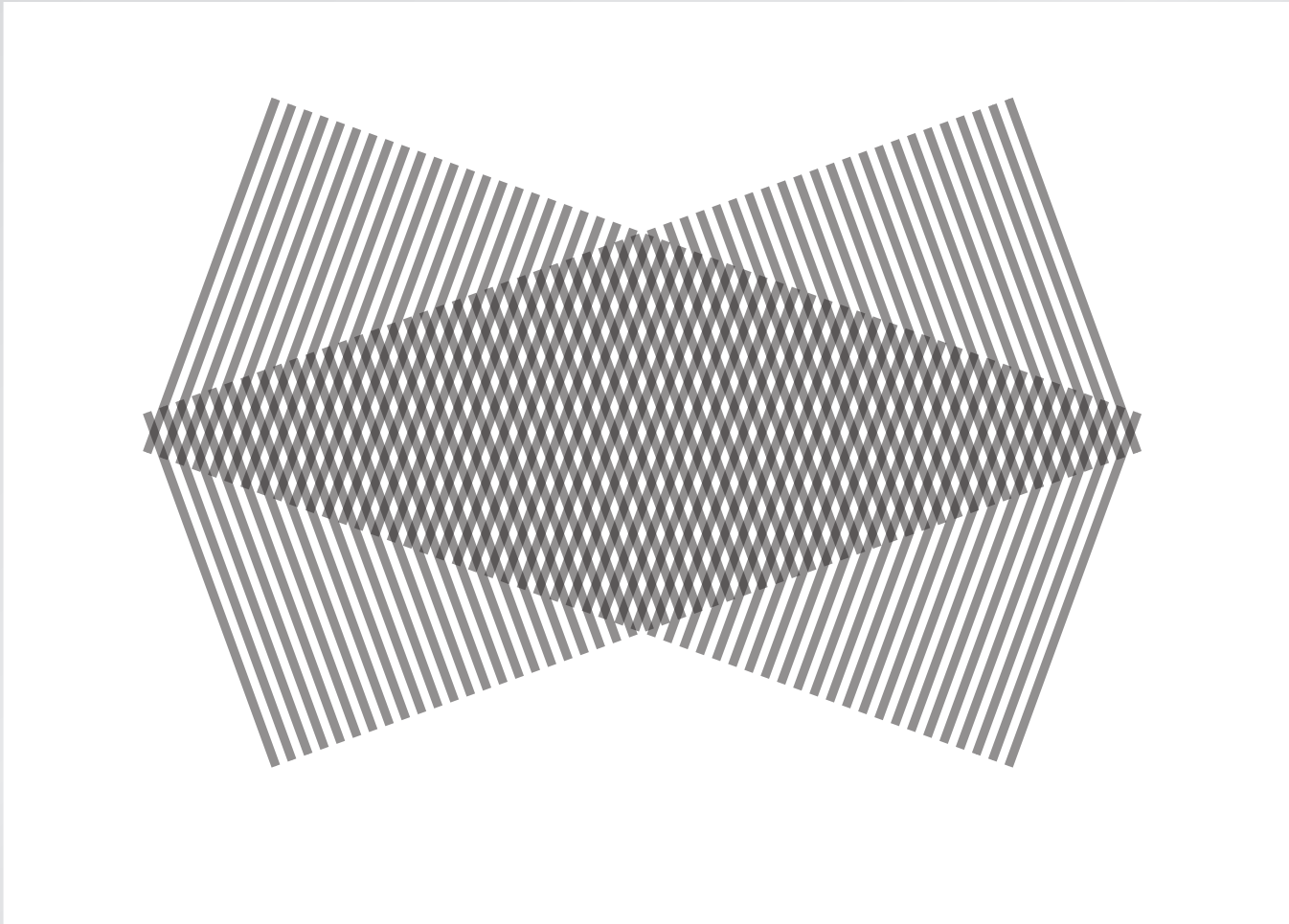
$$\alpha = \alpha_o + \beta I + \gamma I^2 + \dots$$

Outline

- propagation of pulses
- nonlinear optics
- **nanoscale optics**
- nonlinear optics at the nanoscale

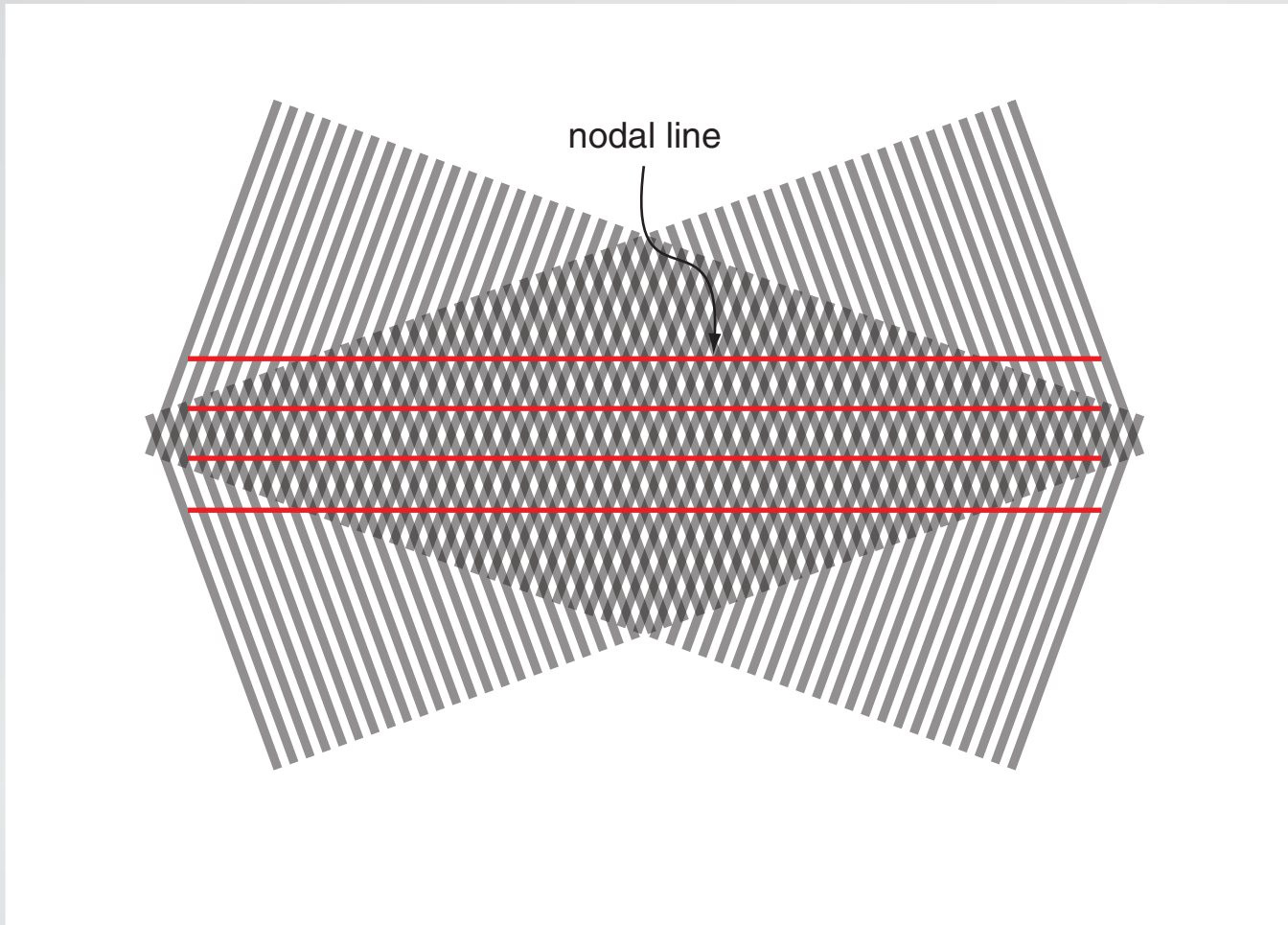
Waveguiding

two crossed planar waves...



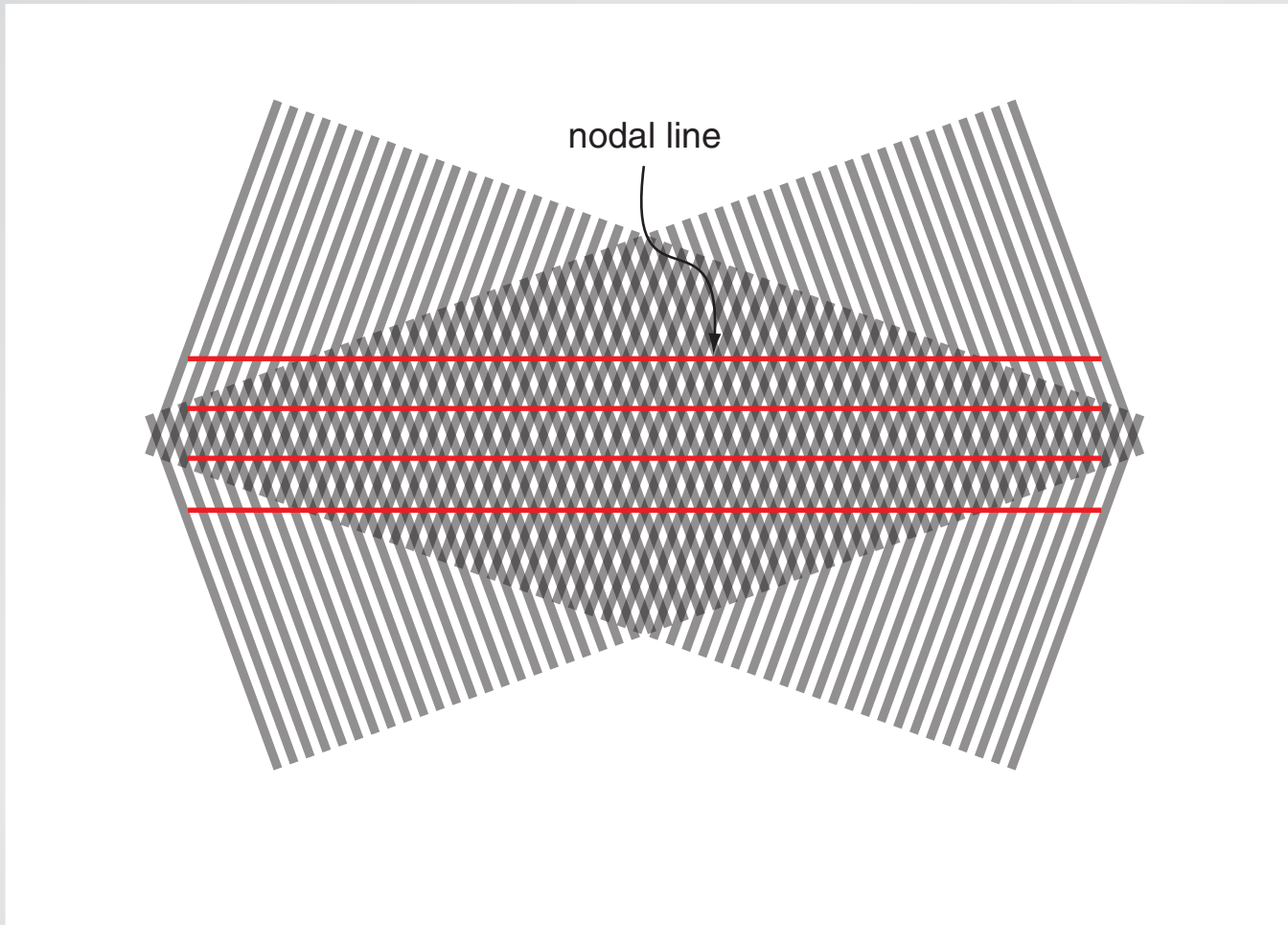
Waveguiding

...cause an interference pattern



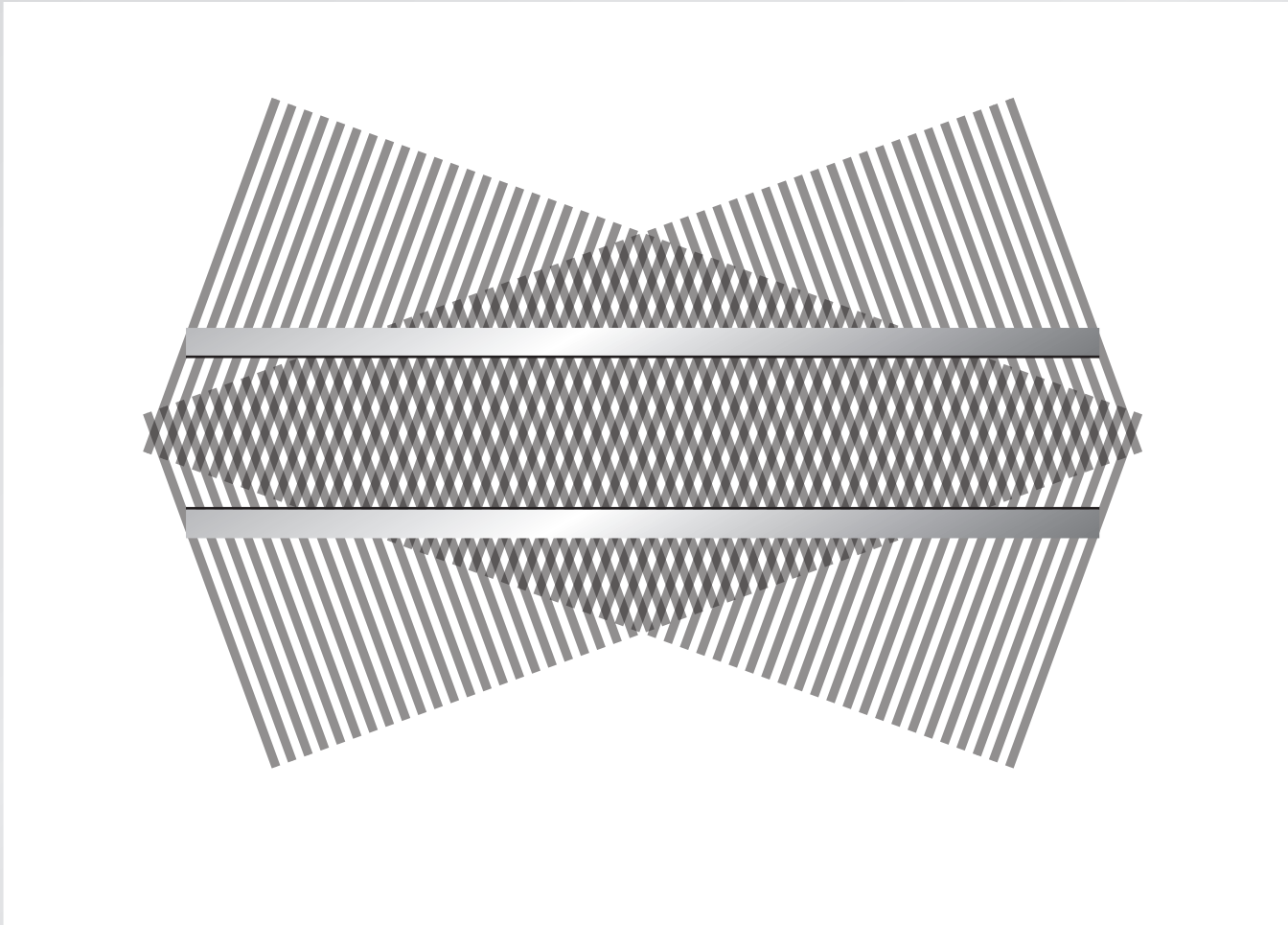
Waveguiding

$E = 0$ on the nodal lines



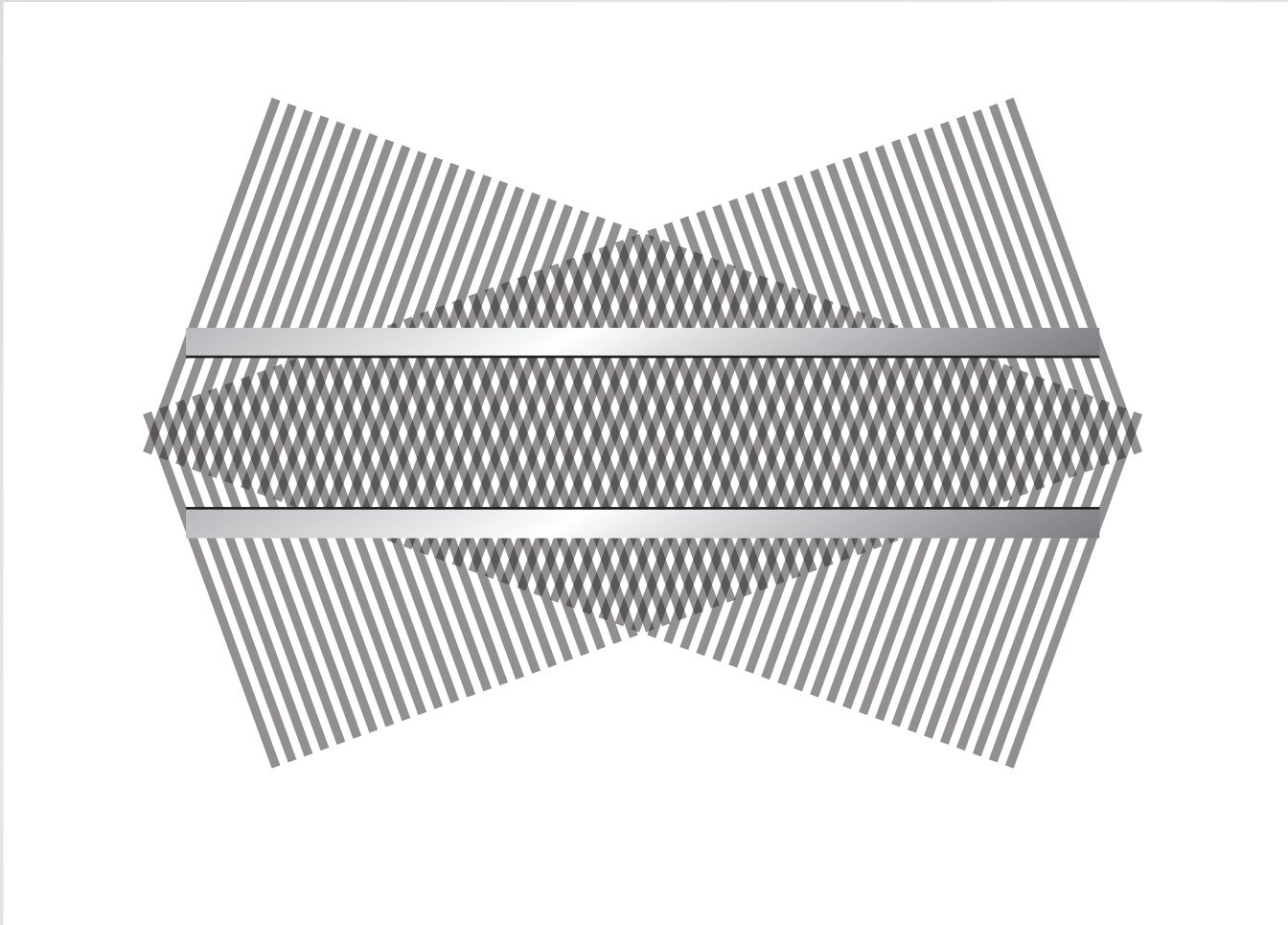
Waveguiding

...satisfying boundary conditions for planar-mirror waveguide



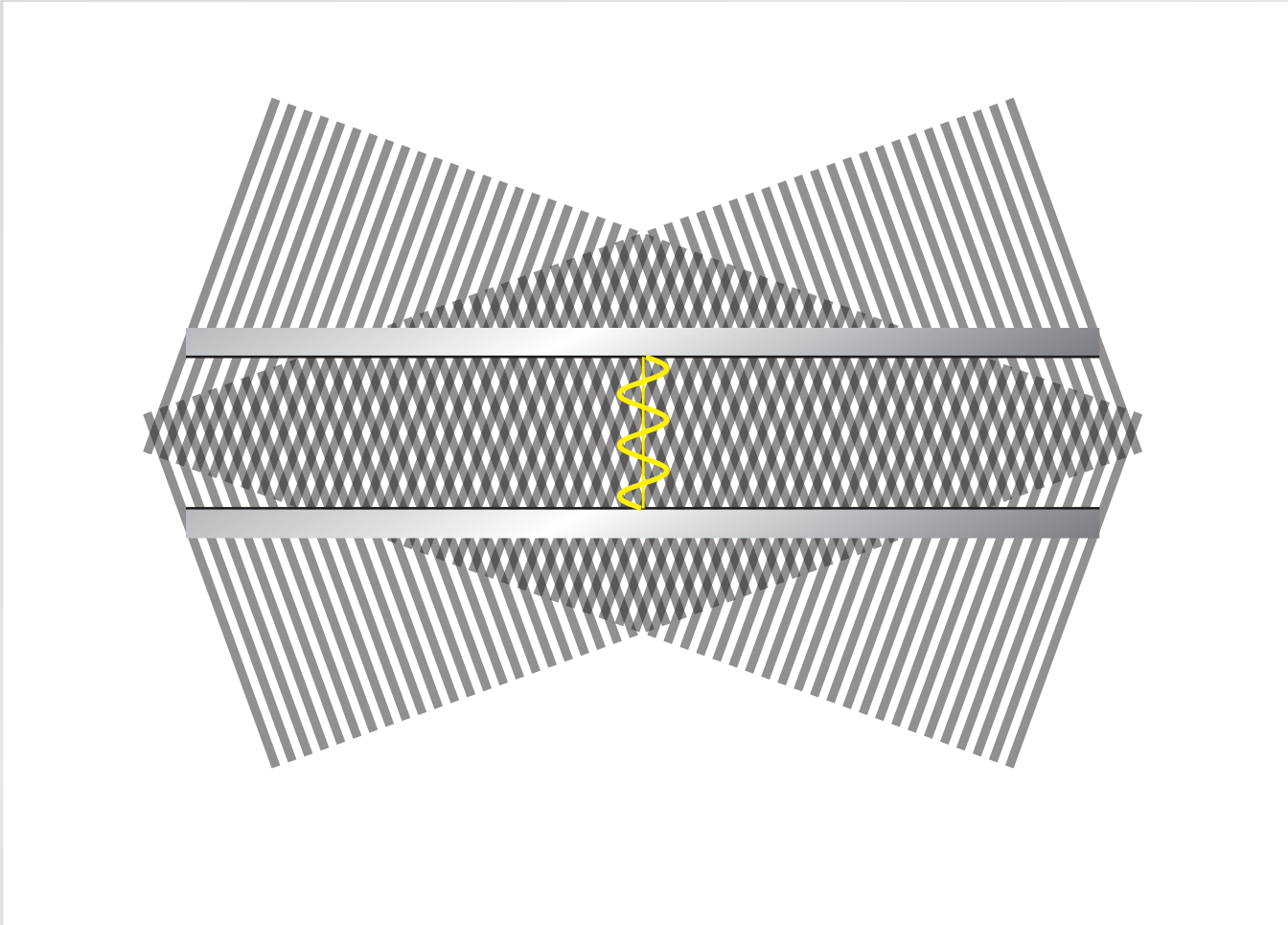
Waveguiding

transverse standing wave, traveling along axis



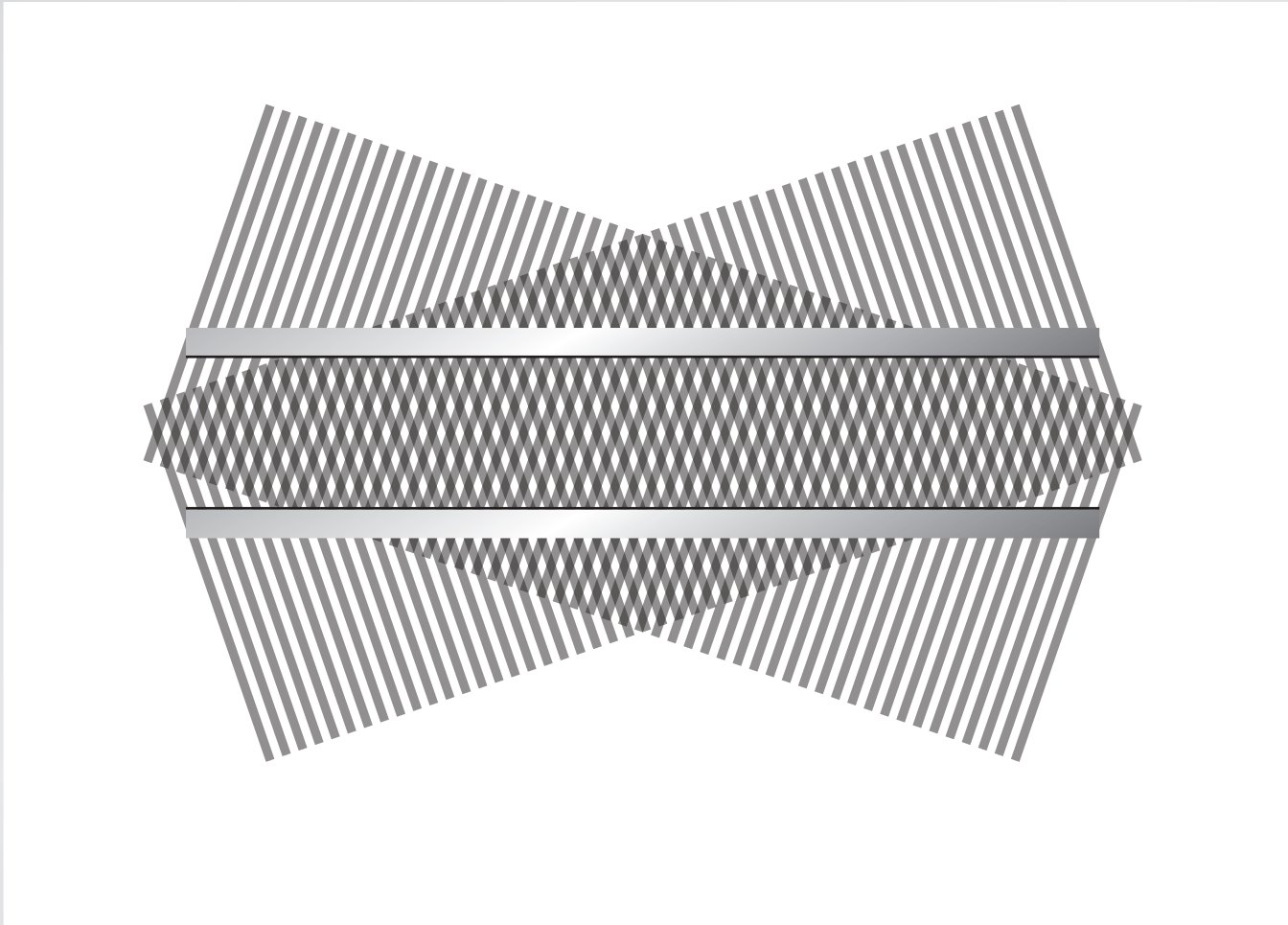
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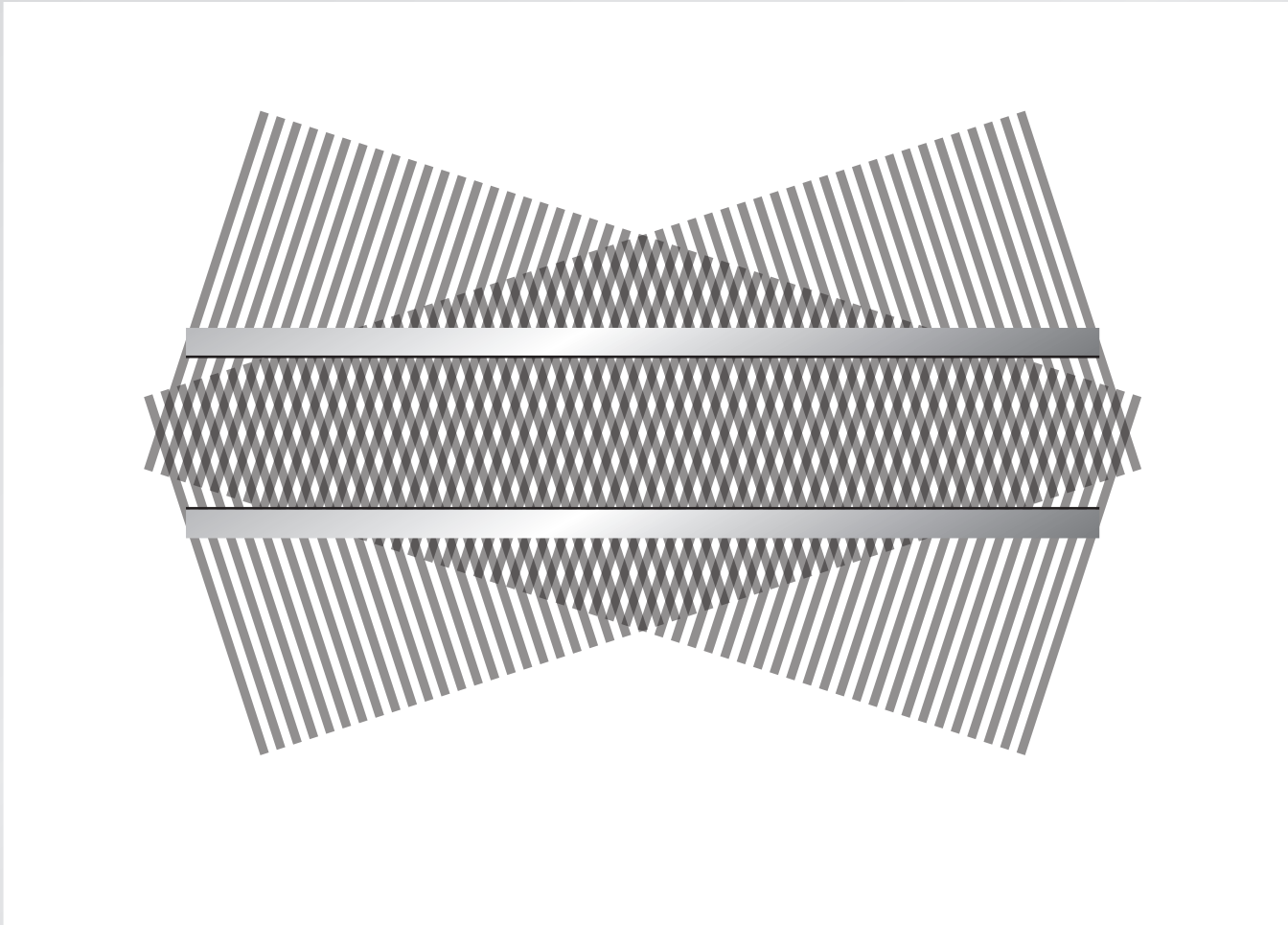
Waveguiding

change angle of incident waves...



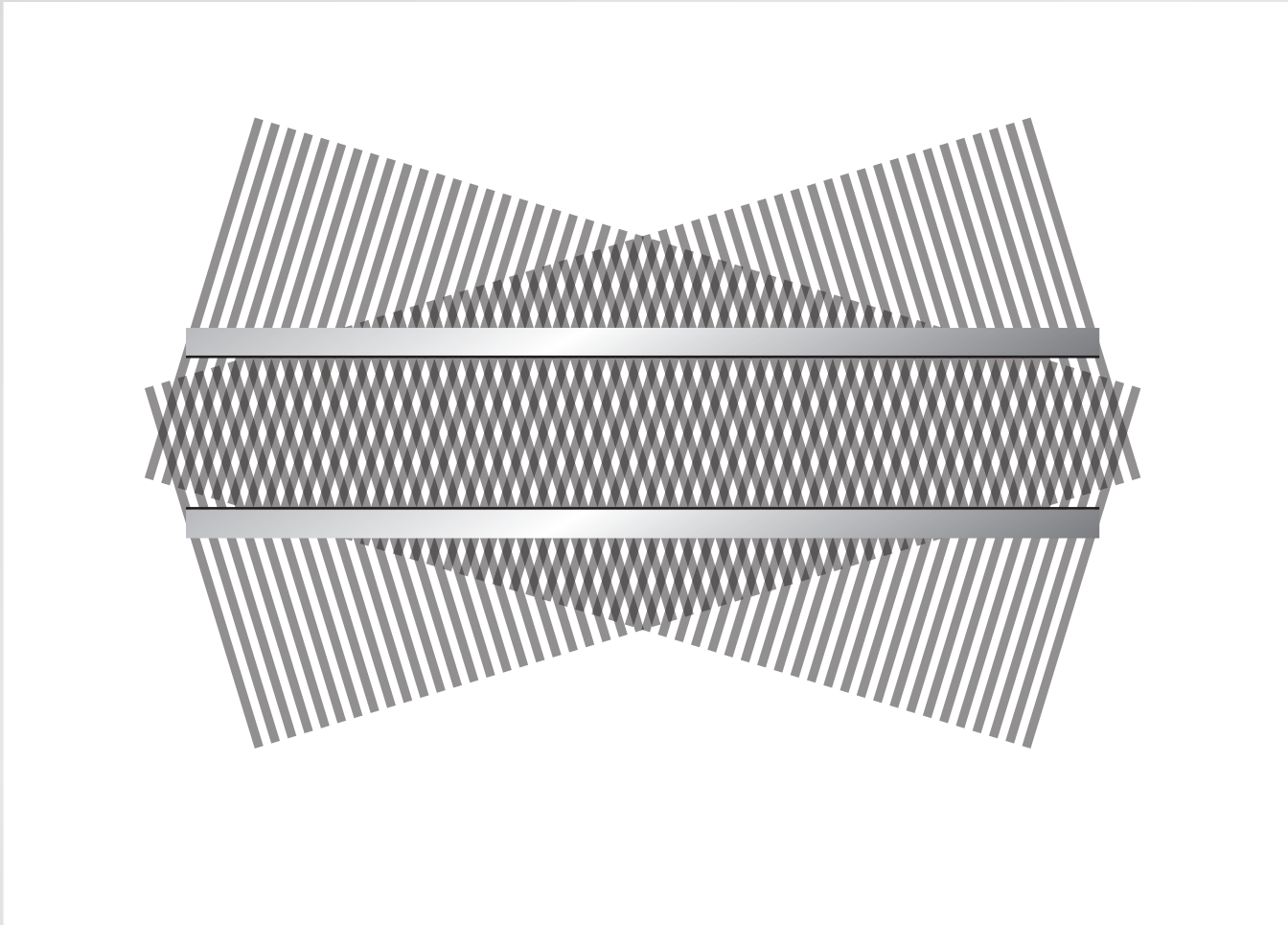
Waveguiding

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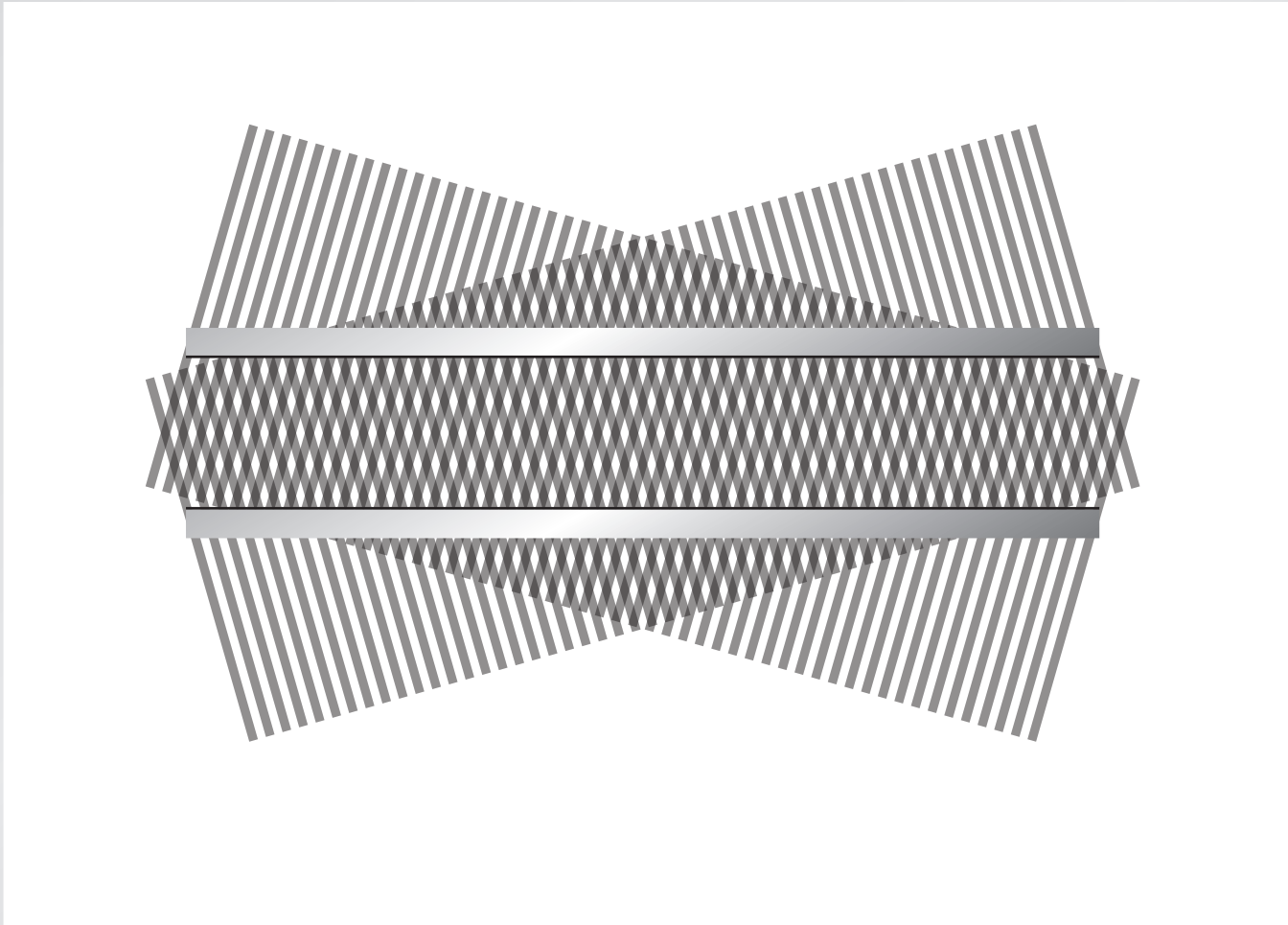
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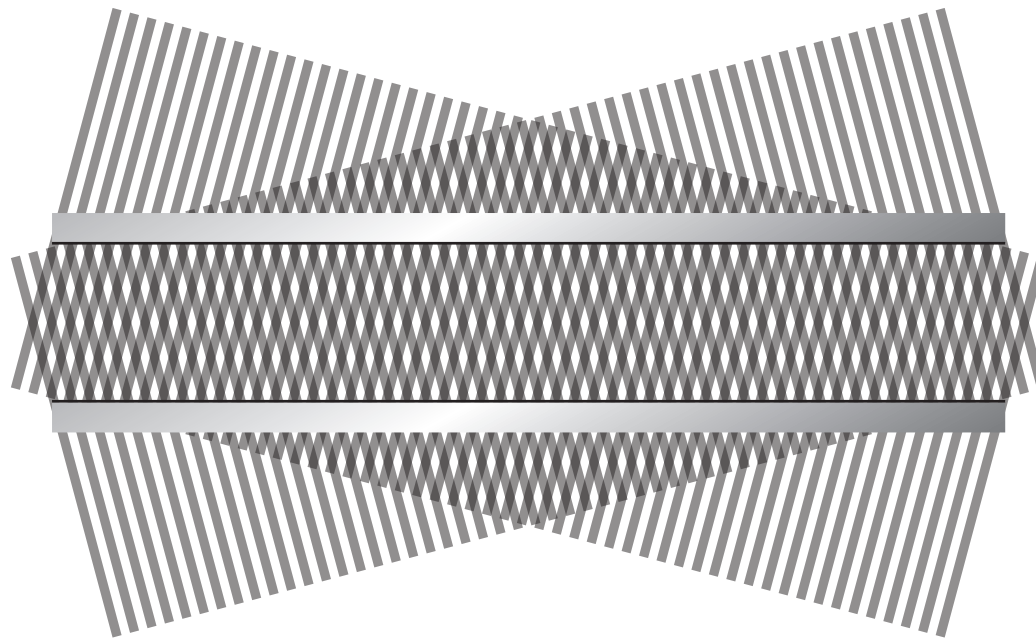
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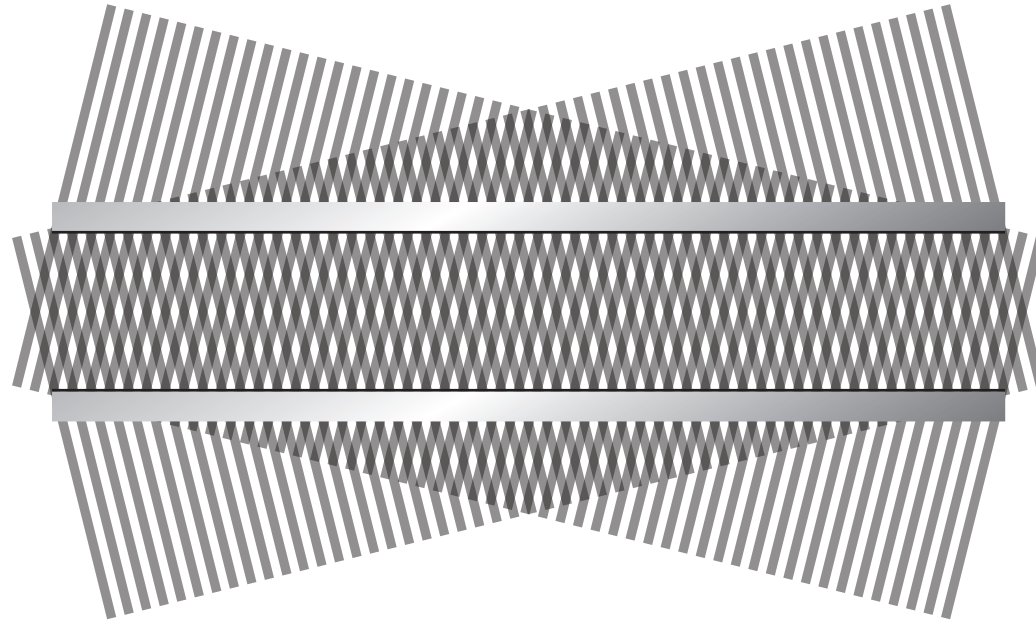
Waveguiding

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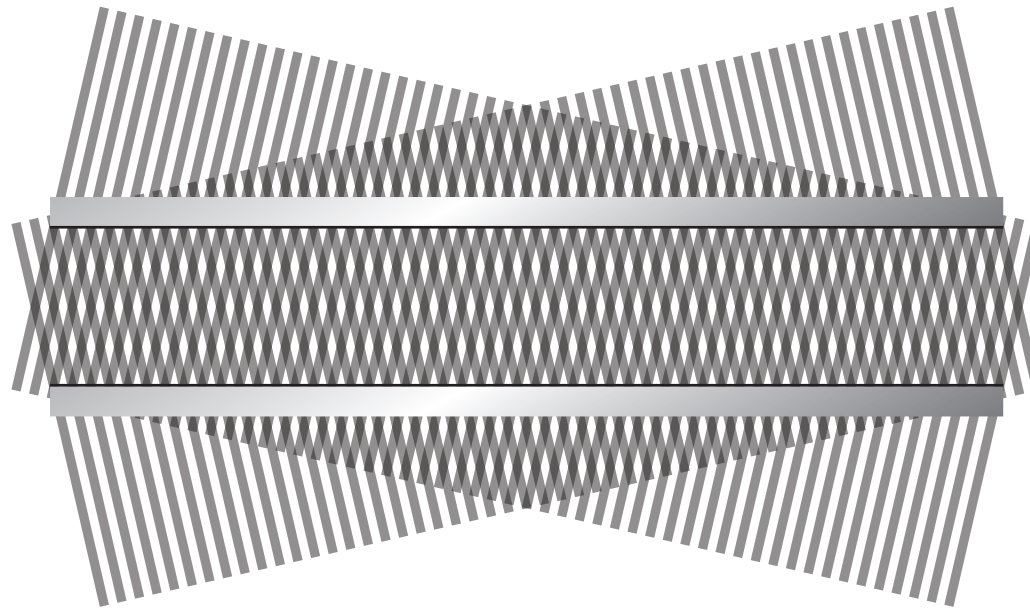
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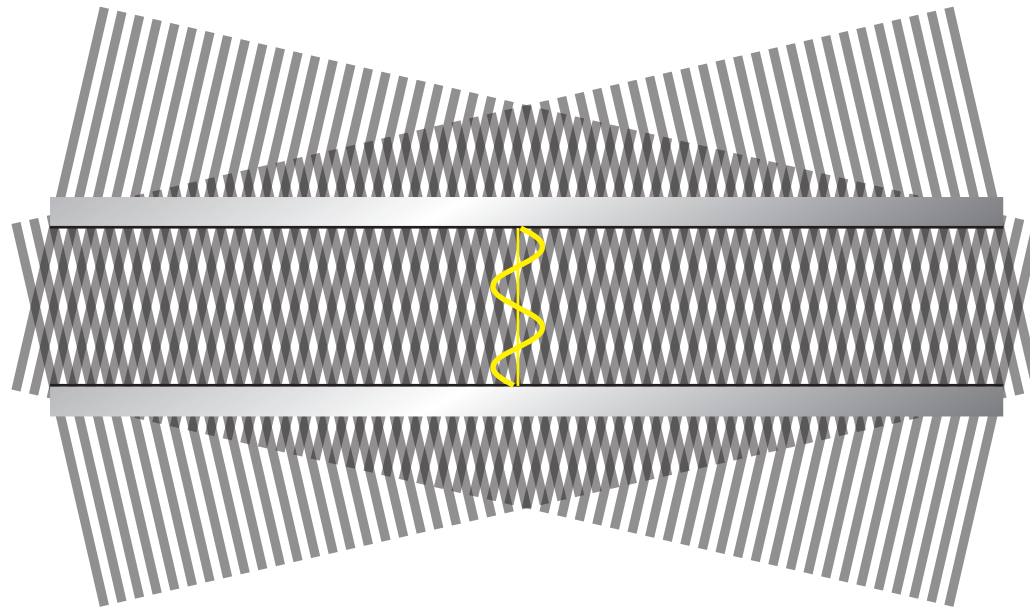
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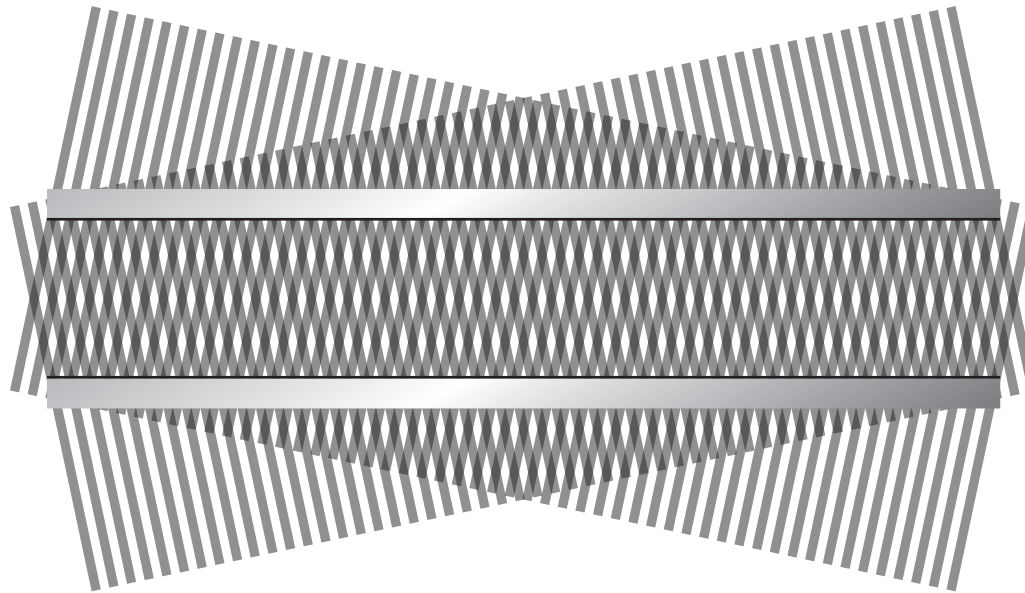
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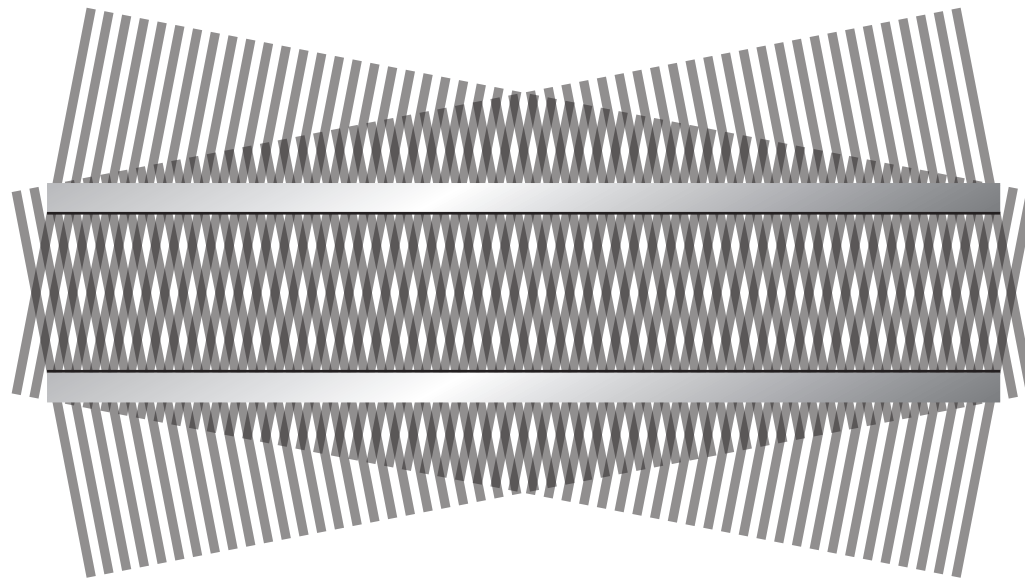
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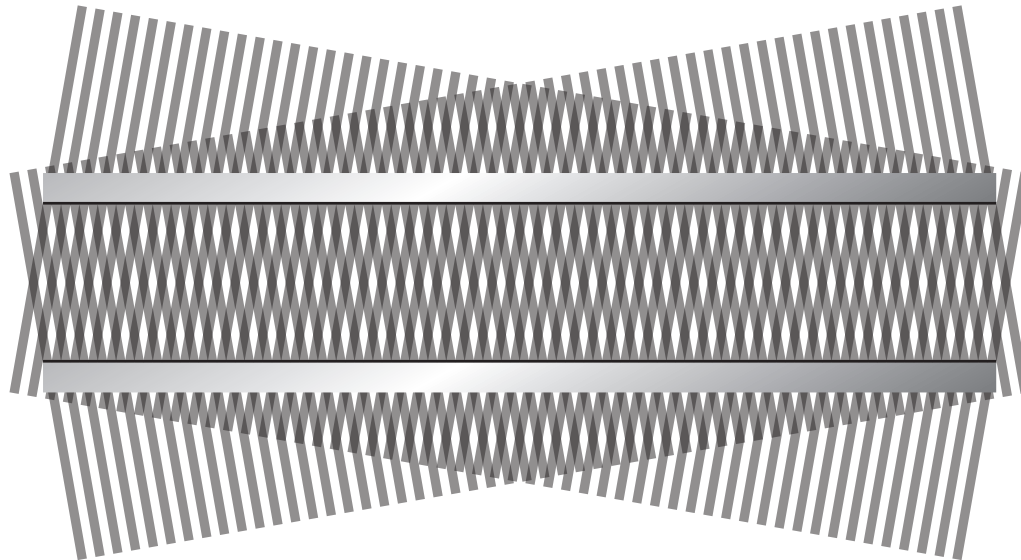
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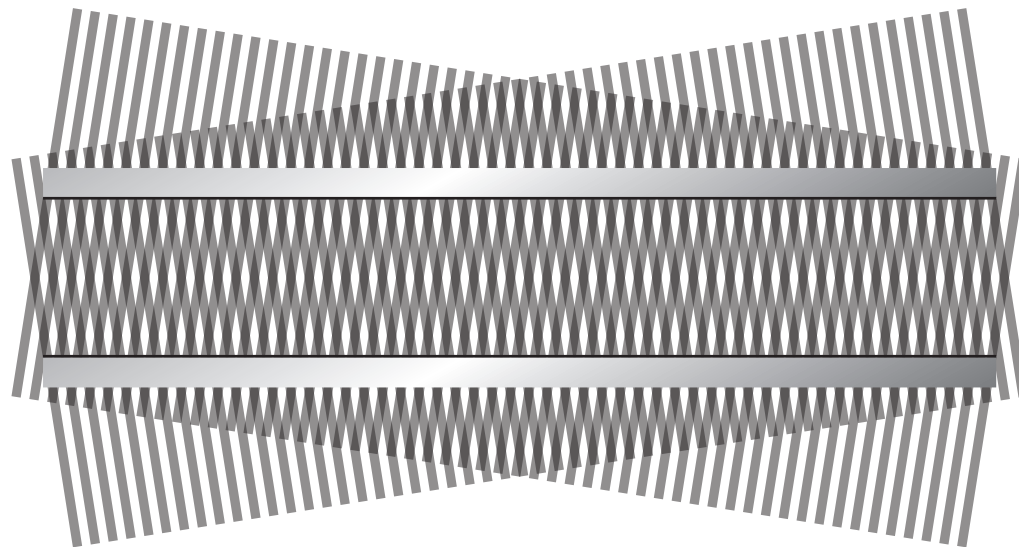
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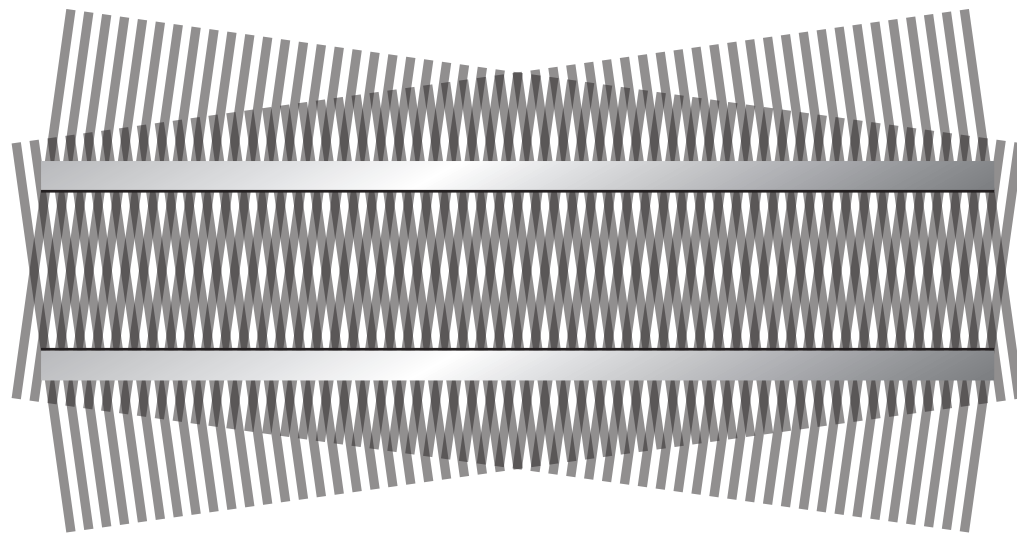
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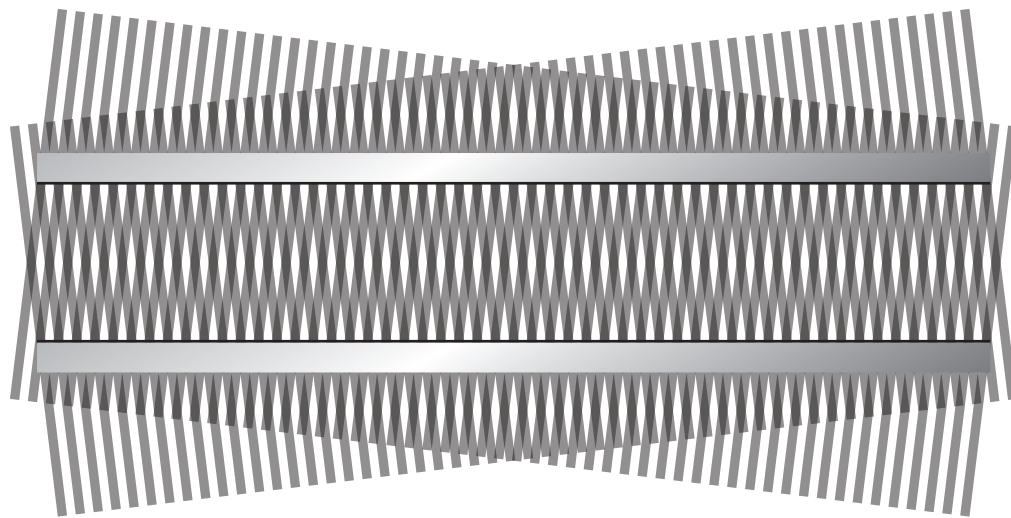
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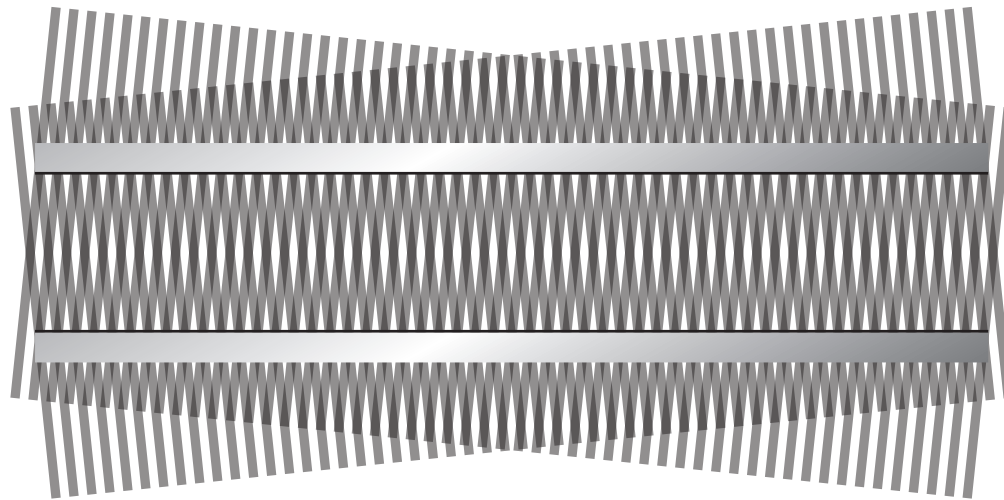
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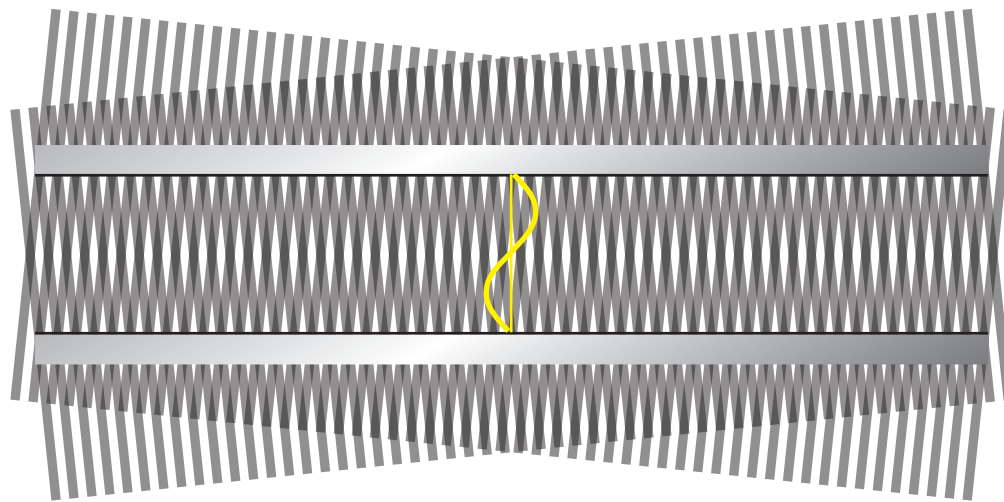
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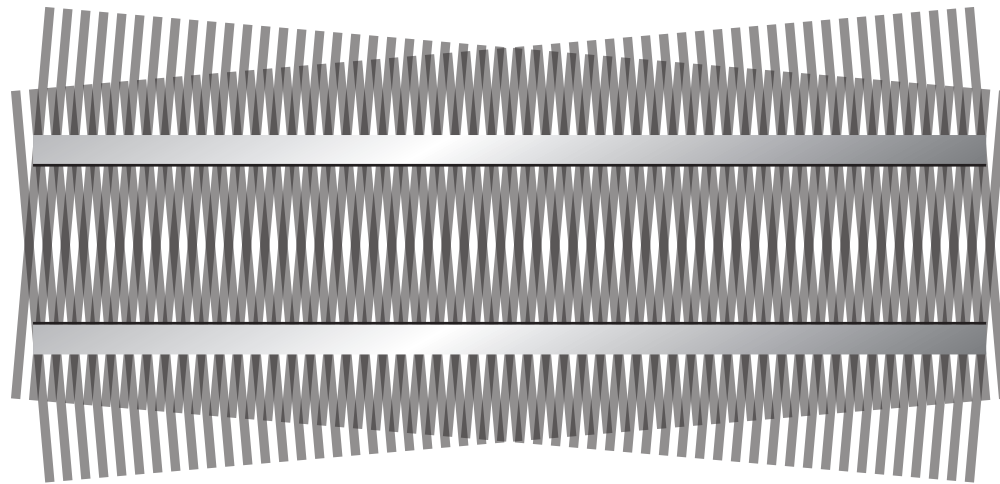
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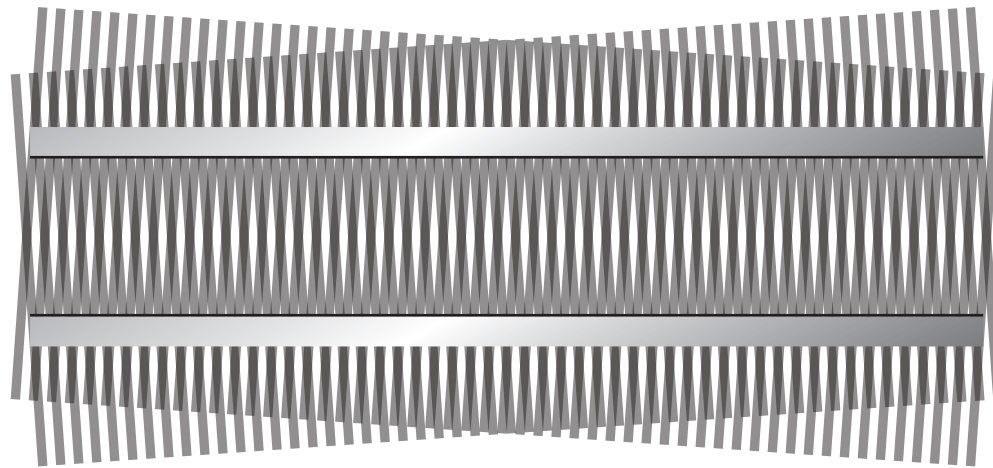
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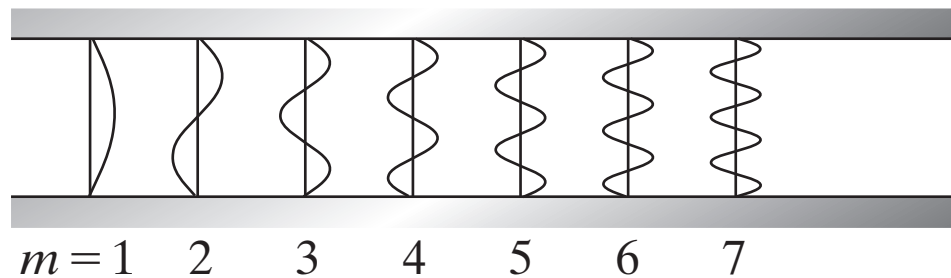
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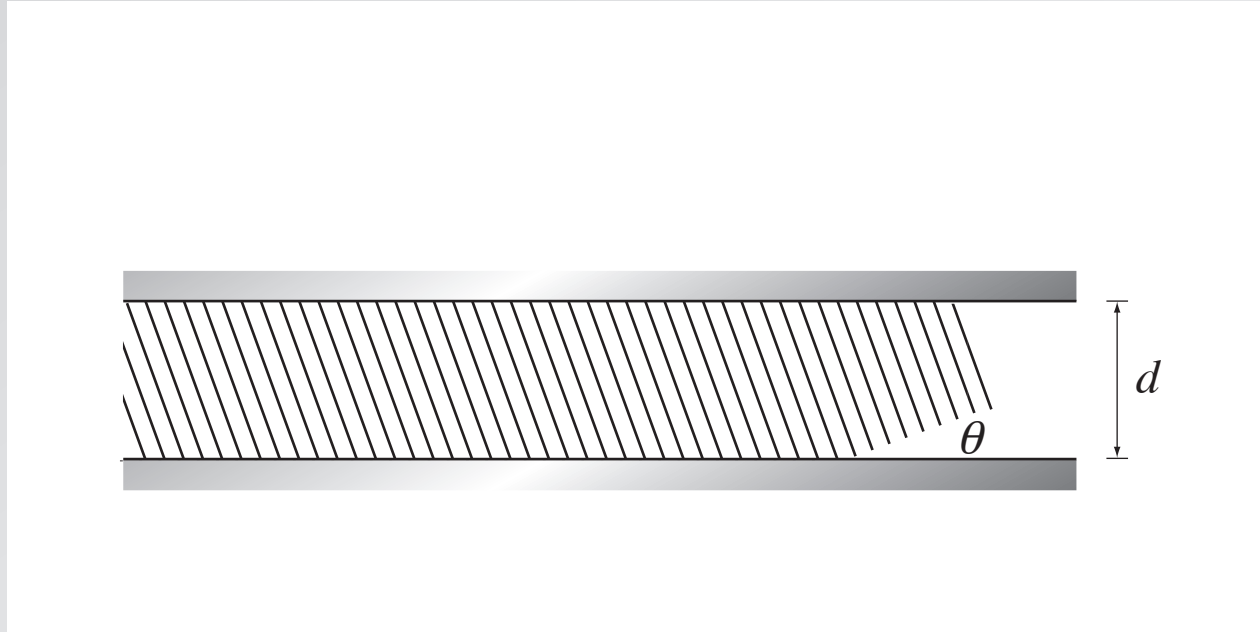
Waveguiding

boundary conditions only satisfied for certain θ



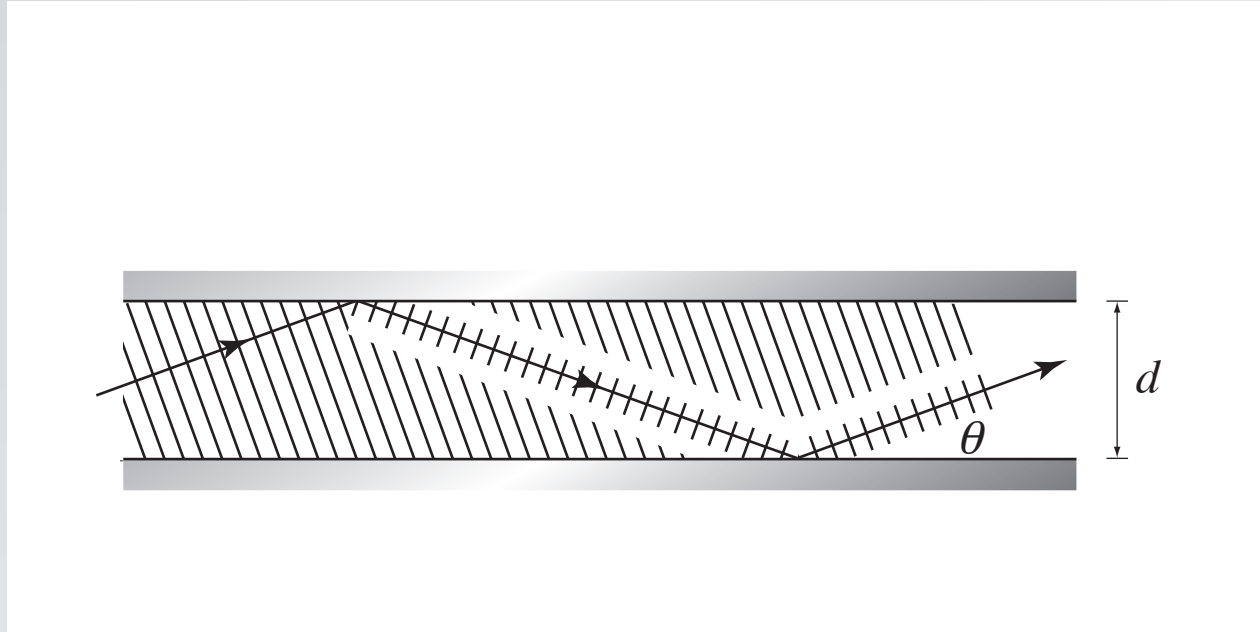
standing wave in y -direction, traveling in z -direction

Waveguiding



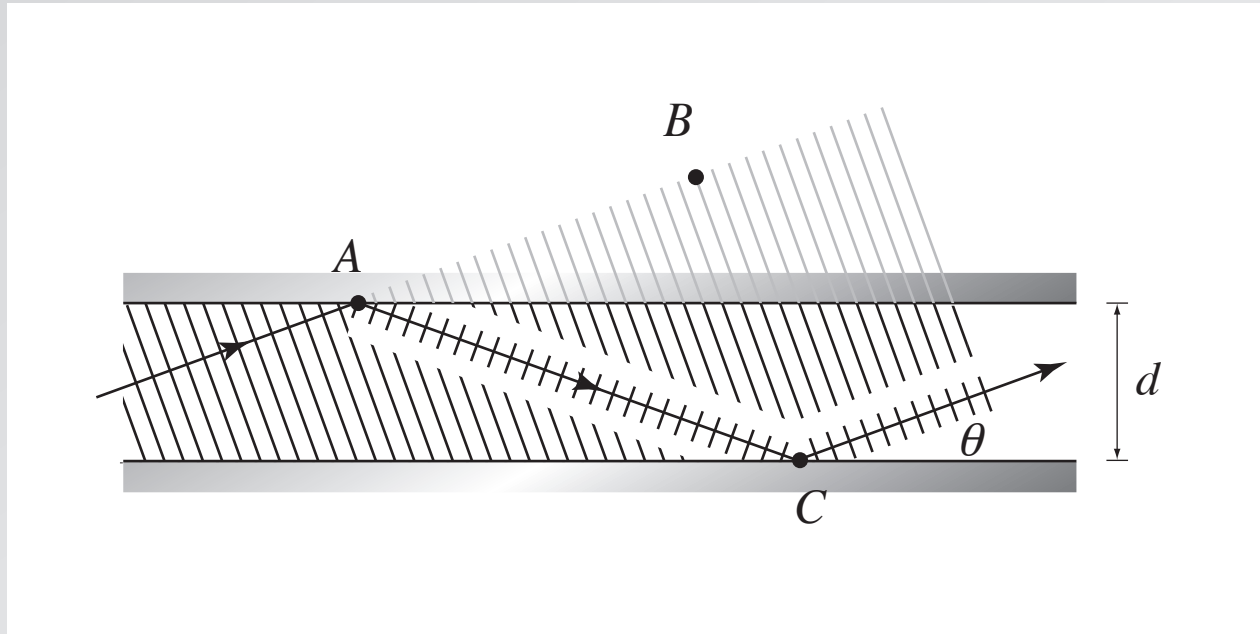
consider wave incident at angle θ

Waveguiding



twice-reflected wave

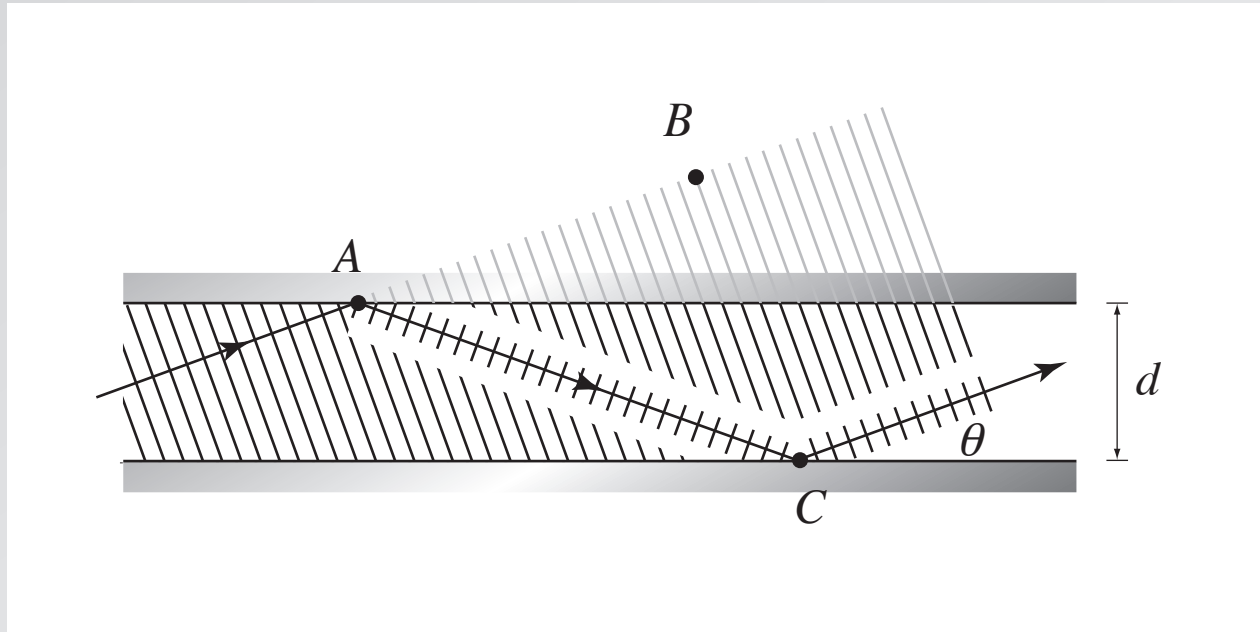
Waveguiding



self consistency:

$$AC - AB = 2d \sin\theta = m\lambda \quad (m = 1, 2, \dots)$$

Waveguiding



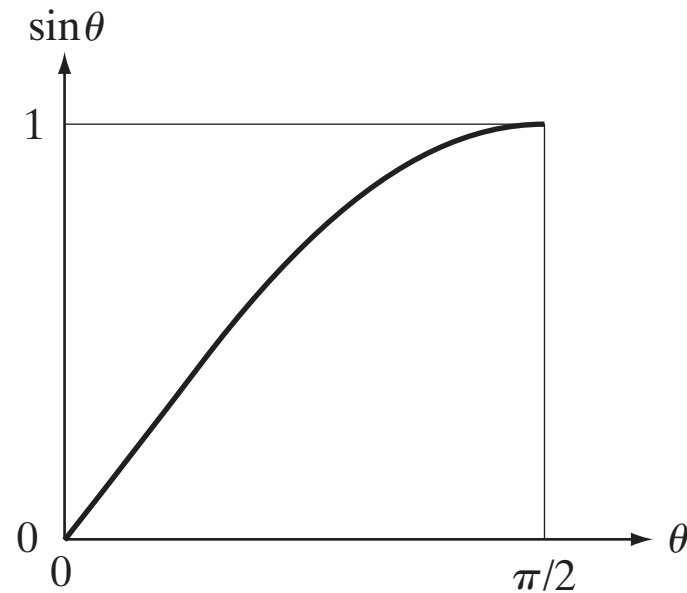
self consistency:

$$AC - AB = 2d \sin\theta = m\lambda \quad (m = 1, 2, \dots)$$

so:

$$\sin\theta_m = m \frac{\lambda}{2d}$$

Waveguiding



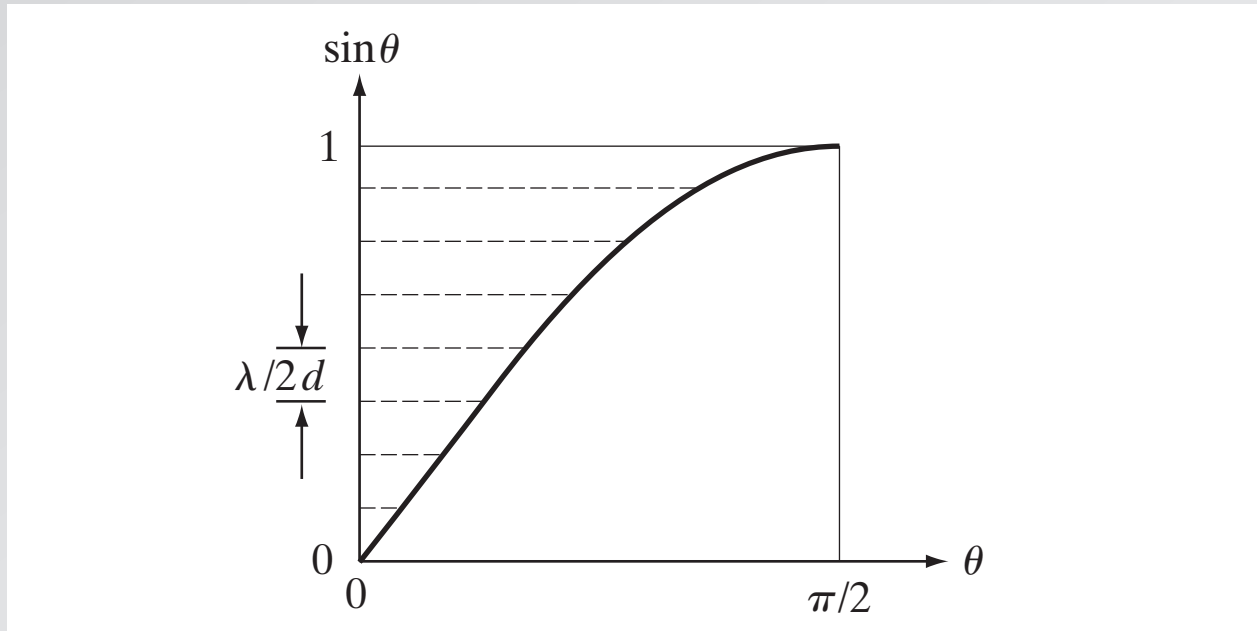
self consistency:

$$AC - AB = 2d \sin \theta = m\lambda \quad (m = 1, 2, \dots)$$

so:

$$\sin \theta_m = m \frac{\lambda}{2d}$$

Waveguiding



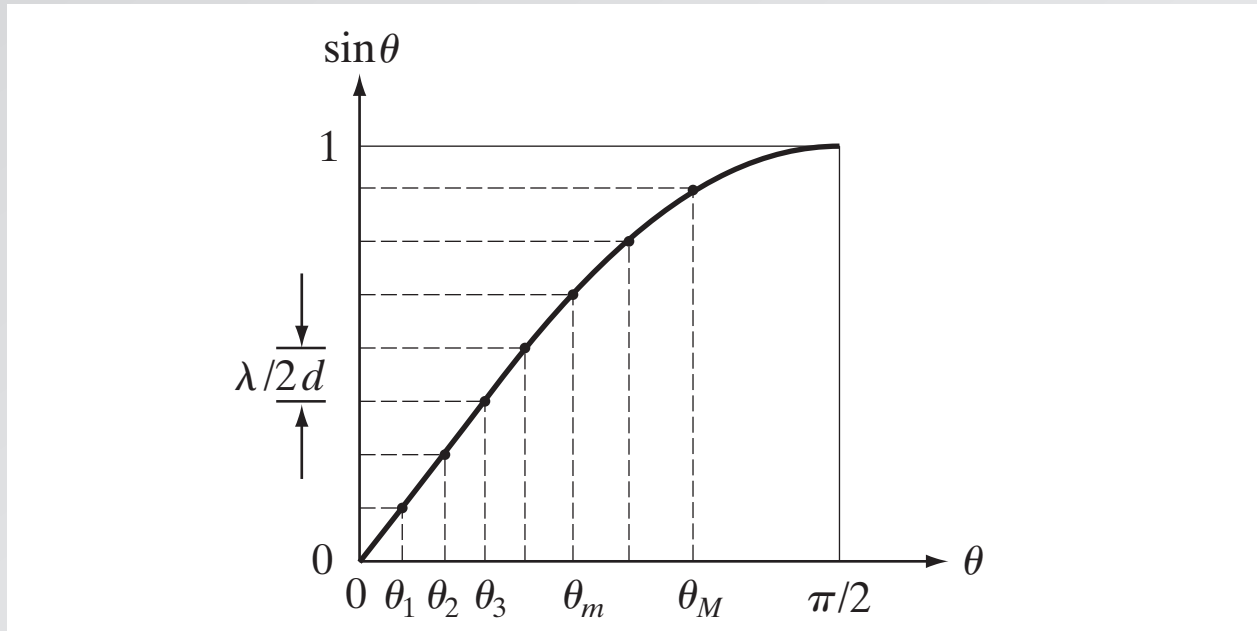
self consistency:

$$AC - AB = 2d \sin \theta = m\lambda \quad (m = 1, 2, \dots)$$

so:

$$\sin \theta_m = m \frac{\lambda}{2d}$$

Waveguiding



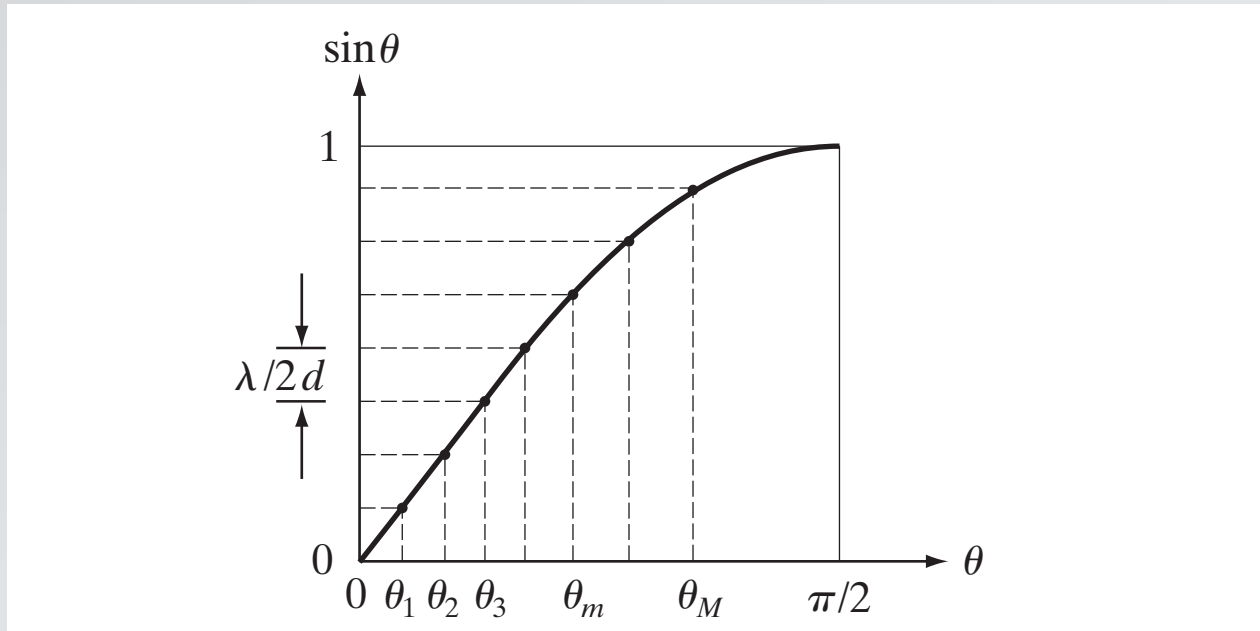
self consistency:

$$AC - AB = 2d \sin \theta = m\lambda \quad (m = 1, 2, \dots)$$

so:

$$\sin \theta_m = m \frac{\lambda}{2d}$$

Waveguiding



number of modes:

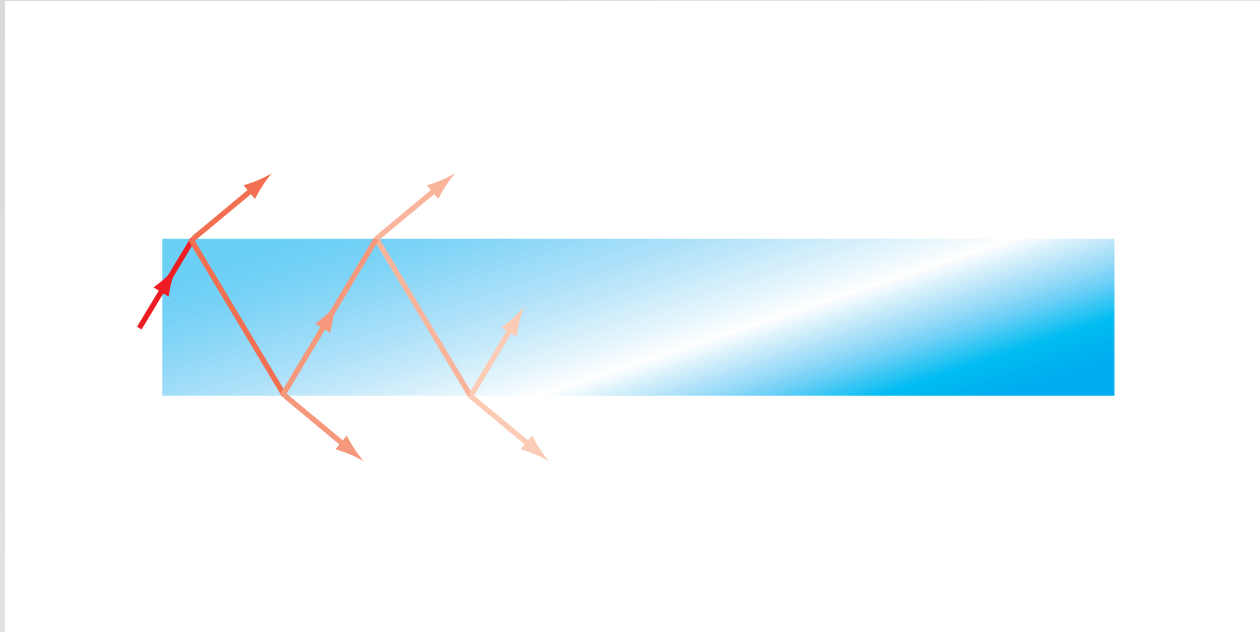
$$M = \frac{2d}{\lambda}$$

Waveguiding



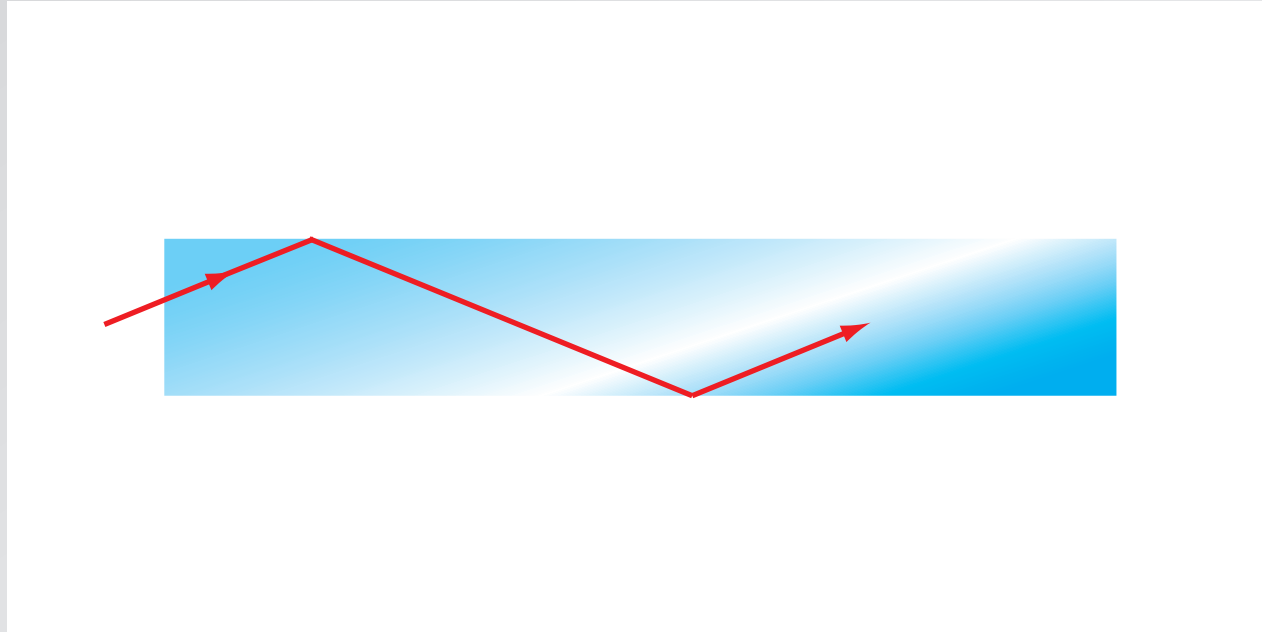
now consider a planar dielectric waveguide

Waveguiding



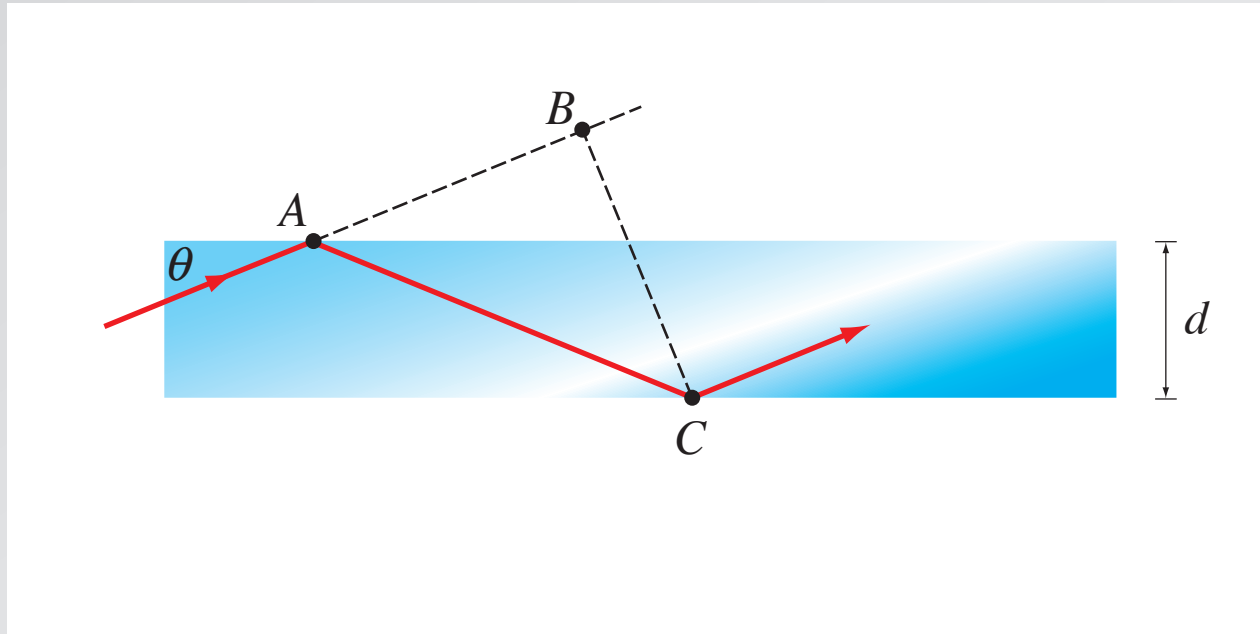
rays incident at angle $\theta > \pi/2 - \theta_c$ are unguided

Waveguiding



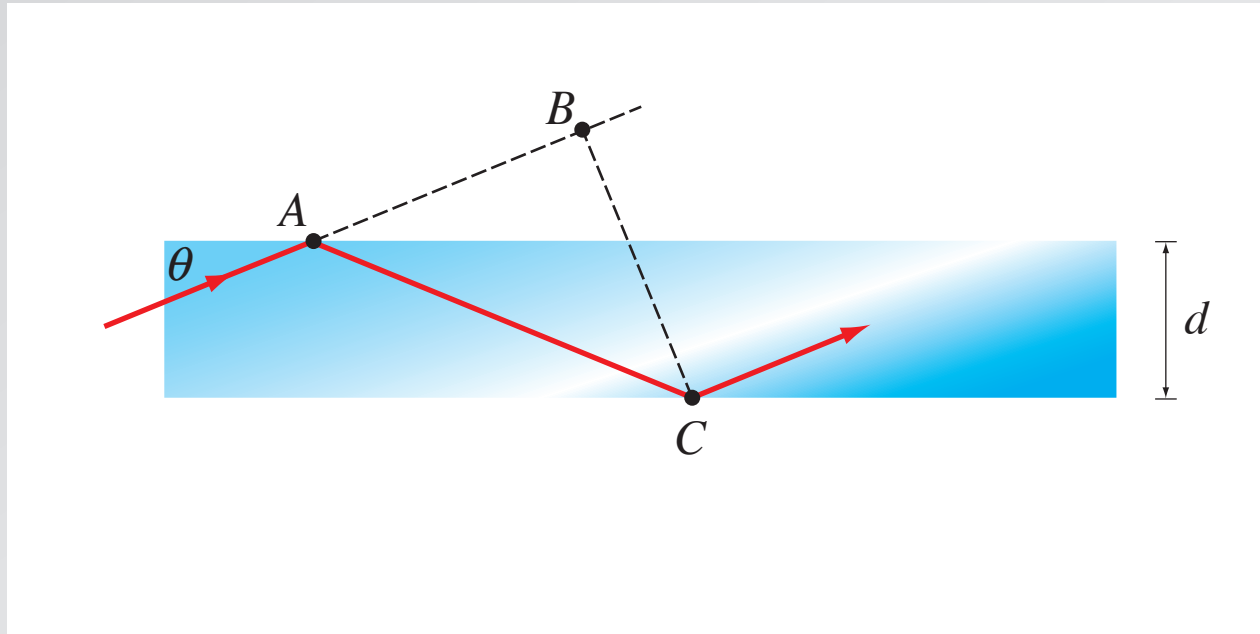
rays incident at angle $\theta < \pi/2 - \theta_c$ are guided

Waveguiding



rays incident at angle $\theta < \pi/2 - \theta_c$ are guided

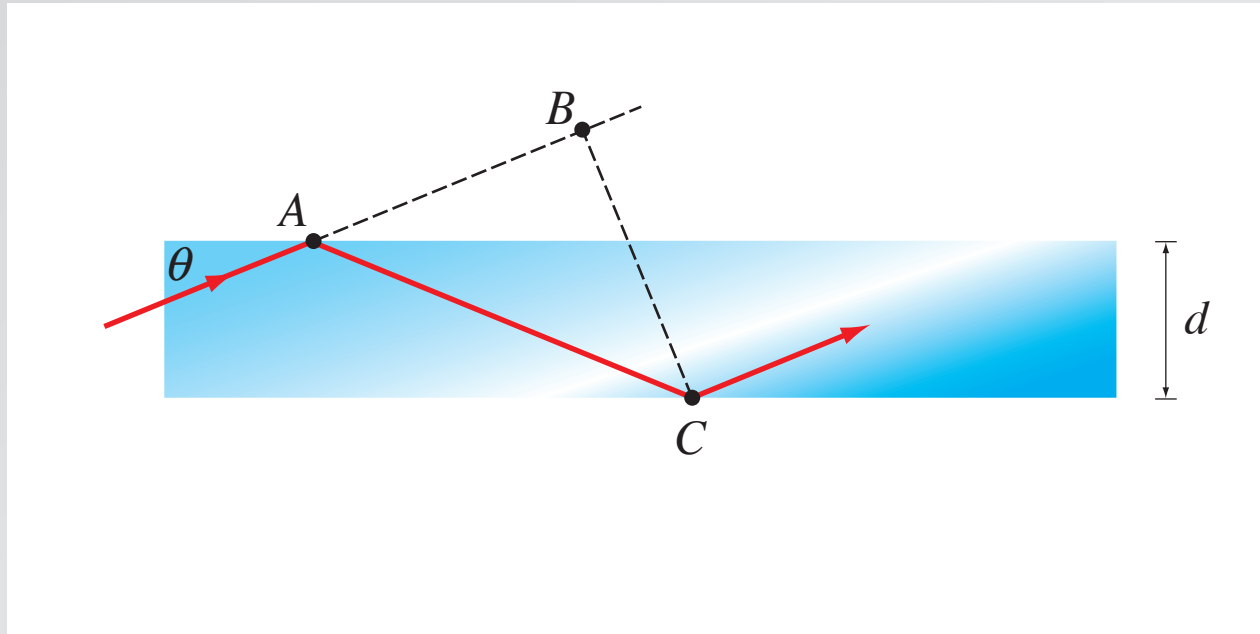
Waveguiding



self consistency:

$$AC - AB = 2d \sin\theta - \frac{\varphi_r}{\pi} \lambda = m\lambda \quad (m = 0, 1, 2\dots)$$

Waveguiding



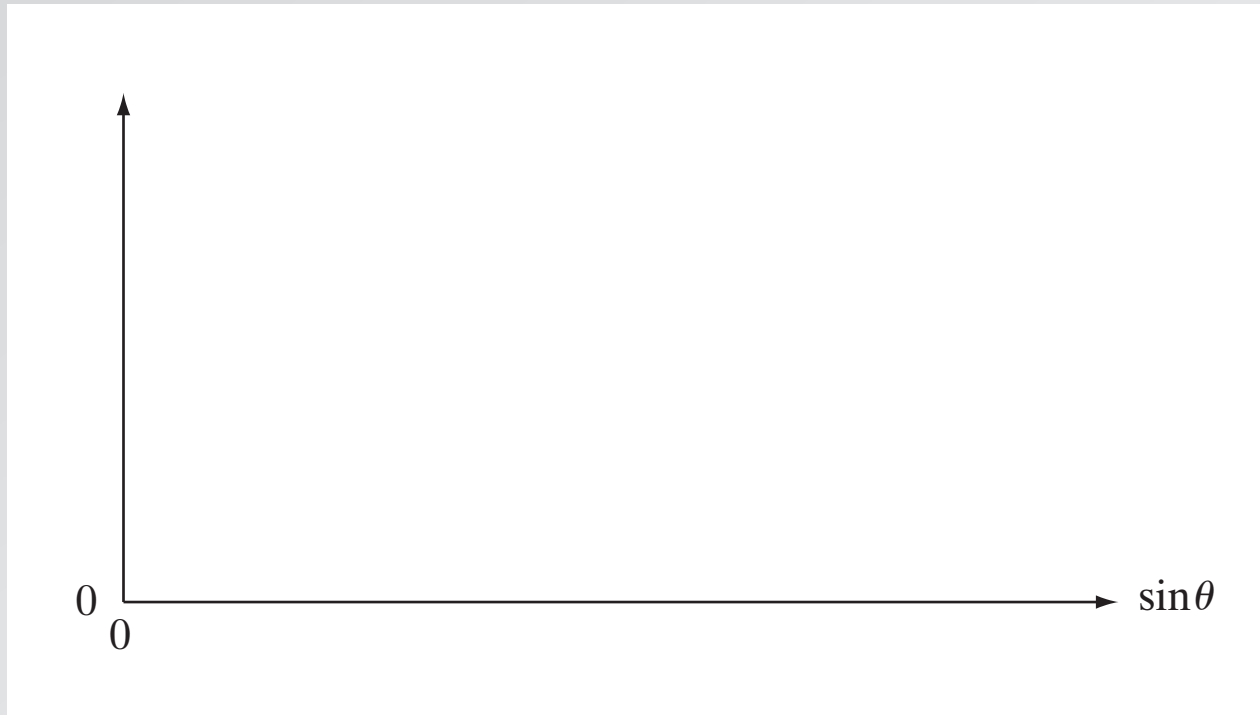
self consistency:

$$AC - AB = 2d \sin\theta - \frac{\varphi_r}{\pi} \lambda = m\lambda \quad (m = 0, 1, 2\dots)$$

SO:

$$\tan\left(\frac{\pi d}{\lambda} \sin\theta - m\frac{\pi}{2}\right) = \left(\frac{\sin^2(\pi/2 - \theta_c)}{\sin^2\theta} - 1\right)^{1/2}$$

Waveguiding



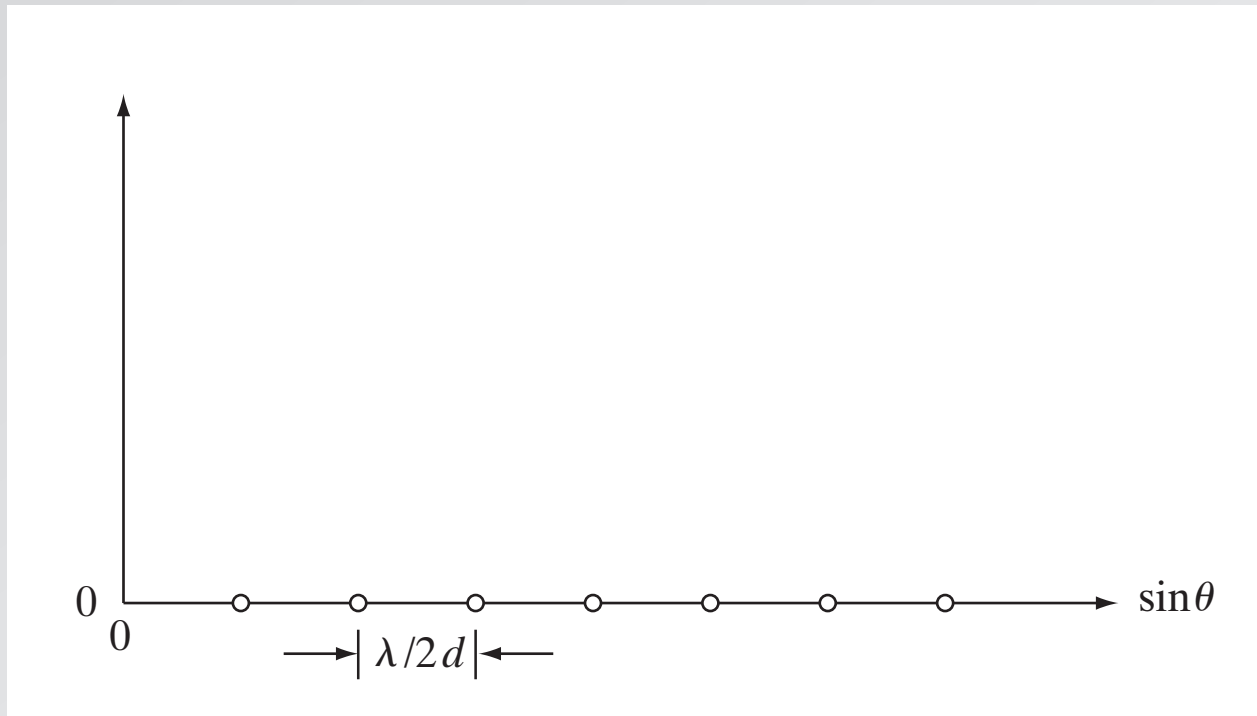
self consistency:

$$AC - AB = 2d \sin \theta - \frac{\varphi_r}{\pi} \lambda = m \lambda \quad (m = 0, 1, 2 \dots)$$

SO:

$$\tan \left(\frac{\pi d}{\lambda} \sin \theta - m \frac{\pi}{2} \right) = \left(\frac{\sin^2(\pi/2 - \theta_c)}{\sin^2 \theta} - 1 \right)^{1/2}$$

Waveguiding



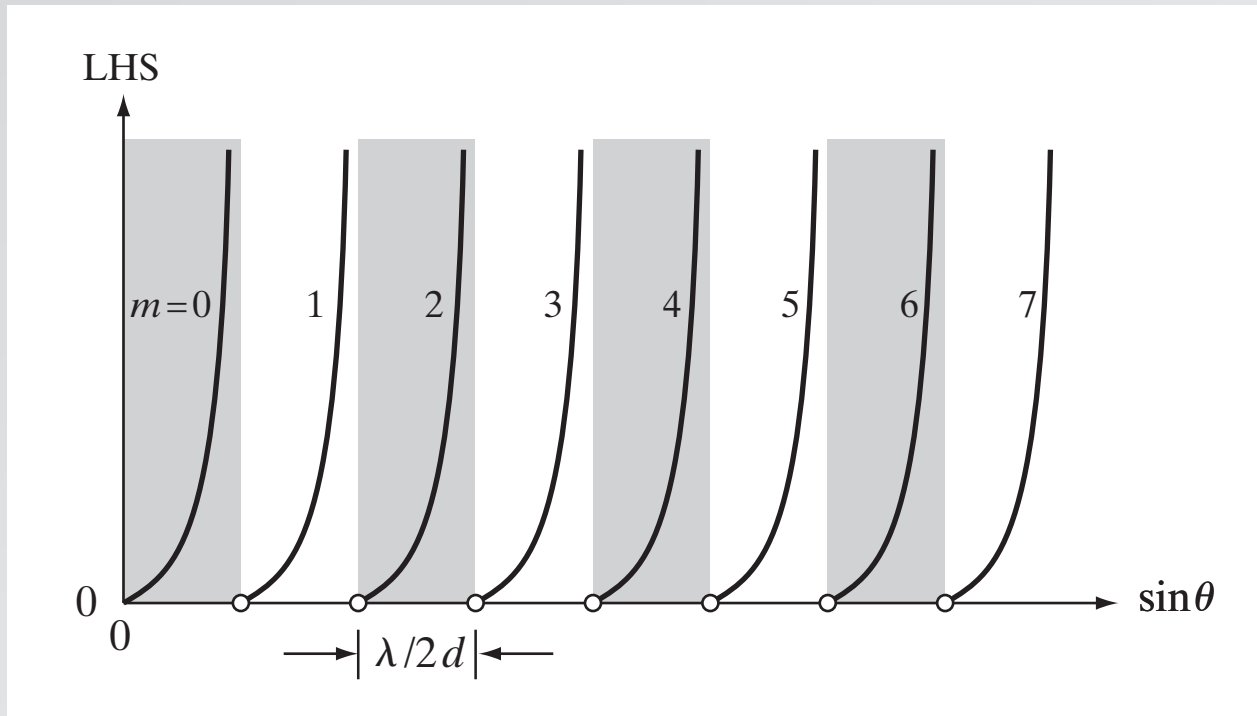
self consistency:

$$AC - AB = 2d \sin \theta - \frac{\varphi_r}{\pi} \lambda = m \lambda \quad (m = 0, 1, 2 \dots)$$

SO:

$$\tan \left(\frac{\pi d}{\lambda} \sin \theta - m \frac{\pi}{2} \right) = \left(\frac{\sin^2(\pi/2 - \theta_c)}{\sin^2 \theta} - 1 \right)^{1/2}$$

Waveguiding



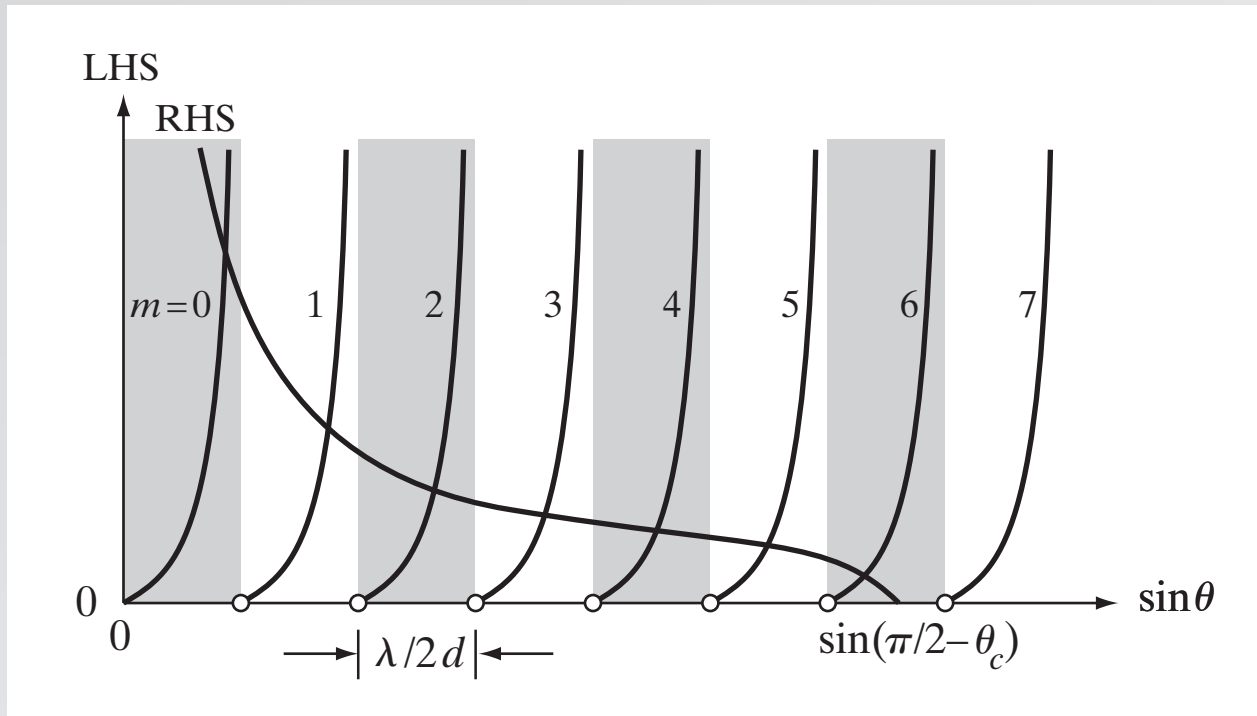
self consistency:

$$AC - AB = 2d \sin \theta - \frac{\varphi_r}{\pi} \lambda = m\lambda \quad (m = 0, 1, 2\dots)$$

SO:

$$\tan \left(\frac{\pi d}{\lambda} \sin \theta - m \frac{\pi}{2} \right) = \left(\frac{\sin^2(\pi/2 - \theta_c)}{\sin^2 \theta} - 1 \right)^{1/2}$$

Waveguiding



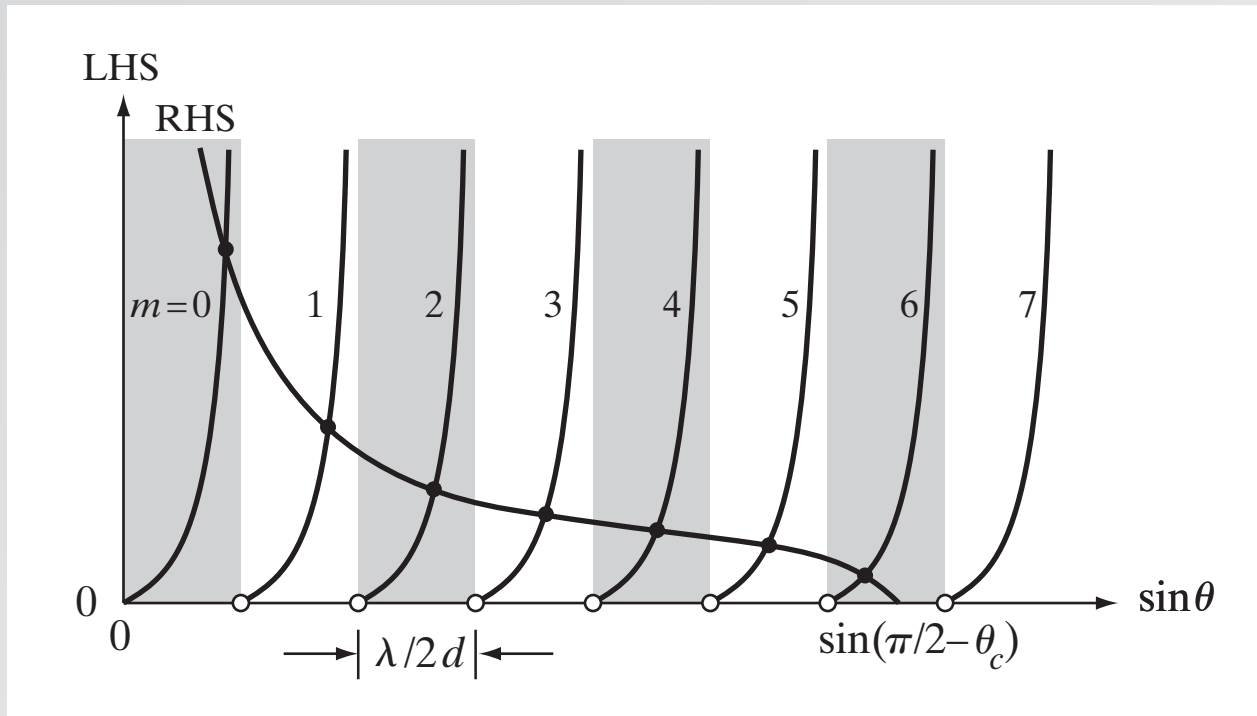
self consistency:

$$AC - AB = 2d \sin \theta - \frac{\varphi_r}{\pi} \lambda = m \lambda \quad (m = 0, 1, 2 \dots)$$

SO:

$$\tan \left(\frac{\pi d}{\lambda} \sin \theta - m \frac{\pi}{2} \right) = \left(\frac{\sin^2(\pi/2 - \theta_c)}{\sin^2 \theta} - 1 \right)^{1/2}$$

Waveguiding



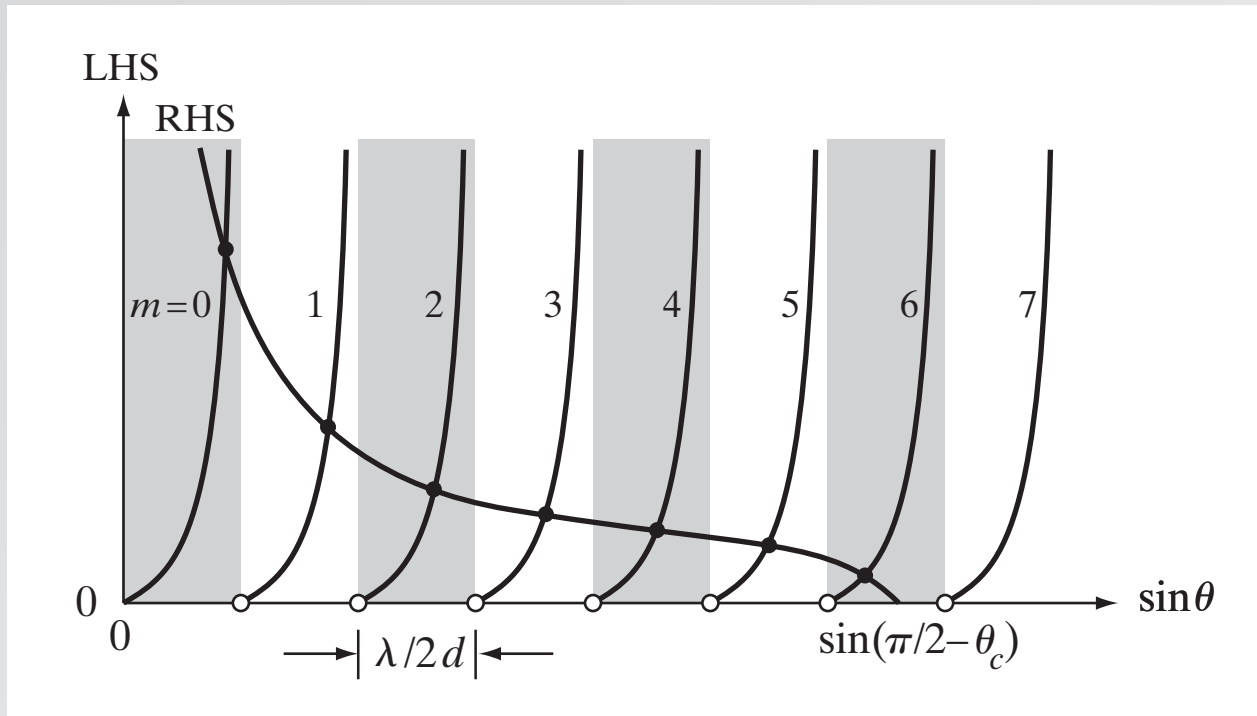
self consistency:

$$AC - AB = 2d \sin \theta - \frac{\varphi_r}{\pi} \lambda = m\lambda \quad (m = 0, 1, 2\dots)$$

SO:

$$\tan\left(\frac{\pi d}{\lambda} \sin \theta - m \frac{\pi}{2}\right) = \left(\frac{\sin^2(\pi/2 - \theta_c)}{\sin^2 \theta} - 1\right)^{1/2}$$

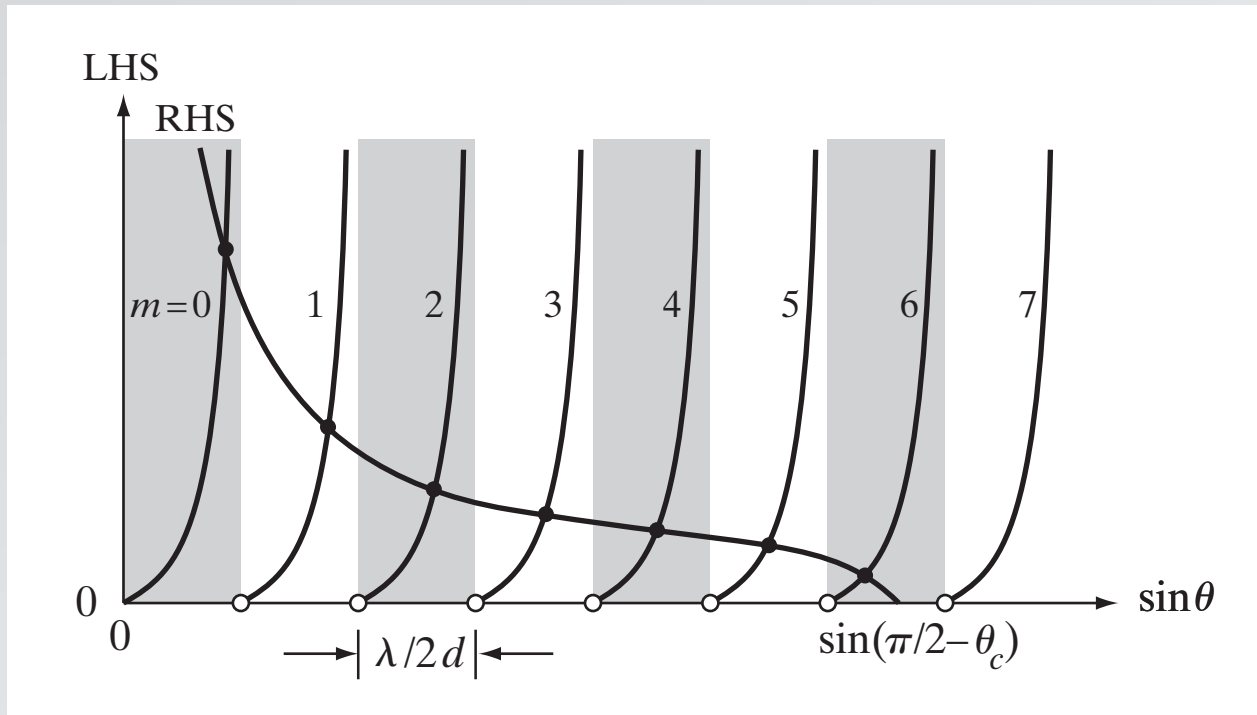
Waveguiding



number of modes:

$$M \doteq \frac{\sin(\pi/2 - \theta_c)}{\lambda/2d}$$

Waveguiding



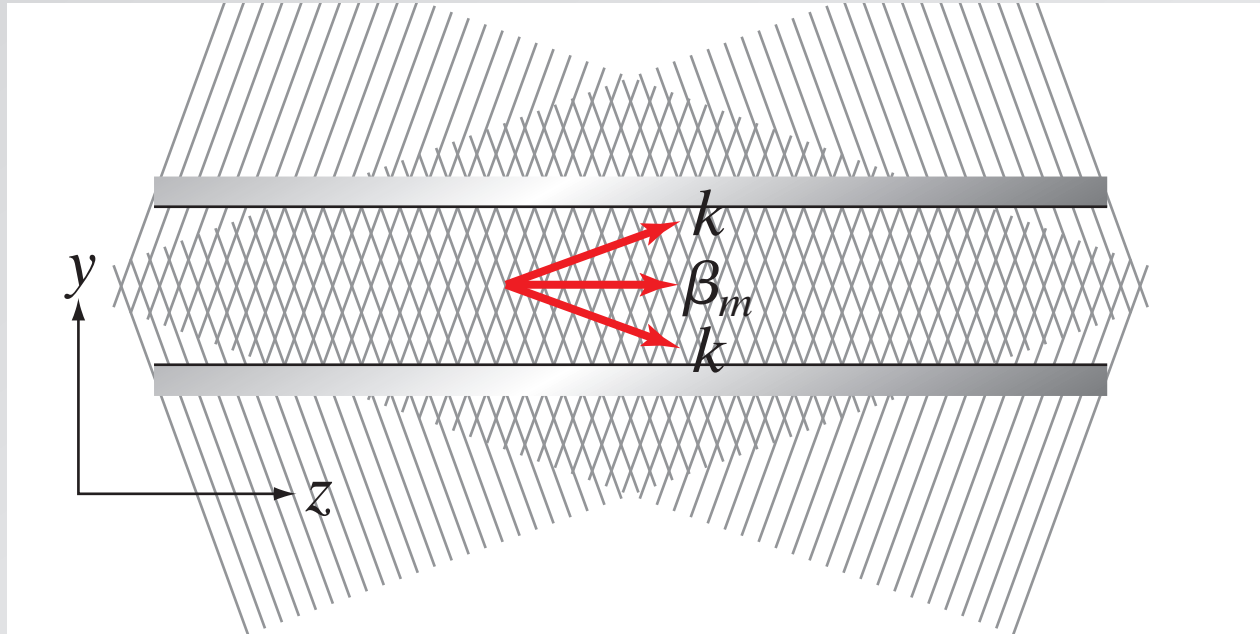
number of modes:

$$M \doteq \frac{\sin(\pi/2 - \theta_c)}{\lambda/2d}$$

or:

$$M \doteq 2 \frac{d}{\lambda} (n_1^2 - n_2^2)^{1/2}$$

Waveguiding



propagation constant of guided wave:

$$\beta_m^2 = k^2 - k_y^2 = k^2 - \frac{m^2 \pi^2}{d^2}$$

group velocity:

$$v_m = c \cos \theta_m$$

Waveguiding

single mode condition for 600-nm light:

planar mirror

$$M \doteq \frac{2d}{\lambda}$$

$$300 < d < 600 \text{ nm}$$

dielectric

$$M \doteq 2 \frac{d}{\lambda} (n_1^2 - n_2^2)^{1/2}$$

$$d < 268 \text{ nm}$$

Waveguiding

single mode condition for 600-nm light:

planar mirror

$$M \doteq \frac{2d}{\lambda}$$

$$300 < d < 600 \text{ nm}$$

dielectric

$$M \doteq 2 \frac{d}{\lambda} (n_1^2 - n_2^2)^{1/2}$$

$$d < 268 \text{ nm}$$

can make d larger by making $n_1 - n_2$ smaller!

Waveguiding

Vector potential obeys:

$$\nabla^2 \vec{A} + \omega^2 \mu_o \epsilon \vec{A} = -i\omega \mu_o \nabla \epsilon \Phi$$

Waveguiding

Vector potential obeys:

$$\nabla^2 \vec{A} + \omega^2 \mu_o \epsilon \vec{A} = 0$$

Waveguiding

Vector potential obeys:

$$\nabla^2 \vec{A} + \omega^2 \mu_o \epsilon \vec{A} = 0$$

Substituting

$$\vec{A} = \hat{y} u(x,y) e^{-i\beta z}$$

Waveguiding

Vector potential obeys:

$$\nabla^2 \vec{A} + \omega^2 \mu_o \epsilon \vec{A} = 0$$

Substituting

$$\vec{A} = \hat{y} u(x,y) e^{-i\beta z}$$

yields:

$$\nabla_T^2 u + [-\beta^2 + \omega^2 \mu \epsilon(r)] u = 0$$

Waveguiding

Vector potential obeys:

$$\nabla^2 \vec{A} + \omega^2 \mu_o \epsilon \vec{A} = 0$$

Substituting

$$\vec{A} = \hat{y} u(x,y) e^{-i\beta z}$$

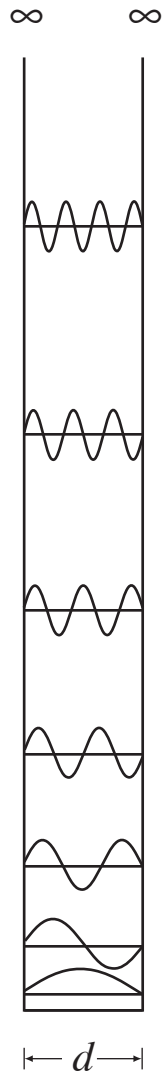
yields:

$$\nabla_T^2 u + [-\beta^2 + \omega^2 \mu \epsilon(r)] u = 0$$

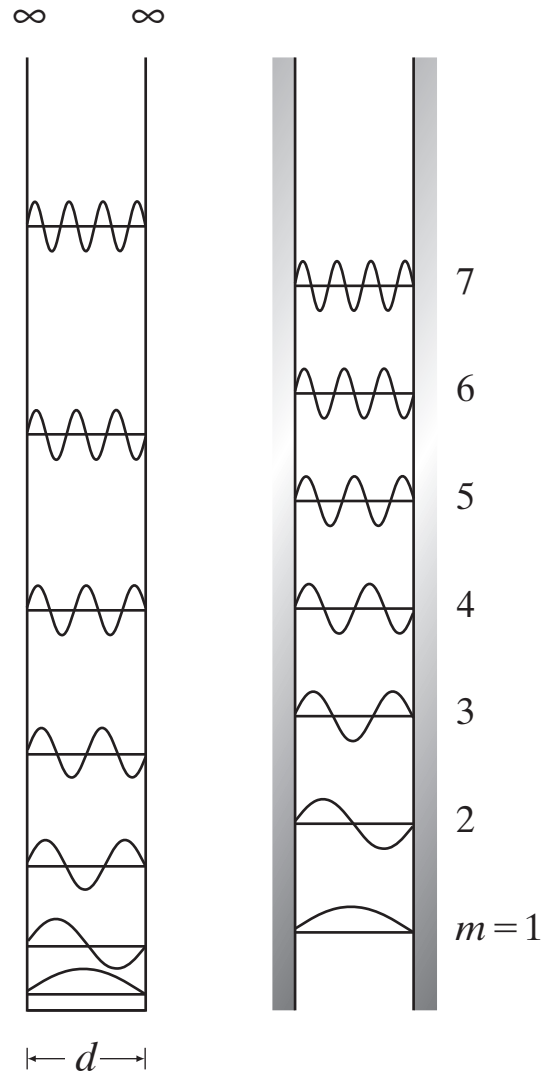
Compare to time-independent Schrödinger equation:

$$\nabla^2 \psi + \frac{2m}{\hbar^2} [E - V(r)] \psi = 0$$

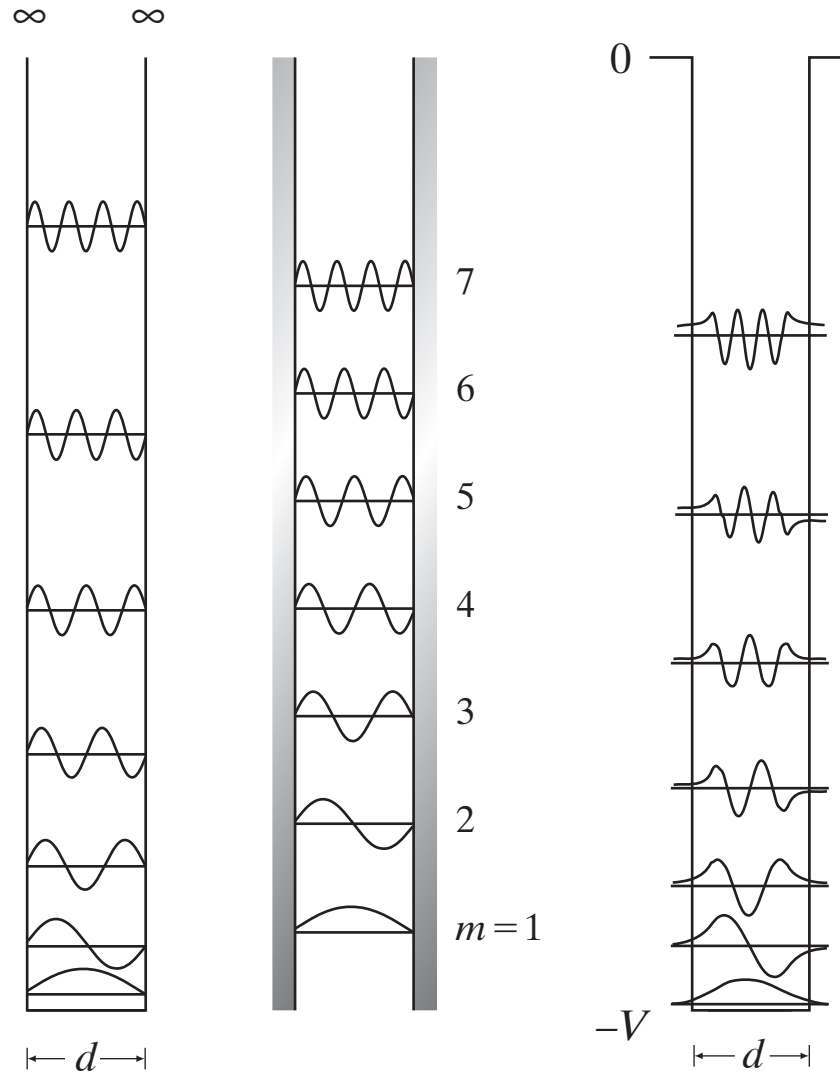
Waveguiding



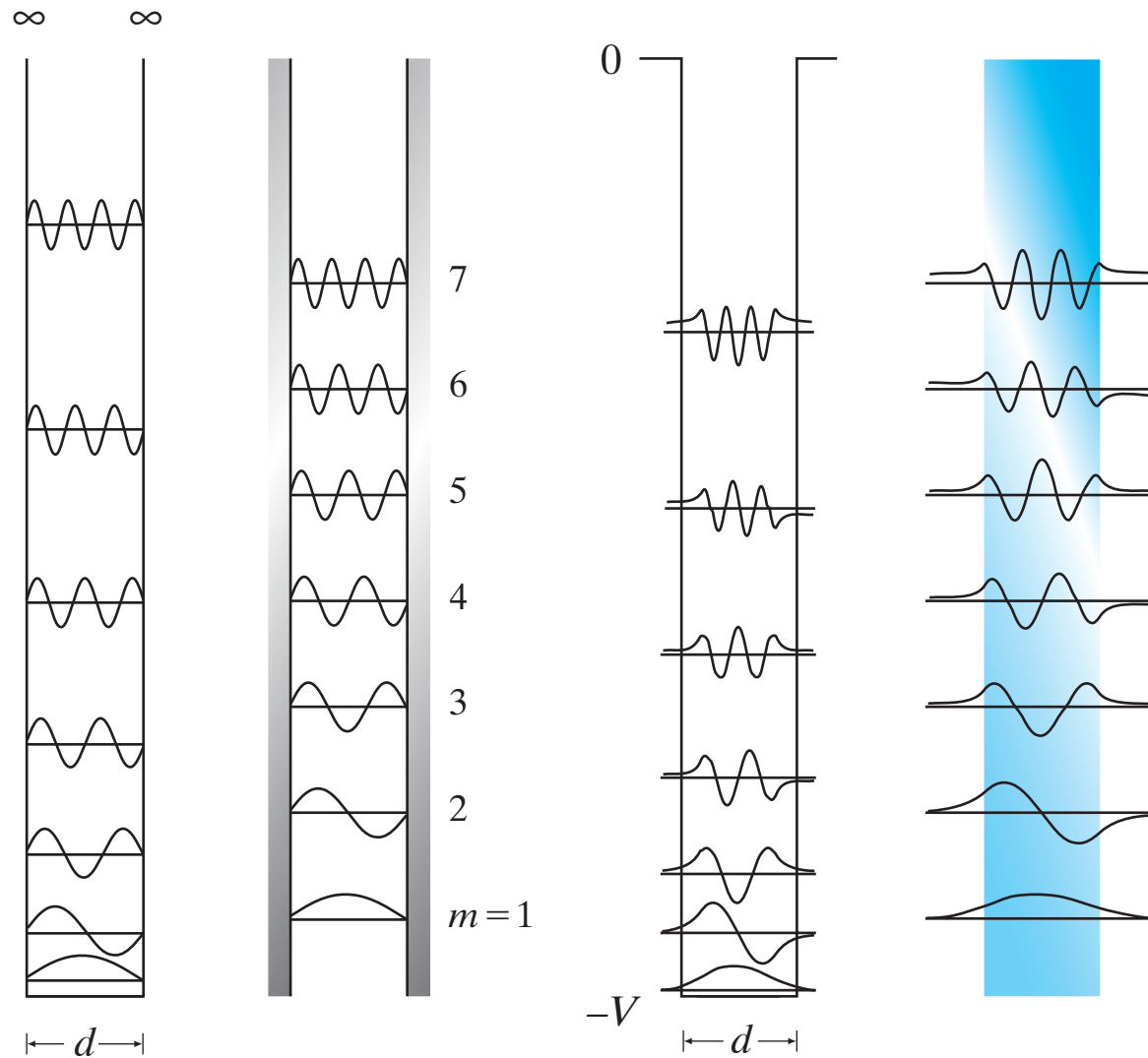
Waveguiding



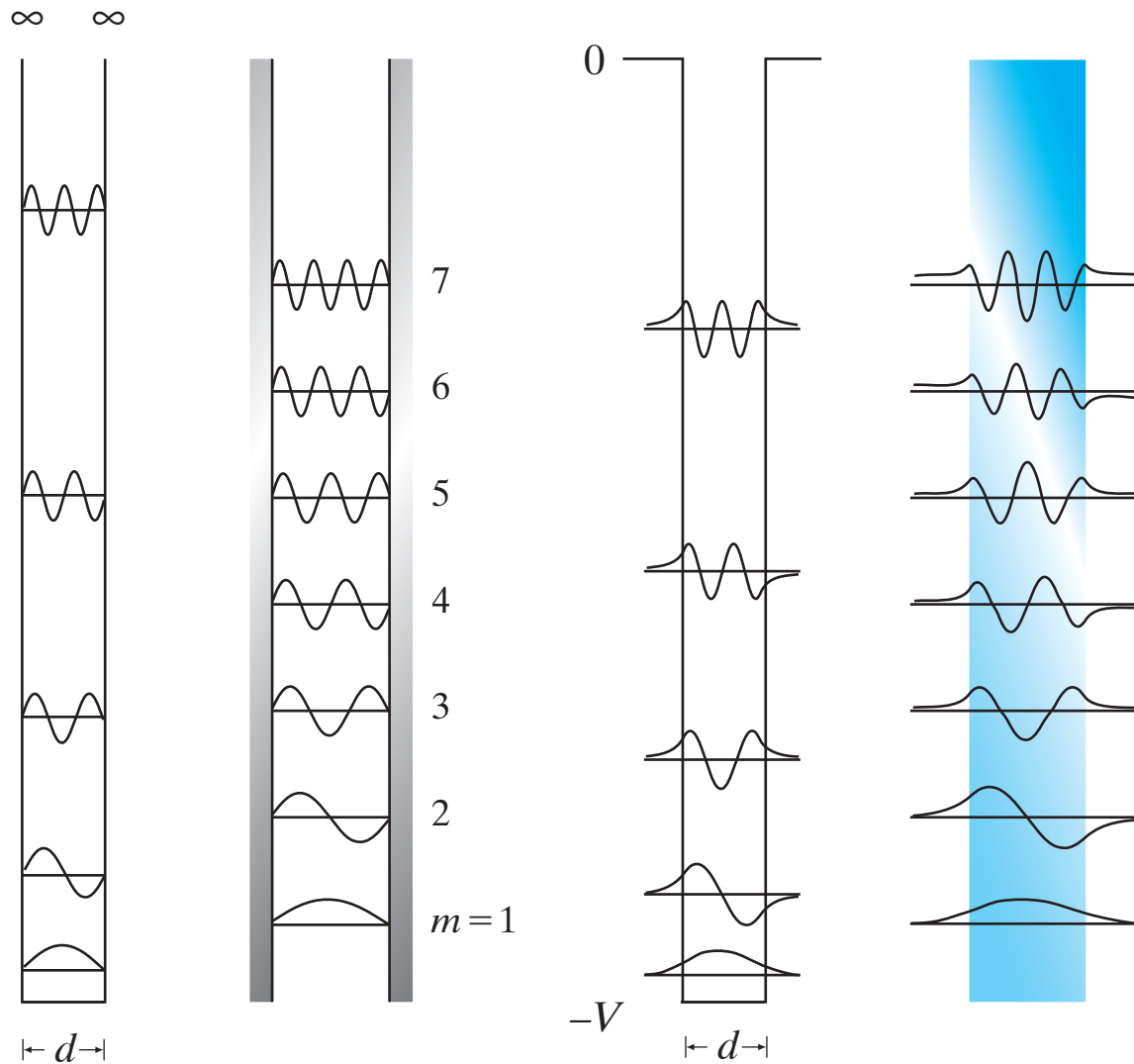
Waveguiding



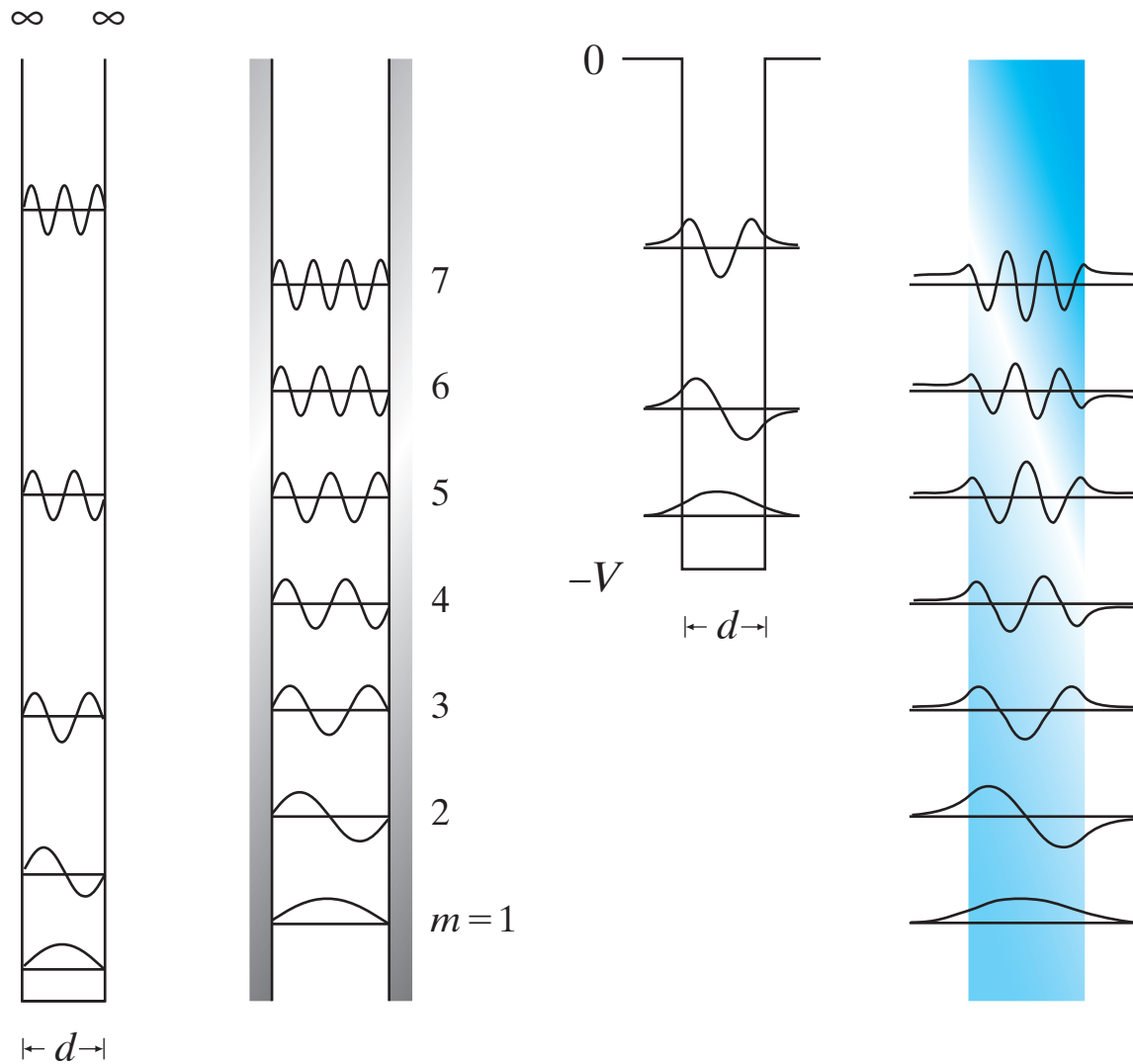
Waveguiding



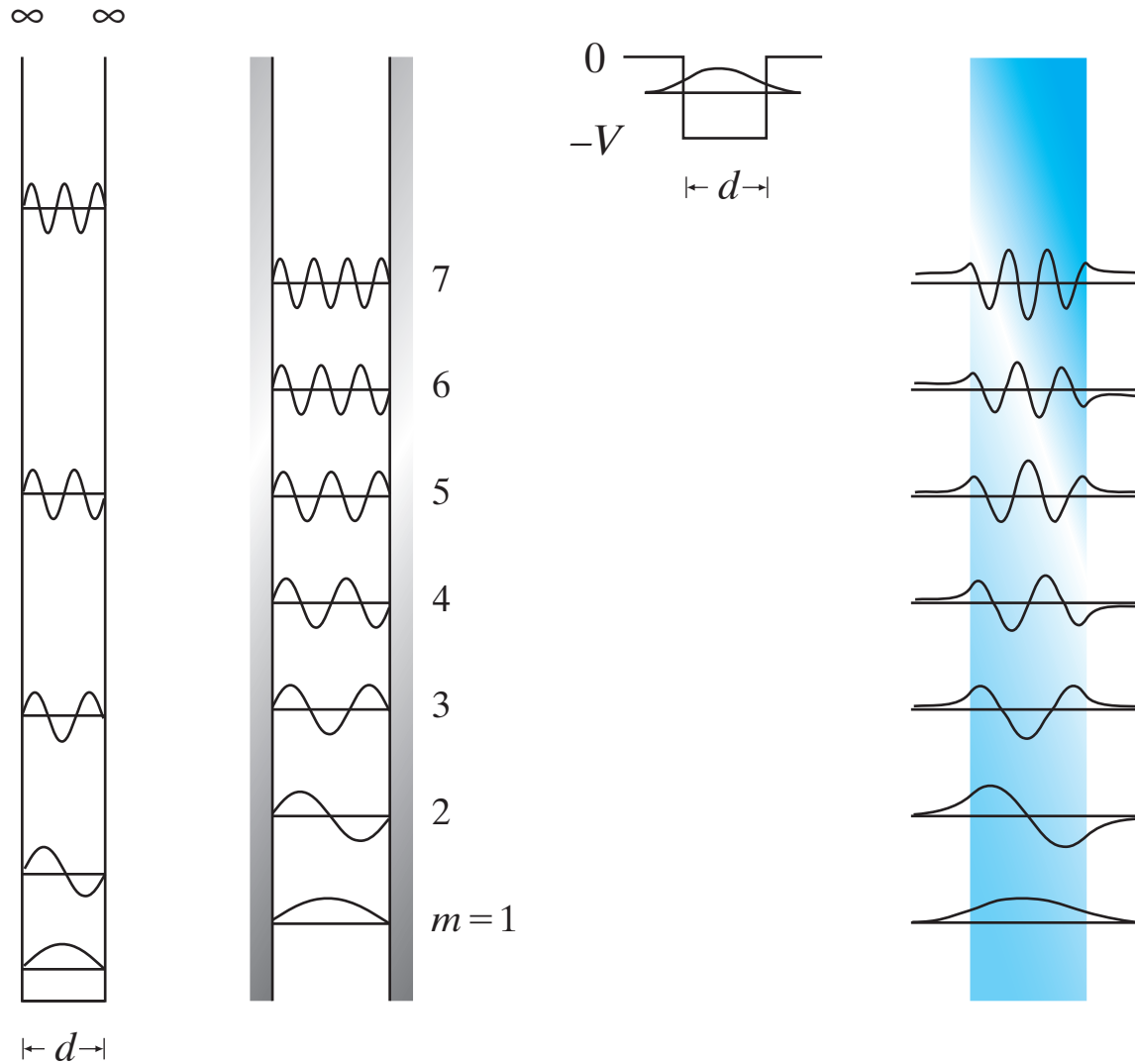
Waveguiding



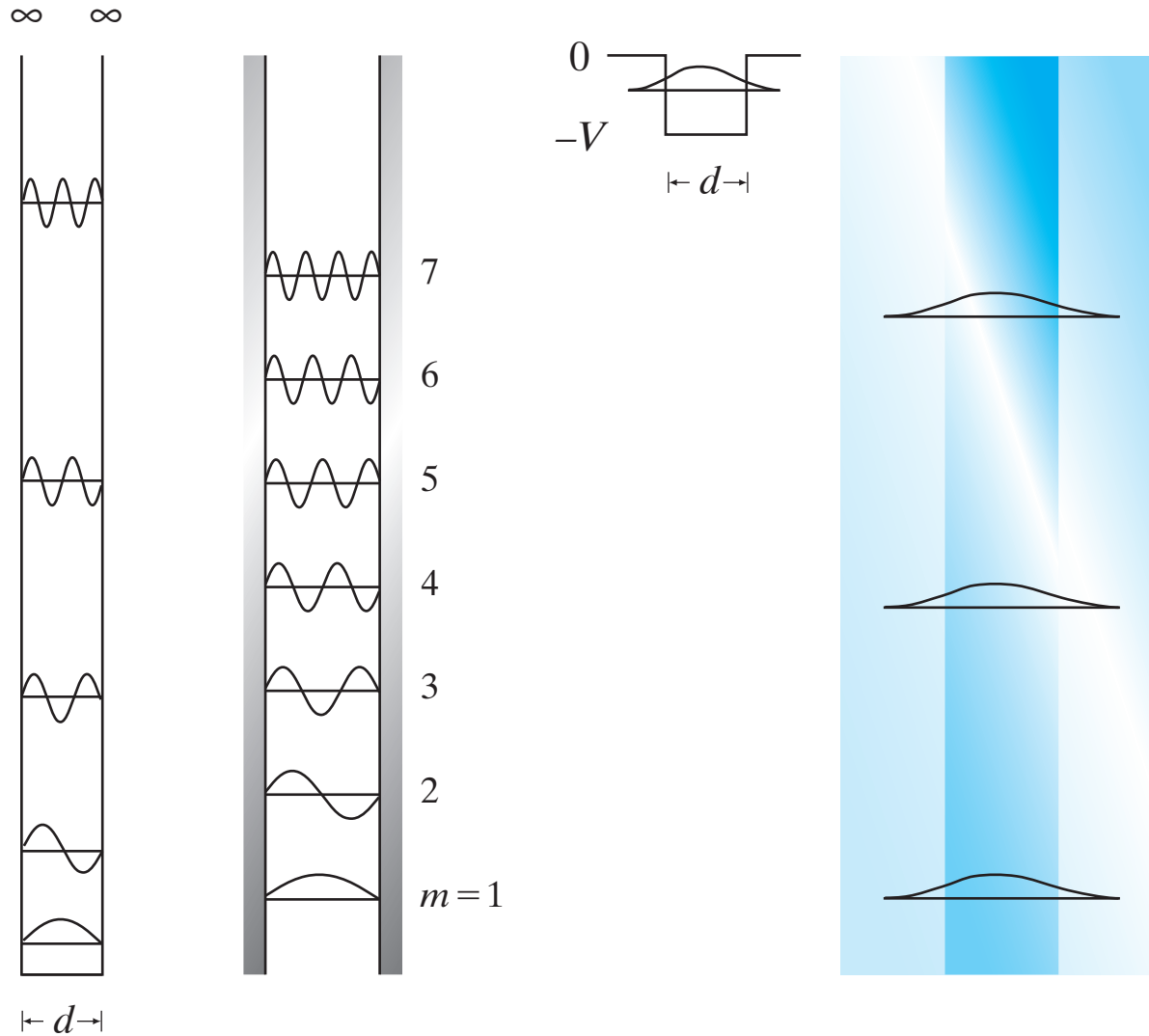
Waveguiding



Waveguiding



Waveguiding



Waveguiding

single mode condition for 600-nm light:

$$M \doteq 2 \frac{d}{\lambda} (n_1^2 - n_2^2)^{1/2}$$

without cladding:

$$d < 268 \text{ nm}$$

Waveguiding

single mode condition for 600-nm light:

$$M \doteq 2 \frac{d}{\lambda} (n_1^2 - n_2^2)^{1/2}$$

without cladding:

$$d < 268 \text{ nm}$$

Add cladding with 0.4% index difference:

$$d < 5 \text{ } \mu\text{m}$$

Waveguiding

commercial single-mode fiber (Corning Titan[®])



	core	cladding
index	$n_1 = 1.468$	$n_2 = 1.462$
diameter:	$8.3 \mu\text{m}$	$125.0 \pm 1.0 \mu\text{m}$

operating wavelength: $\lambda = 1310 \text{ nm}/1550 \text{ nm}$

Waveguiding

drawbacks of clad fibers:

- **weak confinement**
- **no tight bending**
- **coupling requires splicing**

Nanowire fabrication

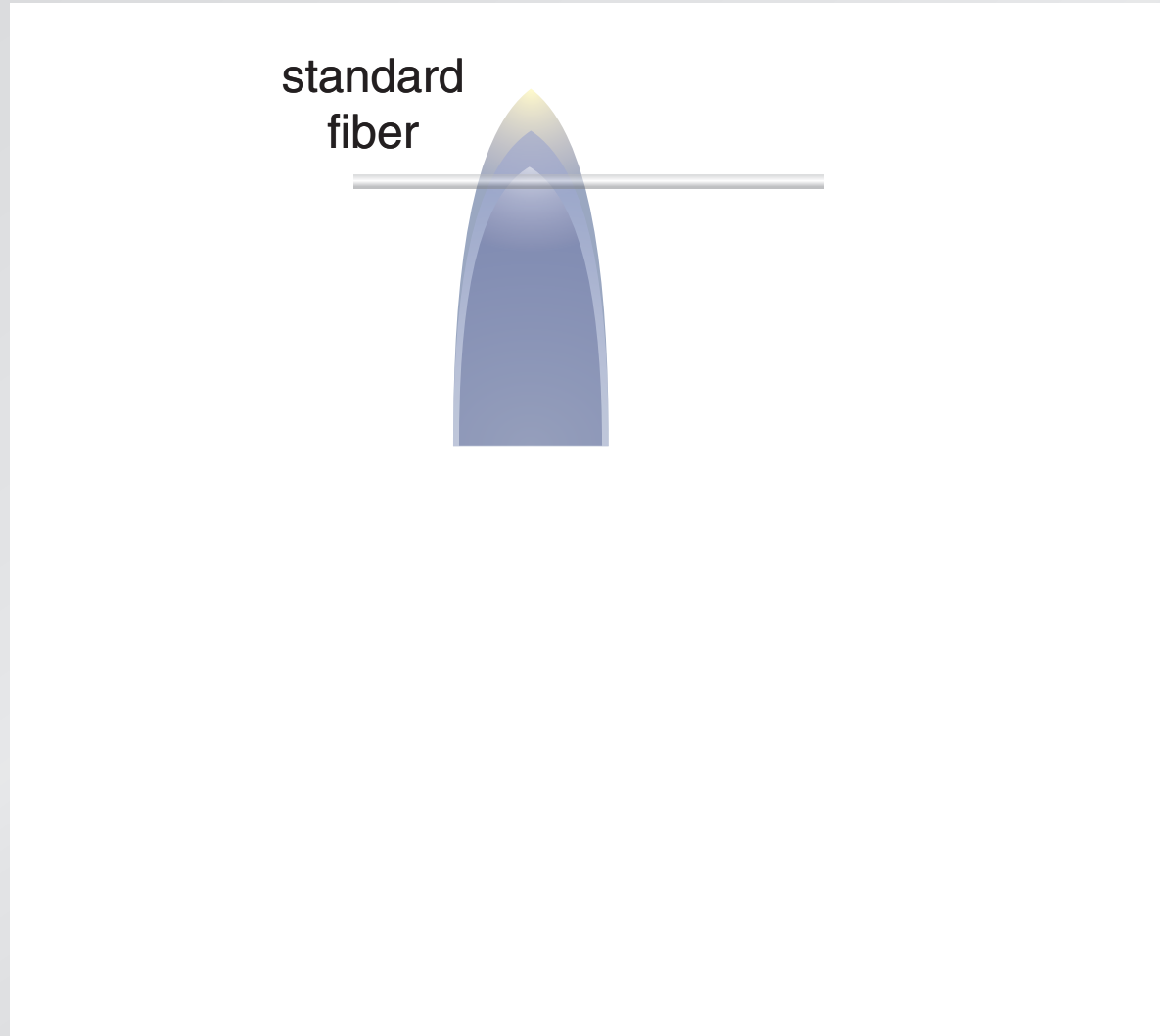
two-step drawing process

standard
fiber



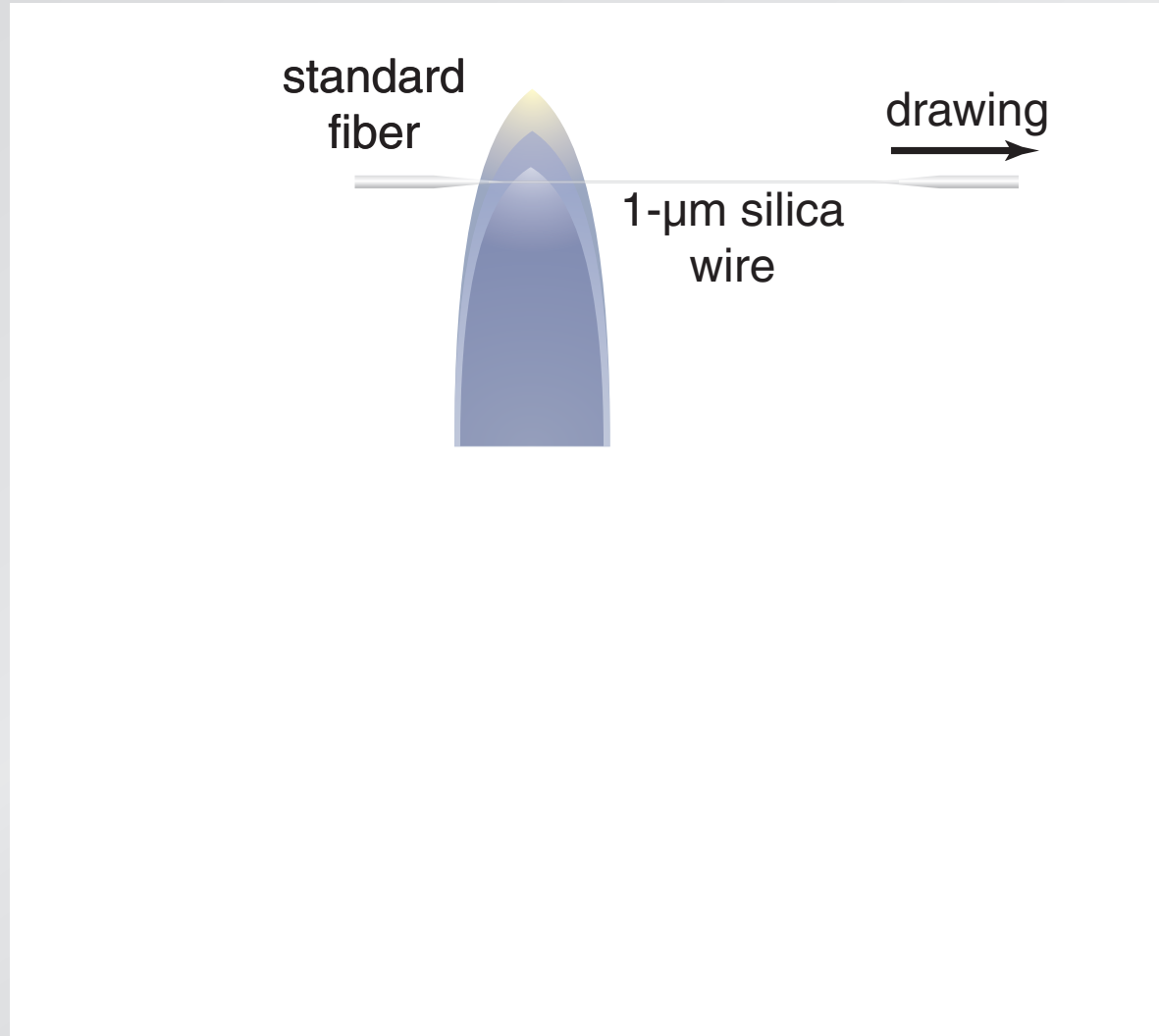
Nanowire fabrication

two-step drawing process



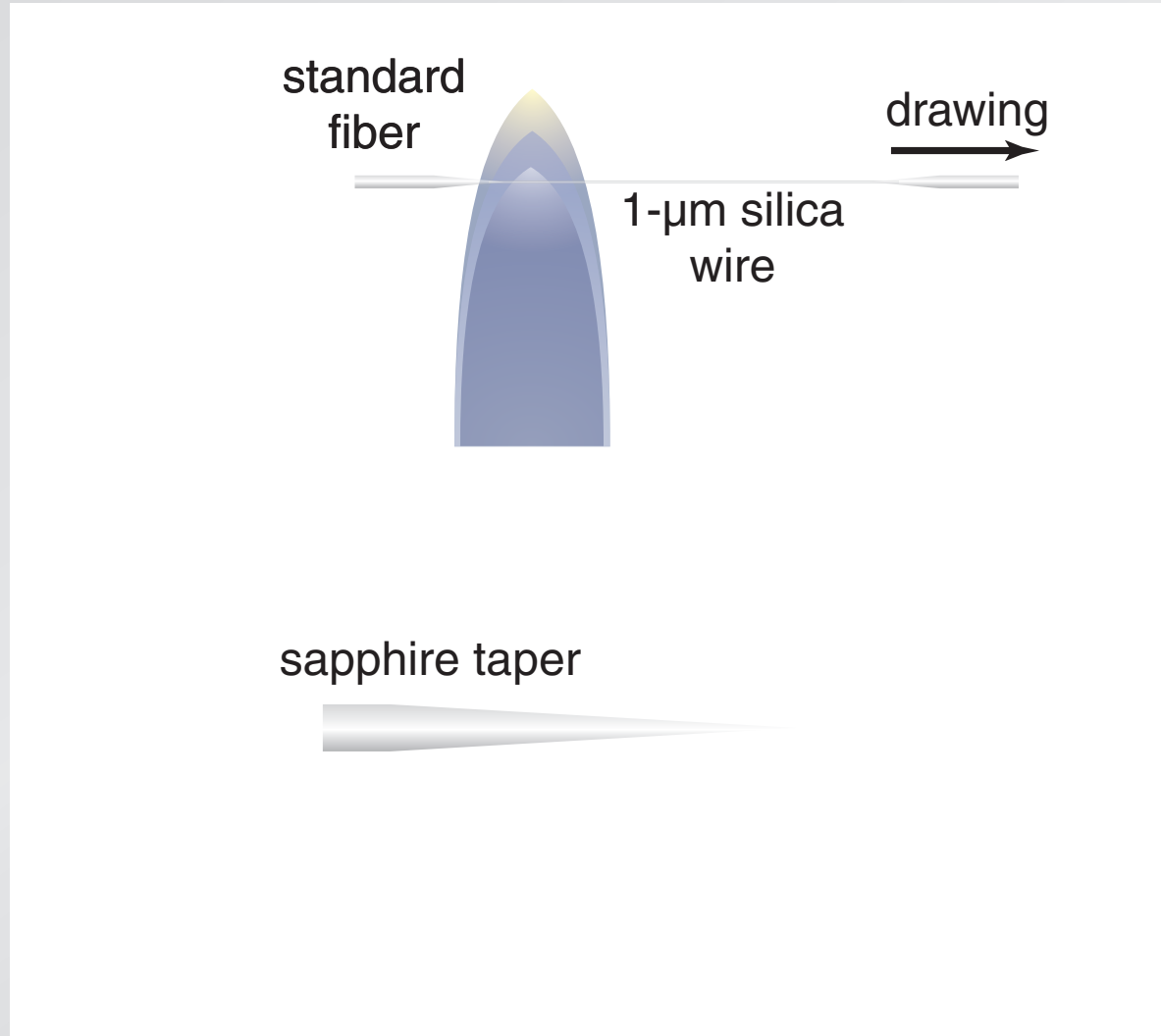
Nanowire fabrication

two-step drawing process



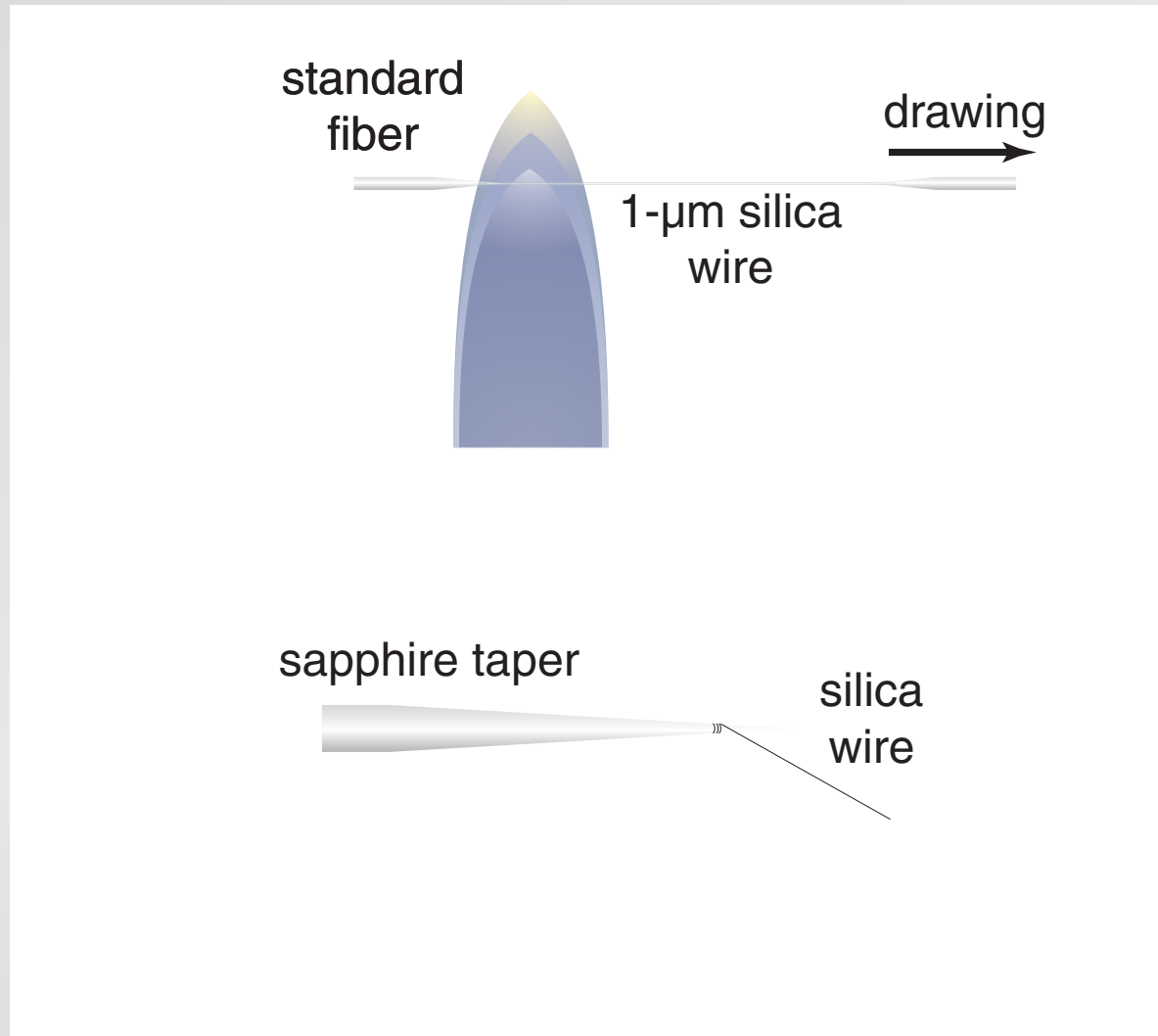
Nanowire fabrication

two-step drawing process



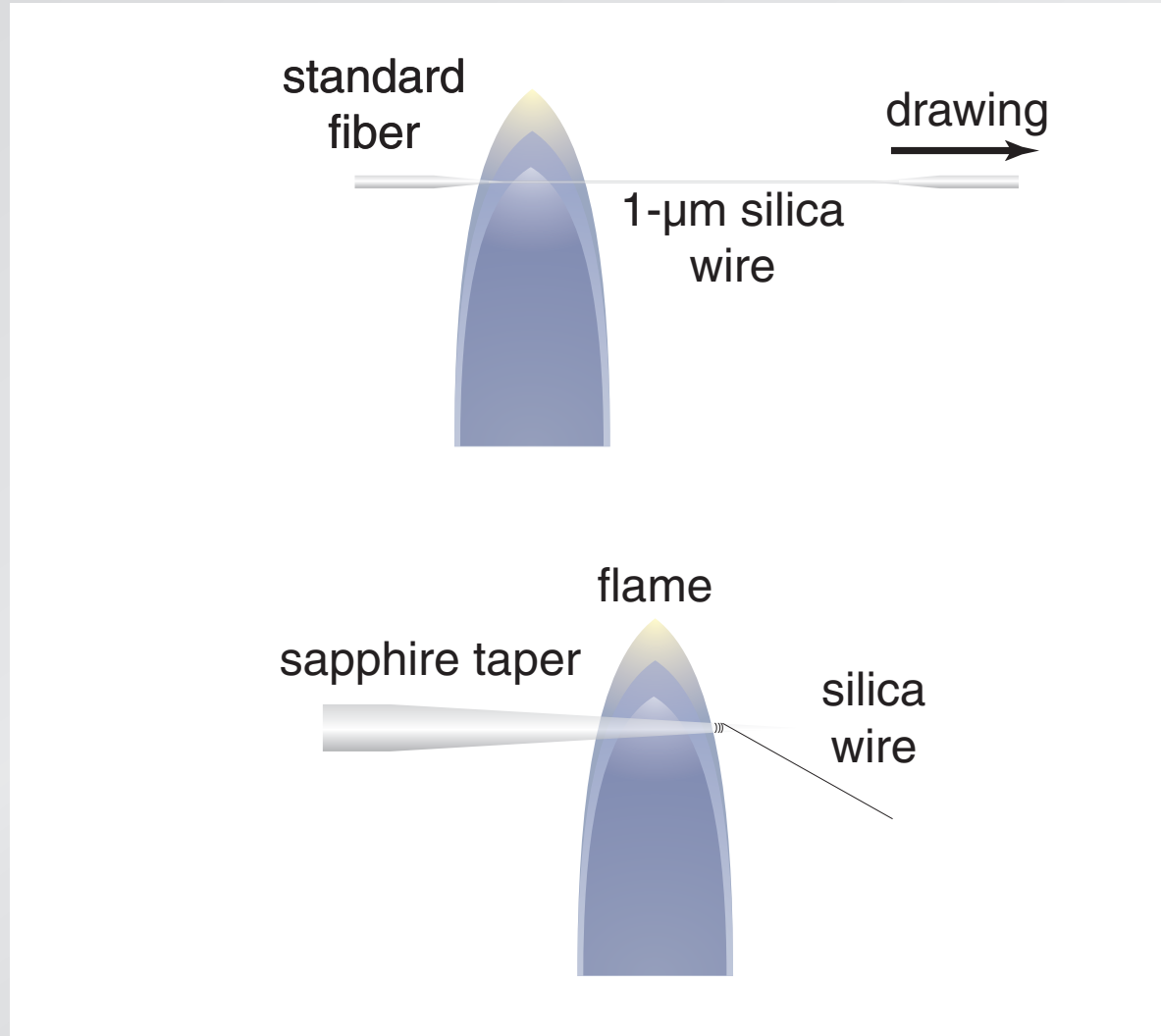
Nanowire fabrication

two-step drawing process



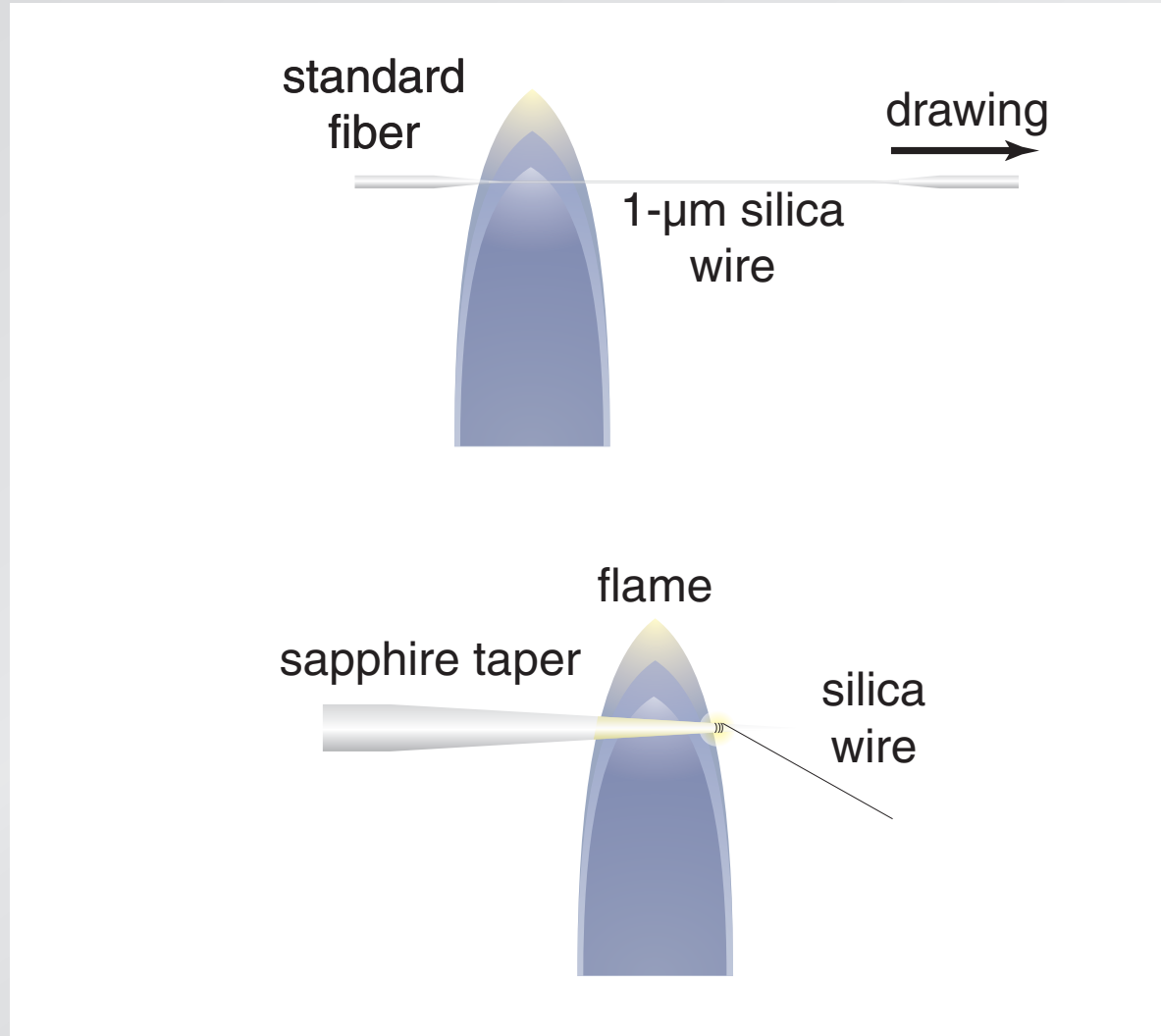
Nanowire fabrication

two-step drawing process



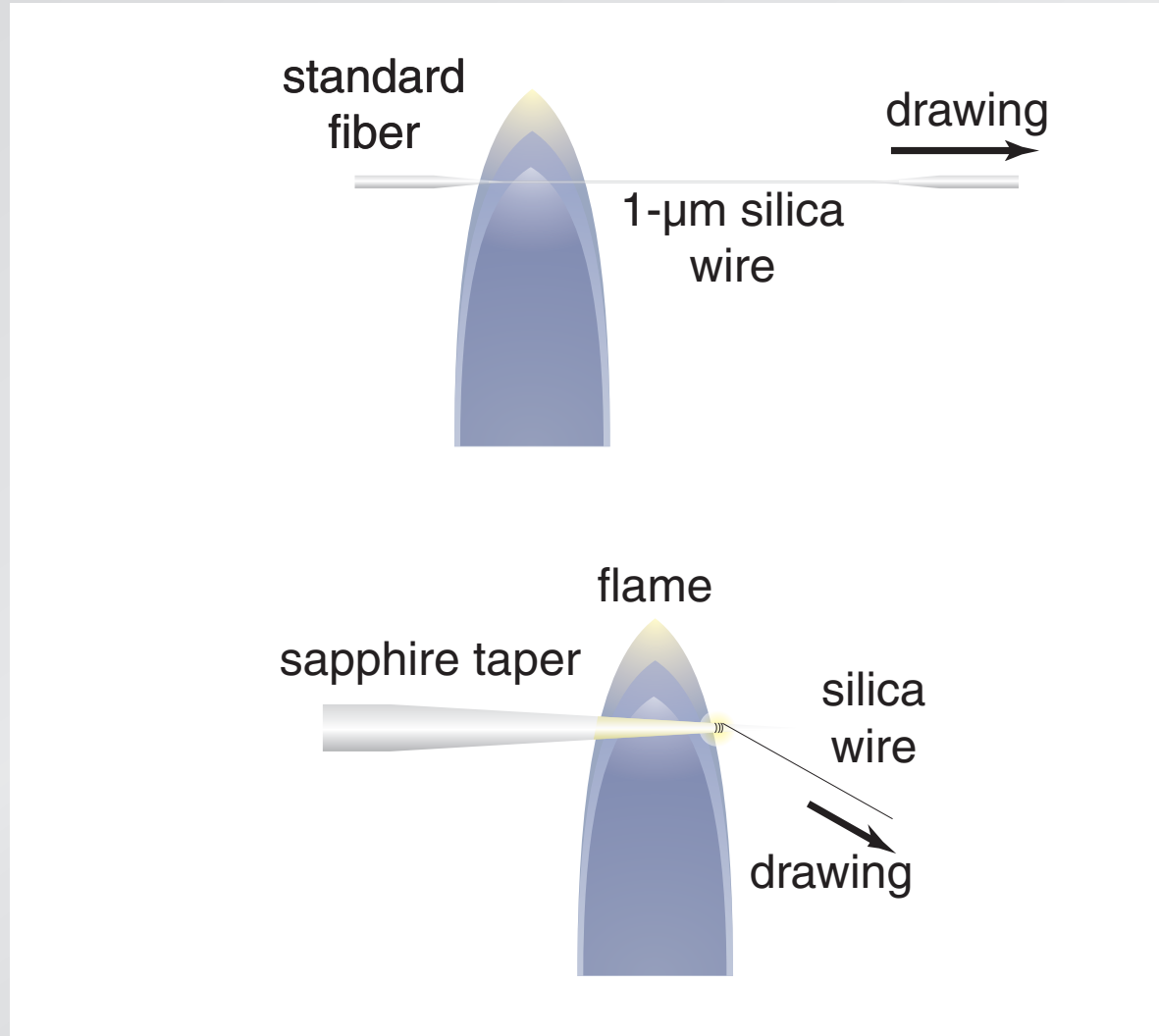
Nanowire fabrication

two-step drawing process



Nanowire fabrication

two-step drawing process



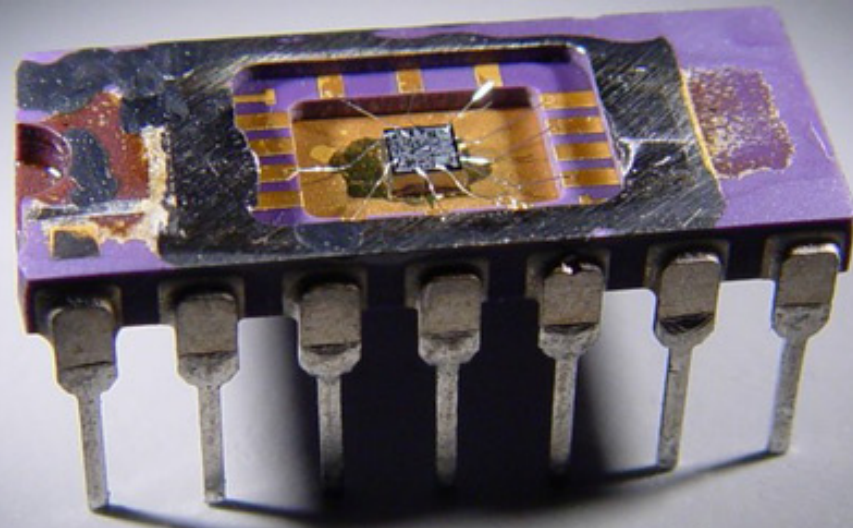
Nanowire fabrication

1 μm

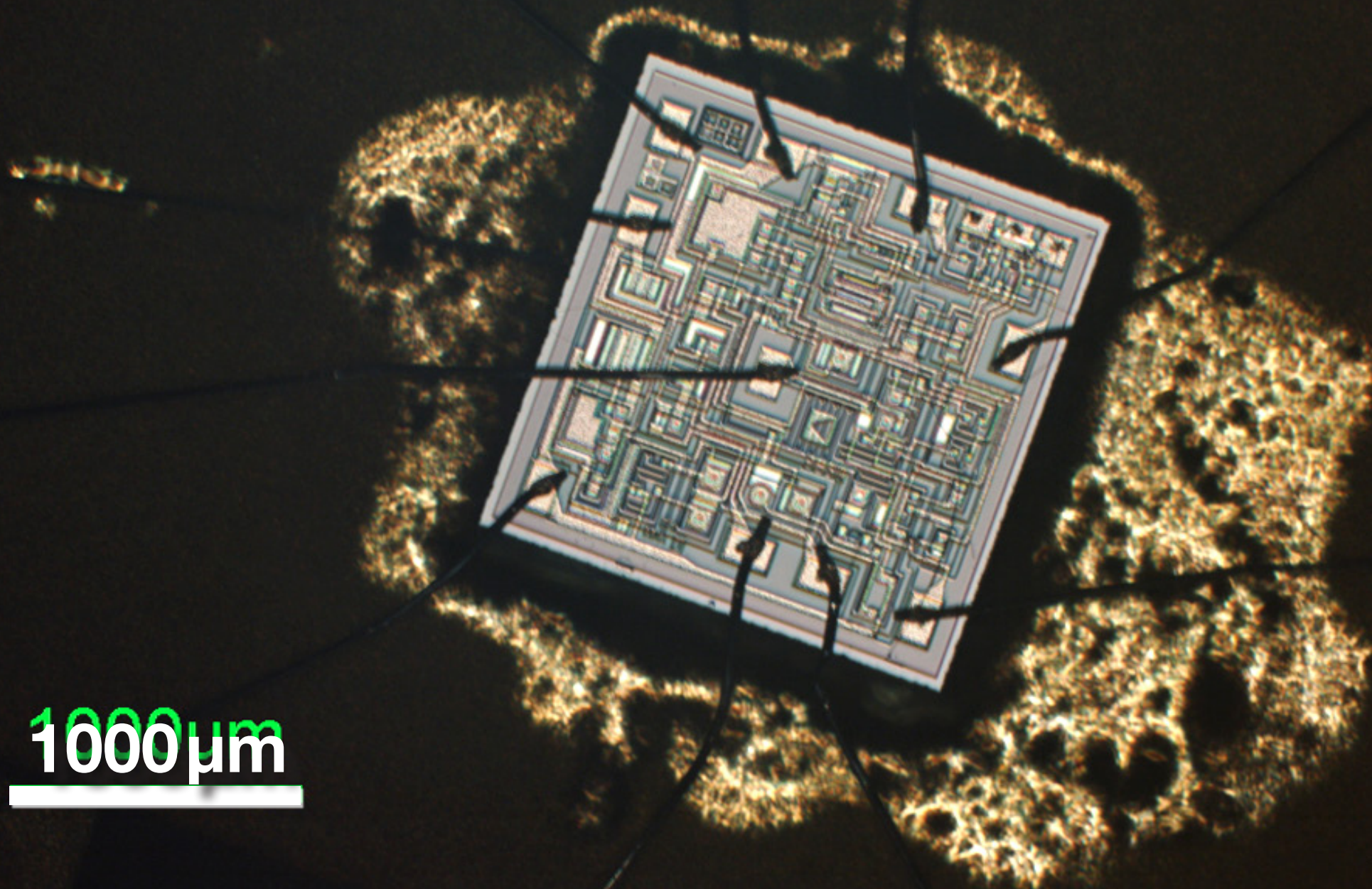


Nature, 426, 816 (2003)

Nanowire fabrication

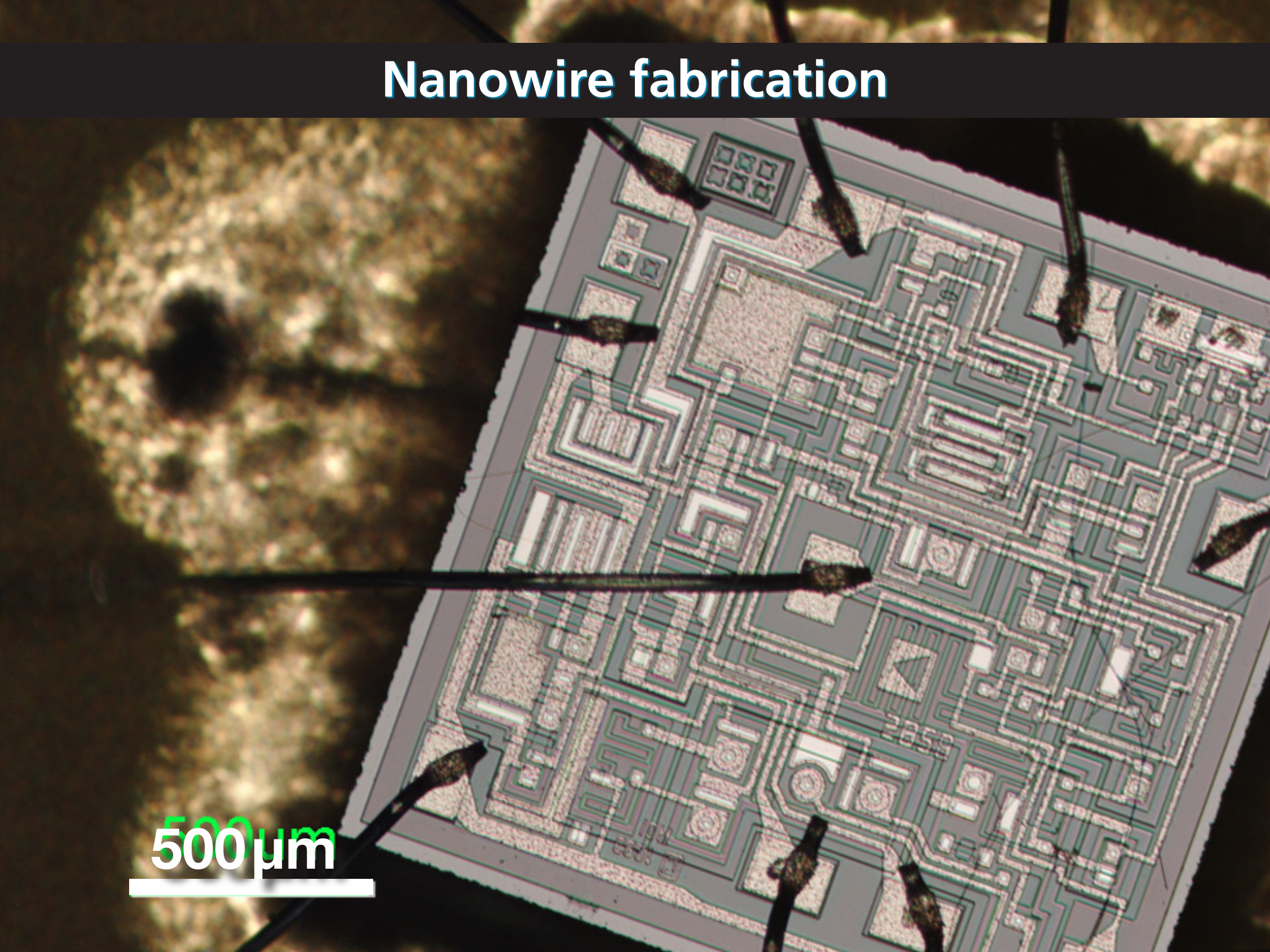
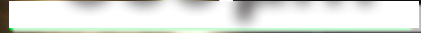


Nanowire fabrication



Nanowire fabrication

500 μm

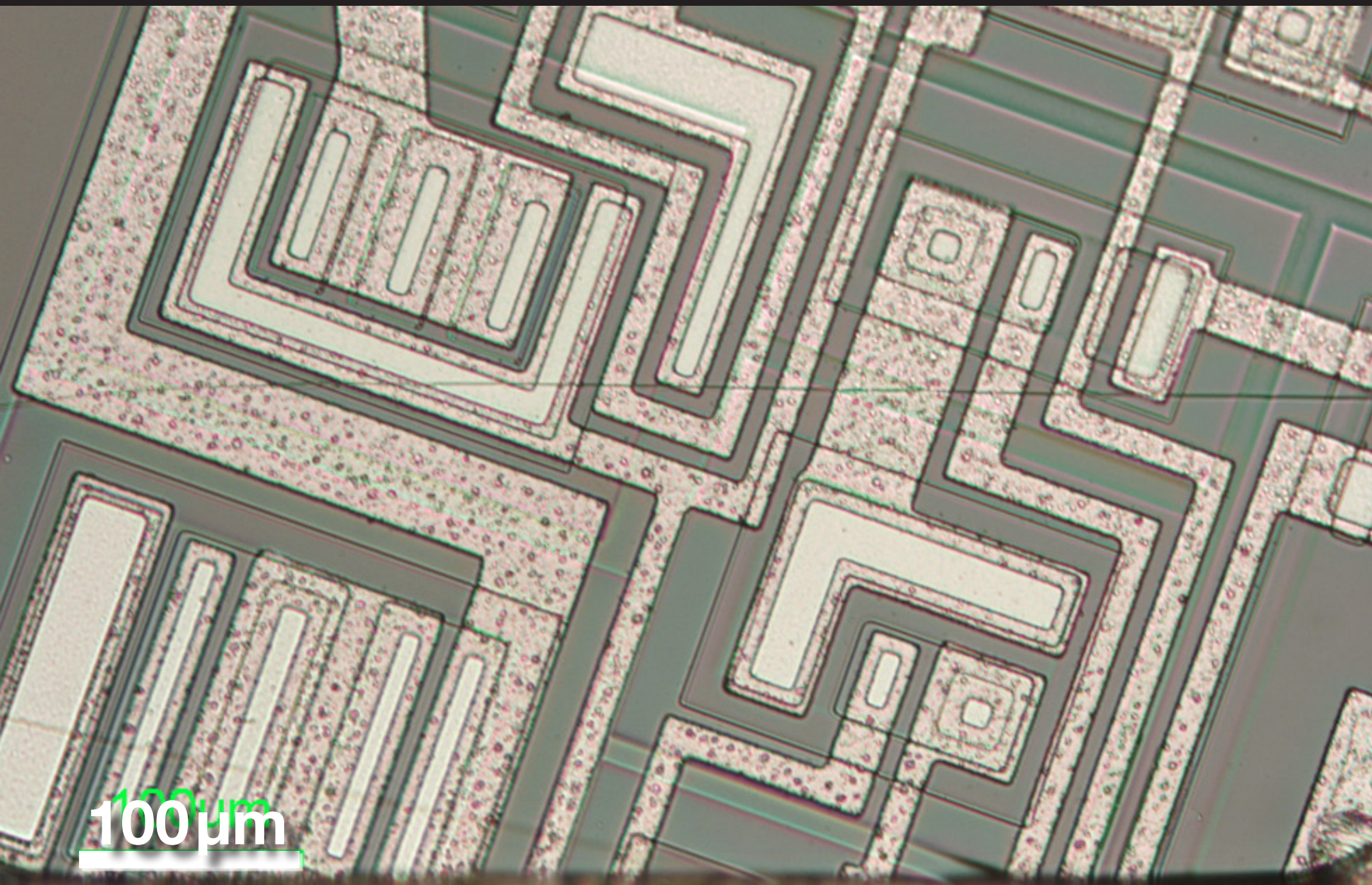


Nanowire fabrication

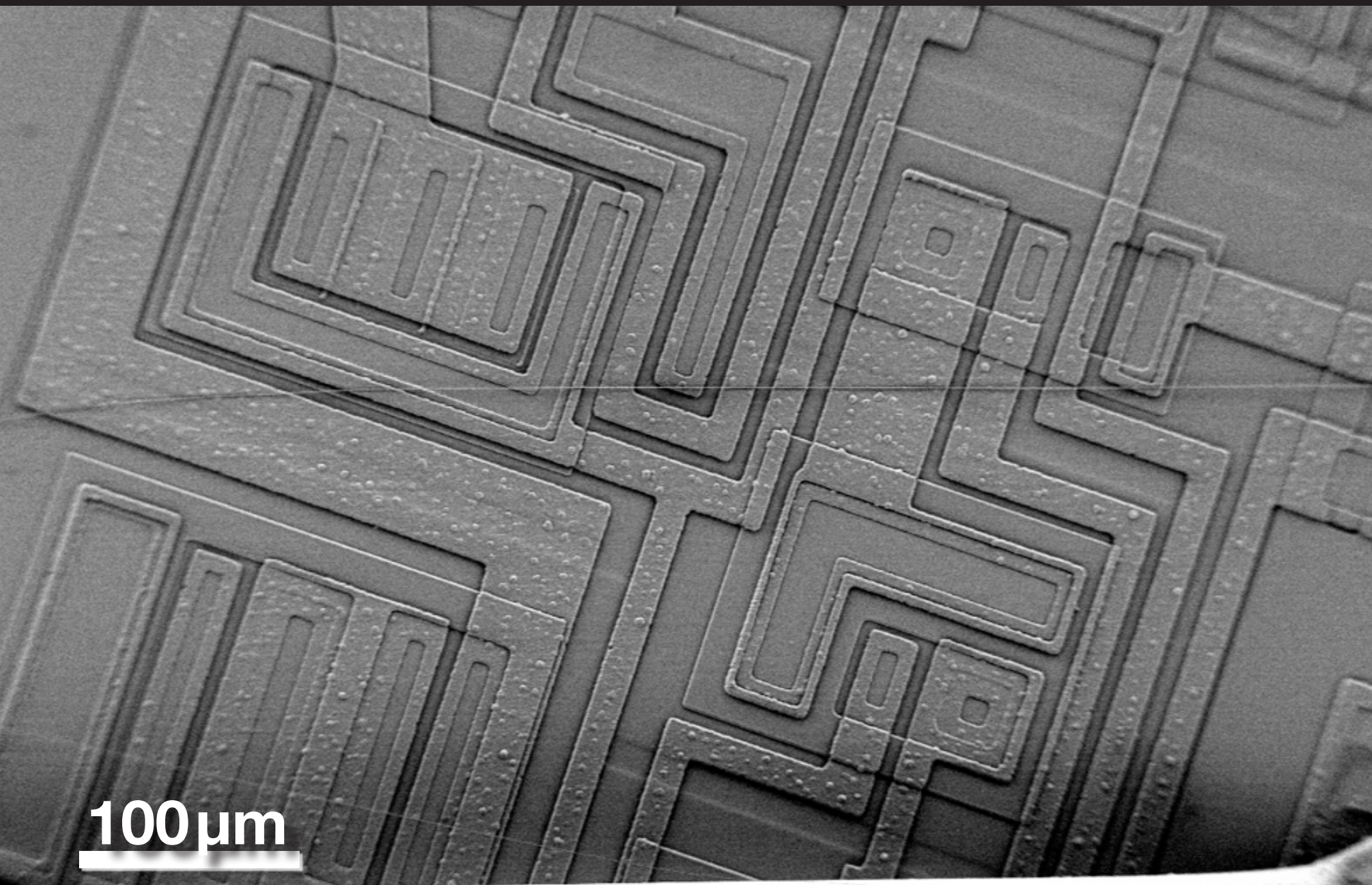
200 μm

A micrograph showing the fabrication of nanowires on a patterned substrate. The substrate features a complex, repeating pattern of rectangular and square structures, likely serving as a template for the nanowire growth. Two dark, elongated structures, representing the nanowires, are visible. One is positioned horizontally across the lower half of the image, and the other is positioned vertically in the upper left quadrant. A scale bar in the bottom left corner indicates a length of 200 micrometers.

Nanowire fabrication

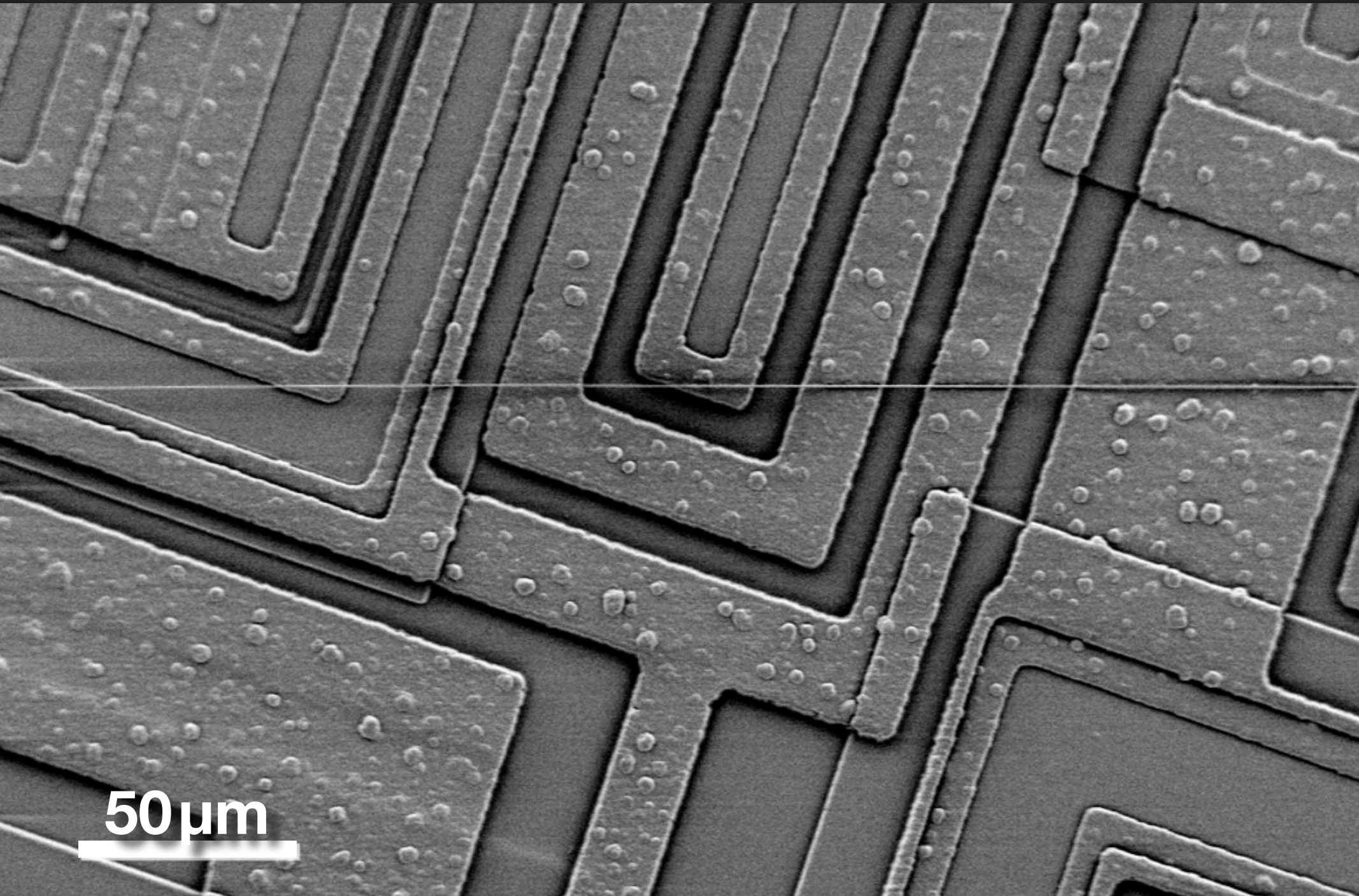


Nanowire fabrication



100 μm

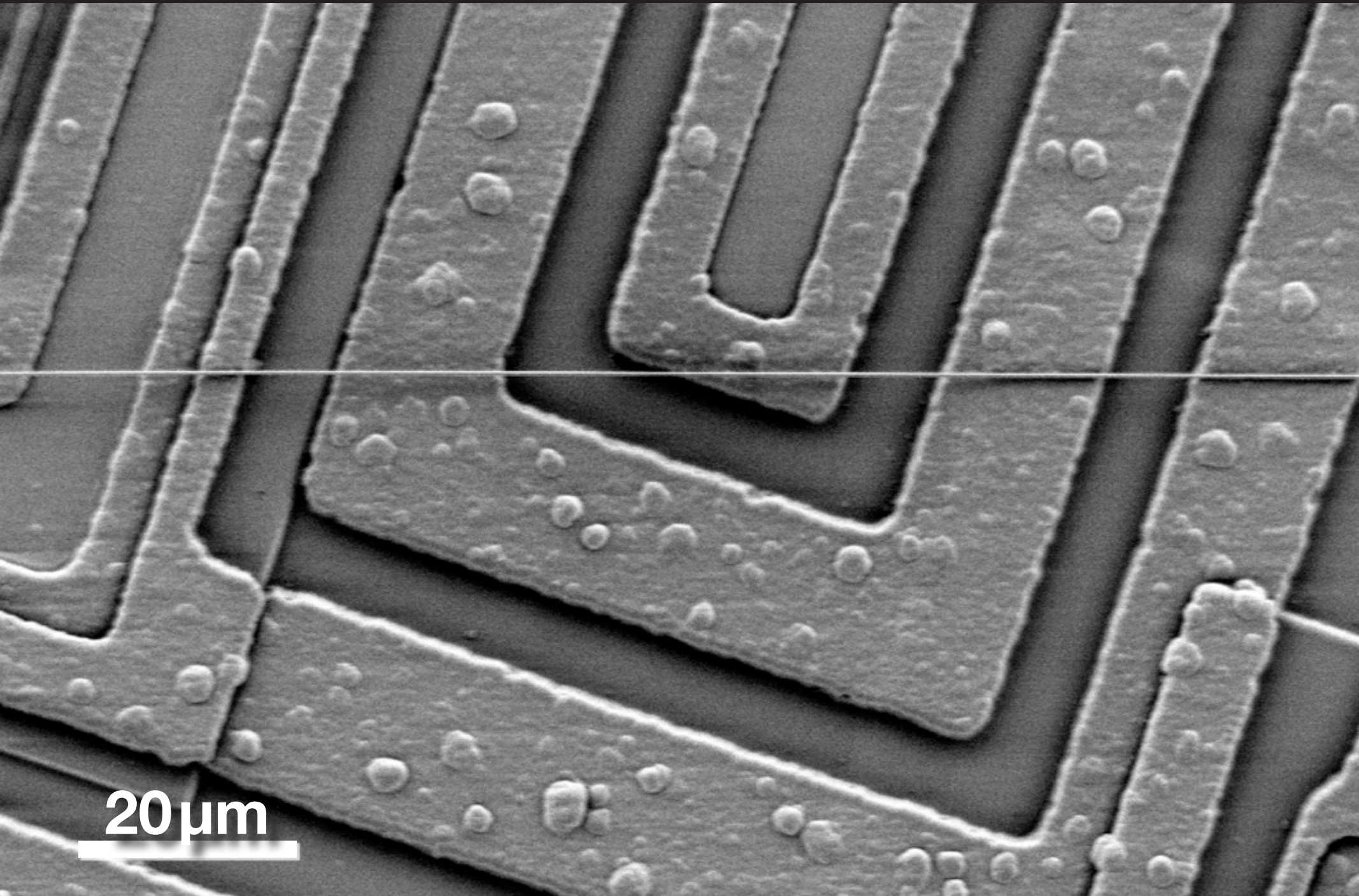
Nanowire fabrication



50 μm

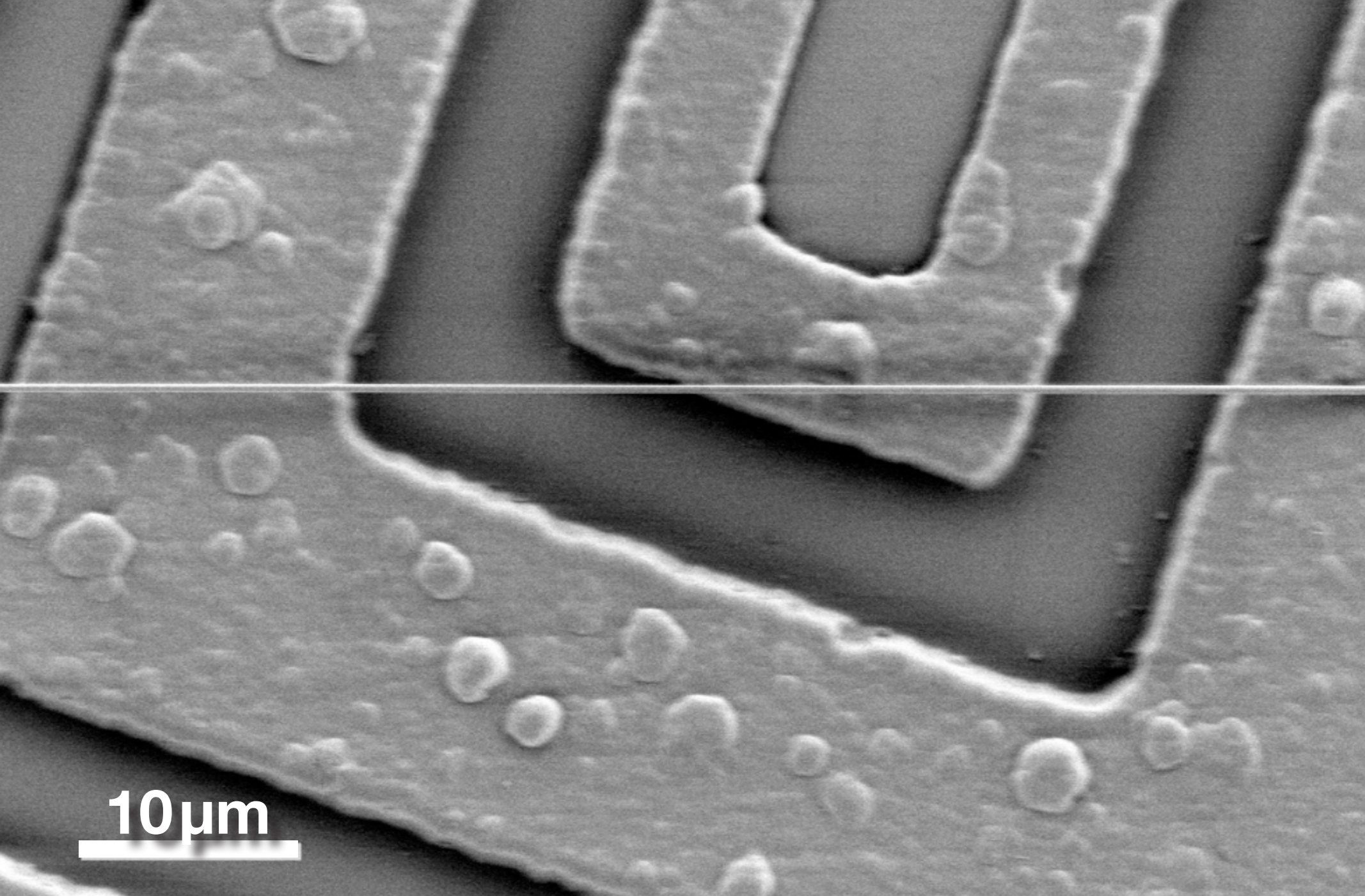


Nanowire fabrication



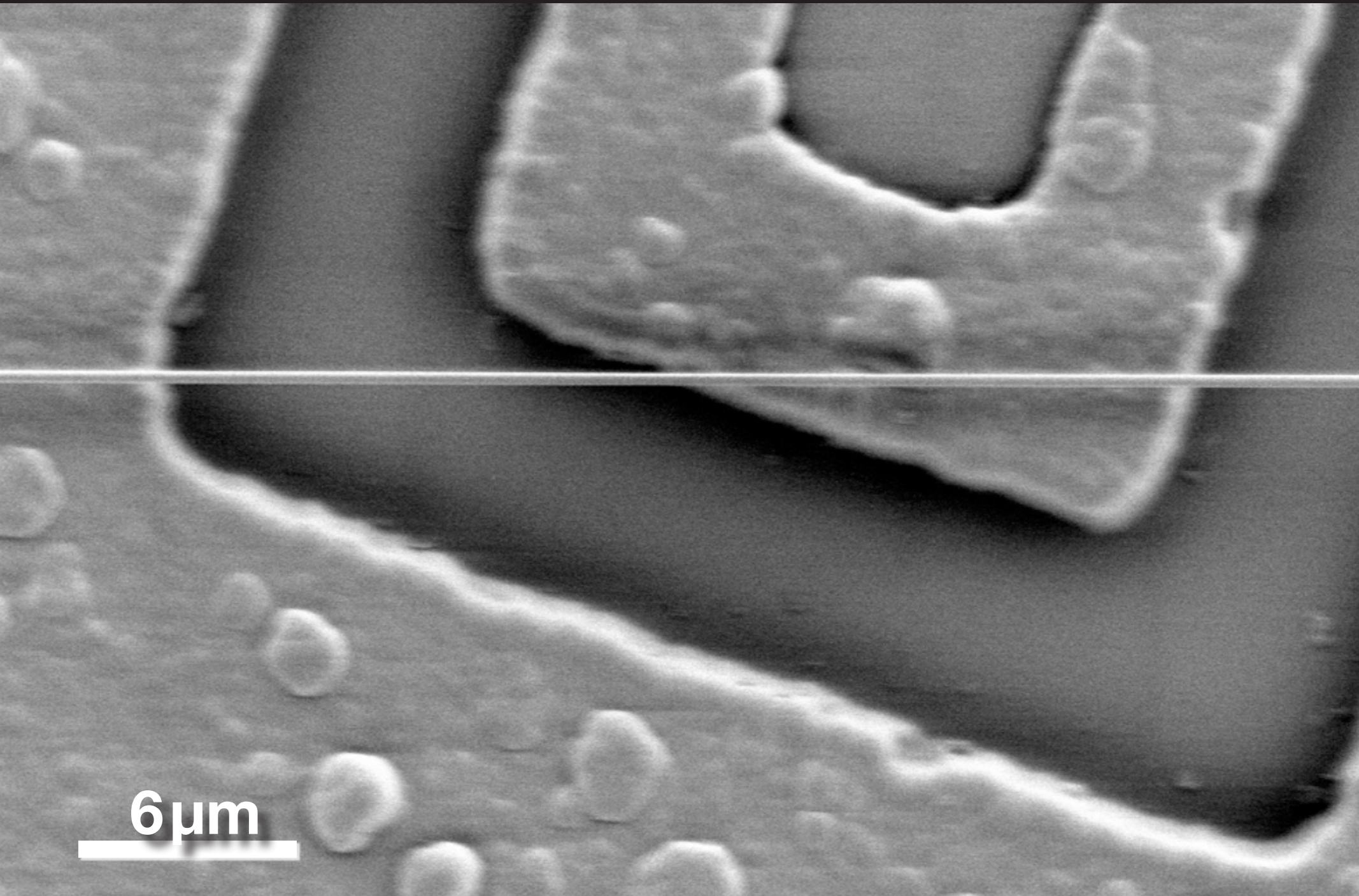
20 μm

Nanowire fabrication



10 μm

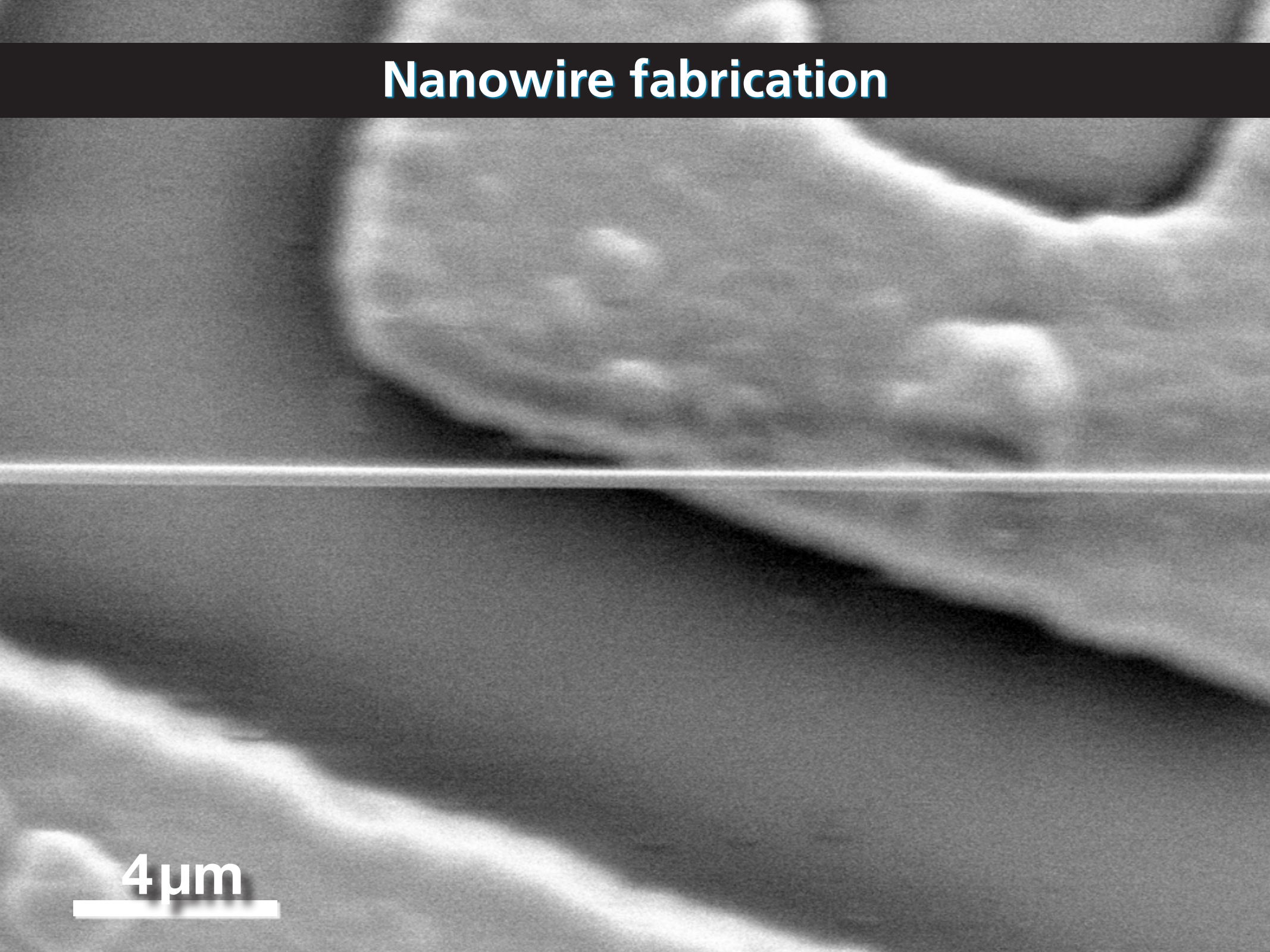
Nanowire fabrication



6 μm



Nanowire fabrication



4 μm



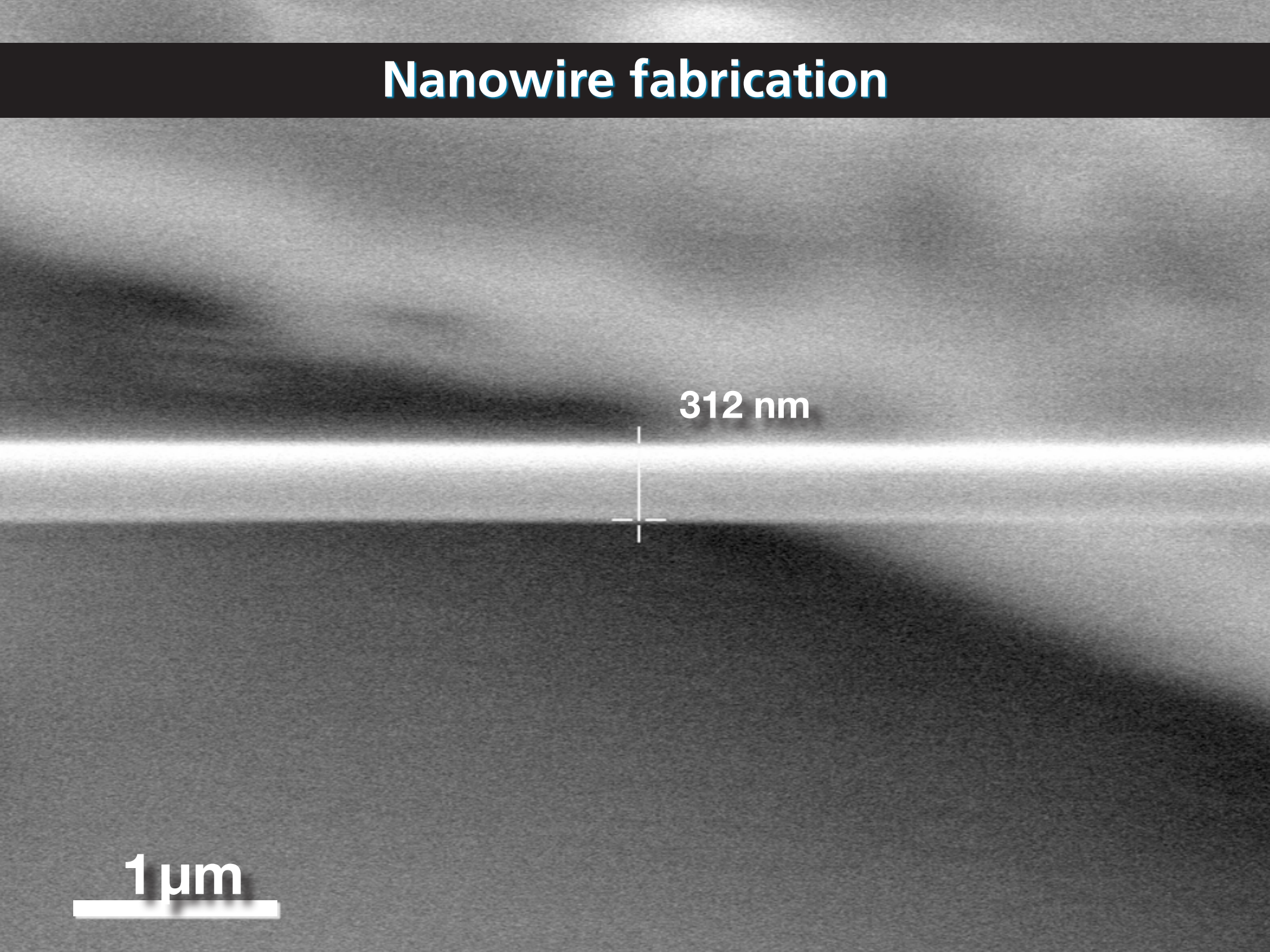
Nanowire fabrication

2 μm

A grayscale micrograph showing a single, long, thin nanowire extending horizontally across the center of the frame. The nanowire is very uniform in thickness and length. In the bottom-left corner, there is a white horizontal scale bar with the text "2 μm" above it.

Nanowire fabrication

312 nm

A transmission electron micrograph (TEM) showing a single, long, cylindrical nanowire oriented horizontally. The nanowire has a uniform diameter. A vertical white line with a crossbar at the bottom is drawn across the center of the nanowire to indicate its diameter. The text "312 nm" is placed to the right of this vertical line. The background is a dark, grainy texture.

1 μm

A horizontal white scale bar is located in the bottom left corner of the image. The text "1 μm " is positioned above the bar.

Waveguiding

Specifications

diameter D : down to 20 nm

length L : up to 90 mm

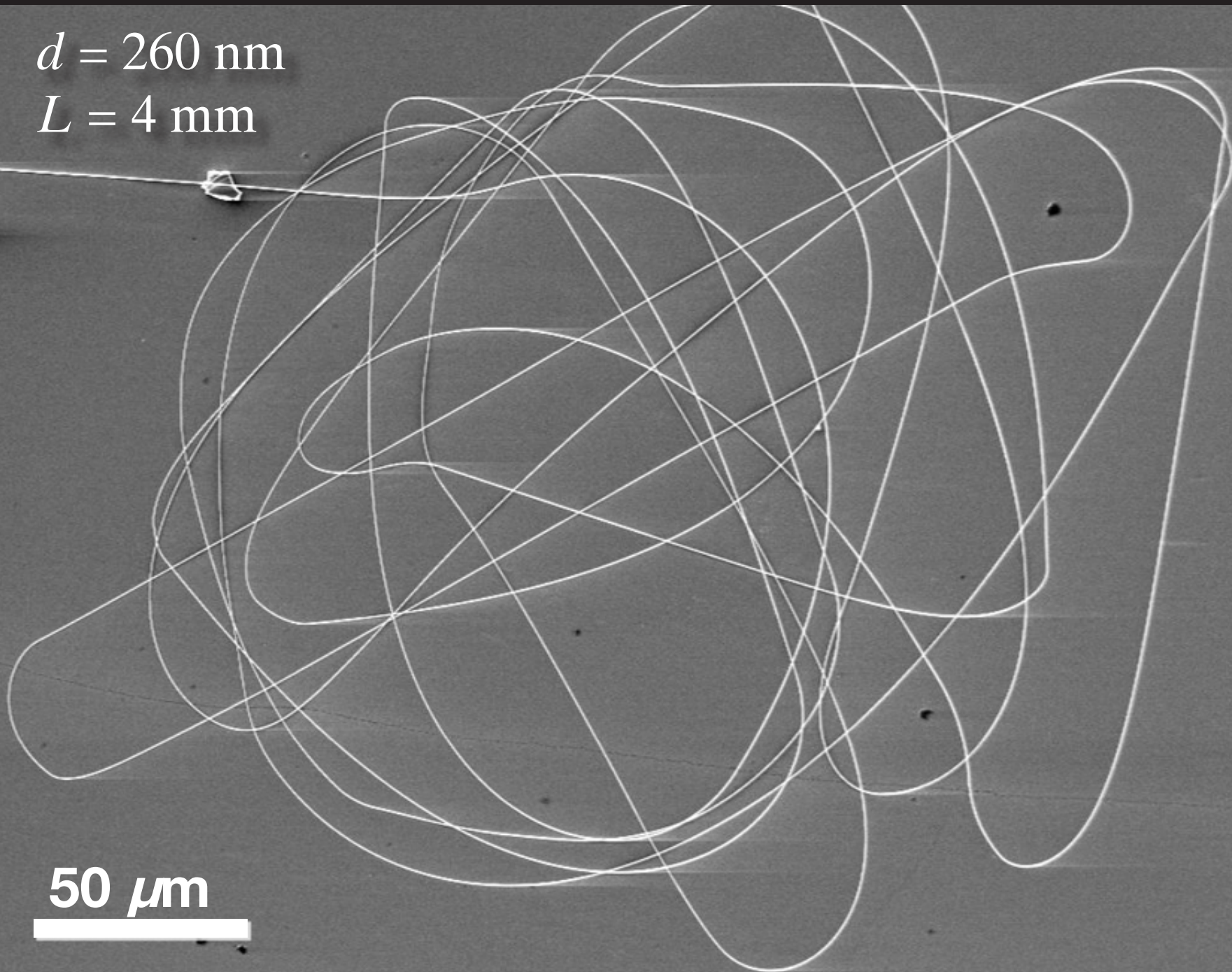
aspect ratio D/L : up to 10^6

diameter uniformity $\Delta D/L$: 2×10^{-6}

Nanowire fabrication

$d = 260 \text{ nm}$

$L = 4 \text{ mm}$



50 μm

Nanowire fabrication

240-nm wire

200 nm



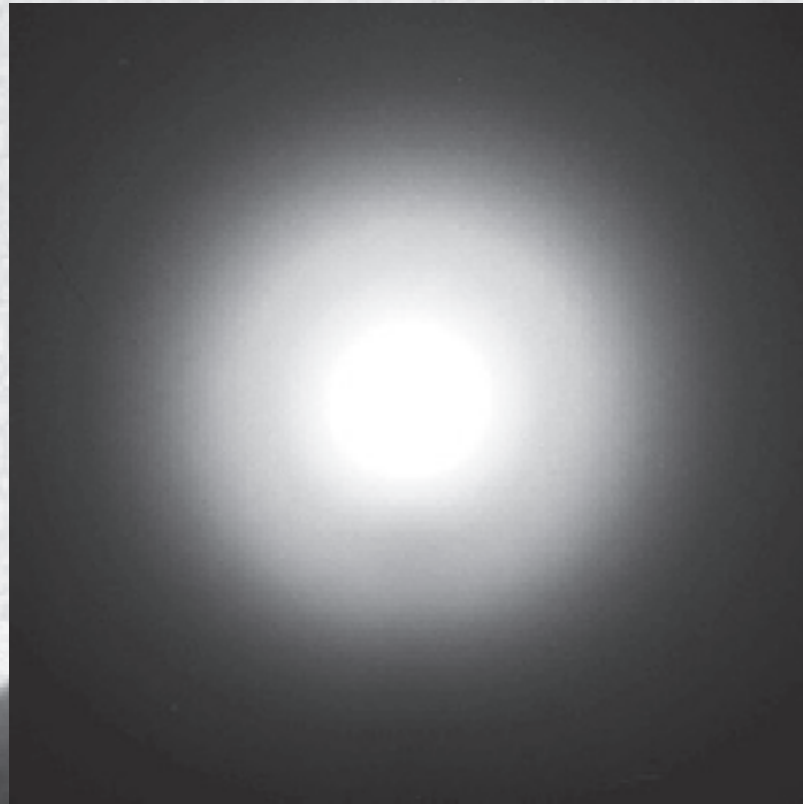
Nanowire fabrication

RMS roughness < 0.5 nm

20 nm



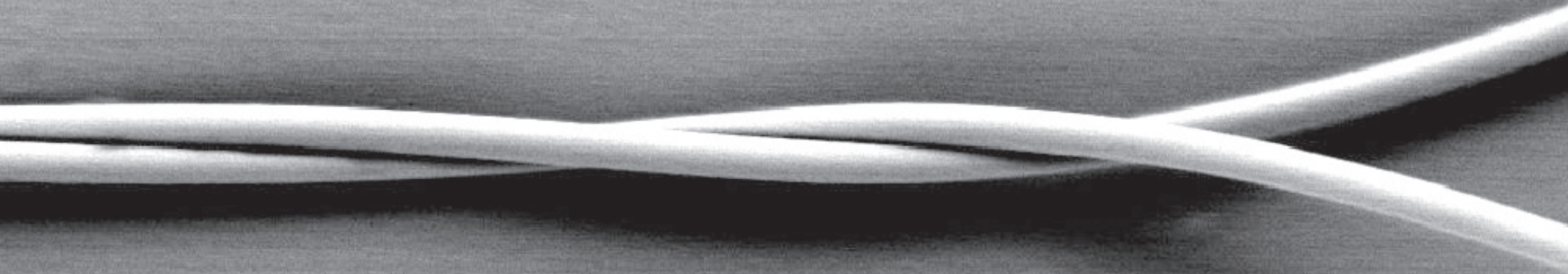
Nanowire fabrication



20 nm



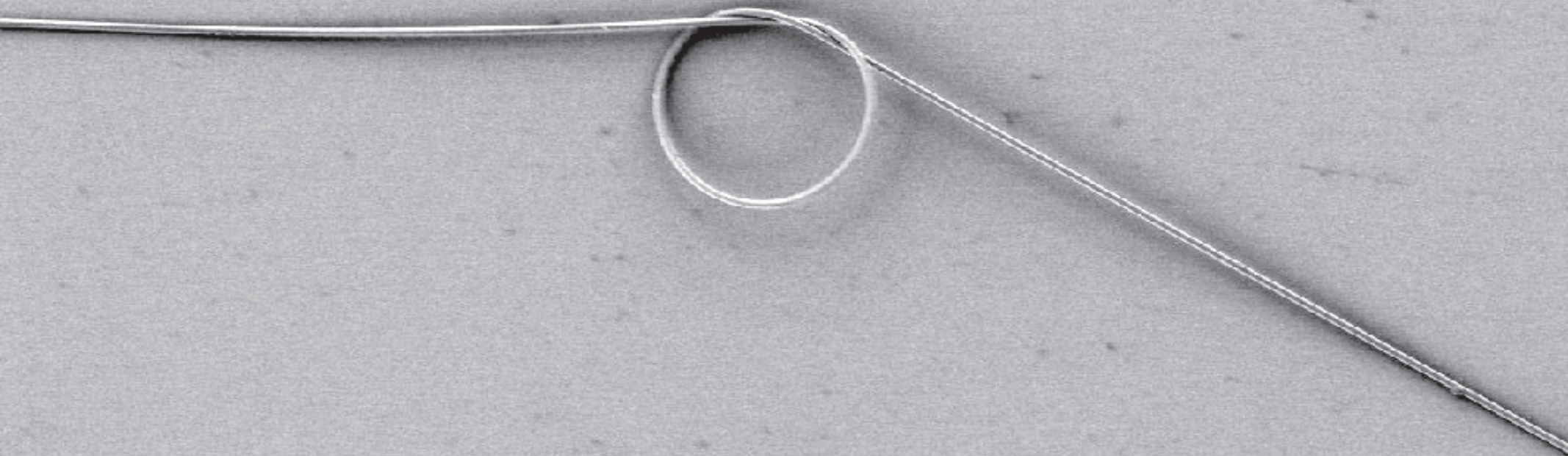
Nanowire fabrication



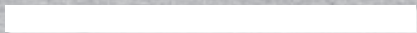
2 μm



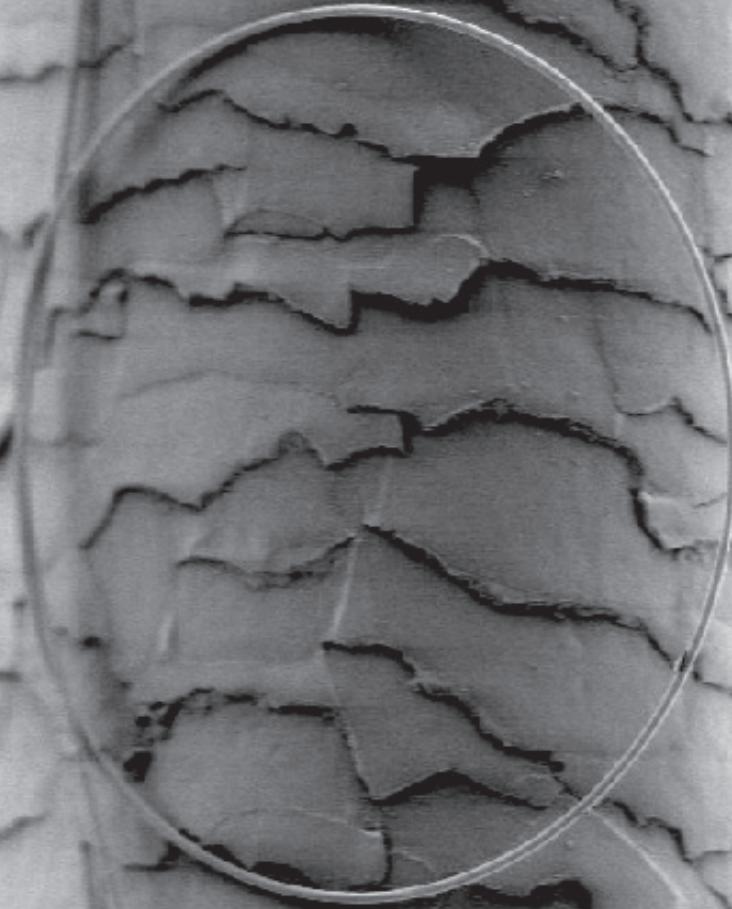
Nanowire fabrication



20 μm



Nanowire fabrication

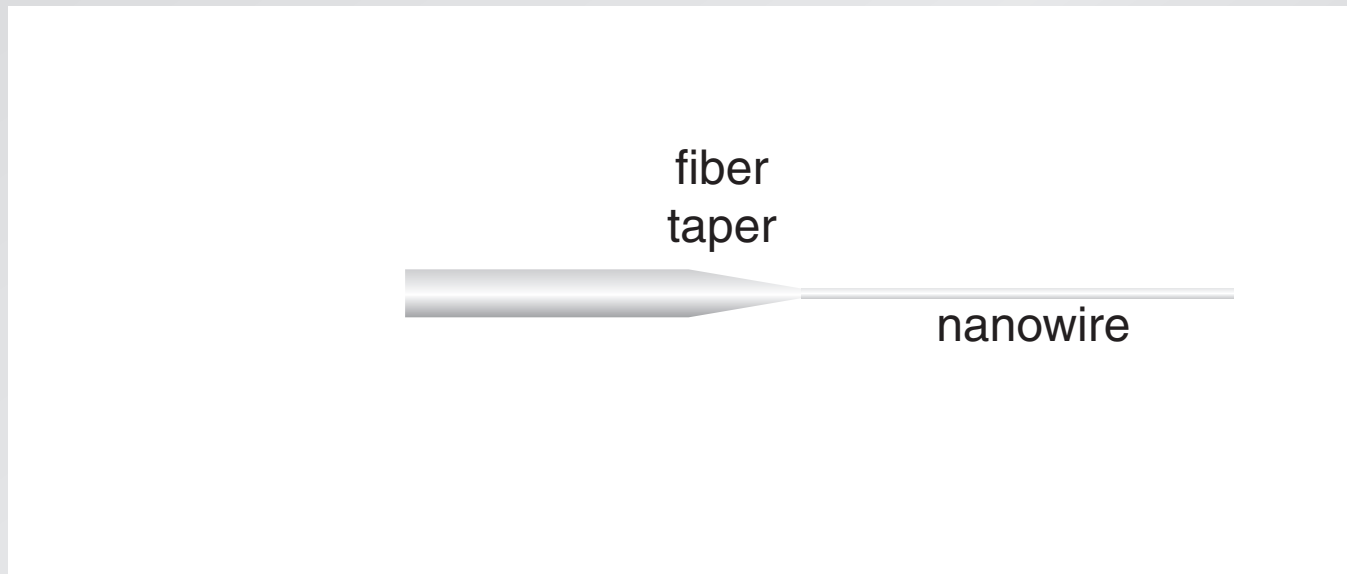


20 μm



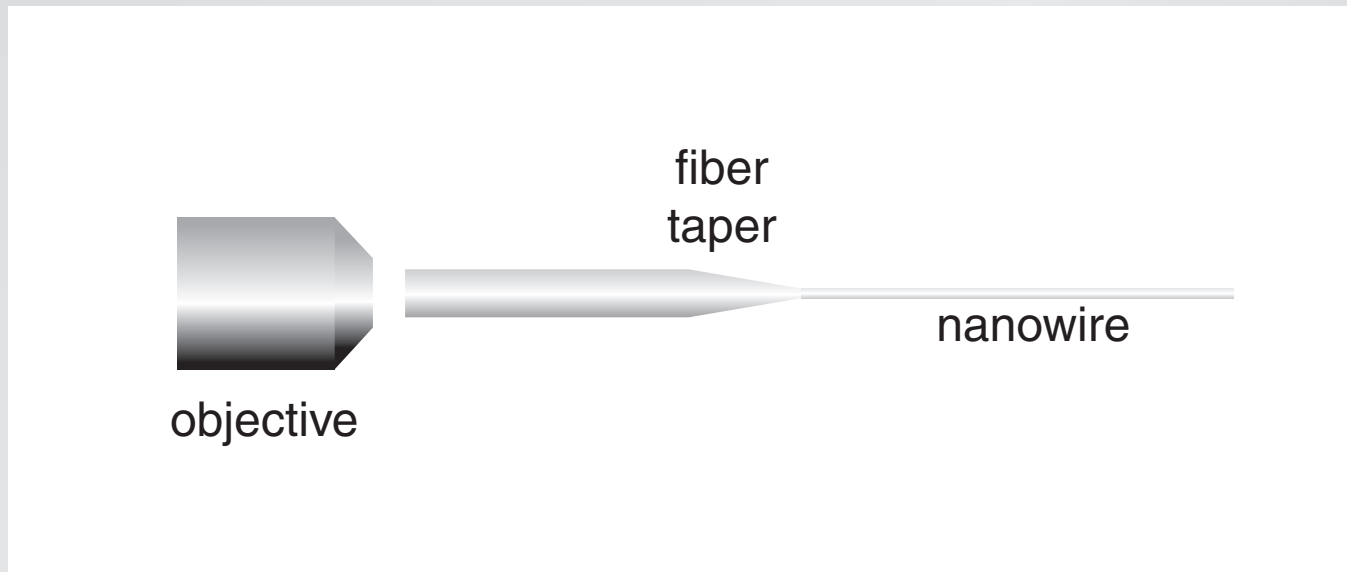
Optical properties

coupling light into nanowires



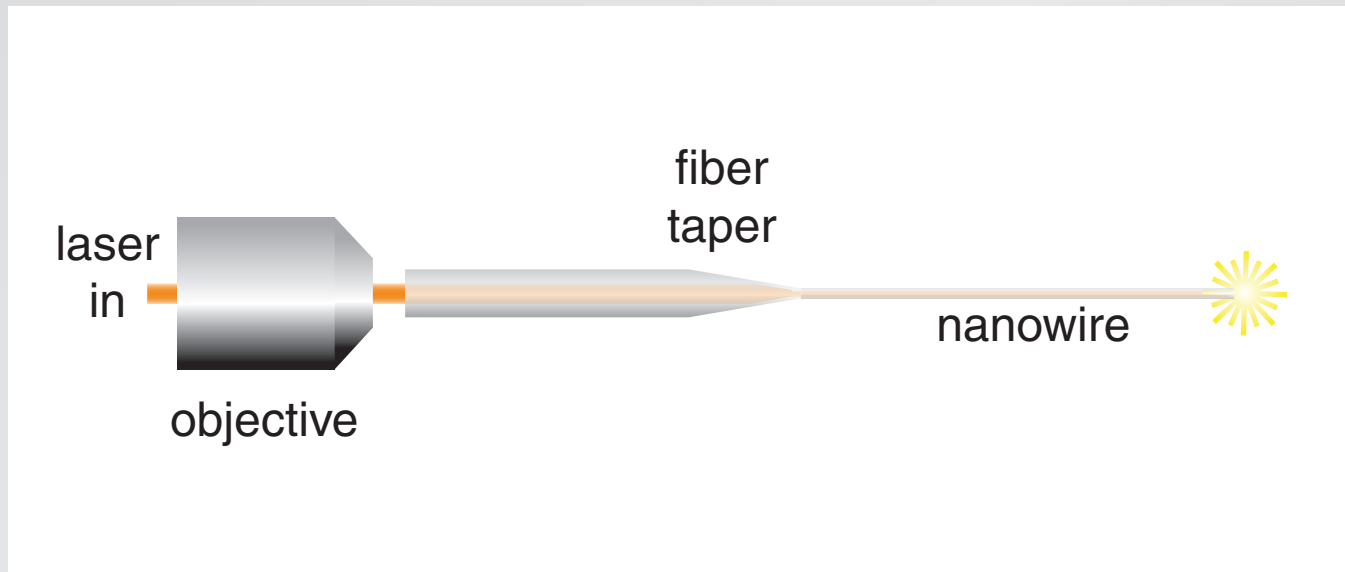
Optical properties

coupling light into nanowires



Optical properties

coupling light into nanowires



Optical properties

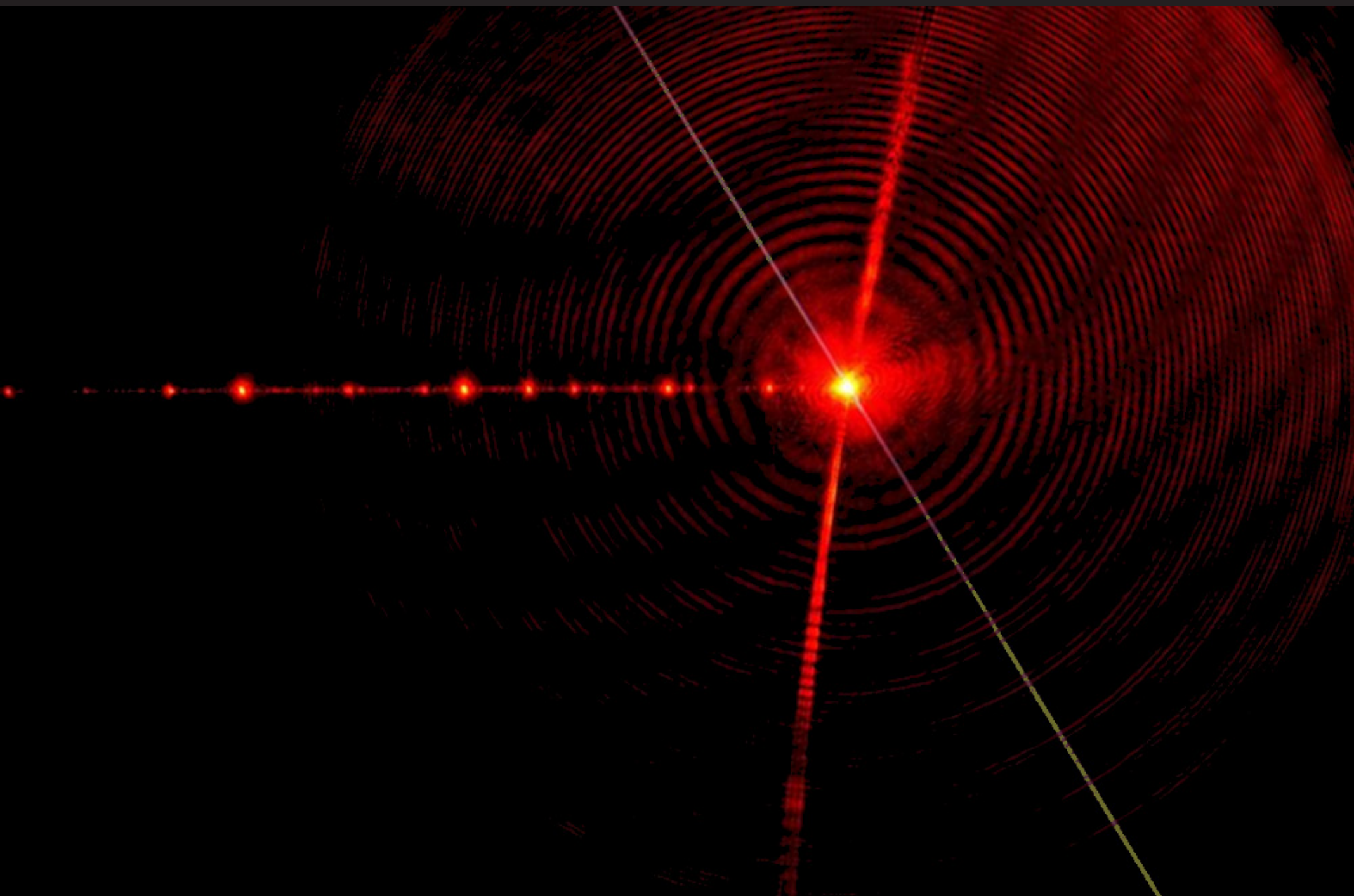
280-nm nanowire



360 nm

450 nm

Optical properties

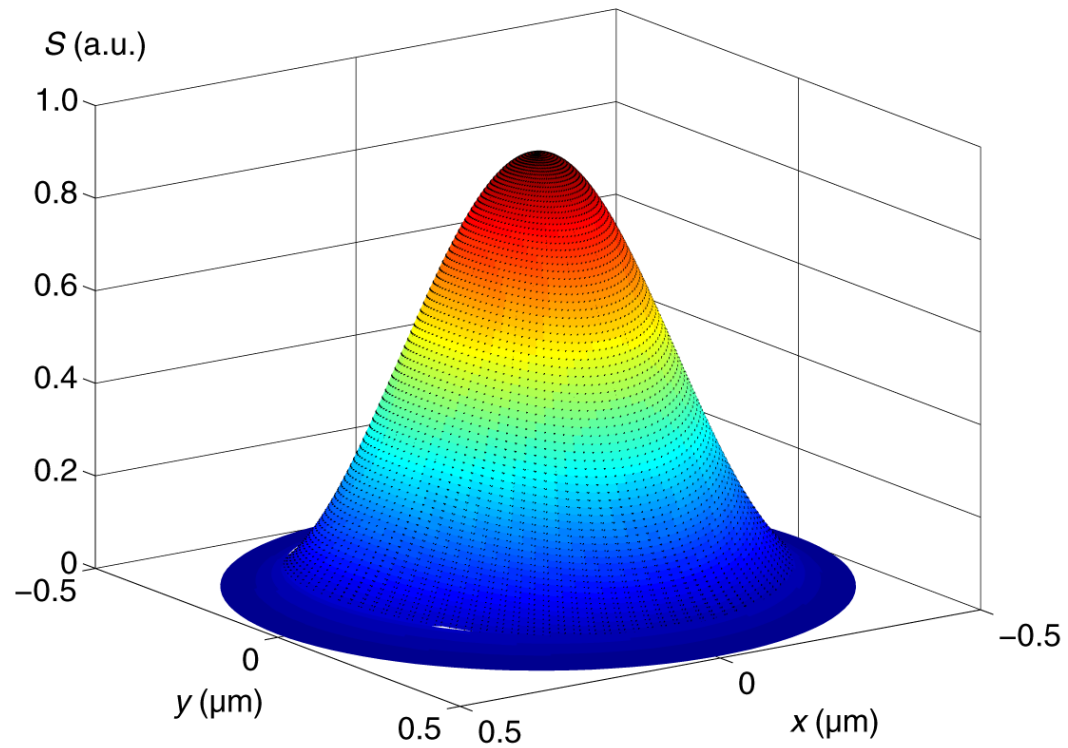


Optical properties



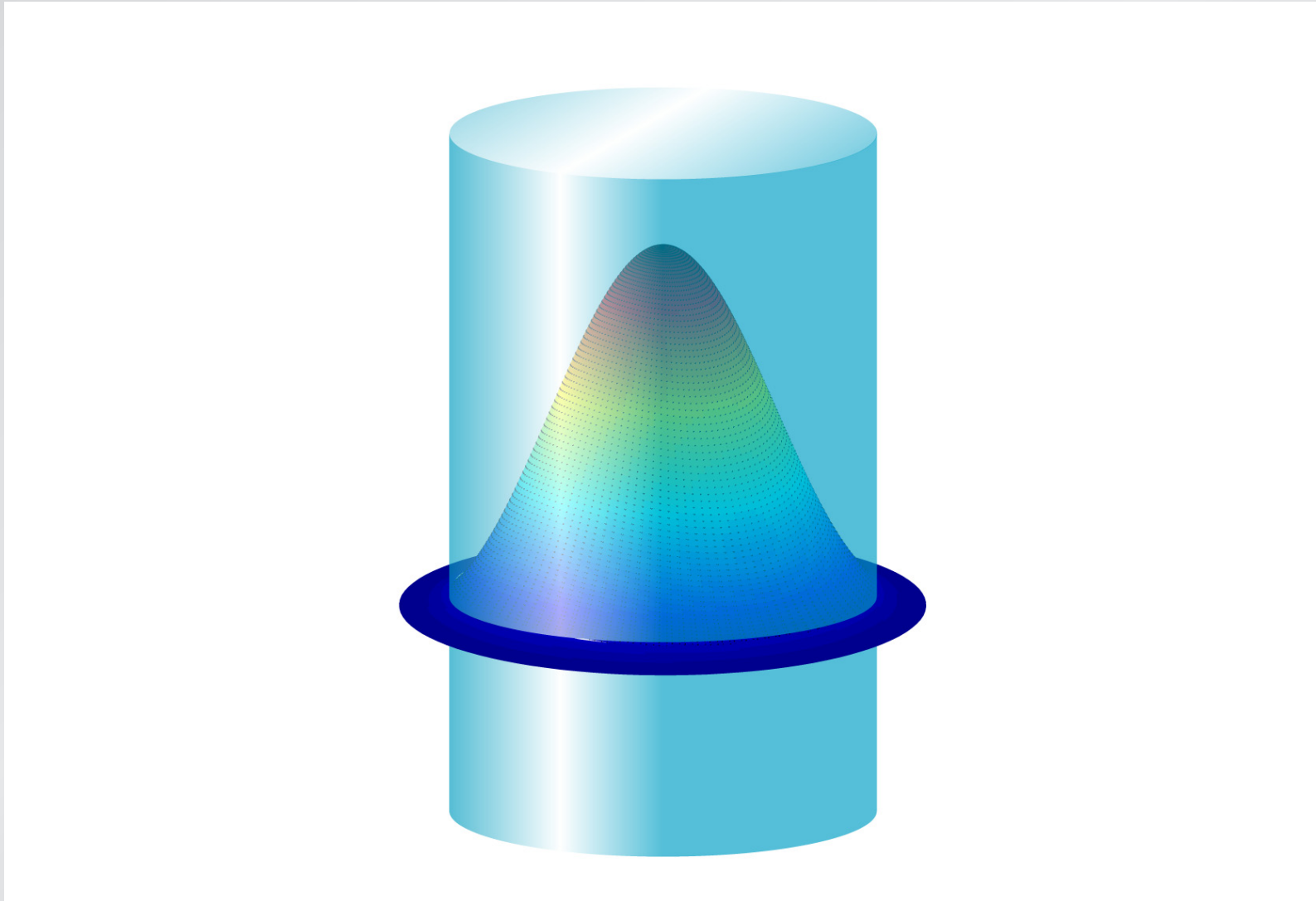
Optical properties

Poynting vector profile for 800-nm nanowire



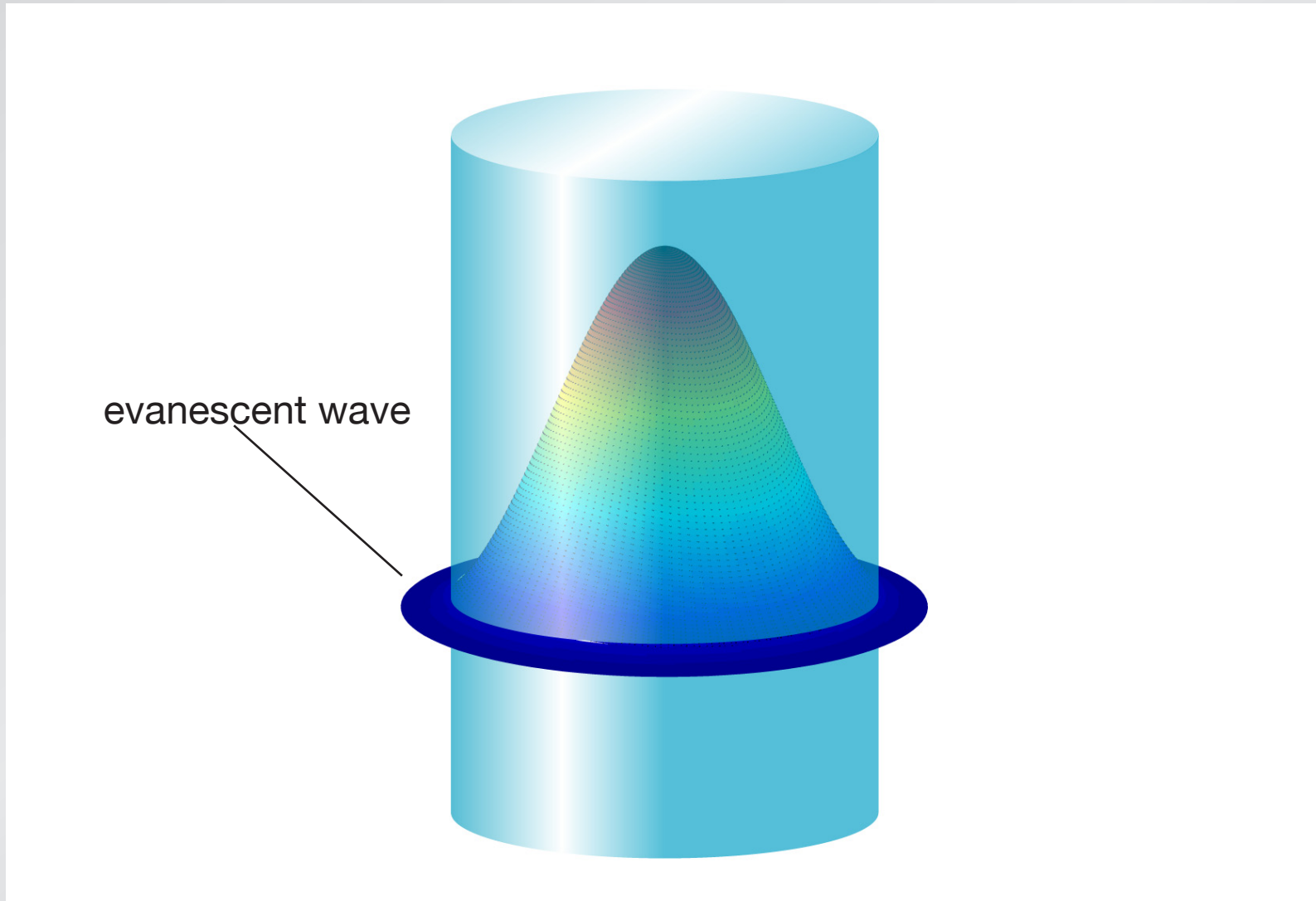
Optical properties

Poynting vector profile for 800-nm nanowire



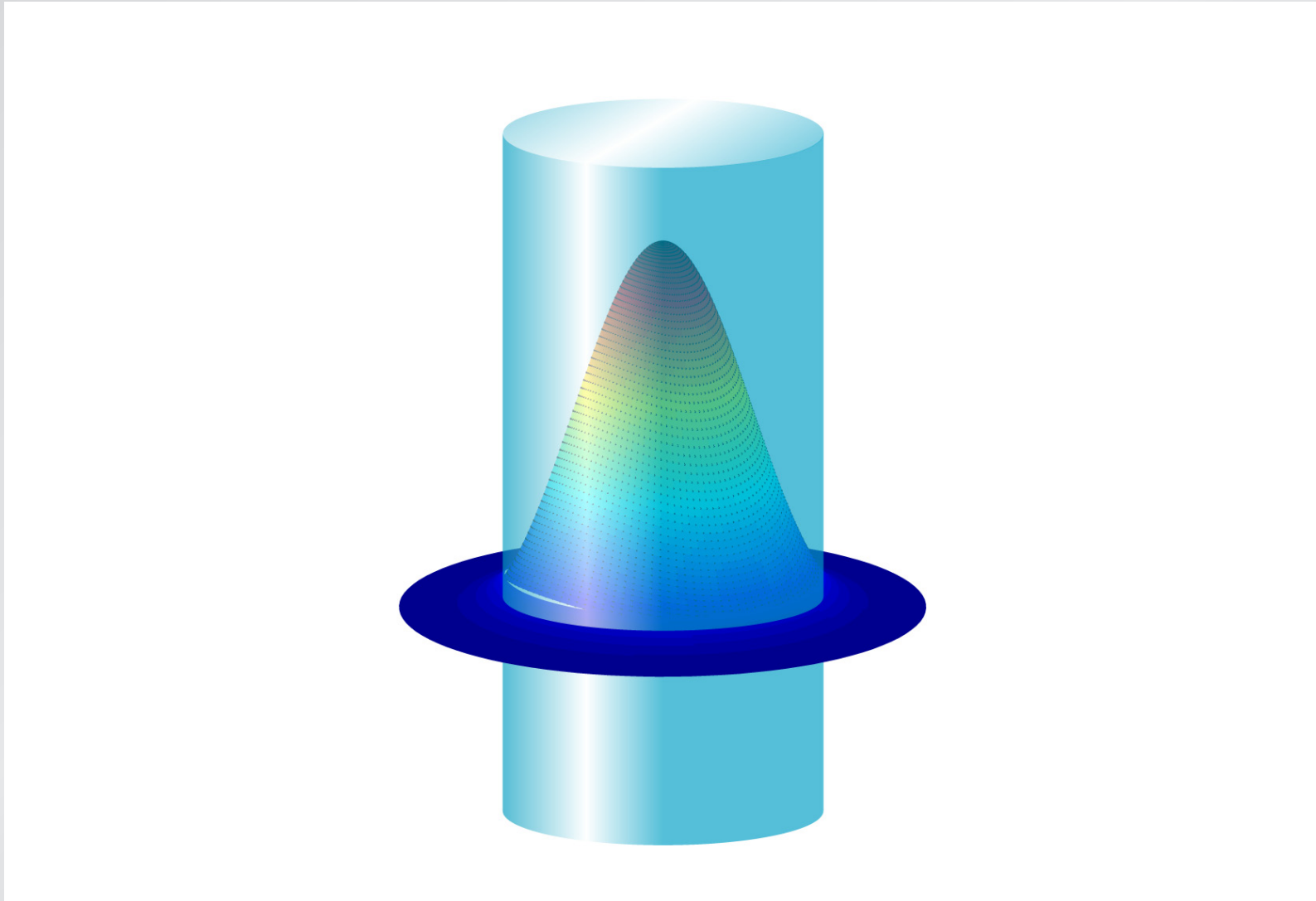
Optical properties

Poynting vector profile for 800-nm nanowire



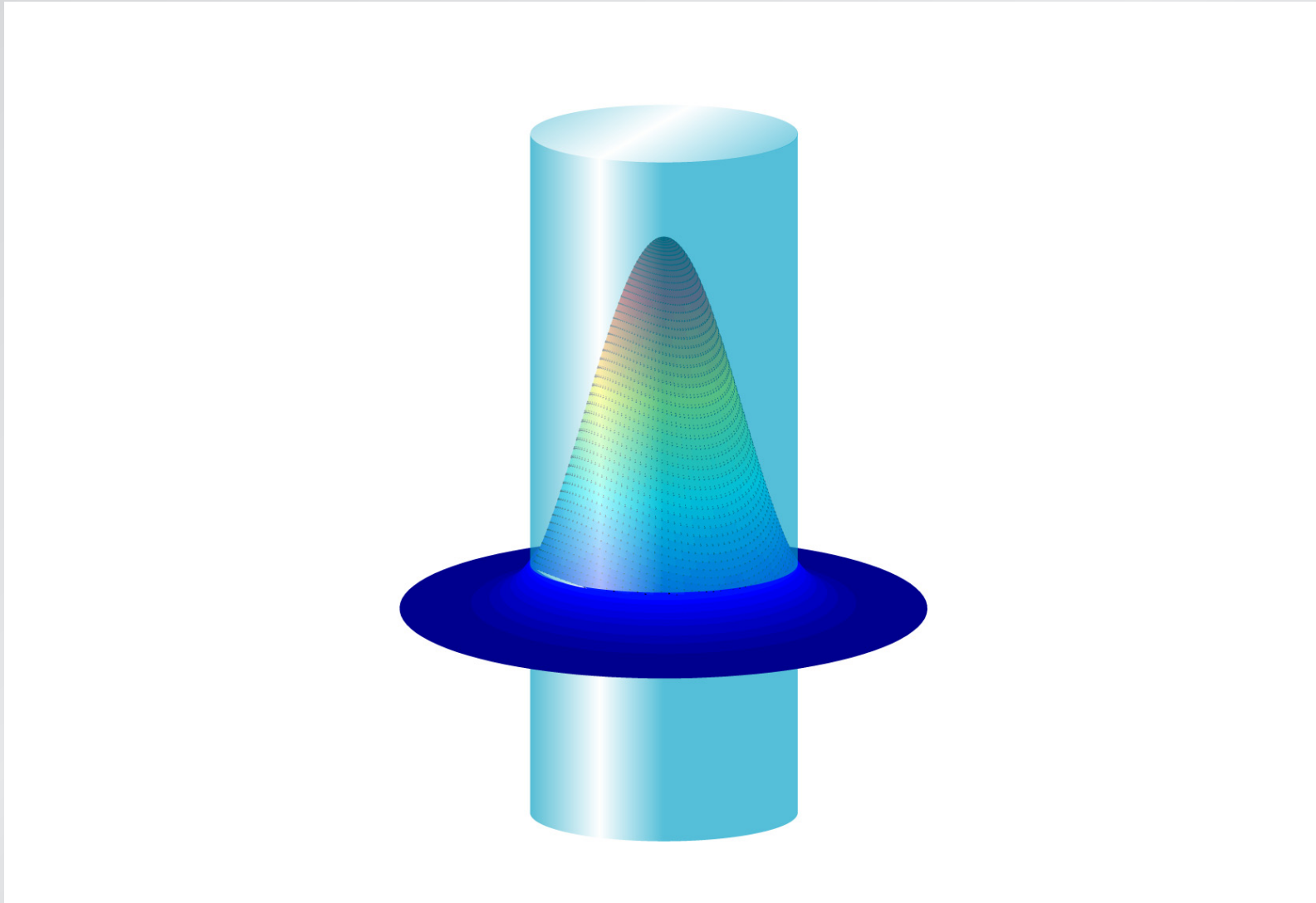
Optical properties

Poynting vector profile for 600-nm nanowire



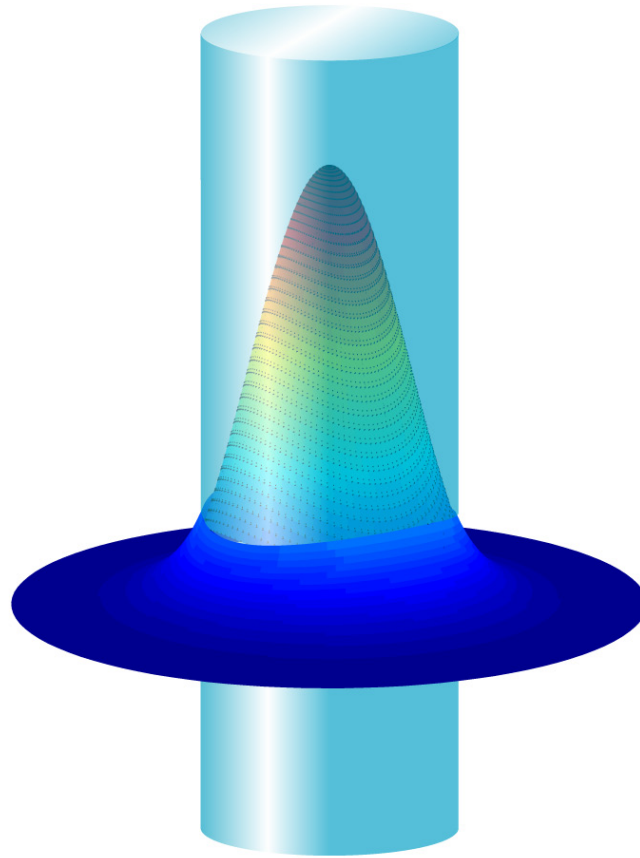
Optical properties

Poynting vector profile for 500-nm nanowire



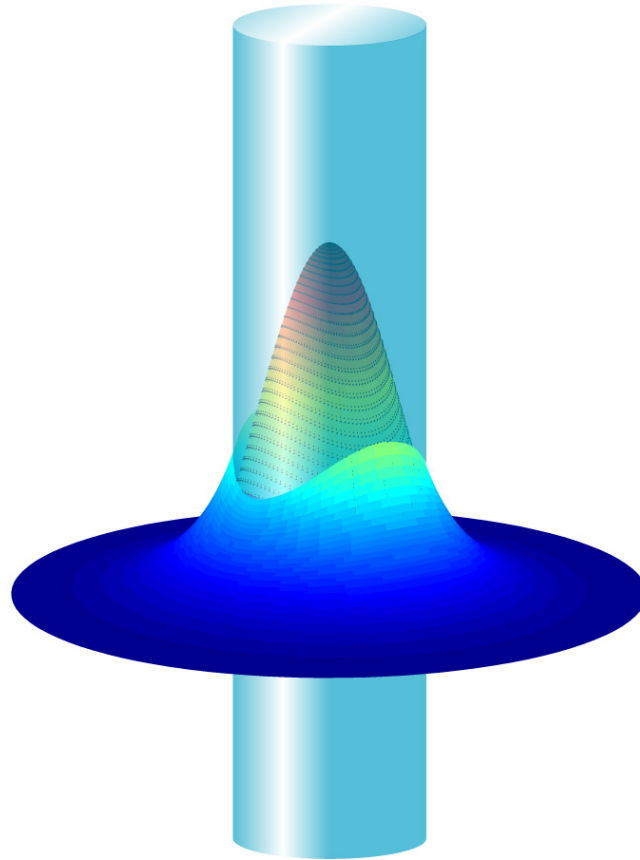
Optical properties

Poynting vector profile for 400-nm nanowire



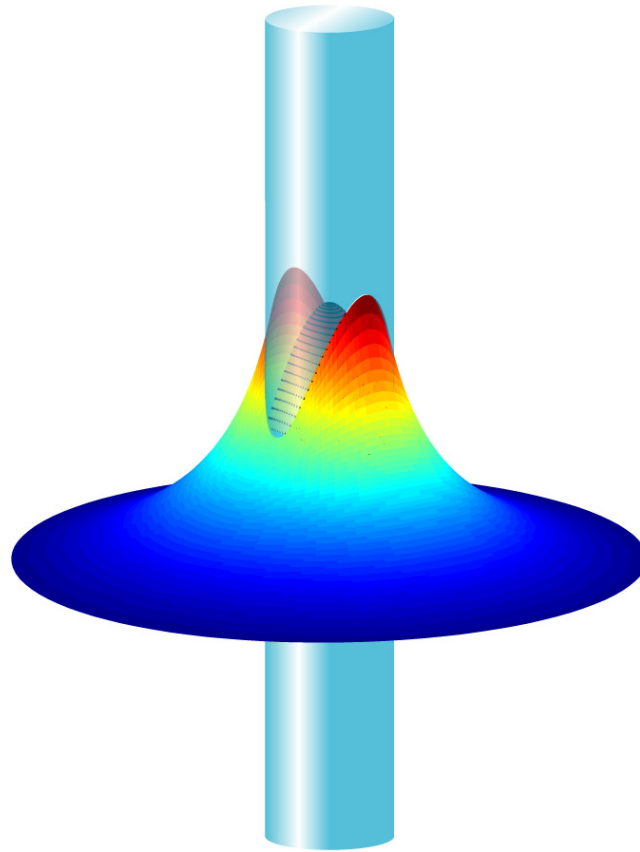
Optical properties

Poynting vector profile for 300-nm nanowire



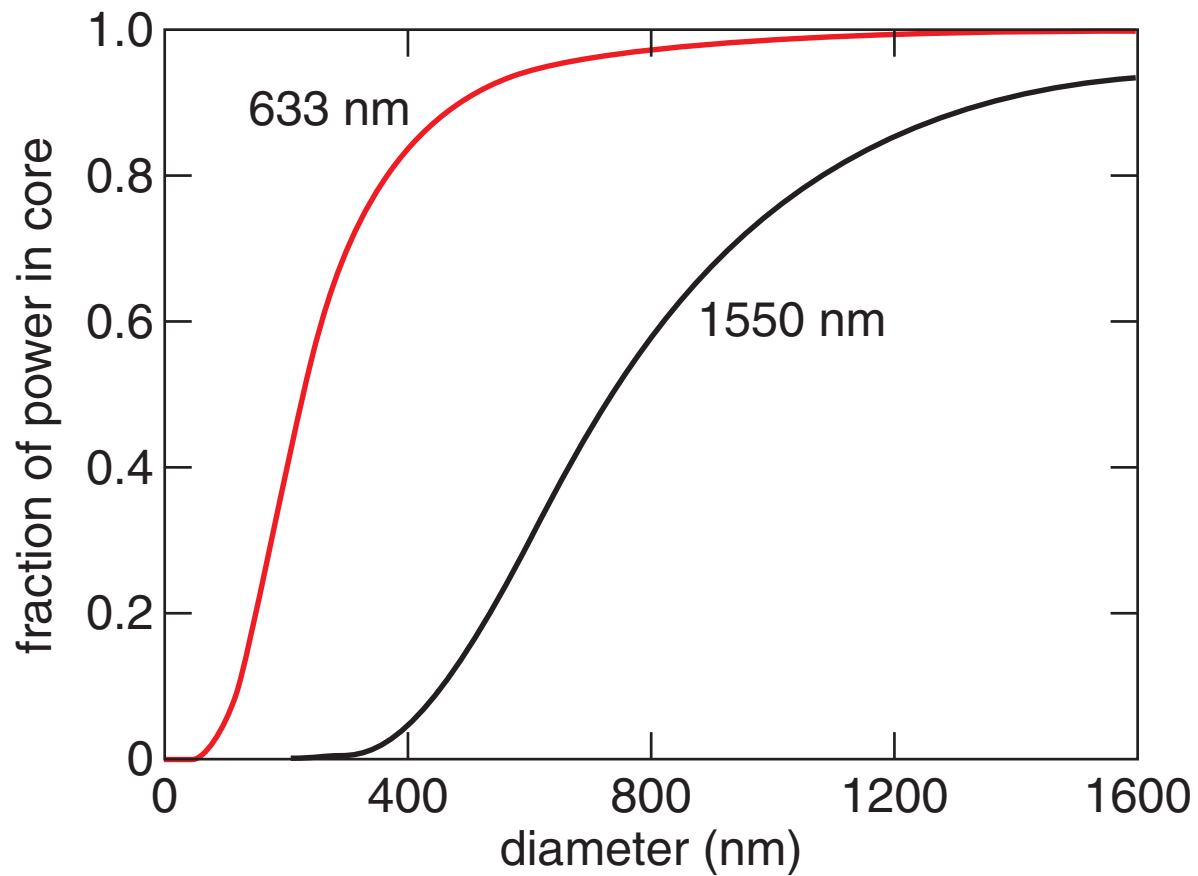
Optical properties

Poynting vector profile for 200-nm nanowire



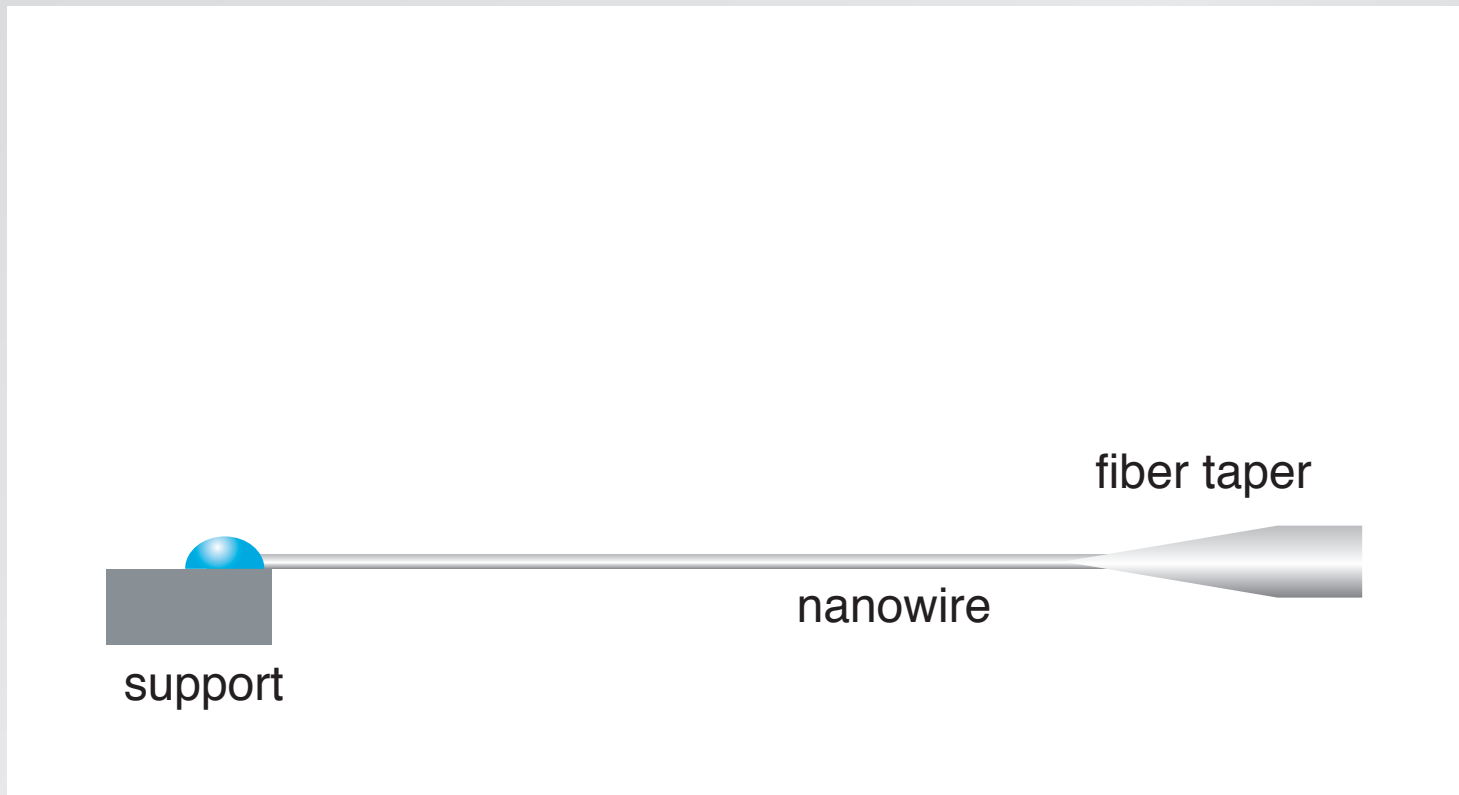
Waveguiding

fraction of power carried in core



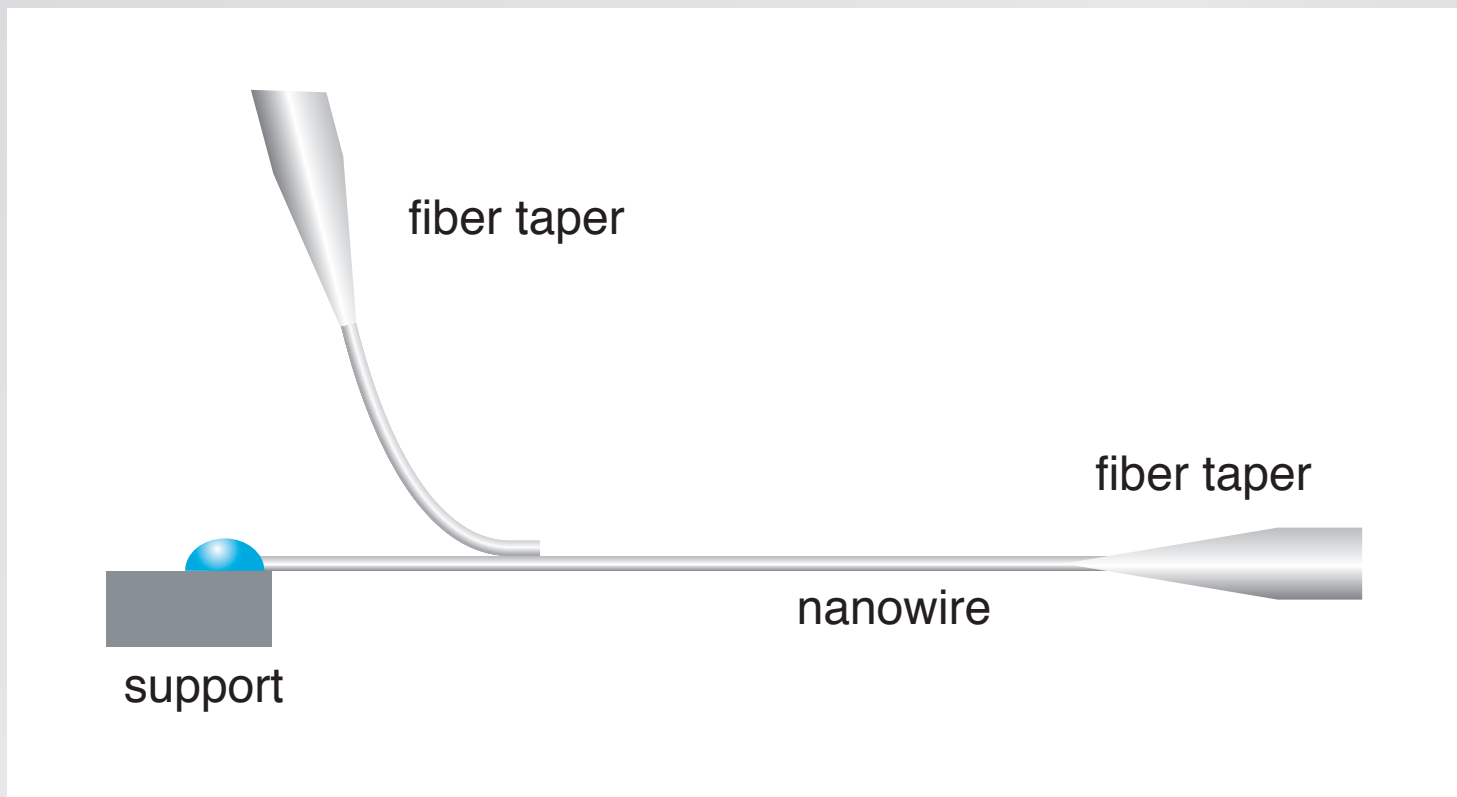
Optical properties

coupling light between nanowires



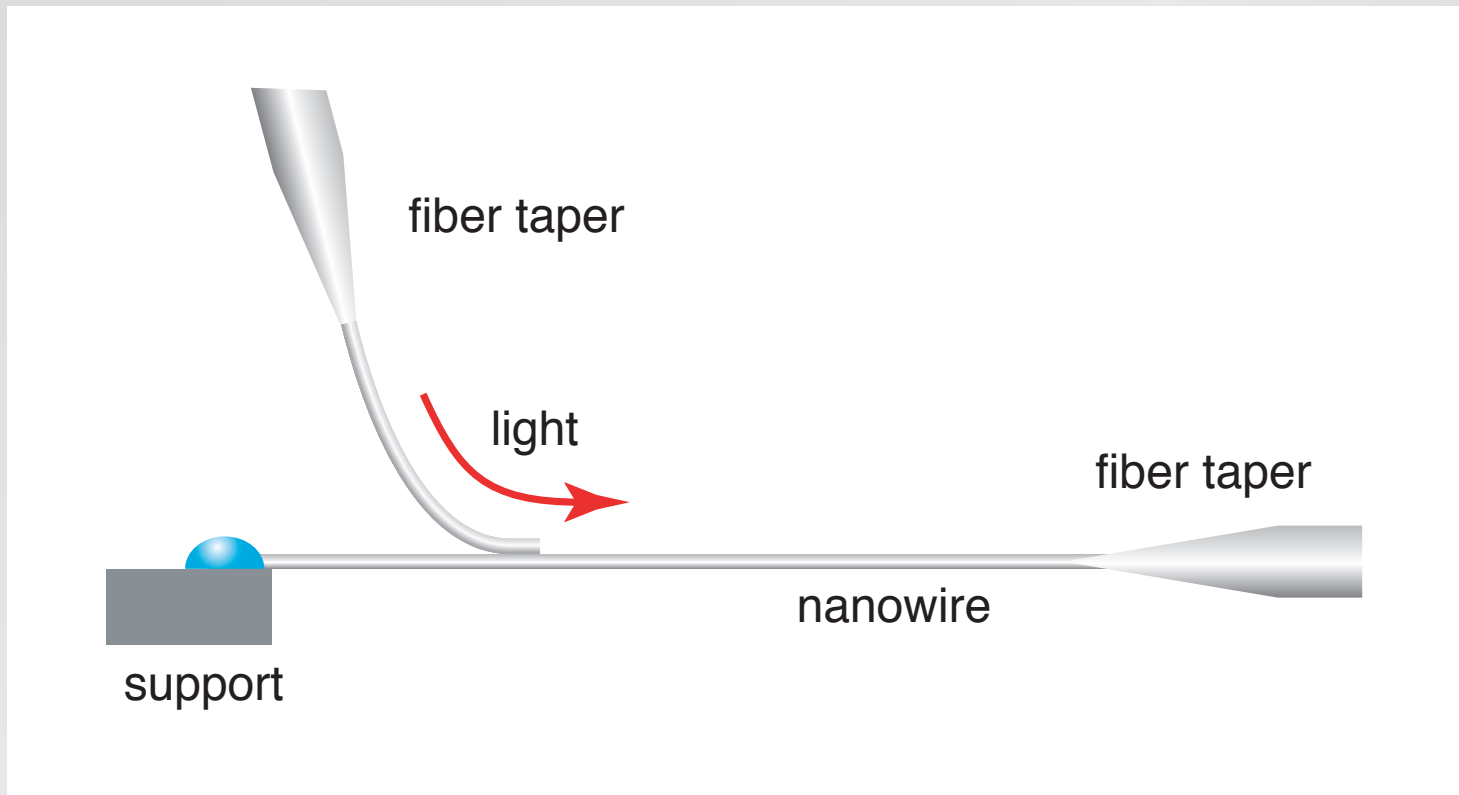
Optical properties

coupling light between nanowires



Optical properties

coupling light between nanowires



Optical properties



50 μm

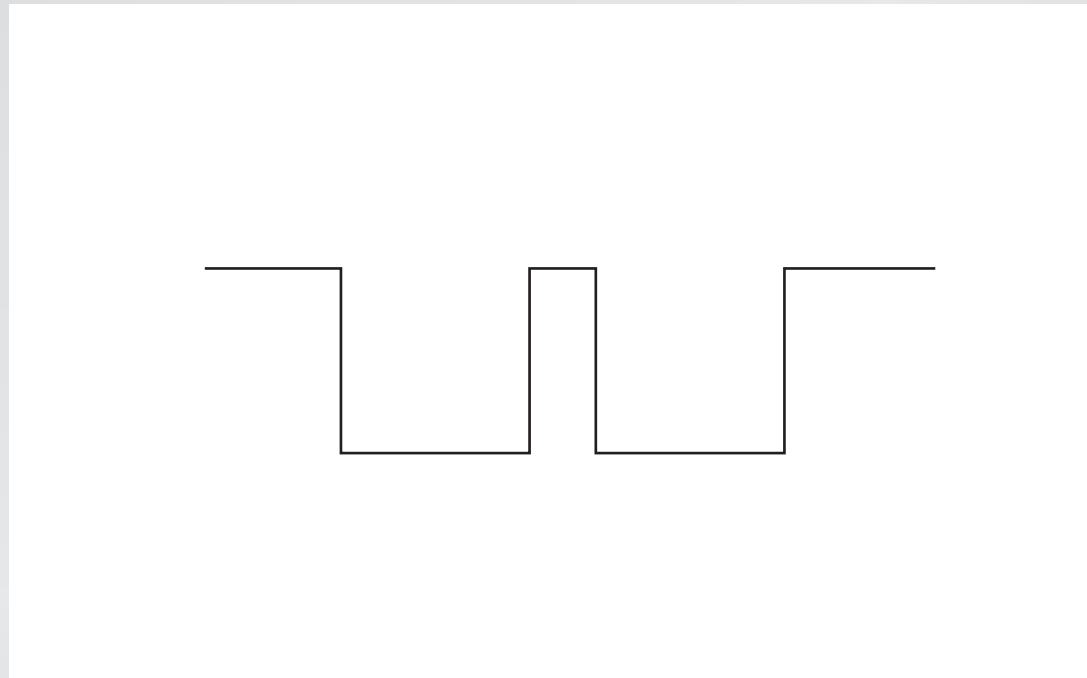
Optical properties



50 μm

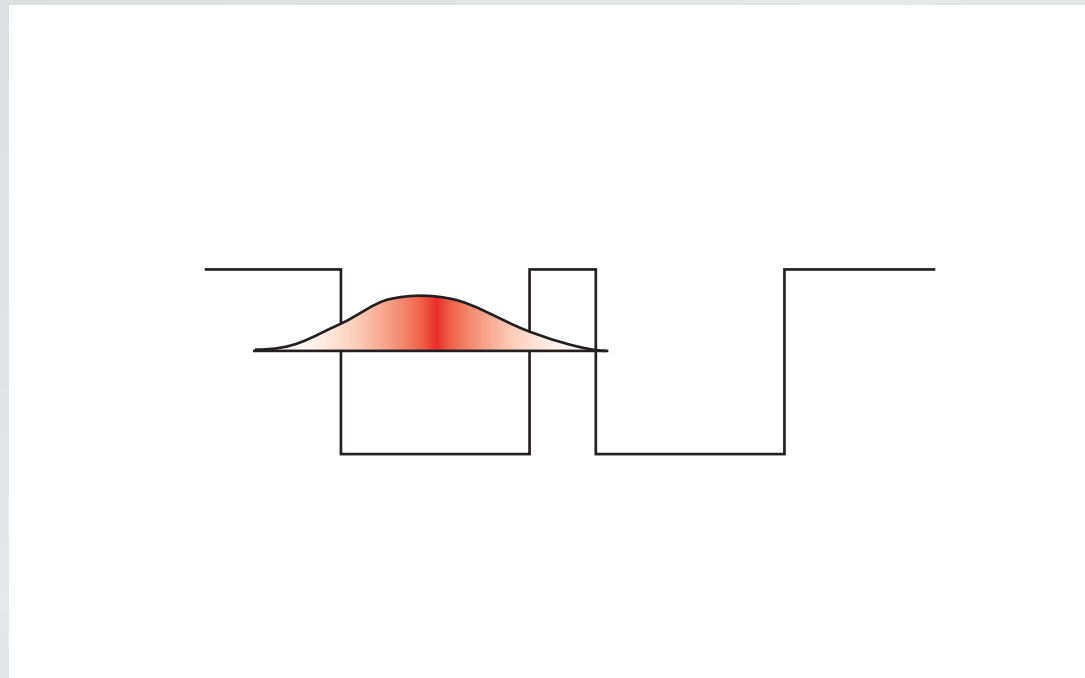
Optical properties

coupling light between nanowires



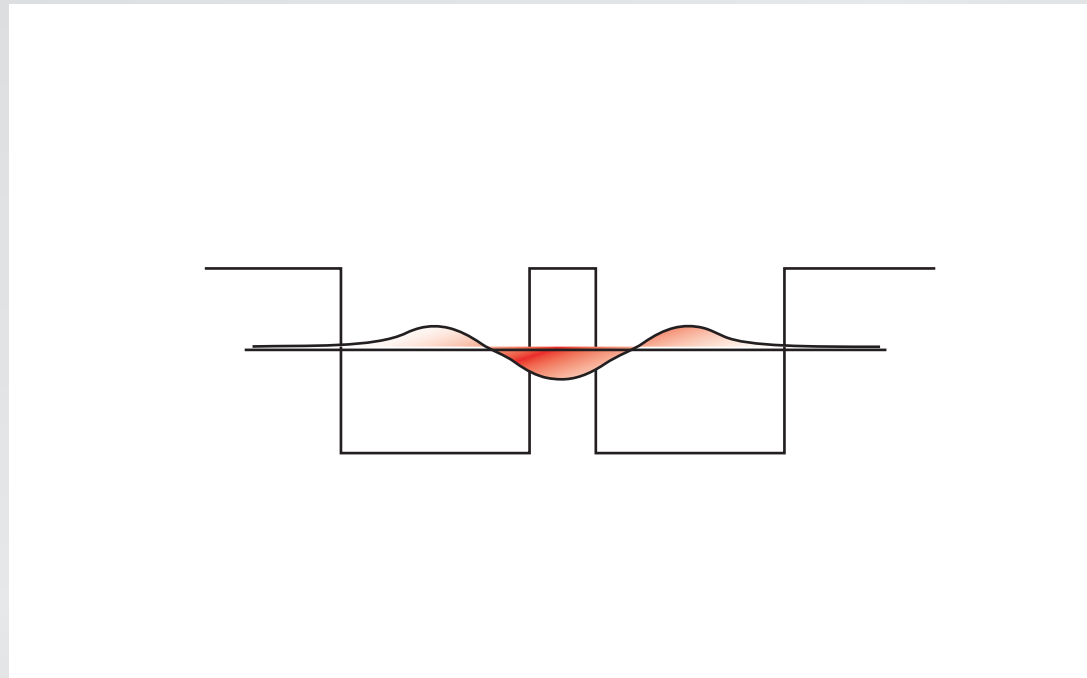
Optical properties

coupling light between nanowires



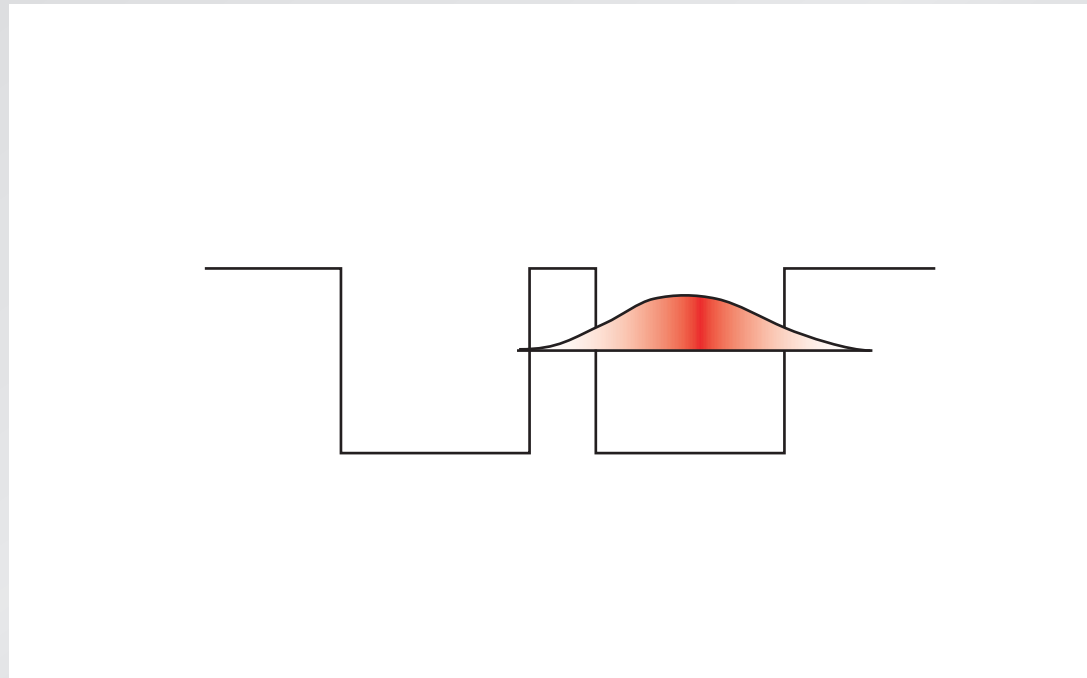
Optical properties

“tunneling” of light



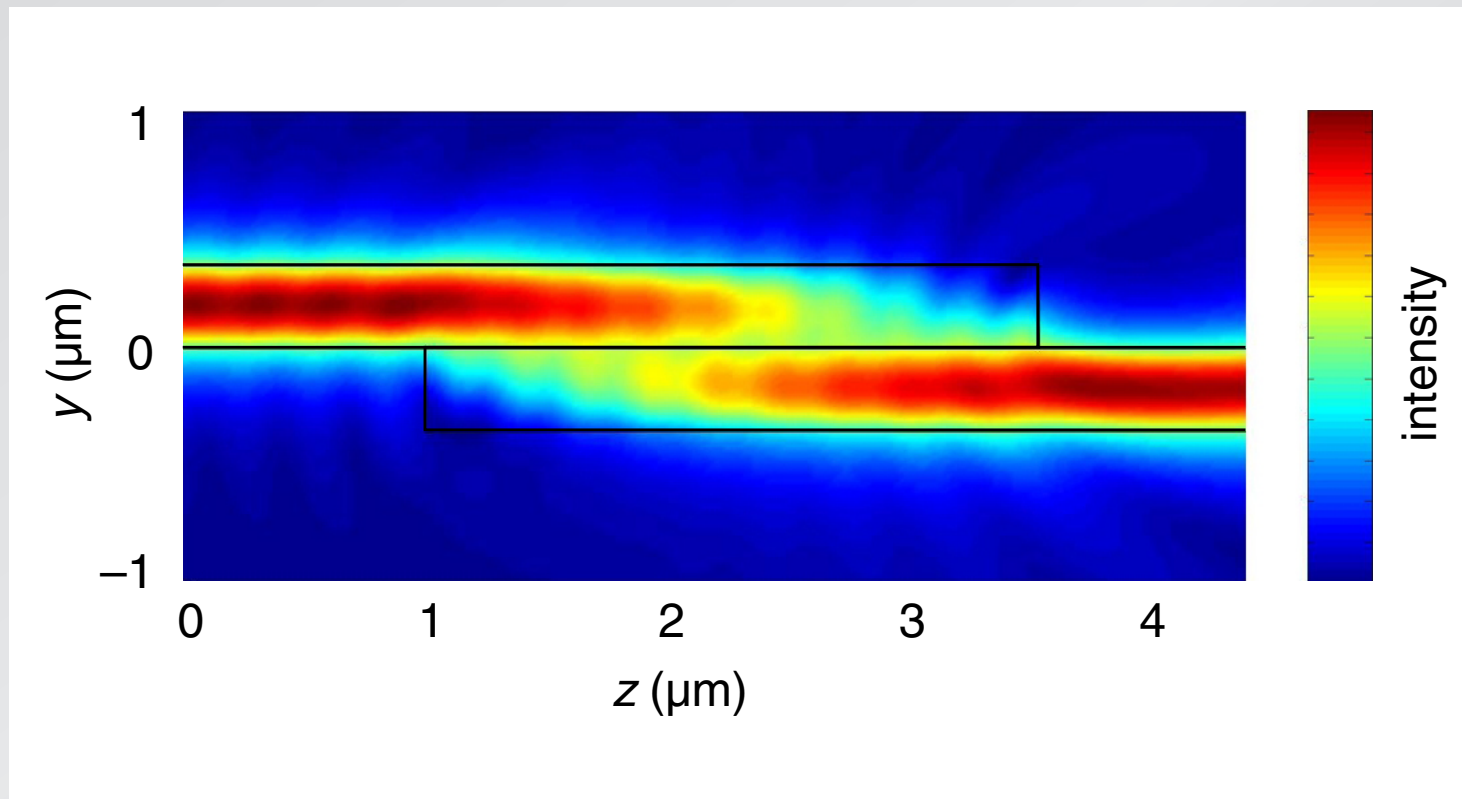
Optical properties

“tunneling” of light

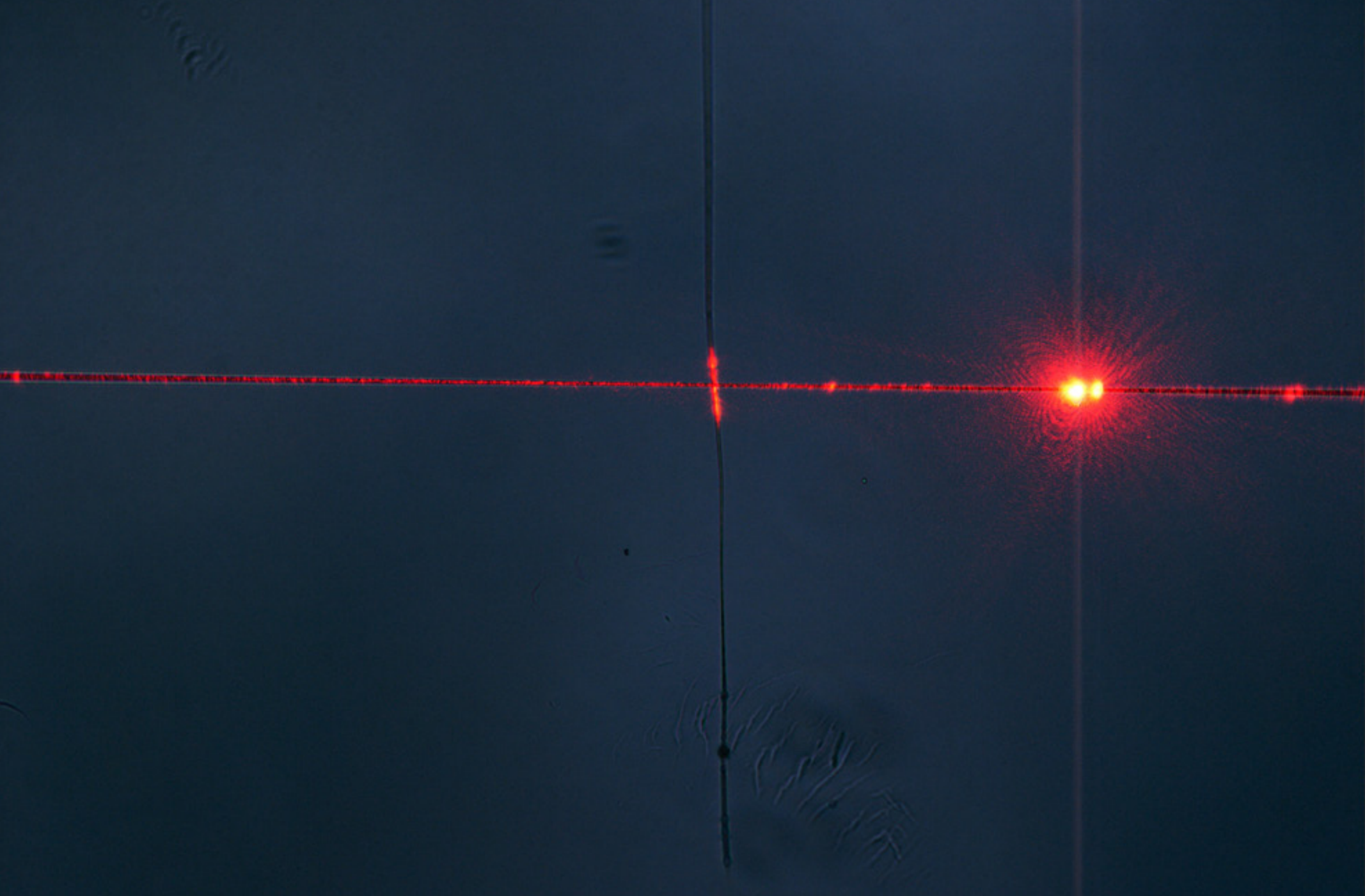


Optical properties

intensity distribution

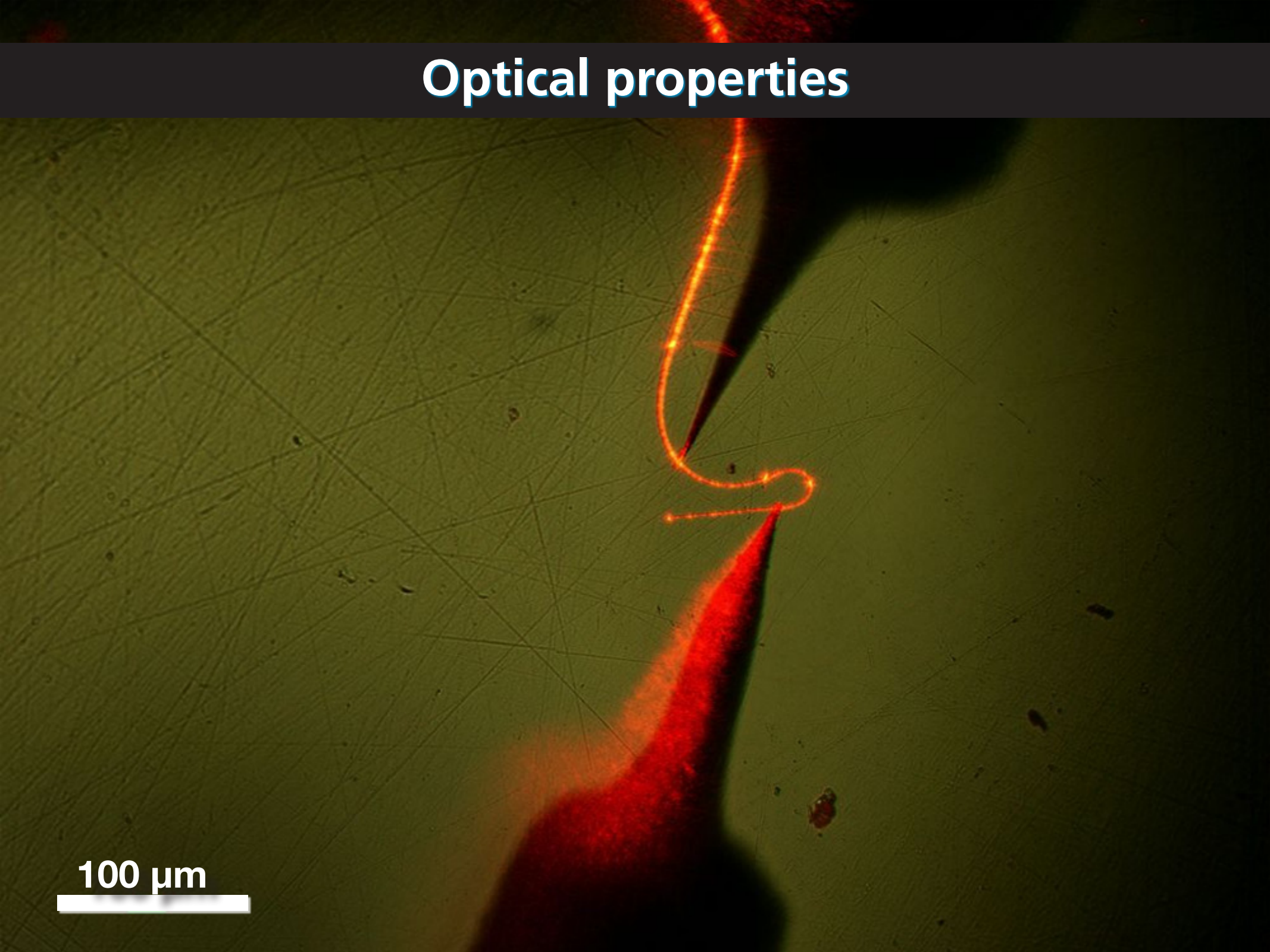


Optical properties

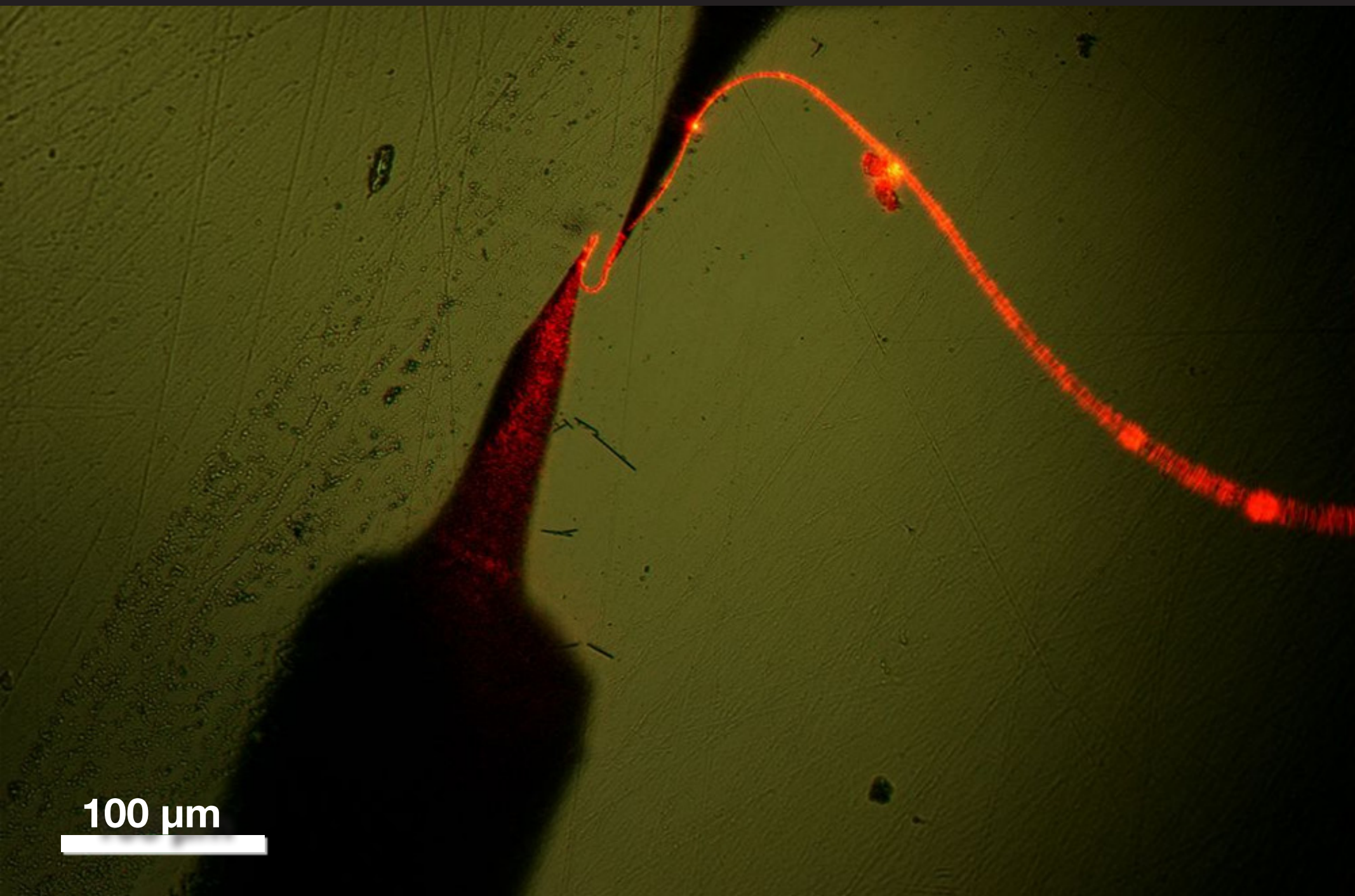


Optical properties

100 μm

An optical micrograph showing a fiber optic tip on the right side, emitting a red laser beam. The beam is directed upwards and then forms a series of loops and curves in the air, demonstrating light confinement and manipulation. The background is a textured, olive-green surface. A white scale bar is located in the bottom left corner.

Optical properties

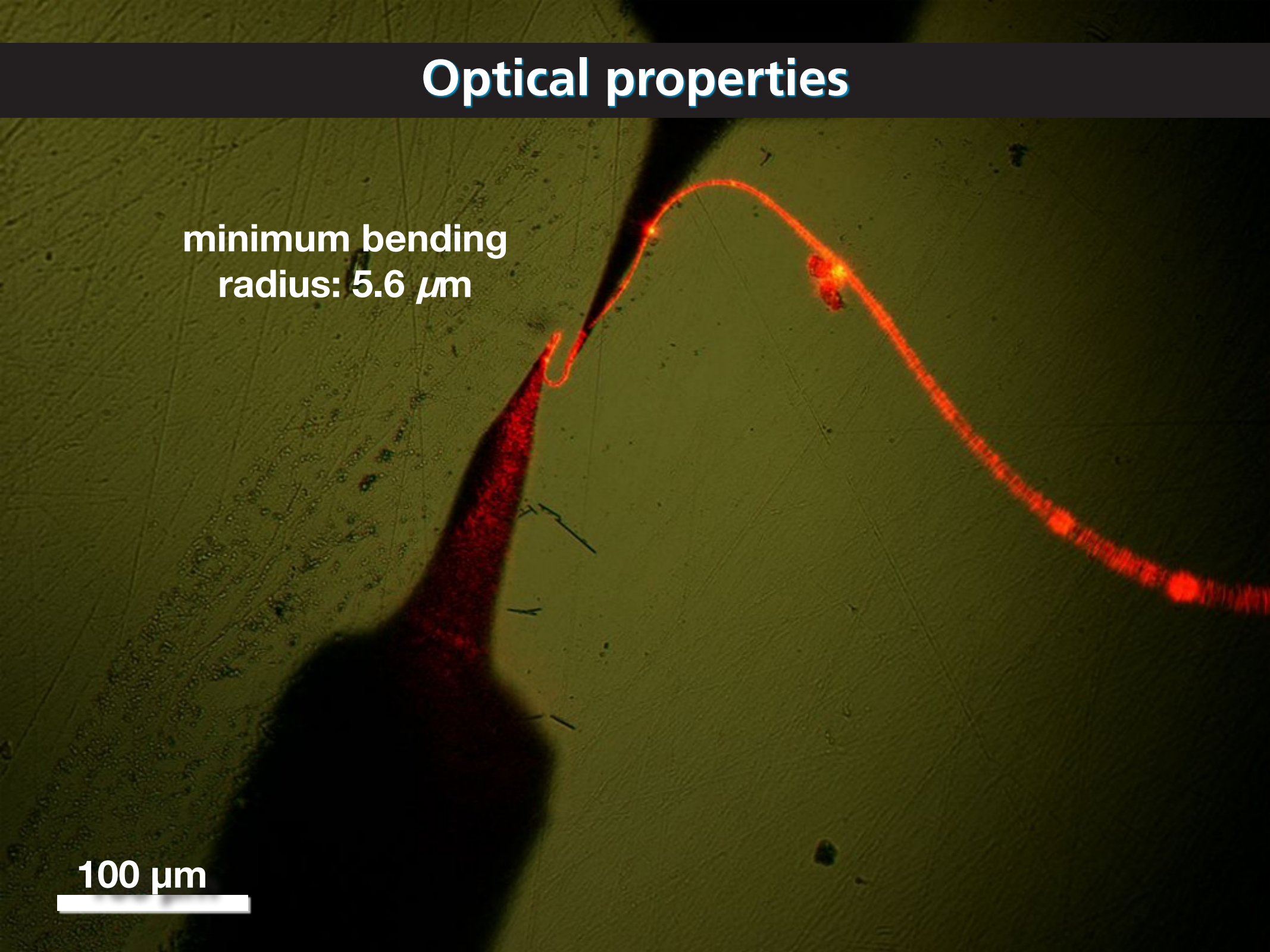


100 μm

Optical properties

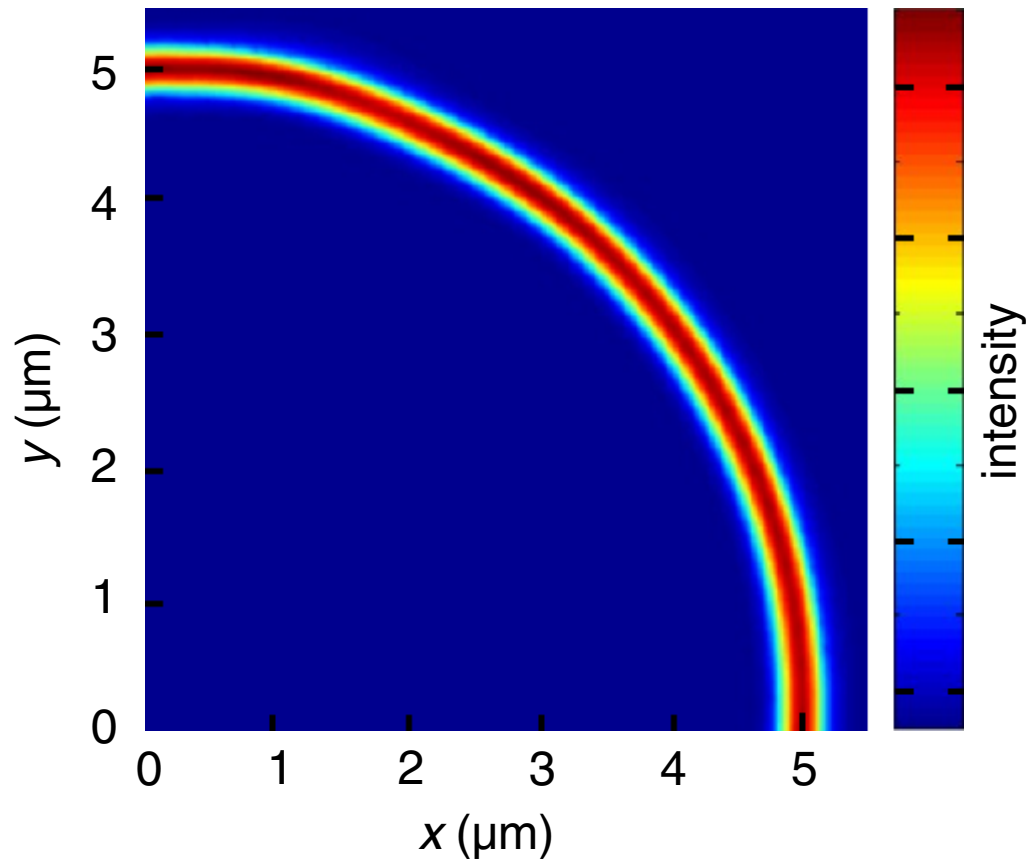
minimum bending
radius: $5.6 \mu\text{m}$

100 μm

An optical micrograph showing a fiber optic cable. A bright red laser spot is focused on the fiber, creating a visible light path that curves through the fiber. The fiber is dark against a light background. A scale bar in the bottom left corner indicates 100 micrometers. Text in the upper left corner states the minimum bending radius is 5.6 micrometers.

Optical properties

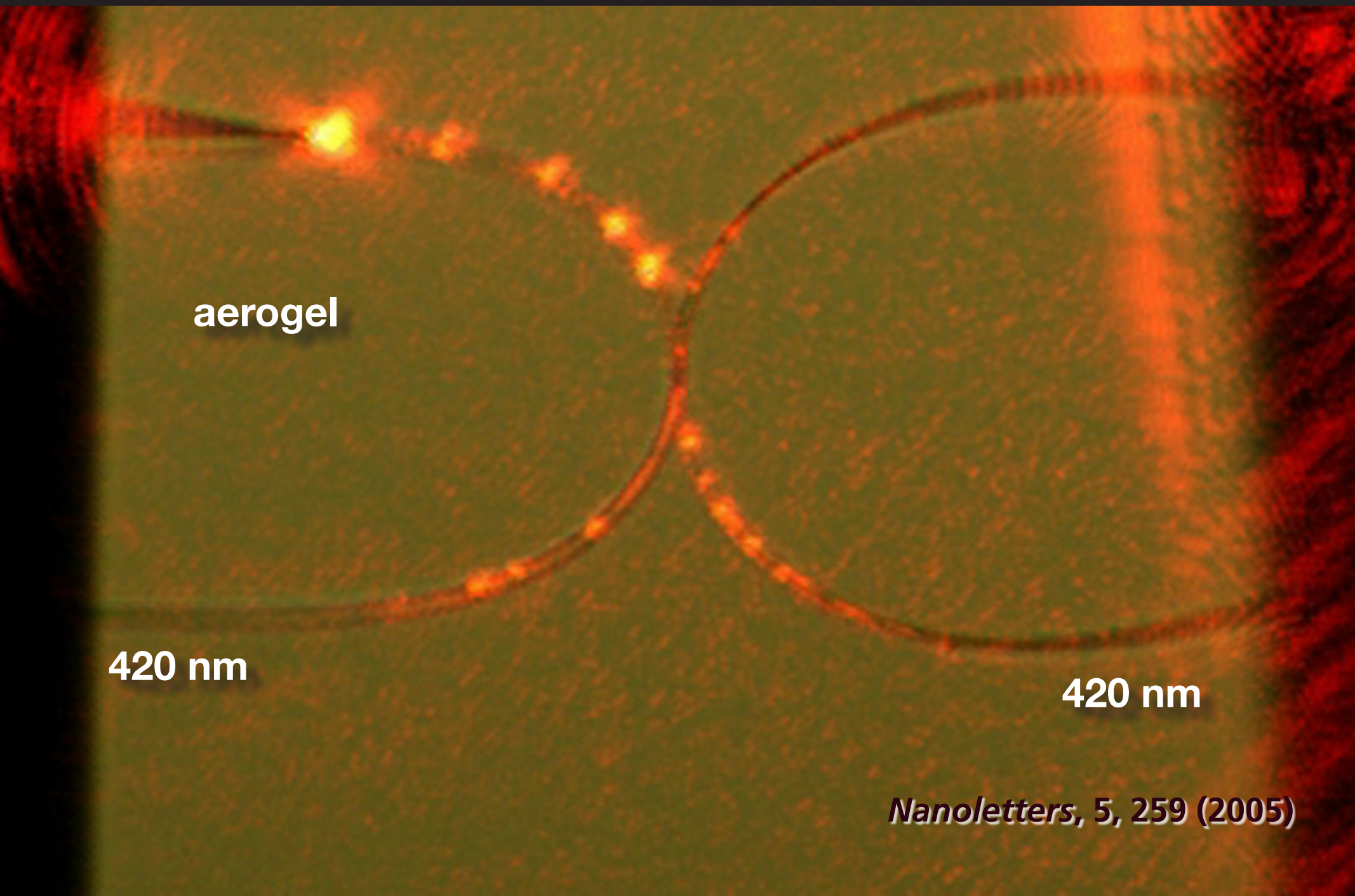
virtually no loss through 5 μm corner!



Manipulating light at the nanoscale



Manipulating light at the nanoscale



aerogel

420 nm

420 nm

Nanoletters, 5, 259 (2005)

Manipulating light at the nanoscale

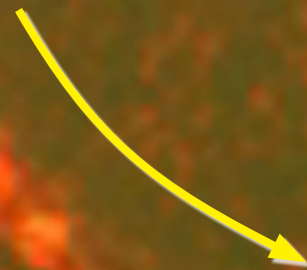
in



out

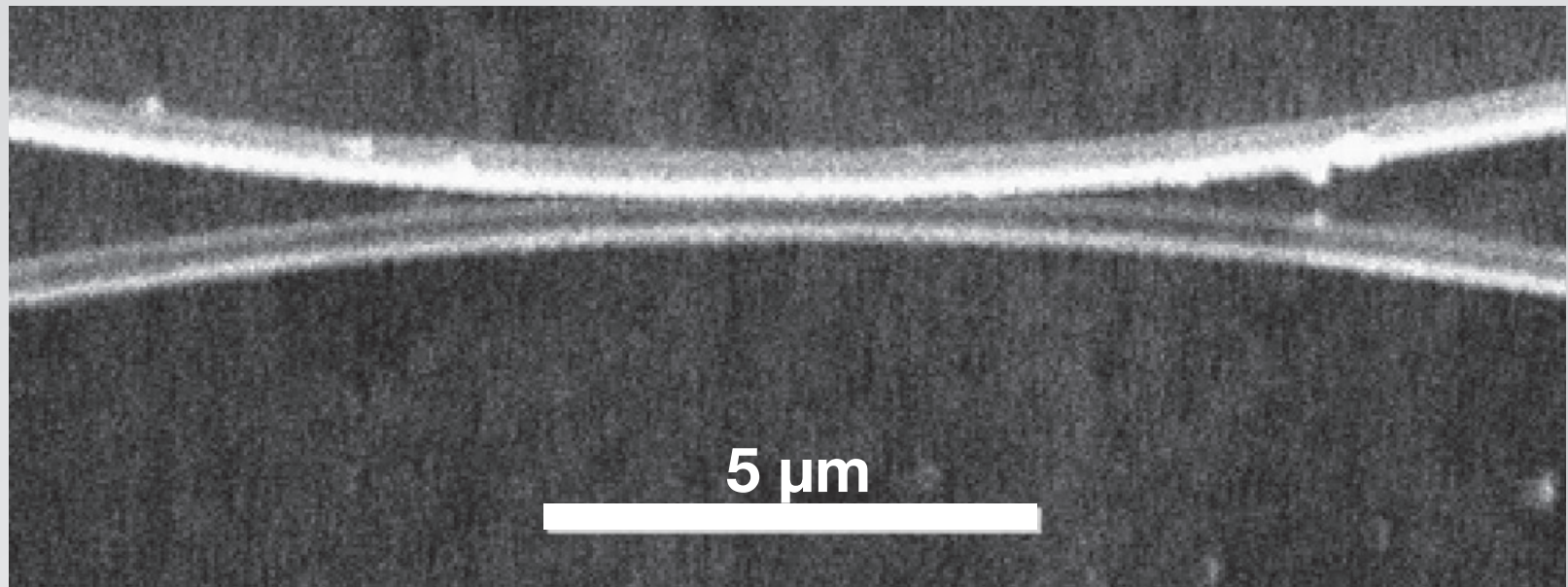


out



Nanoletters, 5, 259 (2005)

Manipulating light at the nanoscale



Nanoletters, 5, 259 (2005)

Manipulating light at the nanoscale

use tapered fibers to couple light to nanoscale objects

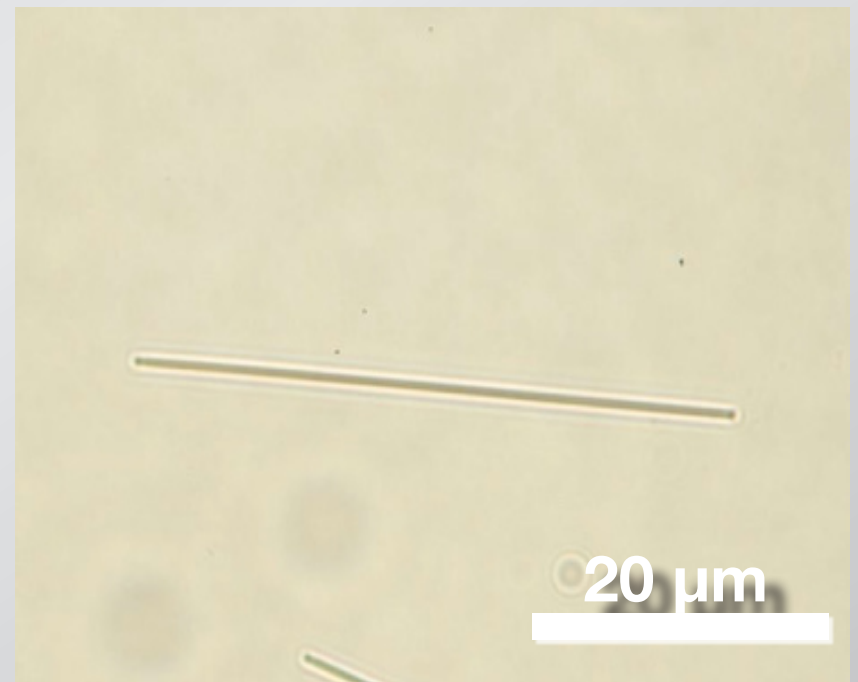
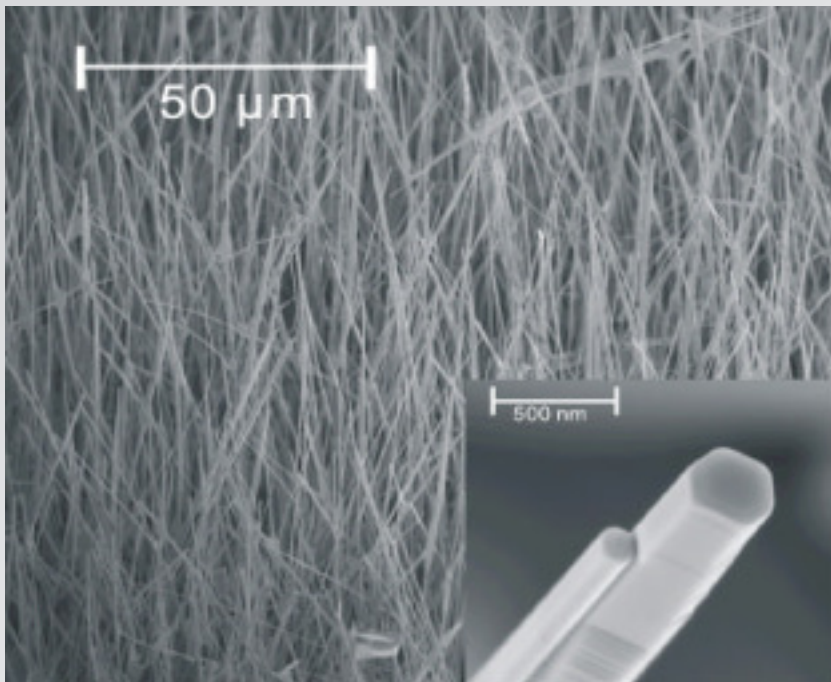
Manipulating light at the nanoscale

ZnO: non-toxic, wide bandgap semiconductor

A petri dish containing a white, powdery substance, likely ZnO nanoparticles, used for light manipulation at the nanoscale.

Manipulating light at the nanoscale

vapor transport grown ZnO nanowires



80–400 nm diameter, up to 80 μm long

Manipulating light at the nanoscale

best of both worlds

ZnO

silica

bottom-up

top-down

semiconductor

glass

active photonic devices

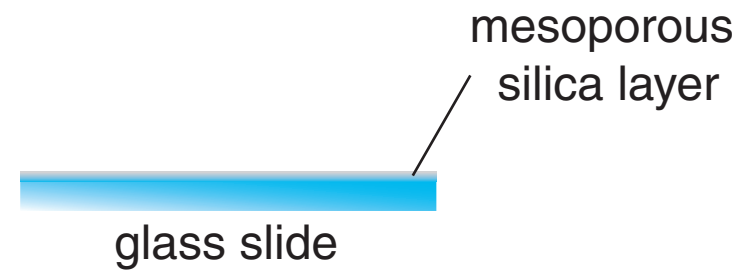
passive waveguides

electrical operation

link to macroworld

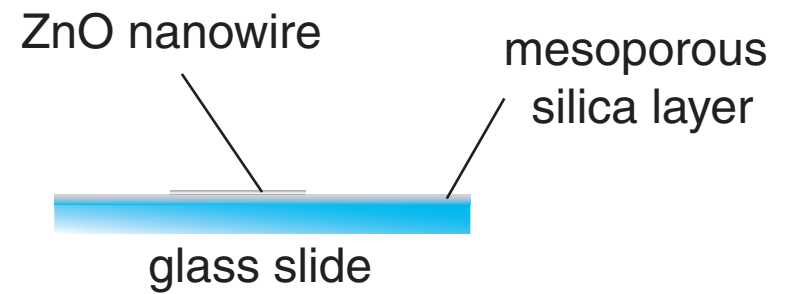
Manipulating light at the nanoscale

coupling to ZnO nanowires



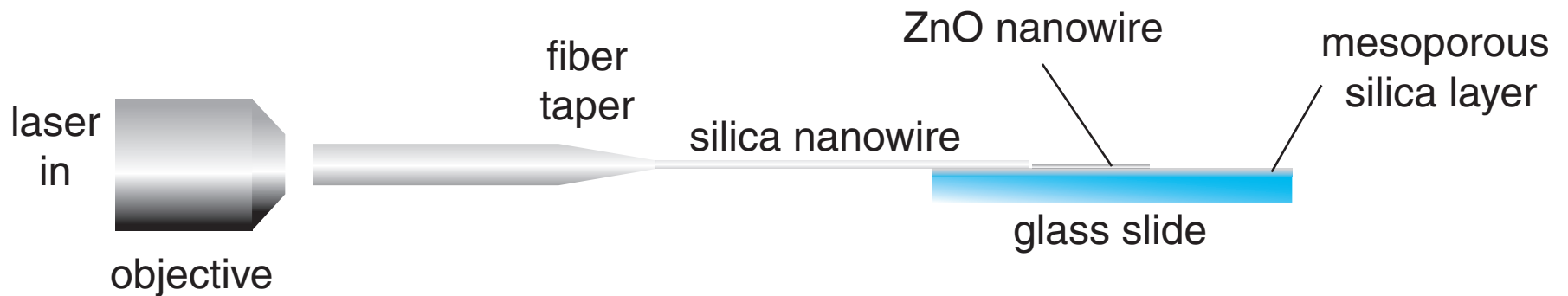
Manipulating light at the nanoscale

coupling to ZnO nanowires



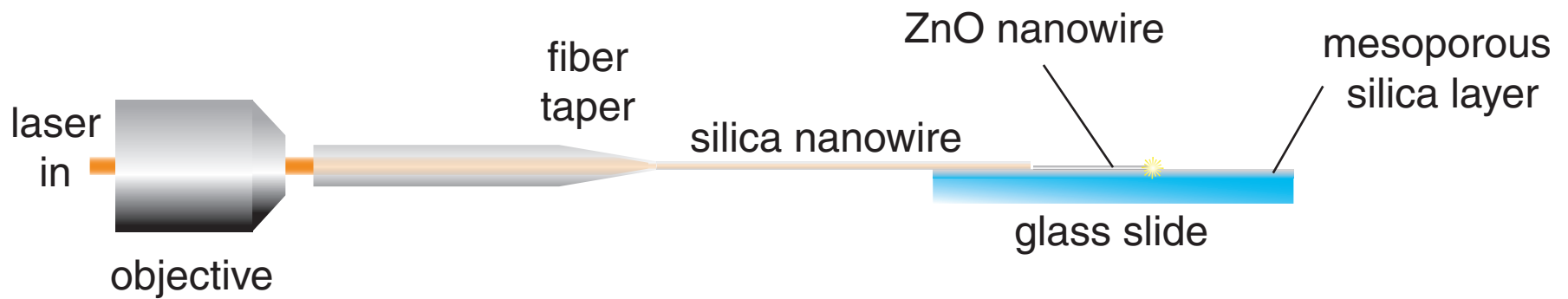
Manipulating light at the nanoscale

coupling to ZnO nanowires

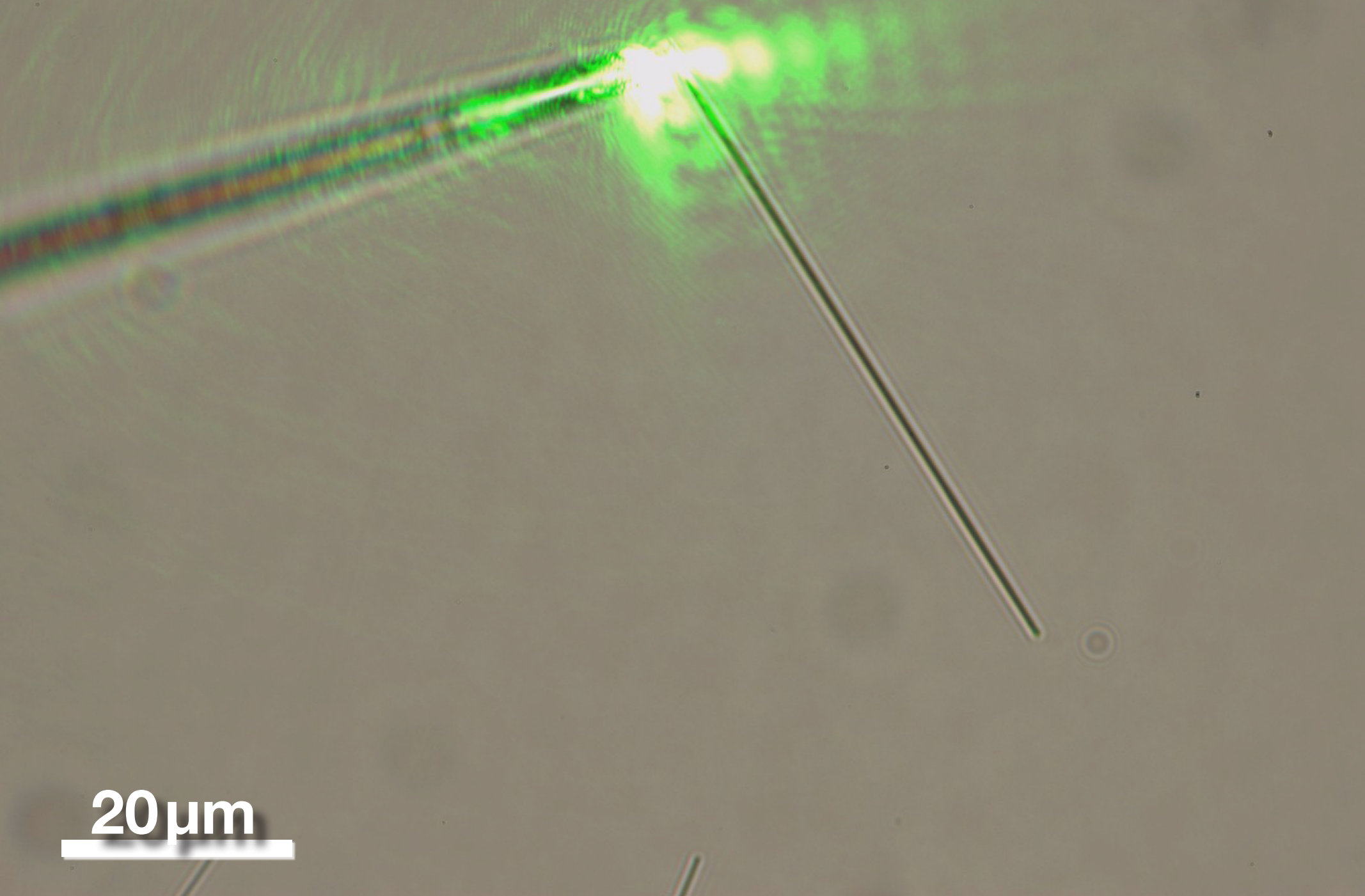


Manipulating light at the nanoscale

coupling to ZnO nanowires

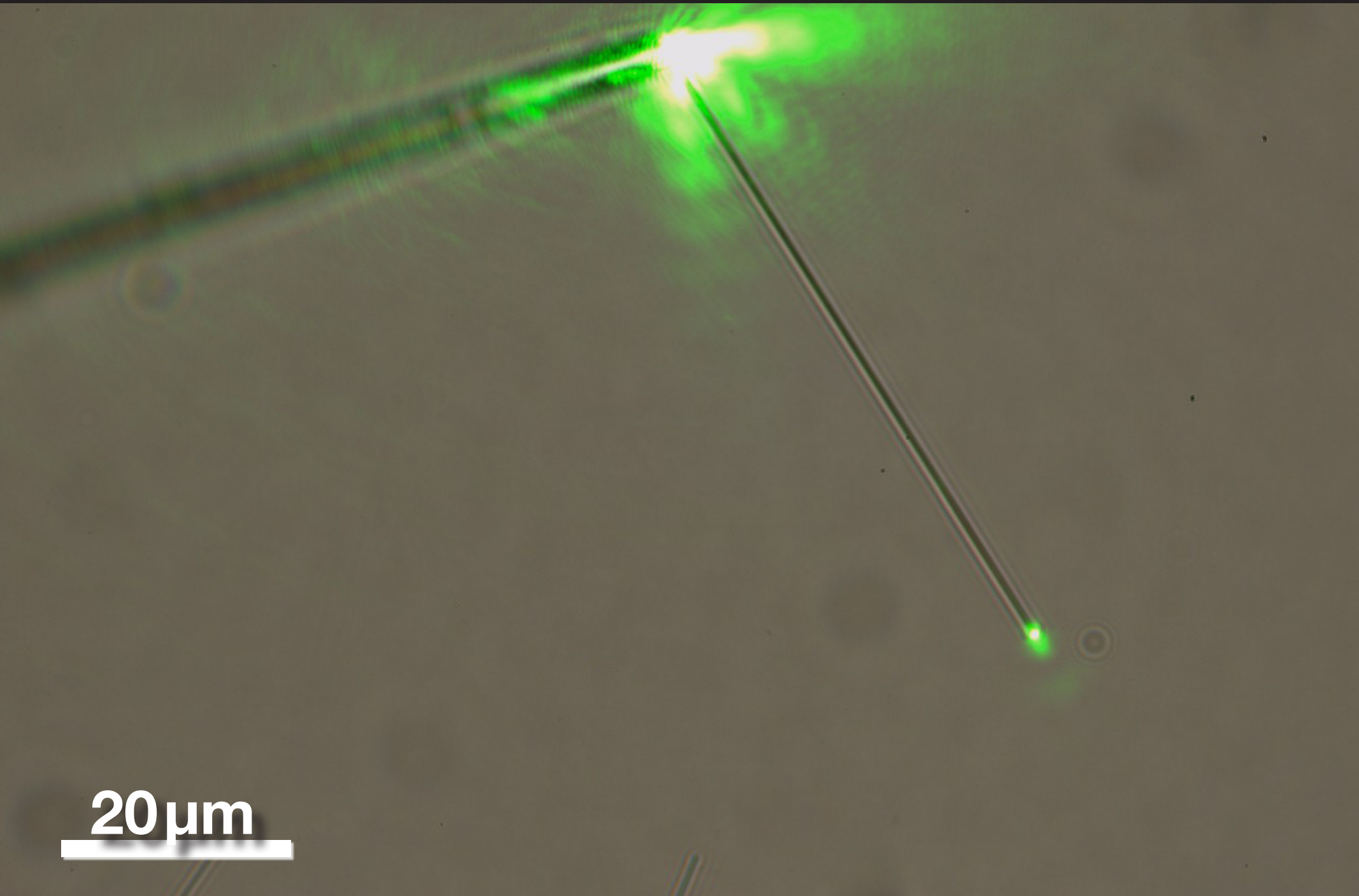


Manipulating light at the nanoscale



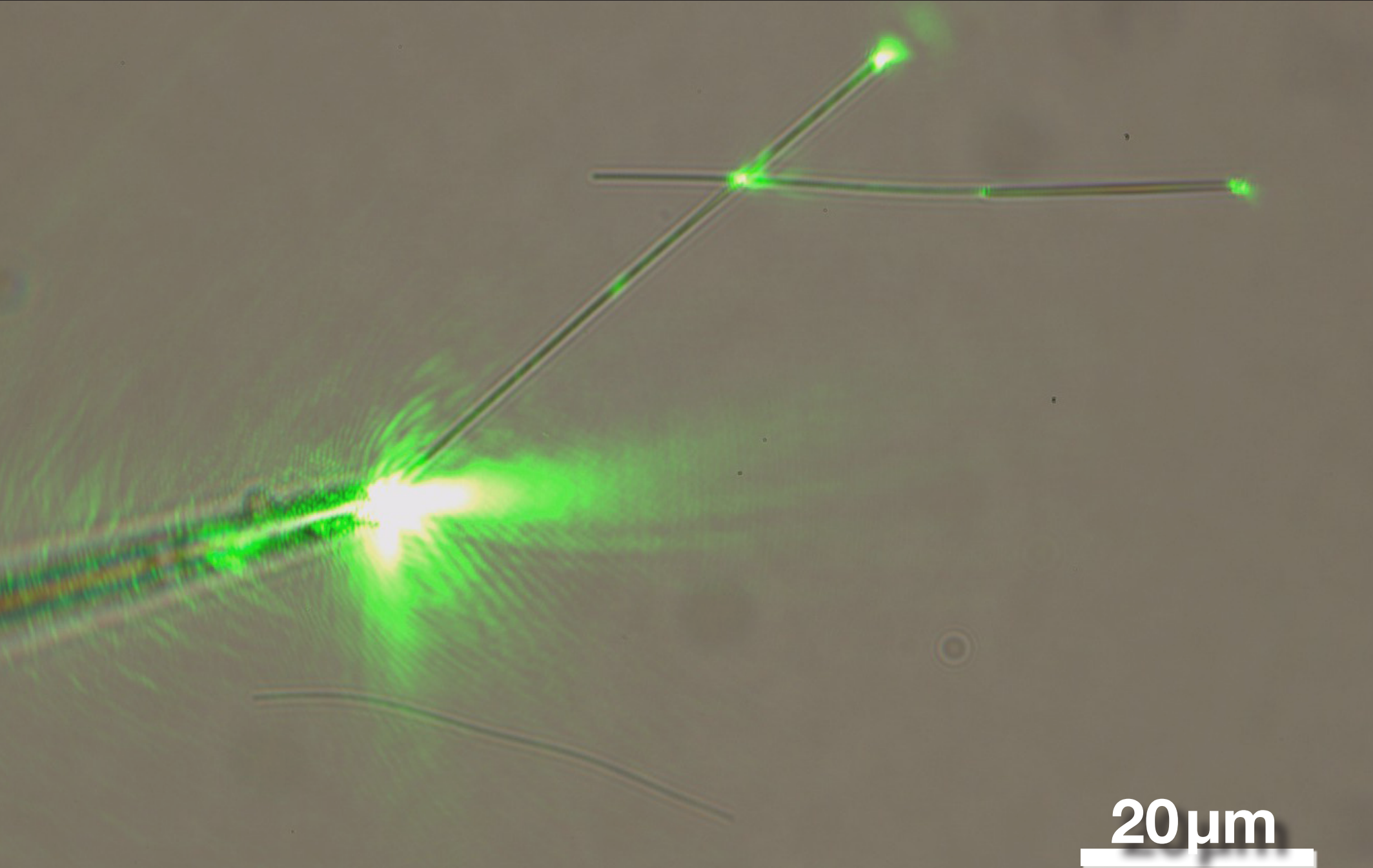
20 μm

Manipulating light at the nanoscale

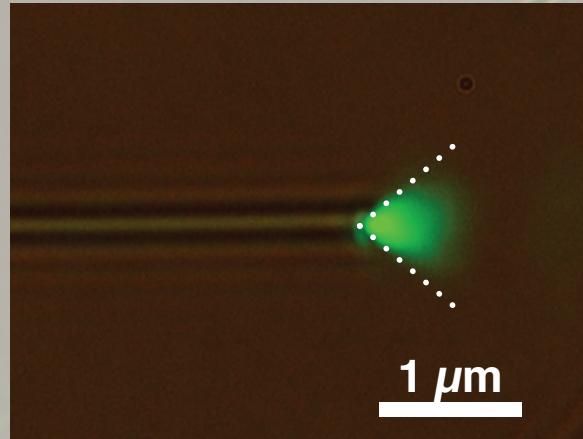


20 μm

Manipulating light at the nanoscale

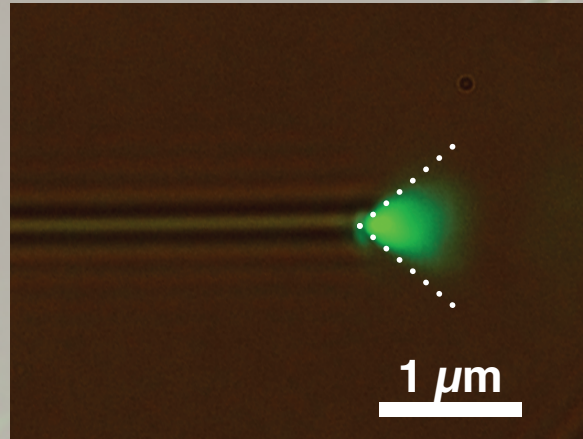


Manipulating light at the nanoscale

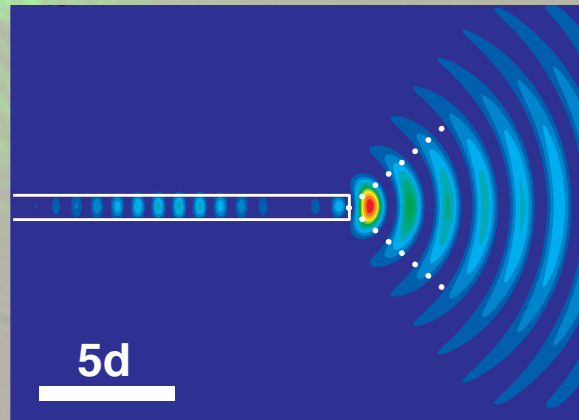


Nano Lett., 7, 3675 (2007)

Manipulating light at the nanoscale

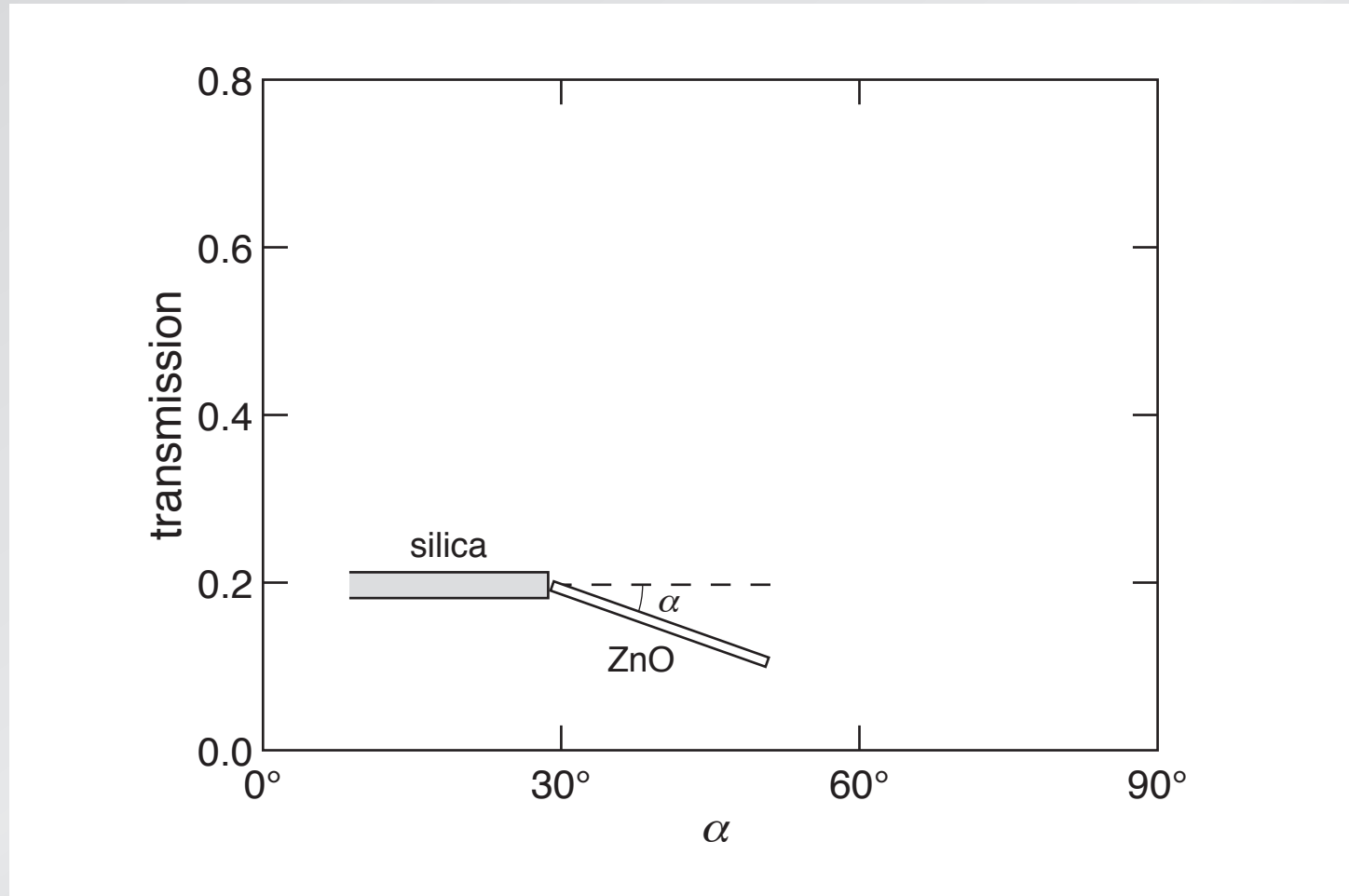


FDTD simulation



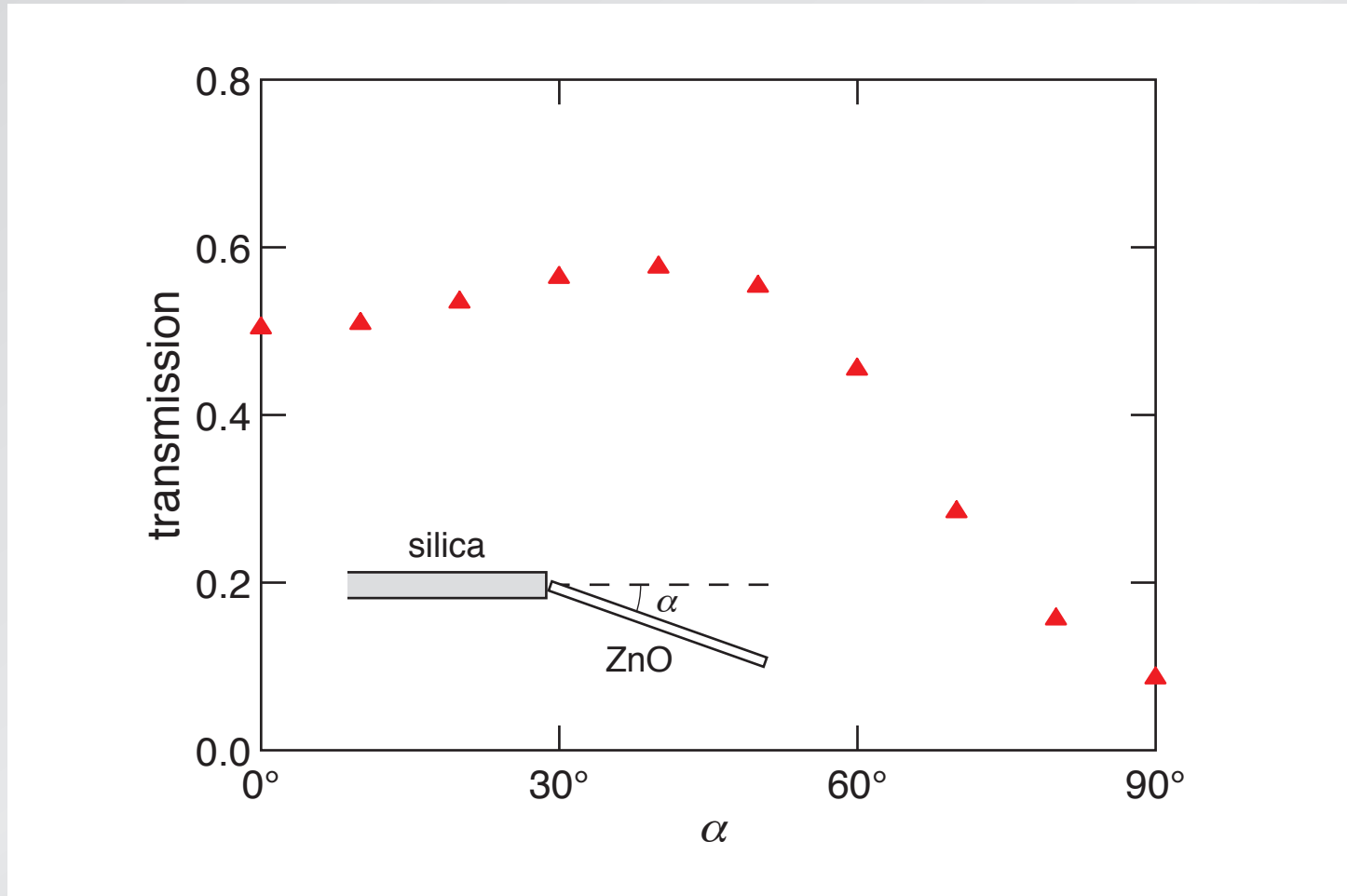
Manipulating light at the nanoscale

coupling efficiency

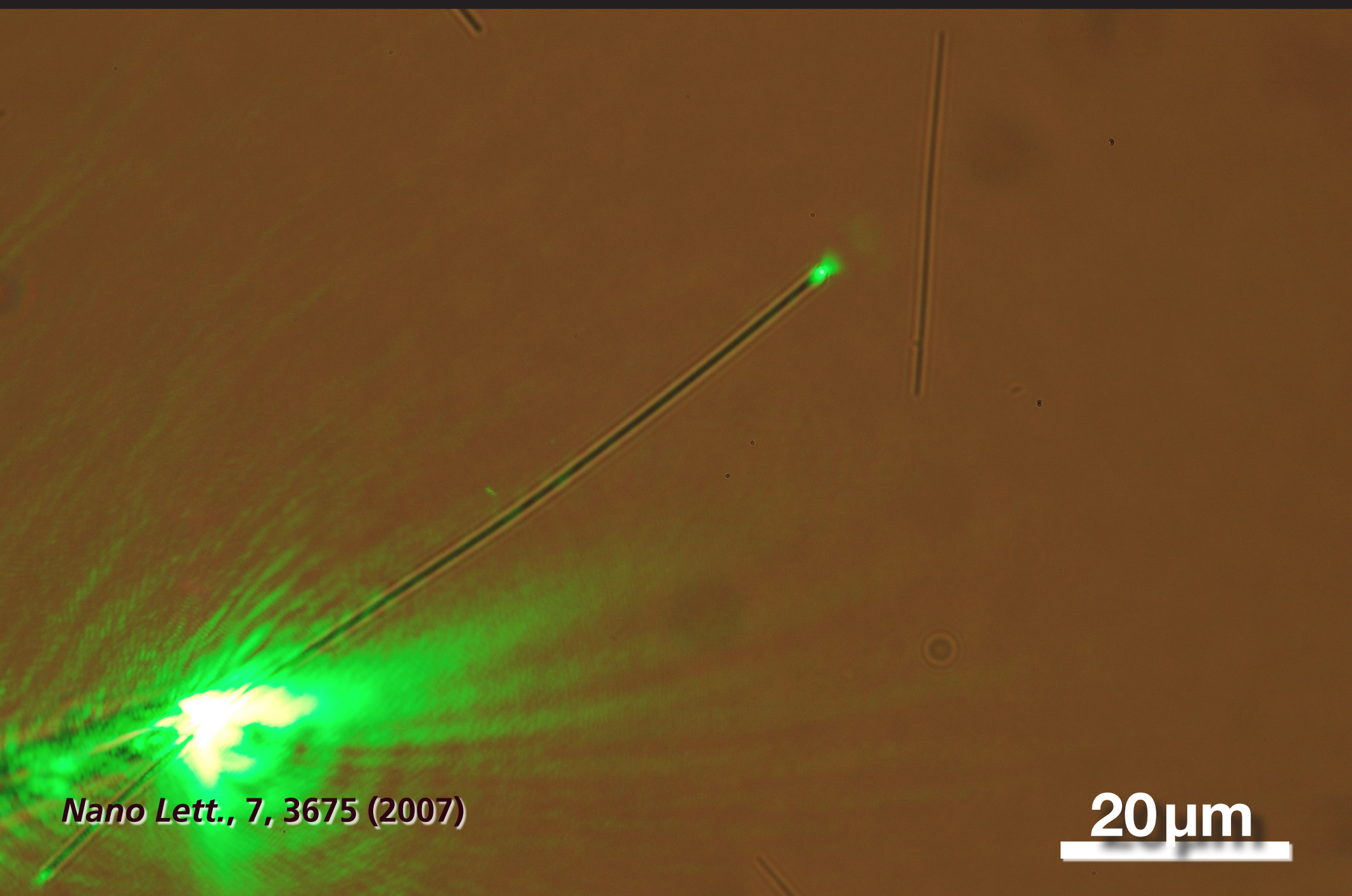


Manipulating light at the nanoscale

coupling efficiency



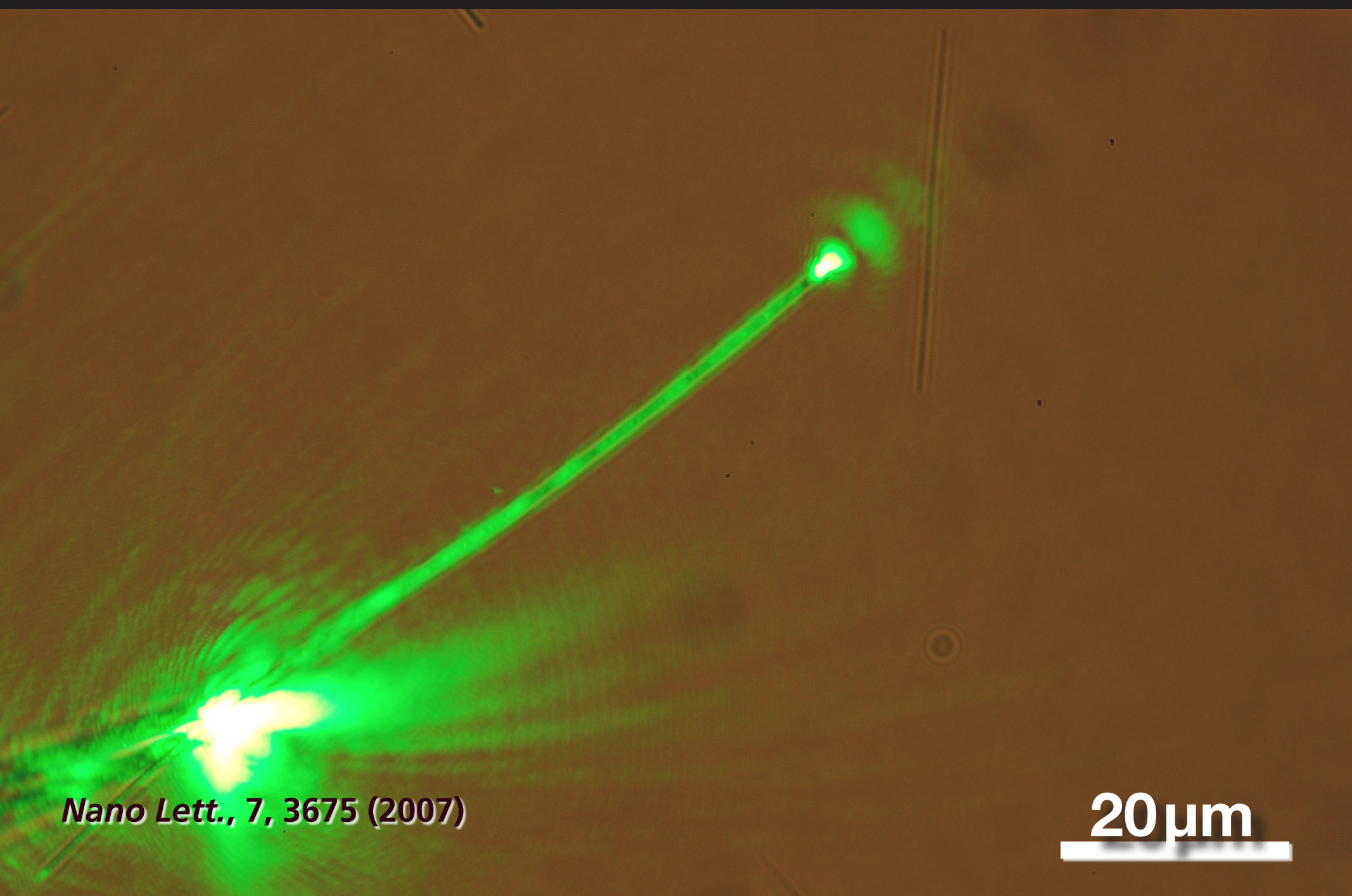
Manipulating light at the nanoscale



Nano Lett., 7, 3675 (2007)

20 μm

Manipulating light at the nanoscale

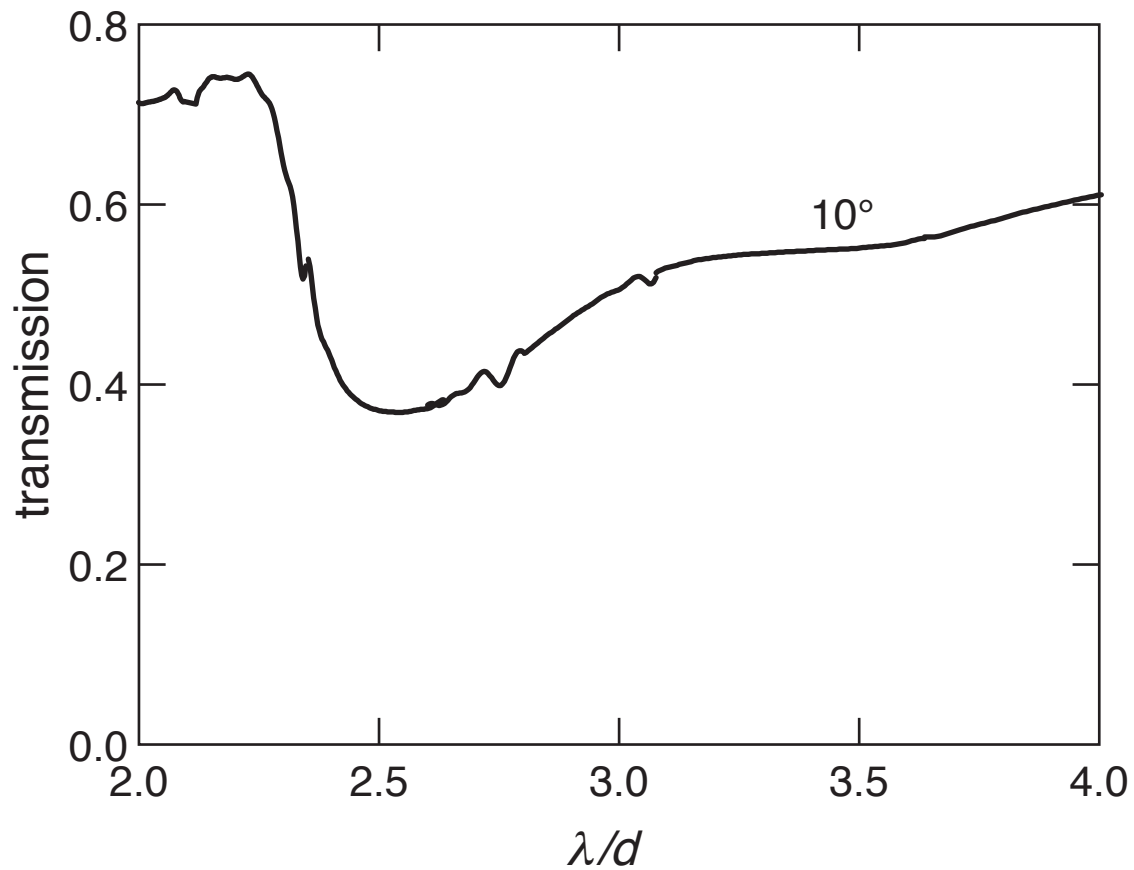


Nano Lett., 7, 3675 (2007)

20 μm

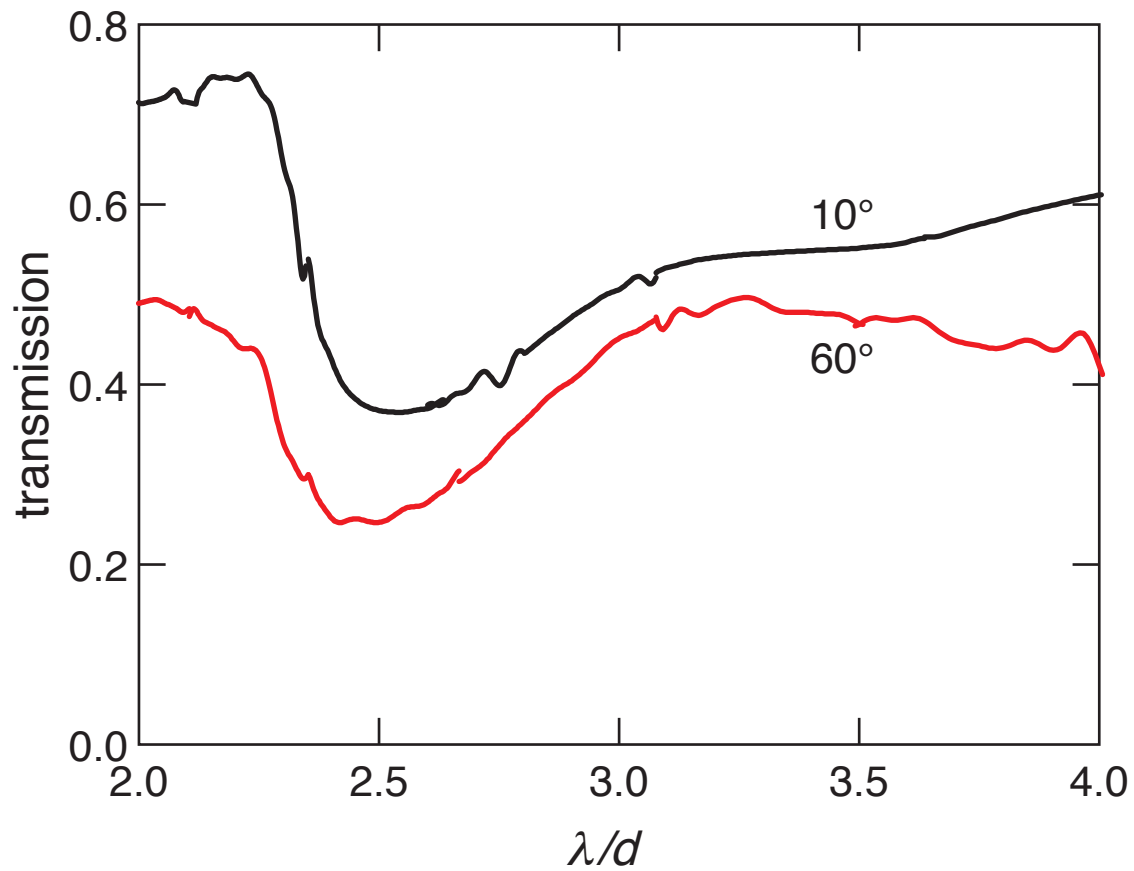
Manipulating light at the nanoscale

transmission spectrum



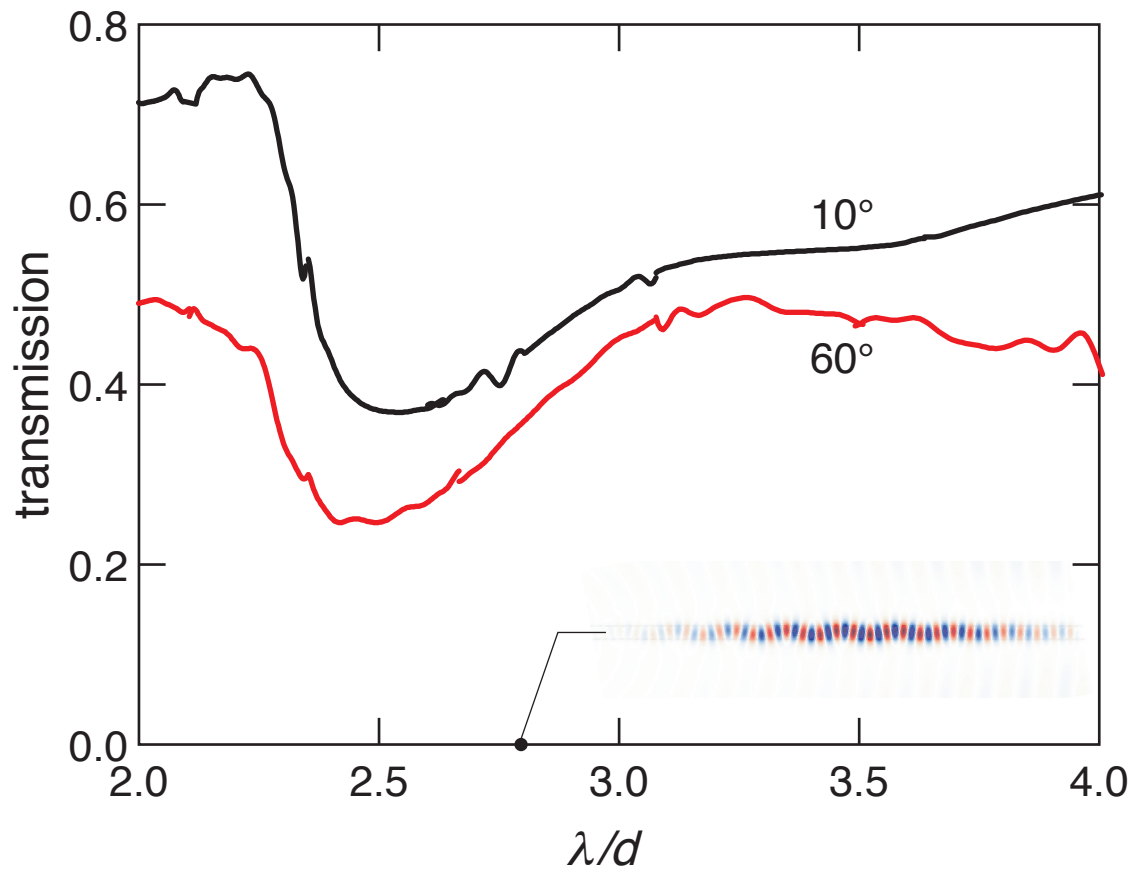
Manipulating light at the nanoscale

transmission spectrum



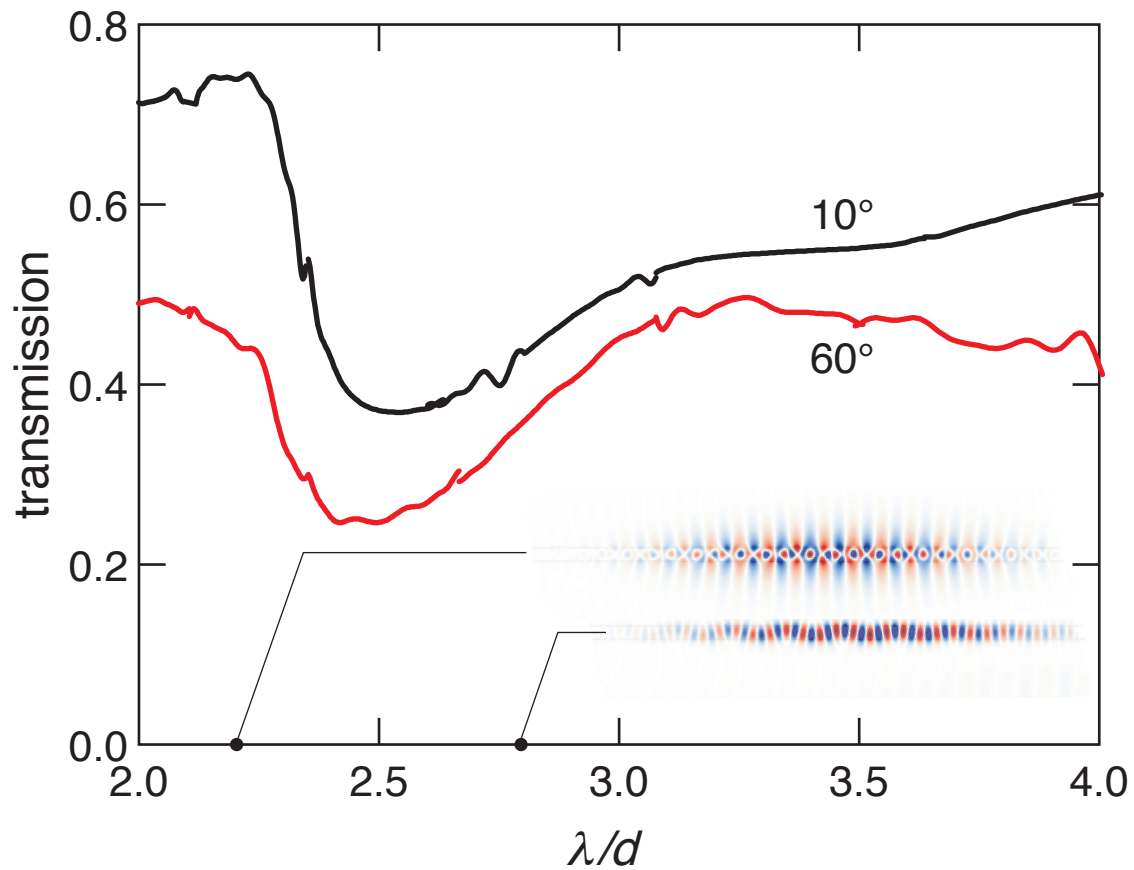
Manipulating light at the nanoscale

transmission spectrum

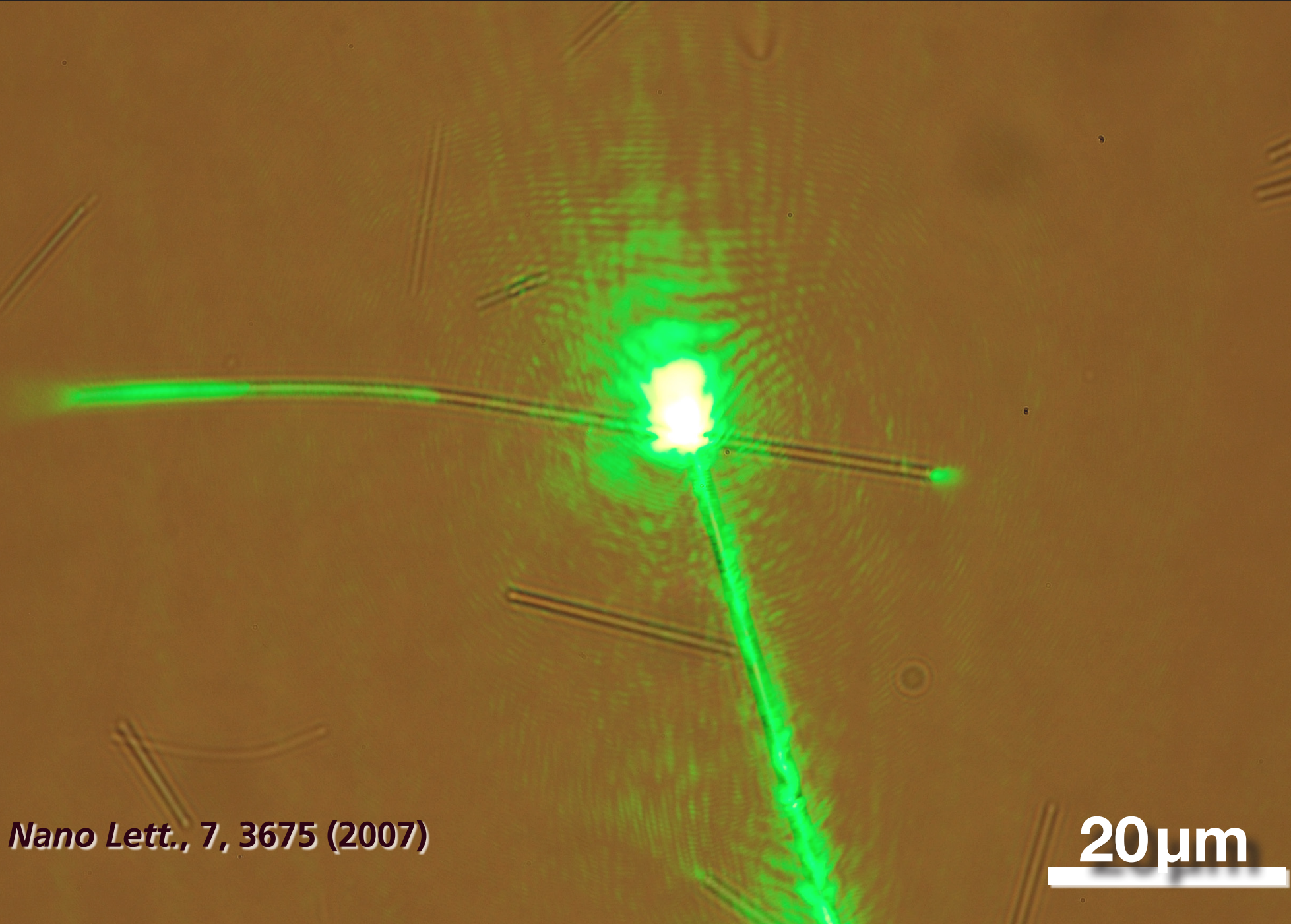


Manipulating light at the nanoscale

transmission spectrum



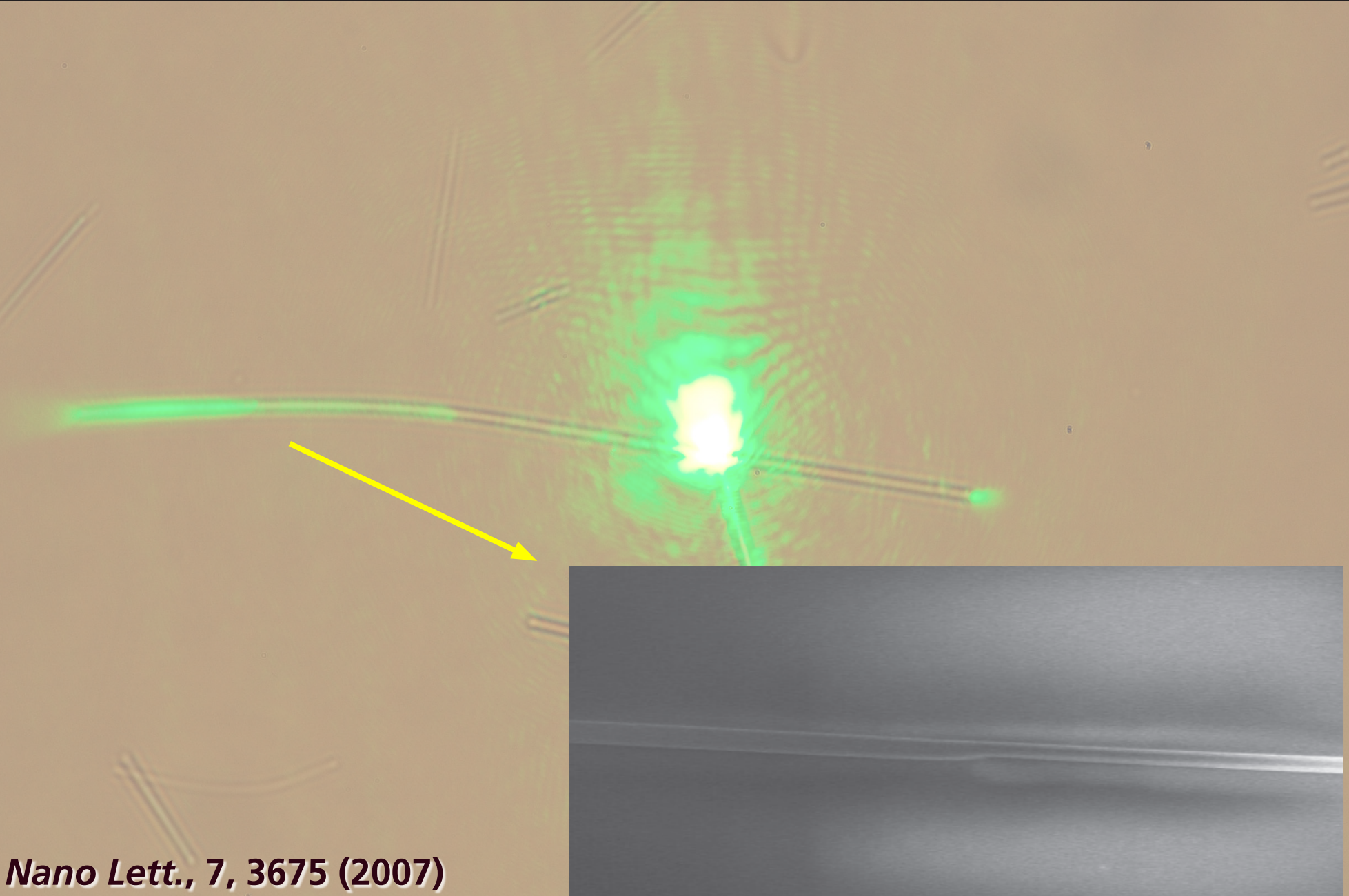
Manipulating light at the nanoscale



Nano Lett., 7, 3675 (2007)

20 μm

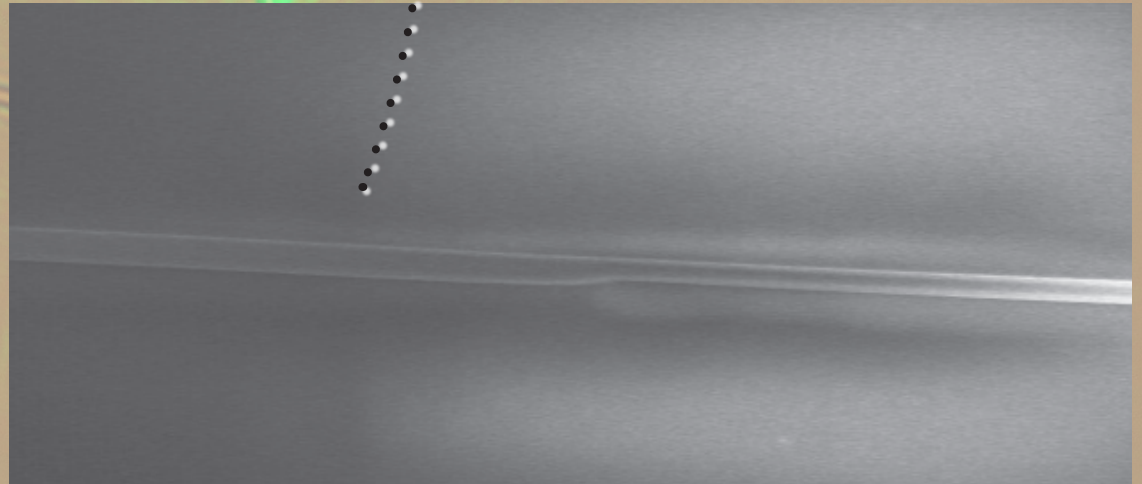
Manipulating light at the nanoscale



Nano Lett., 7, 3675 (2007)

Manipulating light at the nanoscale

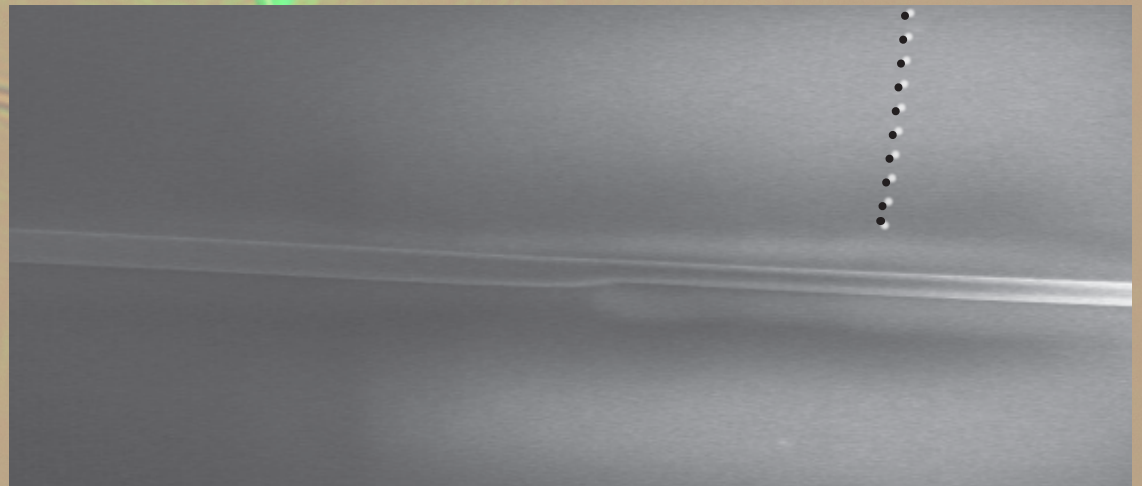
large diameter:
multimode



Nano Lett., 7, 3675 (2007)

Manipulating light at the nanoscale

small diameter:
single mode



Nano Lett., 7, 3675 (2007)

Manipulating light at the nanoscale

Points to keep in mind:

- **low-loss guiding**
- **convenient evanescent coupling**
- **attached to ordinary fiber**

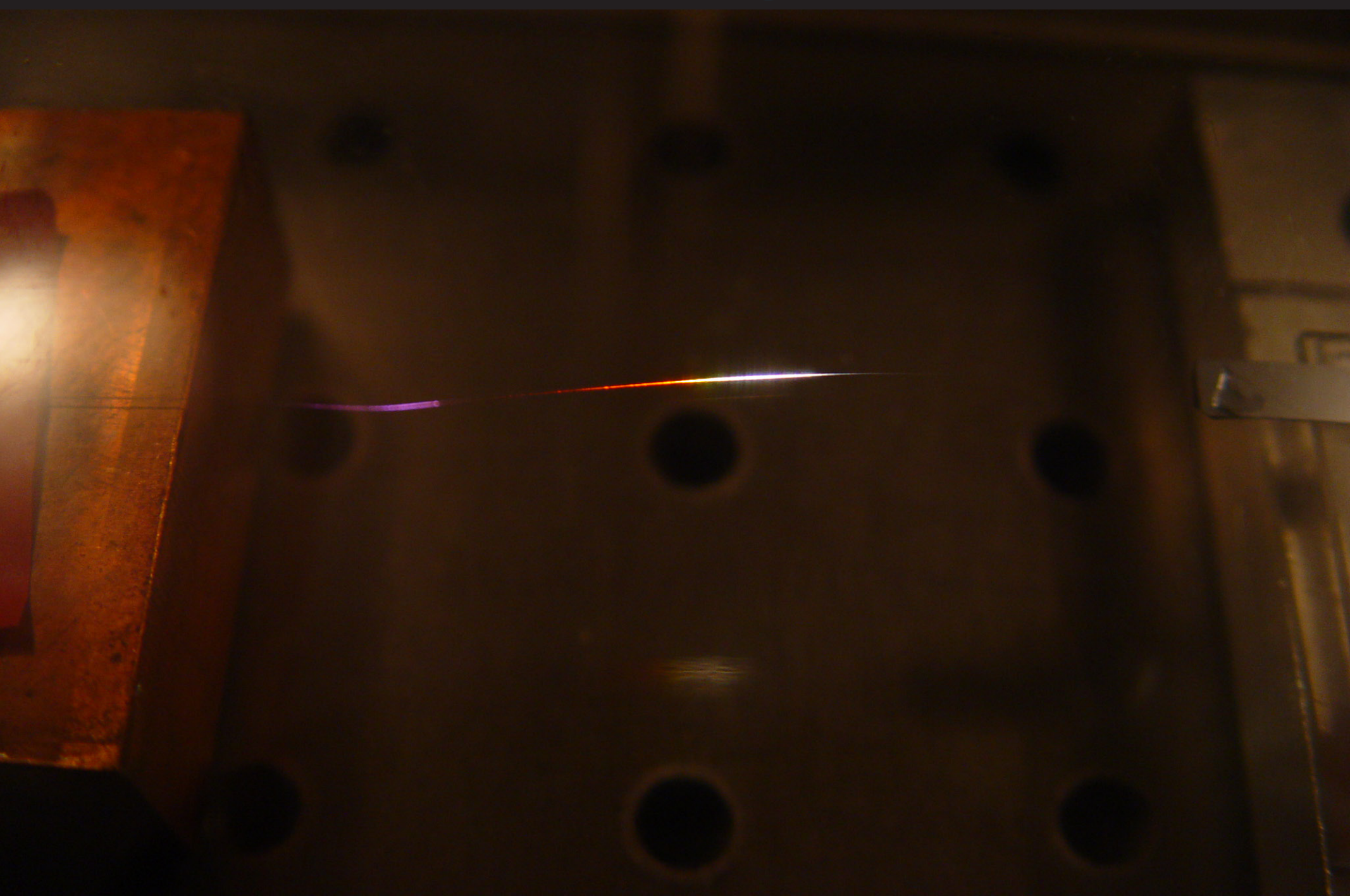
Outline

- propagation of pulses
- nonlinear optics
- nanoscale optics
- **nonlinear optics at the nanoscale**

Supercontinuum generation

strong confinement \longrightarrow high intensity

Supercontinuum generation

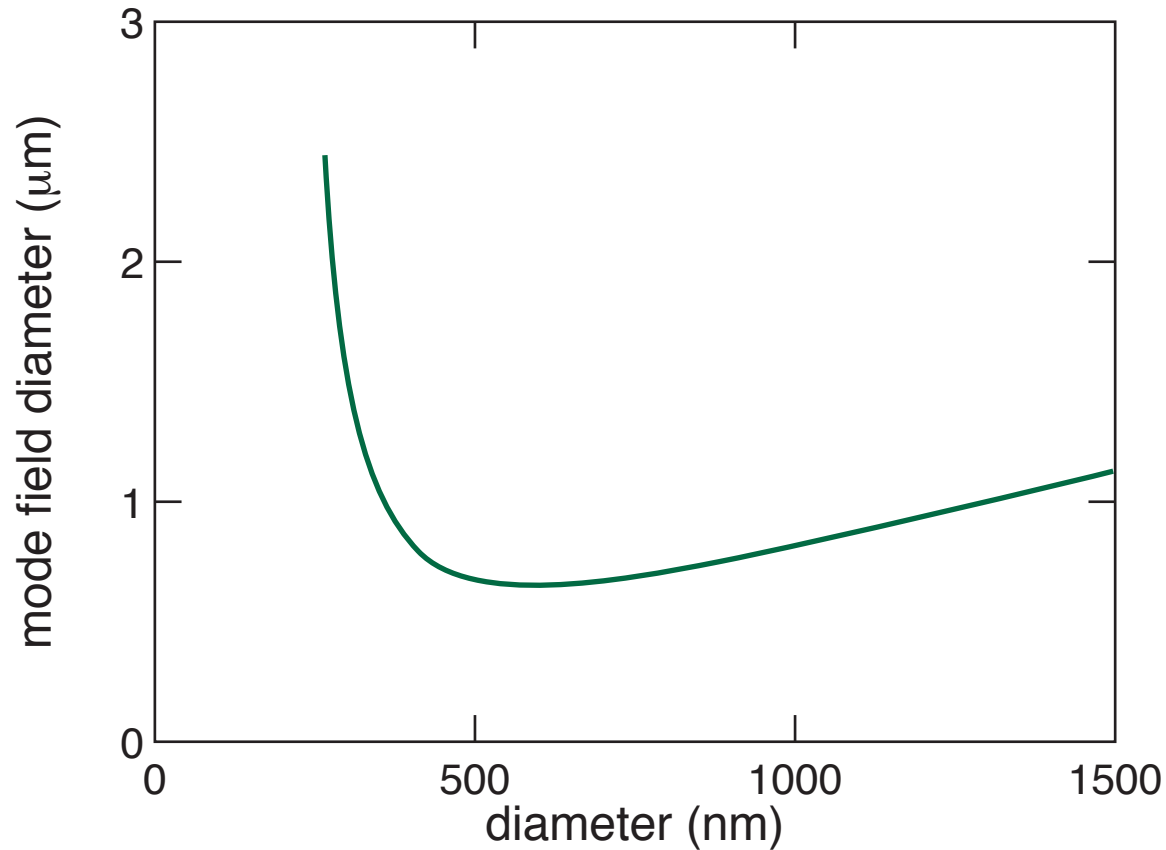


Supercontinuum generation



Supercontinuum generation

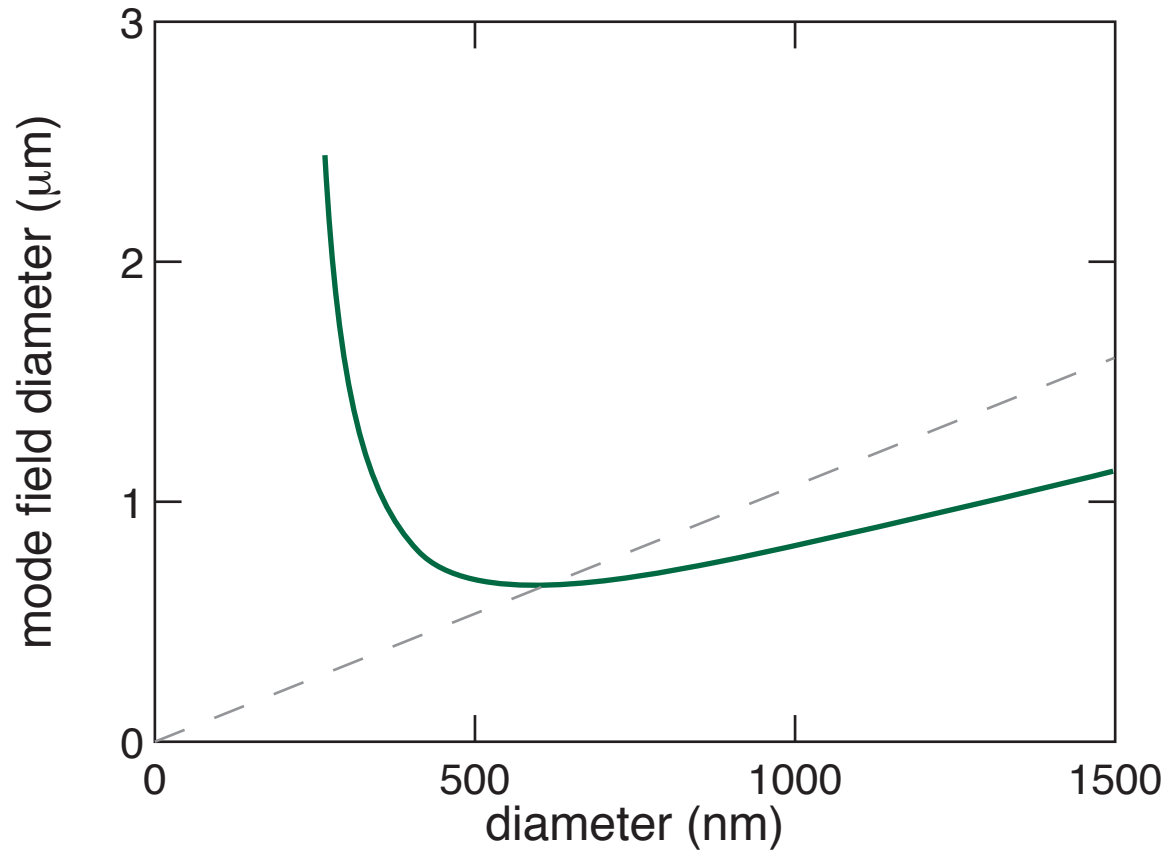
mode field diameter ($\lambda = 800$ nm)



M.A. Foster, *et al.*, *Optics Express*, 12, 2880 (2004)

Supercontinuum generation

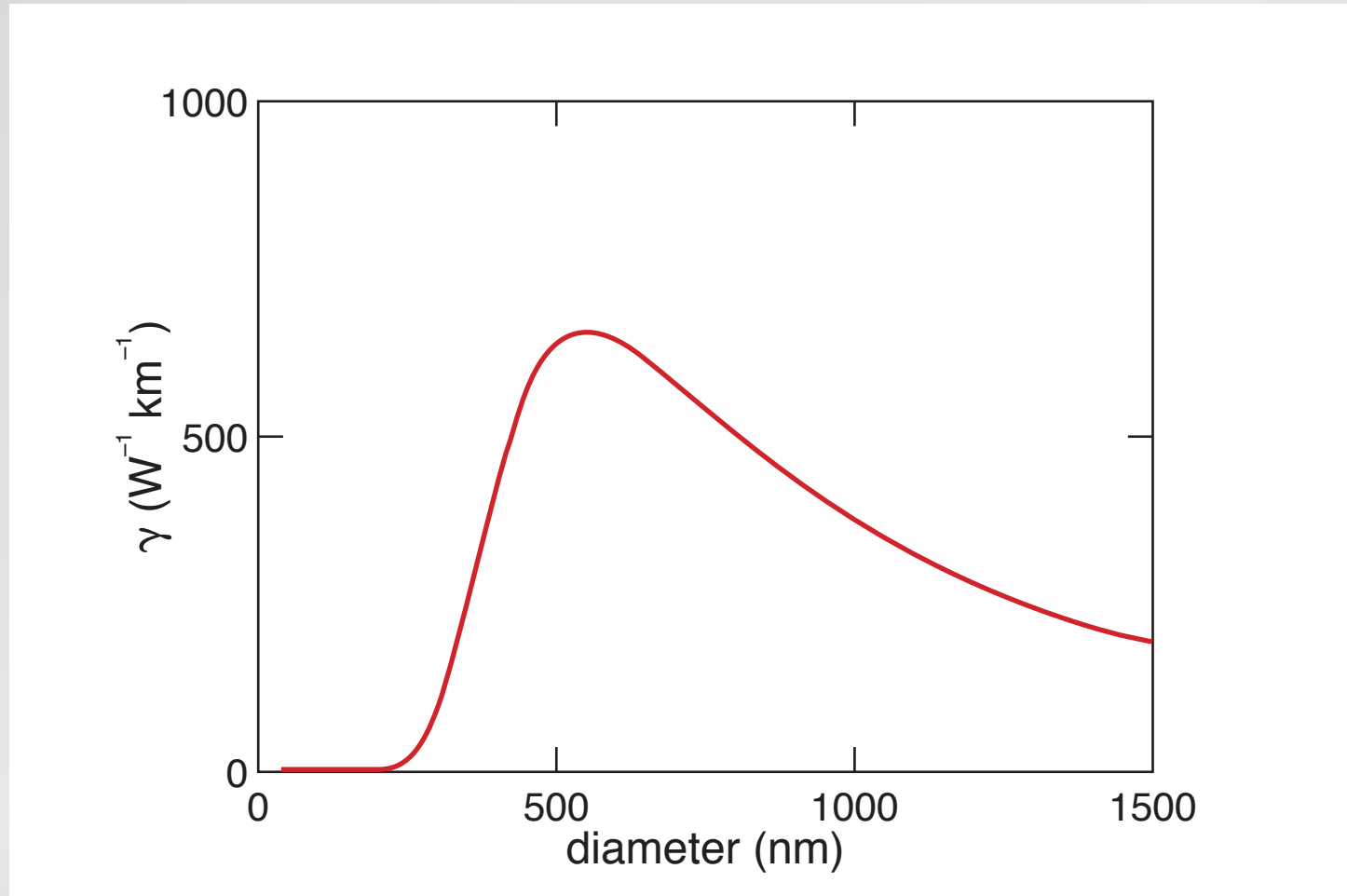
mode field diameter ($\lambda = 800$ nm)



M.A. Foster, et al., *Optics Express*, 12, 2880 (2004)

Supercontinuum generation

nonlinear parameter



M.A. Foster, et al., *Optics Express*, 12, 2880 (2004)

Supercontinuum generation

dispersion important!

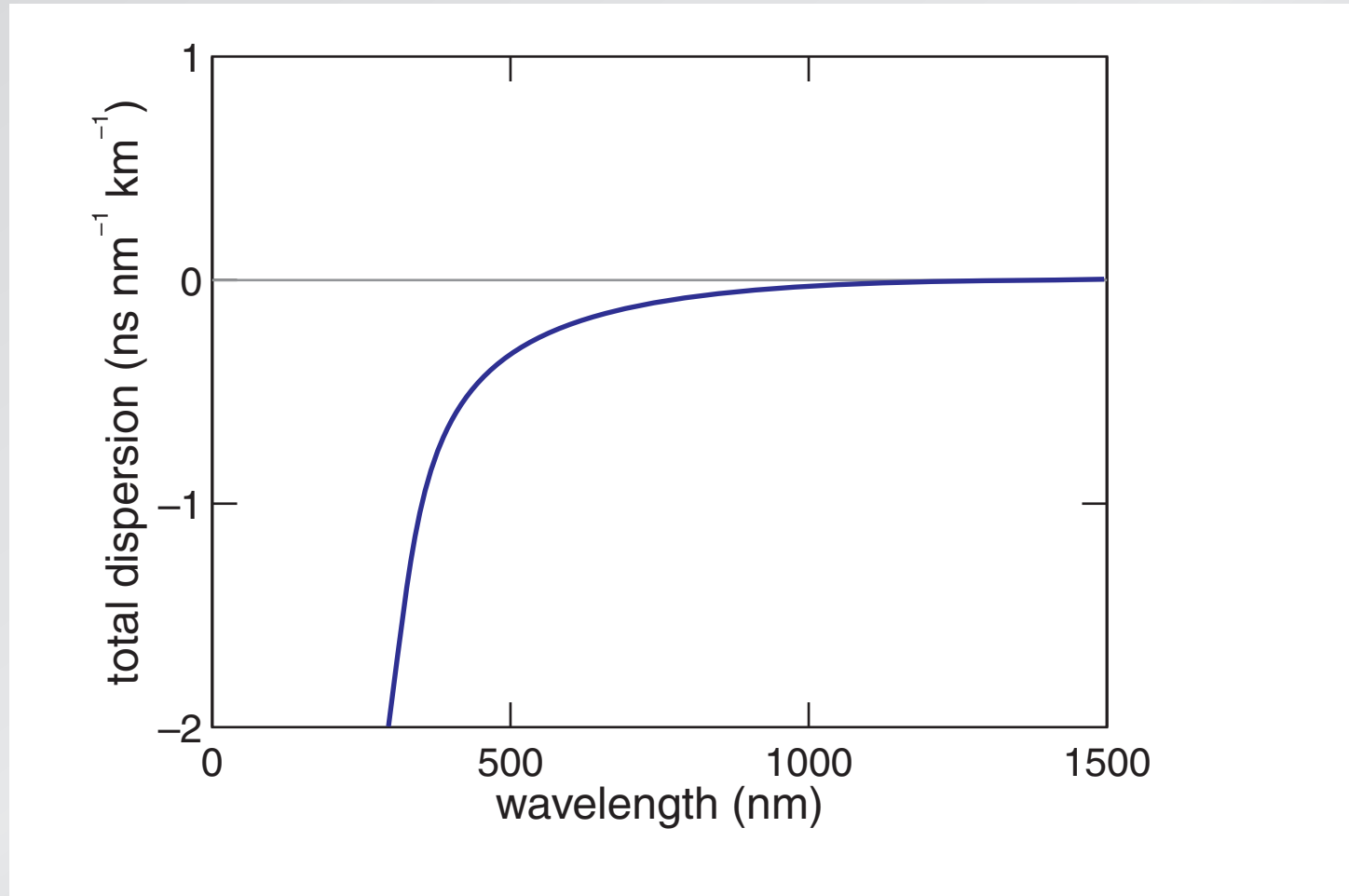
Supercontinuum generation

dispersion:

- modal dispersion
- material dispersion
- waveguide dispersion
- nonlinear dispersion

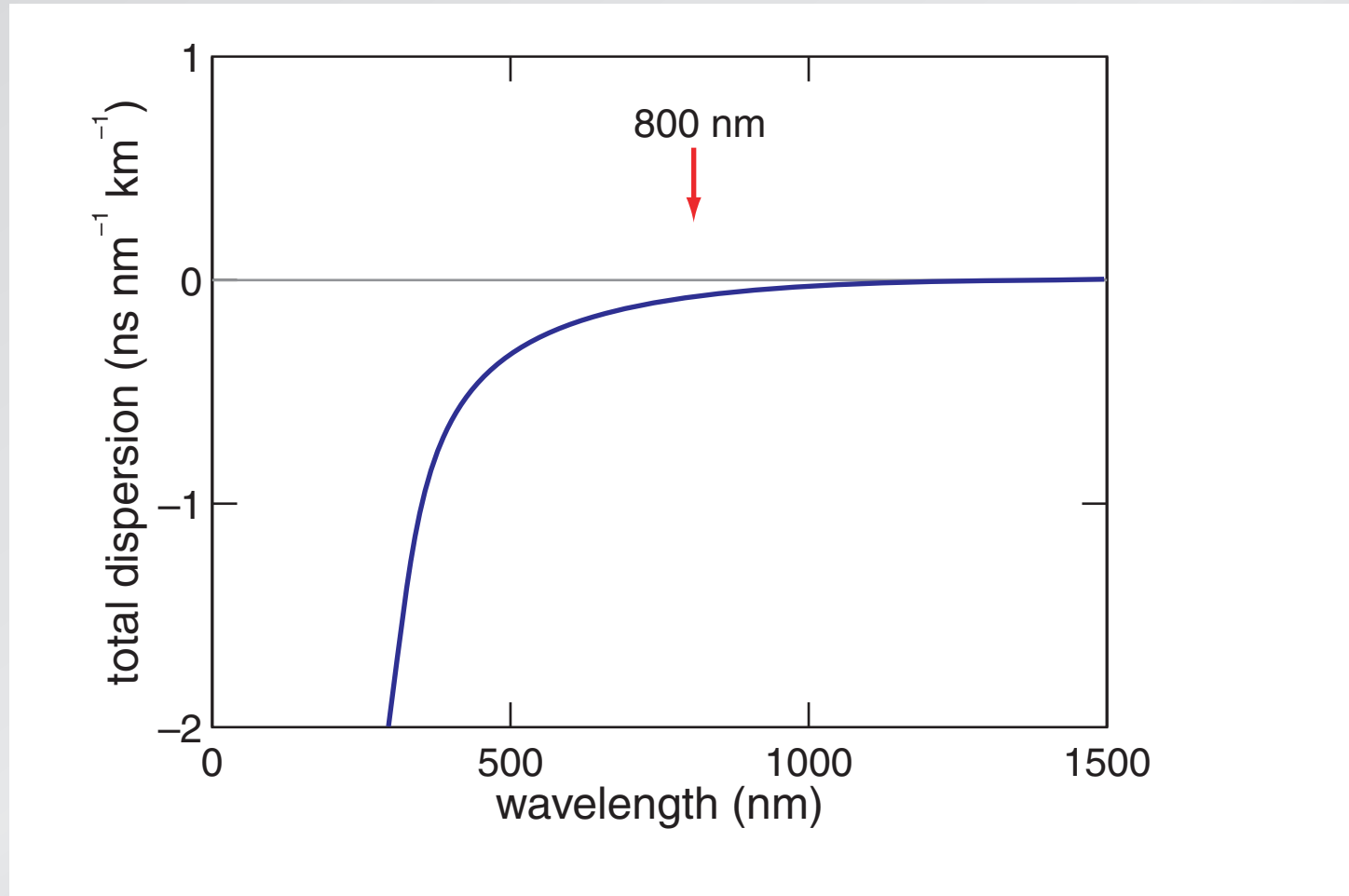
Supercontinuum generation

waveguide dispersion



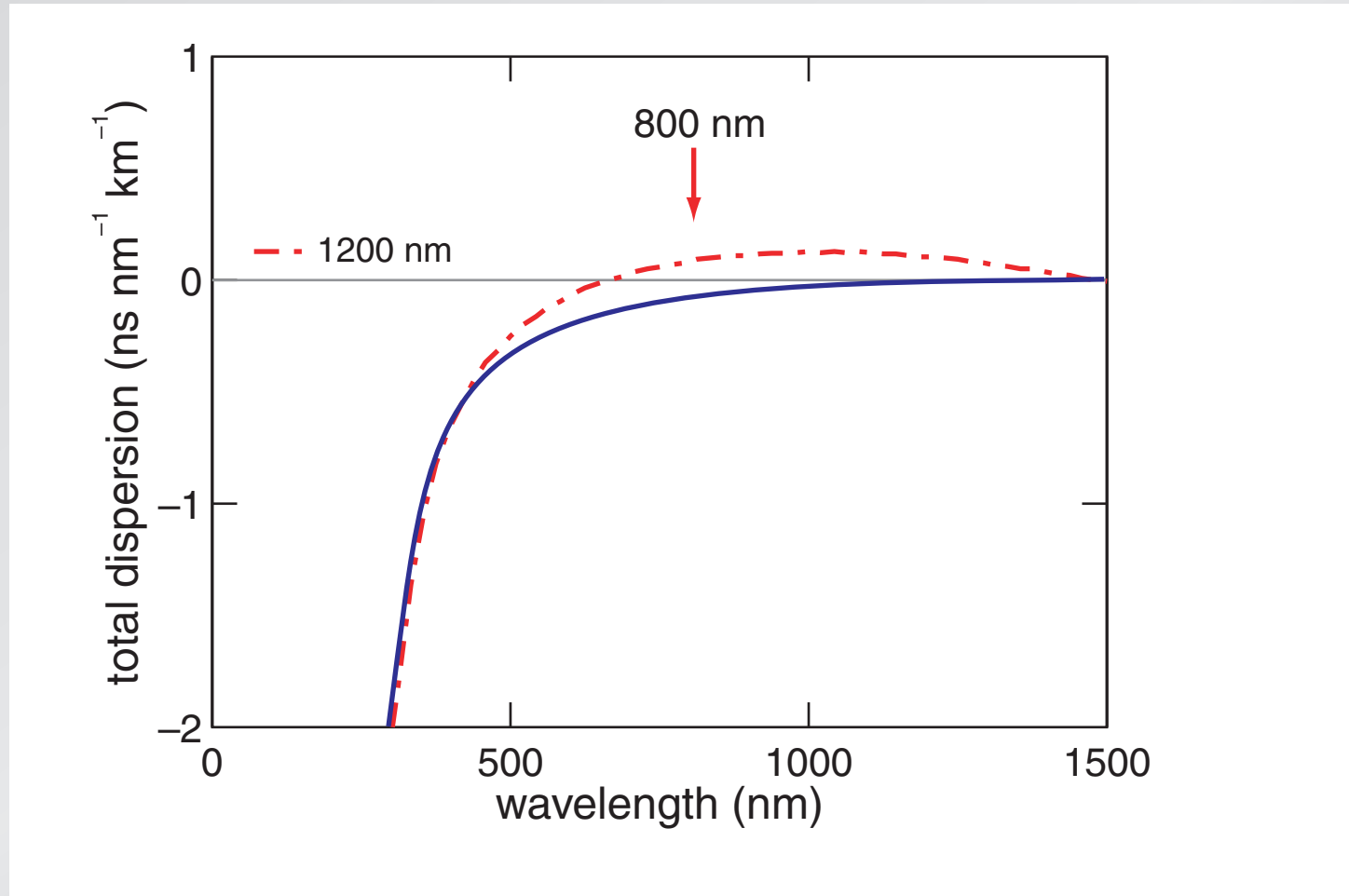
Supercontinuum generation

waveguide dispersion



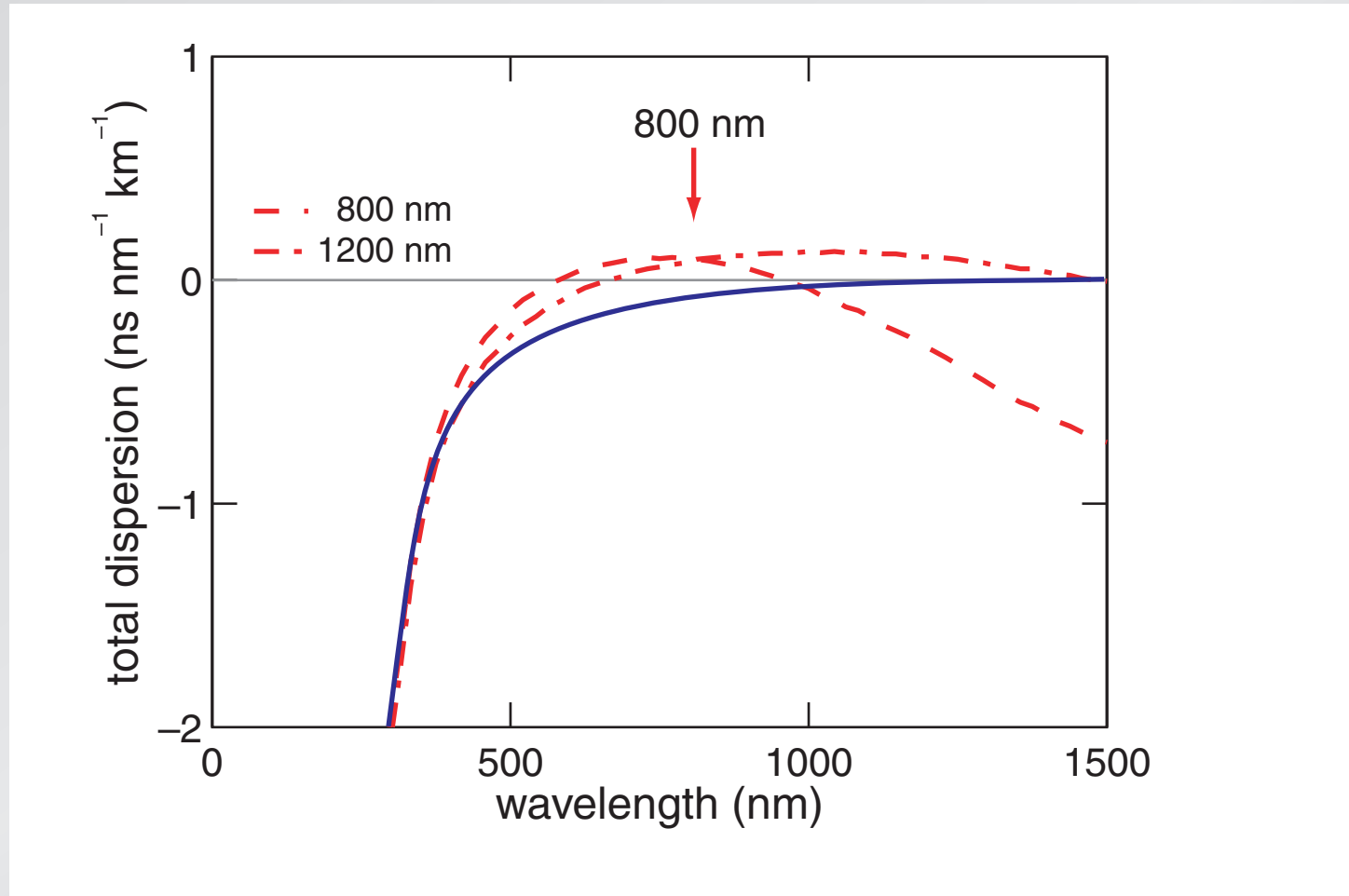
Supercontinuum generation

waveguide dispersion



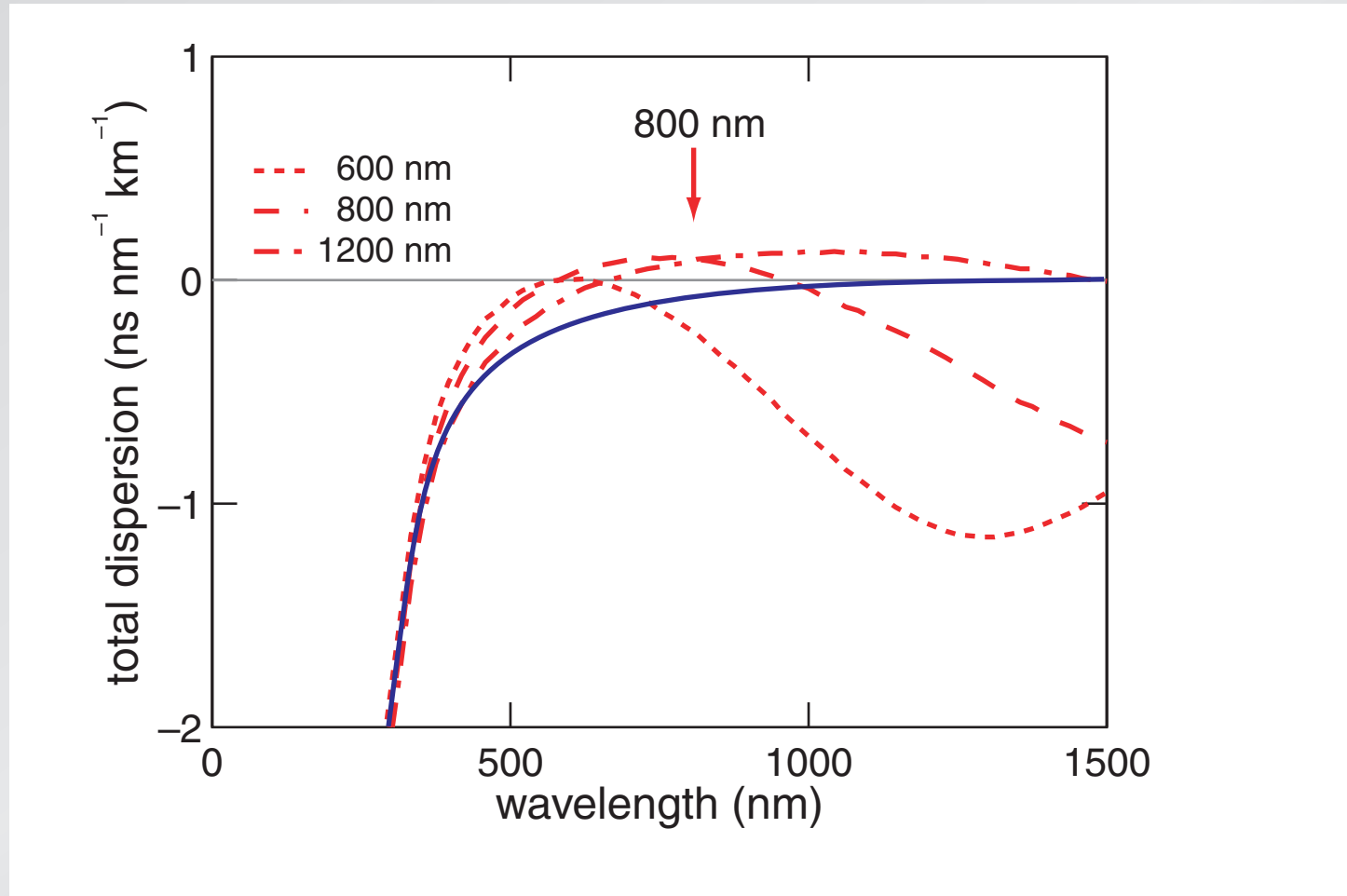
Supercontinuum generation

waveguide dispersion



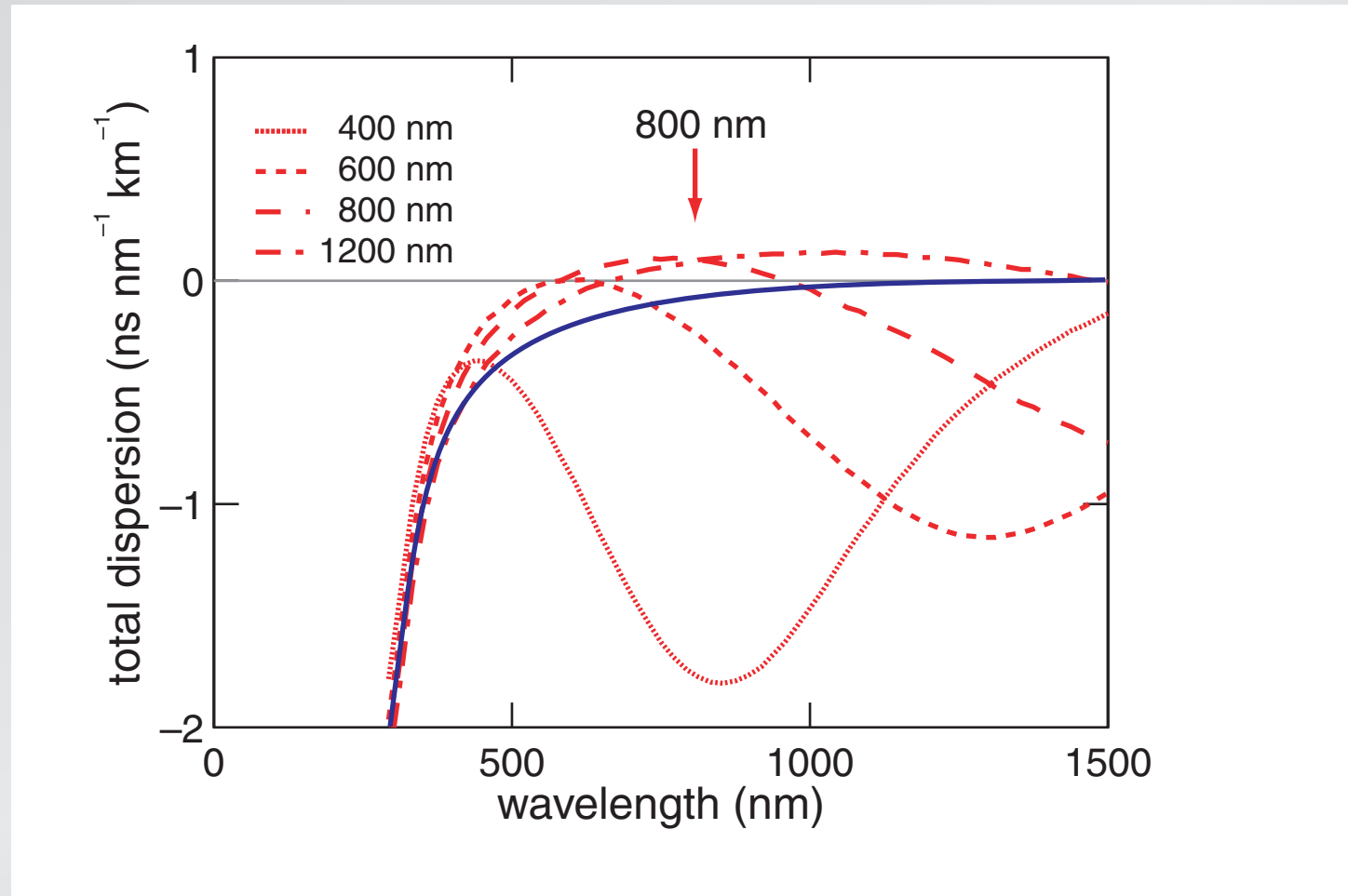
Supercontinuum generation

waveguide dispersion



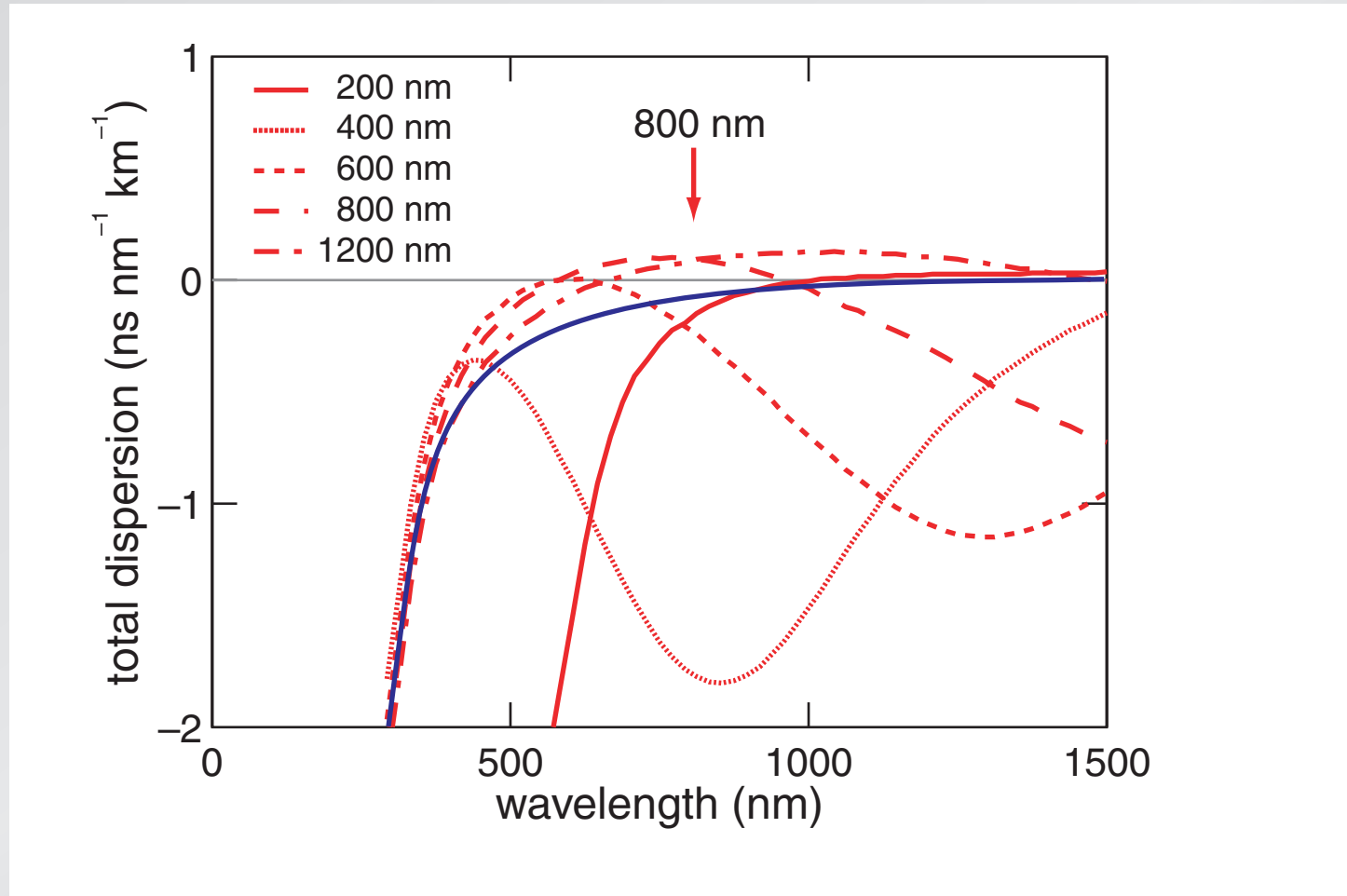
Supercontinuum generation

waveguide dispersion



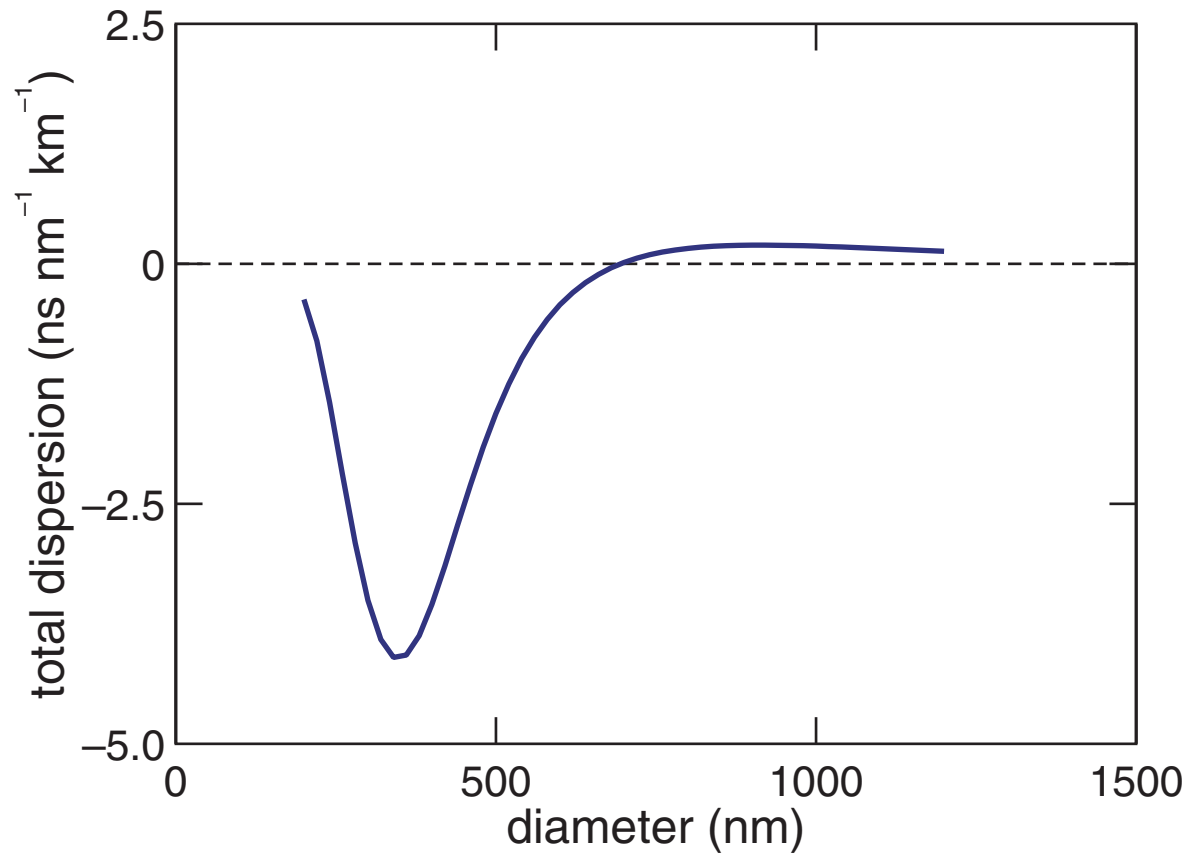
Supercontinuum generation

waveguide dispersion



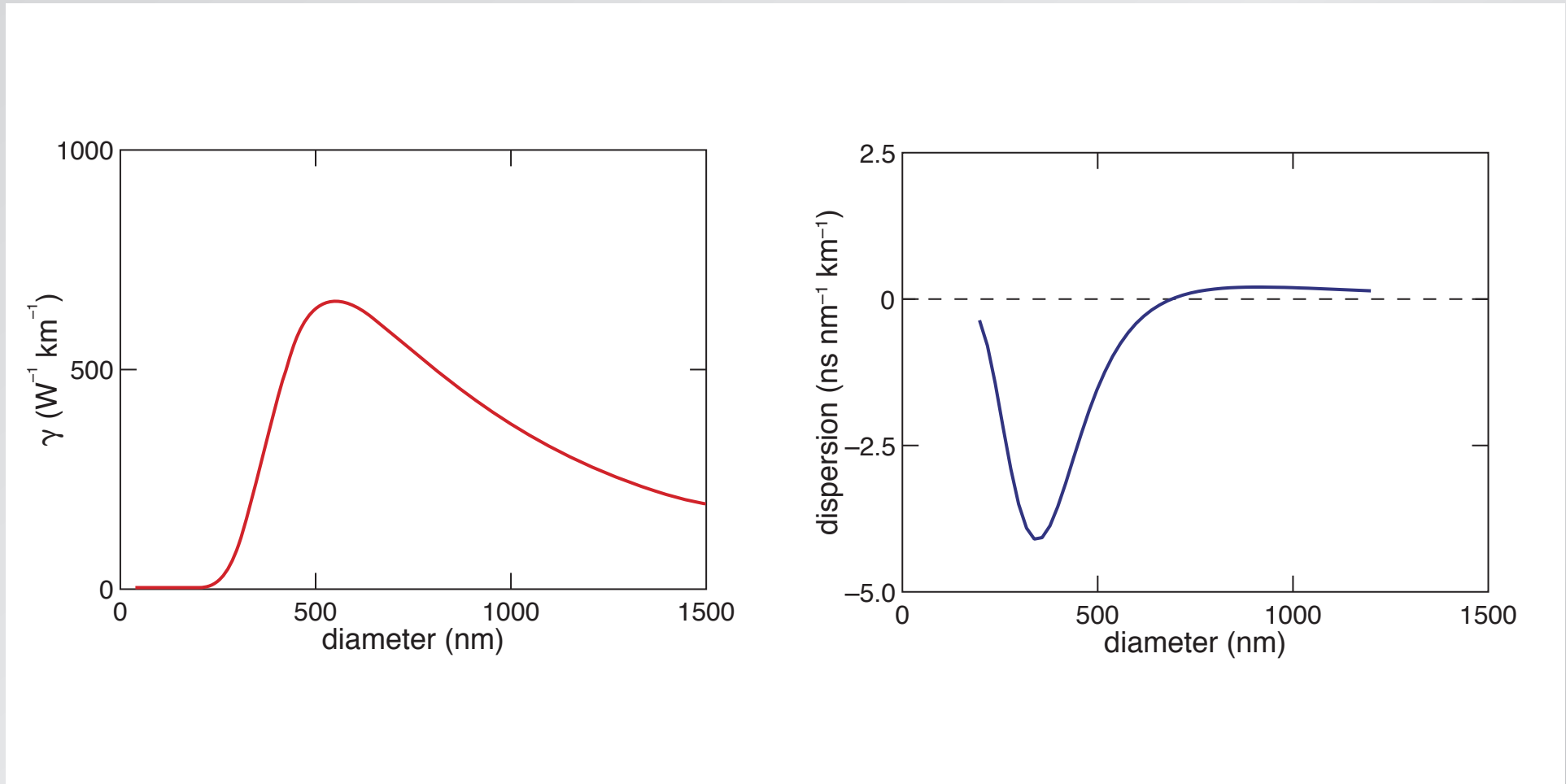
Supercontinuum generation

waveguide dispersion



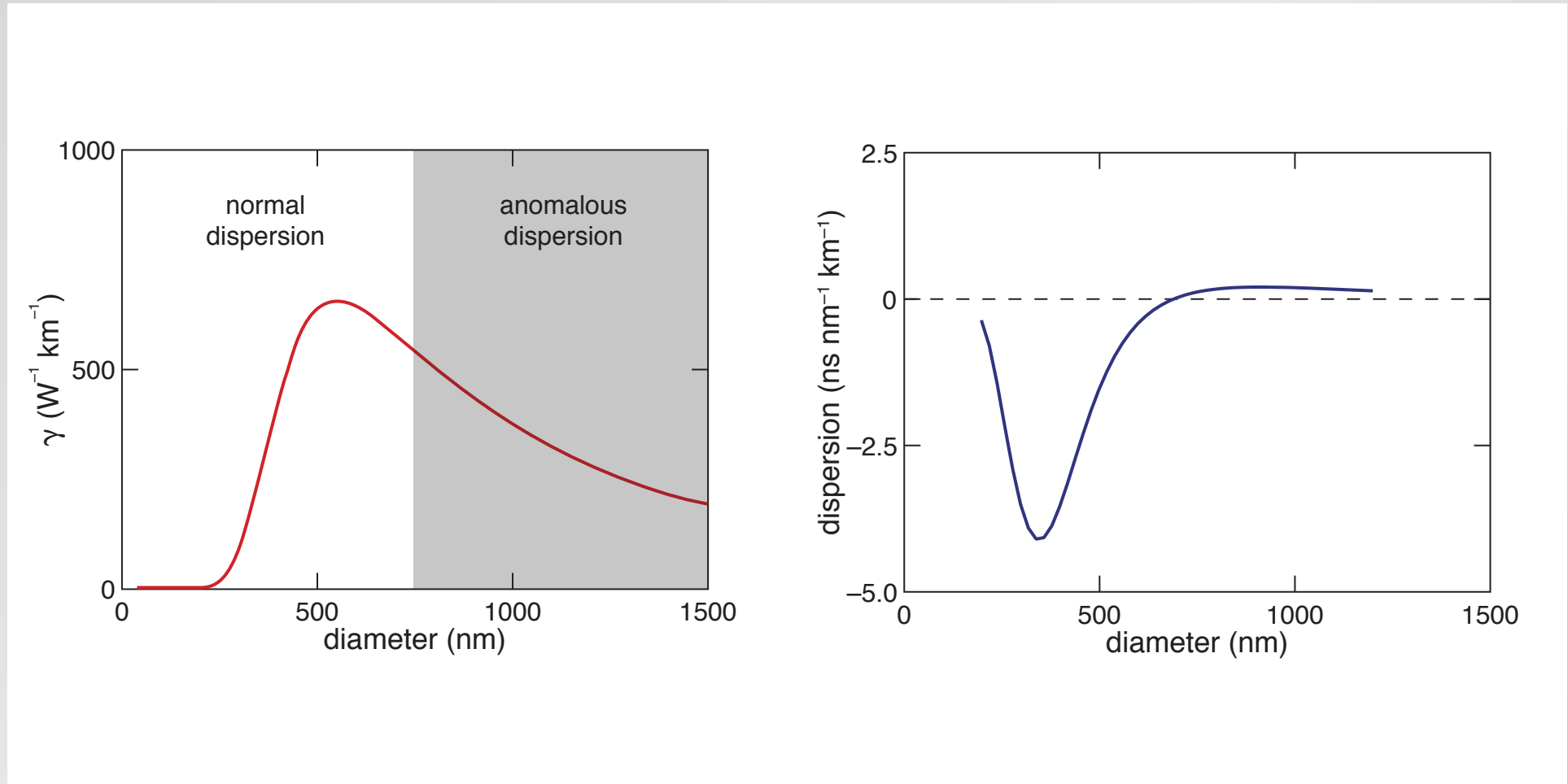
Supercontinuum generation

waveguide dispersion



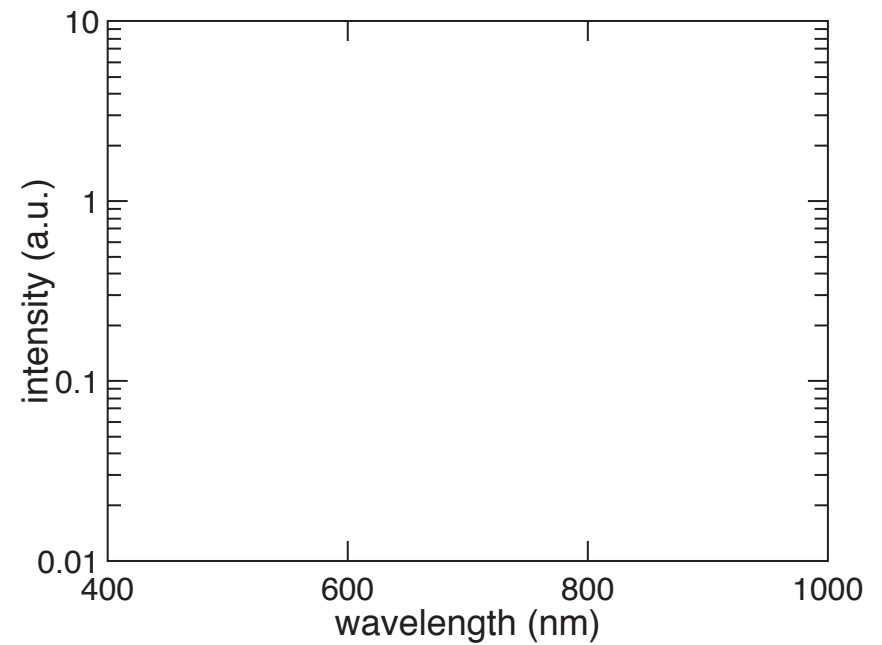
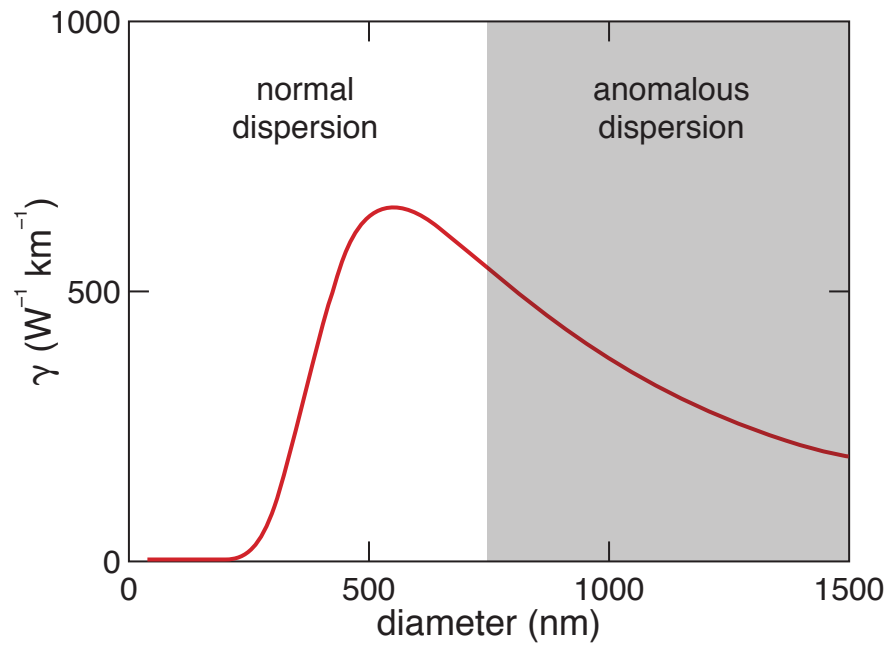
Supercontinuum generation

waveguide dispersion



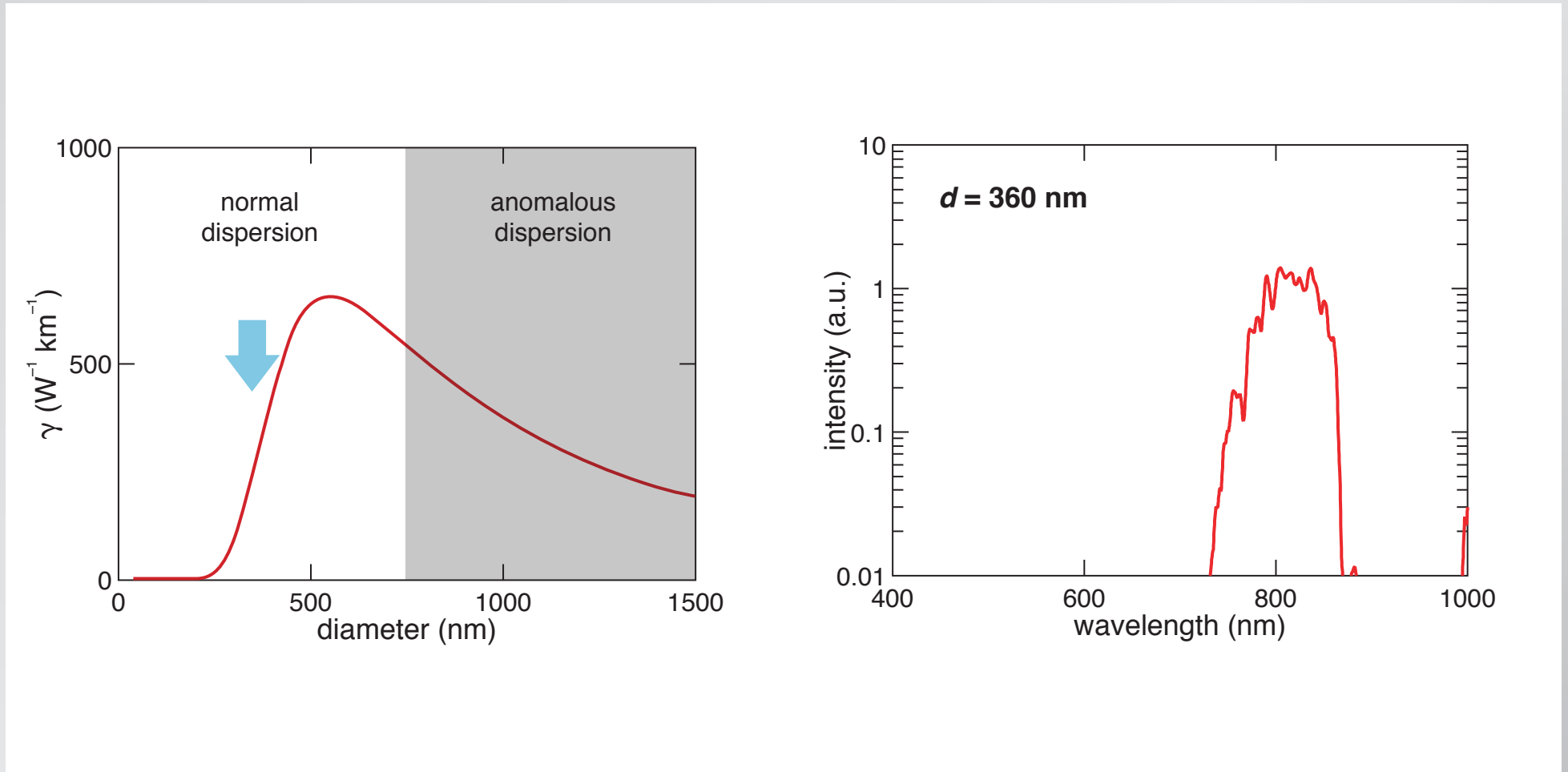
Supercontinuum generation

nanowire continuum generation



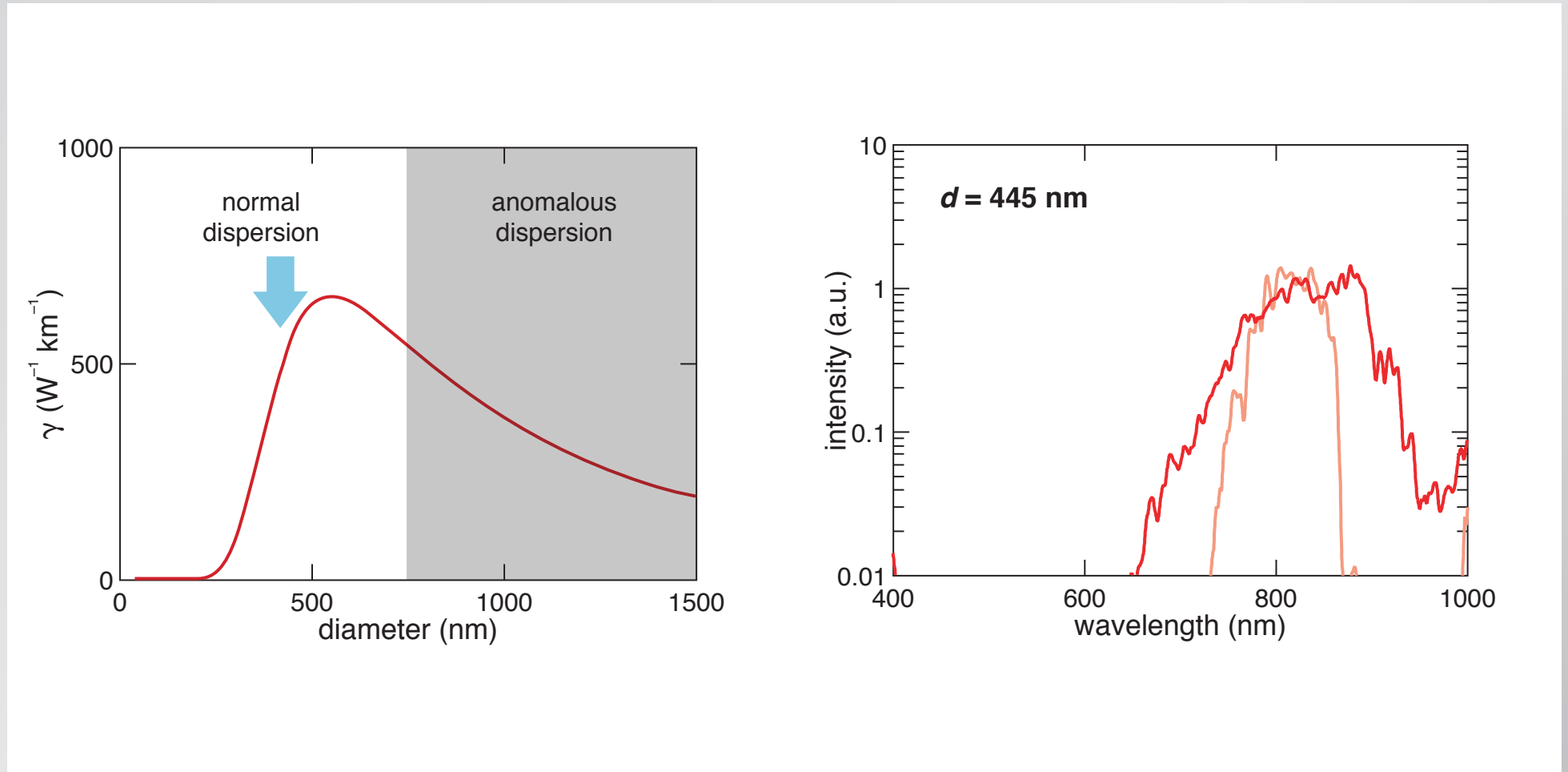
Supercontinuum generation

nanowire continuum generation



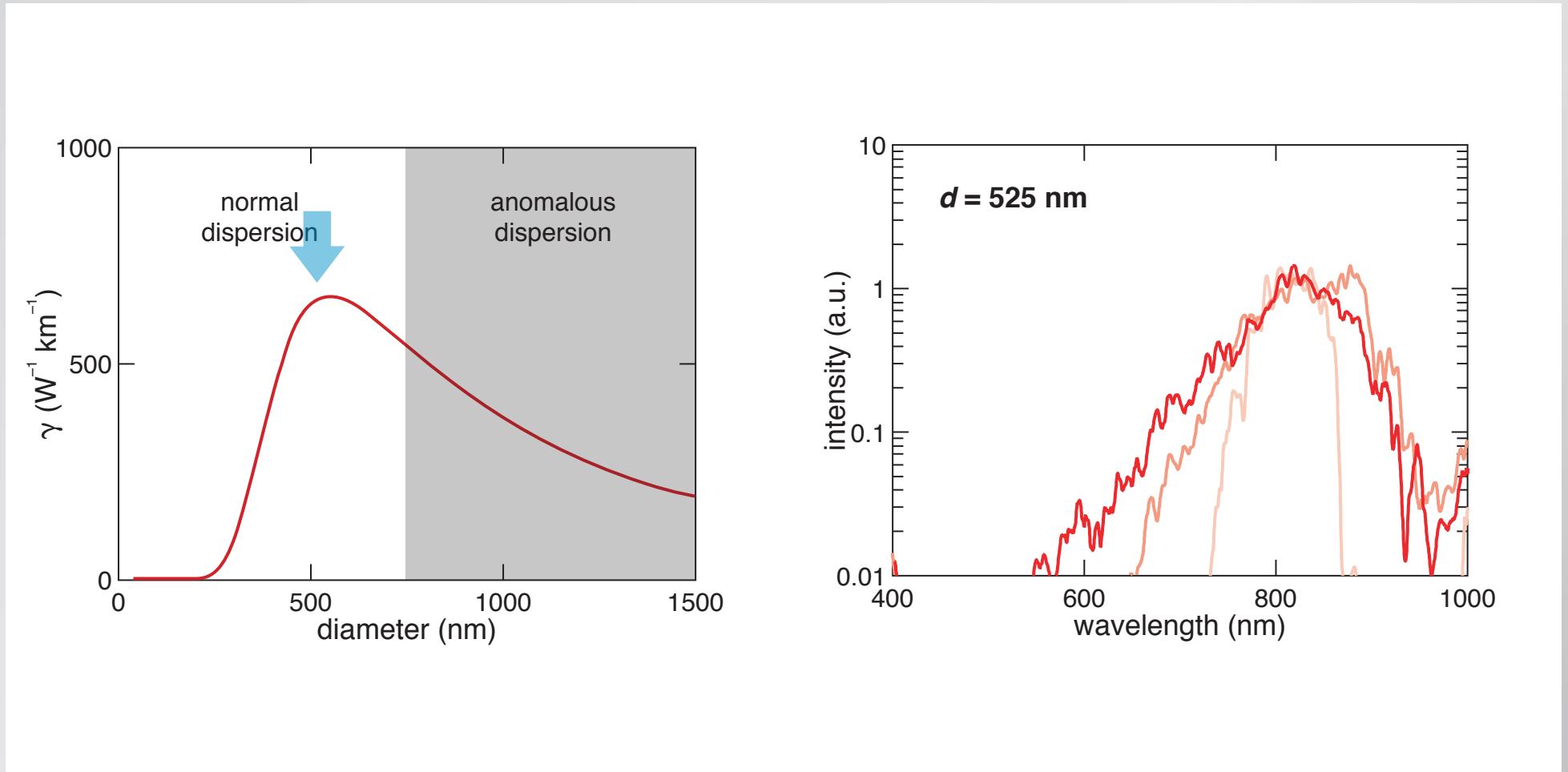
Supercontinuum generation

nanowire continuum generation



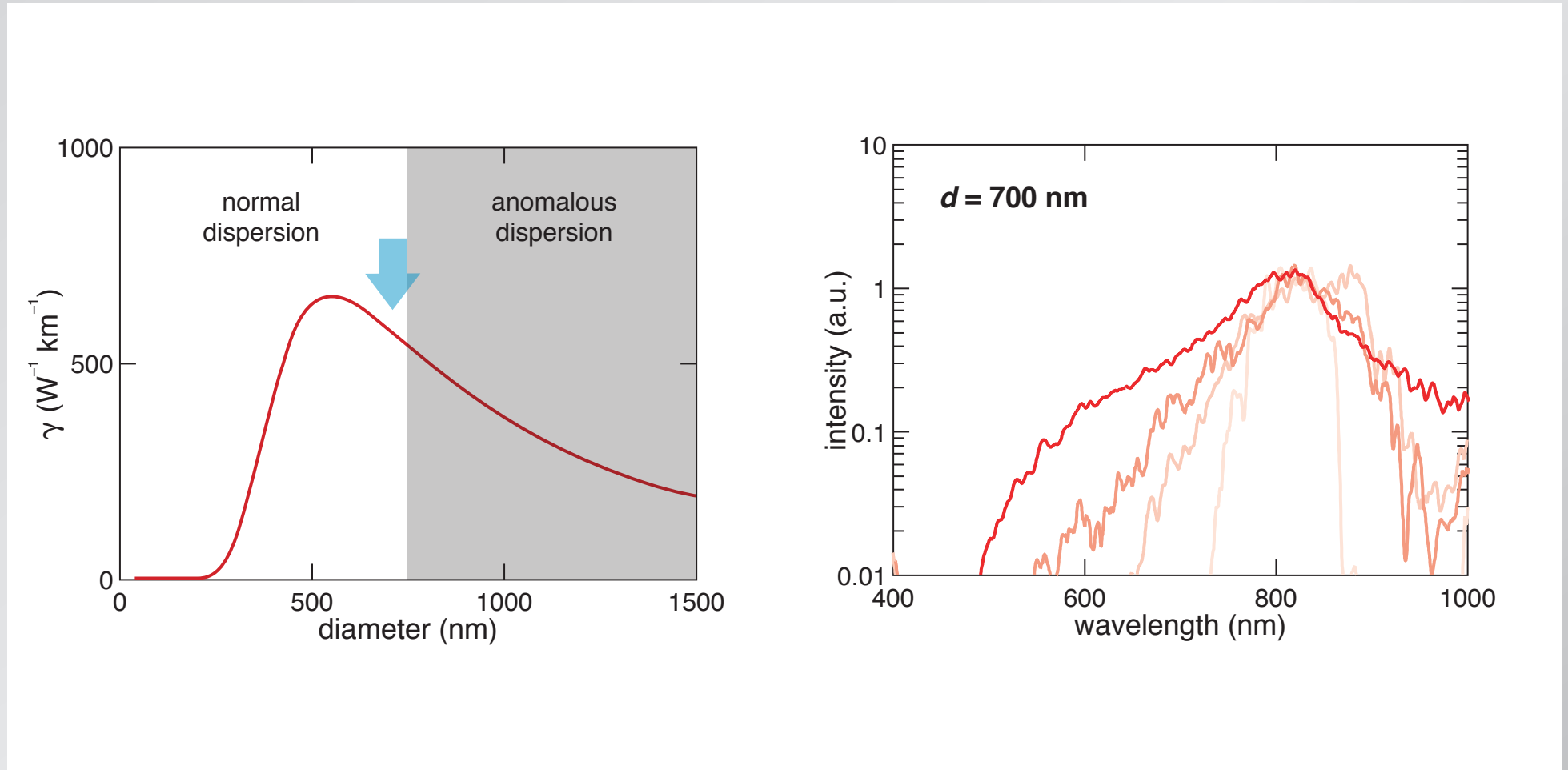
Supercontinuum generation

nanowire continuum generation



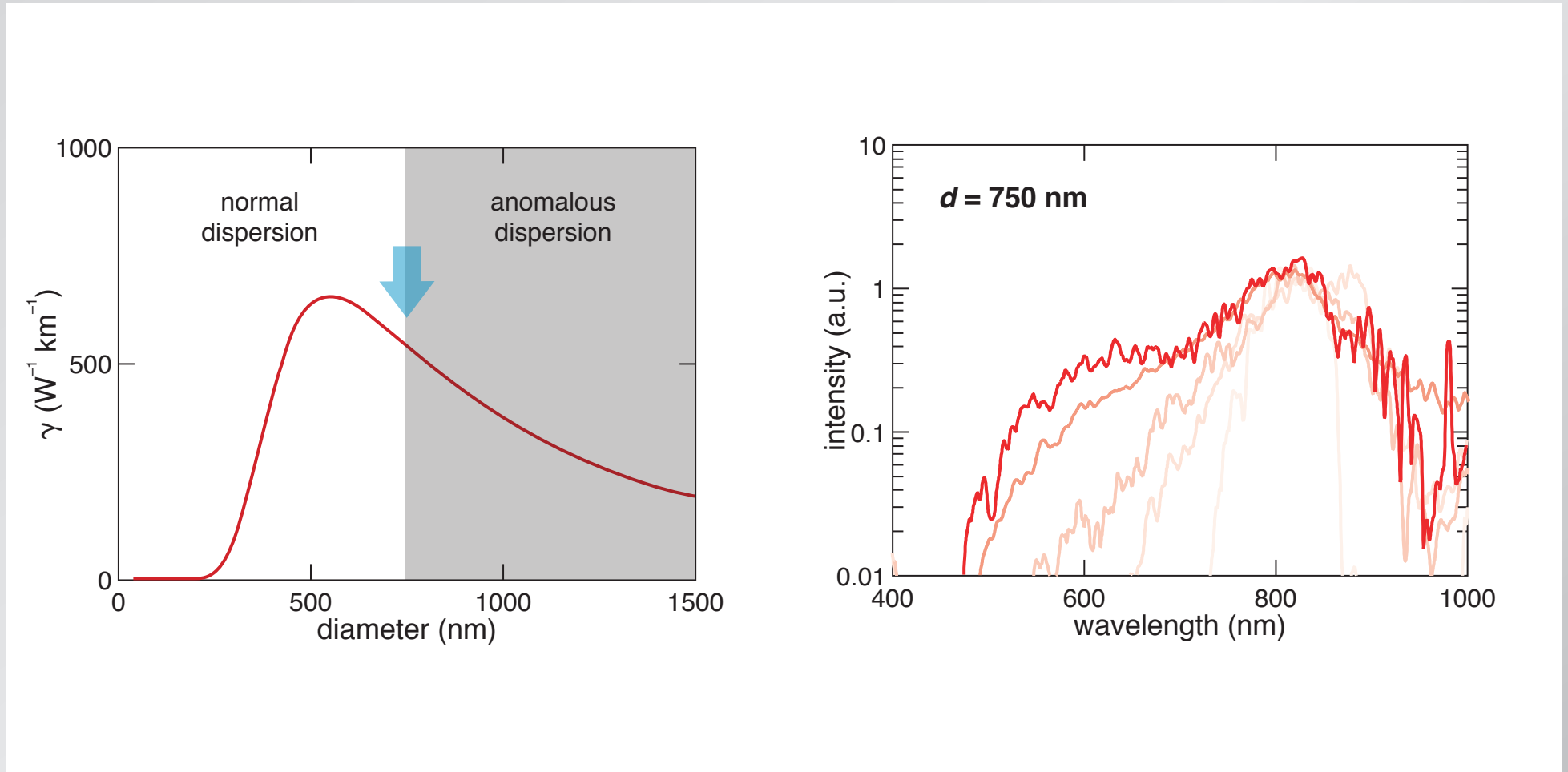
Supercontinuum generation

nanowire continuum generation



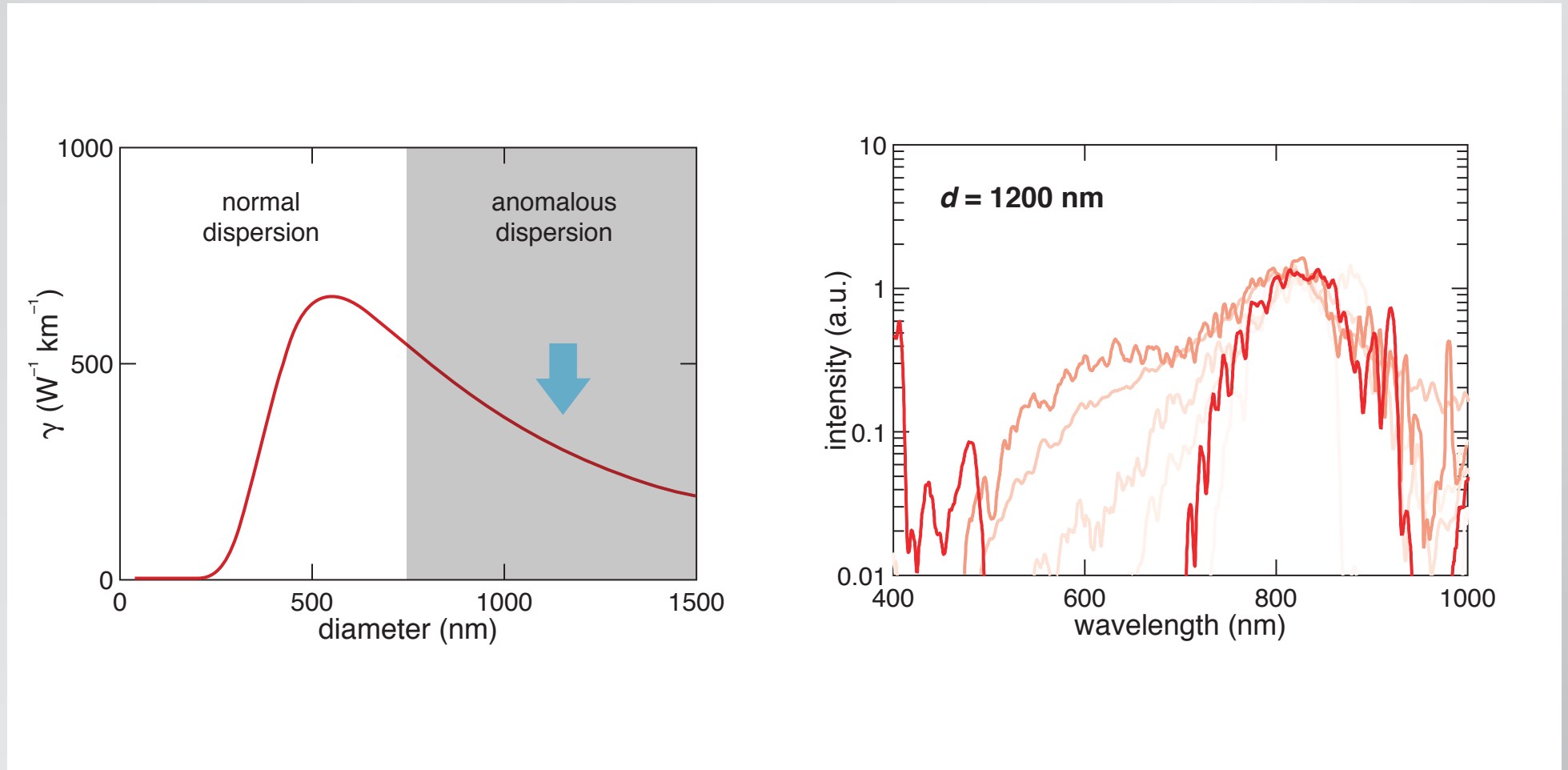
Supercontinuum generation

nanowire continuum generation



Supercontinuum generation

nanowire continuum generation



Supercontinuum generation

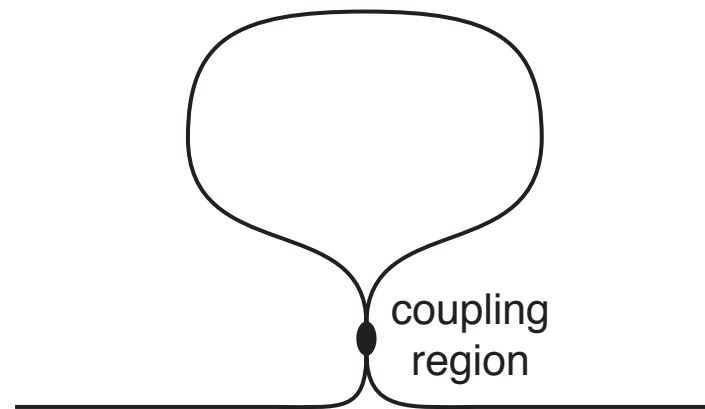
energy in nanowire \approx 1 nJ!

Supercontinuum generation

- **nanojoule nonlinear optics**
- **optimum diameter for silica 500–600 nm**
- **low dispersion**

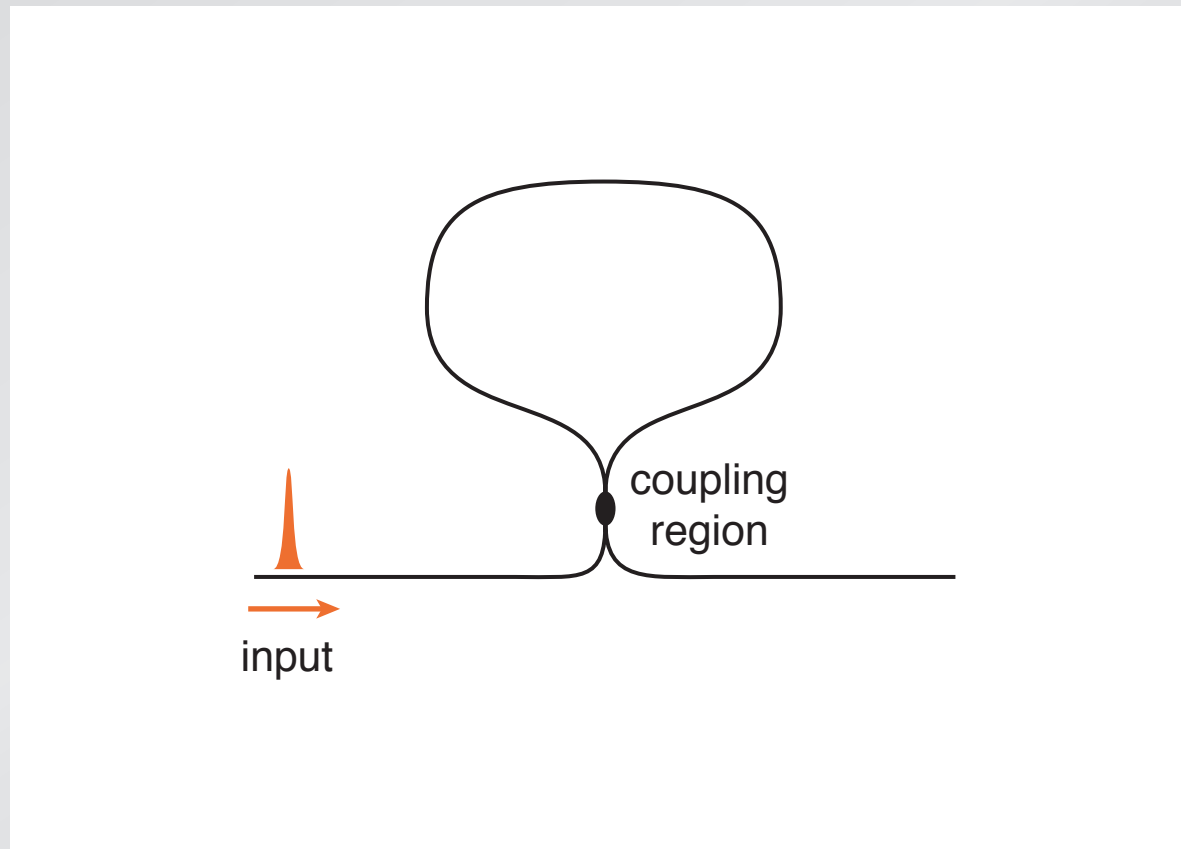
Optical logic gates

nanowire Sagnac interferometer



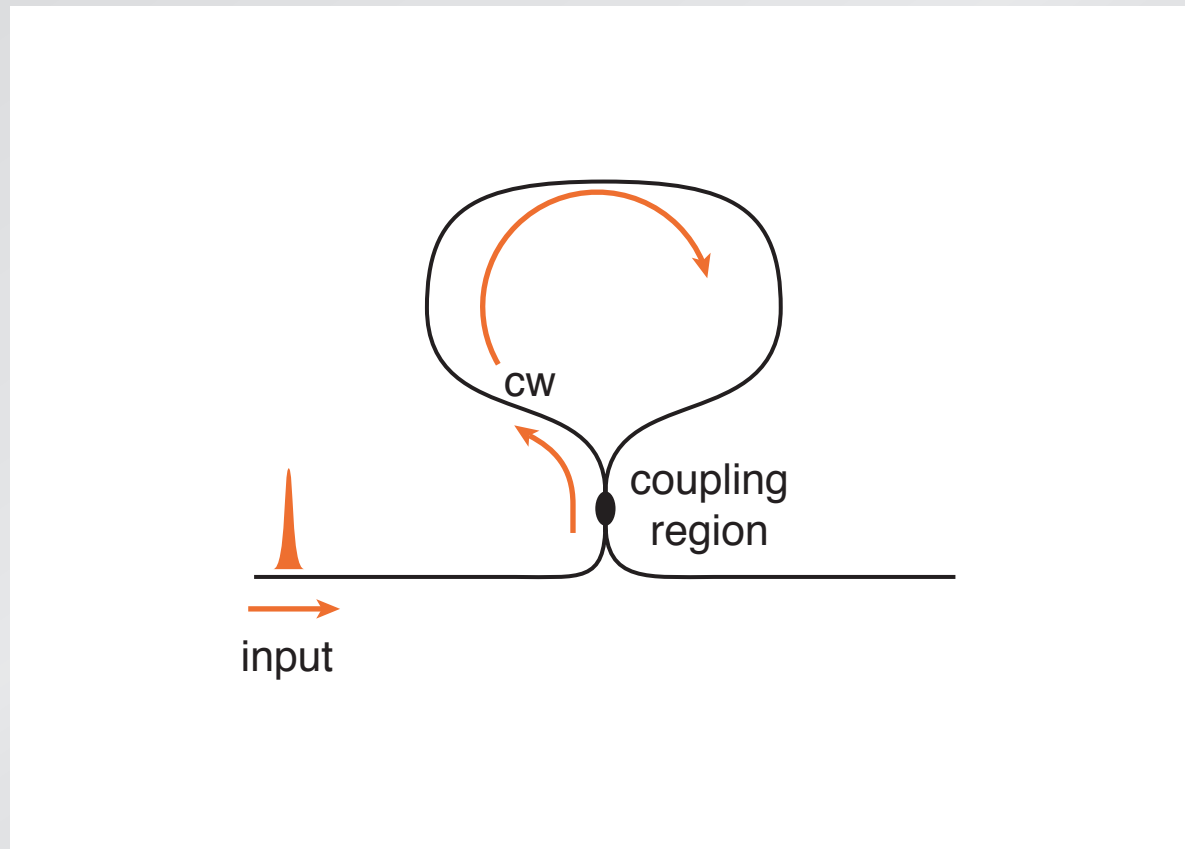
Optical logic gates

nanowire Sagnac interferometer



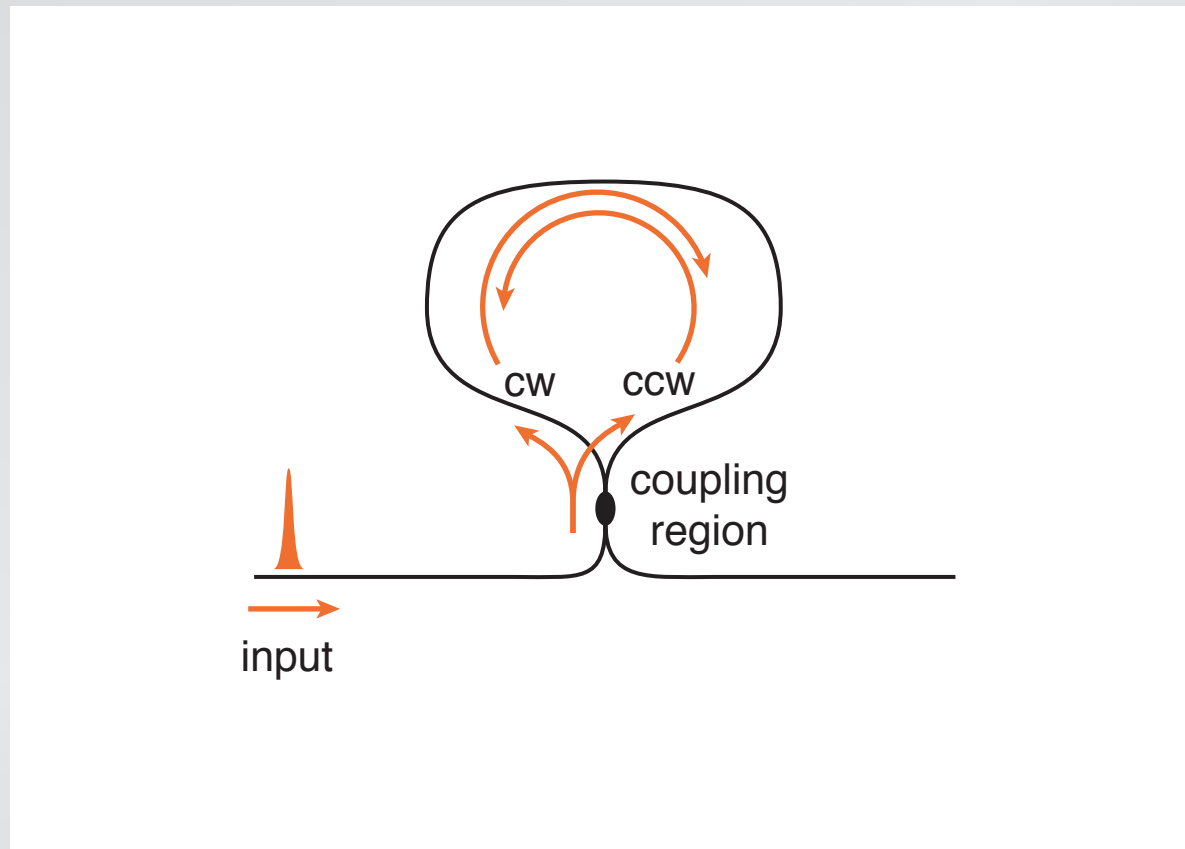
Optical logic gates

nanowire Sagnac interferometer



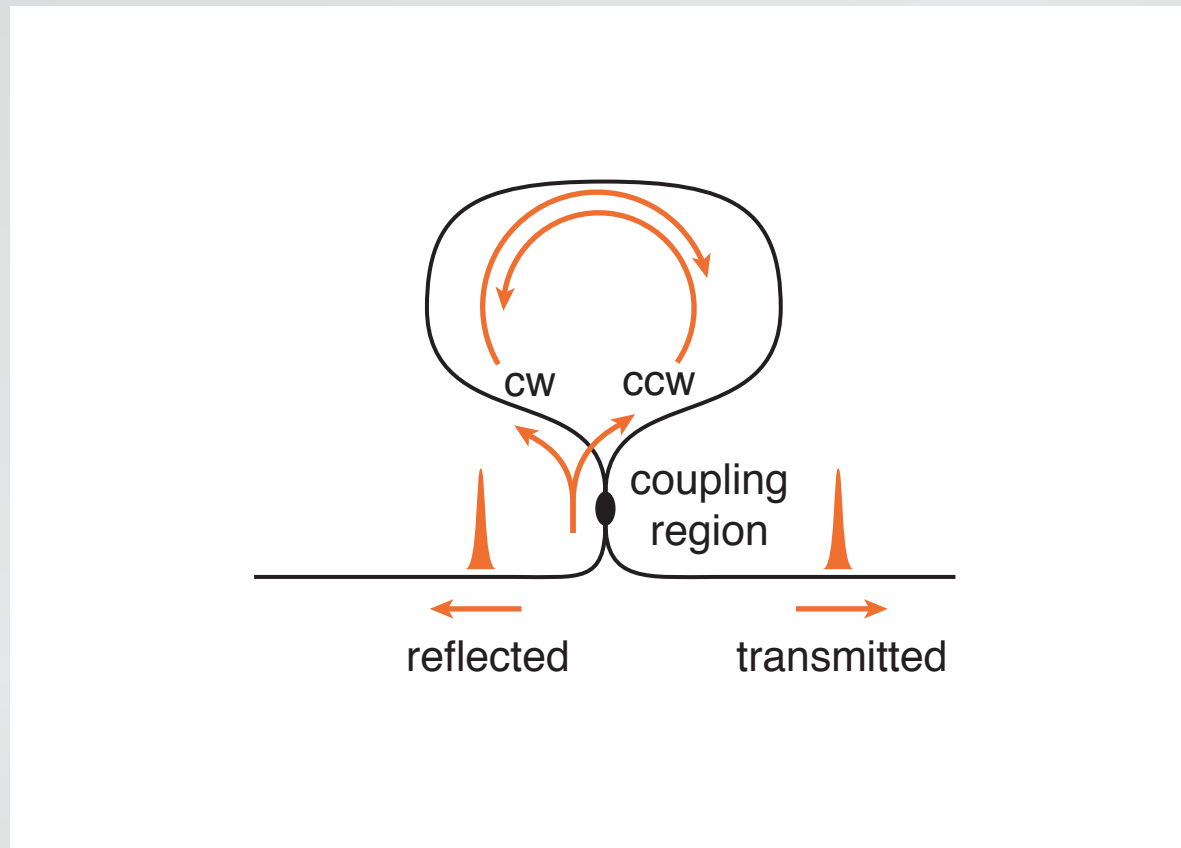
Optical logic gates

nanowire Sagnac interferometer



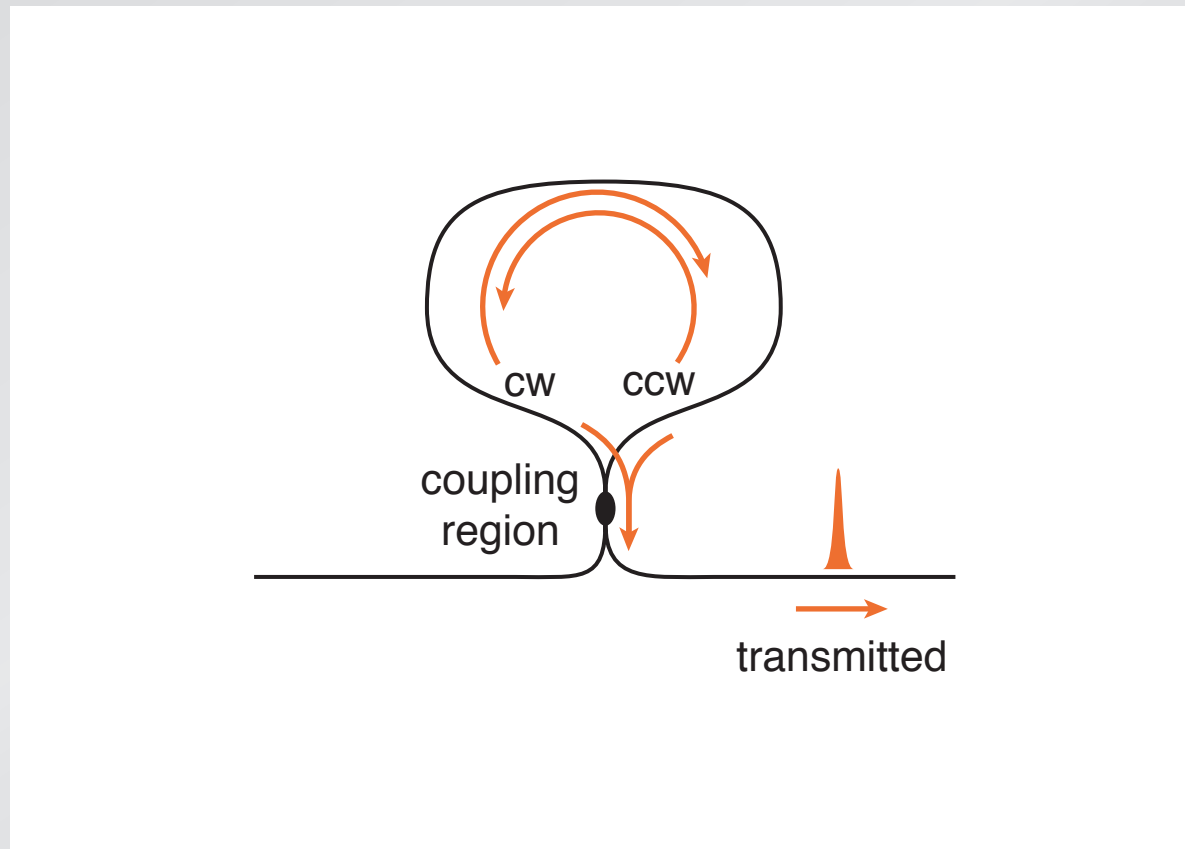
Optical logic gates

nanowire Sagnac interferometer



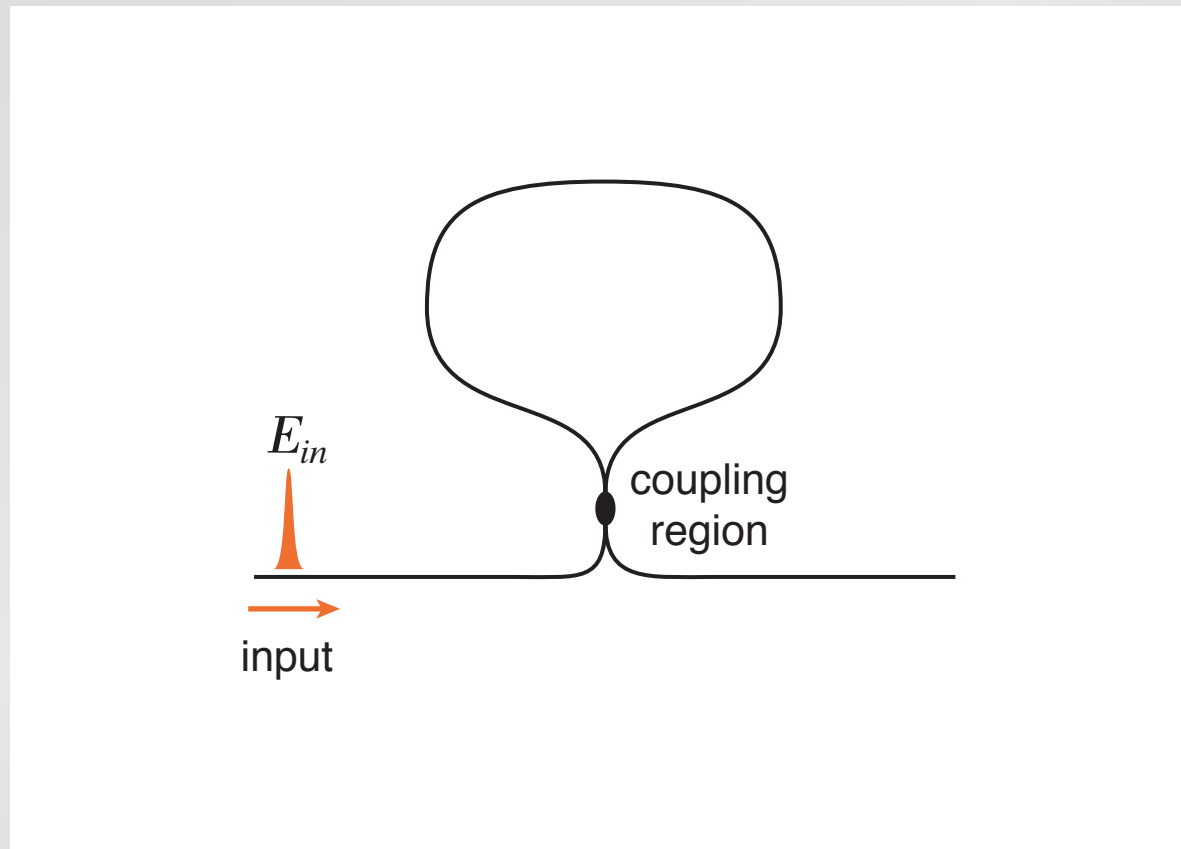
Optical logic gates

output = transmitted cw + ccw power



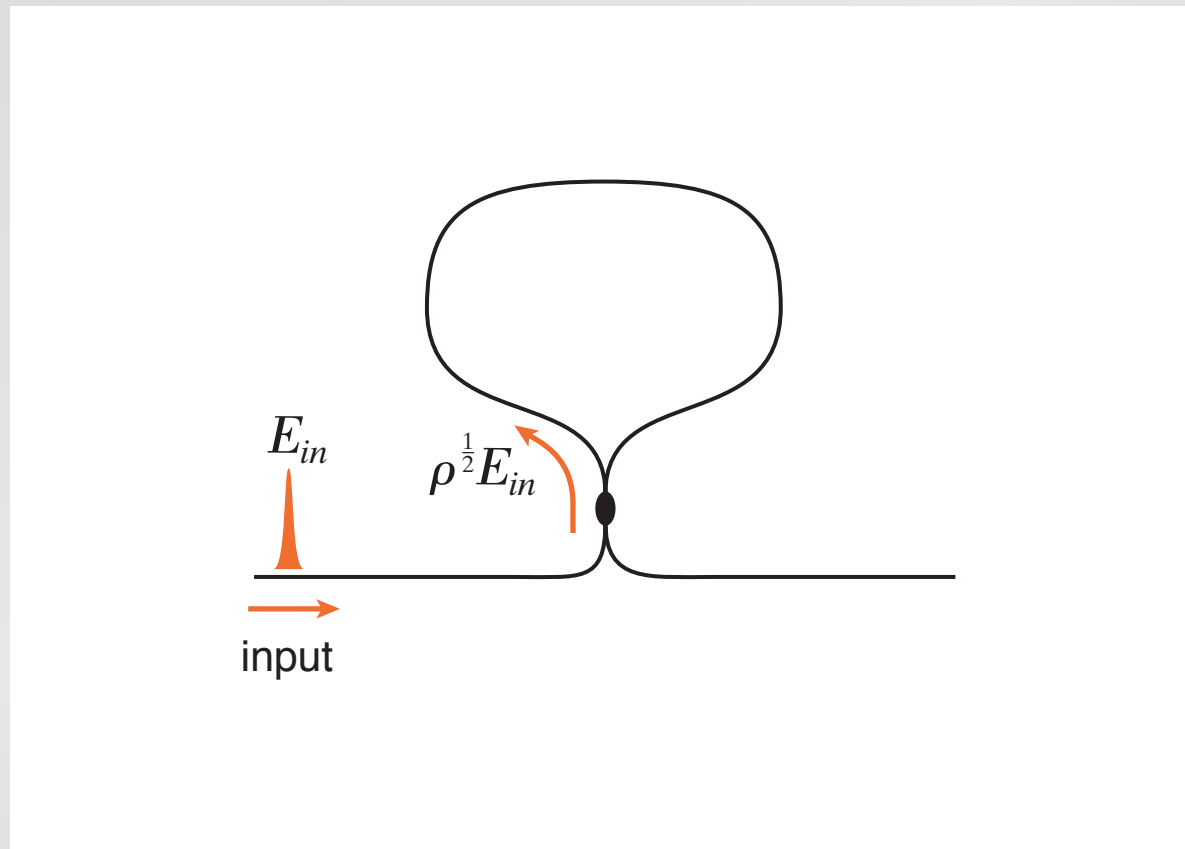
Optical logic gates

input electric field amplitude E_{in}



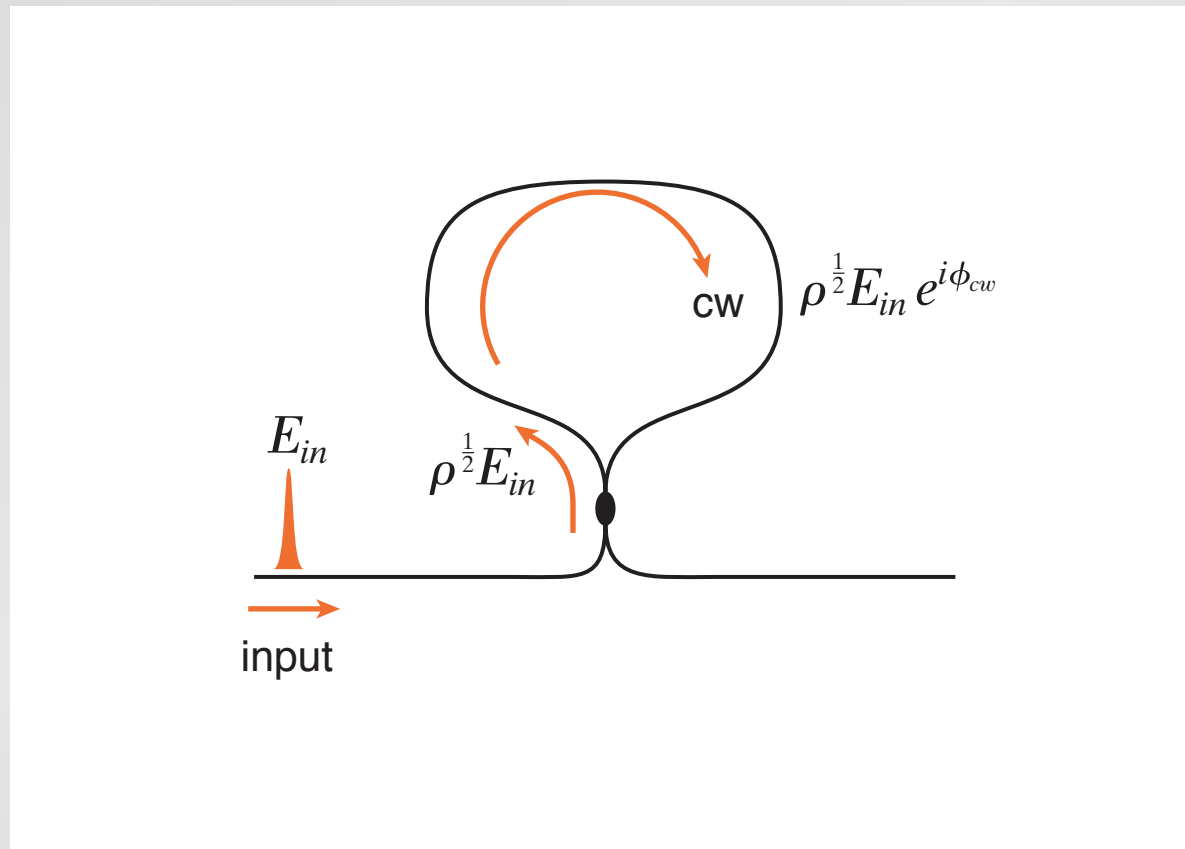
Optical logic gates

coupling parameter: ρ



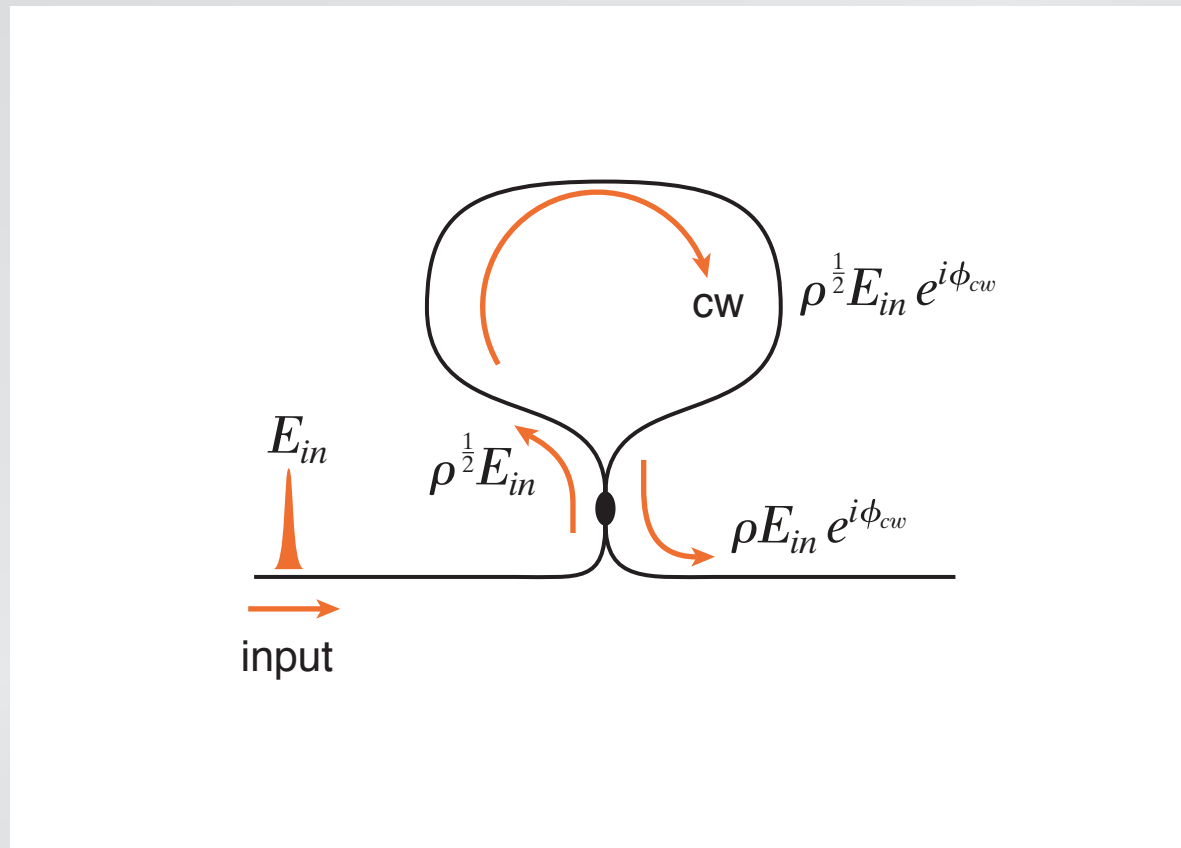
Optical logic gates

phase accumulation over path length of loop L



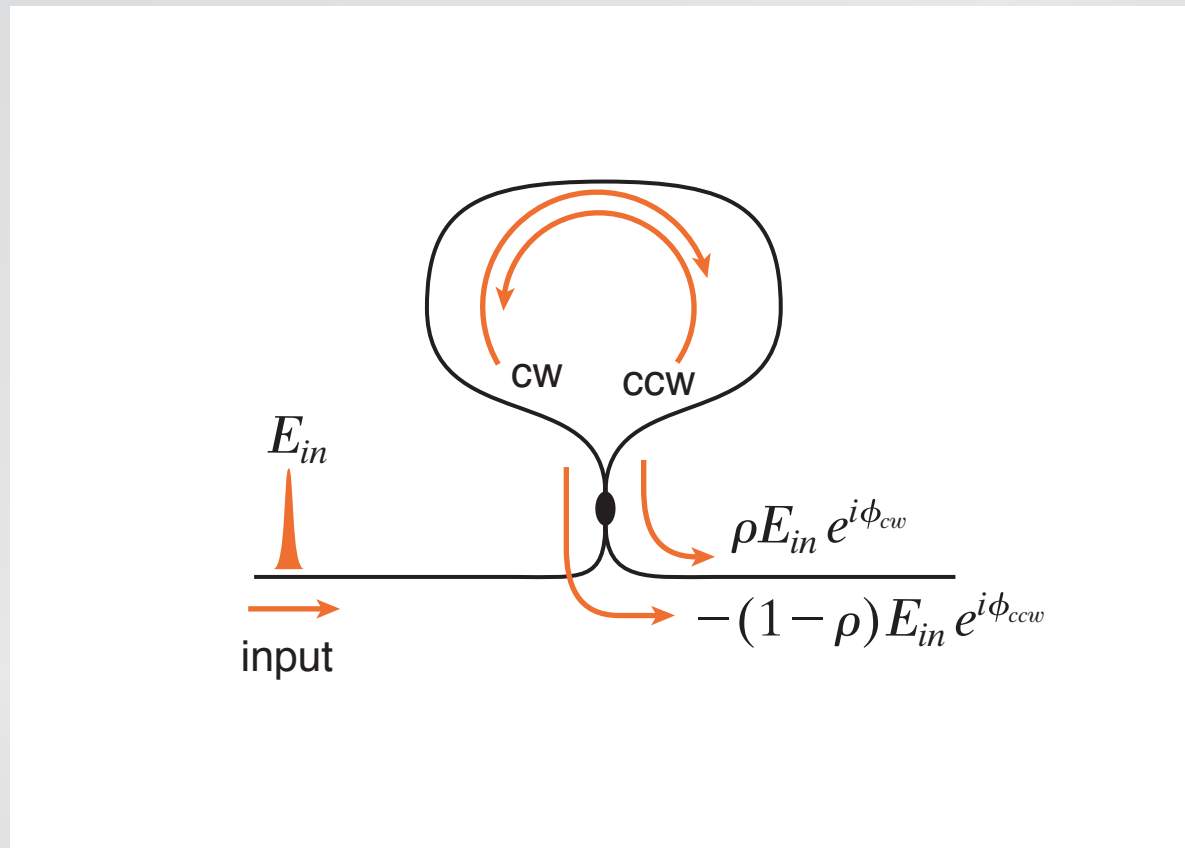
Optical logic gates

coupling parameter: ρ



Optical logic gates

output is sum of transmitted cw and ccw



Manipulating light at the nanoscale

accumulated phase:

$$\phi = k_0 n L$$

Manipulating light at the nanoscale

accumulated phase:

$$\phi = k_o n L$$

nonlinear index:

$$n = n_o + n_2 I = n_o + n_2 \frac{P_i}{A_{eff}}$$

Manipulating light at the nanoscale

accumulated phase:

$$\phi = k_o n L$$

nonlinear index:

$$n = n_o + n_2 I = n_o + n_2 \frac{P_i}{A_{eff}}$$

nonlinear parameter:

$$\gamma = n_2 \frac{k_o}{A_{eff}}$$

Manipulating light at the nanoscale

power-dependent output:

$$\frac{E_{out}^2}{E_{in}^2} = 1 - 2\rho(1 - \rho)\{1 + \cos[(1 - 2\rho)\gamma P_o L]\}$$

Manipulating light at the nanoscale

power-dependent output:

$$\frac{E_{out}^2}{E_{in}^2} = 1 - 2\rho(1 - \rho)\{1 + \cos[(1 - 2\rho)\gamma P_o L]\}$$

Q: What happens for a 50-50 coupler?

- 1. All the light is transmitted.**
- 2. Half the light is transmitted.**
- 3. No light is transmitted.**
- 4. The transmission depends on the input power P_o .**
- 5. Other**

Manipulating light at the nanoscale

power-dependent output:

$$\frac{E_{out}^2}{E_{in}^2} = 1 - 2\rho(1 - \rho)\{1 + \cos[(1 - 2\rho)\gamma P_o L]\}$$

Q: What happens for a 50-50 coupler?

1. All the light is transmitted.
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Manipulating light at the nanoscale

power-dependent output:

$$\frac{E_{out}^2}{E_{in}^2} = 1 - 2\rho(1 - \rho)\{1 + \cos[(1 - 2\rho)\gamma P_o L]\}$$

for 50-50 coupler:

$$\rho = 0.5$$

Manipulating light at the nanoscale

power-dependent output:

$$\frac{E_{out}^2}{E_{in}^2} = 1 - 2\rho(1 - \rho)\{1 + \cos[(1 - 2\rho)\gamma P_o L]\}$$

for 50-50 coupler:

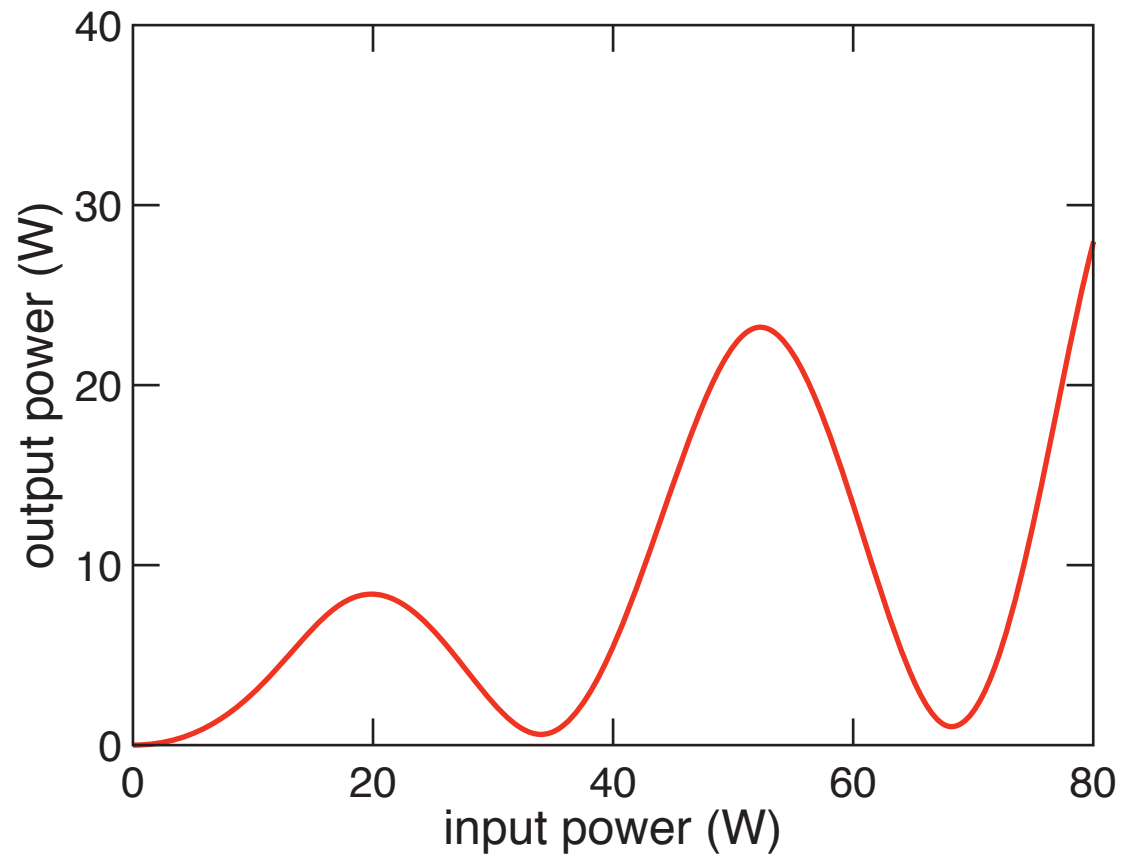
$$\rho = 0.5$$

no transmission:

$$\frac{E_{out}^2}{E_{in}^2} = 0$$

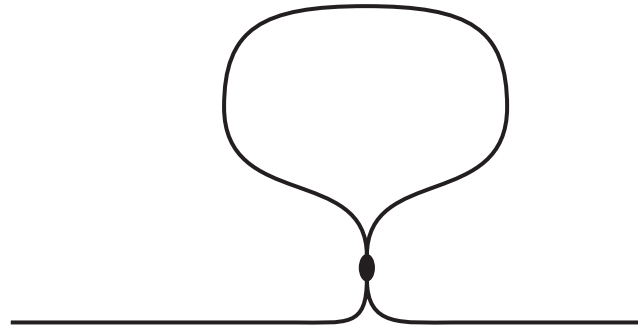
Optical logic gates

when $\rho \neq 0.5$:



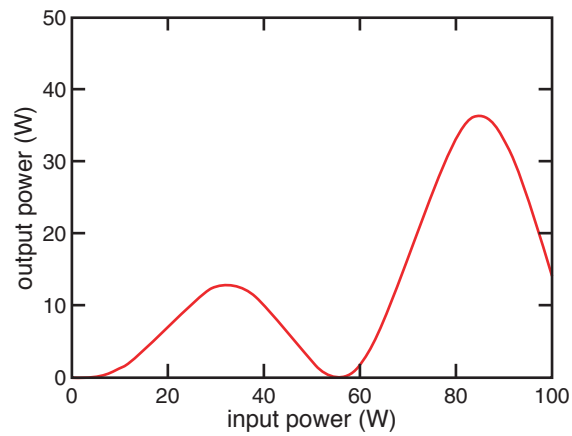
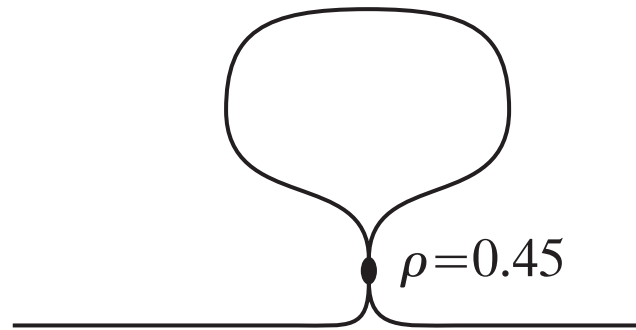
Optical logic gates

nonlinear nanogate



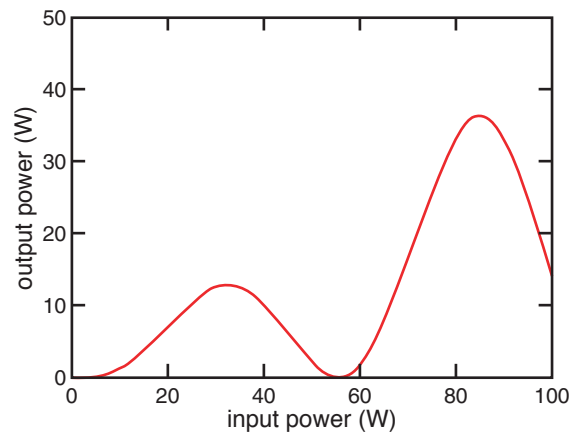
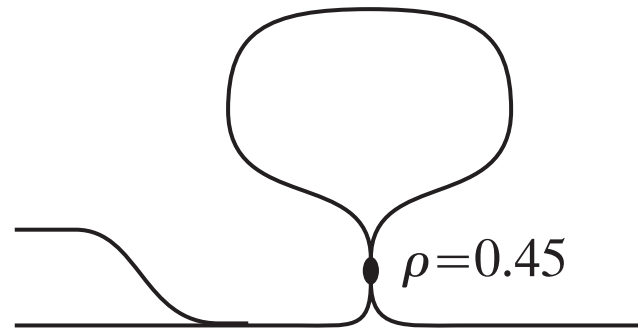
Optical logic gates

nonlinear nanogate



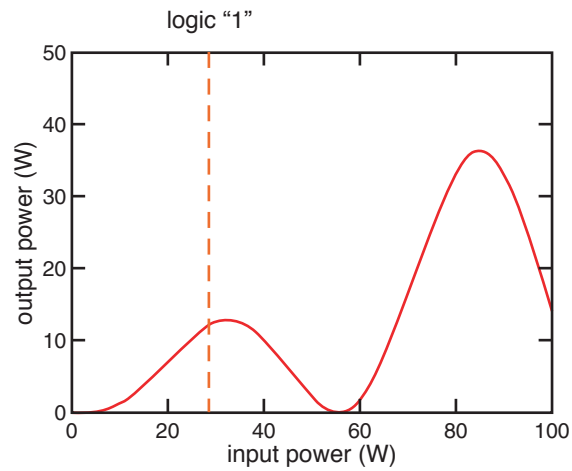
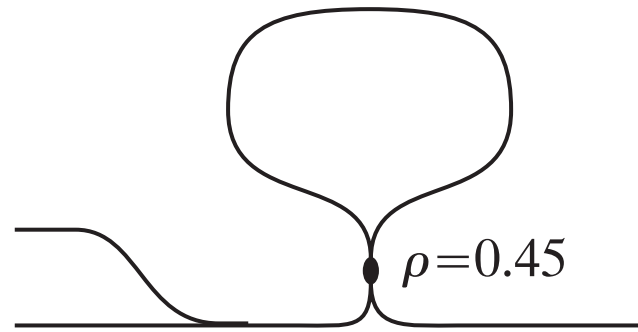
Optical logic gates

nonlinear nanogate



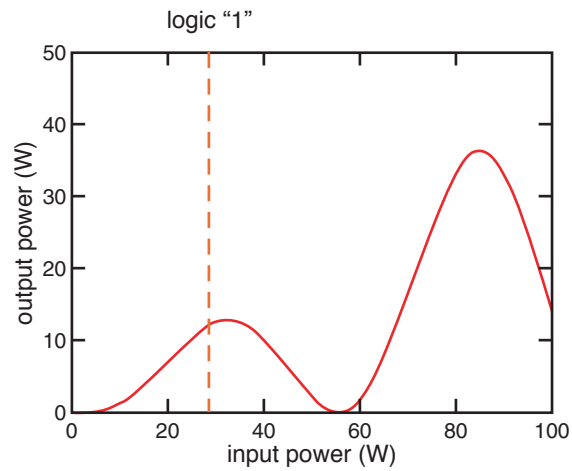
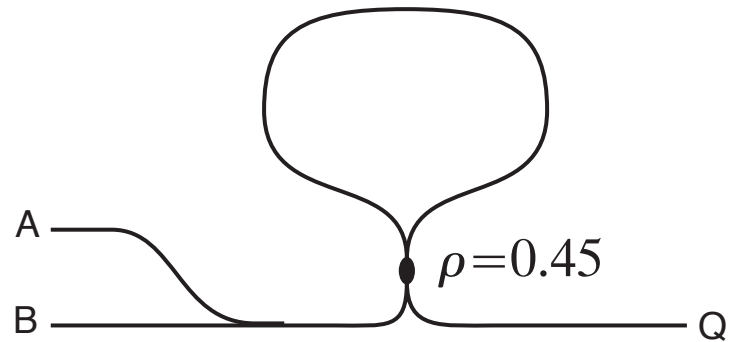
Optical logic gates

nonlinear nanogate



Optical logic gates

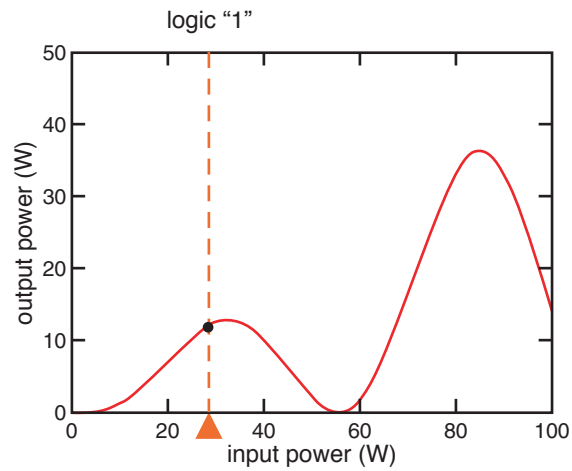
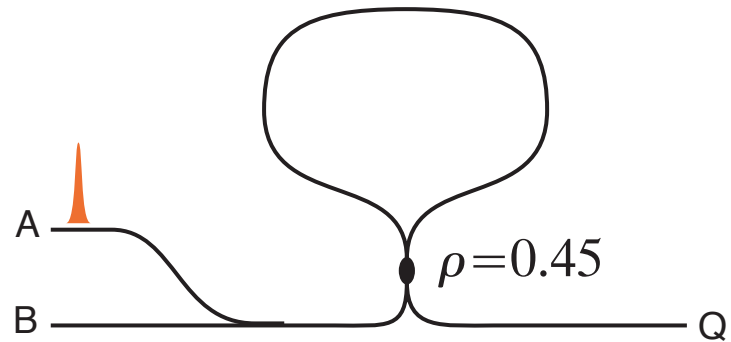
nonlinear nanogate



A	B	Q
0	0	0

Optical logic gates

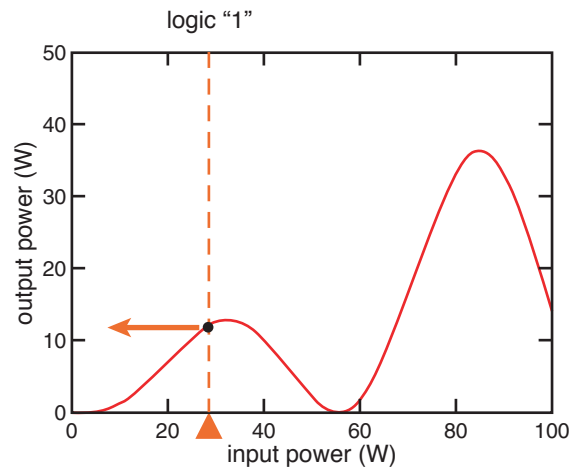
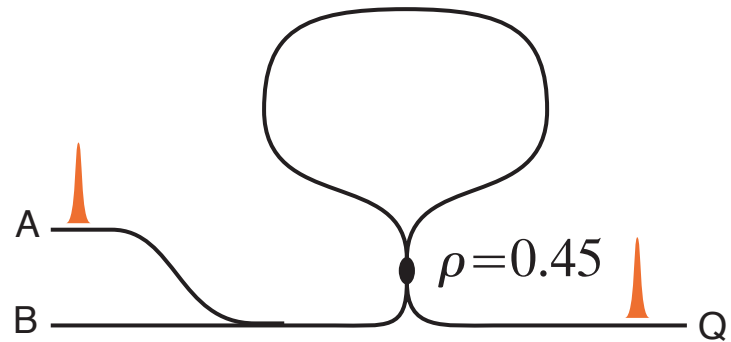
nonlinear nanogate



A	B	Q
0	0	0

Optical logic gates

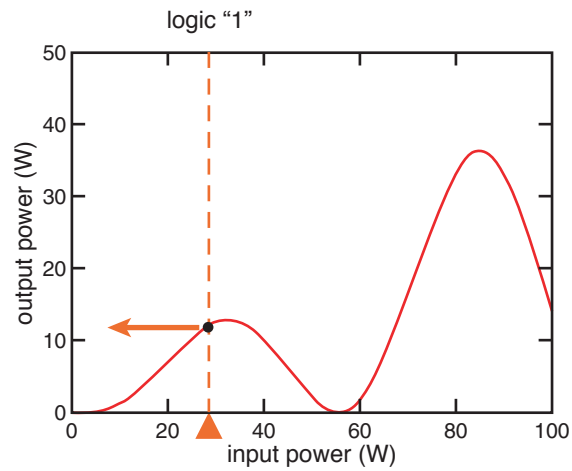
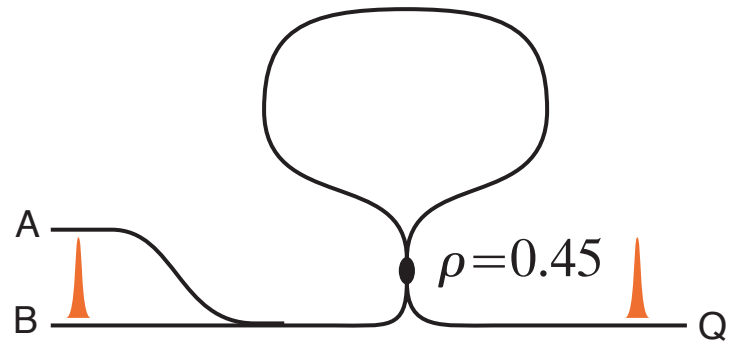
nonlinear nanogate



A	B	Q
0	0	0
1	0	1

Optical logic gates

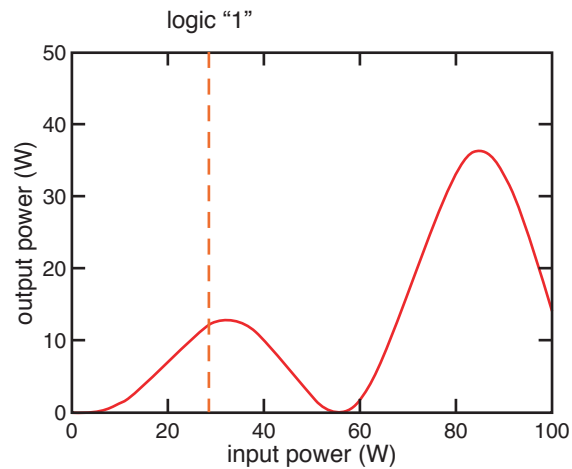
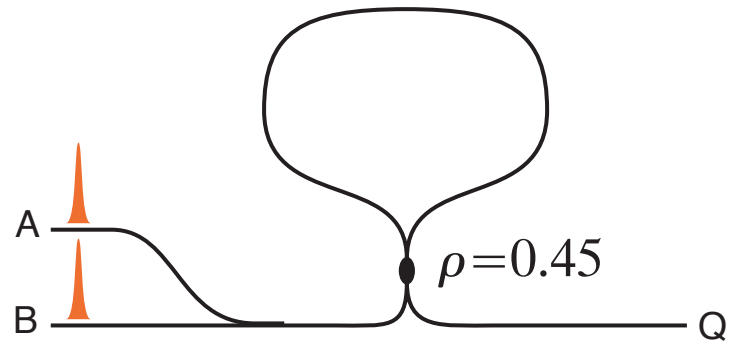
nonlinear nanogate



A	B	Q
0	0	0
1	0	1
0	1	1

Optical logic gates

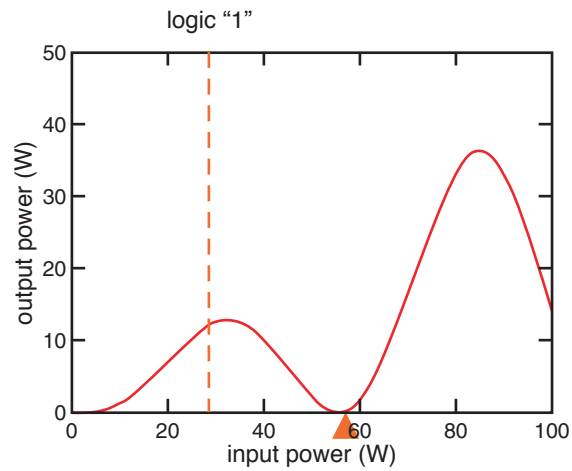
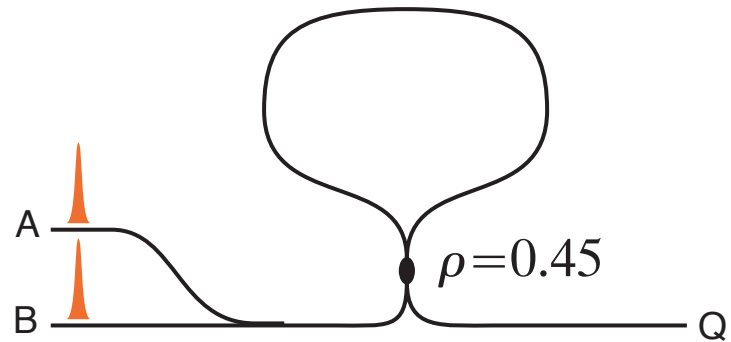
nonlinear nanogate



A	B	Q
0	0	0
1	0	1
0	1	1

Optical logic gates

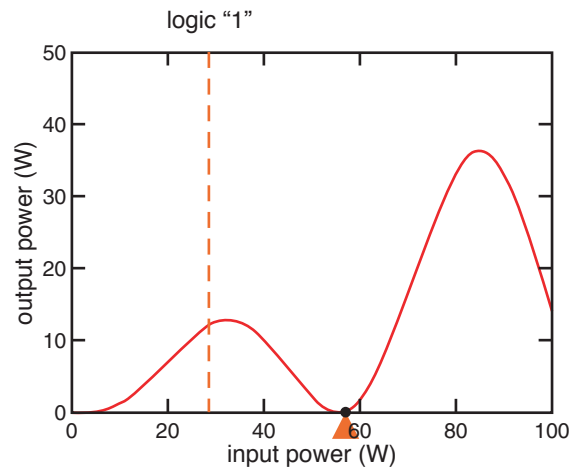
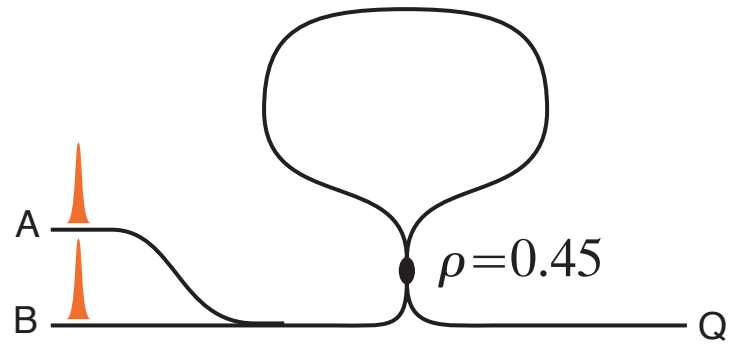
nonlinear nanogate



A	B	Q
0	0	0
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Optical logic gates

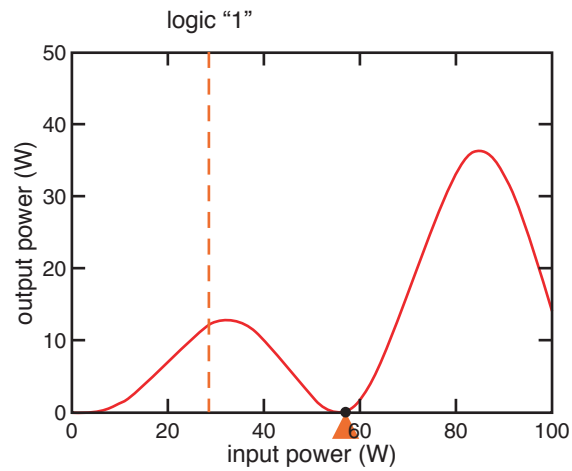
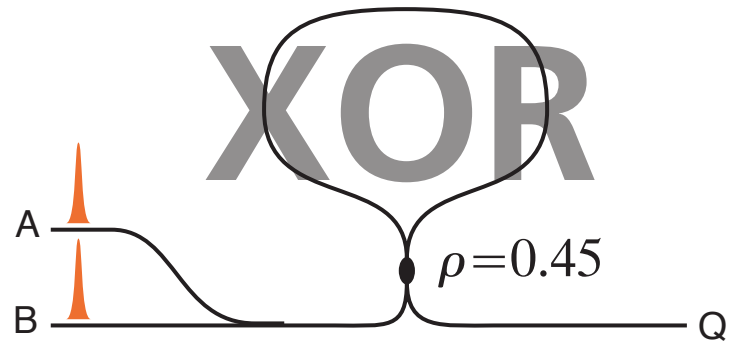
nonlinear nanogate



A	B	Q
0	0	0
1	0	1
0	1	1
1	1	0

Optical logic gates

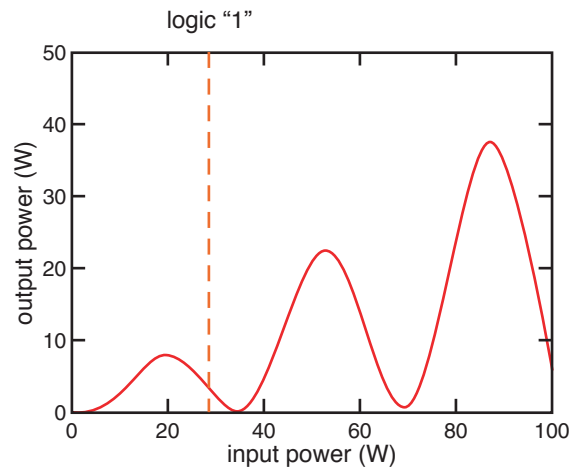
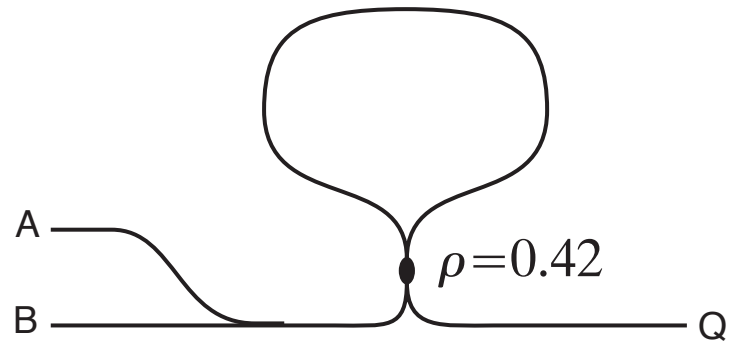
nonlinear nanogate



A	B	Q
0	0	0
1	0	1
0	1	1
1	1	0

Optical logic gates

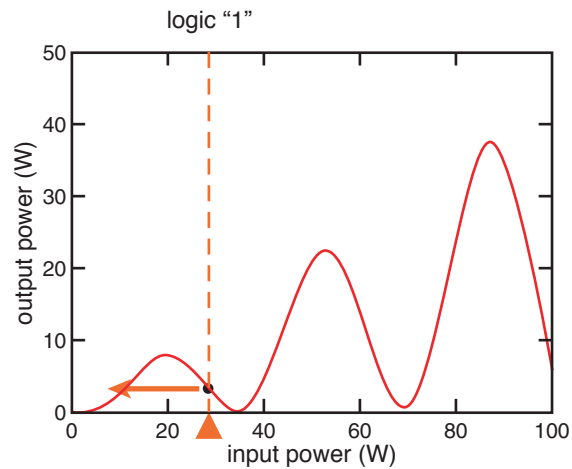
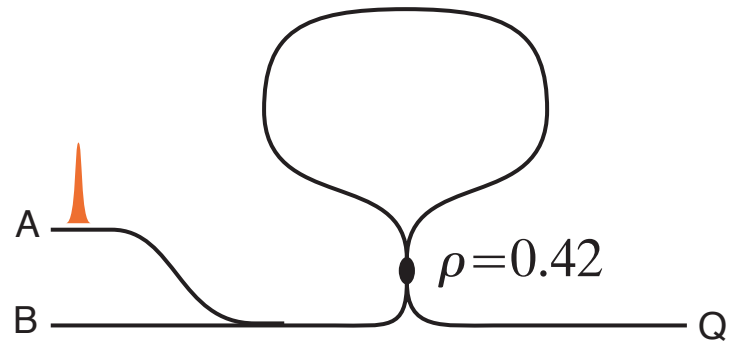
nonlinear nanogate



A	B	Q
0	0	0

Optical logic gates

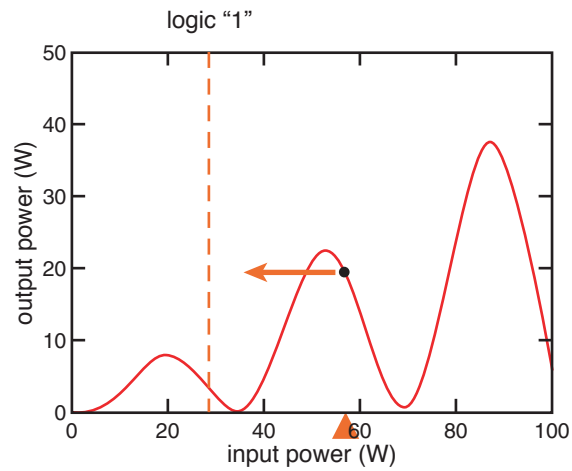
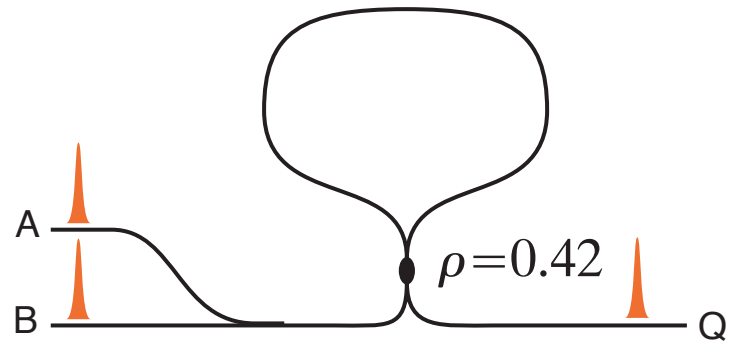
nonlinear nanogate



A	B	Q
0	0	0
1	0	0
0	1	0

Optical logic gates

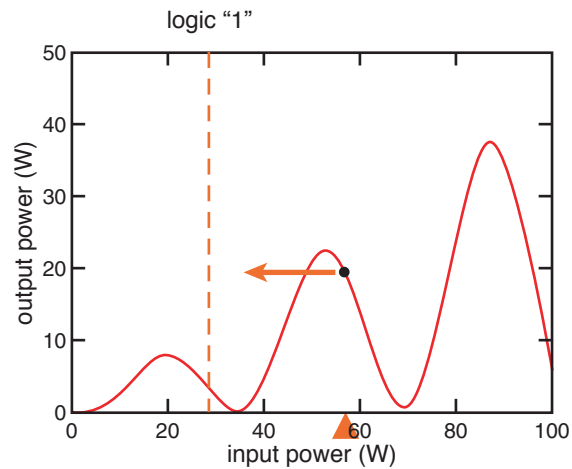
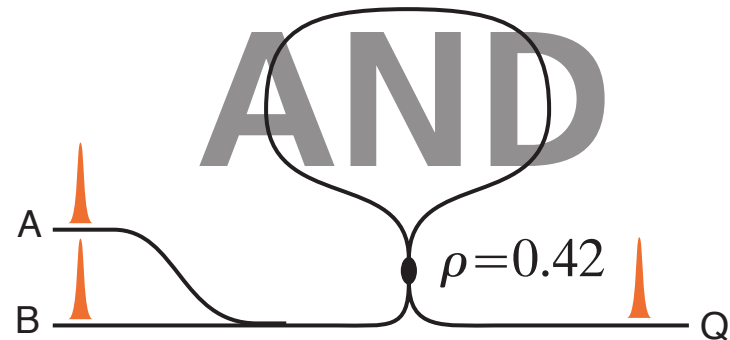
nonlinear nanogate



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1	0	0
0	1	0
1	1	1

Optical logic gates

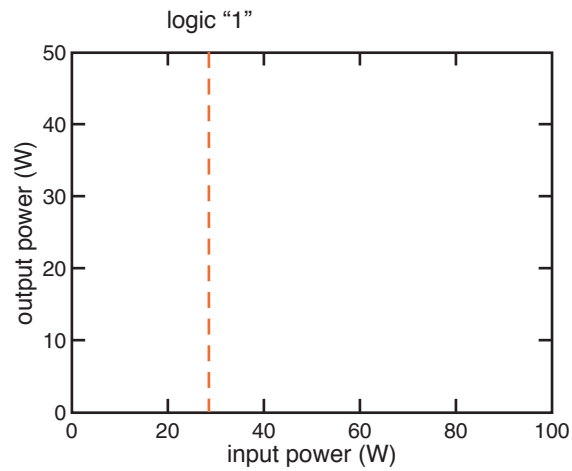
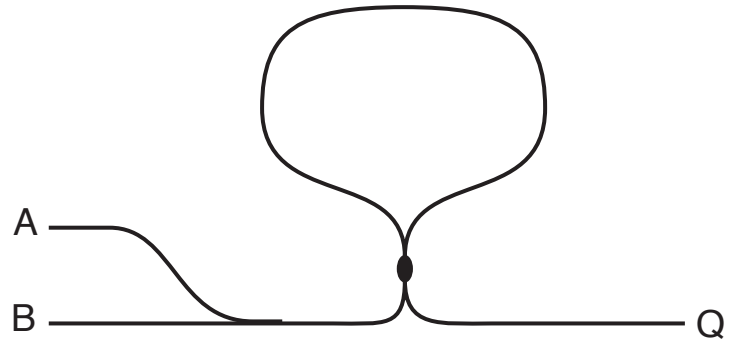
nonlinear nanogate



A	B	Q
0	0	0
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0	1	0
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Optical logic gates

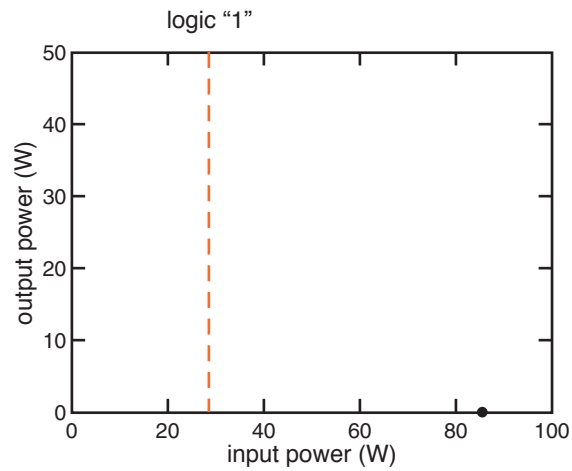
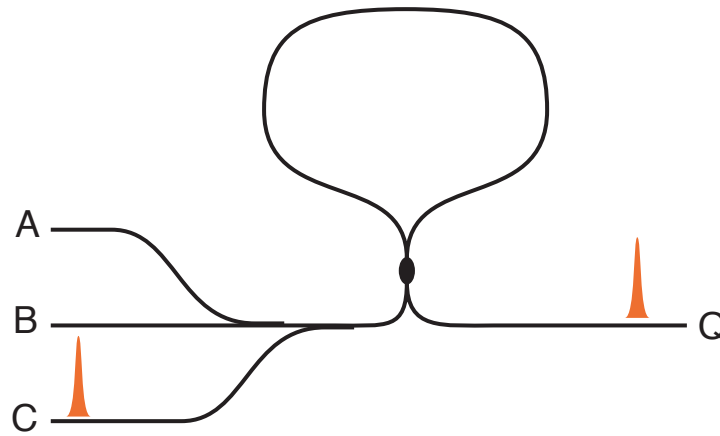
for NAND gate need output with no input



A	B	Q
0	0	1

Optical logic gates

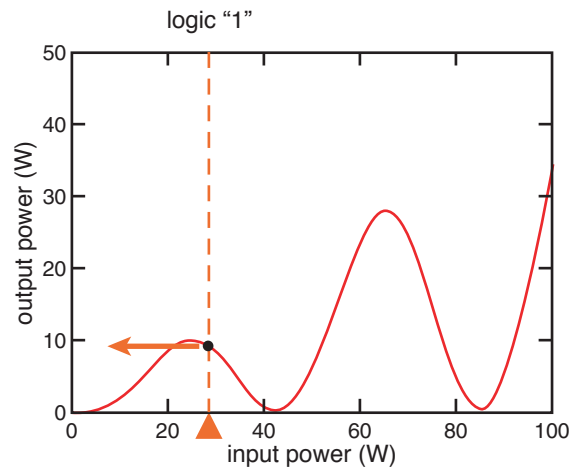
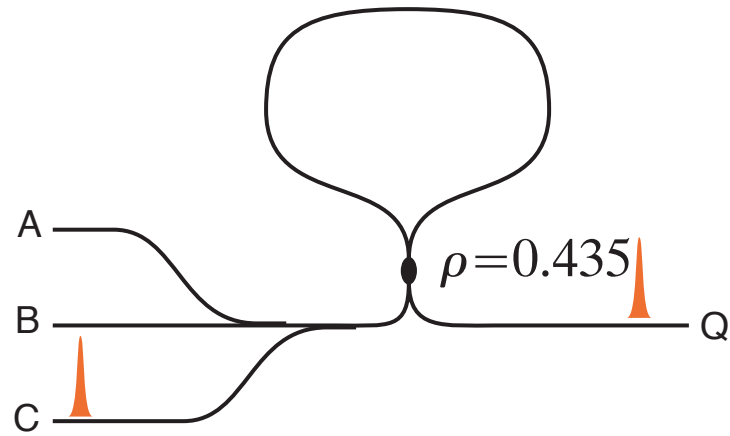
for NAND gate need output with no input



A	B	Q
0	0	1

Optical logic gates

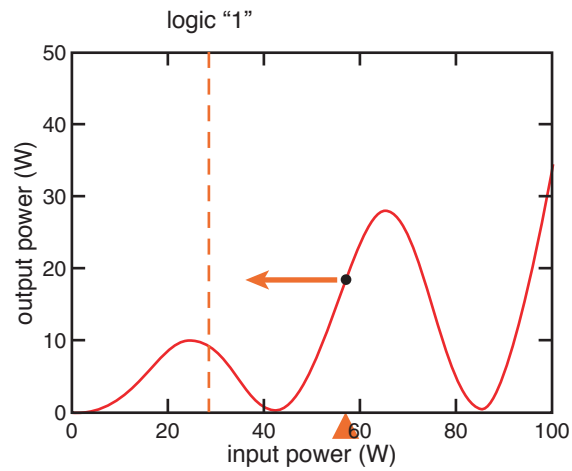
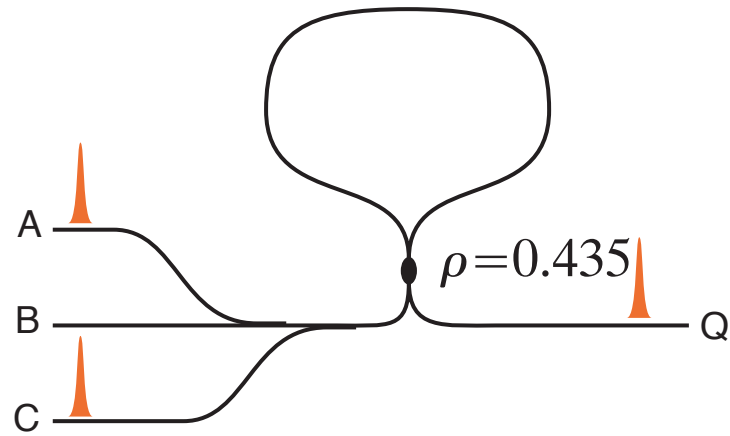
for NAND gate need output with no input



A	B	Q
0	0	1

Optical logic gates

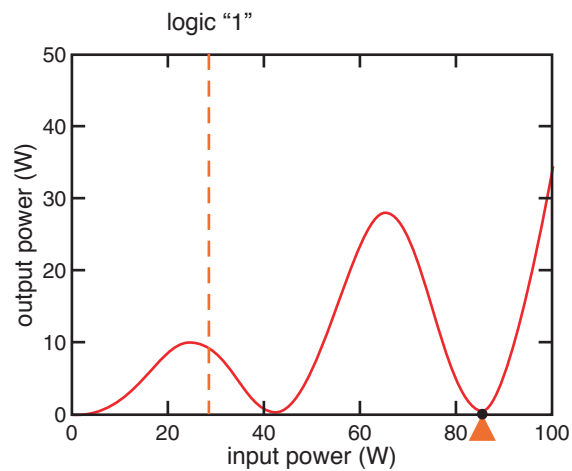
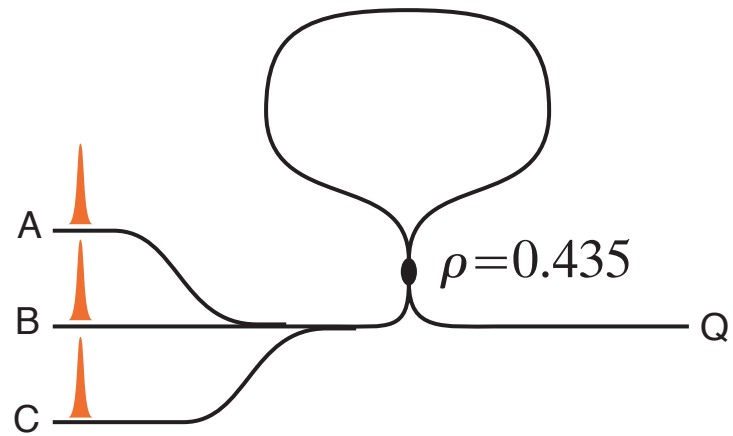
universal NAND gate



A	B	Q
0	0	1
1	0	1
0	1	1

Optical logic gates

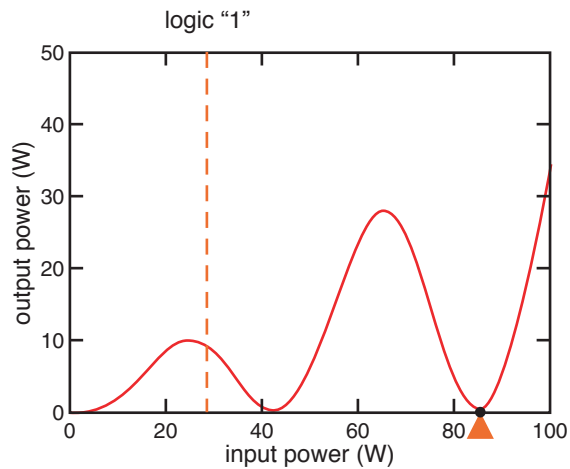
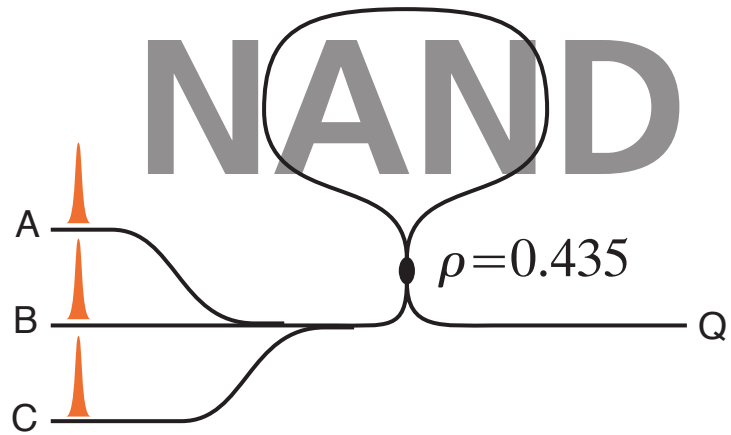
universal NAND gate



A	B	Q
0	0	1
1	0	1
0	1	1
1	1	0

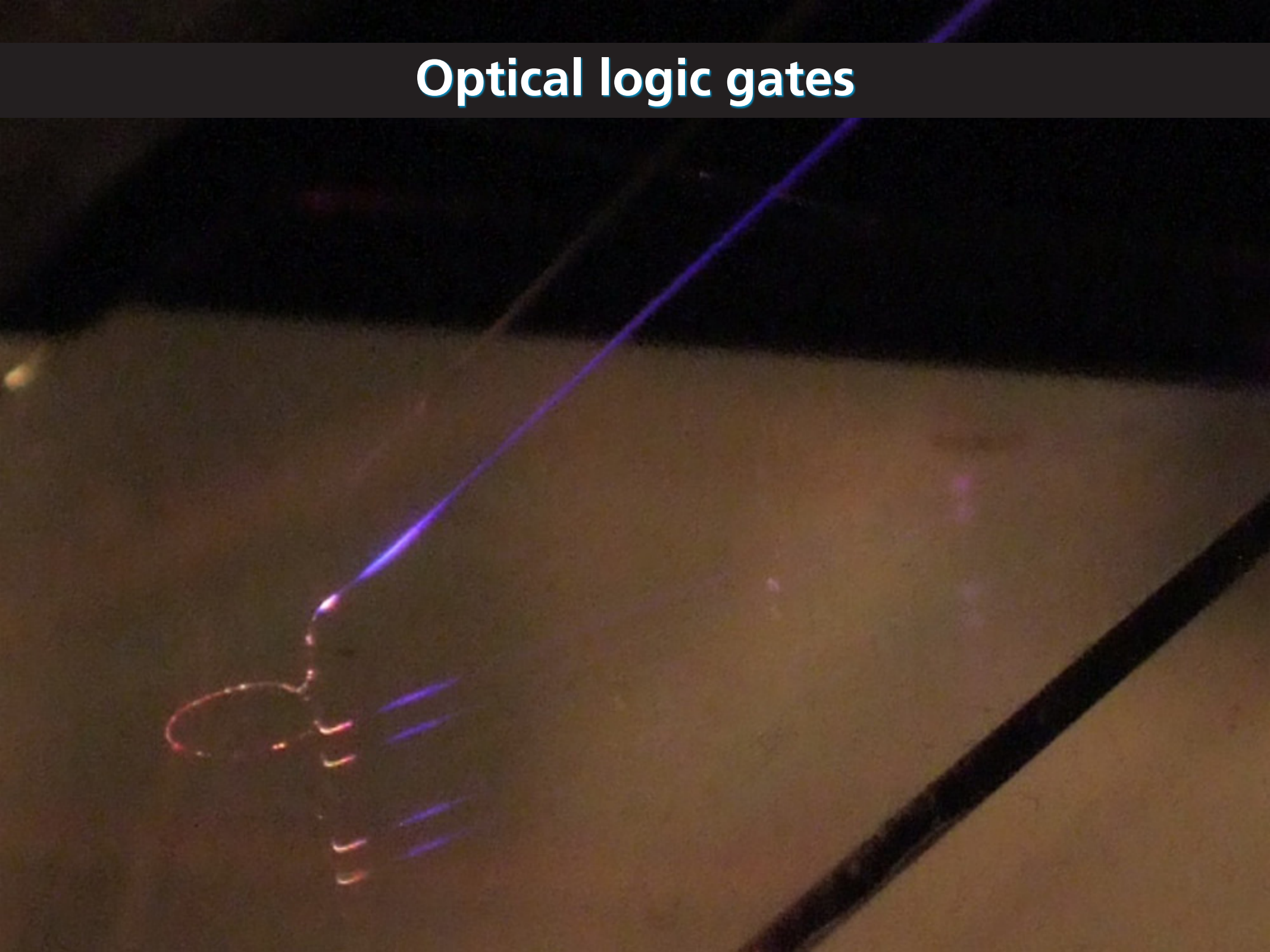
Optical logic gates

universal NAND gate



A	B	Q
0	0	1
1	0	1
0	1	1
1	1	0

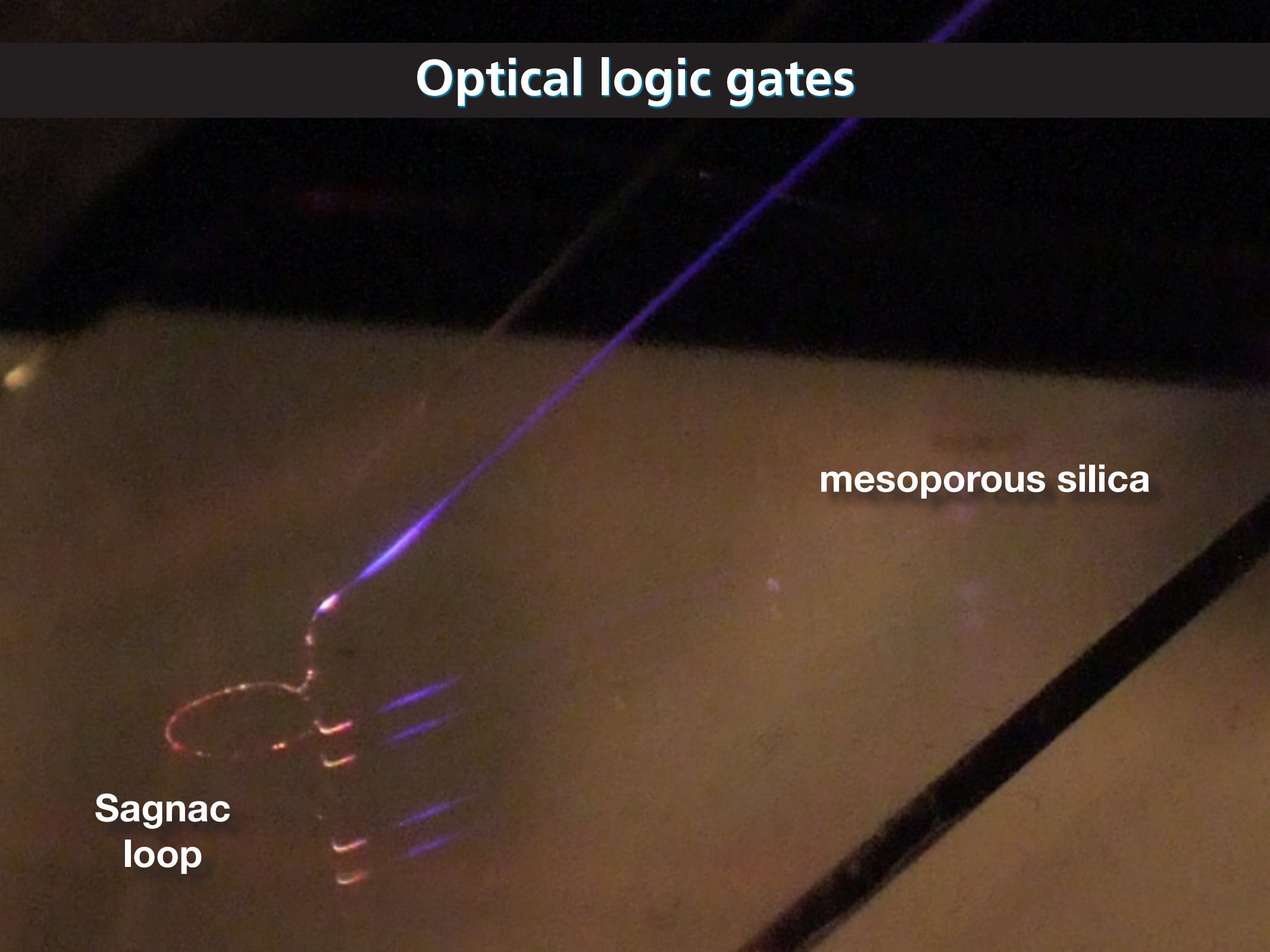
Optical logic gates



Optical logic gates

mesoporous silica

Sagnac
loop



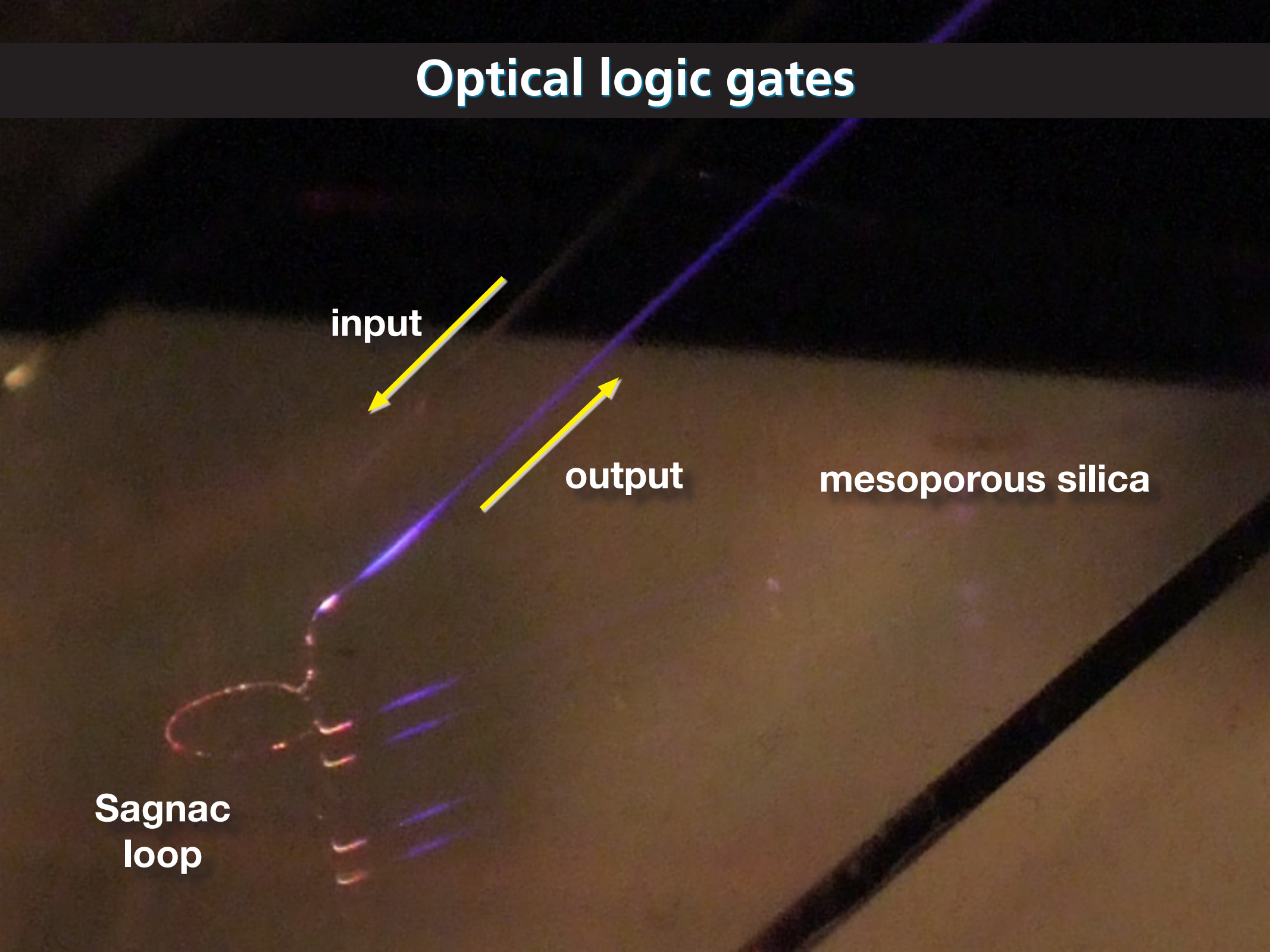
Optical logic gates

input

output

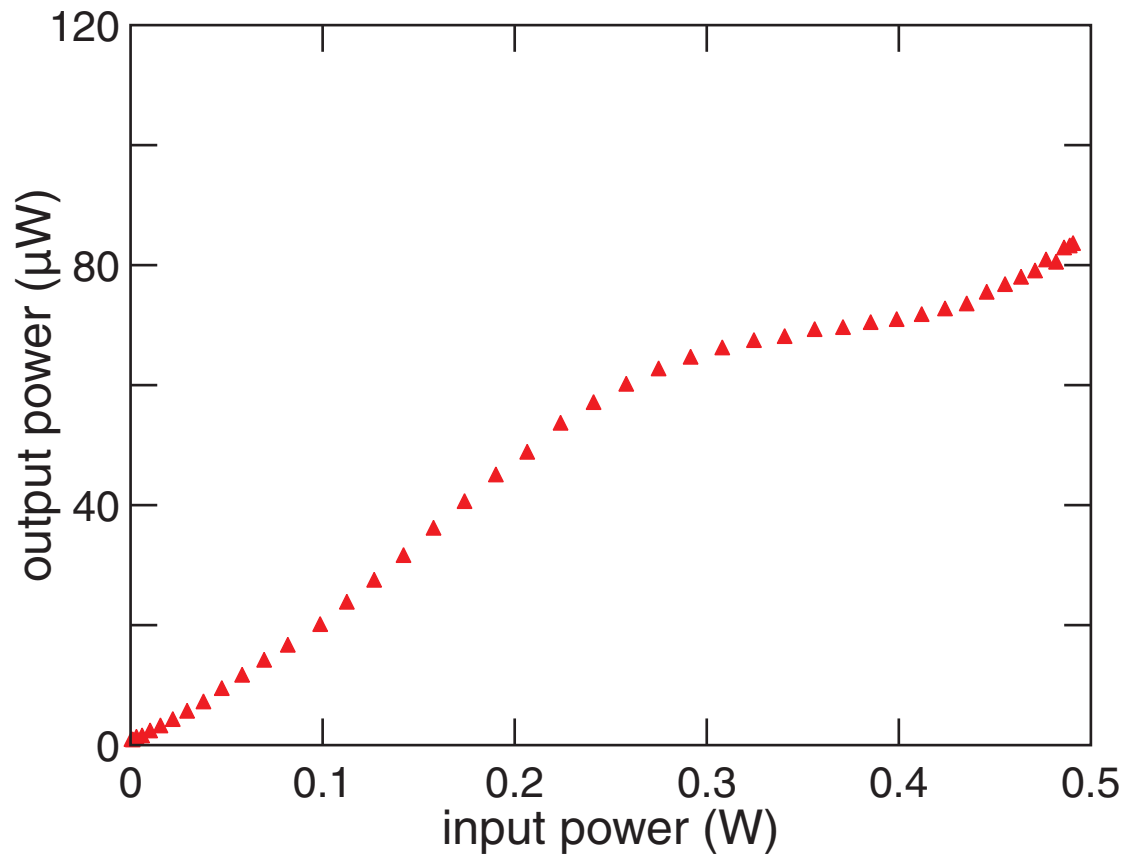
mesoporous silica

Sagnac
loop



Optical logic gates

very preliminary data

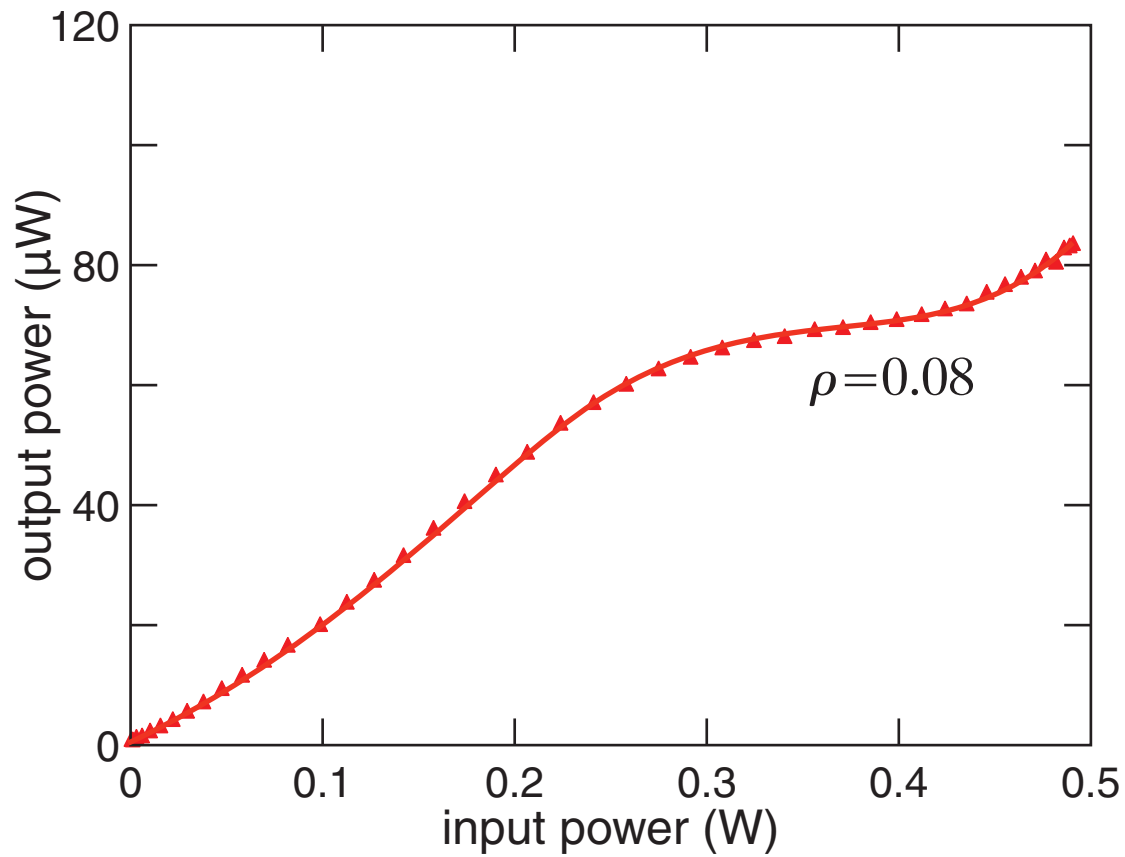


Optical logic gates

light-by-light modulation!

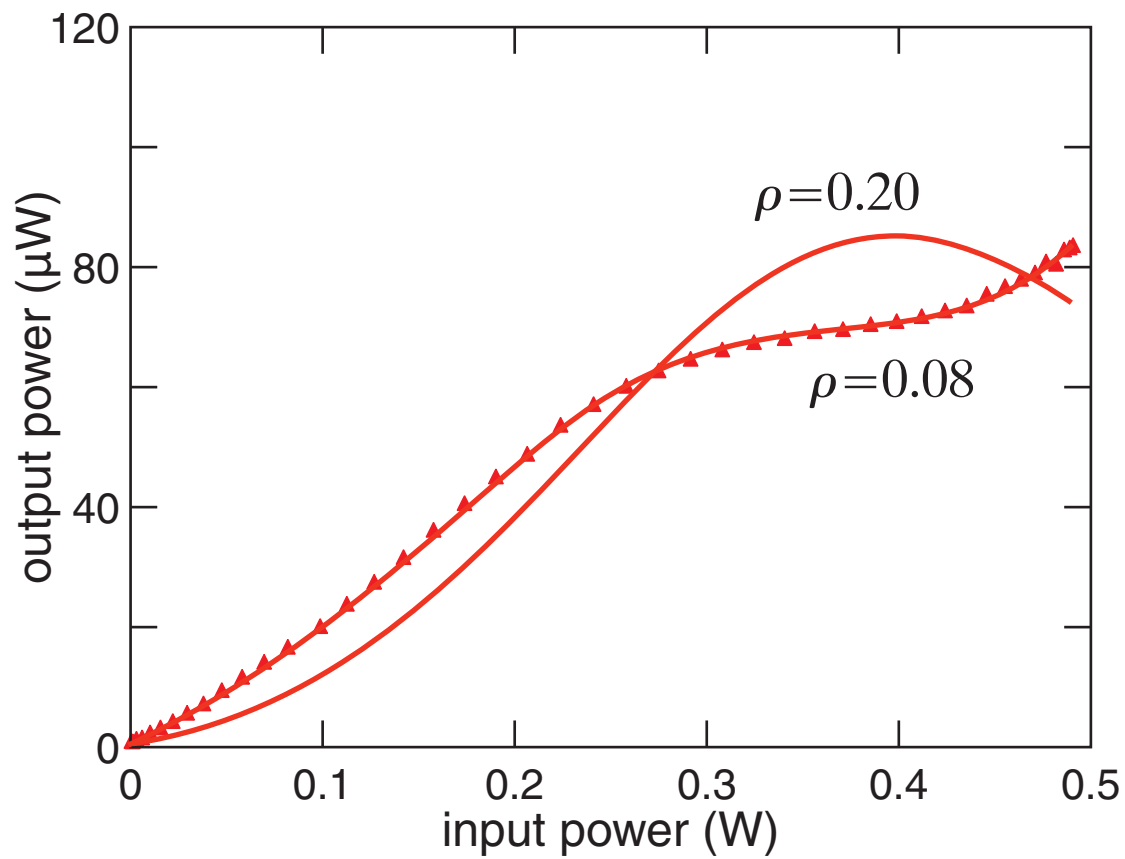
Optical logic gates

very preliminary data

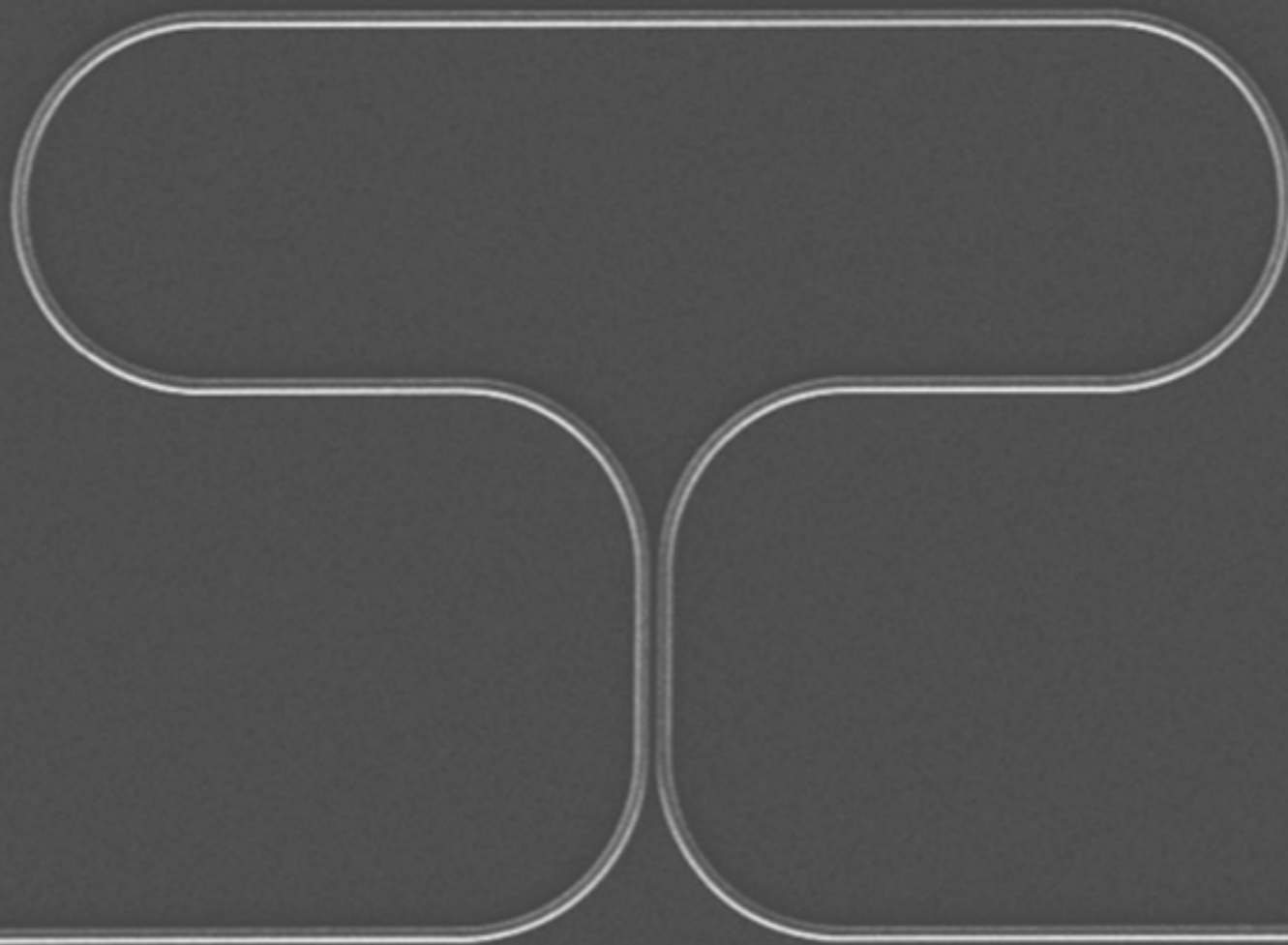


Optical logic gates

very preliminary data



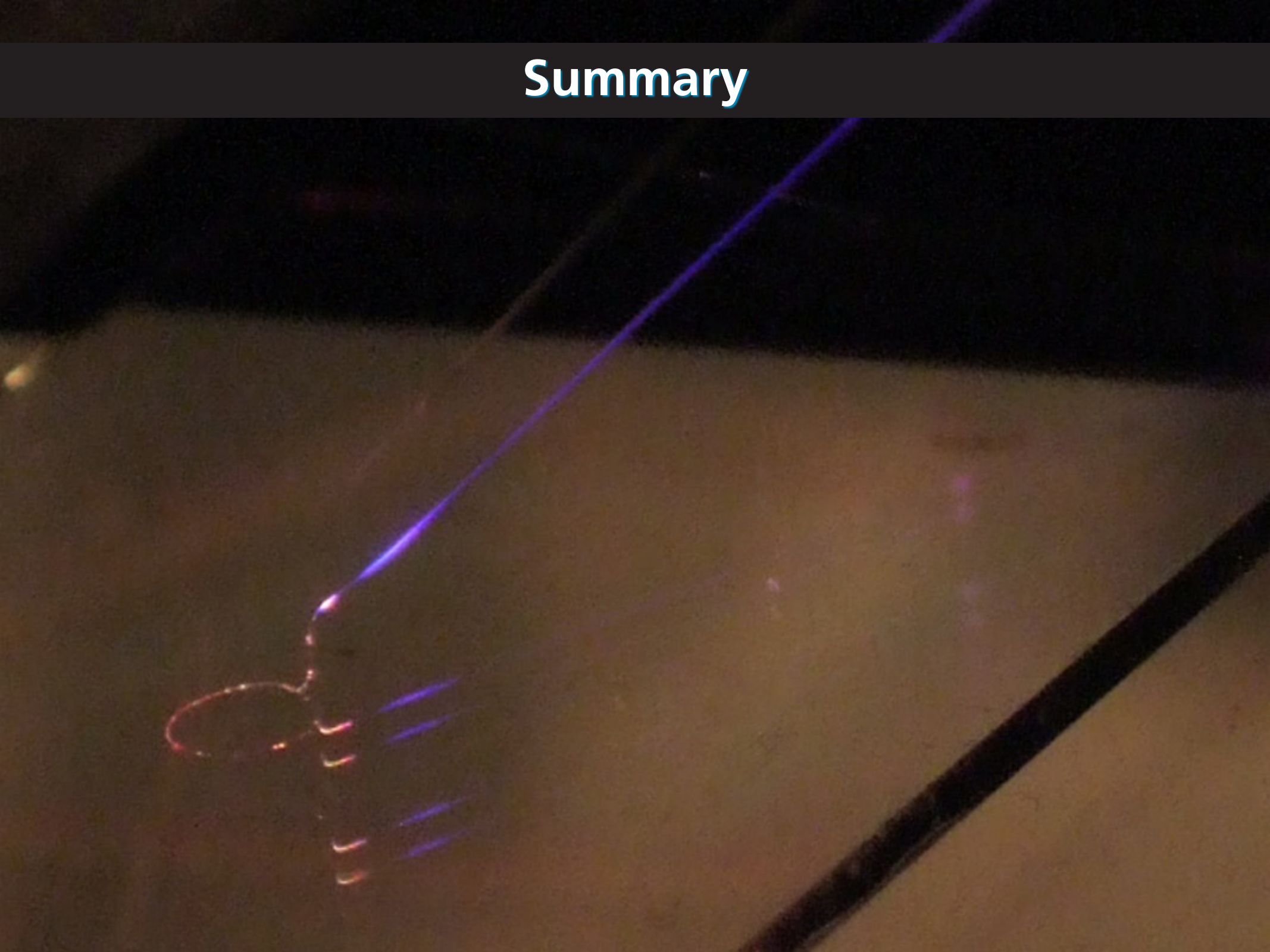
Optical logic gates



10 μm



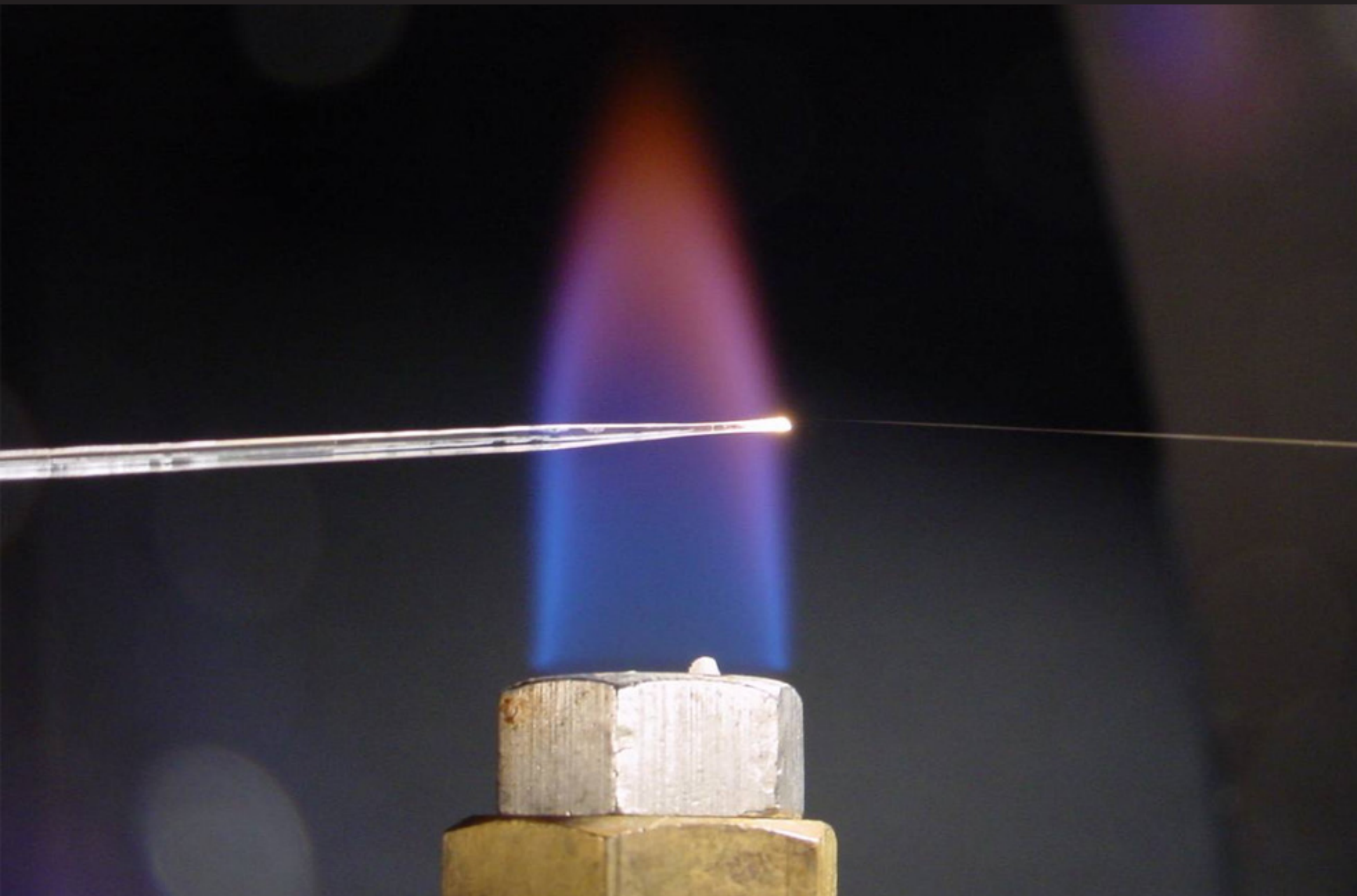
Summary



Summary

- several nanodevices demonstrated
- large γ permits miniature Sagnac loops
- switching energy ≈ 100 pJ

Summary





Funding:

**Harvard Center for Imaging and Mesoscopic Structures
National Science Foundation
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