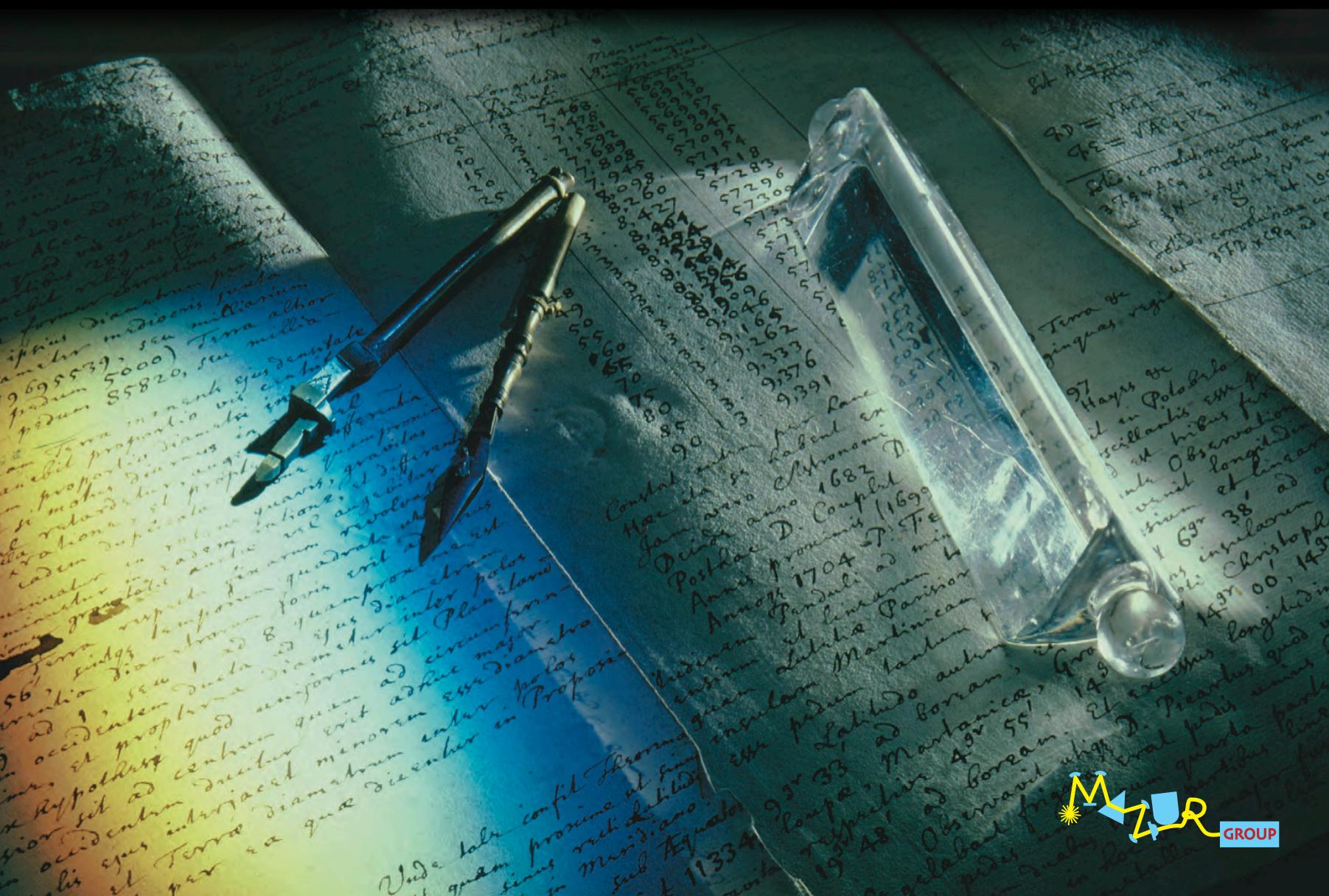


Less is more: Extreme optics with zero refractive index



Propagation of EM waves

Governed by the wave equation:

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

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μ magnetic permeability

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where

$$v_p = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}} c = \frac{1}{n} c$$

v_p

and

$$n = \sqrt{\epsilon\mu}$$

Propagation of EM waves

1.



$$\nu_p$$

$$\nu_p = \frac{1}{n} c$$

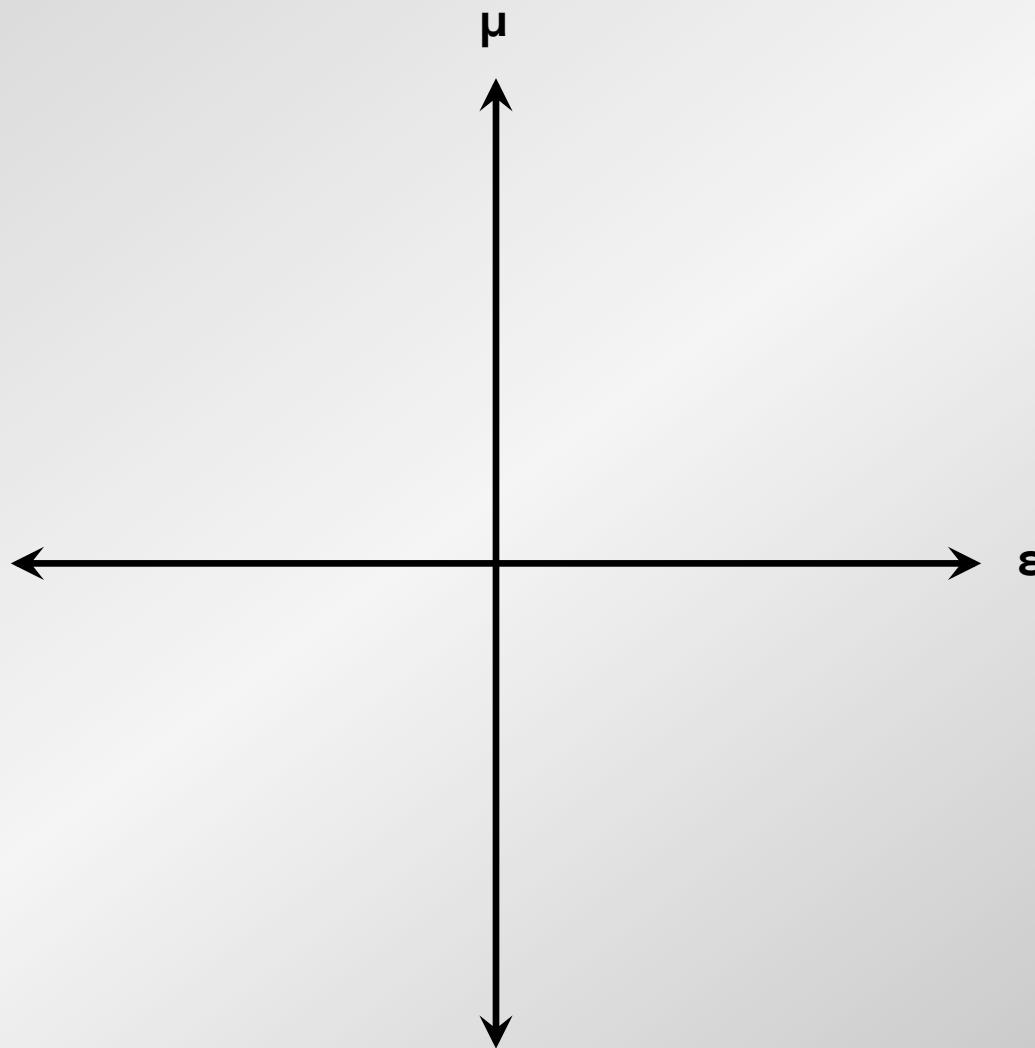
2.

ϵ electric permittivity

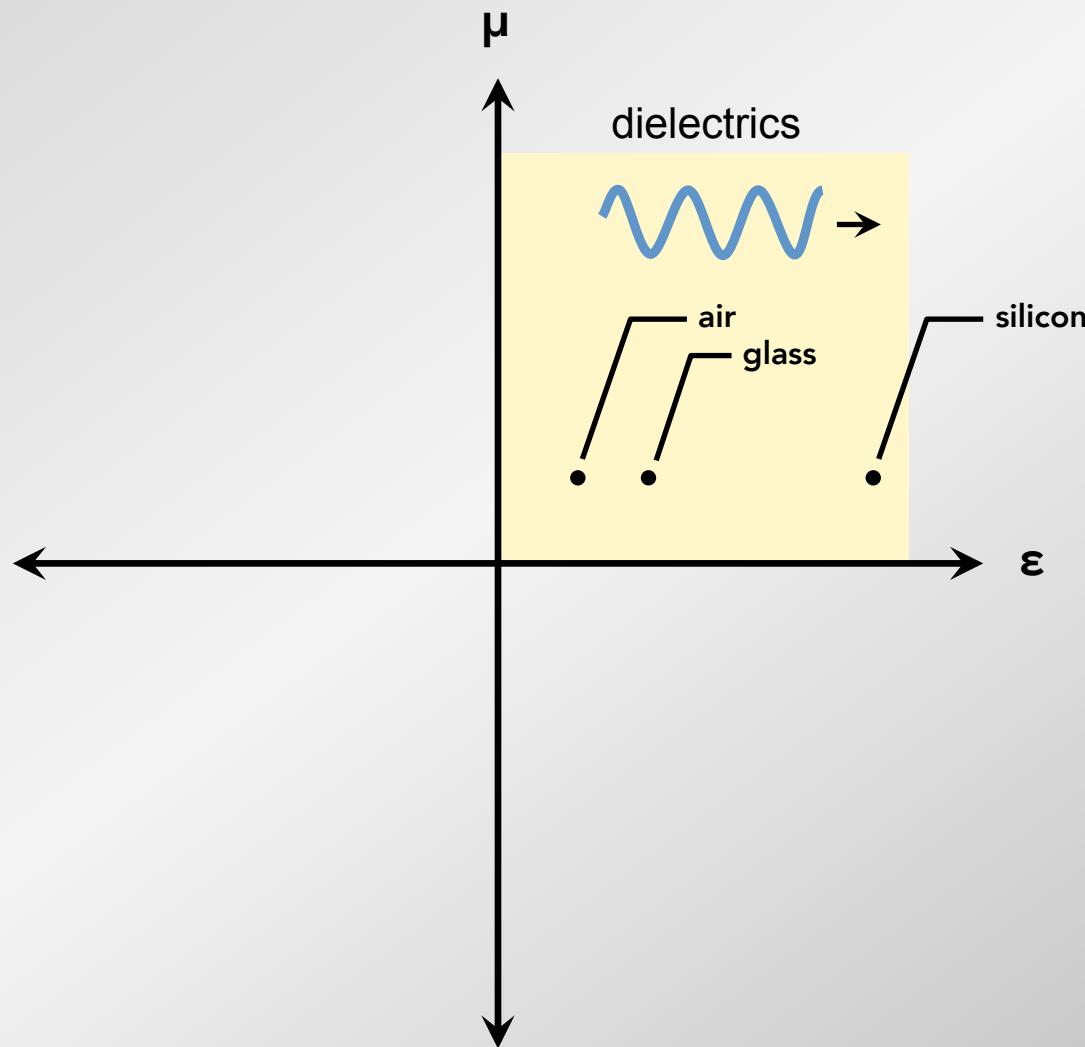
μ magnetic permeability

$$n = \sqrt{\epsilon\mu}$$

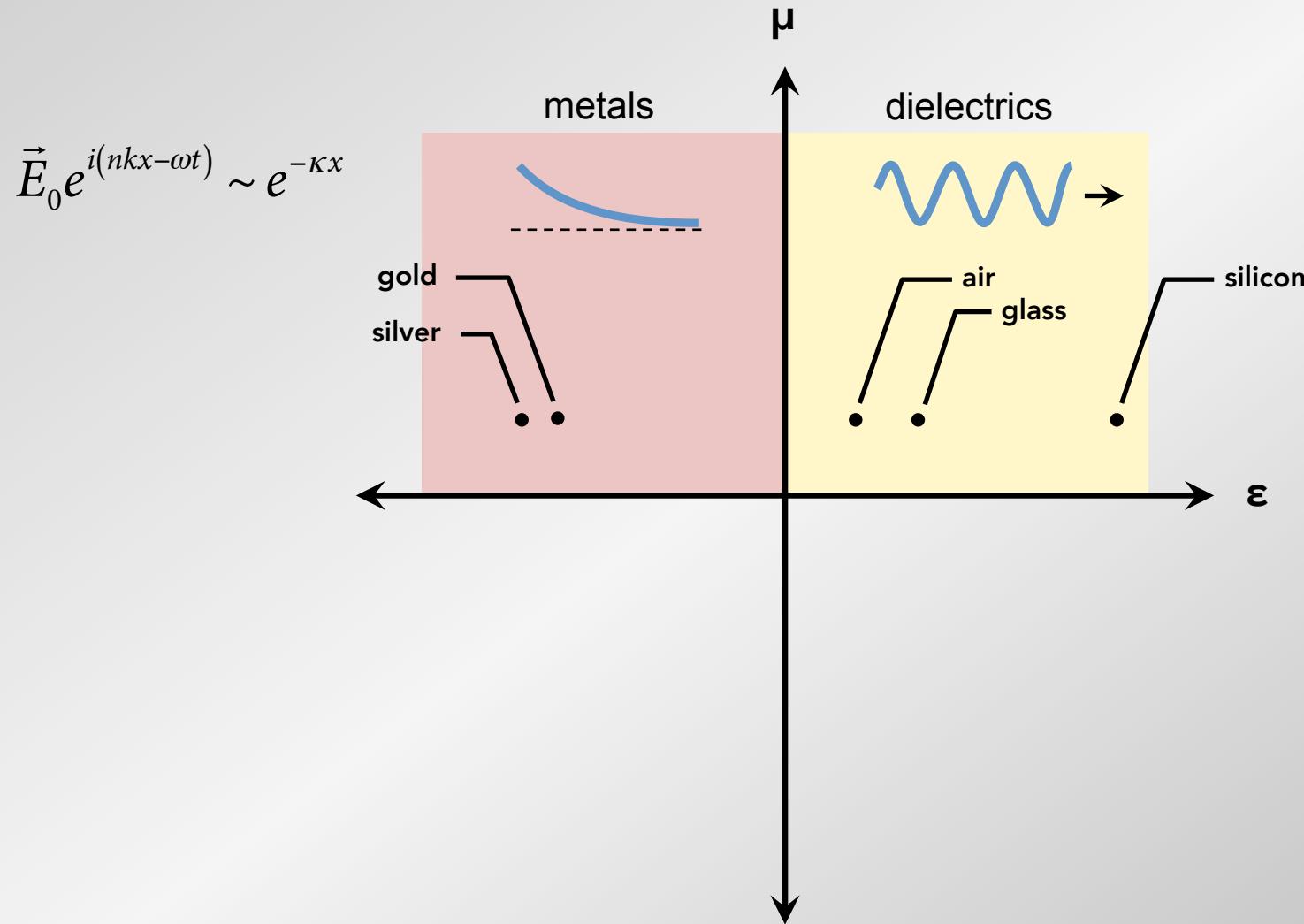
Classification of materials



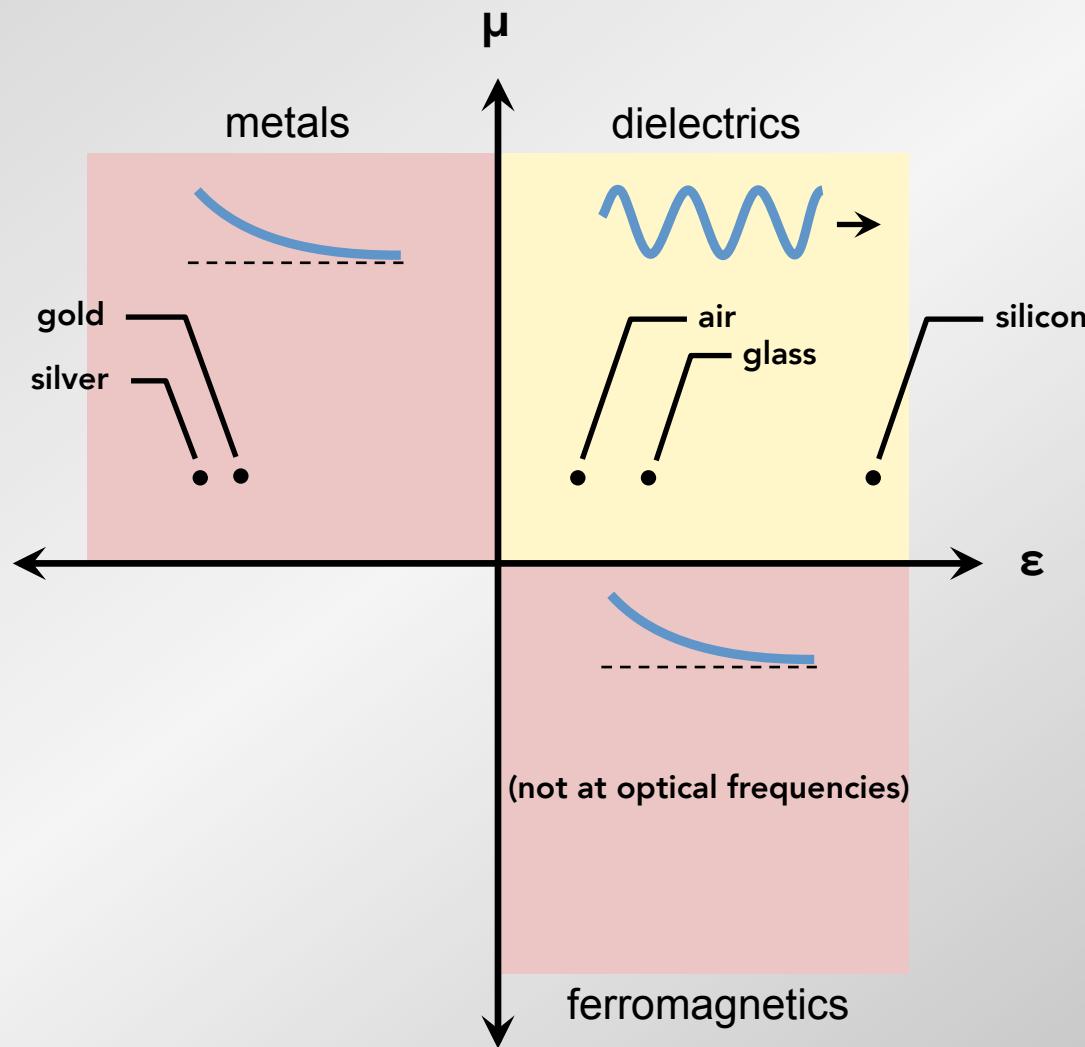
Classification of materials



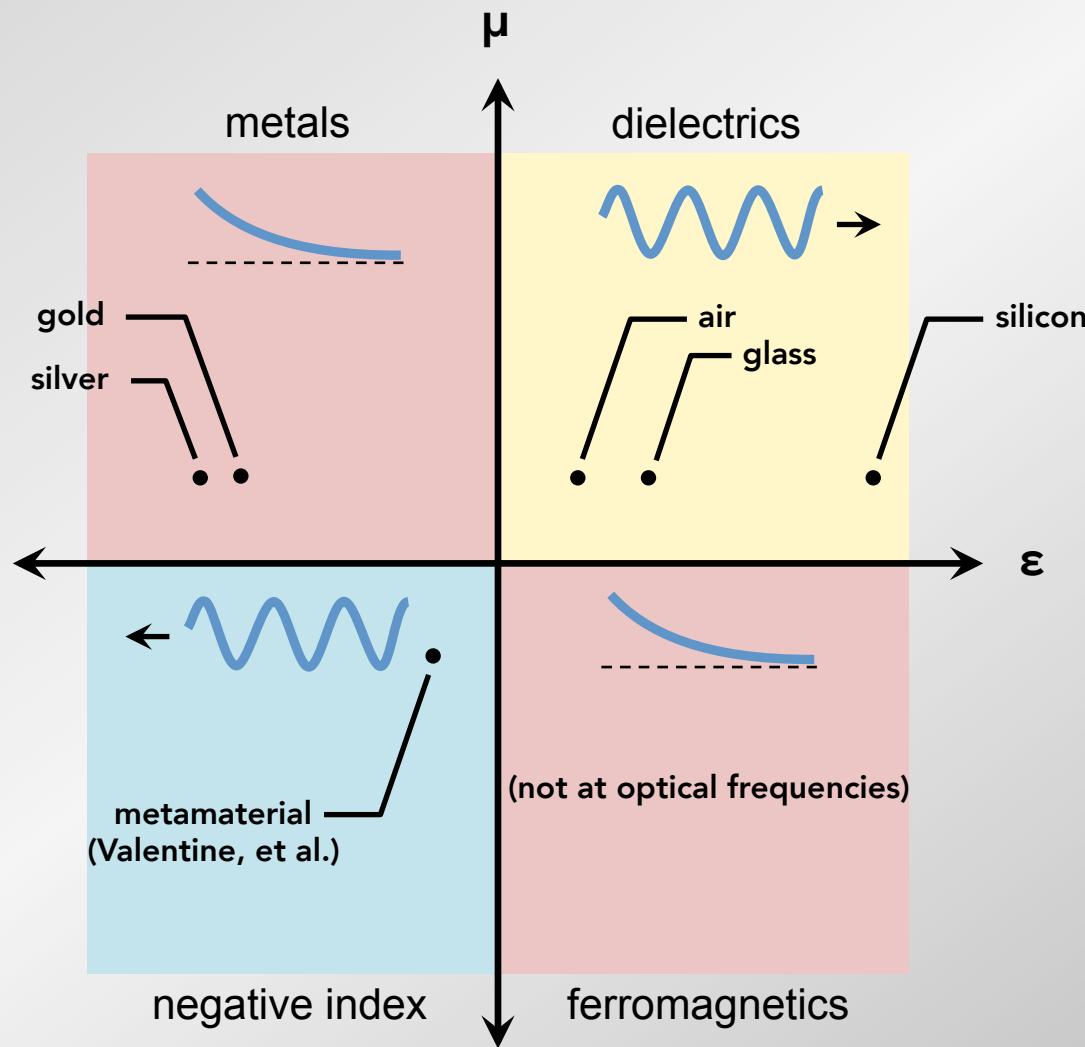
Classification of materials



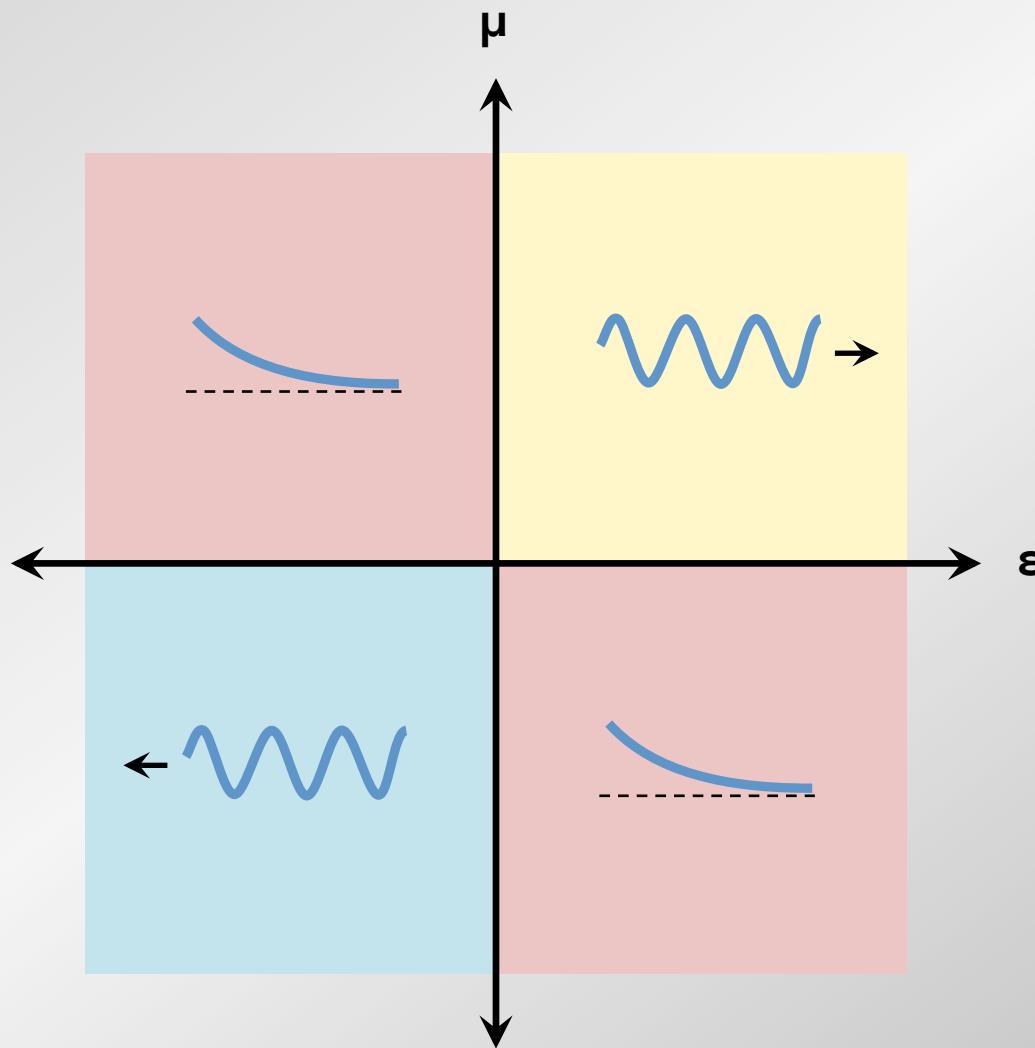
Classification of materials



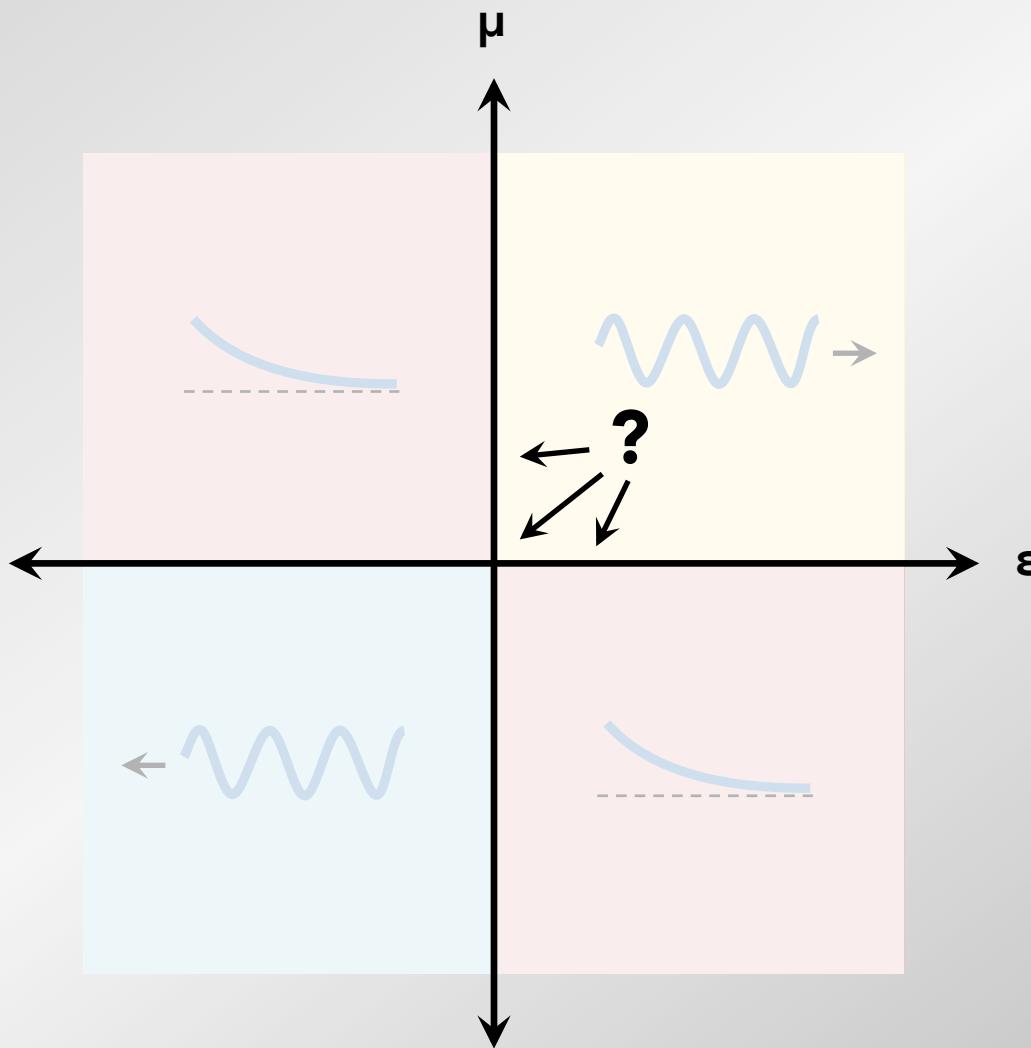
Classification of materials



Classification of materials



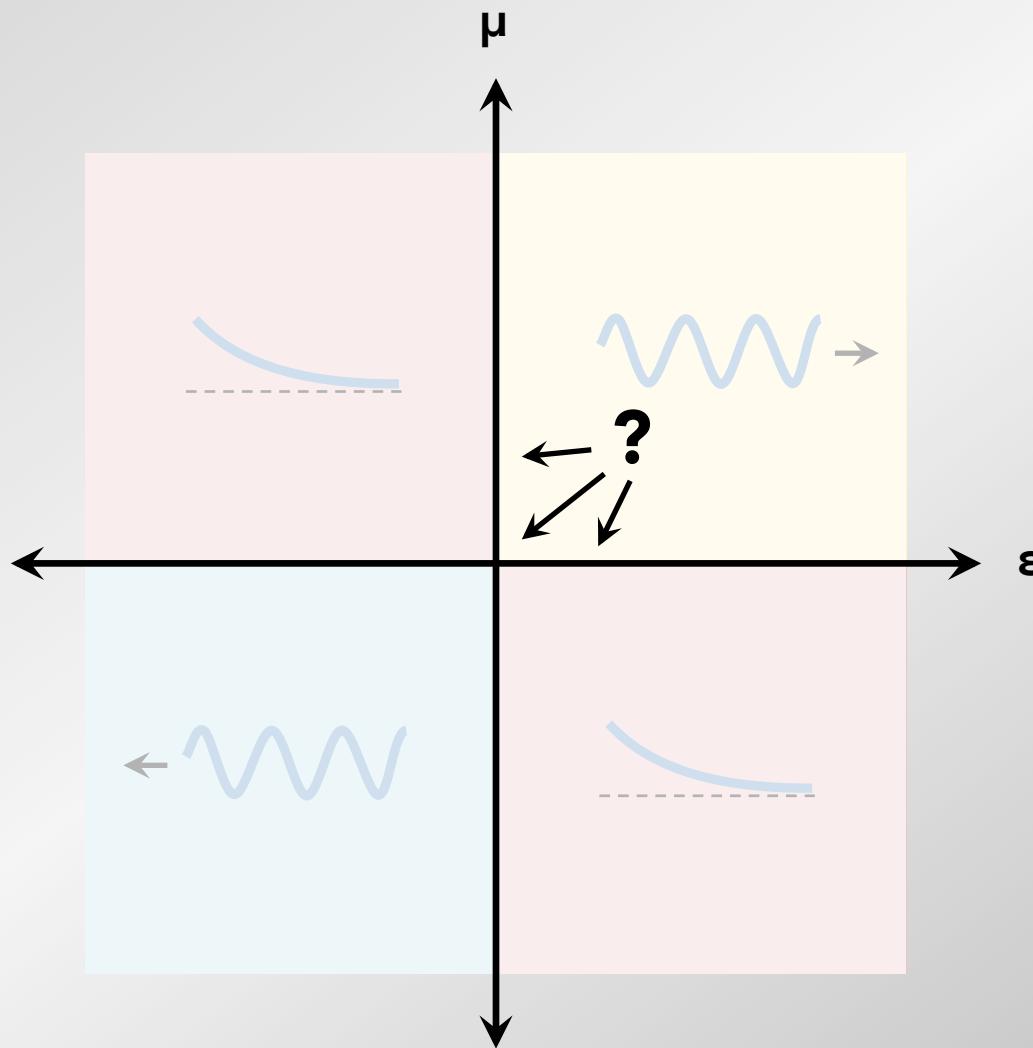
Classification of materials



1

zero index

Classification of materials



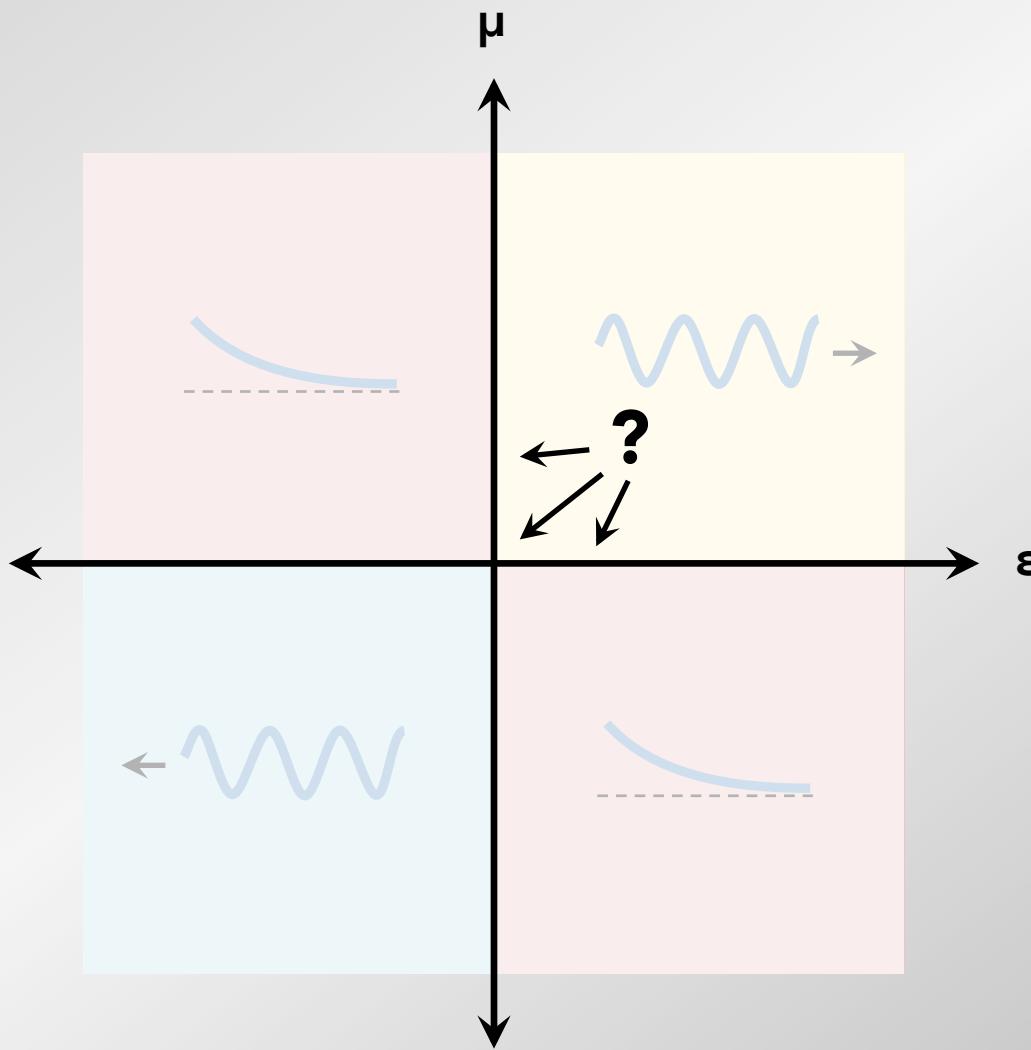
1

zero index

2

metamaterials

Classification of materials

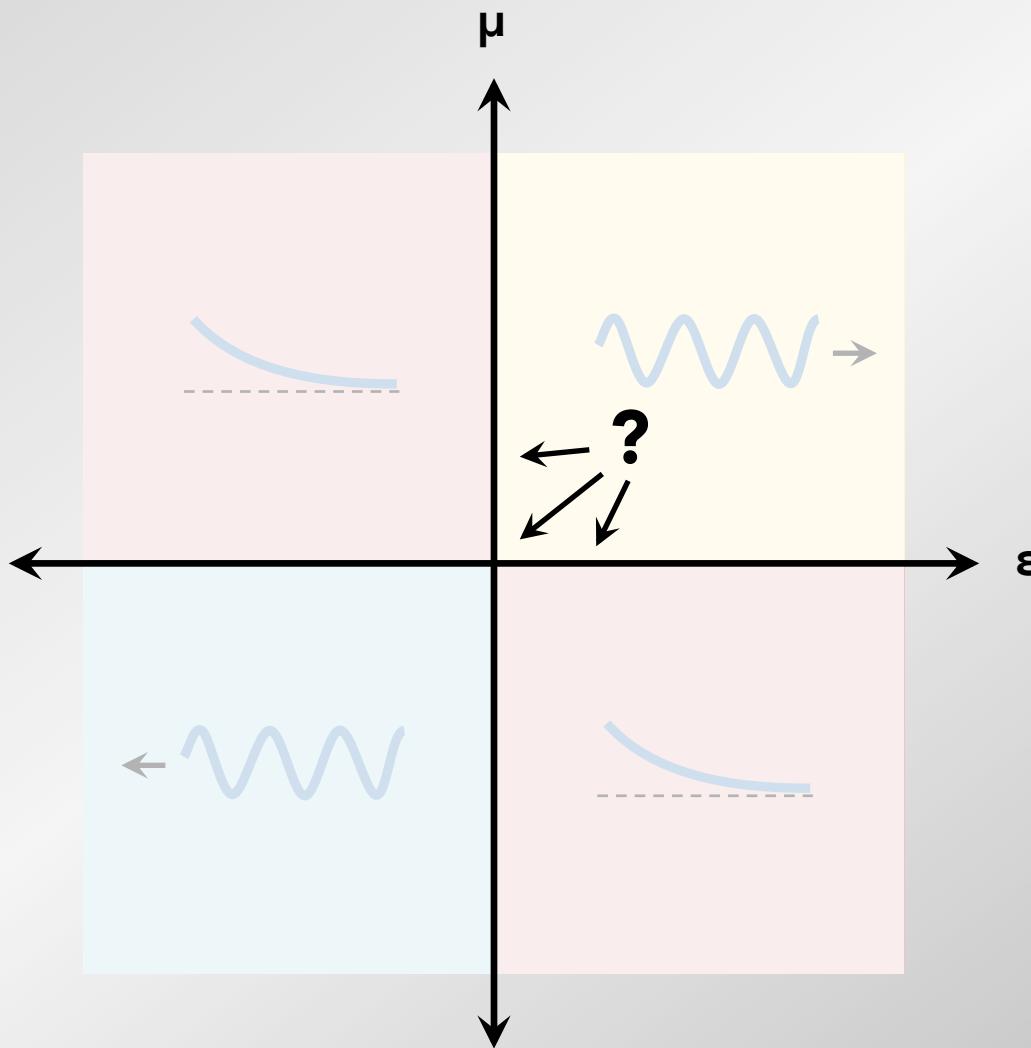


1 zero index

2 metamaterials

3 experiment

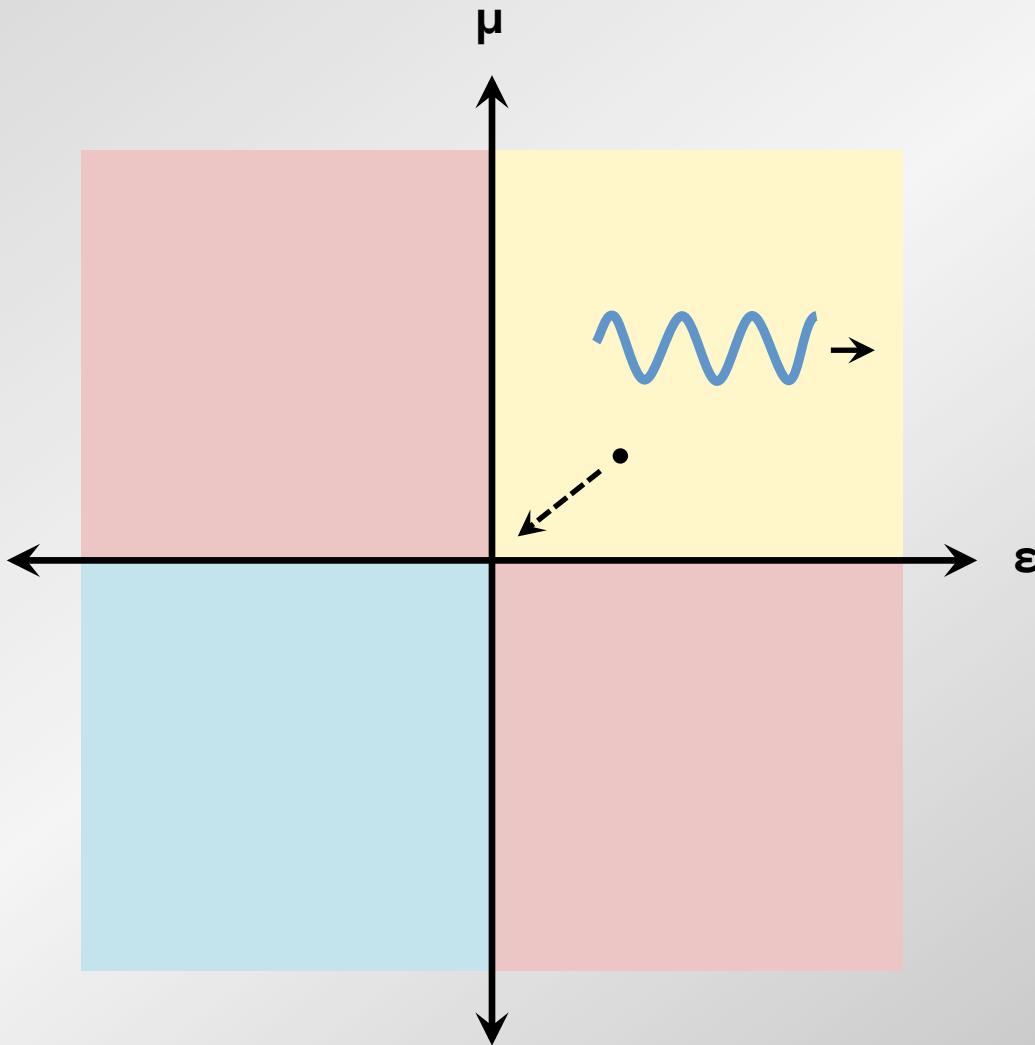
Classification of materials



1

zero index

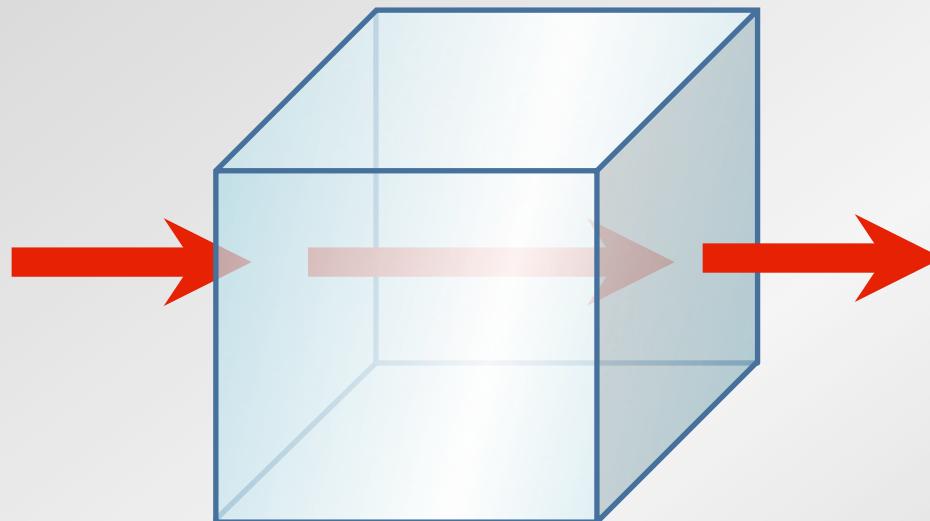
What happens as $n \rightarrow 0$?



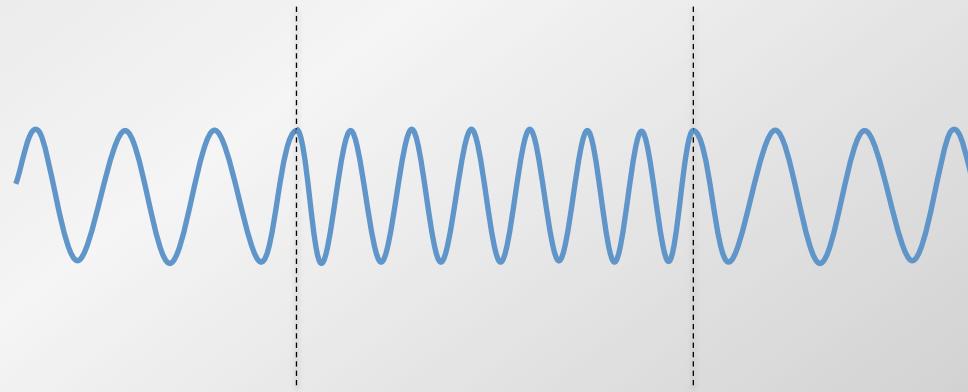
1

zero index

$n = 1.53$



Electric Field

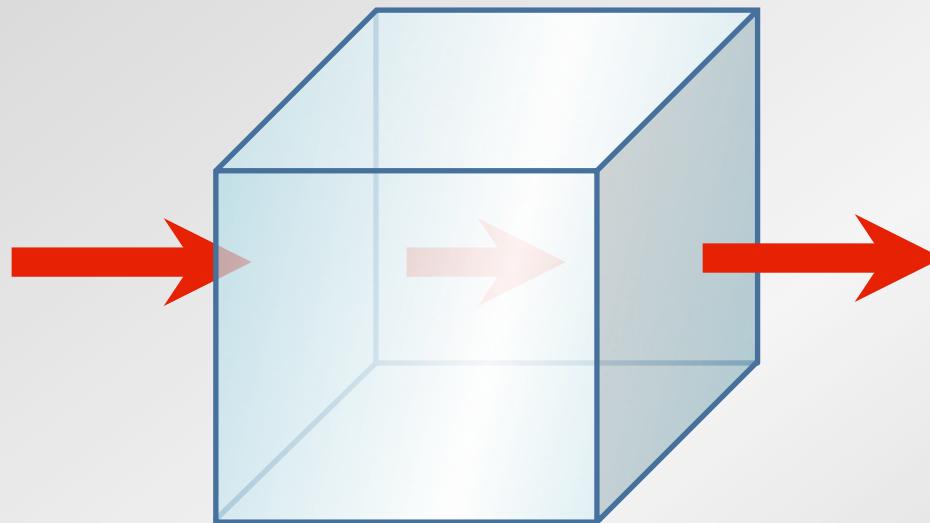


In most materials ($n > 1$) the wavelength is **short**, compared to free space

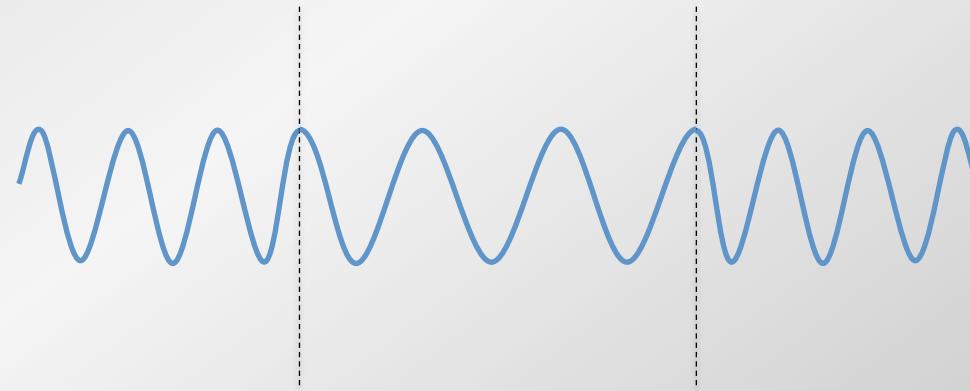
1

zero index

$$n = 0.66$$



Electric Field

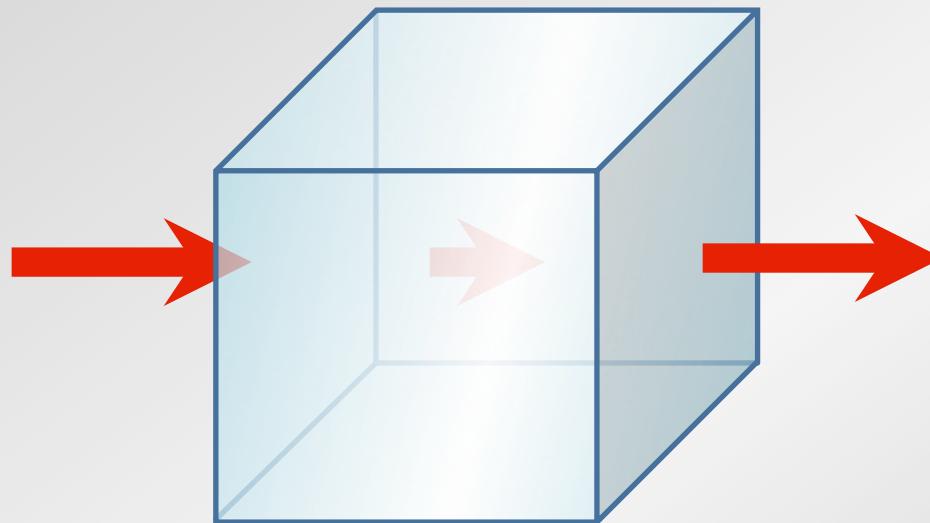


As the index **decreases**, the wavelength **increases**...

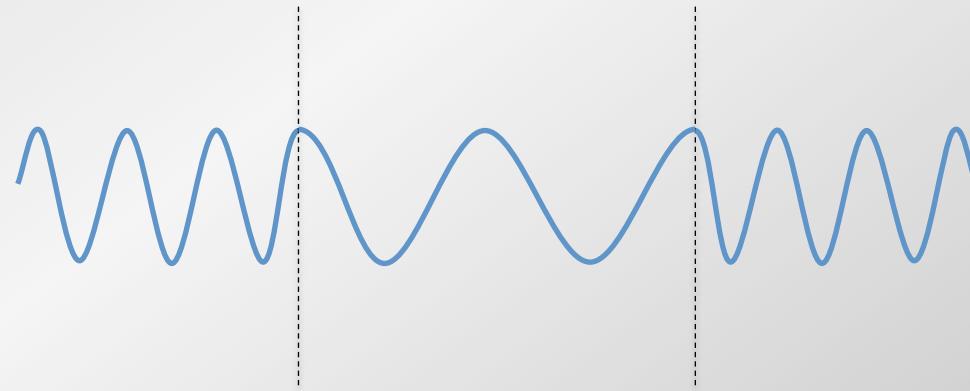
1

zero index

$$n = 0.48$$



Electric Field

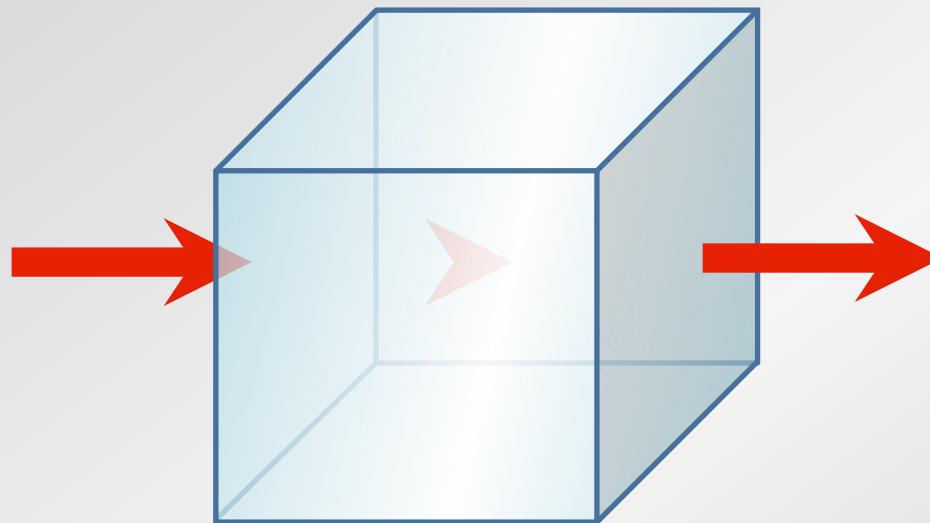


As the index **decreases**, the wavelength **increases**...

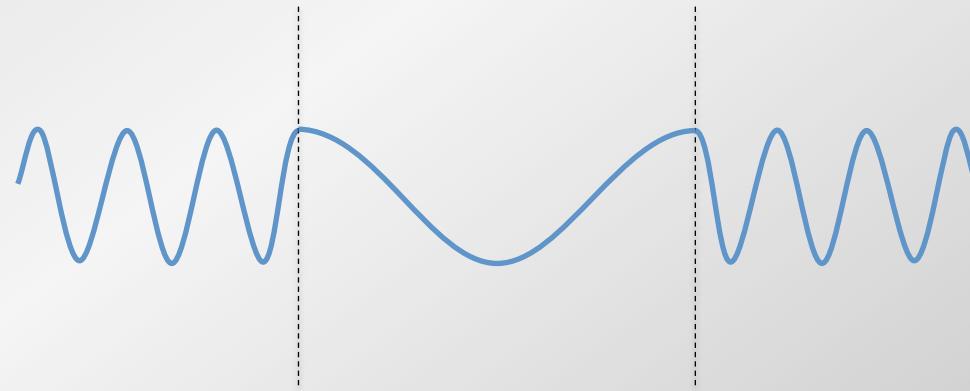
1

zero index

$$n = 0.22$$



Electric Field

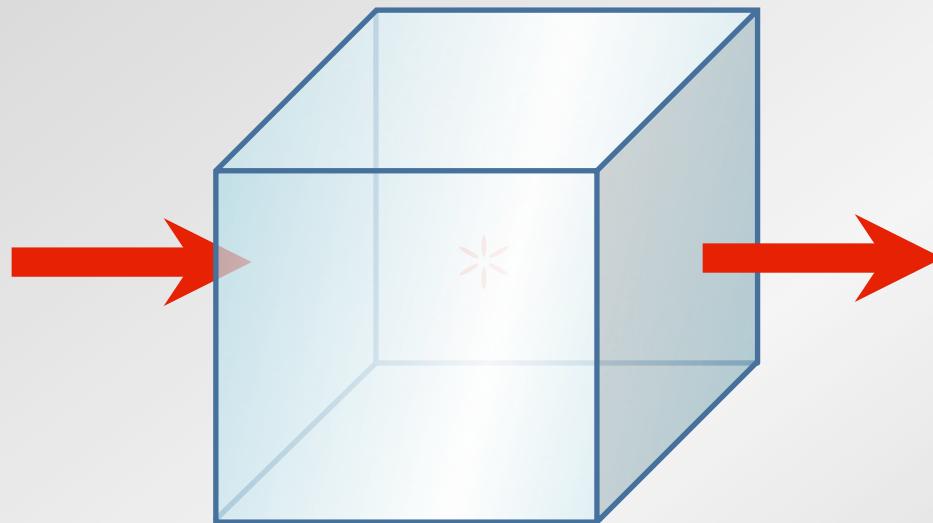


As the index **decreases**, the wavelength **increases**...

1

zero index

$n = 0$



Electric Field

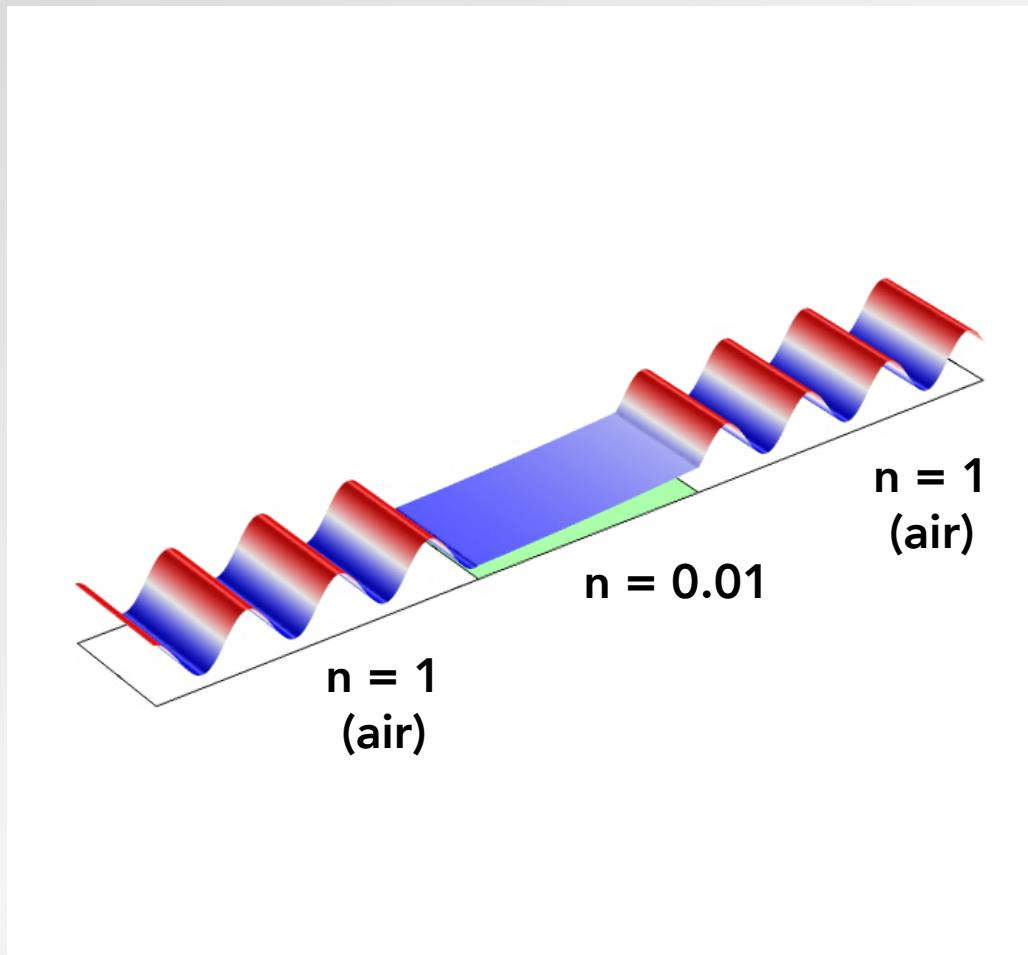


at zero index, wavelength and phase velocity become **infinite**

1

zero index

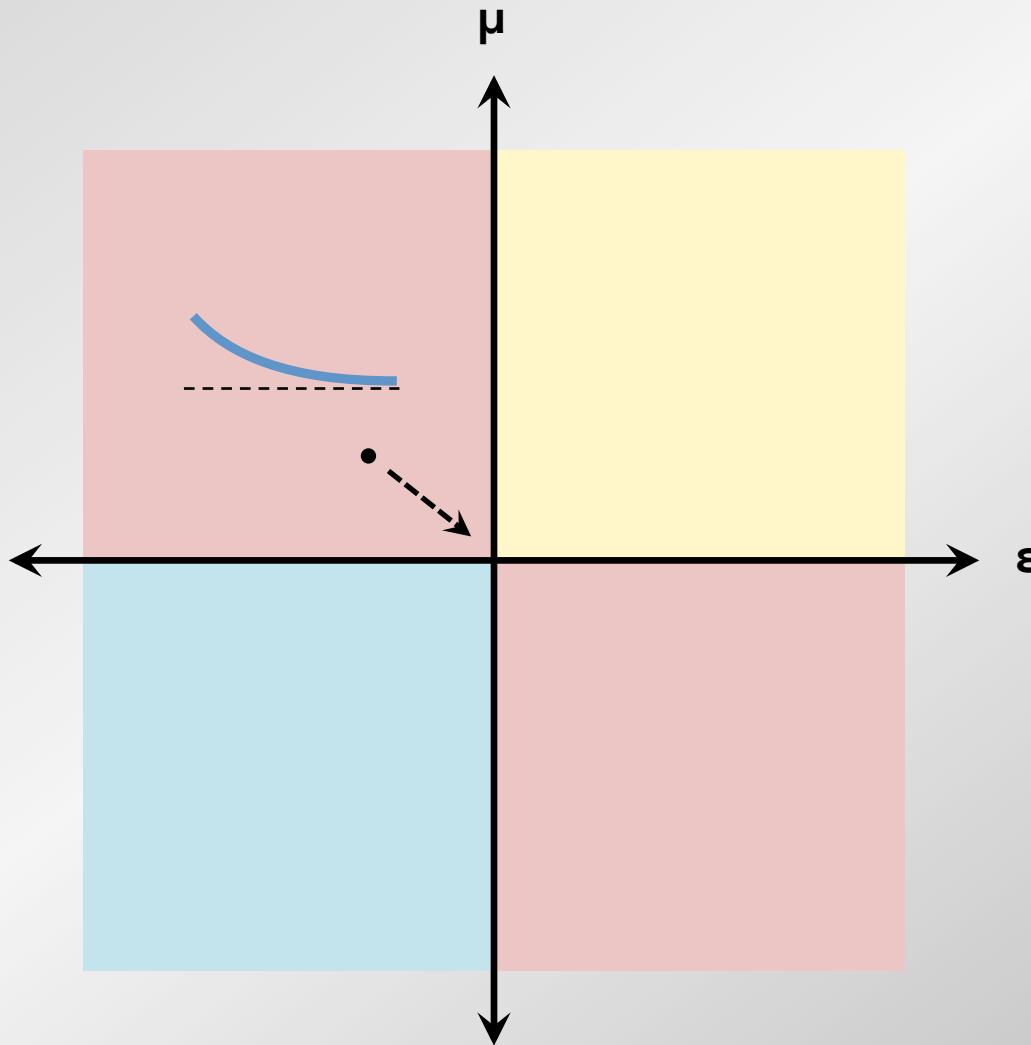
Wave propagation at zero index



1

zero index

What happens as $n \rightarrow 0^+$?

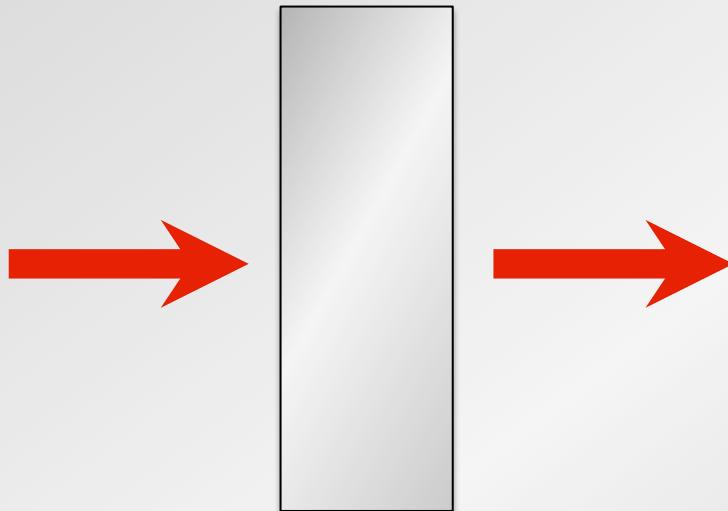


1

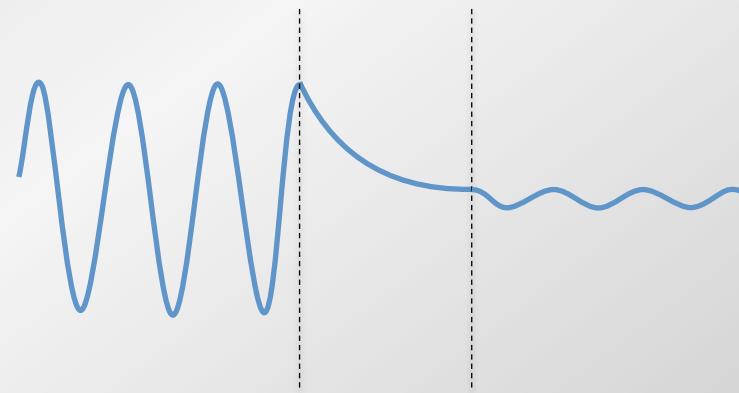
zero index

imaginary index:

$$\vec{E}_0 e^{i(nkx - \omega t)} \sim e^{-\kappa x}$$



Electric Field

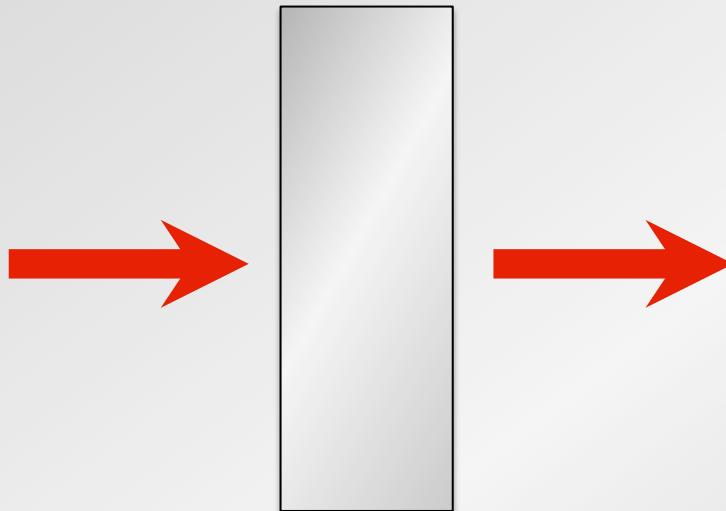


1

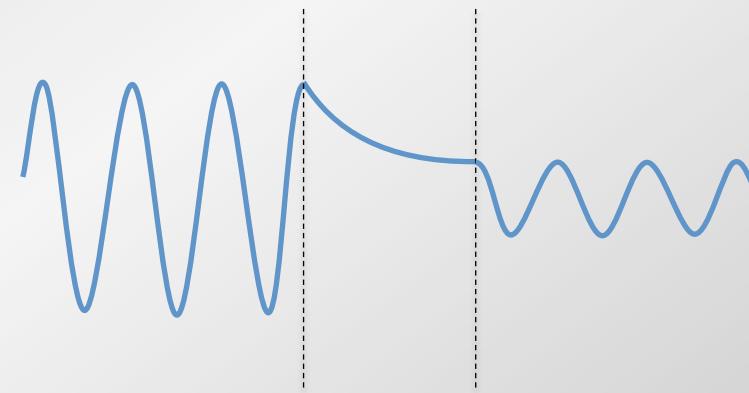
zero index

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$$\vec{E}_0 e^{i(nkx - \omega t)} \sim e^{-\kappa x}$$



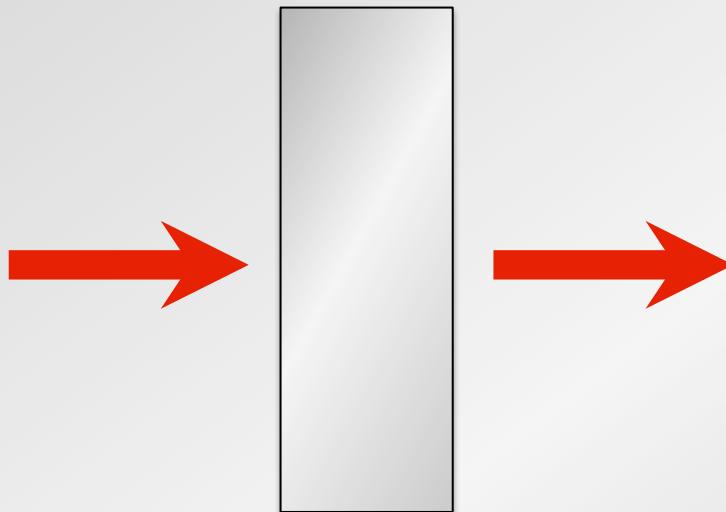
Electric Field



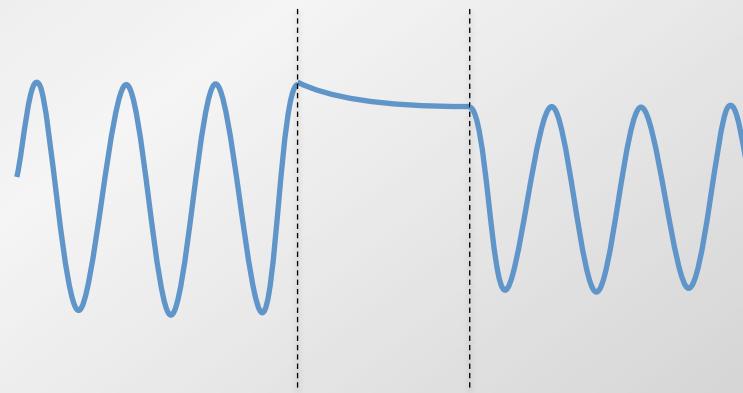
the exponential decay length scales with the index...

imaginary index:

$$\vec{E}_0 e^{i(nkx - \omega t)} \sim e^{-\kappa x}$$

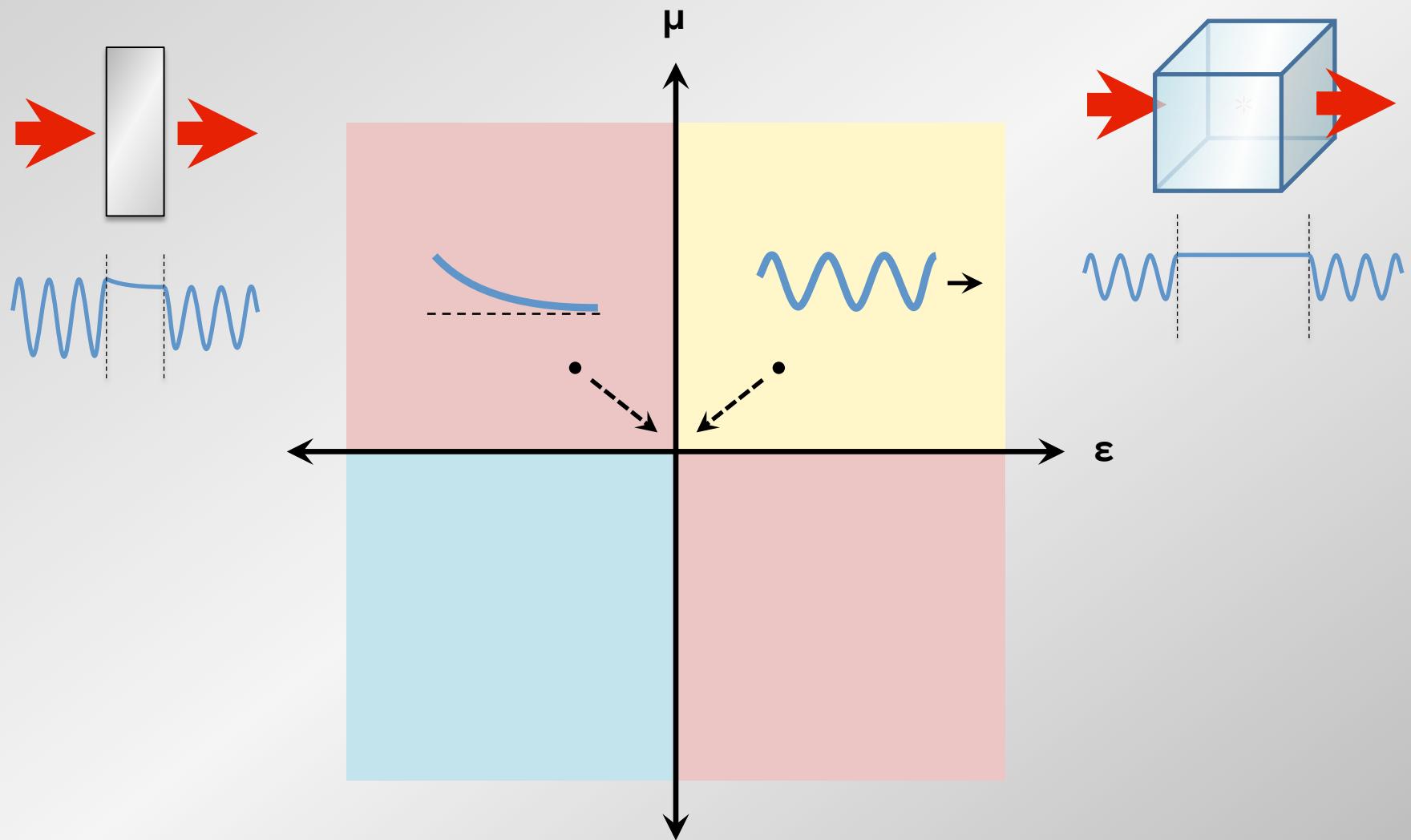


Electric Field



... so at zero index, the decay length becomes infinite

What happens as $n \rightarrow 0$?



1

zero index

Zero index means new physics

wave equation:

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

Zero index means new physics

wave equation:

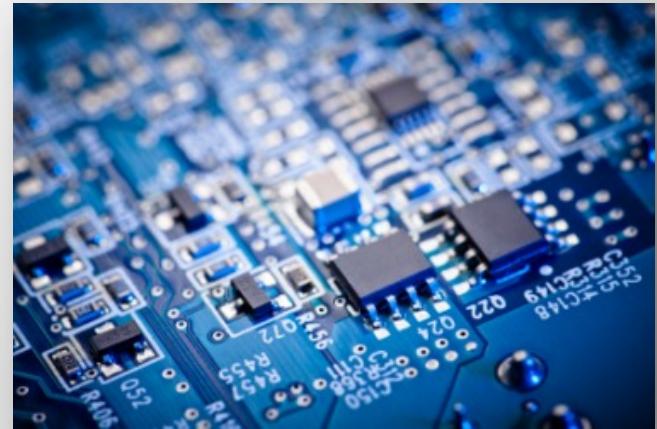
$$\nabla^2 \vec{E} - \frac{\mu \epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

Zero index means new physics

	<u>spatial</u>	<u>temporal</u>
Poisson wave equation:	$\nabla^2 \vec{E} = 0$	$\frac{\partial^2 \vec{E}}{\partial t^2} = 0$

Zero index means new physics

	<u>spatial</u>	<u>temporal</u>
Poisson wave equation:	$\nabla^2 \vec{E} = 0$	$\frac{\partial^2 \vec{E}}{\partial t^2} = 0$
Electrostatics:	$\nabla^2 V = 0$	



Zero index means new physics

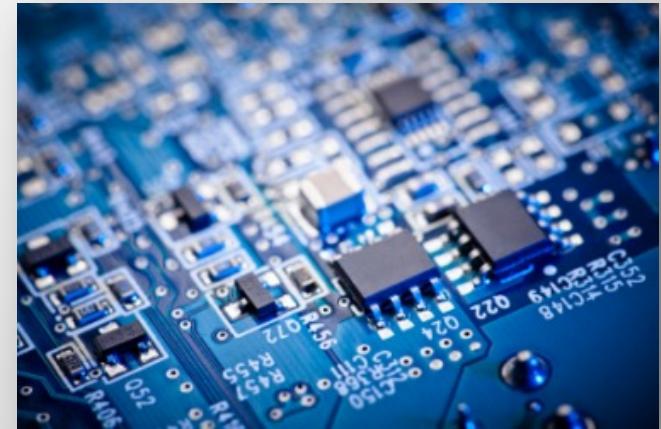
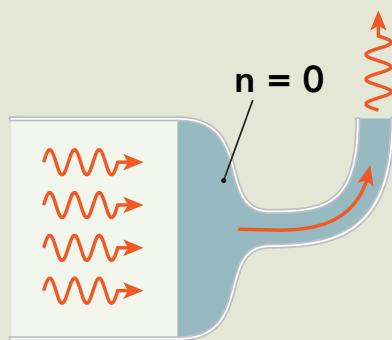
Poisson
wave equation:

spatial

$$\nabla^2 \vec{E} = 0$$

temporal

$$\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$



light flows like electricity!

1

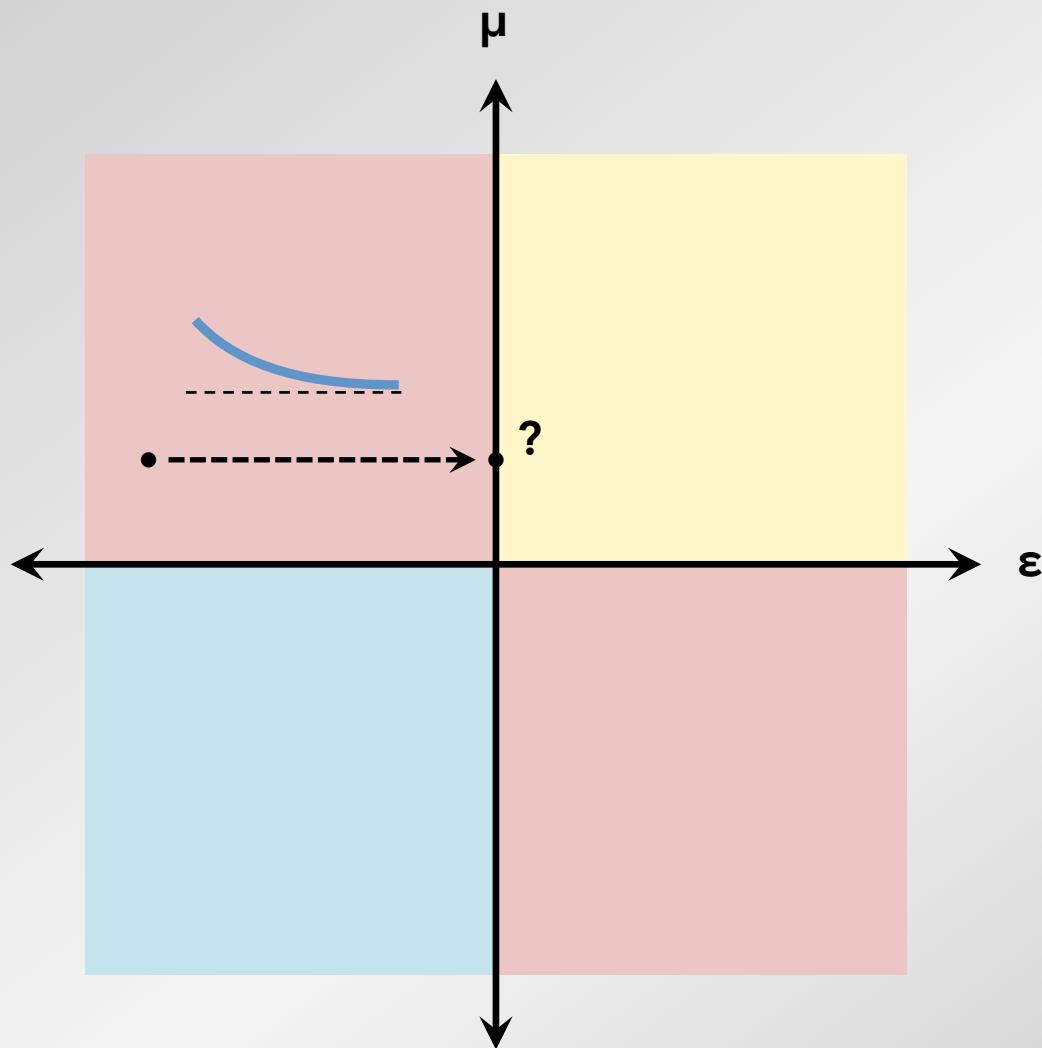
zero index

Which materials exhibit zero index?

1

zero index

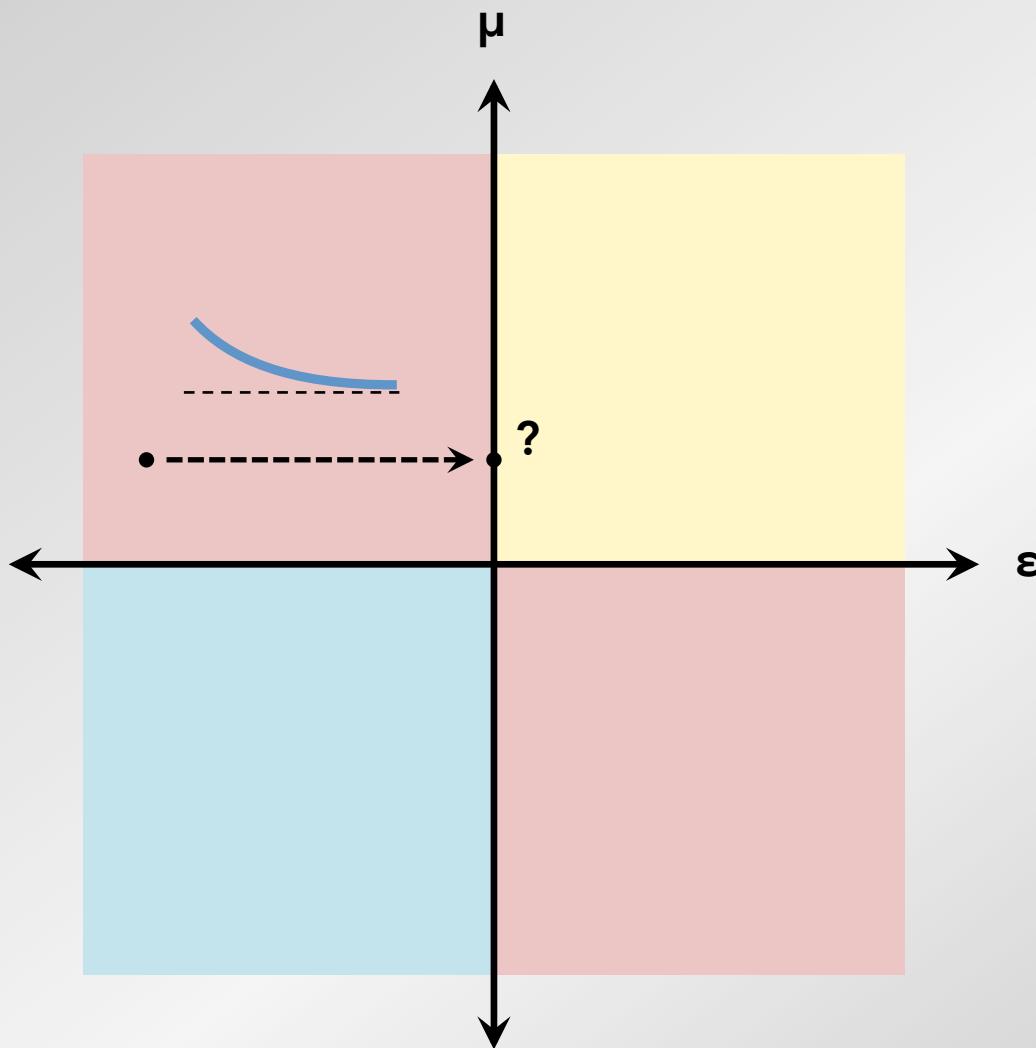
Which materials exhibit zero index?



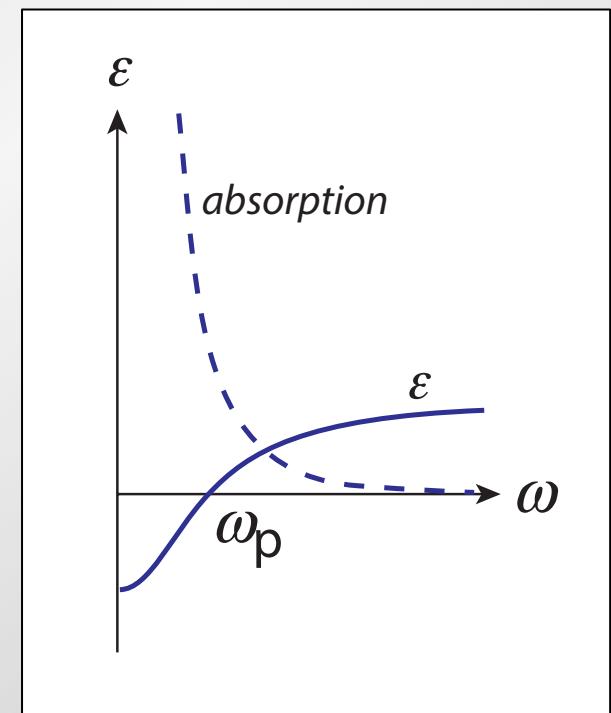
1

zero index

Which materials exhibit zero index?



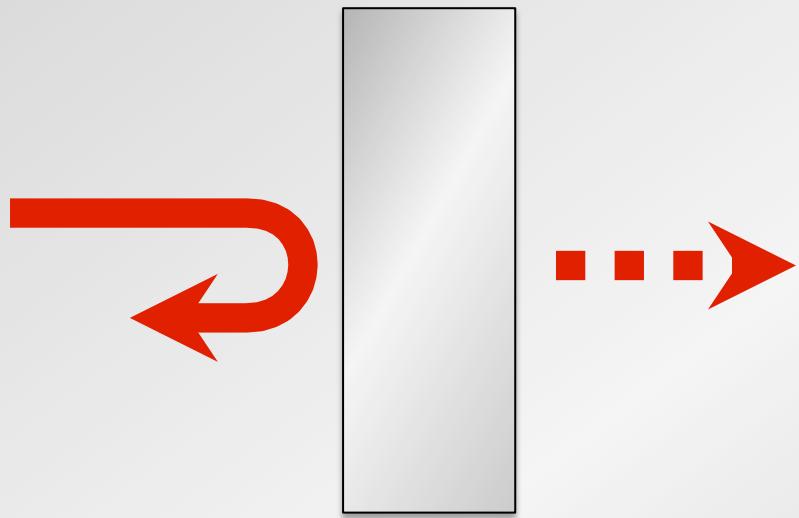
permittivity of metals



1

zero index

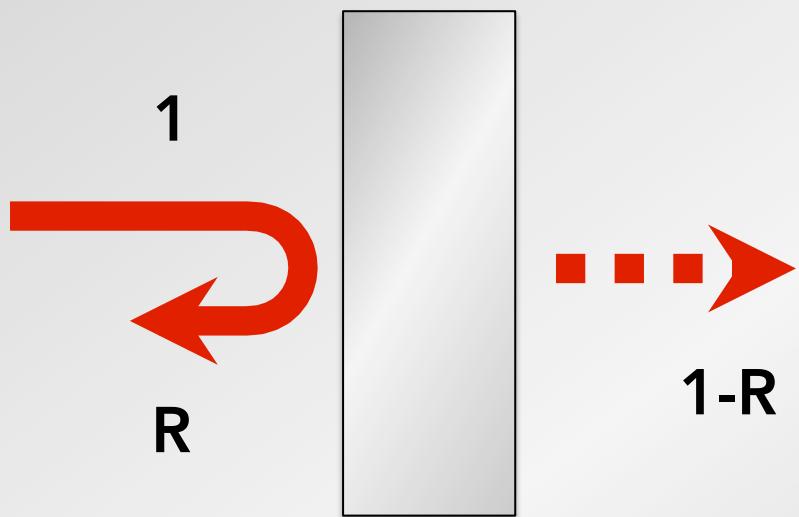
Which materials exhibit zero index?



1

zero index

Which materials exhibit zero index?



Reflection (in air):

$$R = \frac{1-Z}{1+Z}$$

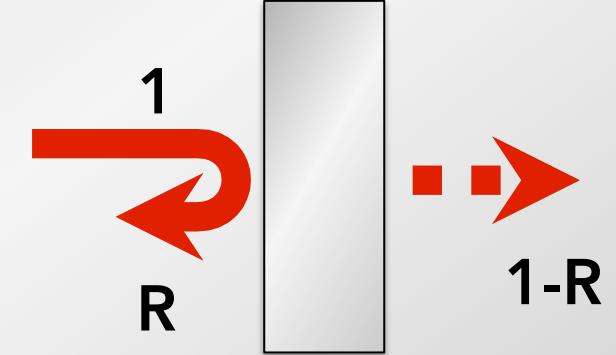
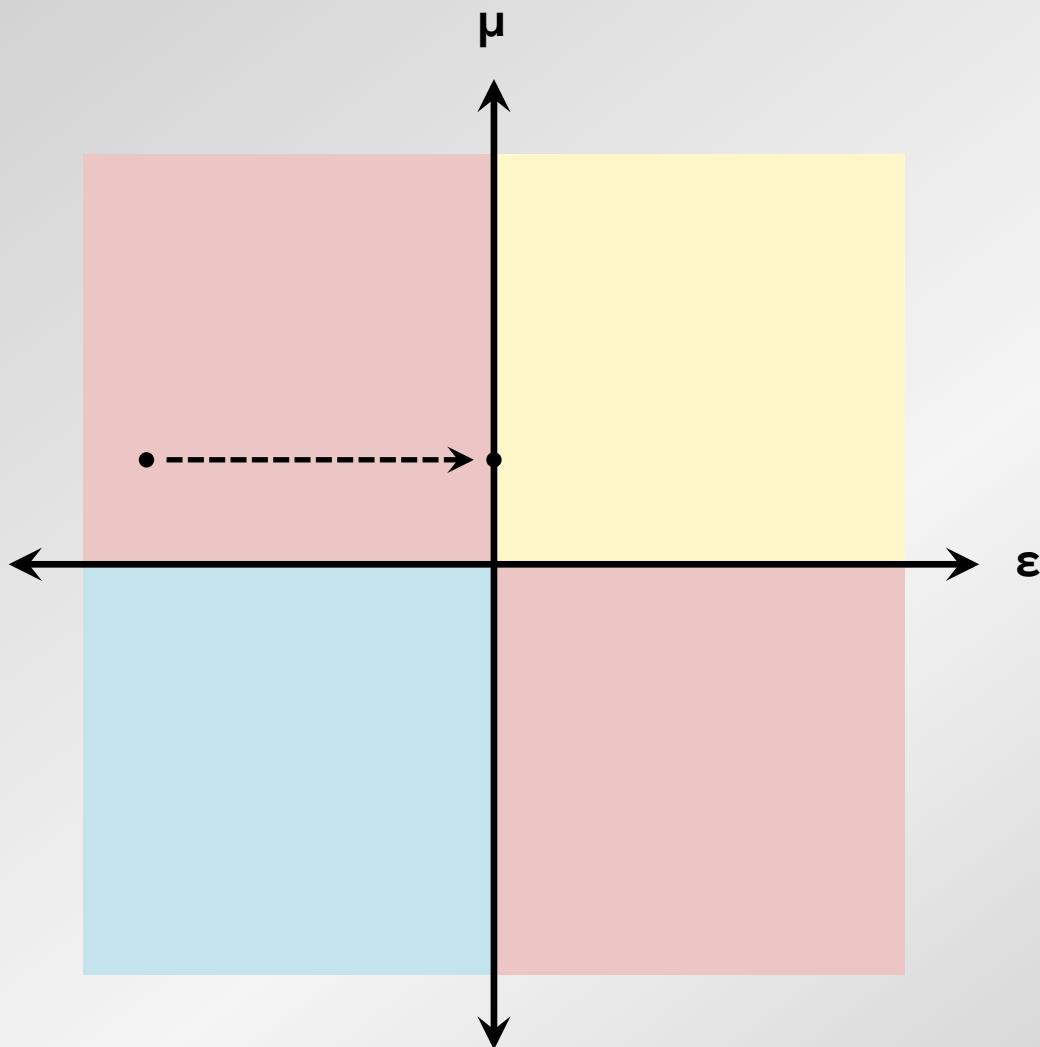
where

$$Z = \sqrt{\frac{\mu}{\epsilon}}$$

1

zero index

Which materials exhibit zero index?

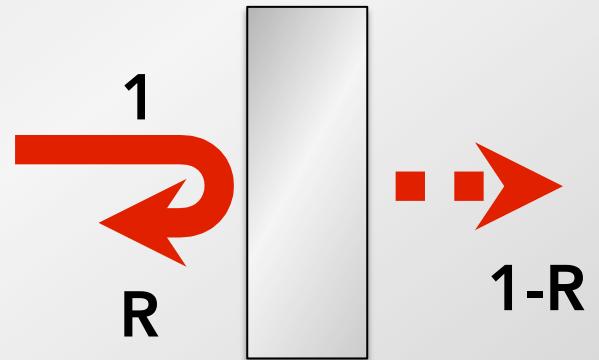
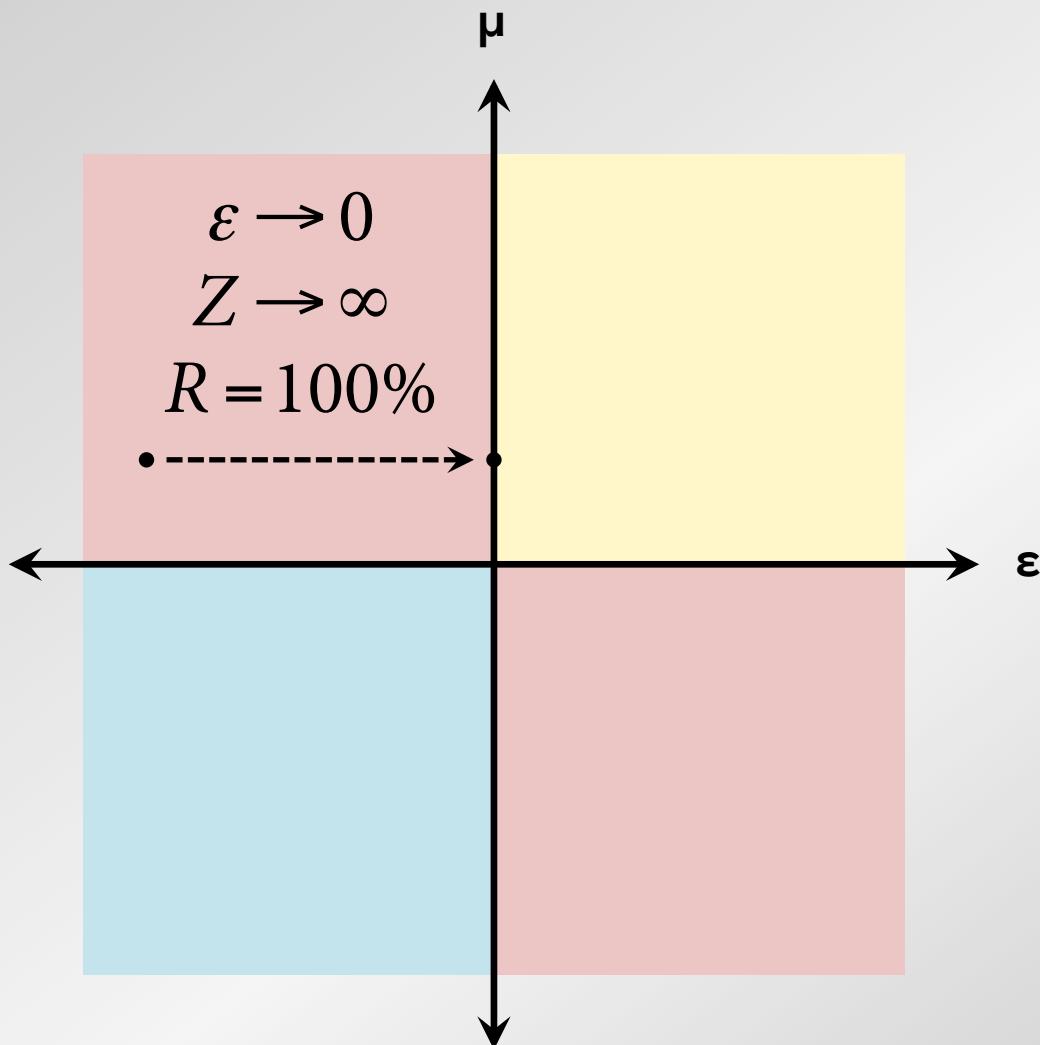


$$R = \frac{1 - Z}{1 + Z} \quad Z = \sqrt{\frac{\mu}{\epsilon}}$$

1

zero index

Which materials exhibit zero index?

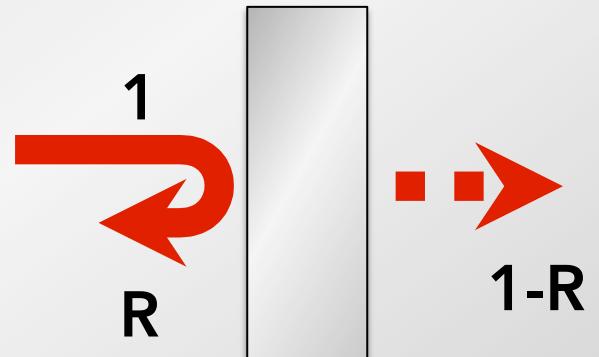
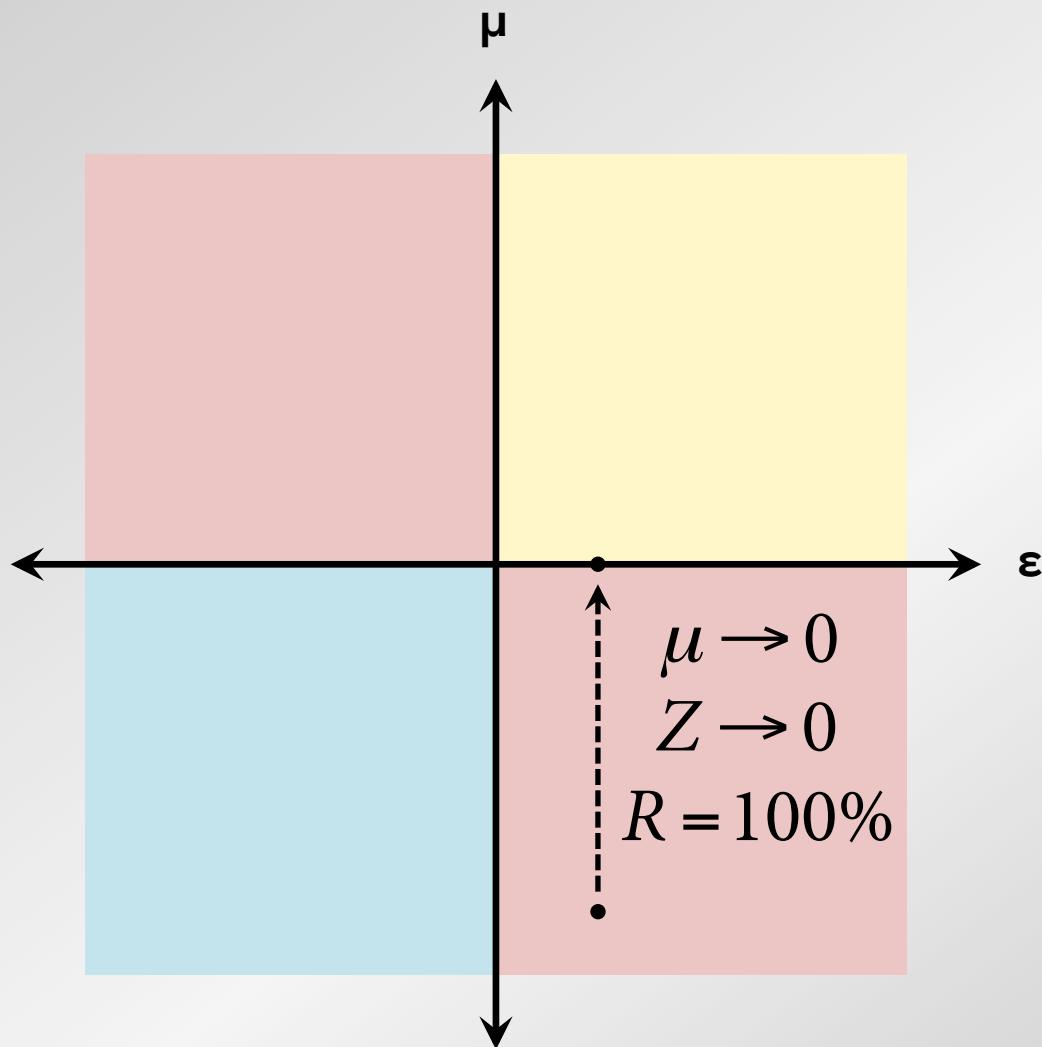


$$R = \frac{1-Z}{1+Z} \quad Z = \sqrt{\frac{\mu}{\epsilon}}$$

1

zero index

Which materials exhibit zero index?

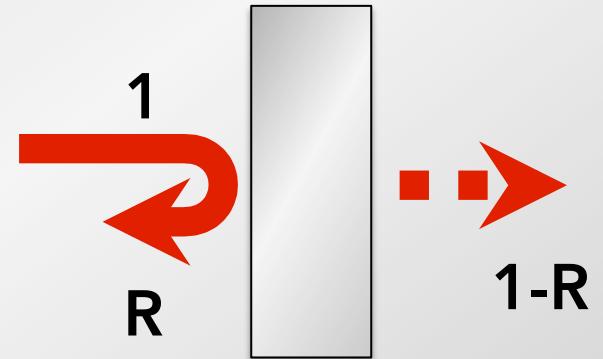
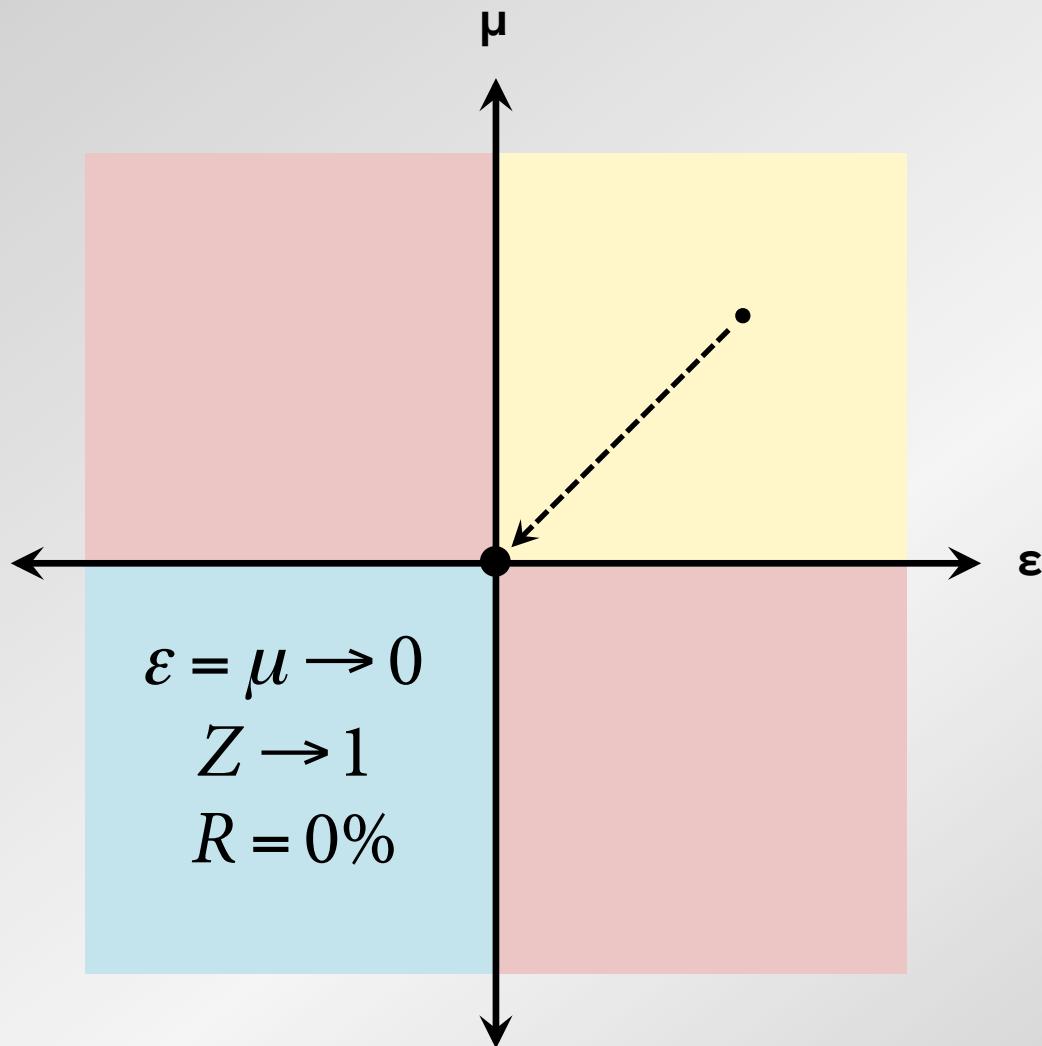


$$R = \frac{1-Z}{1+Z} \quad Z = \sqrt{\frac{\mu}{\epsilon}}$$

1

zero index

Which materials exhibit zero index?

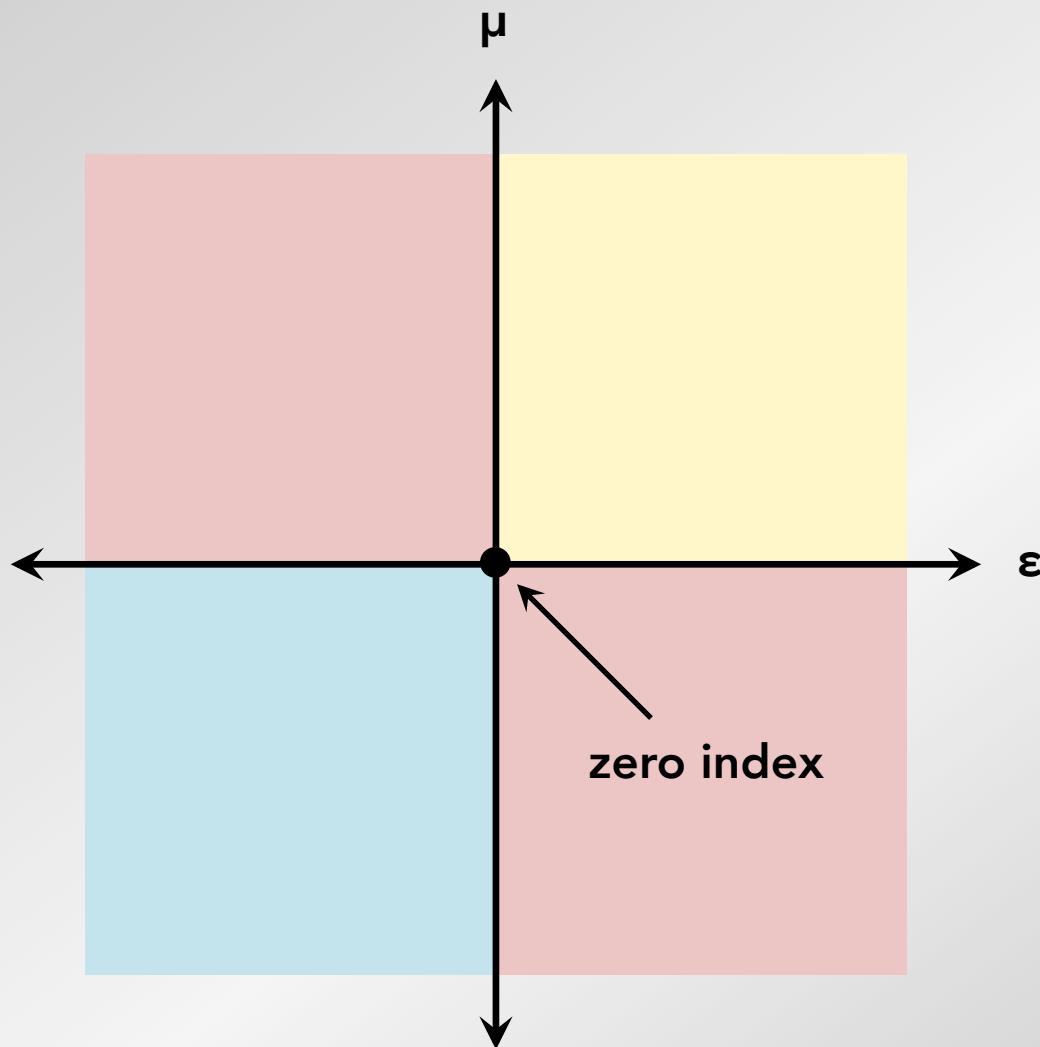


$$R = \frac{1-Z}{1+Z} \quad Z = \sqrt{\frac{\mu}{\epsilon}}$$

1

zero index

Which materials exhibit zero index?



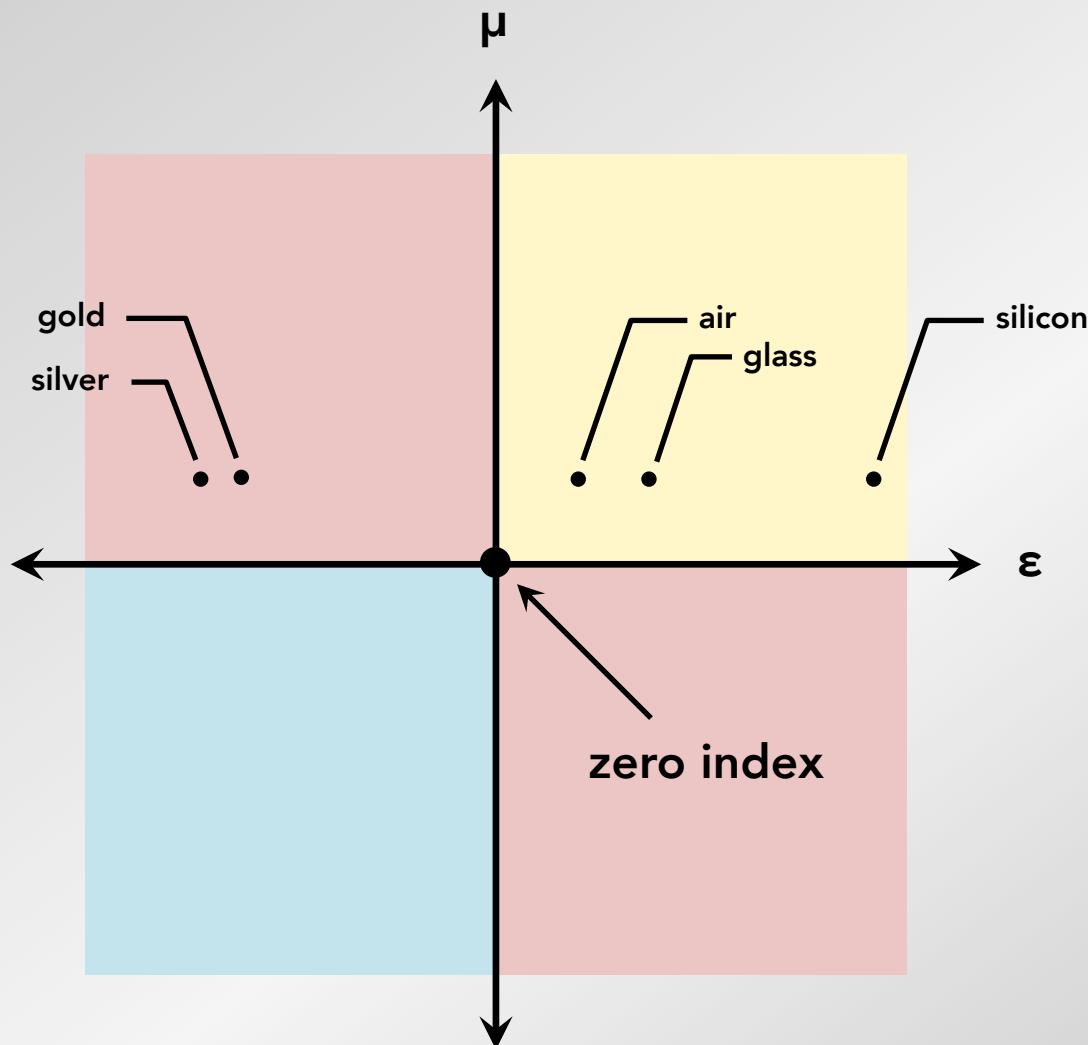
Recipe for zero index:

$$\epsilon = 0 \quad \mu = 0$$

1

zero index

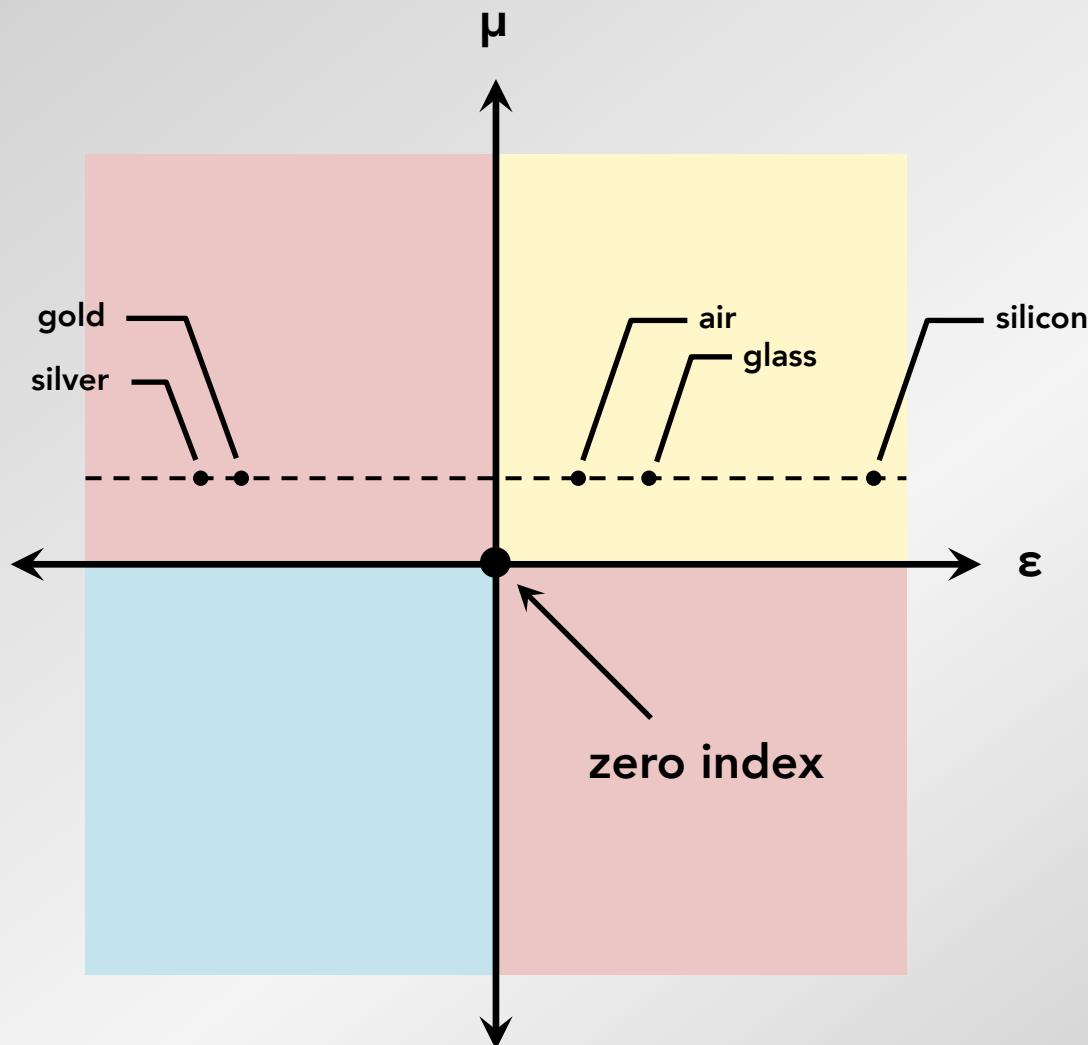
Which materials exhibit zero index?



Recipe for zero index:

$$\epsilon = 0 \quad \mu = 0$$

Which materials exhibit zero index?

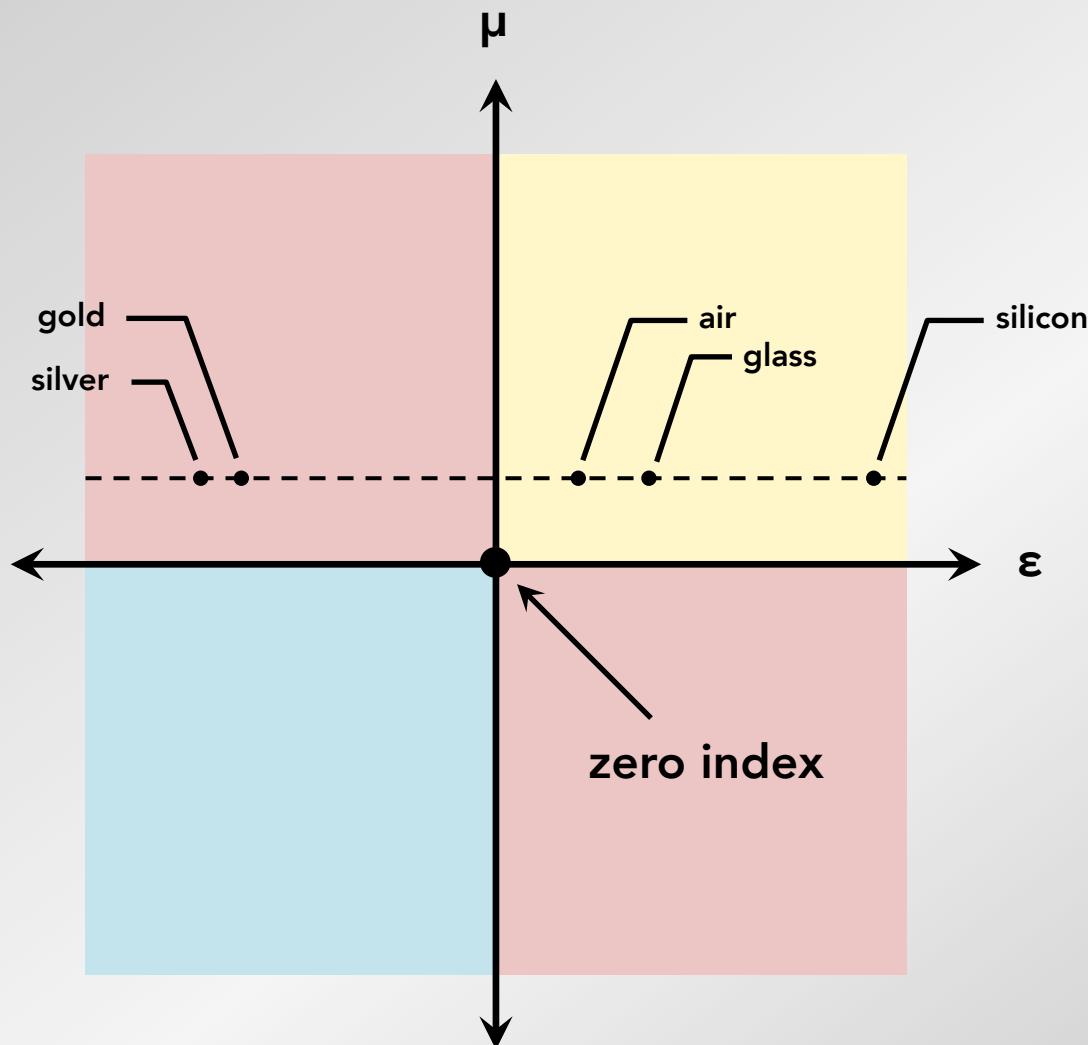


Recipe for zero index:

$$\epsilon = 0 \quad \mu = 0$$

There are no magnetic materials ($\mu = 1$) at optical frequencies

Which materials exhibit zero index?



Recipe for zero index:

$$\epsilon = 0 \quad \mu = 0$$

There are no magnetic materials ($\mu = 1$) at optical frequencies

Solution: composites!

Can we build a new material?

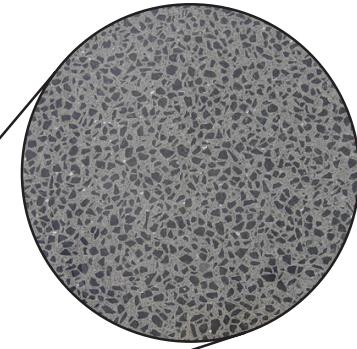
1

zero index

2

metamaterials

Can we build a new material?



geometry/size



material
properties

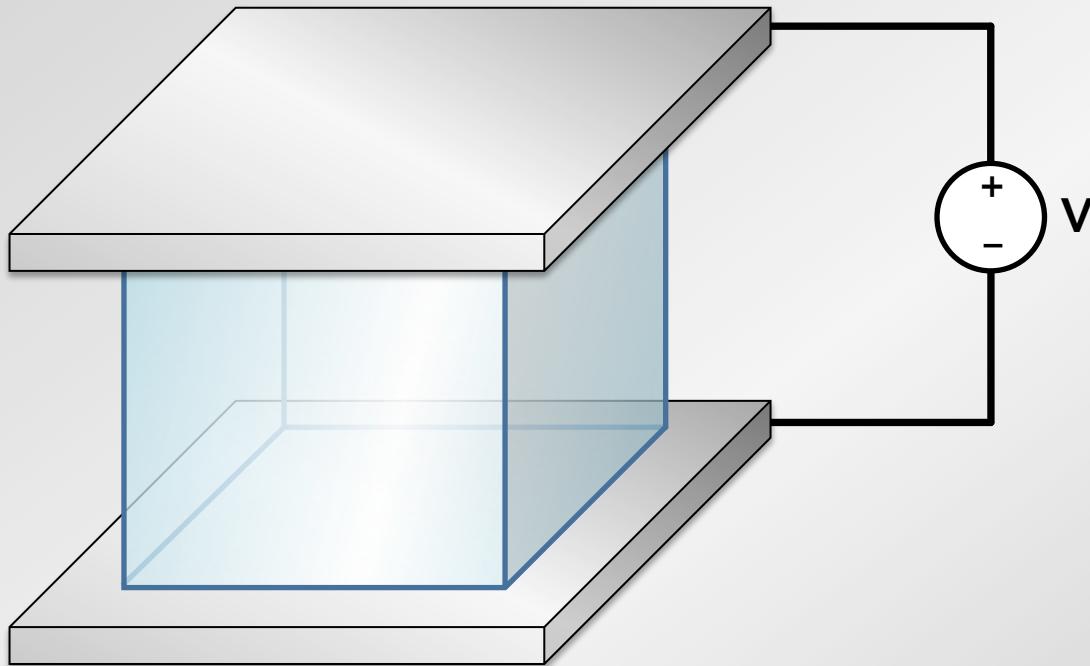
1

zero index

2

metamaterials

Microscopic origins of permittivity



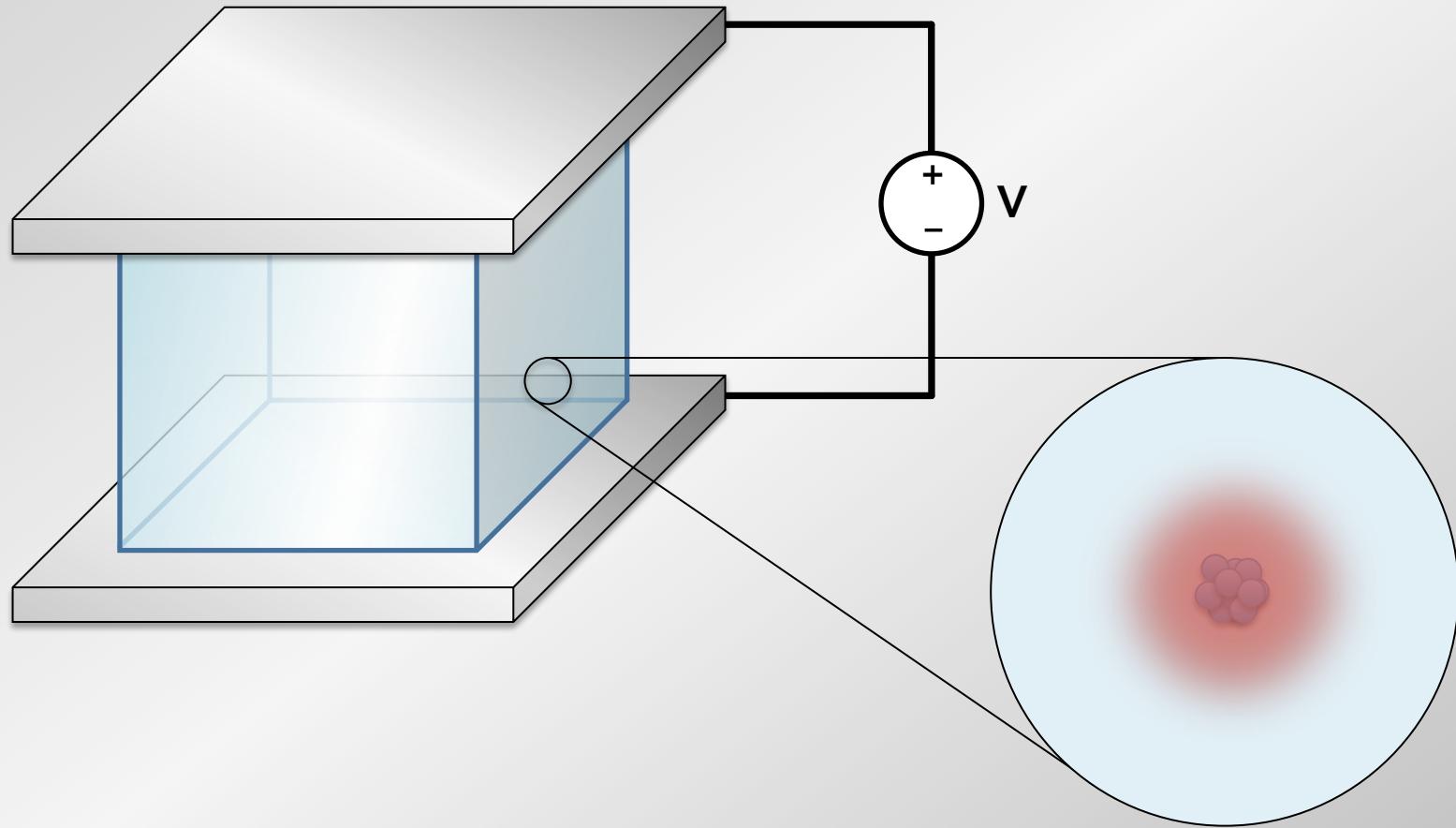
1

zero index

2

metamaterials

Microscopic origins of permittivity



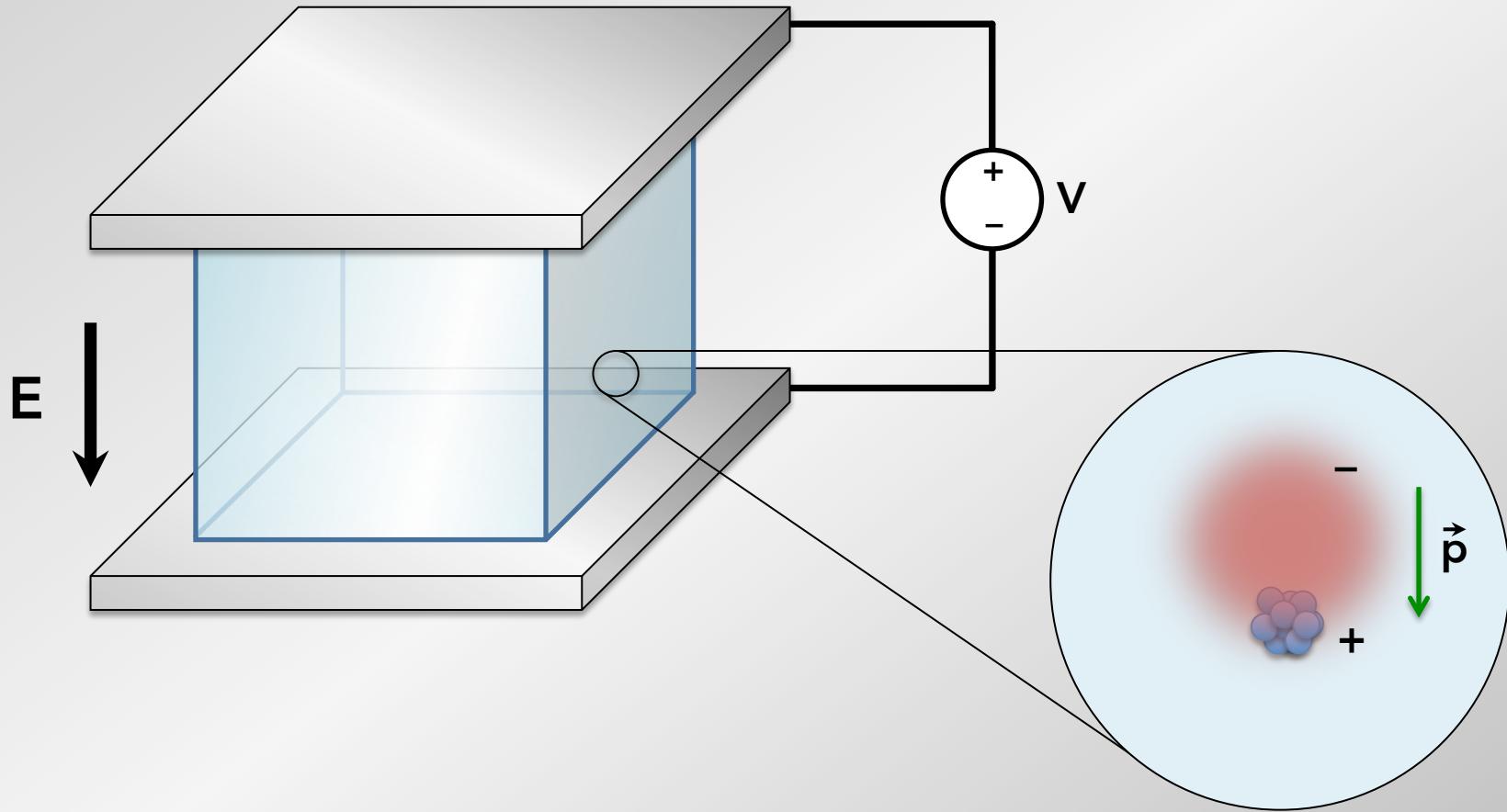
1

zero index

2

metamaterials

Microscopic origins of permittivity



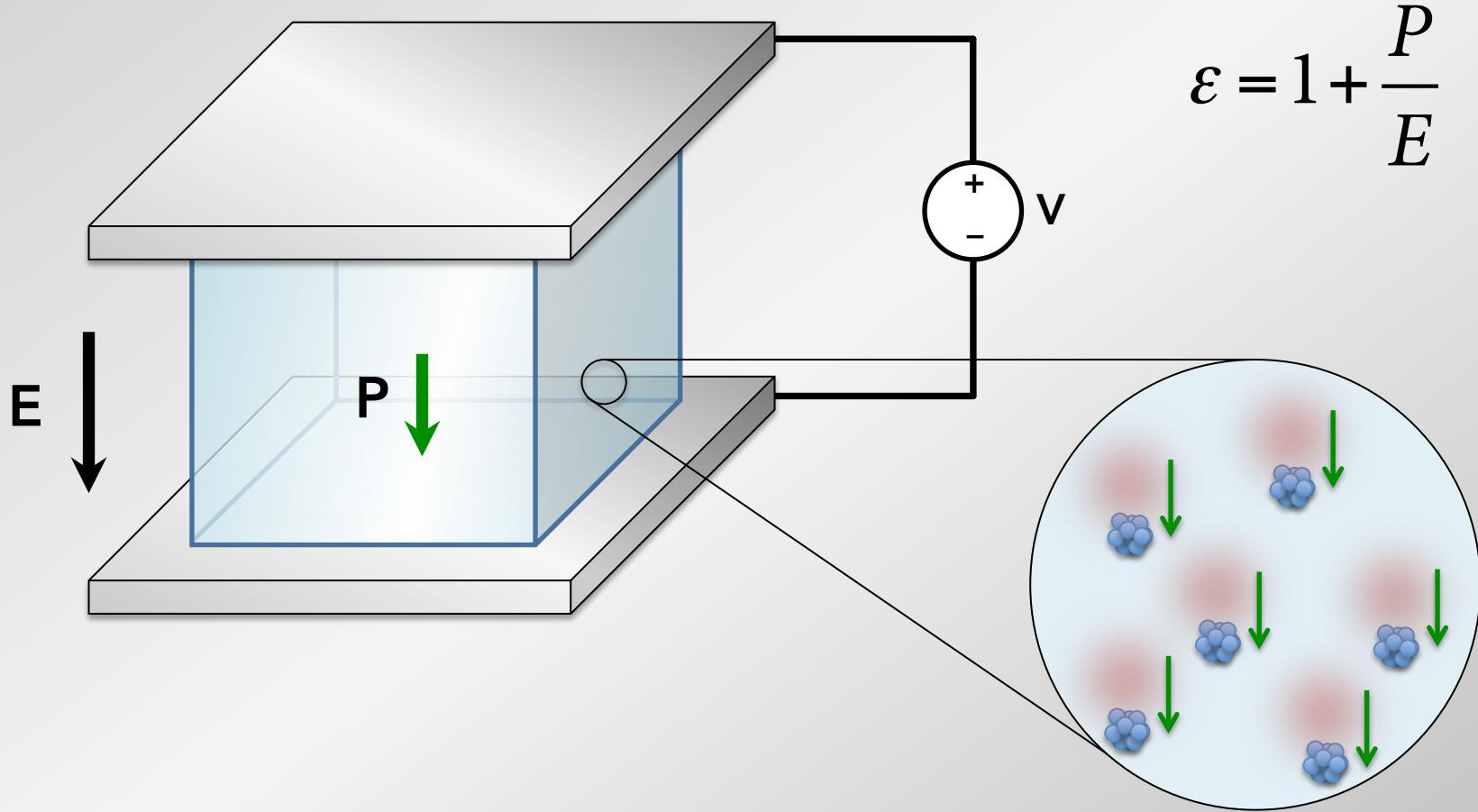
1

zero index

2

metamaterials

Microscopic origins of permittivity



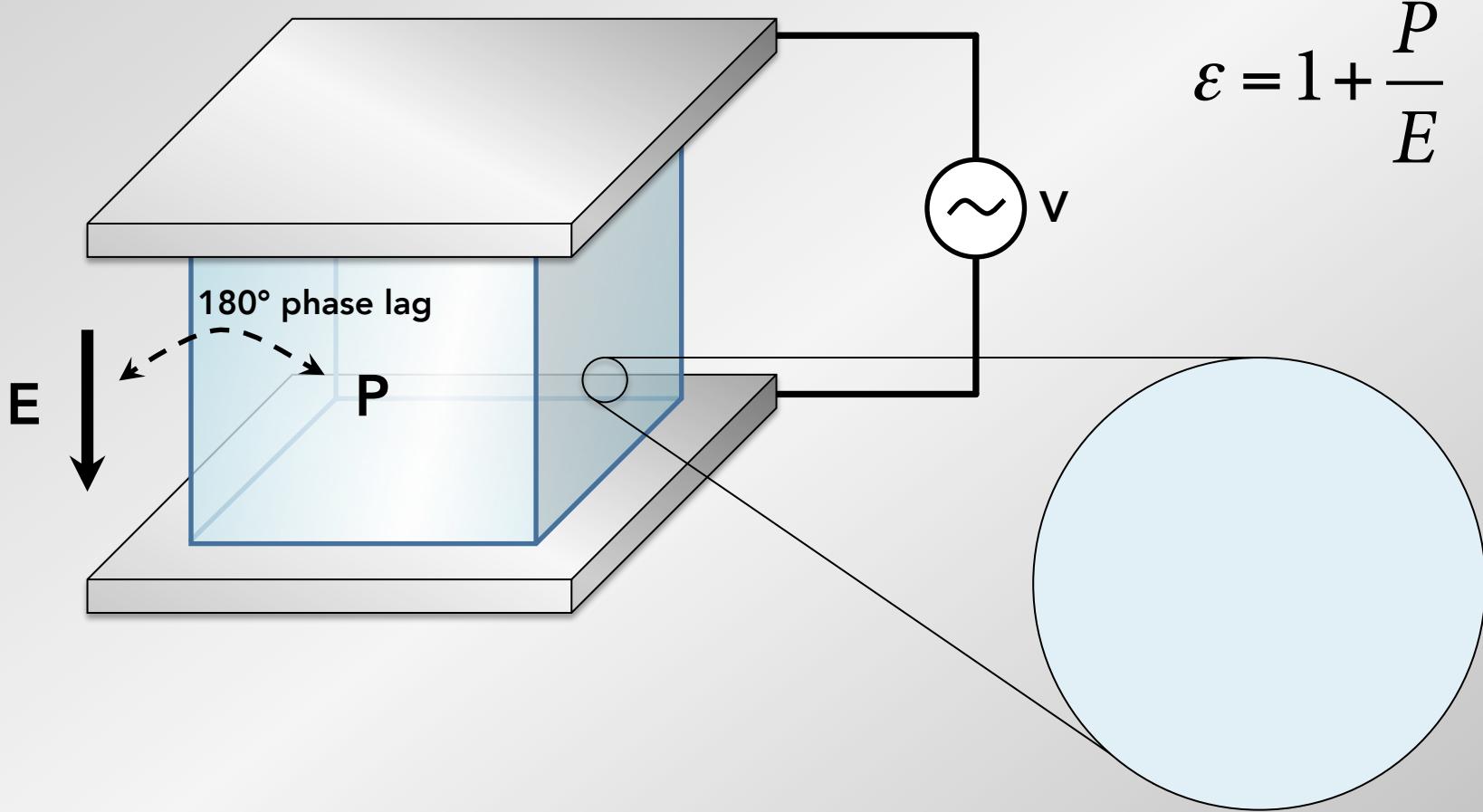
1

zero index

2

metamaterials

Microscopic origins of permittivity



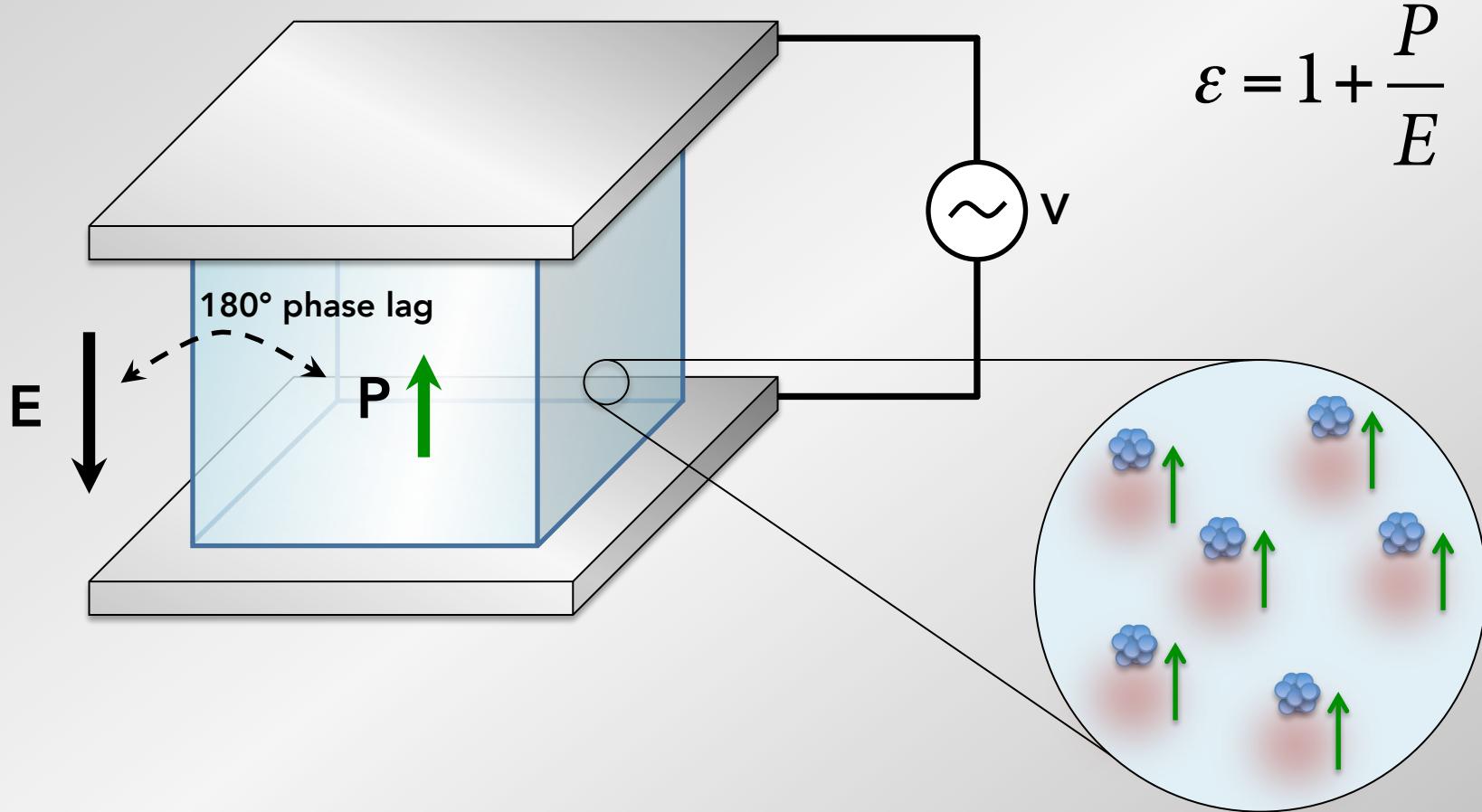
1

zero index

2

metamaterials

Microscopic origins of permittivity



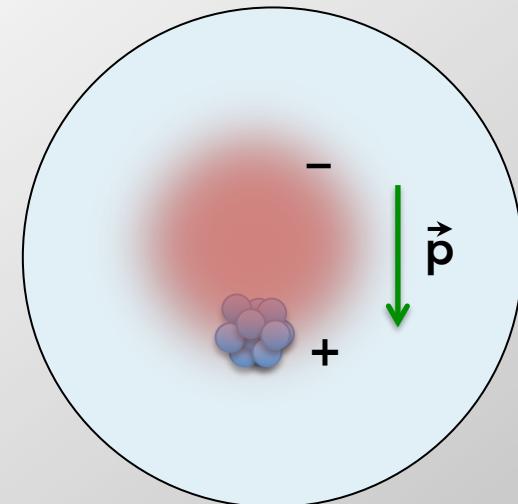
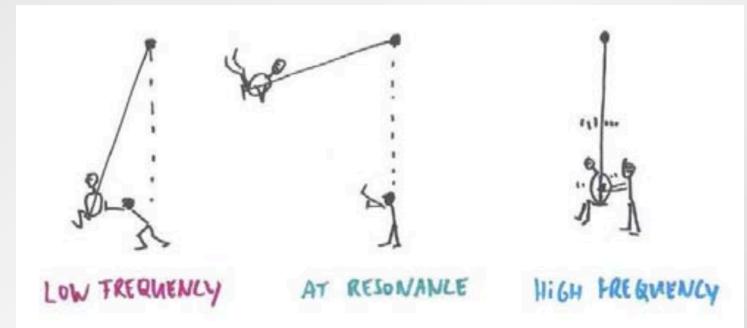
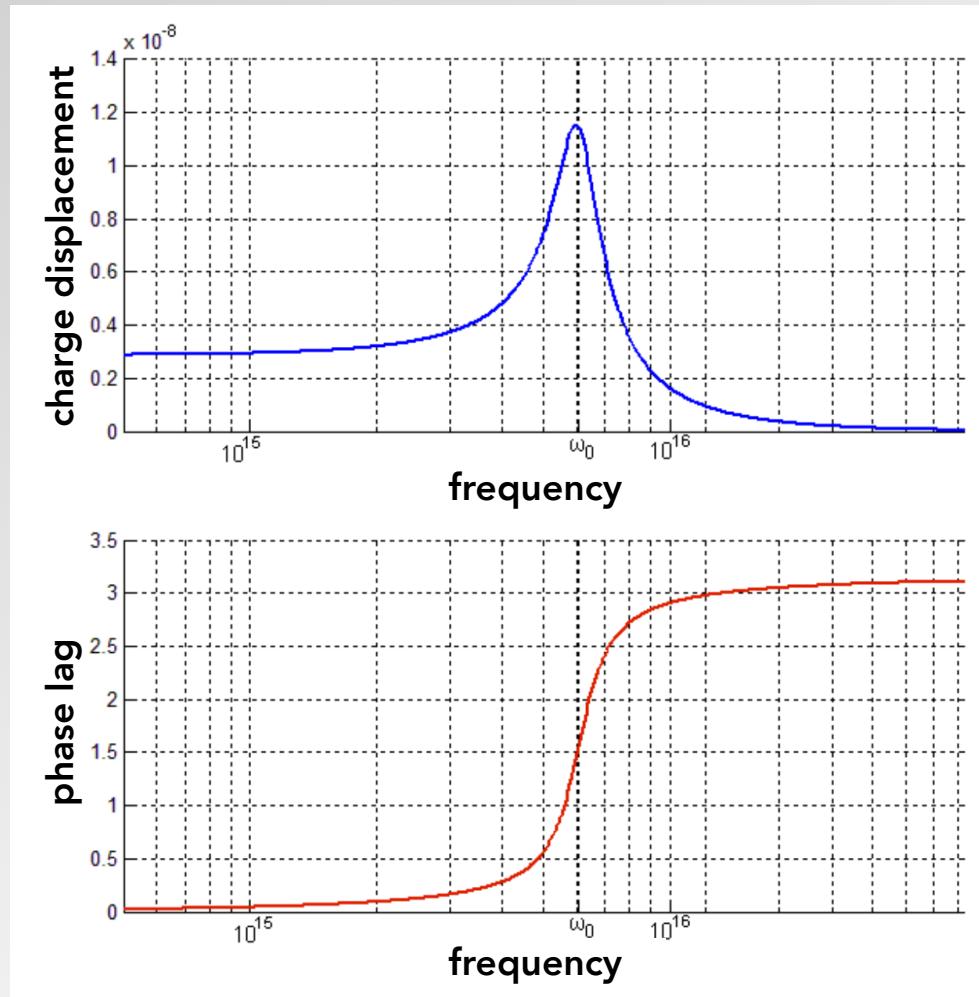
1

zero index

2

metamaterials

Microscopic origins of permittivity



Credit: MIT OCW; Almog, Bradley, Bulovic

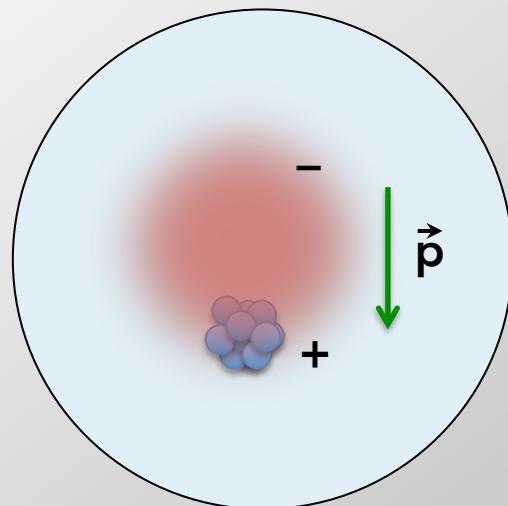
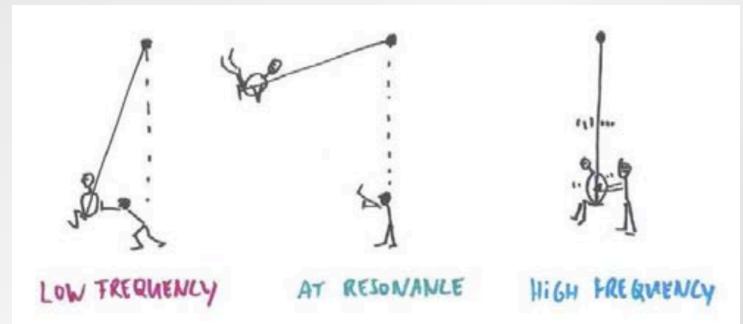
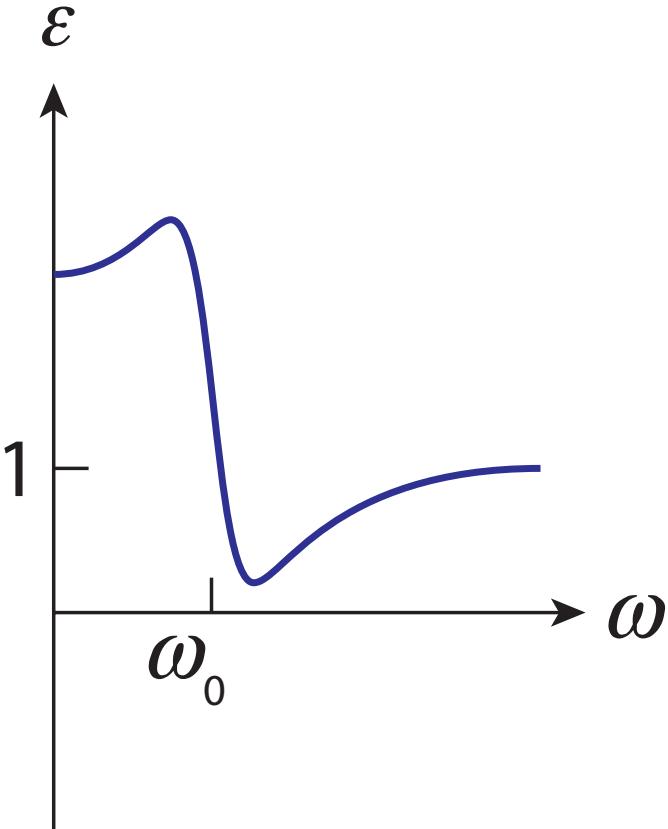
1

zero index

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metamaterials

Microscopic origins of permittivity



Credit: MIT OCW; Almog, Bradley, Bulovic

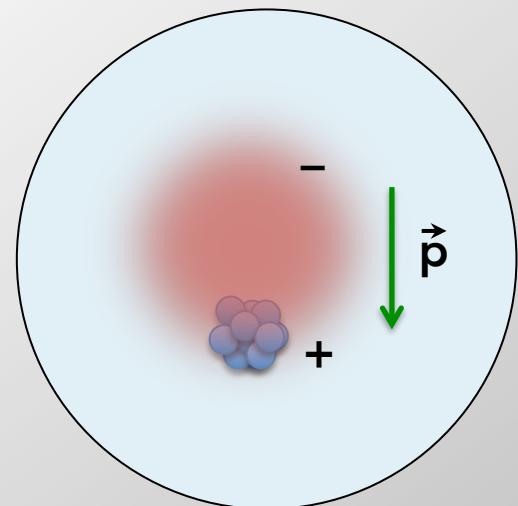
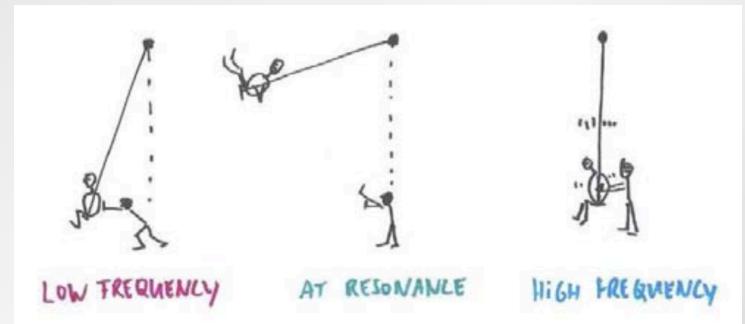
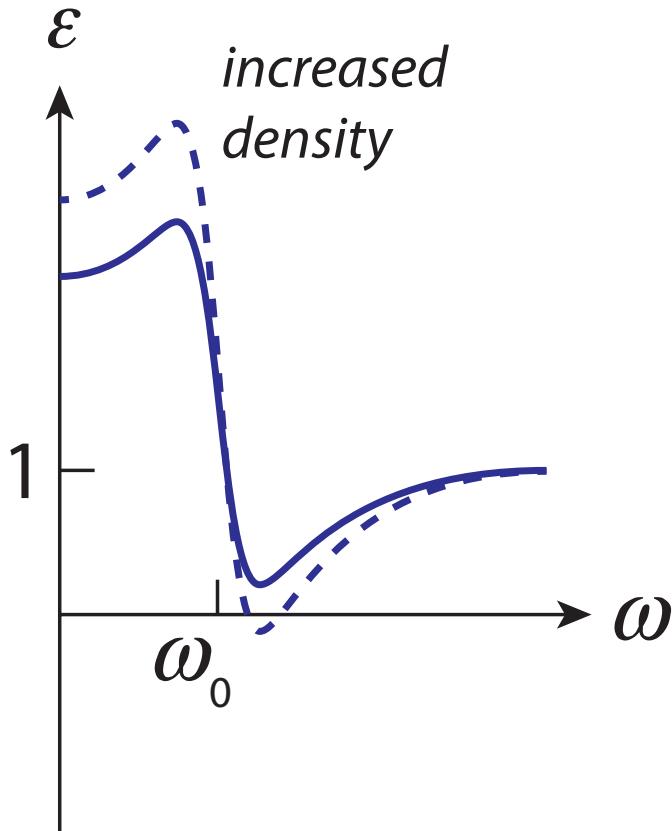
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zero index

2

metamaterials

Microscopic origins of permittivity



Credit: MIT OCW; Almog, Bradley, Bulovic

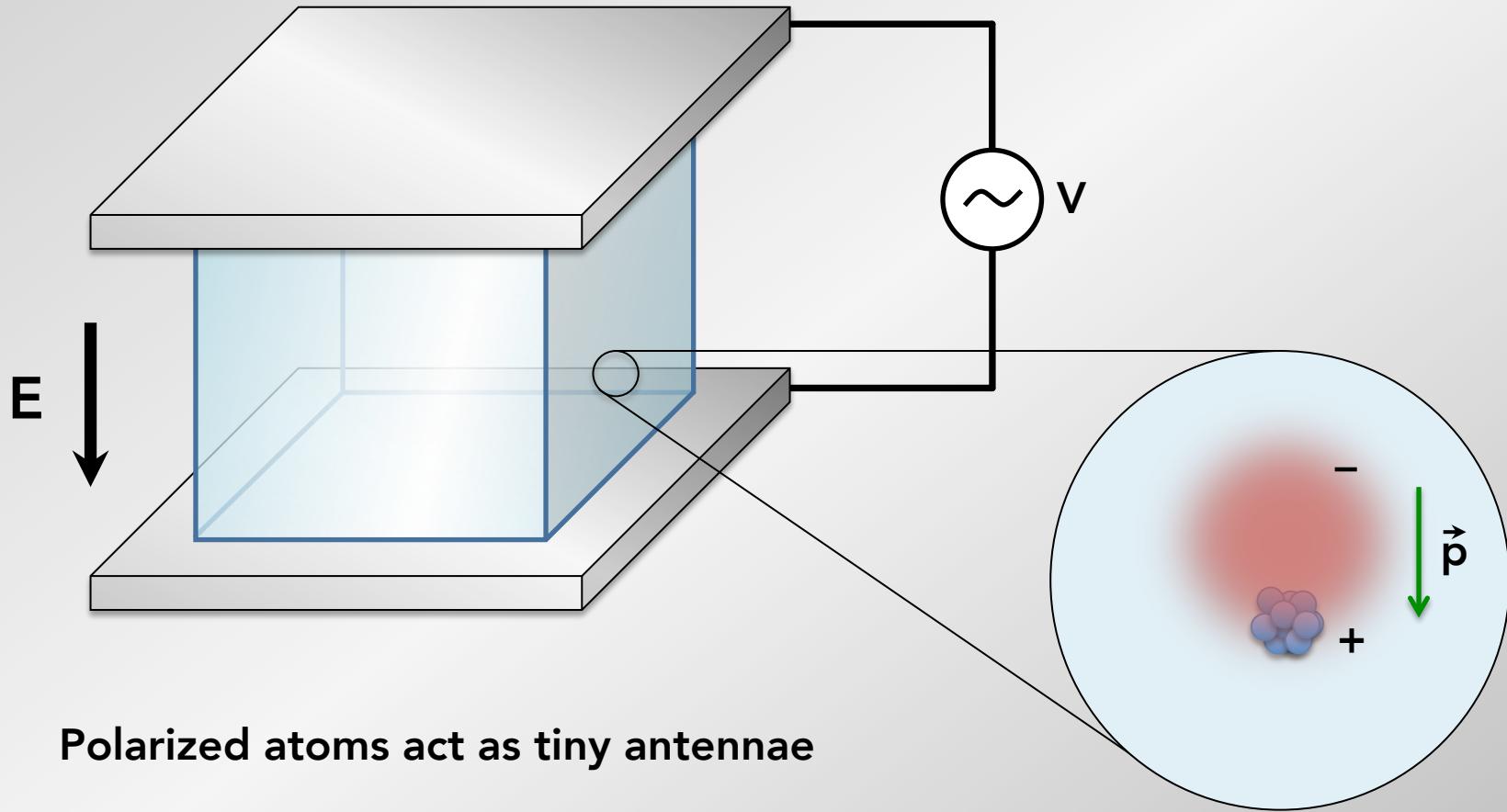
1

zero index

2

metamaterials

Microscopic origins of permittivity



Polarized atoms act as tiny antennae

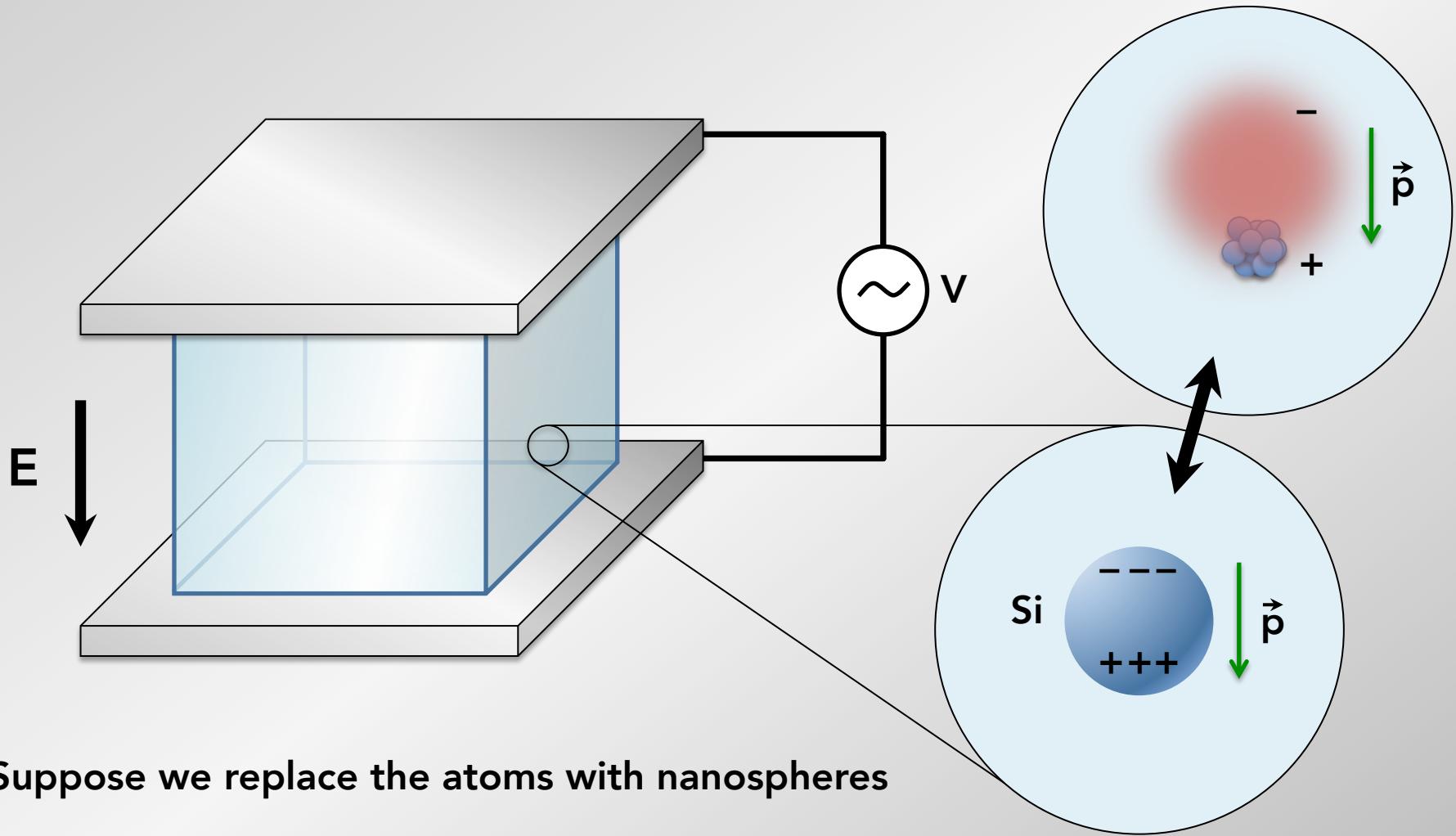
1

zero index

2

metamaterials

Microscopic origins of permittivity



Suppose we replace the atoms with nanospheres

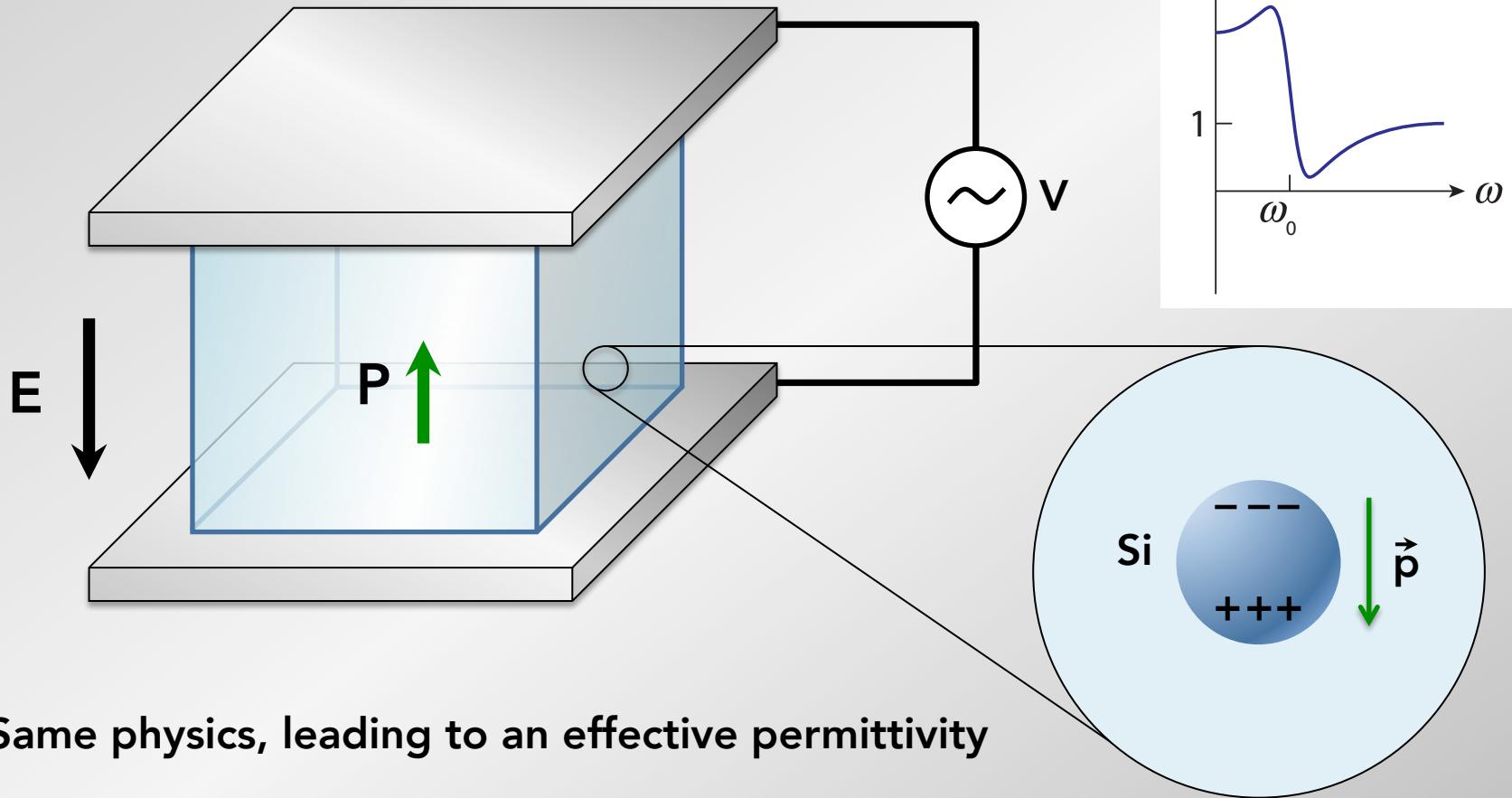
1

zero index

2

metamaterials

Microscopic origins of permittivity



Same physics, leading to an effective permittivity

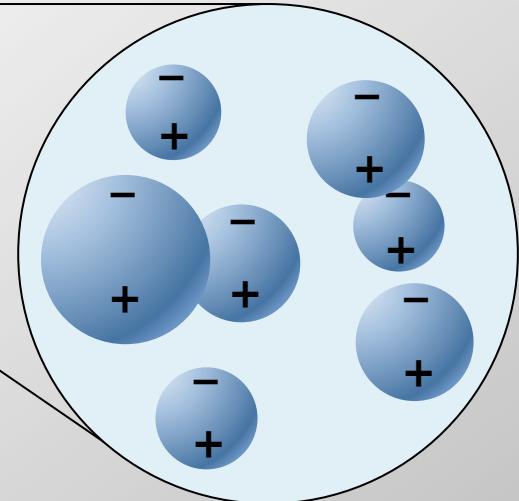
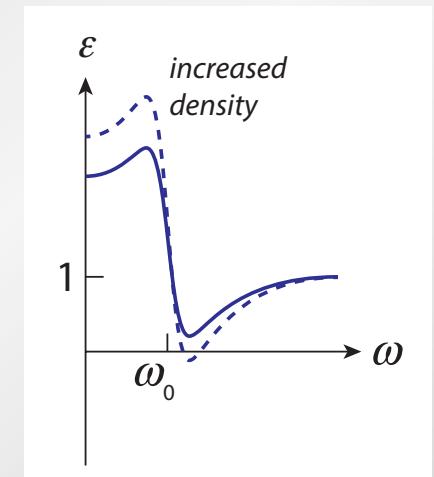
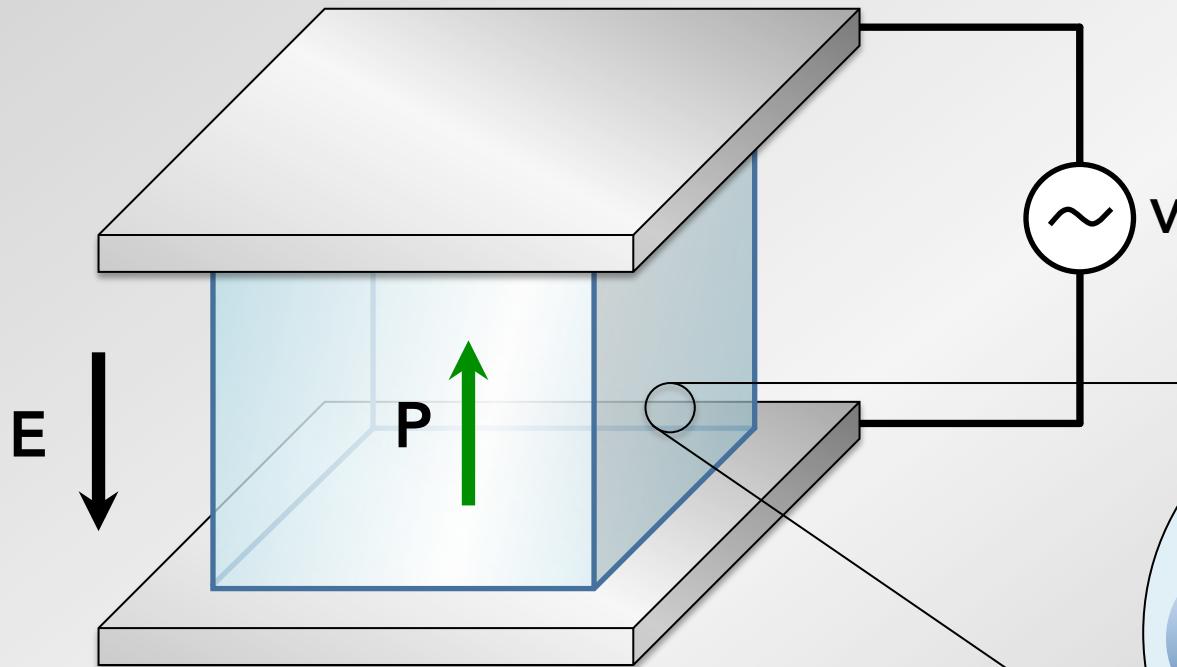
1

zero index

2

metamaterials

Microscopic origins of permittivity



But this time, it's tunable!

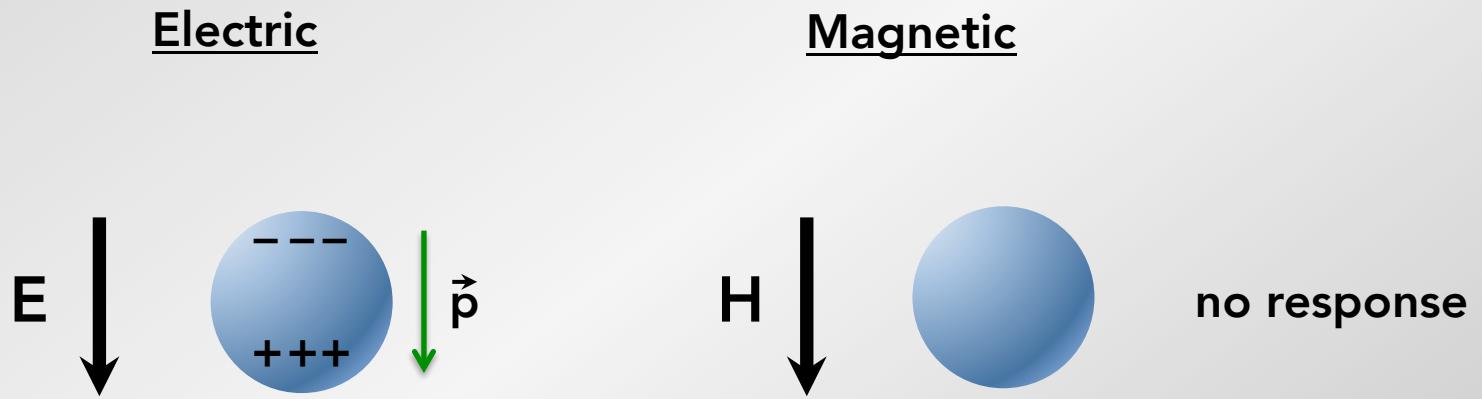
1

zero index

2

metamaterials

Engineering the effective permeability, μ



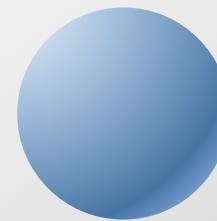
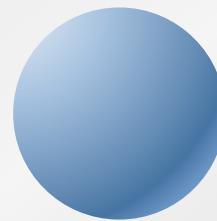
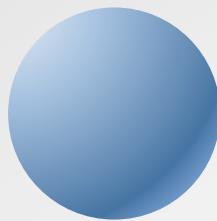
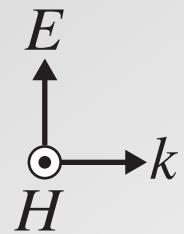
1

zero index

2

metamaterials

Engineering the effective permeability, μ



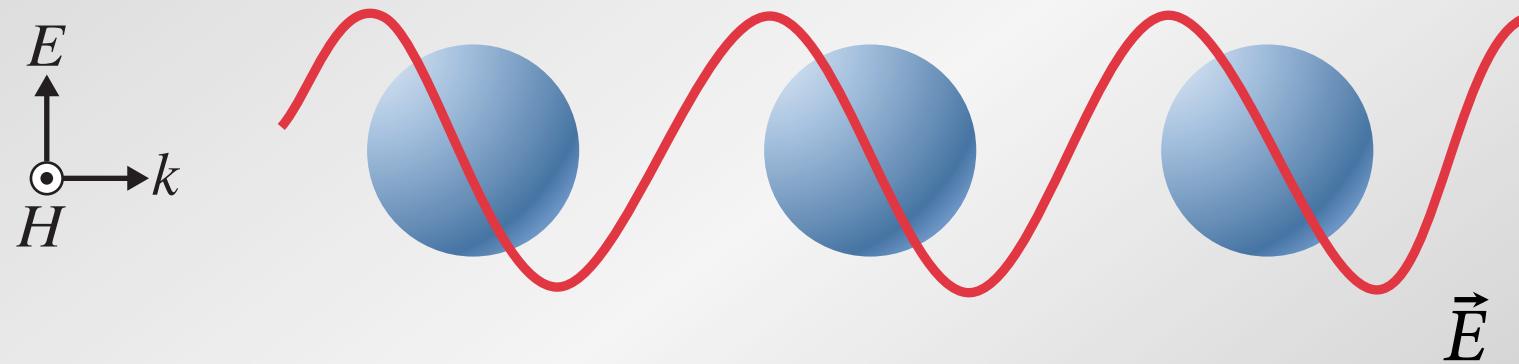
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zero index

2

metamaterials

Engineering the effective permeability, μ



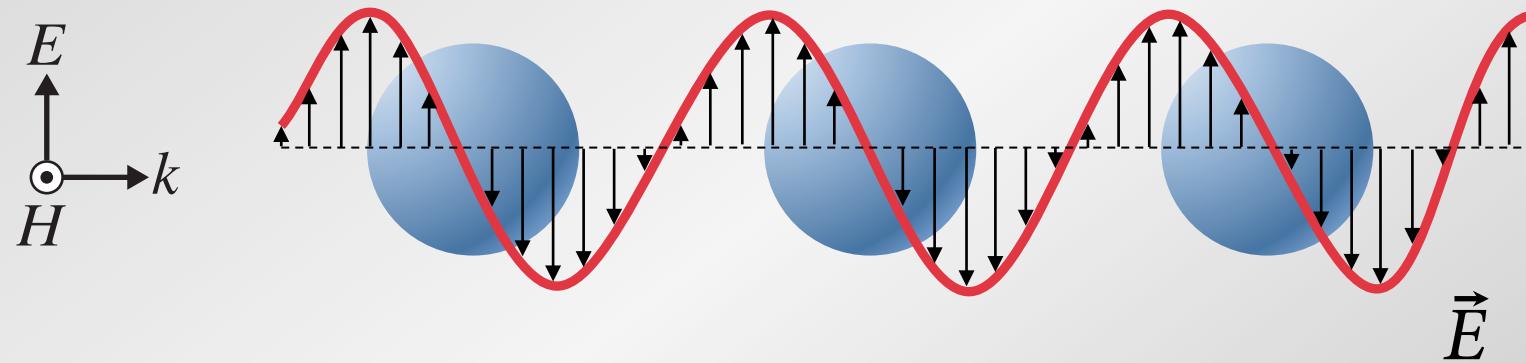
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zero index

2

metamaterials

Engineering the effective permeability, μ



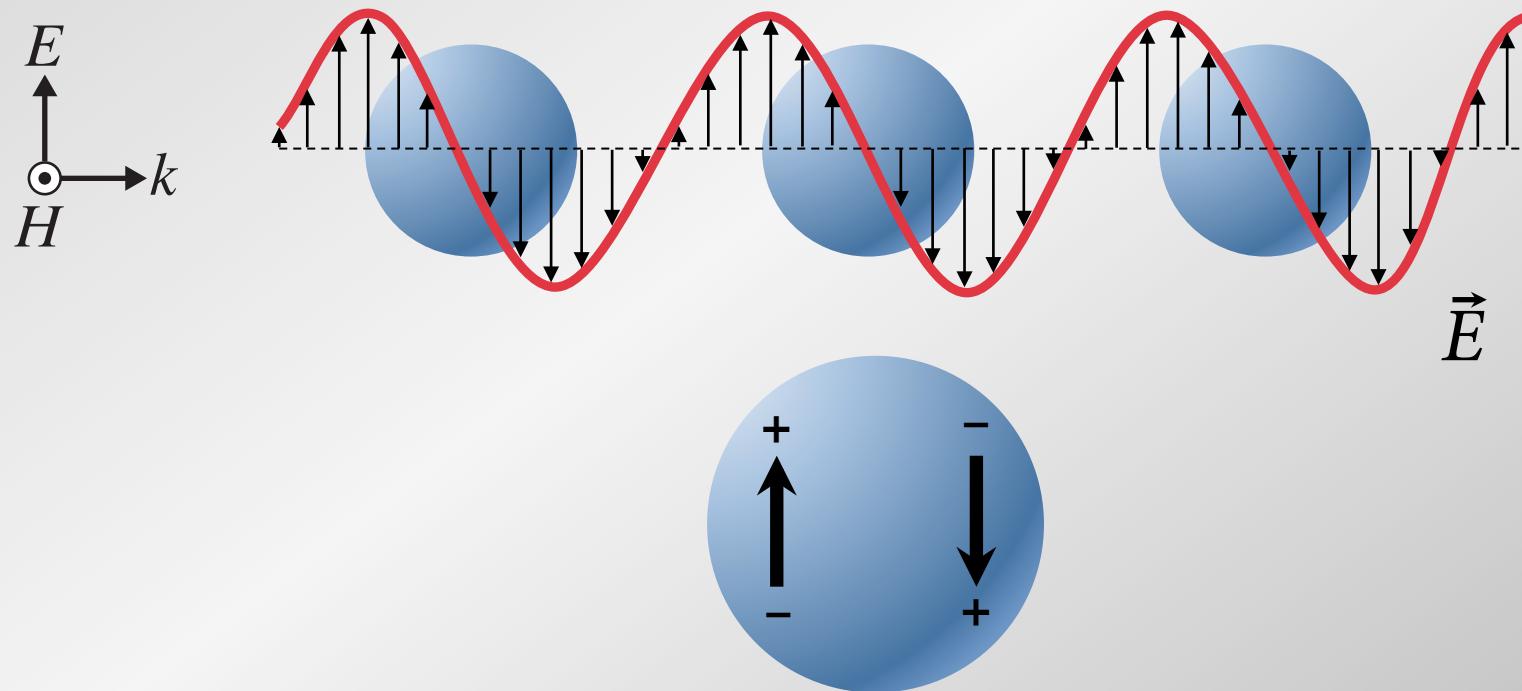
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zero index

2

metamaterials

Engineering the effective permeability, μ



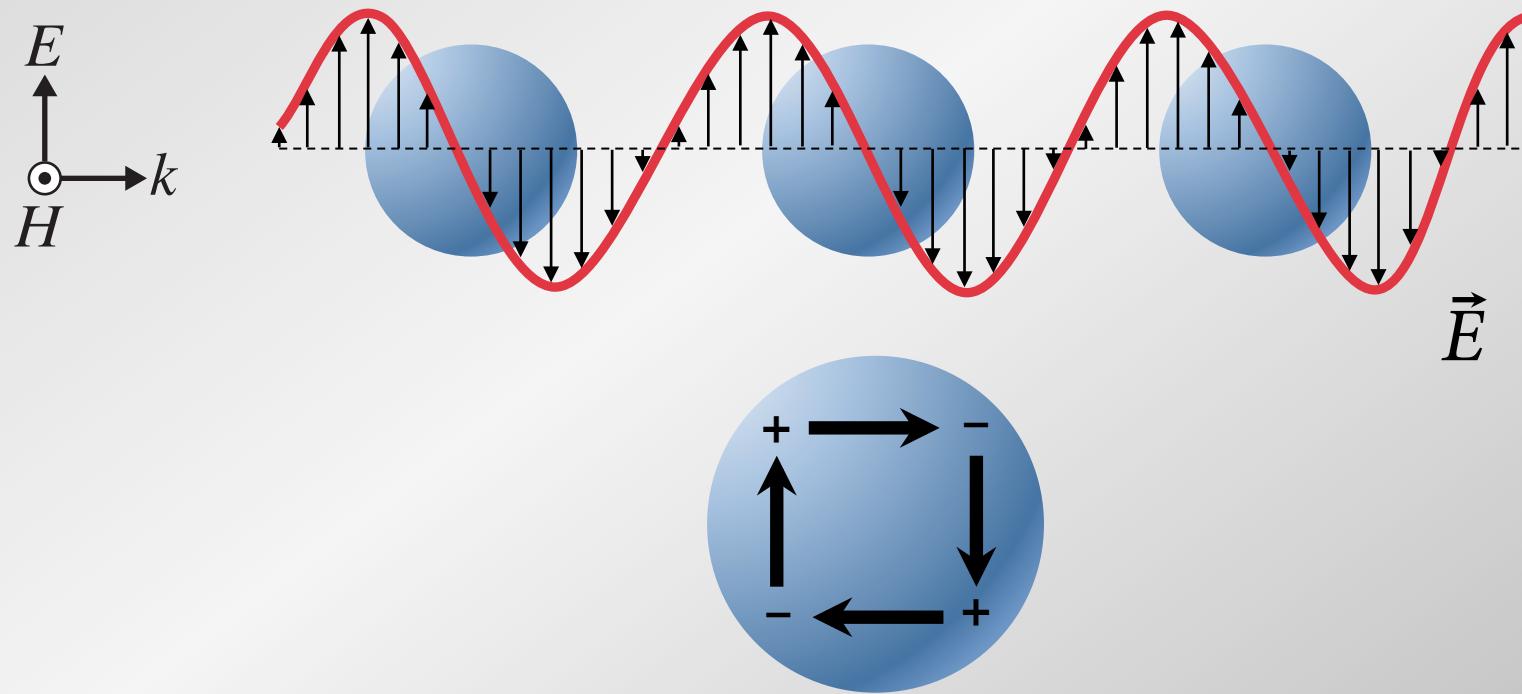
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zero index

2

metamaterials

Engineering the effective permeability, μ



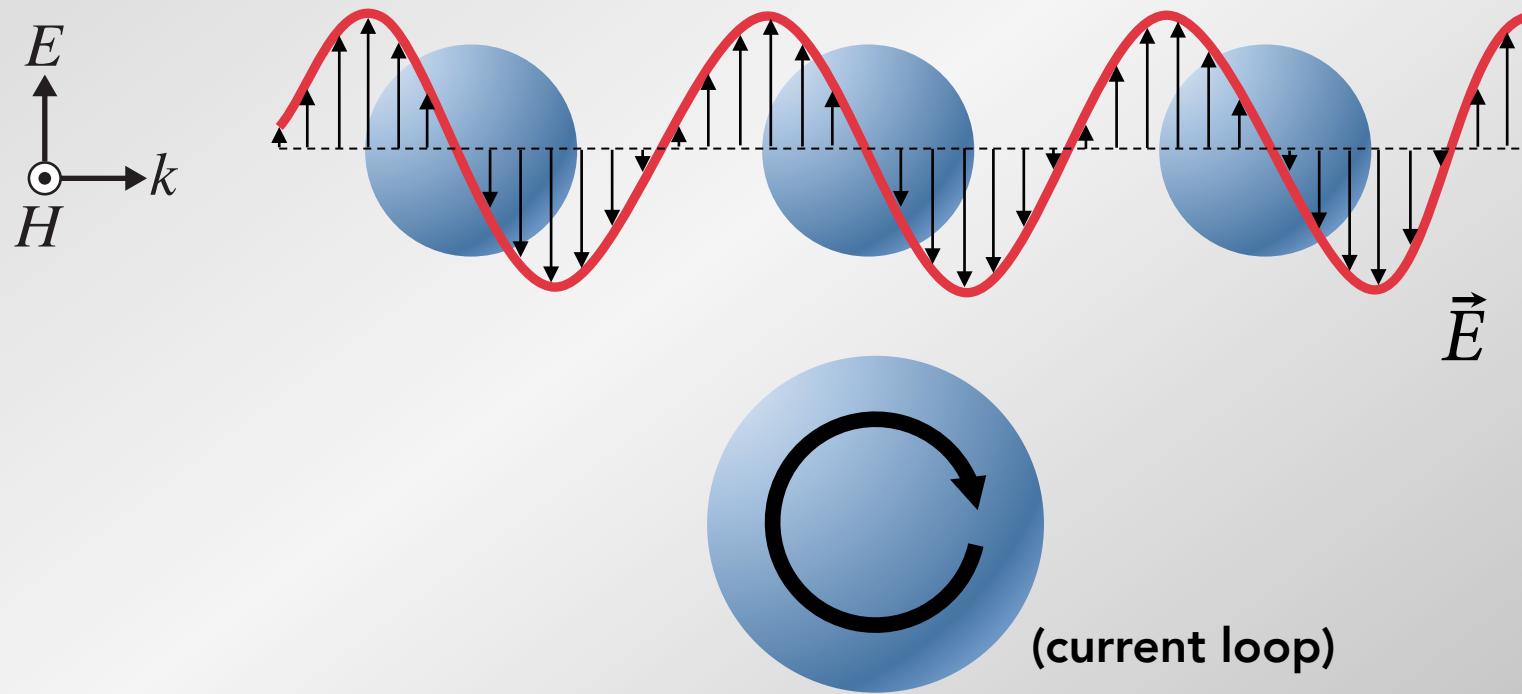
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zero index

2

metamaterials

Engineering the effective permeability, μ



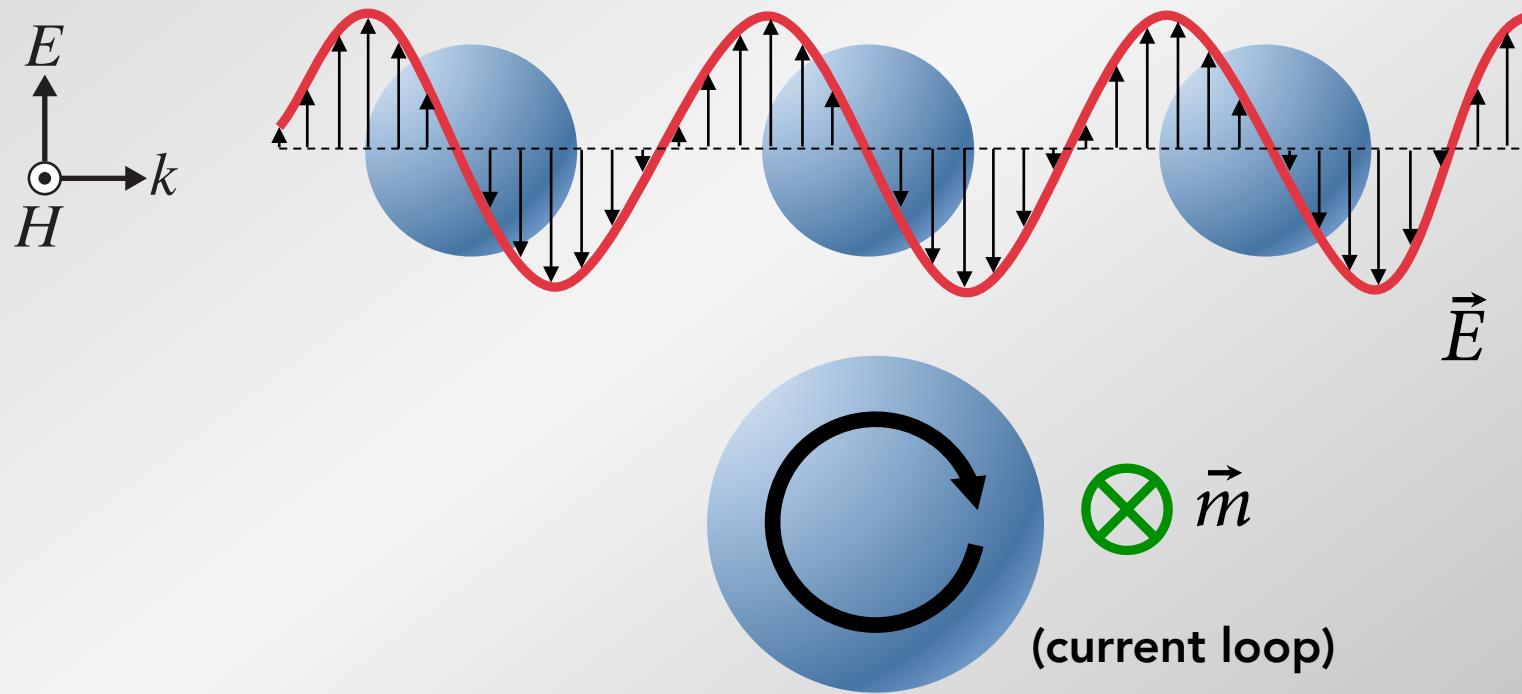
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zero index

2

metamaterials

Engineering the effective permeability, μ



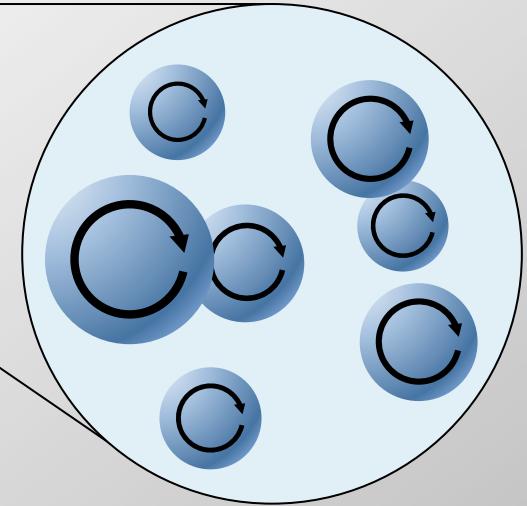
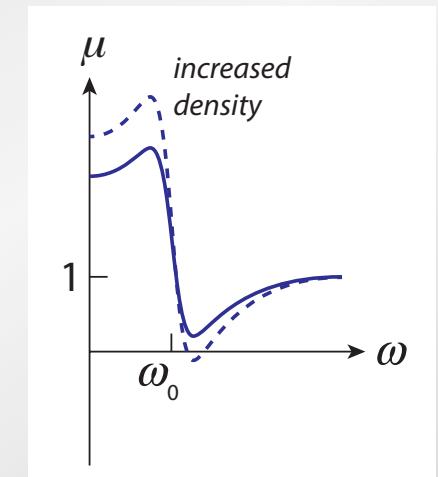
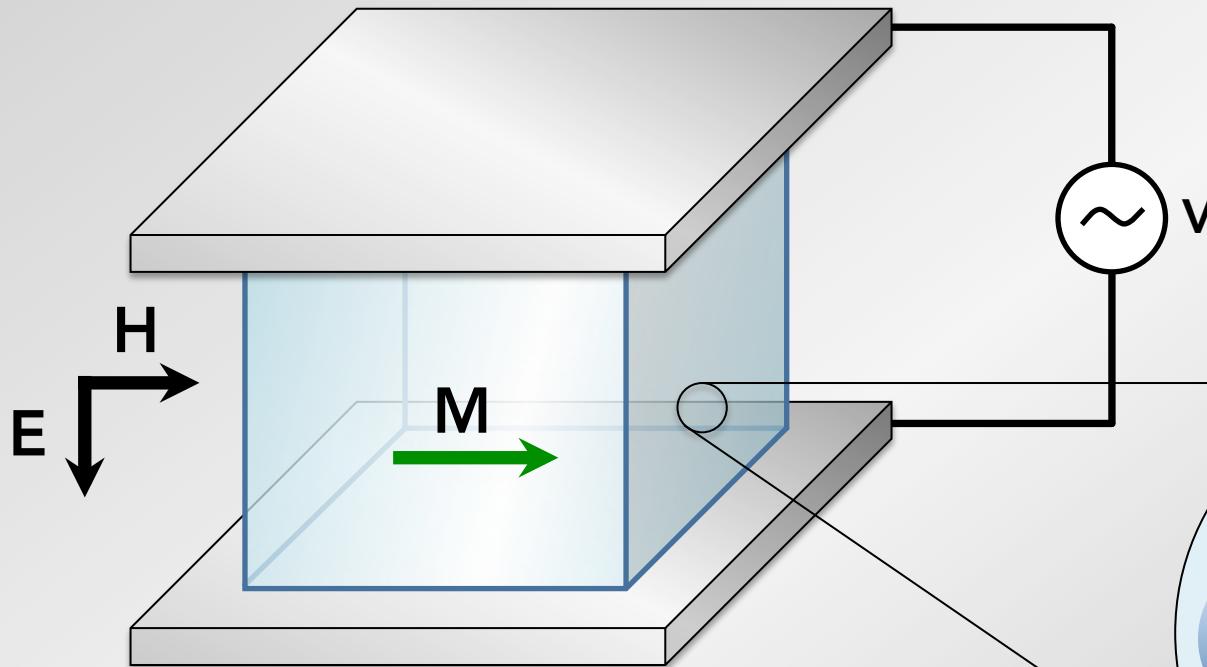
1

zero index

2

metamaterials

Engineering the effective permeability, μ



Dielectric nanoparticles generate
(tunable) optical magnetism!

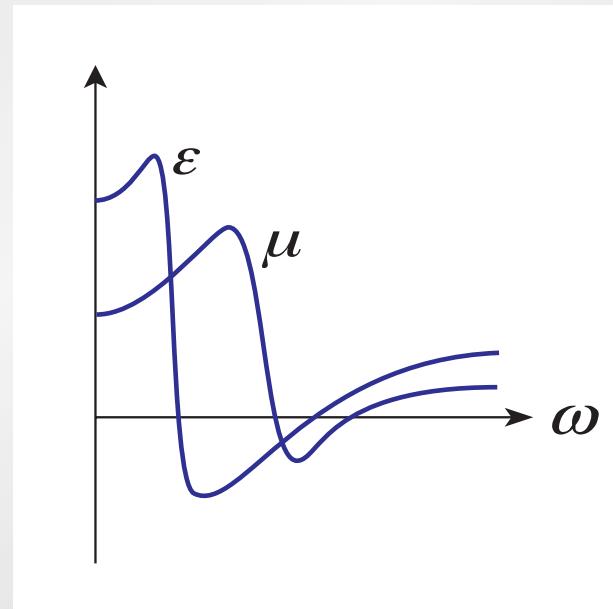
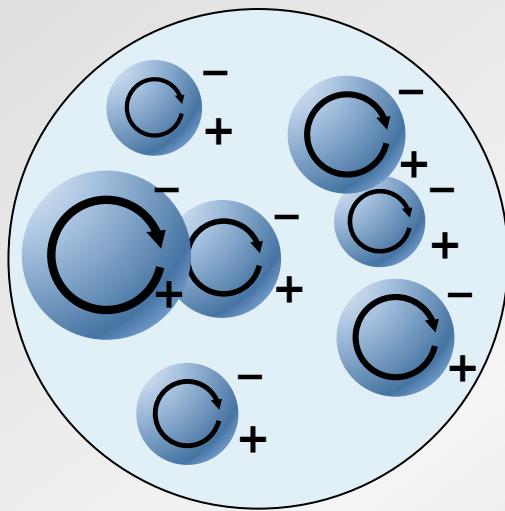
1

zero index

2

metamaterials

Engineering the effective permeability, μ



Both resonances occur simultaneously

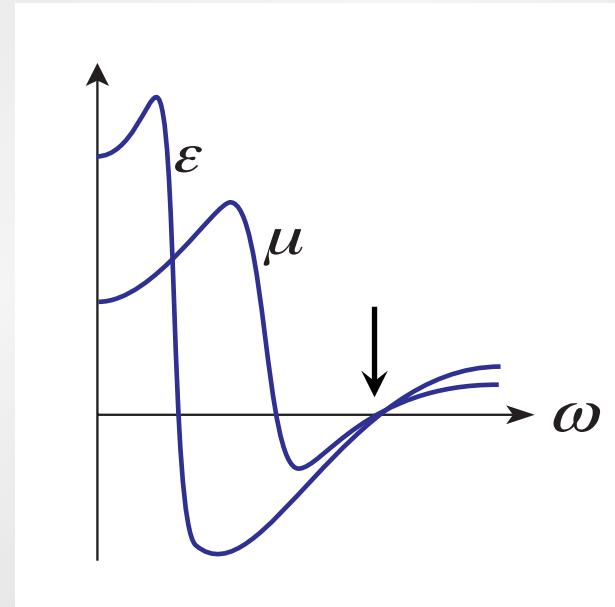
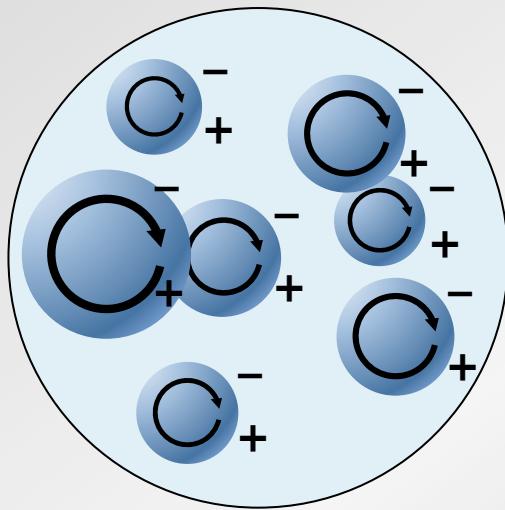
1

zero index

2

metamaterials

Engineering the effective permeability, μ



Both resonances occur simultaneously

By tuning the size and spacing, we can achieve $\epsilon = \mu = 0$

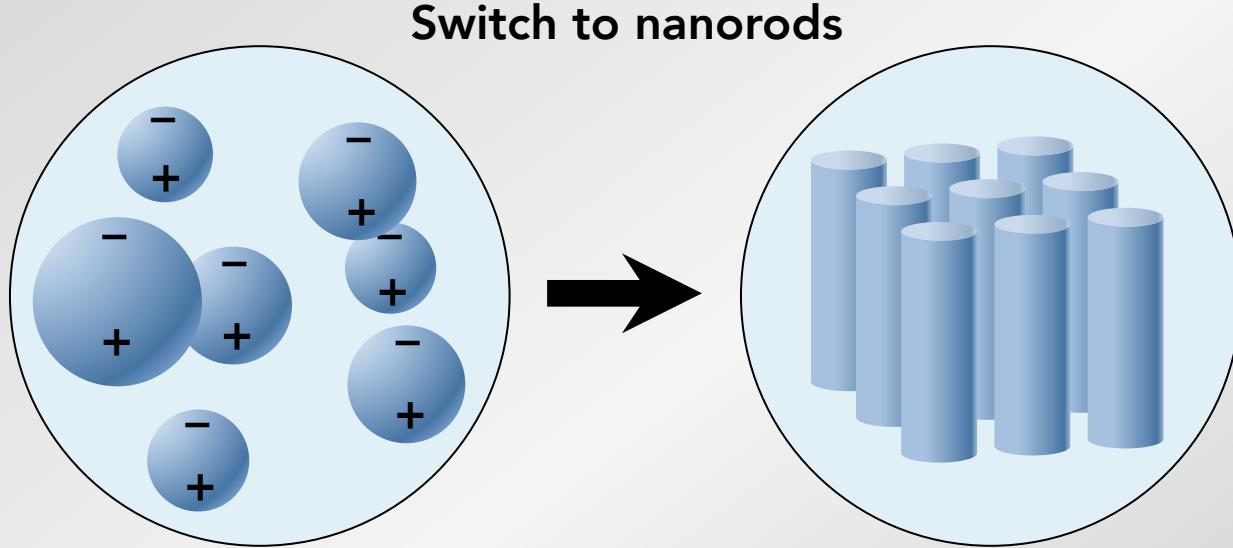
1

zero index

2

metamaterials

Tuning the design toward zero index



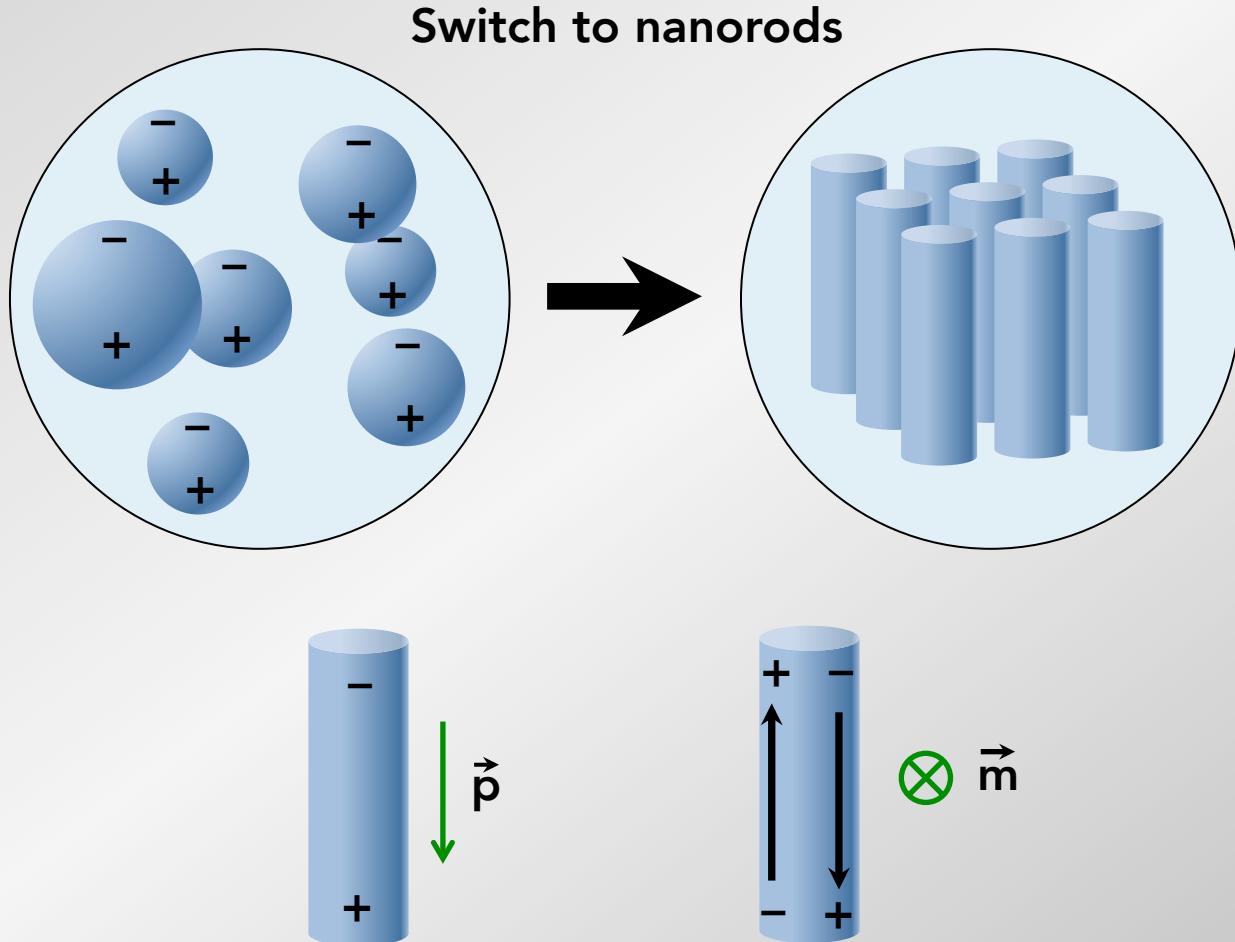
1

zero index

2

metamaterials

Tuning the design toward zero index



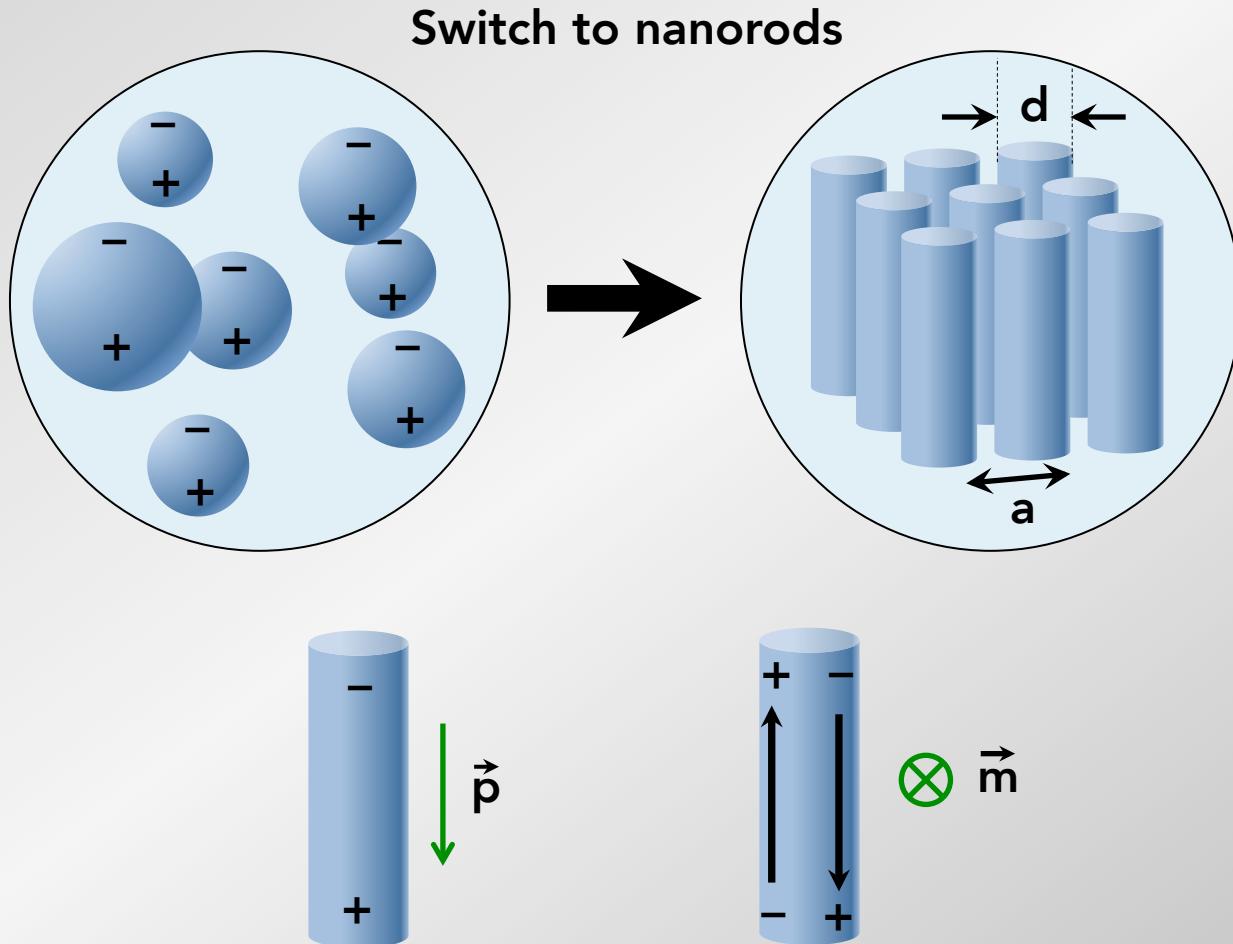
1

zero index

2

metamaterials

Tuning the design toward zero index



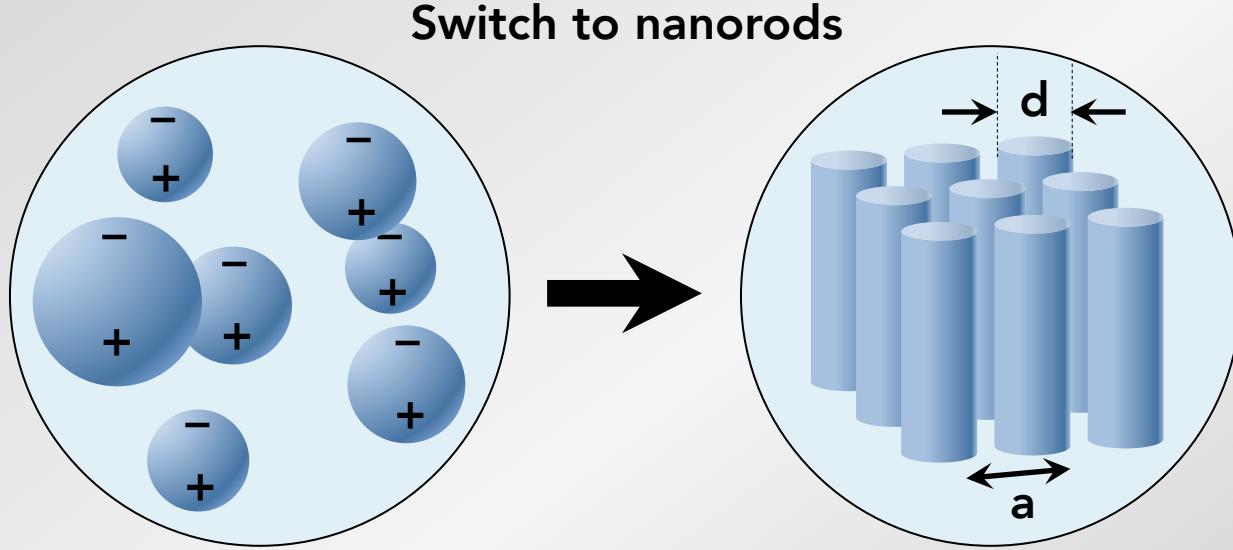
1

zero index

2

metamaterials

Tuning the design toward zero index



Compute the effective index for a range of sizes and spacings

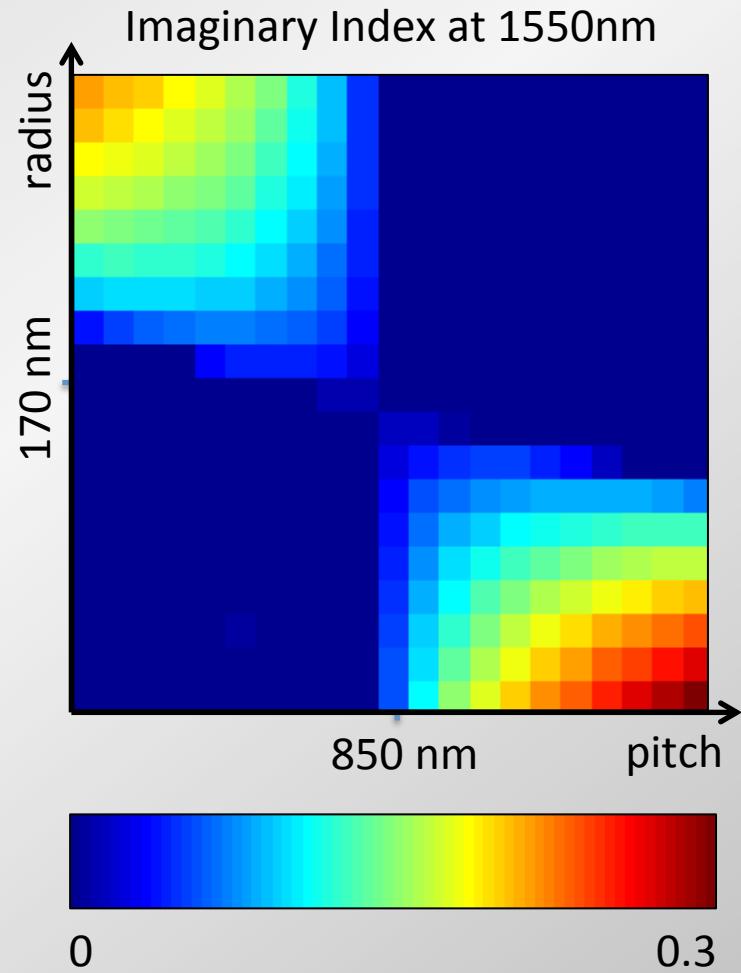
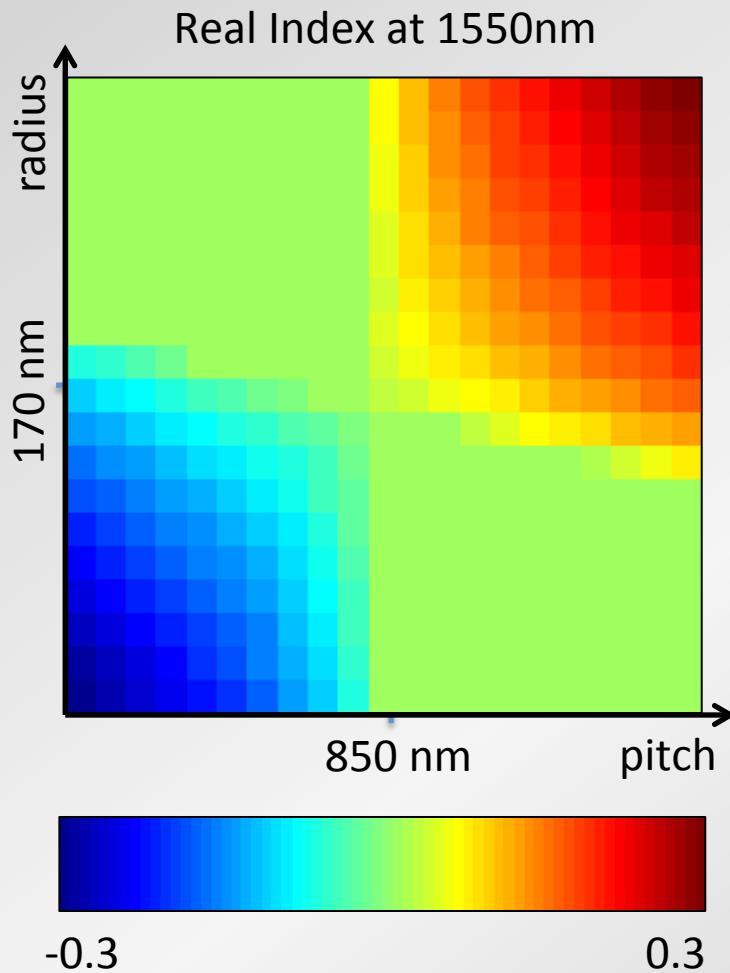
1

zero index

2

metamaterials

Effective refractive index



1

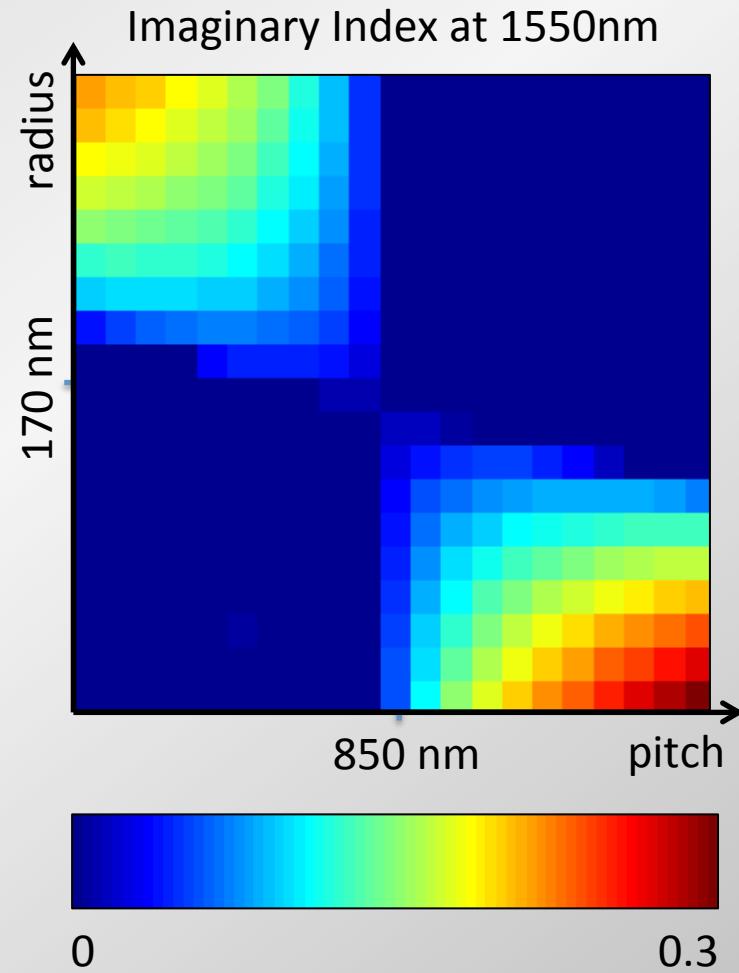
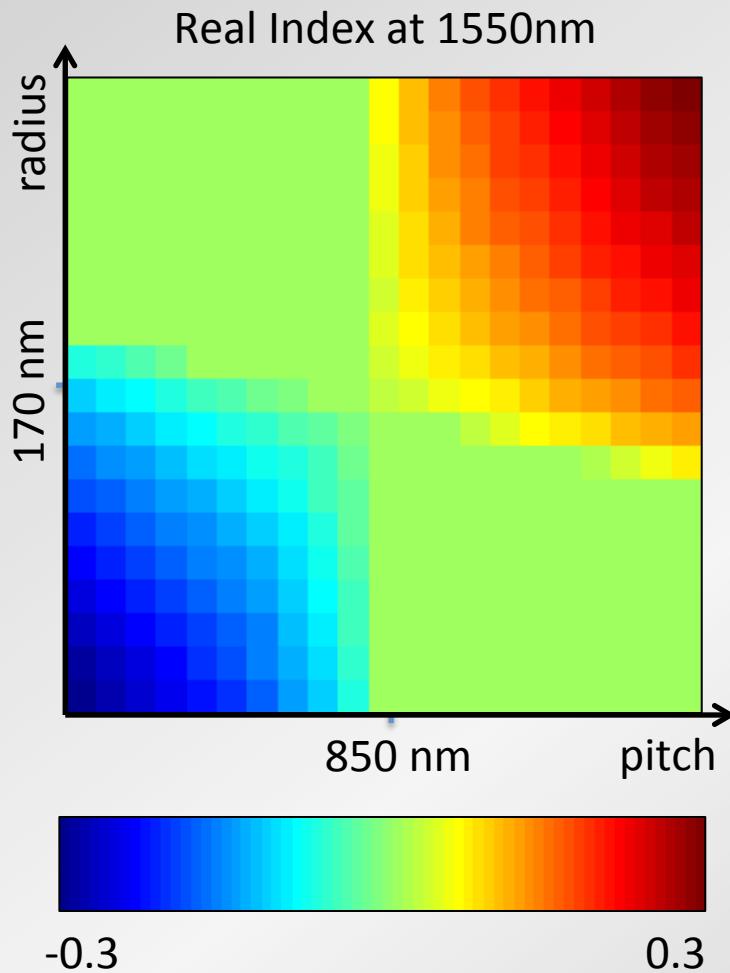
zero index

2

metamaterials

Effective refractive index

$$n = n' + i n''$$



1

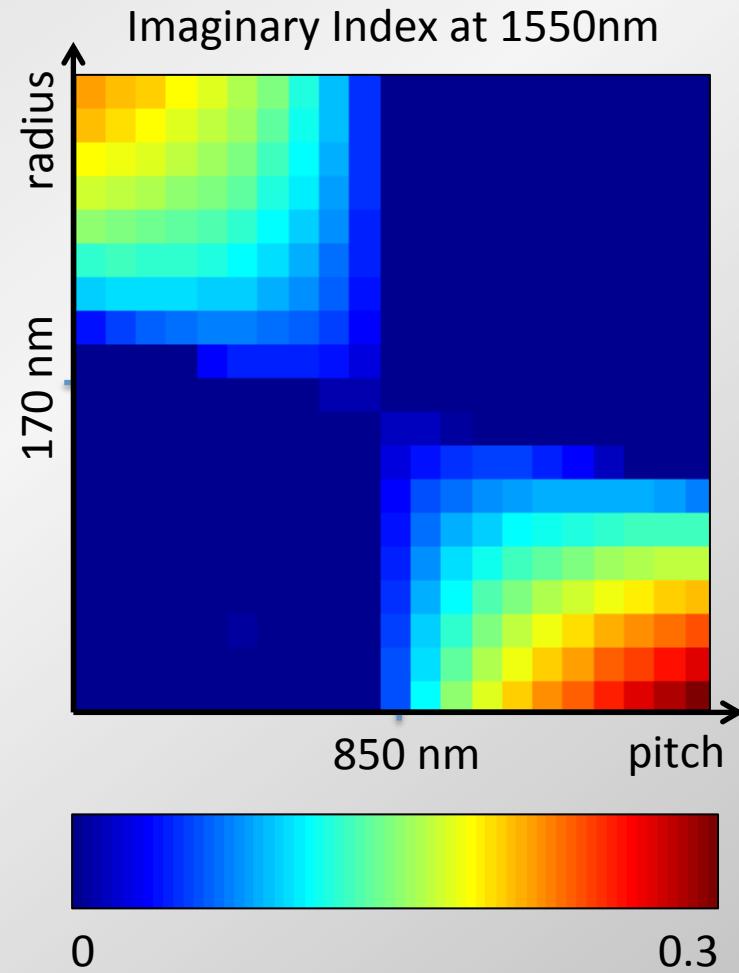
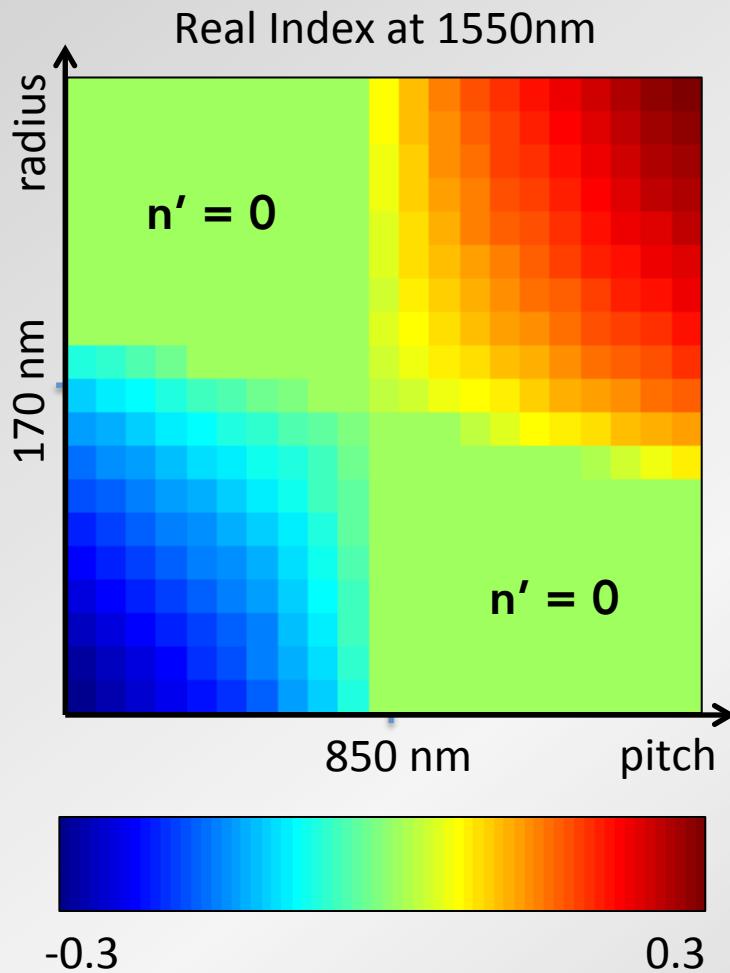
zero index

2

metamaterials

Effective refractive index

$$n = n' + i n''$$



1

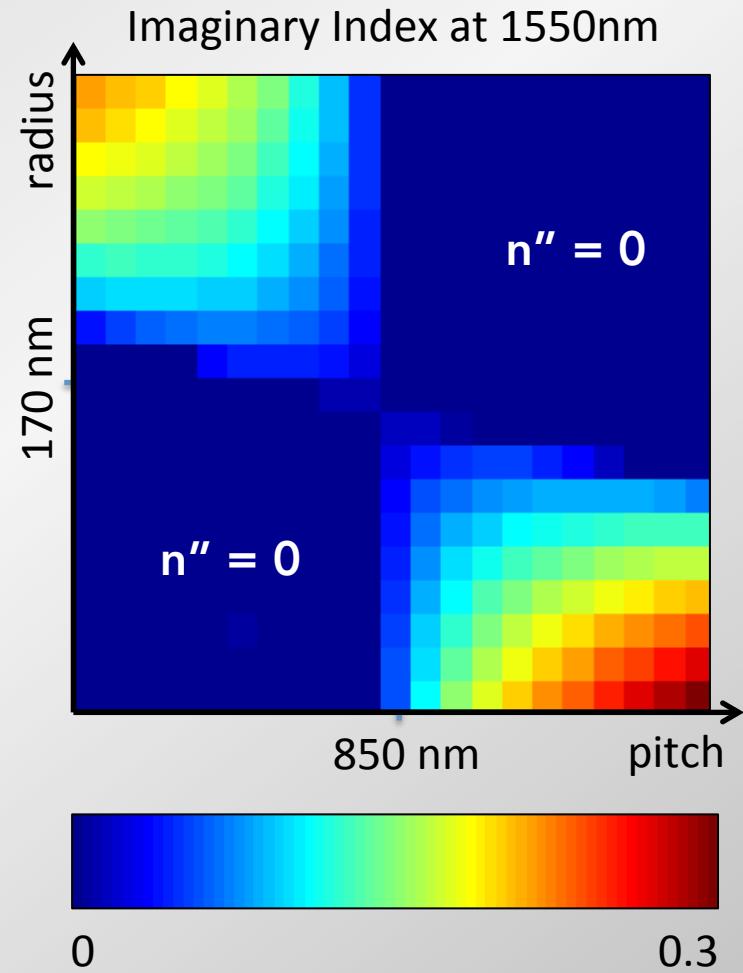
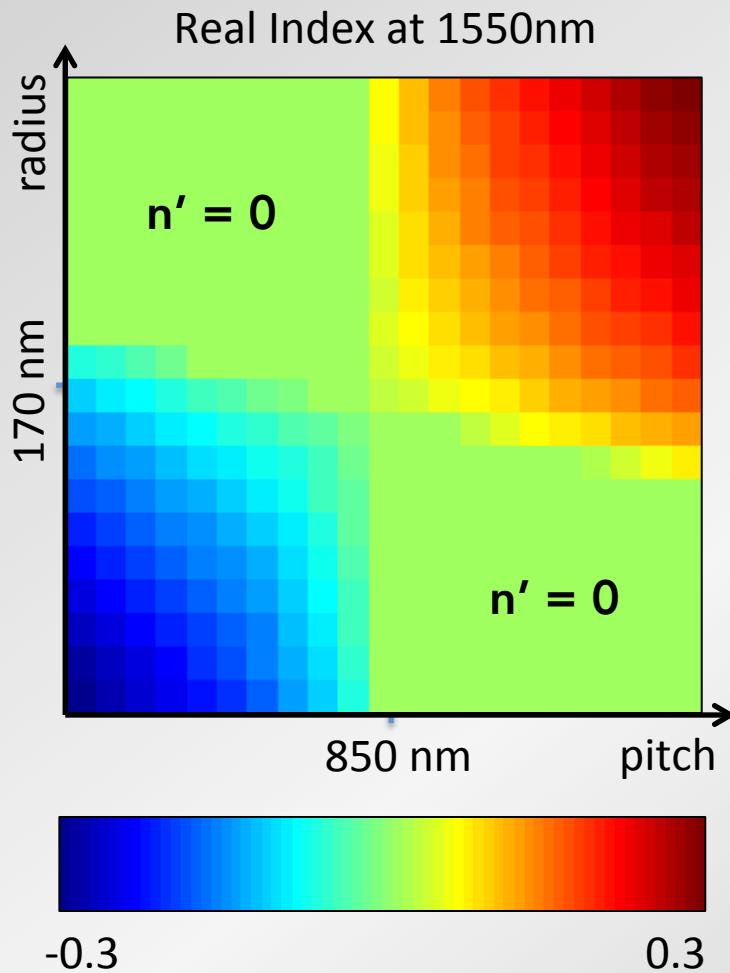
zero index

2

metamaterials

Effective refractive index

$$n = n' + i n''$$



1

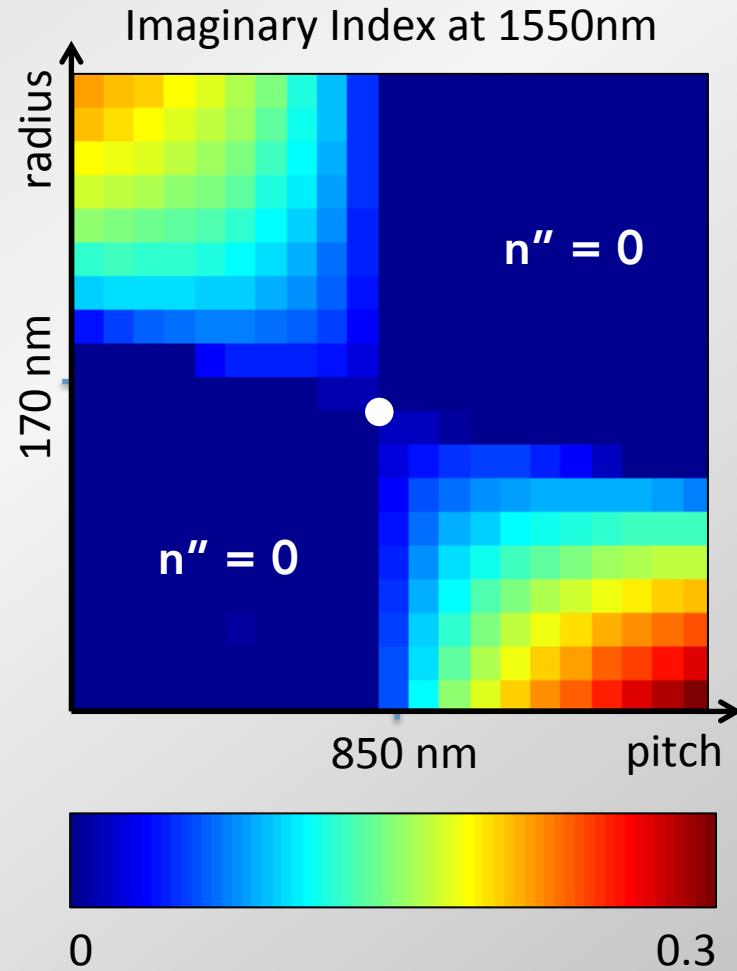
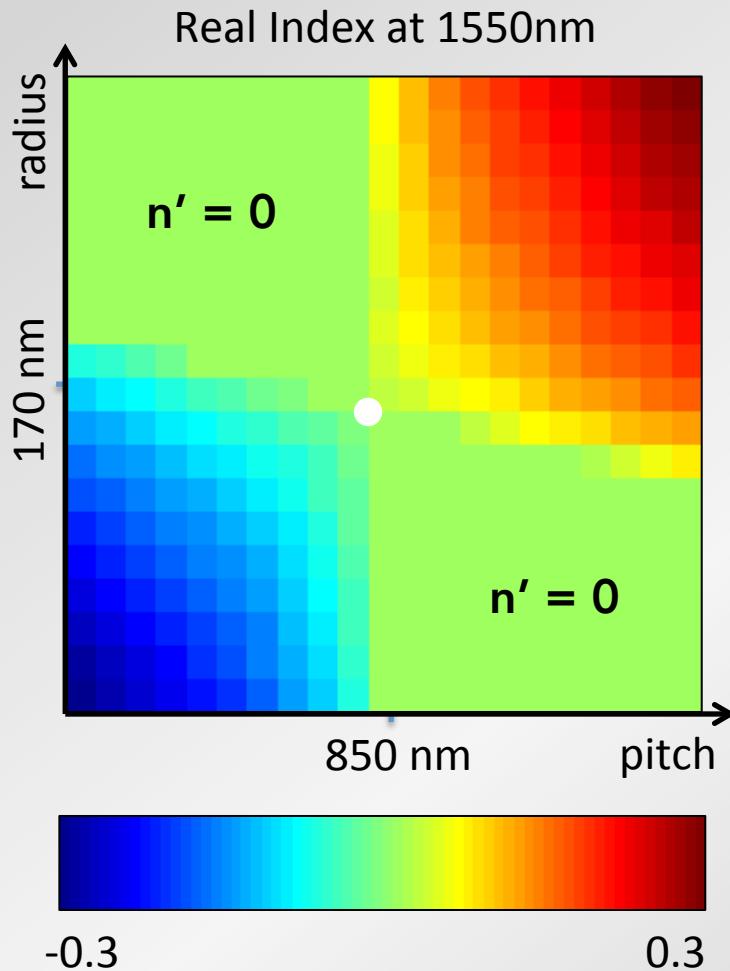
zero index

2

metamaterials

Effective refractive index

$$n = n' + i n''$$



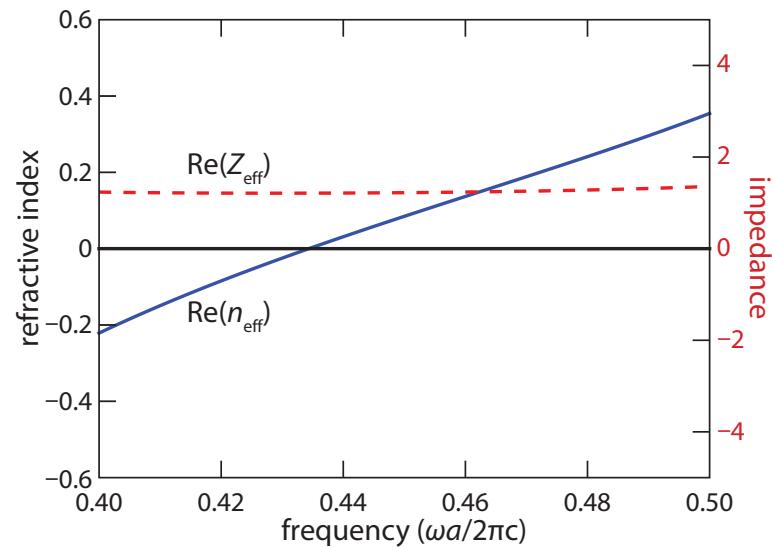
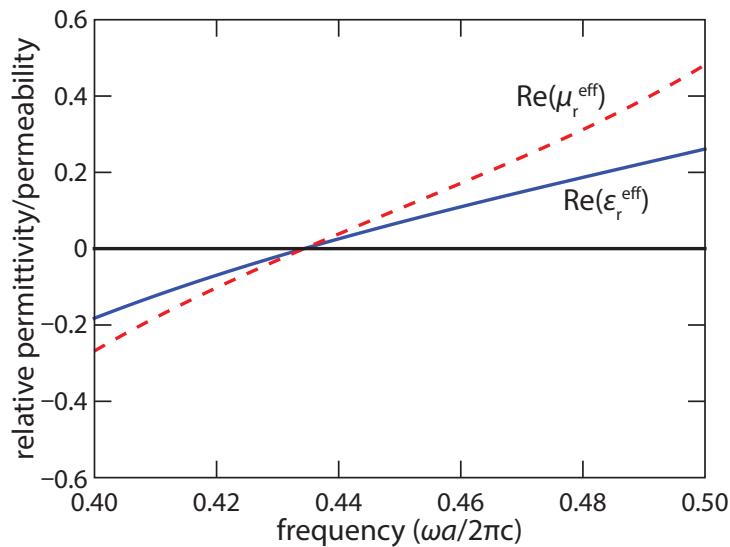
1

zero index

2

metamaterials

Optimized design for zero index



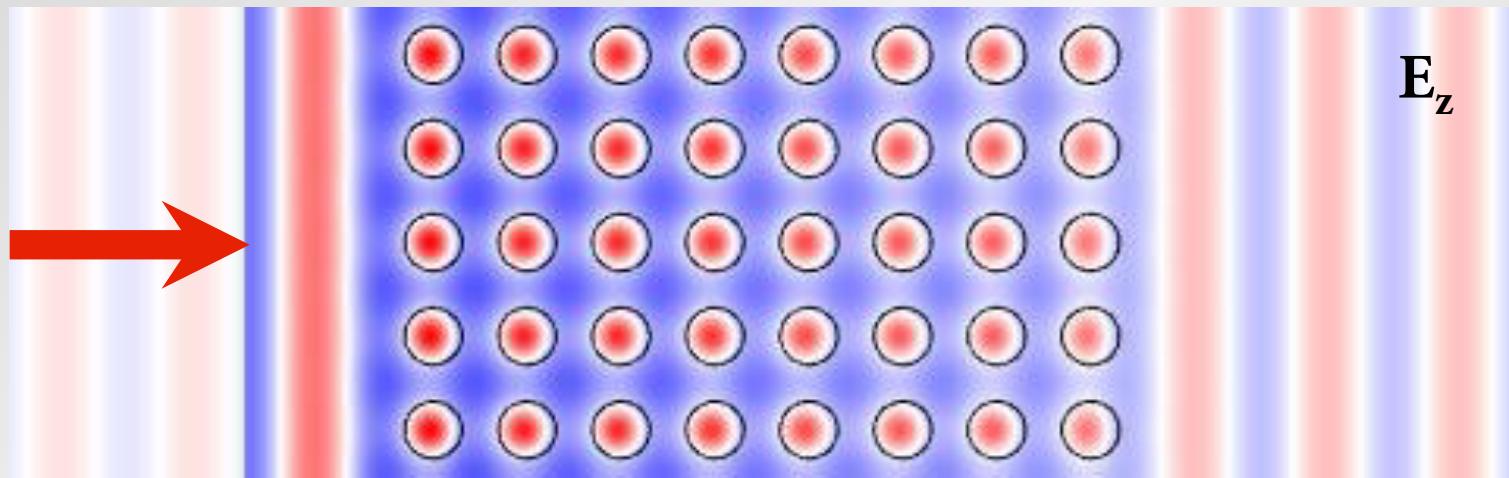
1

zero index

2

metamaterials

Optimized design for zero index

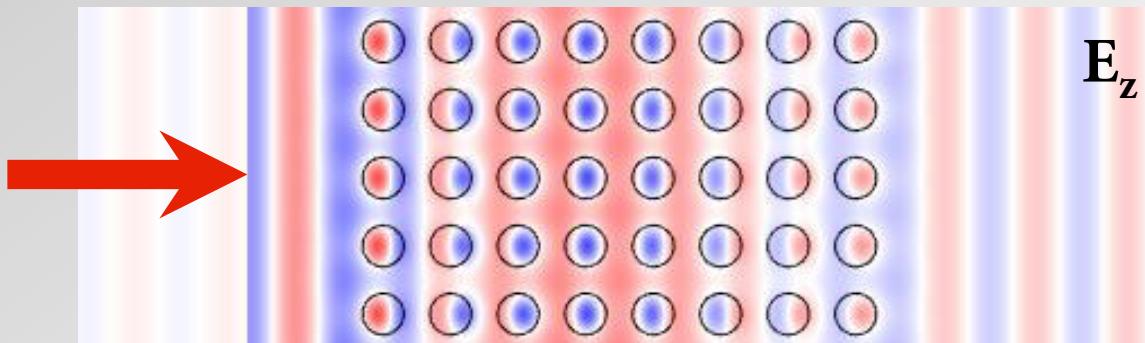


1

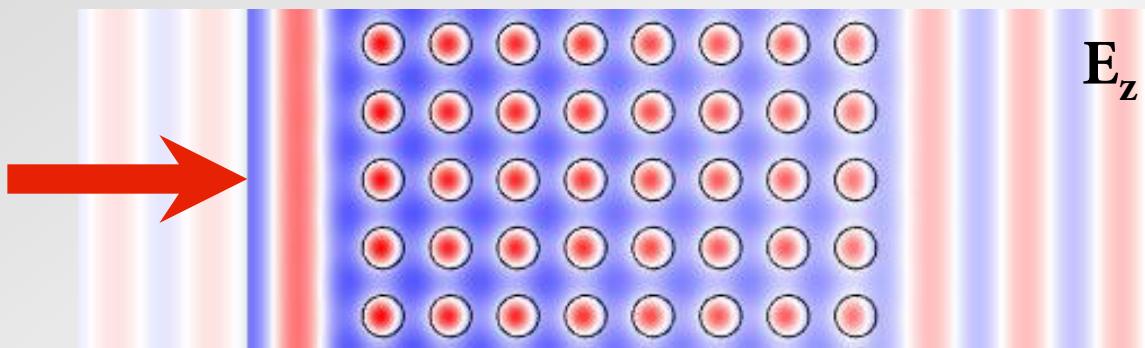
zero index

2

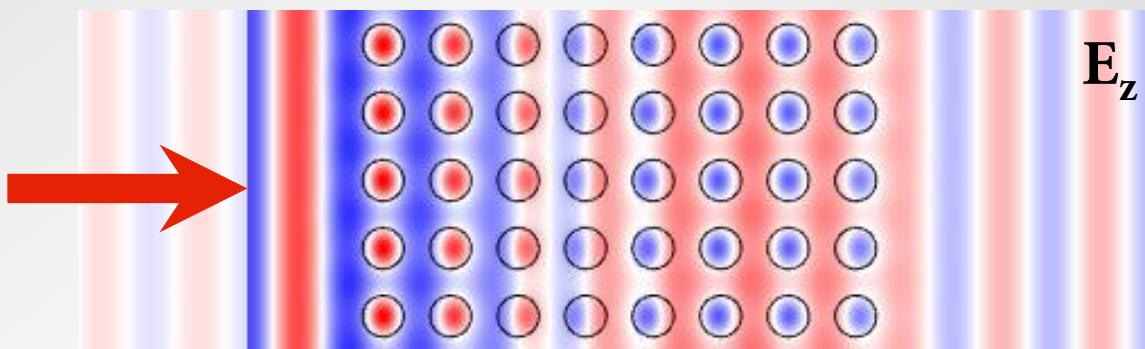
metamaterials



$\lambda_0 = 1530\text{nm}$
 $n = 0.2$



$\lambda_0 = 1590\text{nm}$
 $n = 0$



$\lambda_0 = 1650\text{nm}$
 $n = -0.2$

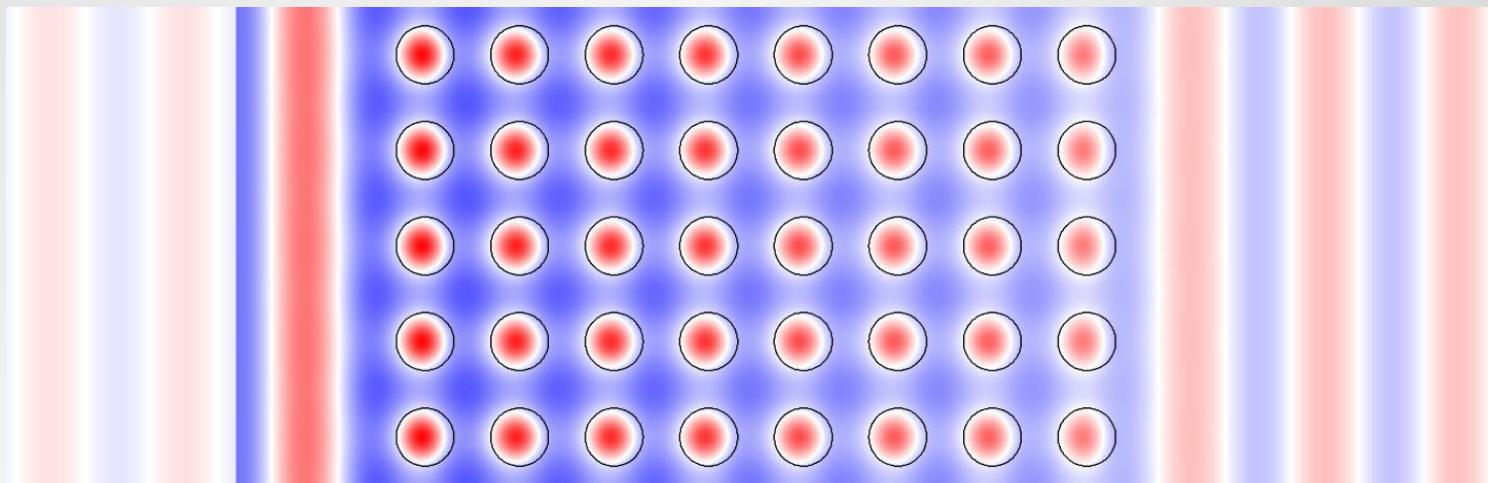
1 zero index

2 metamaterials

ideal zero-index material



zero-index metamaterial



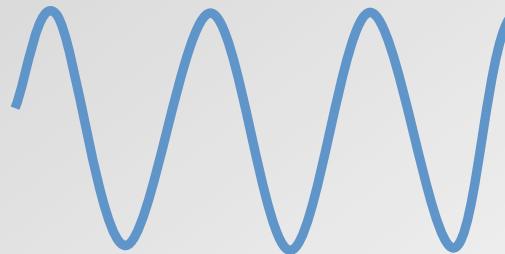
1

zero index

2

metamaterials

Super-luminal velocity



ν_p

$$\nu_p = \frac{1}{n} c$$

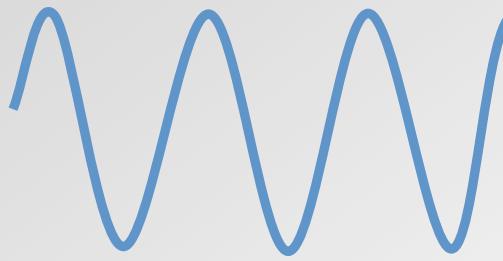
1

zero index

2

metamaterials

Super-luminal velocity



ν_p

$$\nu_p = \frac{1}{n} c \xrightarrow{n \rightarrow 0} \infty$$

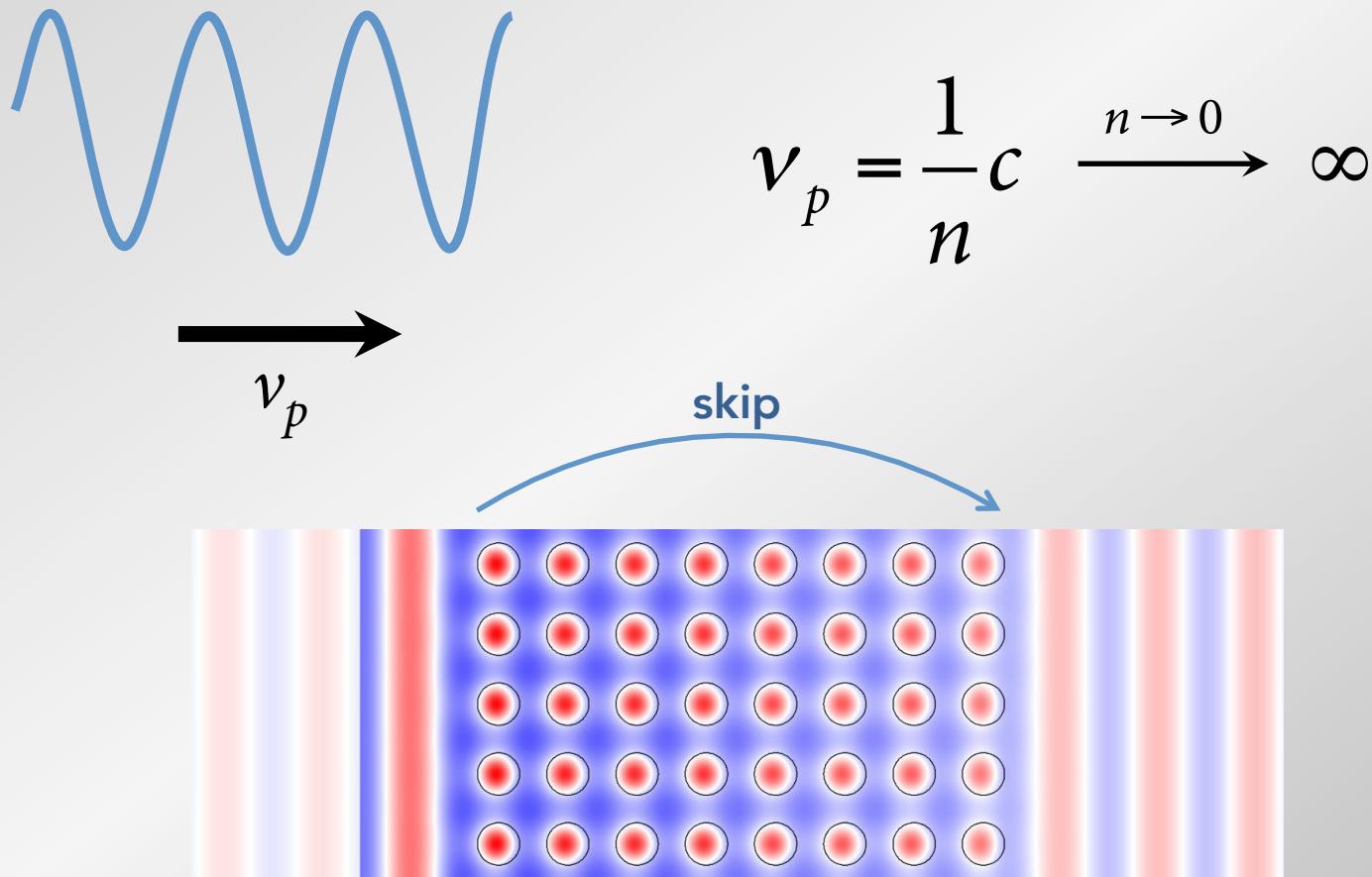
1

zero index

2

metamaterials

Super-luminal velocity



1

zero index

2

metamaterials

1

zero index

2

metamaterials

Can we use this phenomenon ($n = 0$) to enhance the speed of signal transmission?

1

zero index

2

metamaterials

Can we use this phenomenon ($n = 0$) to enhance the speed of signal transmission?

- 1. Yes, because the wave speed is infinite**
- 2. Yes, but only up to the speed of light in a vacuum**
- 3. No, because the signal would get distorted**
- 4. No, because the wave carries no information**

1

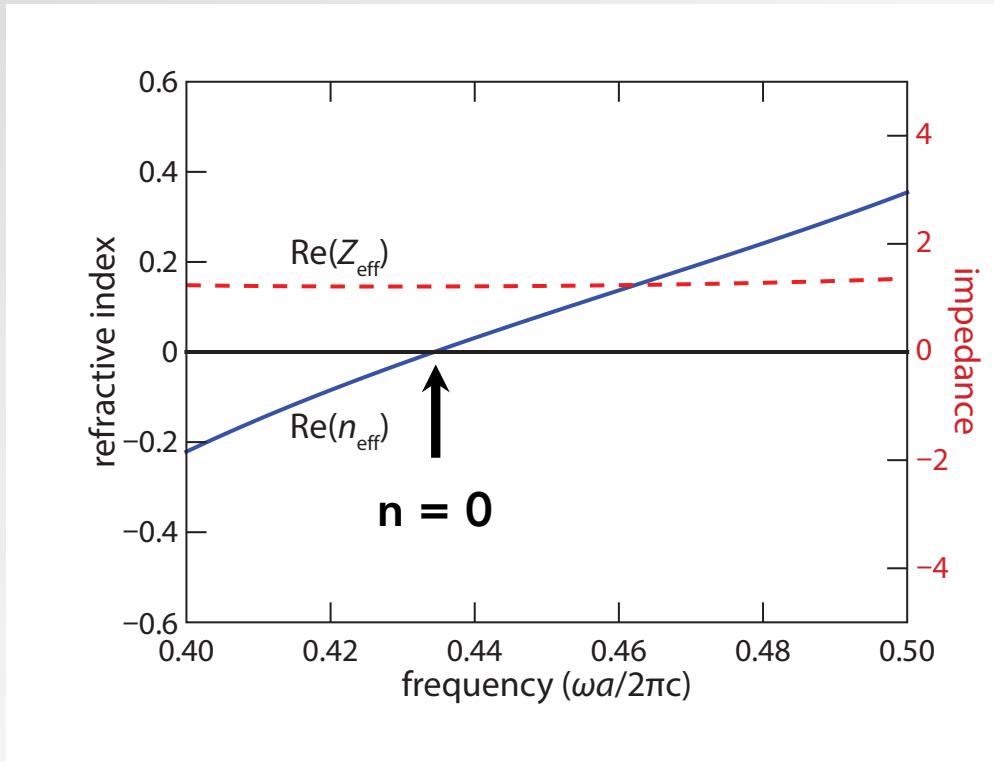
zero index

2

metamaterials

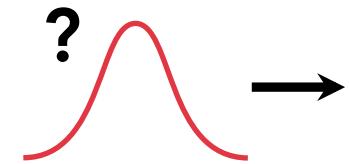
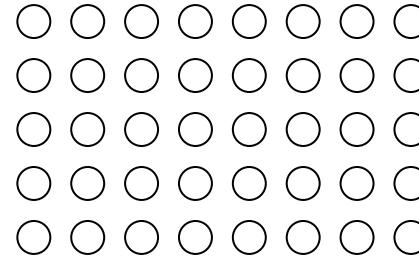
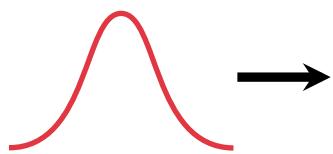
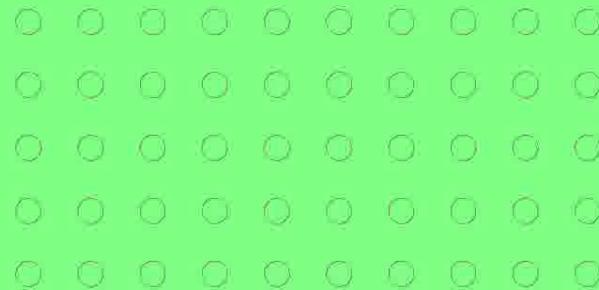
Can we use this phenomenon ($n = 0$) to enhance the speed of signal transmission?

1. Yes, because the wave speed is infinite
2. Yes, but only up to the speed of light in a vacuum
3. No, because the signal would get distorted
4. No, because the wave carries no information 



The phase velocity is infinite for only one wavelength

What happens to a signal pulse?



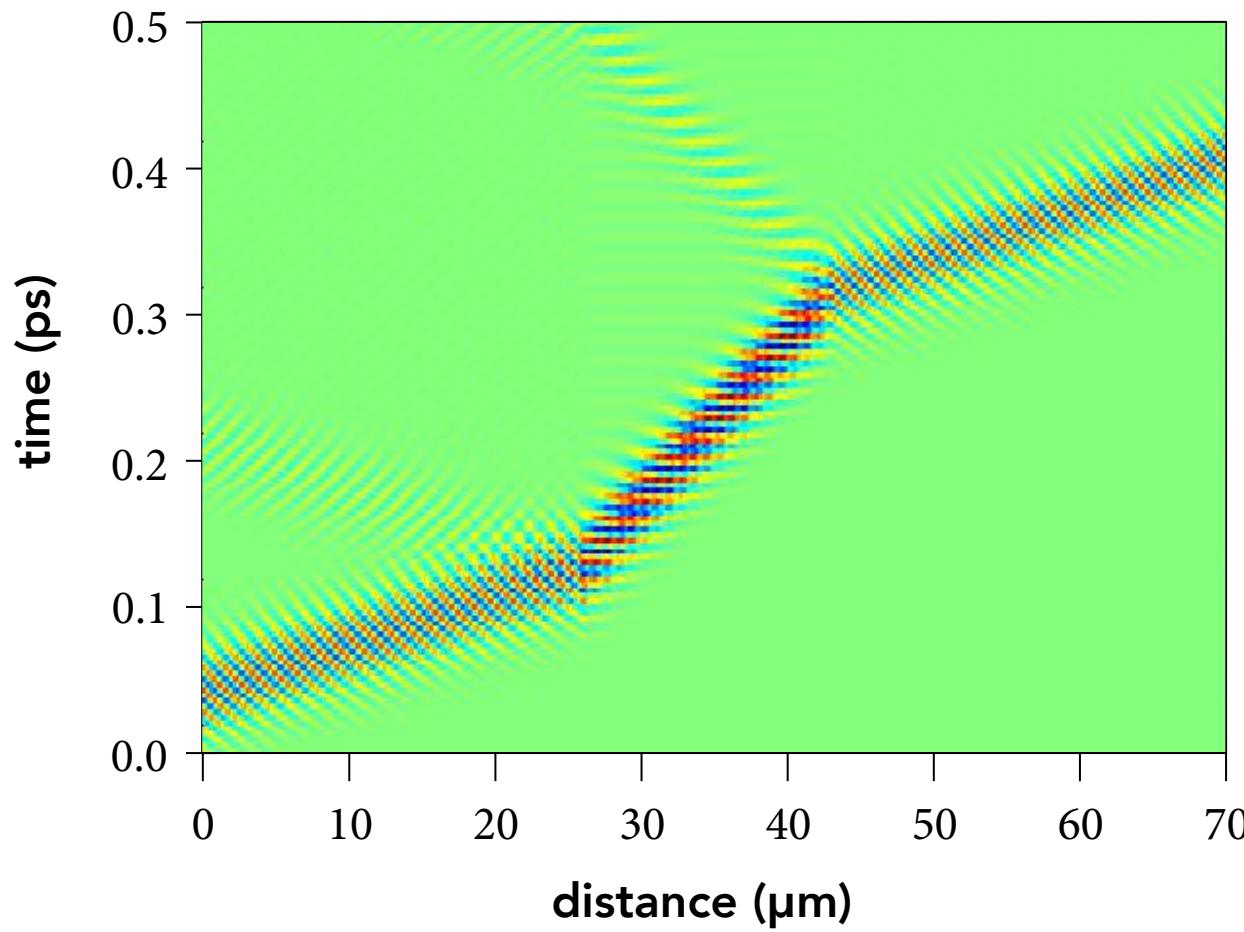
1

zero index

2

metamaterials

What happens to a signal pulse?



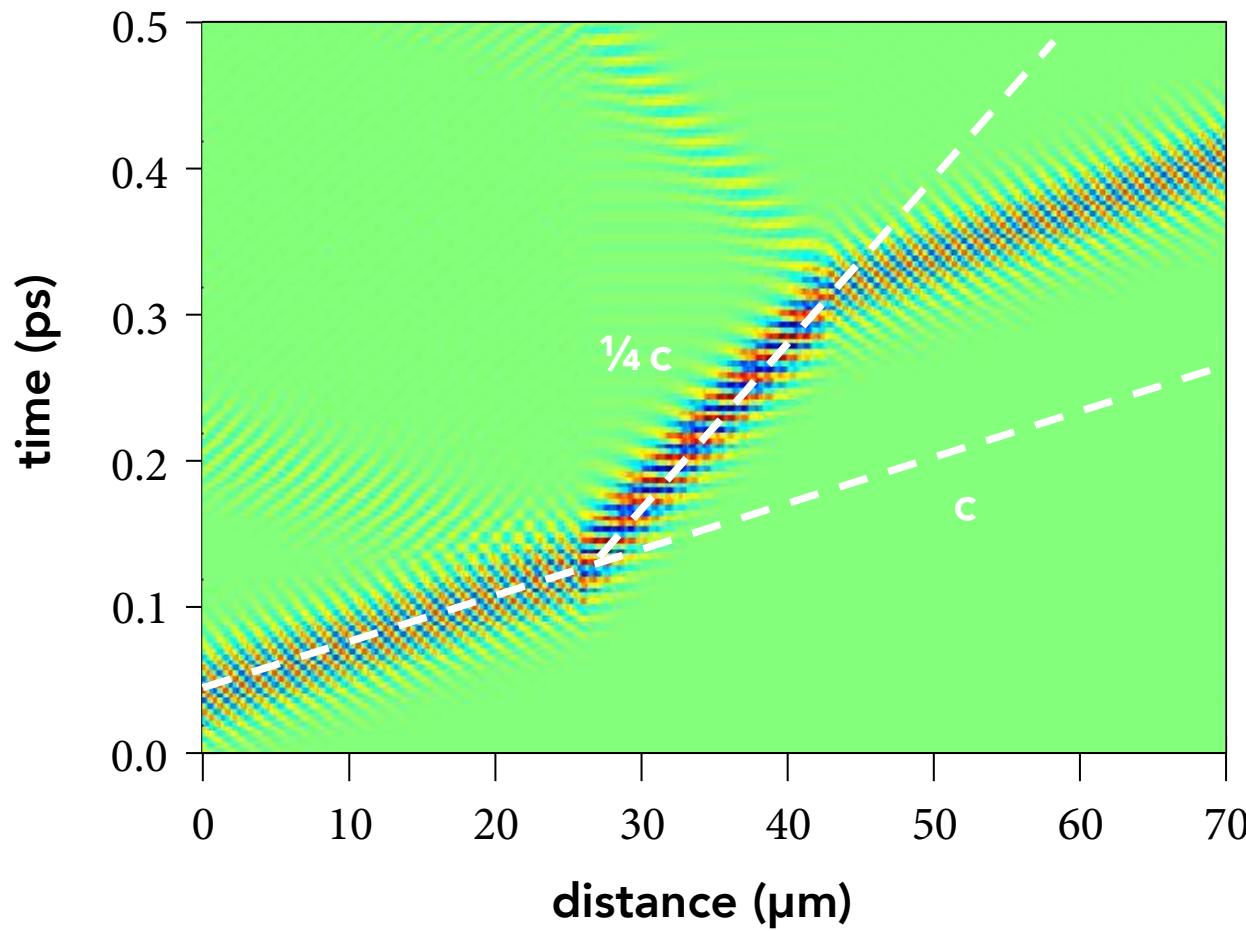
1

zero index

2

metamaterials

What happens to a signal pulse?



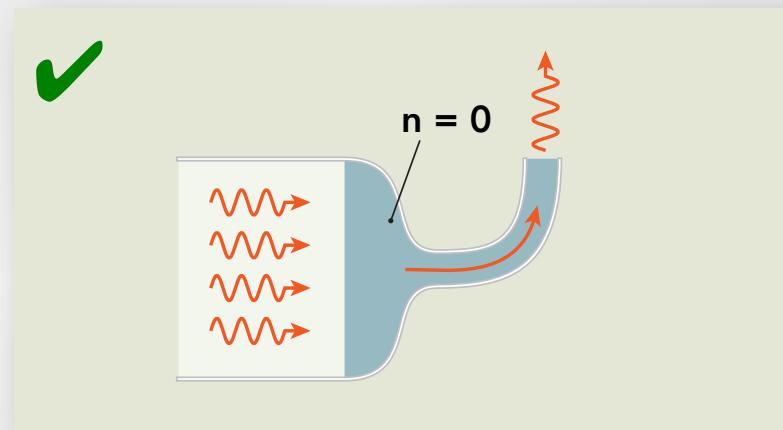
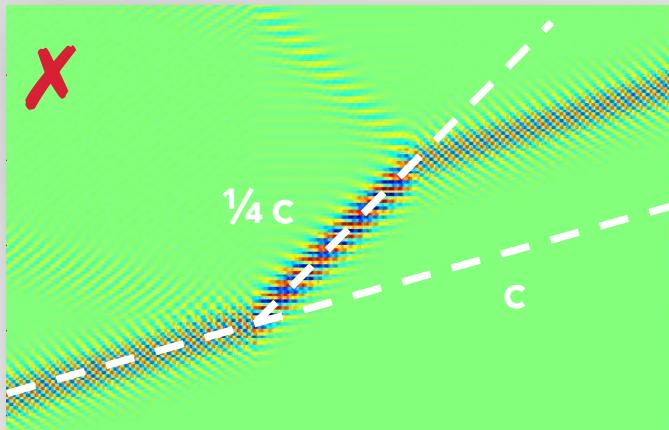
1

zero index

2

metamaterials

What happens to a signal pulse?



Zero index is really about infinite wavelength, not infinite speed

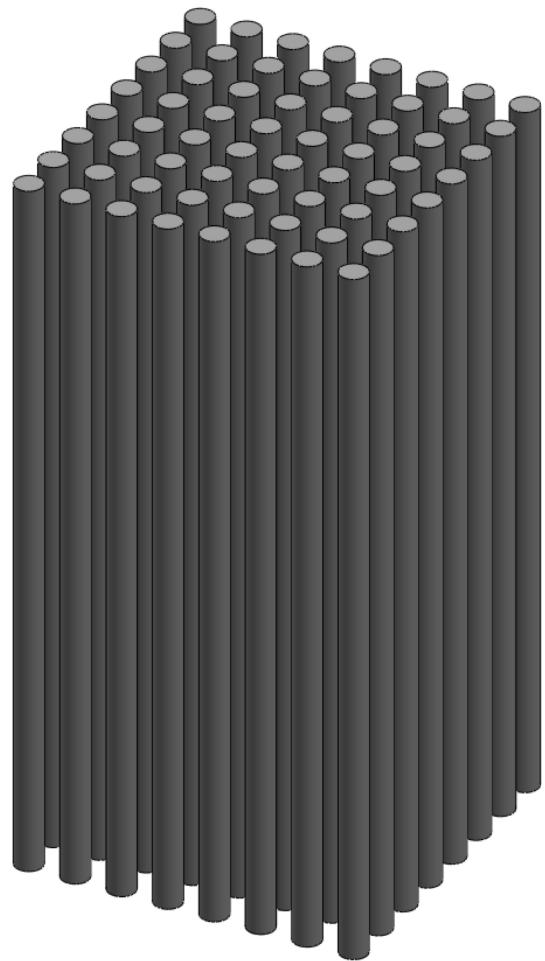
1

zero index

2

metamaterials

On-chip zero-index fabrication



1

zero index

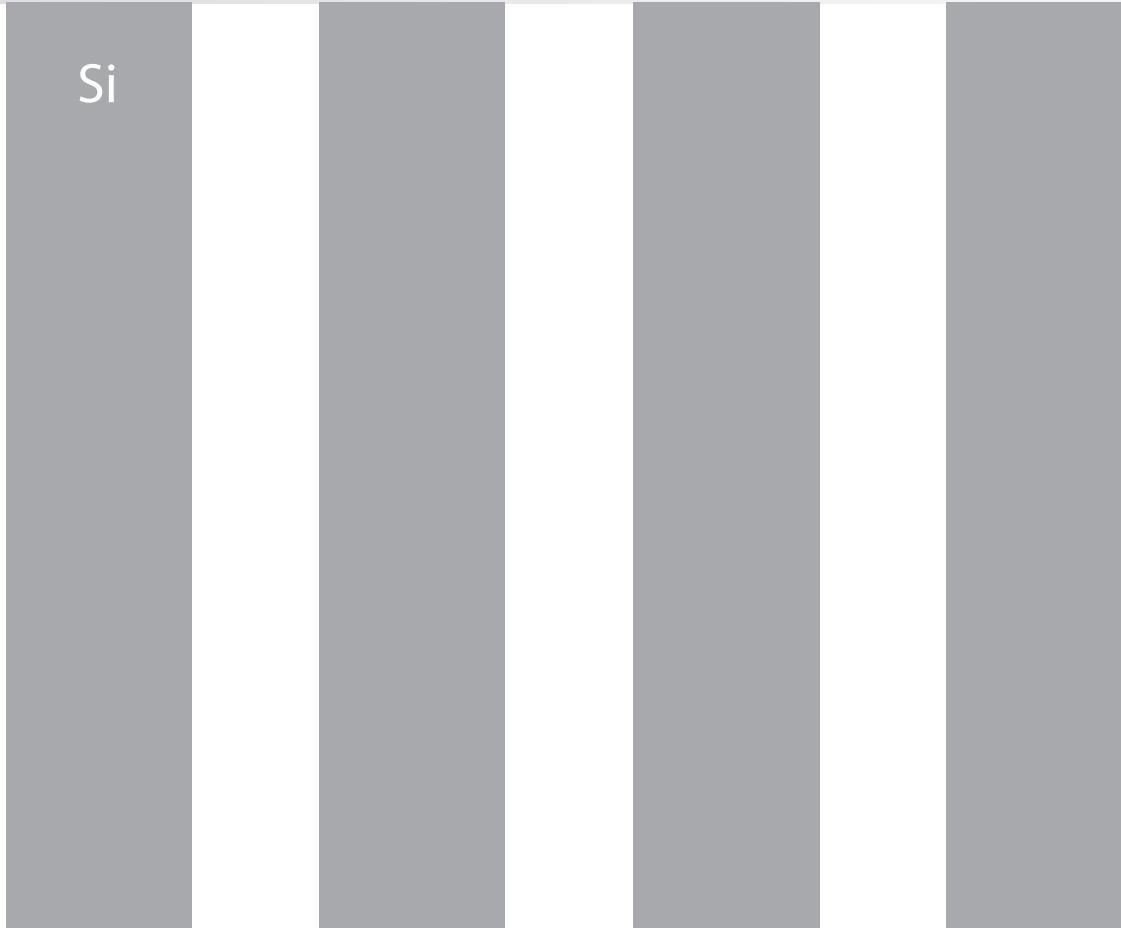
2

metamaterials

3

experiment

On-chip zero-index fabrication



1

zero index

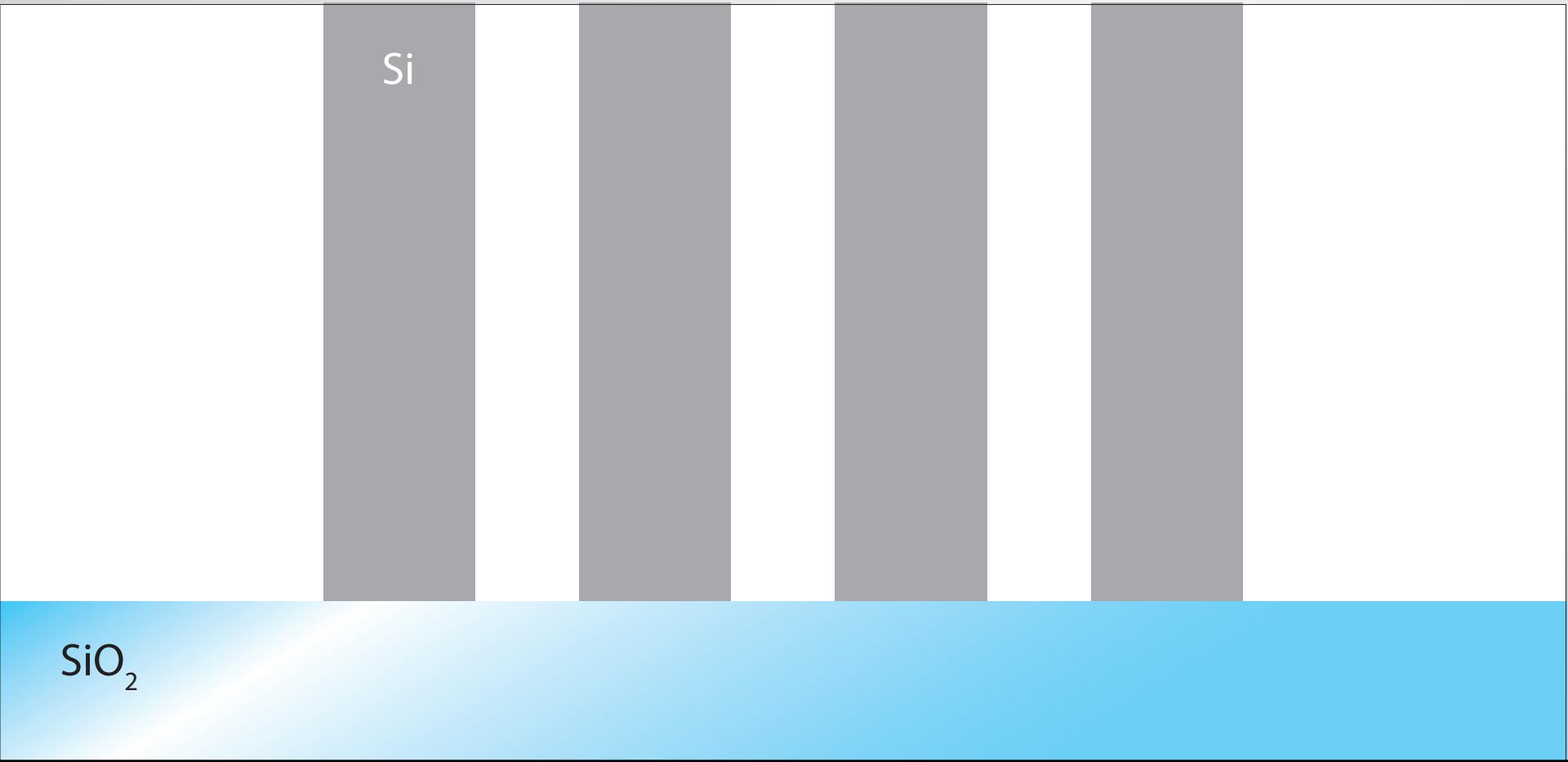
2

metamaterials

3

experiment

On-chip zero-index fabrication



1 zero index

2 metamaterials

3 experiment

On-chip zero-index fabrication

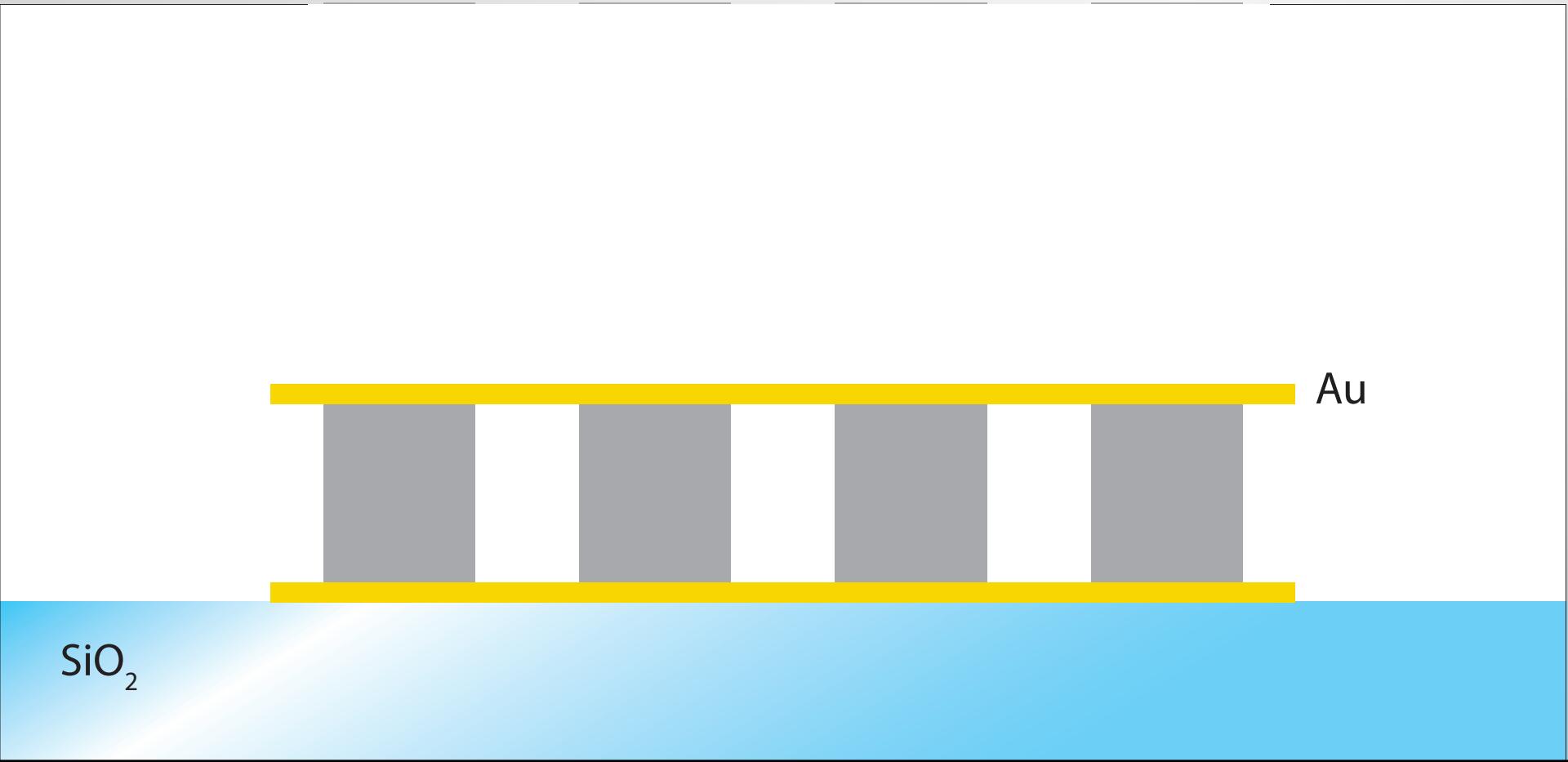


1 zero index

2 metamaterials

3 experiment

On-chip zero-index fabrication

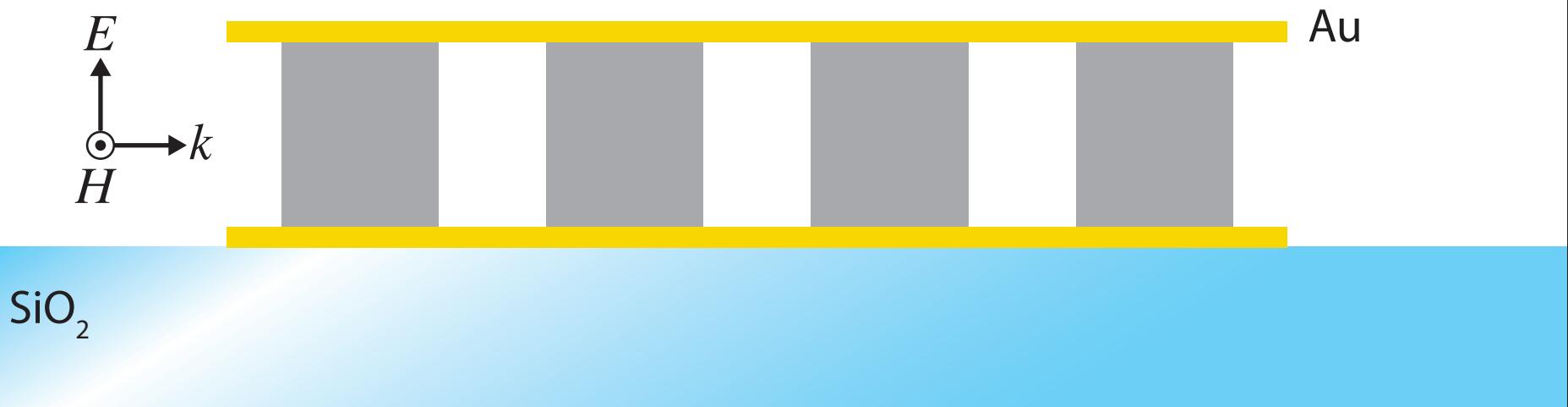


1 zero index

2 metamaterials

3 experiment

On-chip zero-index fabrication

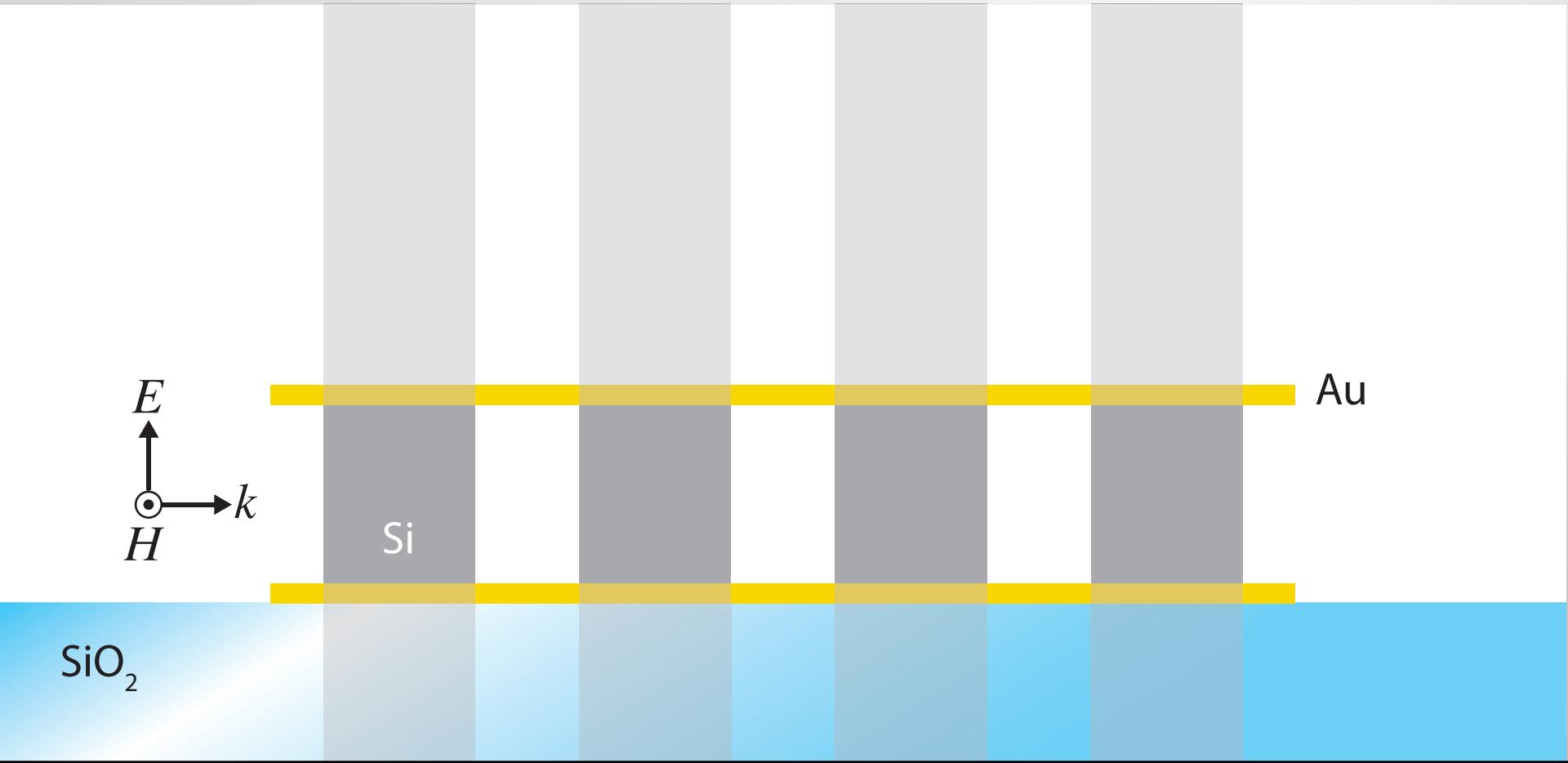


1 zero index

2 metamaterials

3 experiment

On-chip zero-index fabrication

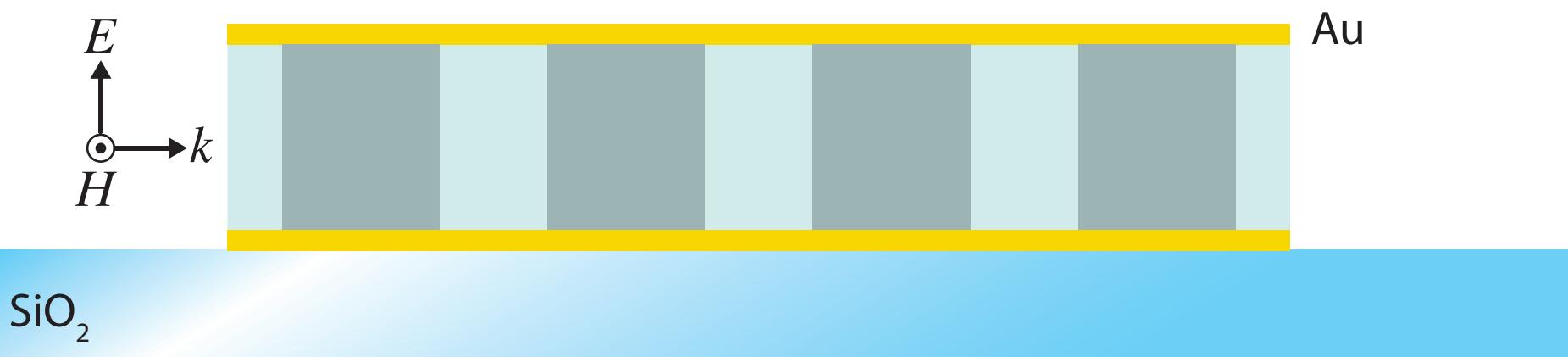


1 zero index

2 metamaterials

3 experiment

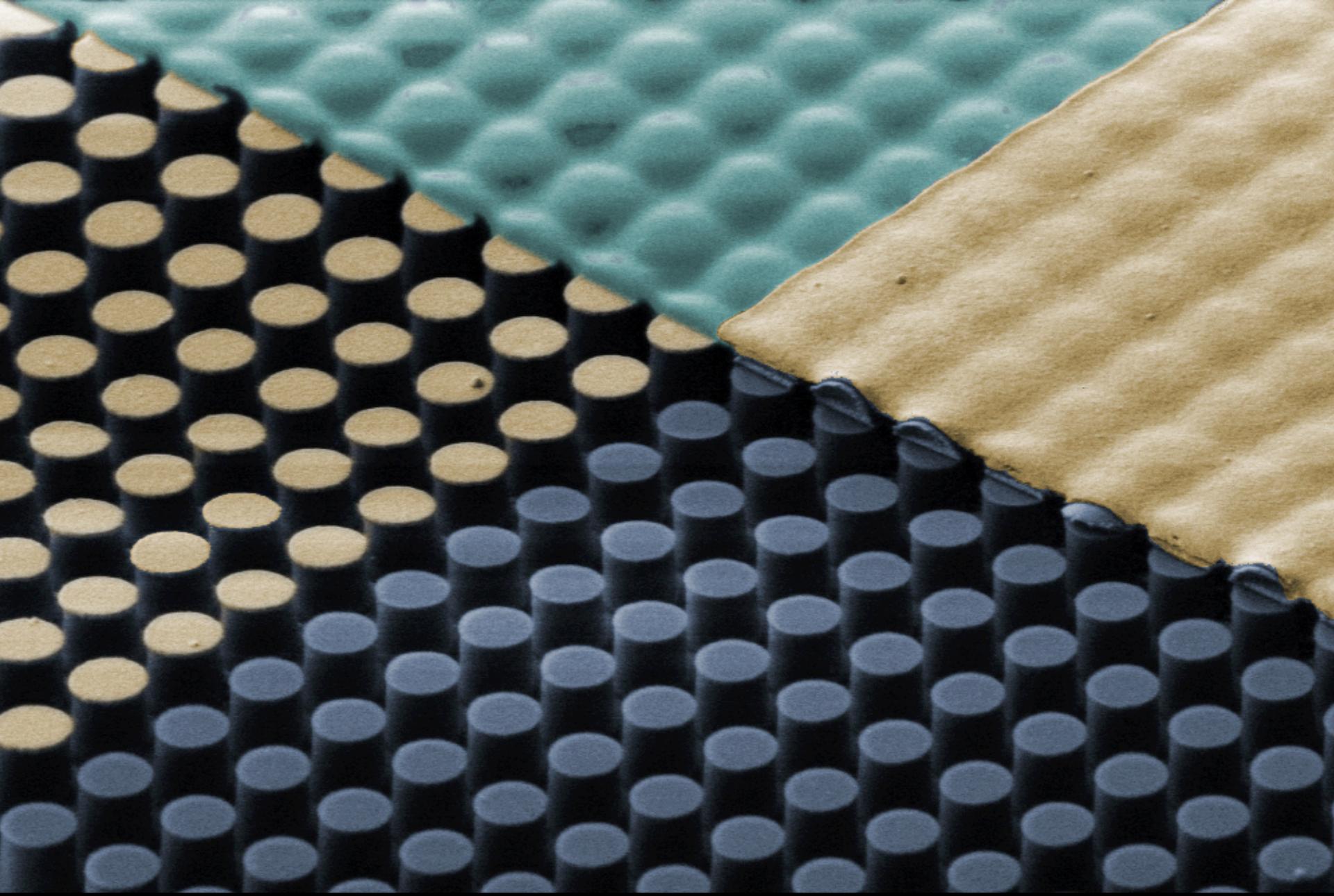
On-chip zero-index fabrication



1 zero index

2 metamaterials

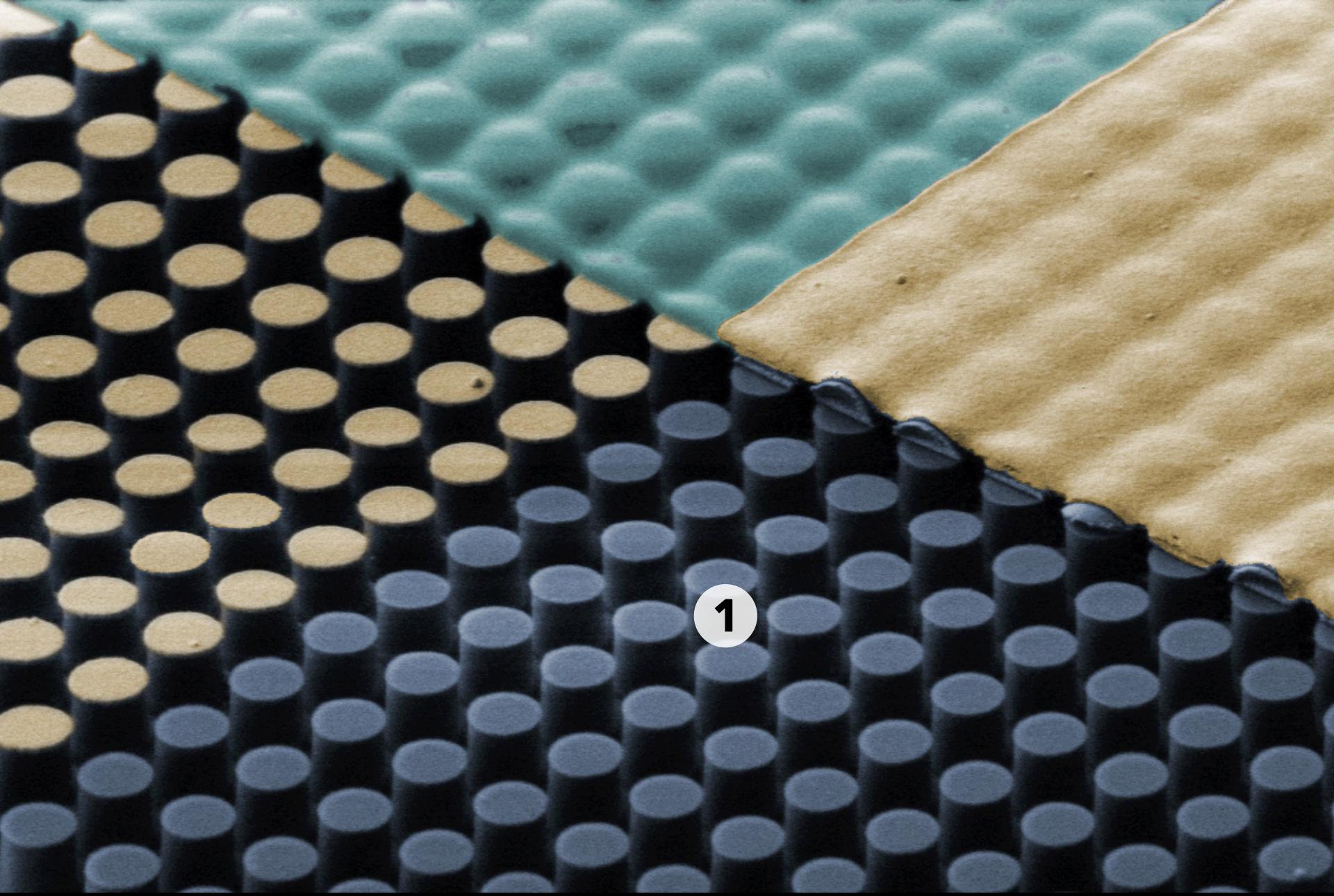
3 experiment



1 zero index

2 metamaterials

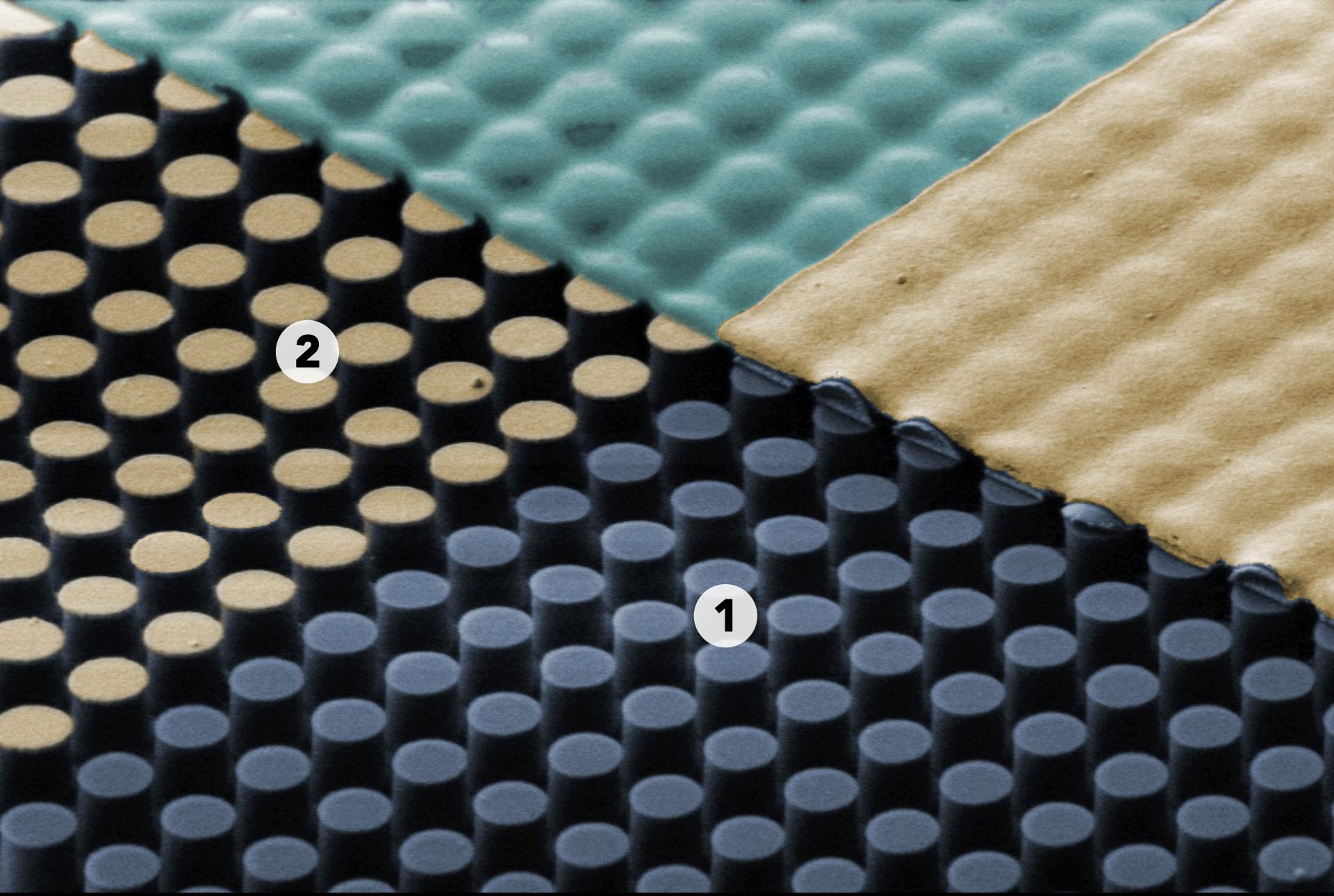
3 experiment



1 zero index

2 metamaterials

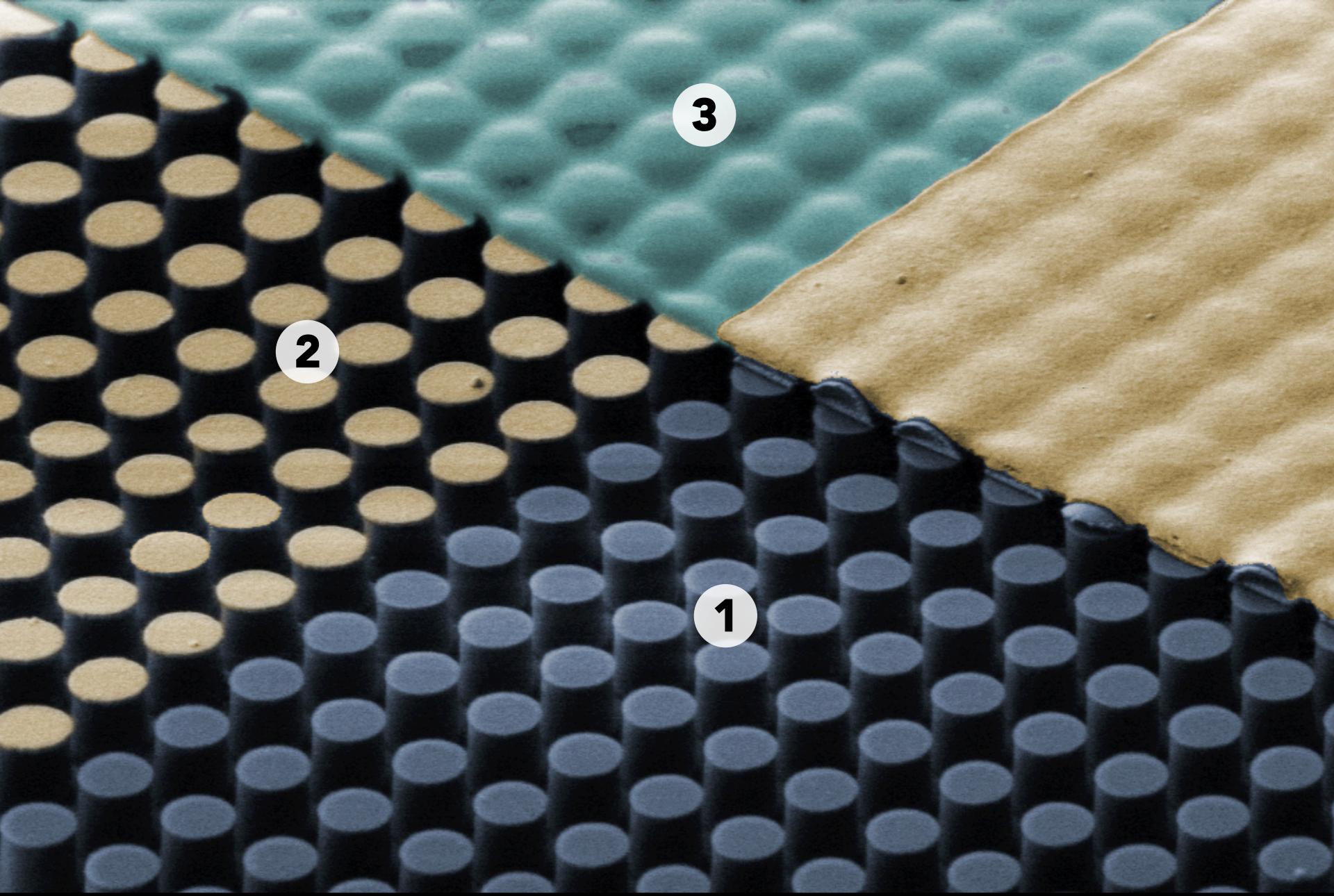
3 experiment



1 zero index

2 metamaterials

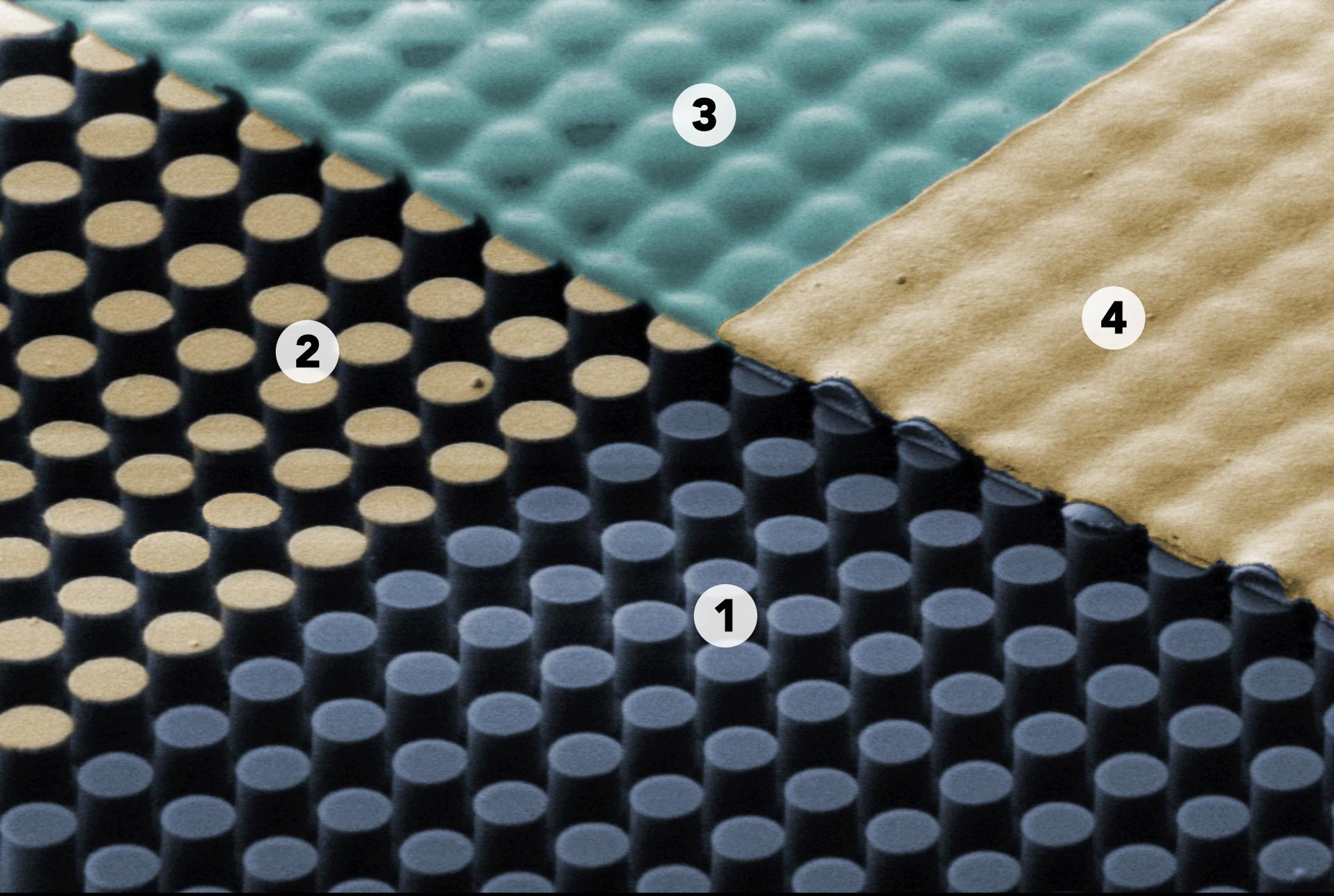
3 experiment



1 zero index

2 metamaterials

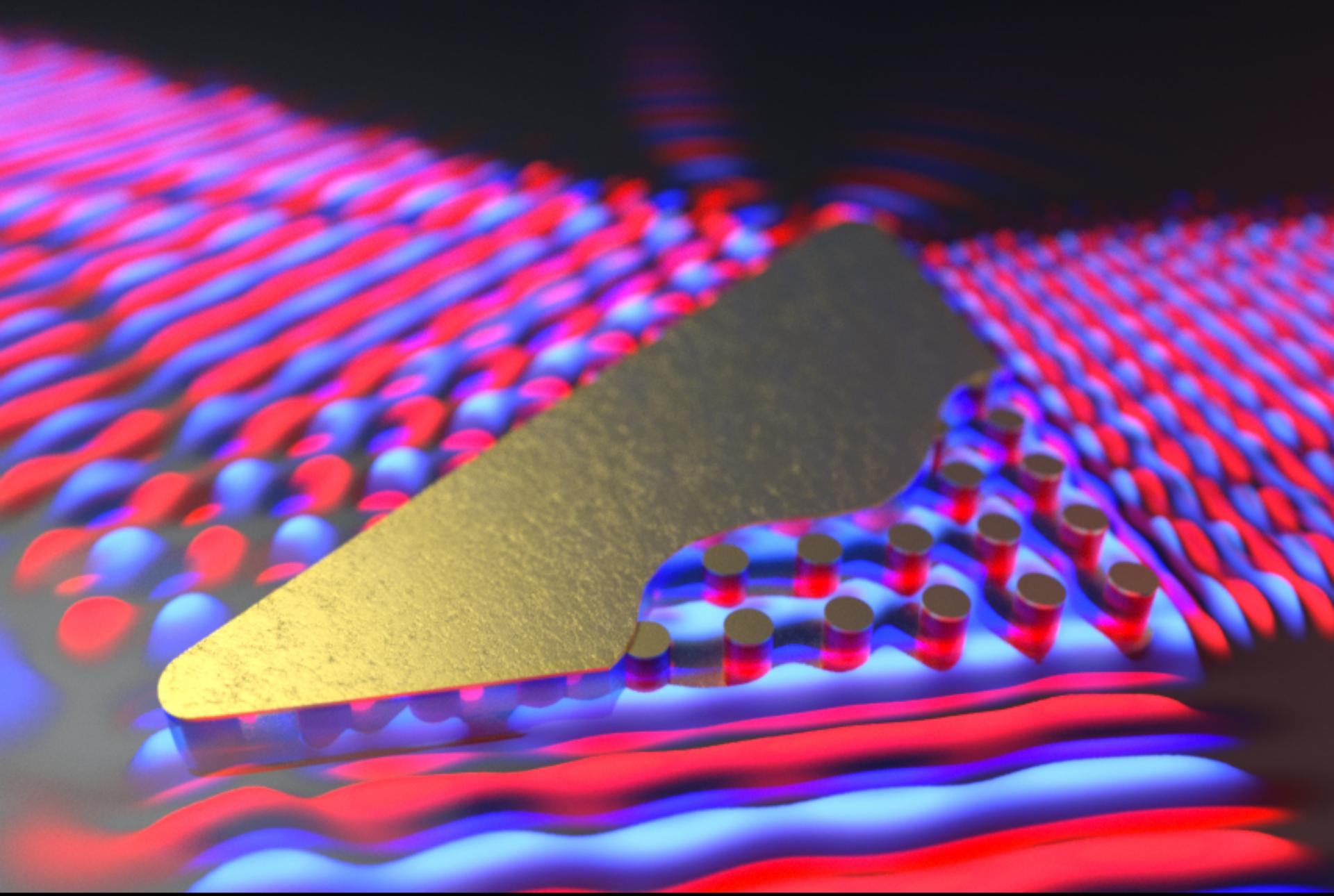
3 experiment



1 zero index

2 metamaterials

3 experiment

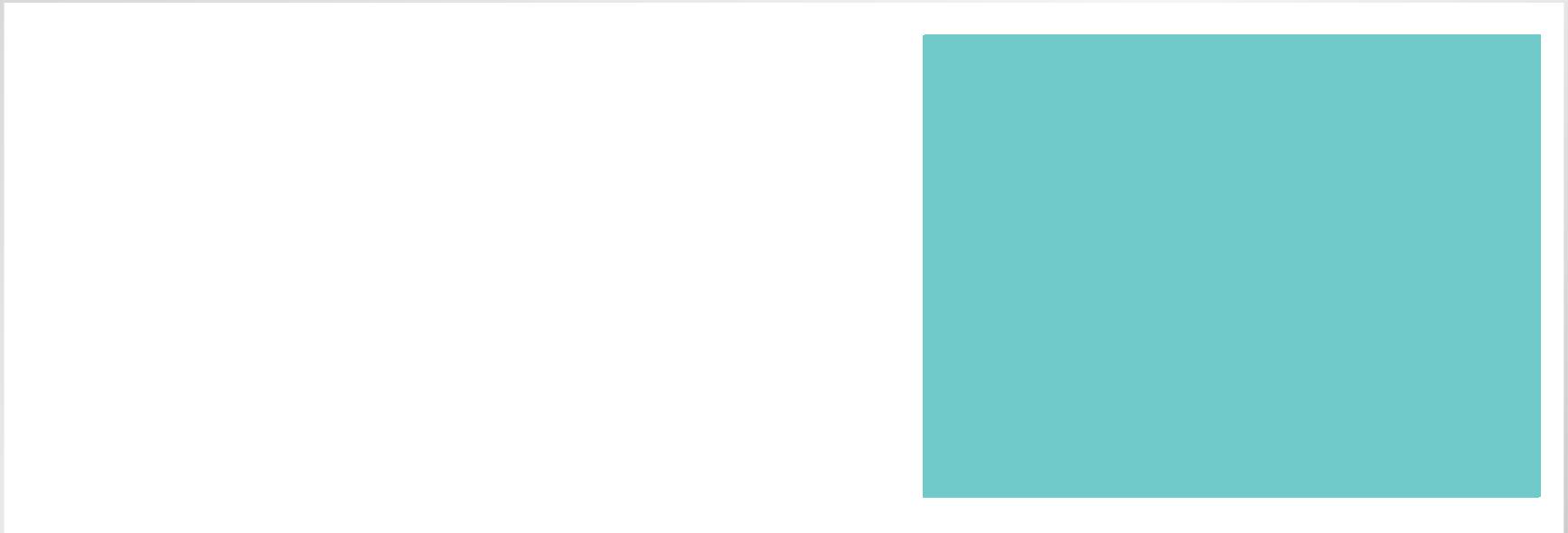


1 zero index

2 metamaterials

3 experiment

On-chip zero-index prism



1

zero index

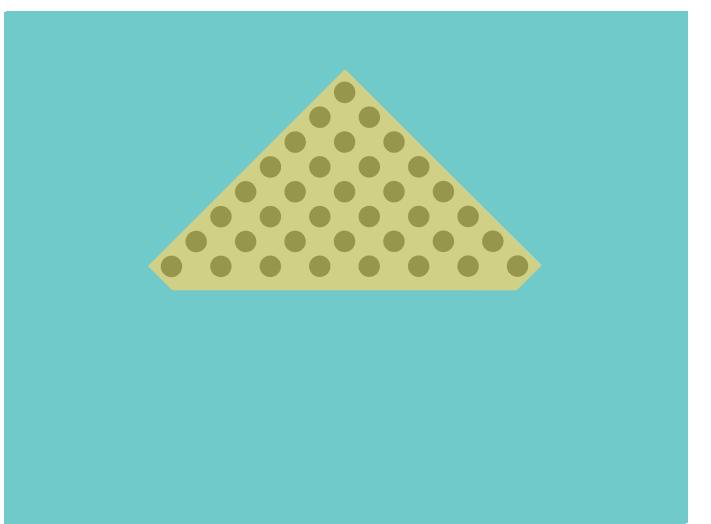
2

metamaterials

3

experiment

On-chip zero-index prism

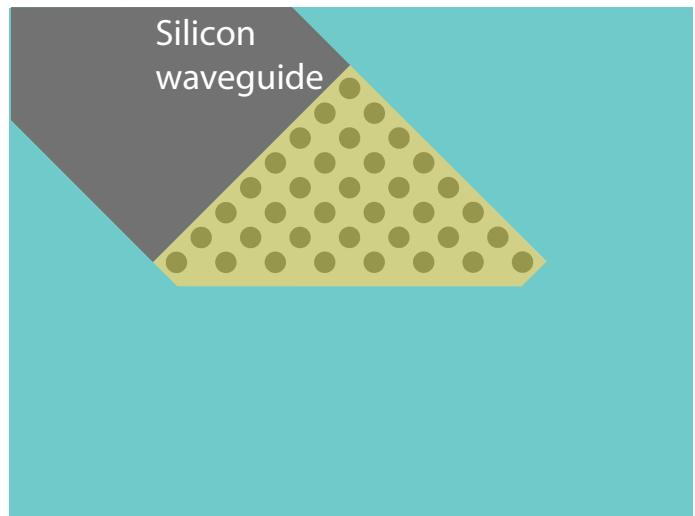


1 zero index

2 metamaterials

3 experiment

On-chip zero-index prism



1

zero index

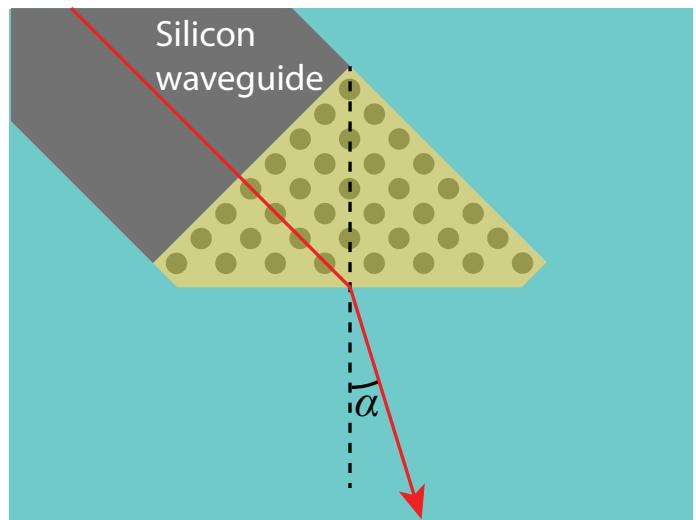
2

metamaterials

3

experiment

On-chip zero-index prism



1

zero index

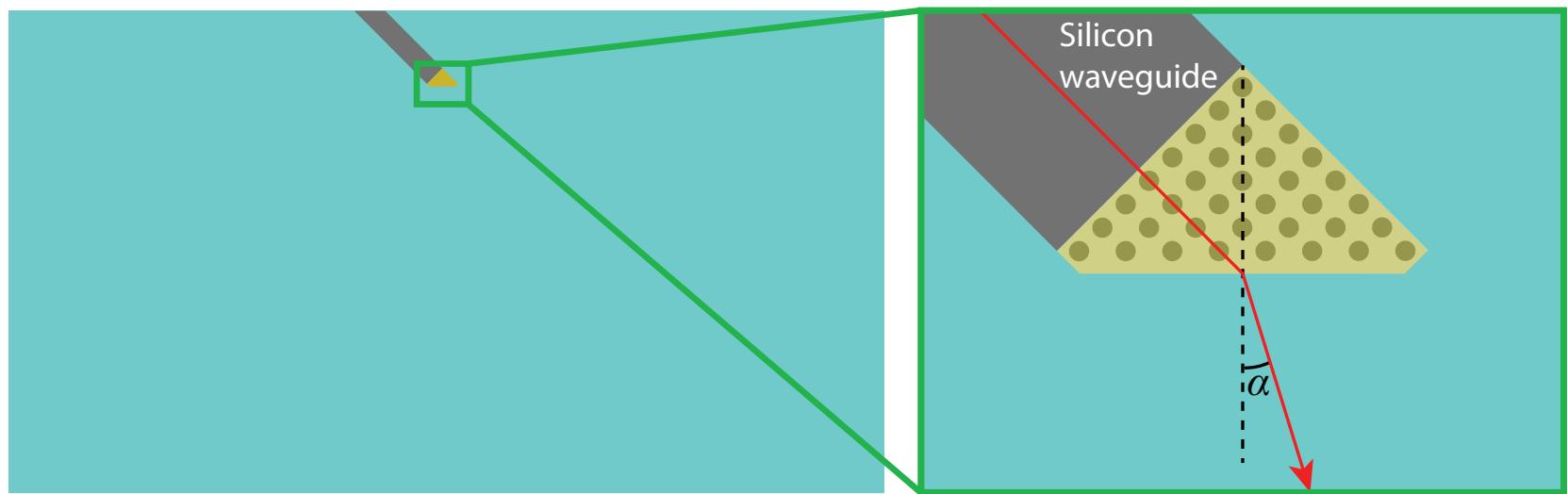
2

metamaterials

3

experiment

On-chip zero-index prism



1

zero index

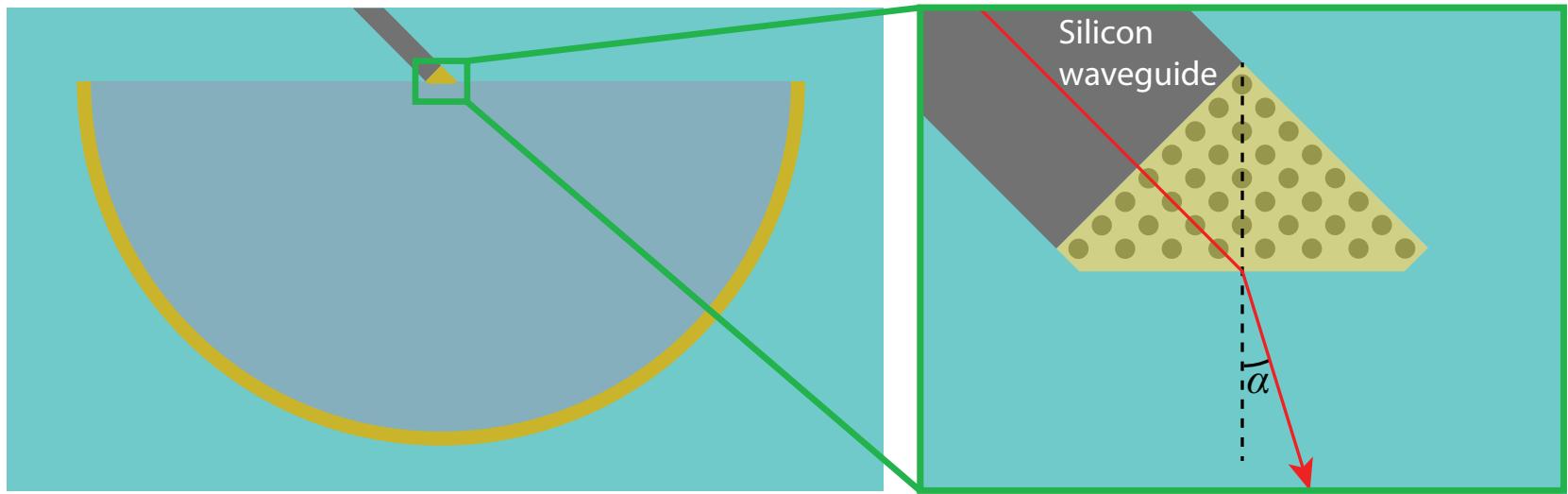
2

metamaterials

3

experiment

On-chip zero-index prism



1

zero index

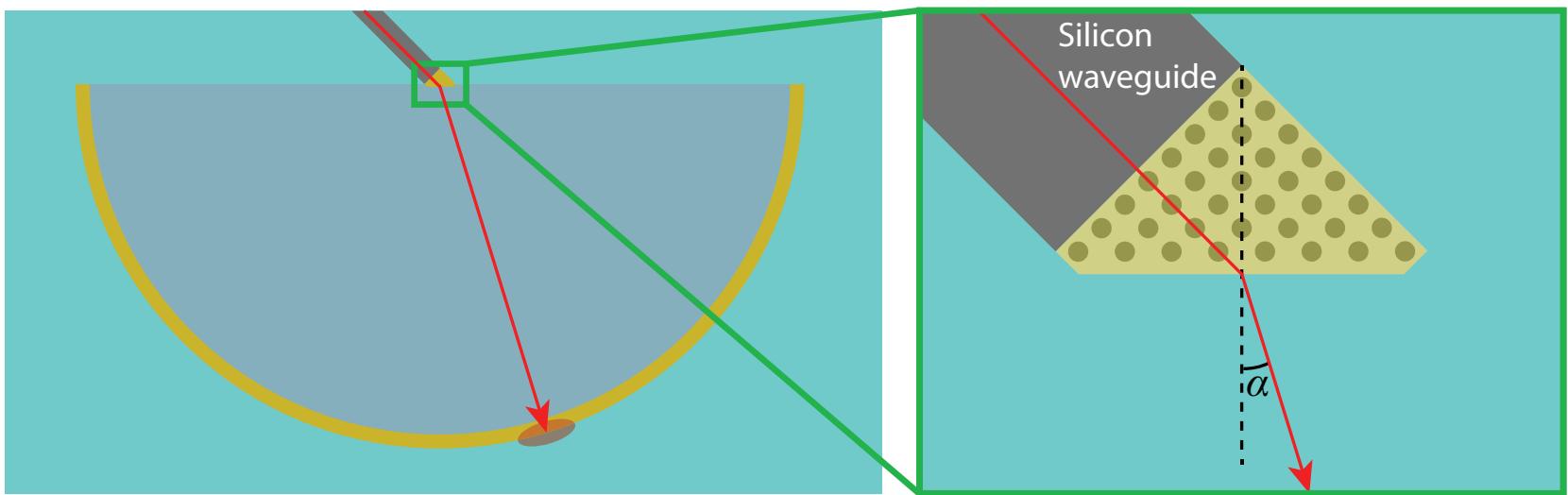
2

metamaterials

3

experiment

On-chip zero-index prism



1

zero index

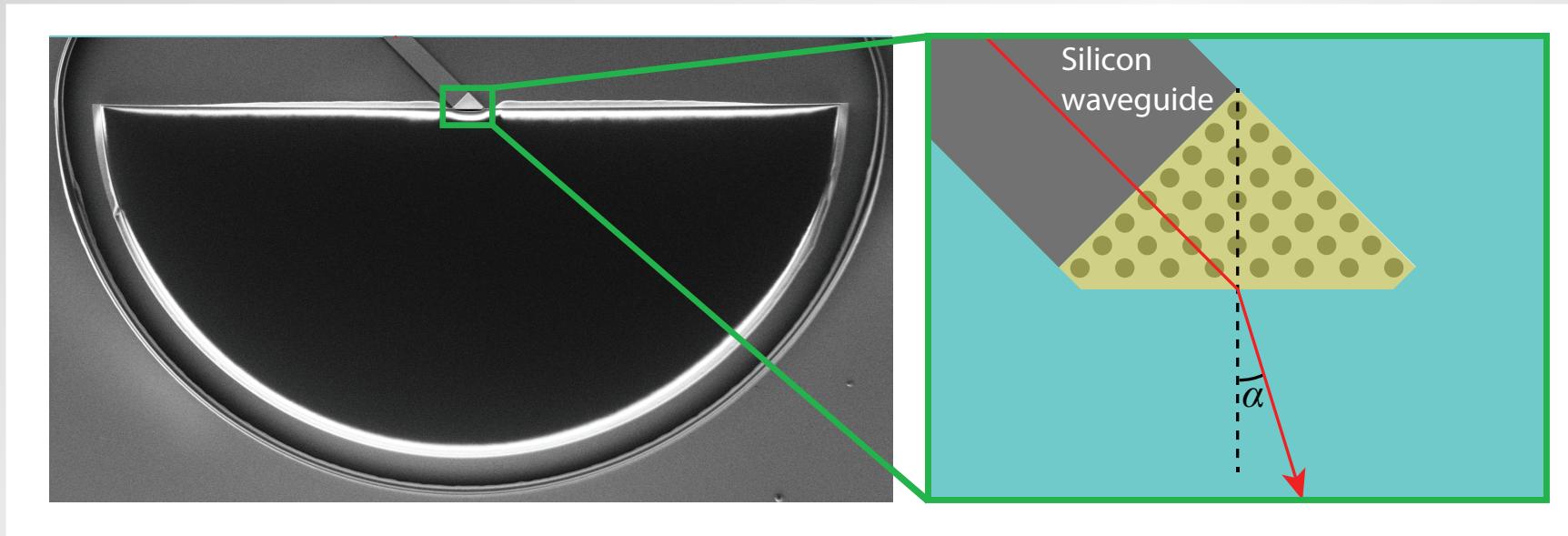
2

metamaterials

3

experiment

On-chip zero-index prism



1

zero index

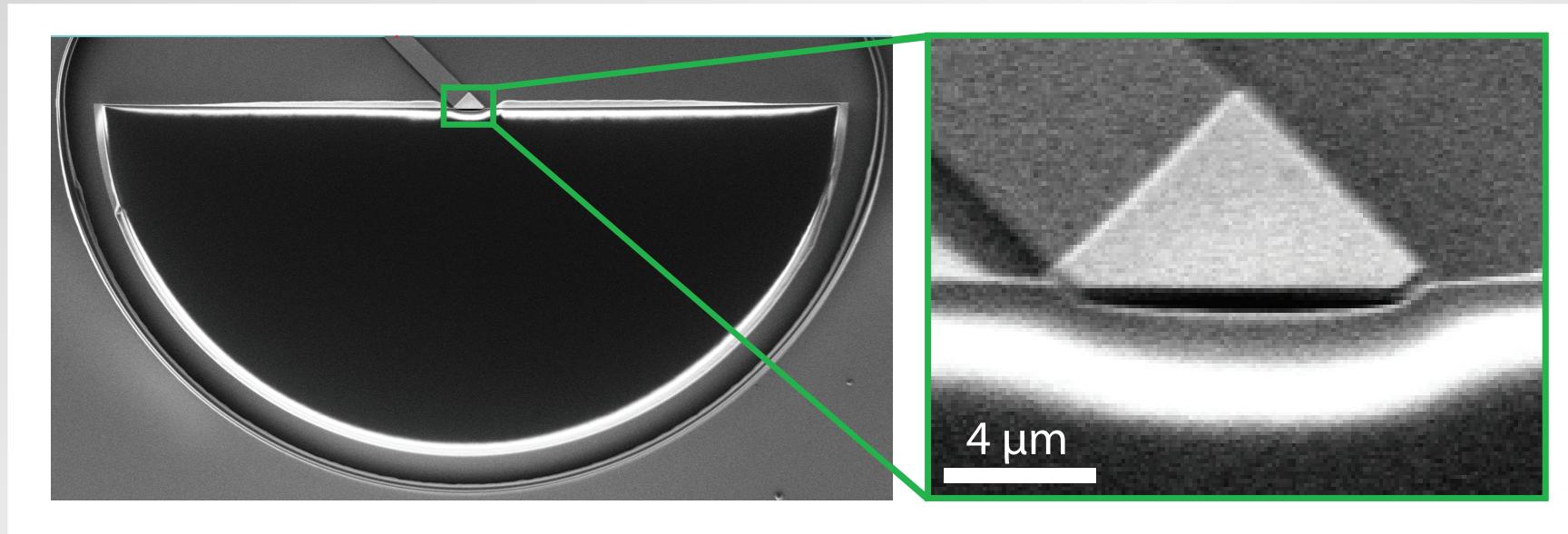
2

metamaterials

3

experiment

On-chip zero-index prism

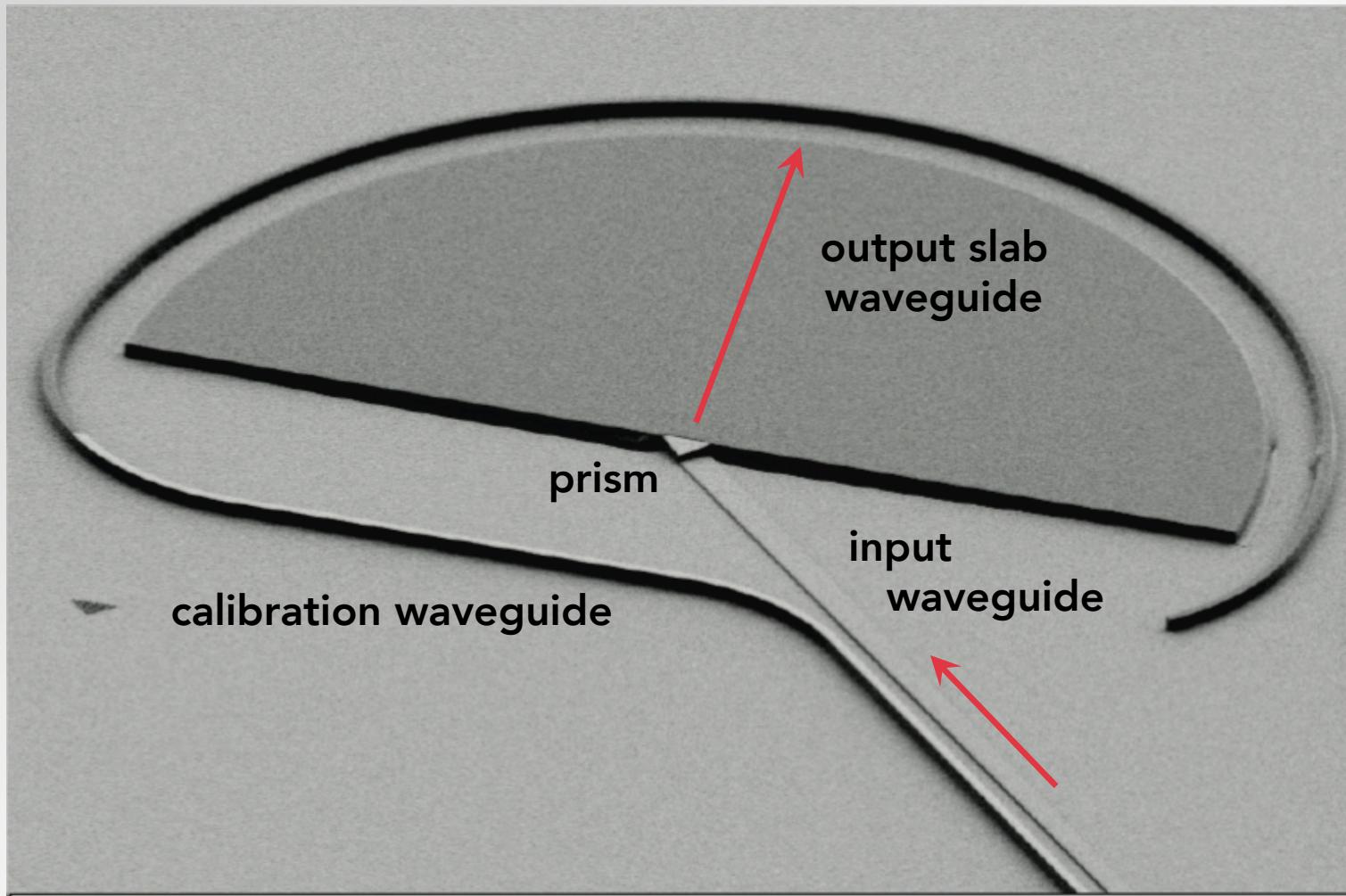


1 zero index

2 metamaterials

3 experiment

On-chip zero-index prism



1

zero index

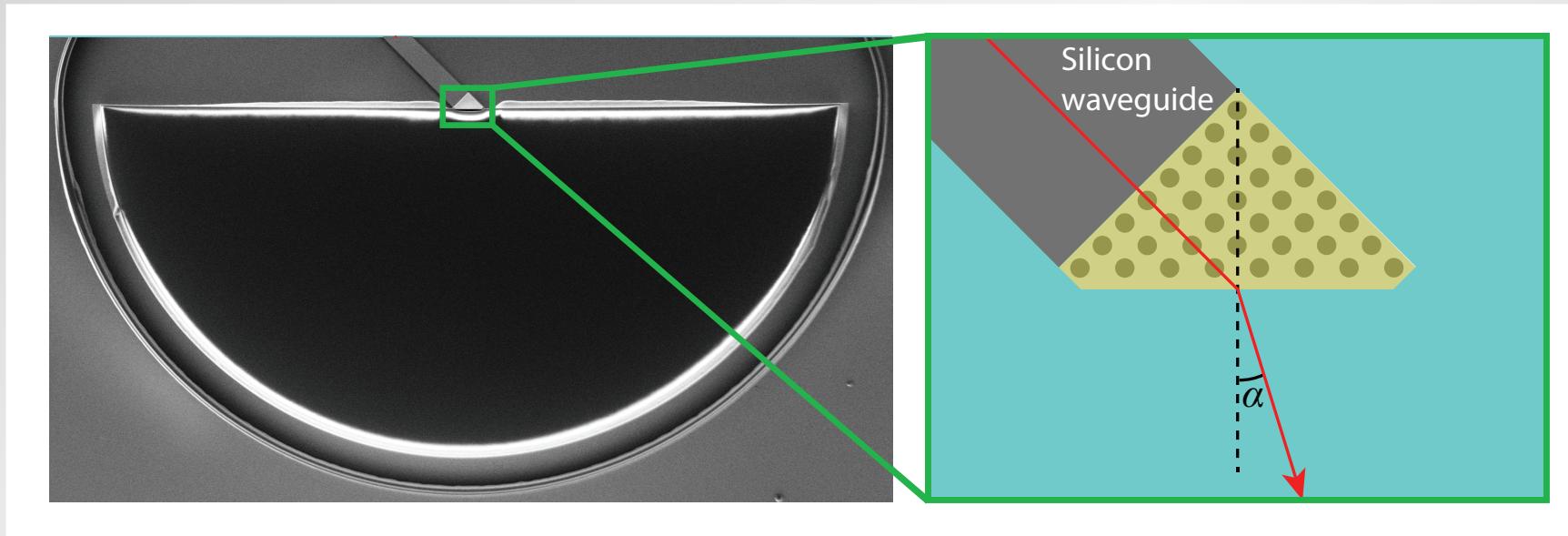
2

metamaterials

3

experiment

On-chip zero-index prism



1

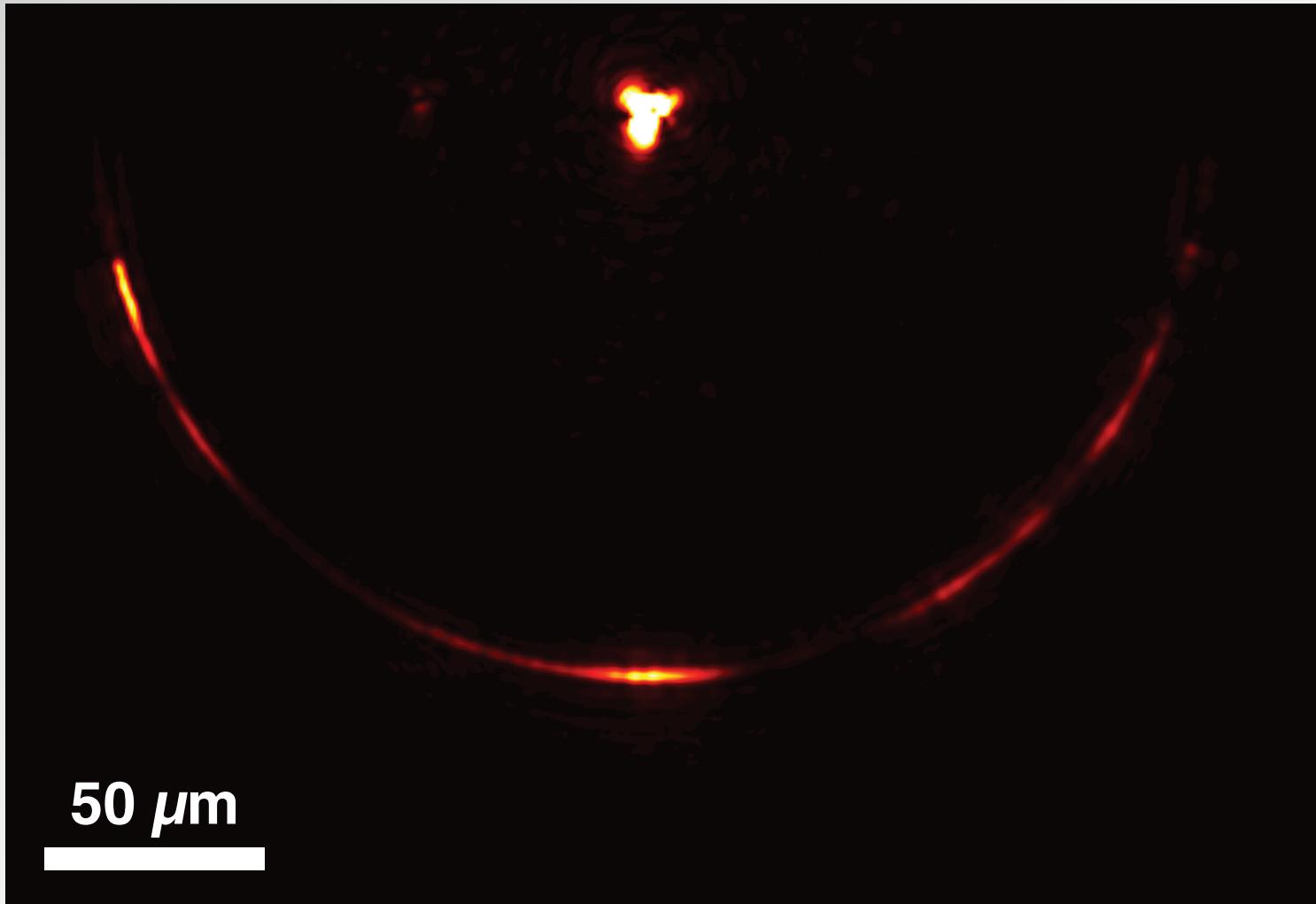
zero index

2

metamaterials

3

experiment



1

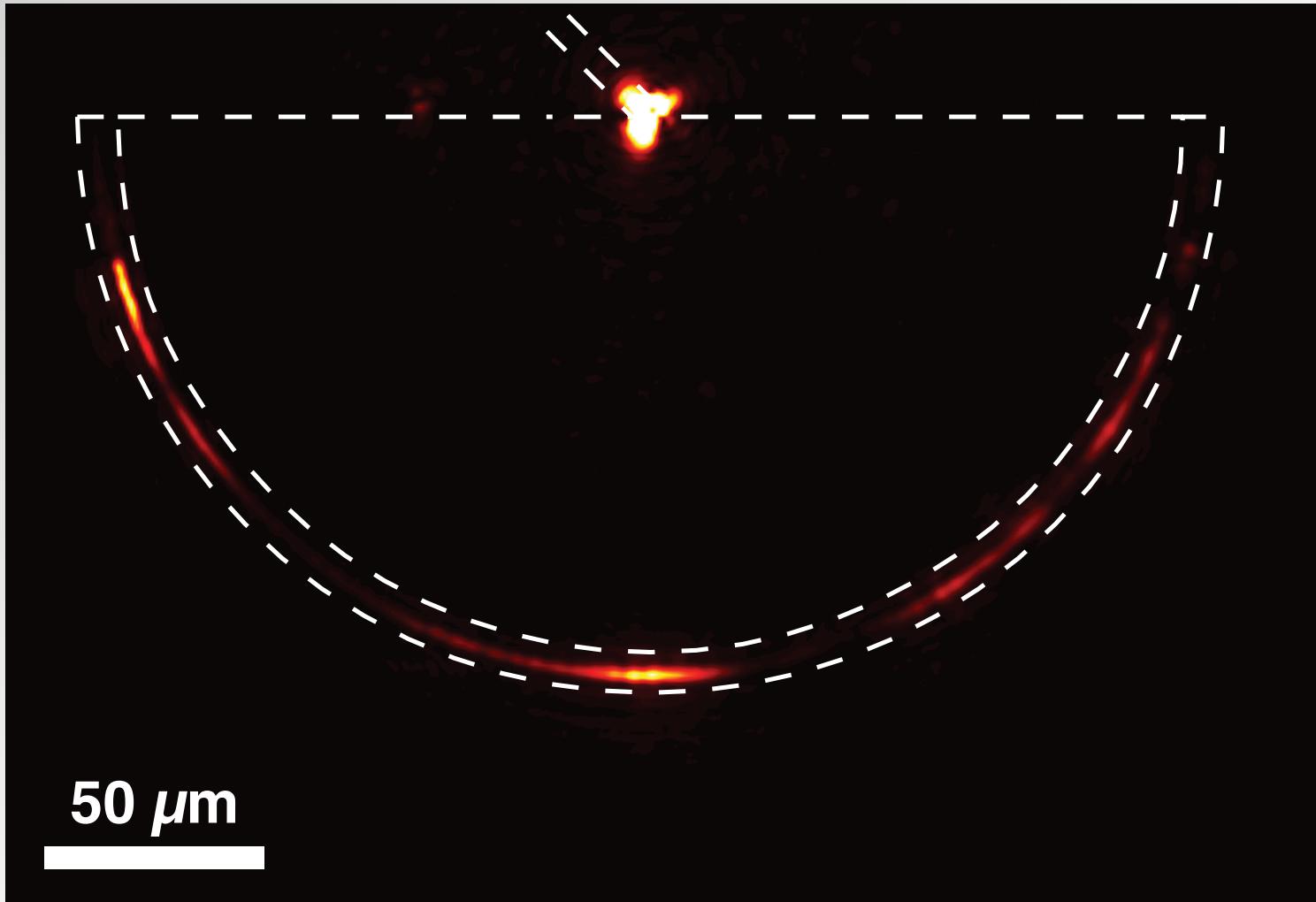
zero index

2

metamaterials

3

experiment



1

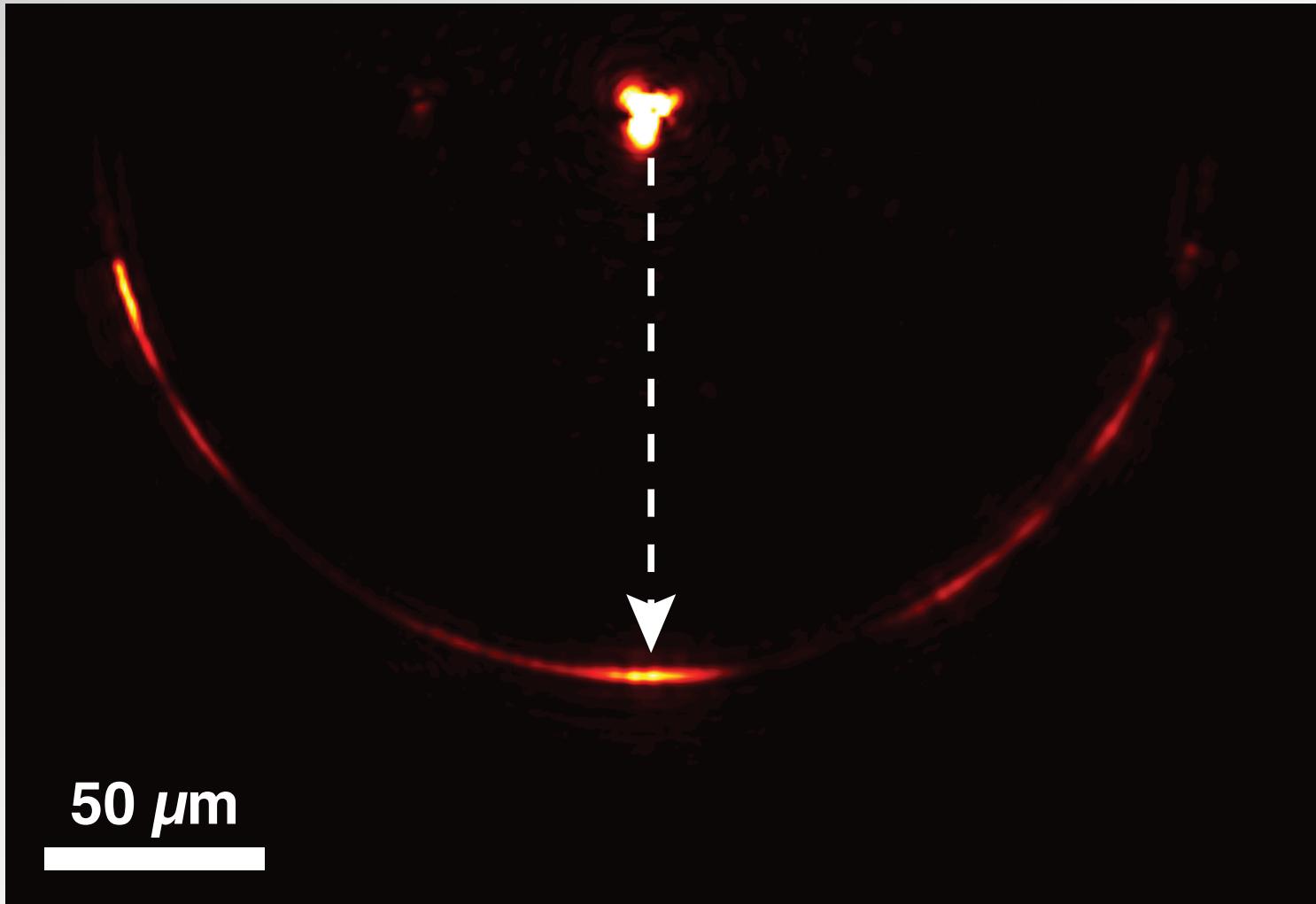
zero index

2

metamaterials

3

experiment



1

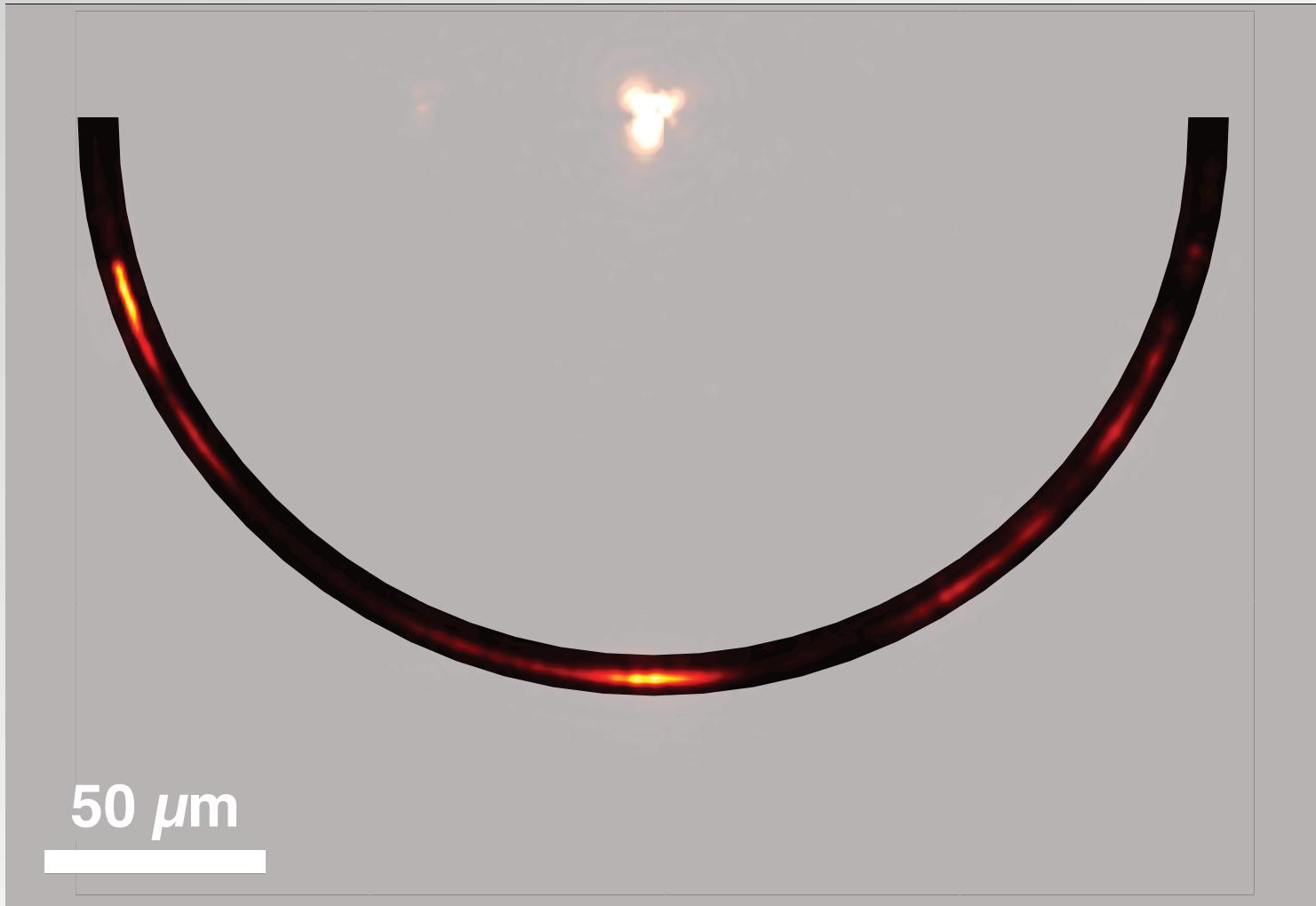
zero index

2

metamaterials

3

experiment

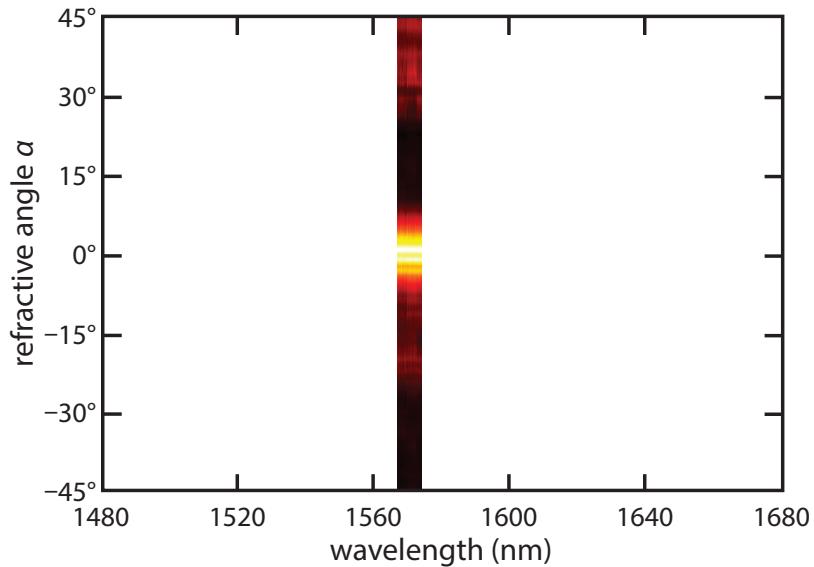


1 zero index

2 metamaterials

3 experiment

Wavelength dependence of refraction angle



1

zero index

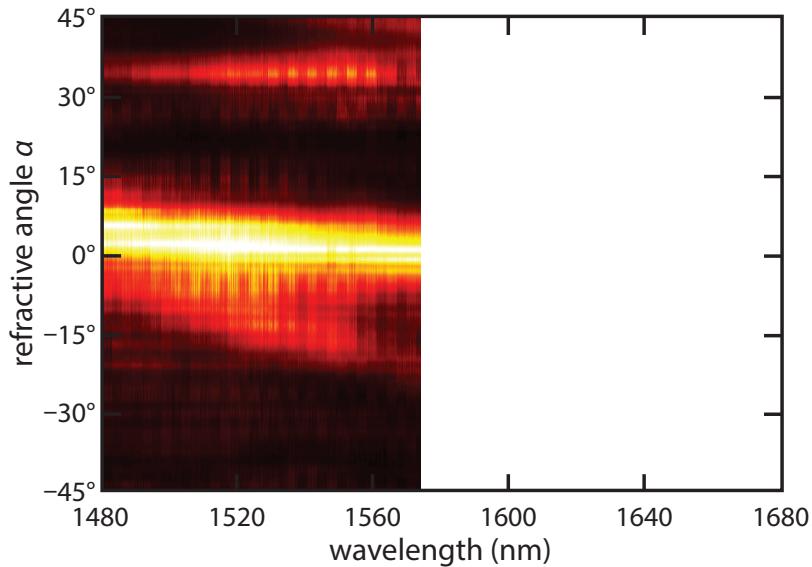
2

metamaterials

3

experiment

Wavelength dependence of refraction angle



1

zero index

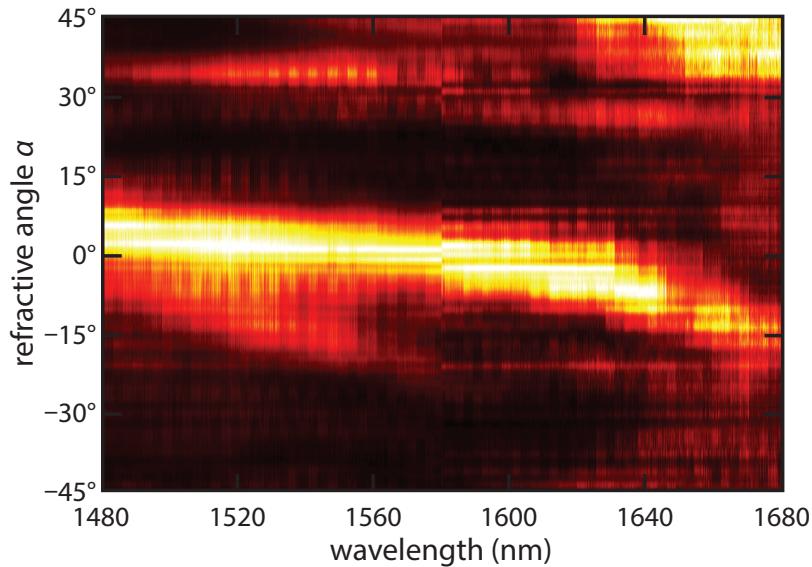
2

metamaterials

3

experiment

Wavelength dependence of refraction angle



1

zero index

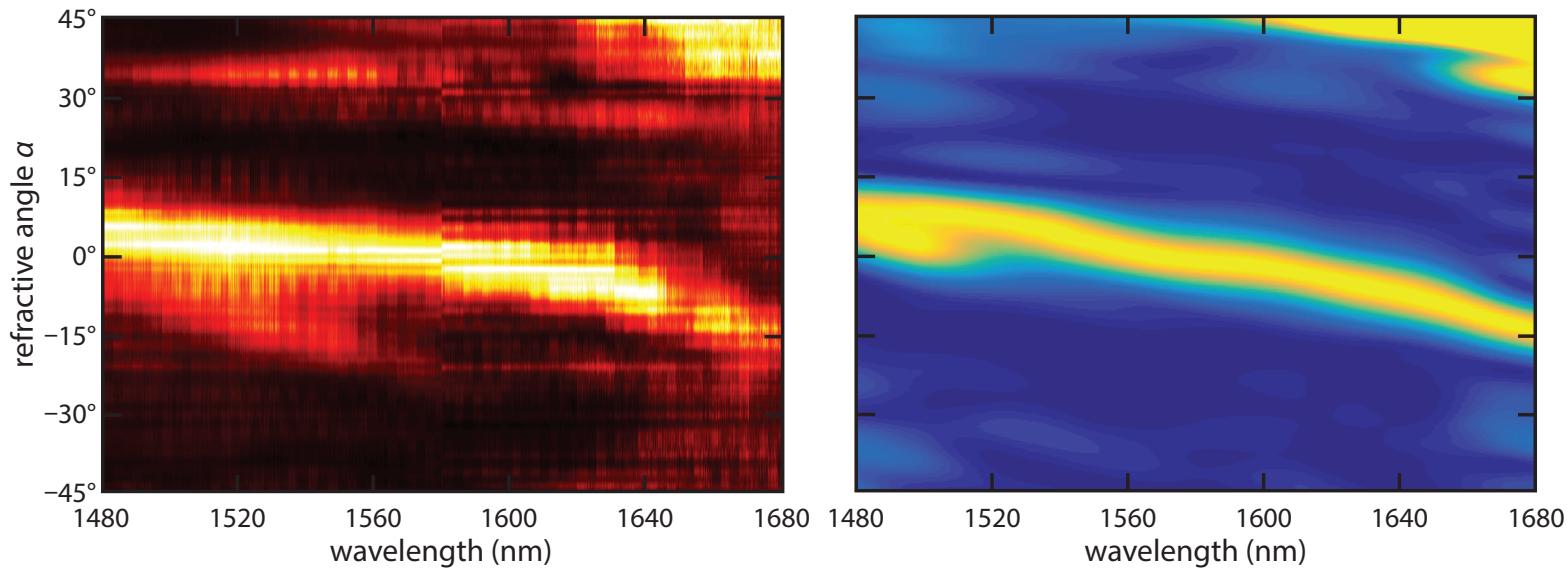
2

metamaterials

3

experiment

Wavelength dependence of refraction angle



1

zero index

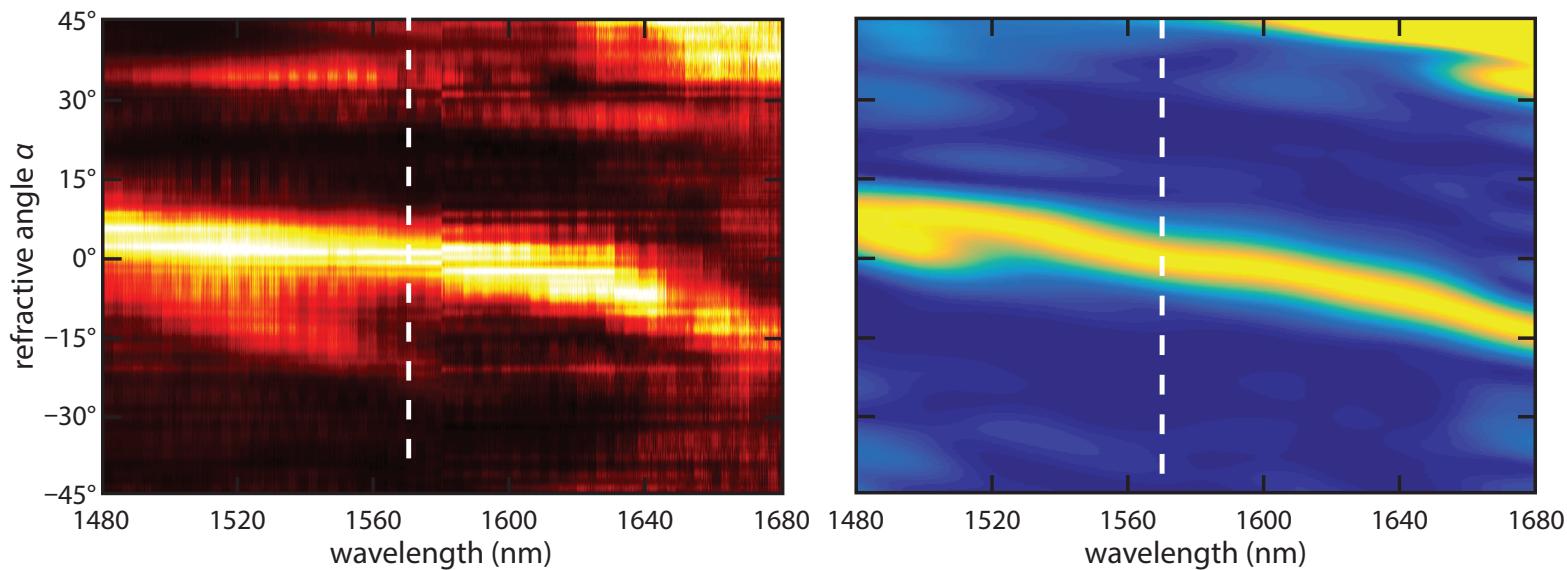
2

metamaterials

3

experiment

Wavelength dependence of refraction angle



1

zero index

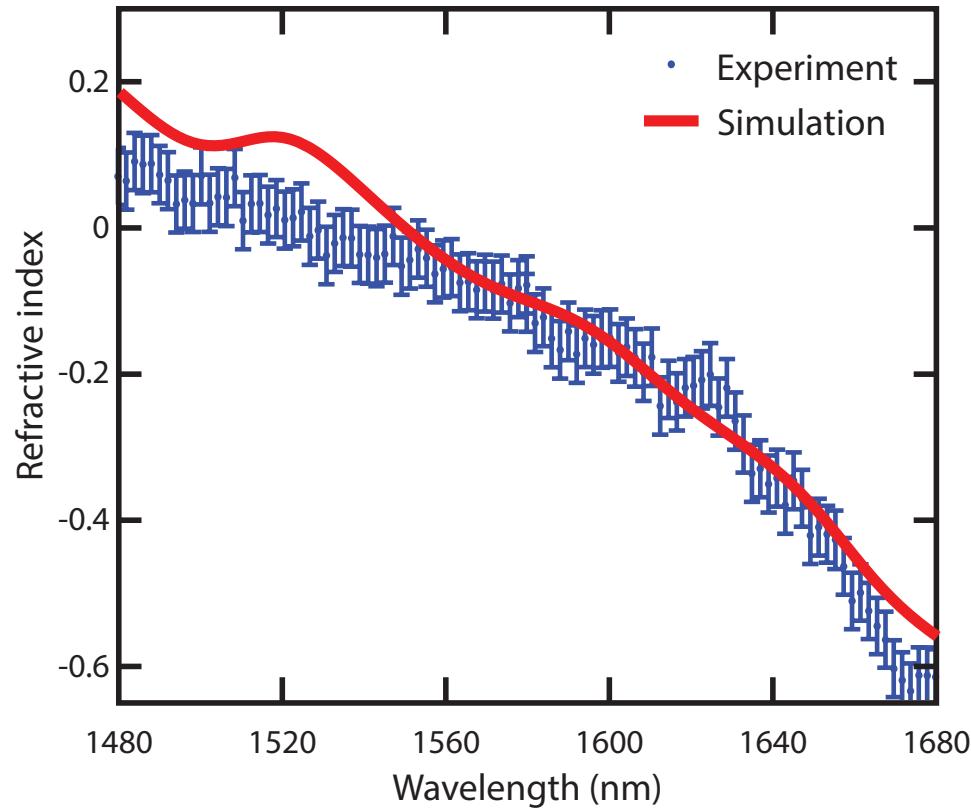
2

metamaterials

3

experiment

Wavelength dependence of refractive index



1

zero index

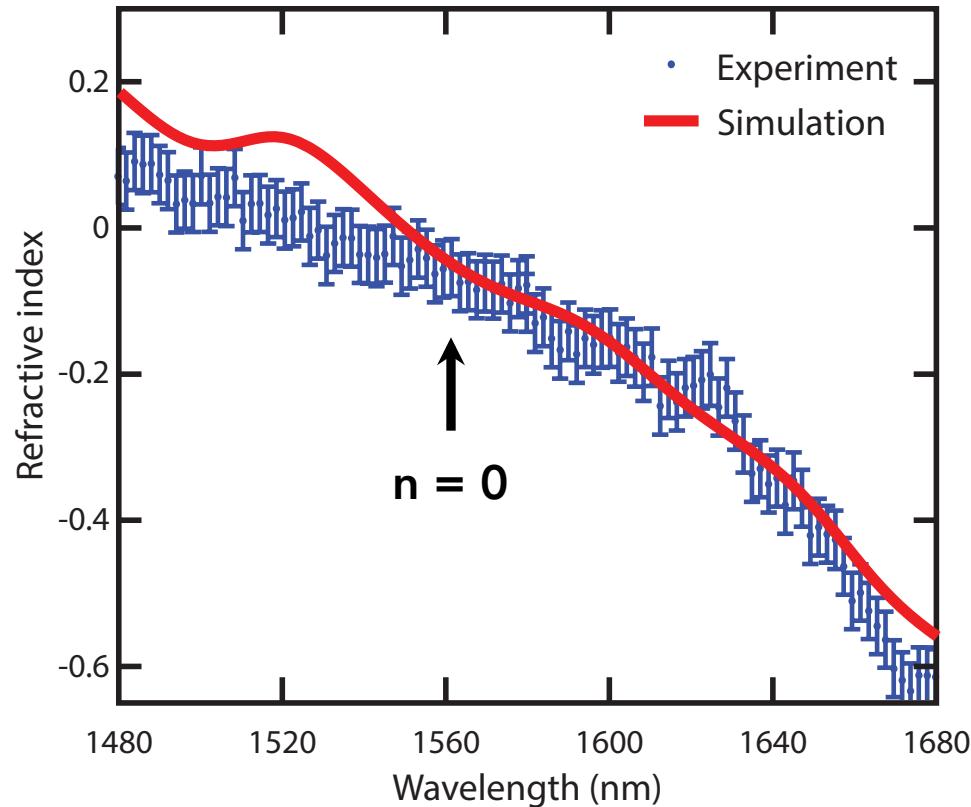
2

metamaterials

3

experiment

Wavelength dependence of refractive index



1

zero index

2

metamaterials

3

experiment

Applications of zero-index metamaterials



1

zero index

2

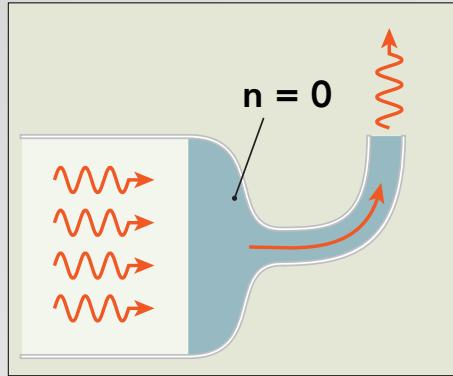
metamaterials

3

experiment

Applications of zero-index metamaterials

zero-index supercoupling



1

zero index

2

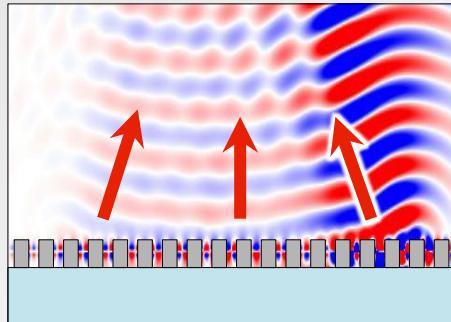
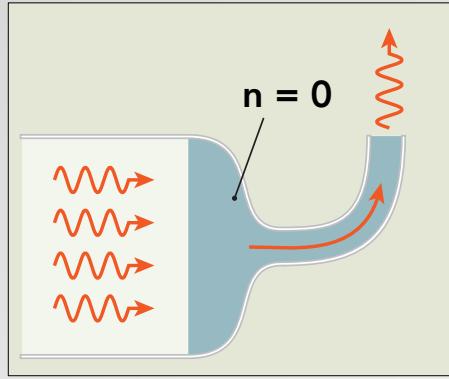
metamaterials

3

experiment

Applications of zero-index metamaterials

zero-index supercoupling



holography and beam steering

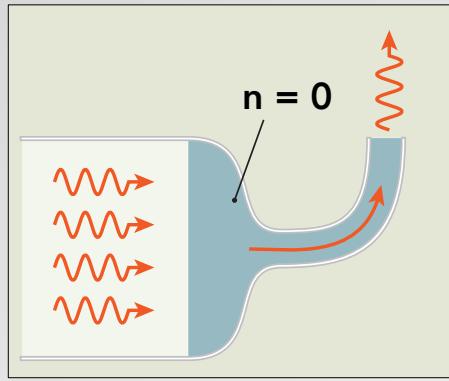
1 zero index

2 metamaterials

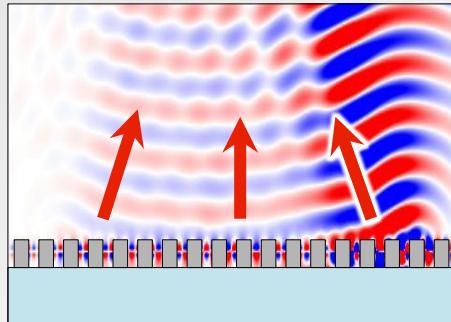
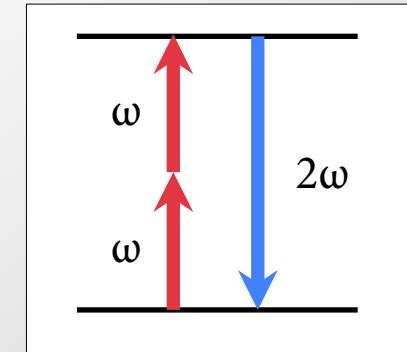
3 experiment

Applications of zero-index metamaterials

zero-index supercoupling



Phase-free
nonlinear optics



holography and beam steering

1

zero index

2

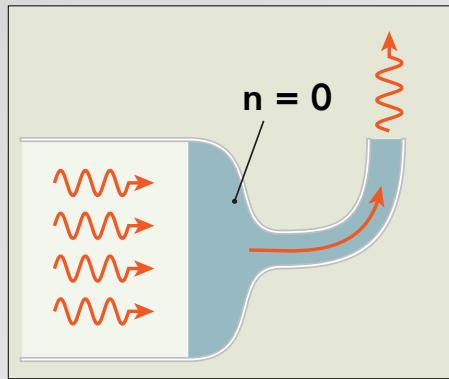
metamaterials

3

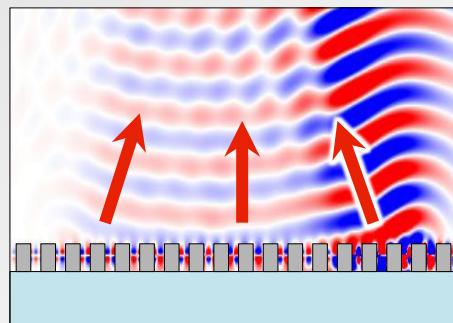
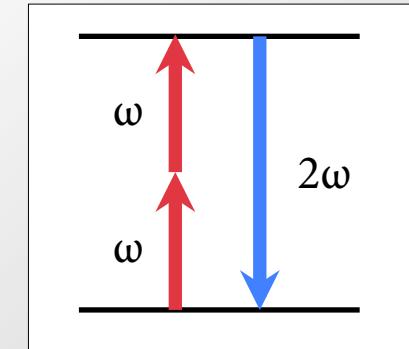
experiment

Applications of zero-index metamaterials

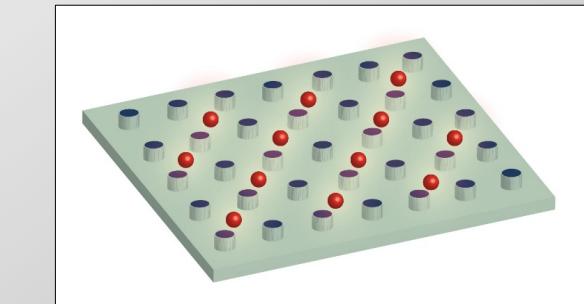
zero-index supercoupling



Phase-free
nonlinear optics



holography and beam steering



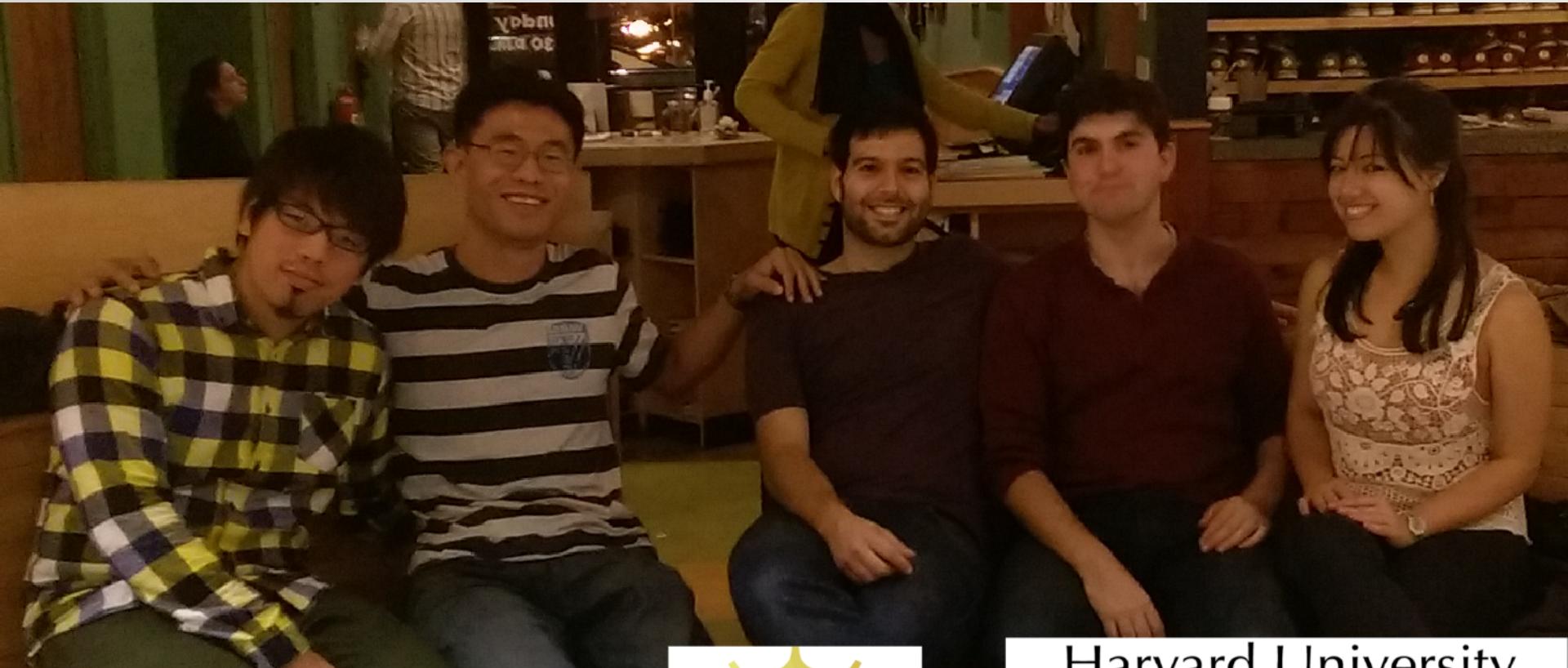
integrated quantum optics

1 zero index

2 metamaterials

3 experiment

Thanks to the zero index team!



Laboratory for Nanoscale
Optics
Prof. Marko Loncar

HARVARD School of Engineering and Applied Sciences

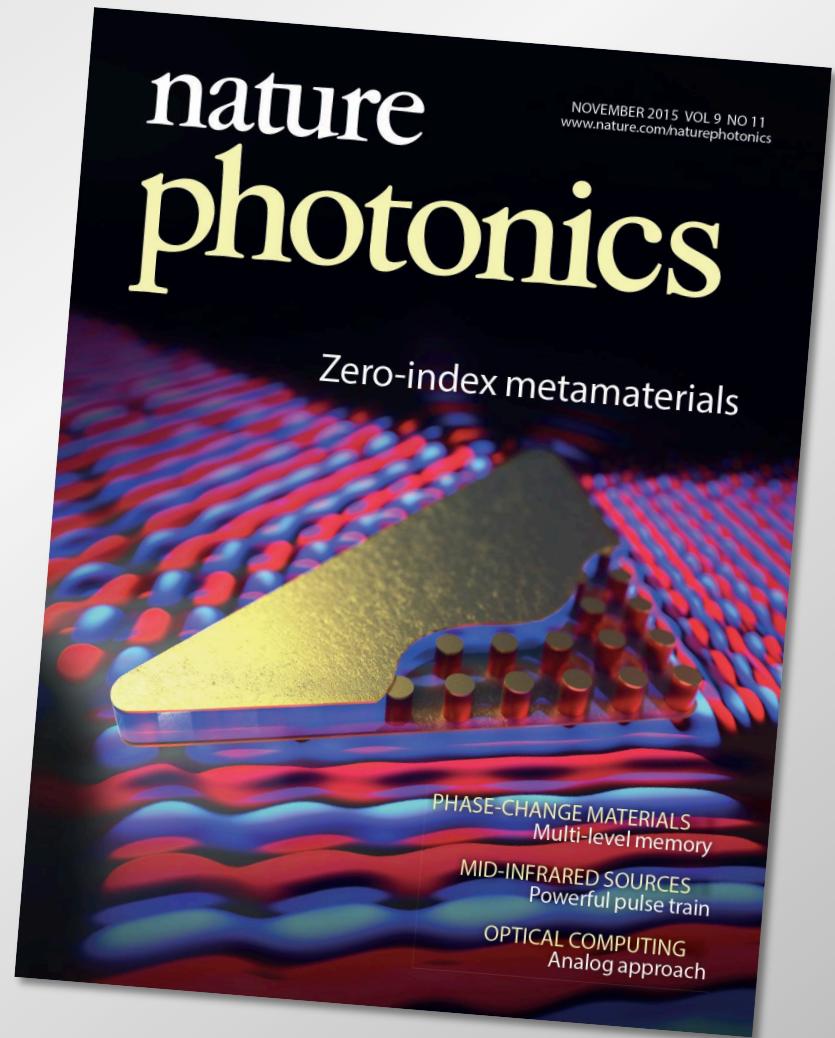


Harvard University
Center for
Nanoscale
Systems

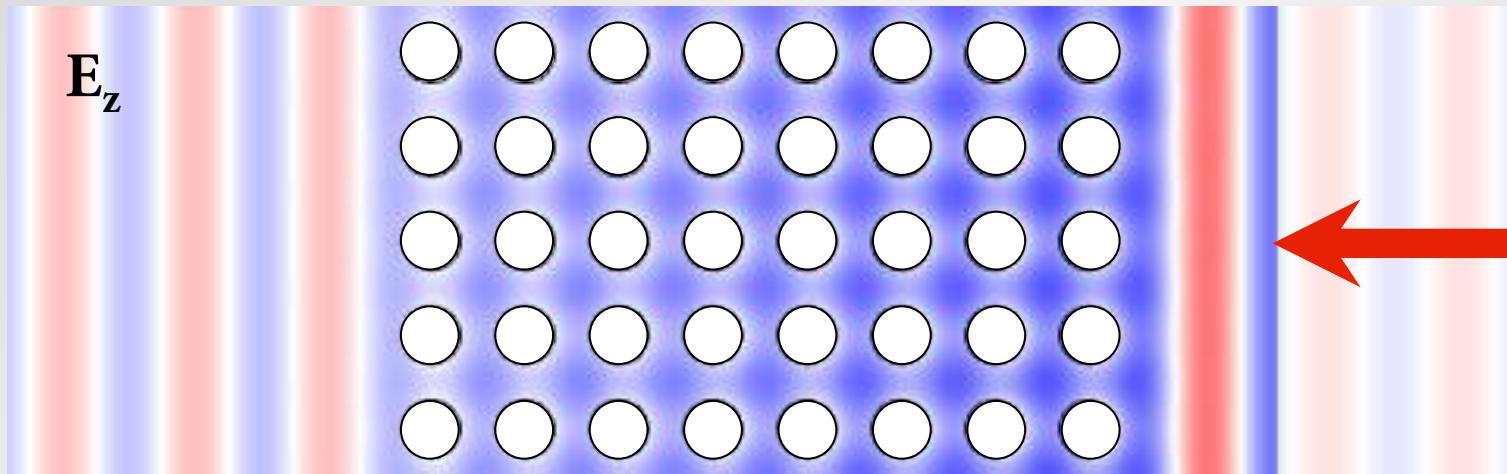
This work is available online:

Li, Y., Kita, S., et al. "On-chip zero-index Metamaterials", *Nature Photon.* 9 (2015)

check us out at:
mazur.harvard.edu



Optimized design for zero index



1

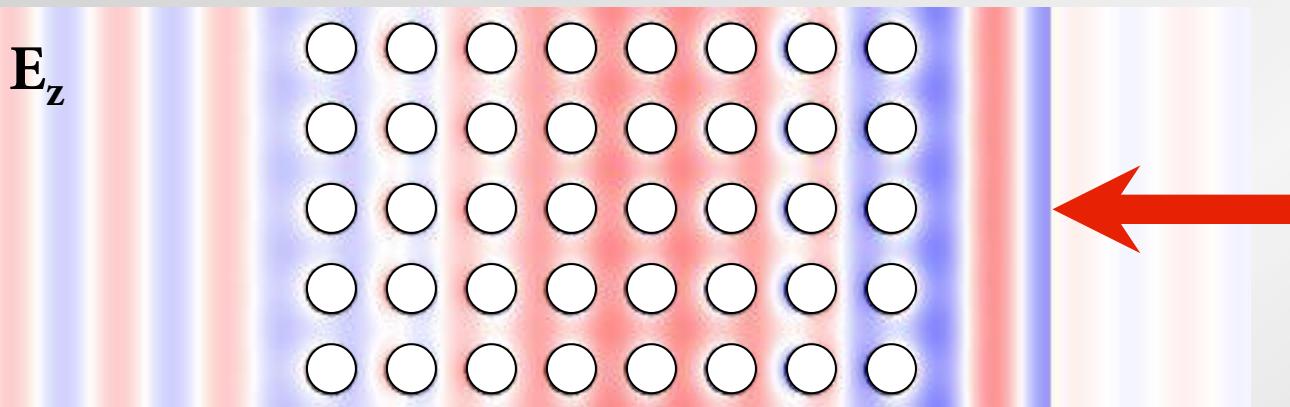
zero index

2

metamaterials

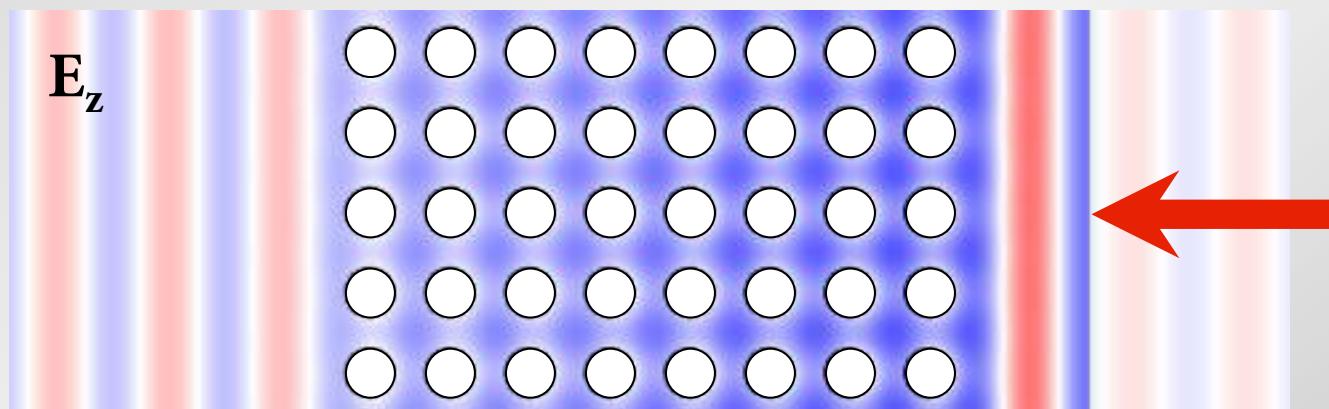
$\lambda_0 = 1530\text{nm}$

$n = 0.2$



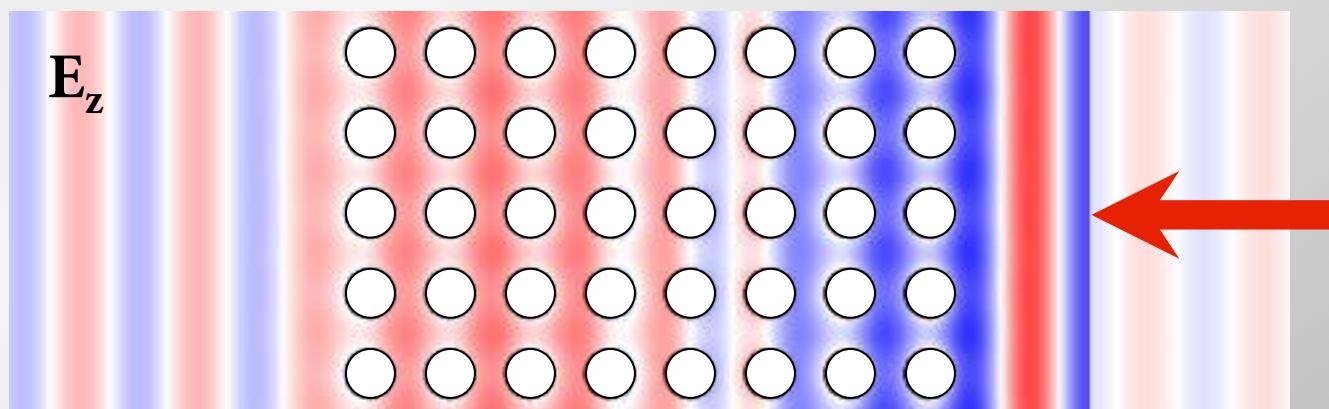
$\lambda_0 = 1590\text{nm}$

$n = 0$



$\lambda_0 = 1650\text{nm}$

$n = -0.2$



1

zero index

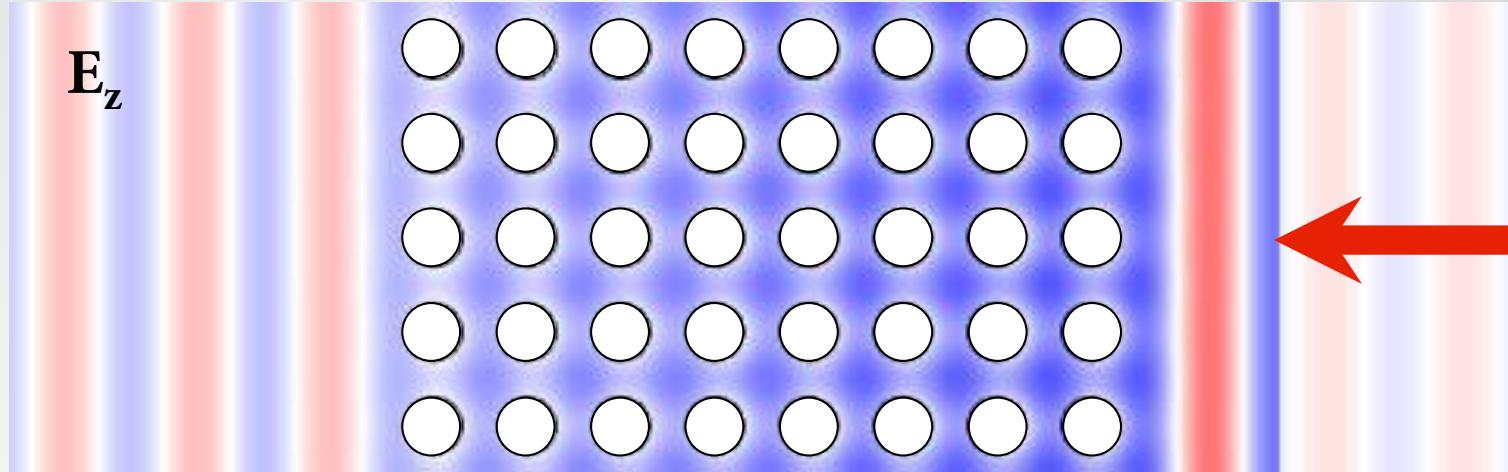
2

metamaterials

- **What about cutoff? Not a sine wave.**
- **Sine wave to the big bang**

Metamaterial Design

Redeem ZIM



1

zero index

2

metamaterials