Guiding and Manipulating Light at the Nanoscale

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NATO-ASI Summer school on
Nanostructures for Optics and Photonics
Centro Ettore Majorana
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Outline

- propagation of pulses
- nonlinear optics
- nanoscale optics
- nonlinear optics at the nanoscale

Propagation of EM wave through medium

Governed by wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

Solution:

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}}c = \frac{1}{n}c$$

where

$$\vec{E} = \vec{E}_o\, e^{i(kx-\omega t)}$$

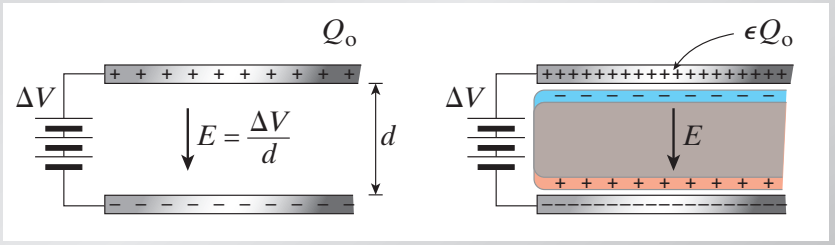
In nonferromagnetic media $\mu \approx 1$, and so $n \approx \sqrt{\epsilon}$.

In dispersive media $n = n(\omega)$.

Propagation of EM wave through medium

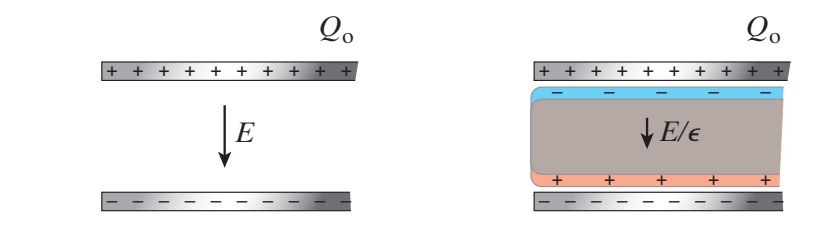
Dielectric constant measures increase in capacitance

$$\epsilon = \frac{C_d}{C_o}$$



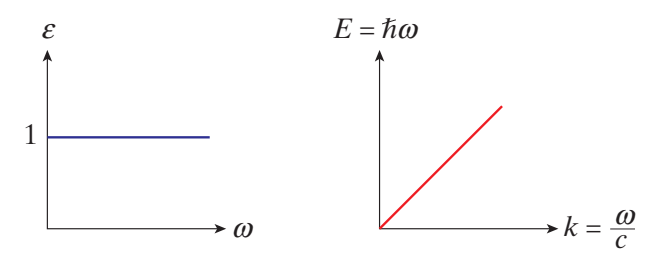
Propagation of EM wave through medium

Alternatively ϵ is measure of attentuation of electric field



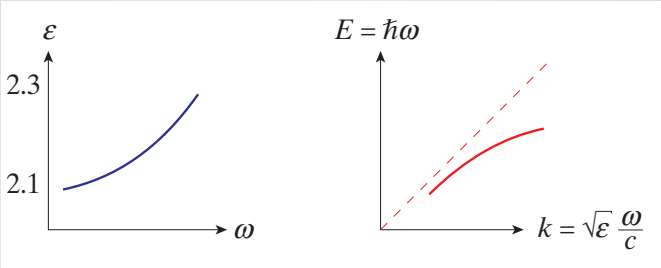
Propagation of EM wave through medium

In vacuum: $$f\lambda = \frac{\omega}{k} = c \quad \Rightarrow \quad \omega = c\,k$$

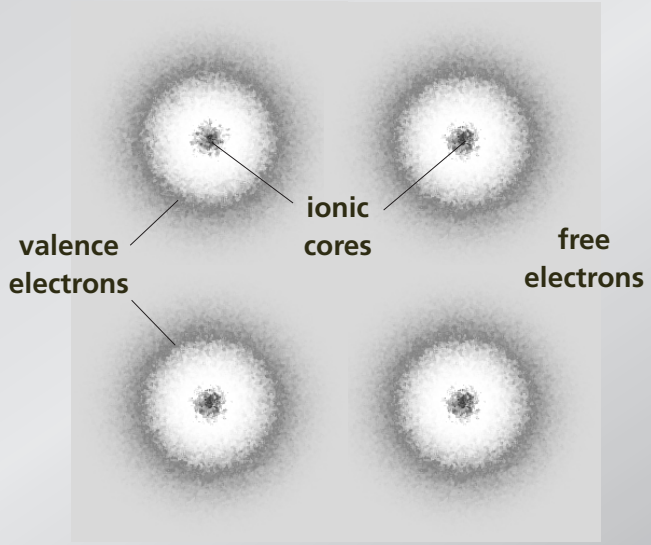


Propagation of EM wave through medium

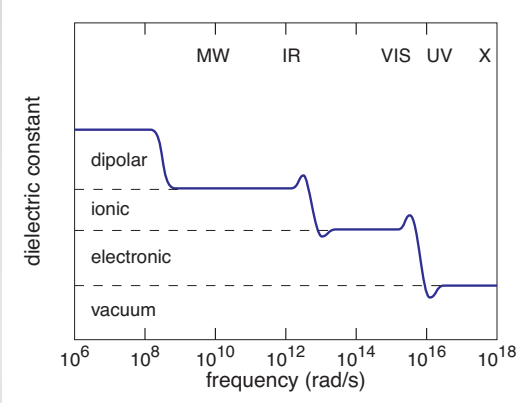
In medium: $$v = \frac{c}{\sqrt{\epsilon(\omega)}} = \frac{c}{n} \quad \Rightarrow \quad \omega = \frac{c}{\sqrt{\epsilon}}\,k$$



Which charges participate?



Dielectric function



Bound electrons

Electron on a string: $F_{binding} = -m_e\omega_o^2 x$

$$F_{damping} = -m_e\gamma\frac{dx}{dt}$$

$$F_{driving} = -eE = -eE_o e^{-i\omega t}$$

Equation of motion: $m\frac{d^2x}{dt^2} = \sum F$

$$m\frac{d^2x}{dt^2} + m\gamma\frac{dx}{dt} + m\omega_o^2 x = -eE$$

Bound electrons

Steady state: electron oscillates at driving frequency

$$x(t) = x_o e^{-i\omega t} \qquad x_o = -\frac{e}{m}\frac{1}{(\omega_o^2 - \omega^2) - i\gamma\omega}E_o$$

Oscillating dipole

$$p(t) = -ex(t) = \frac{e^2}{m}\frac{1}{(\omega_o^2 - \omega^2) - i\gamma\omega}E_o e^{-i\omega t}$$

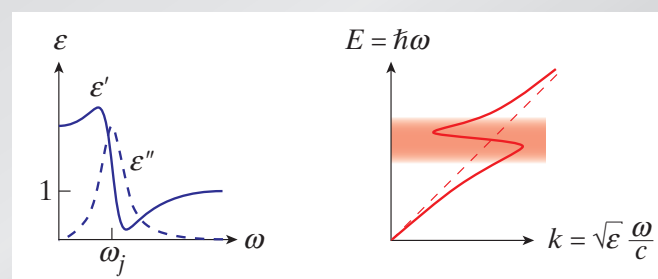
Polarization

$$P(t) = \left(\frac{Ne^2}{m}\right)\sum_j \frac{f_j}{(\omega_j^2 - \omega^2) - i\gamma_j\omega}E(t) \equiv \epsilon_o\chi_e E(t)$$

Bound electrons

Dielectric function

$$\epsilon(\omega) \equiv 1 + \chi_e = 1 + \frac{Ne^2}{\epsilon_o m}\sum_j \frac{f_j}{(\omega_j^2 - \omega^2) - i\gamma_j\omega}$$



Free electrons

No binding: $F_{binding} \approx 0$

Equation of motion: $m\dfrac{d^2x}{dt^2} + m\gamma\dfrac{dx}{dt} = -eE$

Solution: $x(t) = \dfrac{e}{m}\dfrac{1}{\omega^2 + i\gamma\omega}E(t)$ **(no resonance)**

Low frequency ($\omega \ll \gamma$) $\Rightarrow$ current generated

$$J = -Ne\frac{dx}{dt} = \frac{Ne^2}{m}\frac{1}{\gamma - i\omega}E \approx \frac{Ne^2}{m\gamma}E \equiv \sigma E$$

Free electrons

$\omega \gg \gamma$: σ **complex** $\Rightarrow$ J **out of phase with** E

Dipole:

$$p(t) = -ex(t) = -\frac{e^2}{m}\frac{1}{\omega^2 + i\gamma\omega}E(t)$$

Polarization:

$$P(t) = -\frac{Ne^2}{m}\frac{1}{\omega^2 + i\gamma\omega}E(t) \equiv \epsilon_o\chi_e E(t)$$

Dielectric function:

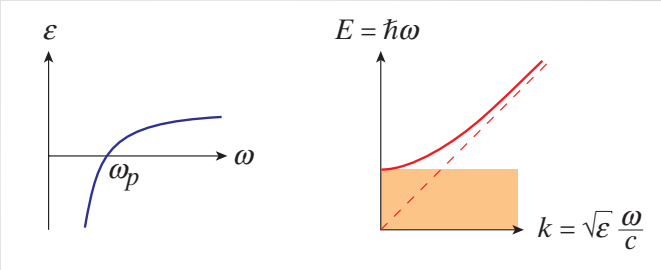
$$\epsilon(\omega) \equiv 1 + \chi_e = 1 - \frac{Ne^2}{m\epsilon_o}\frac{1}{\omega^2 + i\gamma\omega} = \epsilon'(\omega) + i\epsilon''(\omega)$$

Free electrons

$$\epsilon(\omega) \equiv 1 + \chi_e = 1 - \frac{Ne^2}{m\epsilon_o}\frac{1}{\omega^2 + i\gamma\omega} = \epsilon'(\omega) + i\epsilon''(\omega)$$

Little damping: $\gamma \approx 0 \Rightarrow \epsilon'' = 0$

$$\epsilon'(\omega) = 1 - \frac{Ne^2}{m\epsilon_o}\frac{1}{\omega^2} \equiv 1 - \frac{\omega_p^2}{\omega^2}$$

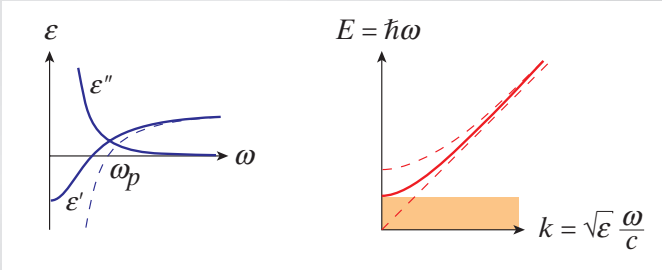


Free electrons

$$\epsilon(\omega) \equiv 1 + \chi_e = 1 - \frac{Ne^2}{m\epsilon_o}\frac{1}{\omega^2 + i\gamma\omega} = \epsilon'(\omega) + i\epsilon''(\omega)$$

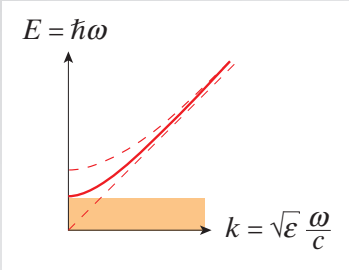
Add damping:

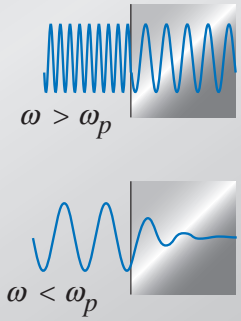
$$\gamma \lesssim \omega_p$$



Free electrons

Plasma acts like a high-pass filter

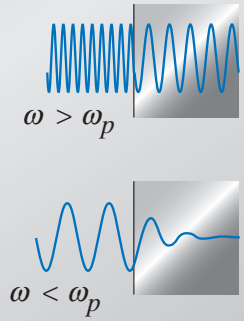




Free electrons

Plasma acts like a high-pass filter

log N (cm^{-3})	ω_p (rad s^{-1})	λ_p
22	6 x 10^{15}	330 nm
18	6 x 10^{13}	33 μm
14	6 x 10^{11}	3.3 mm
10	6 x 10^{9}	0.33 m



Pulse dispersion

Consider two propagating waves:

$$y_1 = A \sin(k_1 x - \omega_1 t) \quad \text{and} \quad y_2 = A \sin(k_2 x - \omega_2 t)$$

propagating at speeds

$$v_1 = \frac{\omega_1}{k_1} = f_1 \lambda_1 \quad \text{and} \quad v_2 = \frac{\omega_2}{k_2} = f_2 \lambda_2 .$$

Superposition:

$$y = A[\sin(k_1 x - \omega_1 t) + \sin(k_2 x - \omega_2 t)]$$

$$\sin\alpha + \sin\beta = 2\cos\left(\frac{\alpha - \beta}{2}\right)\sin\left(\frac{\alpha + \beta}{2}\right)$$

Pulse dispersion

$$y = 2A \cos\tfrac{1}{2}[(k_1 - k_2)x - (\omega_1 - \omega_2)t] \sin\left[\frac{k_1 + k_2}{2}x - \frac{\omega_1 + \omega_2}{2}t\right]$$

Let: $k_1 - k_2 \equiv \Delta k$ **and** $\omega_1 - \omega_2 \equiv \Delta\omega$

$$\frac{k_1 + k_2}{2} \equiv k \quad \text{and} \quad \frac{\omega_1 + \omega_2}{2} \equiv \omega$$

and so:

$$y = 2A \cos\tfrac{1}{2}(x\Delta k - t\Delta\omega) \sin(kx - \omega t)$$

traveling sine wave, with amplitude modulation.

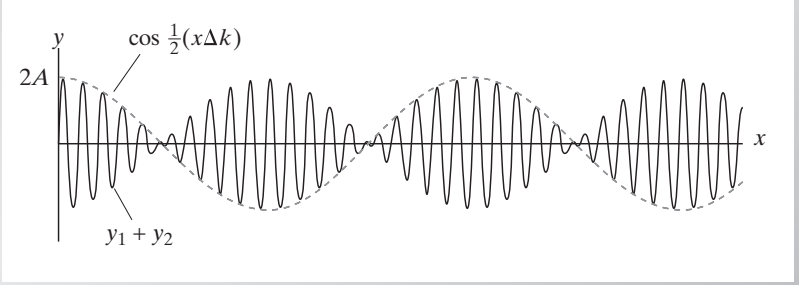
Pulse dispersion

$$y = 2A\cos\tfrac{1}{2}(x\Delta k - t\Delta\omega)\sin(kx - \omega t)$$

At $t = 0$**:**

$$y = 2A\cos\tfrac{1}{2}(x\Delta k)\sin(kx)$$

envelope carrier



Pulse dispersion

$$y = 2A\cos\tfrac{1}{2}(x\Delta k - t\Delta\omega)\sin(kx - \omega t)$$

speed of carrier ('phase velocity'):

$$v_p = \frac{\omega}{k} = f\lambda$$

speed of envelope ('group velocity'):

$$v_g = \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk}$$

Pulse dispersion

$$y = 2A\cos\tfrac{1}{2}(x\Delta k - t\Delta\omega)\sin(kx - \omega t)$$

If no dispersion $\quad v_p = \dfrac{\omega_1}{k_1} = \dfrac{\omega_2}{k_2}$

group velocity:

$$v_g = \frac{\Delta\omega}{\Delta k} = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{\omega_1/k_1 - \omega_2/k_1}{1 - k_2/k_1} = \frac{v_p - \omega_2/k_1}{1 - k_2/k_1}$$

$$v_g = \frac{v_p - \omega_2/k_2(k_2/k_1)}{1 - k_2/k_1} = v_p\frac{1 - k_2/k_1}{1 - k_2/k_1} = v_p$$

Pulse dispersion

$$y = 2A\cos\tfrac{1}{2}(x\Delta k - t\Delta\omega)\sin(kx - \omega t)$$

If no dispersion $\quad v_p = \dfrac{\omega_1}{k_1} = \dfrac{\omega_2}{k_2}$

group and phase velocities are the same:

$$v_g = v_p$$

and so the envelope and carrier travel together

Pulse dispersion

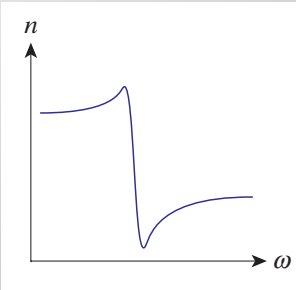
$$y = 2A\cos\tfrac{1}{2}(x\Delta k - t\Delta\omega)\sin(kx - \omega t)$$

Types of dispersion:

$\frac{dn}{d\omega} > 0$ **normal dispersion** $v_g < v_p$

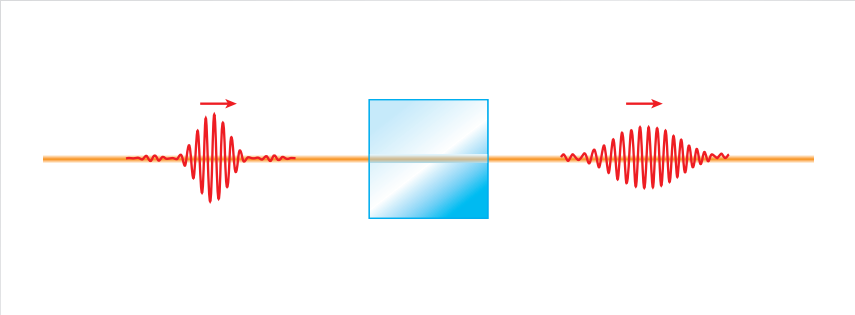
$\frac{dn}{d\omega} = 0$ **no dispersion** $v_g = v_p$

$\frac{dn}{d\omega} < 0$ **anomalous dispersion** $v_g > v_p$



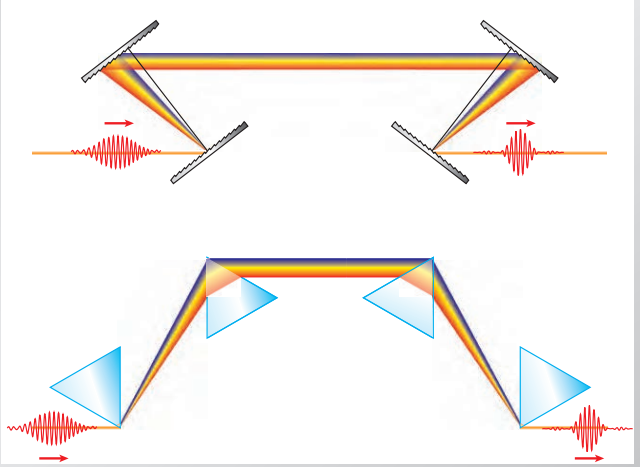
Pulse dispersion

medium causes pulse to stretch



compensate by rearranging spectral components!

Pulse dispersion compensation



Pulse dispersion

Write dispersion as Taylor series:

$$\omega(k) = \omega_o + \left(\frac{d\omega}{dk}\right)_{k=k_o}(k-k_o) + \tfrac{1}{2}\left(\frac{d^2\omega}{dk^2}\right)_{k=k_o}(k-k_o)^2$$

let

$$u \equiv \left(\frac{d\omega}{dk}\right)_{k=k_o} \quad \text{and} \quad w \equiv \left(\frac{d^2\omega}{dk^2}\right)_{k=k_o}.$$

group velocity: $v_g = \frac{d\omega}{dk} = u + wk$

if $w = 0$, then group velocity and pulse shape constant!

Pulse dispersion compensation

So not path length but $\frac{d^2\phi}{d\omega^2}$ matters!

	$\frac{dl_{eff}}{d\omega}$	$\frac{d^2\phi}{d\omega^2}$
dispersion	+	+
gratings	–	–
prisms	+	–

Outline

- propagation of pulses
- nonlinear optics
- nanoscale optics
- nonlinear optics at the nanoscale

Nonlinear optics

Linear optics:

$$\vec{P} = \chi\vec{E}$$

Nonlinear polarization:

$$P = \chi^{(1)}E + \chi^{(2)}E^2 + \chi^{(3)}E^3 + \ldots$$

and so:

$$P = P^{(1)} + P^{(2)} + P^{(3)} + \ldots$$

$P^{(2)} \approx P^{(1)}$ when $E = E_{at} \approx \frac{e}{a}$, and so $\chi^{(n)} \approx \frac{\chi^{(1)}}{E_{at}^{n-1}}$.

Nonlinear optics

Nonlinear polarization can drive new field:

$$\nabla^2\vec{E} + \frac{n^2}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2\vec{P}}{\partial t^2}$$

But even terms disappear in media with inversion symmetry!

$$\vec{P}^{(2)} = \chi^{(2)}:\vec{E}\vec{E}$$

Invert all vectors:

$$-\vec{P}^{(2)} = \chi^{(2)}:(-\vec{E})(-\vec{E})$$

and so $\chi^{(2)} = -\chi^{(2)} = 0$.

Nonlinear optics

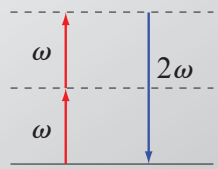
Consider oscillating electric field:

$$E(t) = E\,e^{i\omega t} + \text{c.c.}$$

Second-order polarization:

$$P^{(2)}(t) = \chi^{(2)}E^2(t) = \tfrac{1}{2}\chi^{(2)}EE^* + \tfrac{1}{4}[\chi^{(2)}E^2e^{-2\omega t} + \text{c.c.}]$$

Physical interpretation:



Nonlinear optics

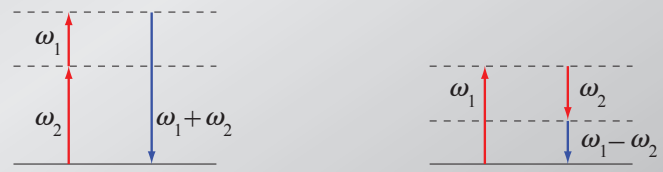
Can also cause frequency mixing! Let

$$E(t) = E_1e^{-i\omega_1 t} + E_2e^{-i\omega_2 t}$$

Second-order polarization will contain terms with

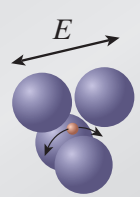
$2\omega_1$ **(SHG),** $2\omega_2$ **(SHG),** $\omega_1 + \omega_2$ **(SFG),** $\omega_1 - \omega_2$ **(DFG)**

Physical interpretation:



Nonlinear optics

Nonlinear response: $P^{(2)} = \chi^{(2)}E^2$



Nonlinear optics

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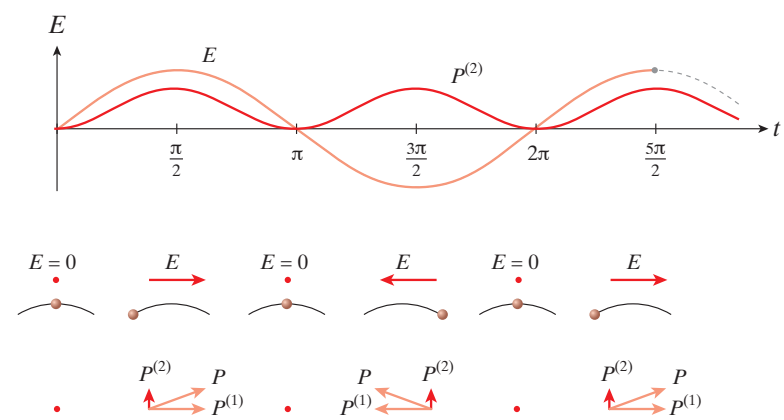


Nonlinear optics

Third-order polarization: $P^{(3)}(t) = \chi^{(3)}E^3(t)$

3 frequencies, 3 terms + c.c.: complicated! In general

$$\cos^3\omega t = \tfrac{1}{4}\cos 3\omega t + \tfrac{3}{4}\cos \omega t$$

Intensity dependent term at fundamental frequency:

$$P^{(3)}(t) = \chi^{(3)}E(t)E^*(t)E(t) = \chi^{(3)}I(t)E(t)$$

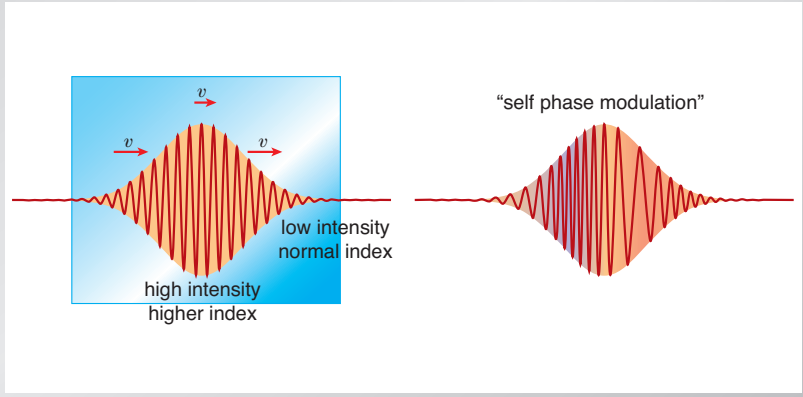
and so $$P = P^{(1)} + P^{(3)} = (\chi^{(1)} + \chi^{(3)}I)E \equiv \chi_{eff}E$$

$$n = \sqrt{\epsilon} = \sqrt{1 + \chi_{eff}} \approx \sqrt{1 + \chi^{(1)}} + \tfrac{1}{2}\frac{\chi^{(3)}I}{\sqrt{1 + \chi^{(1)}}} = n_o + n_2 I$$

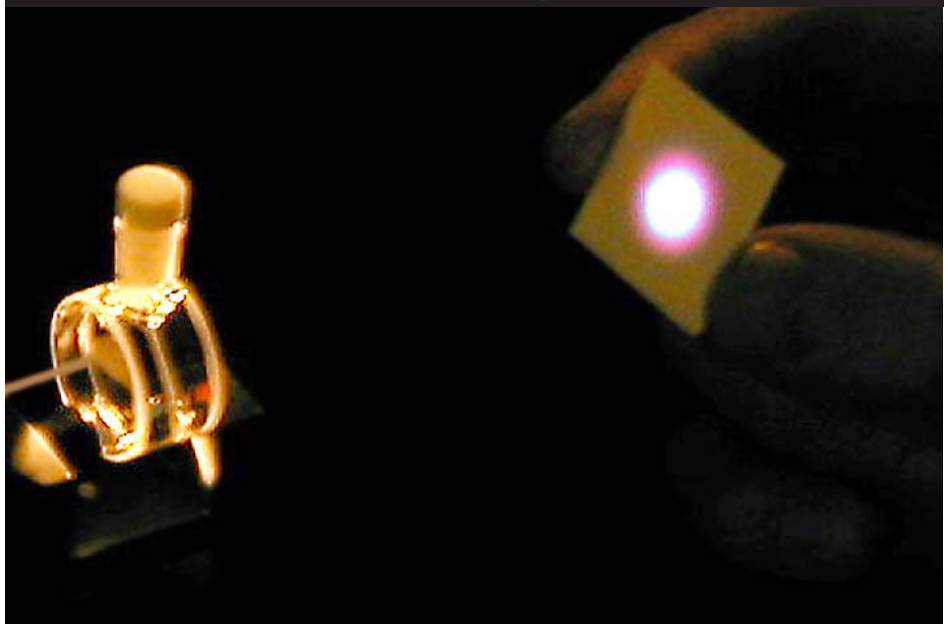
Nonlinear optics

Intensity-dependent index of refraction:

$$n = n_o + n_2 I$$



Nonlinear optics



Nonlinear optics

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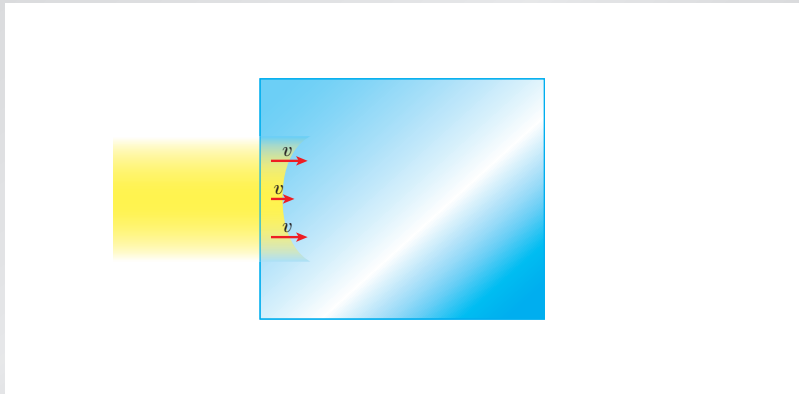
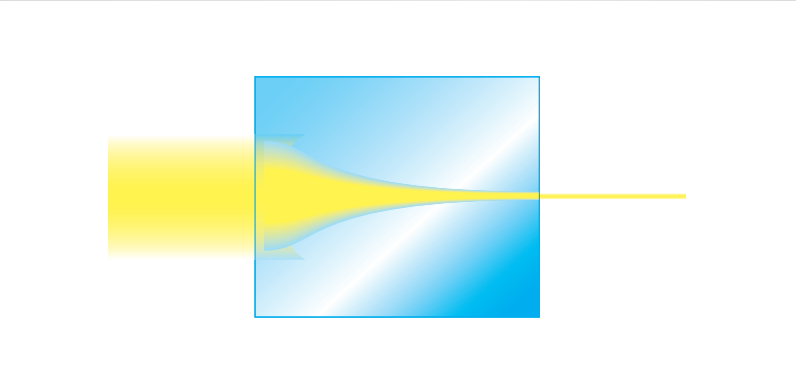


Nonlinear optics

self-focusing



Nonlinear optics

but susceptibility is complex!

susceptibility	real part	imaginary part
linear	refraction	absorption
nonlinear	SHG, SFG, DFG, THG,…	multiphoton absorption

$$\alpha = \alpha_o + \beta I + \gamma I^2 + \ldots$$

Outline

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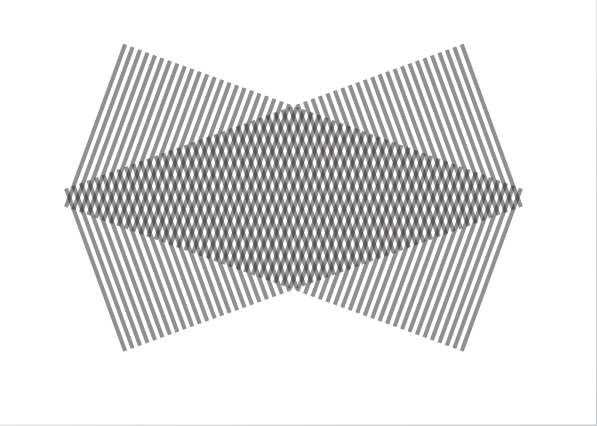
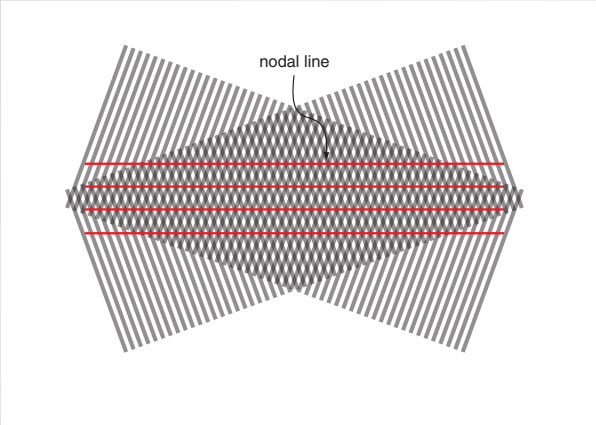
Waveguiding

two crossed planar waves…



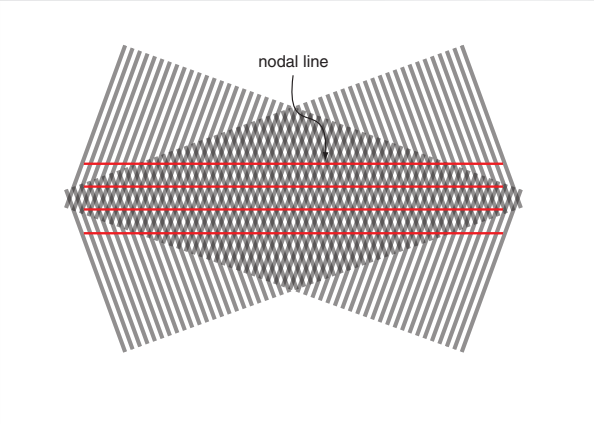
Waveguiding

…cause an interference pattern



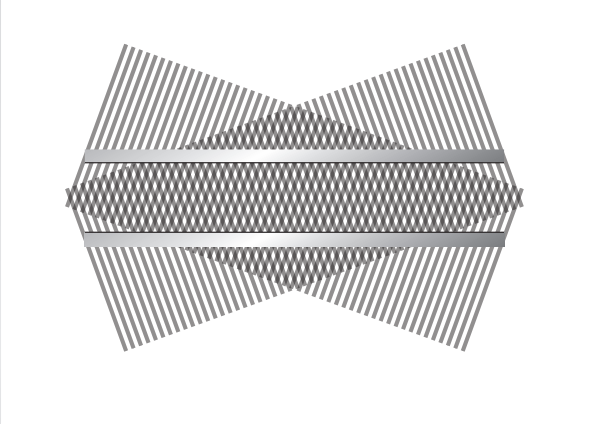
Waveguiding

$E = 0$ on the nodal lines



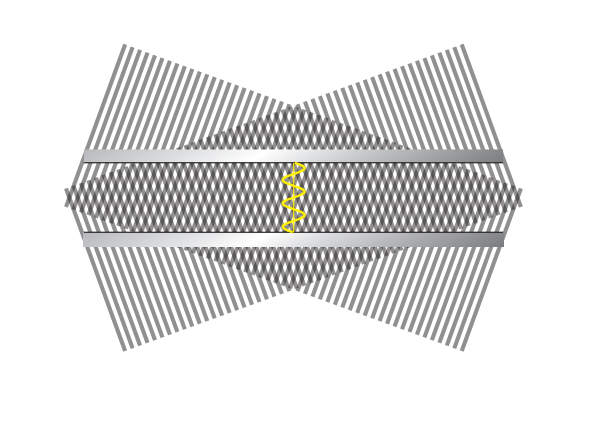
Waveguiding

…satisfying boundary conditions for planar-mirror waveguide



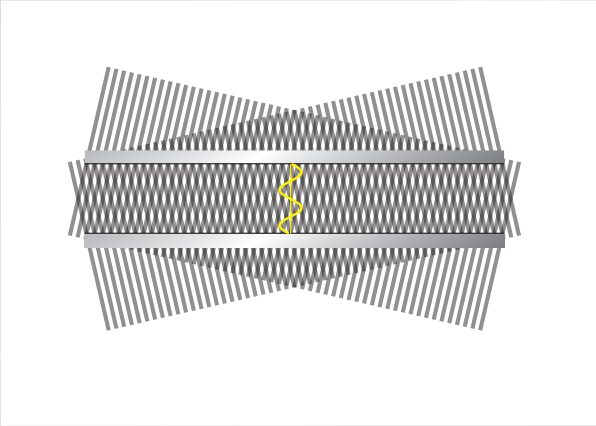
Waveguiding

transverse standing wave, traveling along axis



Waveguiding

change angle of incident waves…



Waveguiding

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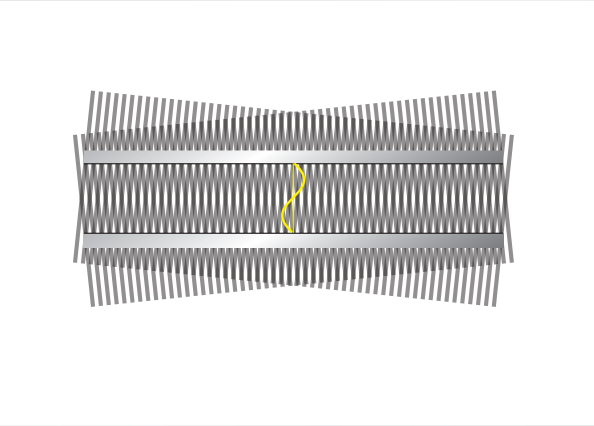
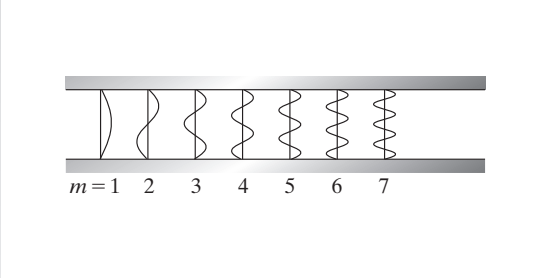


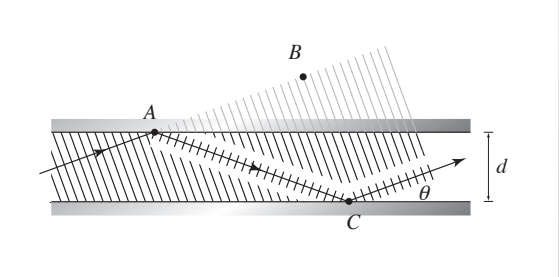
Waveguiding

boundary conditions only satisfied for certain θ



standing wave in *y*-direction, traveling in *z*-direction

Waveguiding



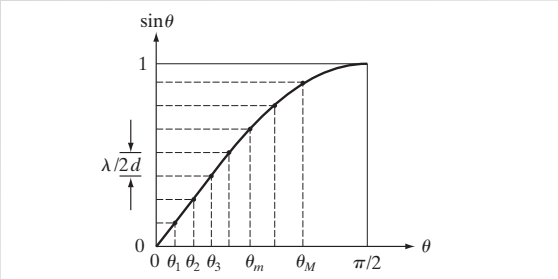
self consistency:

$$AC - AB = 2d\sin\theta = m\lambda \quad (m = 1, 2, ...)$$

so:

$$\sin\theta_m = m\frac{\lambda}{2d}$$

Waveguiding



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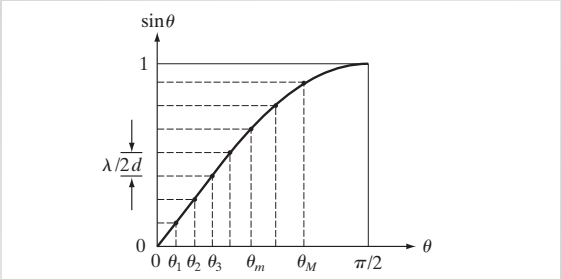
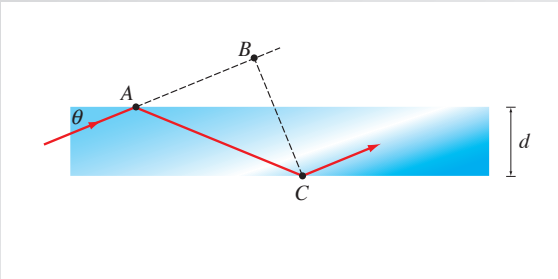
Waveguiding



number of modes:

$$M \doteq \frac{2d}{\lambda}$$

Waveguiding



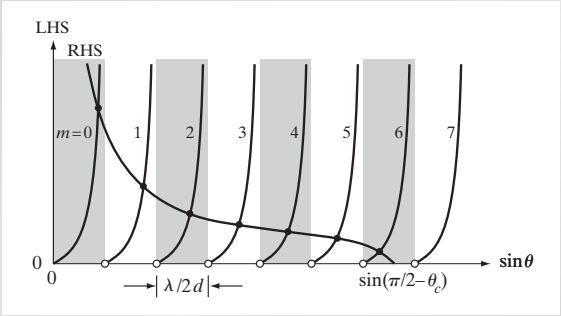
self consistency:

$$AC - AB = 2d\sin\theta - \frac{\varphi_r}{\pi}\lambda = m\lambda \quad (m = 0, 1, 2\ldots)$$

so:

$$\tan\left(\frac{\pi d}{\lambda}\sin\theta - m\frac{\pi}{2}\right) = \left(\frac{\sin^2(\pi/2-\theta_c)}{\sin^2\theta} - 1\right)^{1/2}$$

Waveguiding



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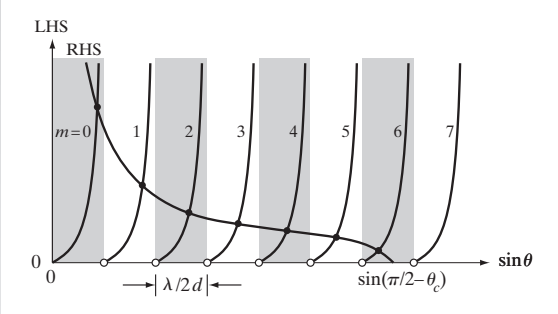
Waveguiding



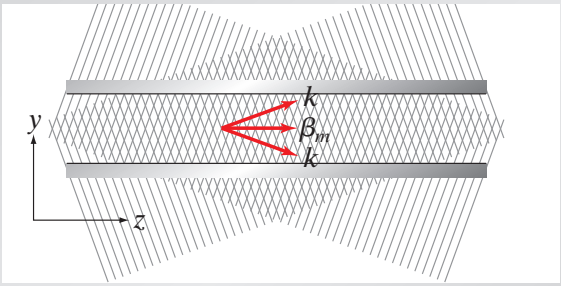
number of modes:

$$M \doteq \frac{\sin(\pi/2 - \theta_c)}{\lambda/2d}$$

or:

$$M \doteq 2\frac{d}{\lambda}(n_1^2 - n_2^2)^{1/2}$$

Waveguiding



propagation constant of guided wave:

$$\beta_m^2 = k^2 - k_y^2 = k^2 - \frac{m^2\pi^2}{d^2}$$

group velocity: $v_m = c\cos\theta_m$

Waveguiding

single mode condition for 600-nm light:

planar mirror $M \doteq \frac{2d}{\lambda}$ $300 < d < 600$ nm

dielectric $M \doteq 2\frac{d}{\lambda}(n_1^2 - n_2^2)^{1/2}$ $d < 268$ nm

Waveguiding

Vector potential obeys:

$$\nabla^2\vec{A} + \omega^2\mu_o\epsilon\vec{A} = 0$$

Substituting $\vec{A} = \hat{y}u(x,y)e^{-i\beta z}$

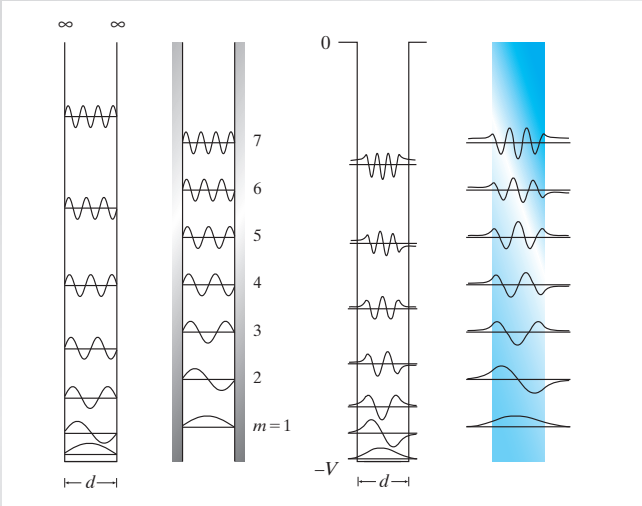
yields:

$$\nabla_T^2 u + [-\beta^2 + \omega^2\mu\epsilon(r)]u = 0$$

Compare to time-independent Schrödinger equation:

$$\nabla^2\psi + \frac{2m}{\hbar^2}[E - V(r)]\psi = 0$$

Waveguiding



Waveguiding

single mode condition for 600-nm light:

$$M \doteq 2\frac{d}{\lambda}(n_1^2 - n_2^2)^{1/2}$$

without cladding: $d < 268$ nm

Add cladding with 0.4% index difference:

$d < 5$ µm

Waveguiding

commercial single-mode fiber (Corning Titan®)

	core	cladding
index	$n_1 = 1.468$	$n_2 = 1.462$
diameter:	8.3 μm	125.0 ± 1.0 μm

operating wavelength: $\lambda = 1310$ nm/1550 nm

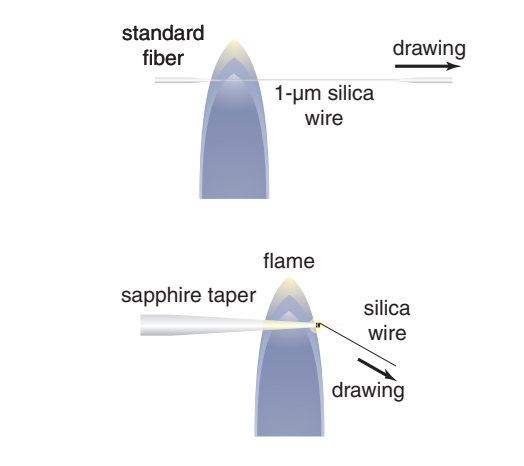
Waveguiding

drawbacks of clad fibers:

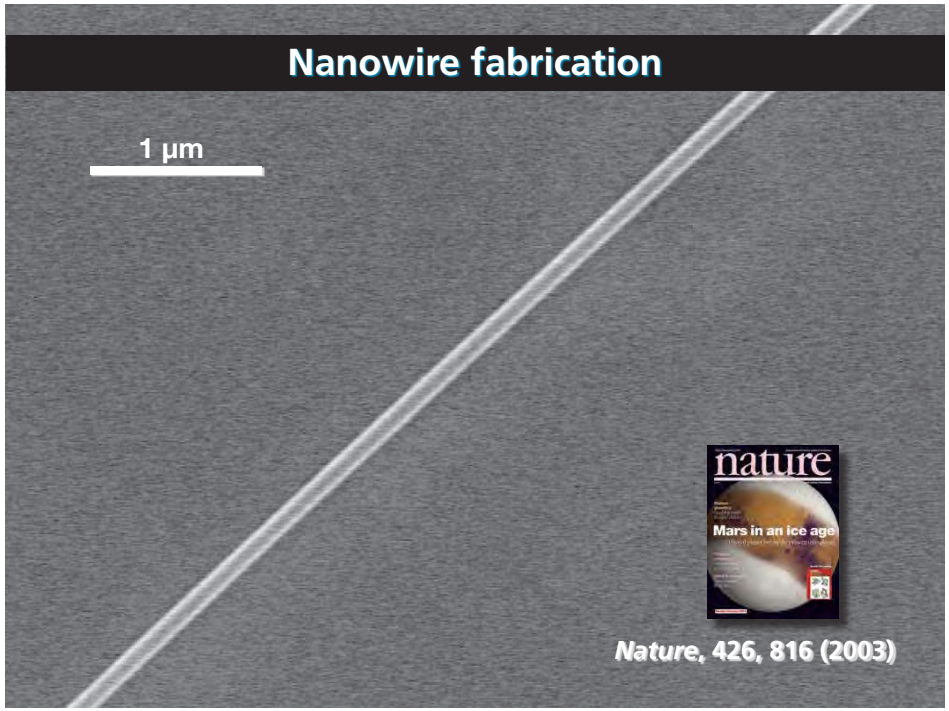
- **weak confinement**
- **no tight bending**
- **coupling requires splicing**

Nanowire fabrication

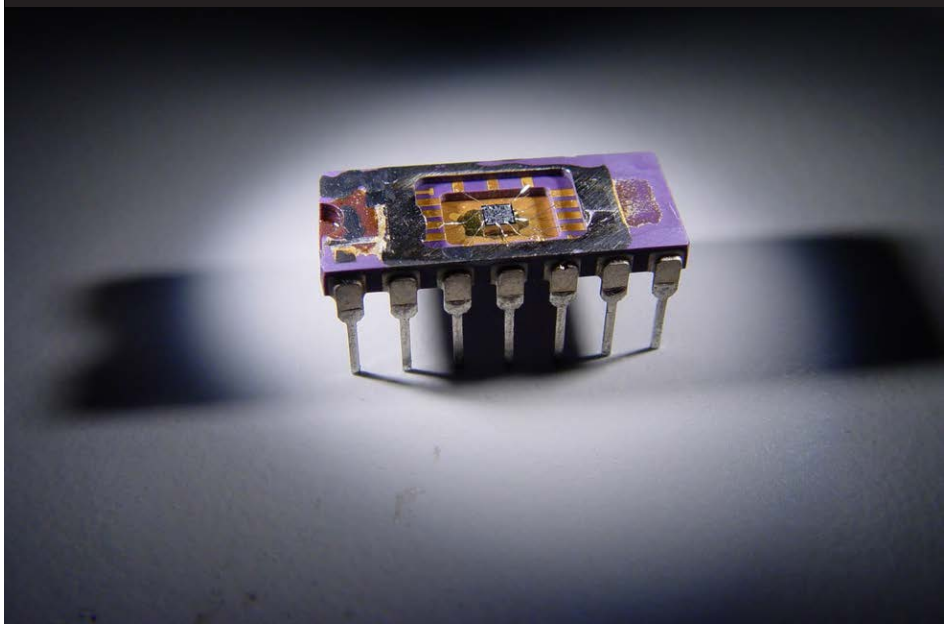
two-step drawing process



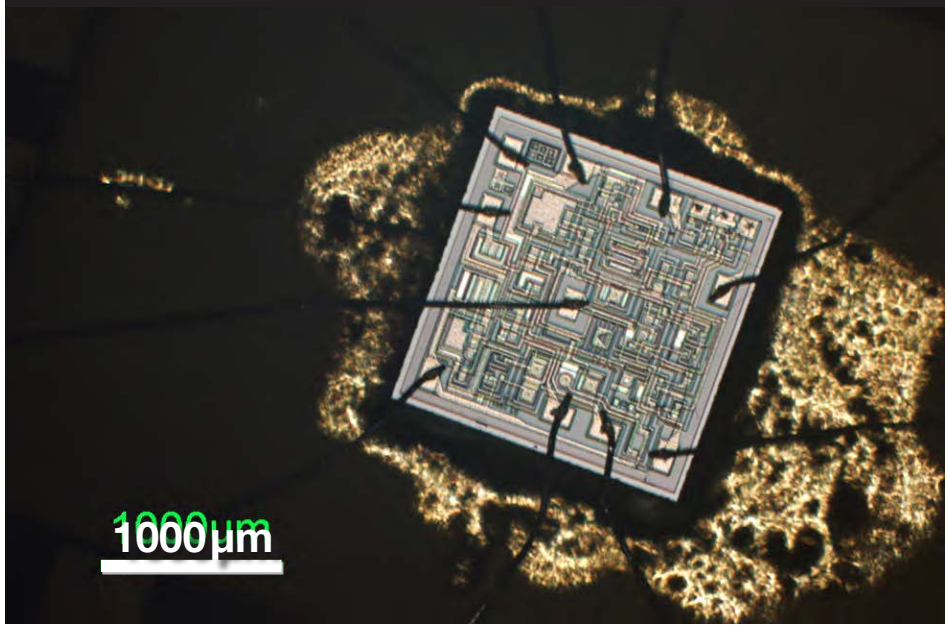
Nanowire fabrication

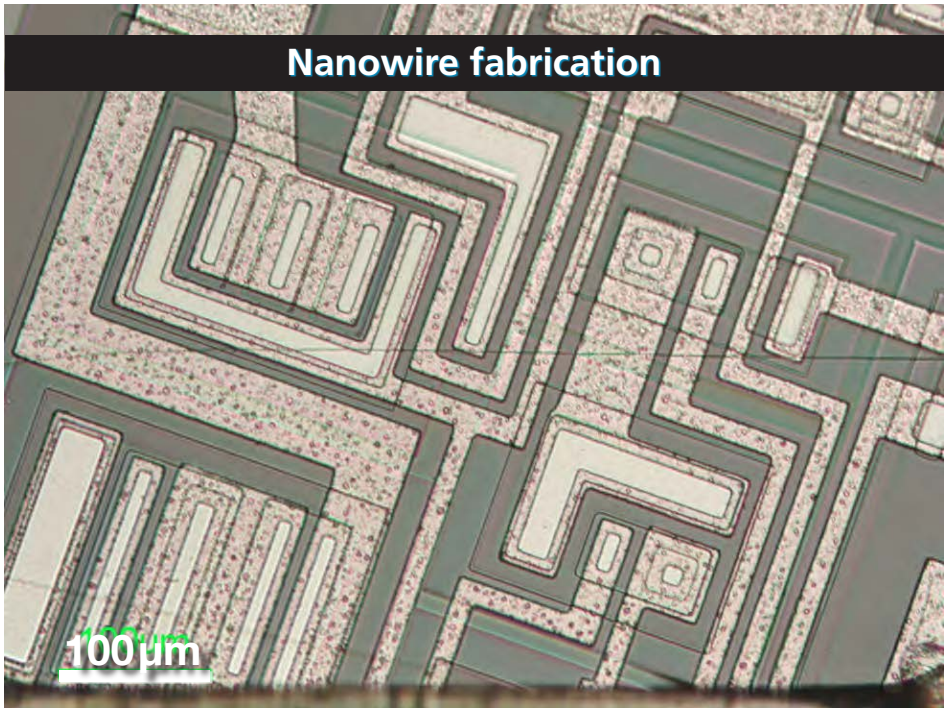


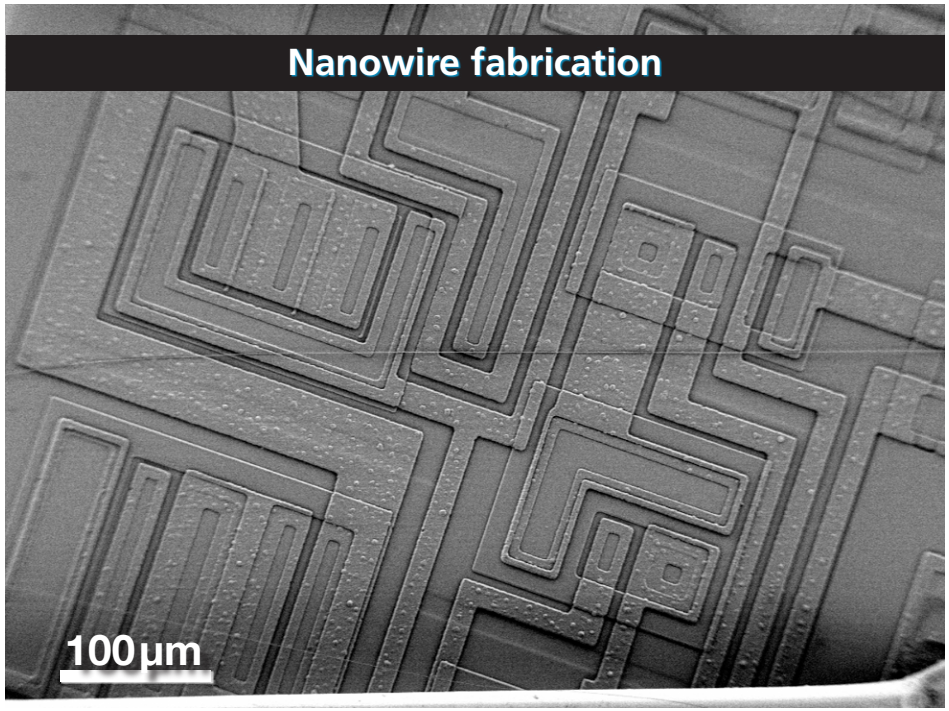
Nanowire fabrication

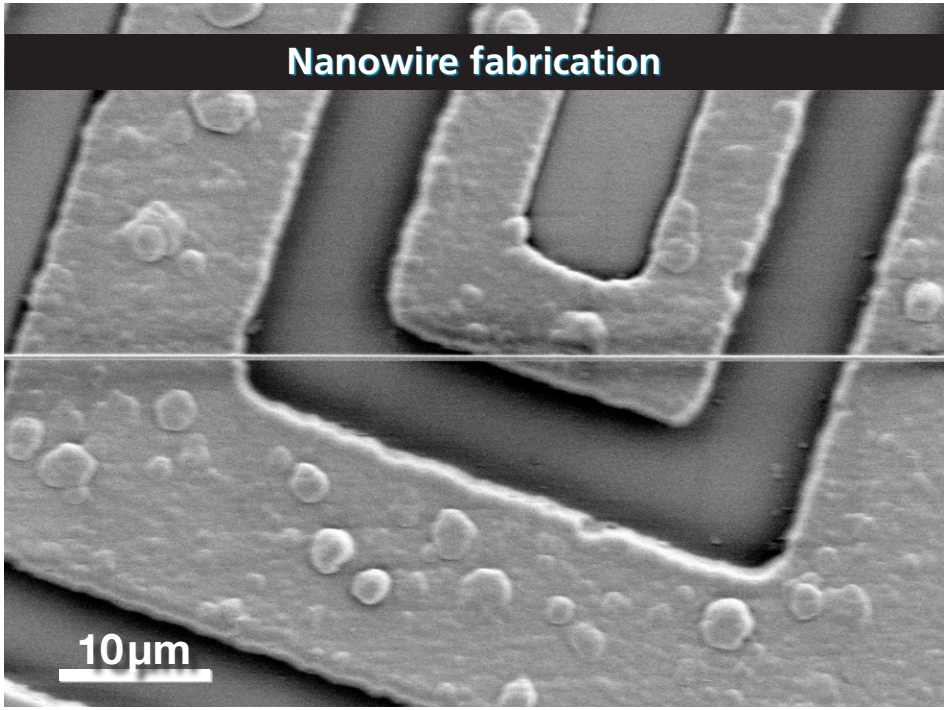


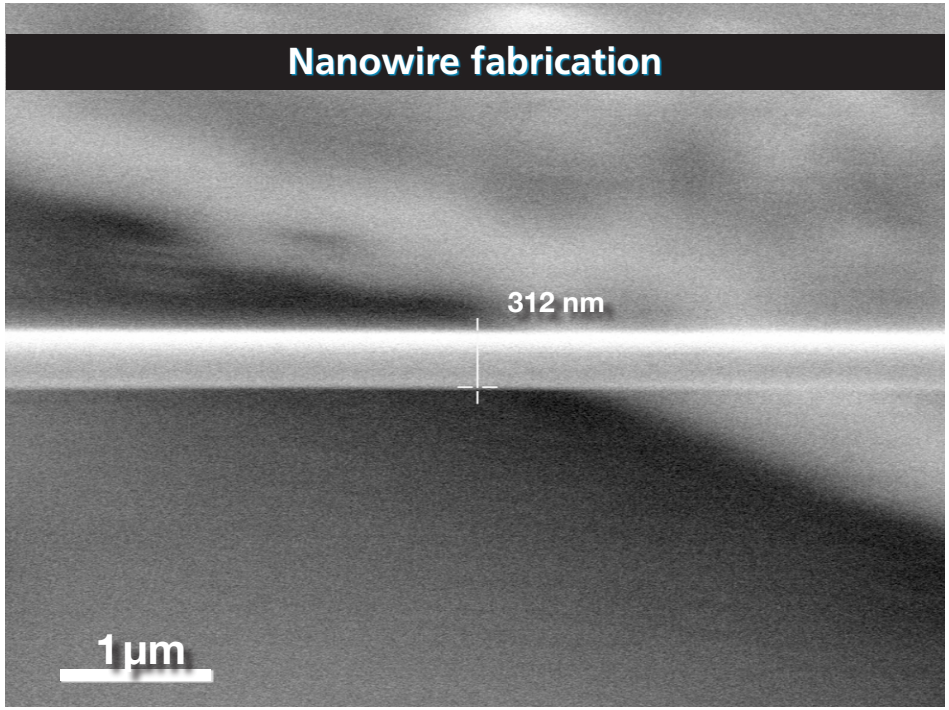
Nanowire fabrication




Nanowire fabrication
100 µm


Nanowire fabrication
100 µm


Nanowire fabrication
10 µm


Nanowire fabrication
312 nm
1 µm

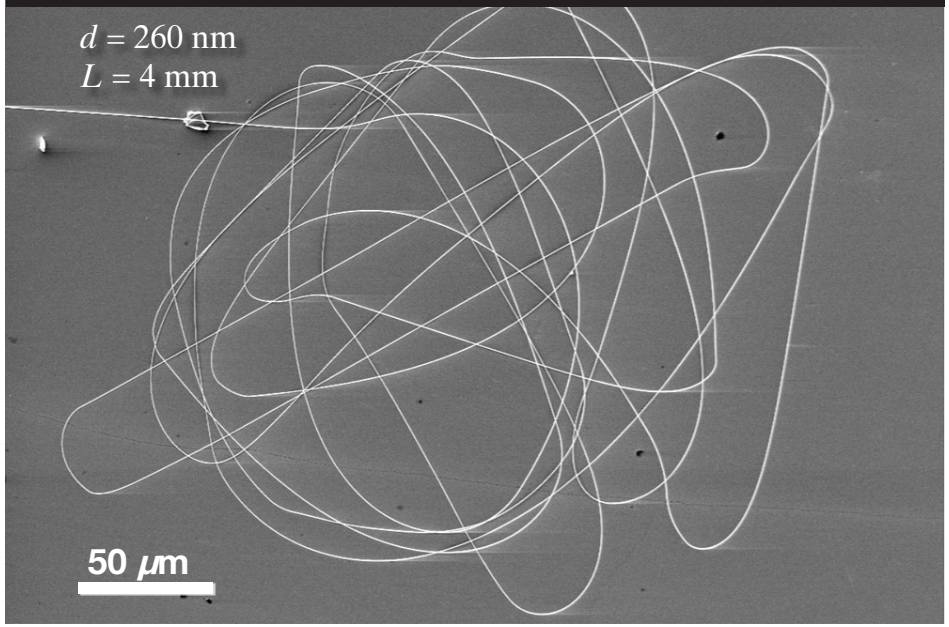
Waveguiding

Specifications

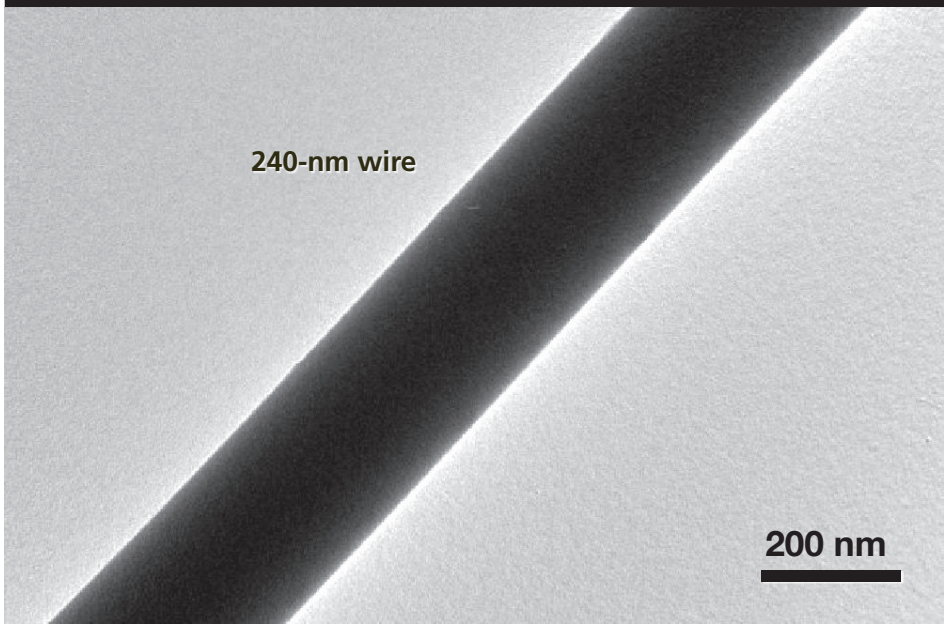
diameter D:	down to 20 nm
length L:	up to 90 mm
aspect ratio D/L:	up to 10^6
diameter uniformity $\Delta D/L$:	2×10^{-6}

Nature, 426, 816 (2003)

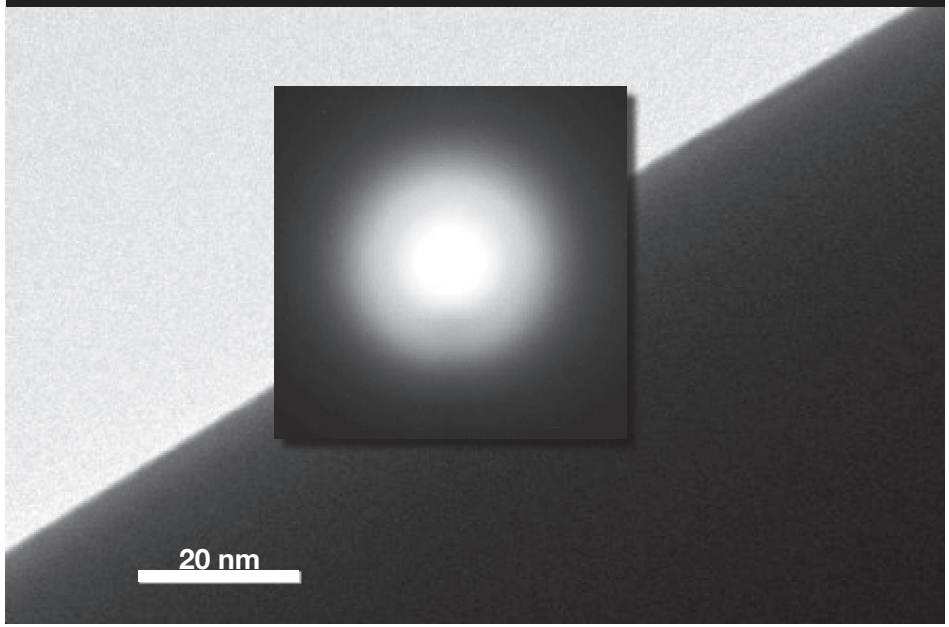
Nanowire fabrication



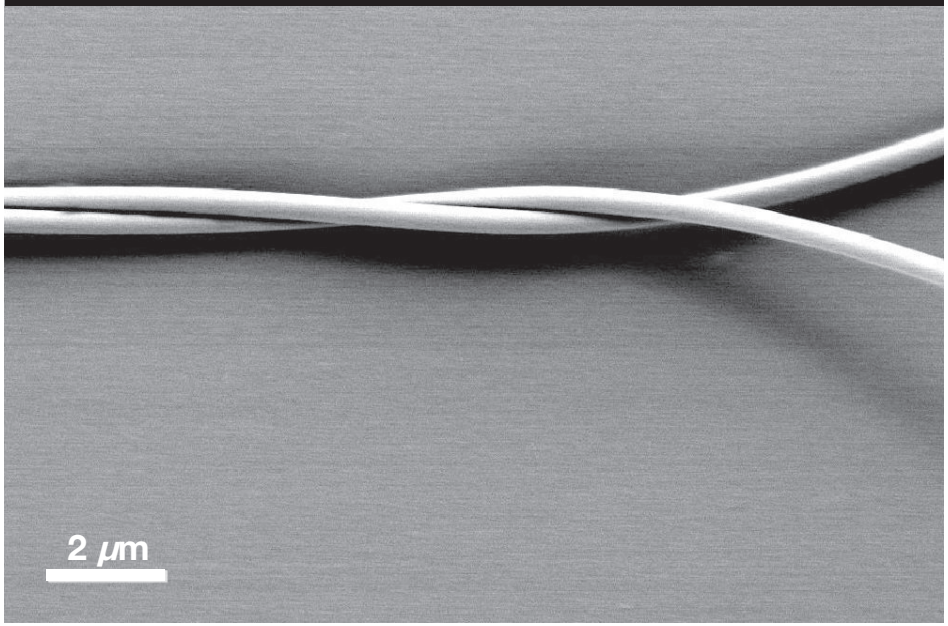
Nanowire fabrication



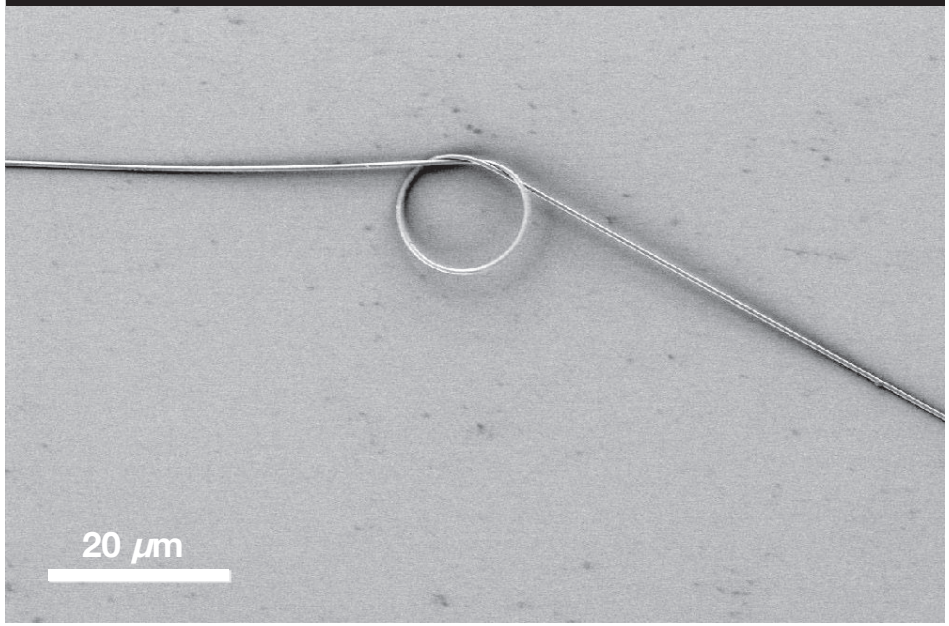
Nanowire fabrication



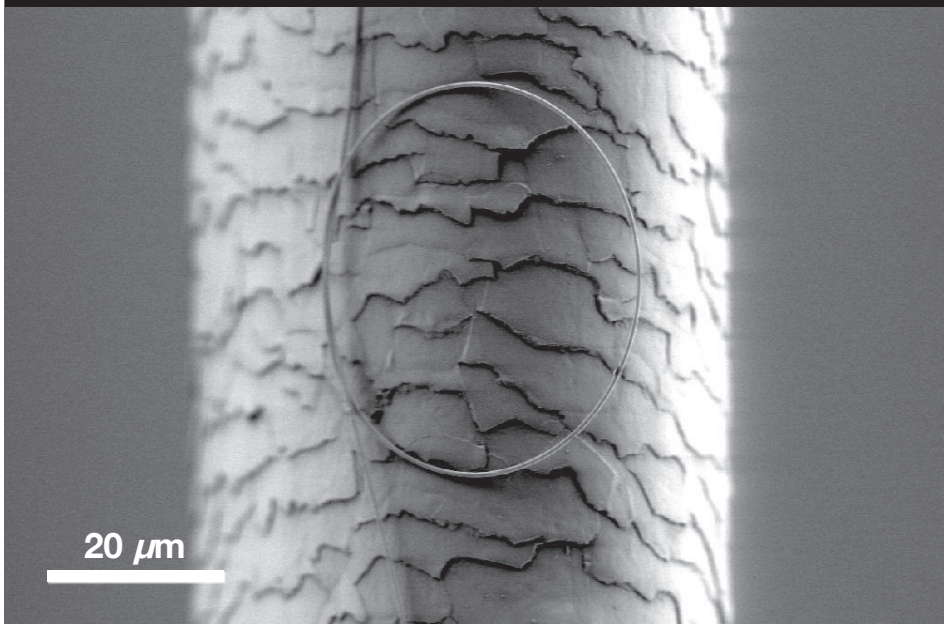
Nanowire fabrication



Nanowire fabrication

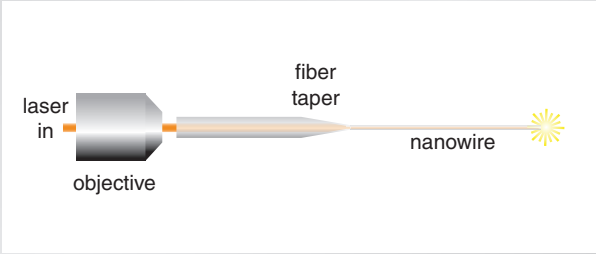


Nanowire fabrication

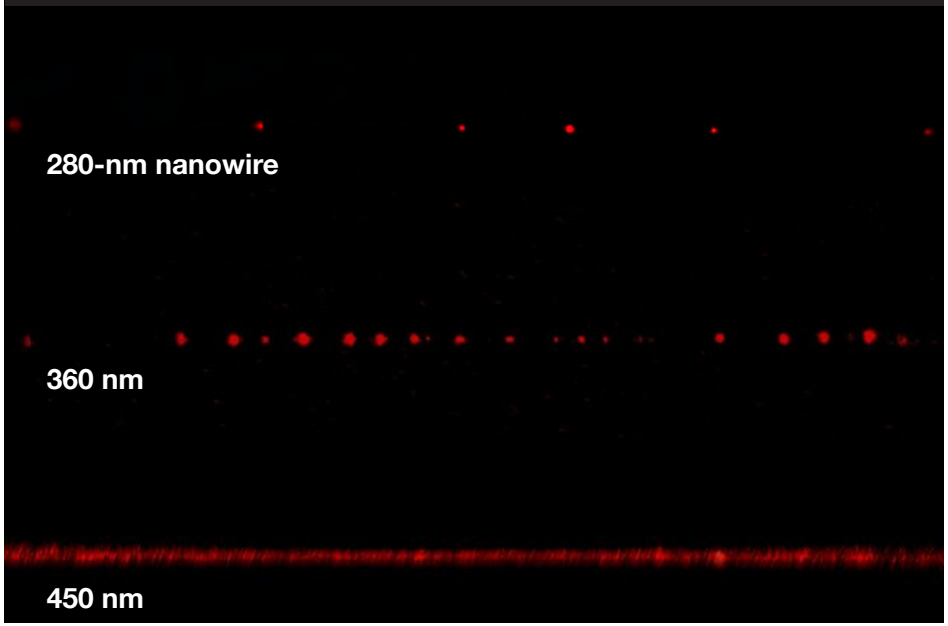


Optical properties

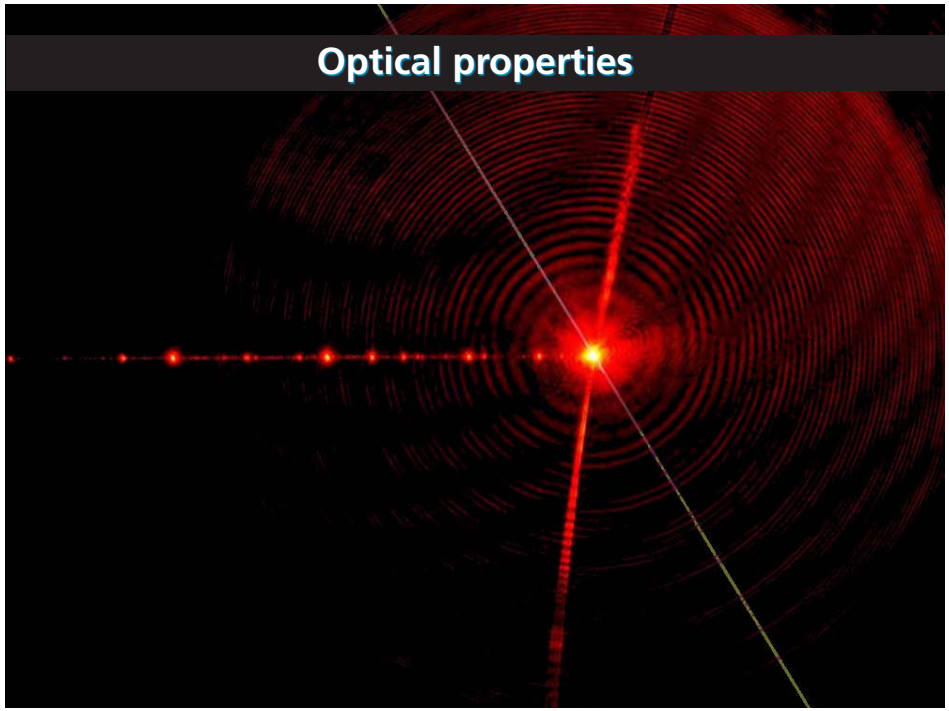
coupling light into nanowires



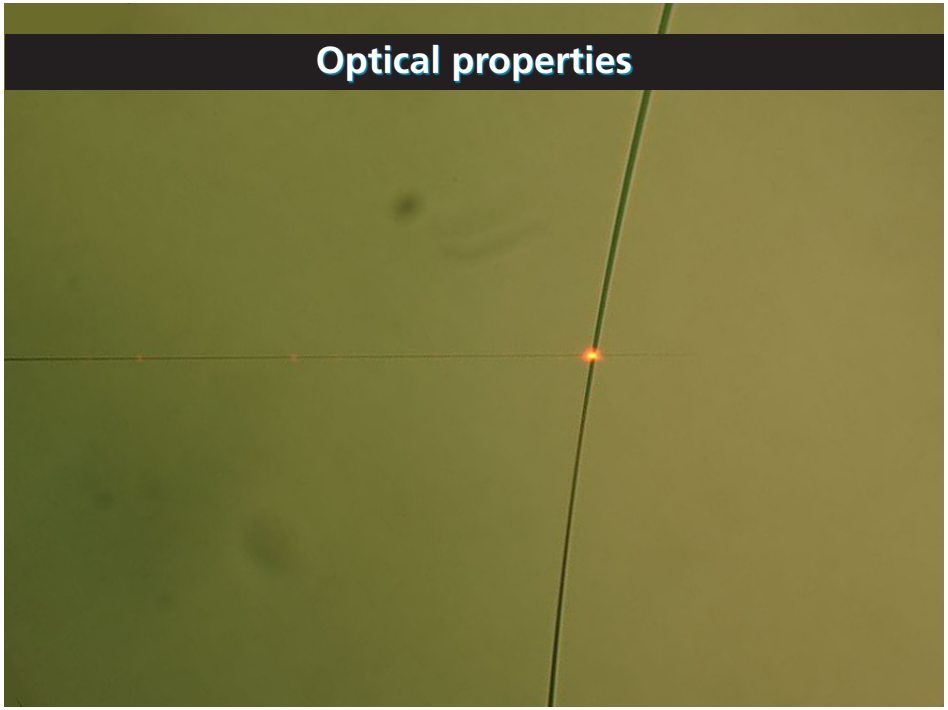
Optical properties



Optical properties

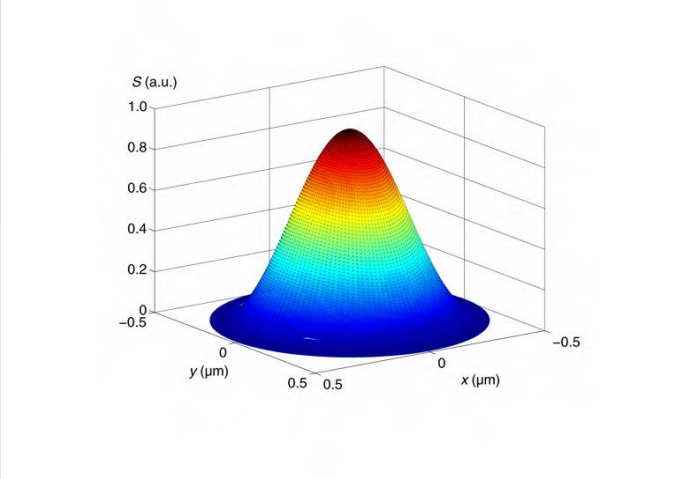


Optical properties



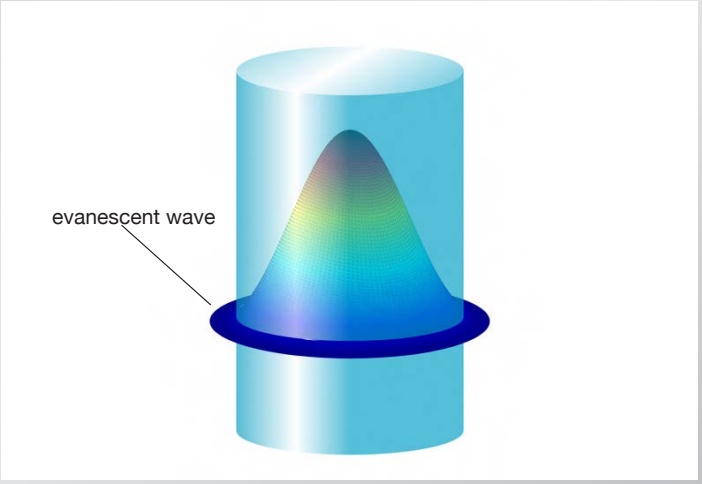
Optical properties

Poynting vector profile for 800-nm nanowire



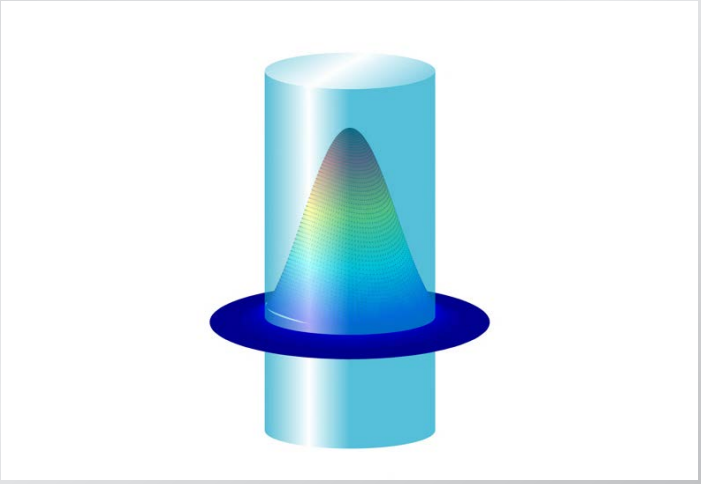
Optical properties

Poynting vector profile for 800-nm nanowire



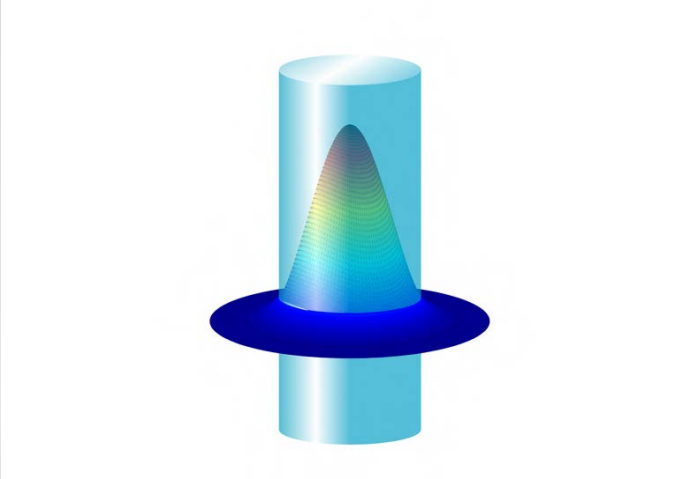
Optical properties

Poynting vector profile for 600-nm nanowire



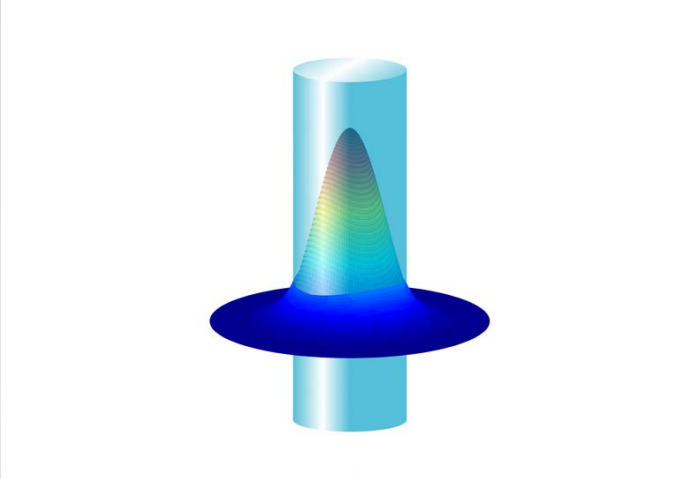
Optical properties

Poynting vector profile for 500-nm nanowire



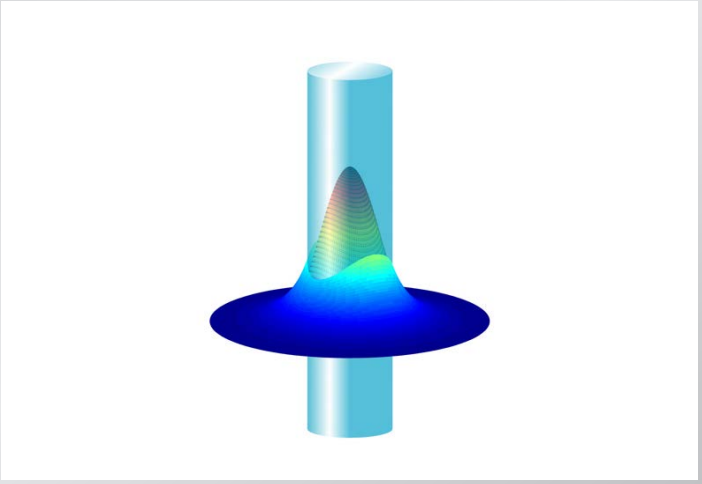
Optical properties

Poynting vector profile for 400-nm nanowire



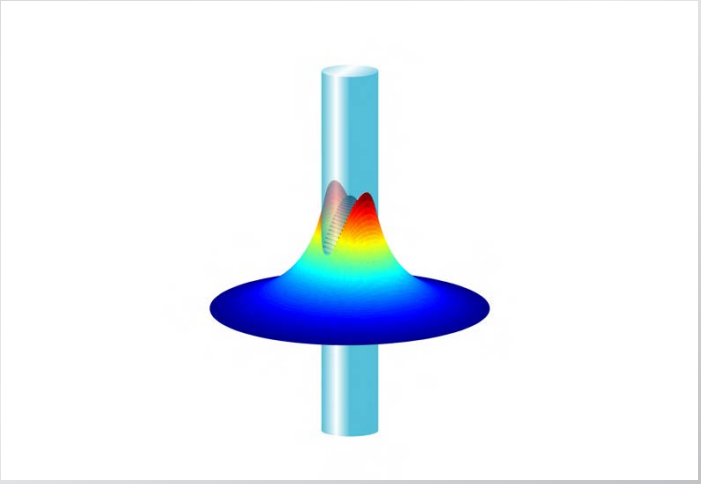
Optical properties

Poynting vector profile for 300-nm nanowire



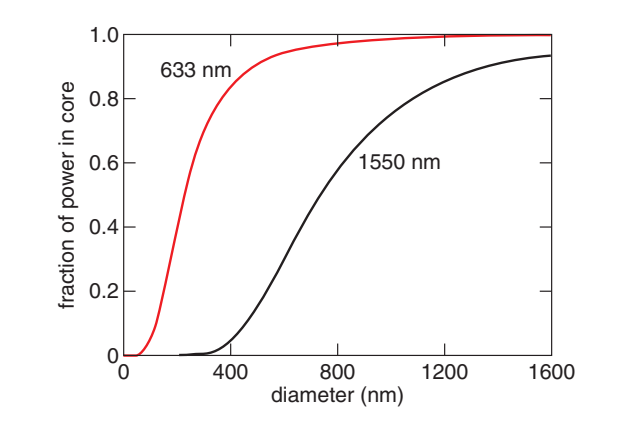
Optical properties

Poynting vector profile for 200-nm nanowire



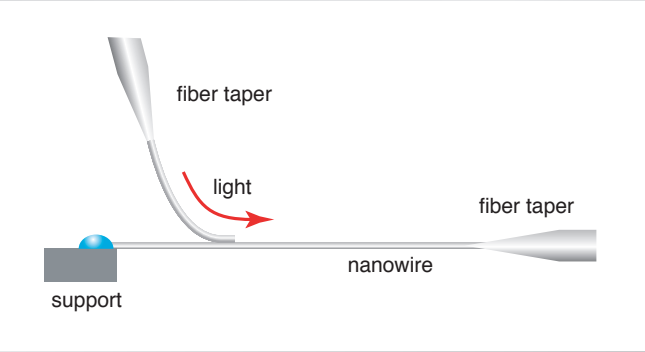
Waveguiding

fraction of power carried in core

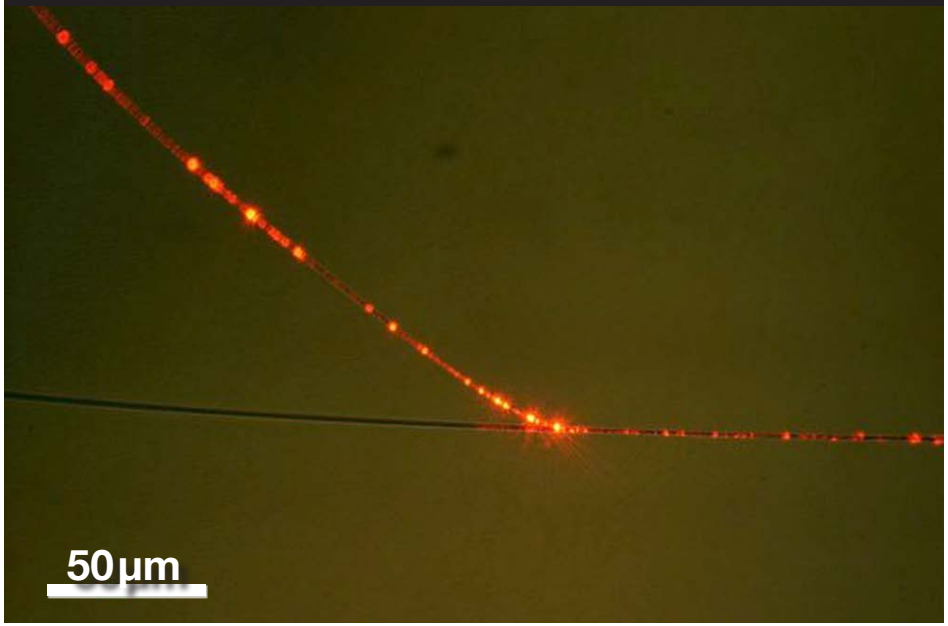


Optical properties

coupling light between nanowires

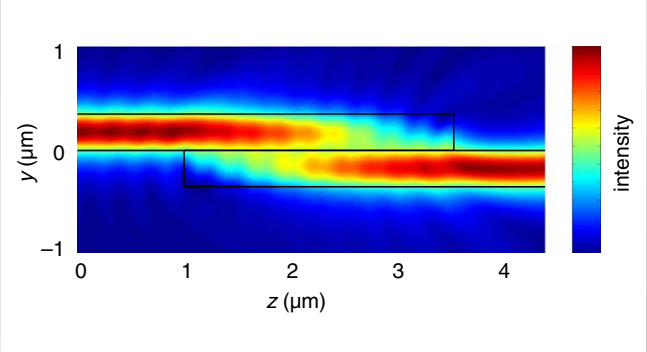


Optical properties

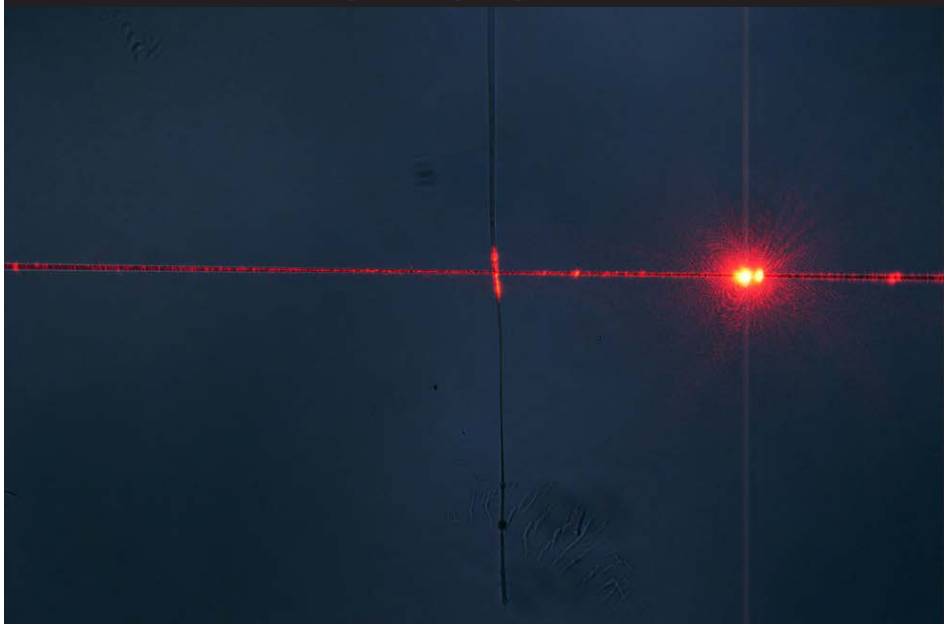


Optical properties

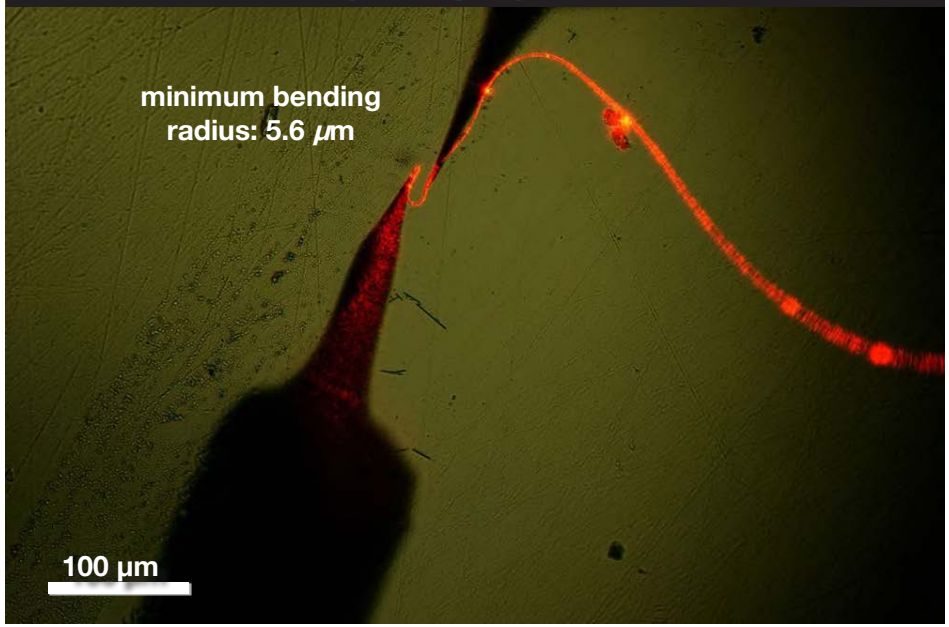
intensity distribution



Optical properties

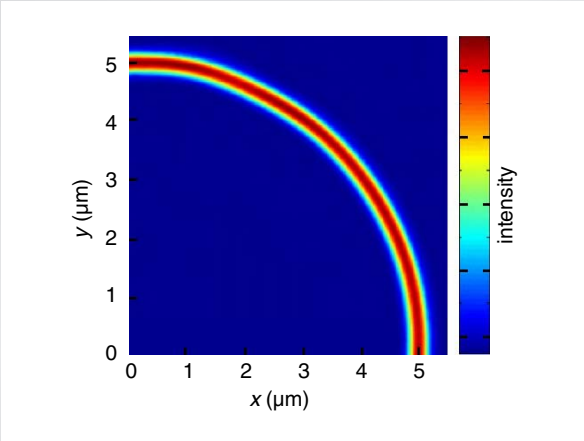


Optical properties



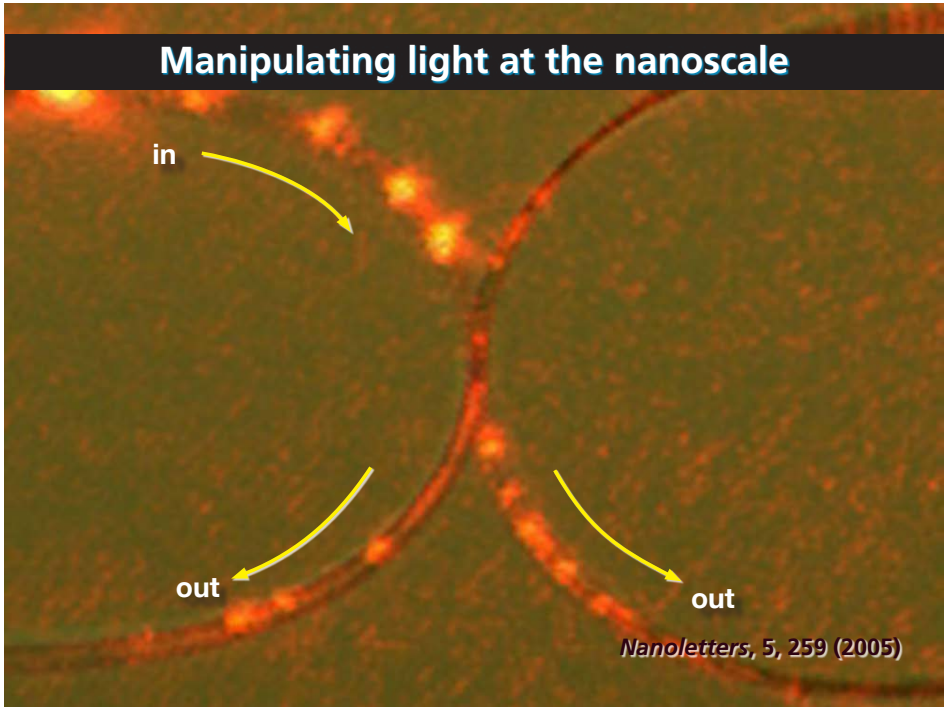
Optical properties

virtually no loss through 5 μm corner!



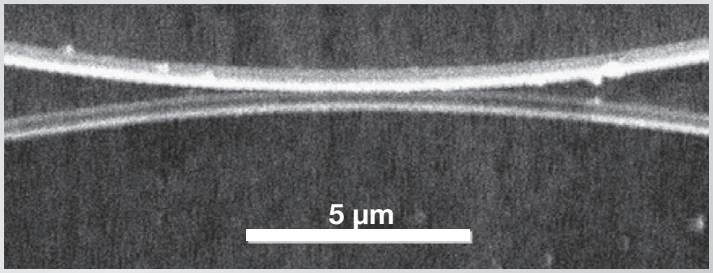
Manipulating light at the nanoscale

Manipulating light at the nanoscale



Nanoletters, 5, 259 (2005)

Manipulating light at the nanoscale



Nanoletters, 5, 259 (2005)

Manipulating light at the nanoscale

Points to keep in mind:

- low-loss guiding
- convenient evanescent coupling
- attached to ordinary fiber

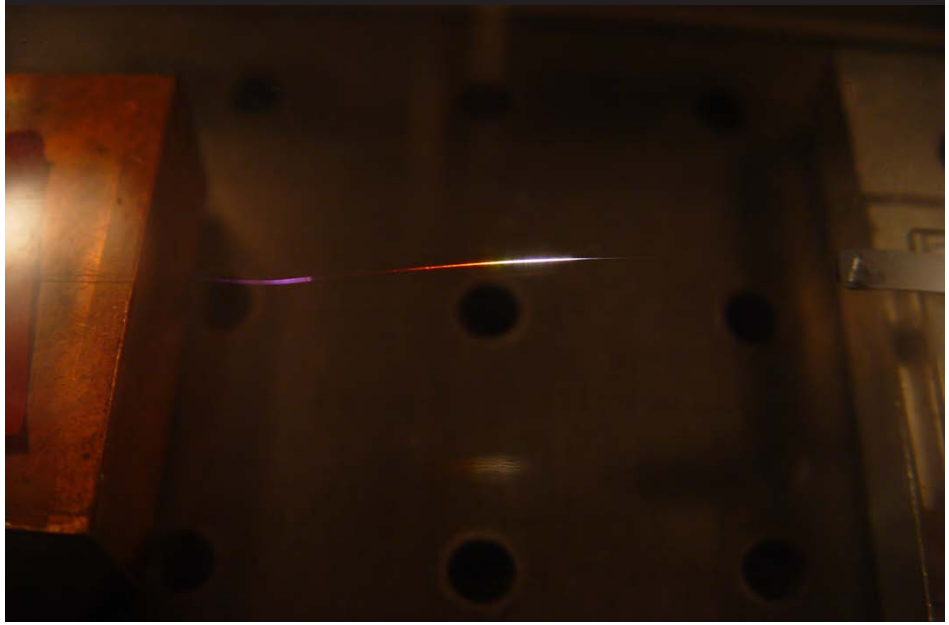
Outline

- propagation of pulses
- nonlinear optics
- nanoscale optics
- nonlinear optics at the nanoscale

Supercontinuum generation

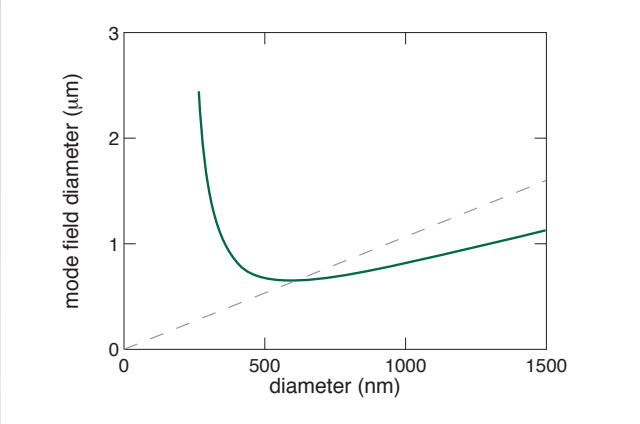
strong confinement $\longrightarrow$ high intensity

Supercontinuum generation



Supercontinuum generation

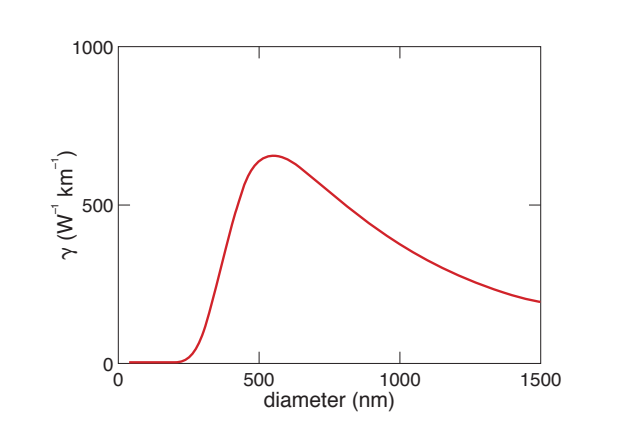
mode field diameter (λ = 800 nm)



M.A. Foster, *et al.*, *Optics Express*, 12, 2880 (2004)

Supercontinuum generation

nonlinear parameter



M.A. Foster, *et al.*, *Optics Express*, 12, 2880 (2004)

Supercontinuum generation

dispersion important!

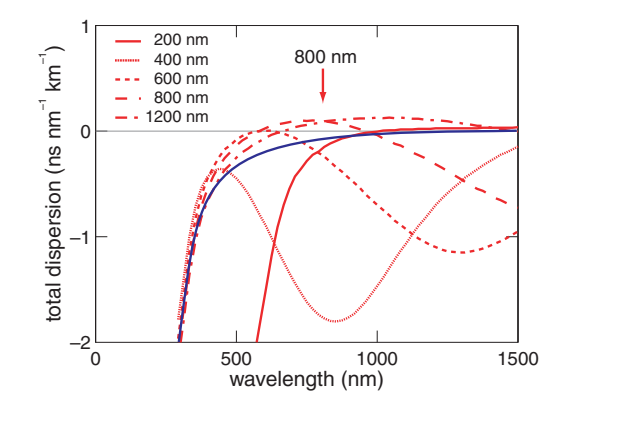
Supercontinuum generation

dispersion:

- modal dispersion
- material dispersion
- waveguide dispersion
- nonlinear dispersion

Supercontinuum generation

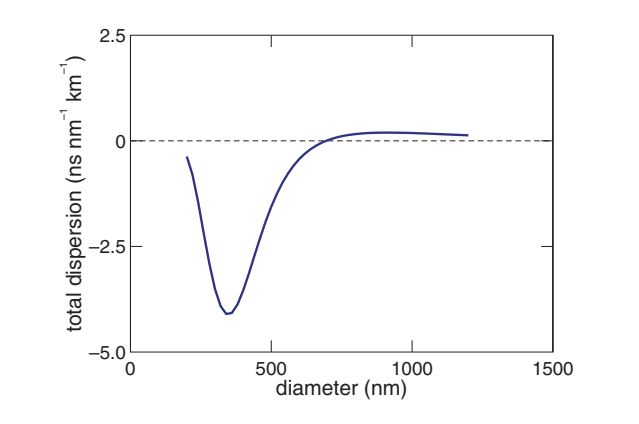
waveguide dispersion



Optics Express, 12, 1025 (2004)

Supercontinuum generation

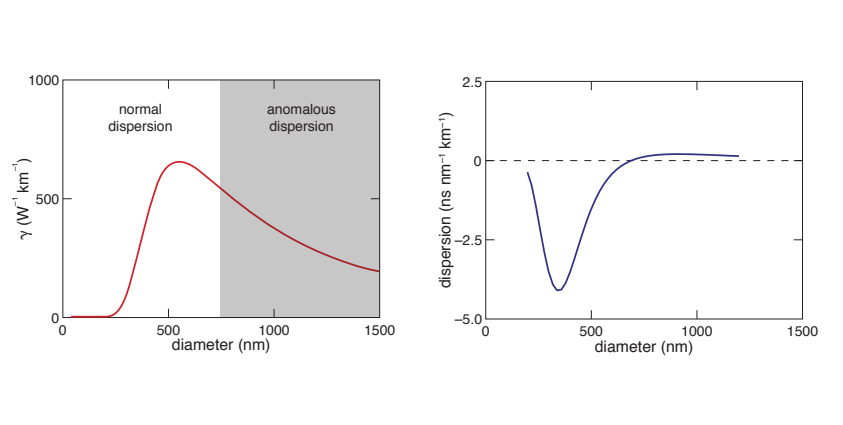
waveguide dispersion



Optics Express, 12, 1025 (2004)

Supercontinuum generation

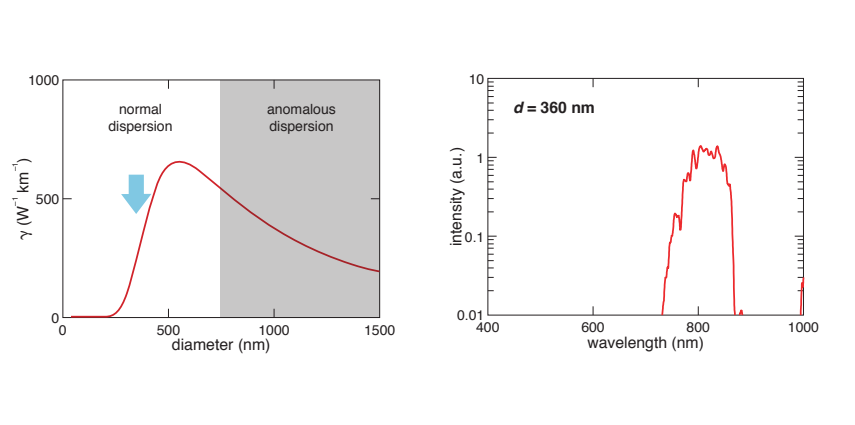
waveguide dispersion



Optics Express, 12, 1025 (2004)

Supercontinuum generation

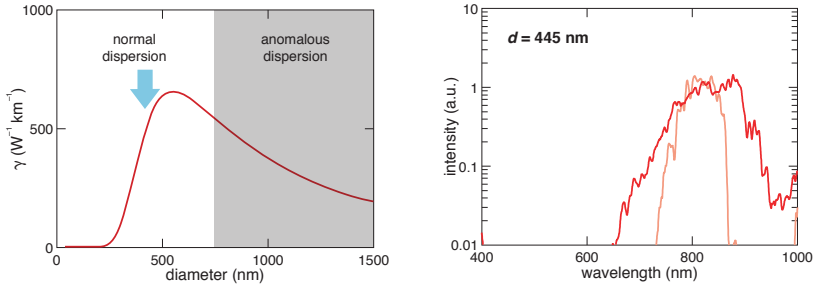
nanowire continuum generation



Optics Express, 14, 9408 (2006)

Supercontinuum generation

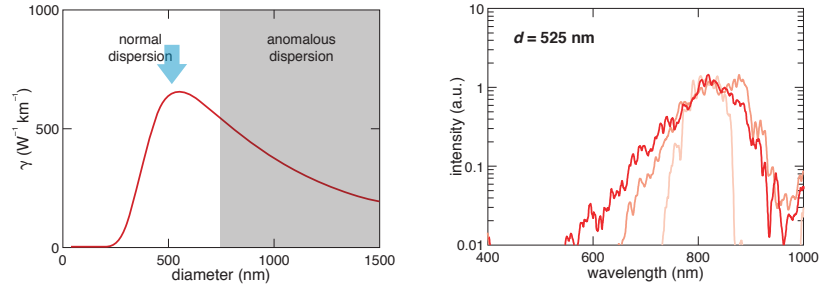
nanowire continuum generation



Optics Express, 14, 9408 (2006)

Supercontinuum generation

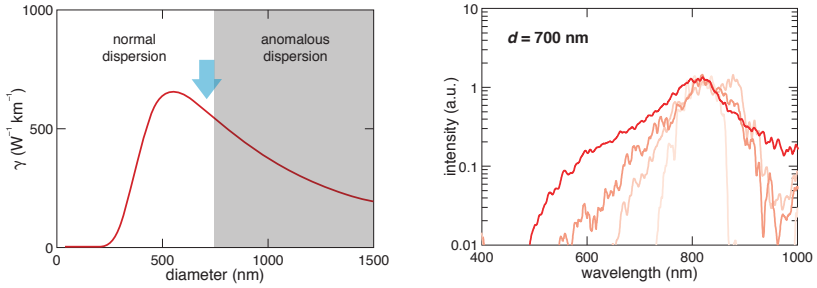
nanowire continuum generation



Optics Express, 14, 9408 (2006)

Supercontinuum generation

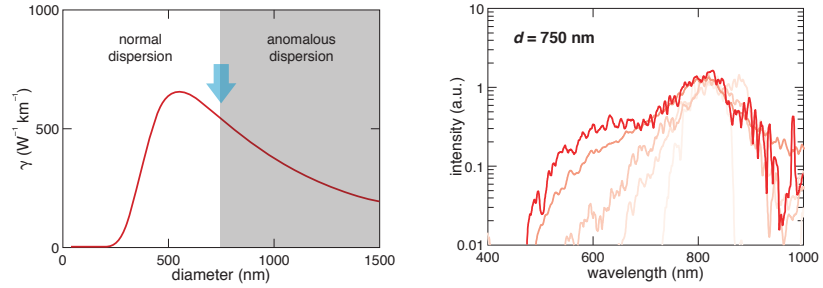
nanowire continuum generation



Optics Express, 14, 9408 (2006)

Supercontinuum generation

nanowire continuum generation



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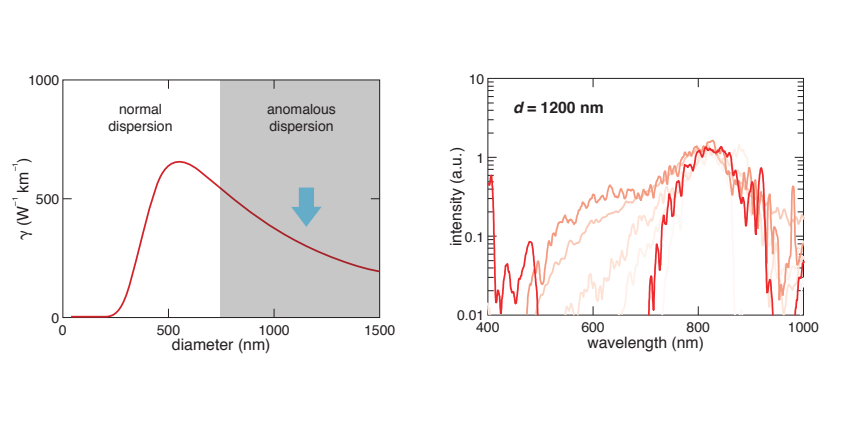
Supercontinuum generation

nanowire continuum generation



Optics Express, 14, 9408 (2006)

Supercontinuum generation

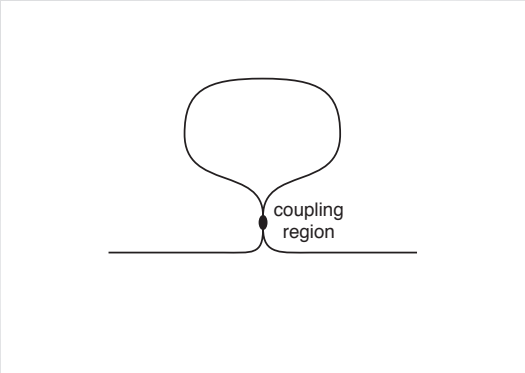
energy in nanowire ≈ 1 nJ!

Supercontinuum generation

- nanojoule nonlinear optics
- optimum diameter for silica 500–600 nm
- low dispersion

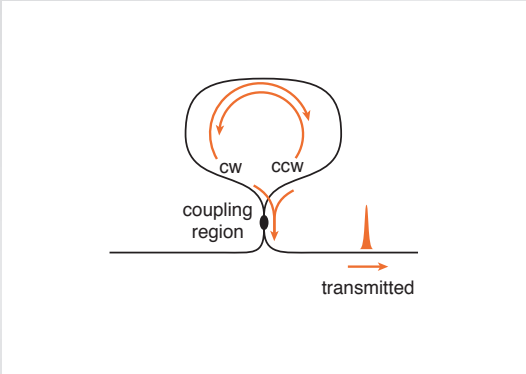
Optical logic gates

nanowire Sagnac interferometer



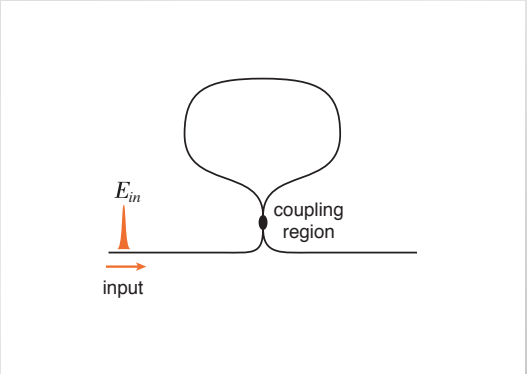
Optical logic gates

output = transmitted cw + ccw power



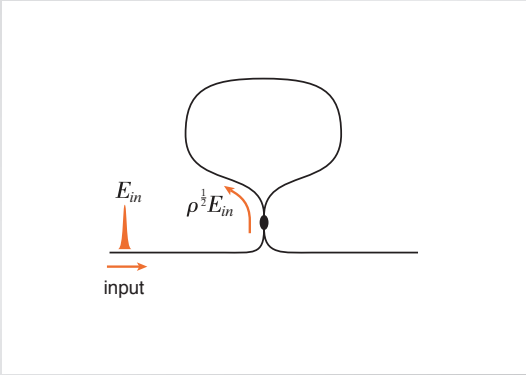
Optical logic gates

input electric field amplitude E_{in}



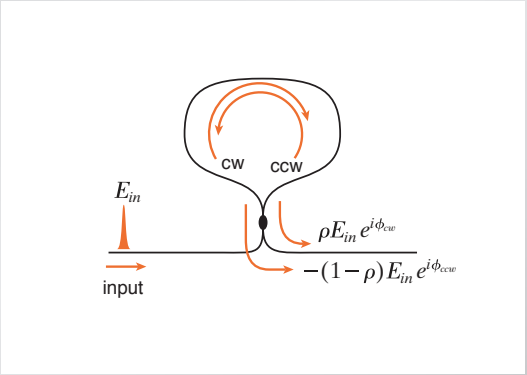
Optical logic gates

coupling parameter: ρ



Optical logic gates

output is sum of transmitted cw and ccw



Manipulating light at the nanoscale

accumulated phase:

$$\phi = k_o n L$$

nonlinear index:

$$n = n_o + n_2 I = n_o + n_2 \frac{P_i}{A_{eff}}$$

nonlinear parameter:

$$\gamma = n_2 \frac{k_o}{A_{eff}}$$

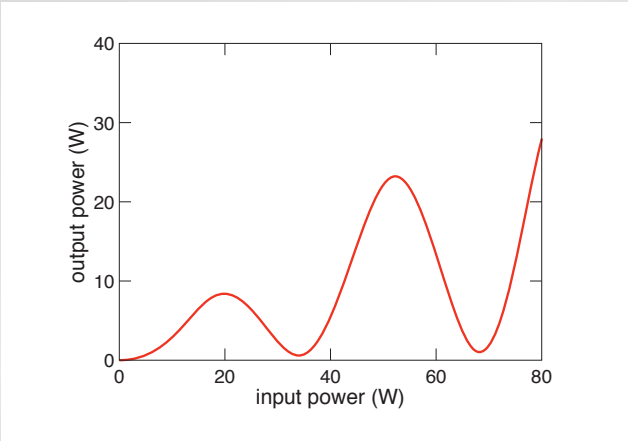
Manipulating light at the nanoscale

power-dependent output:

$$\frac{E_{out}^2}{E_{in}^2} = 1 - 2\rho(1-\rho)\{1 + \cos[(1-2\rho)\gamma P_o L]\}$$

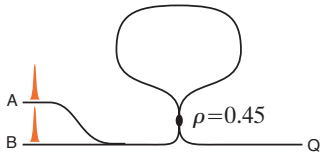
Optical logic gates

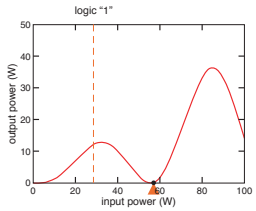
when $\rho \neq 0.5$:



Optical logic gates

nonlinear nanogate

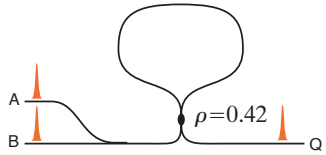


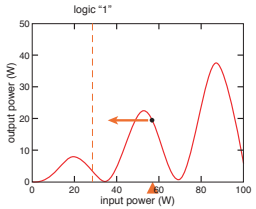


A	B	Q
0	0	0
1	0	1
0	1	1
1	1	0

Optical logic gates

nonlinear nanogate

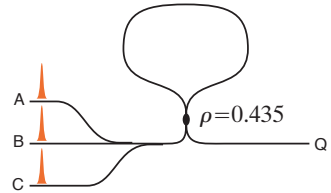


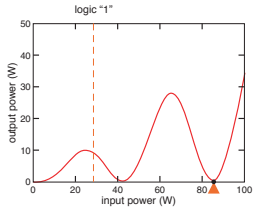


A	B	Q
0	0	0
1	0	0
0	1	0
1	1	1

Optical logic gates

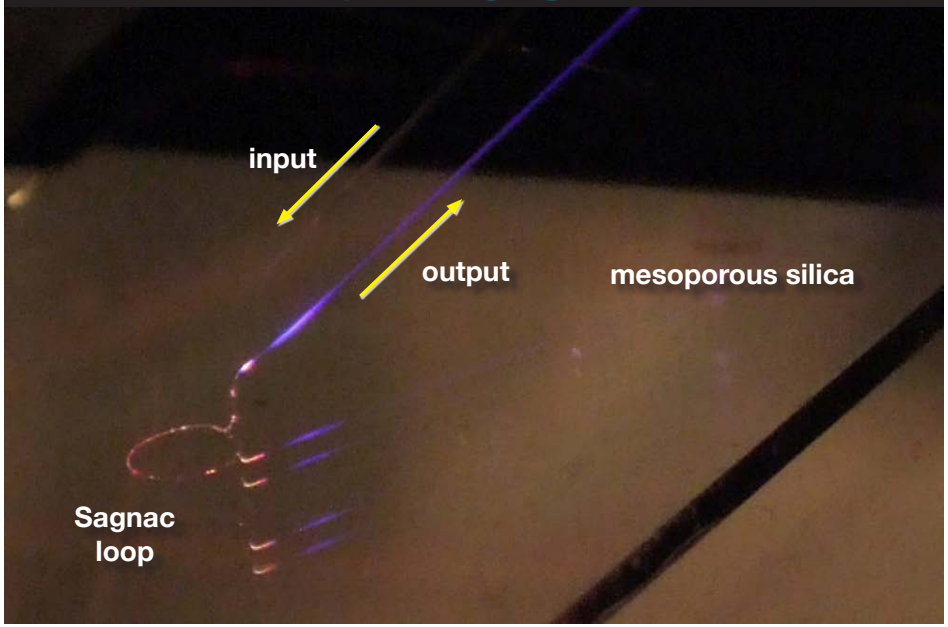
universal NAND gate





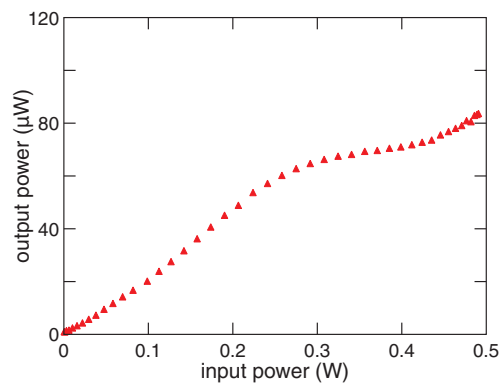
A	B	Q
0	0	1
1	0	1
0	1	1
1	1	0

Optical logic gates



Optical logic gates

very preliminary data

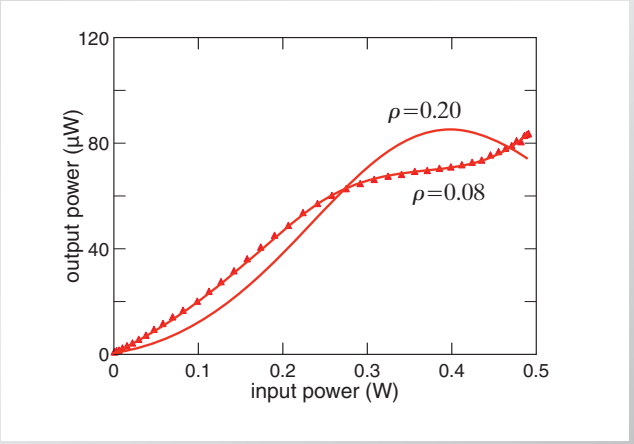


Optical logic gates

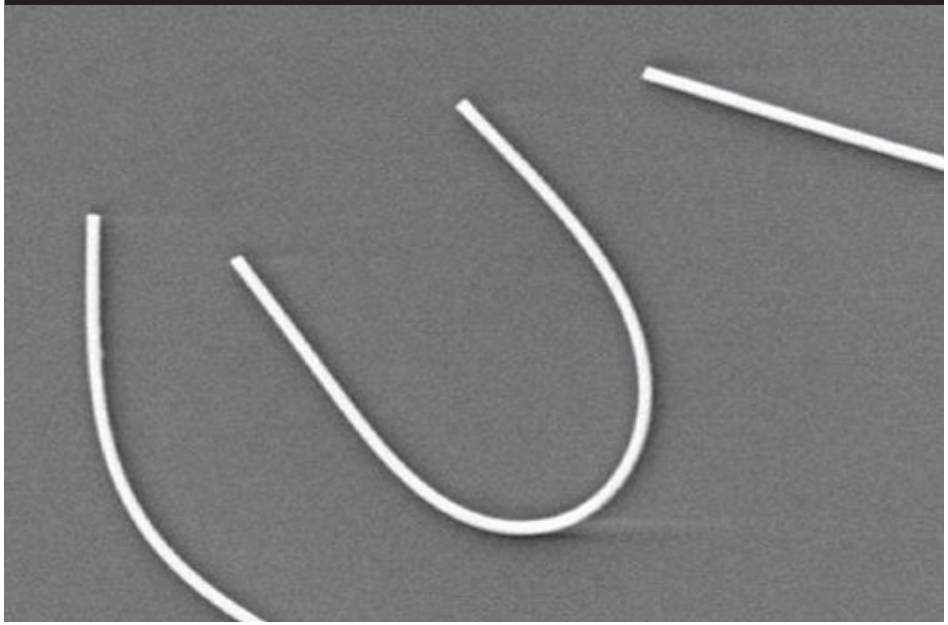
light-by-light modulation!

Optical logic gates

very preliminary data



Optical logic gates

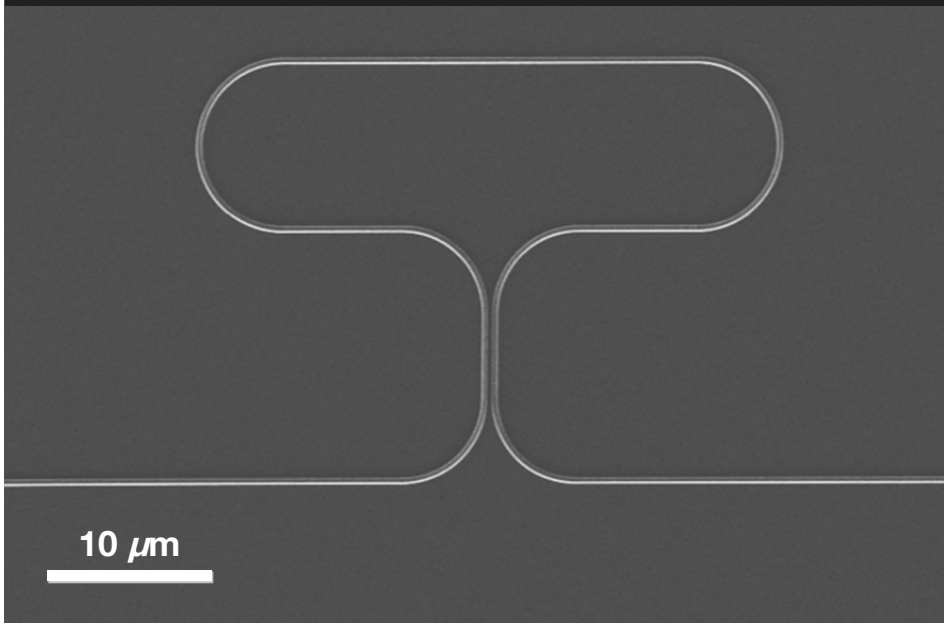


Optical logic gates

need a different approach!

- lithographic fabrication
- greater index
- greater nonlinearity

Optical logic gates



Optical logic gates

TiO_2 properties

large nonlinearity	30x silica
high index of refraction	2.4
wide bandgap	3.1 eV
low two-photon absorption	> 800 nm
effective nonlinearity	50,000 W^{-1} km^{-1}

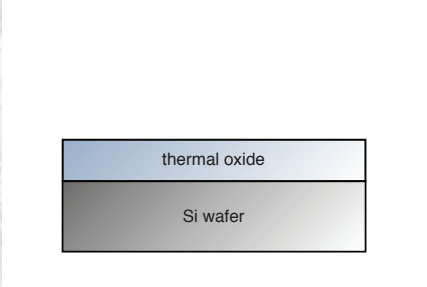
Optical logic gates

reactive sputtering of titanium with oxygen

Optics Express, 20, 23821 (2012)

Optical logic gates

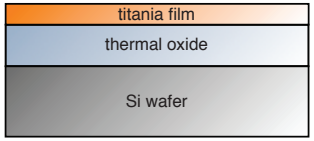
begin with silicon wafer with thermal oxide



Optics Express, 20, 23821 (2012)

Optical logic gates

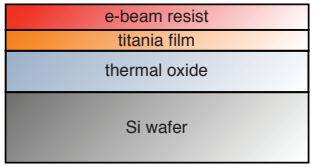
deposit titania using reactive sputtering



Optics Express, 20, 23821 (2012)

Optical logic gates

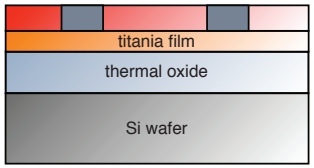
spin on e-beam resist



Optics Express, 20, 23821 (2012)

Optical logic gates

write pattern using e-beam



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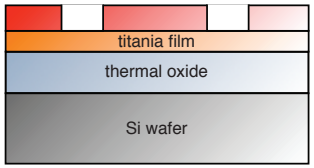
Optical logic gates

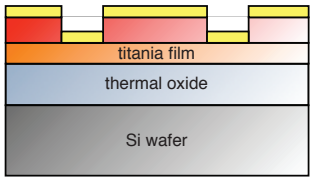
develop to remove exposed regions



Optics Express, 20, 23821 (2012)

Optical logic gates

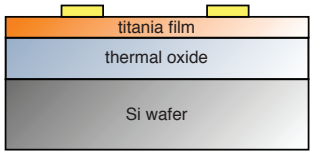
deposit thin metal film



Optics Express, 20, 23821 (2012)

Optical logic gates

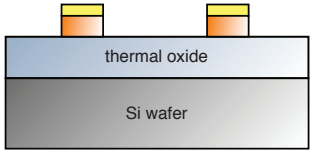
dissolve resist, lift off metal film



Optics Express, 20, 23821 (2012)

Optical logic gates

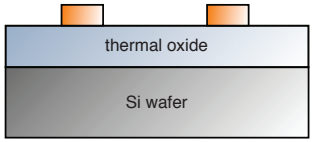
reactive ion etch through titania film



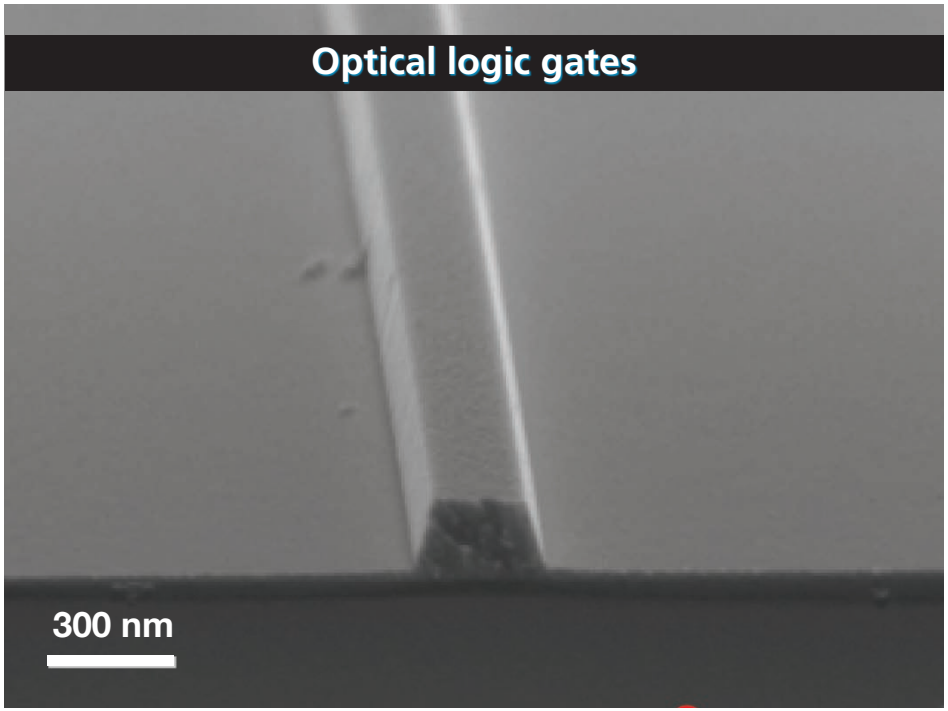
Optics Express, 20, 23821 (2012)

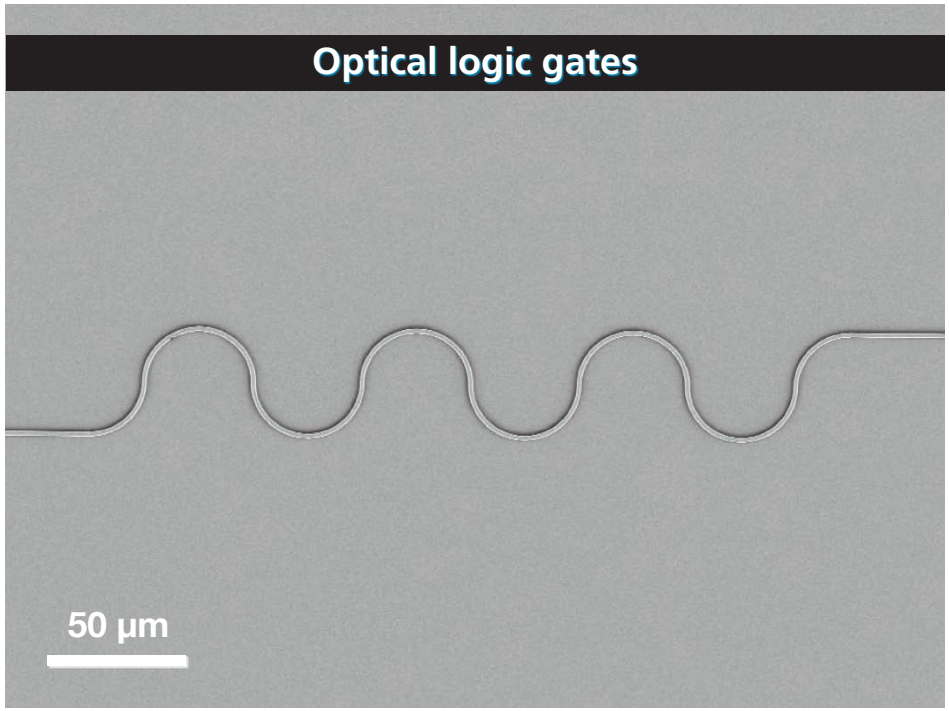
Optical logic gates

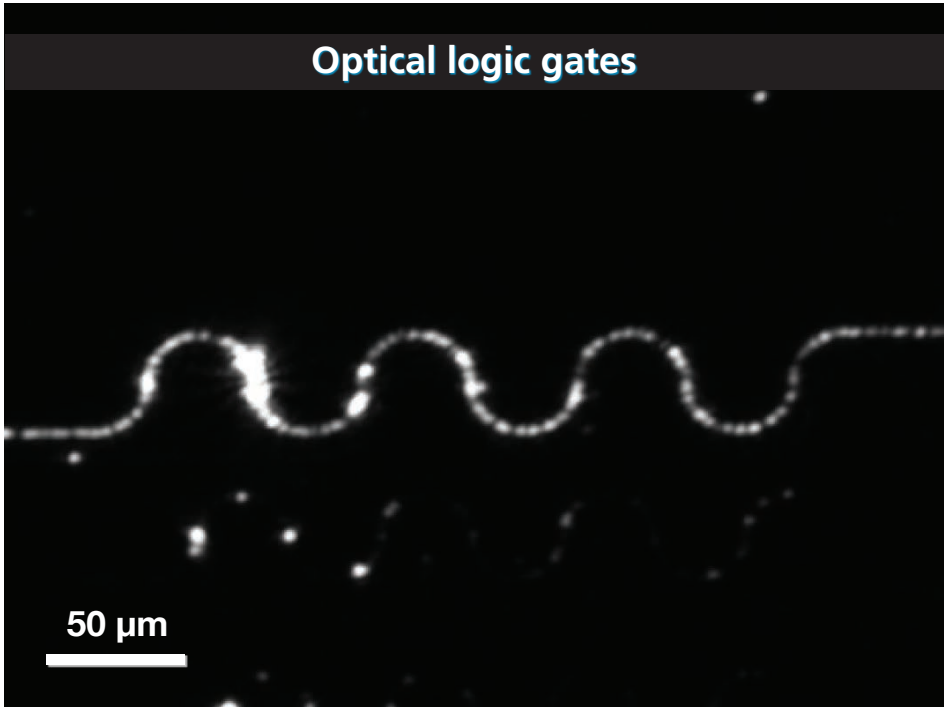
remove remaining metal

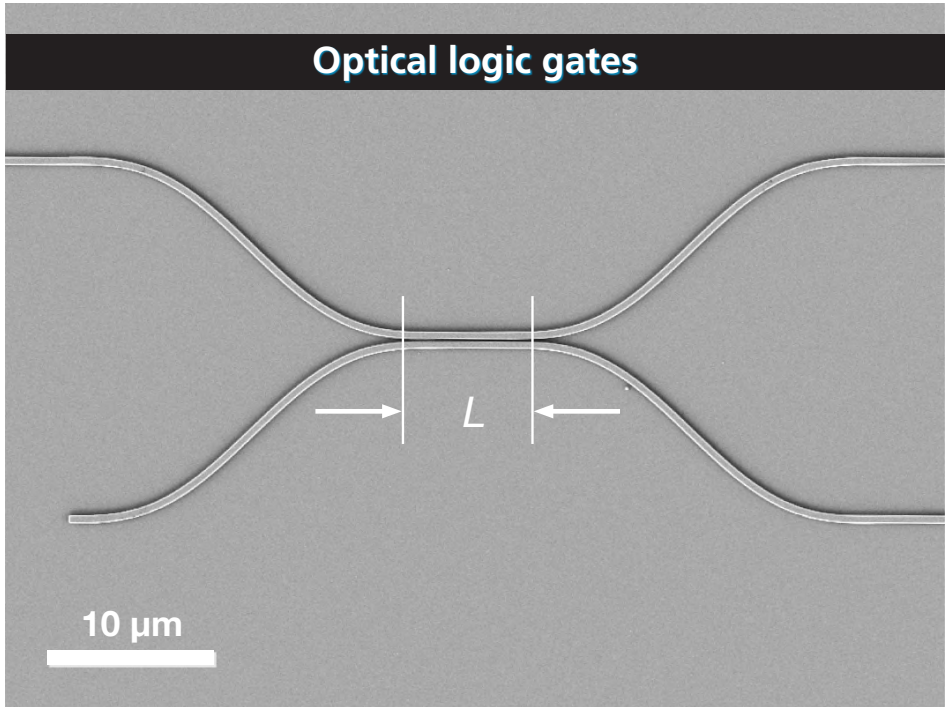


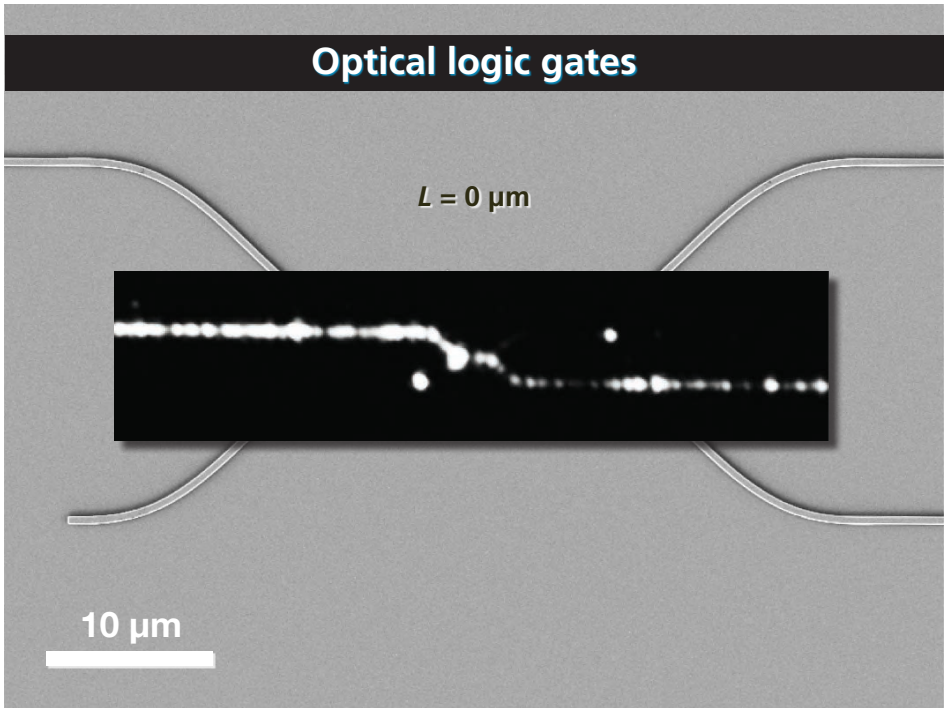
Optics Express, 20, 23821 (2012)

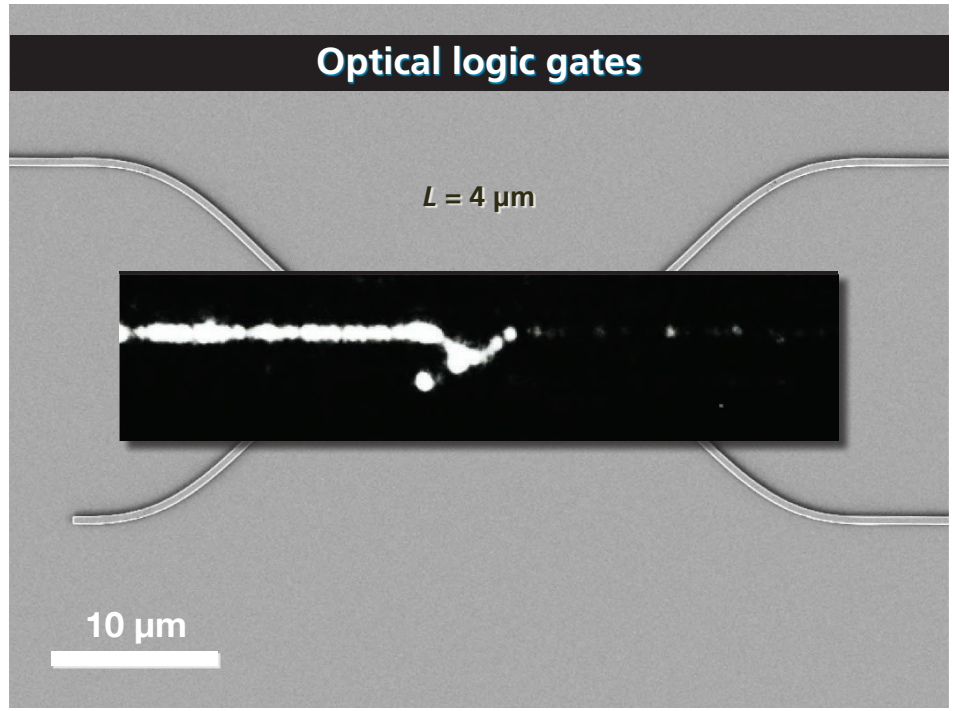

Optical logic gates
300 nm

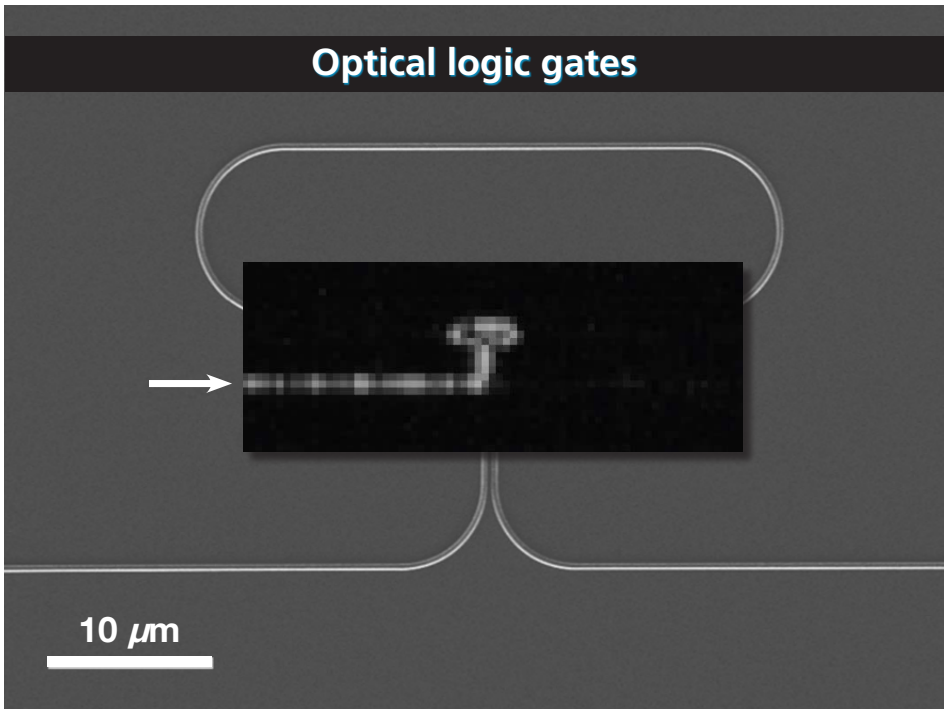

Optical logic gates
50 µm

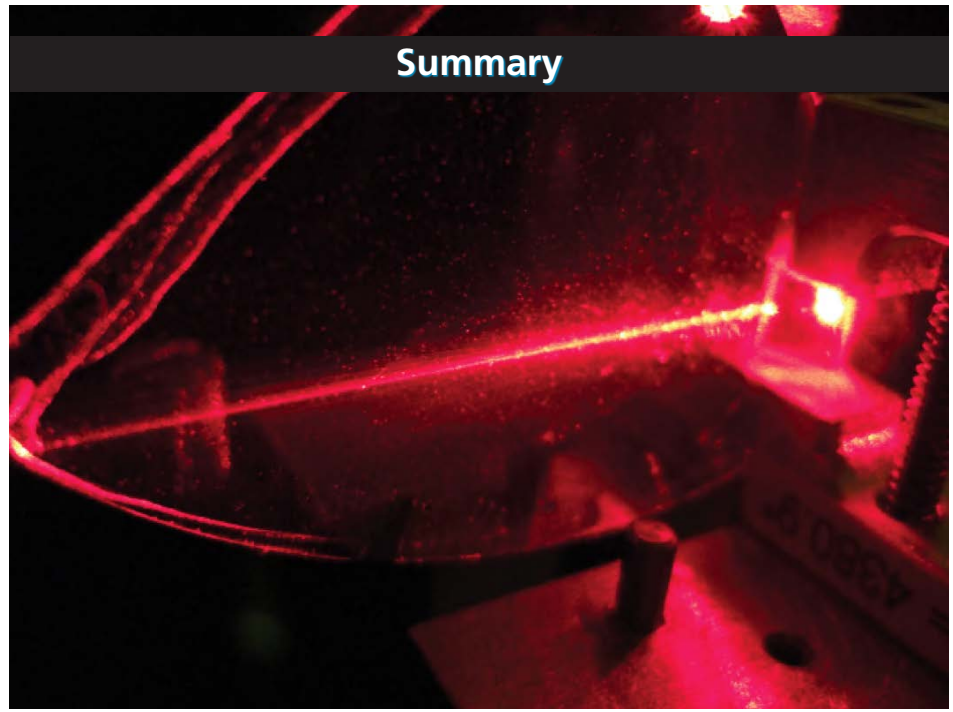

Optical logic gates
50 µm


Optical logic gates
L
10 µm


Optical logic gates
L = 0 μm
10 μm


Optical logic gates
L = 4 μm
10 μm

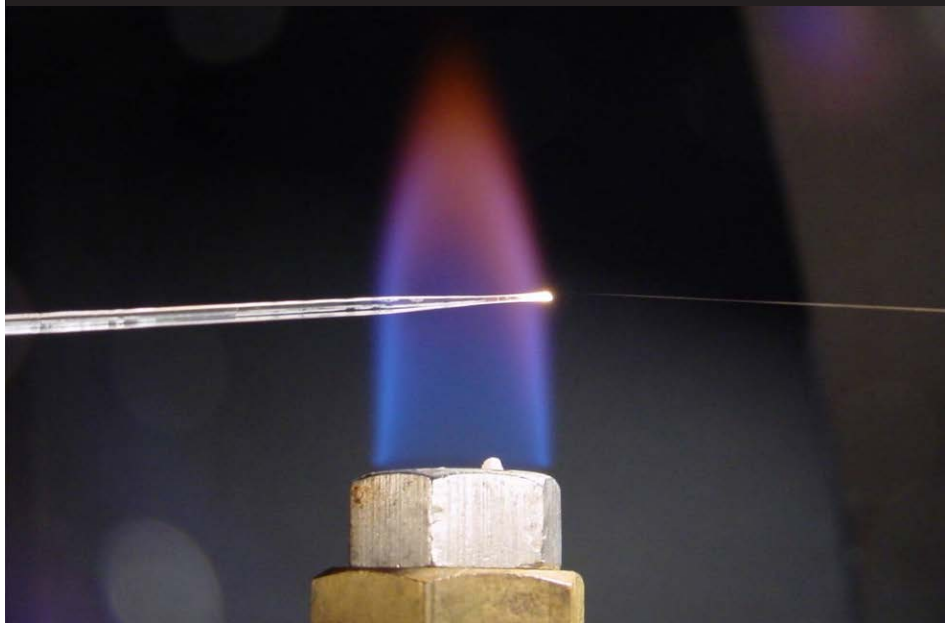

Optical logic gates
10 μm


Summary

Summary

- several nanodevices demonstrated
- large γ permits miniature Sagnac loops
- switching energy $\approx$ 100 pJ

Summary



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