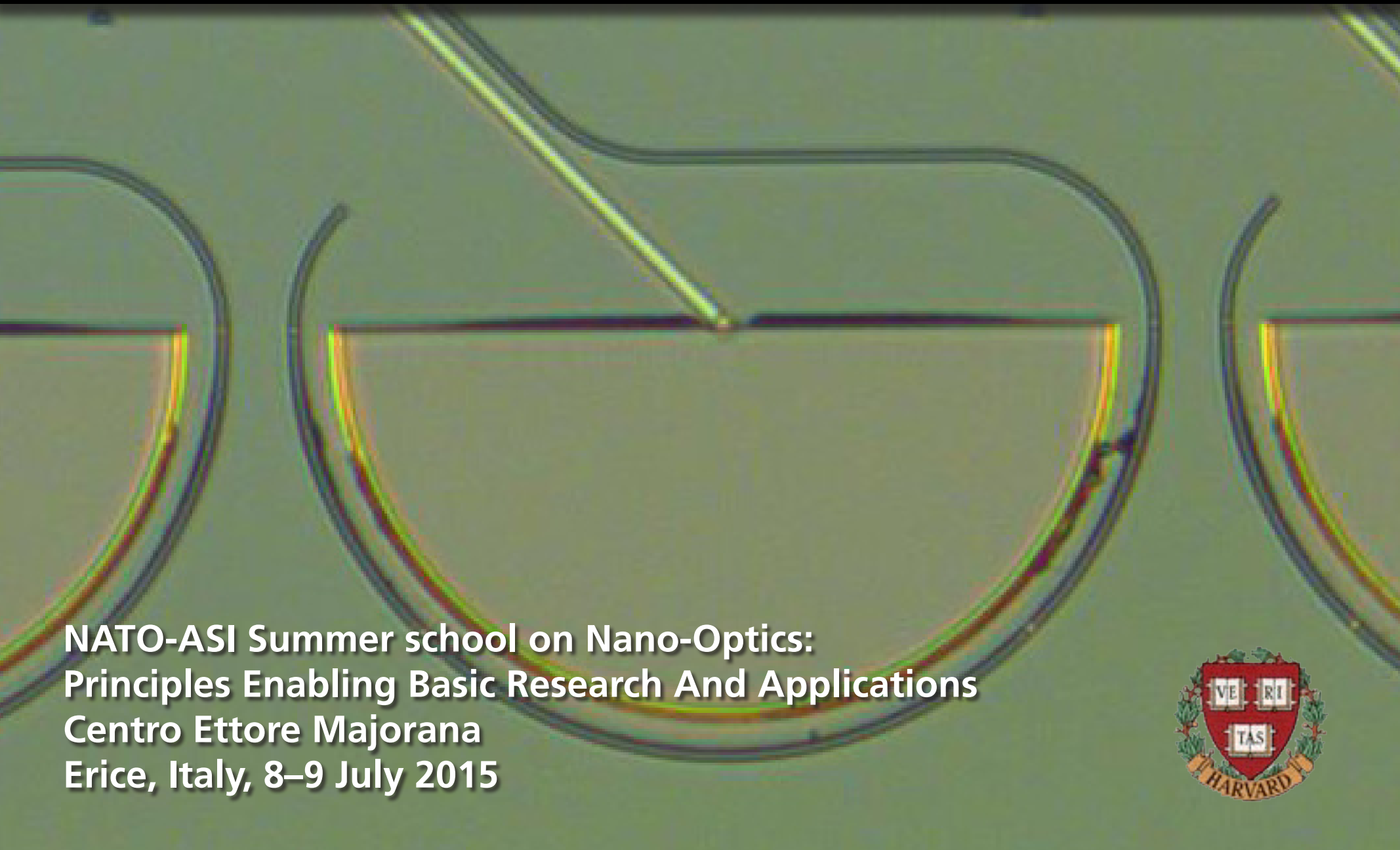
Manipulating Light at the Nanoscale



NATO-ASI Summer school on Nano-Optics:
Principles Enabling Basic Research And Applications
Centro Ettore Majorana
Erice, Italy, 8–9 July 2015

Manipulating Light at the Nanoscale

@eric__mazur

NATO-ASI Summer school on Nano-Optics:
Principles Enabling Basic Research And Applications
Centro Ettore Majorana
Erice, Italy, 8–9 July 2015

for a copy of these slides:

http://ericmazur.com

Follow me! eric_mazur

Outline

- optical properties of materials
- dispersion of pulses
- **nonlinear optics**
- waveguiding
- engineering the index

Nonlinear optics

Linear optics:

$$\vec{P} = \chi \vec{E}$$

Nonlinear optics

Linear optics:

$$\vec{P} = \chi \vec{E}$$

Nonlinear polarization:

$$P = \chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \ldots$$

Nonlinear optics

Linear optics:

$$\vec{P} = \chi \vec{E}$$

Nonlinear polarization:

$$P = \chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \ldots$$

and so:

$$P = P^{(1)} + P^{(2)} + P^{(3)} + \ldots$$

Nonlinear optics

Linear optics:

$$\vec{P} = \chi \vec{E}$$

Nonlinear polarization:

$$P = \chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \ldots$$

and so:

$$P = P^{(1)} + P^{(2)} + P^{(3)} + \ldots$$

$P^{(2)} \approx P^{(1)}$ **when** $E = E_{at} \approx \dfrac{e}{a}$**, and so** $\chi^{(n)} \approx \dfrac{\chi^{(1)}}{E_{at}^{n-1}}$**.**

Nonlinear optics

Nonlinear polarization can drive new field:

$$\nabla^2 \vec{E} + \frac{n^2}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 \vec{P}}{\partial t^2}$$

Nonlinear optics

Nonlinear polarization can drive new field:

$$\nabla^2 \vec{E} + \frac{n^2}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 \vec{P}}{\partial t^2}$$

But even terms disappear in media with inversion symmetry!

$$\vec{P}^{(2)} = \chi^{(2)} : \vec{E}\vec{E}$$

Nonlinear optics

Nonlinear polarization can drive new field:

$$\nabla^2 \vec{E} + \frac{n^2}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 \vec{P}}{\partial t^2}$$

But even terms disappear in media with inversion symmetry!

$$\vec{P}^{(2)} = \chi^{(2)} : \vec{E}\vec{E}$$

Invert all vectors:

$$-\vec{P}^{(2)} = \chi^{(2)} : (-\vec{E})(-\vec{E})$$

Nonlinear optics

Nonlinear polarization can drive new field:

$$\nabla^2 \vec{E} + \frac{n^2}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 \vec{P}}{\partial t^2}$$

But even terms disappear in media with inversion symmetry!

$$\vec{P}^{(2)} = \chi^{(2)} : \vec{E}\vec{E}$$

Invert all vectors:

$$-\vec{P}^{(2)} = \chi^{(2)} : (-\vec{E})(-\vec{E})$$

and so $\chi^{(2)} = -\chi^{(2)} = 0$.

Nonlinear optics

Consider oscillating electric field:

$$E(t) = E\, e^{i\omega t} + \text{c.c.}$$

Nonlinear optics

Consider oscillating electric field:

$$E(t) = E\, e^{i\omega t} + \text{c.c.}$$

Second-order polarization:

$$P^{(2)}(t) = \chi^{(2)} E^2(t) = \tfrac{1}{2}\chi^{(2)} E E^* + \tfrac{1}{4}[\chi^{(2)} E^2 e^{-2\omega t} + \text{c.c.}]$$

Nonlinear optics

Consider oscillating electric field:

$$E(t) = E\, e^{i\omega t} + \text{c.c.}$$

Second-order polarization:

$$P^{(2)}(t) = \chi^{(2)} E^2(t) = \tfrac{1}{2}\chi^{(2)} E E^* + \tfrac{1}{4}[\chi^{(2)} E^2 e^{-2\omega t} + \text{c.c.}]$$

Nonlinear optics

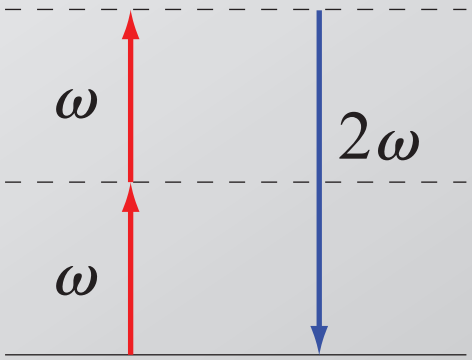
Consider oscillating electric field:

$$E(t) = E\, e^{i\omega t} + \text{c.c.}$$

Second-order polarization:

$$P^{(2)}(t) = \chi^{(2)} E^2(t) = \tfrac{1}{2}\chi^{(2)} E E^* + \tfrac{1}{4}[\chi^{(2)} E^2 e^{-2\omega t} + \text{c.c.}]$$

Physical interpretation:



Nonlinear optics

Can also cause frequency mixing!

Nonlinear optics

Can also cause frequency mixing! Let

$$E(t) = E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t}$$

Nonlinear optics

Can also cause frequency mixing! Let

$$E(t) = E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t}$$

Second-order polarization will contain terms with

$2\omega_1$ **(SHG),** $2\omega_2$ **(SHG),** $\omega_1 + \omega_2$ **(SFG),** $\omega_1 - \omega_2$ **(DFG)**

Nonlinear optics

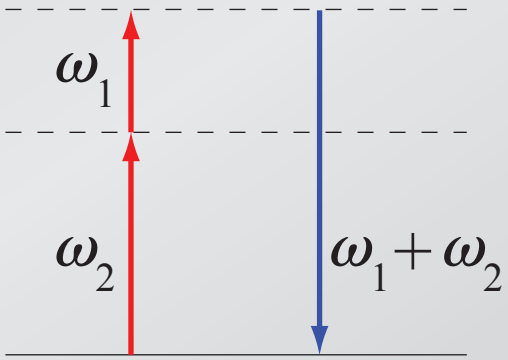
Can also cause frequency mixing! Let

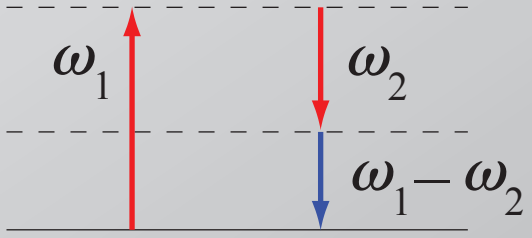
$$E(t) = E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t}$$

Second-order polarization will contain terms with

$2\omega_1$ **(SHG),** $2\omega_2$ **(SHG),** $\omega_1 + \omega_2$ **(SFG),** $\omega_1 - \omega_2$ **(DFG)**

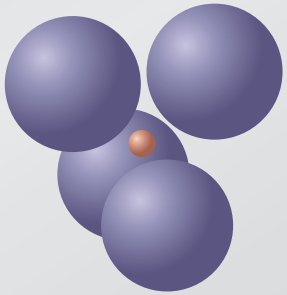
Physical interpretation:





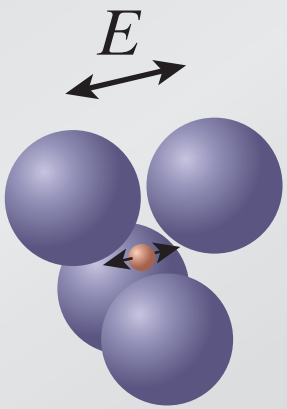
Nonlinear optics

Linear response: $\vec{P} = \chi \vec{E}$



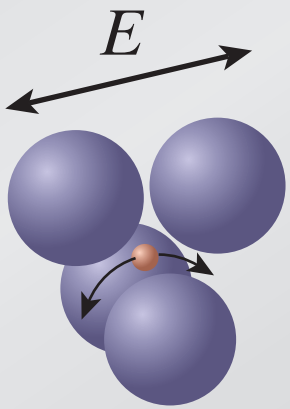
Nonlinear optics

Linear response: $\vec{P} = \chi \vec{E}$



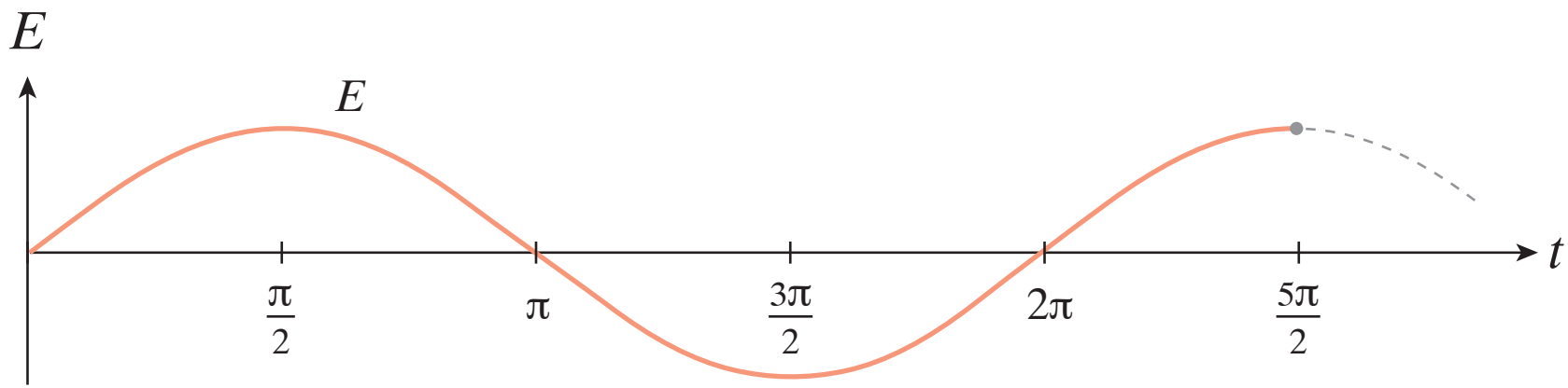
Nonlinear optics

Nonlinear response: $P^{(2)} = \chi^{(2)} E^2$



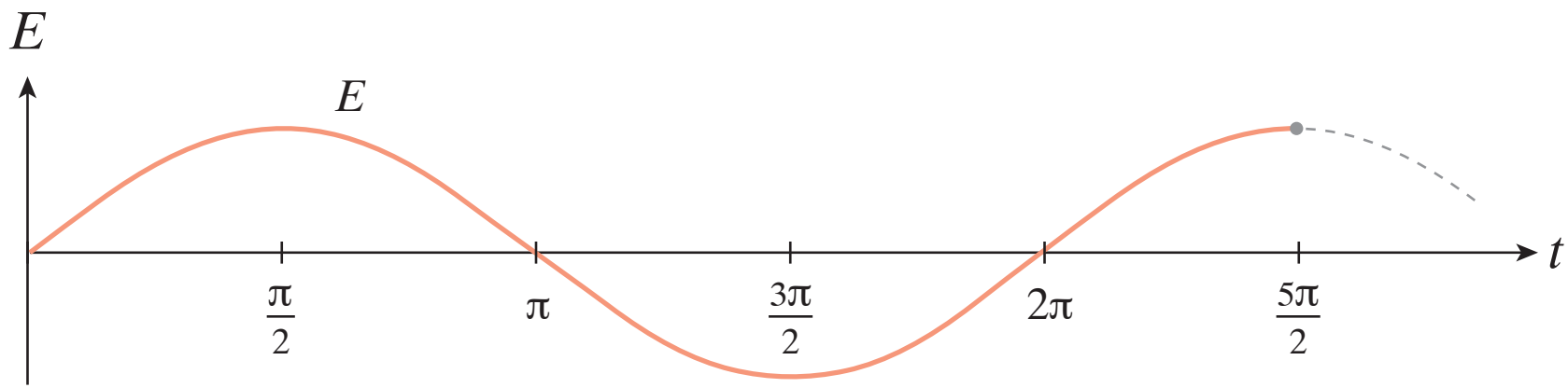
Nonlinear optics

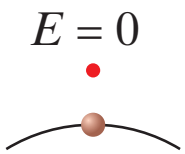
Nonlinear response: $$P^{(2)} = \chi^{(2)} E^2$$



Nonlinear optics

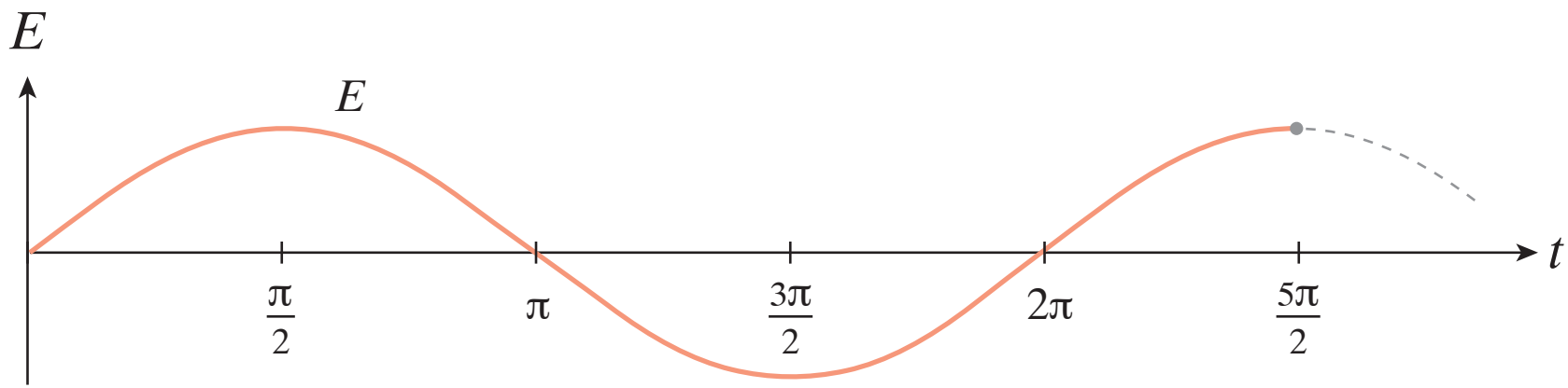
Nonlinear response: $P^{(2)} = \chi^{(2)} E^2$





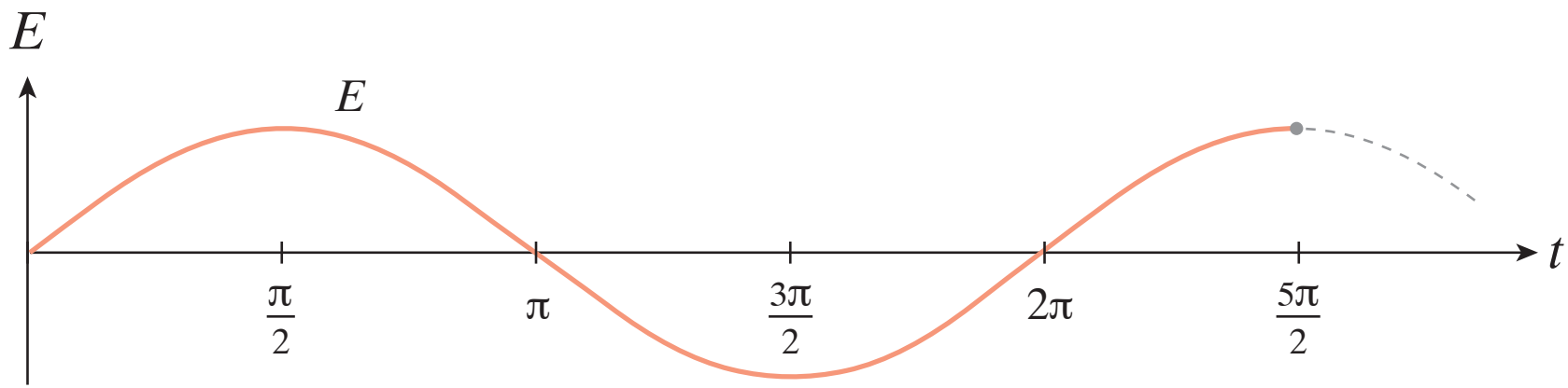
Nonlinear optics

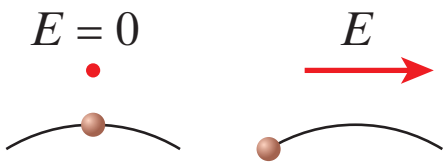
Nonlinear response: $P^{(2)} = \chi^{(2)} E^2$



Nonlinear optics

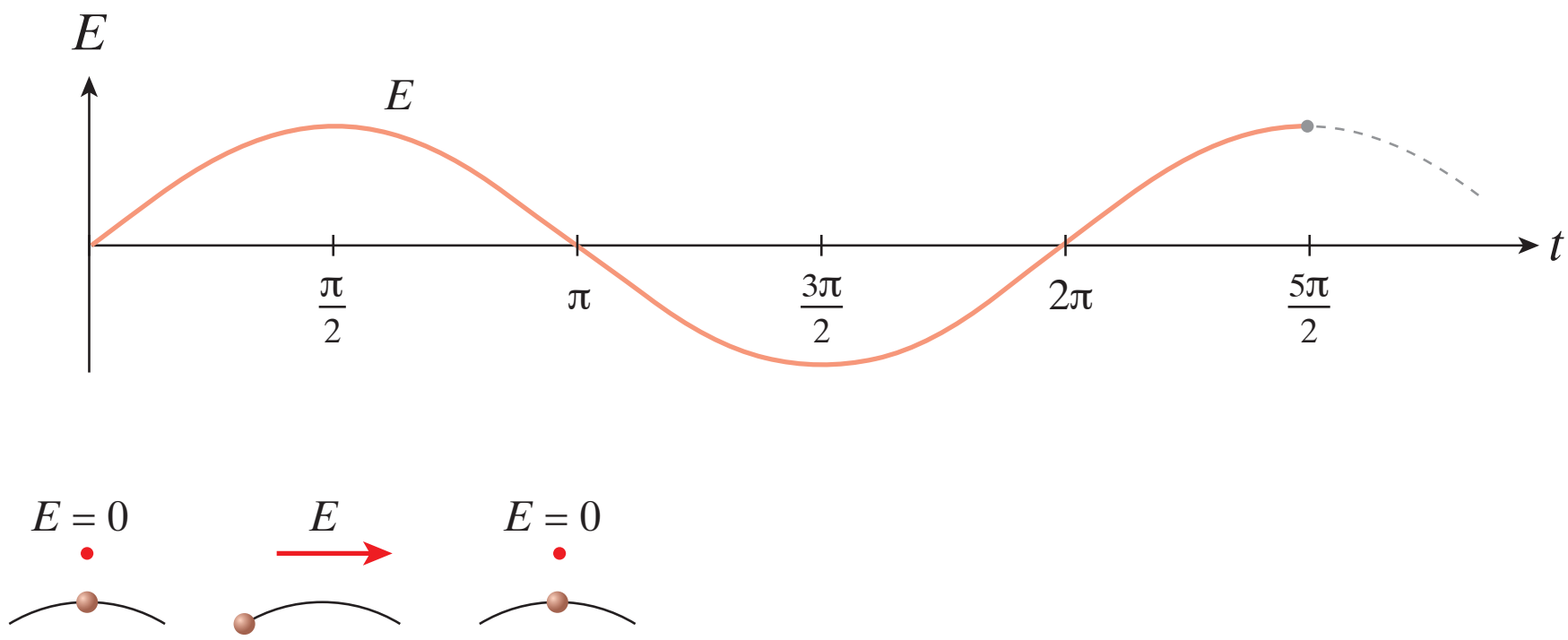
Nonlinear response: $P^{(2)} = \chi^{(2)} E^2$





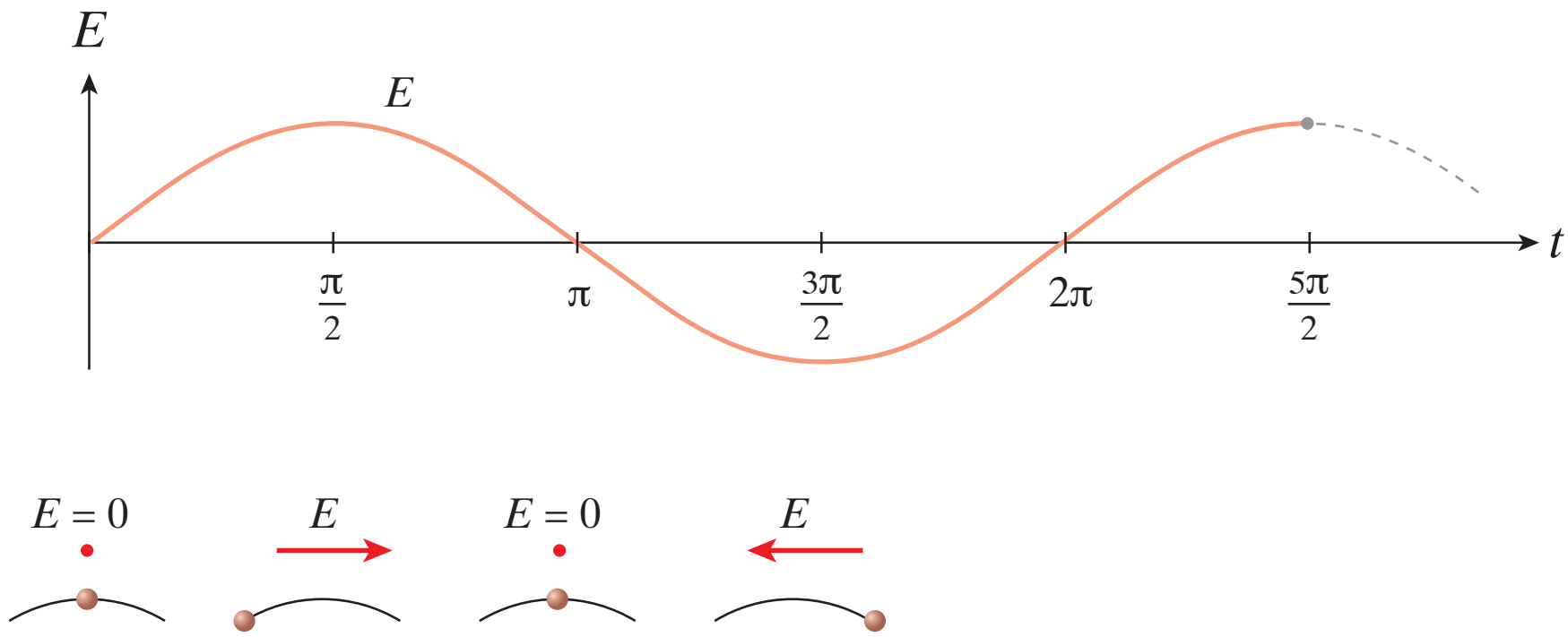
Nonlinear optics

Nonlinear response: $P^{(2)} = \chi^{(2)} E^2$



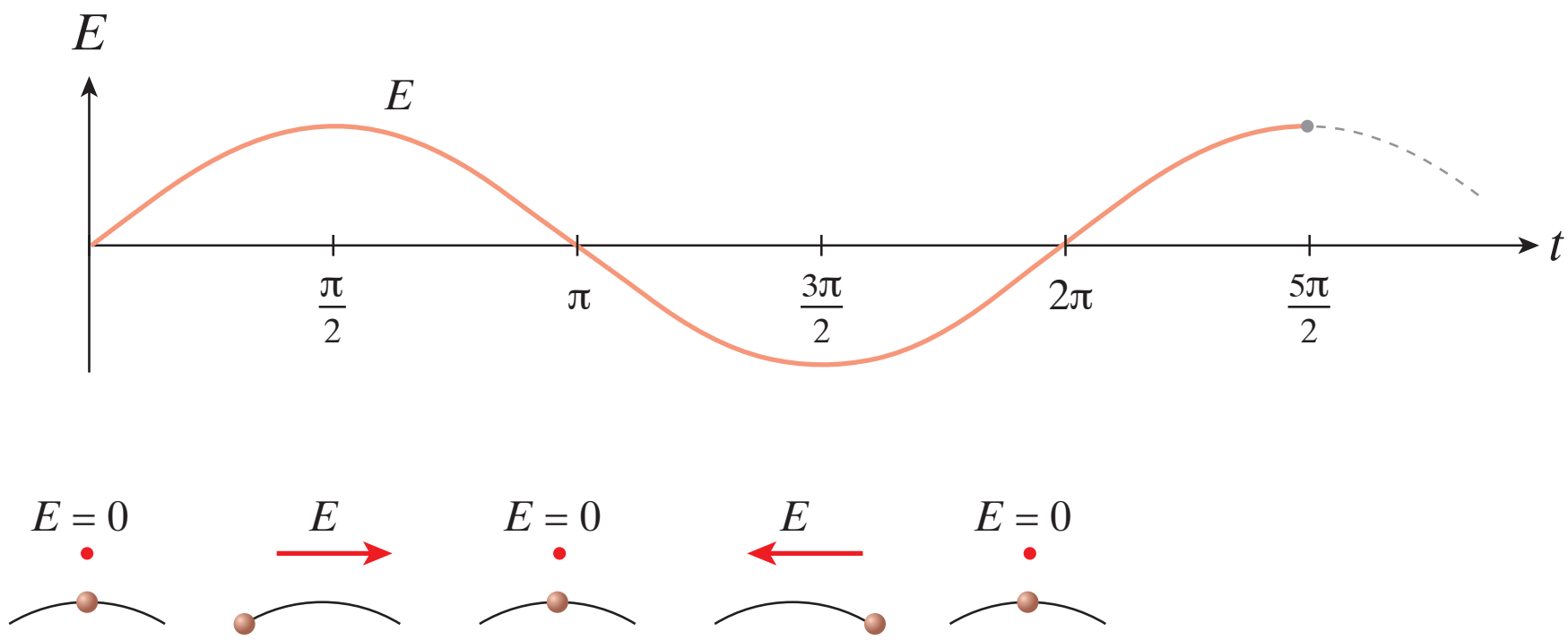
Nonlinear optics

Nonlinear response: $P^{(2)} = \chi^{(2)} E^2$



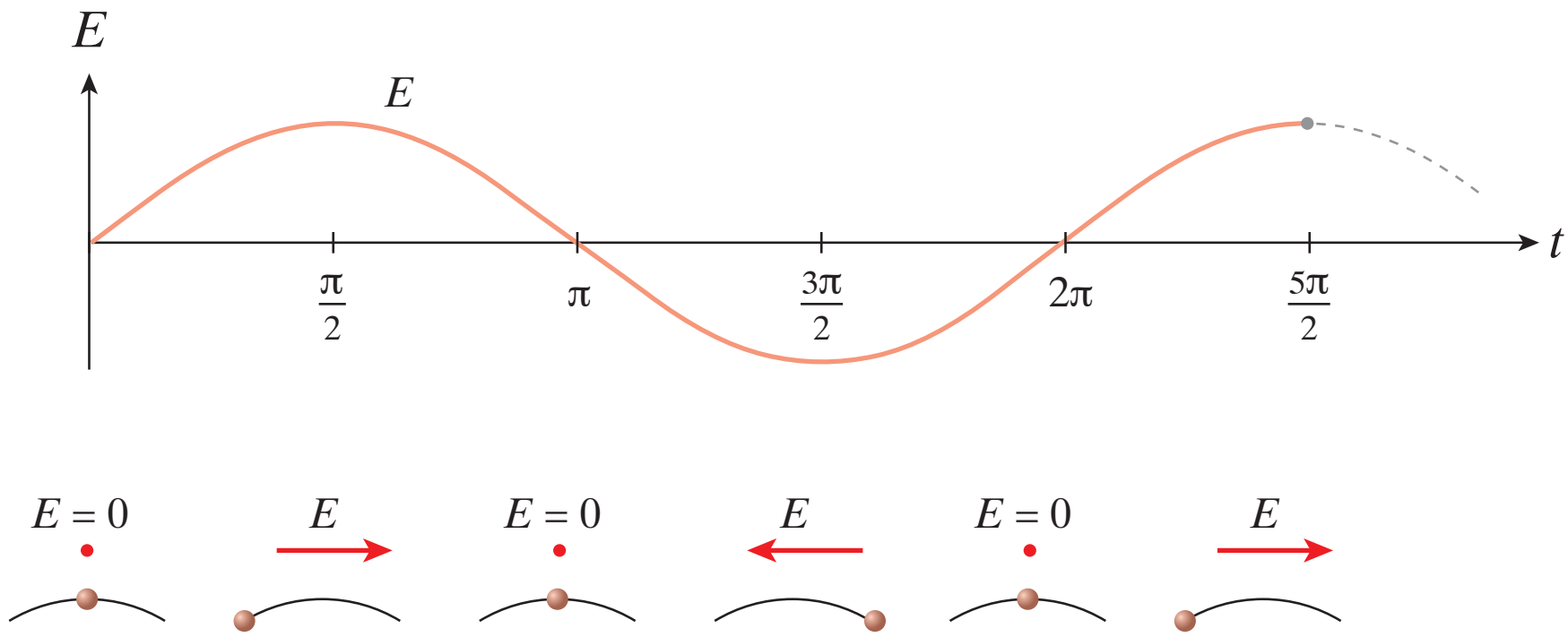
Nonlinear optics

Nonlinear response: $P^{(2)} = \chi^{(2)} E^2$



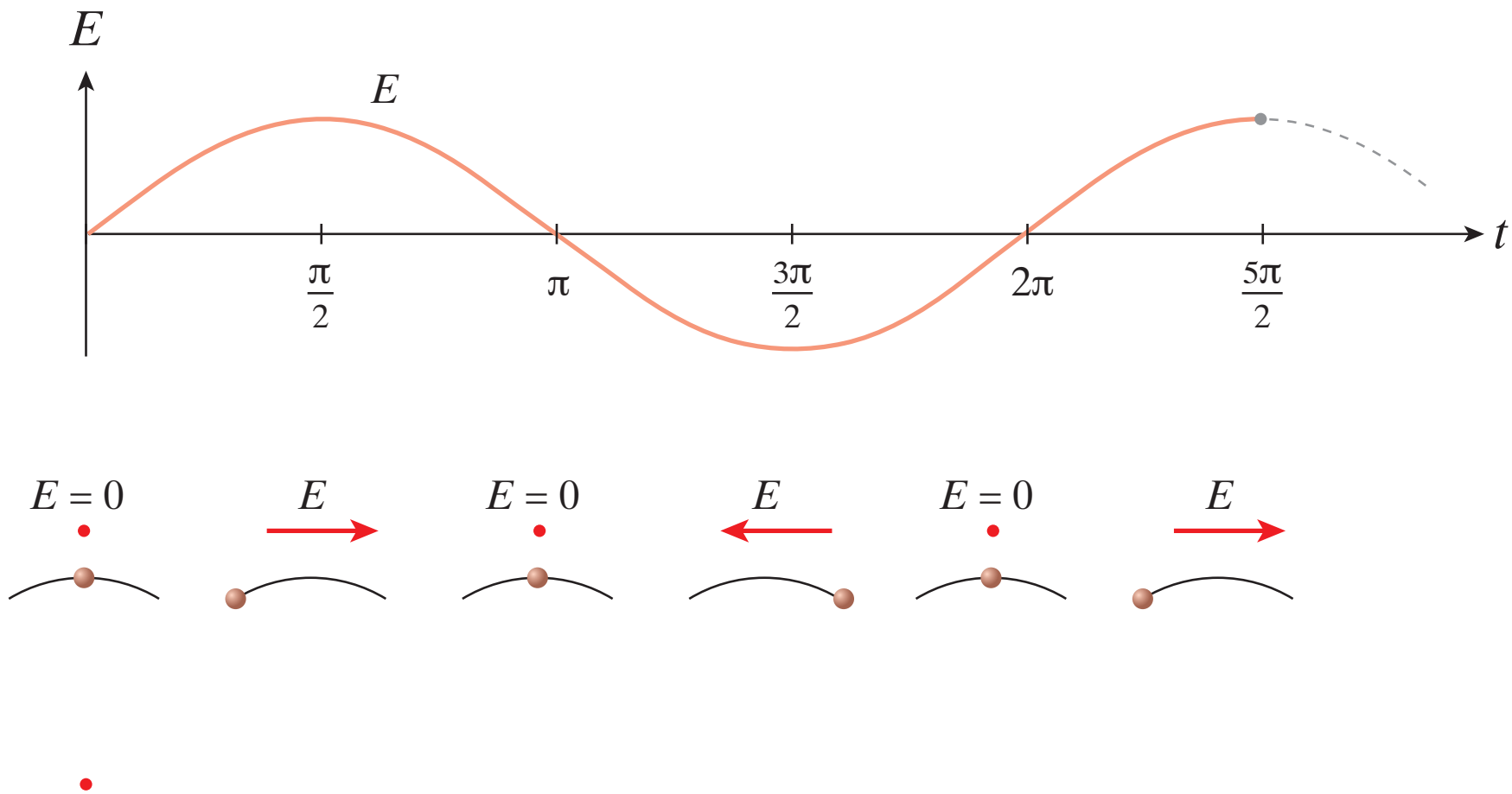
Nonlinear optics

Nonlinear response: $P^{(2)} = \chi^{(2)} E^2$



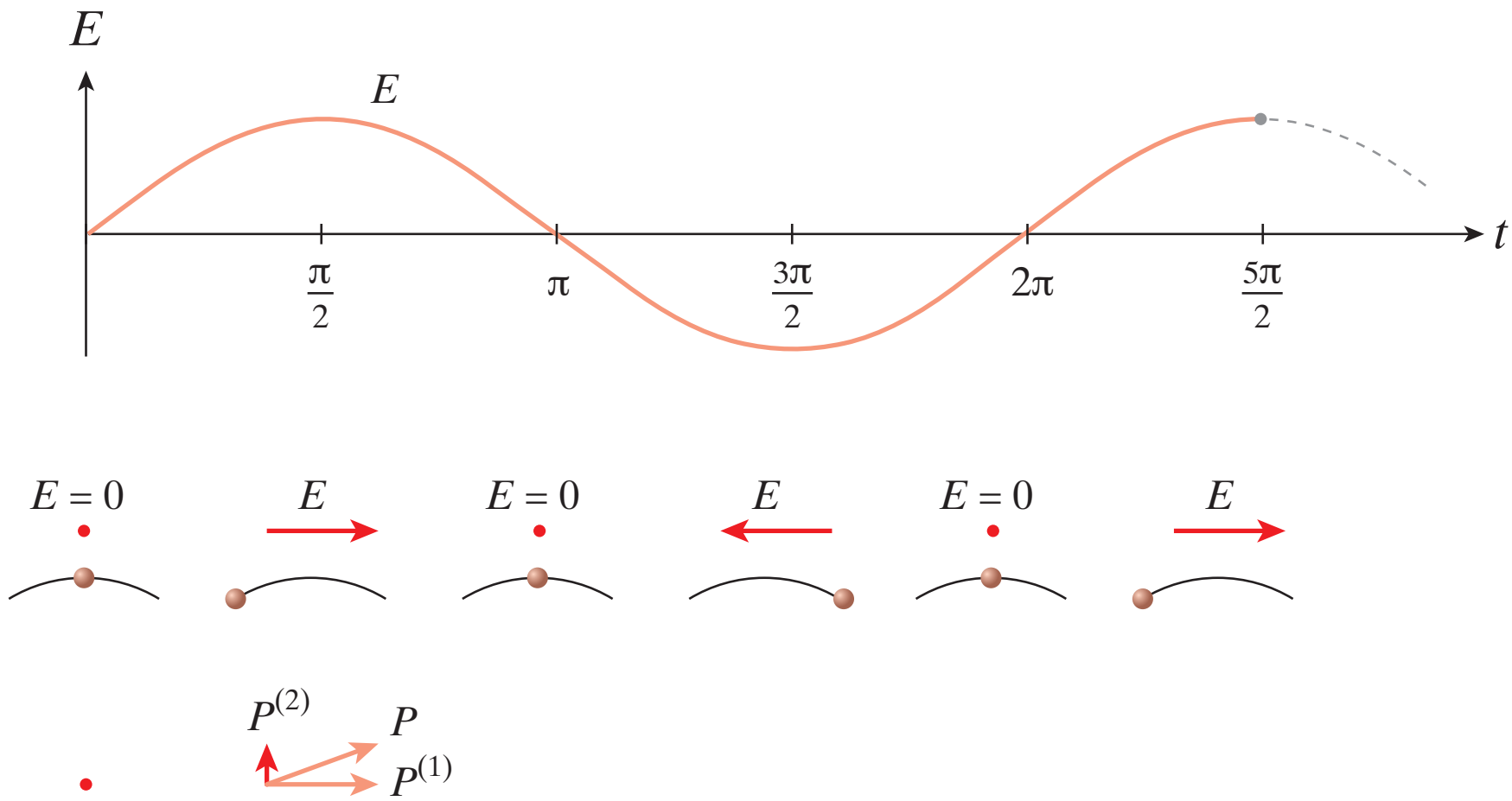
Nonlinear optics

Nonlinear response: $P^{(2)} = \chi^{(2)} E^2$



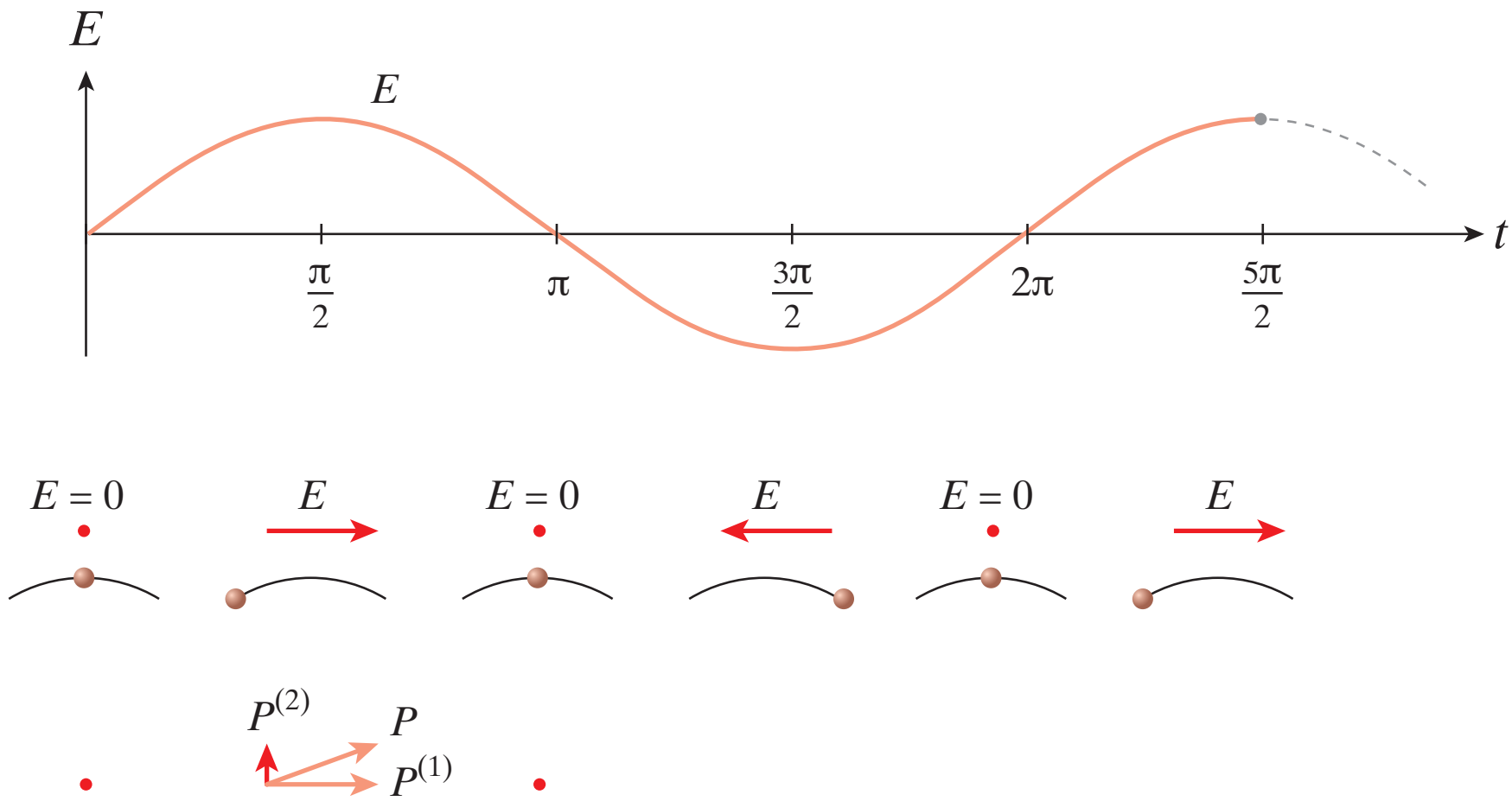
Nonlinear optics

Nonlinear response: $P^{(2)} = \chi^{(2)} E^2$



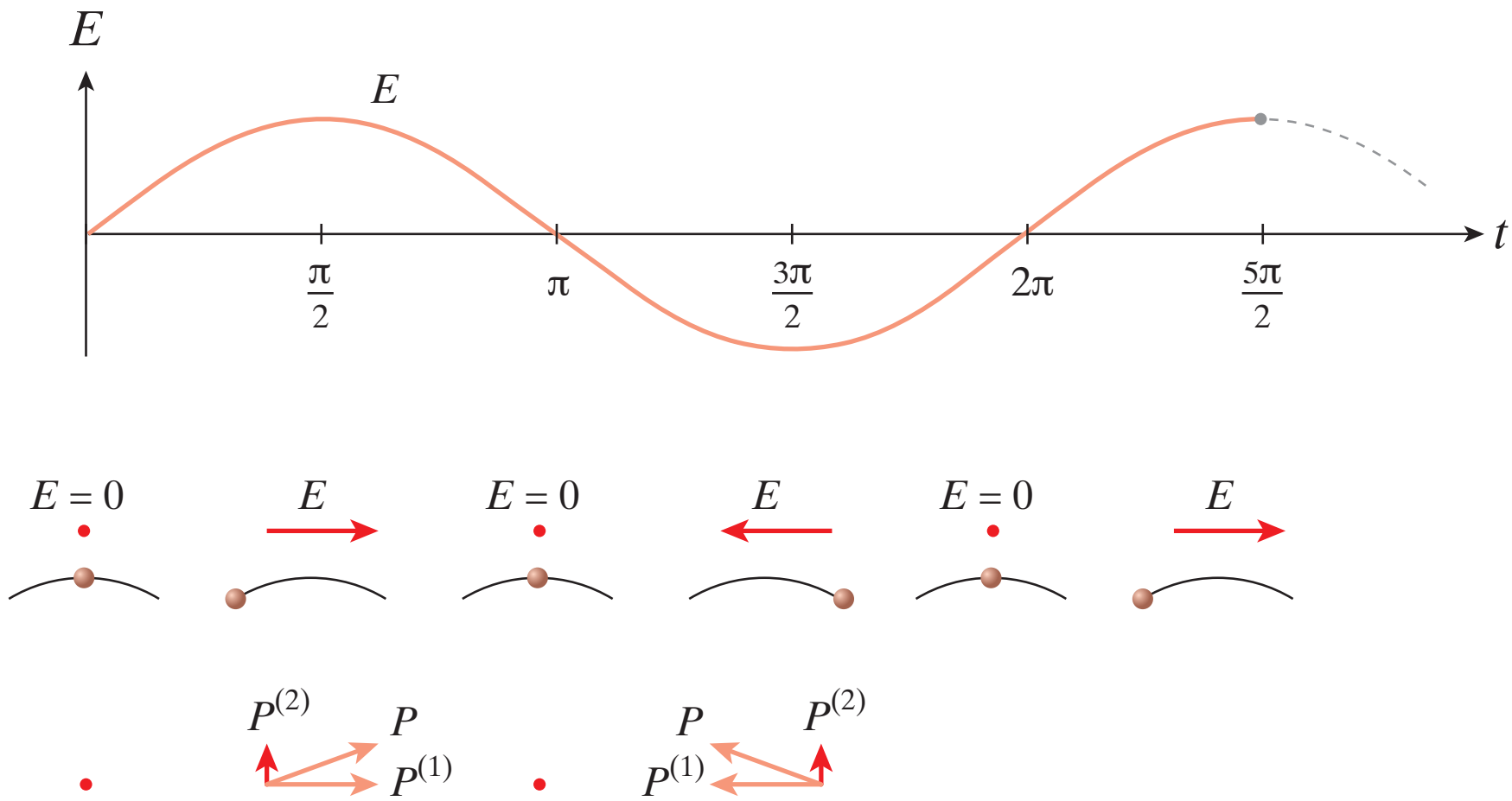
Nonlinear optics

Nonlinear response: $P^{(2)} = \chi^{(2)} E^2$



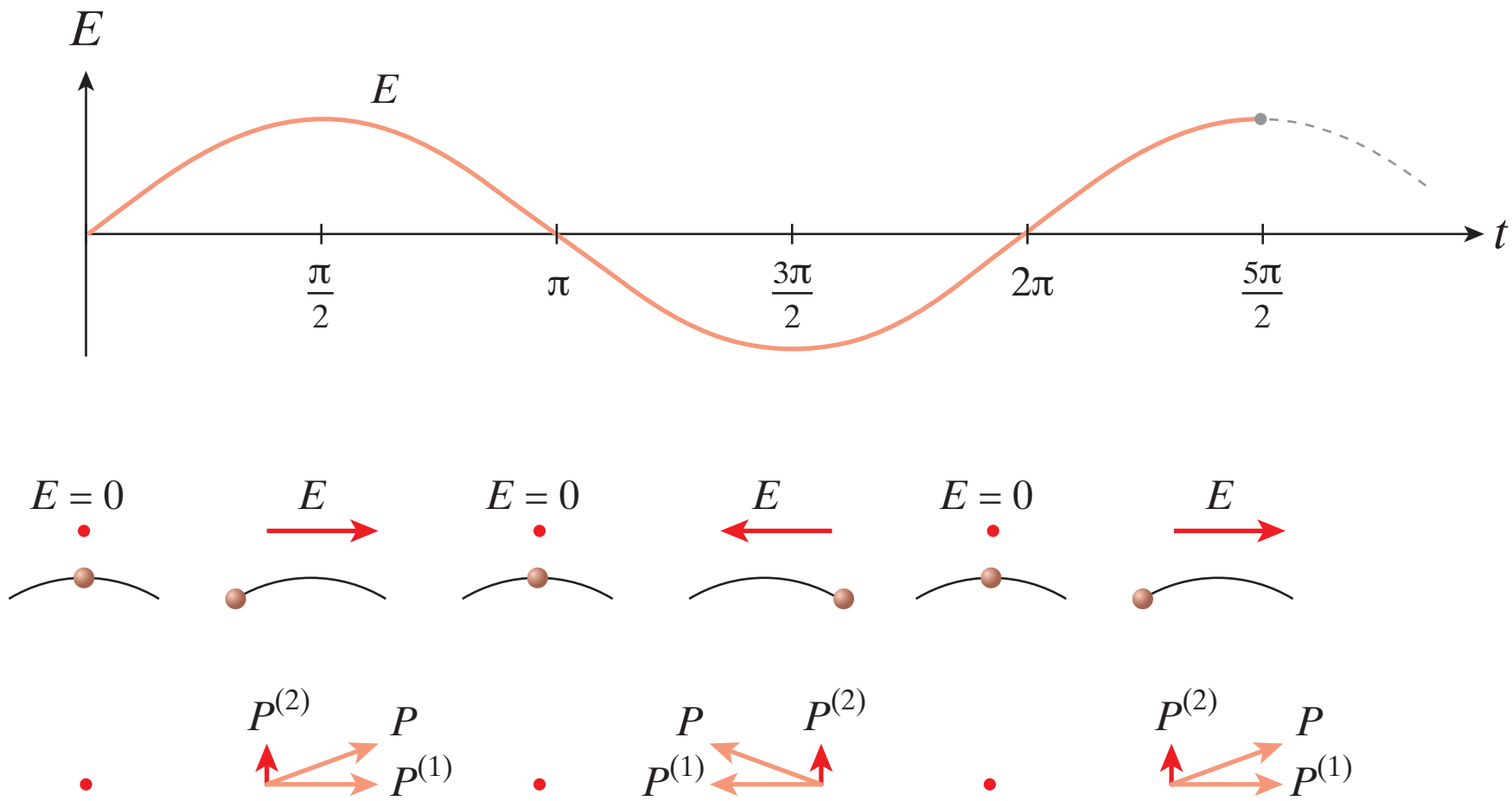
Nonlinear optics

Nonlinear response: $P^{(2)} = \chi^{(2)} E^2$



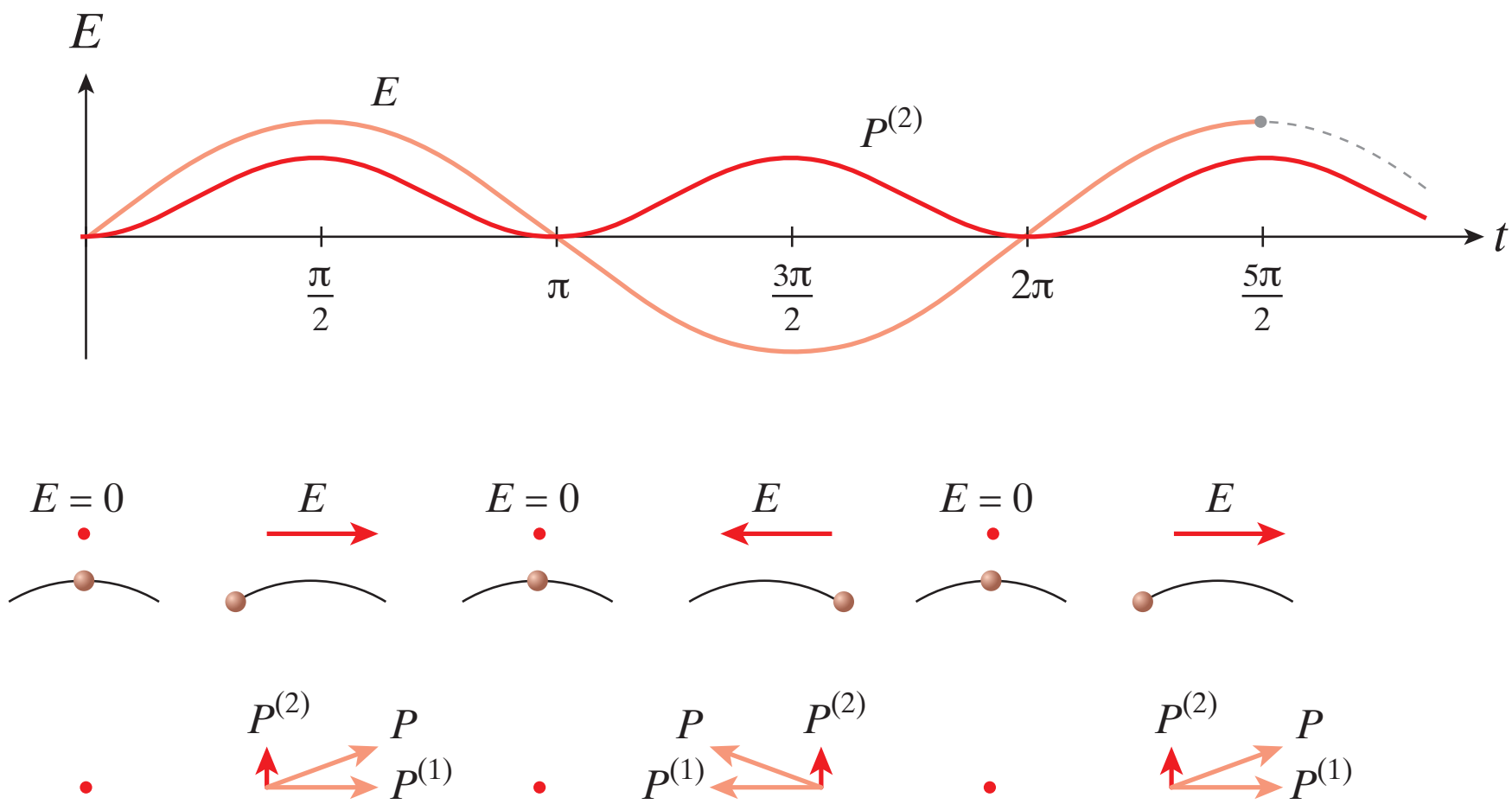
Nonlinear optics

Nonlinear response: $P^{(2)} = \chi^{(2)} E^2$



Nonlinear optics

Nonlinear response: $P^{(2)} = \chi^{(2)} E^2$



Nonlinear optics

Nonlinear response: $P^{(2)} = \chi^{(2)} E^2$

Q: Silicon atoms are arranged in this way. Does bulk silicon generate second harmonic?

1. Yes, silicon is not centrosymmetric (as the unit cell shows)
2. No, the crystal as a whole is centrosymmetric
3. No, any radiation at the second harmonic is absorbed
4. Other

Nonlinear optics

Nonlinear response: $P^{(2)} = \chi^{(2)} E^2$

Q: Silicon atoms are arranged in this way. Does bulk silicon generate second harmonic?

1. Yes, silicon is not centrosymmetric (as the unit cell shows)
2. **No, the crystal as a whole is centrosymmetric** ✔
3. No, any radiation at the second harmonic is absorbed
4. Other

Nonlinear optics

Nonlinear optics

Third-order polarization: $P^{(3)}(t) = \chi^{(3)} E^3(t)$

Nonlinear optics

Third-order polarization: $P^{(3)}(t) = \chi^{(3)} E^3(t)$

3 frequencies, 3 terms + c.c.: complicated! In general

$$\cos^3 \omega t = \tfrac{1}{4} \cos 3\omega t + \tfrac{3}{4} \cos \omega t$$

Nonlinear optics

Third-order polarization: $P^{(3)}(t) = \chi^{(3)} E^3(t)$

3 frequencies, 3 terms + c.c.: complicated! In general

$$\cos^3 \omega t = \tfrac{1}{4} \cos 3\omega t + \tfrac{3}{4} \cos \omega t$$

Intensity dependent term at fundamental frequency:

$$P^{(3)}(t) = \chi^{(3)} E(t) E^*(t) E(t) = \chi^{(3)} I(t) E(t)$$

Nonlinear optics

Third-order polarization: $P^{(3)}(t) = \chi^{(3)} E^3(t)$

3 frequencies, 3 terms + c.c.: complicated! In general

$$\cos^3 \omega t = \tfrac{1}{4} \cos 3\omega t + \tfrac{3}{4} \cos \omega t$$

Intensity dependent term at fundamental frequency:

$$P^{(3)}(t) = \chi^{(3)} E(t) E^*(t) E(t) = \chi^{(3)} I(t) E(t)$$

and so $$P = P^{(1)} + P^{(3)} = (\chi^{(1)} + \chi^{(3)} I) E \equiv \chi_{eff} E$$

Nonlinear optics

Third-order polarization: $P^{(3)}(t) = \chi^{(3)} E^3(t)$

3 frequencies, 3 terms + c.c.: complicated! In general

$$\cos^3 \omega t = \tfrac{1}{4} \cos 3\omega t + \tfrac{3}{4} \cos \omega t$$

Intensity dependent term at fundamental frequency:

$$P^{(3)}(t) = \chi^{(3)} E(t) E^*(t) E(t) = \chi^{(3)} I(t) E(t)$$

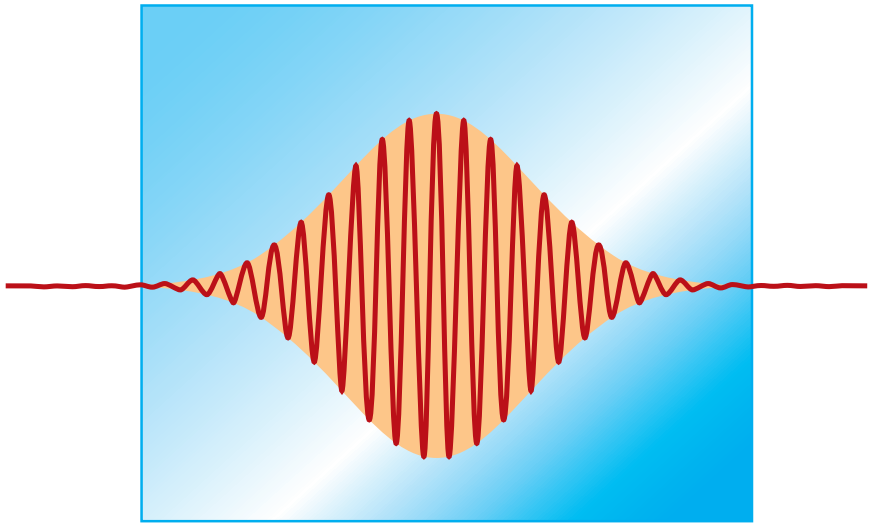
and so $$P = P^{(1)} + P^{(3)} = (\chi^{(1)} + \chi^{(3)} I) E \equiv \chi_{eff} E$$

$$n = \sqrt{\epsilon} = \sqrt{1 + \chi_{eff}} \approx \sqrt{1 + \chi^{(1)}} + \tfrac{1}{2} \frac{\chi^{(3)} I}{\sqrt{1 + \chi^{(1)}}} = n_o + n_2 I$$

Nonlinear optics

Intensity-dependent index of refraction:

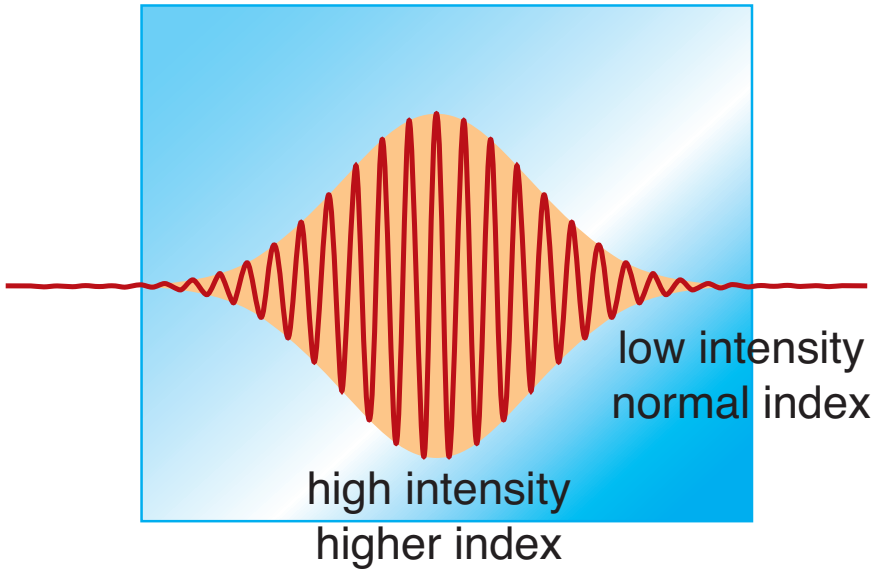
$$n = n_o + n_2 I$$



Nonlinear optics

Intensity-dependent index of refraction:

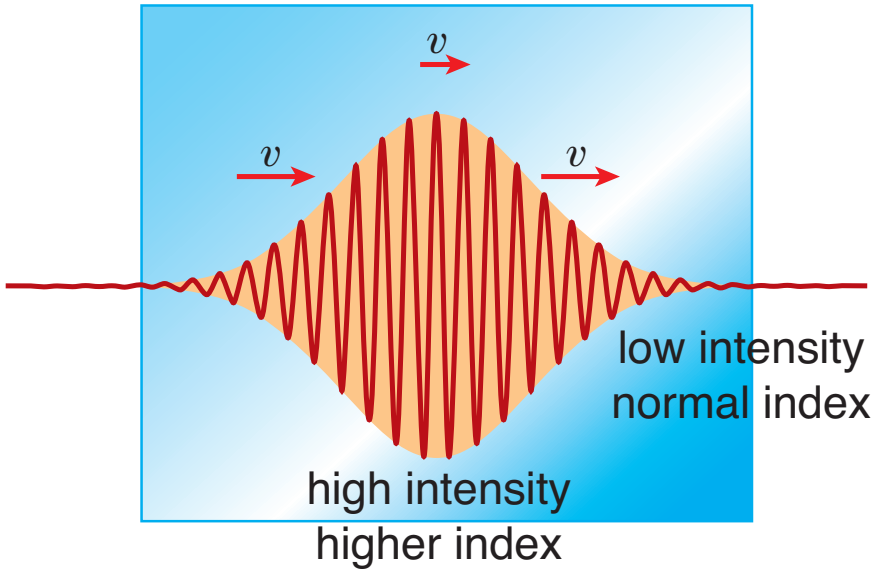
$$n = n_o + n_2 I$$



Nonlinear optics

Intensity-dependent index of refraction:

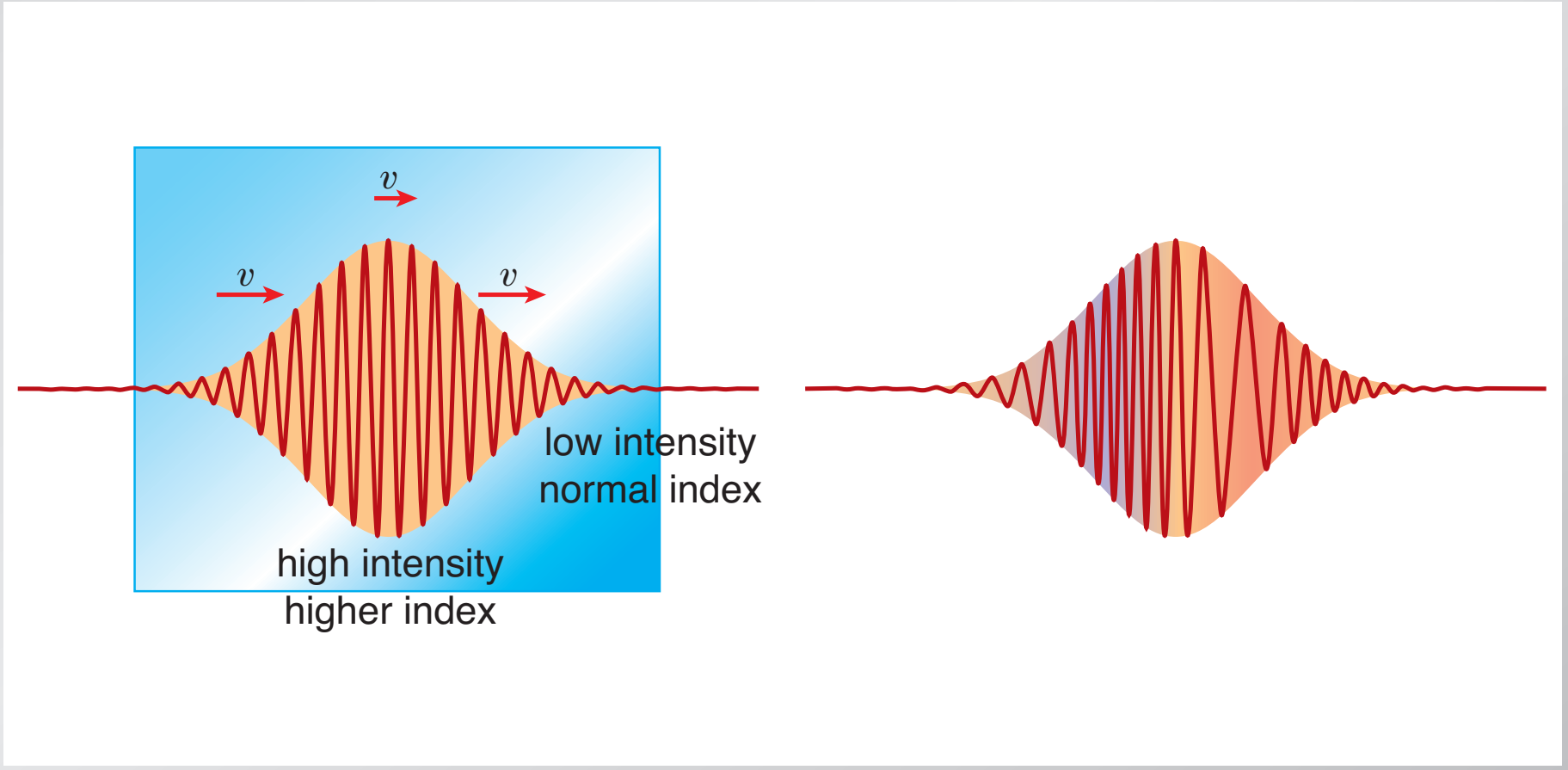
$$n = n_o + n_2 I$$



Nonlinear optics

Intensity-dependent index of refraction:

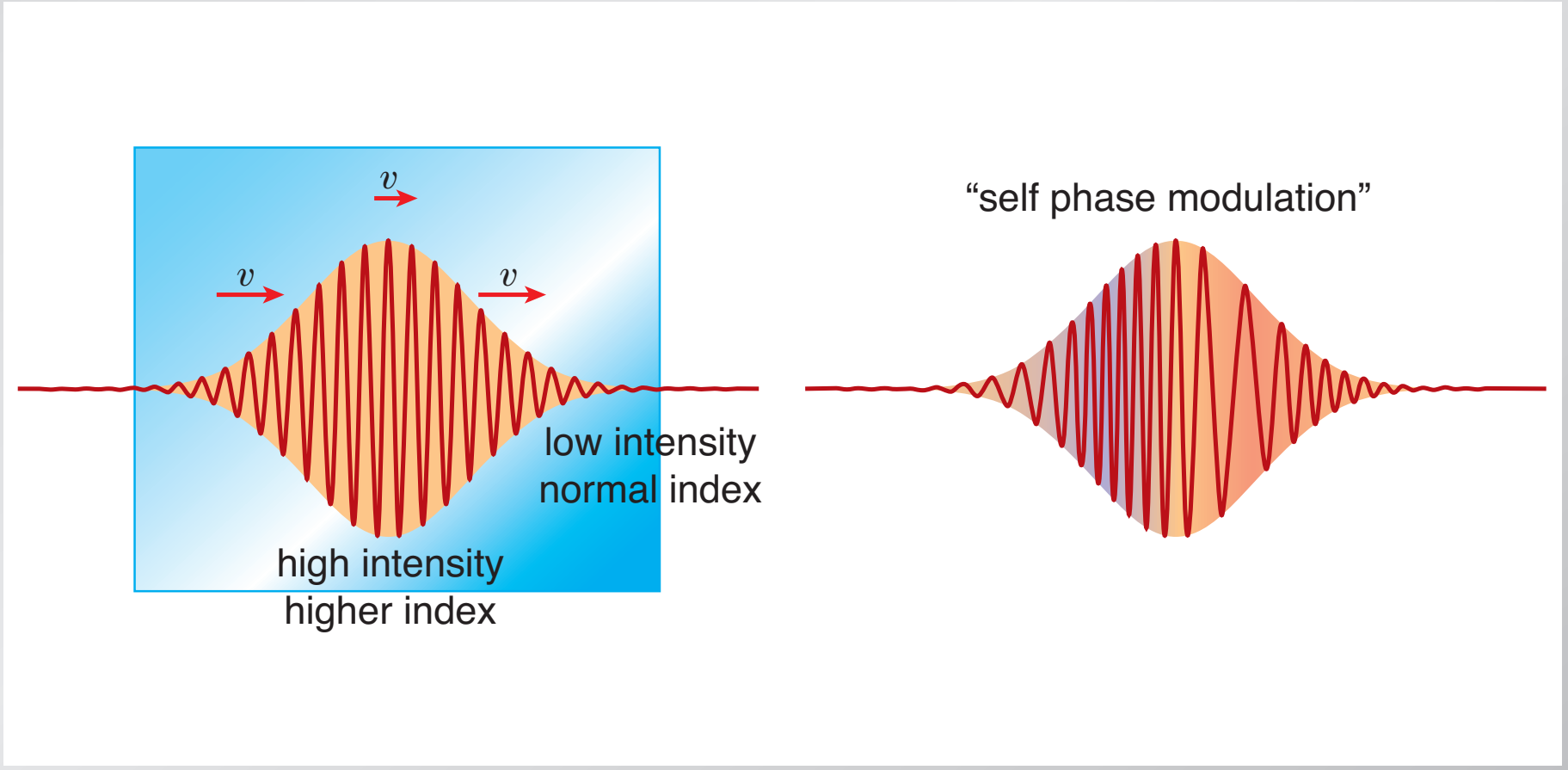
$$n = n_o + n_2 I$$



Nonlinear optics

Intensity-dependent index of refraction:

$$n = n_o + n_2 I$$



Nonlinear optics

Phase:

$$\frac{\phi}{2\pi} = \frac{nL}{\lambda}$$

Nonlinear optics

Phase:

$$\frac{\phi}{2\pi} = \frac{nL}{\lambda}$$

$$\phi = \frac{2\pi}{\lambda} L(n_o + n_2 I)$$

Nonlinear optics

Phase:

$$\frac{\phi}{2\pi} = \frac{nL}{\lambda} \qquad \phi = \frac{2\pi}{\lambda} L(n_o + n_2 I)$$

Frequency change:

$$\Delta\omega = -\frac{d\phi}{dt}$$

Nonlinear optics

Phase: $$\frac{\phi}{2\pi} = \frac{nL}{\lambda} \qquad \phi = \frac{2\pi}{\lambda} L(n_o + n_2 I)$$

Frequency change: $$\Delta\omega = -\frac{d\phi}{dt}$$

Q: Sketch the time dependence of the frequency shift for a Gaussian pulse and determine which is true (assume $n_2 > 0$):

1. Leading edge is blue shifted, trailing edge red shifted
2. Leading and trailing edge blue shifted, center red shifted
3. Leading edge is red shifted, trailing edge blue shifted
4. Leading and trailing edge red shifted, center blue shifted
5. Other

Nonlinear optics

Phase: $$\frac{\phi}{2\pi} = \frac{nL}{\lambda} \qquad \phi = \frac{2\pi}{\lambda} L(n_o + n_2 I)$$

Frequency change: $$\Delta\omega = -\frac{d\phi}{dt}$$

Q: Sketch the time dependence of the frequency shift for a Gaussian pulse and determine which is true (assume $n_2 > 0$):

1. Leading edge is blue shifted, trailing edge red shifted
2. Leading and trailing edge blue shifted, center red shifted
3. **Leading edge is red shifted, trailing edge blue shifted** ✔
4. Leading and trailing edge red shifted, center blue shifted
5. Other

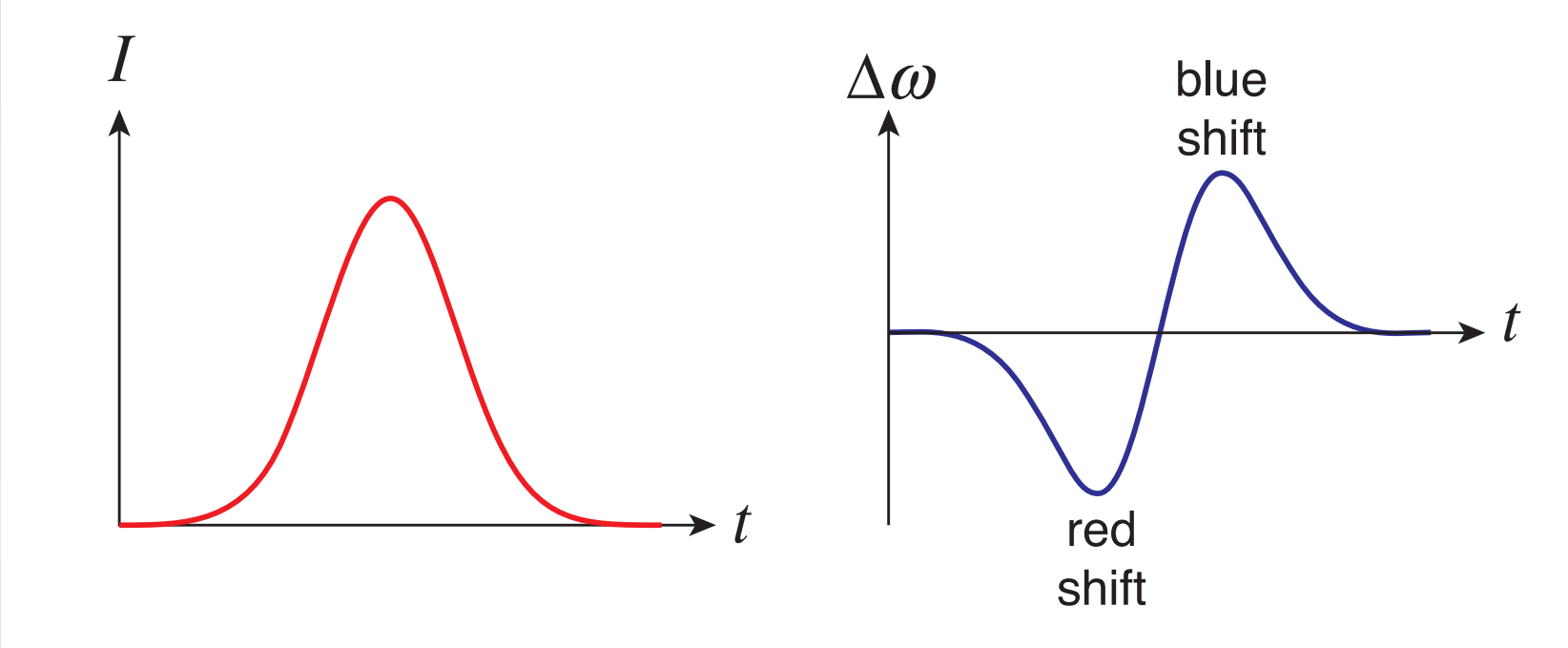
Nonlinear optics

Phase:

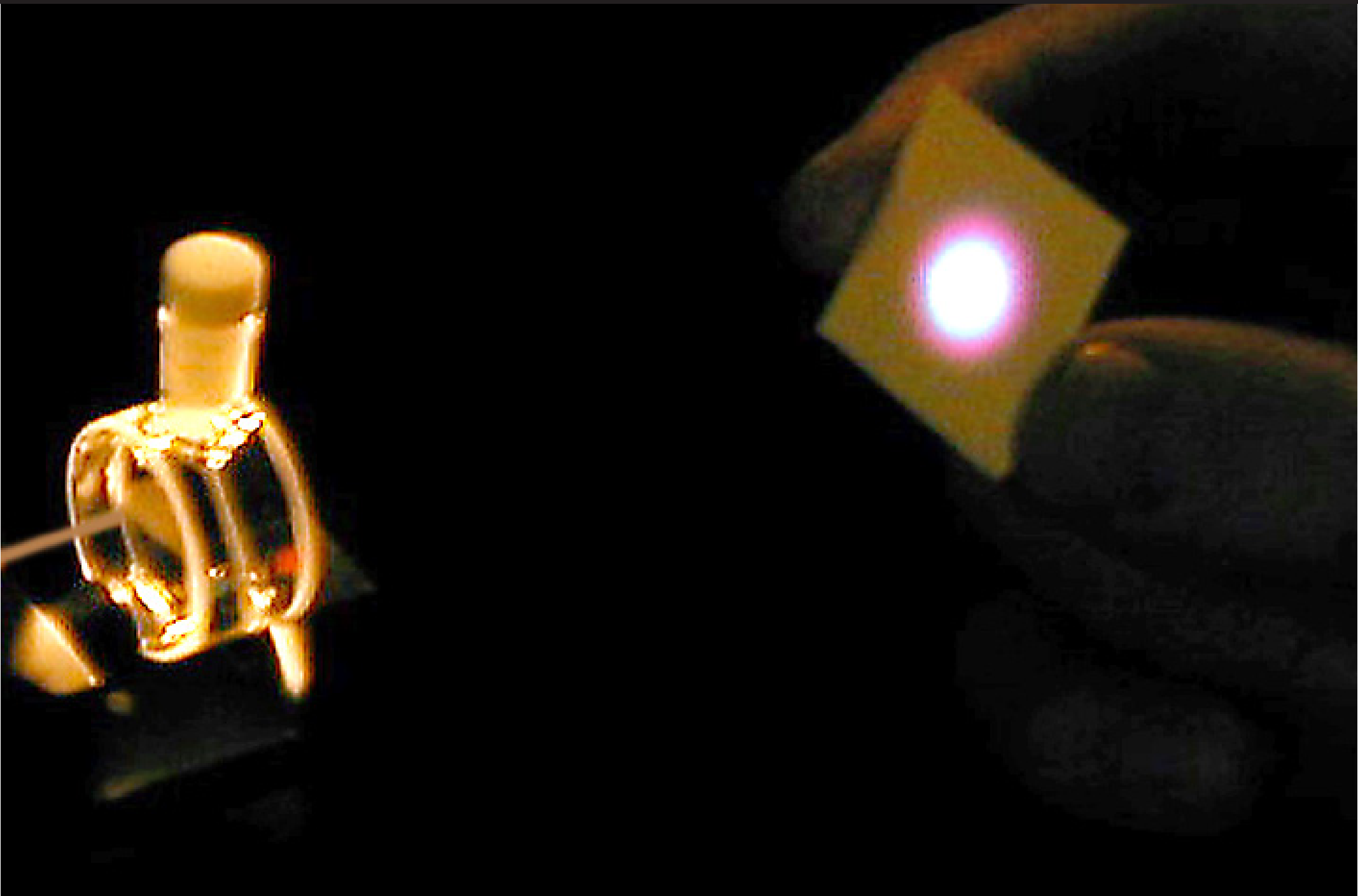
$$\frac{\phi}{2\pi} = \frac{nL}{\lambda} \qquad \phi = \frac{2\pi}{\lambda} L(n_o + n_2 I)$$

Frequency change:

$$\Delta\omega = -\frac{d\phi}{dt} = \frac{-2\pi}{\lambda} L n_2 \frac{dI}{dt}$$



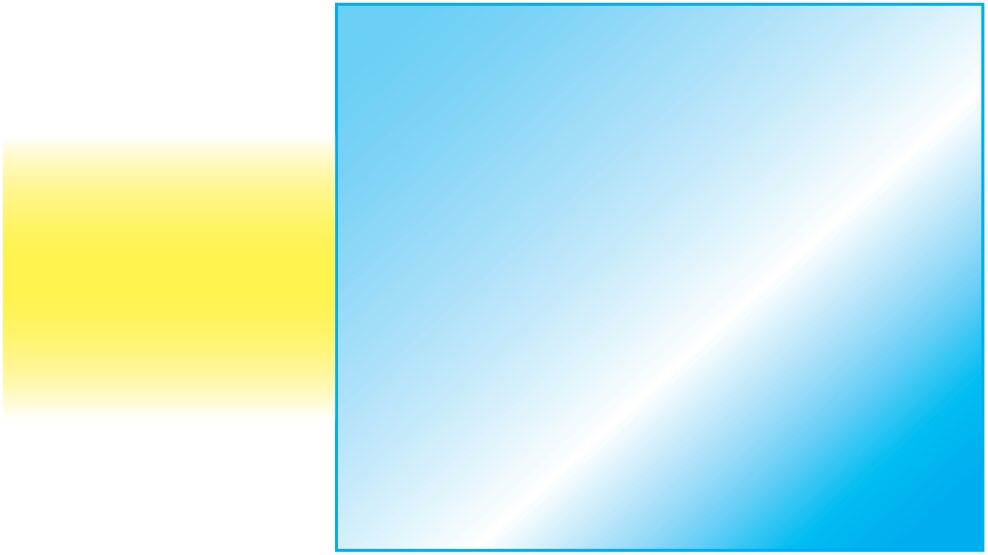
Nonlinear optics



Nonlinear optics

Intensity-dependent index of refraction:

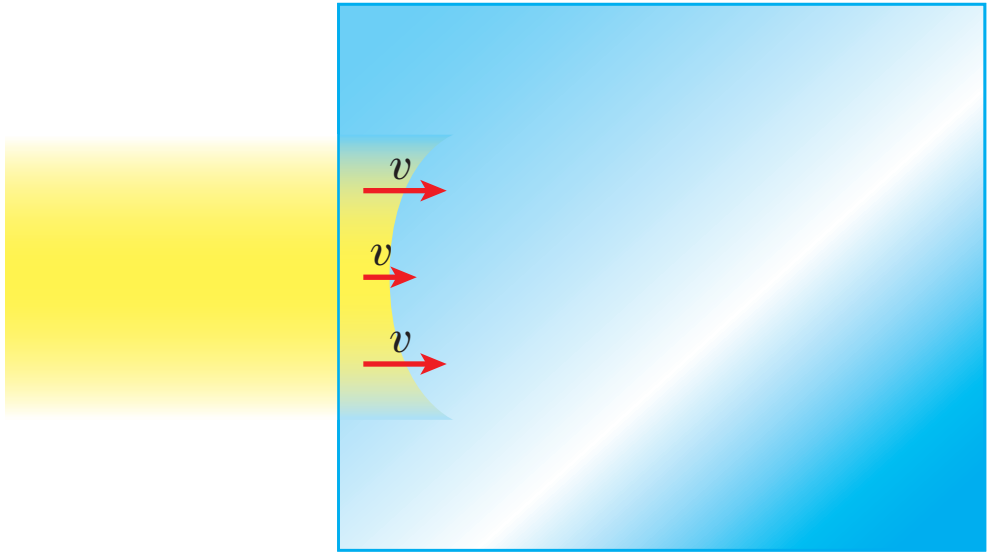
$$n = n_o + n_2 I$$



Nonlinear optics

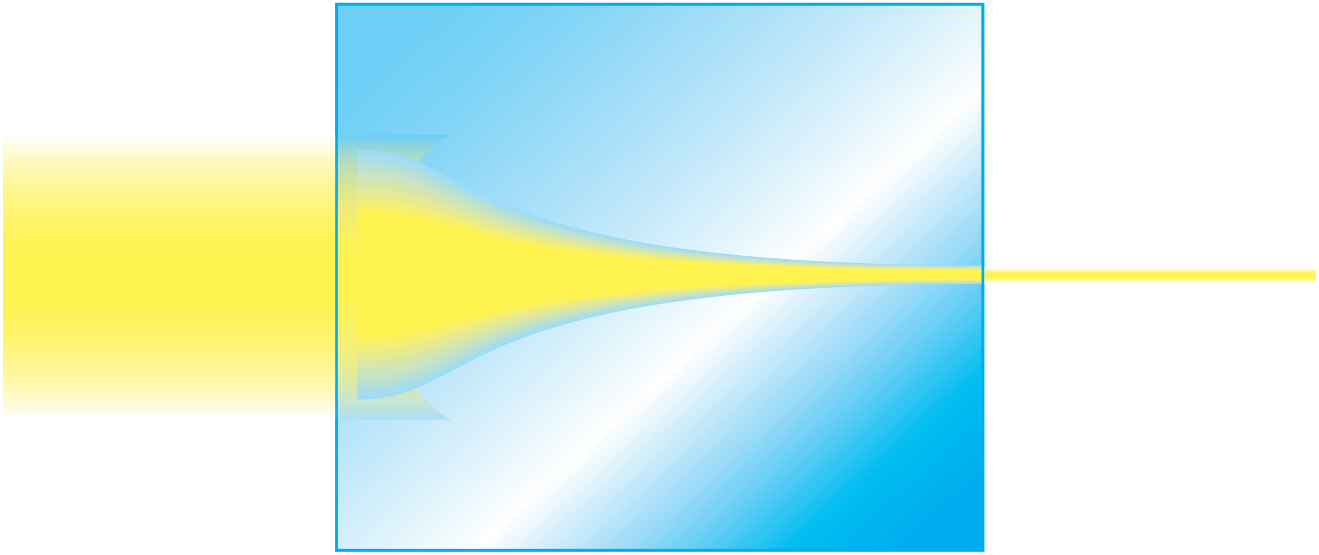
Intensity-dependent index of refraction:

$$n = n_o + n_2 I$$



Nonlinear optics

self-focusing



Nonlinear optics

but susceptibility is complex!

susceptibility	real part	imaginary part
linear	refraction	absorption
nonlinear	SHG, SFG, DFG, THG,…	multiphoton absorption

$$\alpha = \alpha_o + \beta I + \gamma I^2 + \ldots$$

Nonlinear optics

Key points

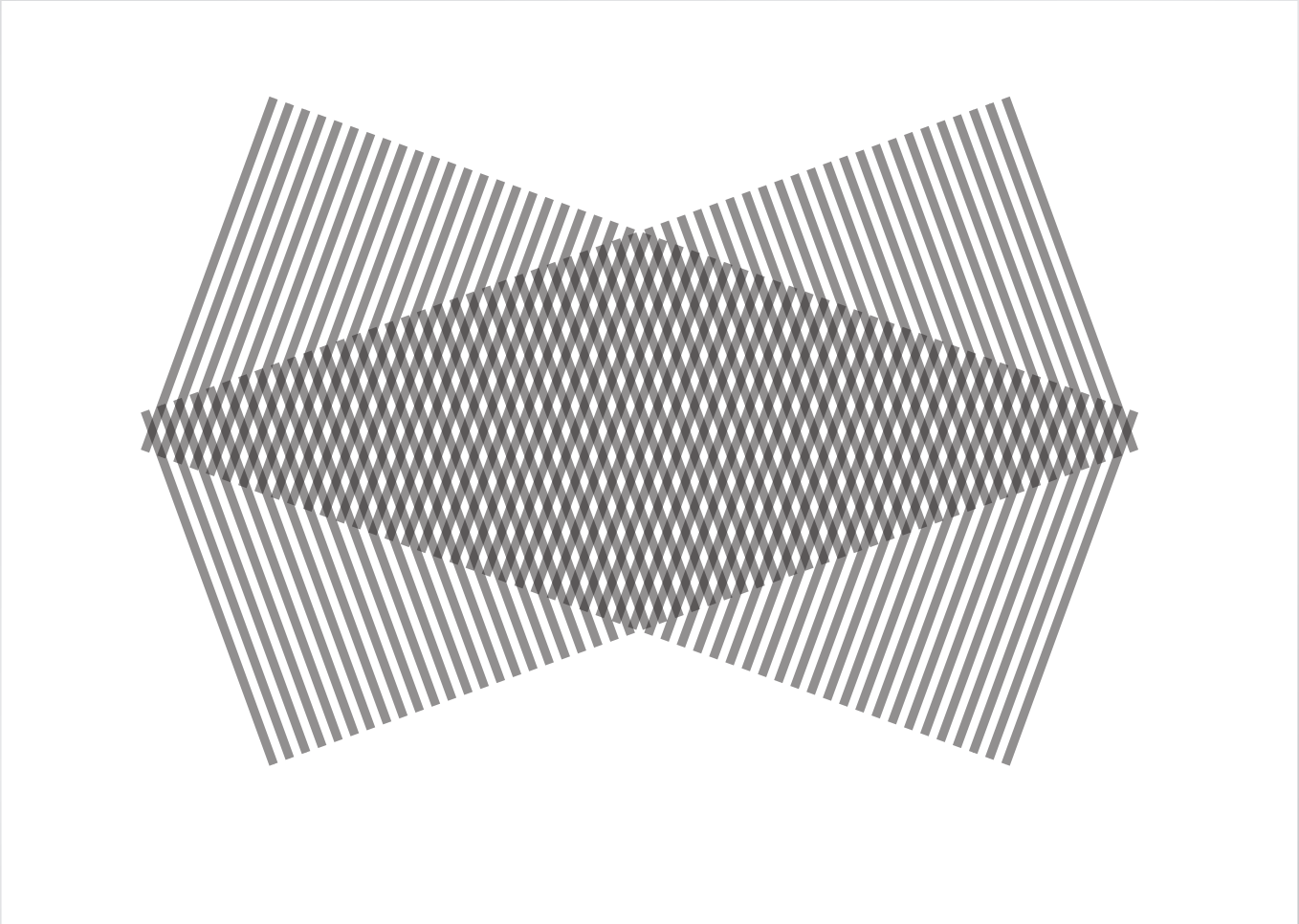
- at high intensities, polarization no longer proportional to E
- nonlinearity can produce radiation at new frequencies
- nonlinearity causes index to depend on instensity of pulse

Outline

- optical properties of materials
- dispersion of pulses
- nonlinear optics
- **waveguiding**
- engineering the index

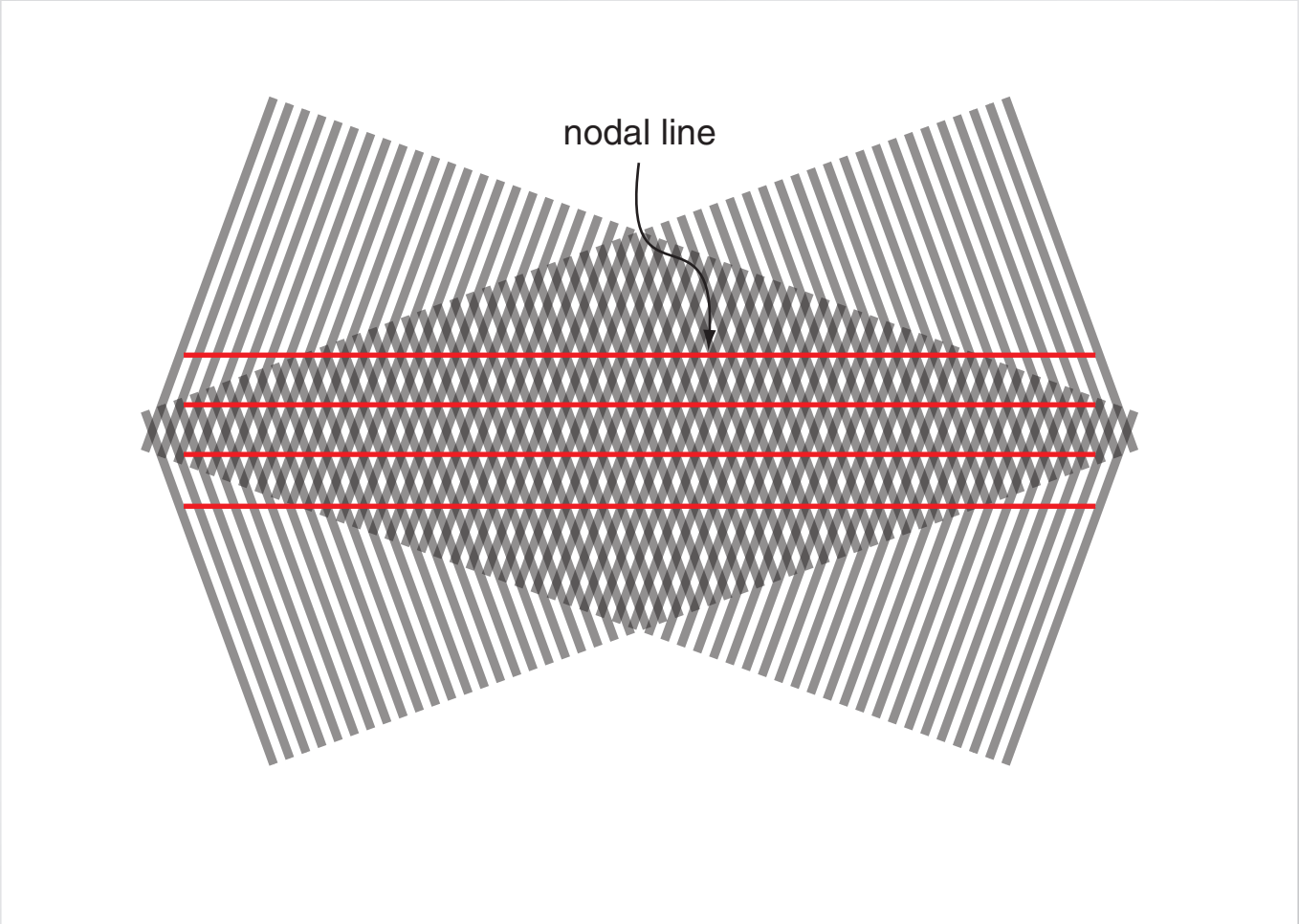
Waveguiding

two crossed planar waves…



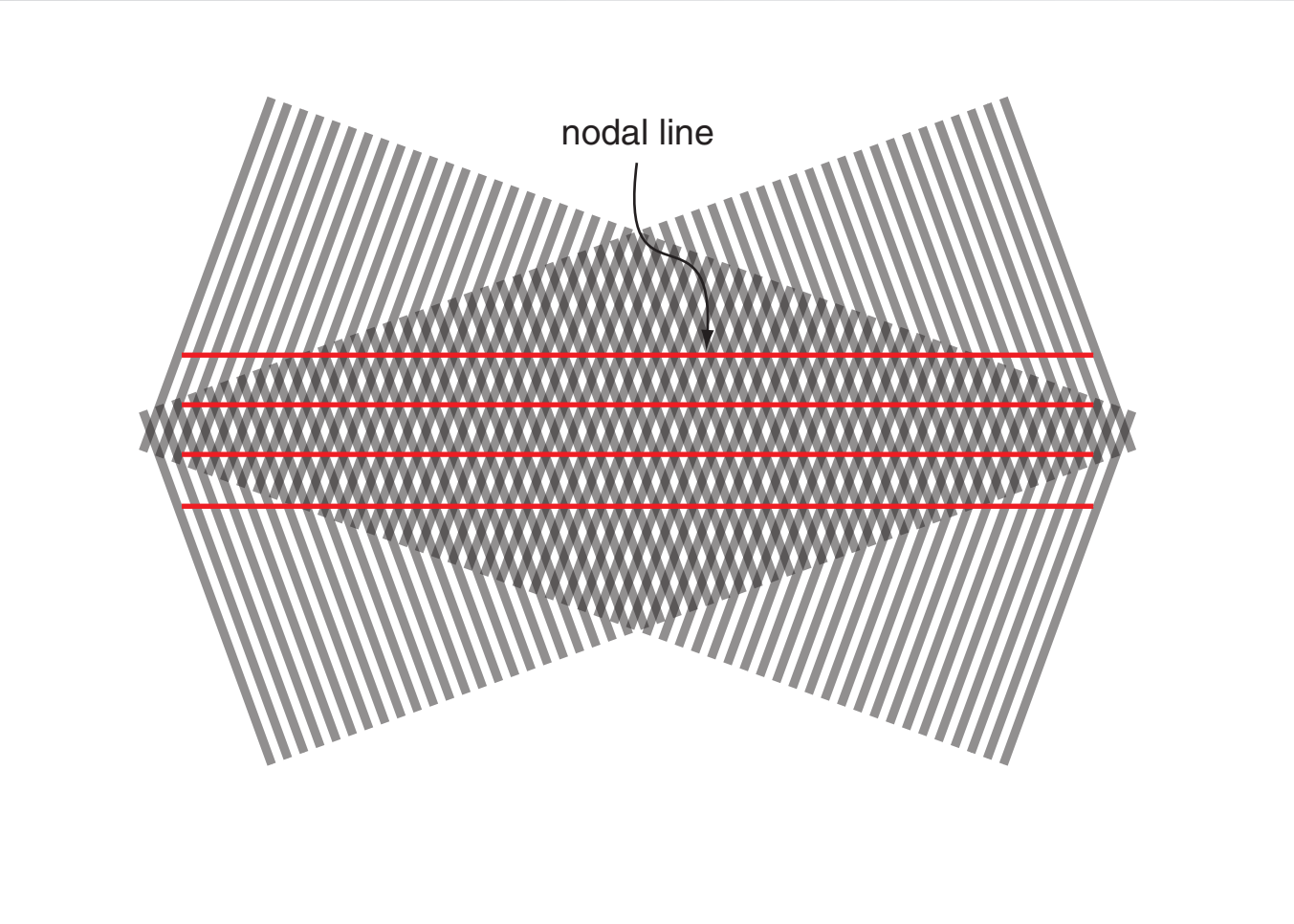
Waveguiding

…cause an interference pattern



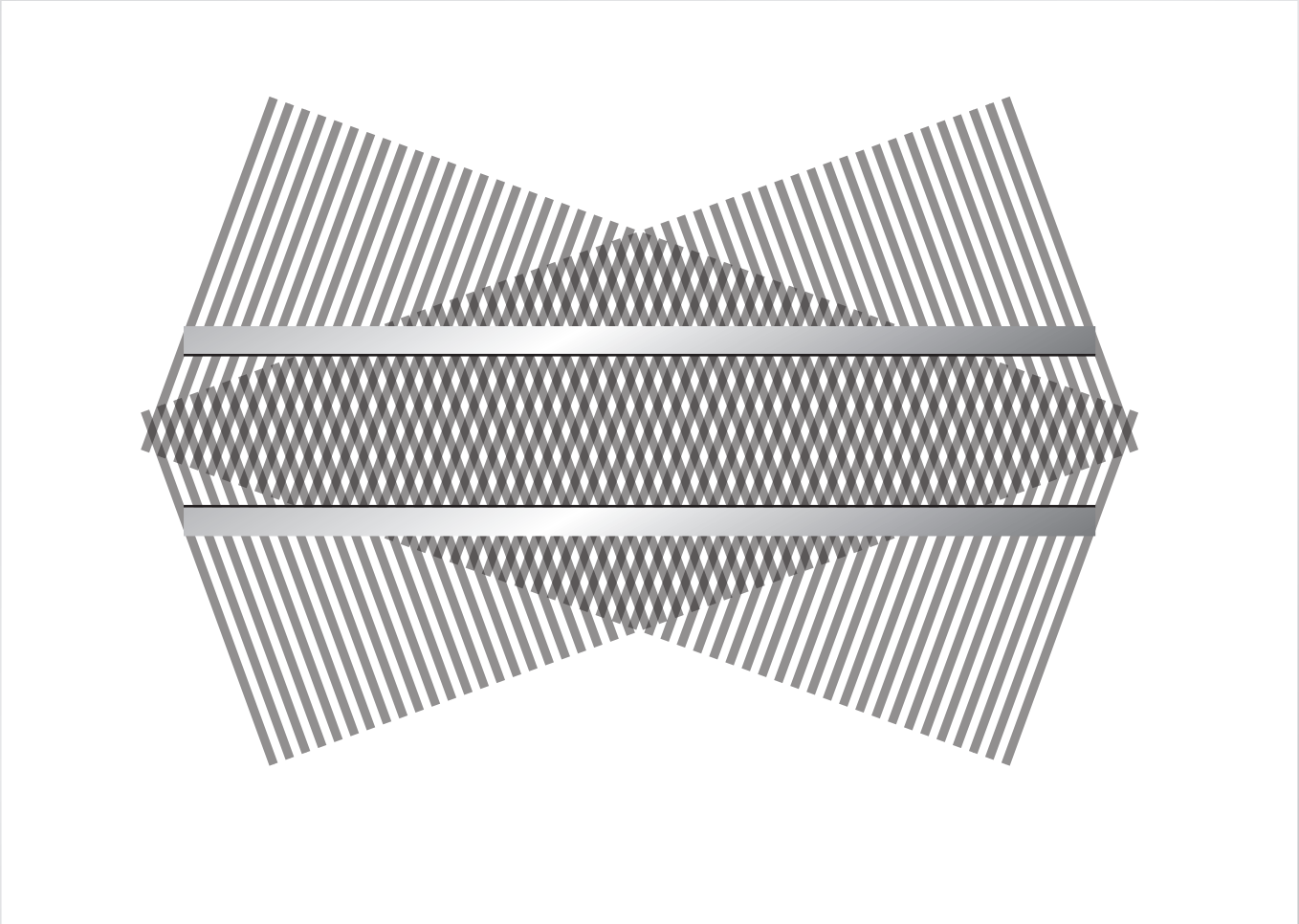
Waveguiding

$E = 0$ **on the nodal lines**



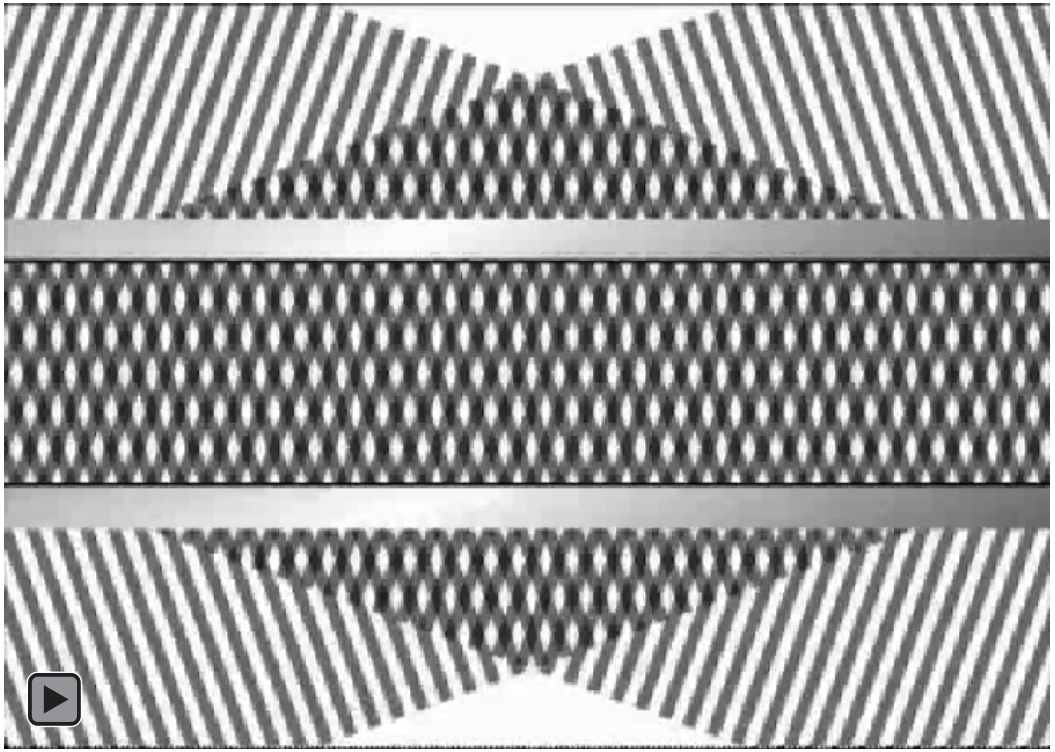
Waveguiding

…satisfying boundary conditions for planar-mirror waveguide



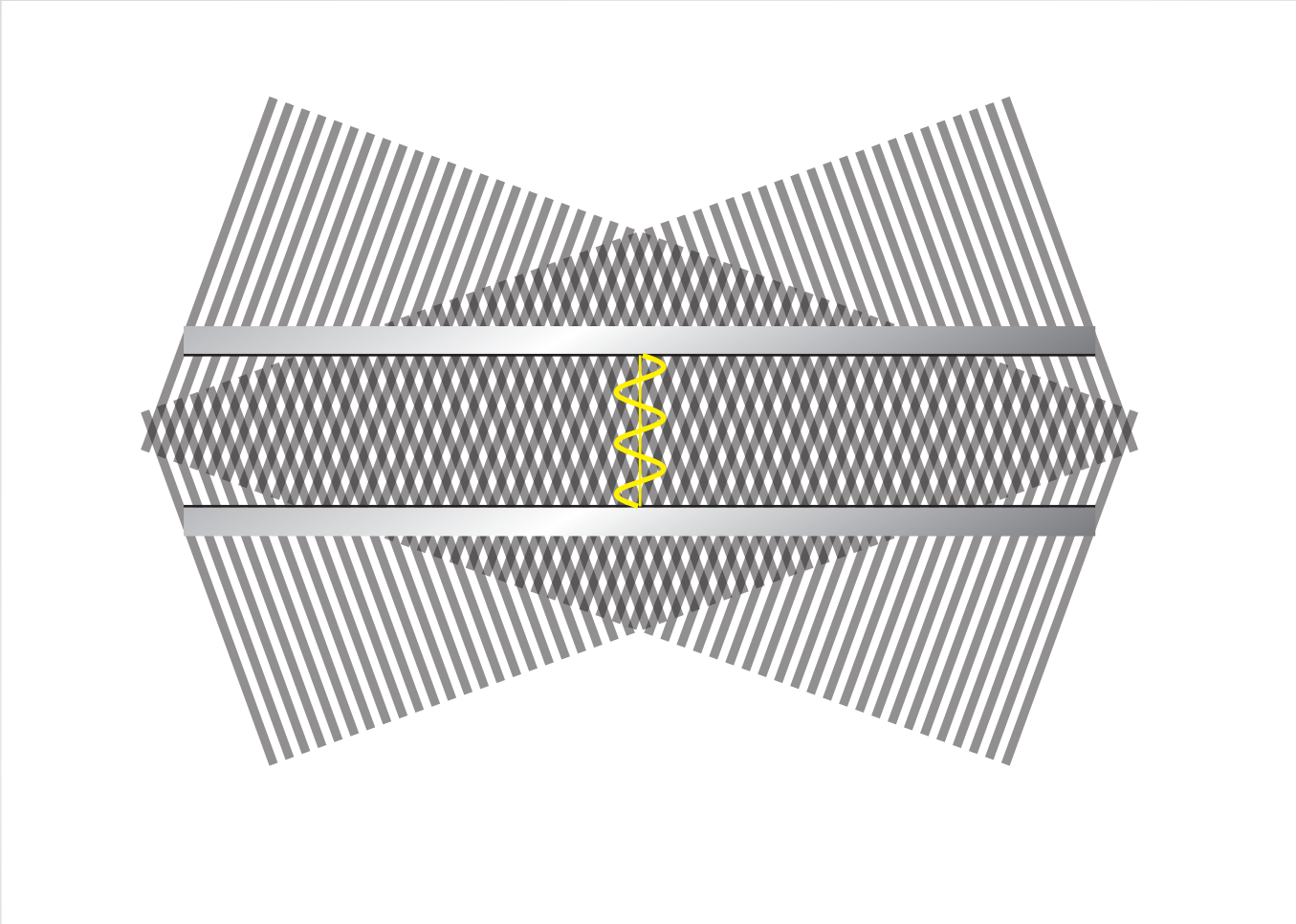
Waveguiding

transverse standing wave, traveling along axis



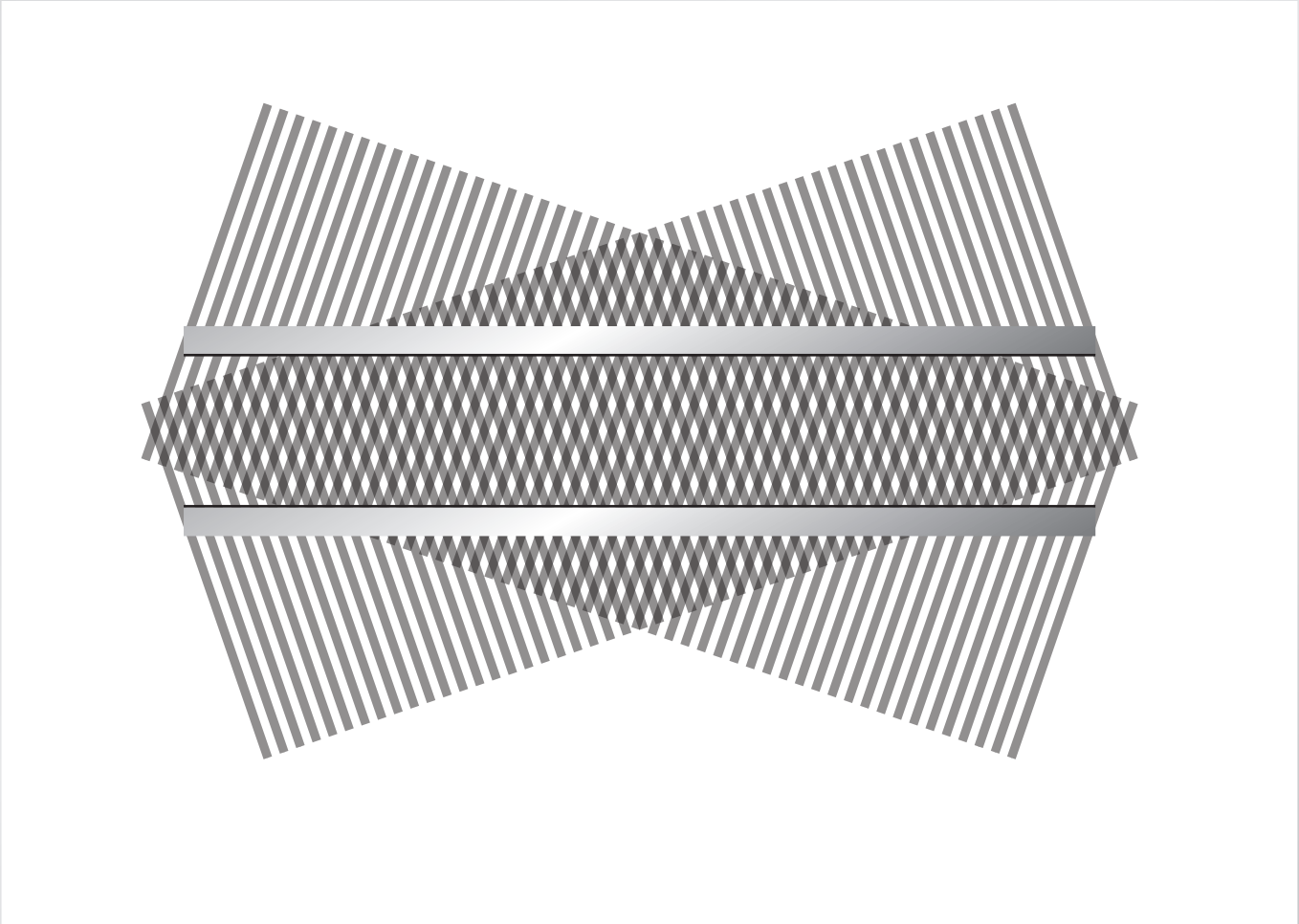
Waveguiding

transverse standing wave, traveling along axis



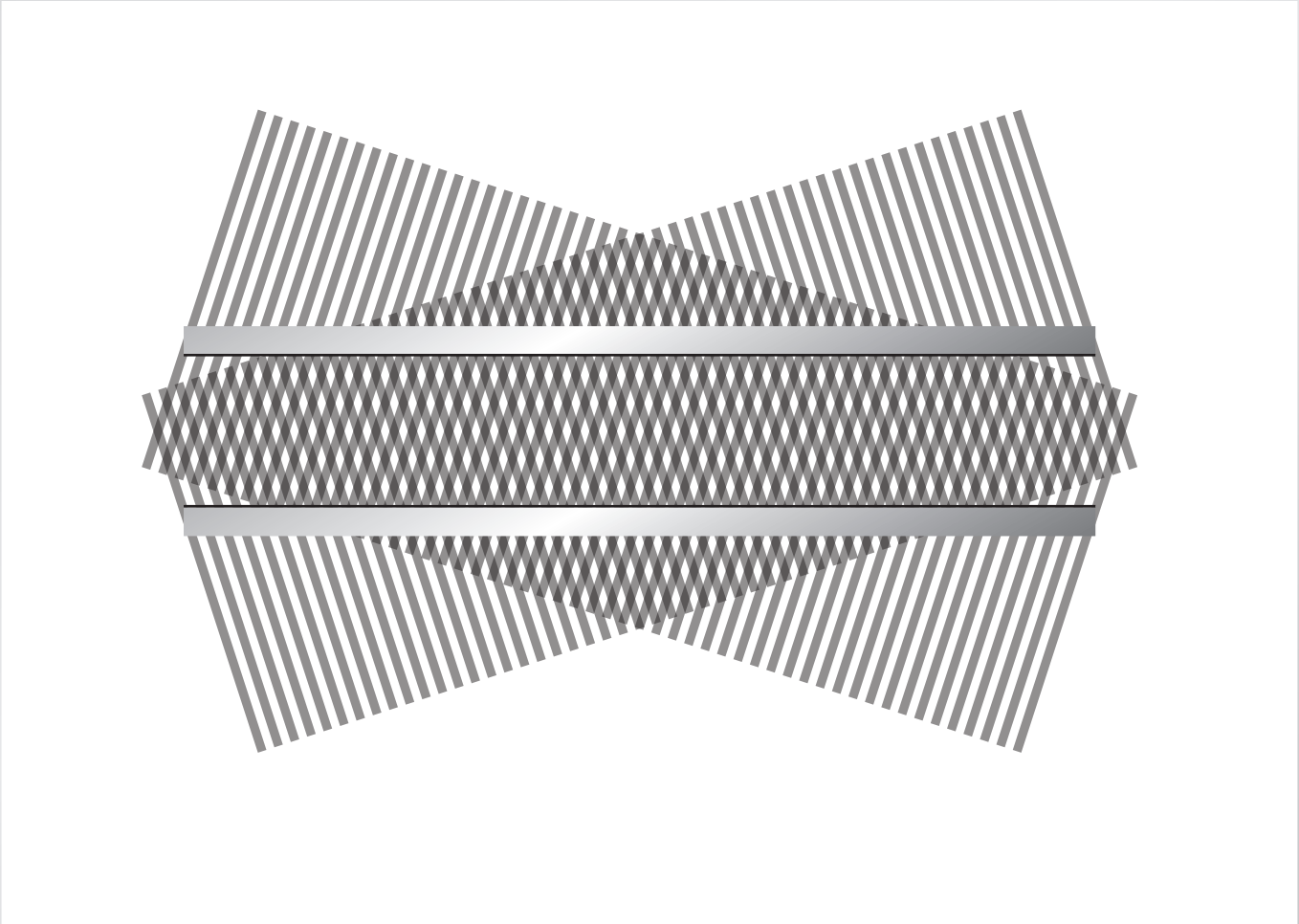
Waveguiding

change angle of incident waves…



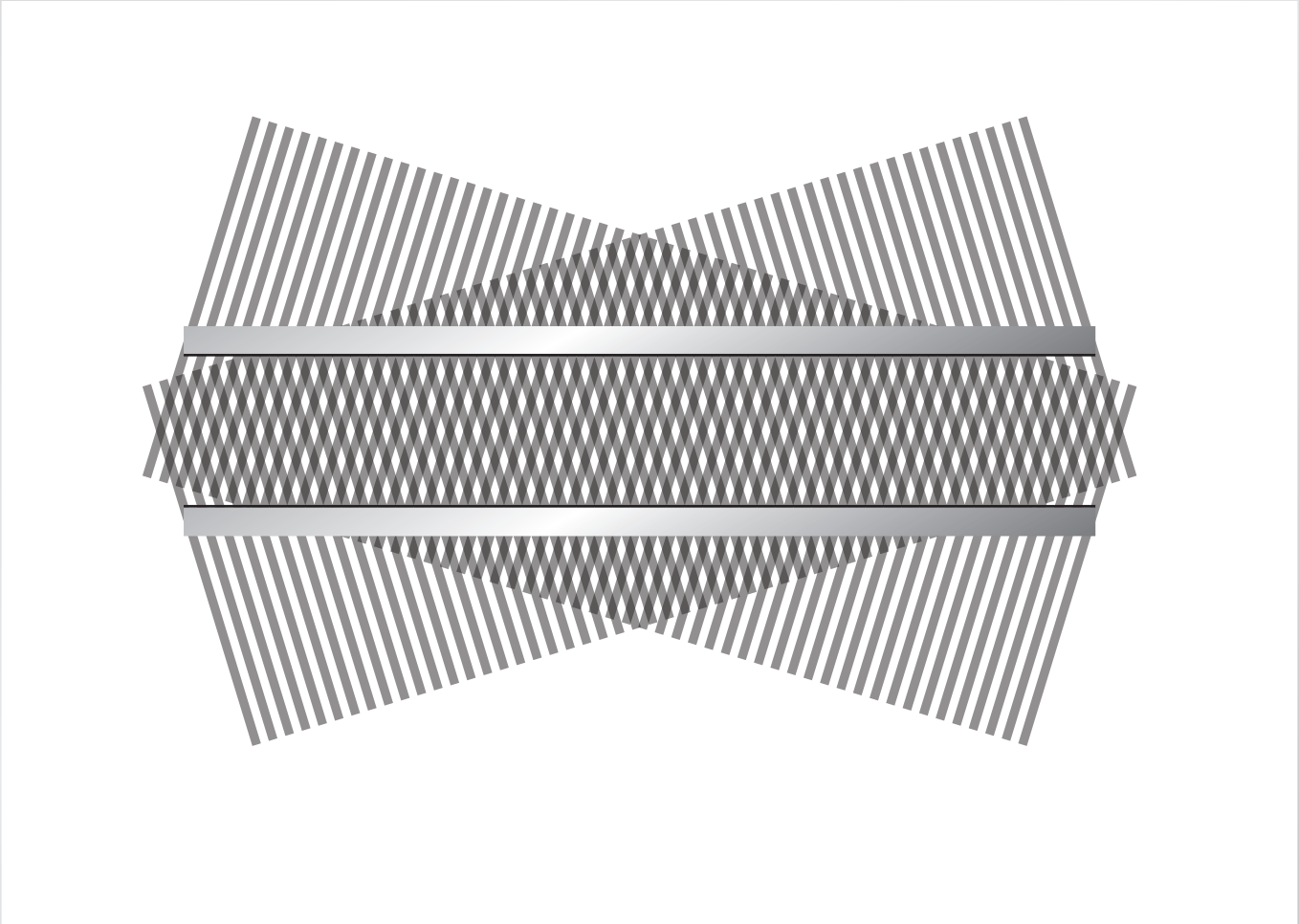
Waveguiding

change angle of incident waves…



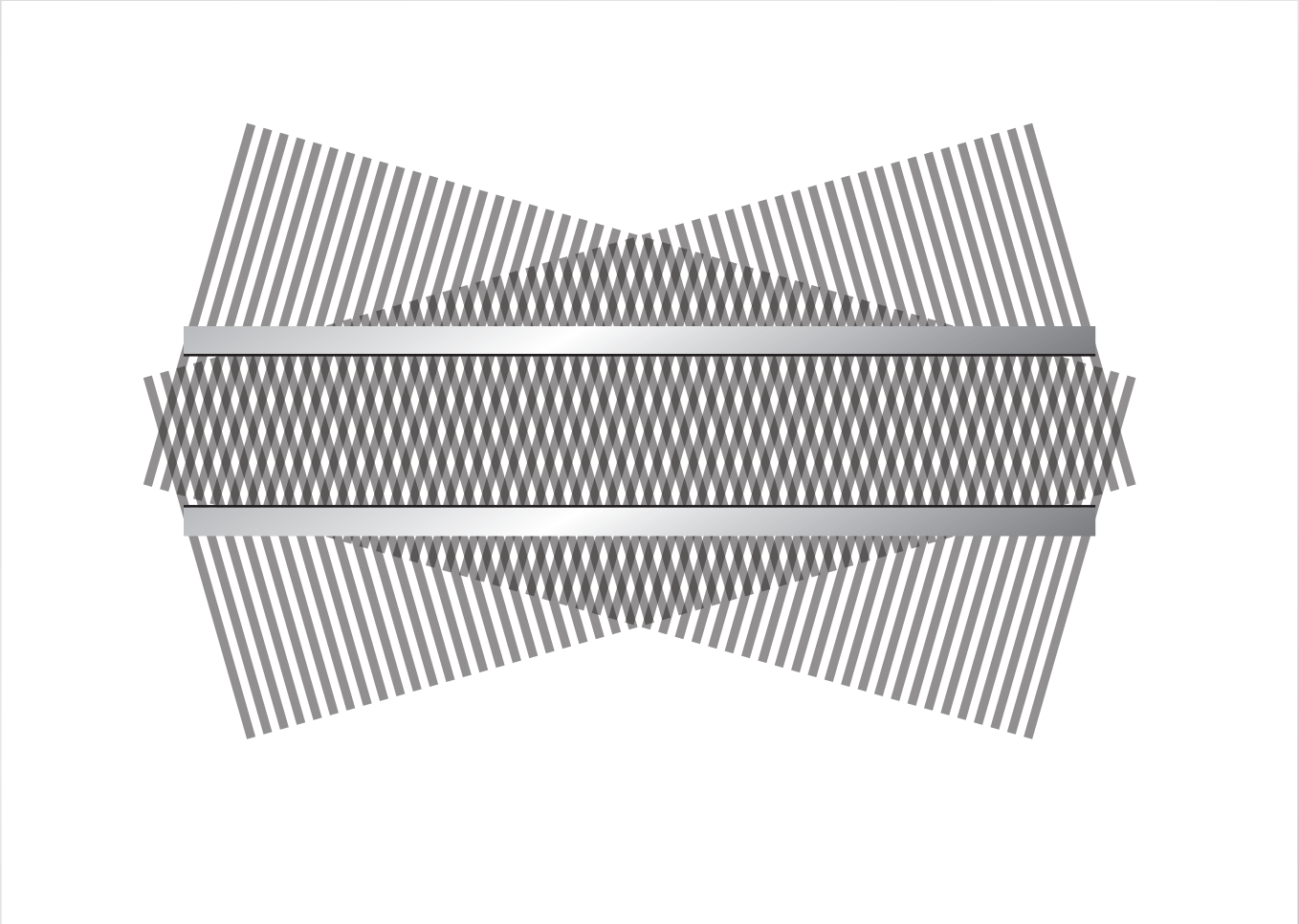
Waveguiding

change angle of incident waves…



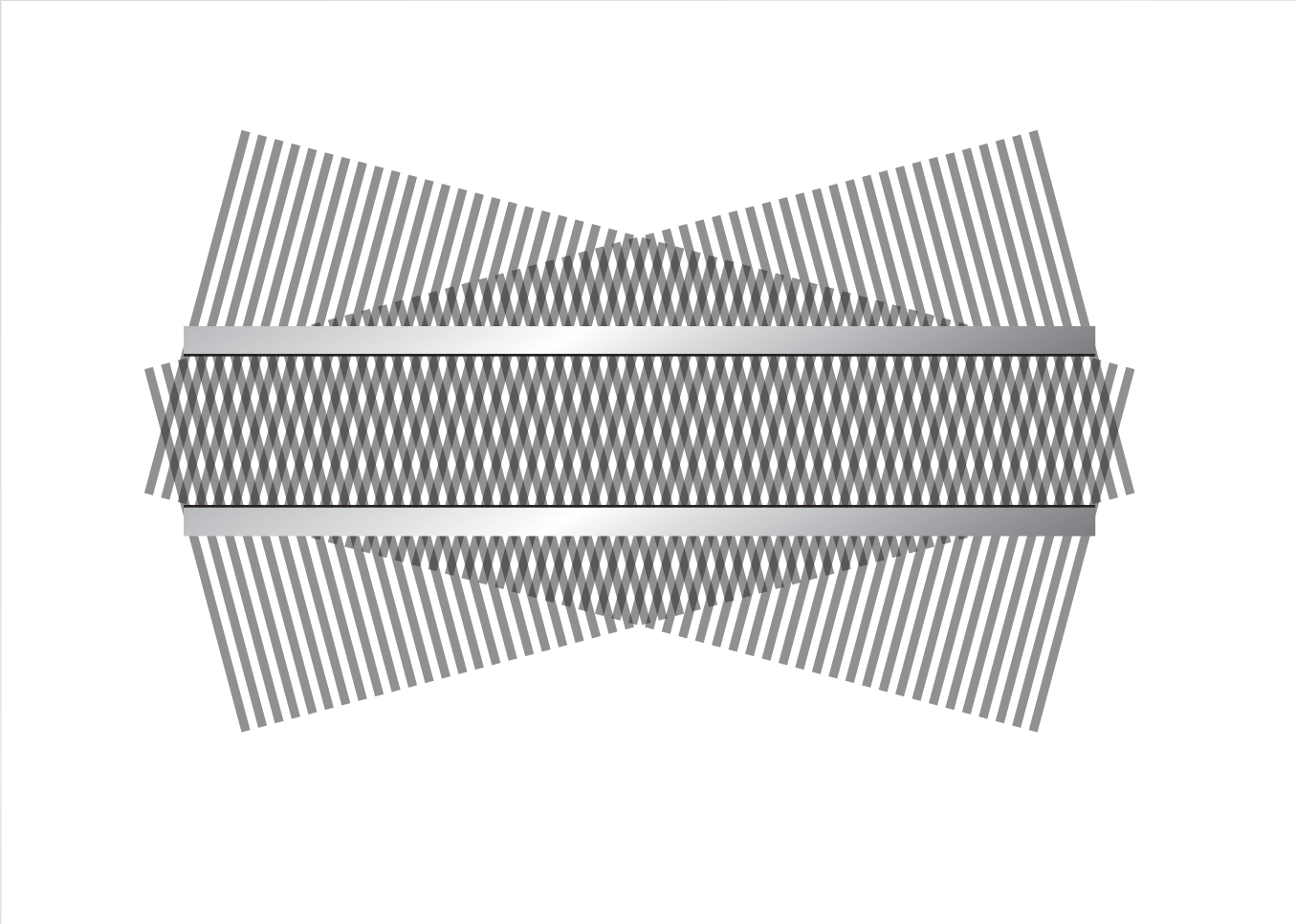
Waveguiding

change angle of incident waves…



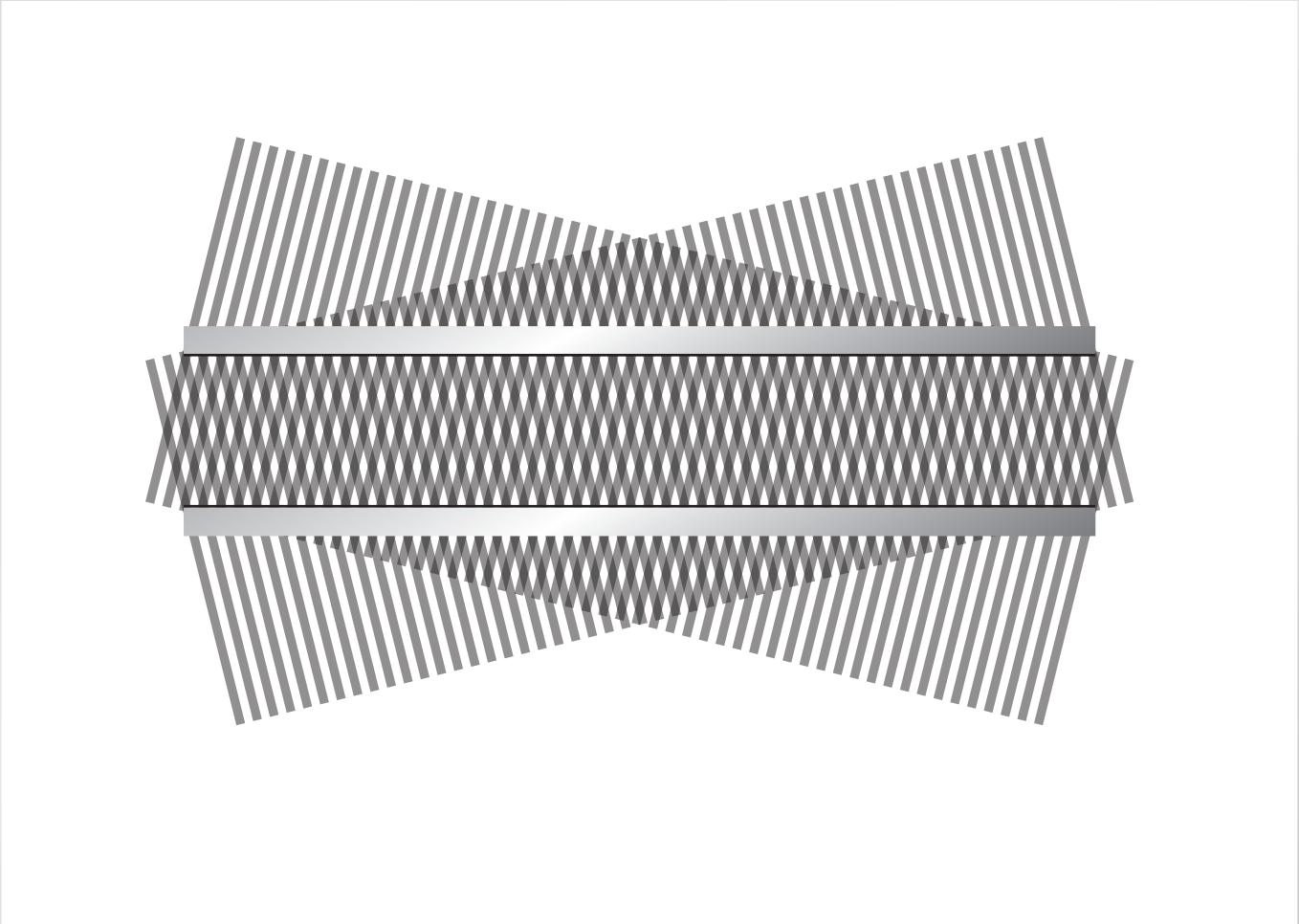
Waveguiding

change angle of incident waves…



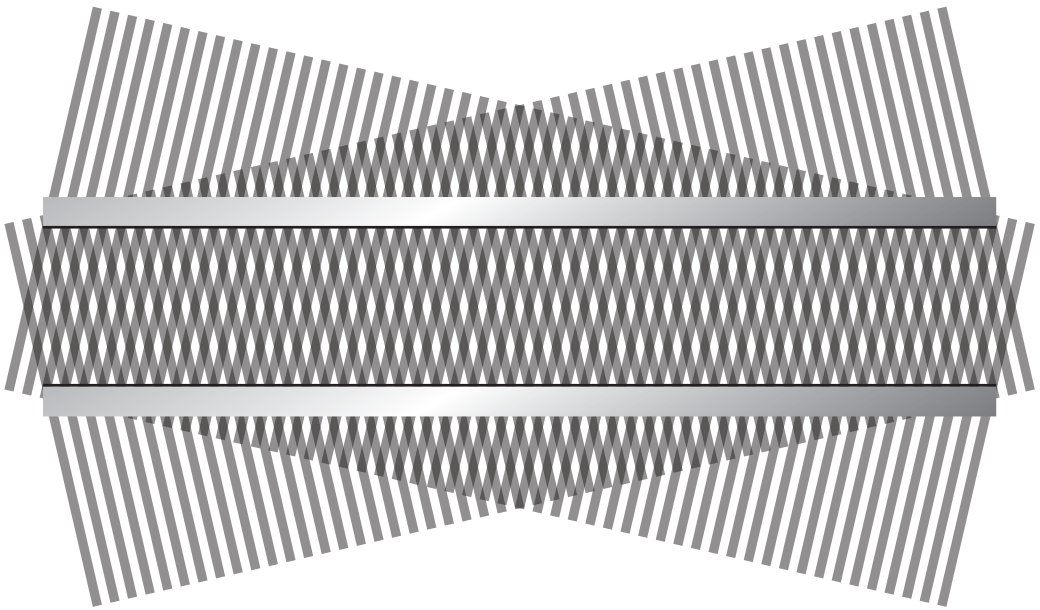
Waveguiding

change angle of incident waves…



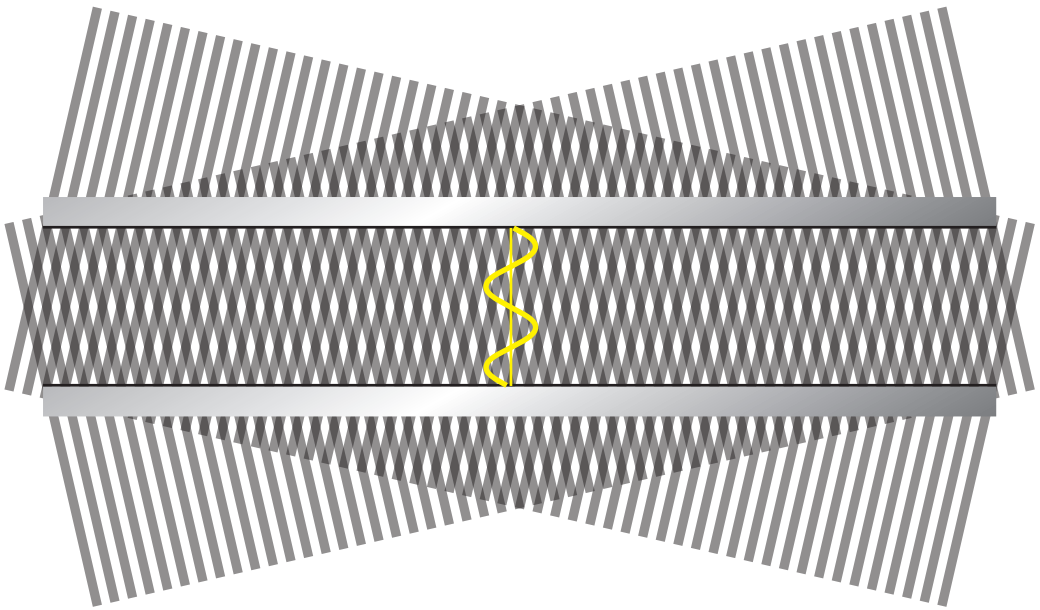
Waveguiding

change angle of incident waves…



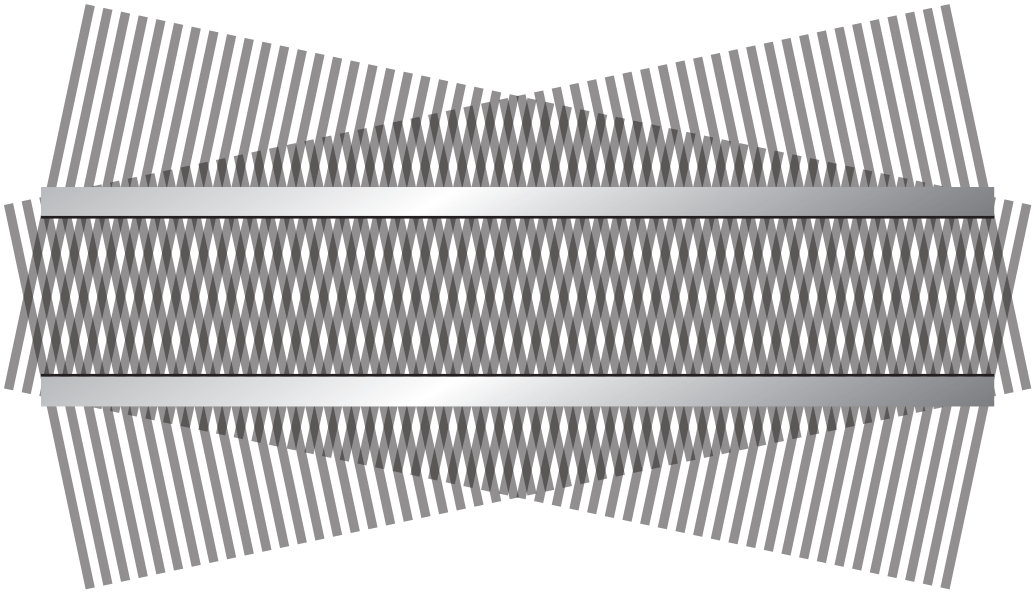
Waveguiding

change angle of incident waves…



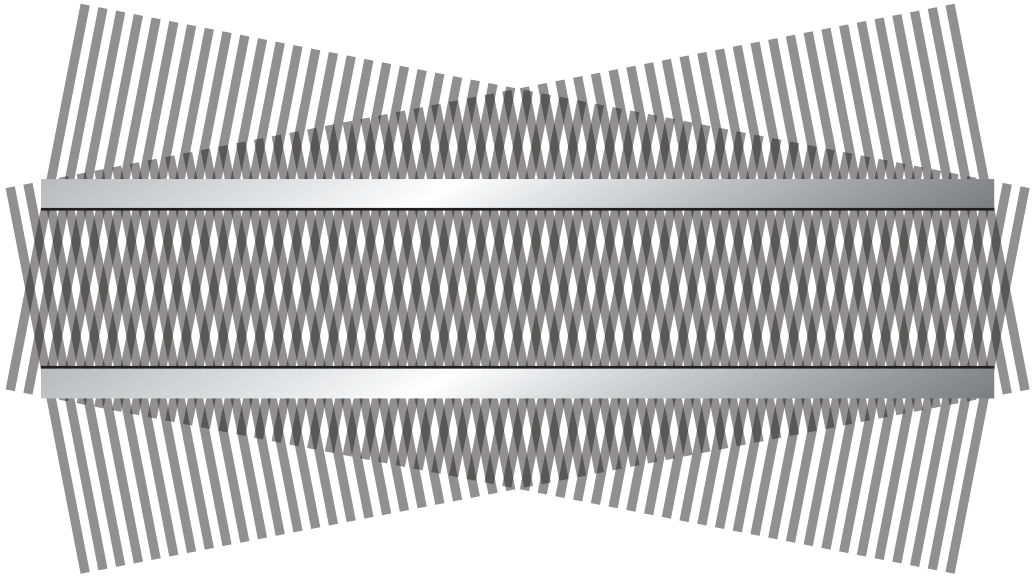
Waveguiding

change angle of incident waves…



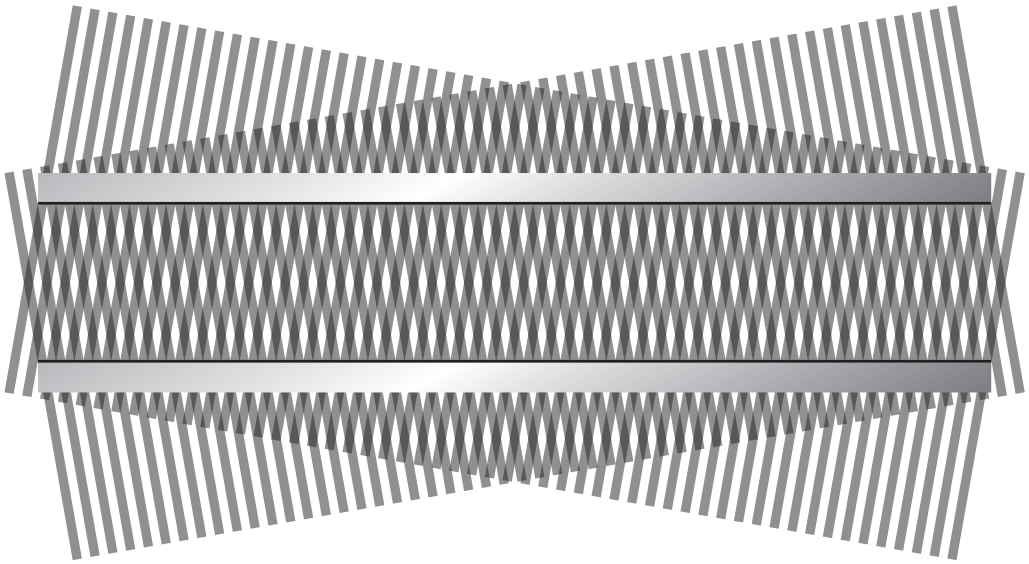
Waveguiding

change angle of incident waves…



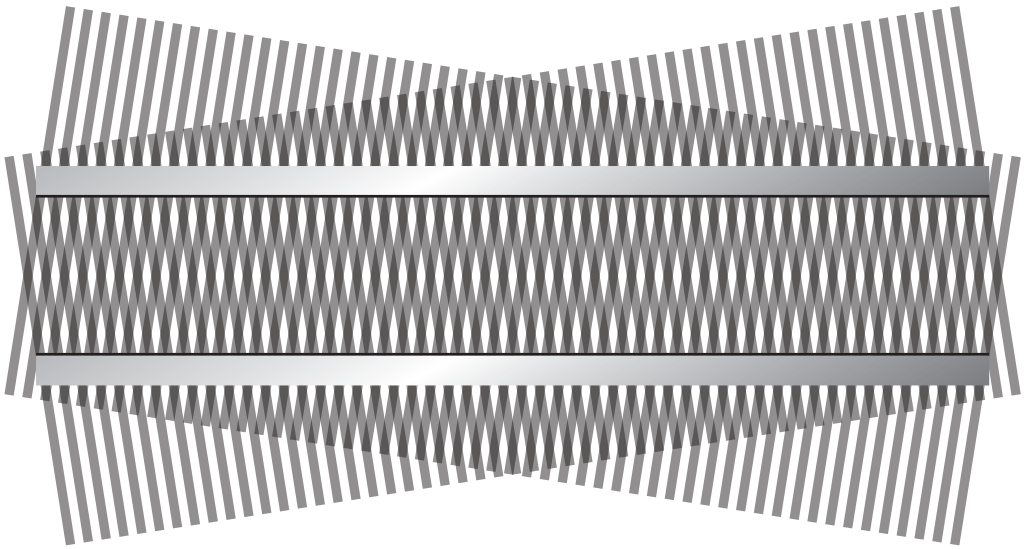
Waveguiding

change angle of incident waves…



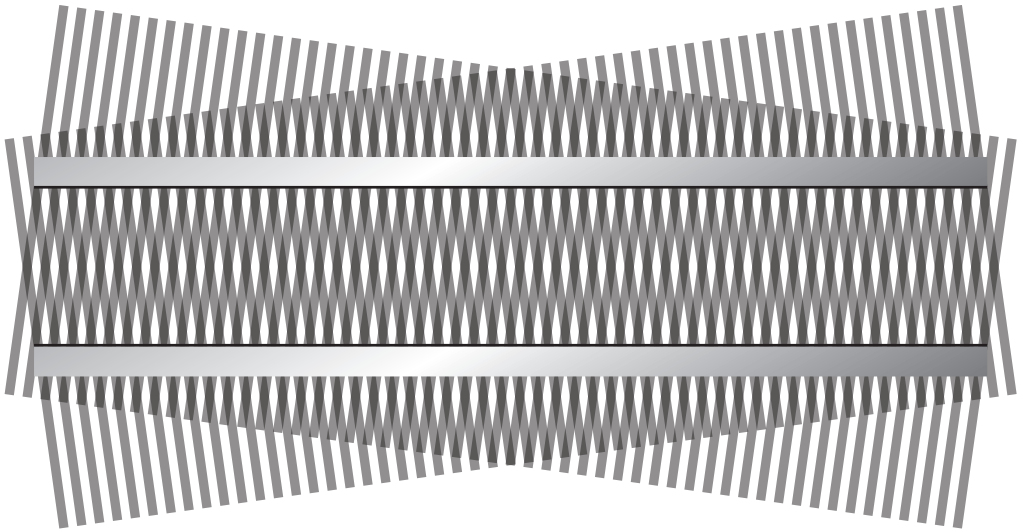
Waveguiding

change angle of incident waves…



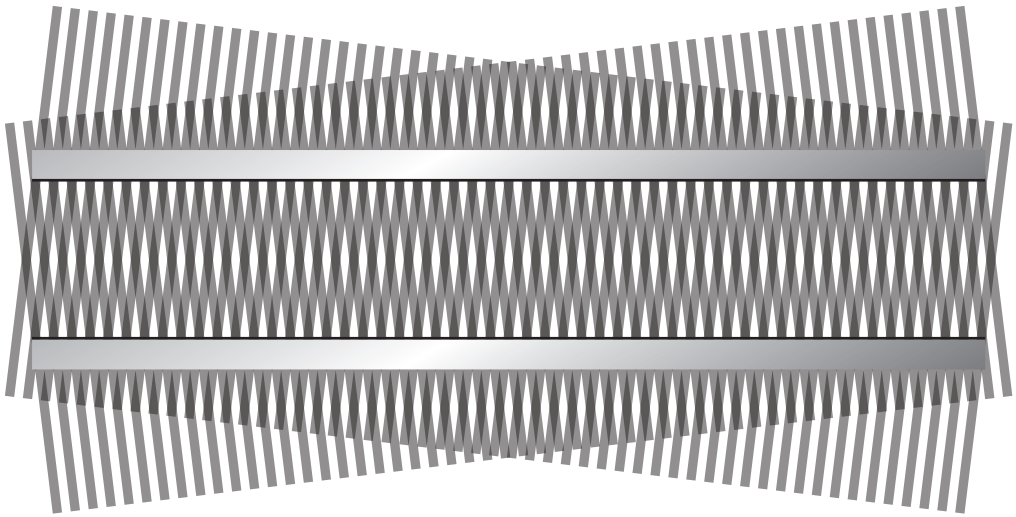
Waveguiding

change angle of incident waves…



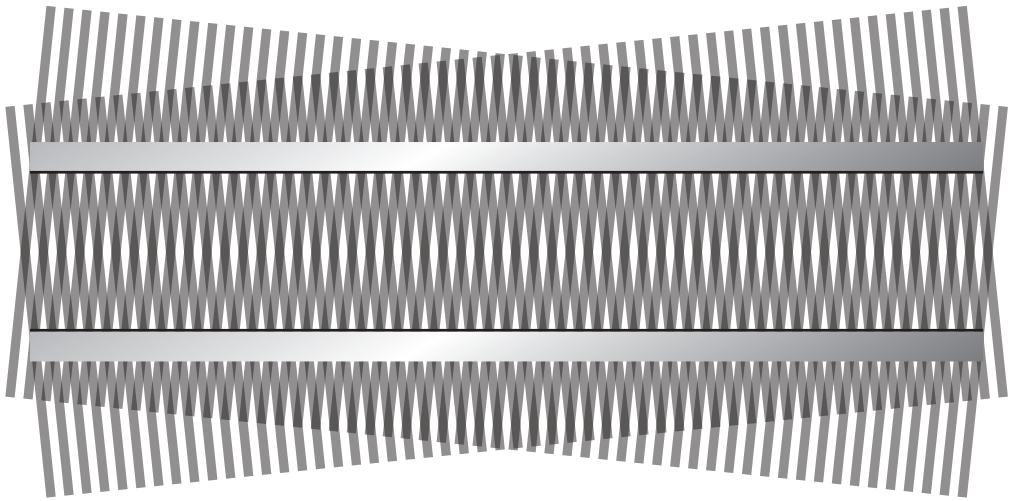
Waveguiding

change angle of incident waves…



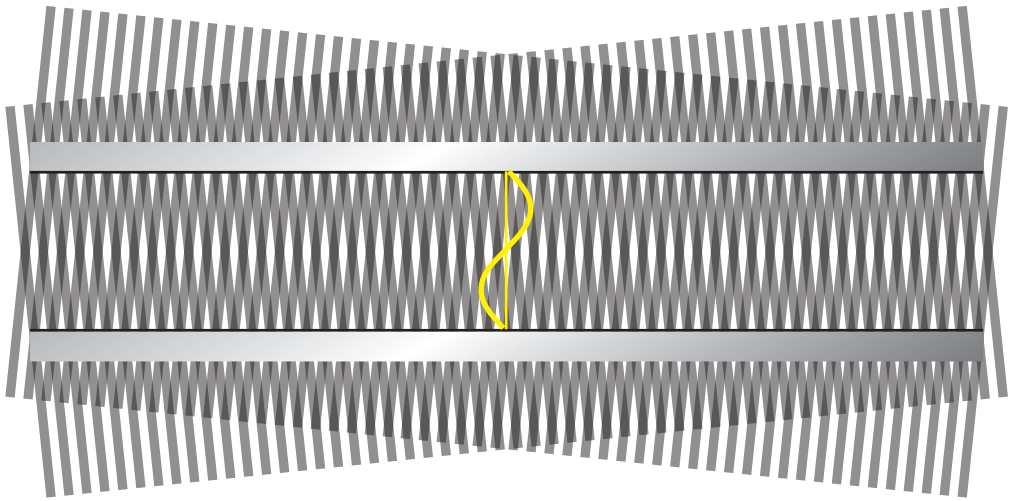
Waveguiding

change angle of incident waves…



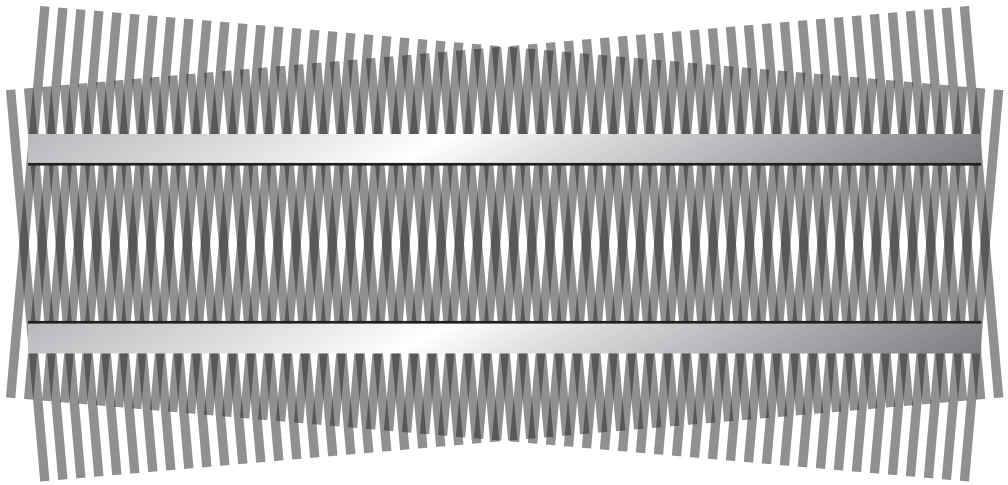
Waveguiding

change angle of incident waves…



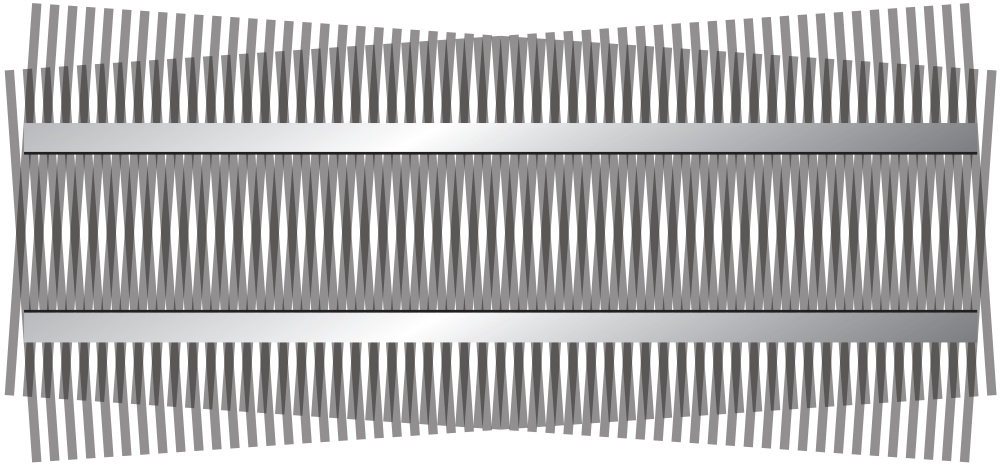
Waveguiding

change angle of incident waves…



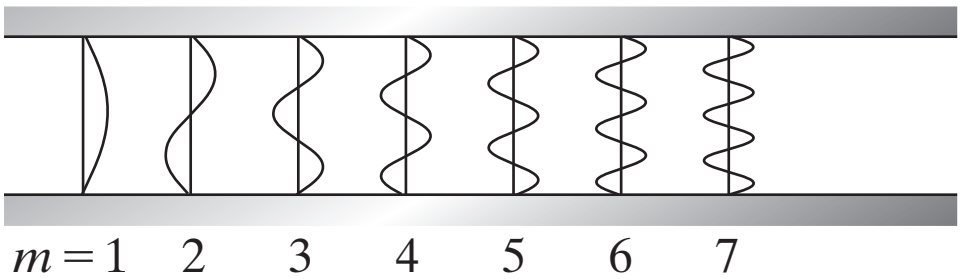
Waveguiding

change angle of incident waves…



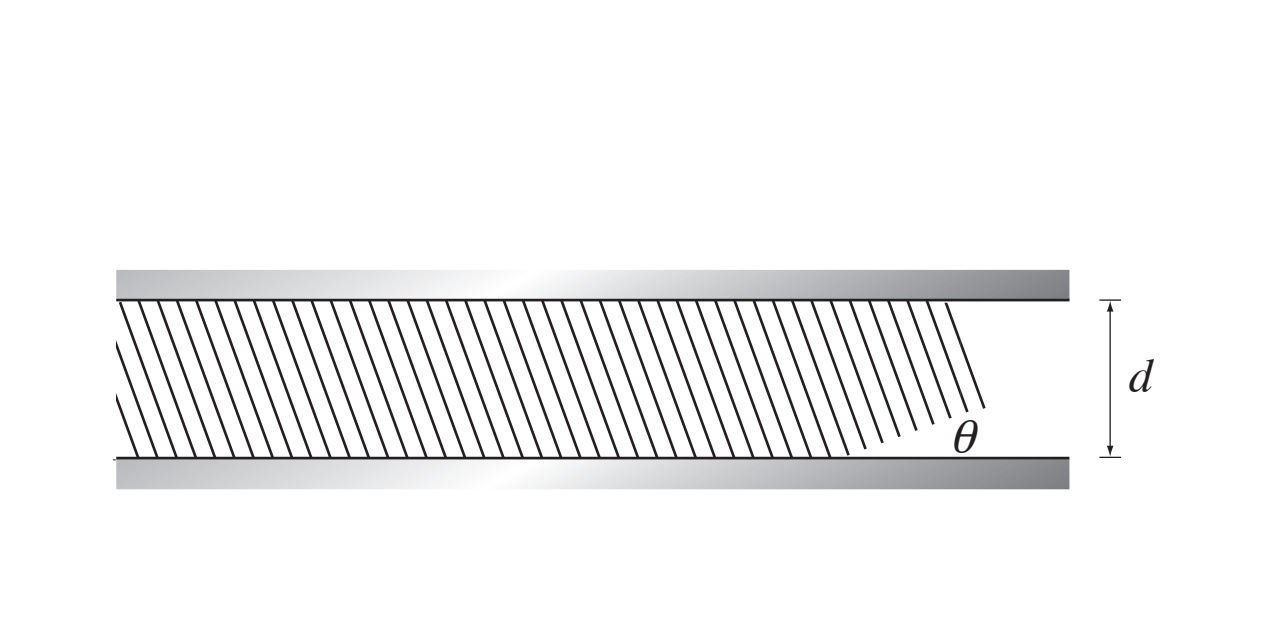
Waveguiding

boundary conditions only satisfied for certain θ



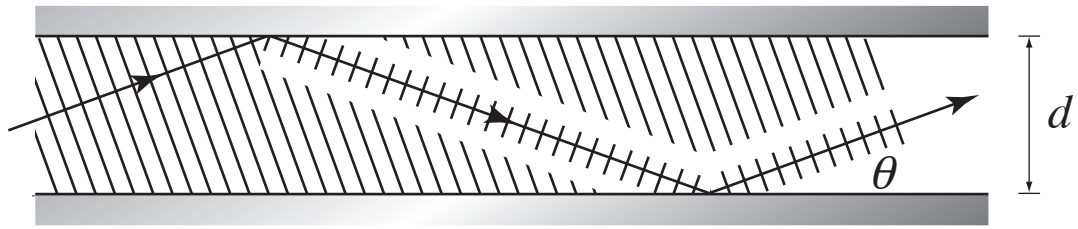
standing wave in *y*-direction, traveling in *z*-direction

Waveguiding



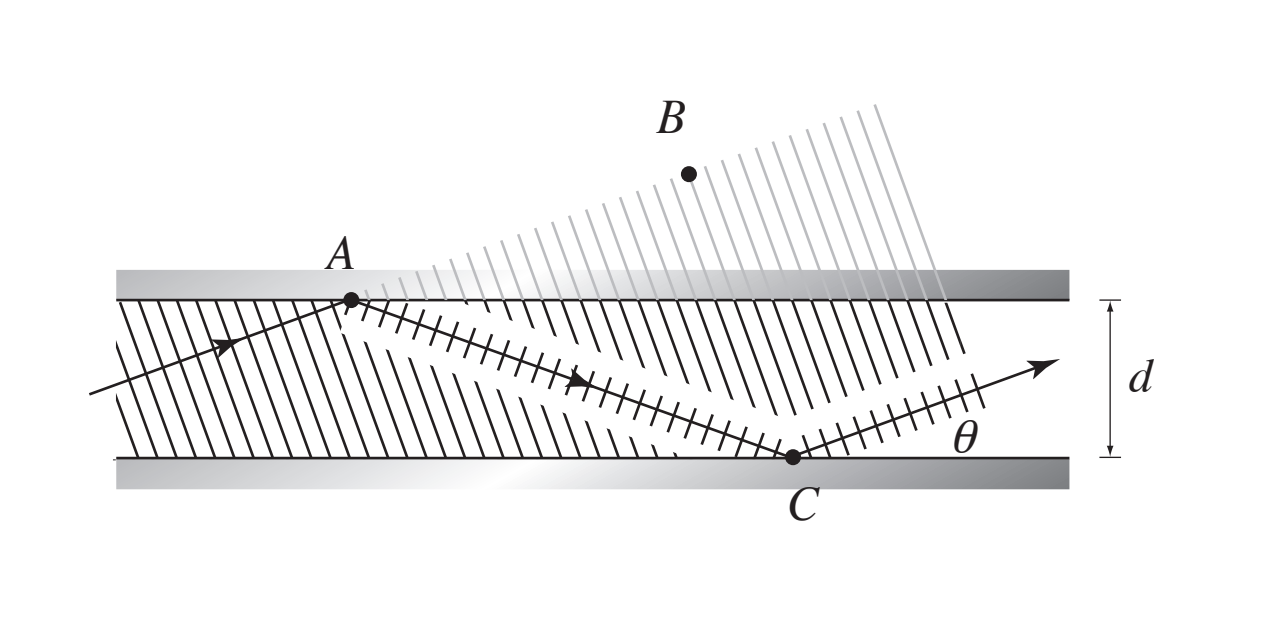
consider wave incident at angle θ

Waveguiding



twice-reflected wave

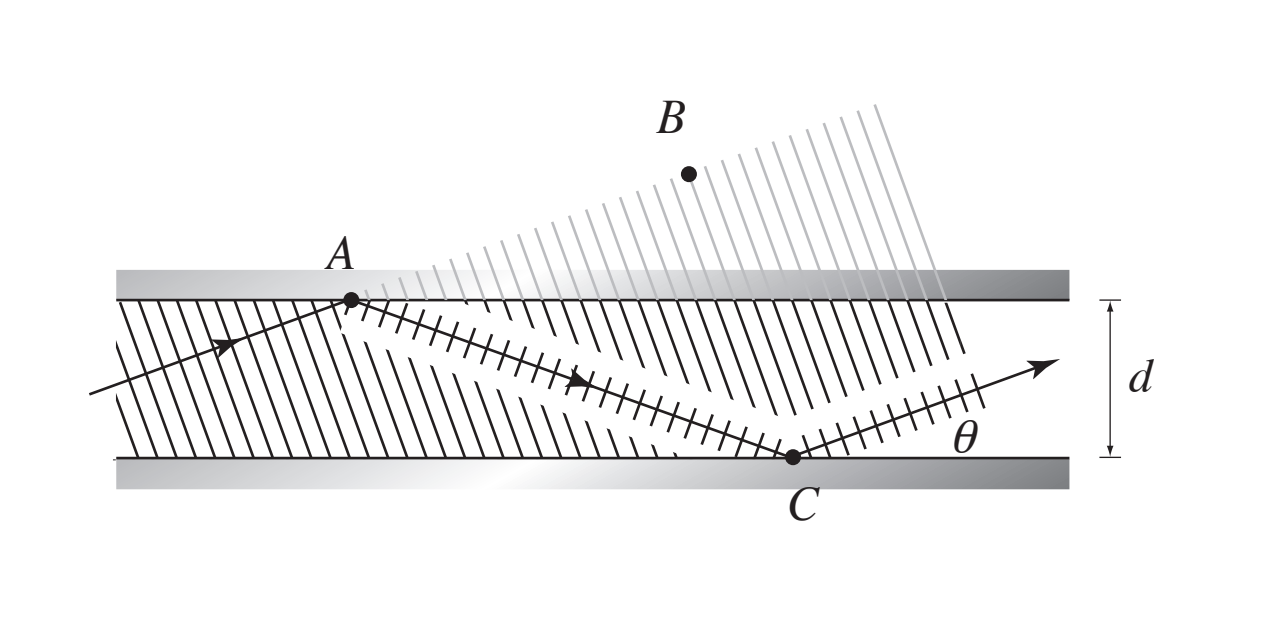
Waveguiding



self consistency:

$$AC - AB = 2d\sin\theta = m\lambda \quad (m = 1, 2, ...)$$

Waveguiding



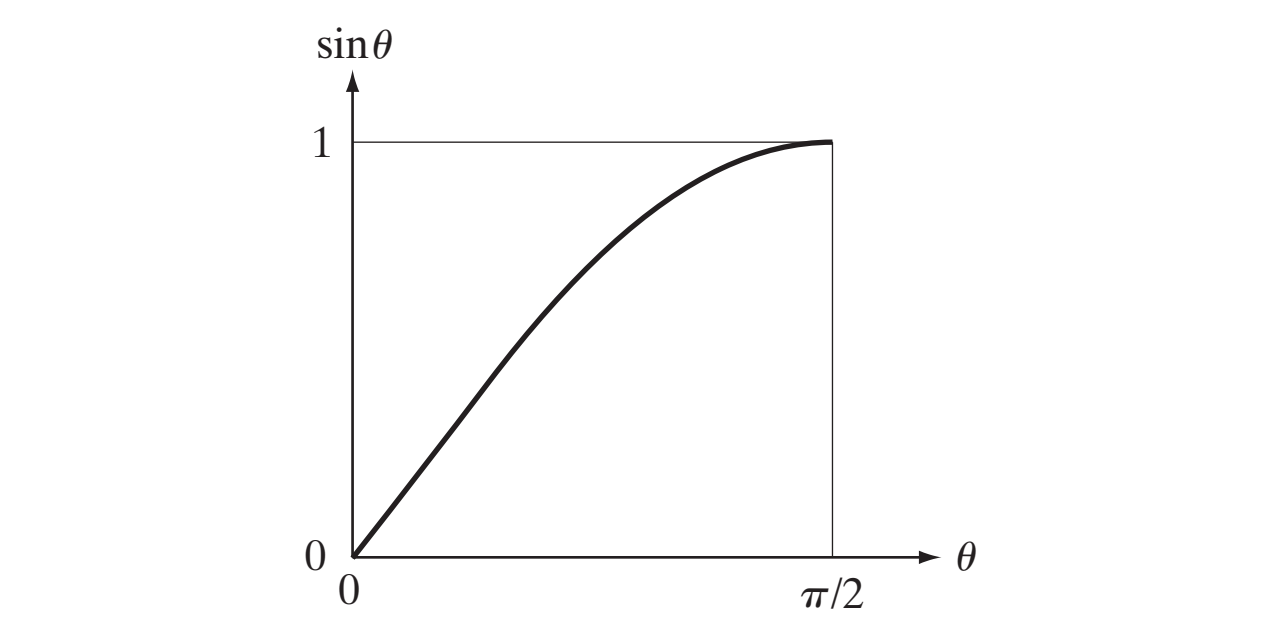
self consistency:

$$AC - AB = 2d\sin\theta = m\lambda \quad (m = 1, 2, ...)$$

so:

$$\sin\theta_m = m\frac{\lambda}{2d}$$

Waveguiding



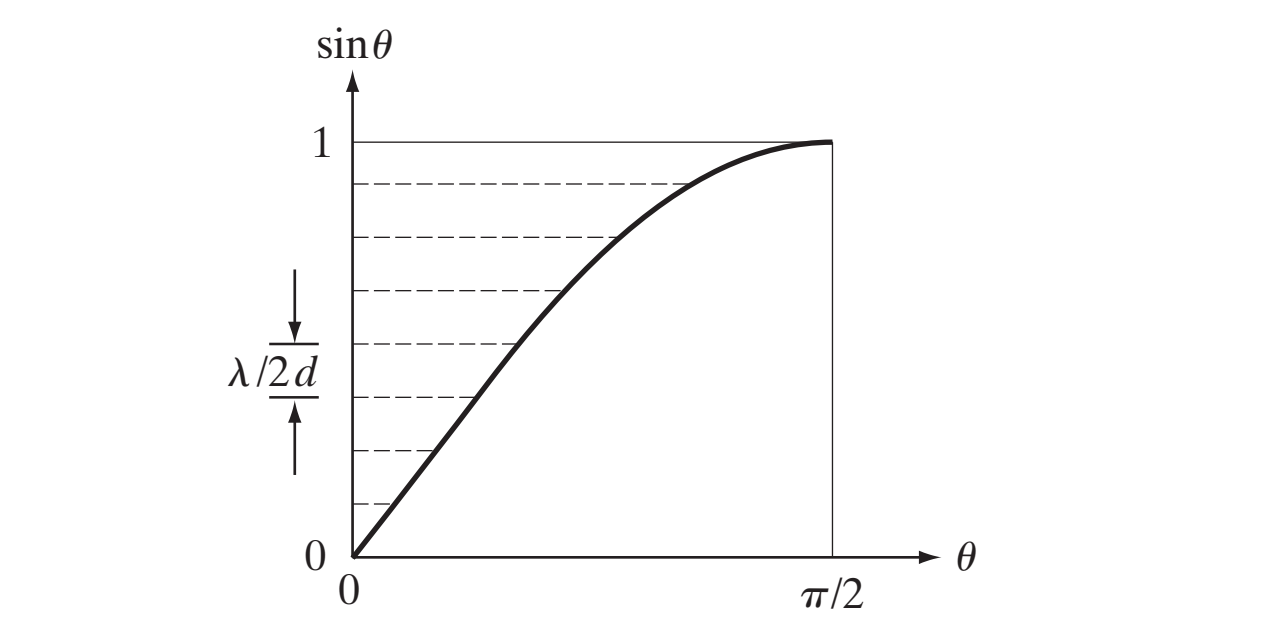
self consistency:

$$AC - AB = 2d\sin\theta = m\lambda \quad (m = 1, 2, ...)$$

so:

$$\sin\theta_m = m\frac{\lambda}{2d}$$

Waveguiding



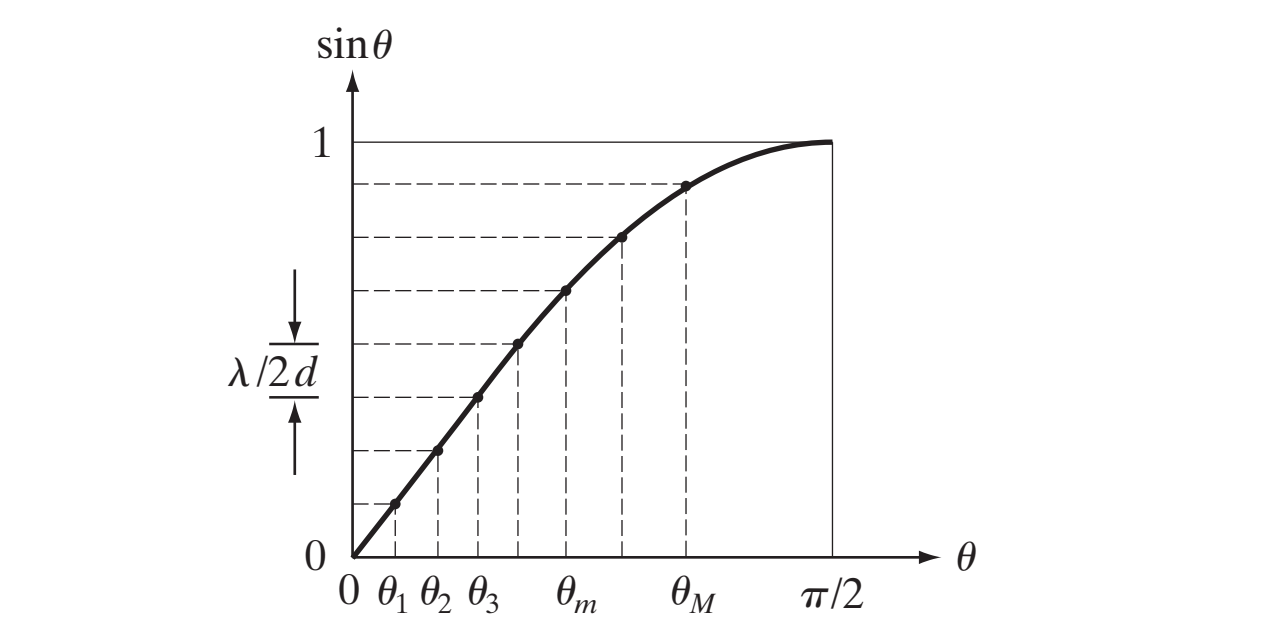
self consistency:

$$AC - AB = 2d\sin\theta = m\lambda \quad (m = 1, 2, ...)$$

so:

$$\sin\theta_m = m\frac{\lambda}{2d}$$

Waveguiding



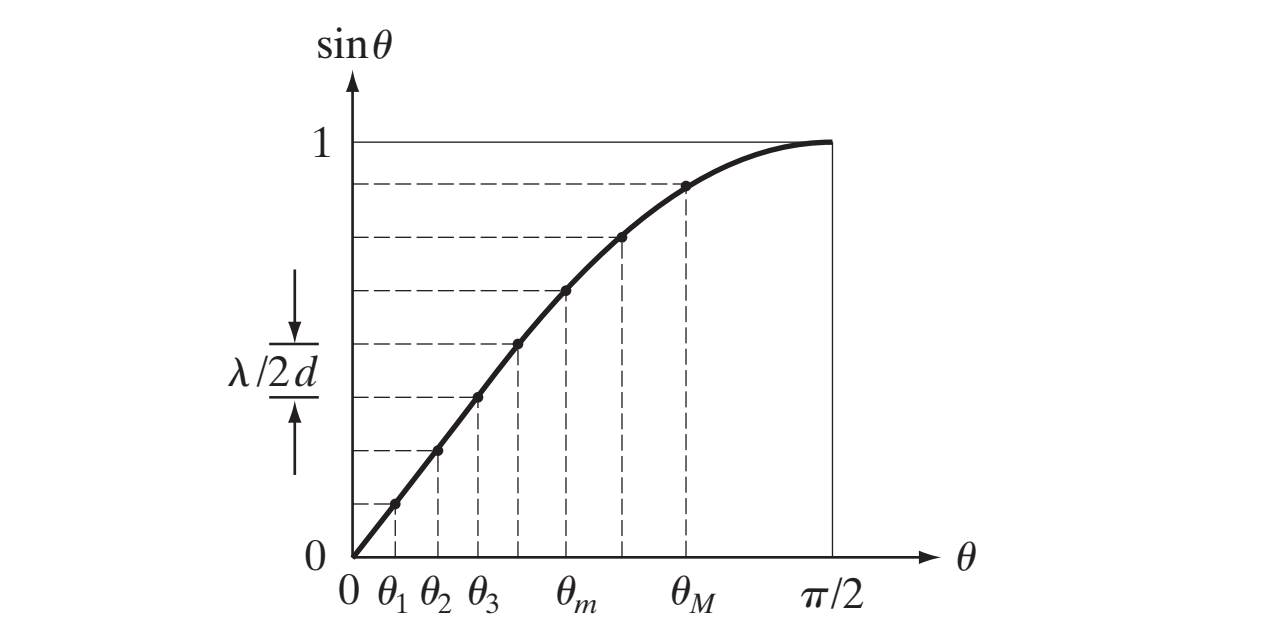
self consistency:

$$AC - AB = 2d\sin\theta = m\lambda \quad (m = 1, 2, ...)$$

so:

$$\sin\theta_m = m\frac{\lambda}{2d}$$

Waveguiding



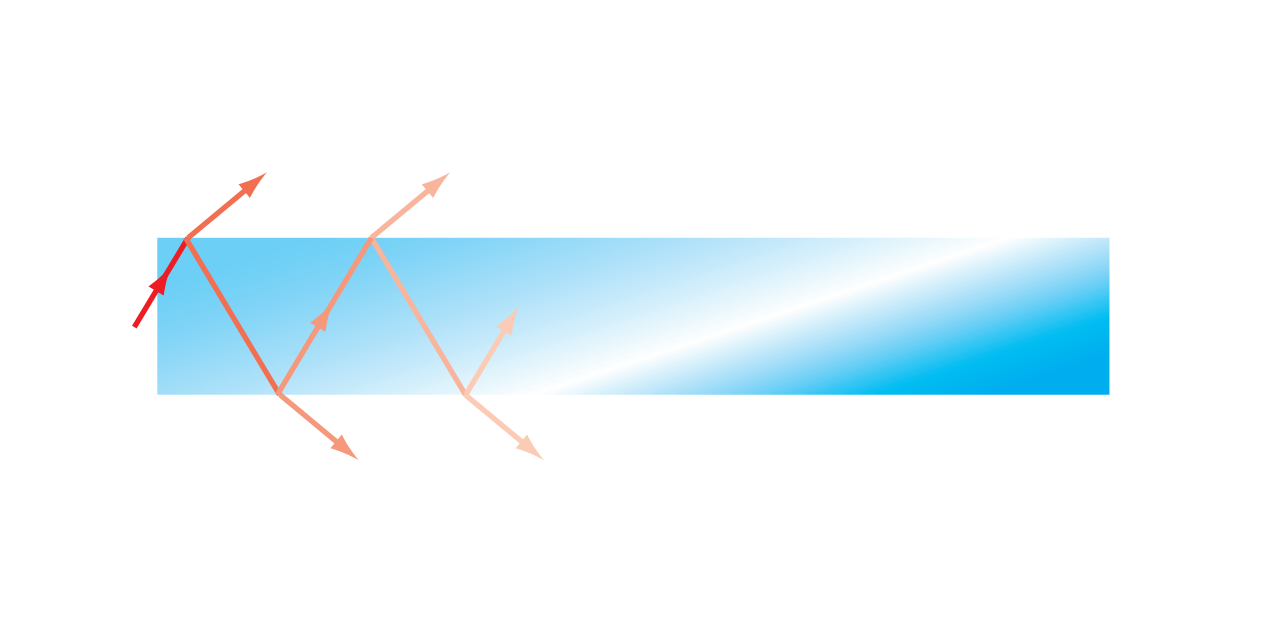
number of modes:

$$M \doteq \frac{2d}{\lambda}$$

Waveguiding

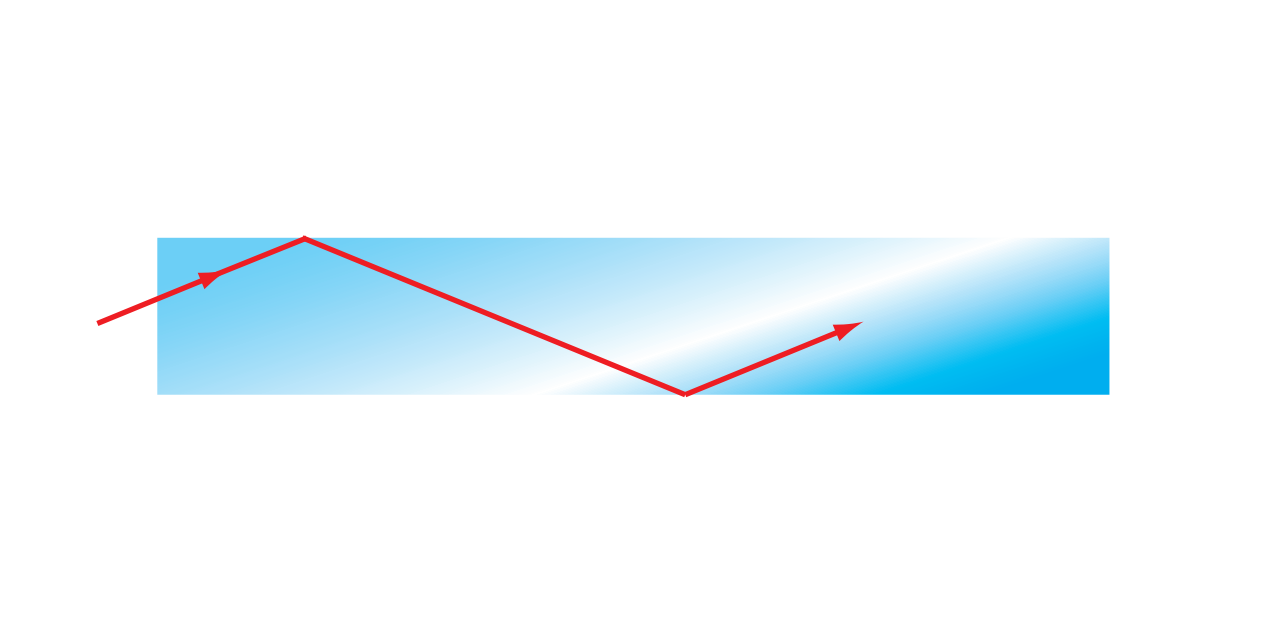
now consider a planar dielectric waveguide

Waveguiding



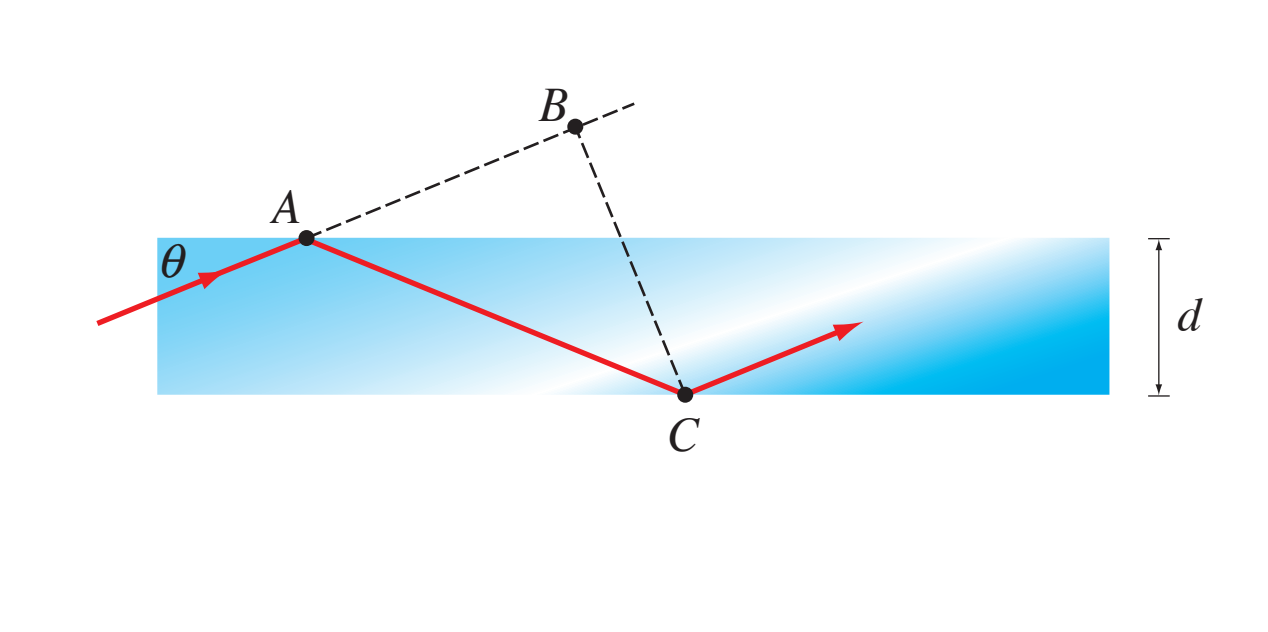
rays incident at angle $\theta > \pi/2 - \theta_c$ are unguided

Waveguiding



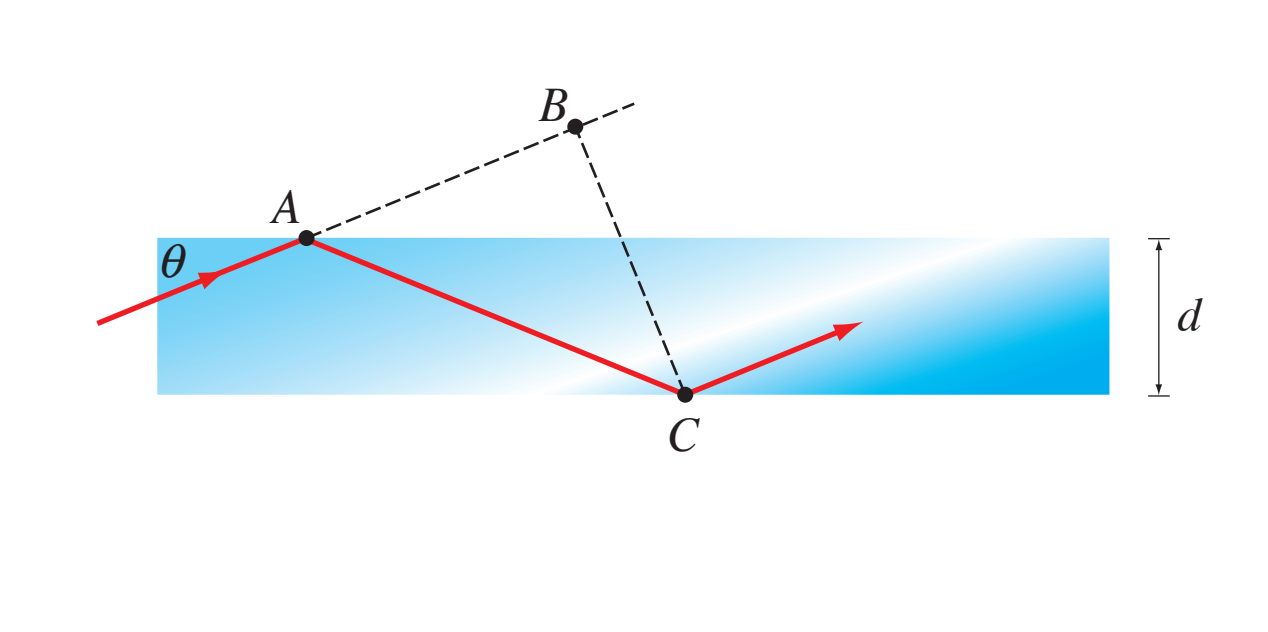
rays incident at angle $\theta < \pi/2 - \theta_c$ are guided

Waveguiding



rays incident at angle $\theta < \pi/2 - \theta_c$ are guided

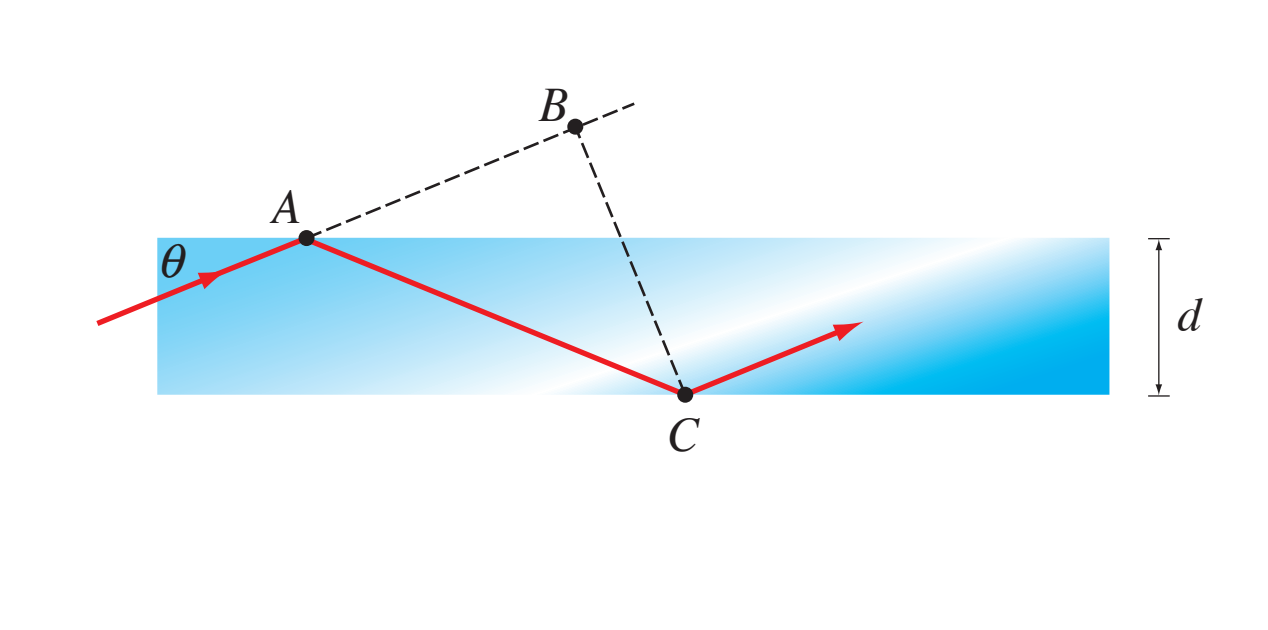
Waveguiding



self consistency:

$$AC - AB = 2d\sin\theta - \frac{\varphi_r}{\pi}\lambda = m\lambda \quad (m = 0, 1, 2\ldots)$$

Waveguiding



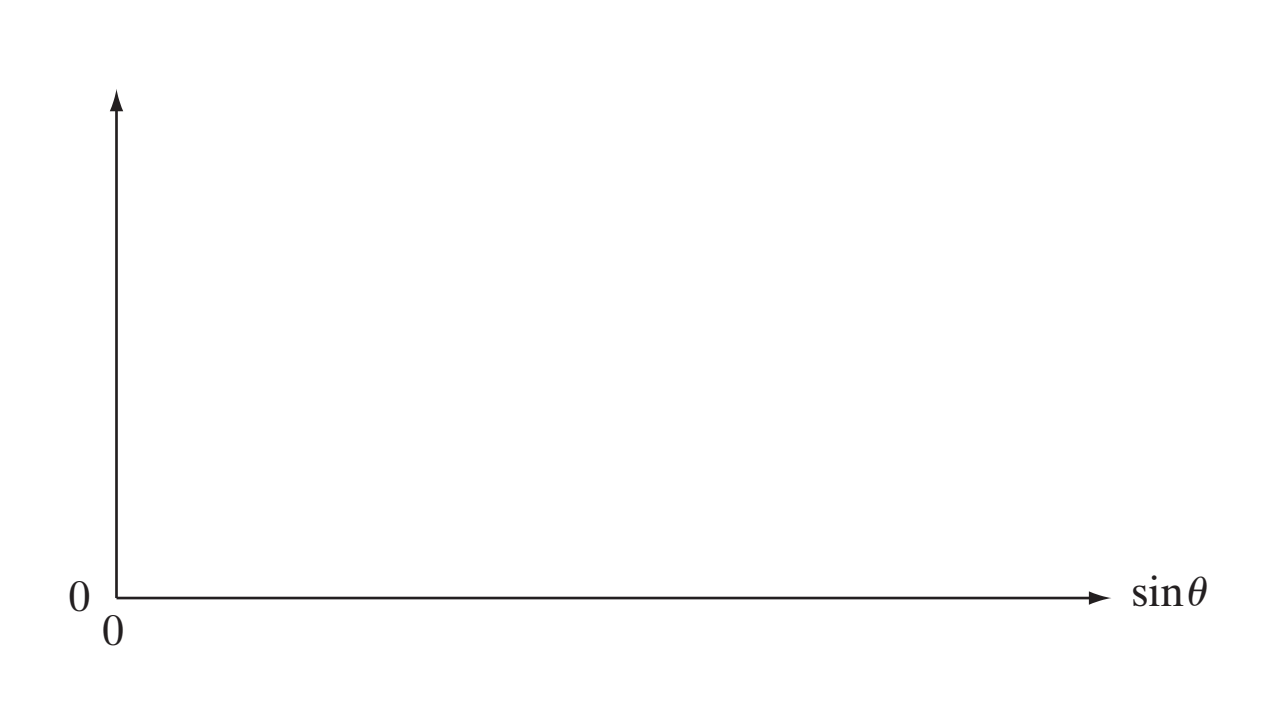
self consistency:

$$AC - AB = 2d\sin\theta - \frac{\varphi_r}{\pi}\lambda = m\lambda \quad (m = 0, 1, 2\ldots)$$

so:

$$\tan\left(\frac{\pi d}{\lambda}\sin\theta - m\frac{\pi}{2}\right) = \left(\frac{\sin^2(\pi/2 - \theta_c)}{\sin^2\theta} - 1\right)^{1/2}$$

Waveguiding



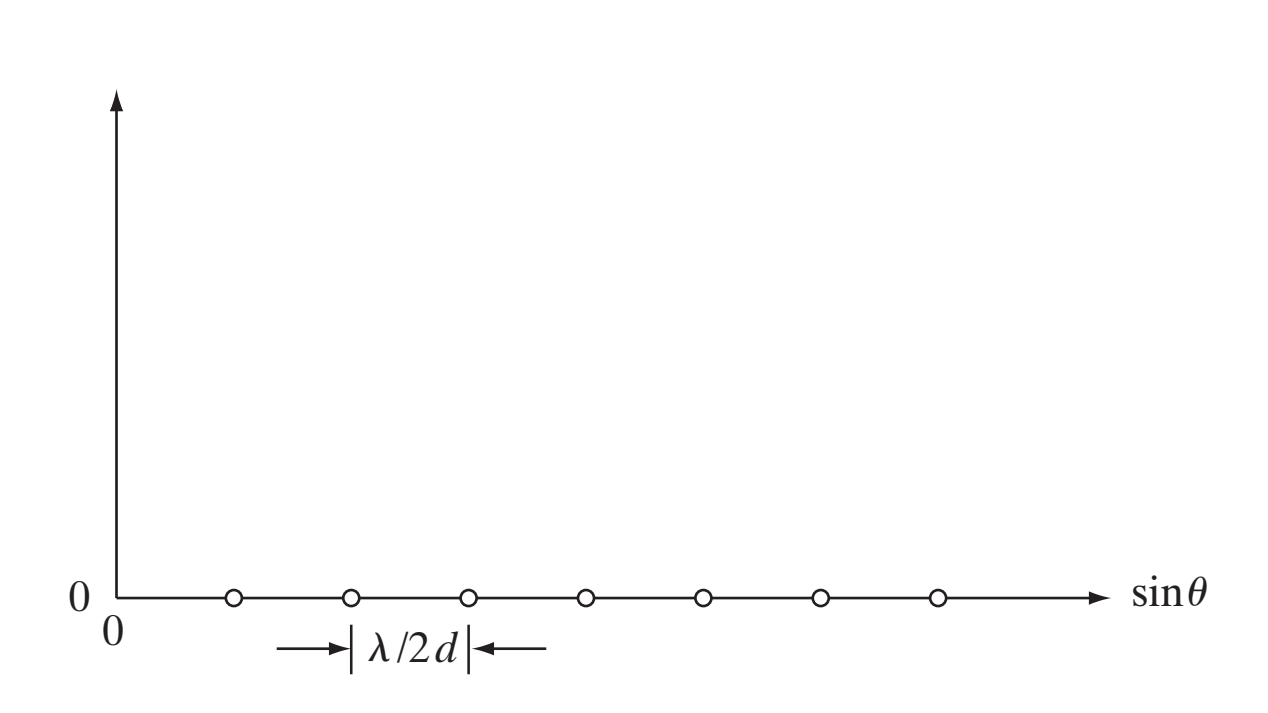
self consistency:

$$AC - AB = 2d\sin\theta - \frac{\varphi_r}{\pi}\lambda = m\lambda \quad (m = 0, 1, 2\ldots)$$

so:

$$\tan\left(\frac{\pi d}{\lambda}\sin\theta - m\frac{\pi}{2}\right) = \left(\frac{\sin^2(\pi/2 - \theta_c)}{\sin^2\theta} - 1\right)^{1/2}$$

Waveguiding



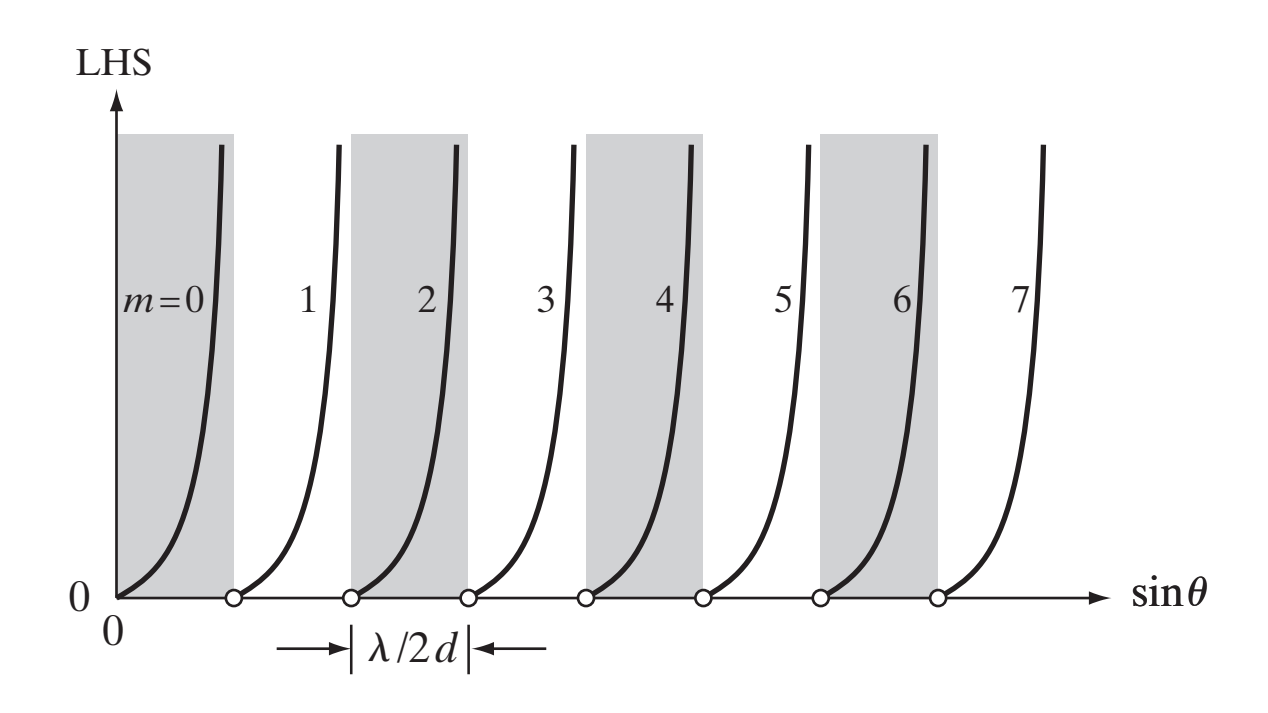
self consistency:

$$AC - AB = 2d\sin\theta - \frac{\varphi_r}{\pi}\lambda = m\lambda \quad (m = 0, 1, 2\ldots)$$

so:

$$\tan\left(\frac{\pi d}{\lambda}\sin\theta - m\frac{\pi}{2}\right) = \left(\frac{\sin^2(\pi/2 - \theta_c)}{\sin^2\theta} - 1\right)^{1/2}$$

Waveguiding



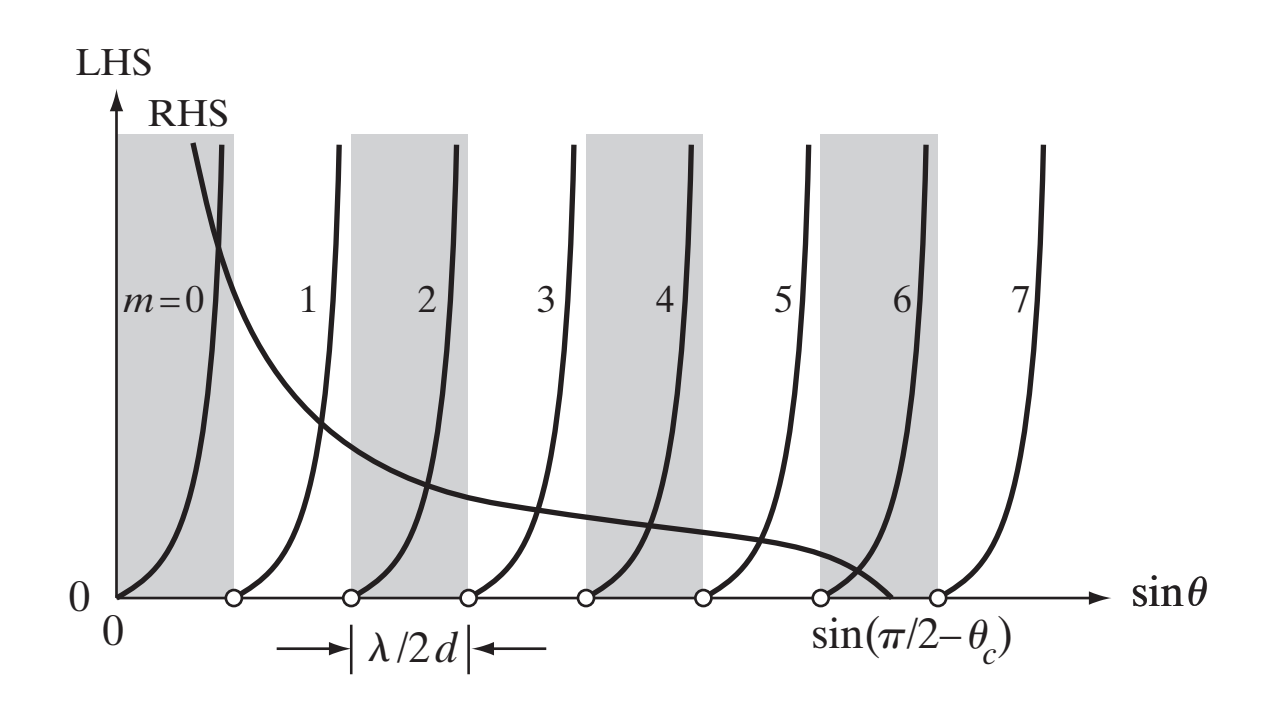
self consistency:

$$AC - AB = 2d\sin\theta - \frac{\varphi_r}{\pi}\lambda = m\lambda \quad (m = 0, 1, 2\ldots)$$

so:

$$\tan\left(\frac{\pi d}{\lambda}\sin\theta - m\frac{\pi}{2}\right) = \left(\frac{\sin^2(\pi/2 - \theta_c)}{\sin^2\theta} - 1\right)^{1/2}$$

Waveguiding



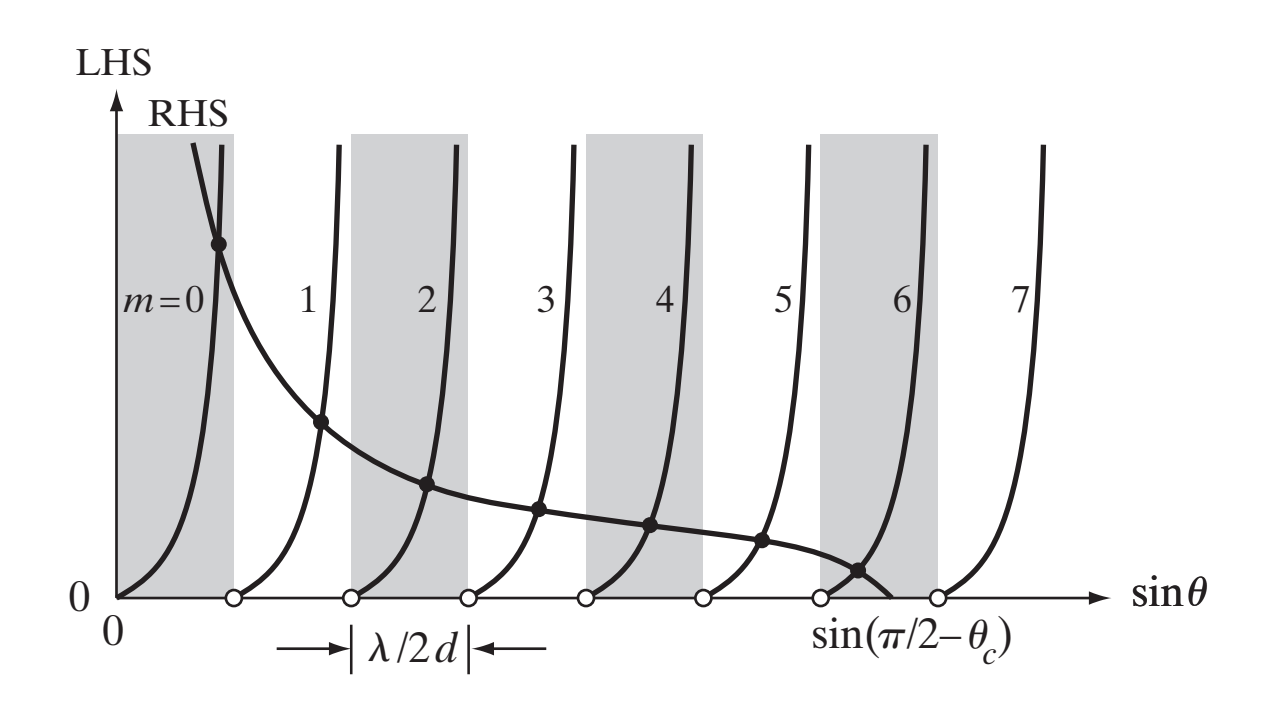
self consistency:

$$AC - AB = 2d\sin\theta - \frac{\varphi_r}{\pi}\lambda = m\lambda \quad (m = 0, 1, 2\ldots)$$

so:

$$\tan\left(\frac{\pi d}{\lambda}\sin\theta - m\frac{\pi}{2}\right) = \left(\frac{\sin^2(\pi/2-\theta_c)}{\sin^2\theta} - 1\right)^{1/2}$$

Waveguiding



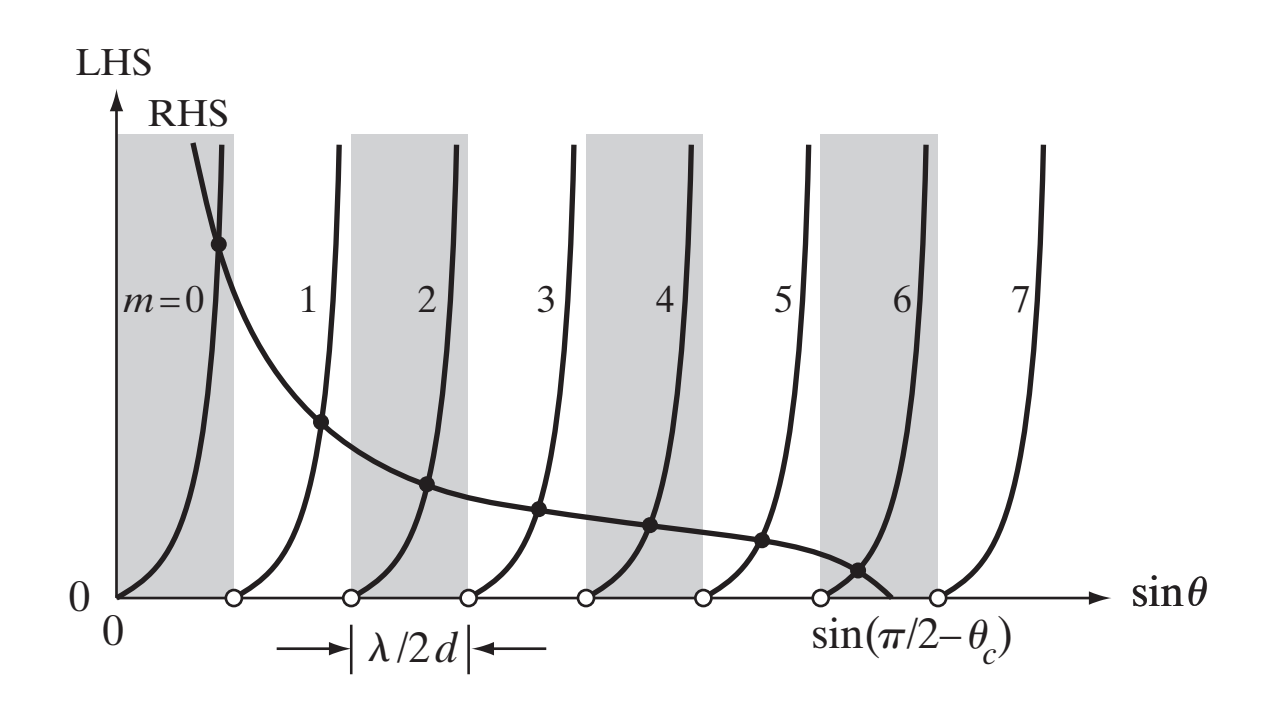
self consistency:

$$AC - AB = 2d\sin\theta - \frac{\varphi_r}{\pi}\lambda = m\lambda \quad (m = 0, 1, 2\ldots)$$

so:

$$\tan\left(\frac{\pi d}{\lambda}\sin\theta - m\frac{\pi}{2}\right) = \left(\frac{\sin^2(\pi/2-\theta_c)}{\sin^2\theta} - 1\right)^{1/2}$$

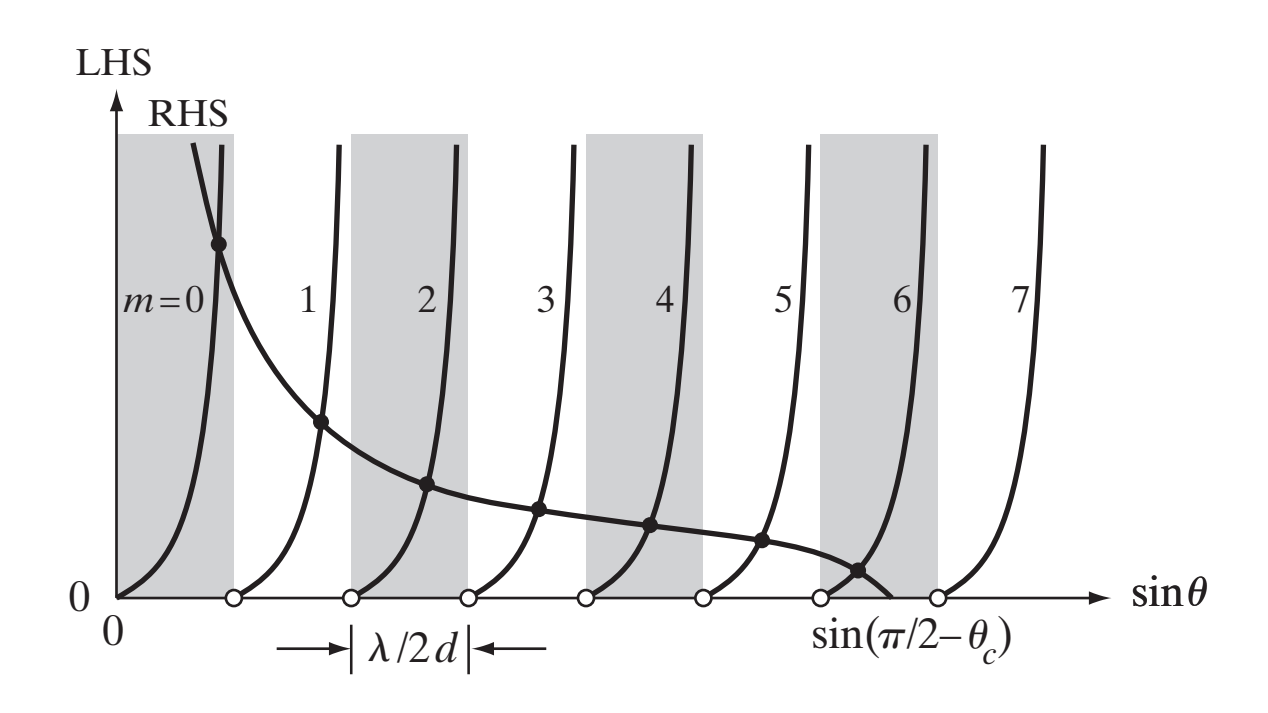
Waveguiding



number of modes:

$$M \doteq \frac{\sin(\pi/2 - \theta_c)}{\lambda/2d}$$

Waveguiding



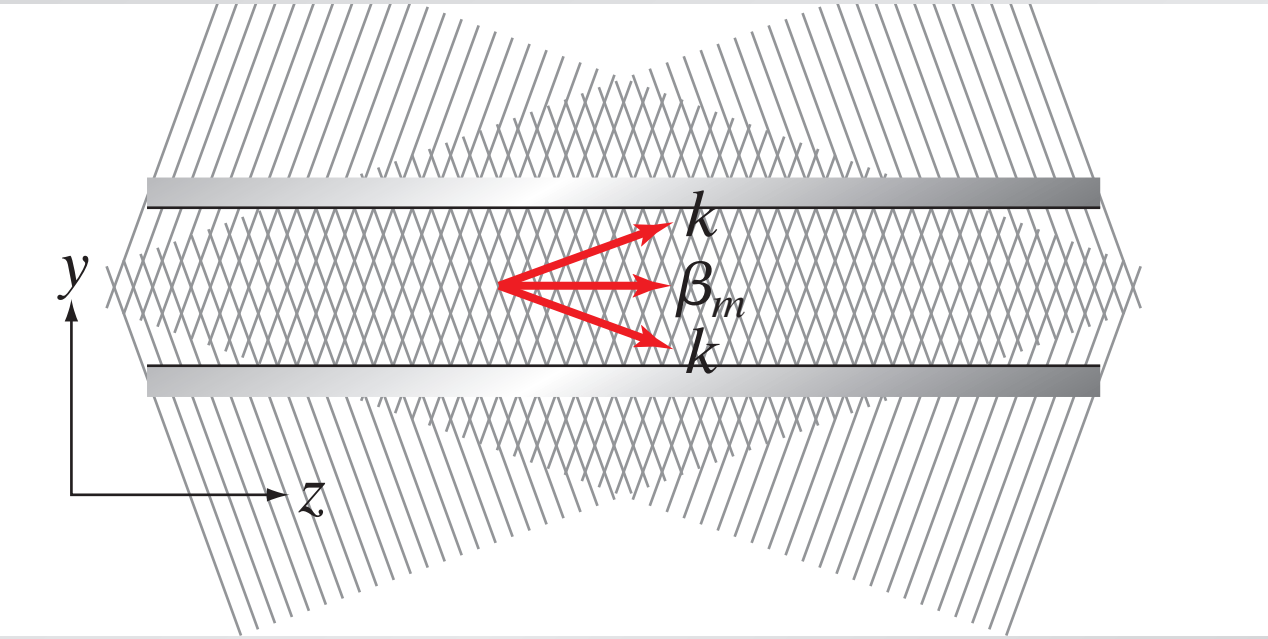
number of modes:

$$M \doteq \frac{\sin(\pi/2 - \theta_c)}{\lambda/2d}$$

or:

$$M \doteq 2\frac{d}{\lambda}(n_1^2 - n_2^2)^{1/2}$$

Waveguiding



propagation constant of guided wave:

$$\beta_m^2 = k^2 - k_y^2 = k^2 - \frac{m^2\pi^2}{d^2}$$

group velocity:

$$v_m = c\cos\theta_m$$

Waveguiding

single mode condition for 600-nm light:

planar mirror $$M \doteqdot \frac{2d}{\lambda}$$ $$300 < d < 600 \text{ nm}$$

dielectric $$M \doteq 2\frac{d}{\lambda}(n_1^2 - n_2^2)^{1/2}$$ $$d < 268 \text{ nm}$$

Waveguiding

single mode condition for 600-nm light:

planar mirror $$M \doteqdot \frac{2d}{\lambda}$$ $$300 < d < 600 \text{ nm}$$

dielectric $$M \doteq 2\frac{d}{\lambda}(n_1^2 - n_2^2)^{1/2}$$ $$d < 268 \text{ nm}$$

can make d **larger by making** $n_1 - n_2$ **smaller!**

Waveguiding

Vector potential obeys:

$$\nabla^2 \vec{A} + \omega^2 \mu_o \epsilon \vec{A} = -i\omega\mu_o \nabla \epsilon \Phi$$

Waveguiding

Vector potential obeys:

$$\nabla^2 \vec{A} + \omega^2 \mu_o \epsilon \vec{A} = 0$$

Waveguiding

Vector potential obeys:

$$\nabla^2 \vec{A} + \omega^2 \mu_o \epsilon \vec{A} = 0$$

Substituting $$\vec{A} = \hat{y} u(x,y) e^{-i\beta z}$$

Waveguiding

Vector potential obeys:

$$\nabla^2 \vec{A} + \omega^2 \mu_o \epsilon \vec{A} = 0$$

Substituting $$\vec{A} = \hat{y} u(x,y) e^{-i\beta z}$$

yields:

$$\nabla_T^2 u + [-\beta^2 + \omega^2 \mu \epsilon(r)] u = 0$$

Waveguiding

Vector potential obeys:

$$\nabla^2 \vec{A} + \omega^2 \mu_o \epsilon \vec{A} = 0$$

Substituting $$\vec{A} = \hat{y} u(x,y) e^{-i\beta z}$$

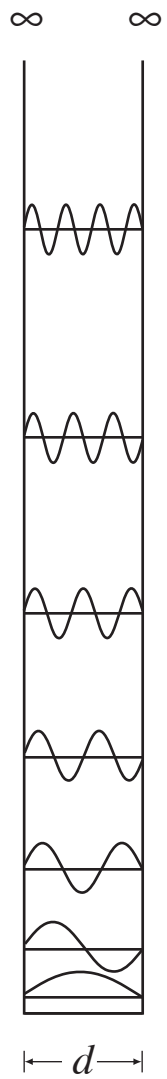
yields:

$$\nabla_T^2 u + [-\beta^2 + \omega^2 \mu \epsilon(r)] u = 0$$

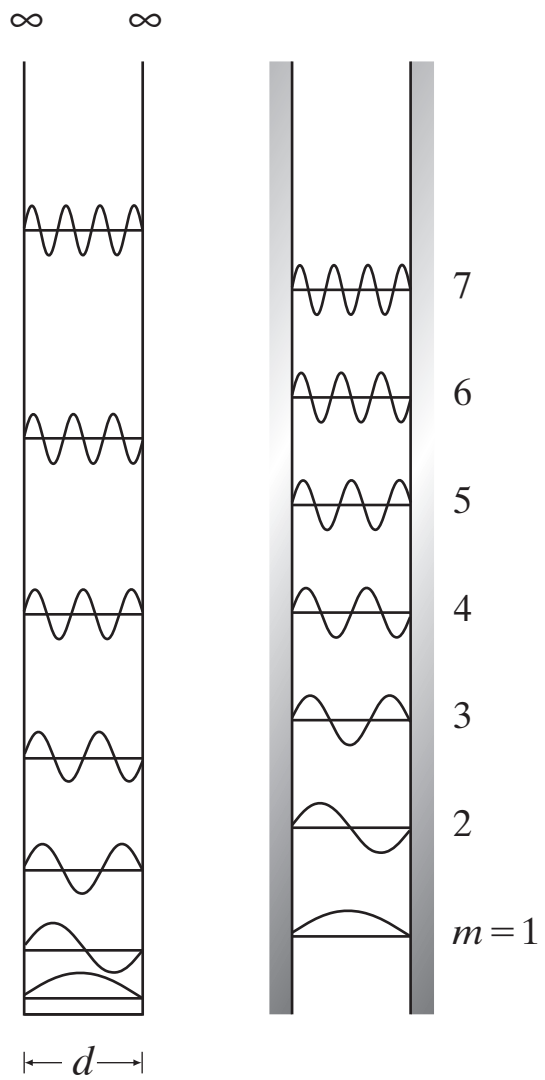
Compare to time-independent Schrödinger equation:

$$\nabla^2 \psi + \frac{2m}{\hbar^2}[E - V(r)]\psi = 0$$

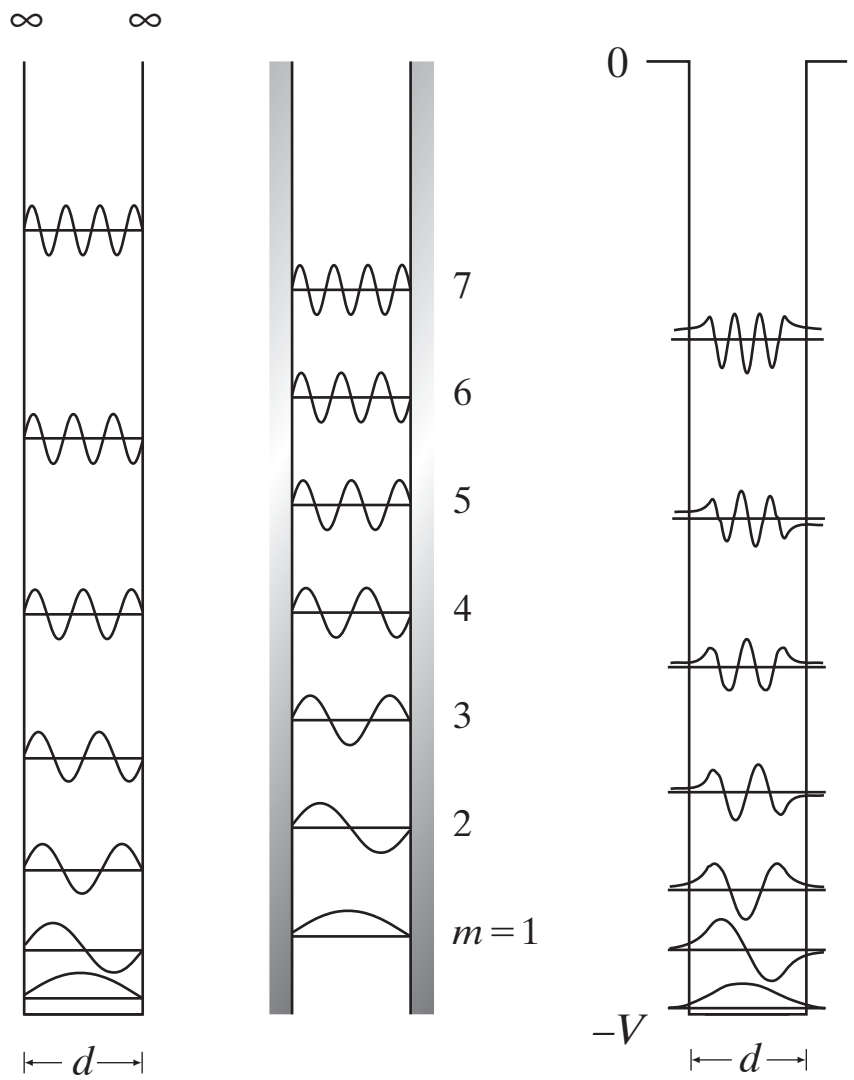
Waveguiding



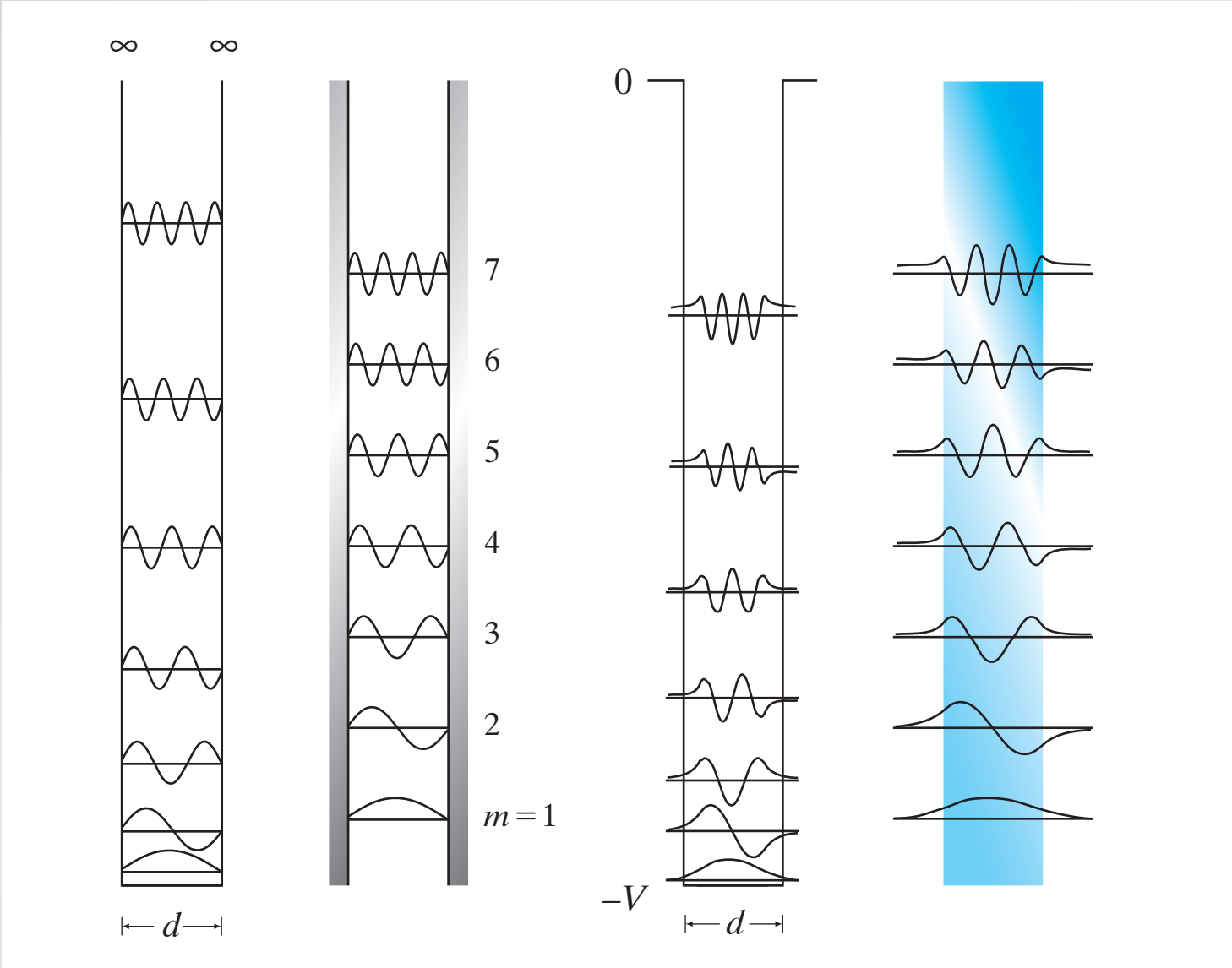
Waveguiding



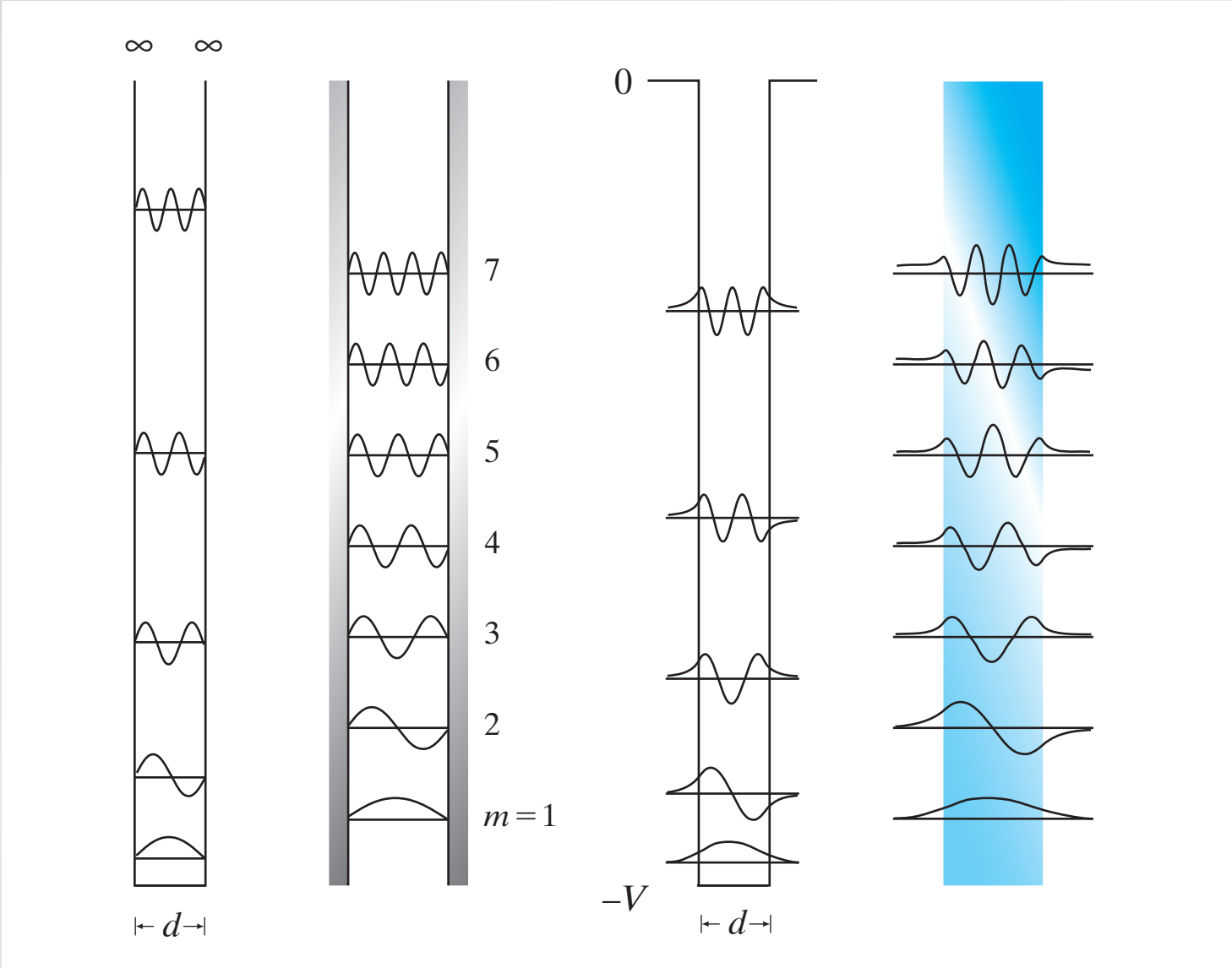
Waveguiding



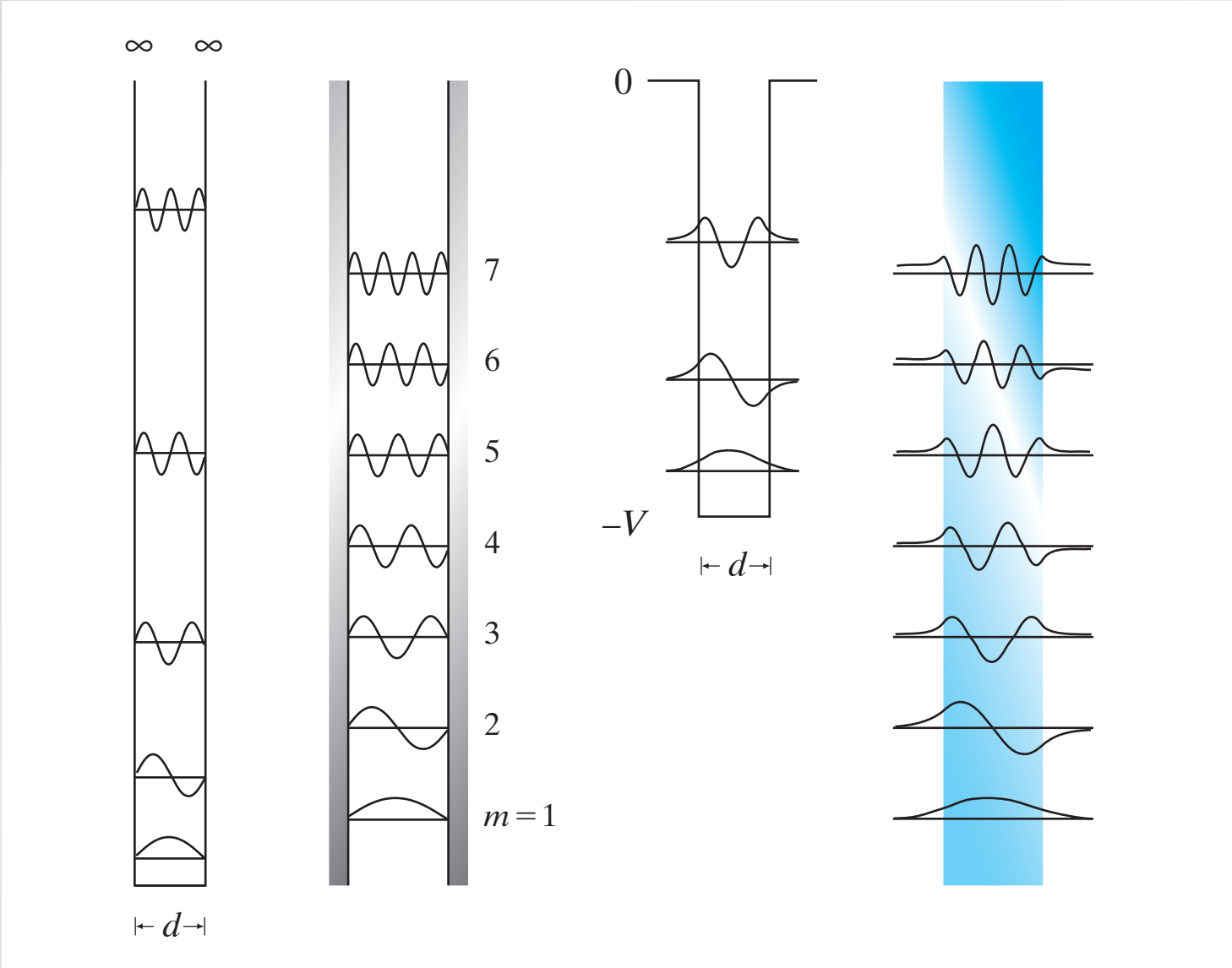
Waveguiding



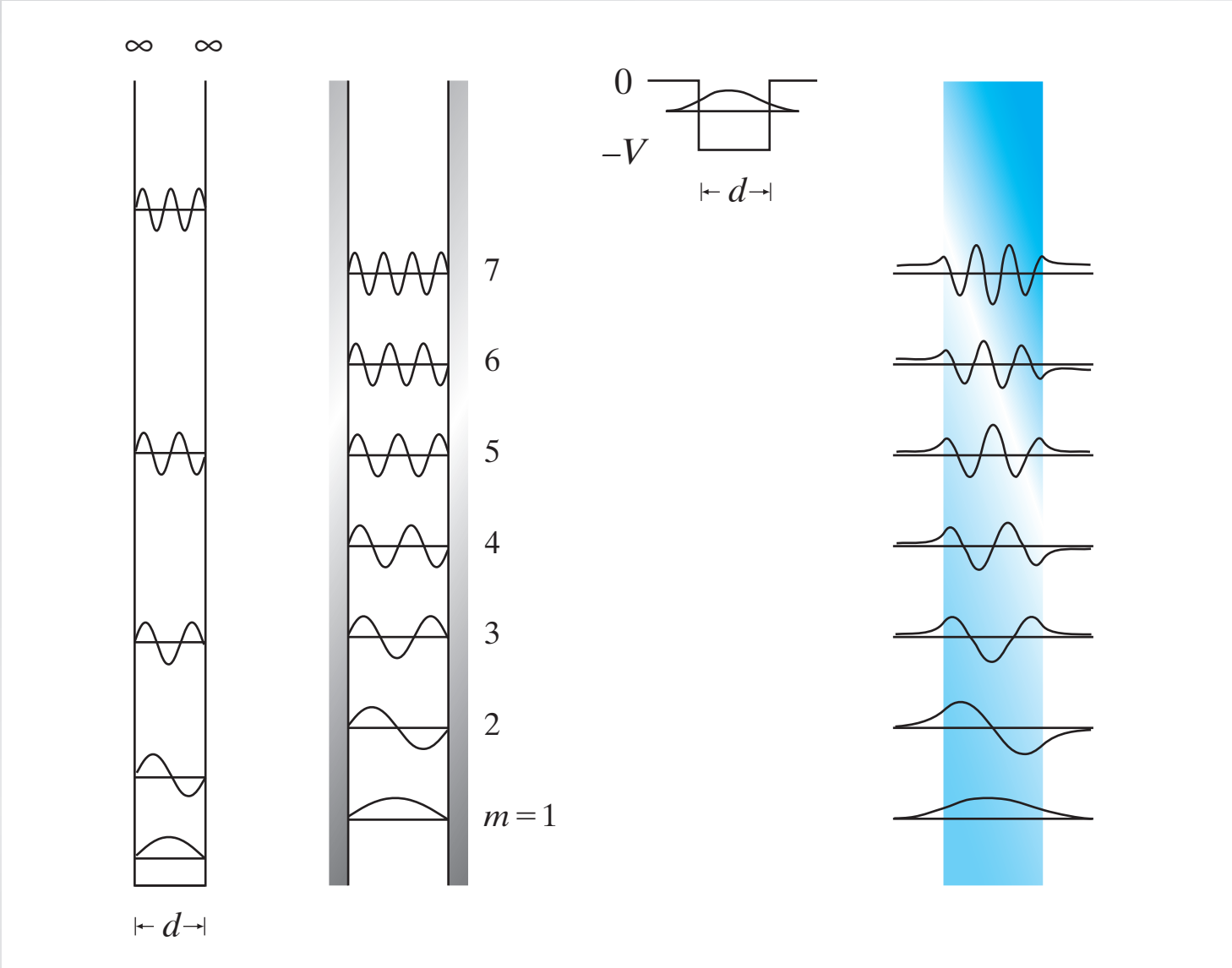
Waveguiding



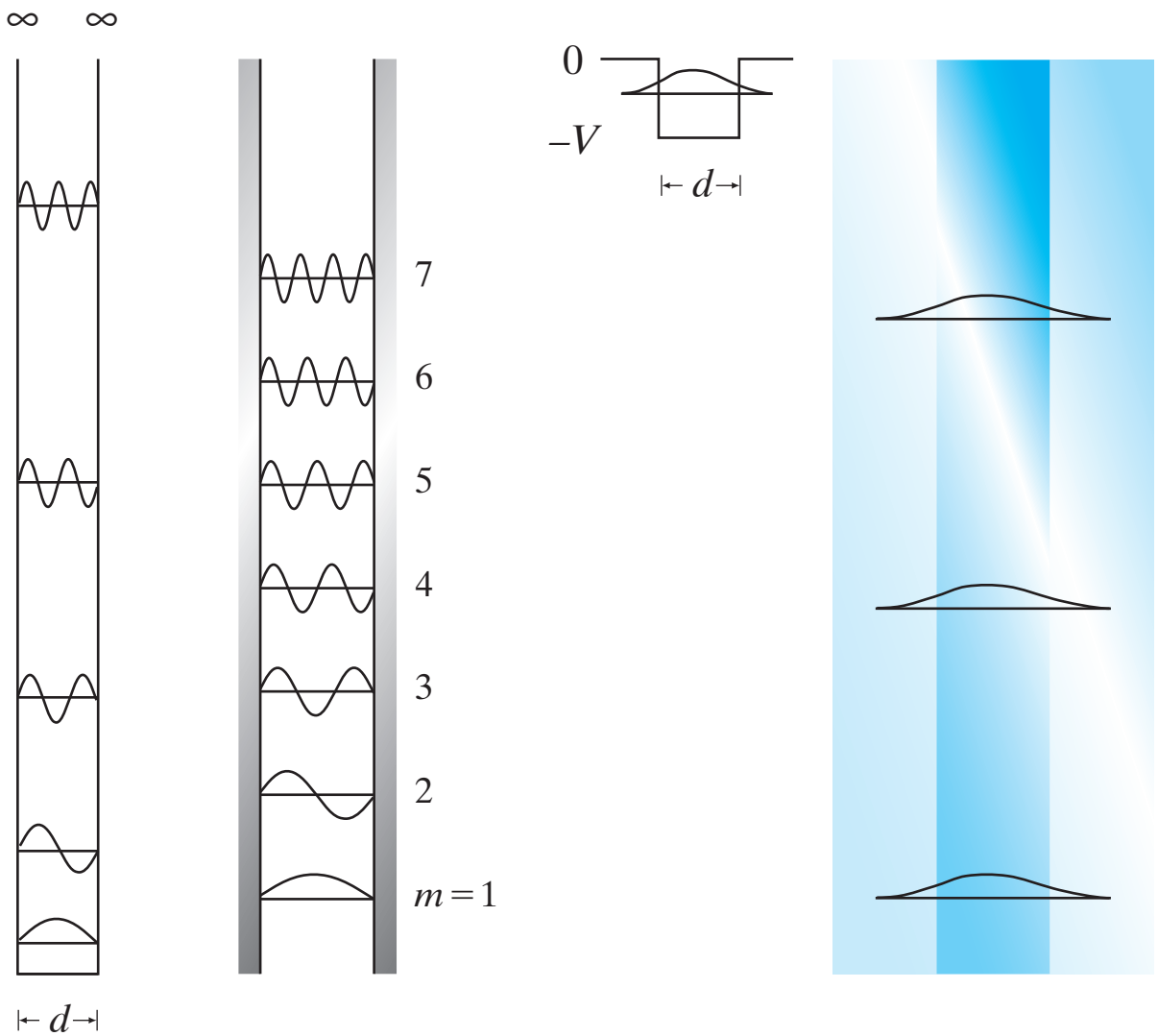
Waveguiding



Waveguiding



Waveguiding



Waveguiding

single mode condition for 600-nm light:

$$M \doteq 2\,\frac{d}{\lambda}(n_1^2 - n_2^2)^{1/2}$$

without cladding: $d < 268\ \mathrm{nm}$

Waveguiding

single mode condition for 600-nm light:

$$M \doteq 2\,\frac{d}{\lambda}(n_1^2 - n_2^2)^{1/2}$$

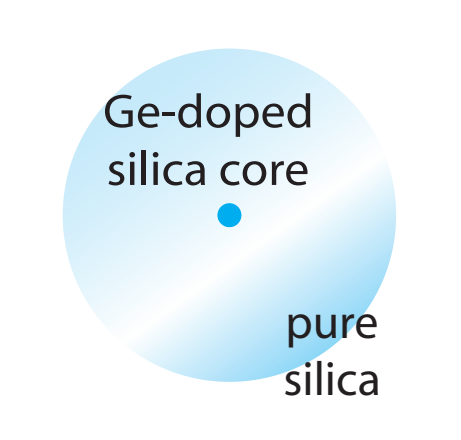
without cladding: $d < 268\ \mathrm{nm}$

Add cladding with 0.4% index difference:

$$d < 5\ \mu\mathrm{m}$$

Waveguiding

commercial single-mode fiber (Corning Titan®)



	core	cladding
index	$n_1 = 1.468$	$n_2 = 1.462$
diameter:	$8.3\ \mu\text{m}$	$125.0 \pm 1.0\ \mu\text{m}$

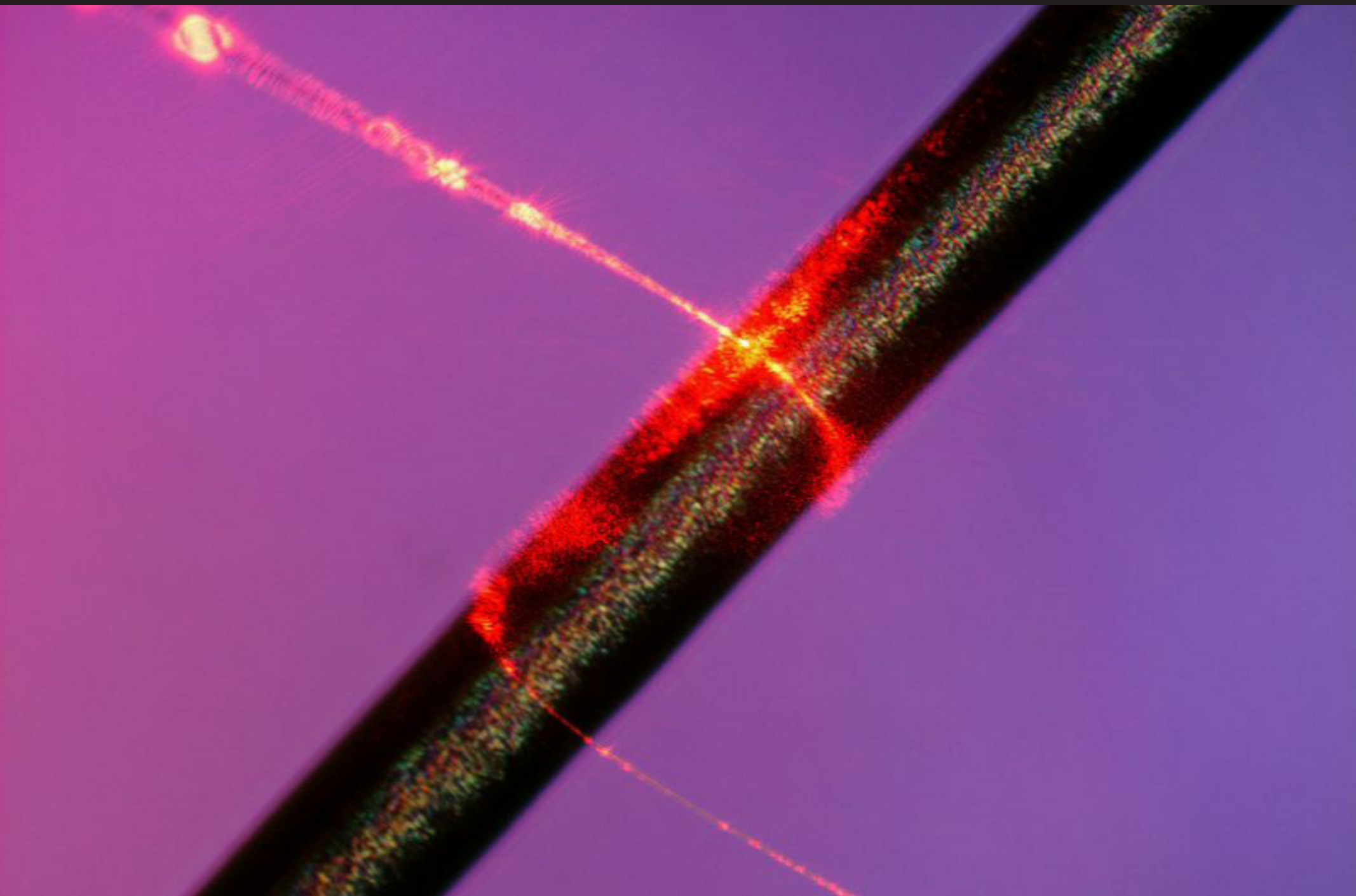
operating wavelength: $\lambda = 1310\ \text{nm}/1550\ \text{nm}$

Waveguiding

drawbacks of clad fibers:

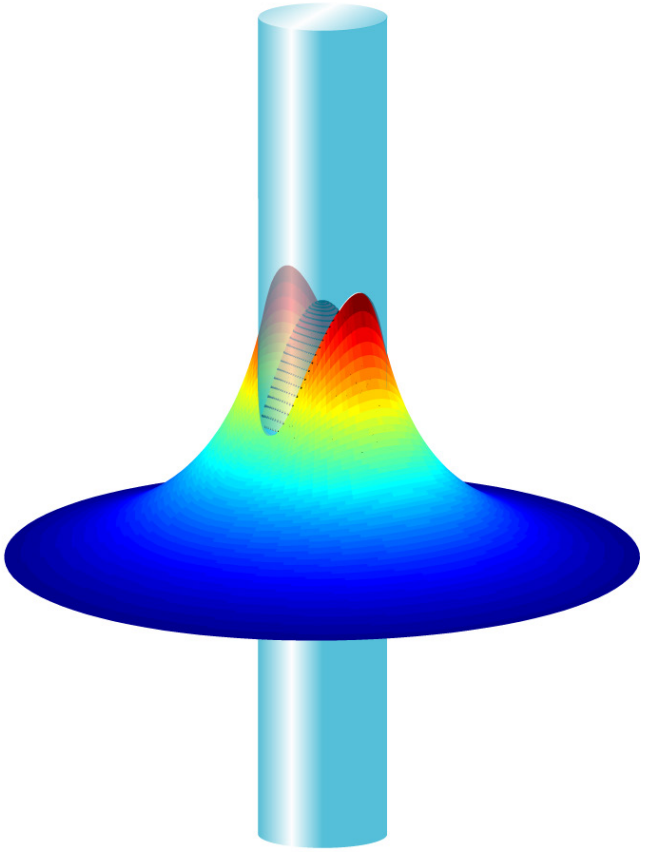
- weak confinement
- no tight bending

Waveguiding at the nanoscale

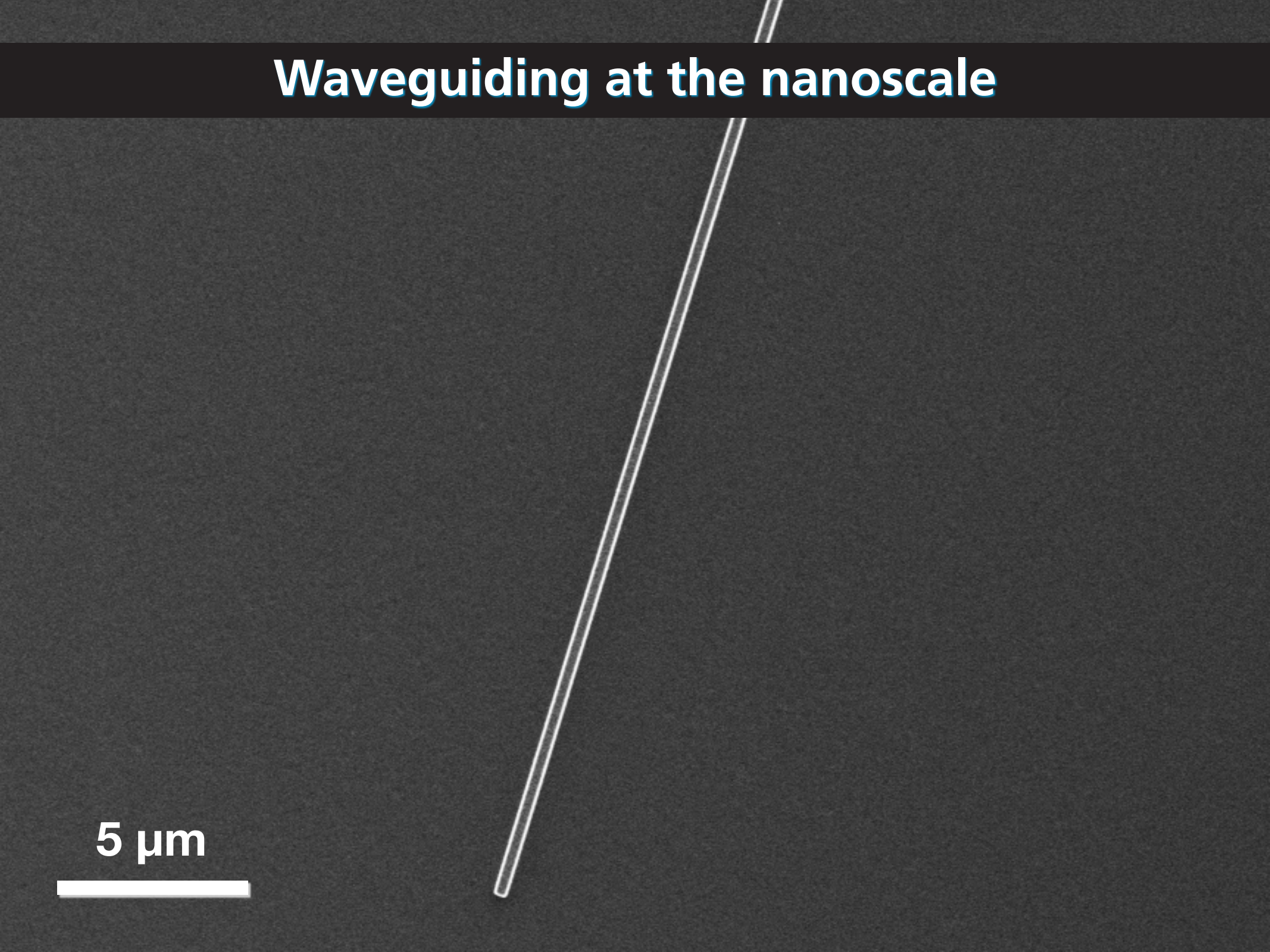


Waveguiding at the nanoscale

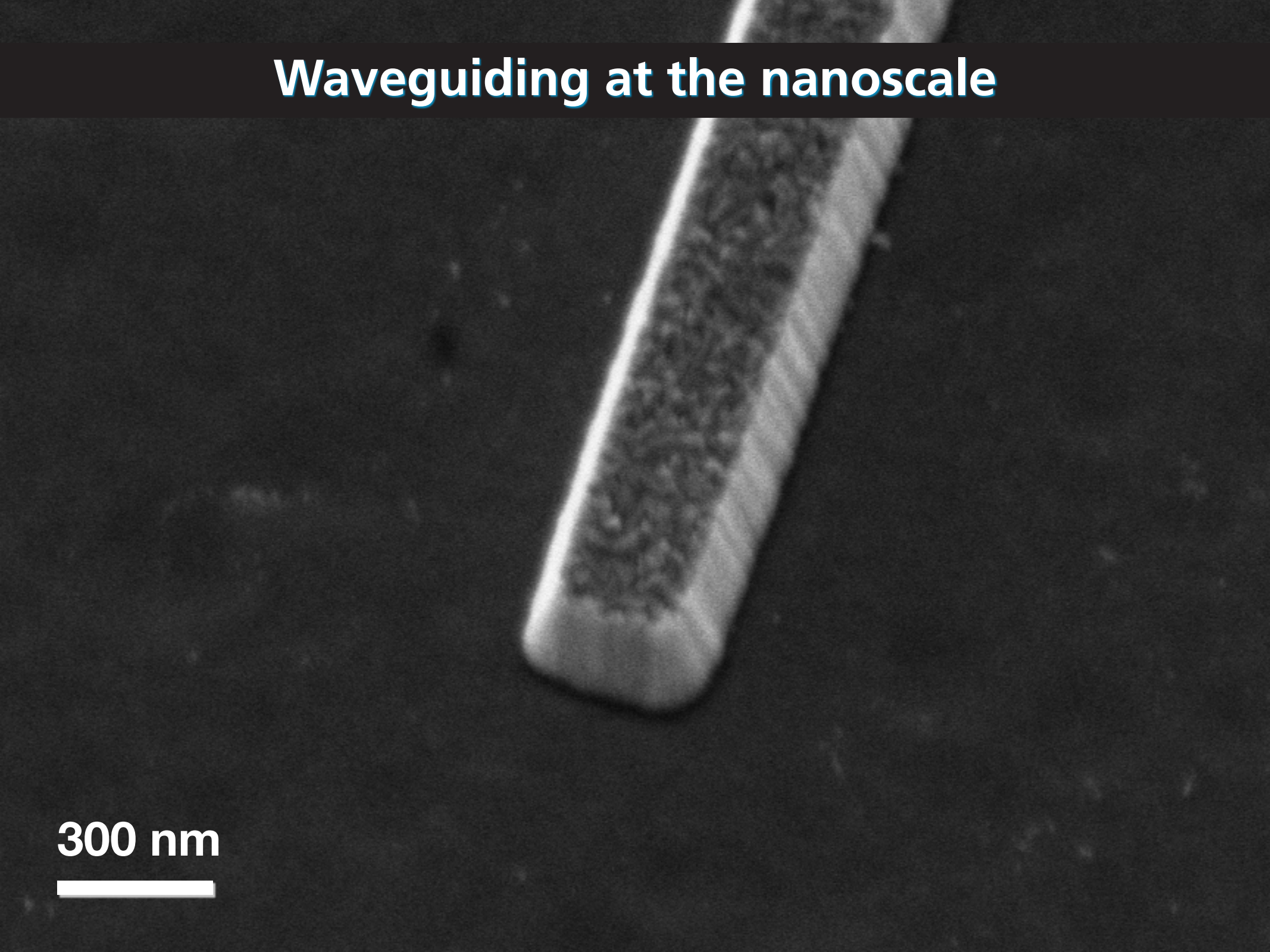
Poynting vector profile for 200-nm nanowire



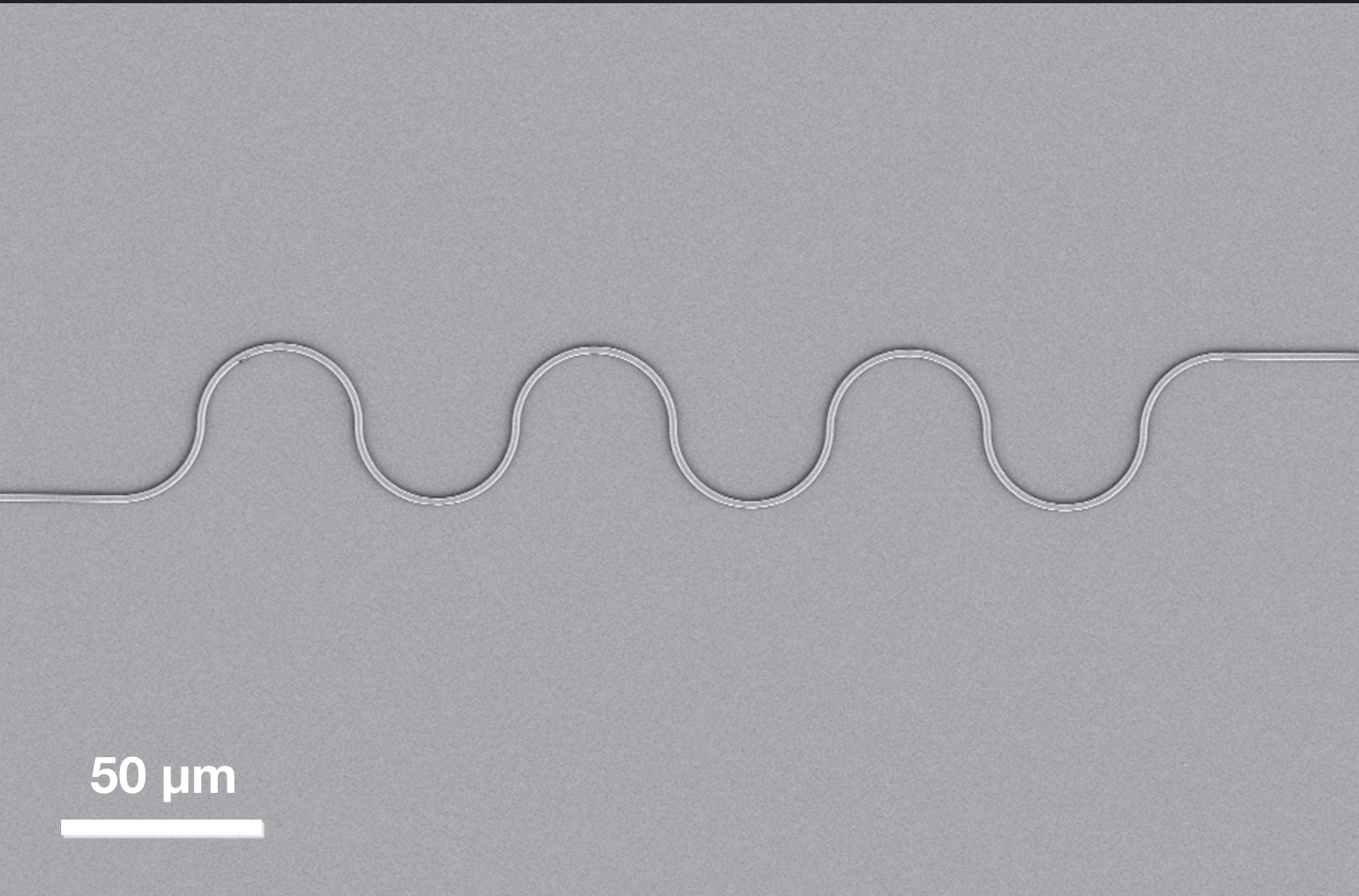
Waveguiding at the nanoscale



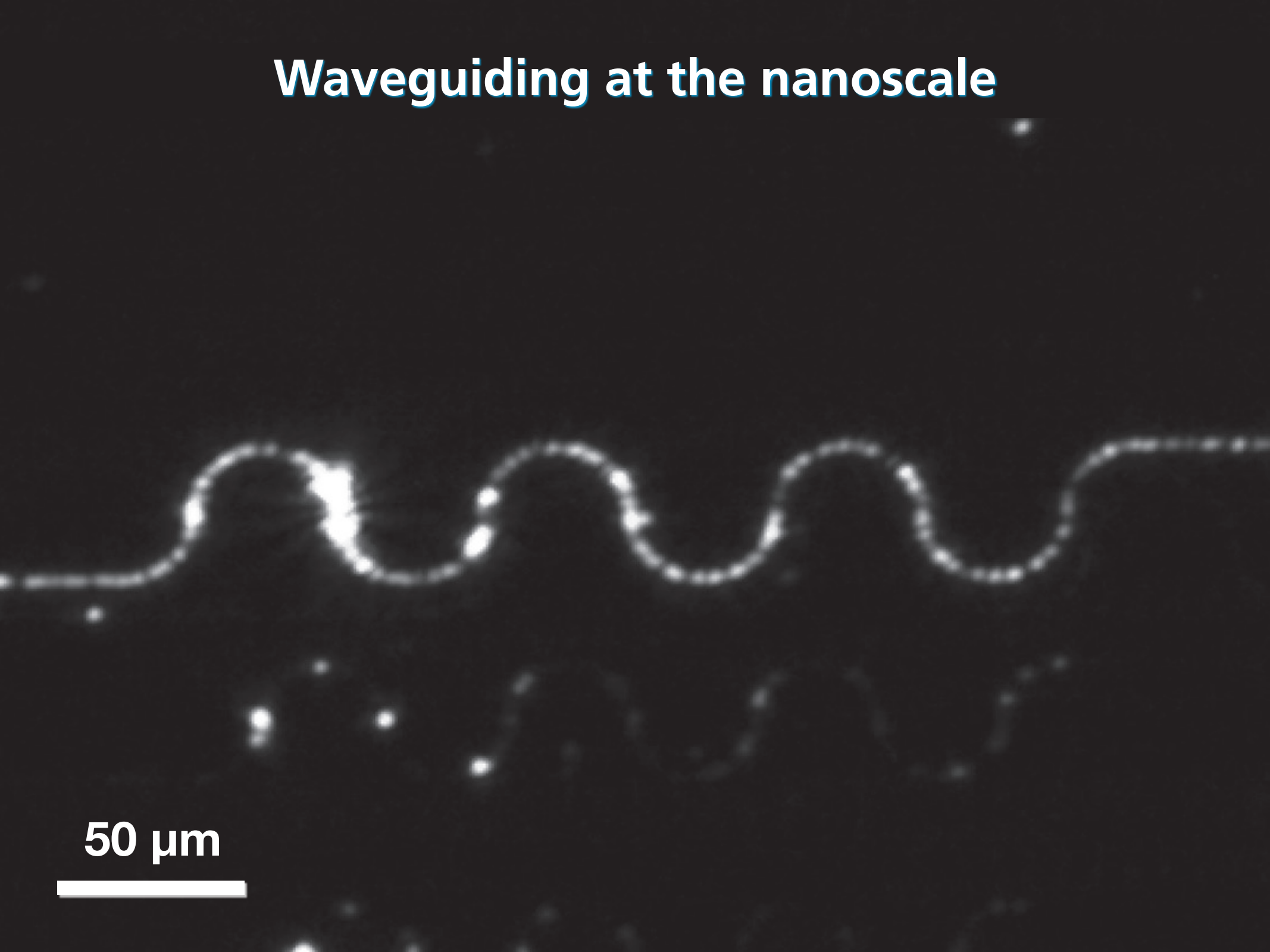
Waveguiding at the nanoscale



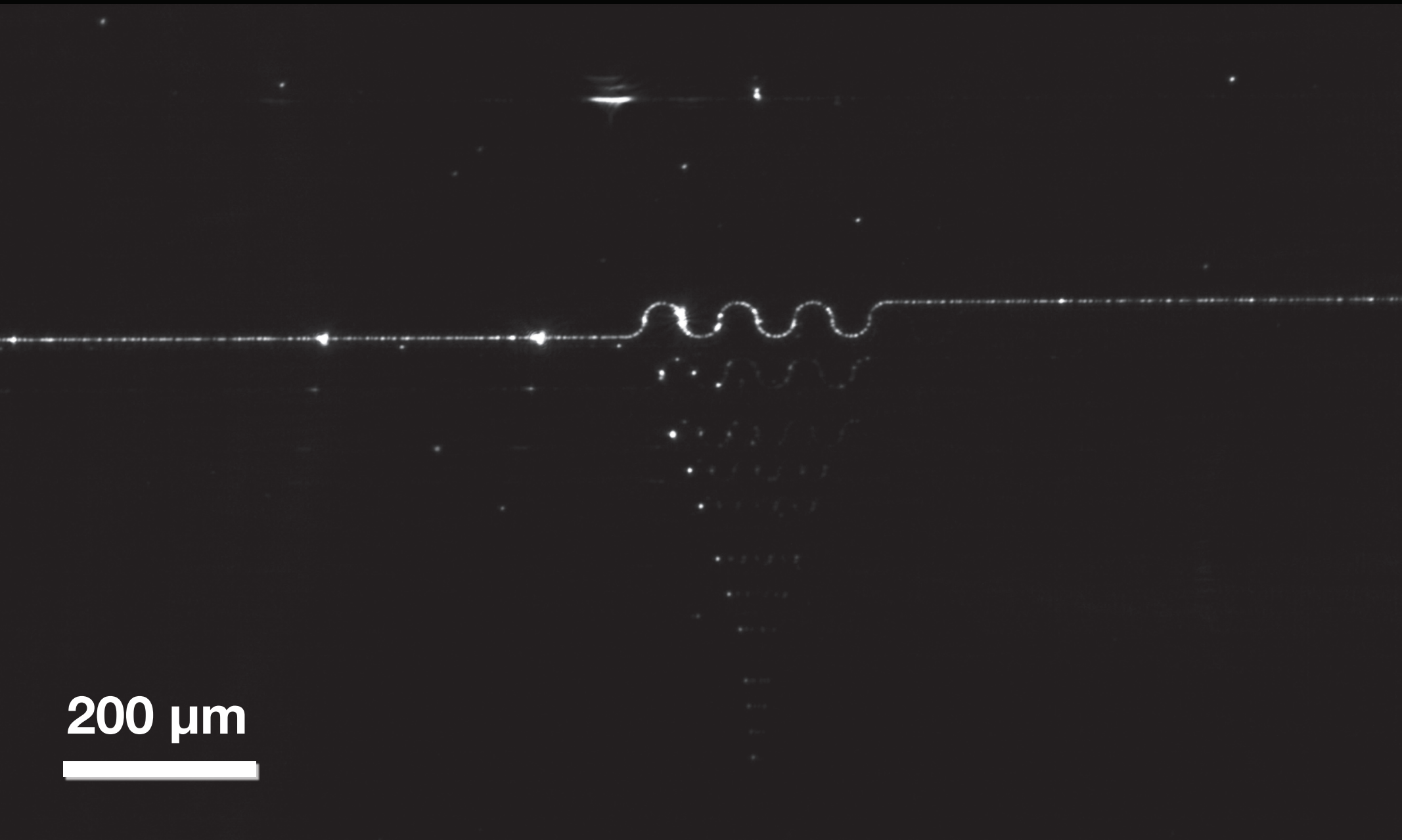
Waveguiding at the nanoscale



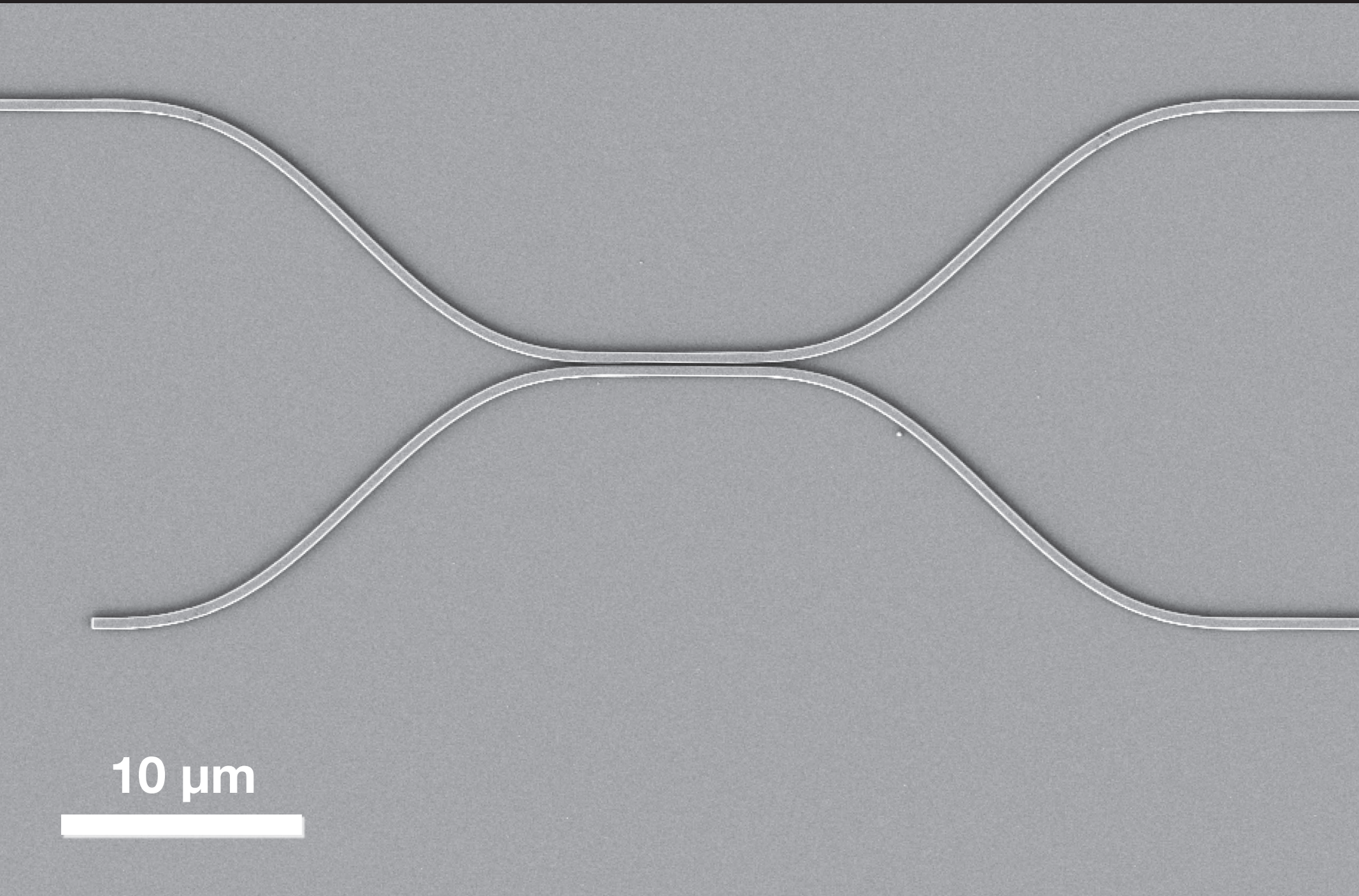
Waveguiding at the nanoscale



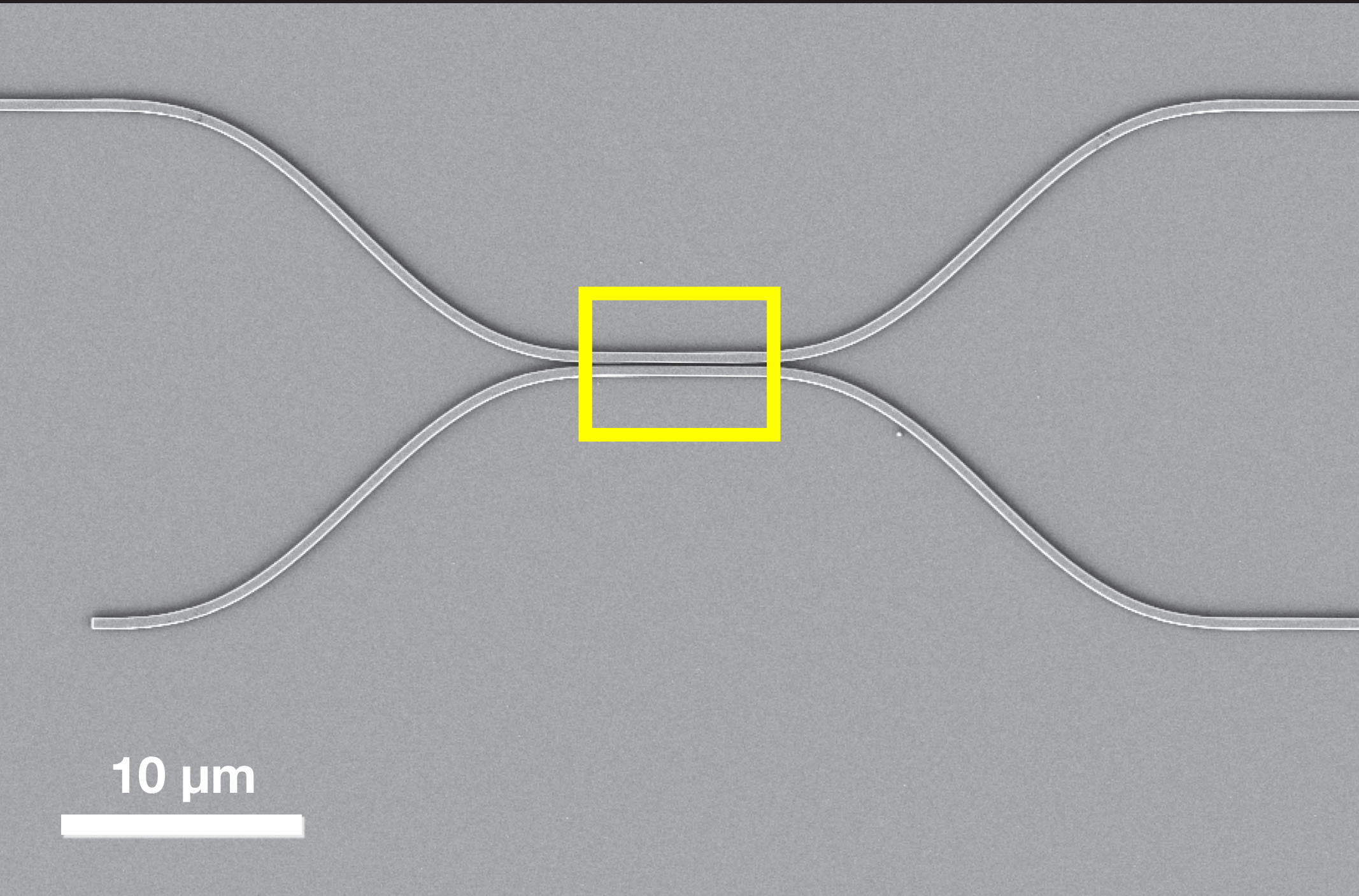
Waveguiding at the nanoscale



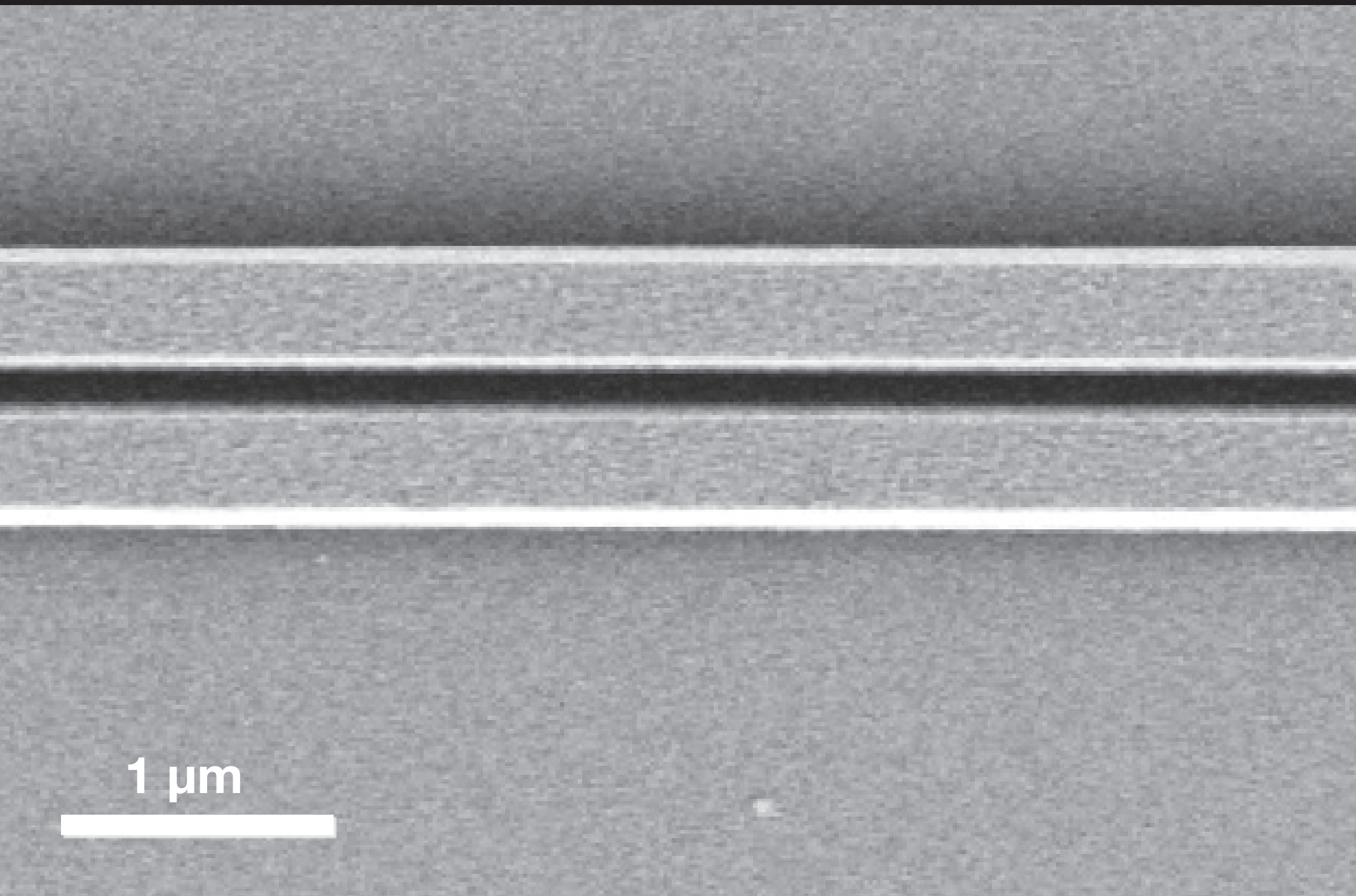
Waveguiding at the nanoscale



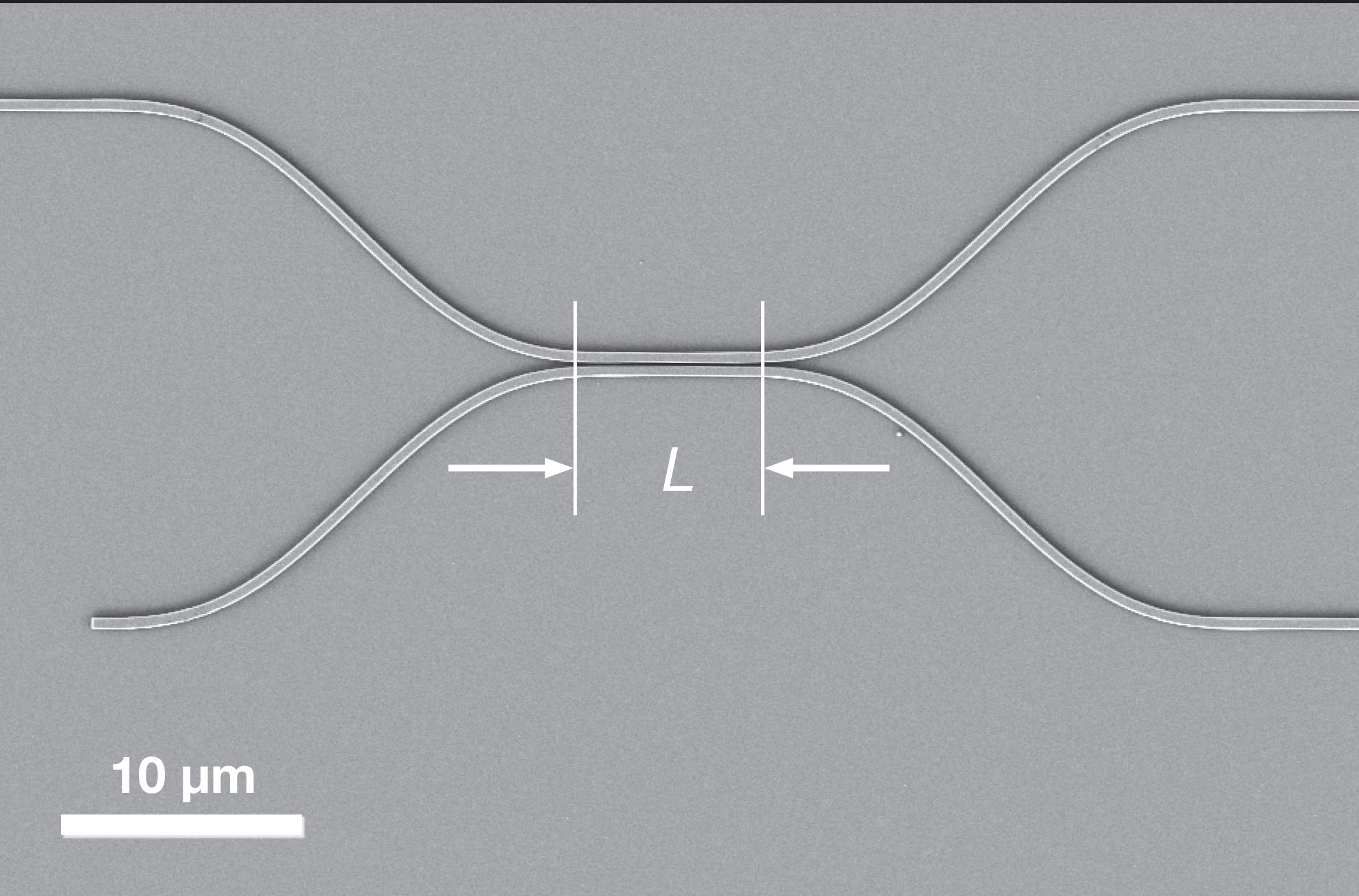
Waveguiding at the nanoscale



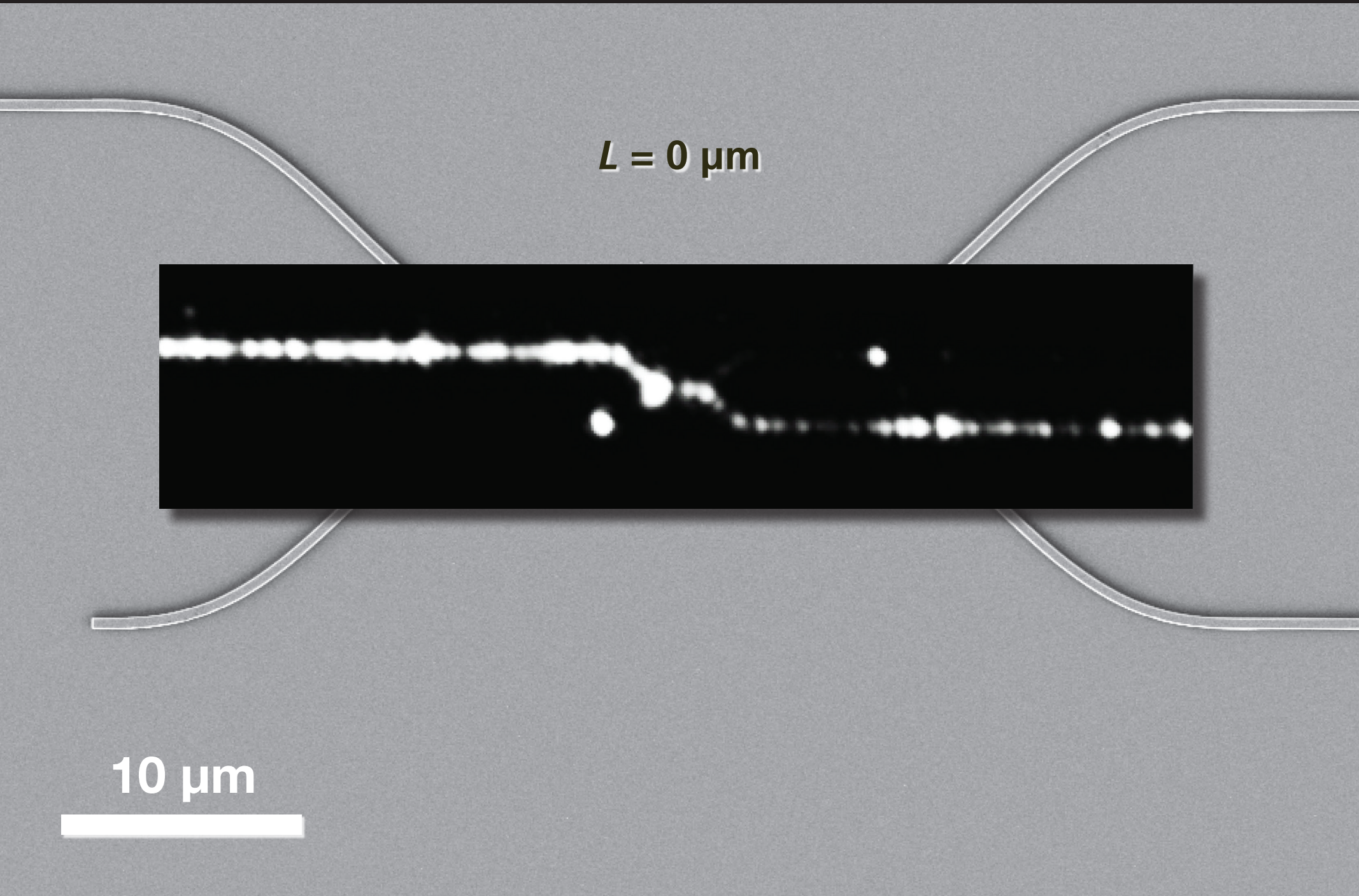
Waveguiding at the nanoscale



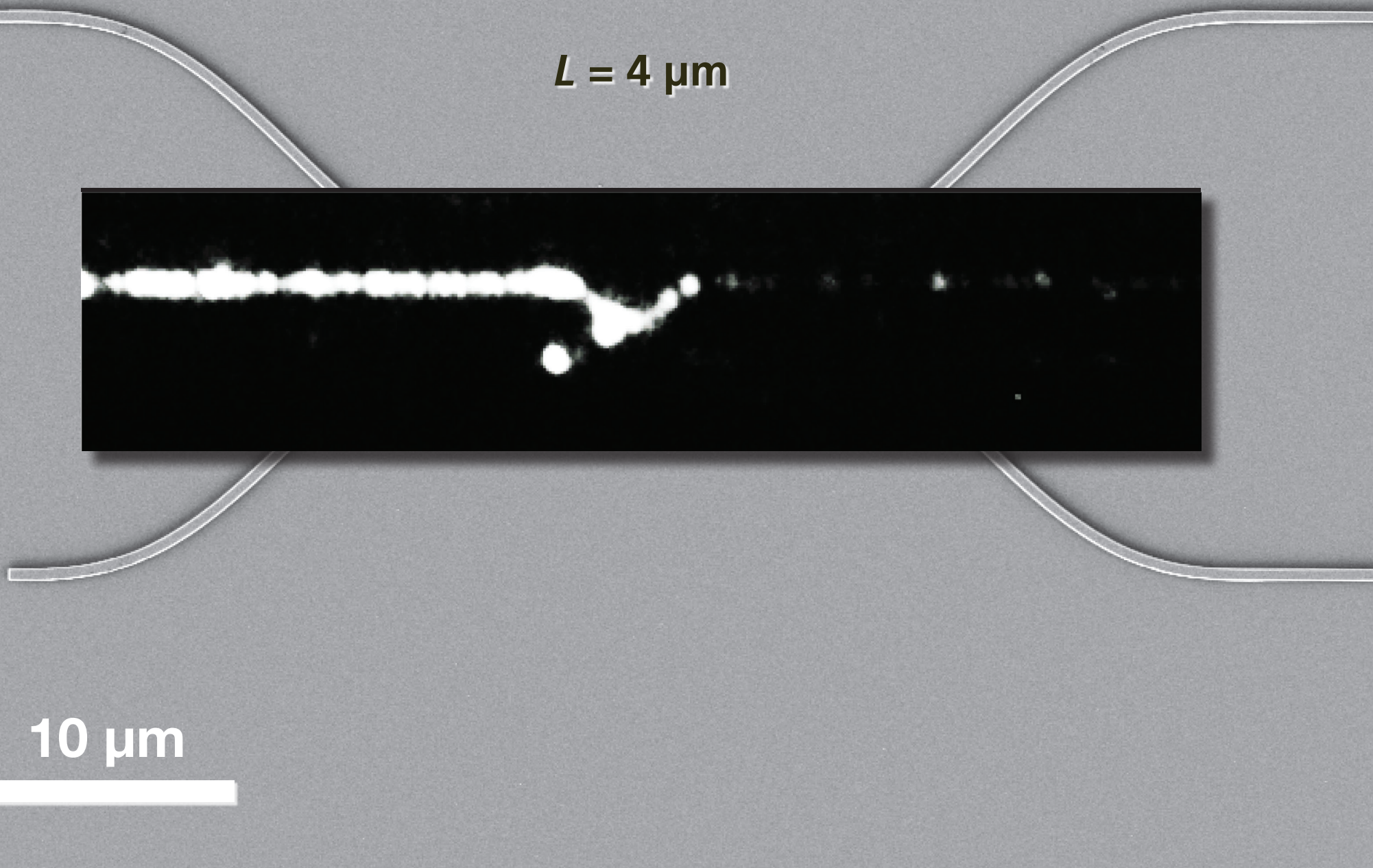
Waveguiding at the nanoscale



Waveguiding at the nanoscale



Waveguiding at the nanoscale



Waveguiding

Key points

- finite structures support a discrete set of modes
- each mode determined by boundary condition and extent
- each mode has unique field distribution
- modes unchanged as they propagage

Outline

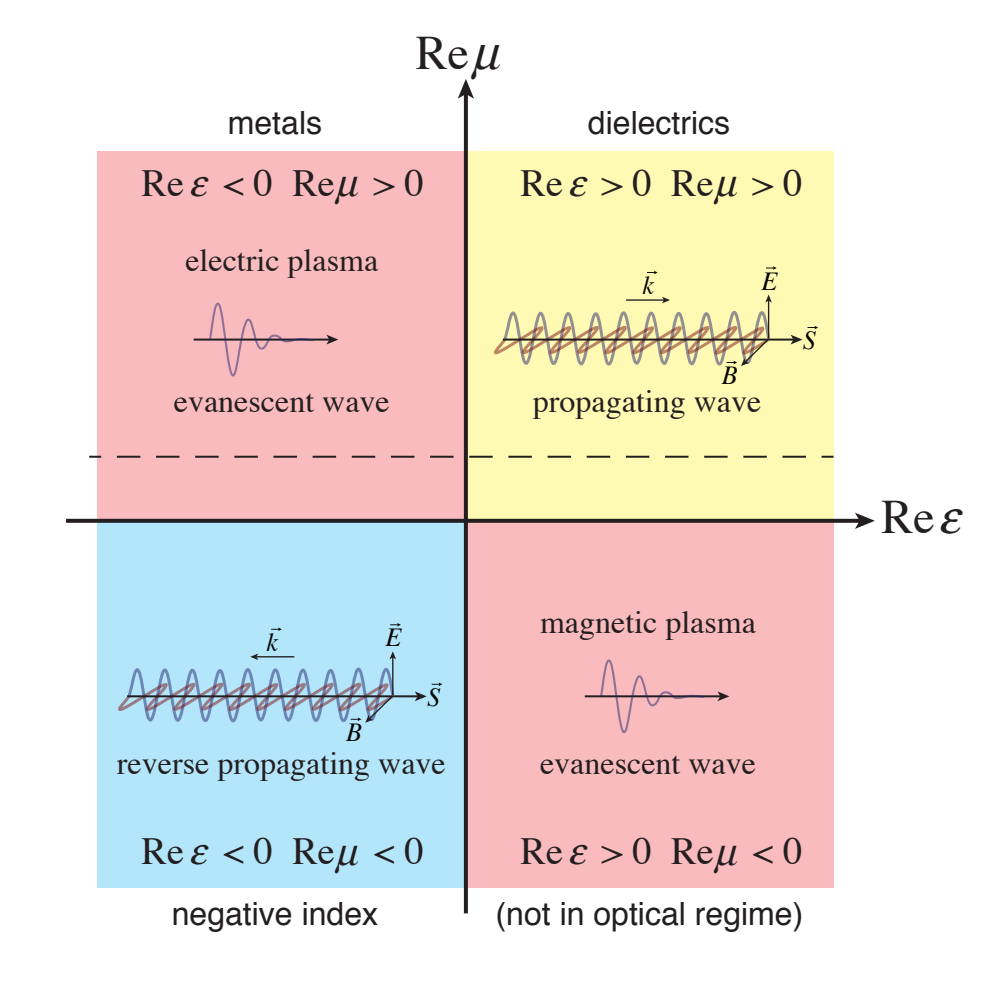
- optical properties of materials
- dispersion of pulses
- nonlinear optics
- waveguiding
- **engineering the index**

Engineering the index

how to optimize manipulation of light at nanoscale?

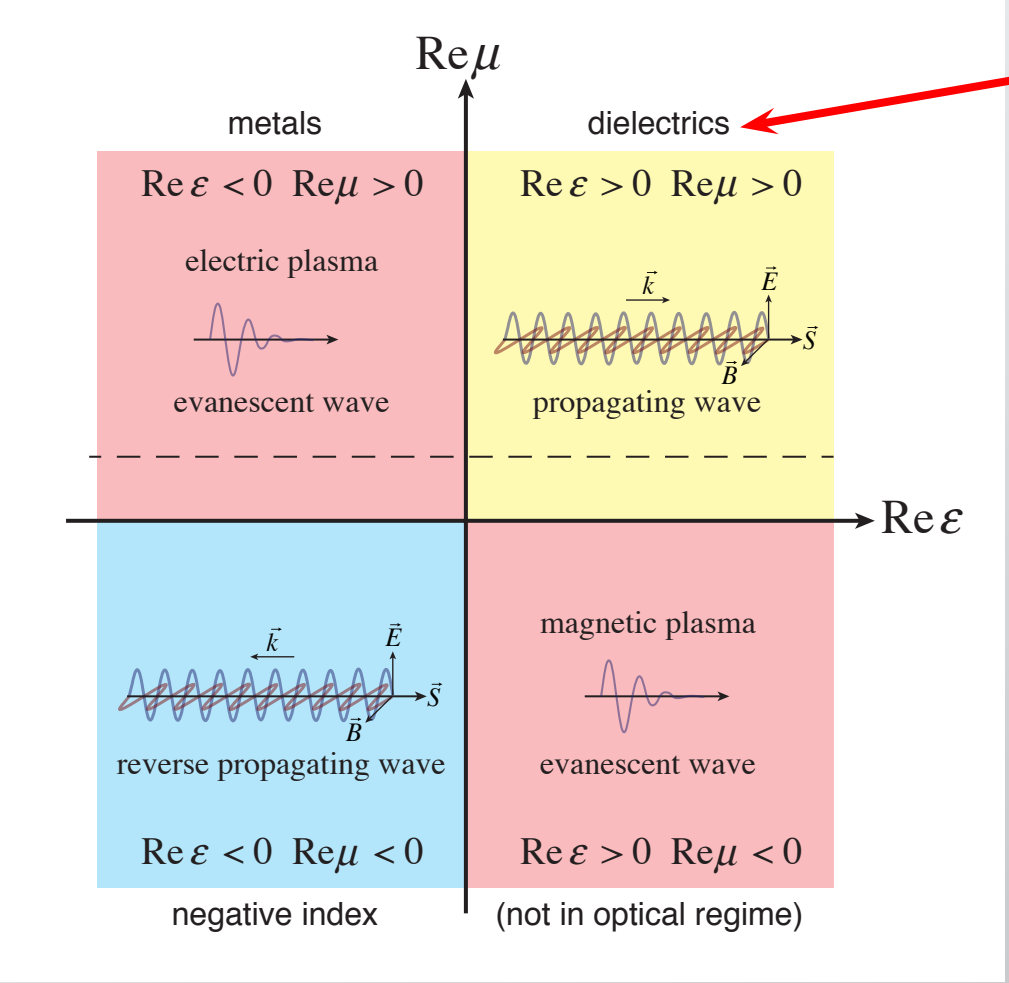
Engineering the index

common materials very limited



Engineering the index

common materials very limited

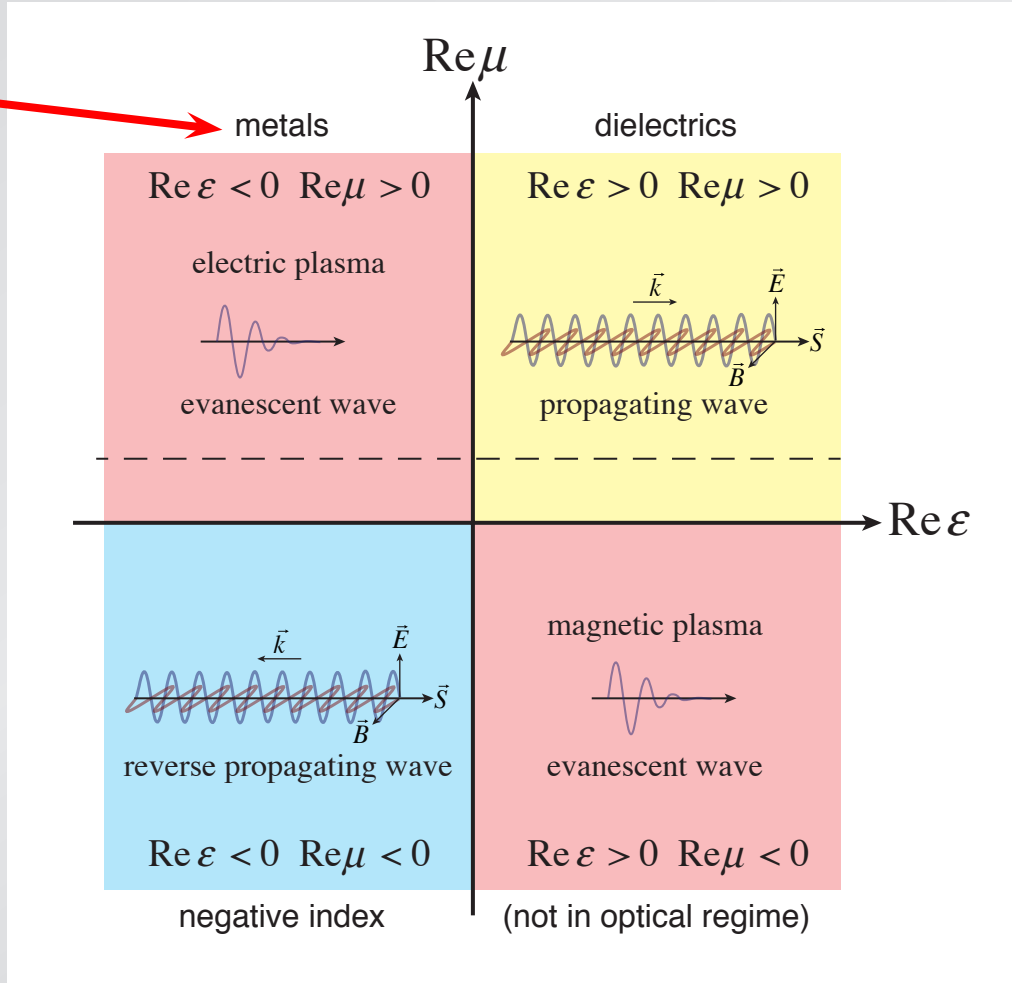


limited by diffraction

Engineering the index

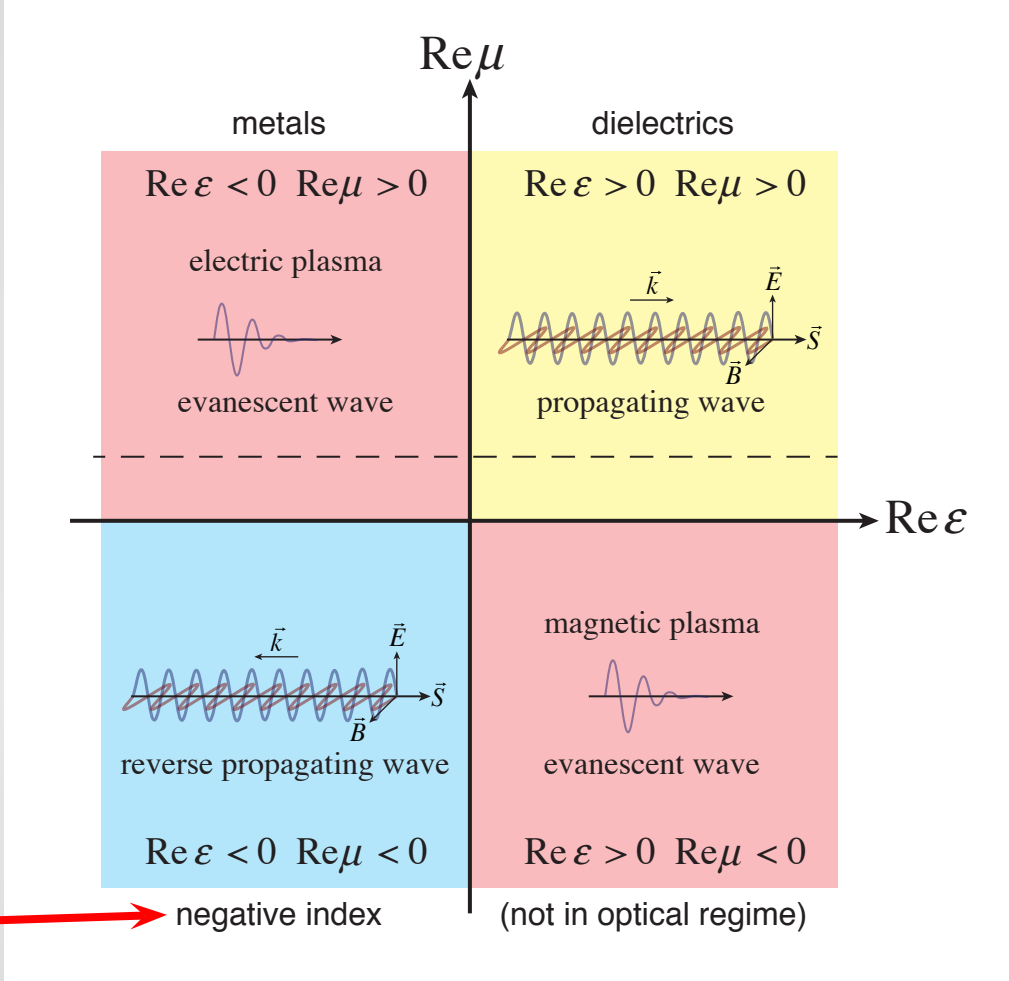
common materials very limited

lossy & no propagation



Engineering the index

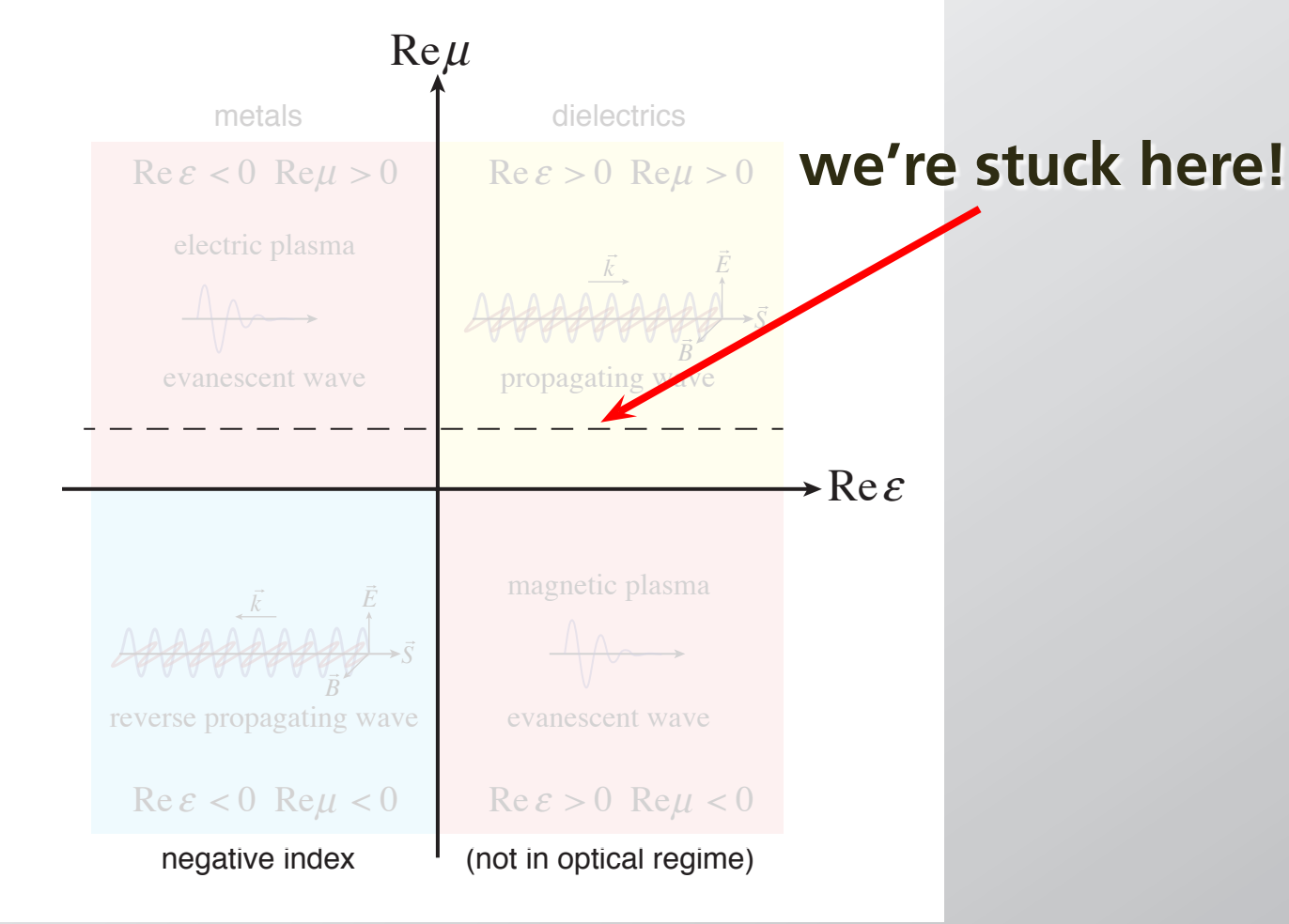
common materials very limited



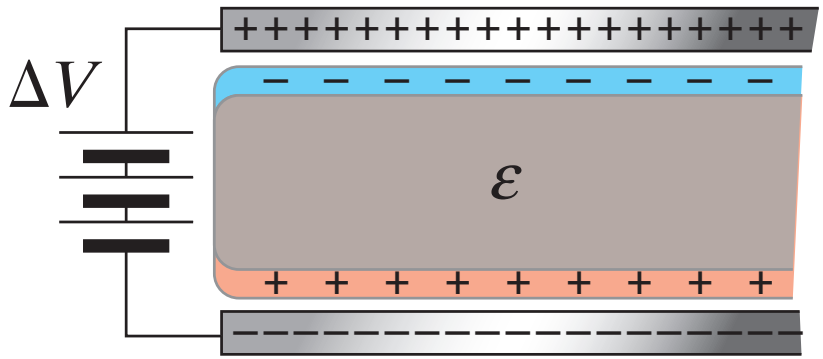
superlensing but…

Engineering the index

common materials very limited

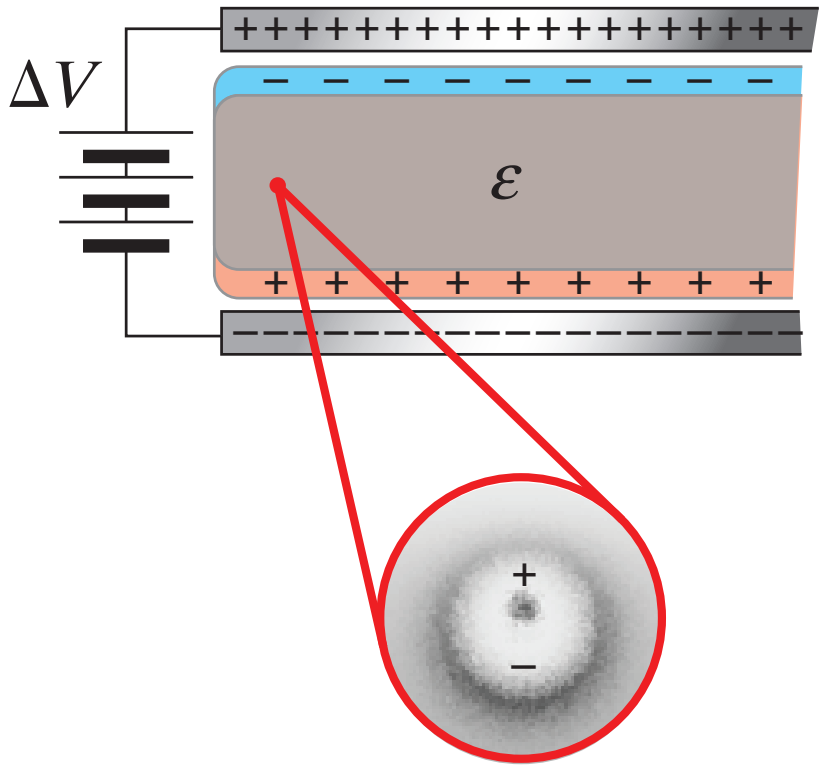


Engineering the index



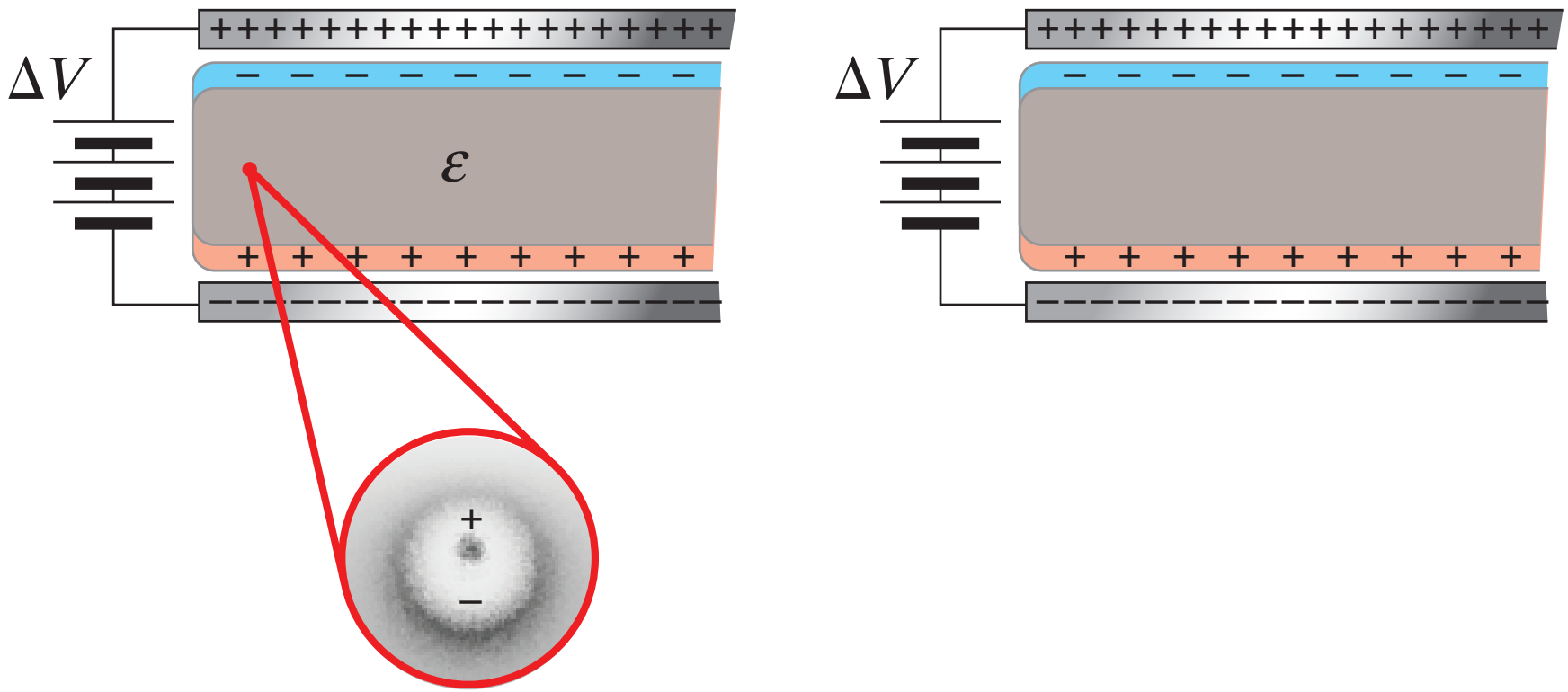
Engineering the index

dielectric constant due to polarization of atoms



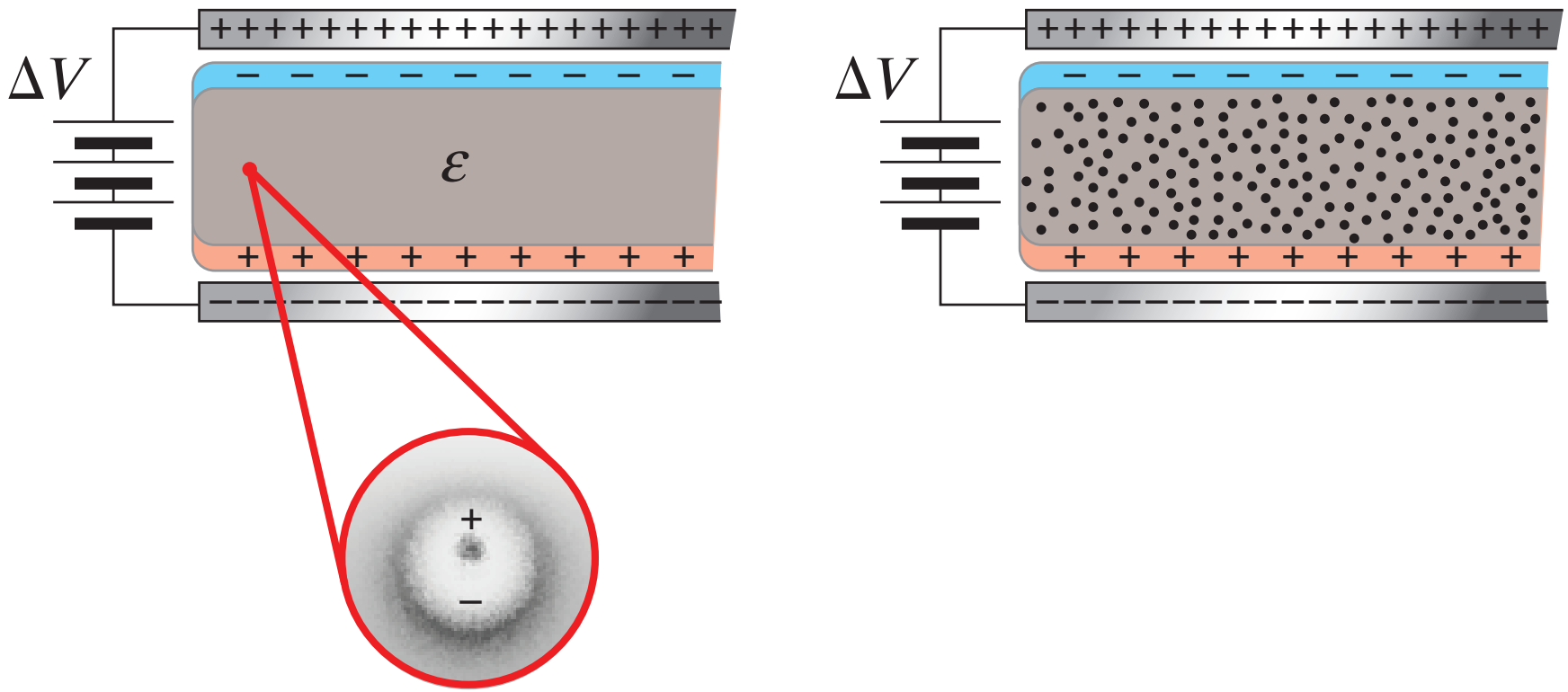
Engineering the index

metal-dielectric composite



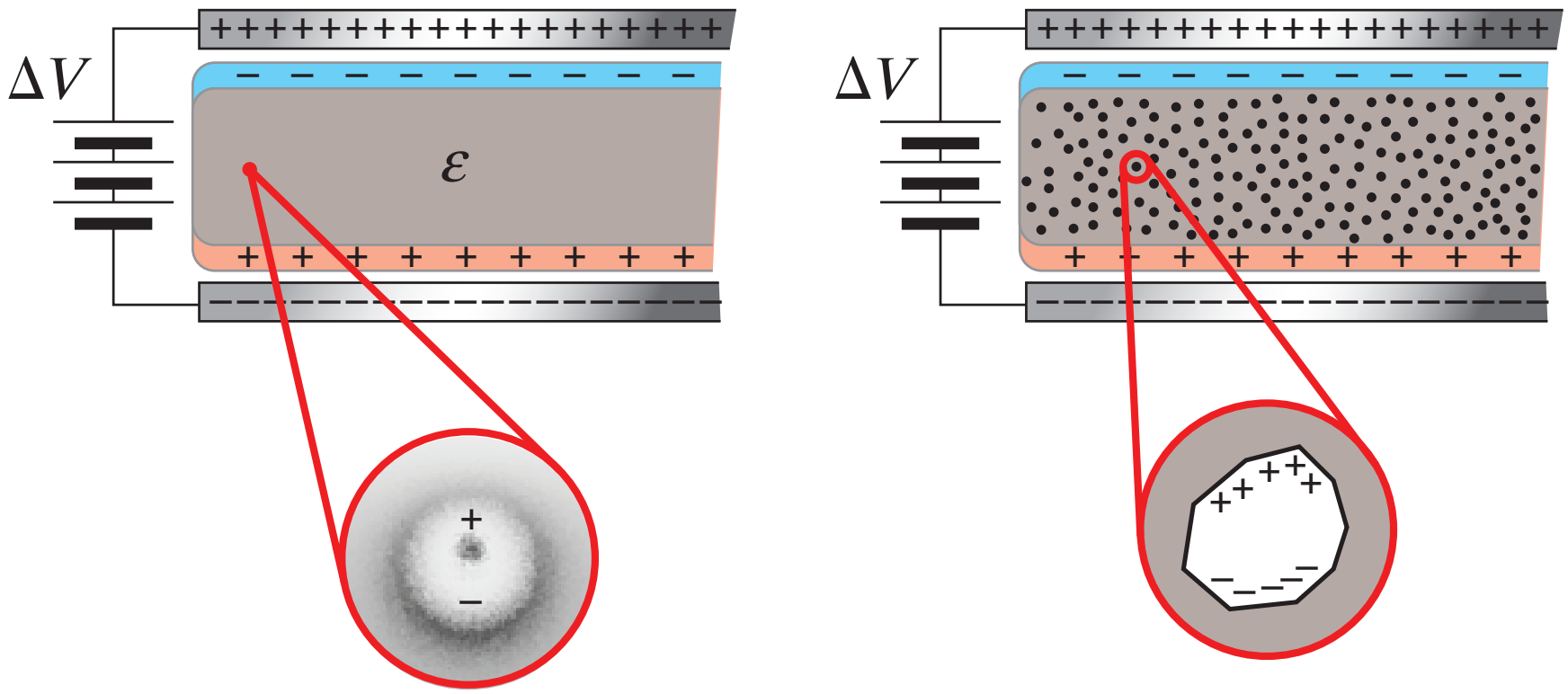
Engineering the index

metal-dielectric composite



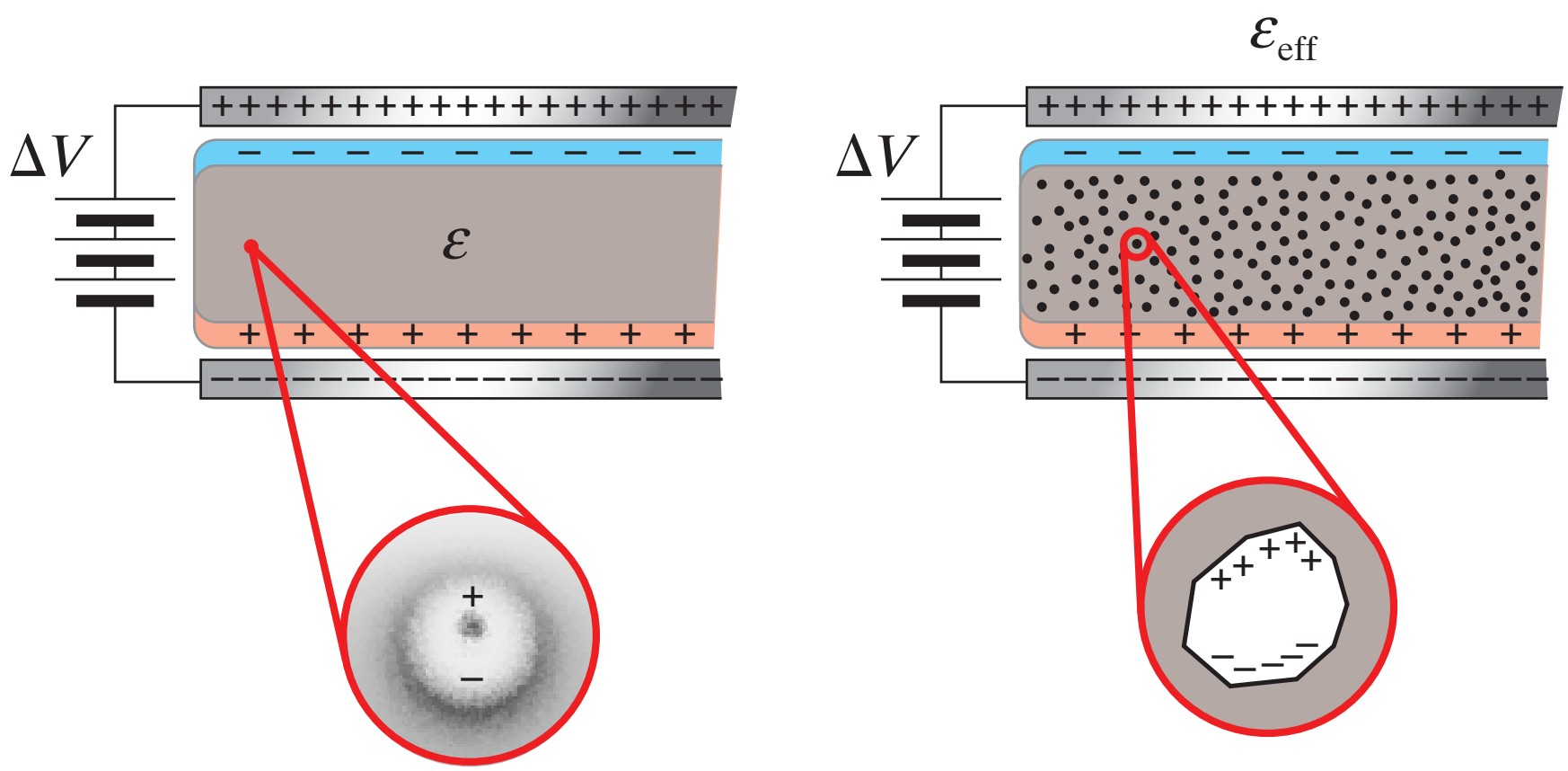
Engineering the index

polarization of metal particles increases dielectric constant



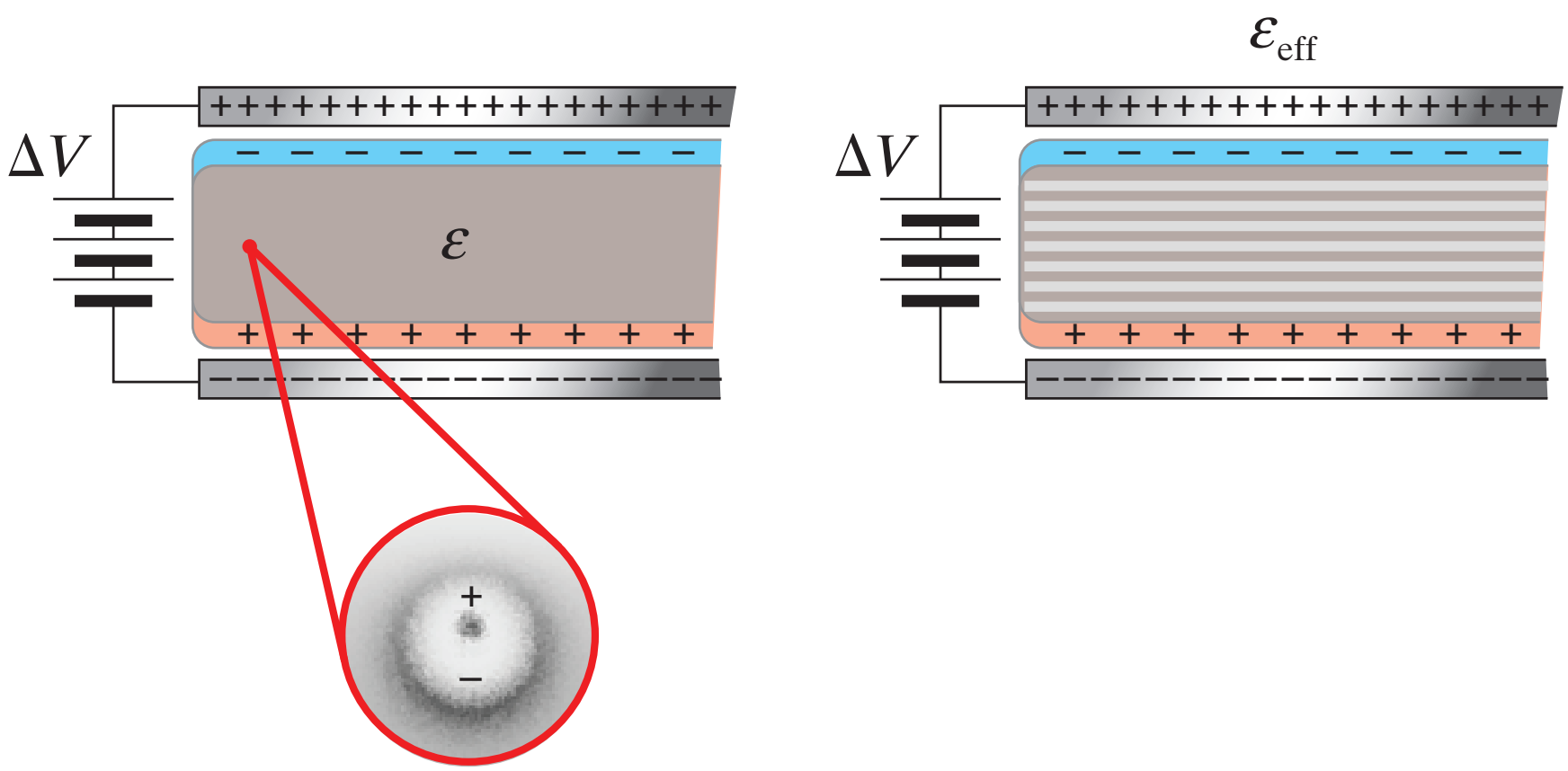
Engineering the index

provided $d \leq \lambda_{\text{eff}}$ can use effective dielectric constant



Engineering the index

can also do this with dielectric composite



Engineering the index

what if we let $\varepsilon = 0$?

Engineering the index

what if we let $\varepsilon = 0$?

if $\varepsilon = 0$, then $n = 0$!

Zero index

Q: If $n = 0$, which of the following is true?

1. the frequency goes to zero.
2. the phase velocity becomes infinite.
3. both of the above.
4. neither of the above.

Zero index

wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

Zero index

wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

solution

$$\vec{E} = \vec{E}_o \, e^{i\,(kx - \omega t)}$$

Zero index

wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

solution

$$\vec{E} = \vec{E}_o \, e^{i\,(kx - \omega t)}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}}\,c = \frac{1}{n}\,c$$

Zero index

wave equation

$$\nabla^2 \vec{E} - \cancel{\frac{\mu \epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}} = 0$$

solution

$$\vec{E} = \vec{E}_o \, e^{i\,(kx - \omega t)}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon \mu}} c = \frac{1}{n} c$$

Zero index

wave equation

$$\nabla^2 \vec{E} - \cancel{\frac{\mu \epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}} = 0$$

solution

$$\vec{E} = \vec{E}_o \, e^{i\,(kx - \omega t)} \longrightarrow \vec{E} = \vec{E}_o \, e^{-i\omega t}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon \mu}} c = \frac{1}{n} c$$

Zero index

wave equation

$$\nabla^2 \vec{E} - \cancel{\frac{\mu \epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}} = 0$$

solution

$$\vec{E} = \vec{E}_o \, e^{i\,(kx - \omega t)} \longrightarrow \vec{E} = \vec{E}_o \, e^{-i\omega t}$$

where

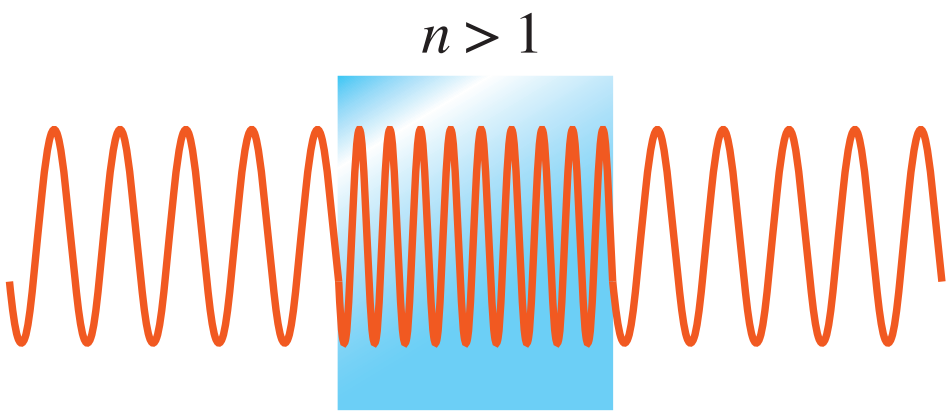
$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}} c = \frac{1}{n} c \longrightarrow \infty$$

Zero index

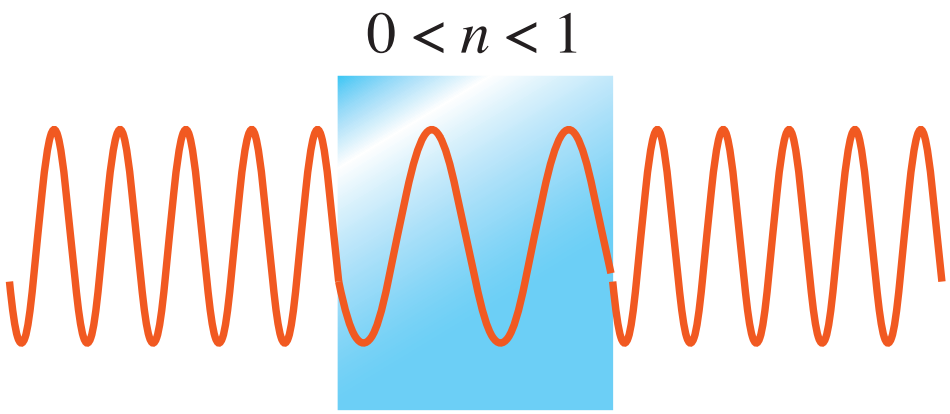
Q: If $n = 0$, which of the following is true?

1. the frequency goes to zero.
2. the phase velocity becomes infinite. ✔
3. both of the above.
4. neither of the above.

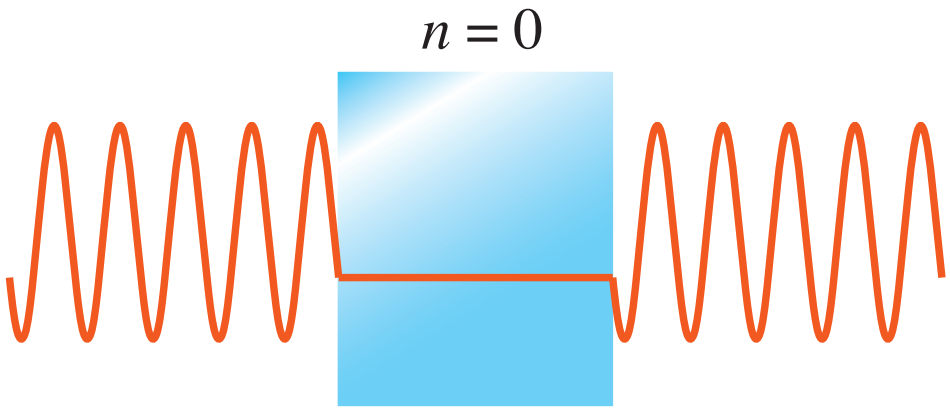
Zero index



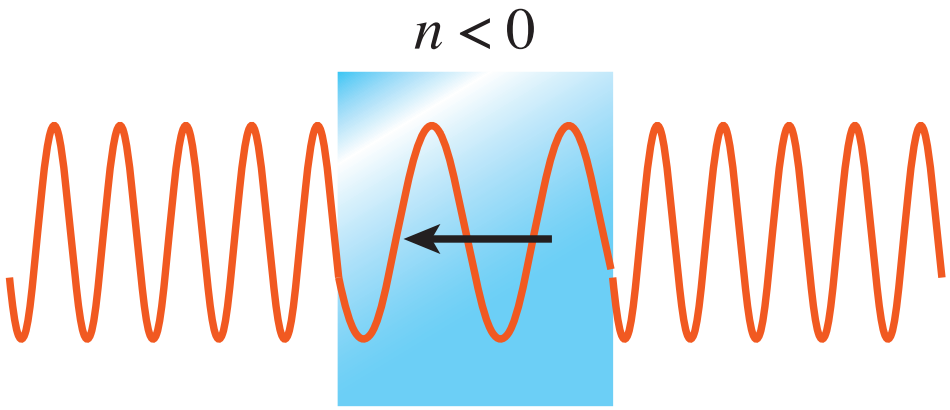
Zero index



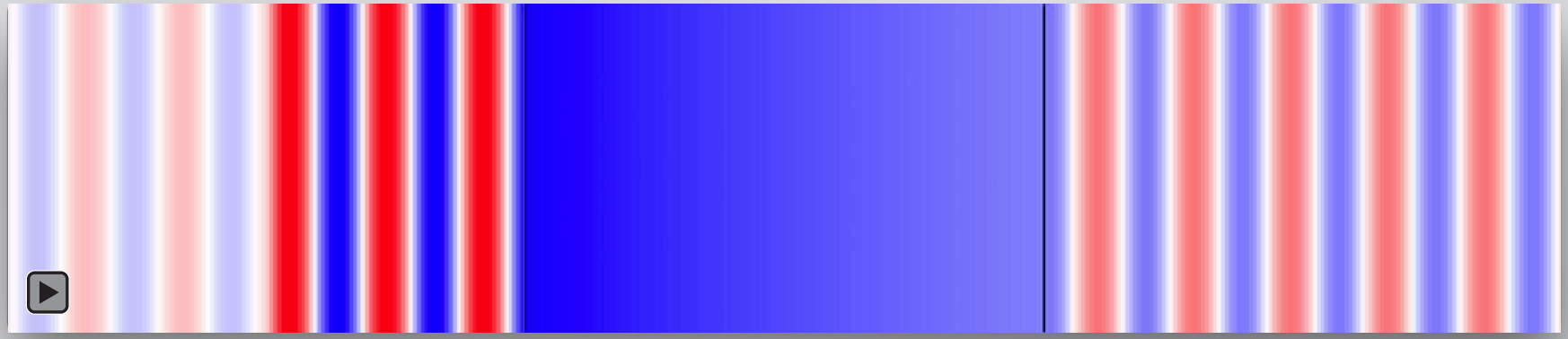
Zero index



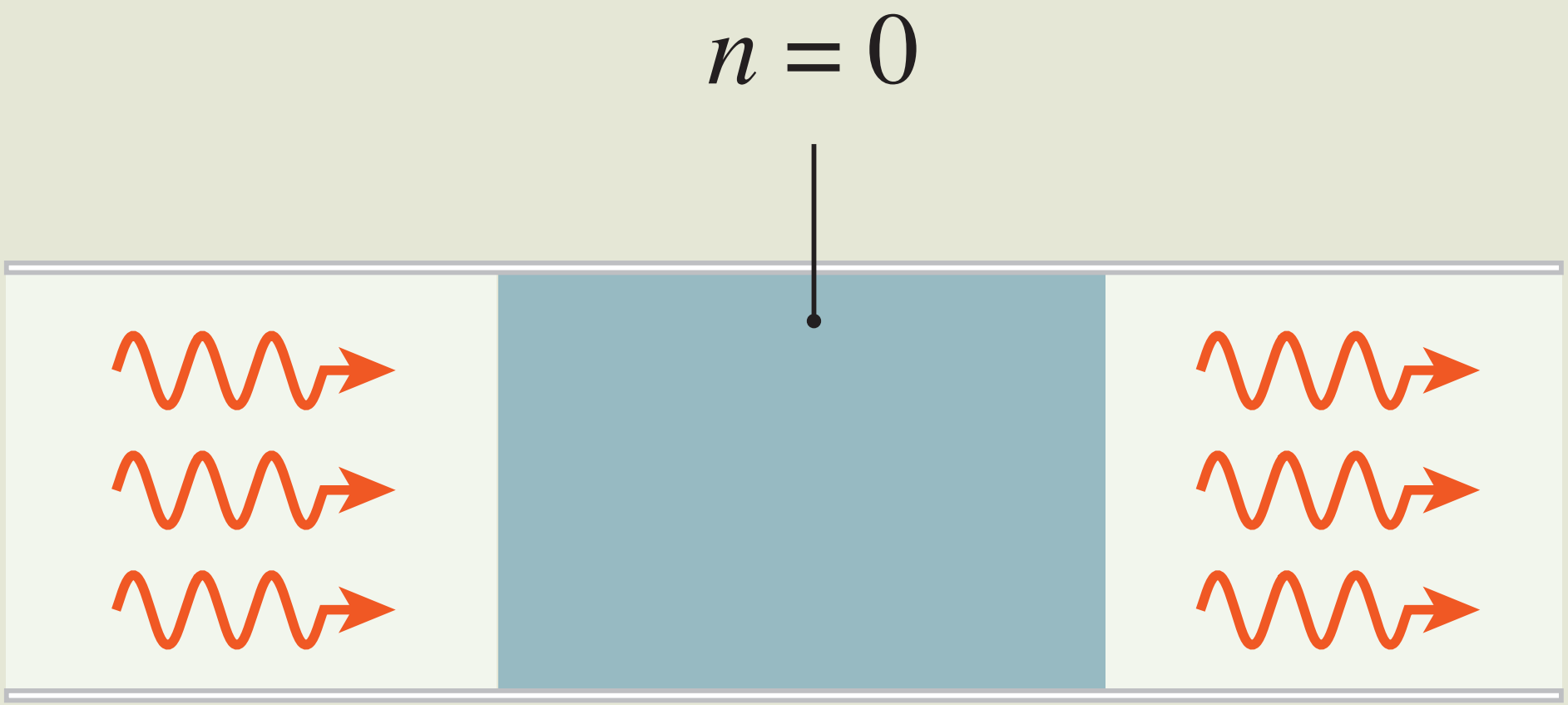
Zero index



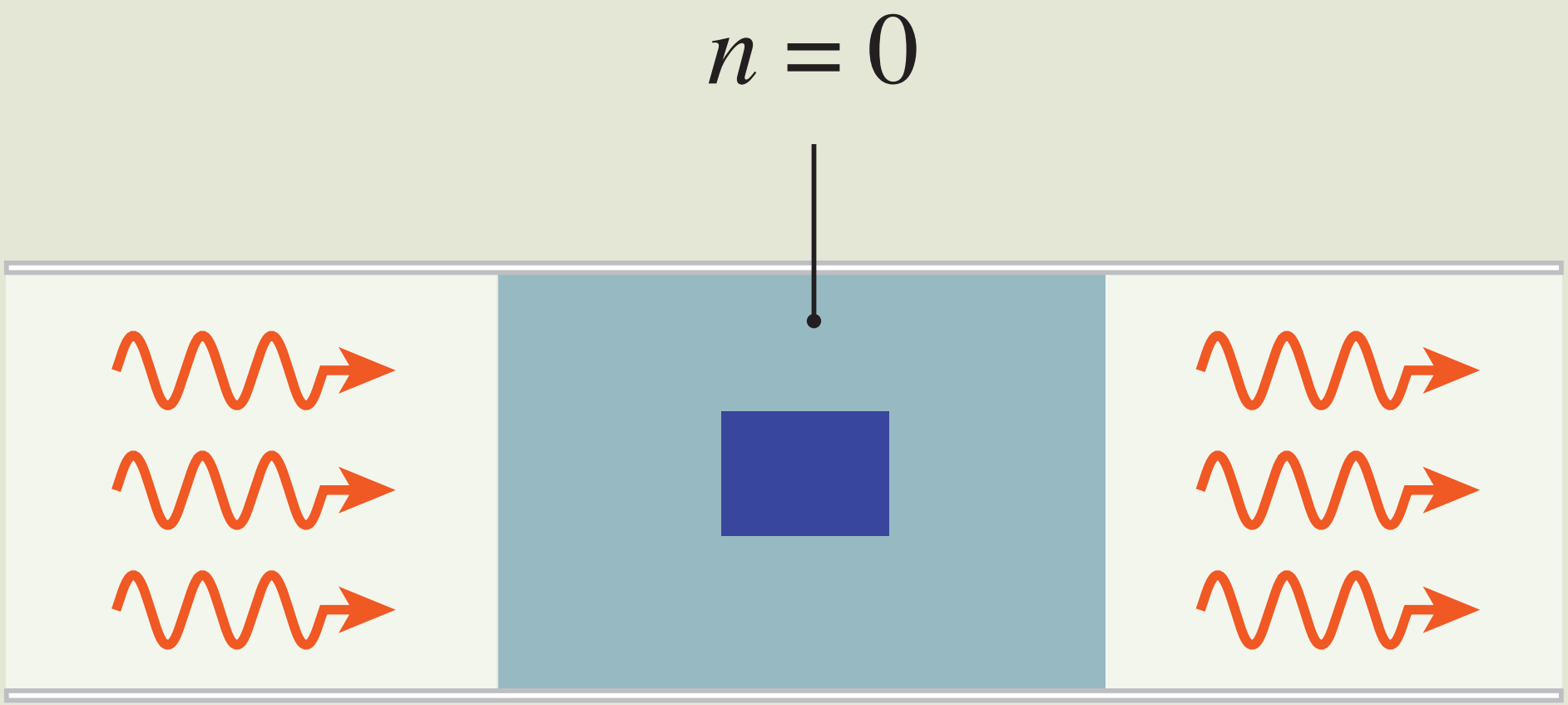
Zero index



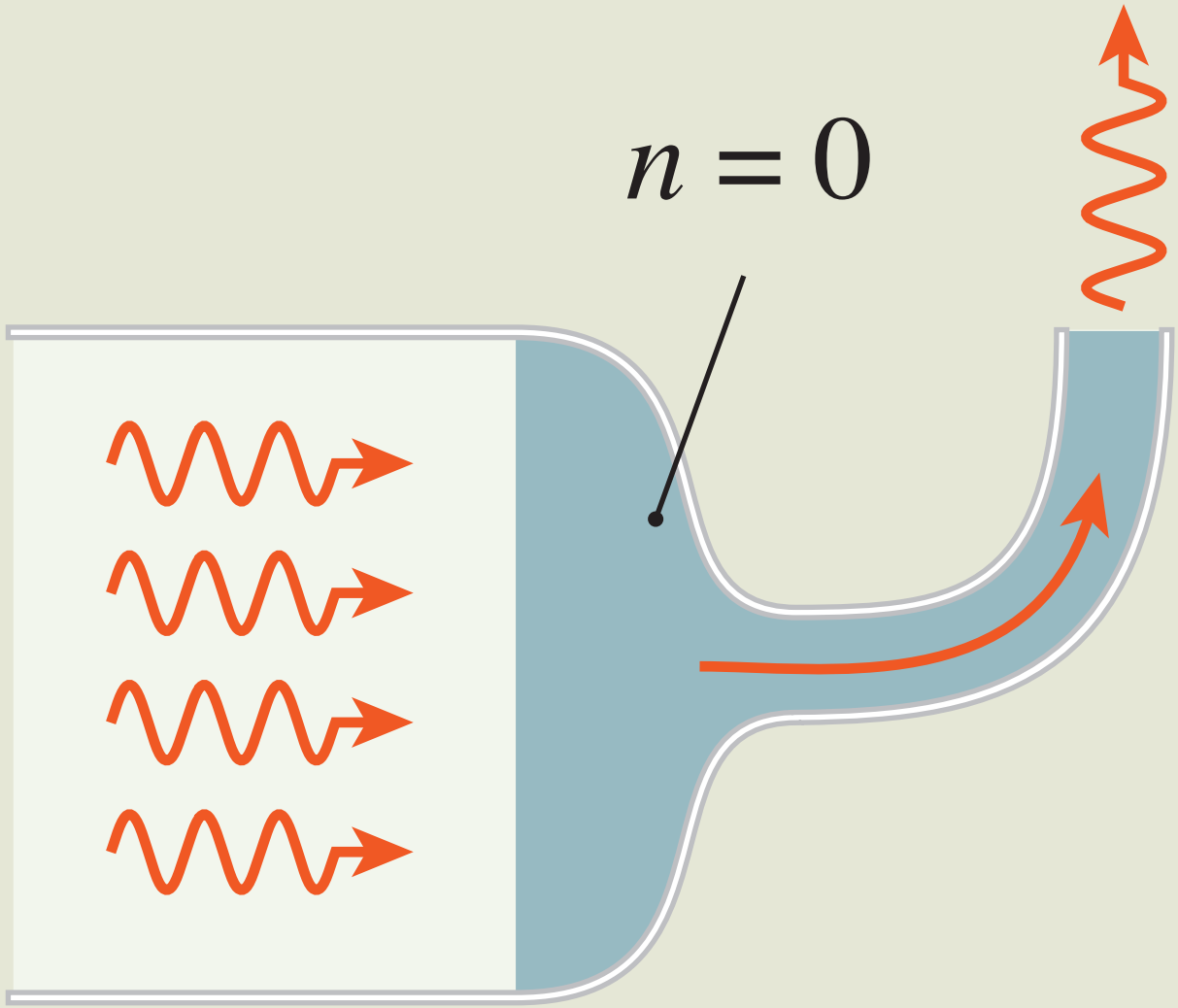
Zero index



Zero index



Zero index



Zero index

how?

$$n = \sqrt{\varepsilon\mu}$$

Zero index

how?

$$n = \sqrt{\varepsilon\mu}$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

Zero index

how?

$$n = \sqrt{\varepsilon\mu}$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

Zero index

how?

$$\varepsilon \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

Zero index

how?

$$\varepsilon \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \;\to\; 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \;\to\; \infty$$

Zero index

how?

$$\varepsilon \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \to 1$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to \infty$$

Zero index

how?

$$\mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

Zero index

how?

$$\mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to 0$$

Zero index

how?

$$\mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \to -1$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to 0$$

Zero index

how?

$$\varepsilon, \mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

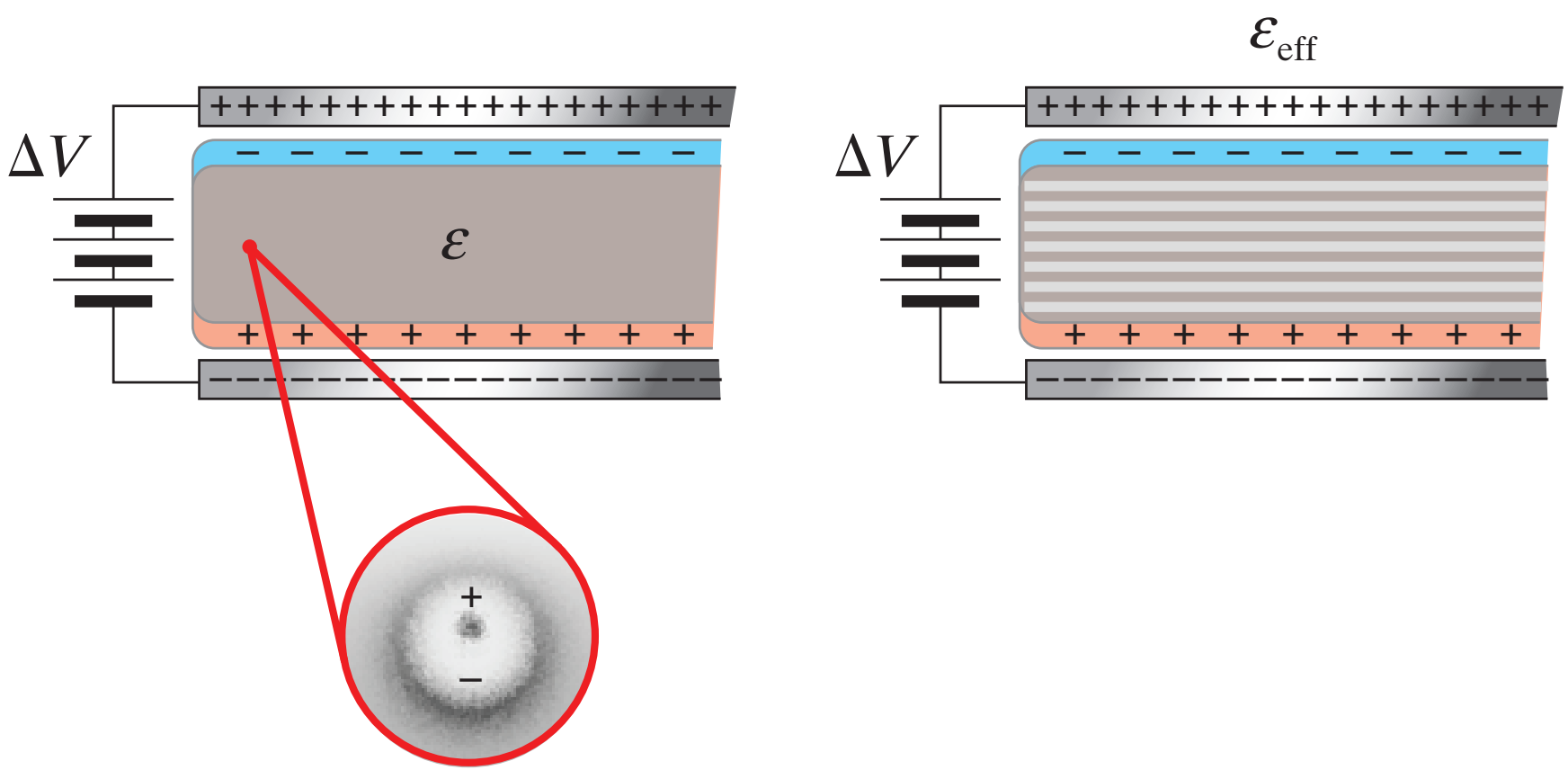
where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

finite!

Zero index

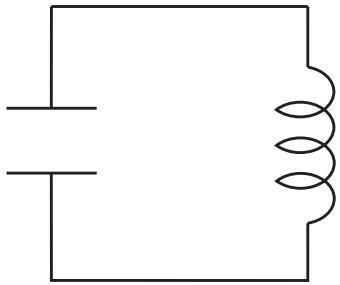
but $\mu \neq 1$ requires a magnetic response!



Engineering a magnetic response

How can we produce coupled E and B-fields?

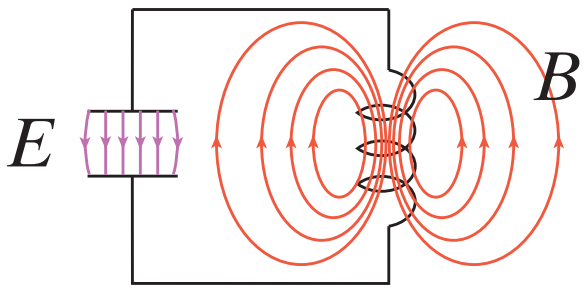
LC circuit



Engineering a magnetic response

How can we produce coupled E and B-fields?

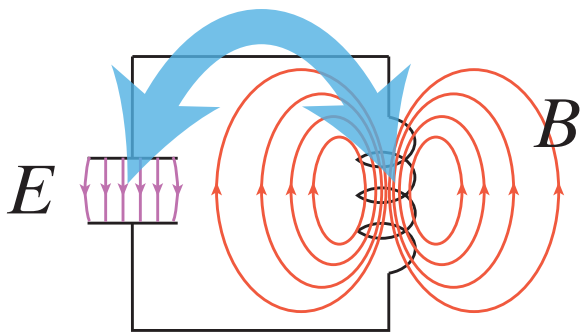
LC circuit



Engineering a magnetic response

How can we produce coupled E and B-fields?

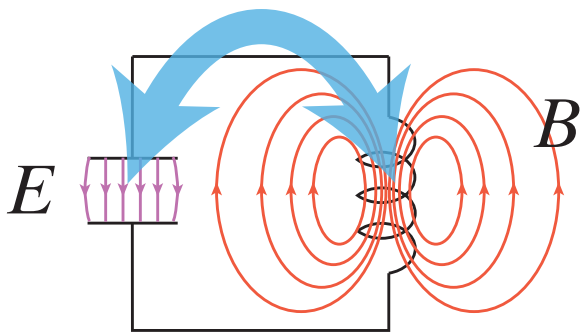
LC circuit



Engineering a magnetic response

How can we produce coupled E and B-fields?

LC circuit

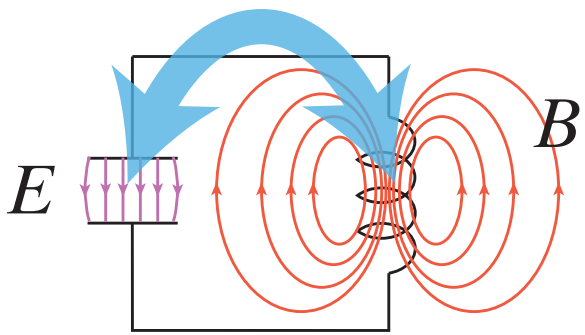


1 GHz

Engineering a magnetic response

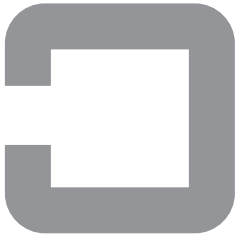
How can we produce coupled E and B-fields?

LC circuit



1 GHz

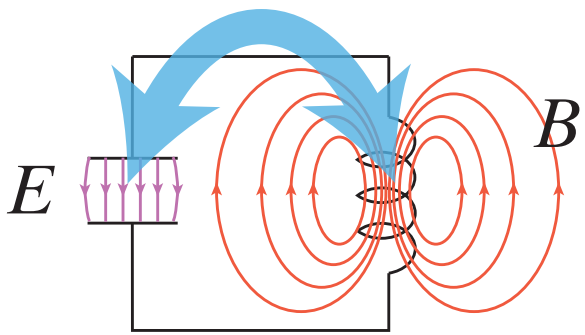
split ring



Engineering a magnetic response

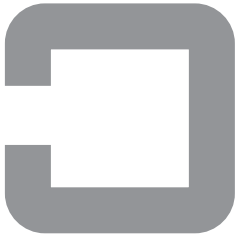
How can we produce coupled E and B-fields?

LC circuit



1 GHz

split ring

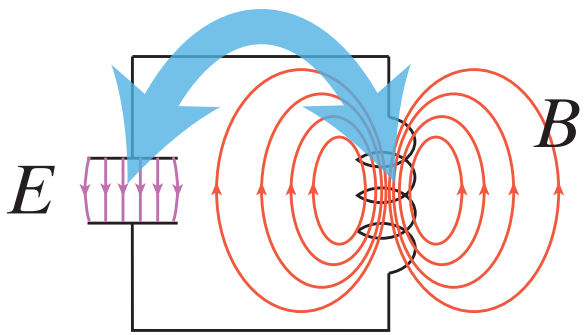


10 THz

Engineering a magnetic response

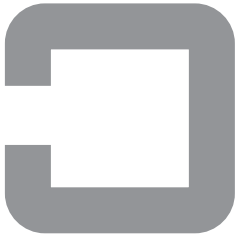
How can we produce coupled E and B-fields?

LC circuit



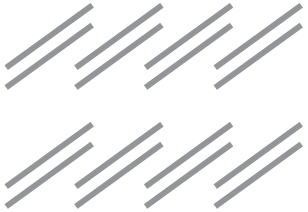
1 GHz

split ring



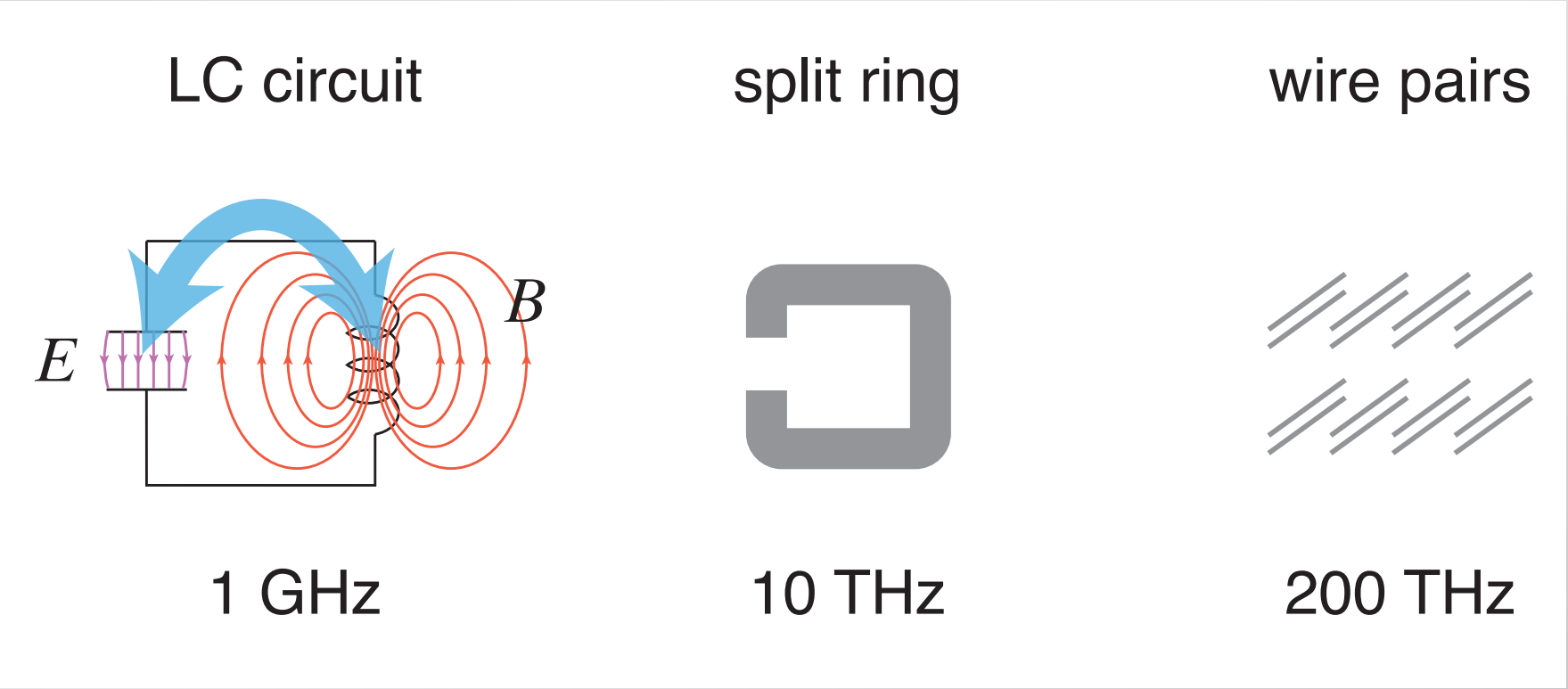
10 THz

wire pairs



Engineering a magnetic response

How can we produce coupled E and B-fields?



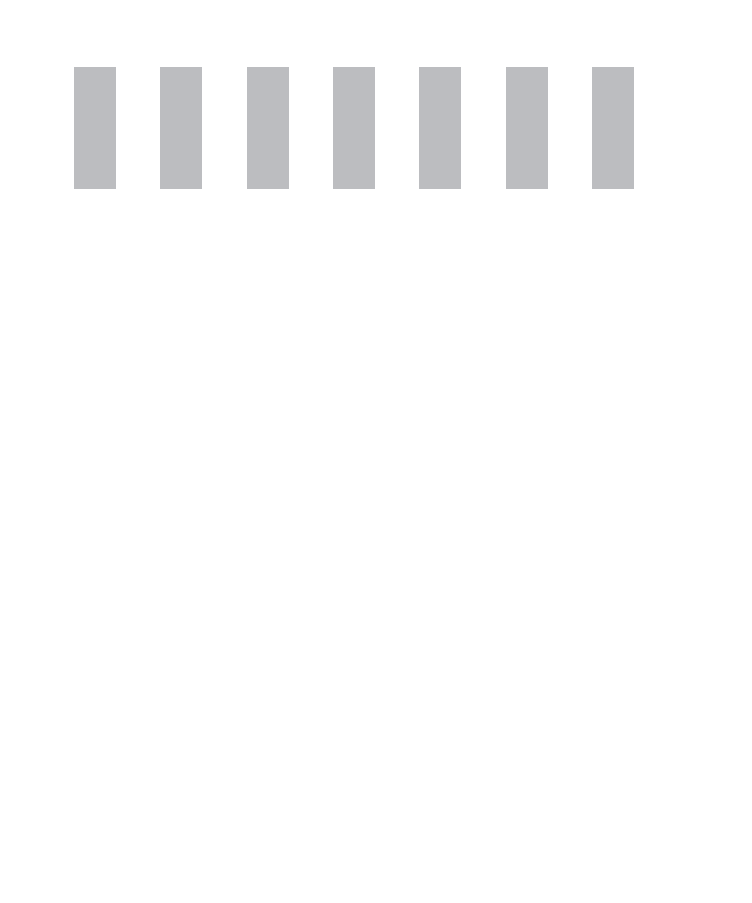
Engineering a magnetic response

How can we produce coupled E and B-fields?

but… metallic losses & not easily made in 3D

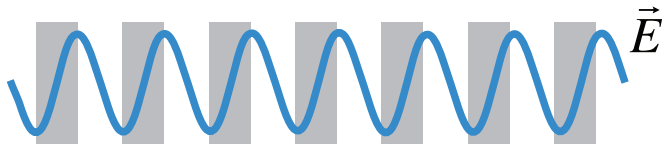
Engineering a magnetic response

instead, use array of dielectric rods



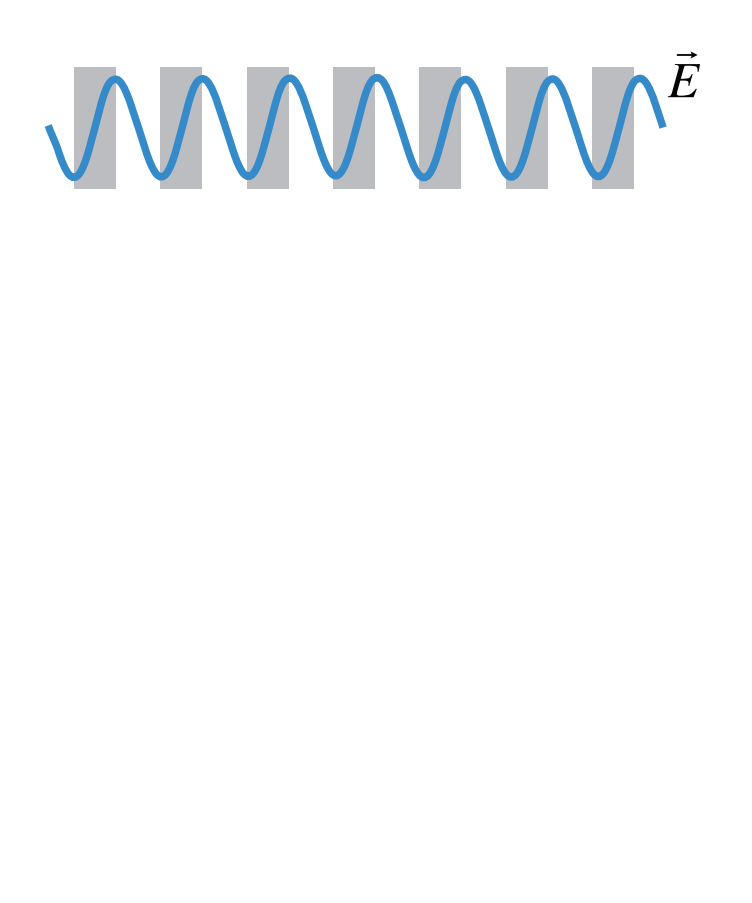
Engineering a magnetic response

incident electromagnetic wave ($\lambda_{\text{eff}} \approx d$)



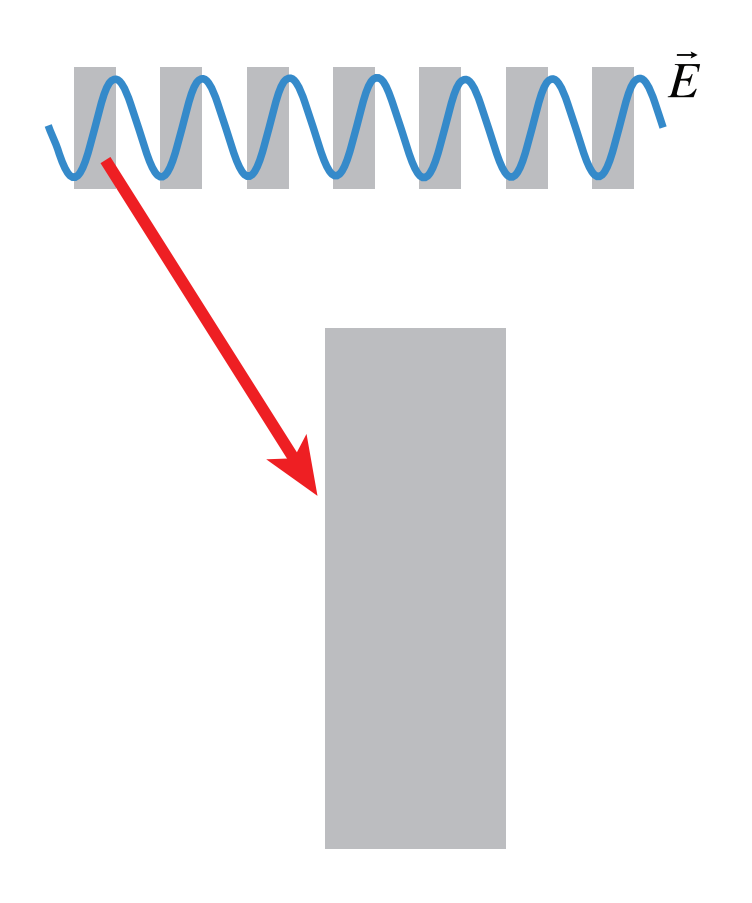
Engineering a magnetic response

produces an electric response…



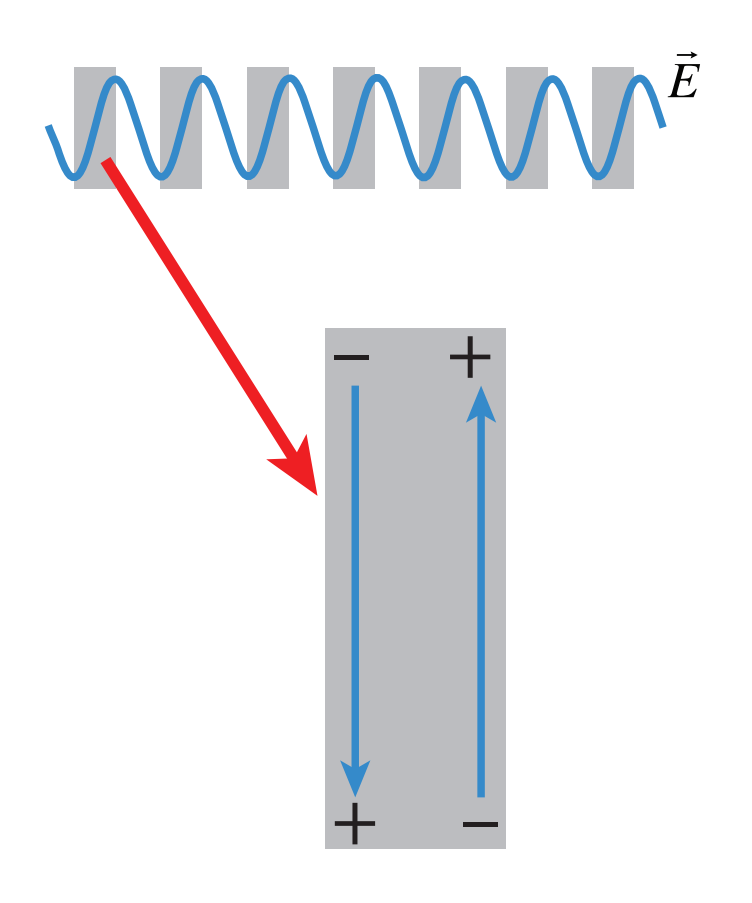
Engineering a magnetic response

… but different electric fields front and back…



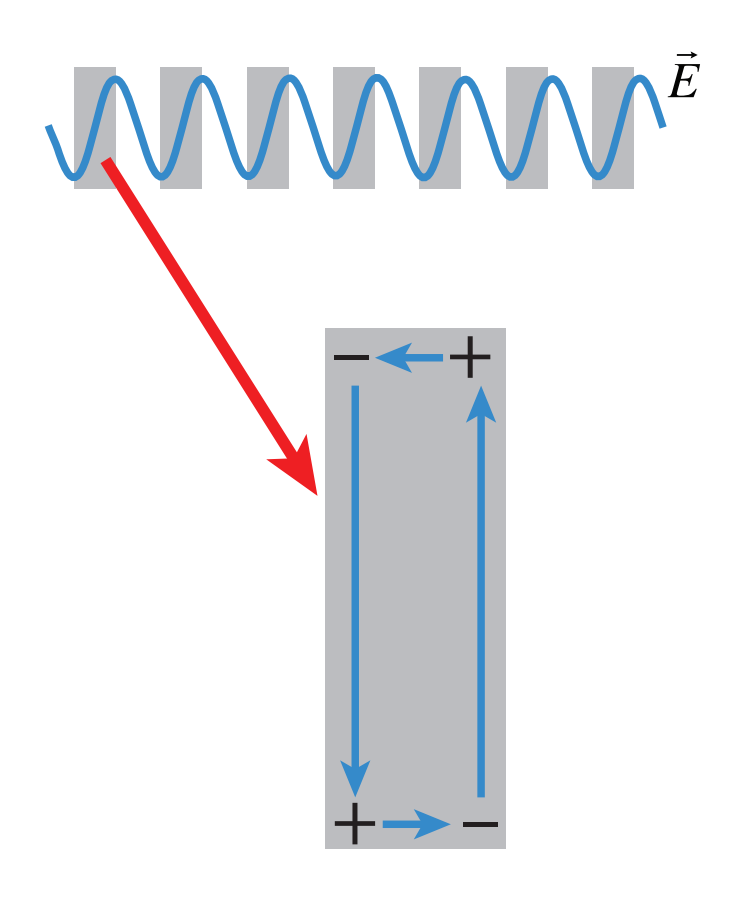
Engineering a magnetic response

…induce different polarizations on opposite sides…



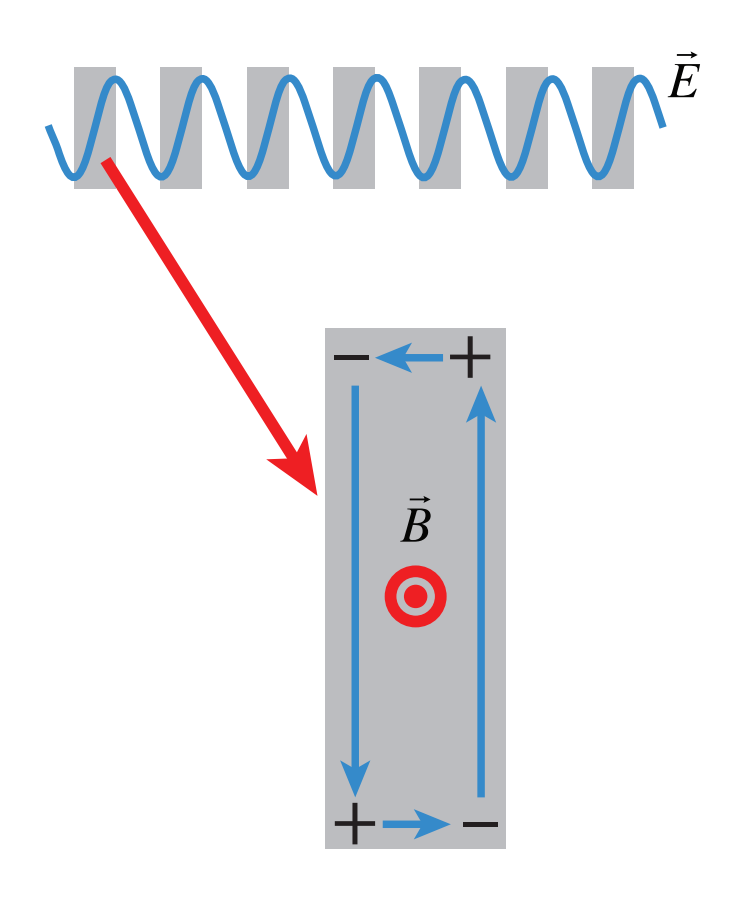
Engineering a magnetic response

…causing a current loop…



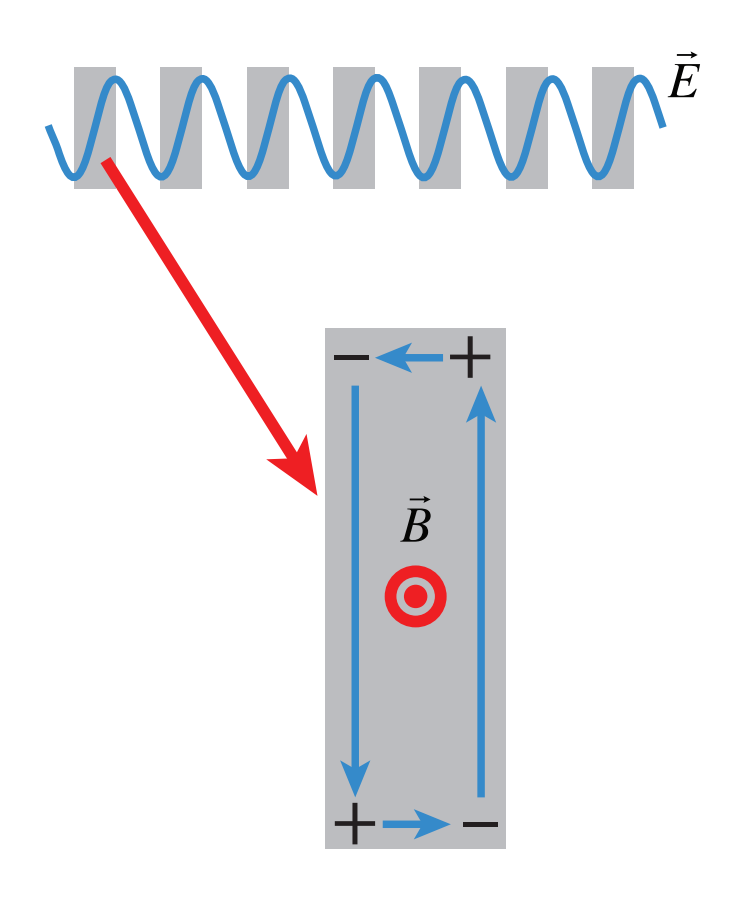
Engineering a magnetic response

…which, in turn, produces an induced magnetic field



Engineering a magnetic response

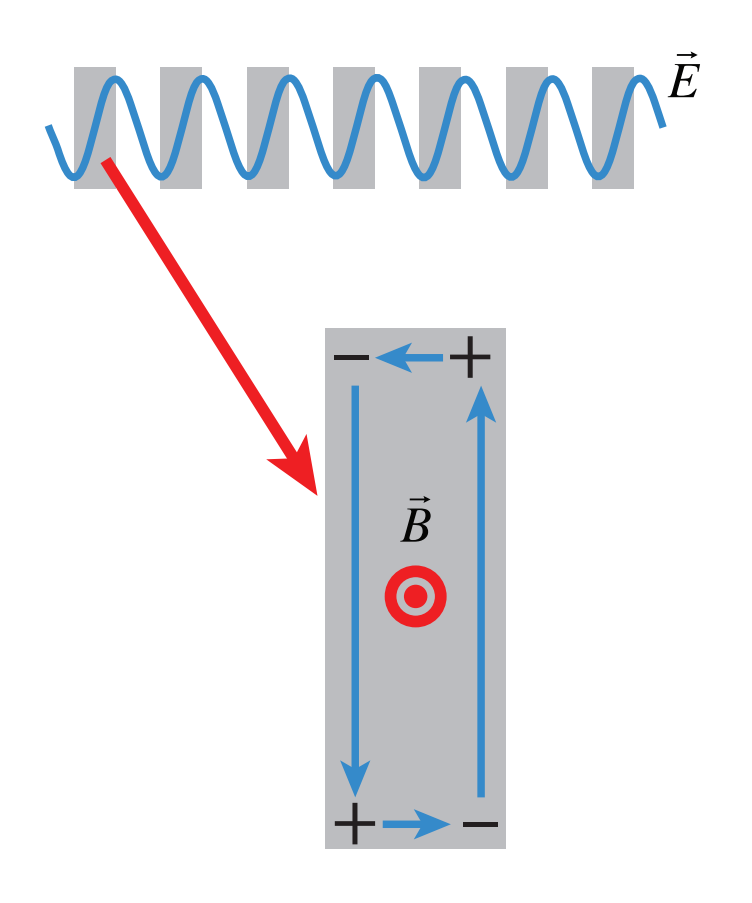
…which, in turn, produces an induced magnetic field



(but it's still a dielectric, so there's an electric response too!)

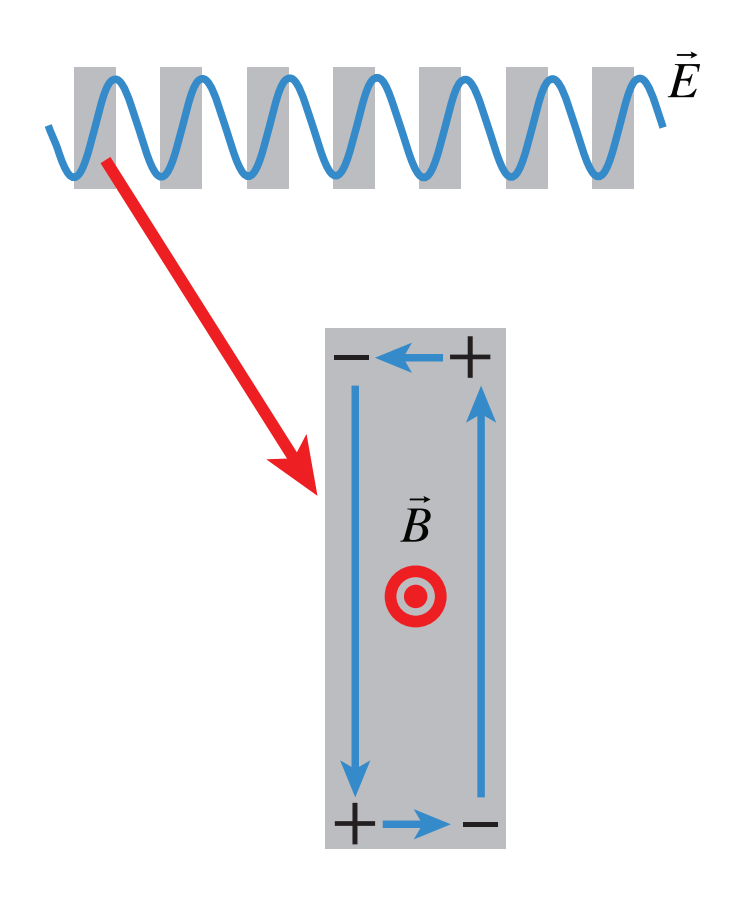
Engineering a magnetic response

adjust design so electrical and magnetic resonances coincide



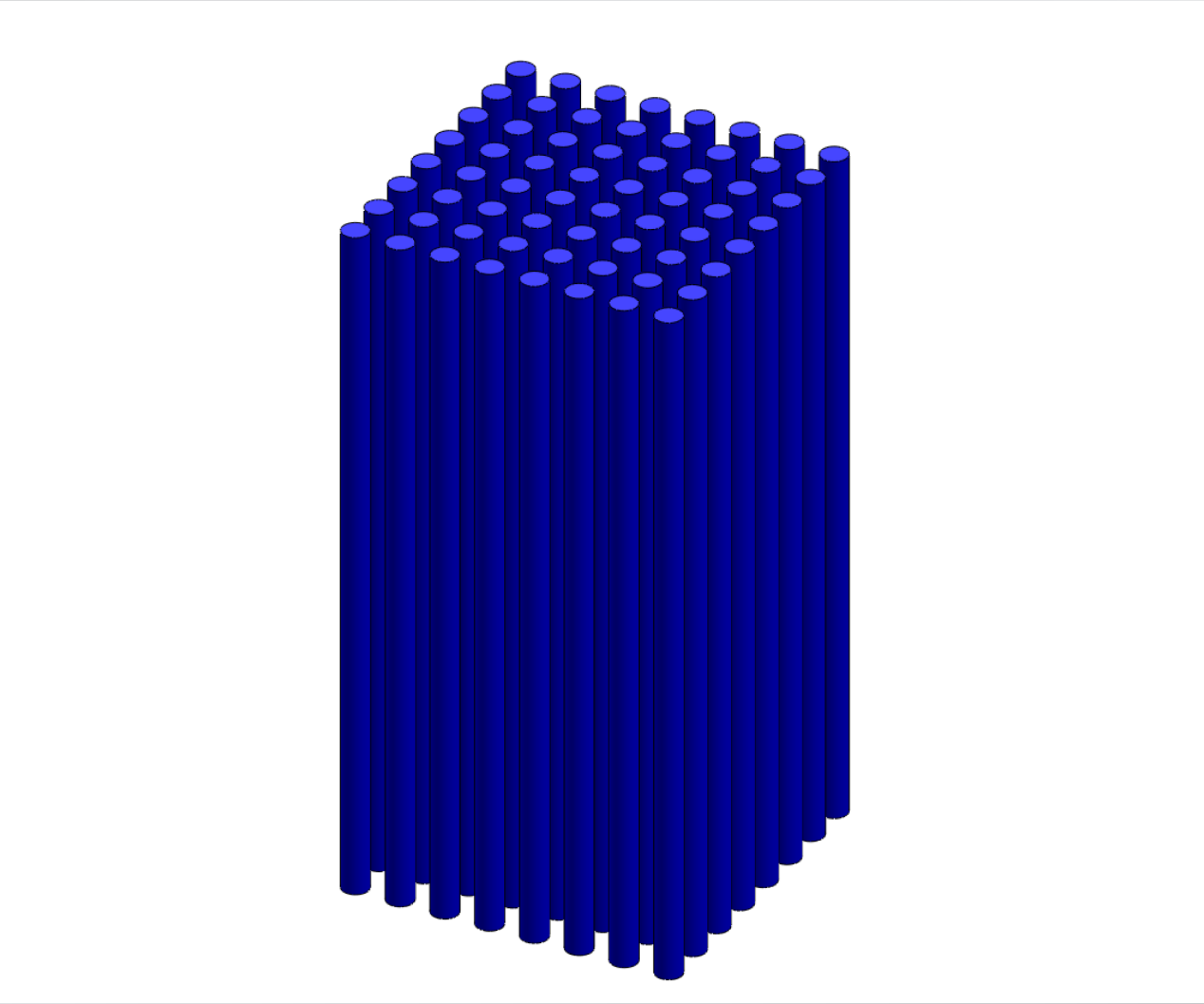
Engineering a magnetic response

adjust design so electrical and magnetic resonances coincide

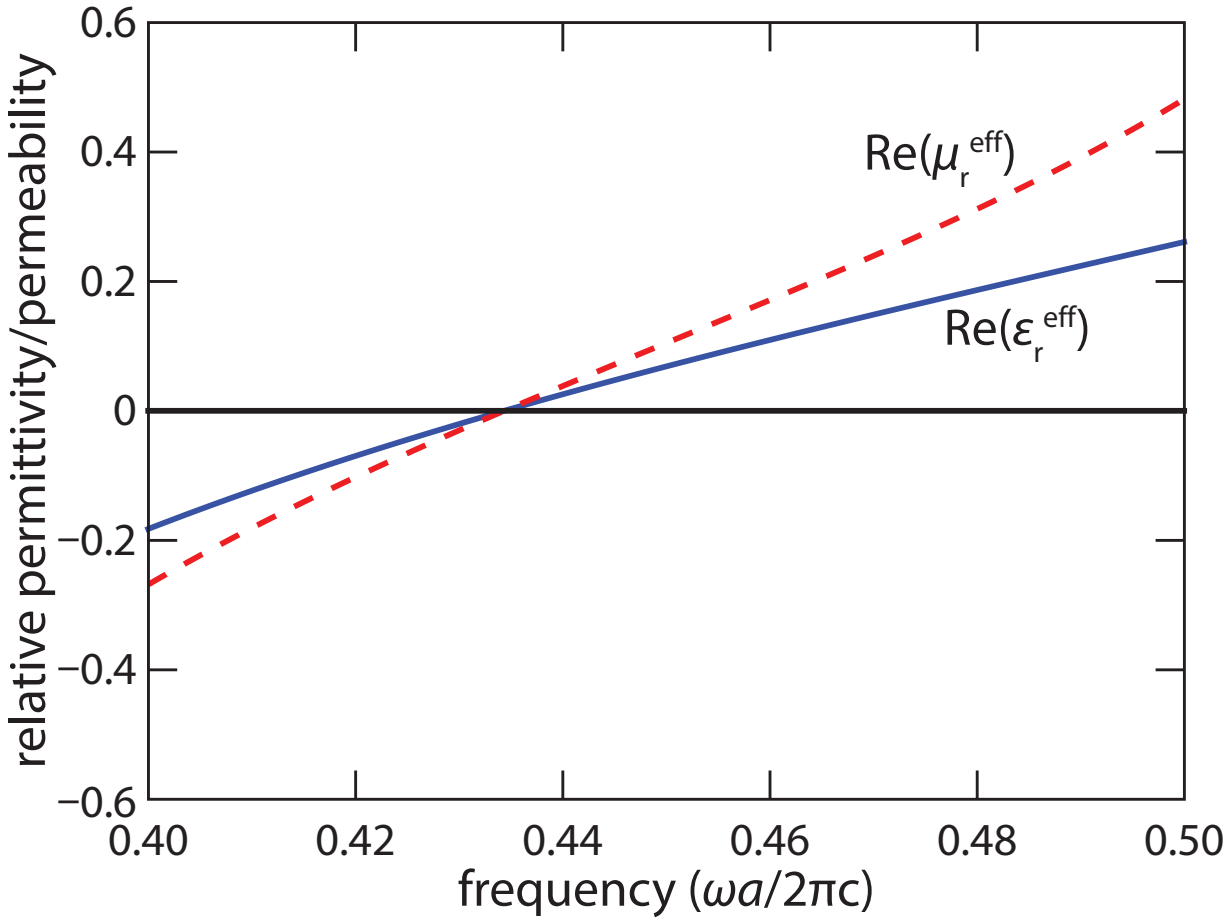


(adjustable parameters: n, d, and a)

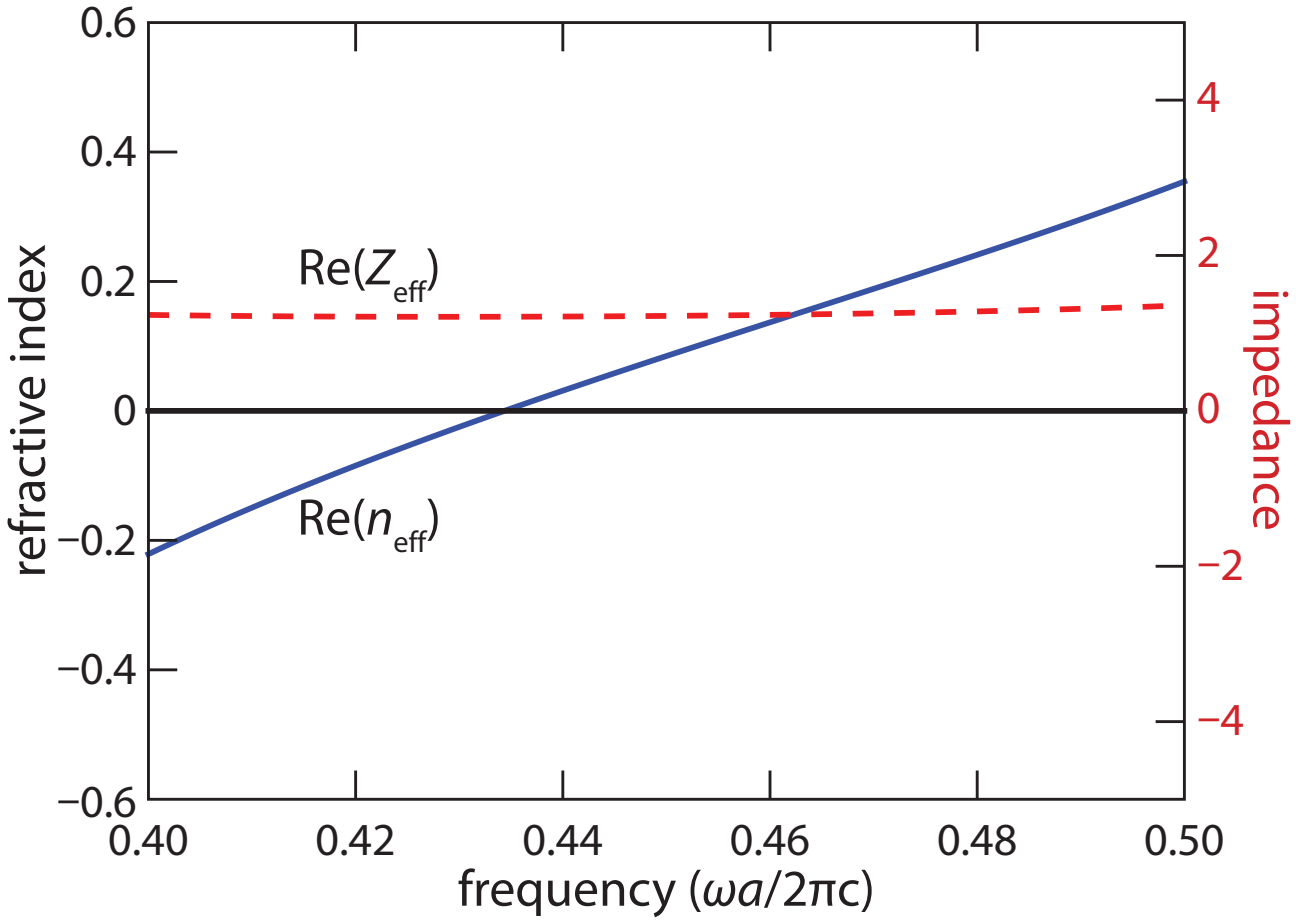
Zero-index materials



Zero-index materials

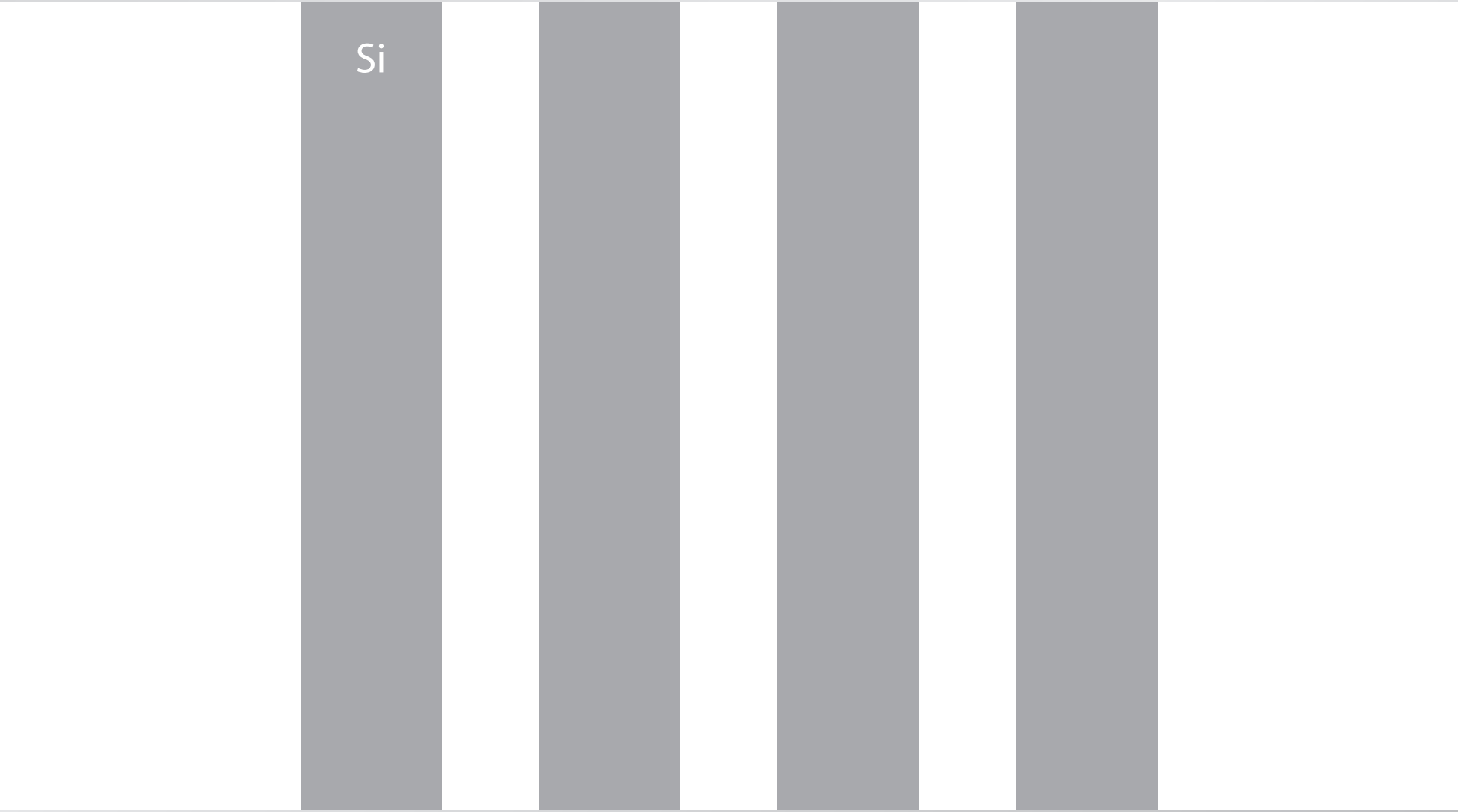


Zero-index materials



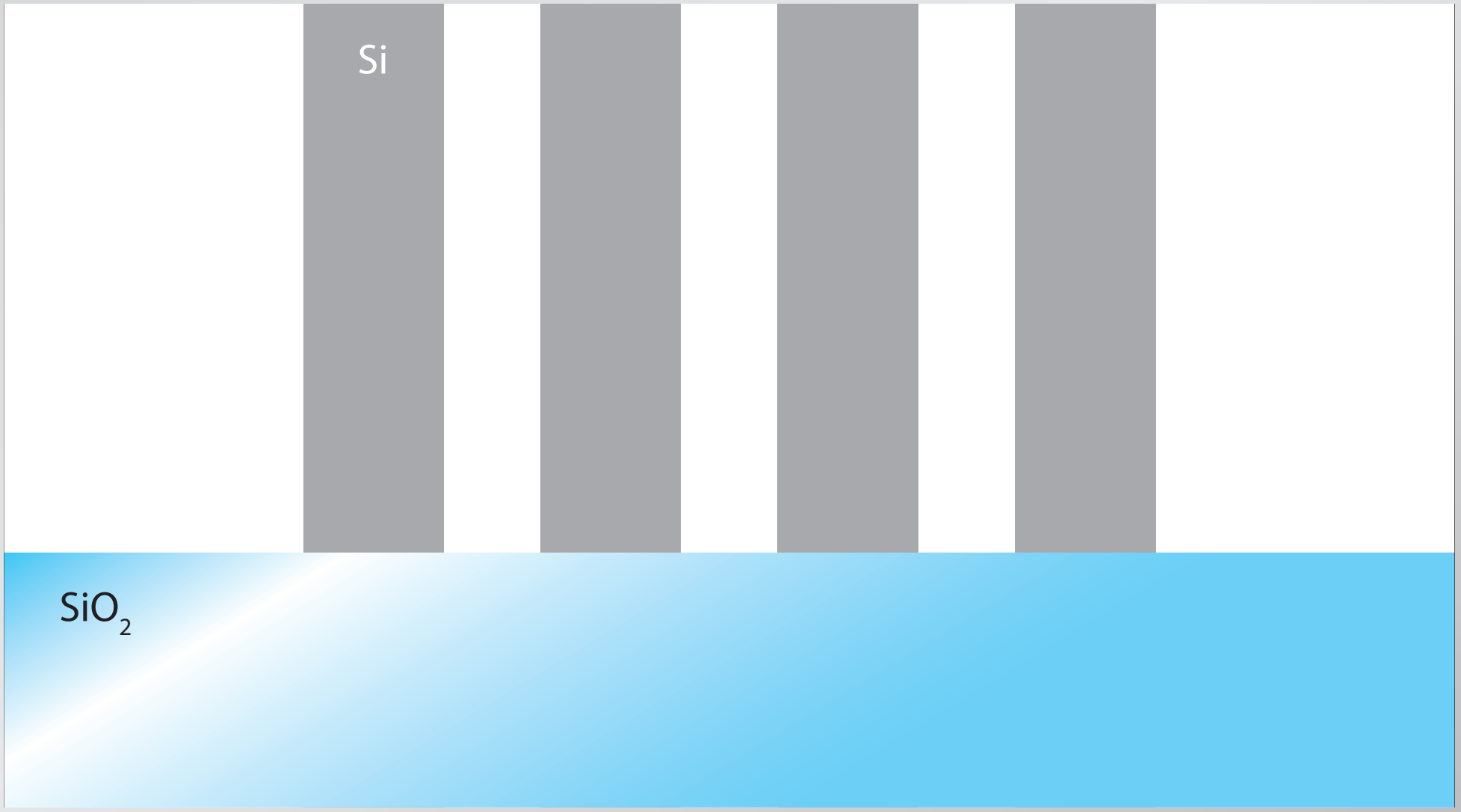
Zero-index materials

On-chip zero-index design



Zero-index materials

On-chip zero-index design

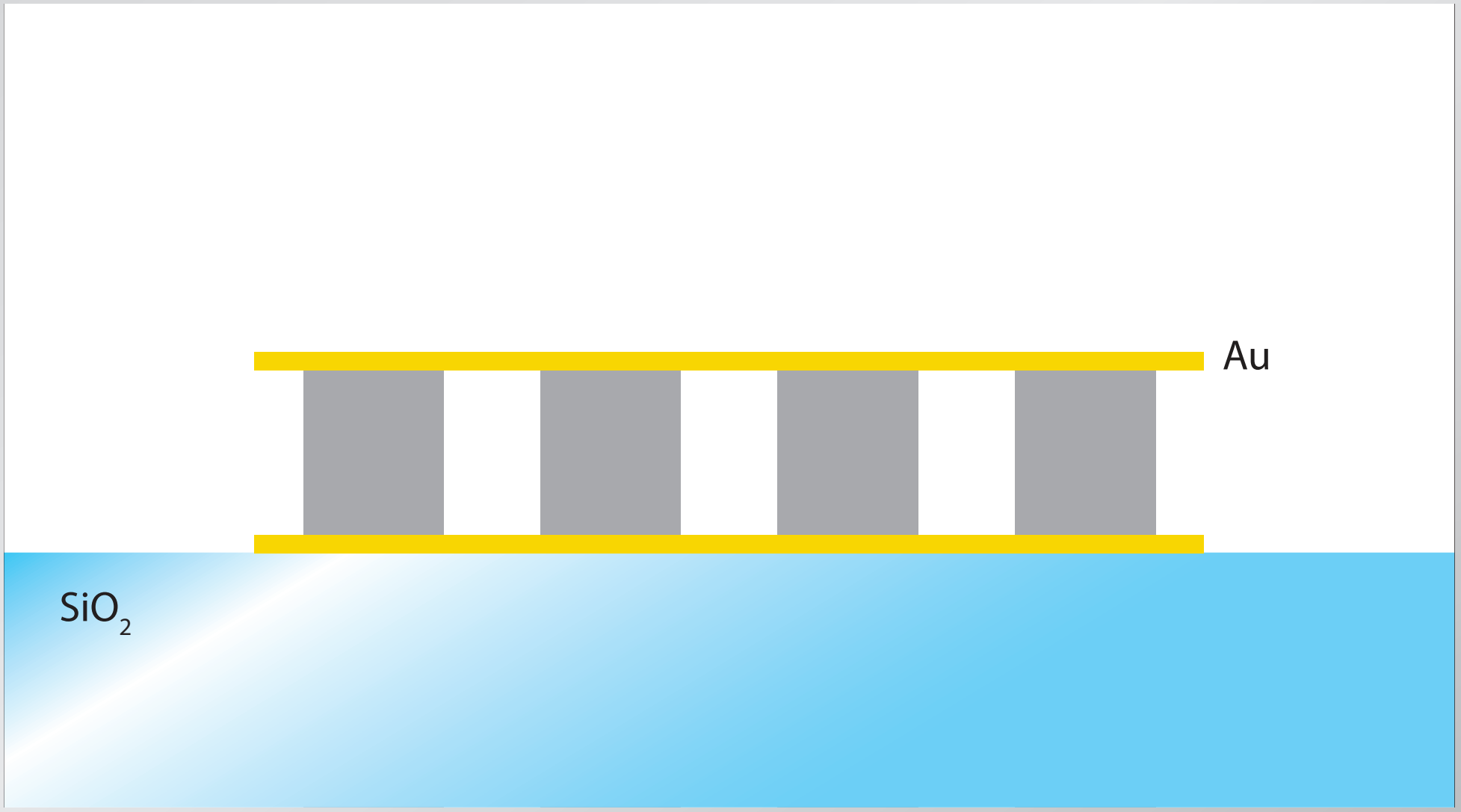


Zero-index materials

On-chip zero-index design

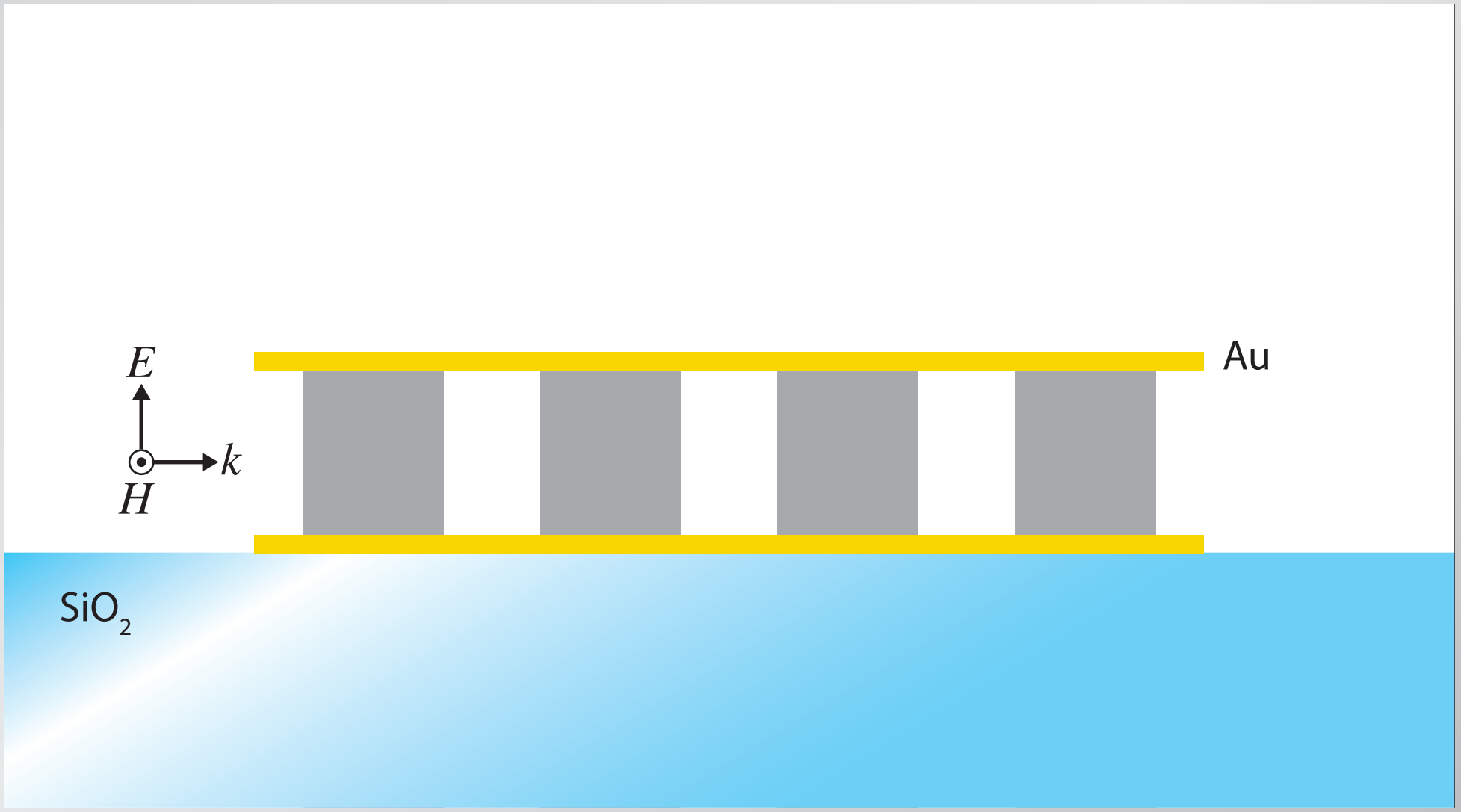
Zero-index materials

On-chip zero-index design



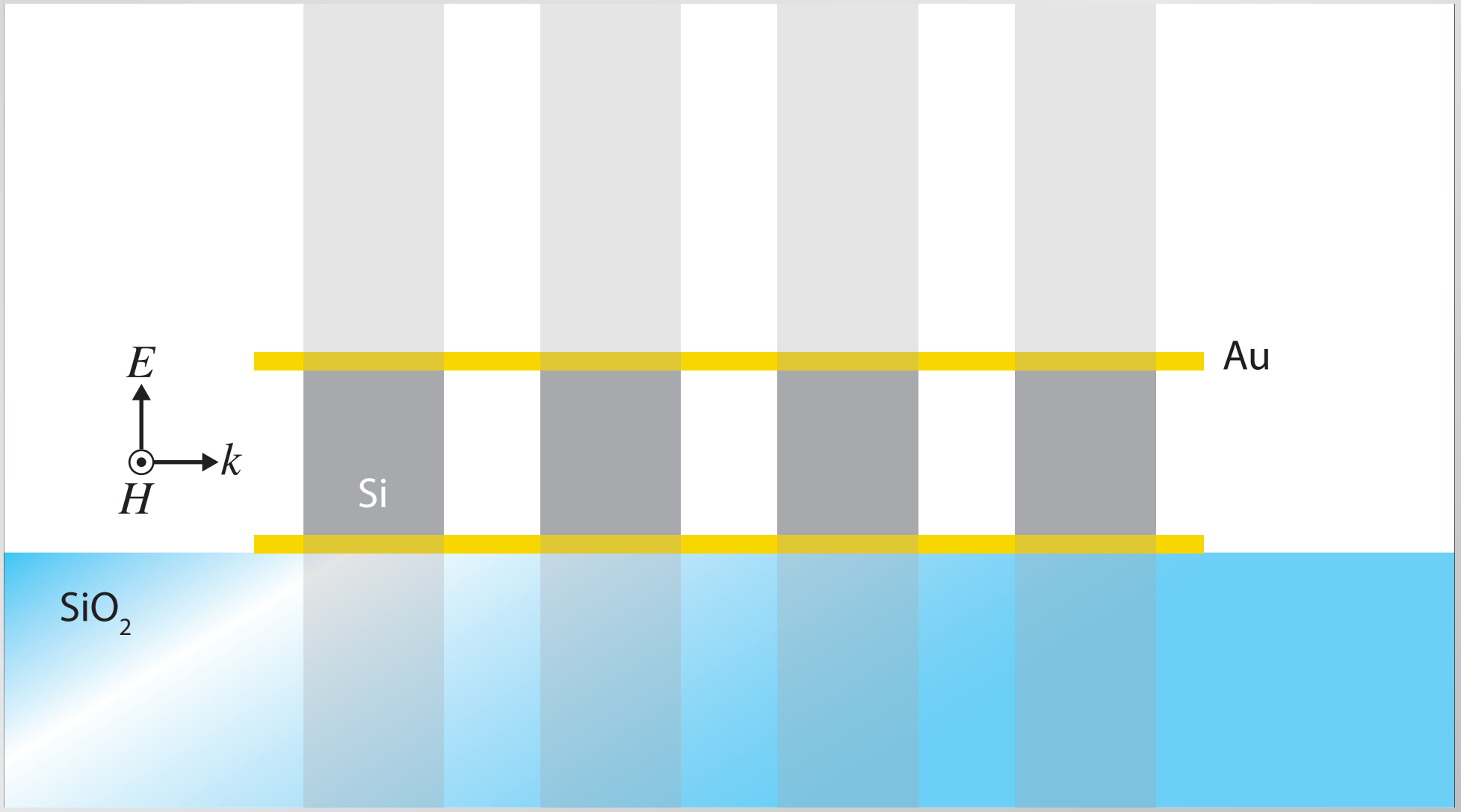
Zero-index materials

On-chip zero-index design



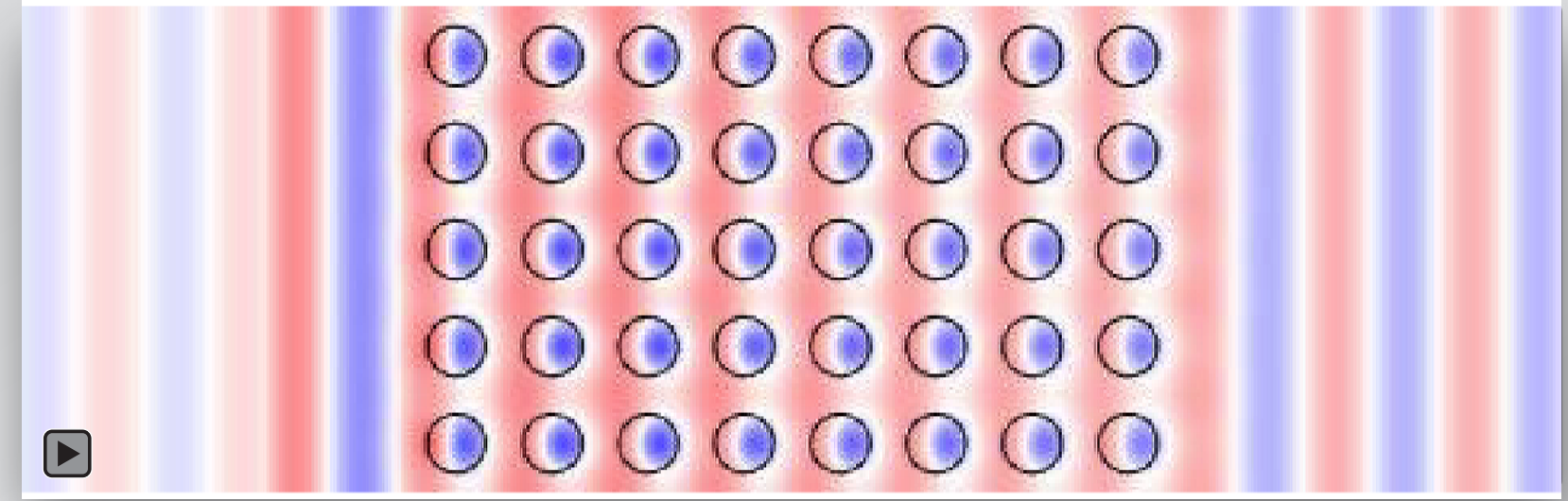
Zero-index materials

On-chip zero-index design



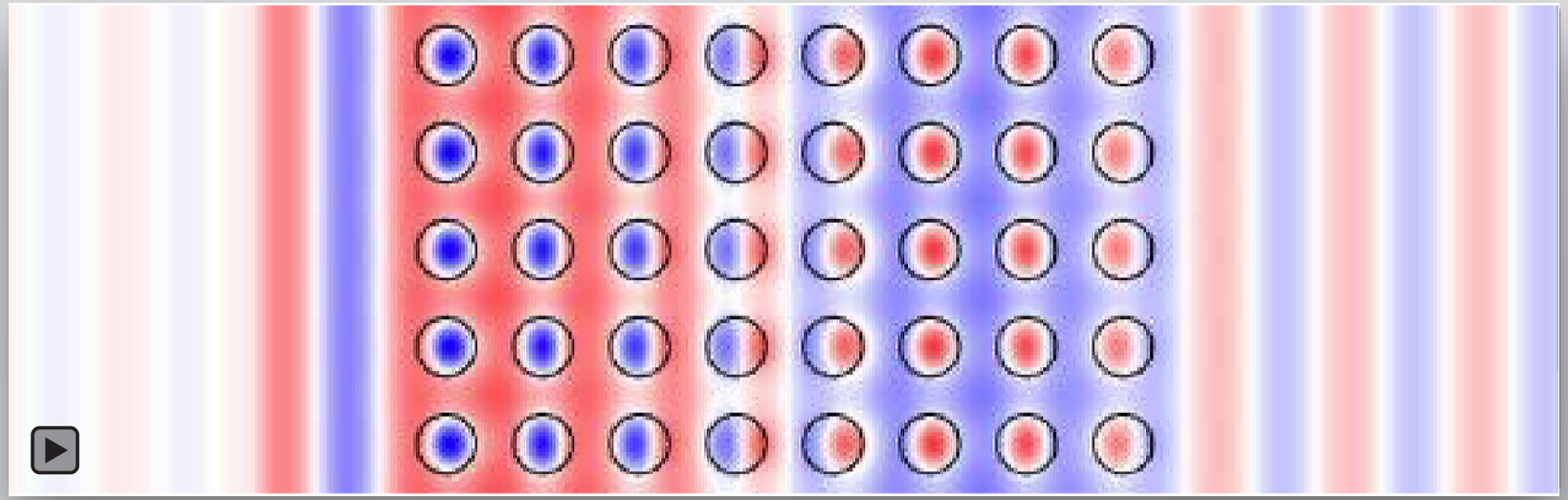
Zero index

at design wavelength (1590 nm)



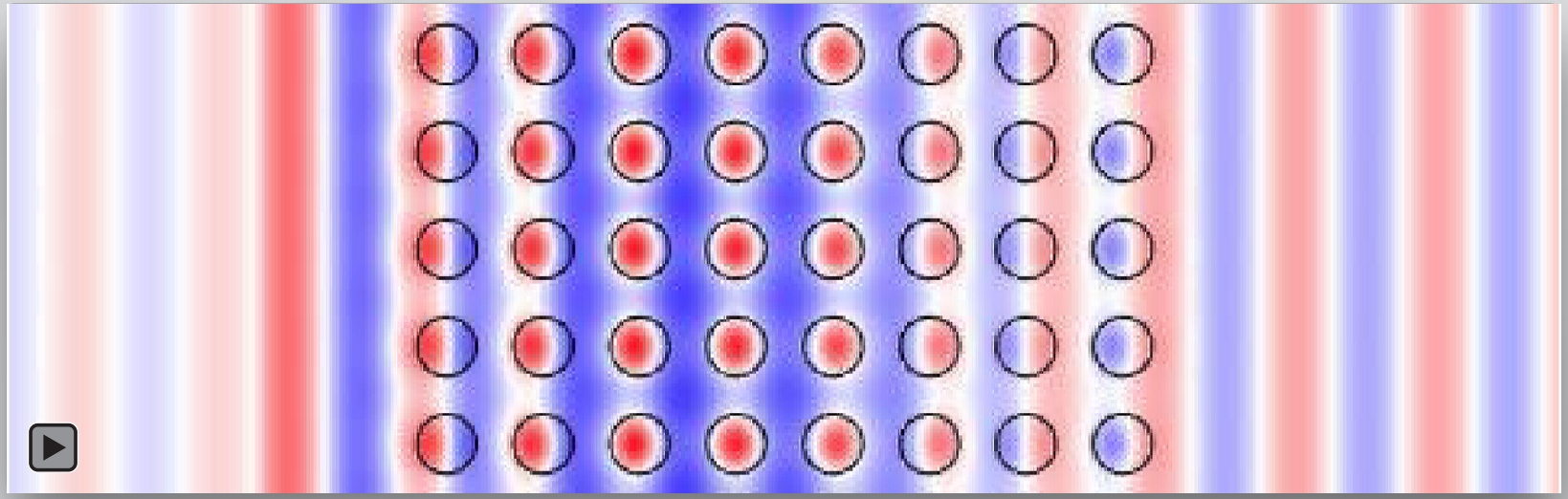
Zero index

below design wavelength (1530 nm)

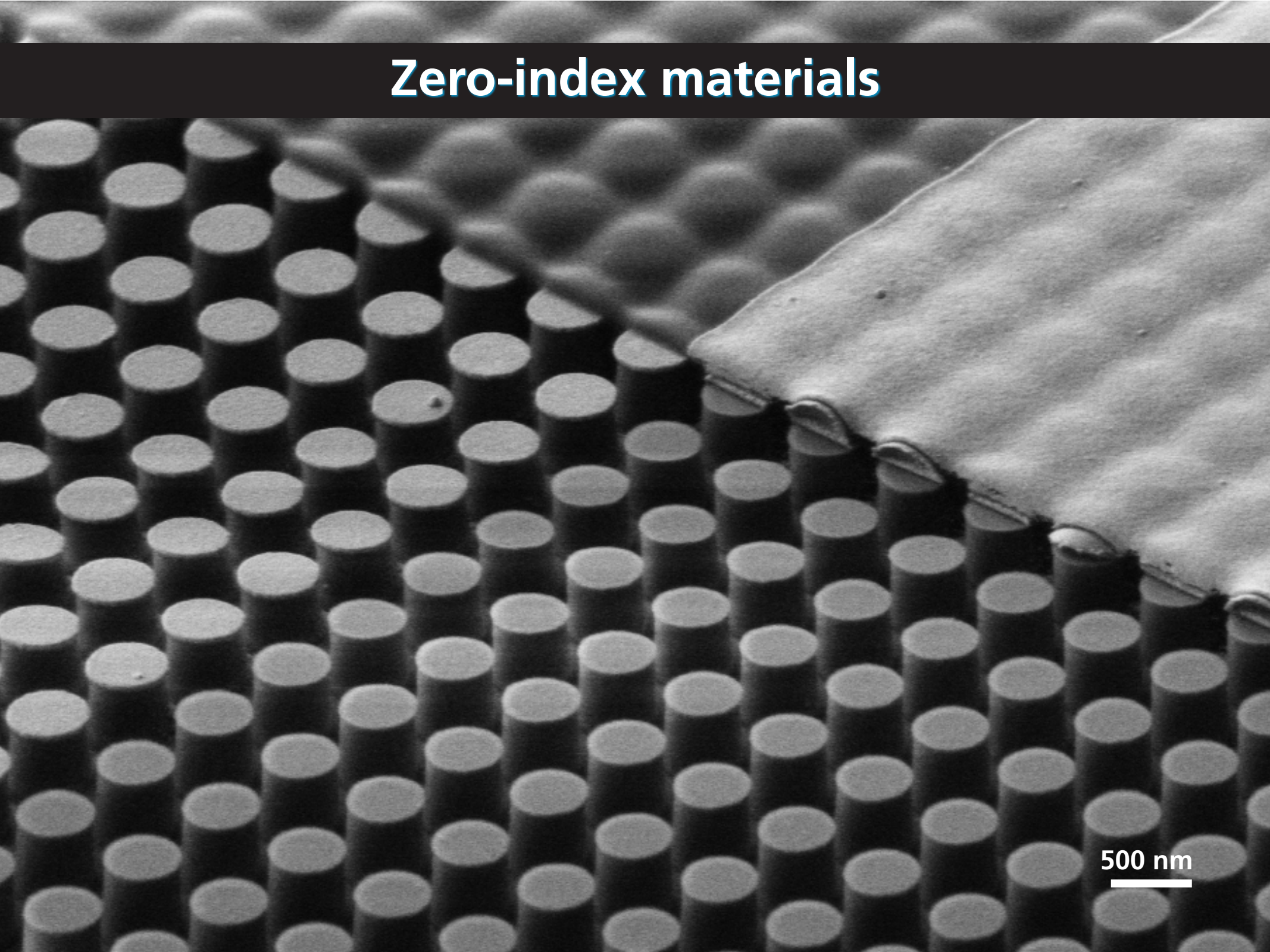


Zero index

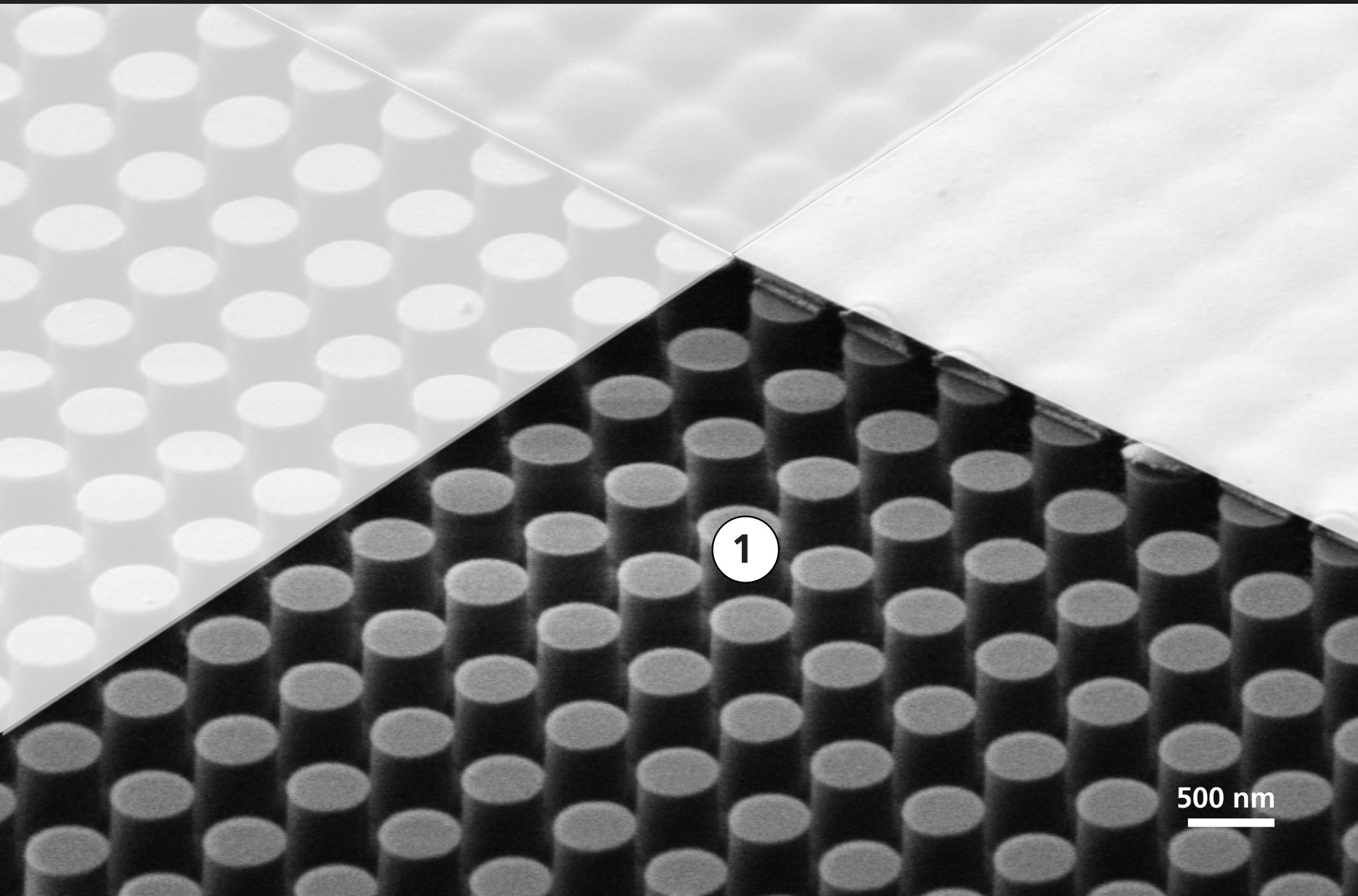
above design wavelength (1650 nm)



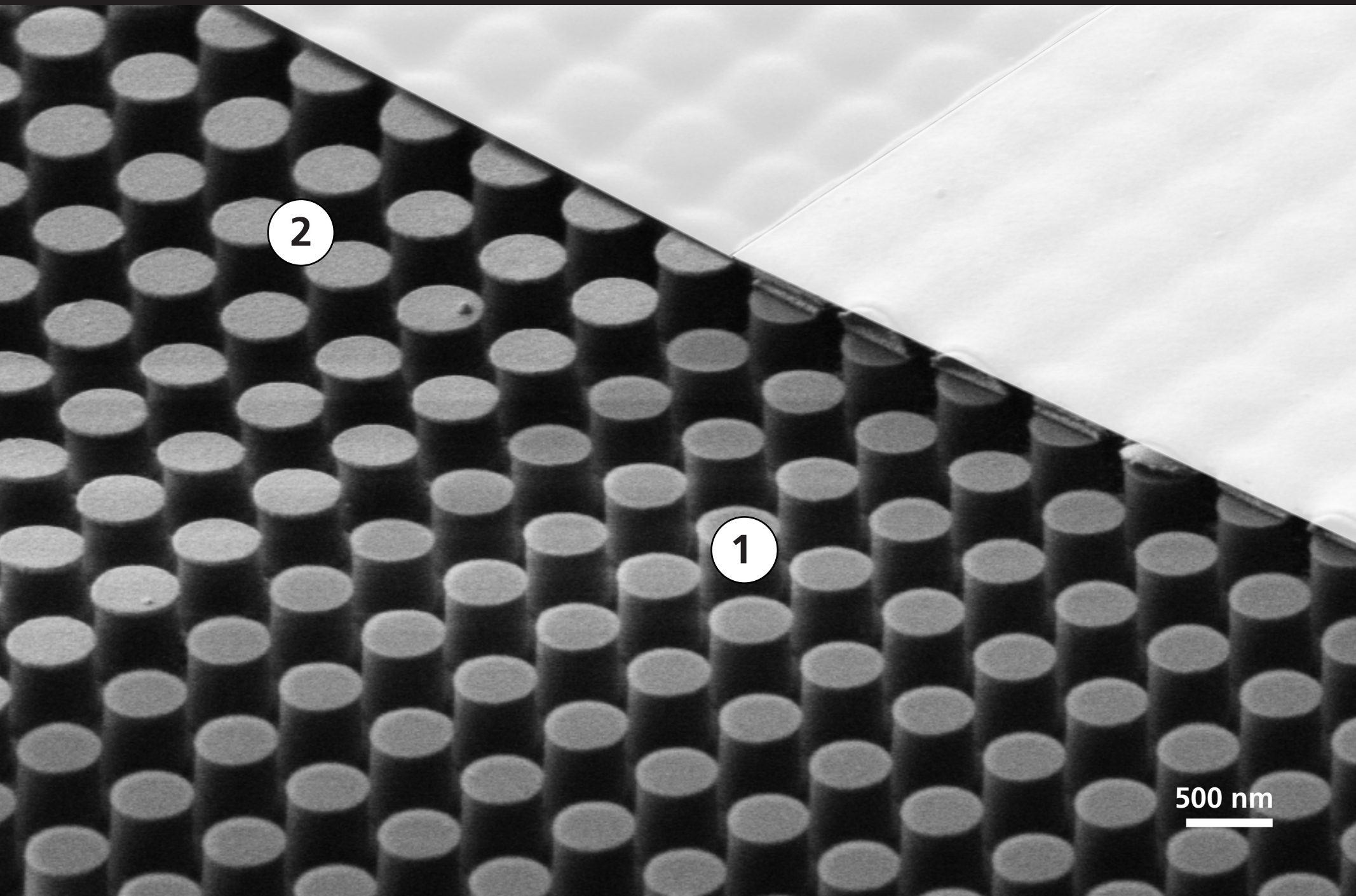
Zero-index materials



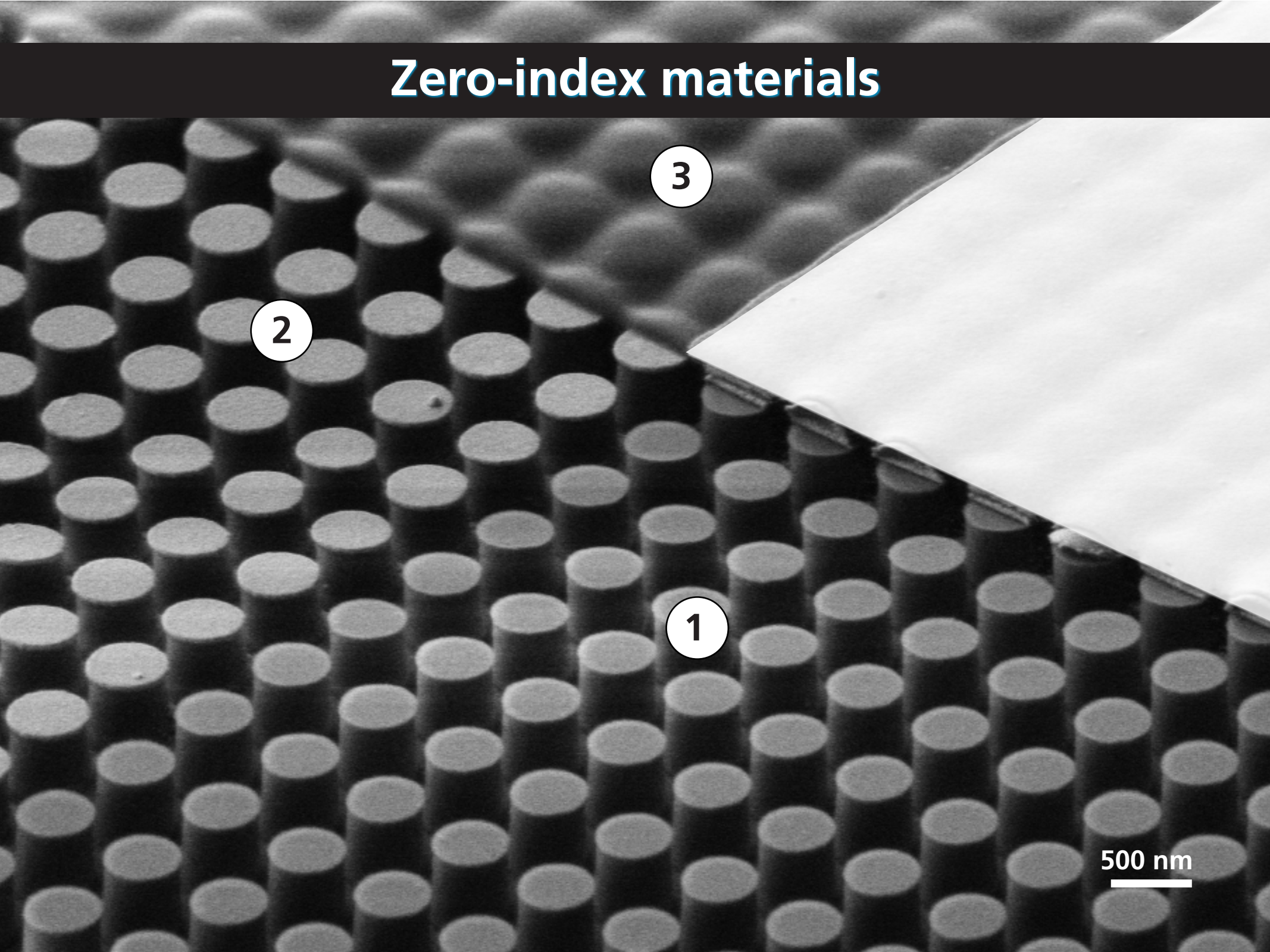
Zero-index materials



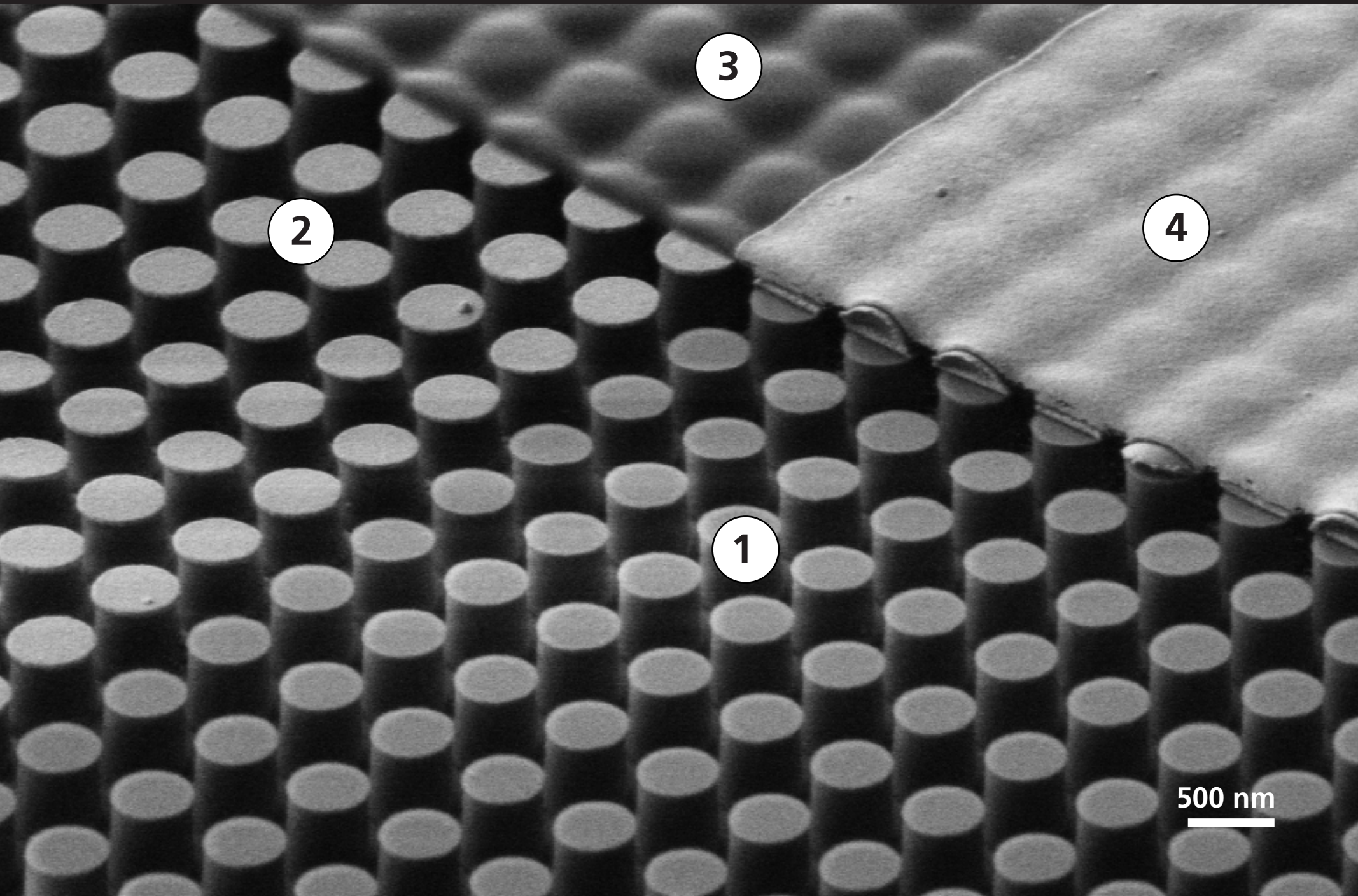
Zero-index materials



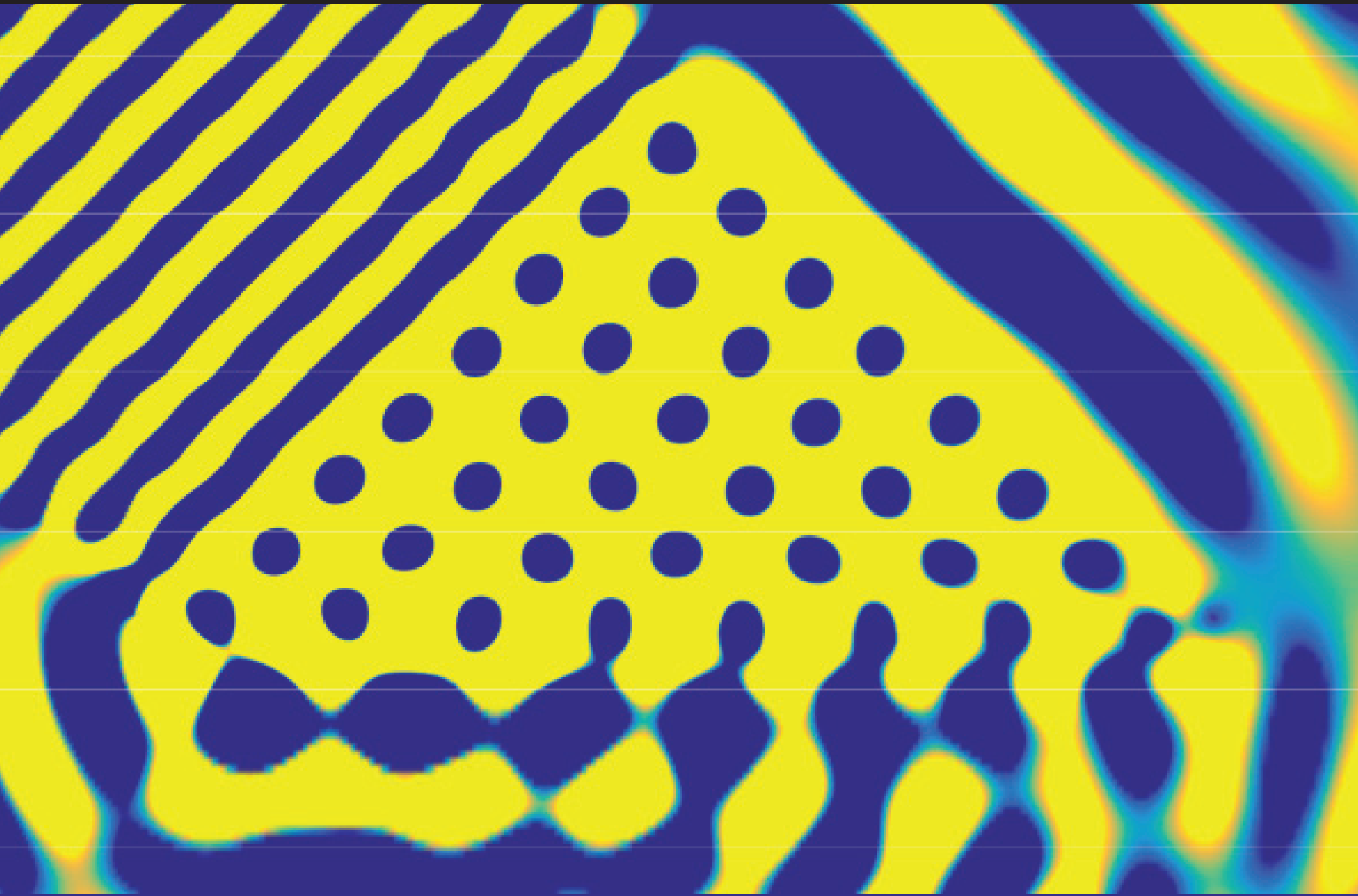
Zero-index materials



Zero-index materials



Zero-index materials

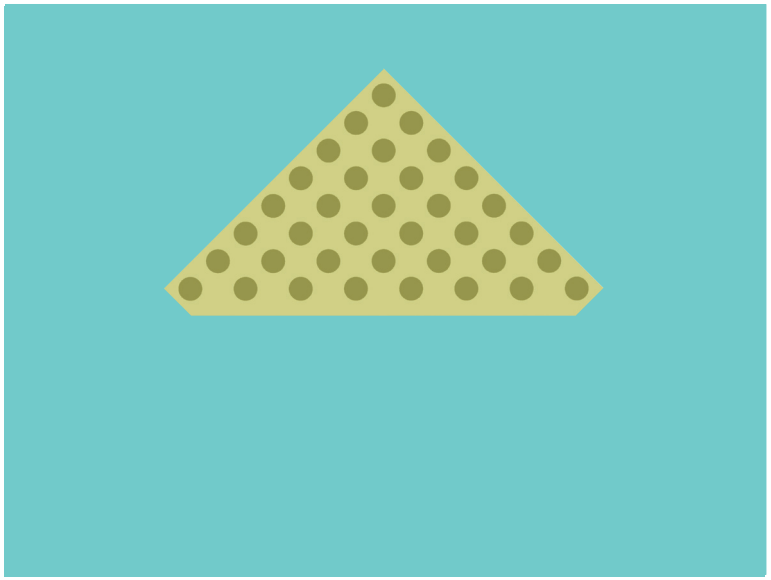


Zero-index materials

On-chip zero-index prism

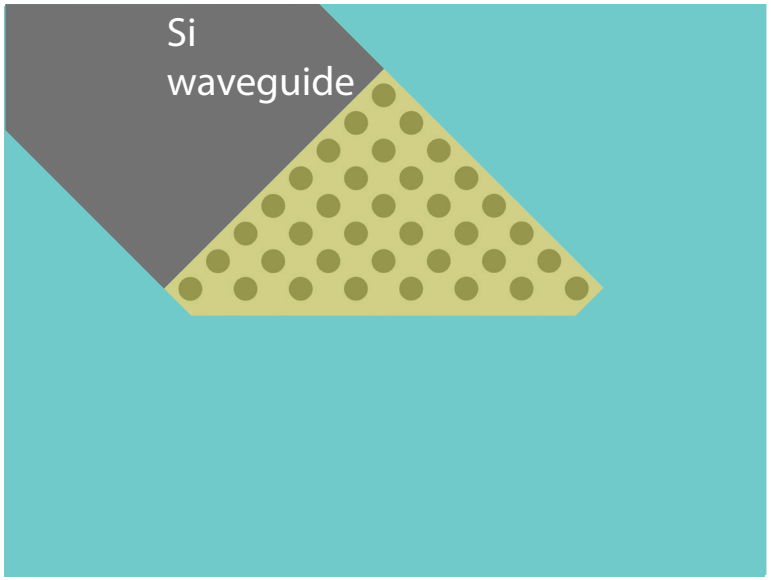
Zero-index materials

On-chip zero-index prism



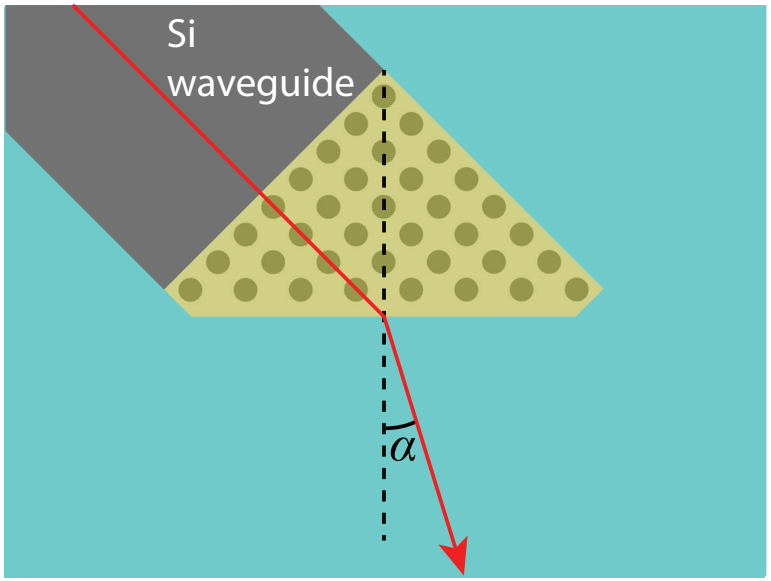
Zero-index materials

On-chip zero-index prism



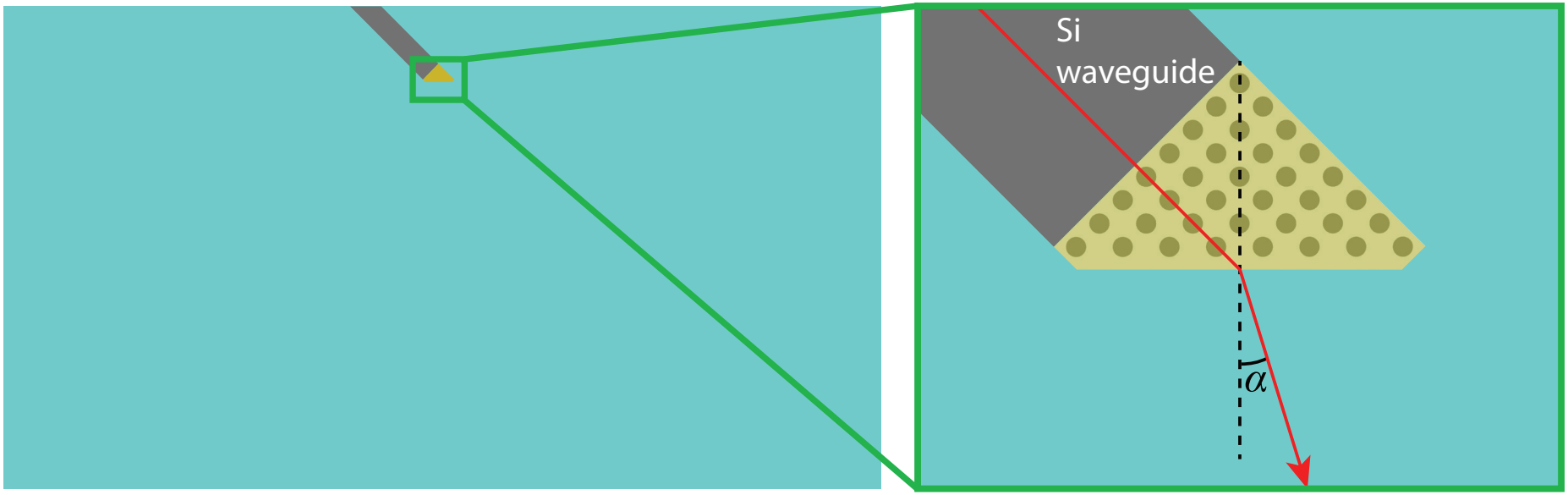
Zero-index materials

On-chip zero-index prism



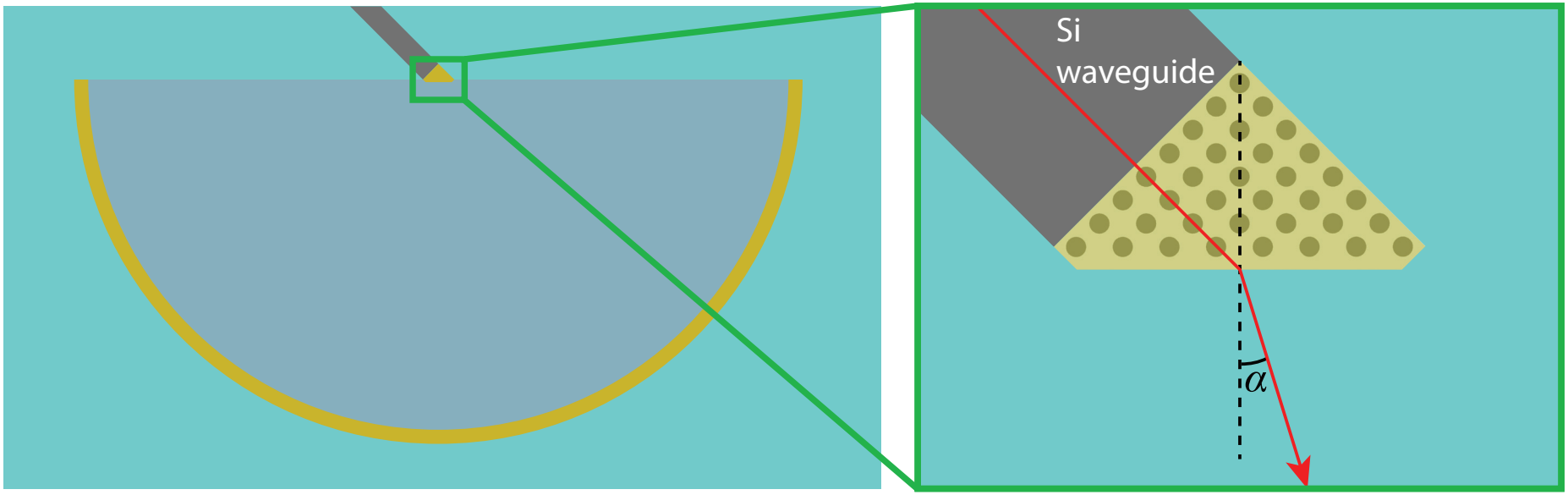
Zero-index materials

On-chip zero-index prism



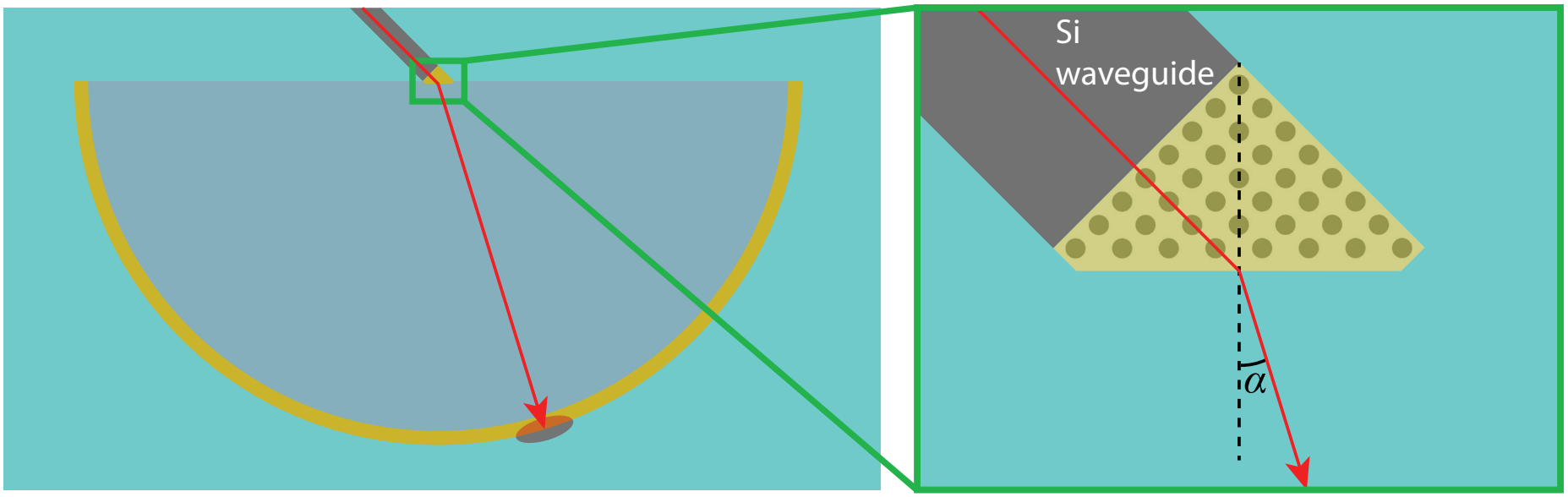
Zero-index materials

On-chip zero-index prism



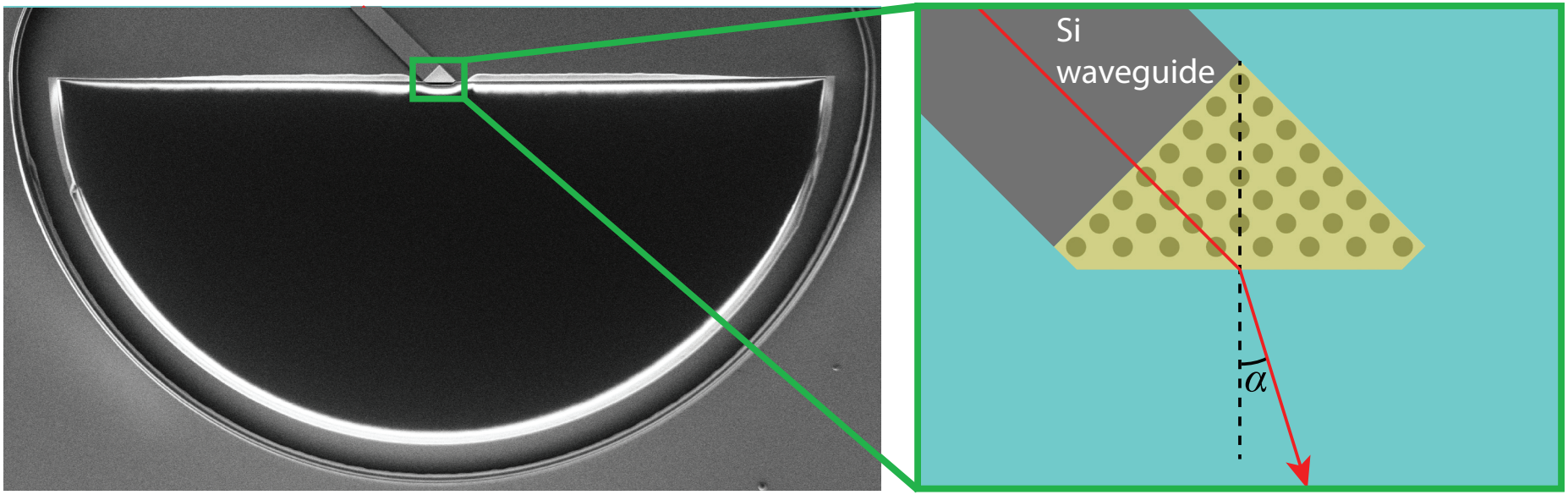
Zero-index materials

On-chip zero-index prism



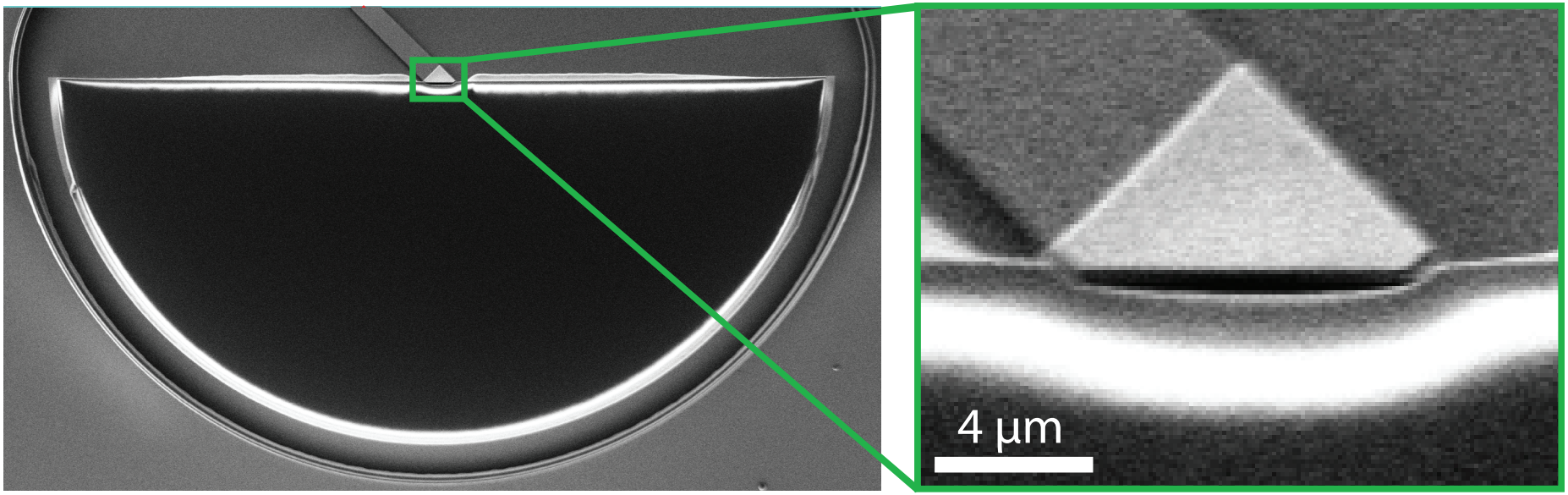
Zero-index materials

On-chip zero-index prism

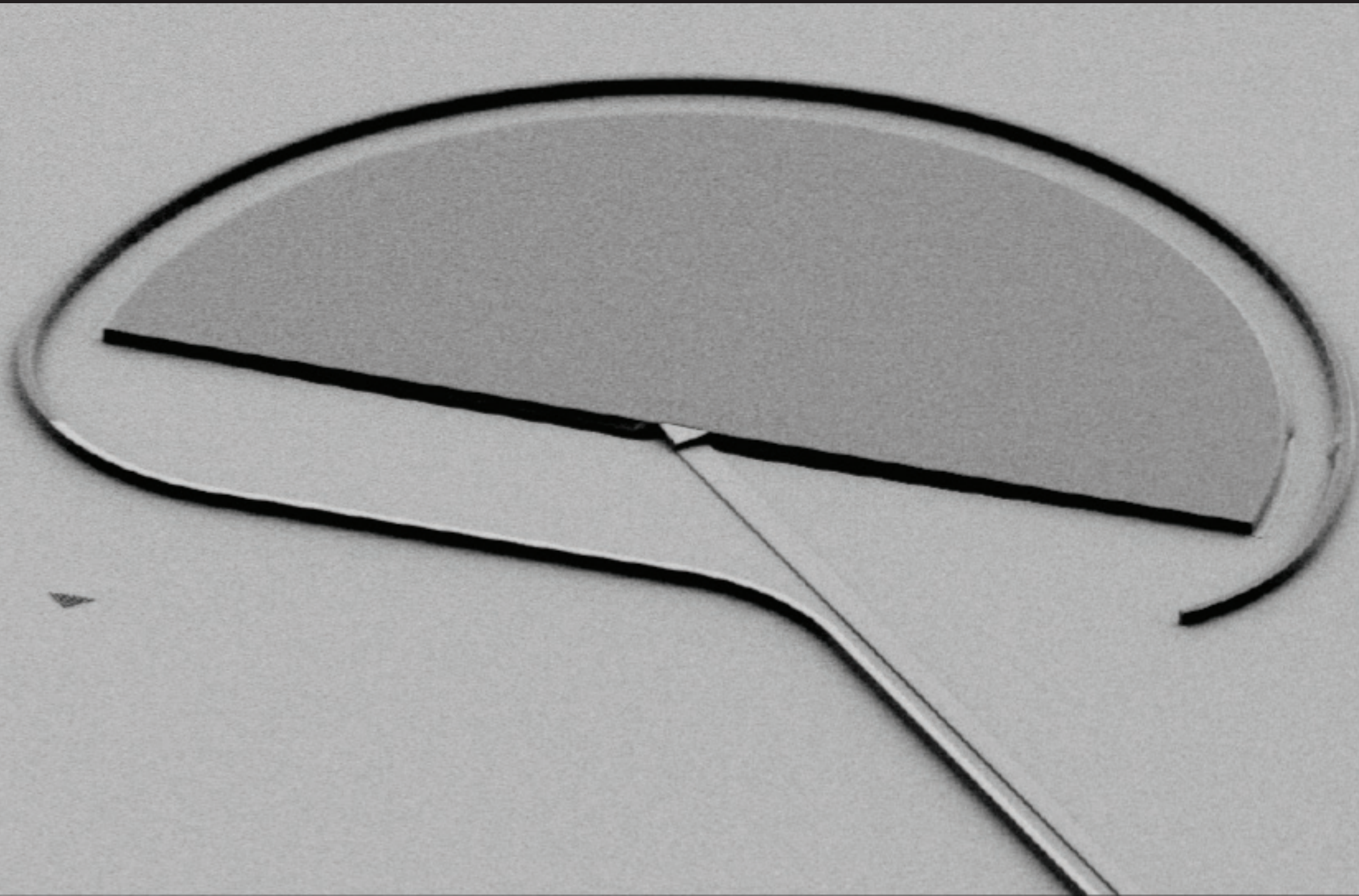


Zero-index materials

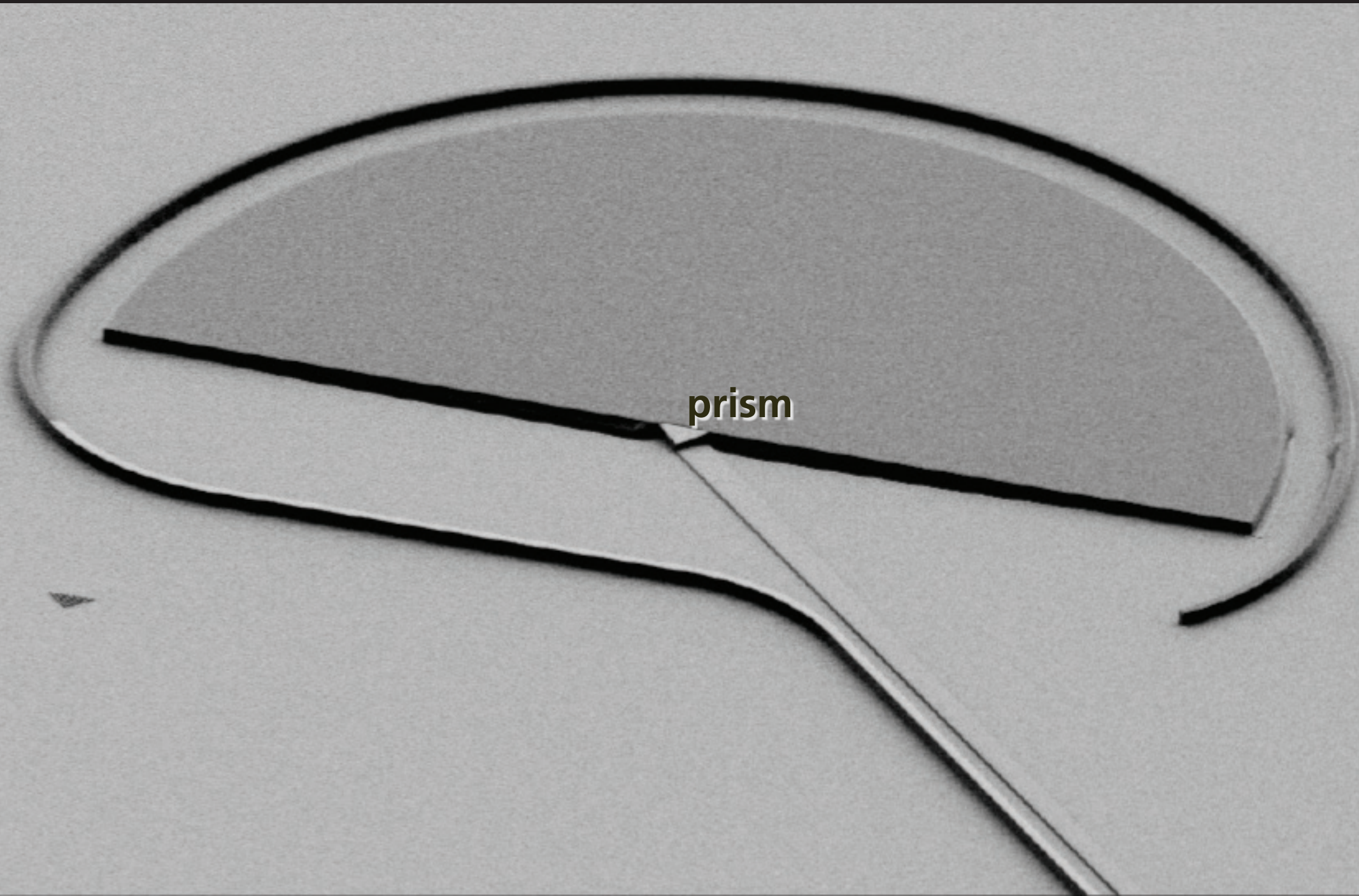
On-chip zero-index prism



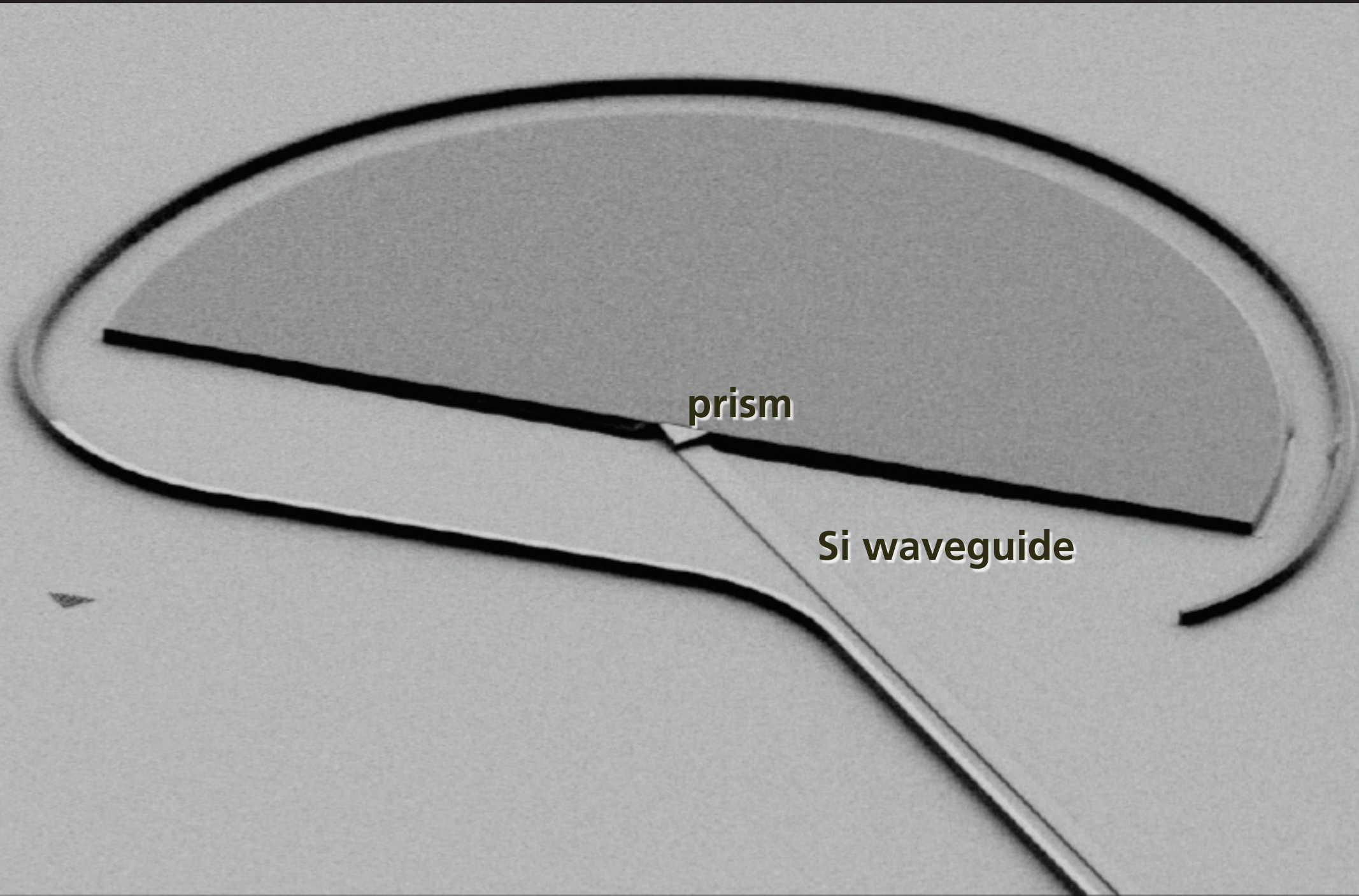
Zero-index materials



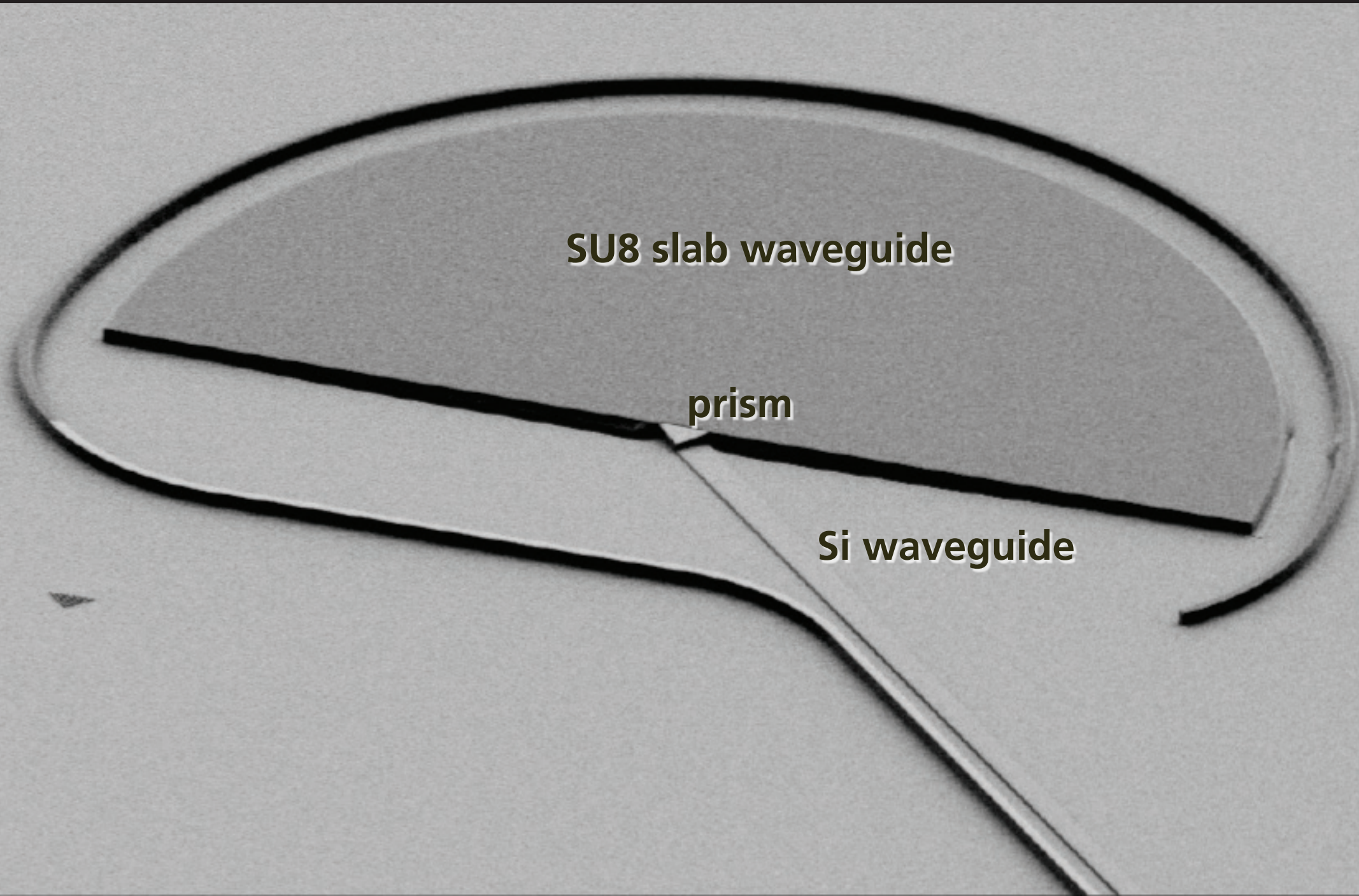
Zero-index materials



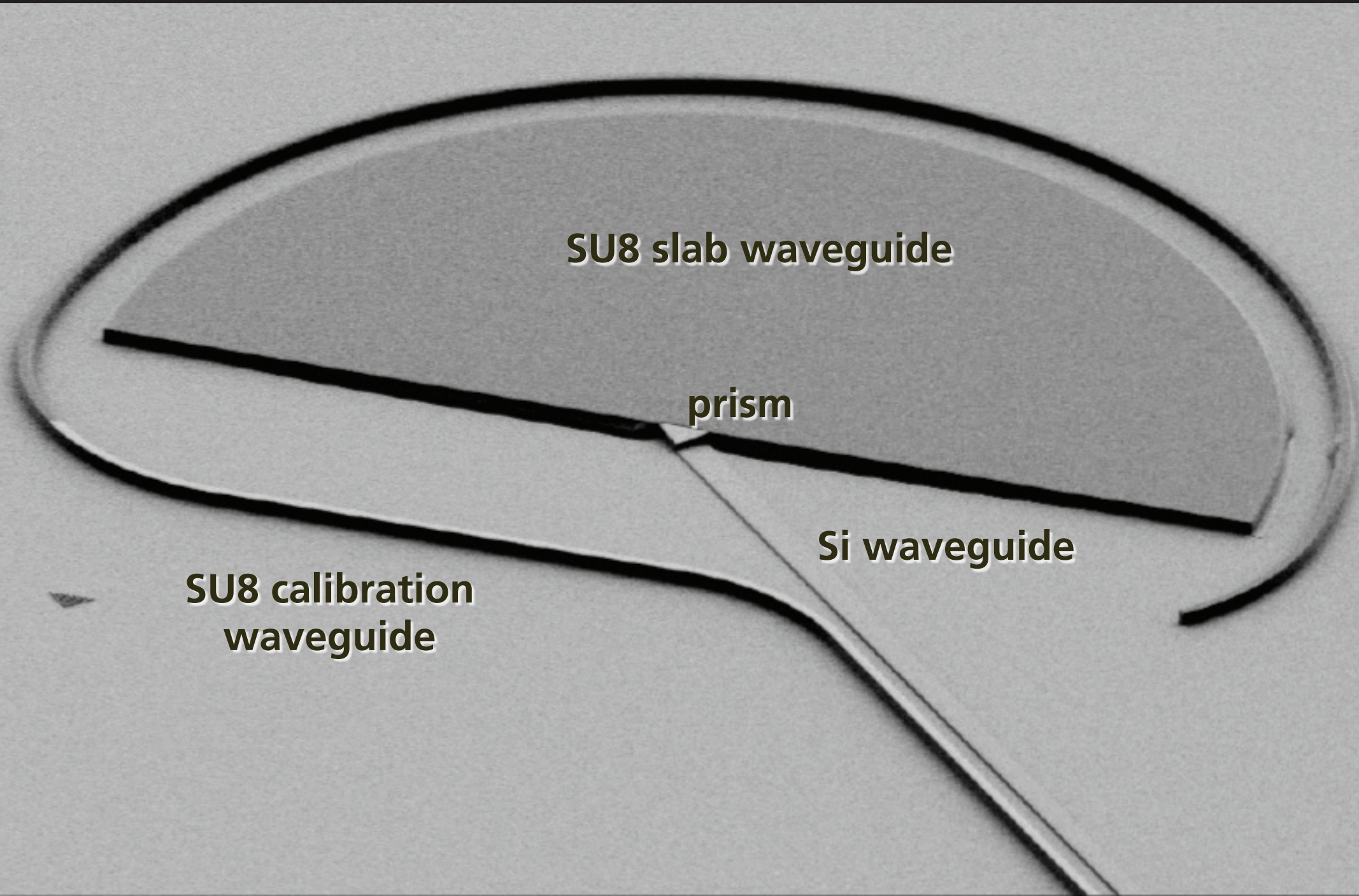
Zero-index materials



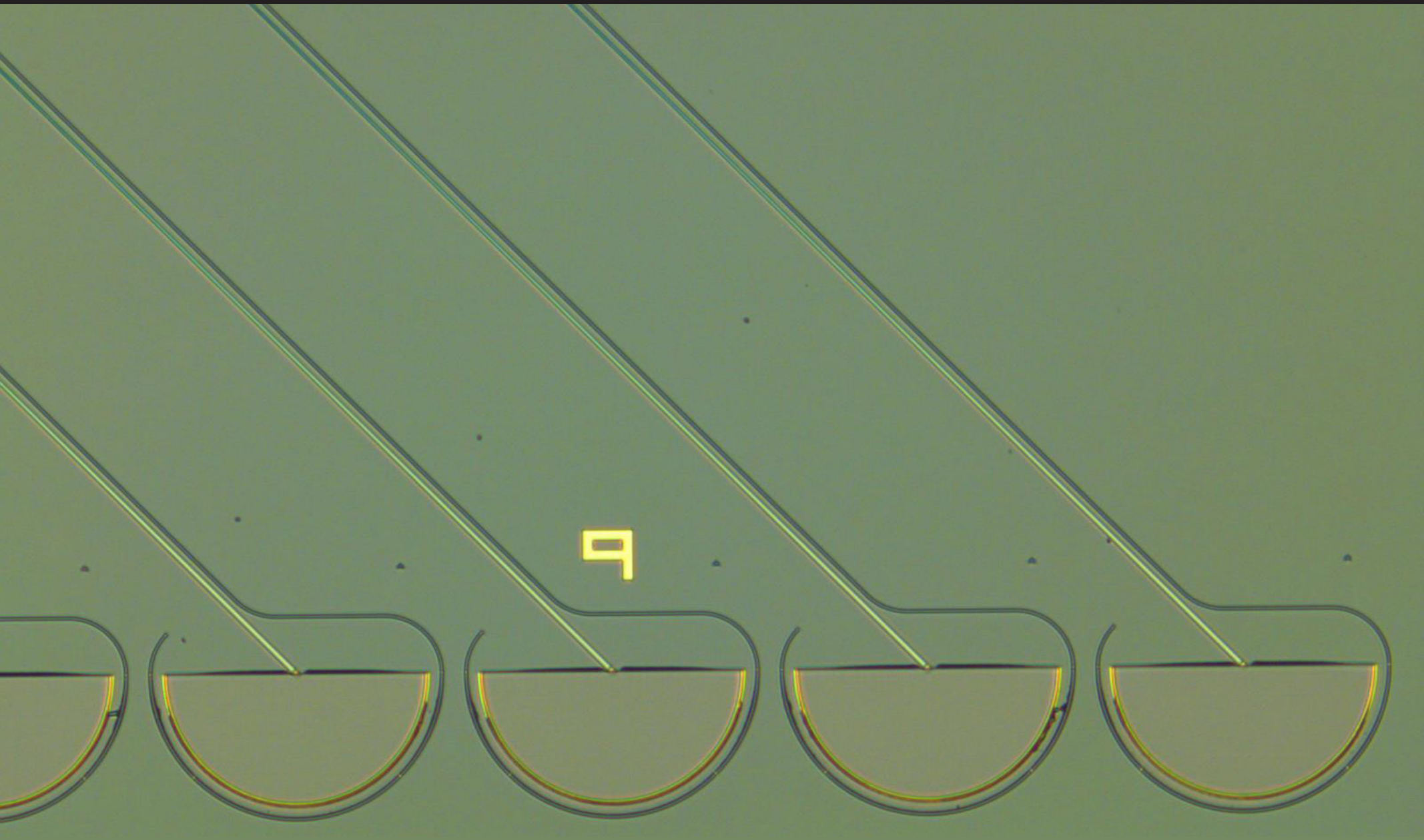
Zero-index materials



Zero-index materials

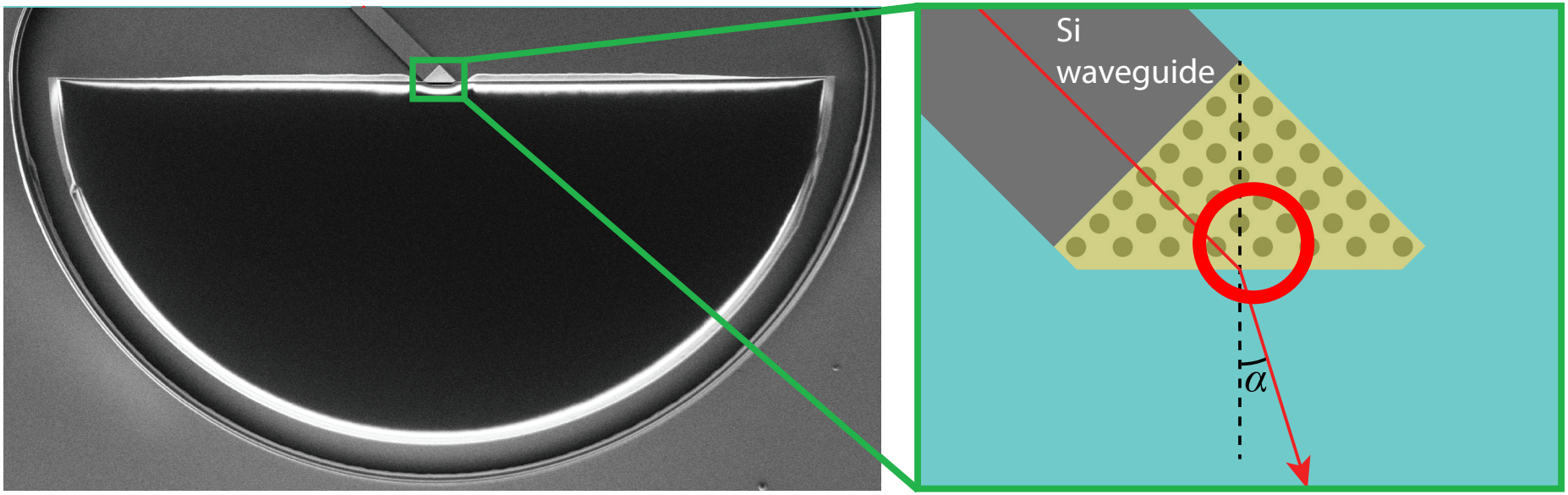


Zero-index materials

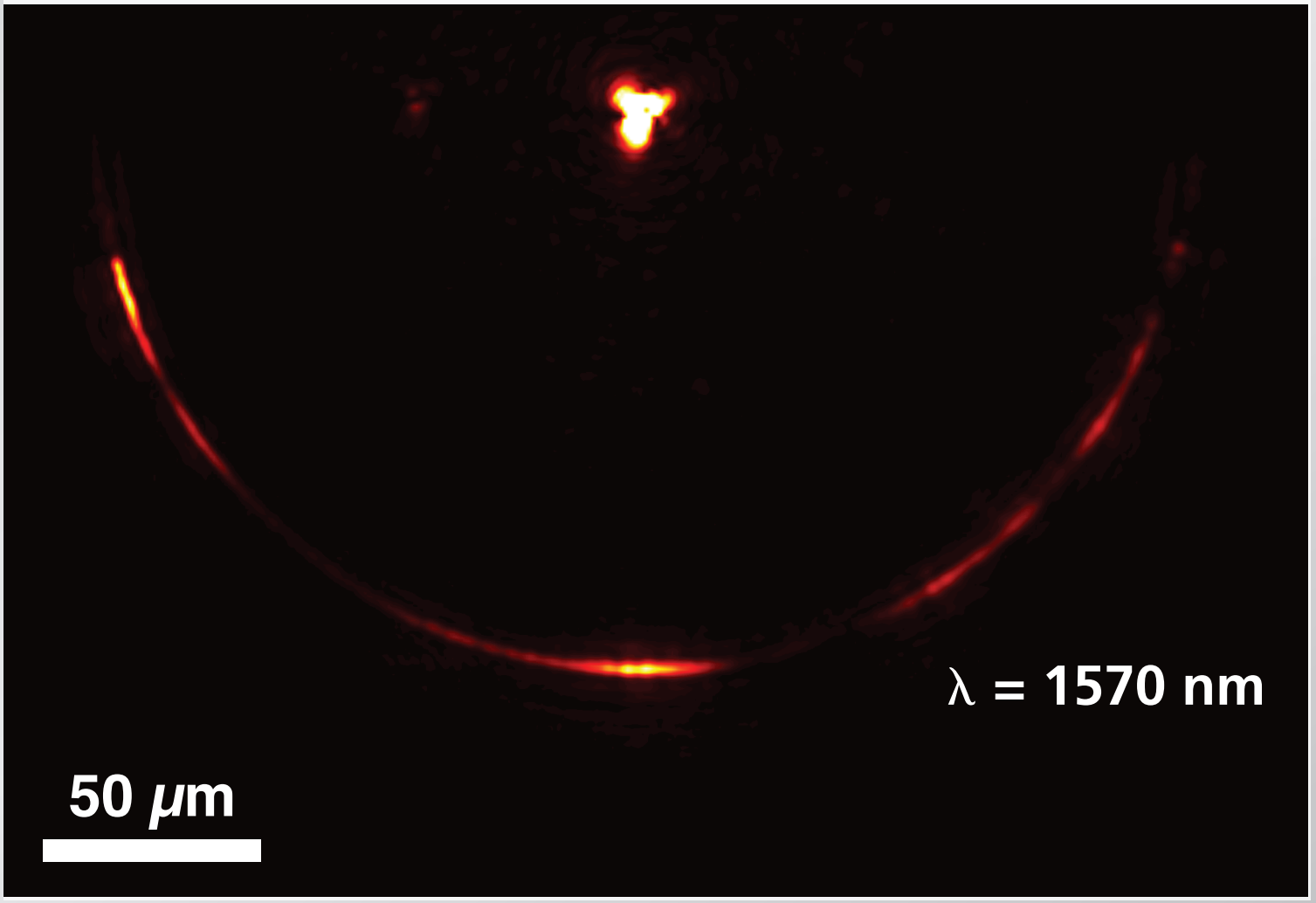


Zero-index materials

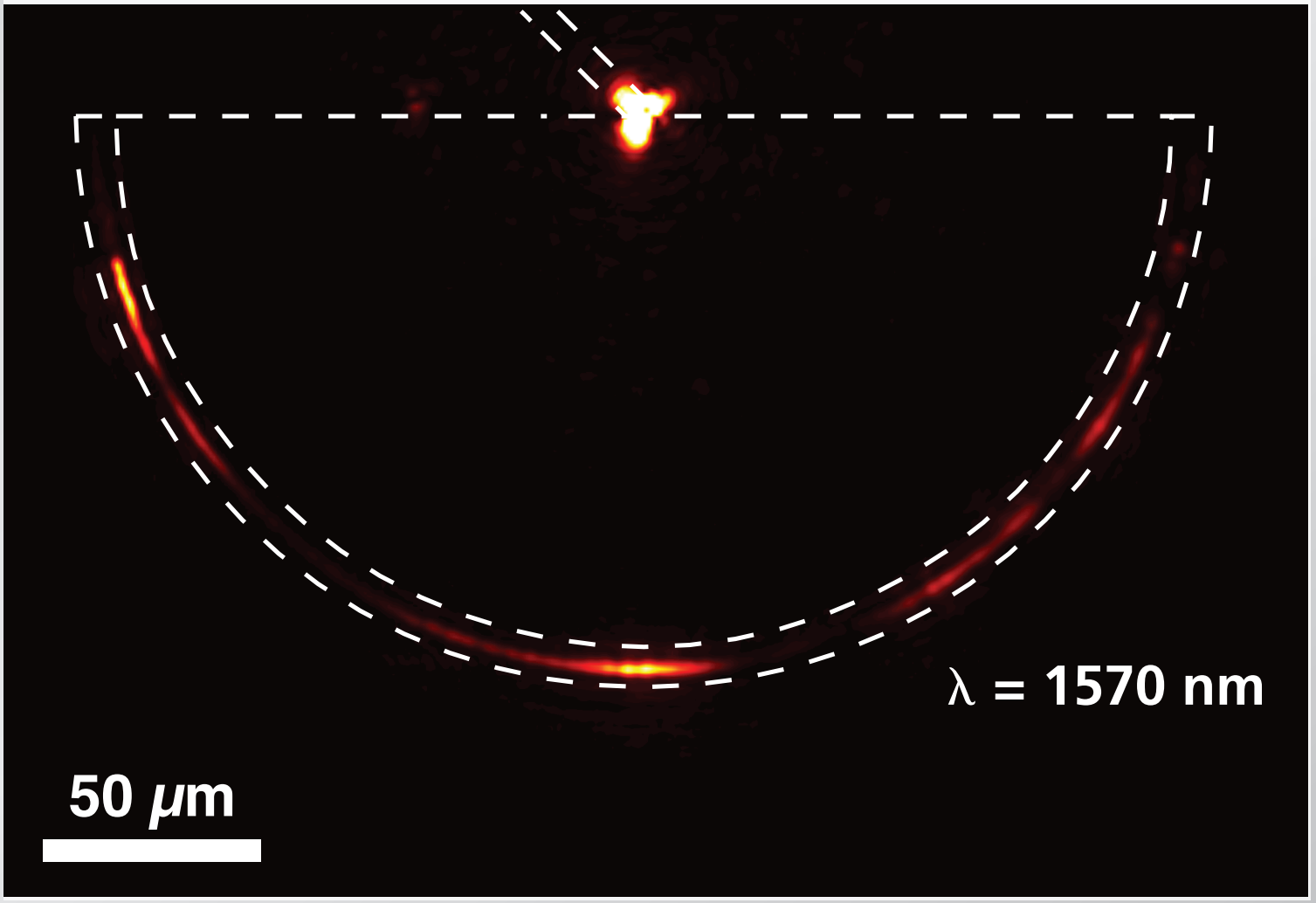
On-chip zero-index prism



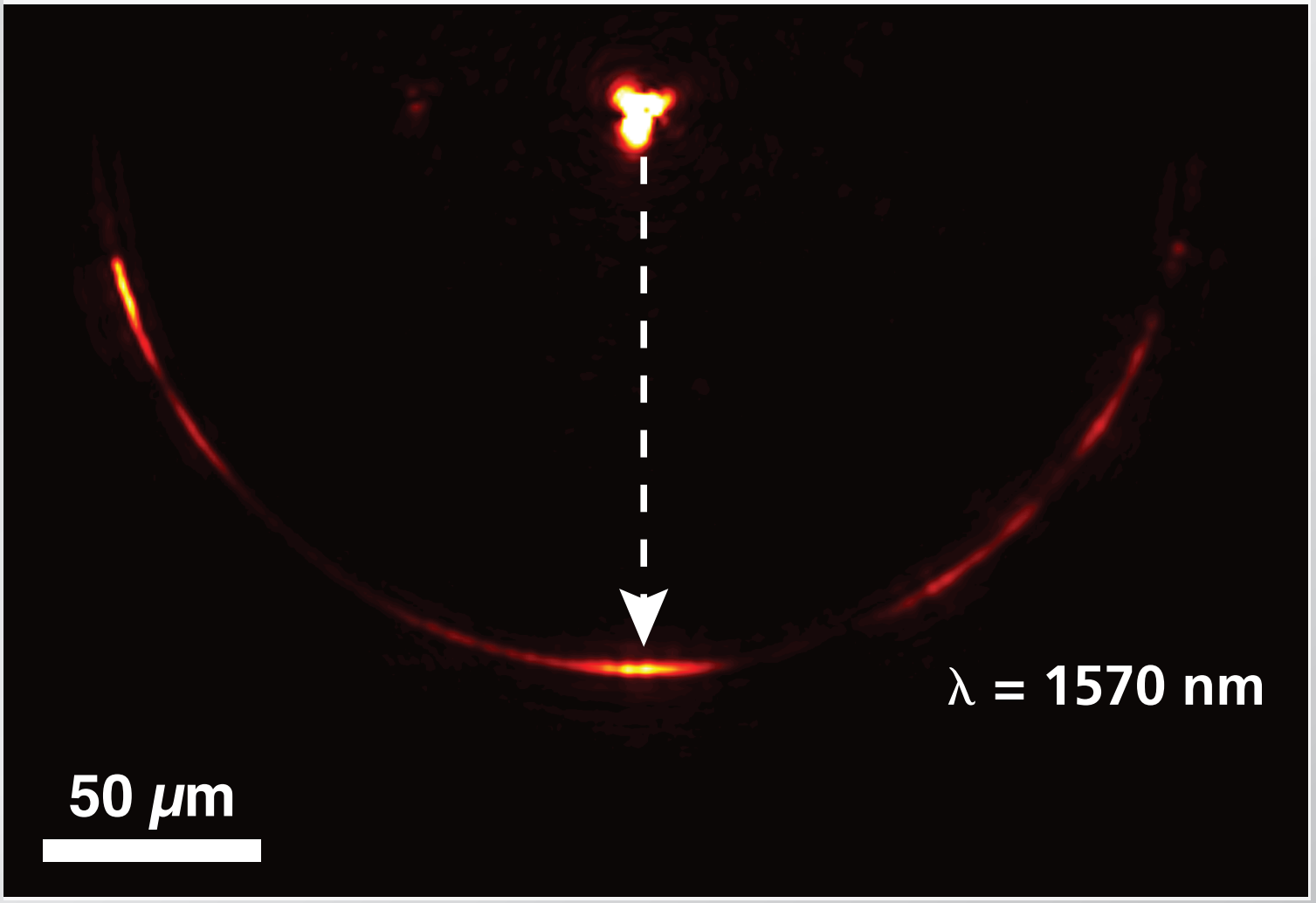
Zero-index materials



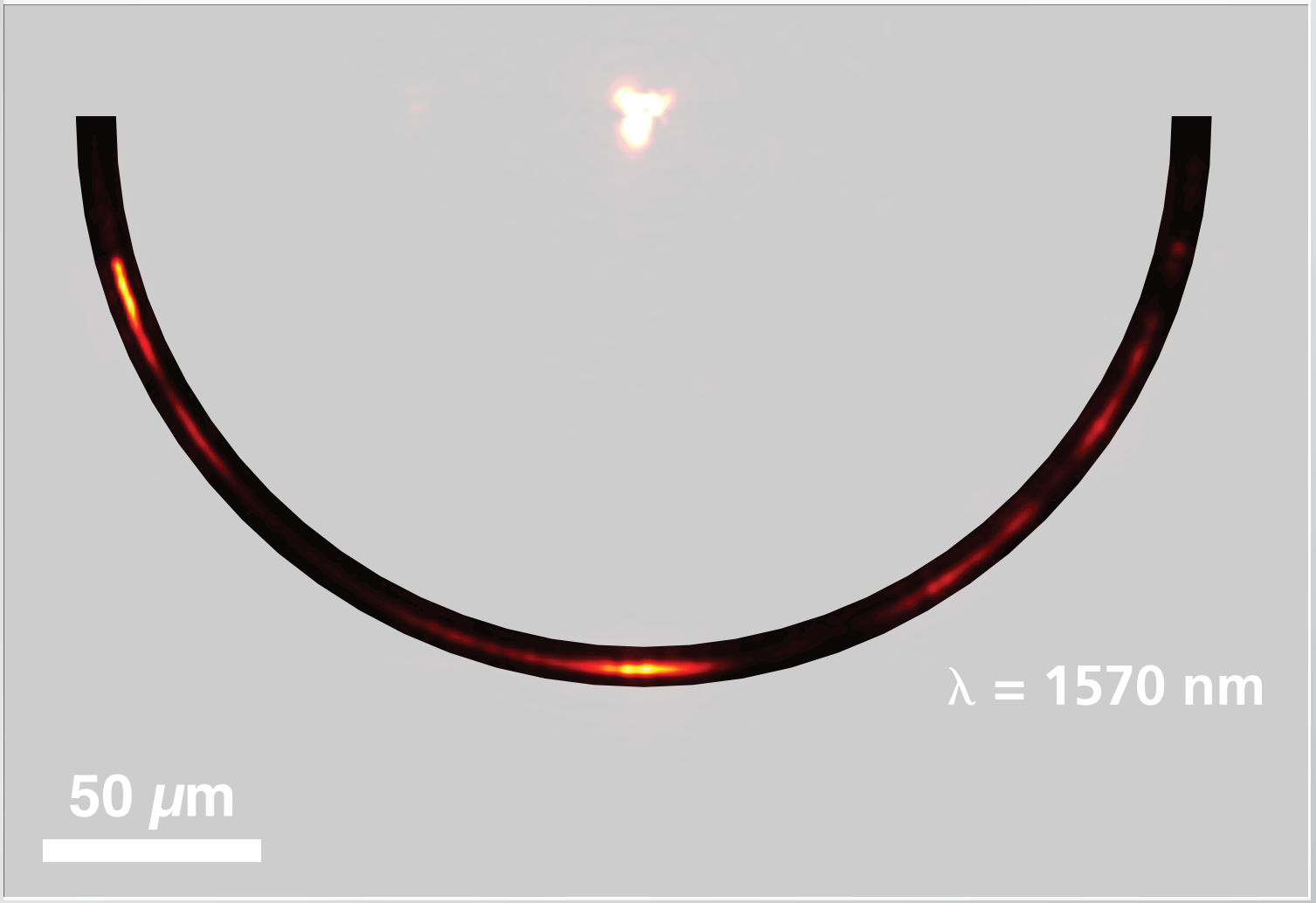
Zero-index materials



Zero-index materials

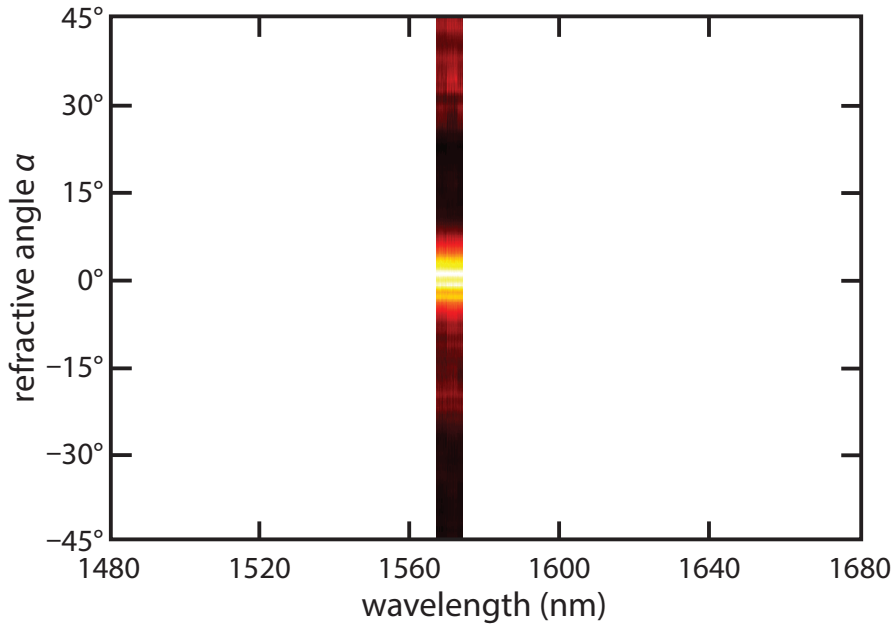


Zero-index materials



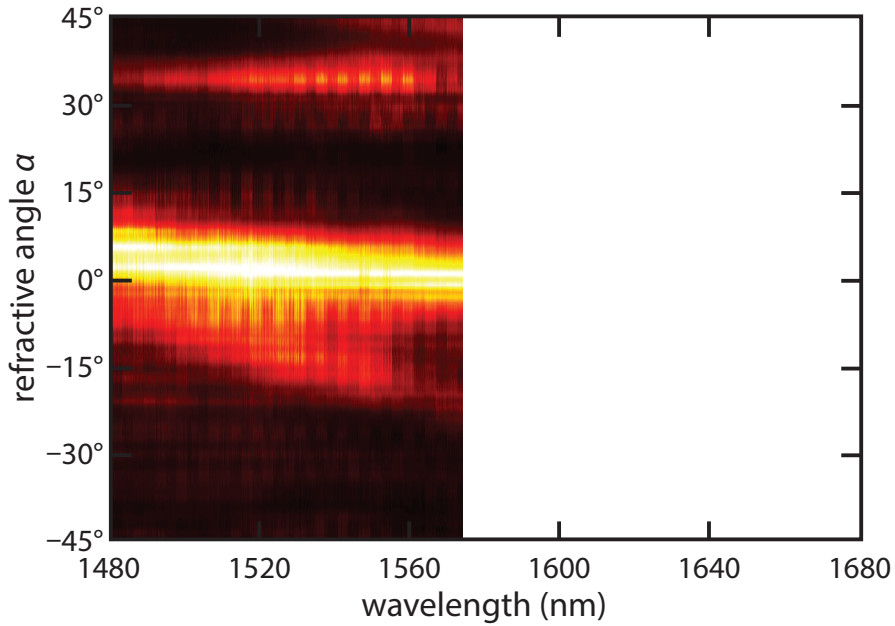
Zero-index materials

Wavelength dependence of refraction angle



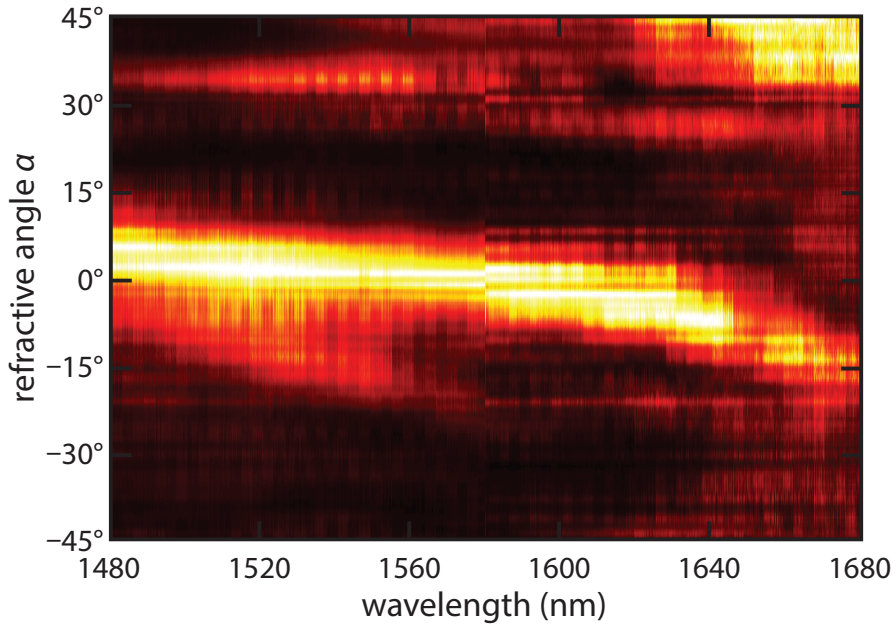
Zero-index materials

Wavelength dependence of refraction angle



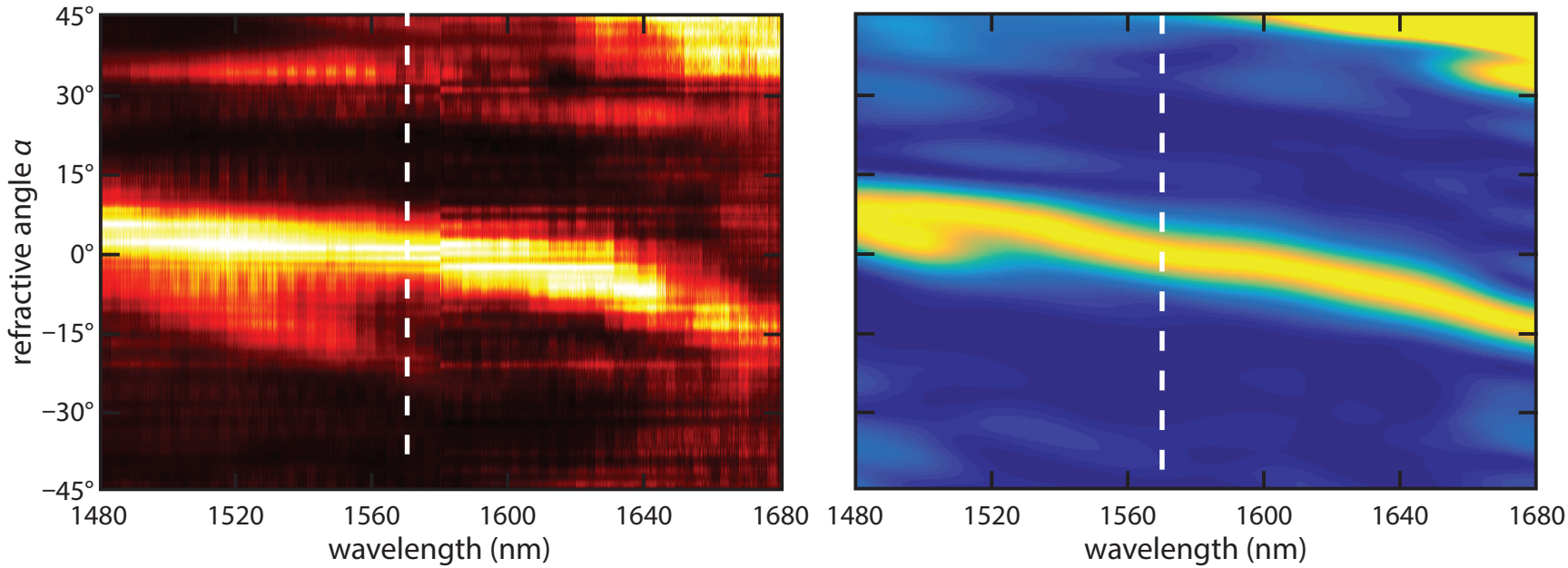
Zero-index materials

Wavelength dependence of refraction angle



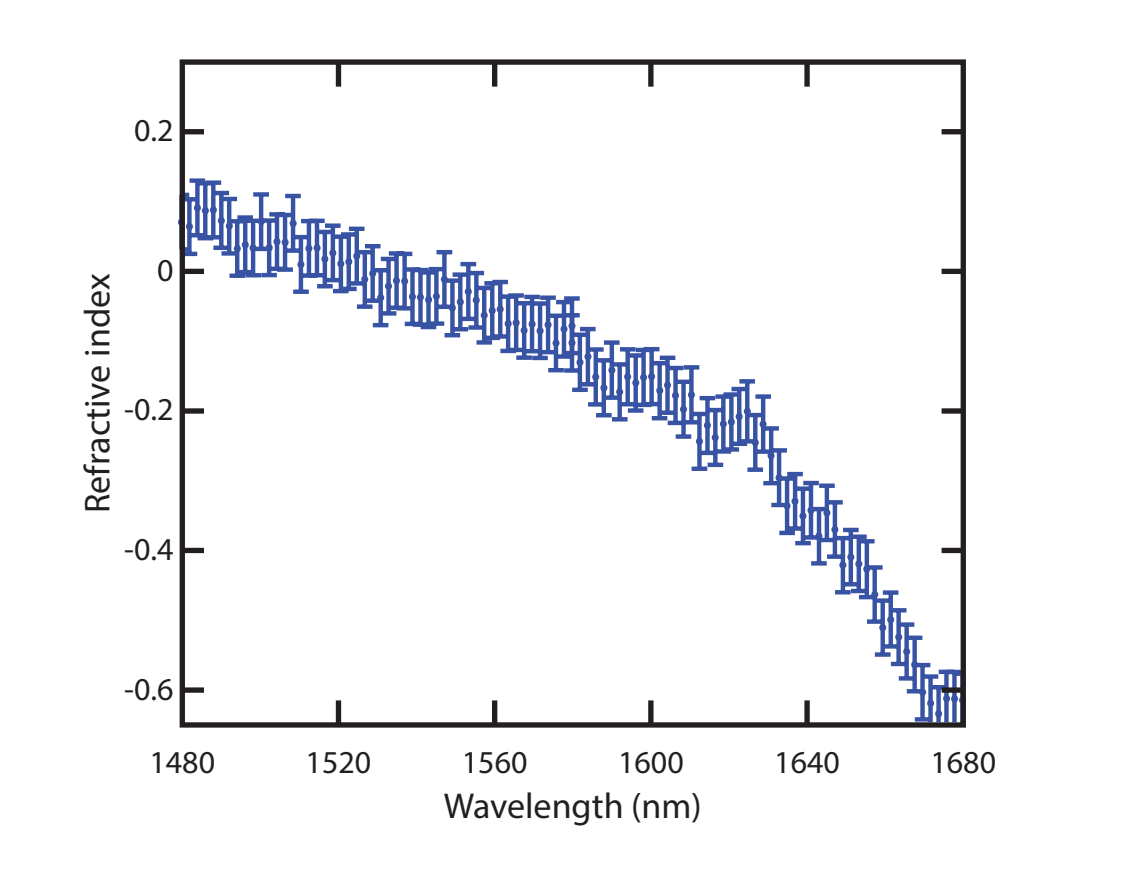
Zero-index materials

Wavelength dependence of refraction angle



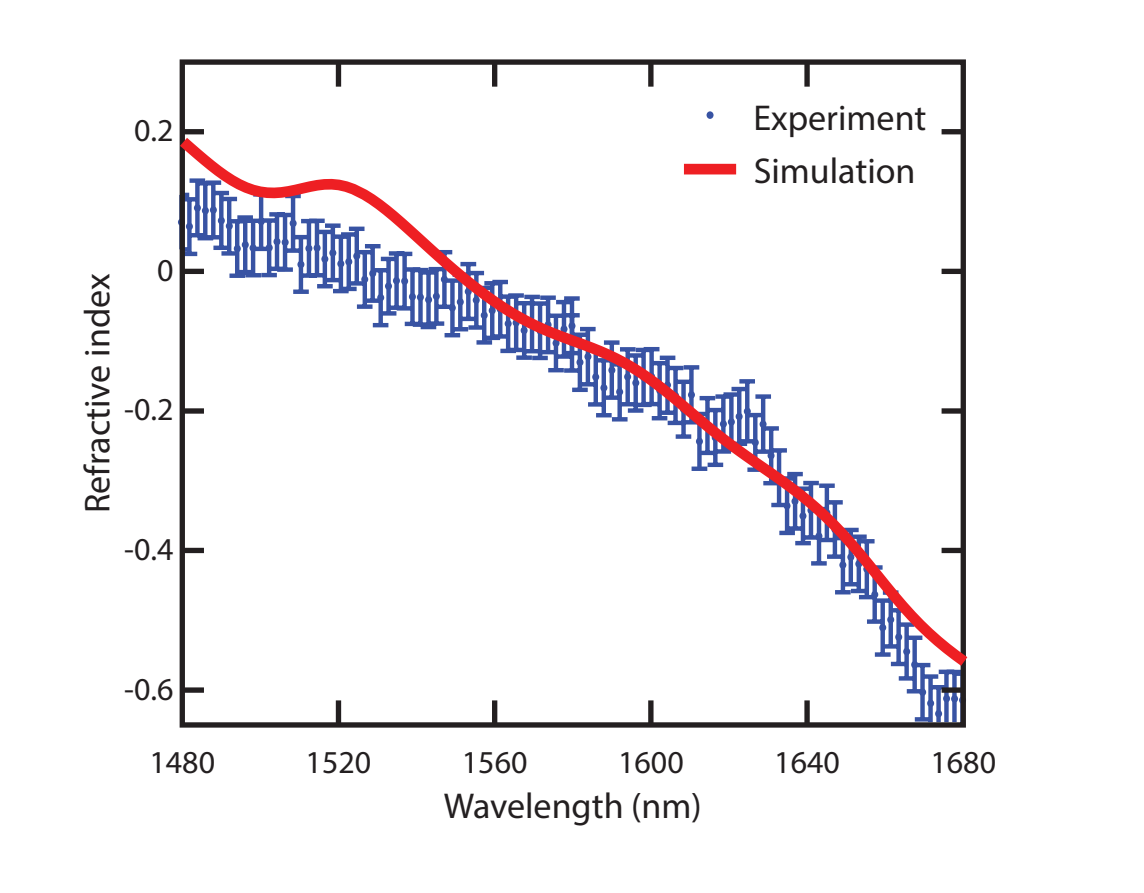
Zero-index materials

Wavelength dependence of index



Zero-index materials

Wavelength dependence of index

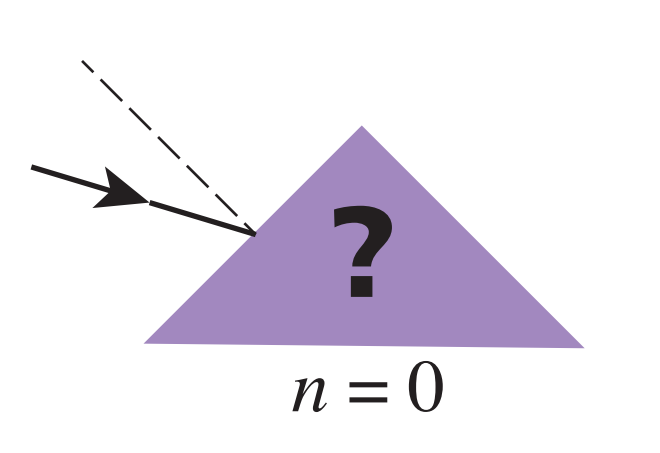


Zero-index materials

unambiguous demonstration of on-chip zero-index material!

Zero-index materials

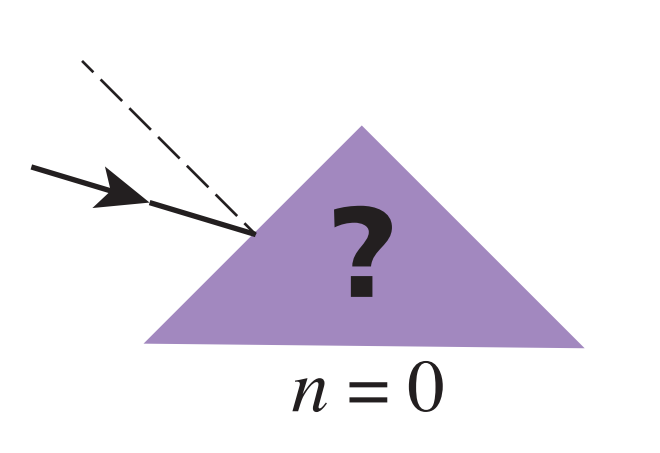
Q: What happens when a beam of light at the wavelength for which $n = 0$ strikes a side of a zero-index prism at an angle away from the normal?



1. beats occur inside and around the prism.
2. the beam comes out at the same angle on the other facets.
3. the beam is perfectly reflected.
4. the beam is transmitted only for certain (nonzero) angles.
5. it couples perfectly, regardless of angle.

Zero-index materials

Q: What happens when a beam of light at the wavelength for which $n = 0$ strikes a side of a zero-index prism at an angle away from the normal?



1. beats occur inside and around the prism.
2. the beam comes out at the same angle on the other facets.
3. the beam is perfectly reflected. ✔
4. the beam is transmitted only for certain (nonzero) angles.
5. it couples perfectly, regardless of angle.

Engineering the index

Key points

- tune optical properties using composite materials
- zero index requires a magnetic response
- produce magnetic response in dielectrics
- demonstrated on-chip impedance-matched $n = 0$

A very special thanks to

Phil Muñoz

Yang Li

Orad Reshef

Funding:

Air Force Office of Scientific Research
Natural Sciences and Engineering Research Council of Canada
Harvard Quantum Optics Center

for a copy of this presentation:

http://ericmazur.com

Follow me! **eric_mazur**