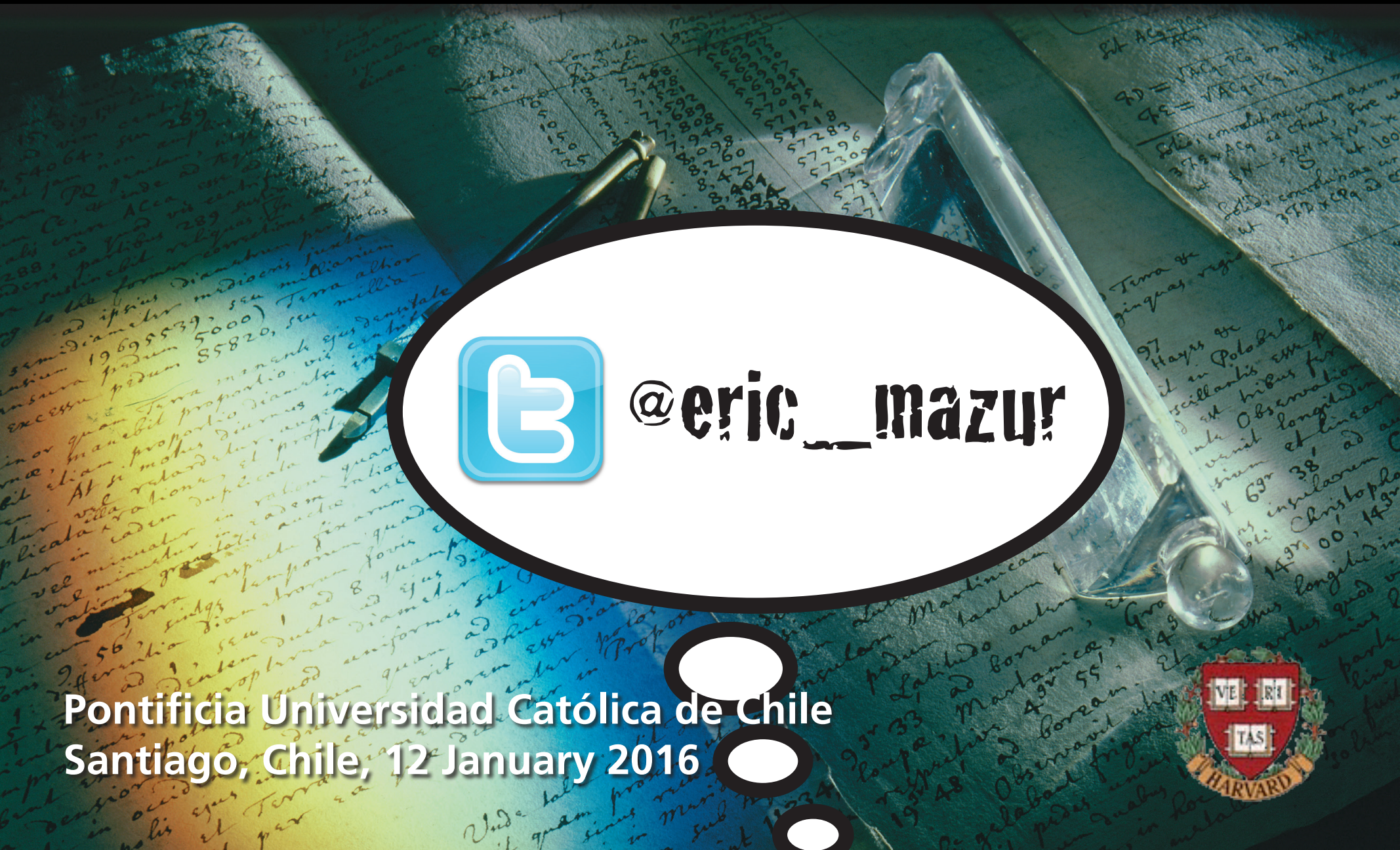
Less is more:
Extreme optics with zero refractive index



Pontificia Universidad Católica de Chile
Santiago, Chile, 12 January 2016

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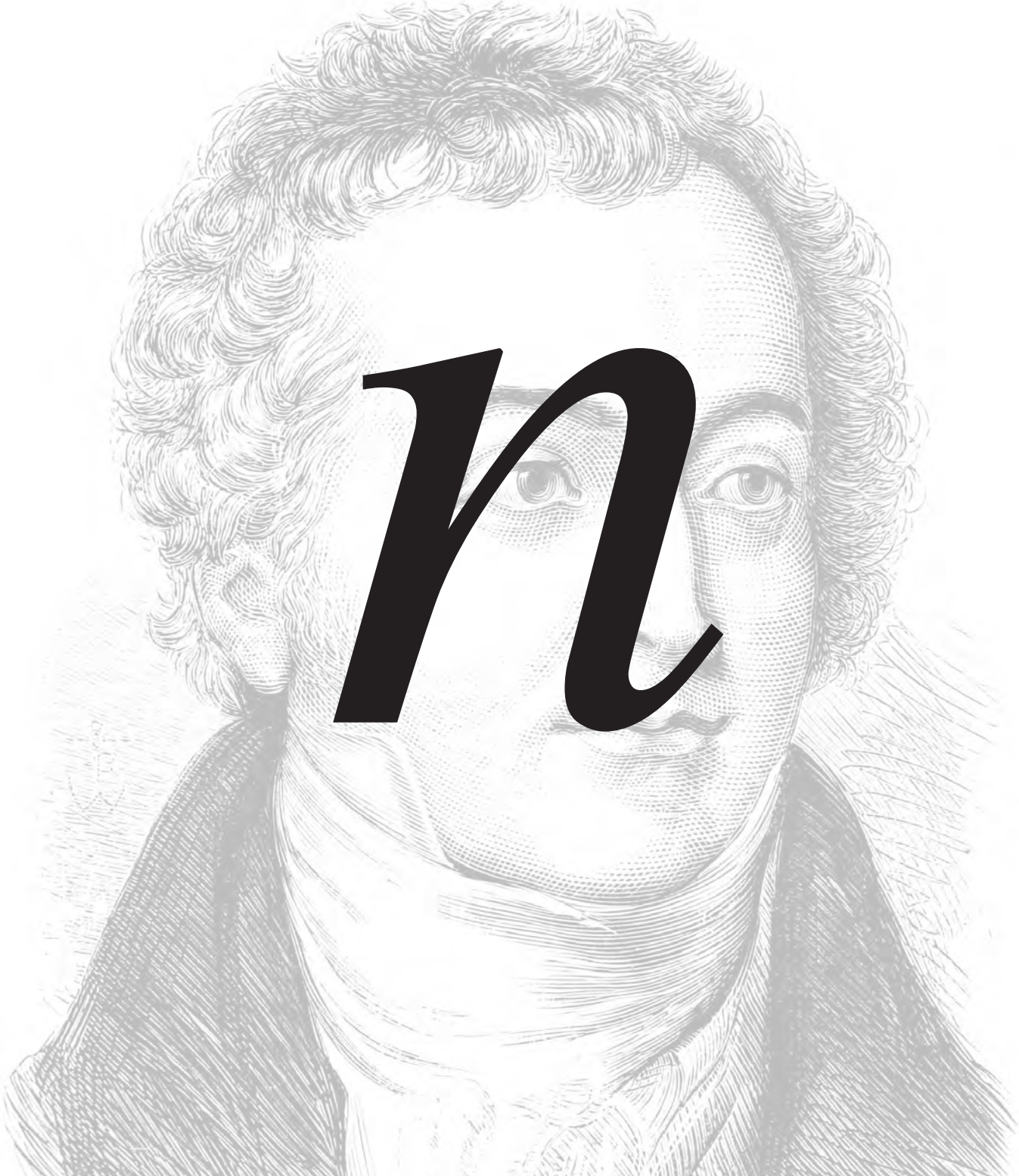
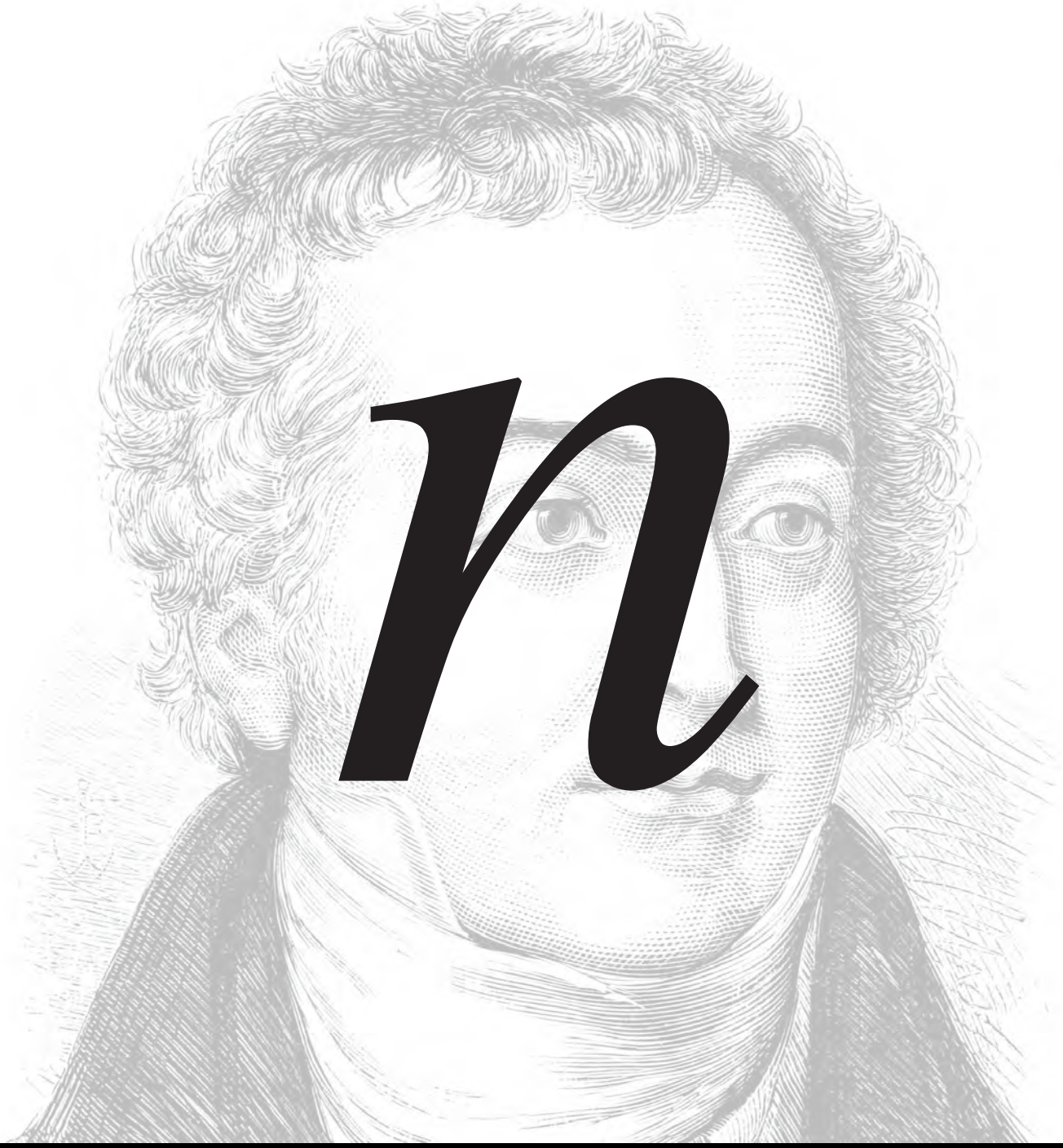
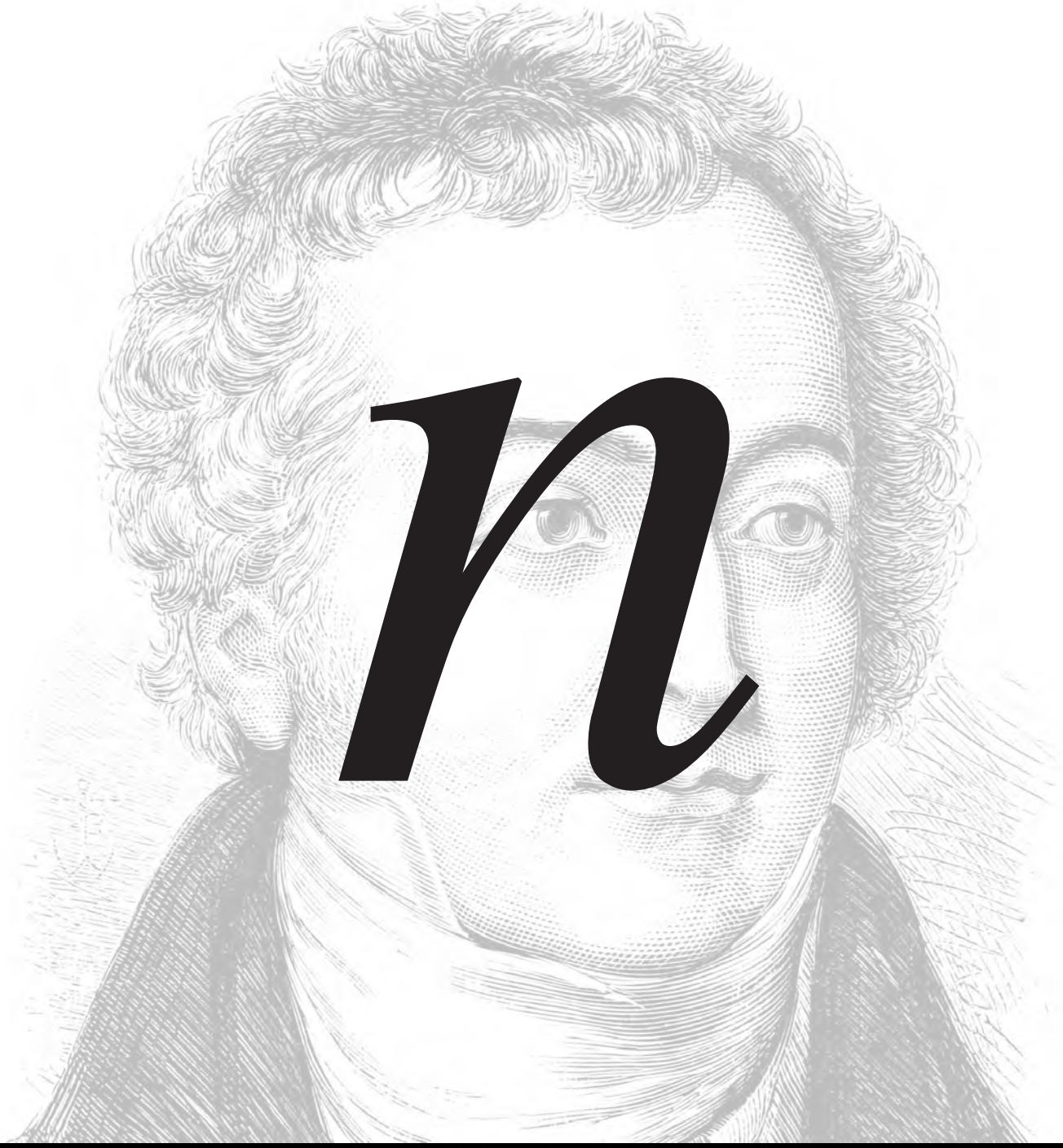


n



n

1 index



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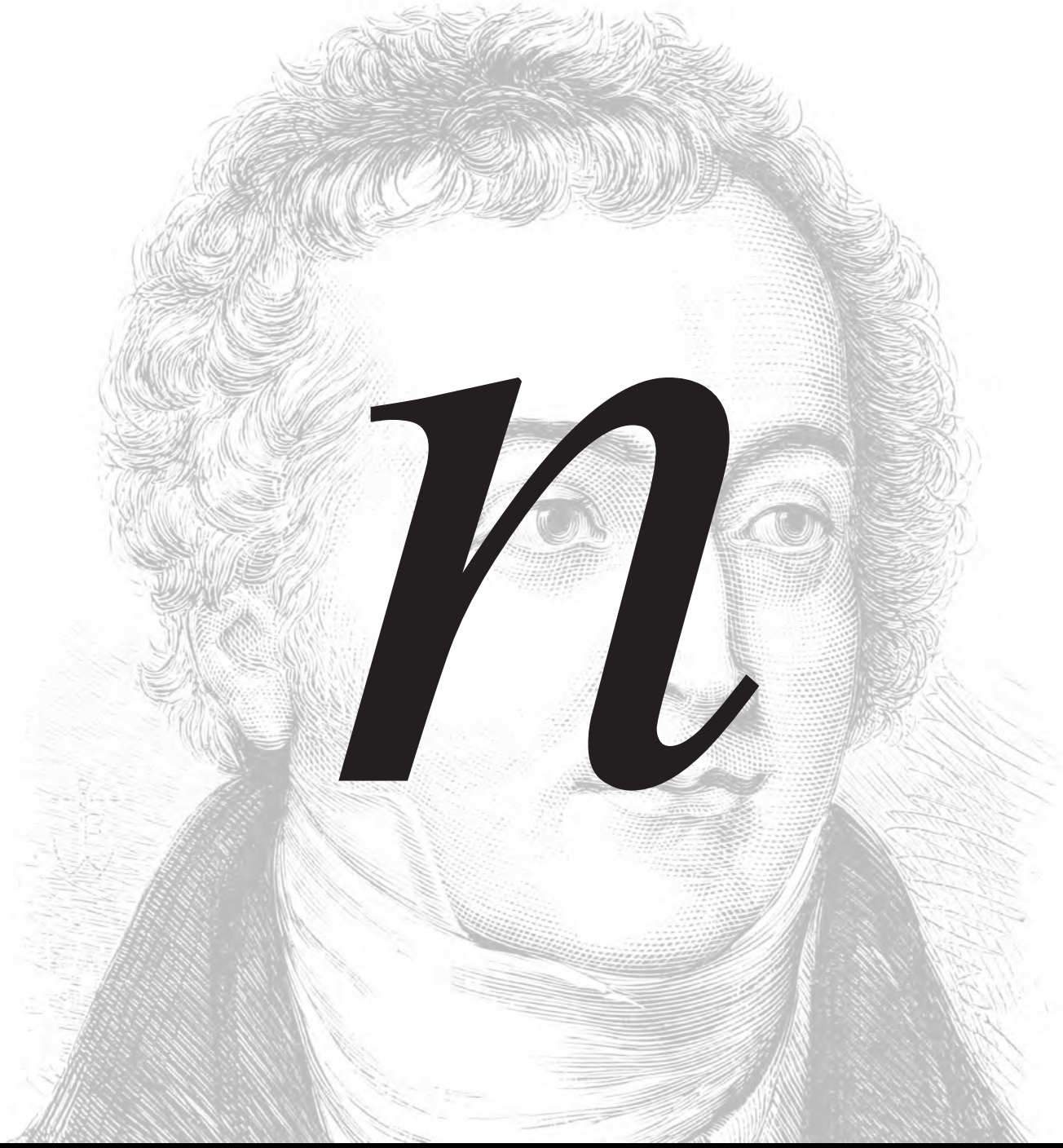

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3 experiments

Propagation of EM wave

Propagation of EM wave

governed by wave equation

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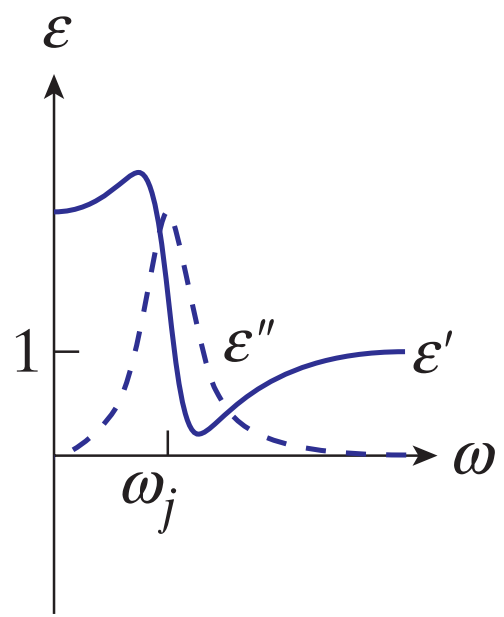
In dispersive media $n = n\,(\omega)$.

Dielectric constant

Lorentz oscillator



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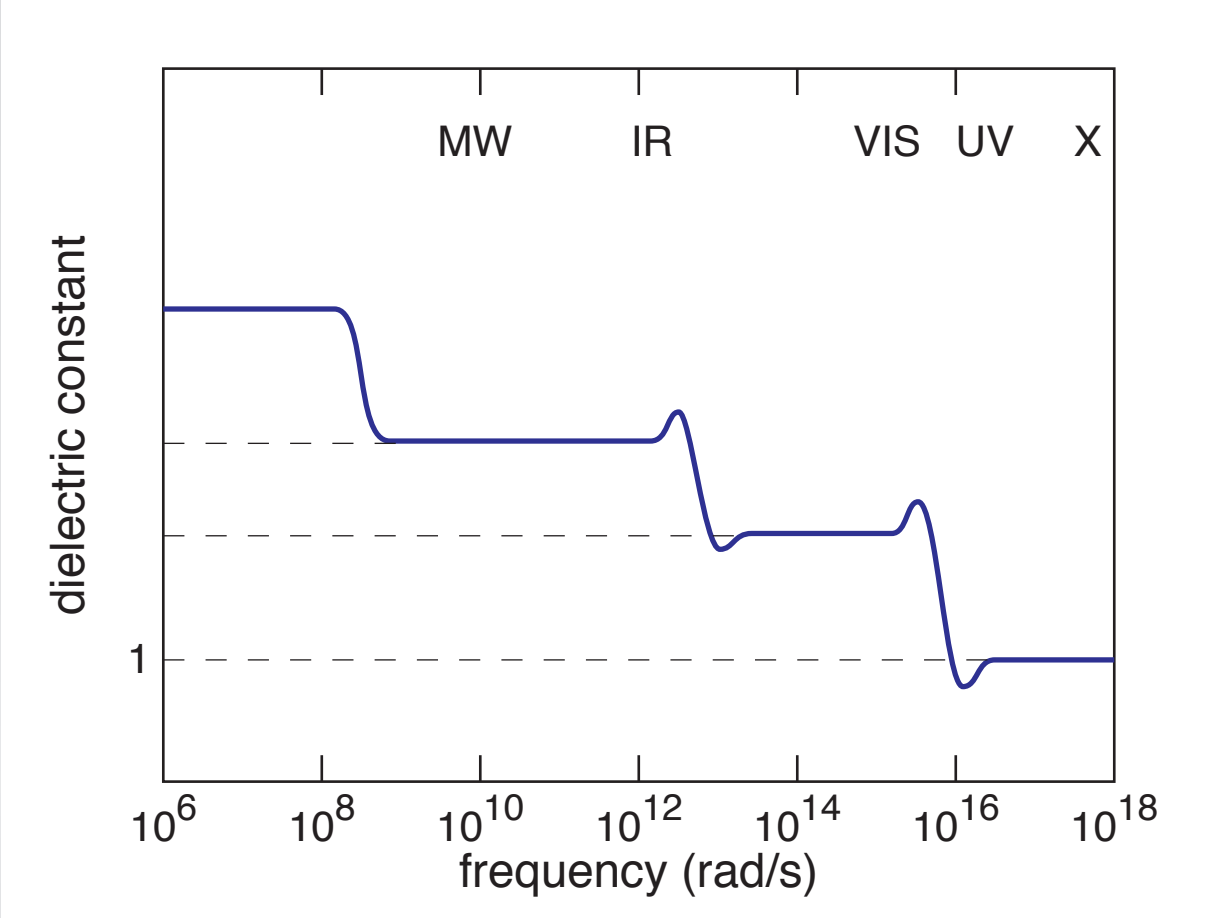

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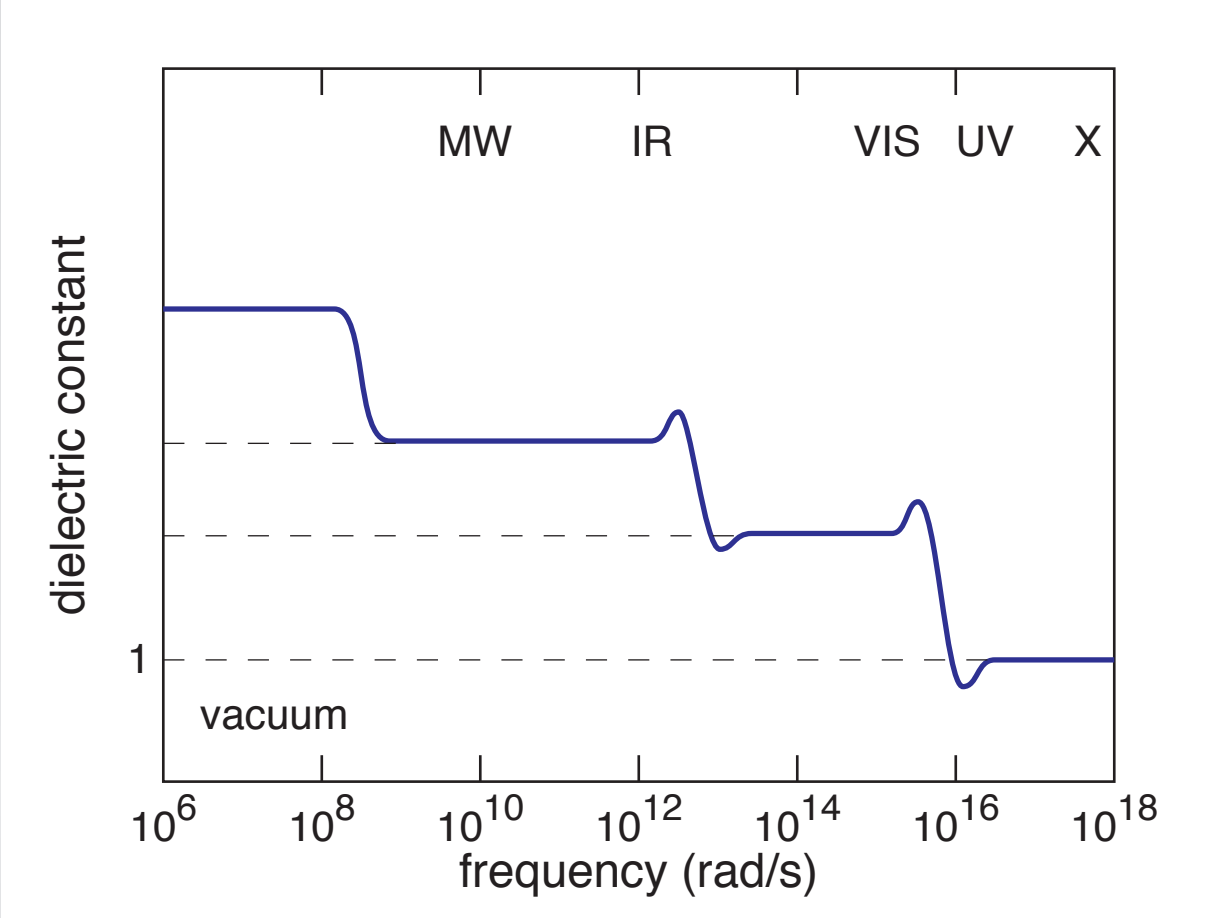

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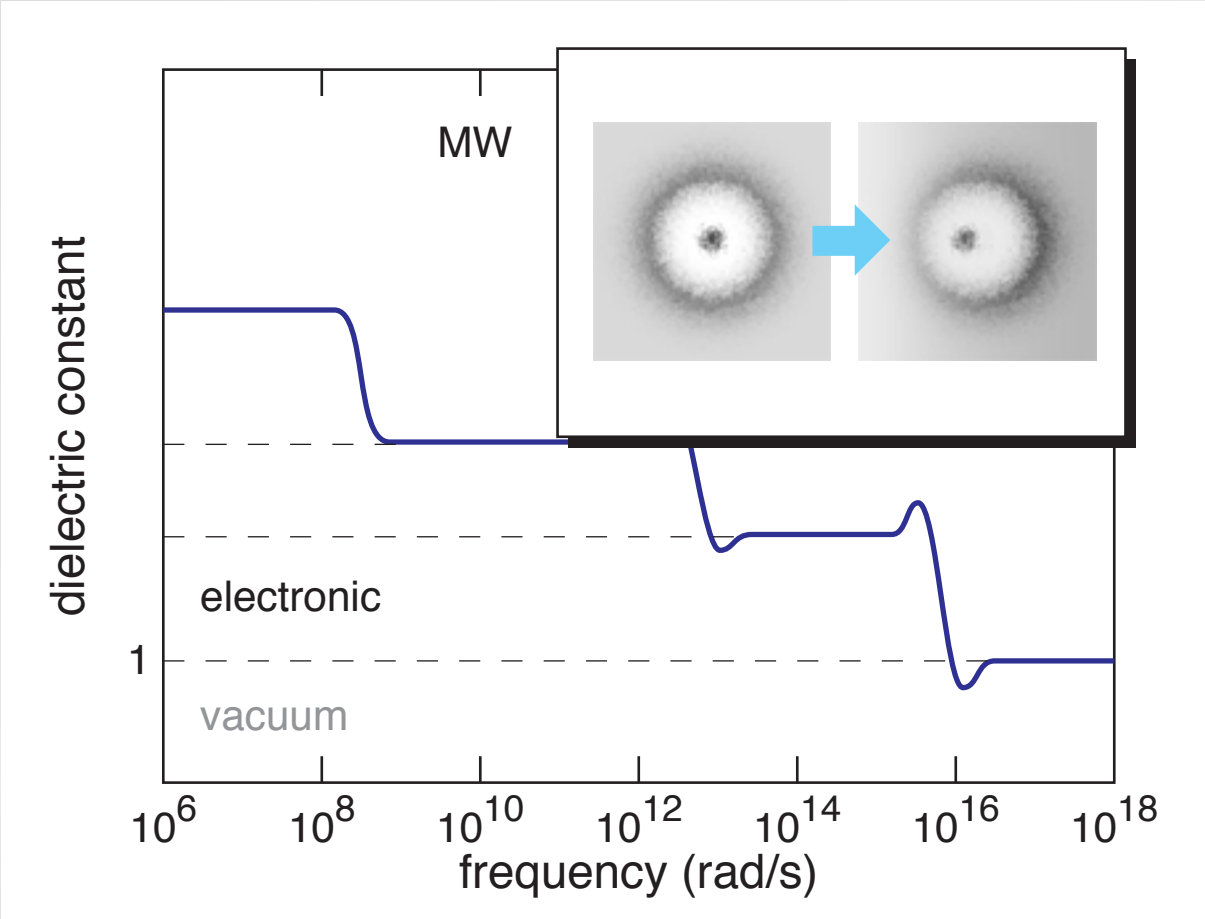

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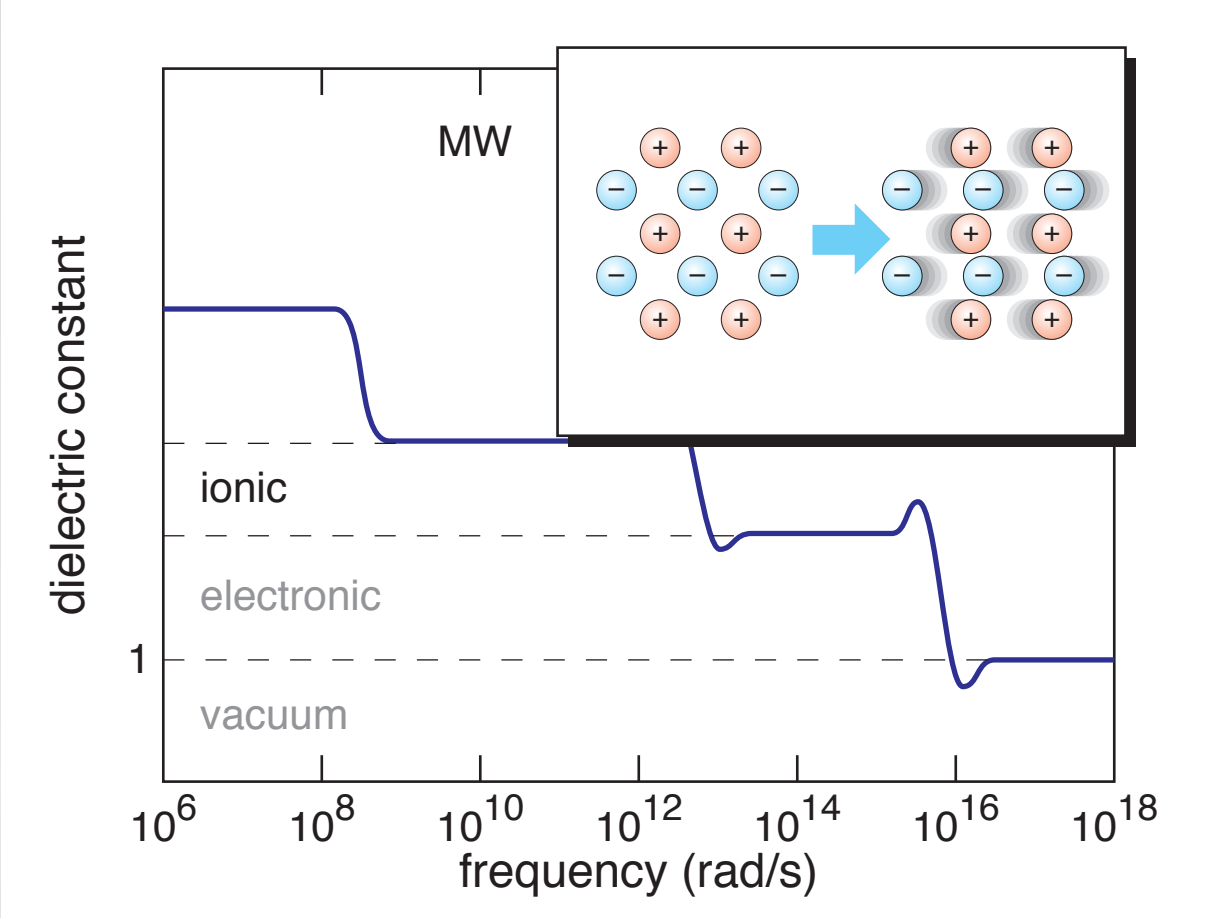

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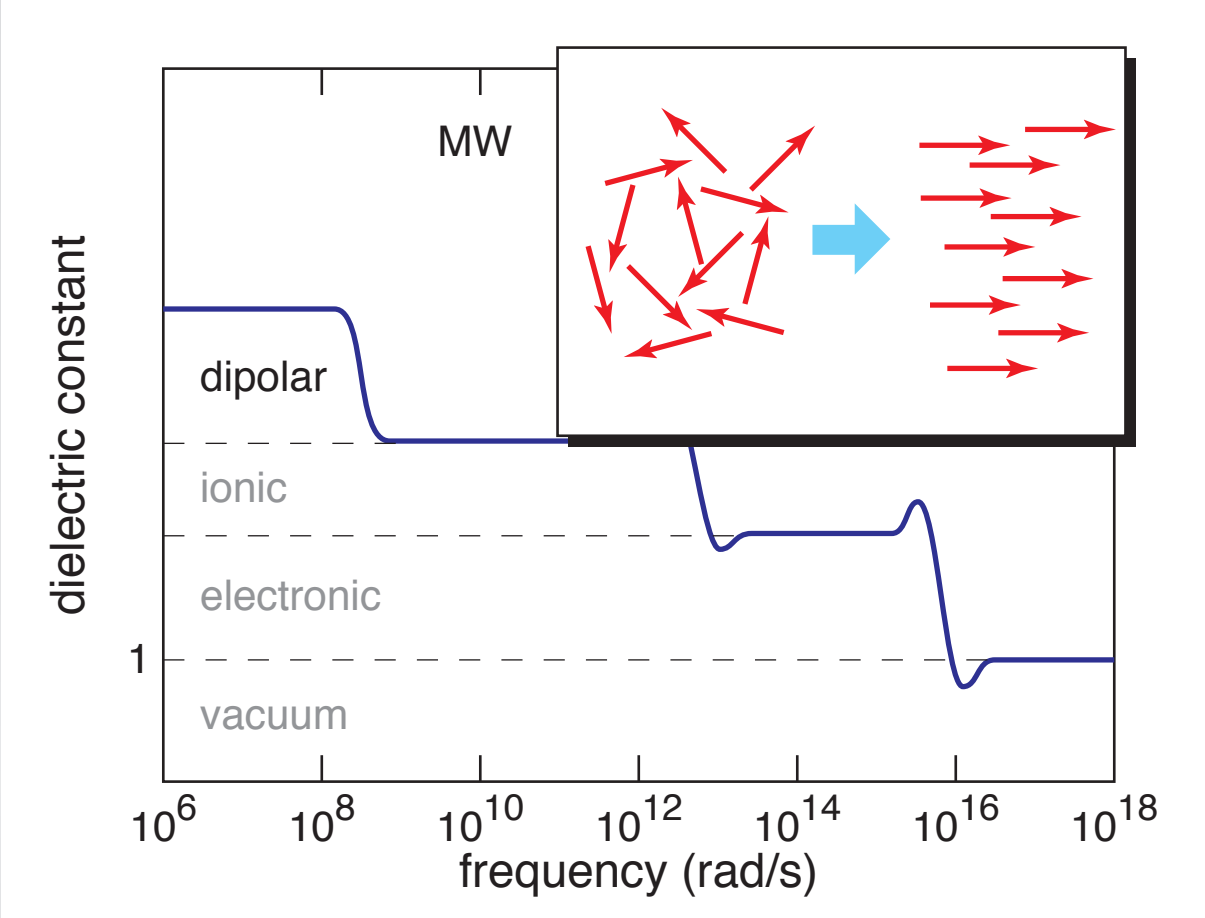

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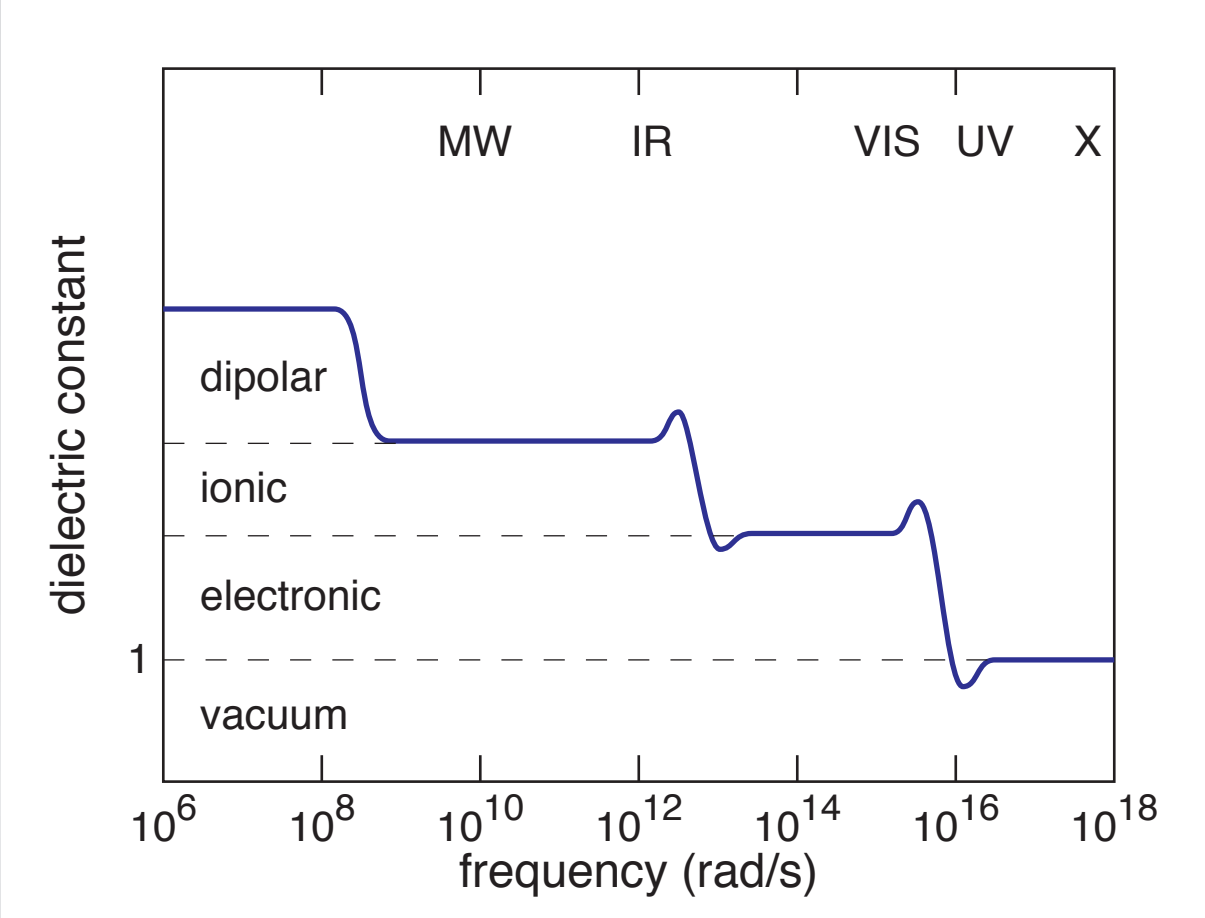
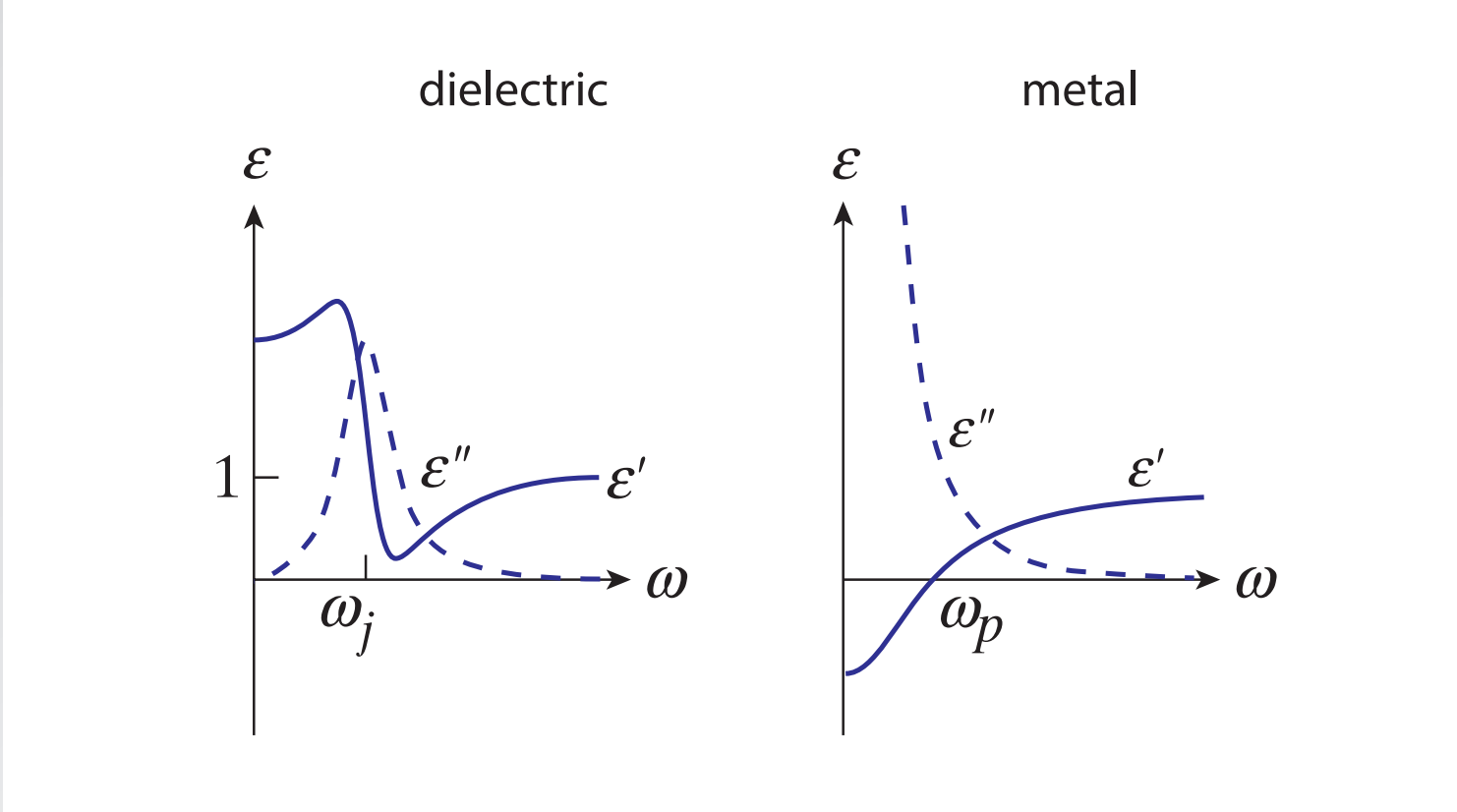
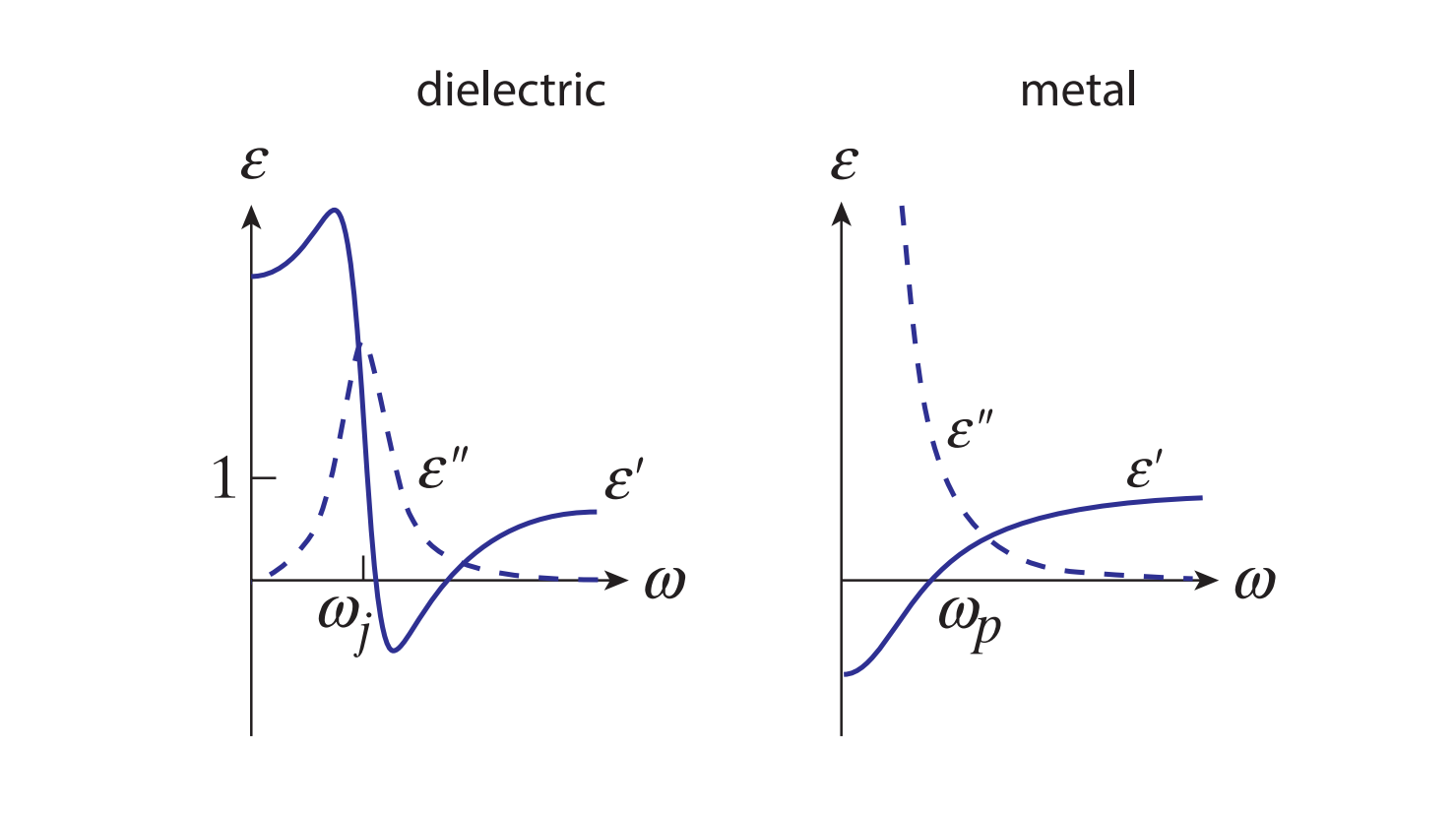


Lorentz and Drude models



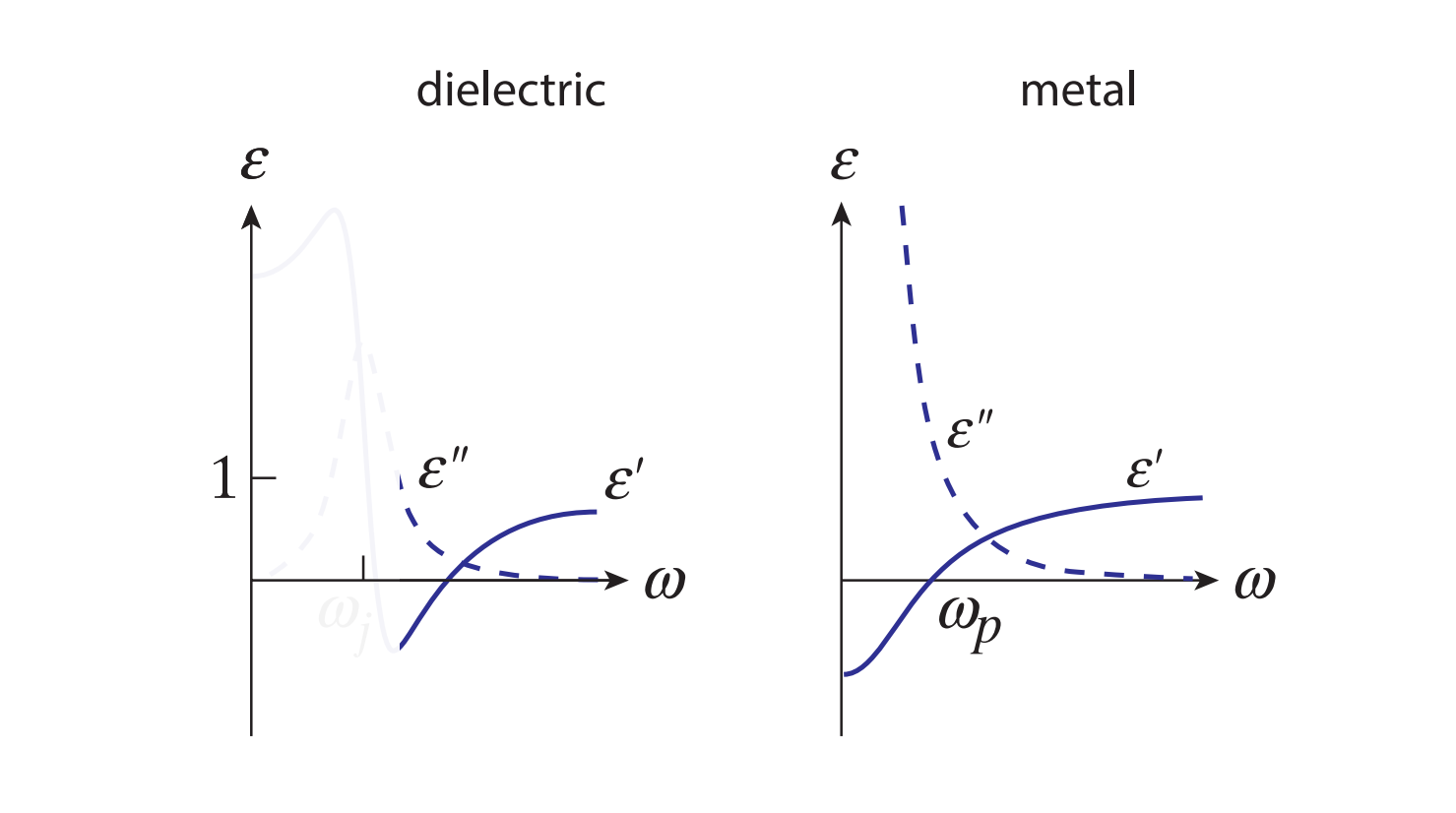
Lorentz and Drude models

for a strong (dielectric) resonance ε can become negative



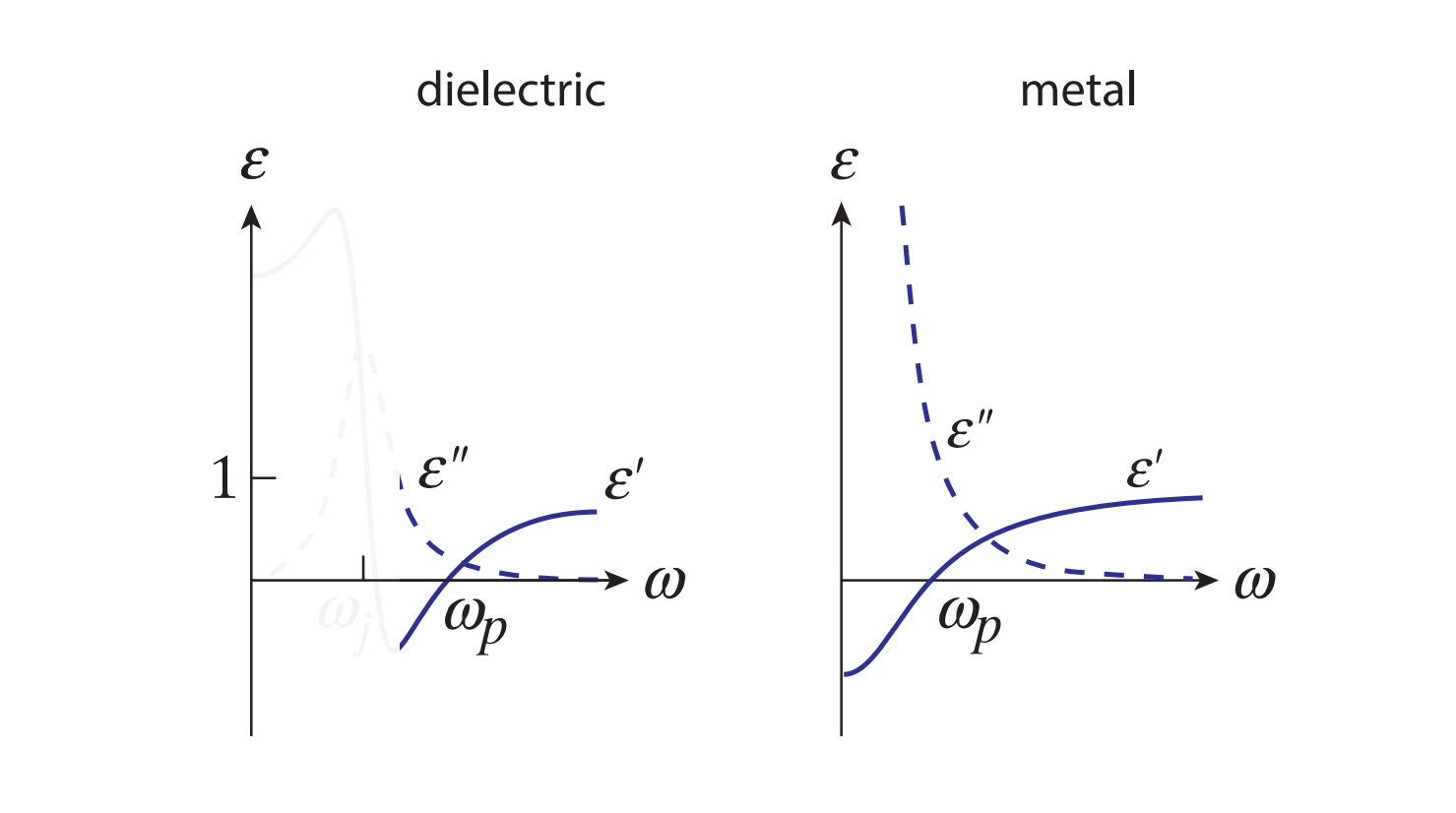
Lorentz and Drude models

valence electrons in dielectric then behave like a plasma



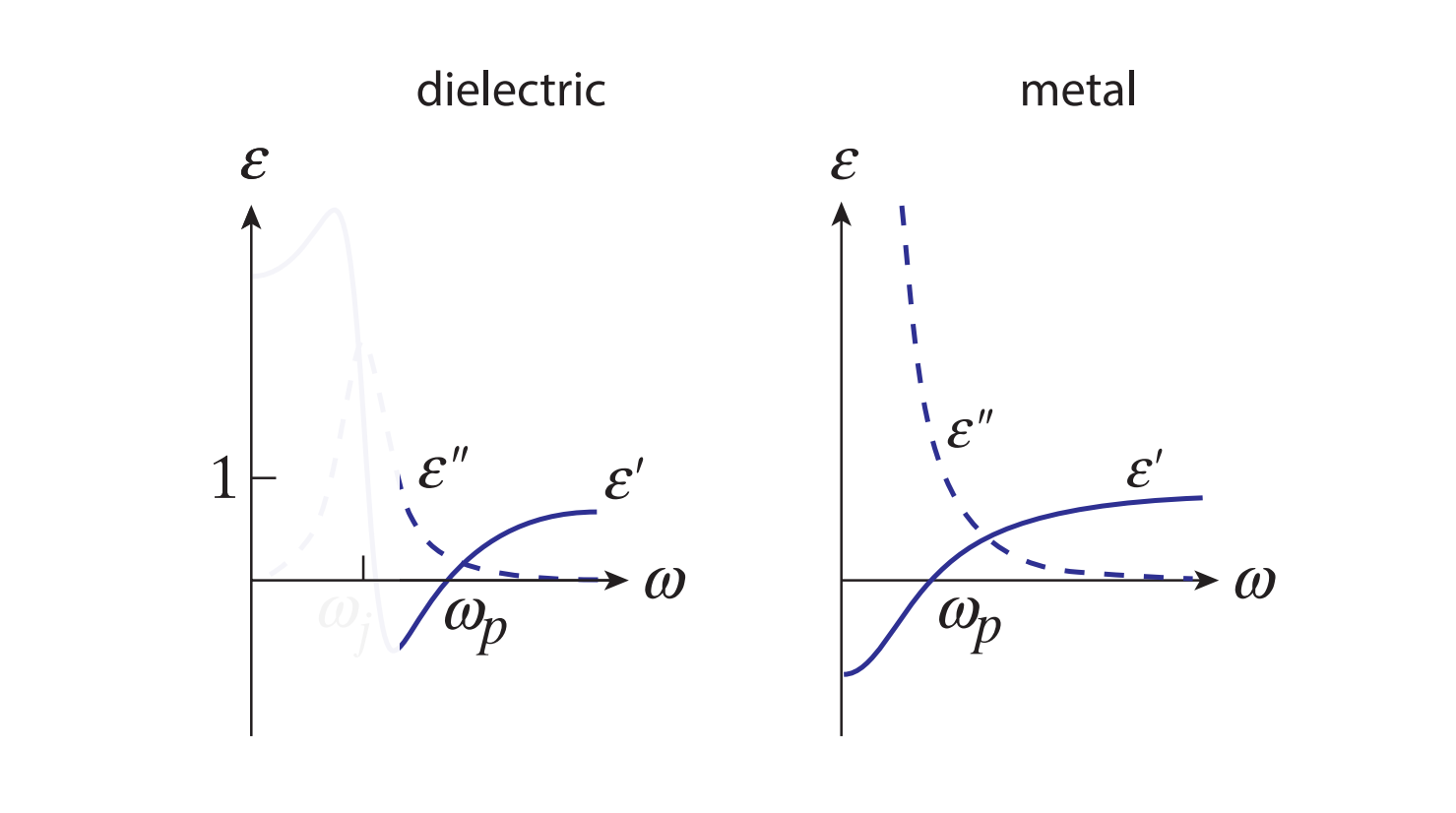
Lorentz and Drude models

with plasma frequency above the resonance



Lorentz and Drude models

(and far below the UV region)



Index also determined by magnetic response

$$n = \sqrt{\epsilon\mu}$$

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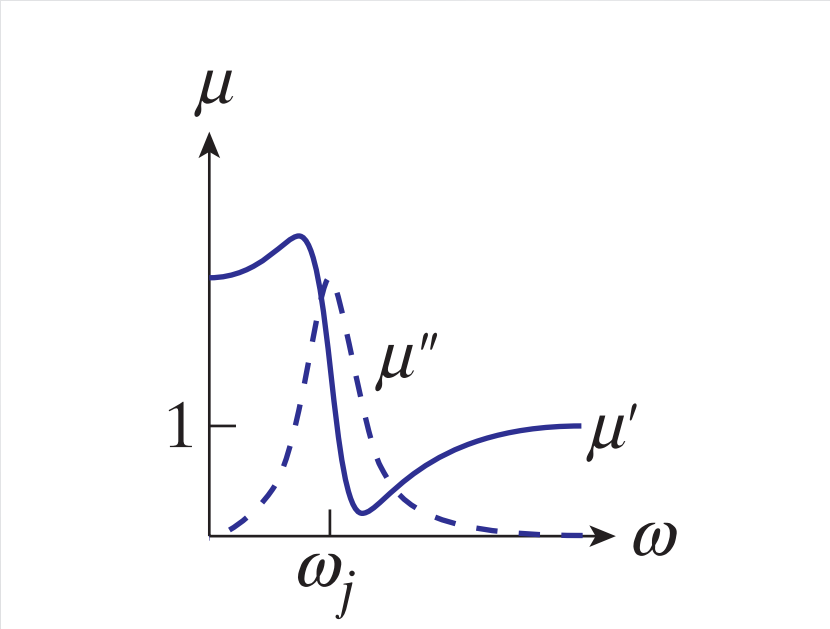
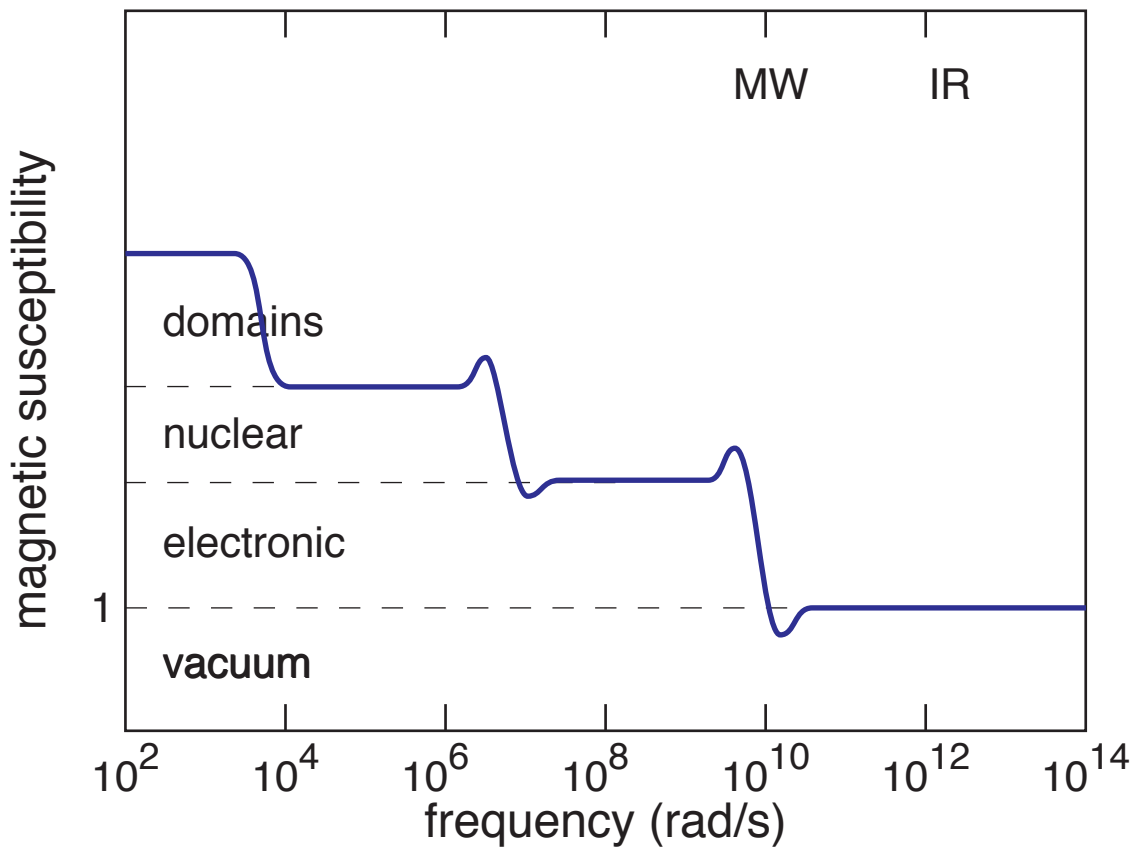
and magnetic response shows similar resonances

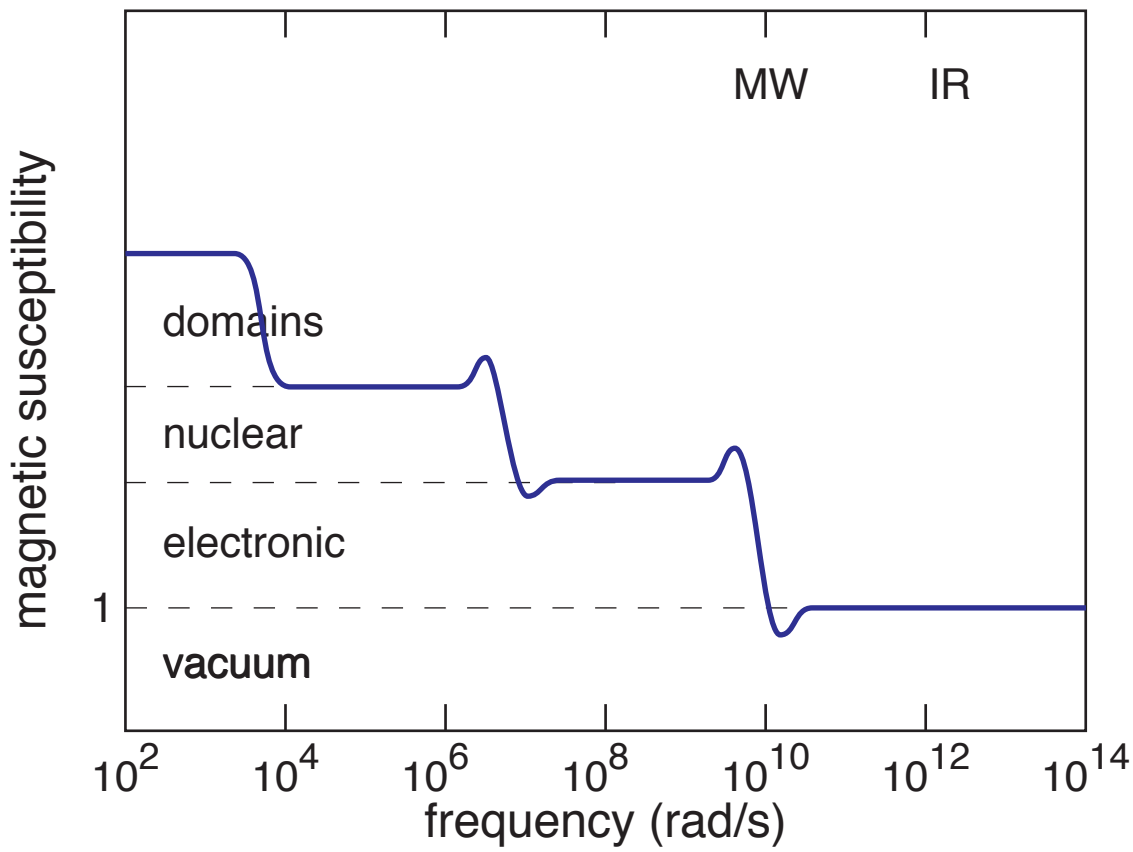


Magnetic response



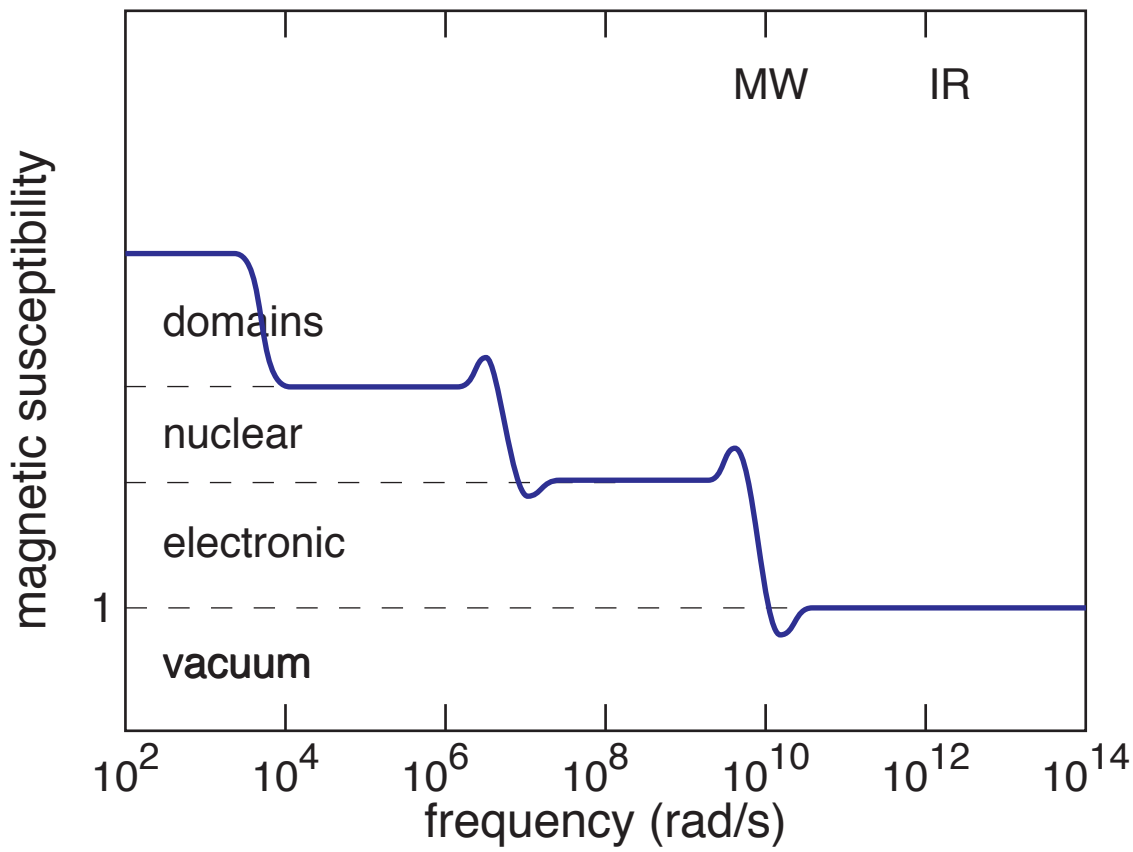
Magnetic response

but magnetic resonances occur below optical frequencies



Magnetic response

so, in optical regime, $\mu \approx 1$



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What happens when $\mathrm{Re}\epsilon$ and/or $\mathrm{Re}\mu$ is negative?

Write complex quantities as

$$\varepsilon = |\varepsilon| e^{i\theta} \qquad \mu = |\mu| e^{i\phi}$$

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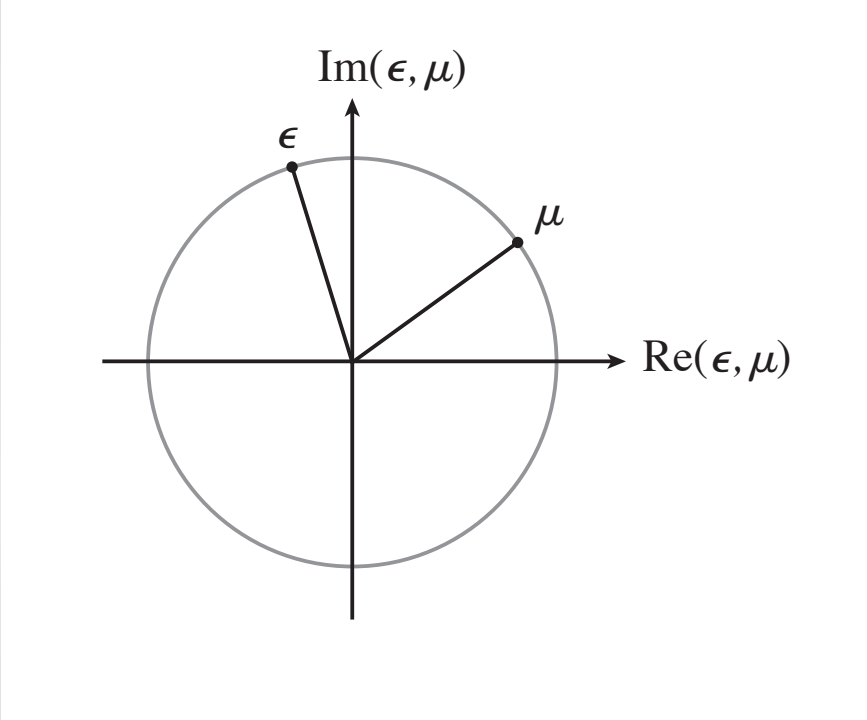

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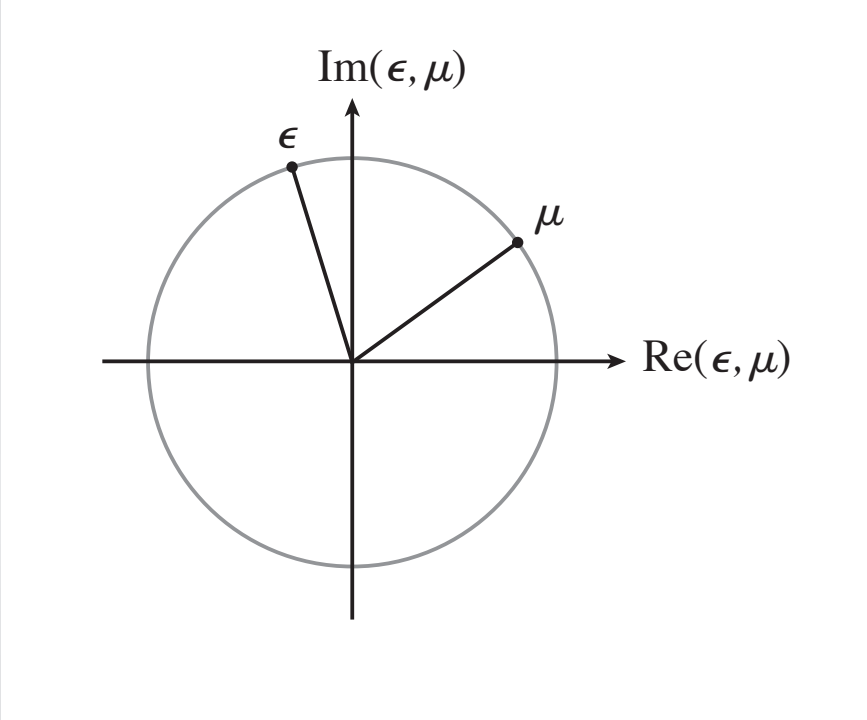

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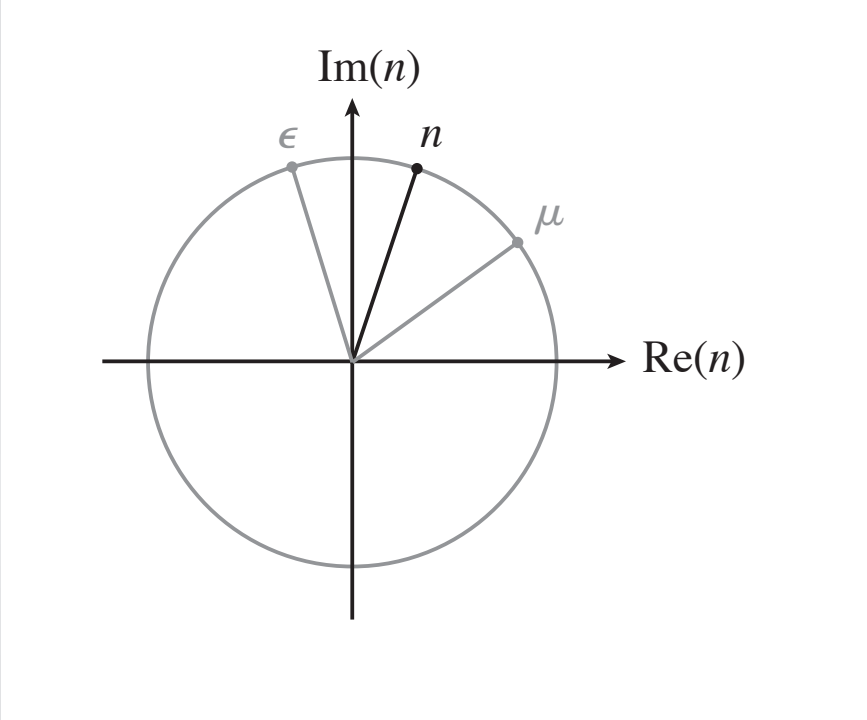

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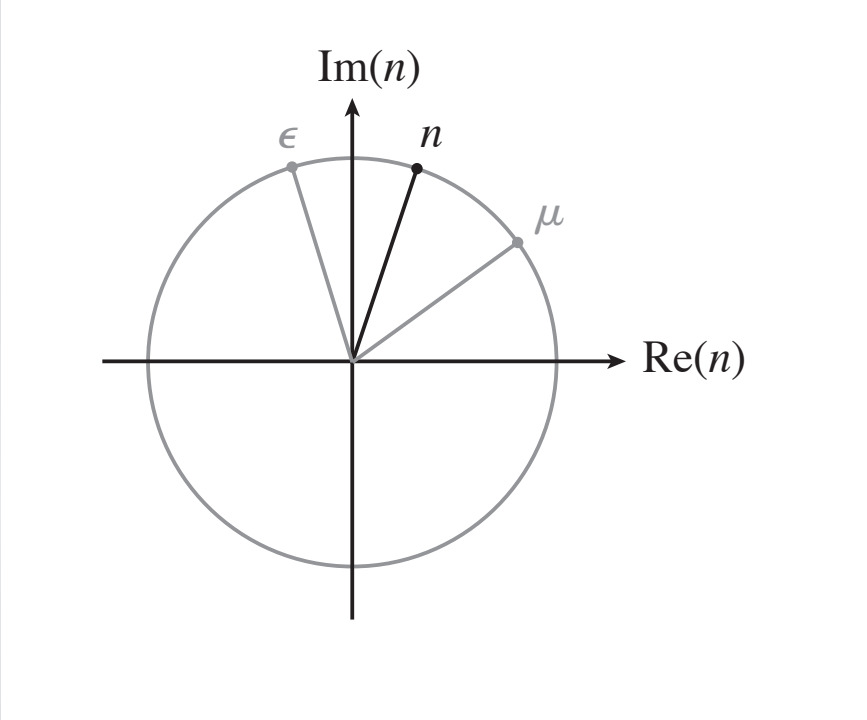

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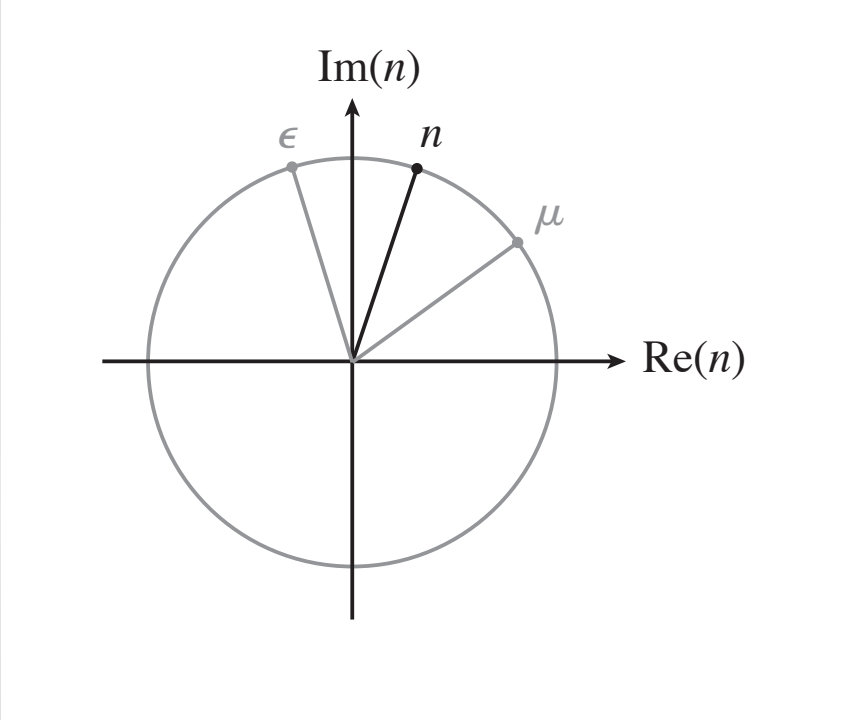

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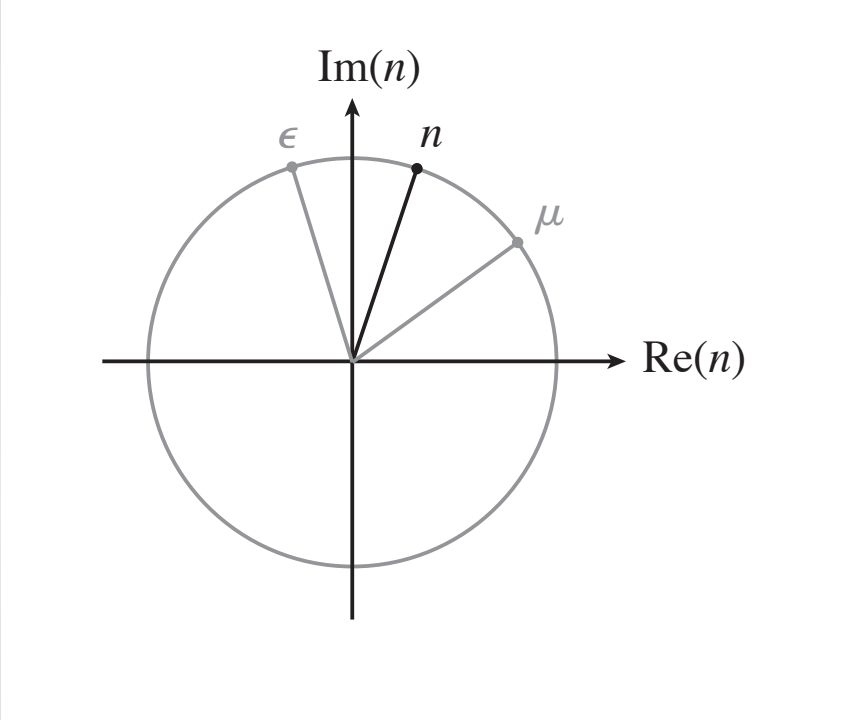
Can add 2π to exponent

$$e^{+i(\theta+\phi)} = e^{+i[\theta+\phi+2\pi]}$$



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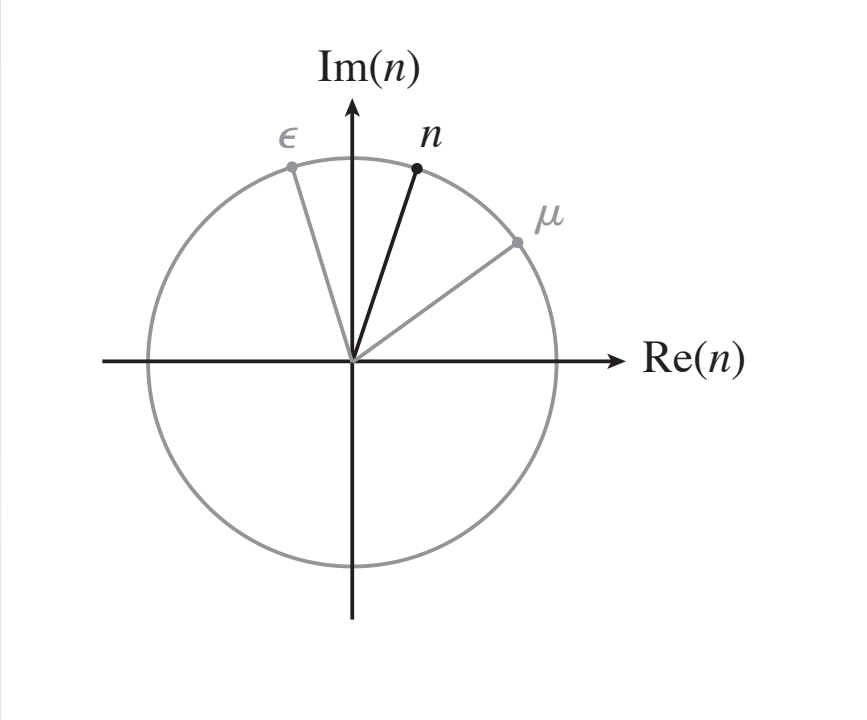

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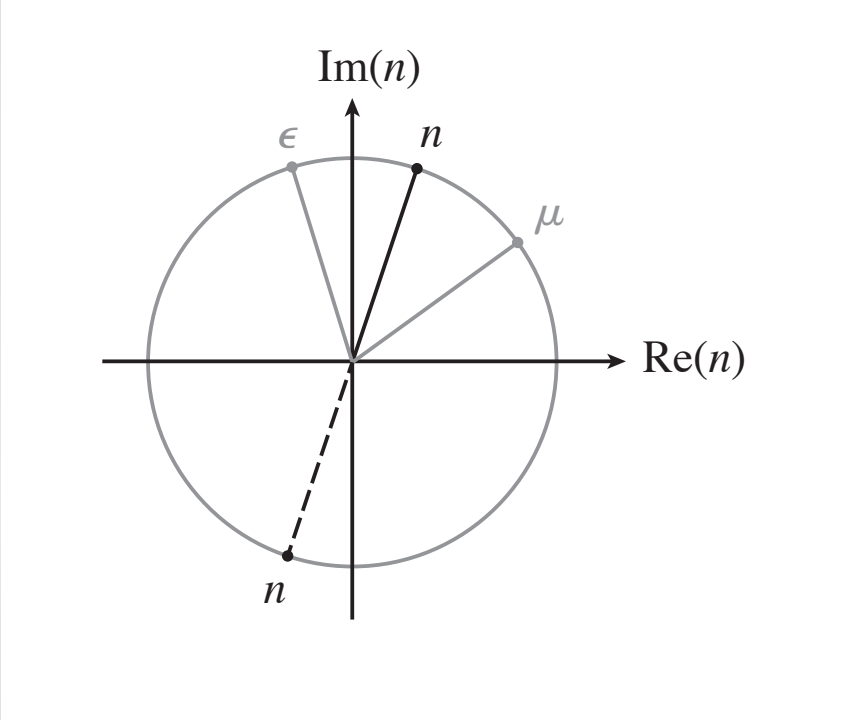

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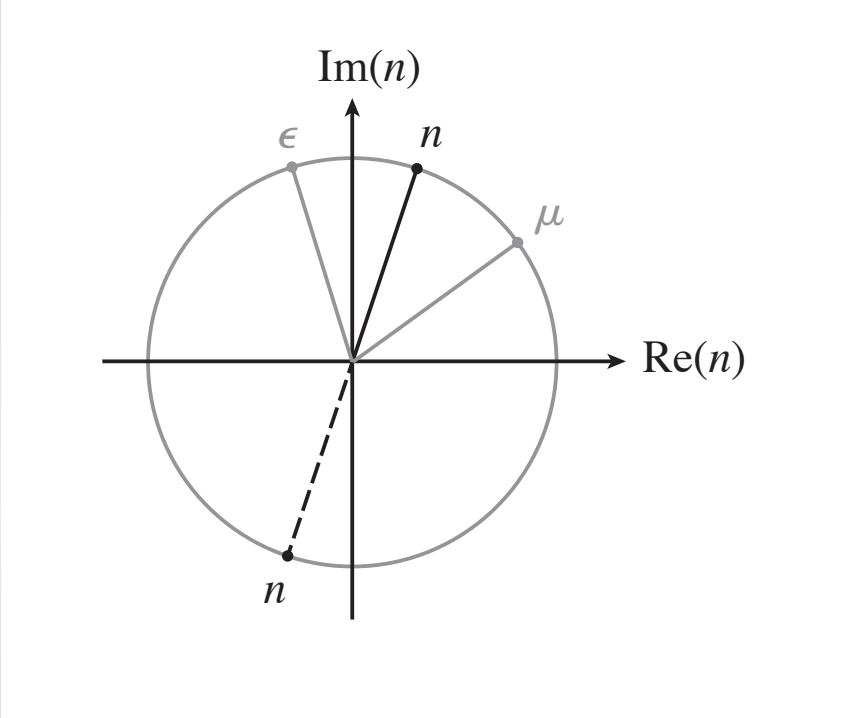

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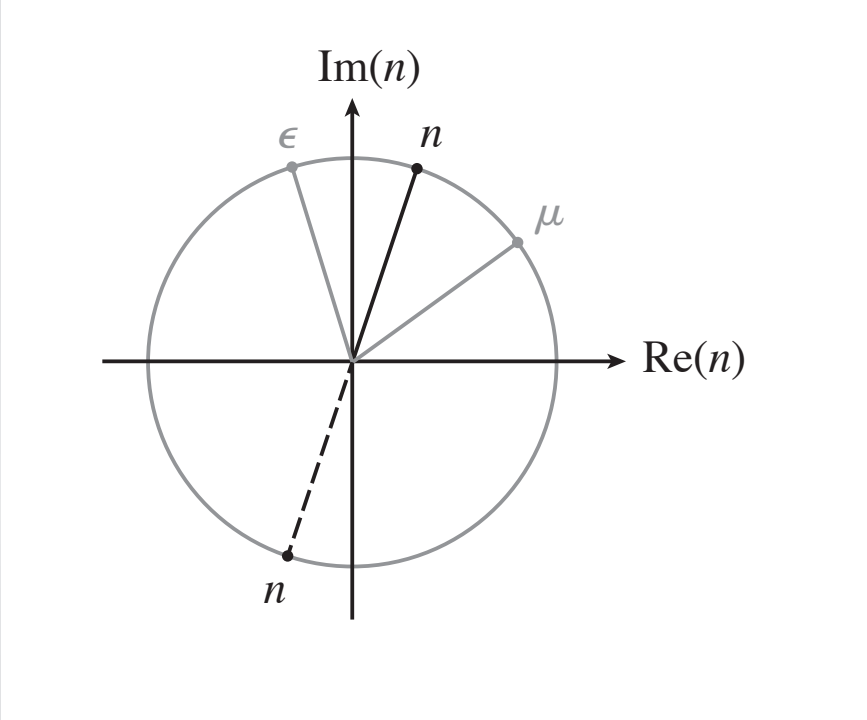

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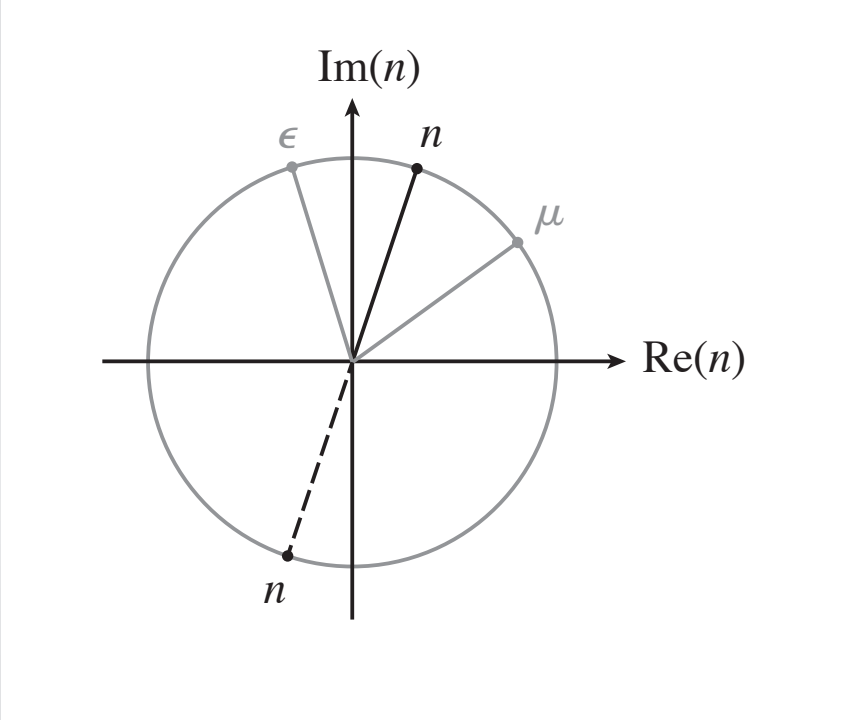

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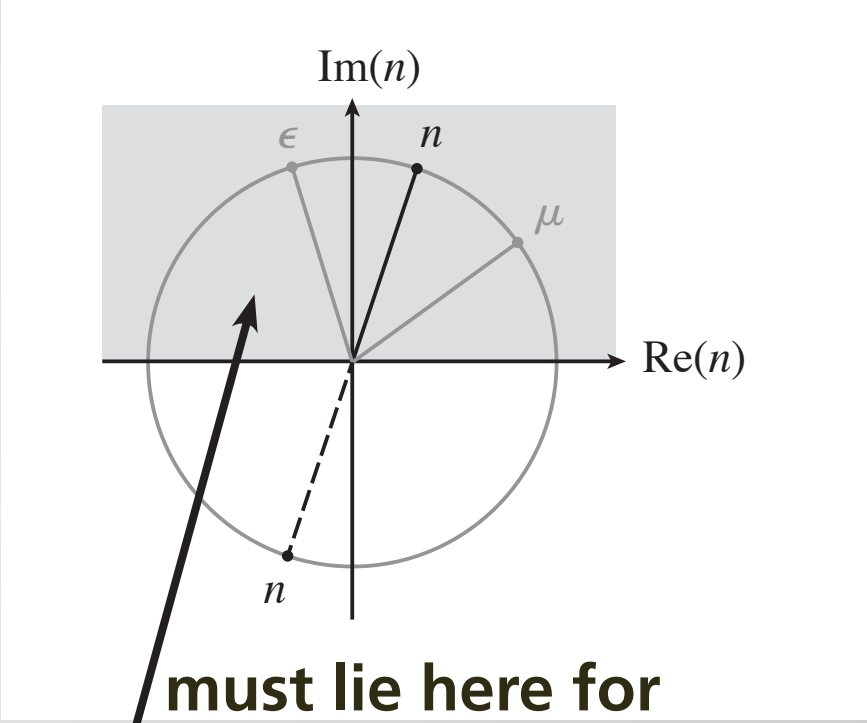

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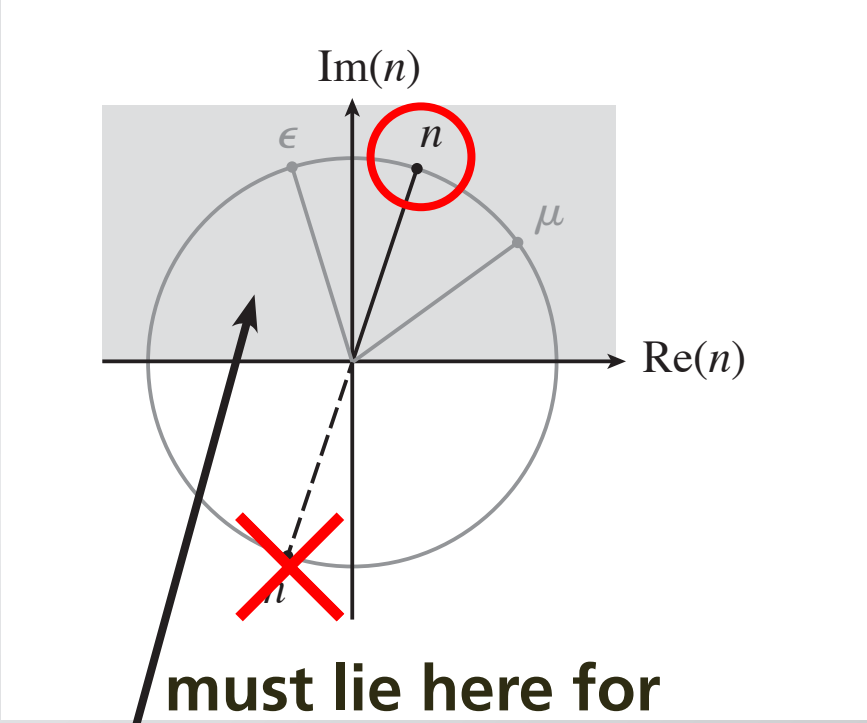

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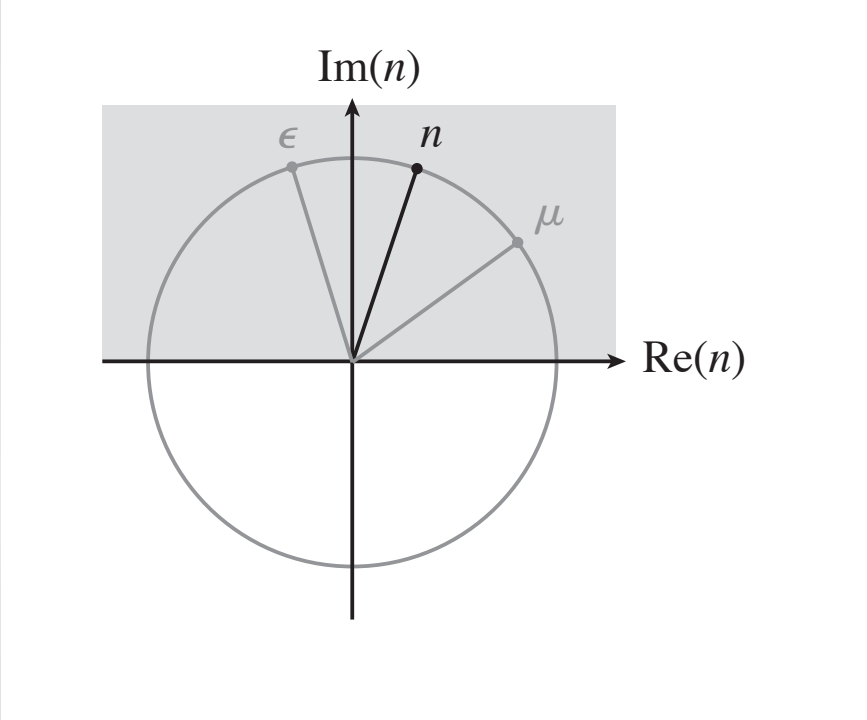
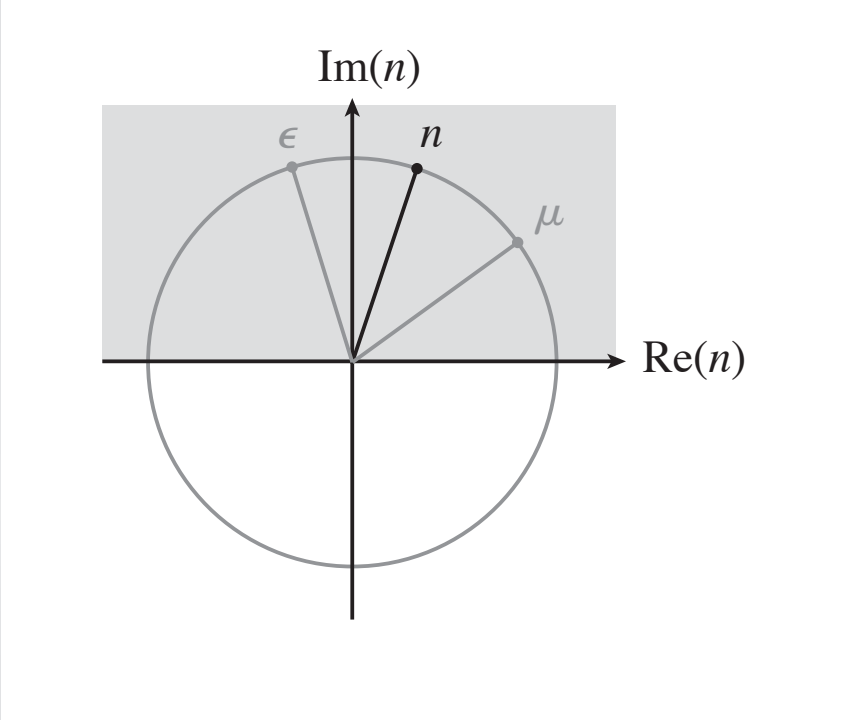


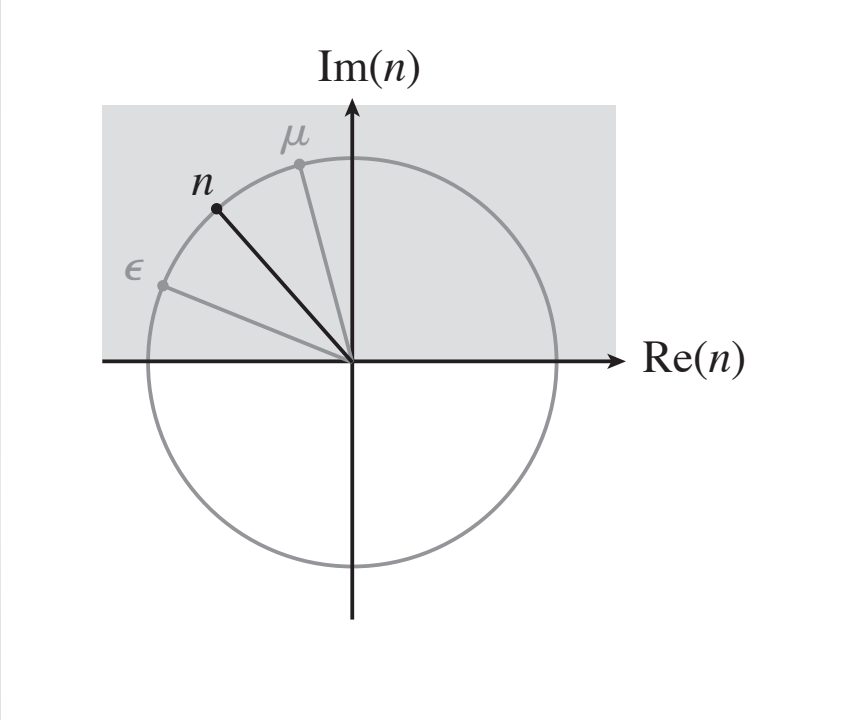
To find n (passive materials):

1. **Draw line that bisects ϵ and μ**
2. **Choose upper branch**



For certain values of ϵ and μ

we can get a *negative* $\mathrm{Re}(n)$!



Q: Must both $\mathrm{Re}\epsilon < 0$ and $\mathrm{Re}\mu < 0$ to get a negative index?

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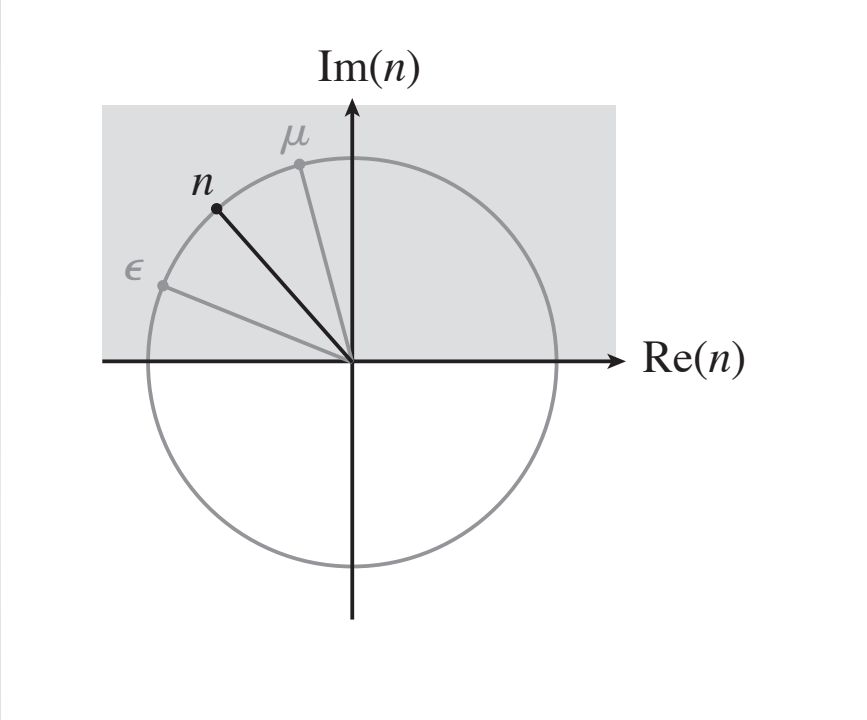

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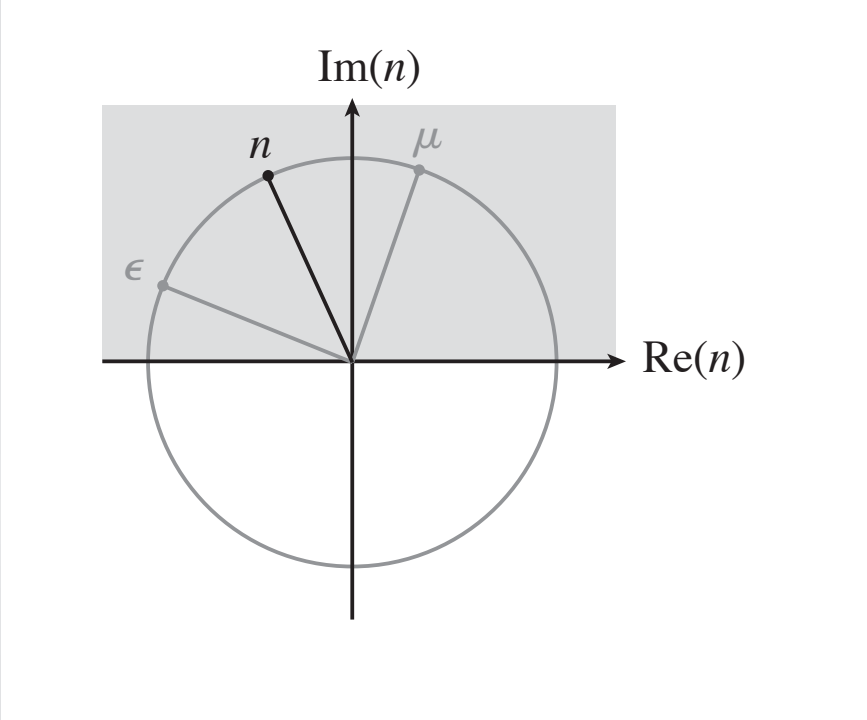
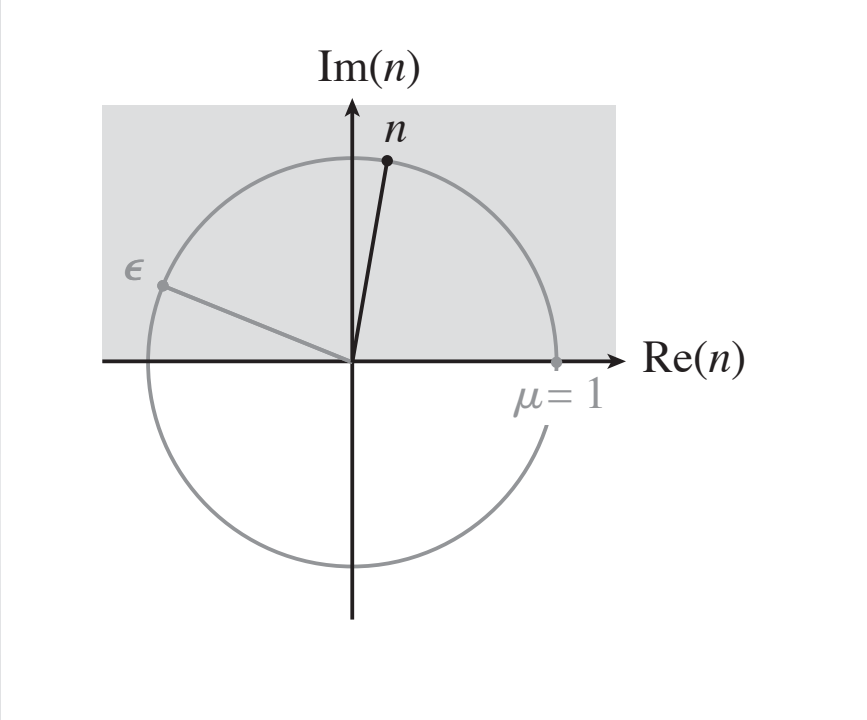


Note: need magnetic response

to achieve $n \leq 0$!



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$$E = E_o e^{i(kx - \omega t)} = E_o e^{-k''x} e^{i(k'x - \omega t)}$$

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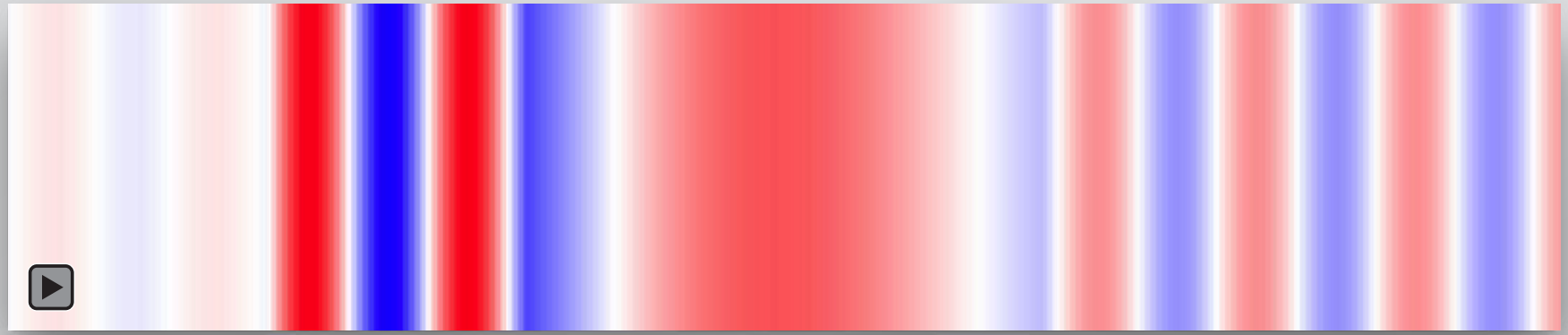
When $\mathrm{Re}(n) < 0,\ k' < 0$, **and so phase velocity reversed!**



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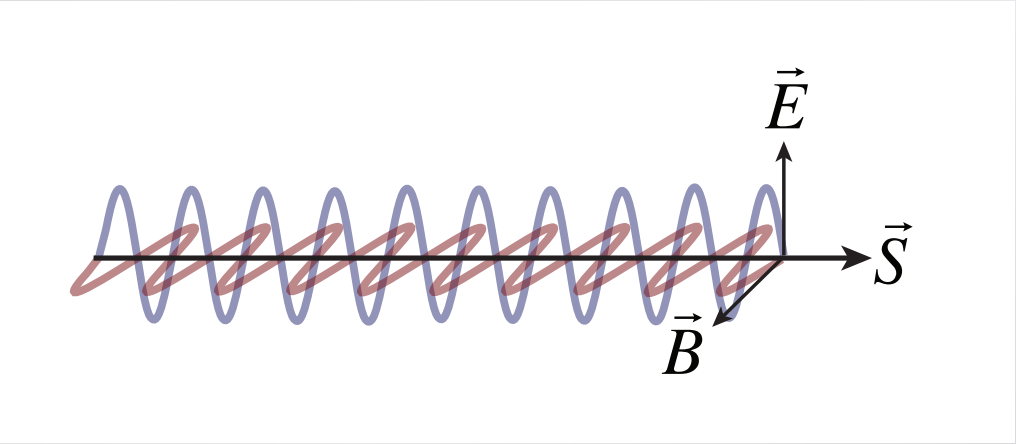

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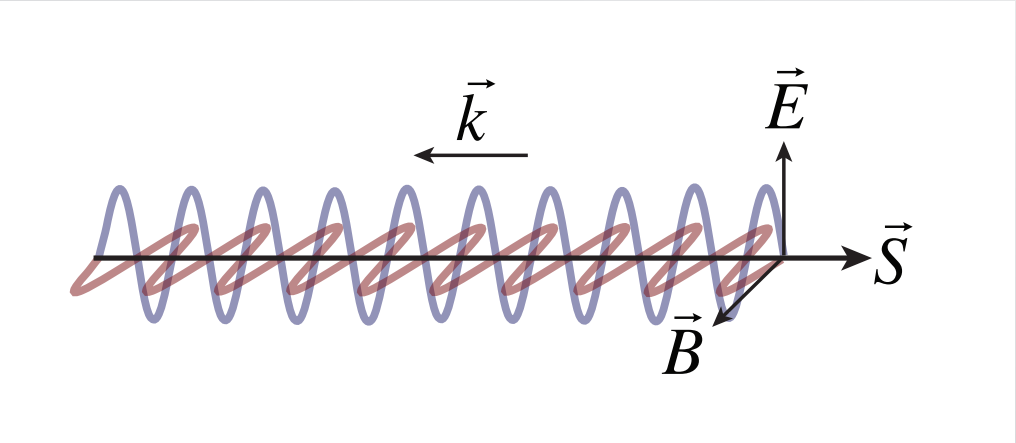
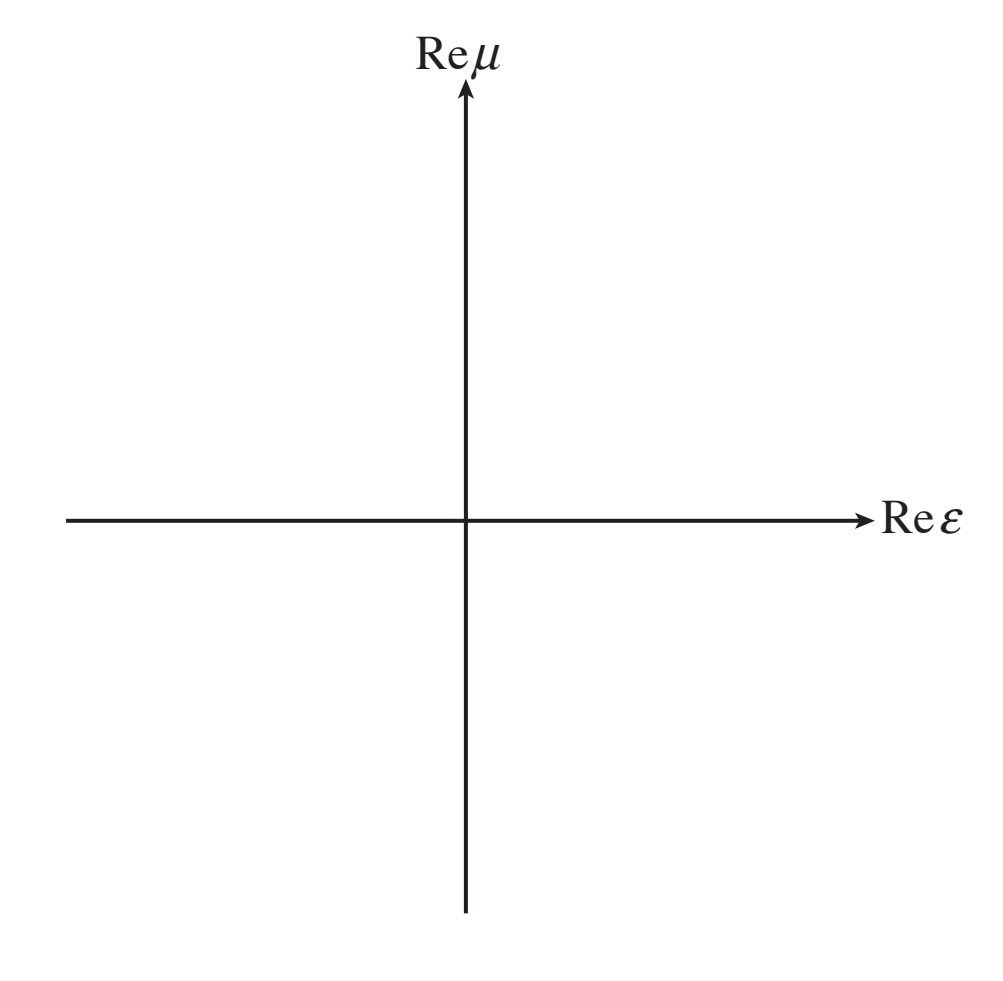
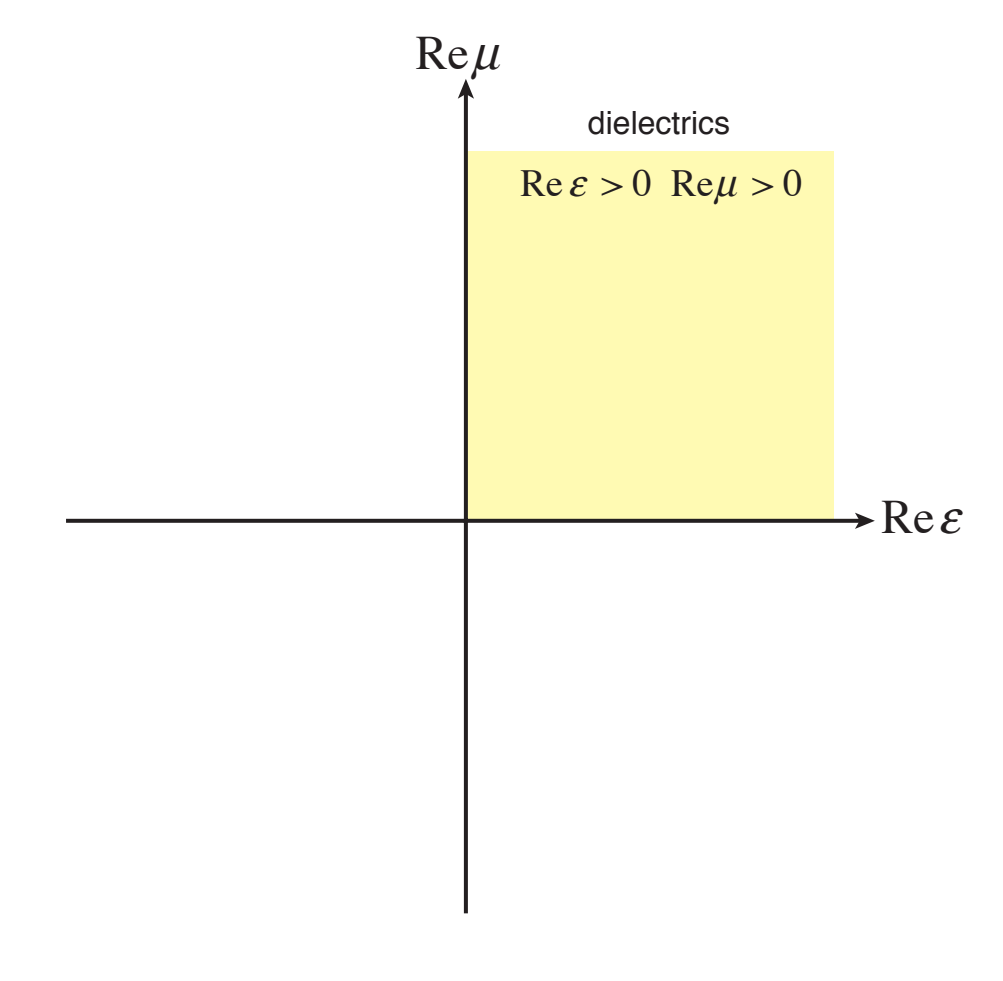


When $\mathrm{Re}(n) < 0,\ k' < 0$, **and so phase velocity reversed!**

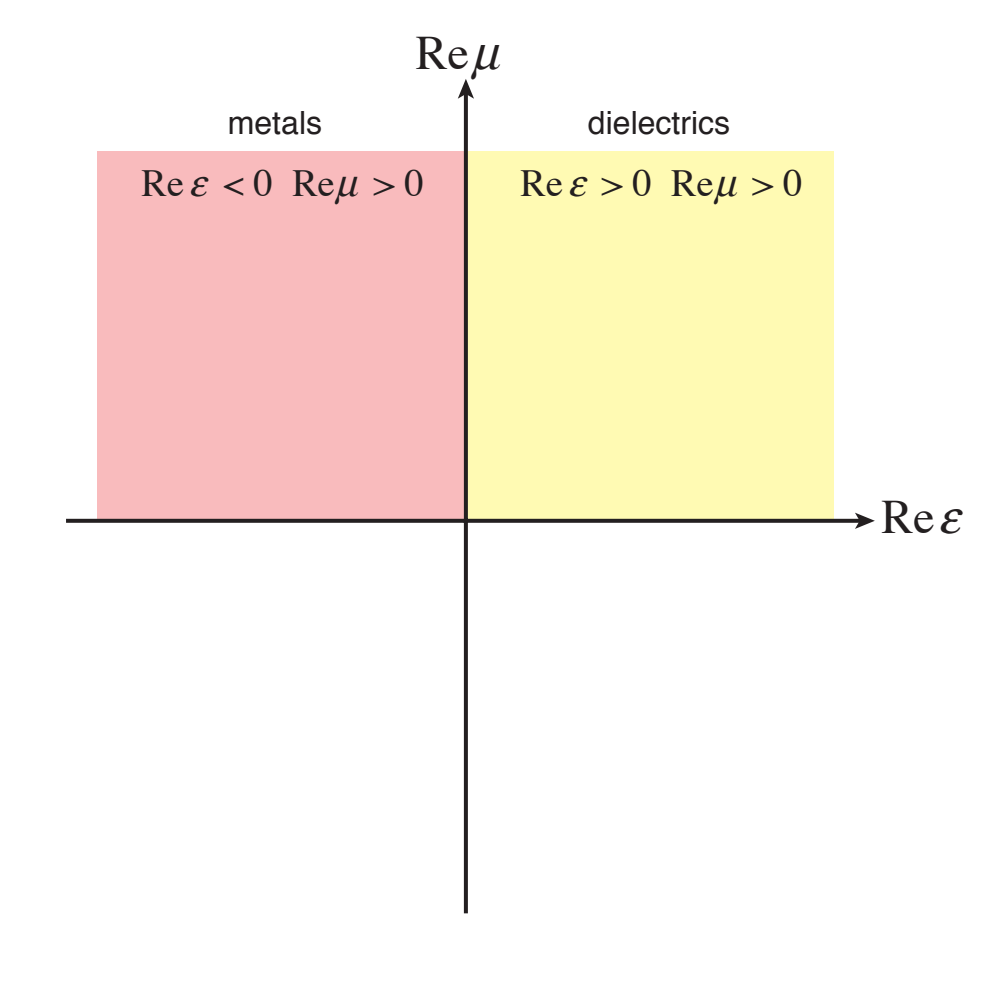
classification of (non-lossy) materials



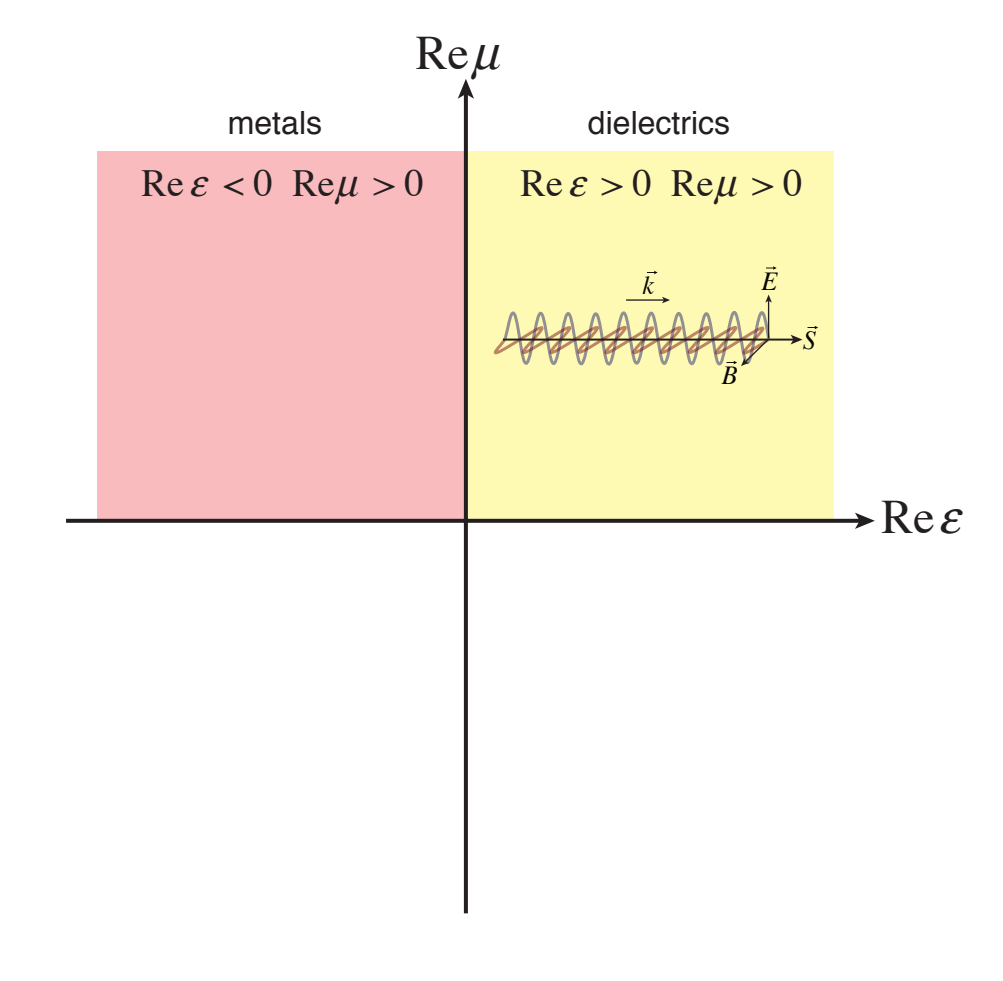
classification of (non-lossy) materials



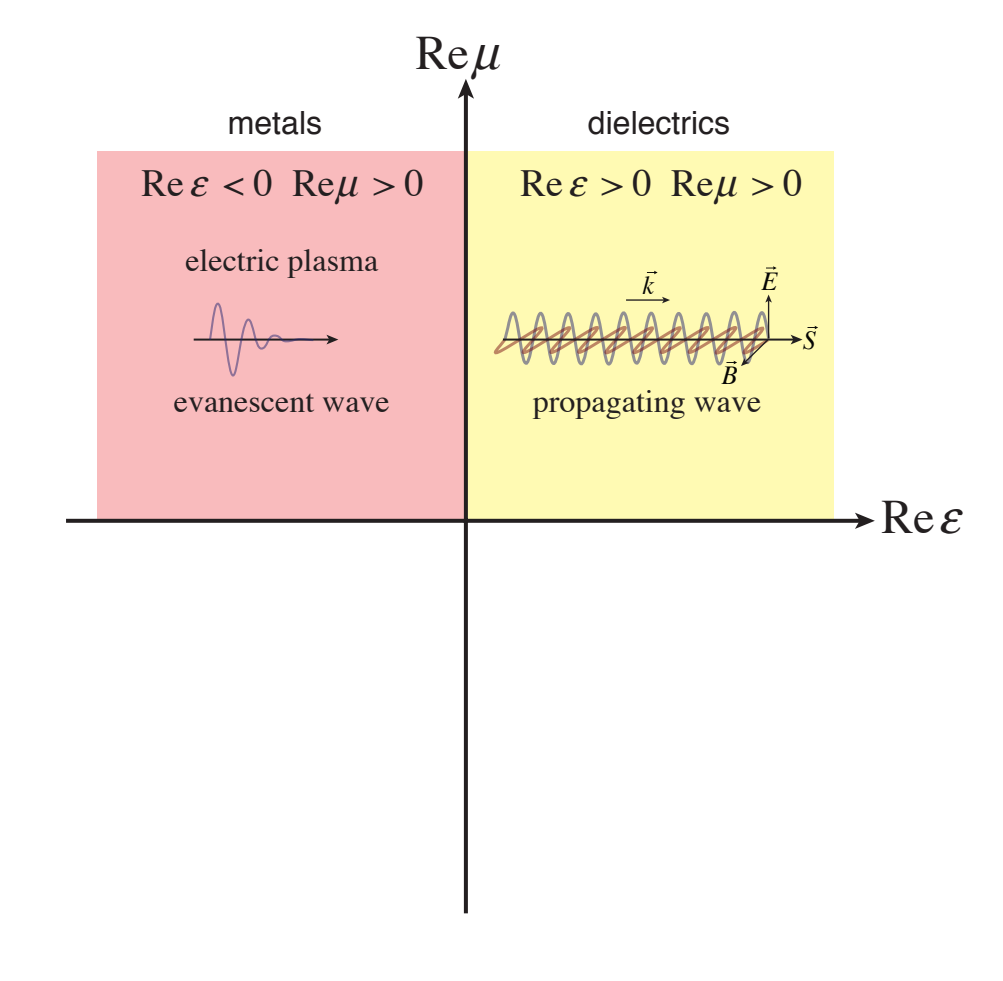
classification of (non-lossy) materials



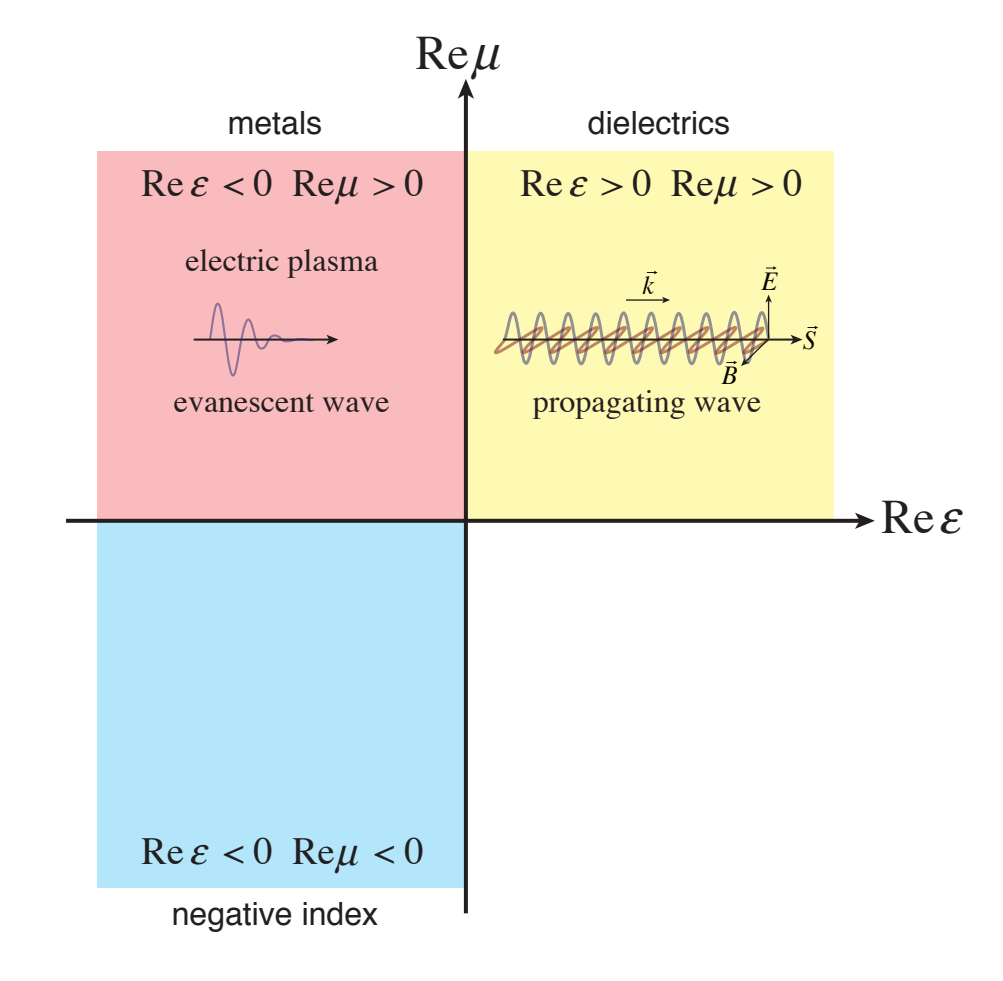
classification of (non-lossy) materials



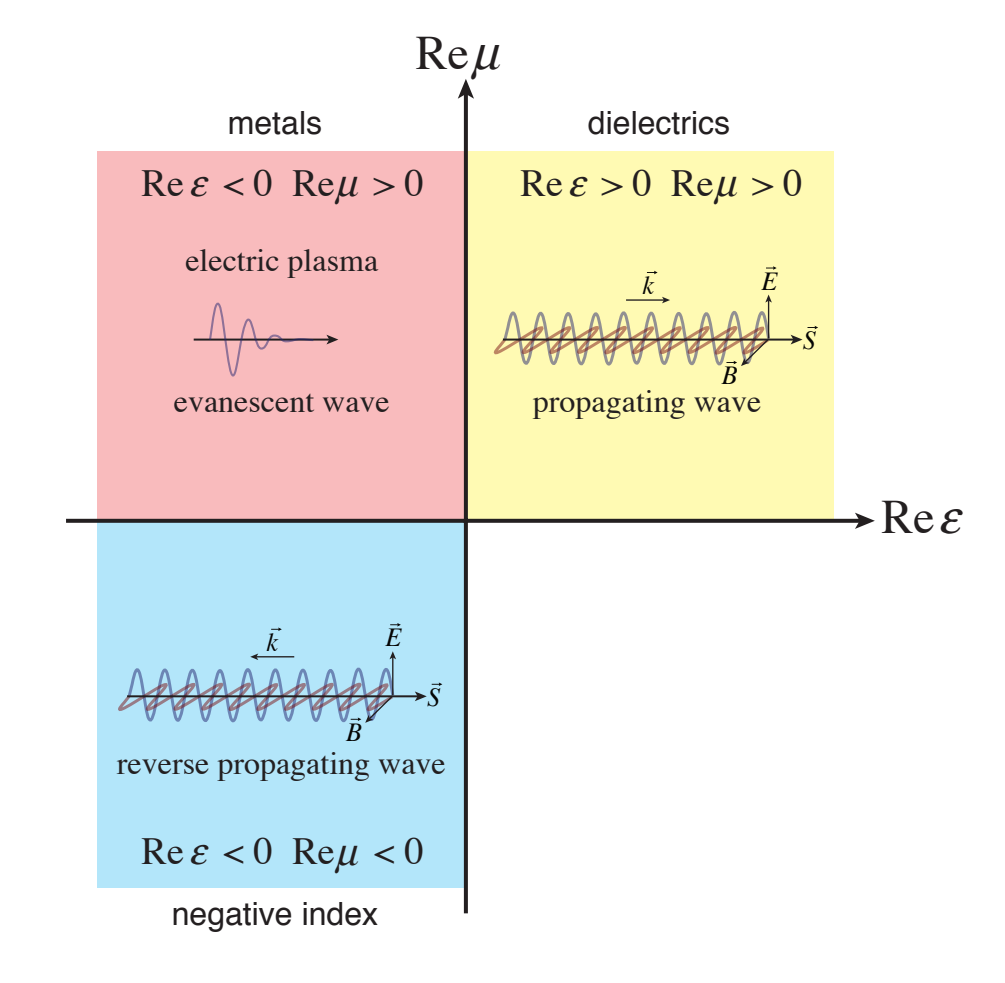
classification of (non-lossy) materials



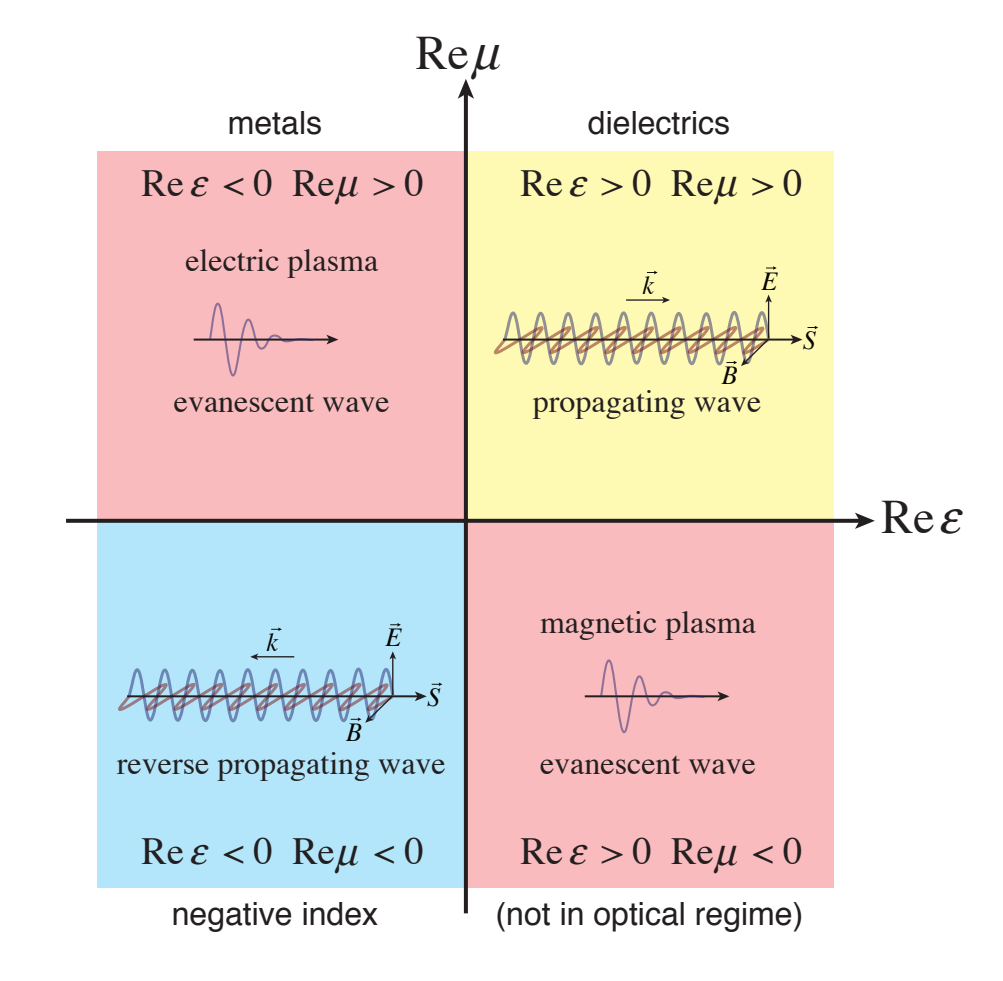
classification of (non-lossy) materials



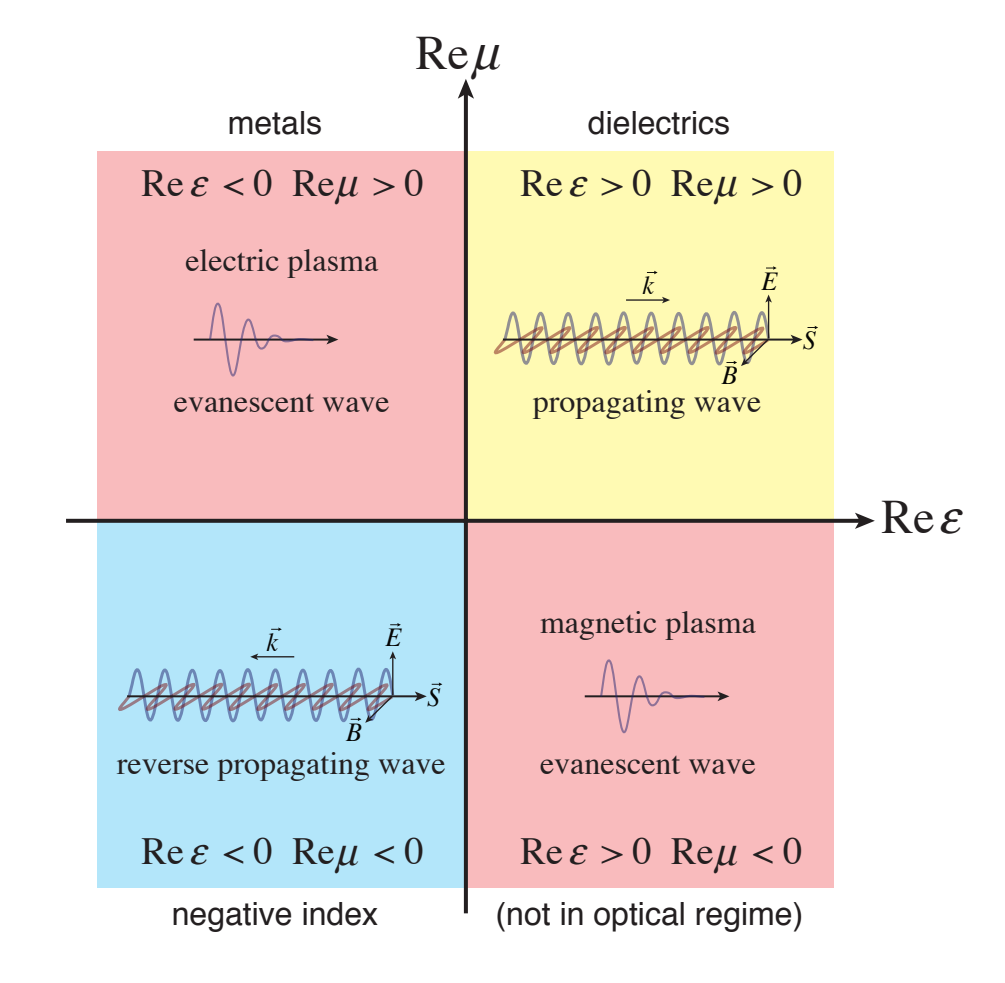
classification of (non-lossy) materials



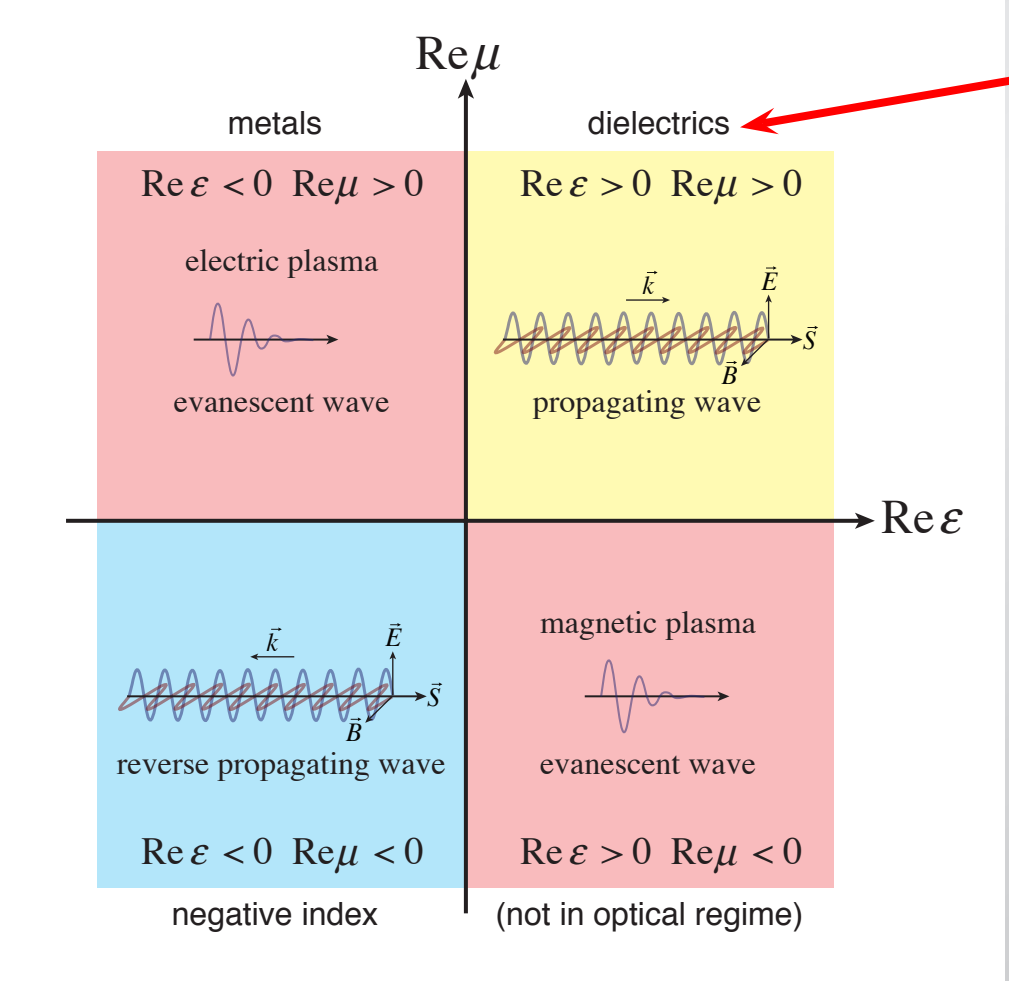
classification of (non-lossy) materials



common materials very limited



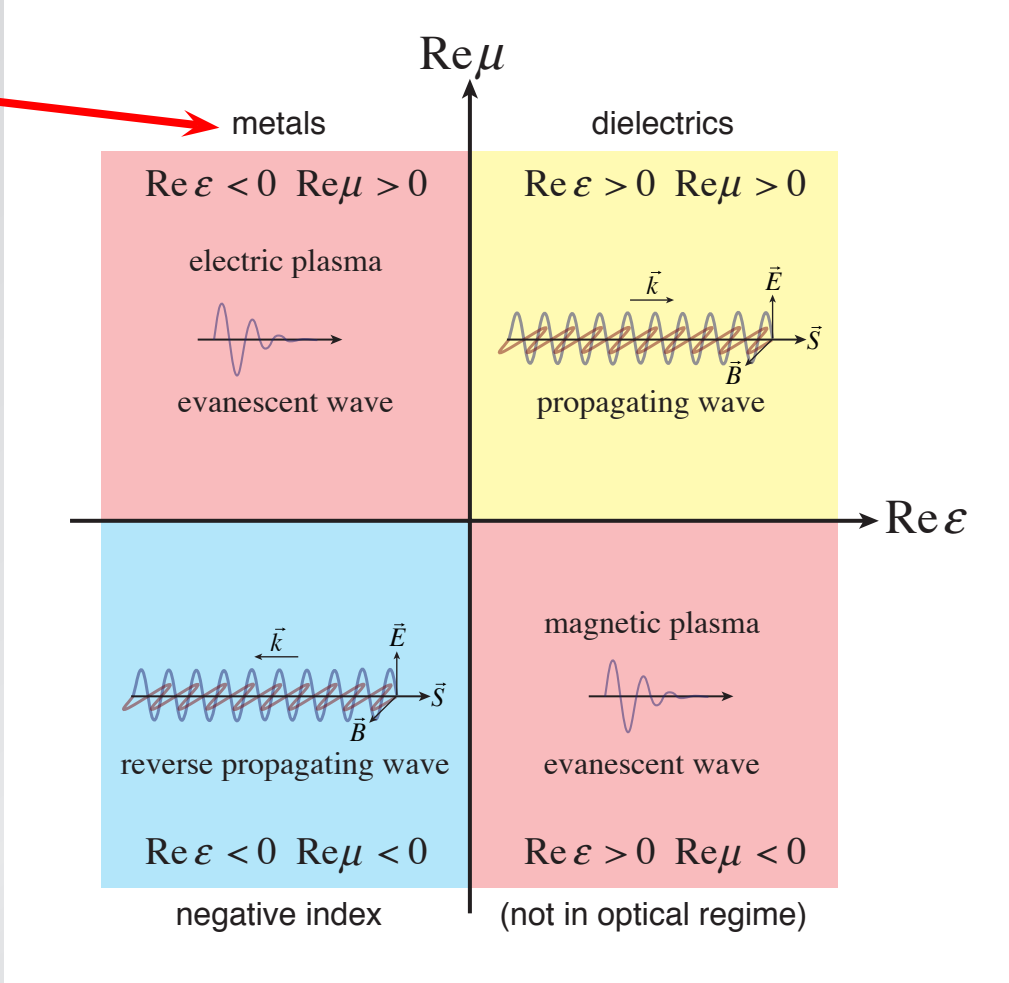
common materials very limited



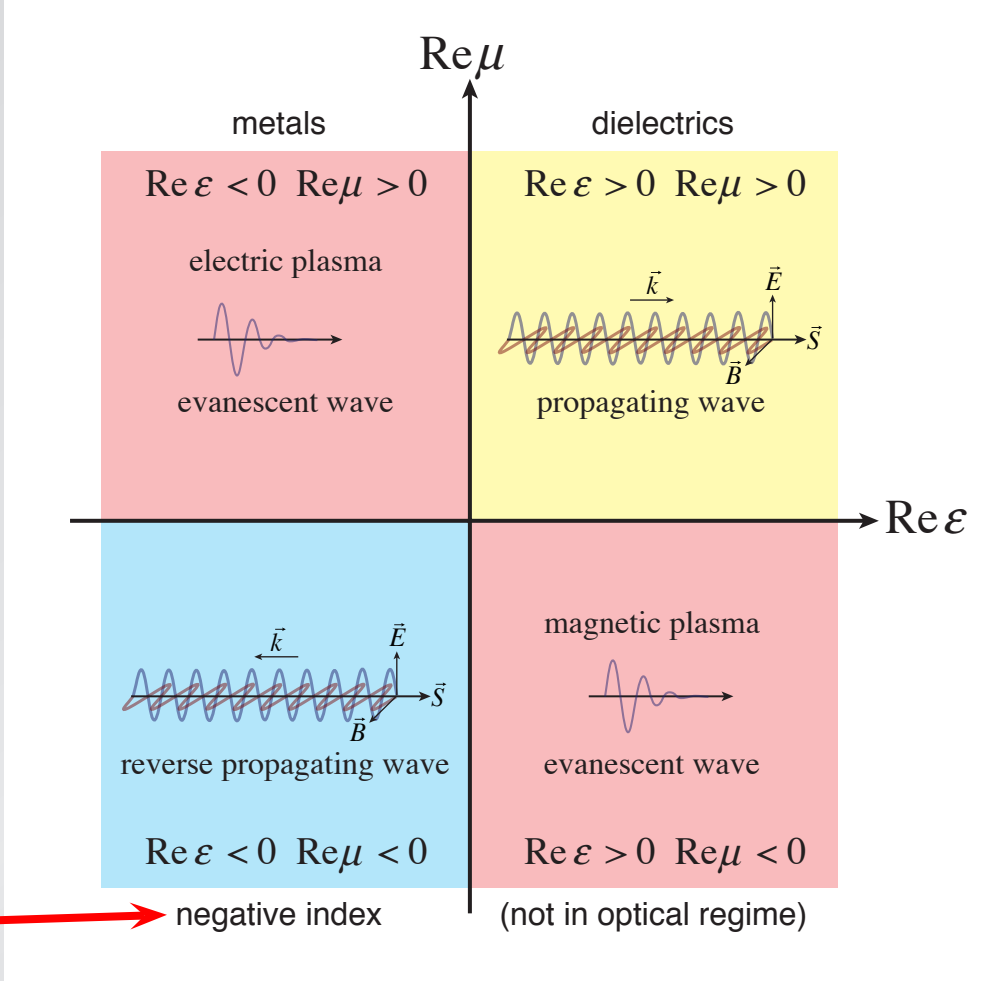
limited by diffraction

common materials very limited

lossy & no propagation

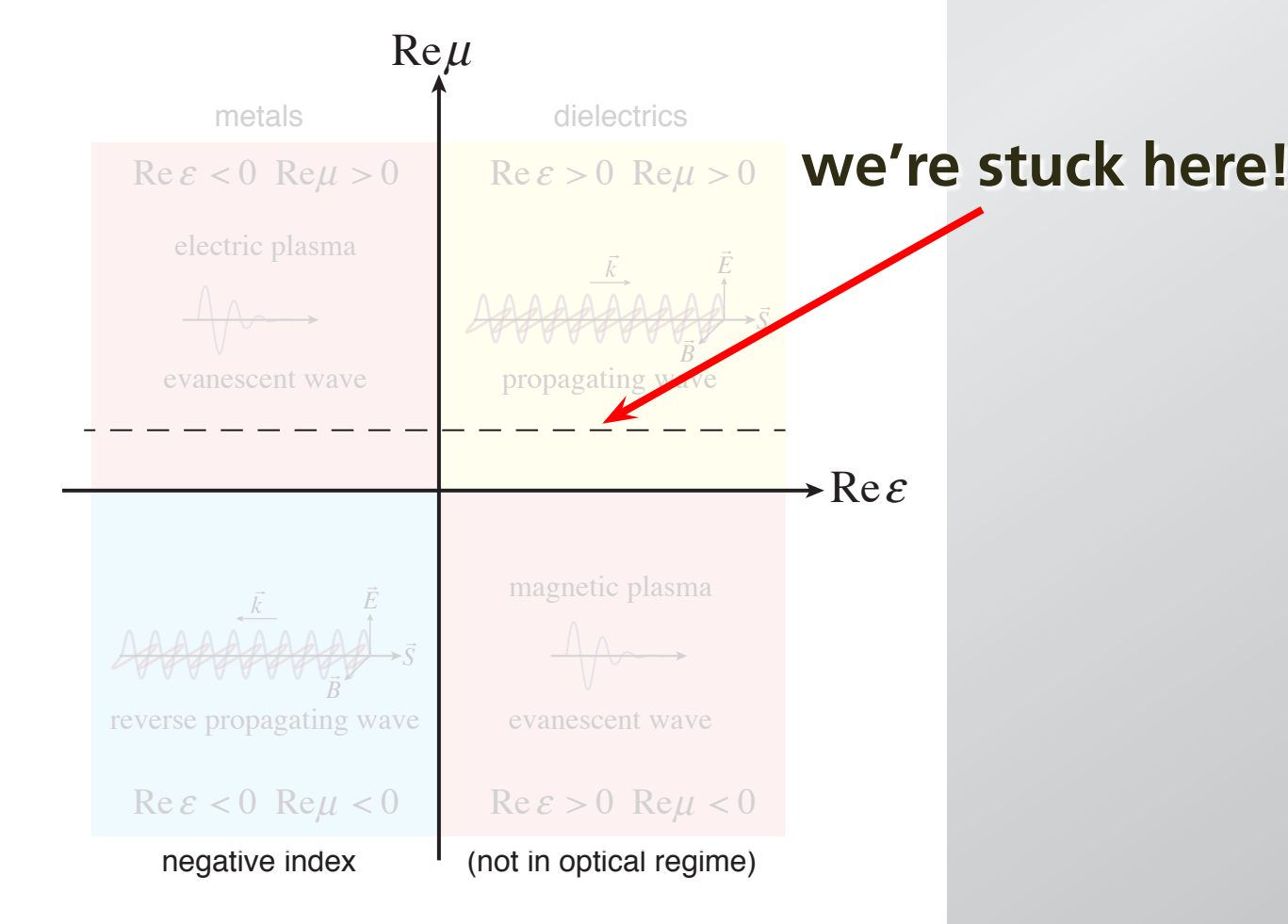


common materials very limited

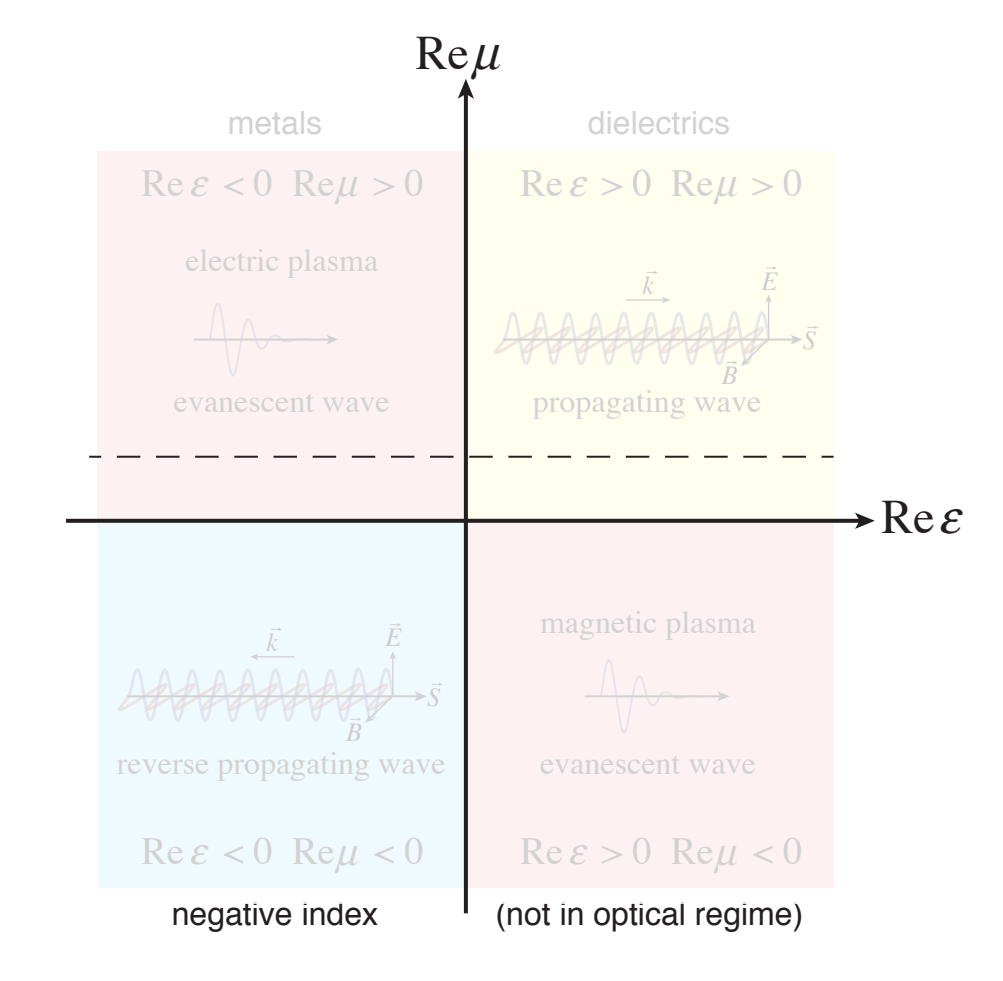


superlensing but…

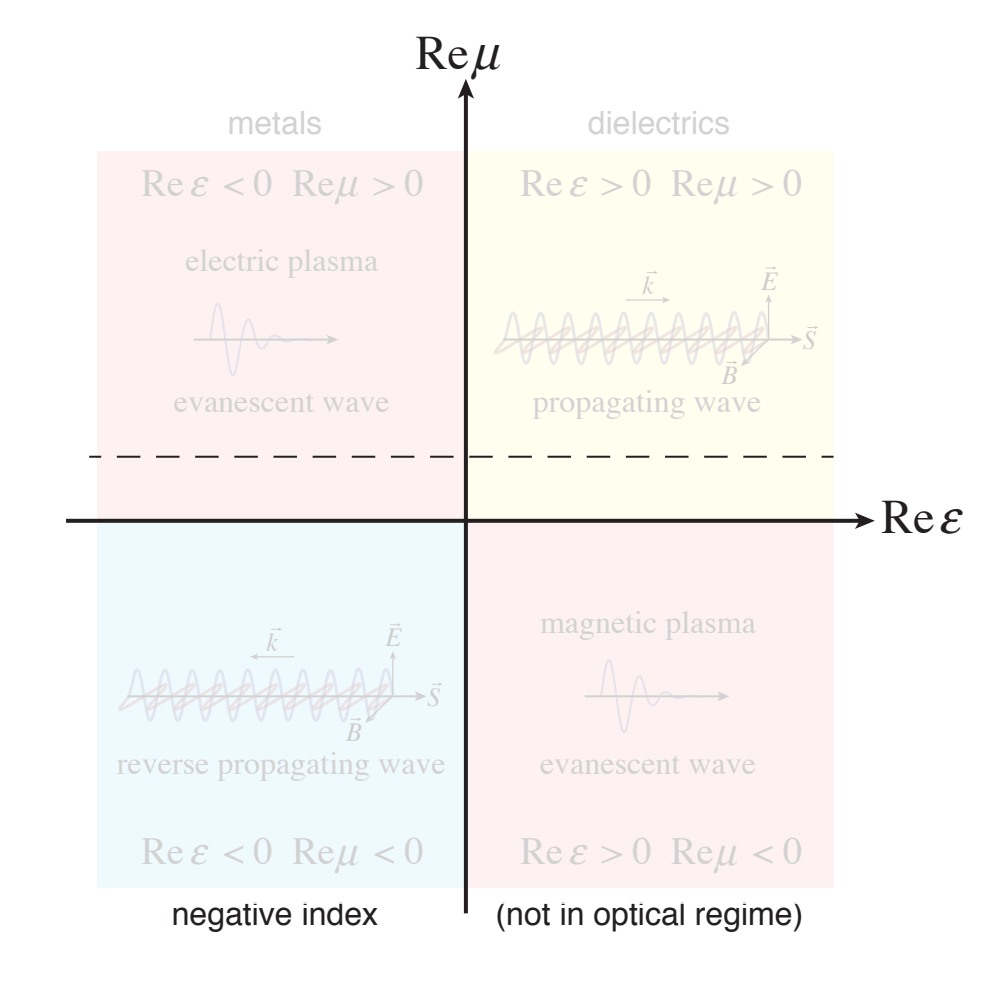
common materials very limited



What happens on the axes?



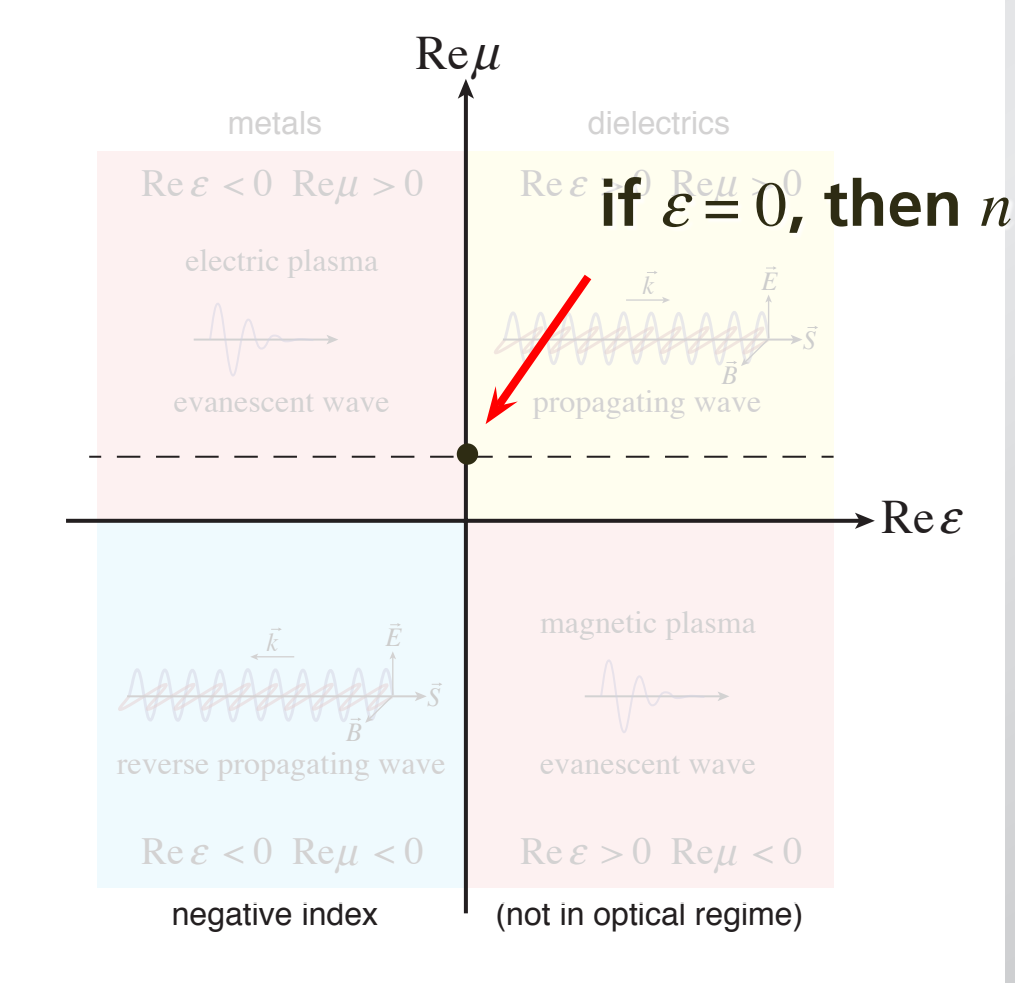
what if we let $\varepsilon = 0$?



1 index

2 zero index

what if we let $\varepsilon = 0$?



if $\varepsilon = 0$, then $n = 0$!

Q: If $n = 0$, which of the following is true?

1. the frequency goes to zero.
2. the phase velocity becomes infinite.
3. both of the above.
4. neither of the above.

1 index

2 zero index

wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

solution

$$\vec{E} = \vec{E}_o \, e^{i\,(kx - \omega t)}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}}\, c = \frac{1}{n}\, c$$

wave equation

$$\nabla^2 \vec{E} - \cancel{\frac{\mu \epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}} = 0$$

solution

$$\vec{E} = \vec{E}_o \, e^{i\,(kx - \omega t)}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon \mu}} c = \frac{1}{n} c$$

1 index

2 zero index

wave equation

$$\nabla^2 \vec{E} - \cancel{\frac{\mu \epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}} = 0$$

solution

$$\vec{E} = \vec{E}_o \, e^{i(kx - \omega t)} \longrightarrow \vec{E} = \vec{E}_o \, e^{-i\omega t}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}} c = \frac{1}{n} c$$

1 index

2 zero index

wave equation

$$\nabla^2 \vec{E} - \cancel{\frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}} = 0$$

solution

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where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}}\, c = \frac{1}{n}\, c \longrightarrow \infty$$

1 index

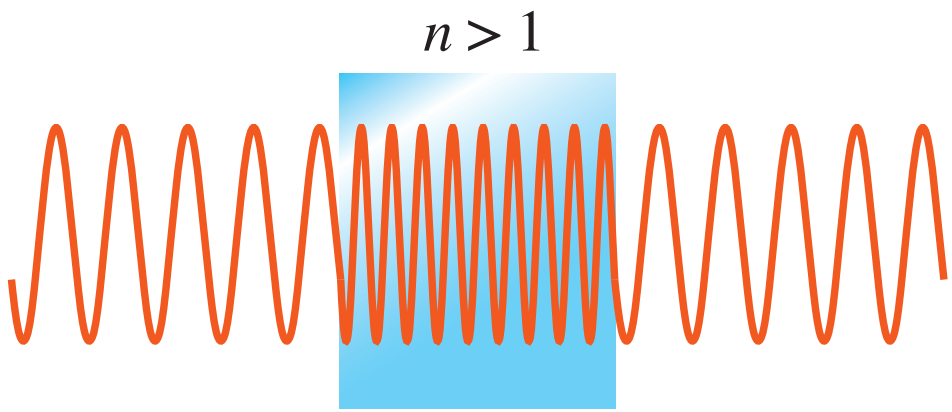
2 zero index

Q: If $n = 0$, which of the following is true?

1. the frequency goes to zero.
2. the phase velocity becomes infinite. ✔
3. both of the above.
4. neither of the above.

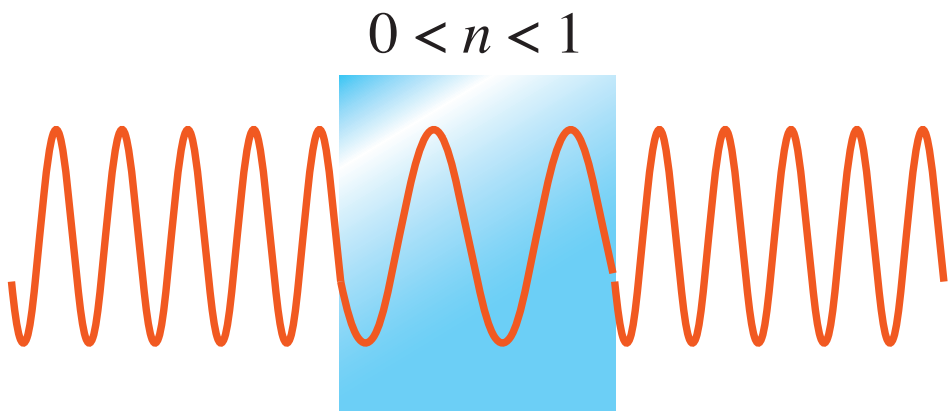
1 index

2 zero index



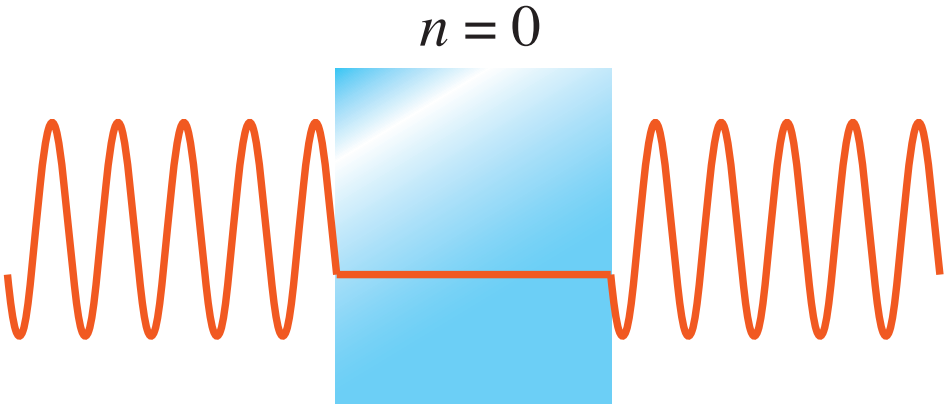
1 index

2 zero index



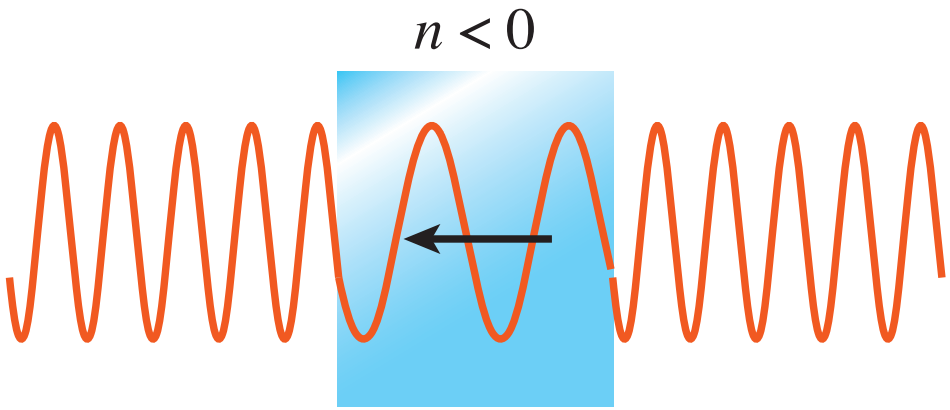
1 index

2 zero index



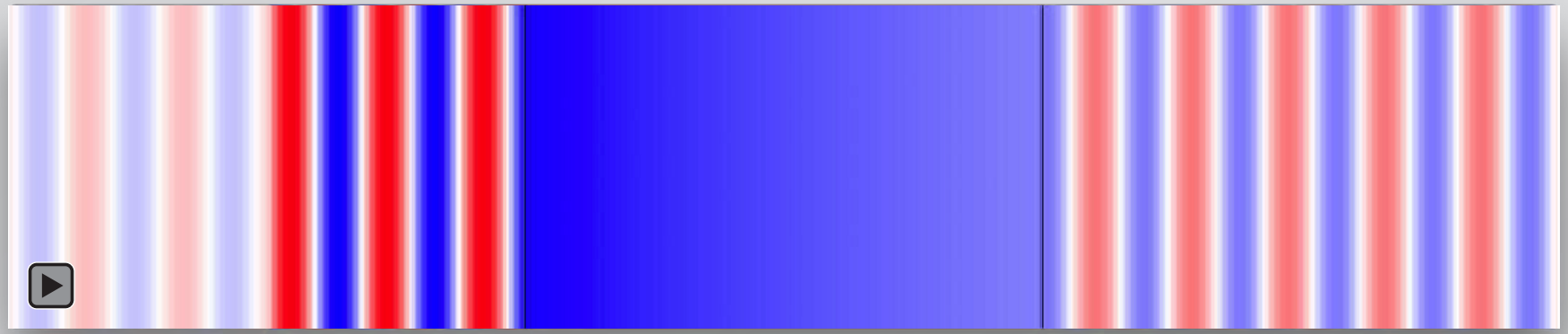
1 index

2 zero index



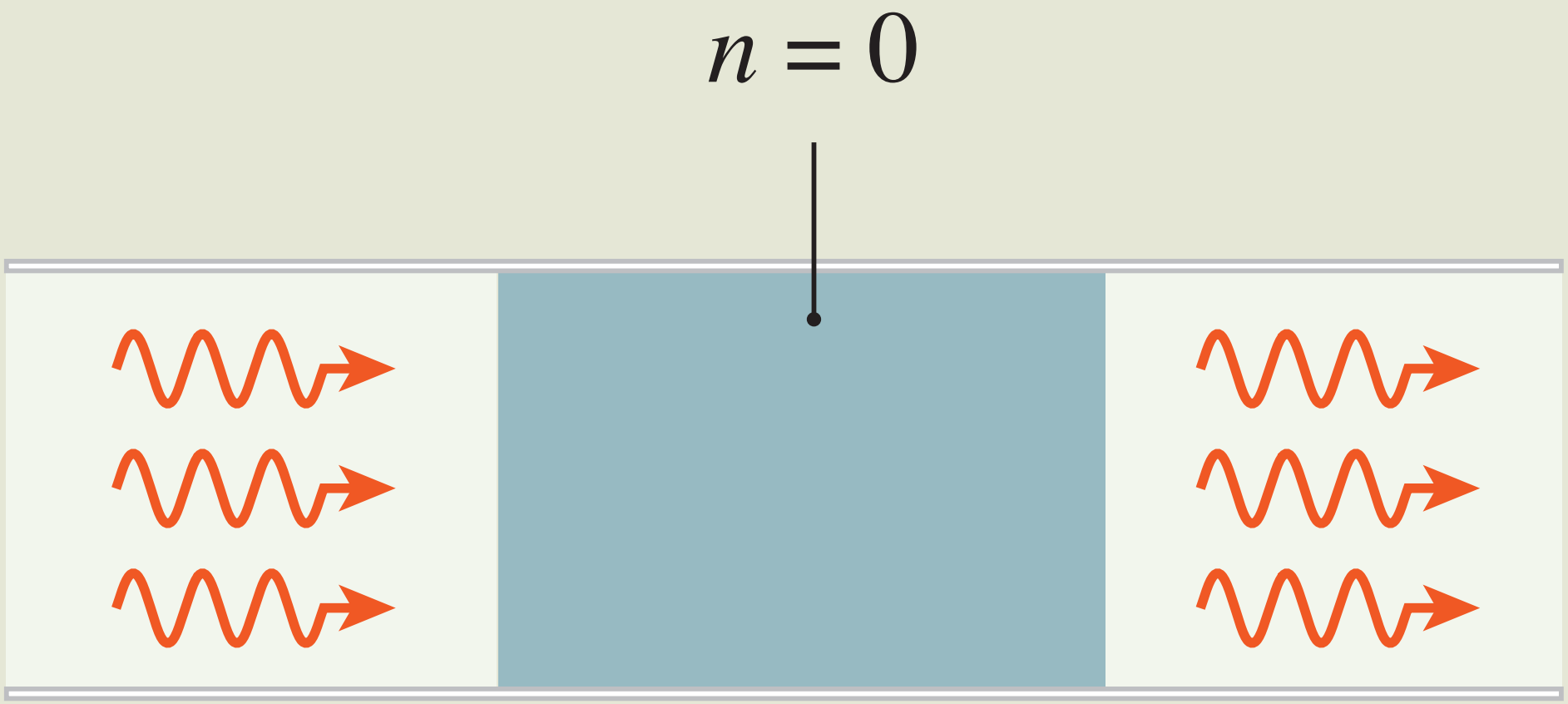
1 index

2 zero index



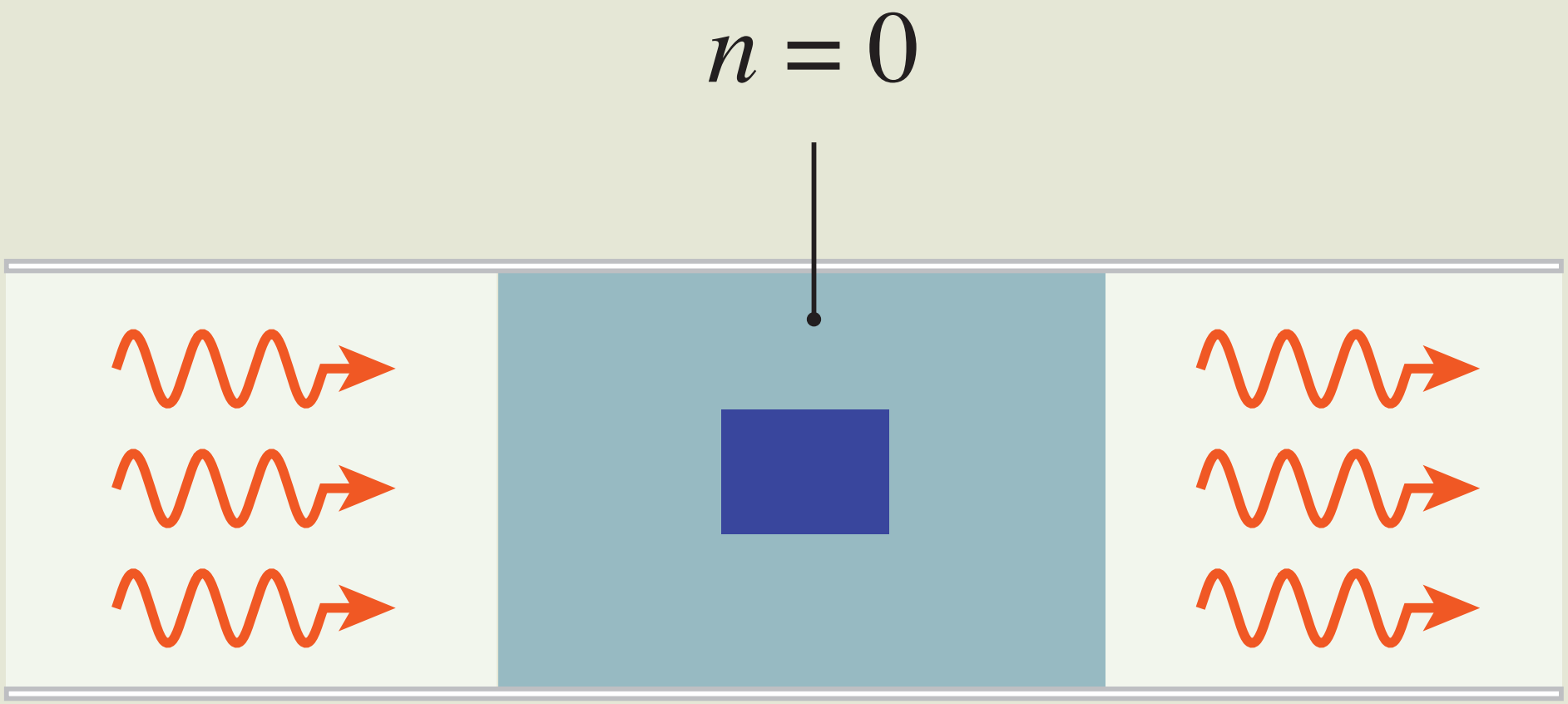
1 index

2 zero index



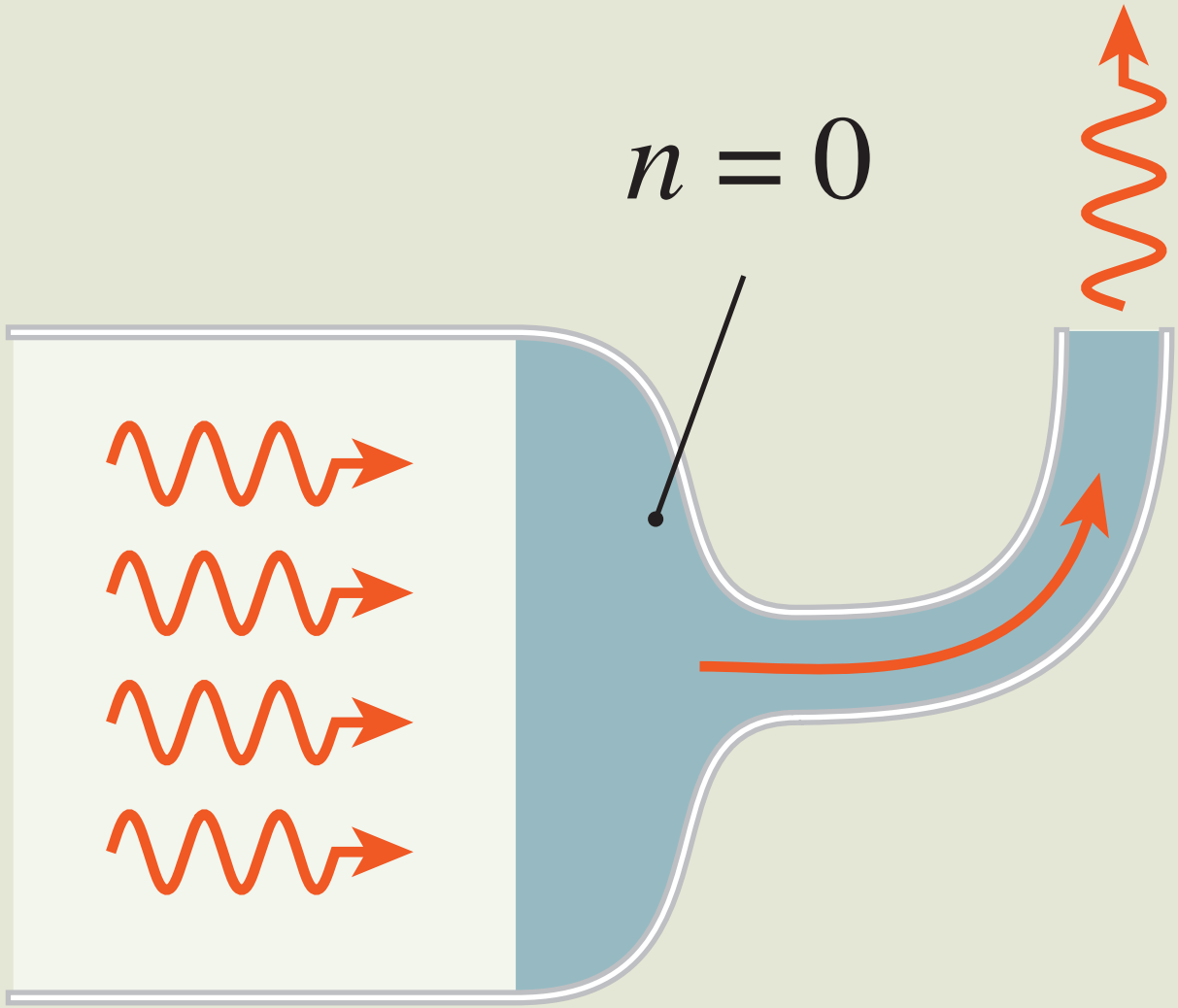
1 index

2 zero index



1 index

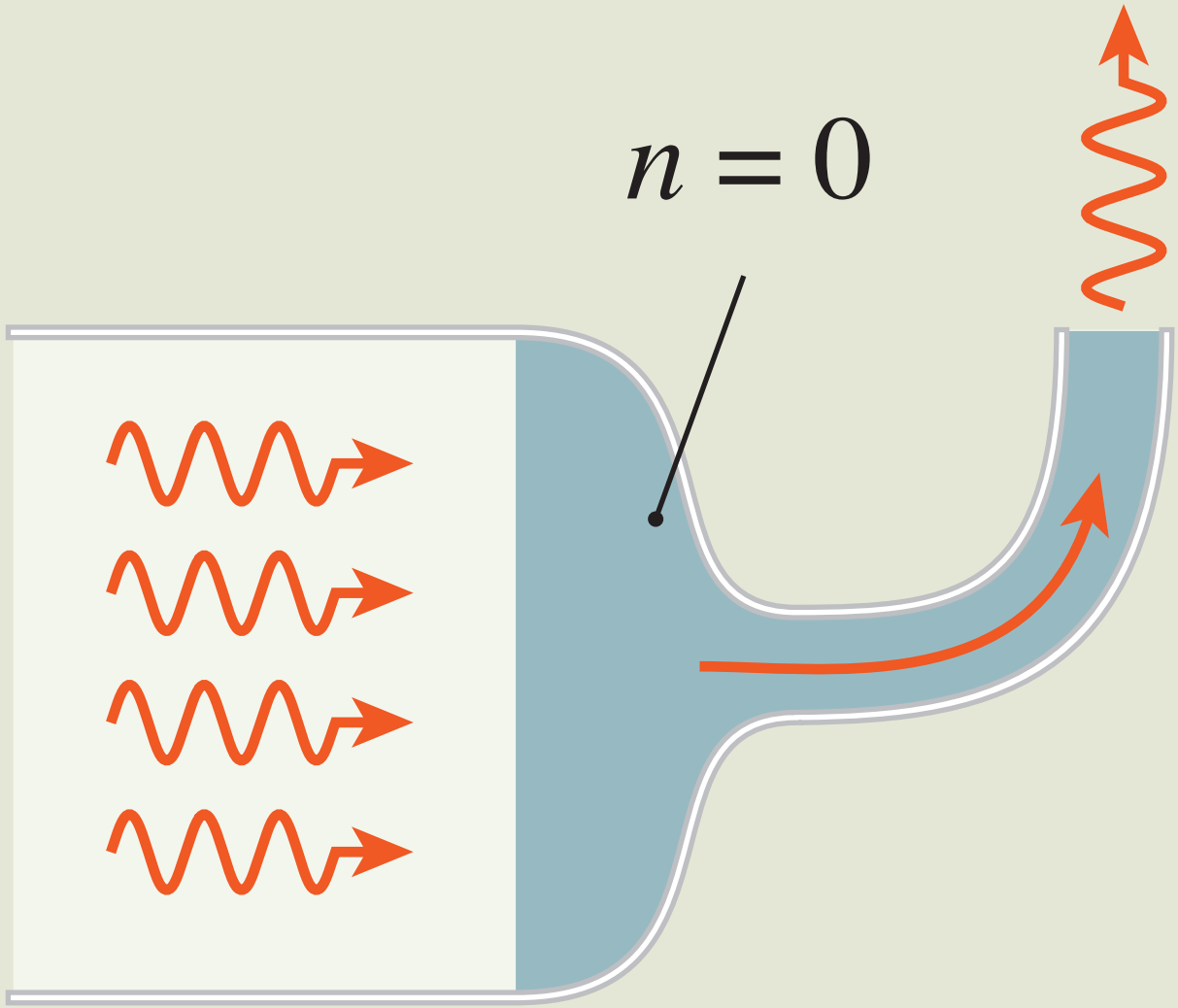
2 zero index



1 index

2 zero index

“tunneling with infinite decay length”



1 index

2 zero index

how?

$$n = \sqrt{\varepsilon\mu}$$

1 index

2 zero index

how?

$$n = \sqrt{\varepsilon\mu}$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

1 index

2 zero index

how?

$$n = \sqrt{\varepsilon\mu}$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

how?

$$\varepsilon \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

1 index

2 zero index

how?

$$\varepsilon \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to \infty$$

1 index

2 zero index

how?

$$\varepsilon \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \to 1$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to \infty$$

1 index

2 zero index

how?

$$\mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

how?

$$\mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to 0$$

1 index 2 zero index

how?

$$\mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \to -1$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to 0$$

1 index

2 zero index

how?

$$\varepsilon, \mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

finite!

1 index

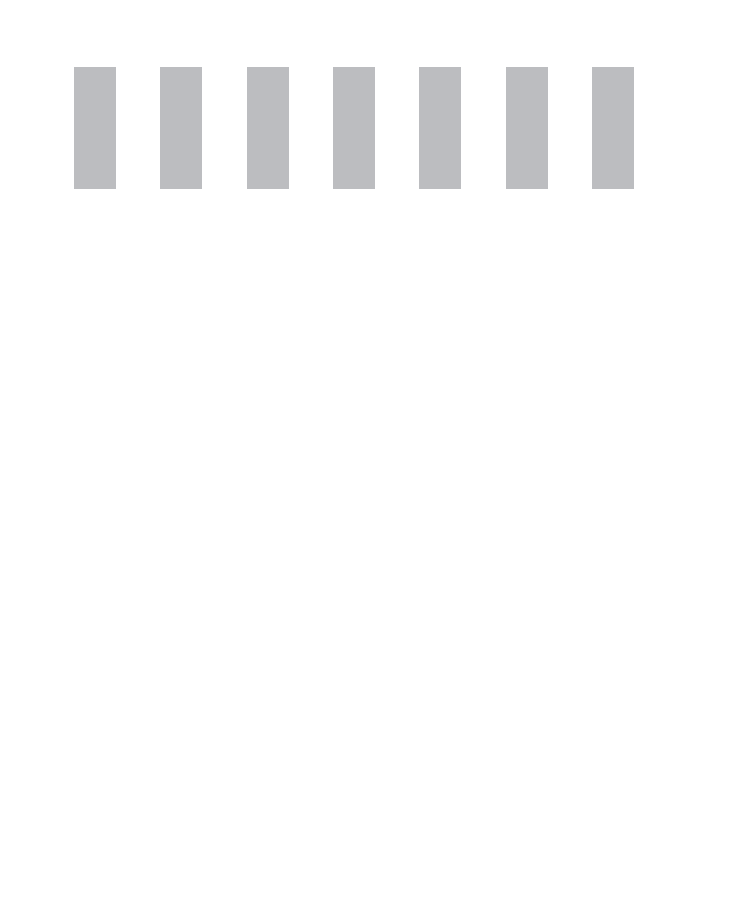
2 zero index

but $\mu \neq 1$ requires a magnetic response!

Engineering a magnetic response

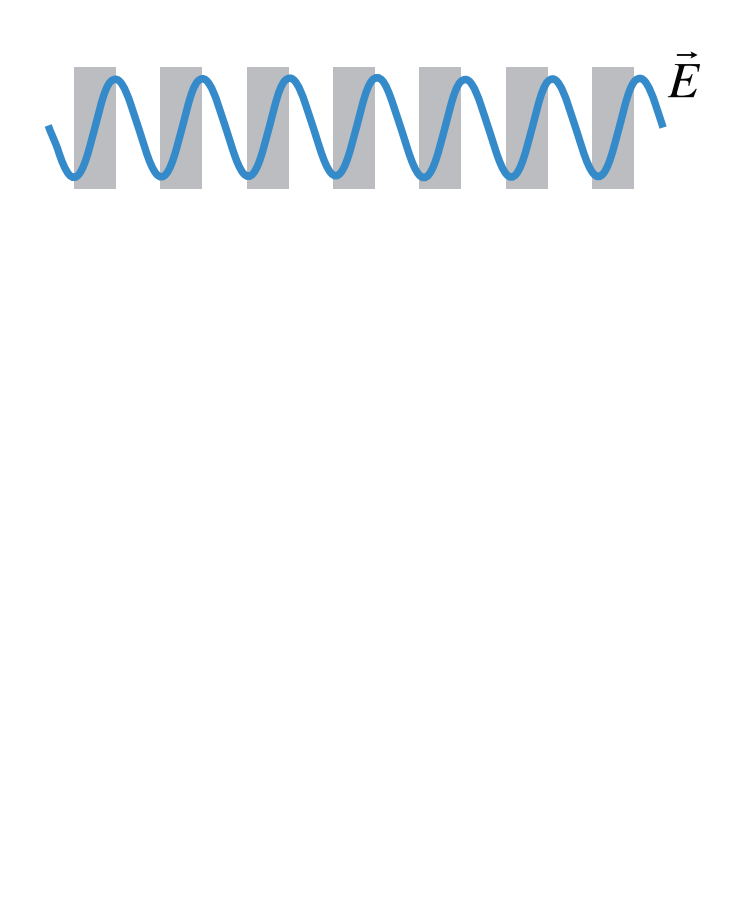
Engineering a magnetic response

use array of dielectric rods



Engineering a magnetic response

incident electromagnetic wave ($\lambda_{\text{eff}} \approx a$)

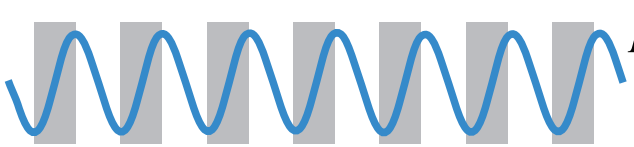


1 index

2 zero index

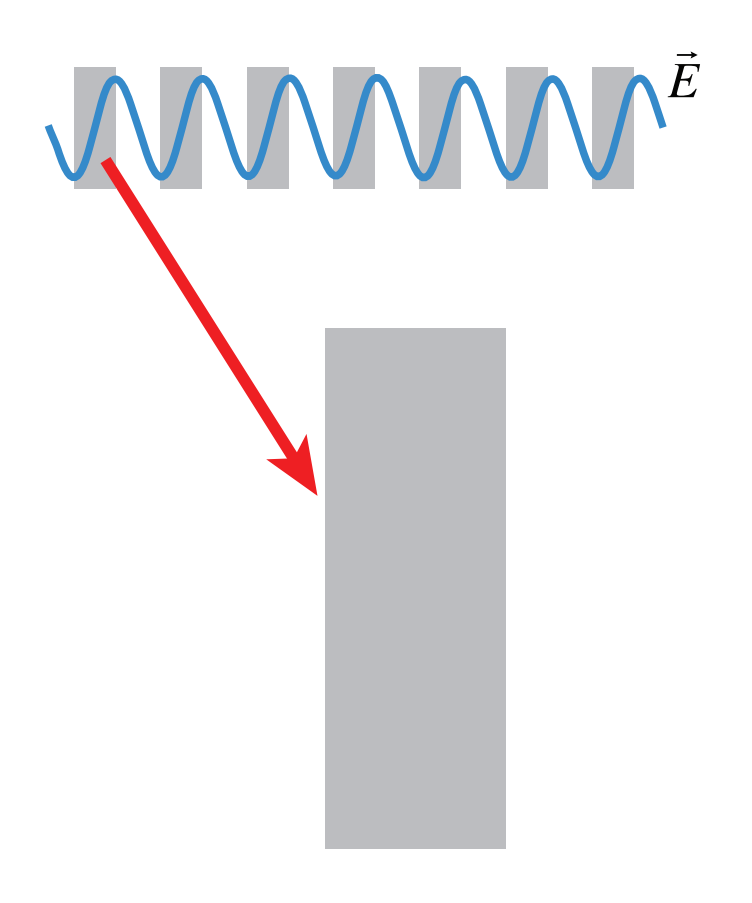
Engineering a magnetic response

produces an electric response…



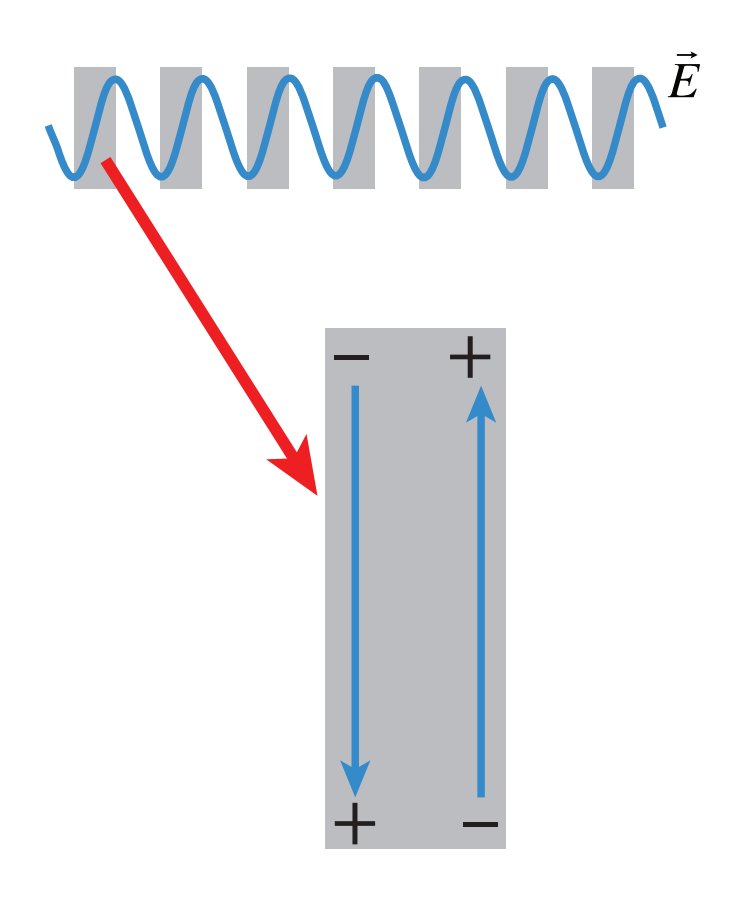
Engineering a magnetic response

… but different electric fields front and back…



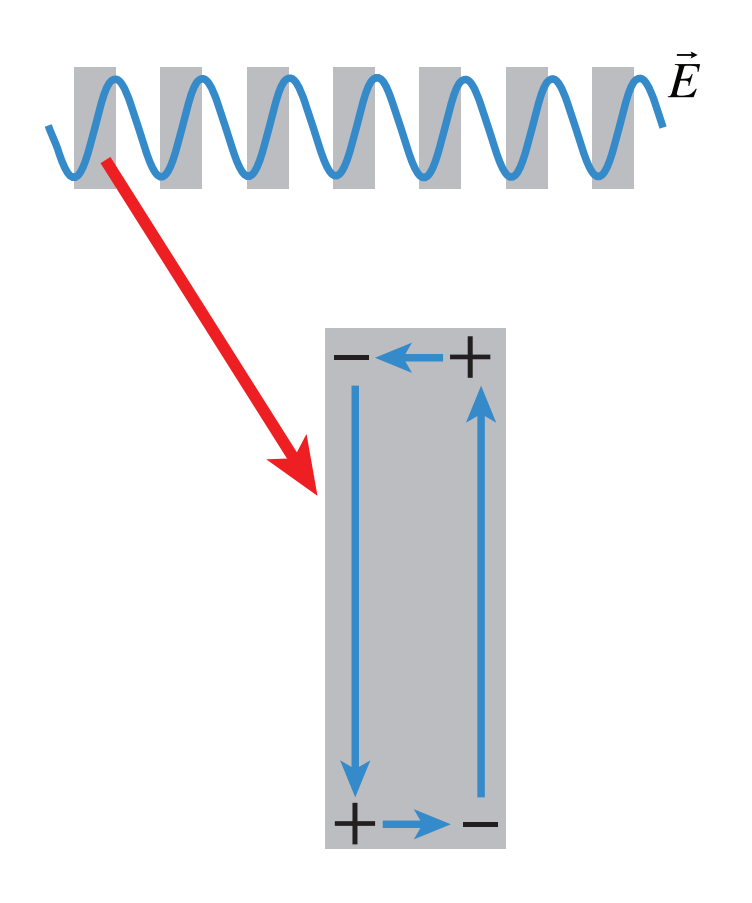
Engineering a magnetic response

…induce different polarizations on opposite sides…



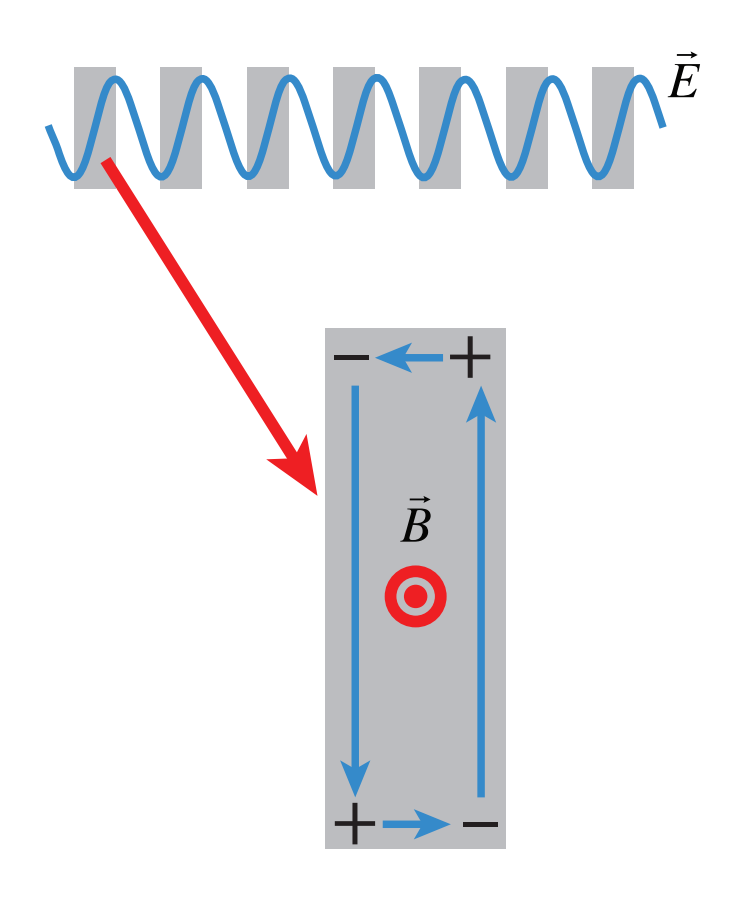
Engineering a magnetic response

…causing a current loop…



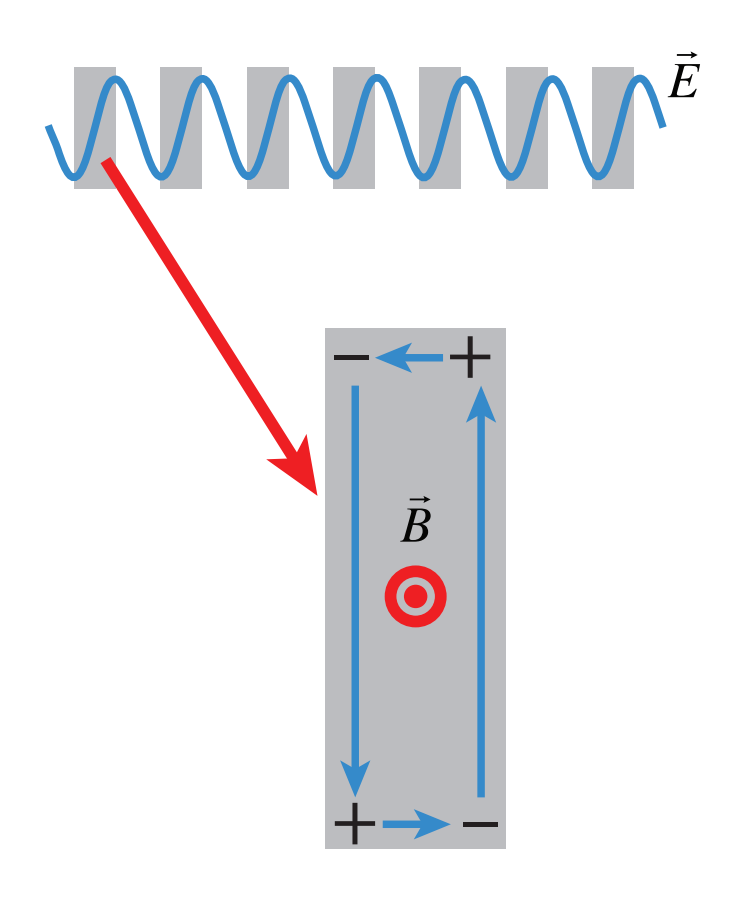
Engineering a magnetic response

…which, in turn, produces an induced magnetic field



Engineering a magnetic response

adjust design so electrical and magnetic resonances coincide



Engineering a magnetic response

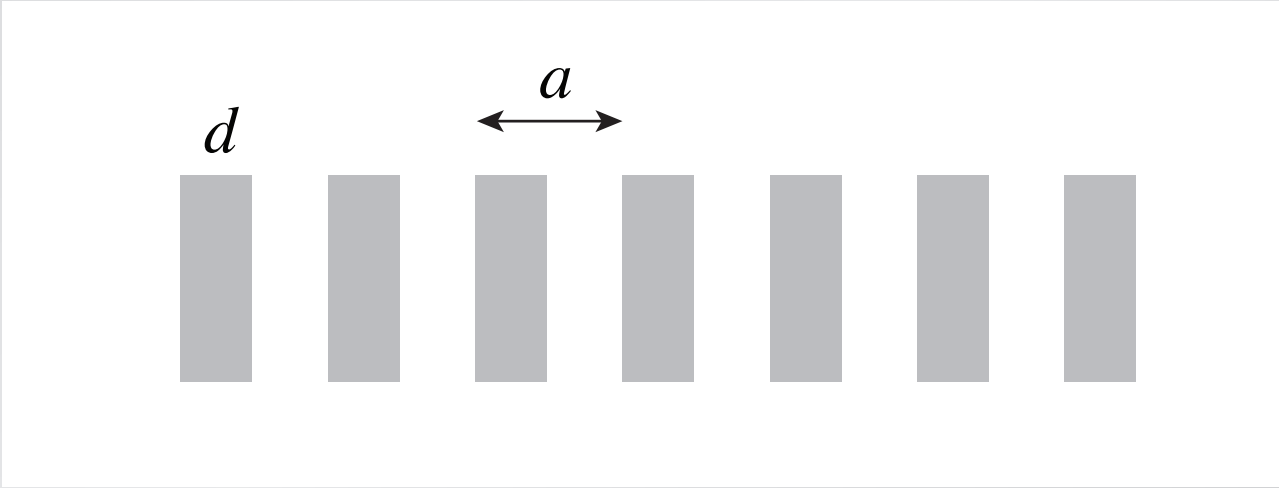
adjustable parameters

Engineering a magnetic response

adjustable parameters

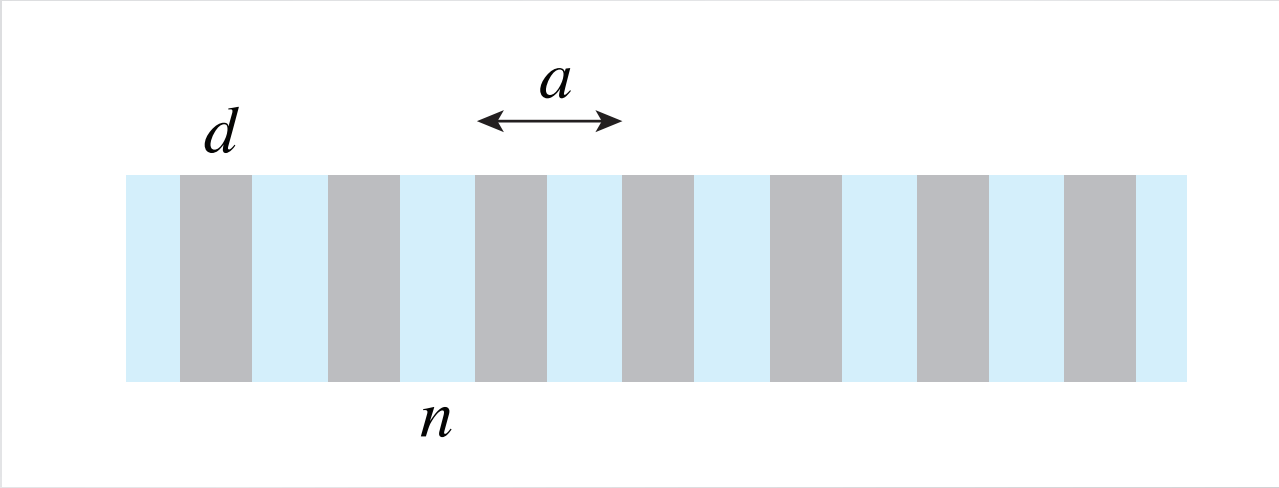
Engineering a magnetic response

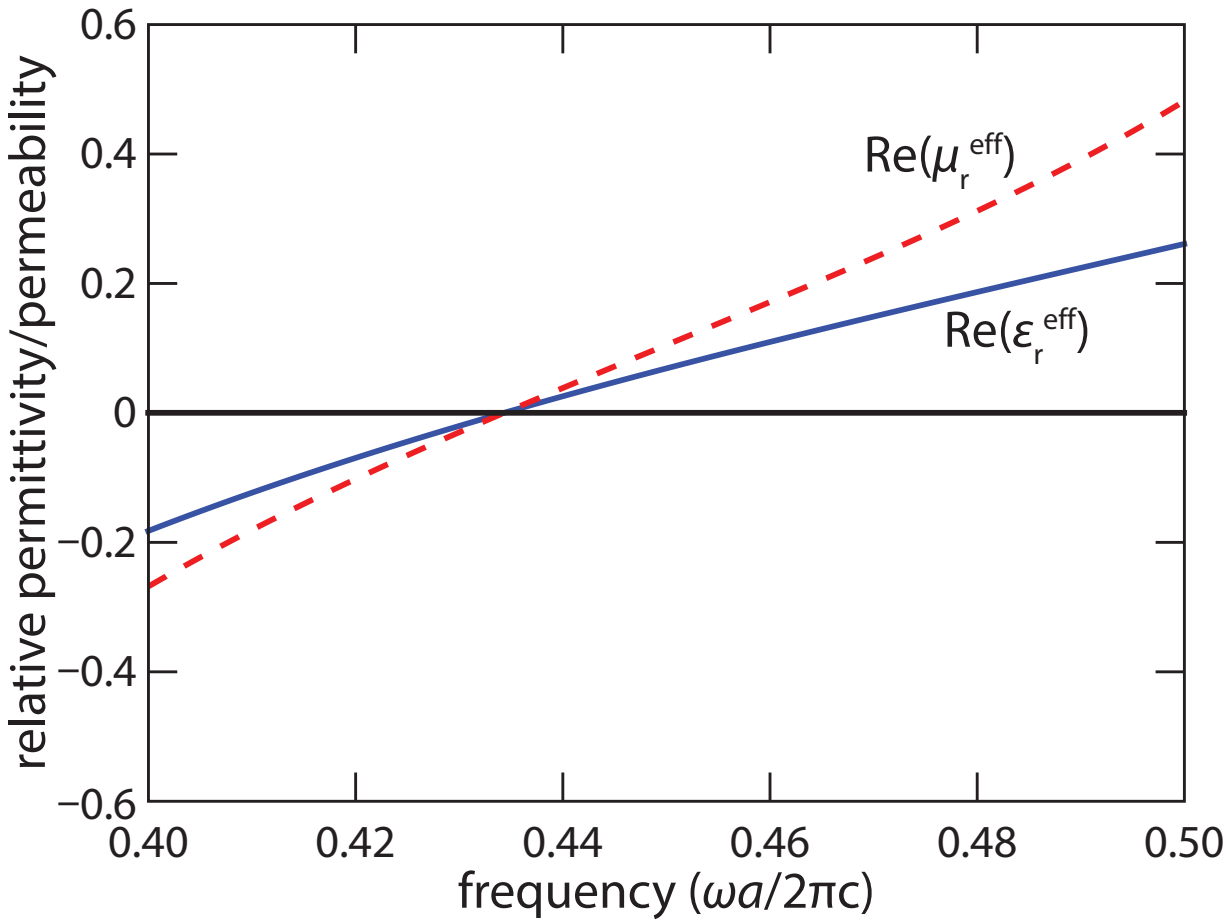
adjustable parameters

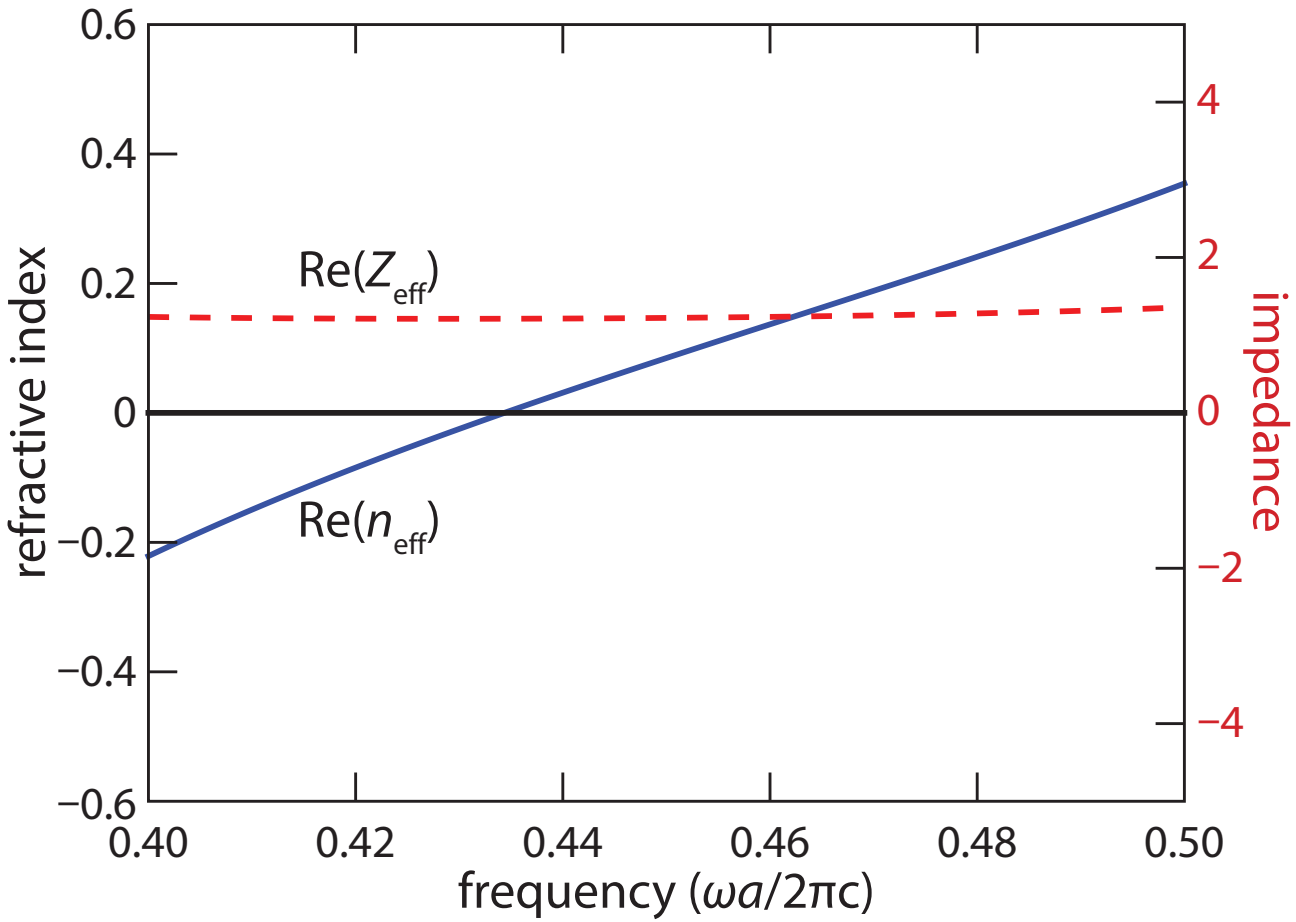


Engineering a magnetic response

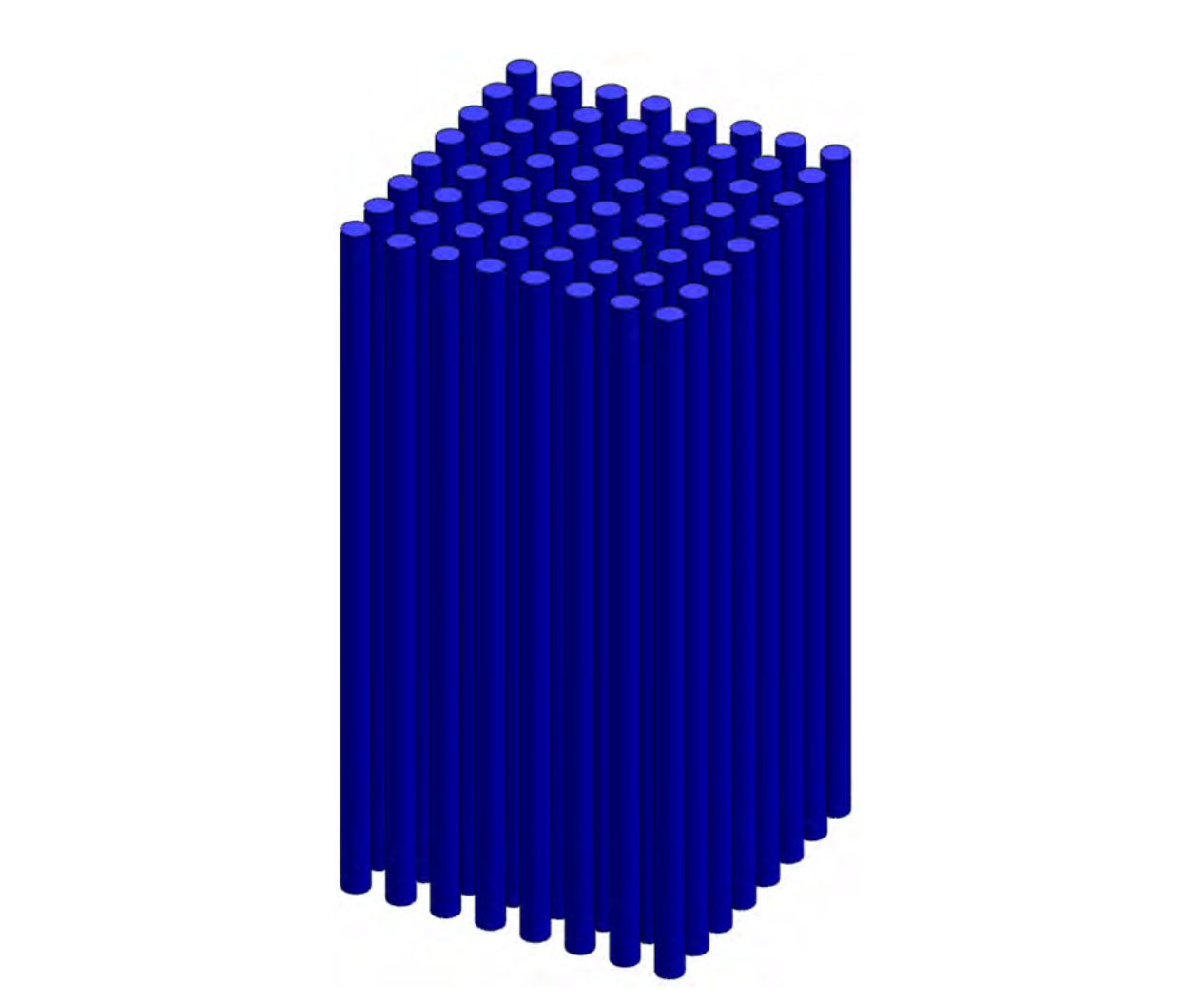
adjustable parameters




0.6
0.4
0.2
0
−0.2
−0.4
−0.6
relative permittivity/permeability
Re(μ_r^{eff})
Re(ε_r^{eff})
0.40
0.42
0.44
0.46
0.48
0.50
frequency ($\omega a/2\pi c$)


0.6
0.4
0.2
0
−0.2
−0.4
−0.6
refractive index
4
2
0
−2
−4
impedance
Re(Z_{eff})
Re(n_{eff})
0.40
0.42
0.44
0.46
0.48
0.50
frequency ($\omega a/2\pi c$)

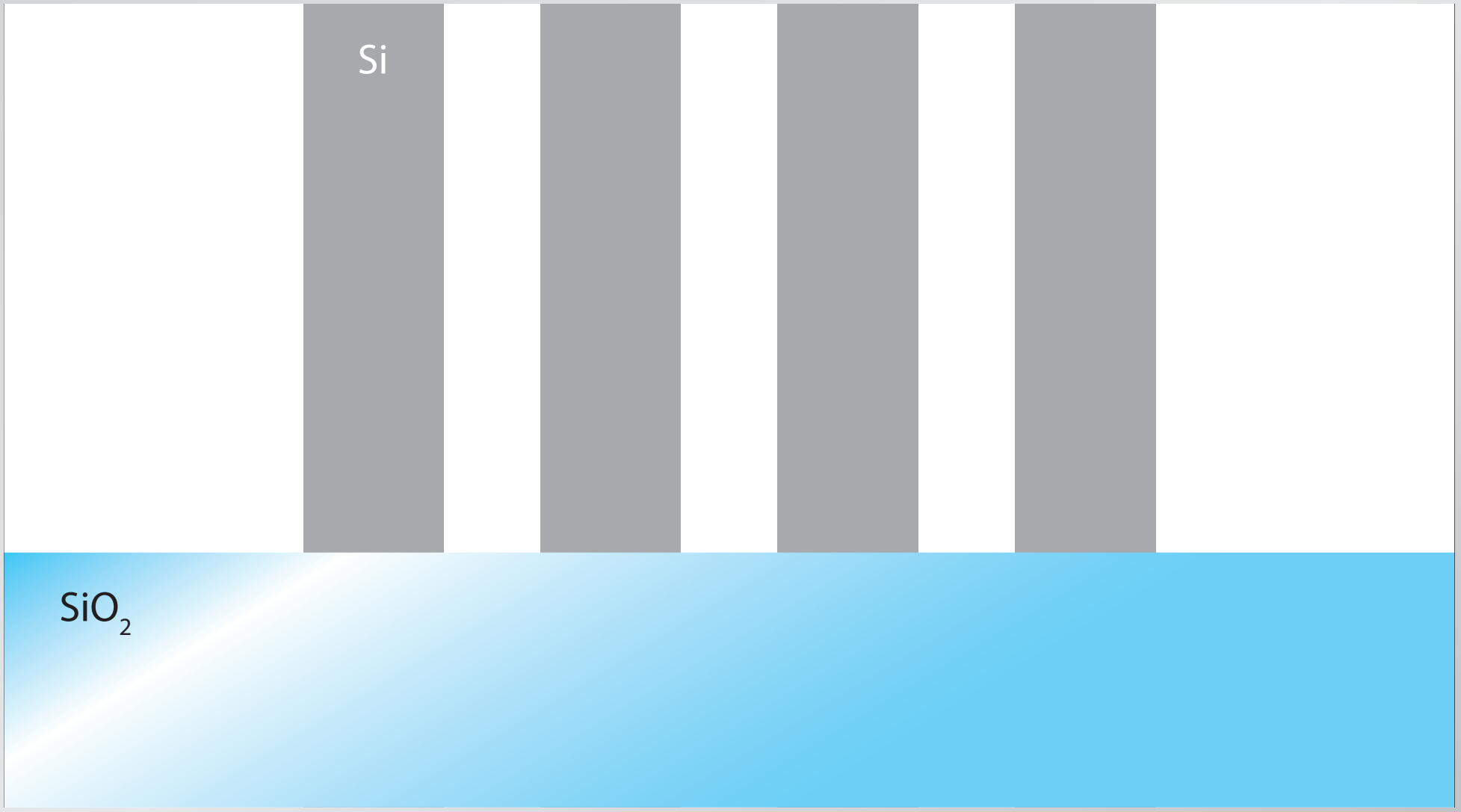
How to fabricate?



1 index 2 zero index 3 experiments

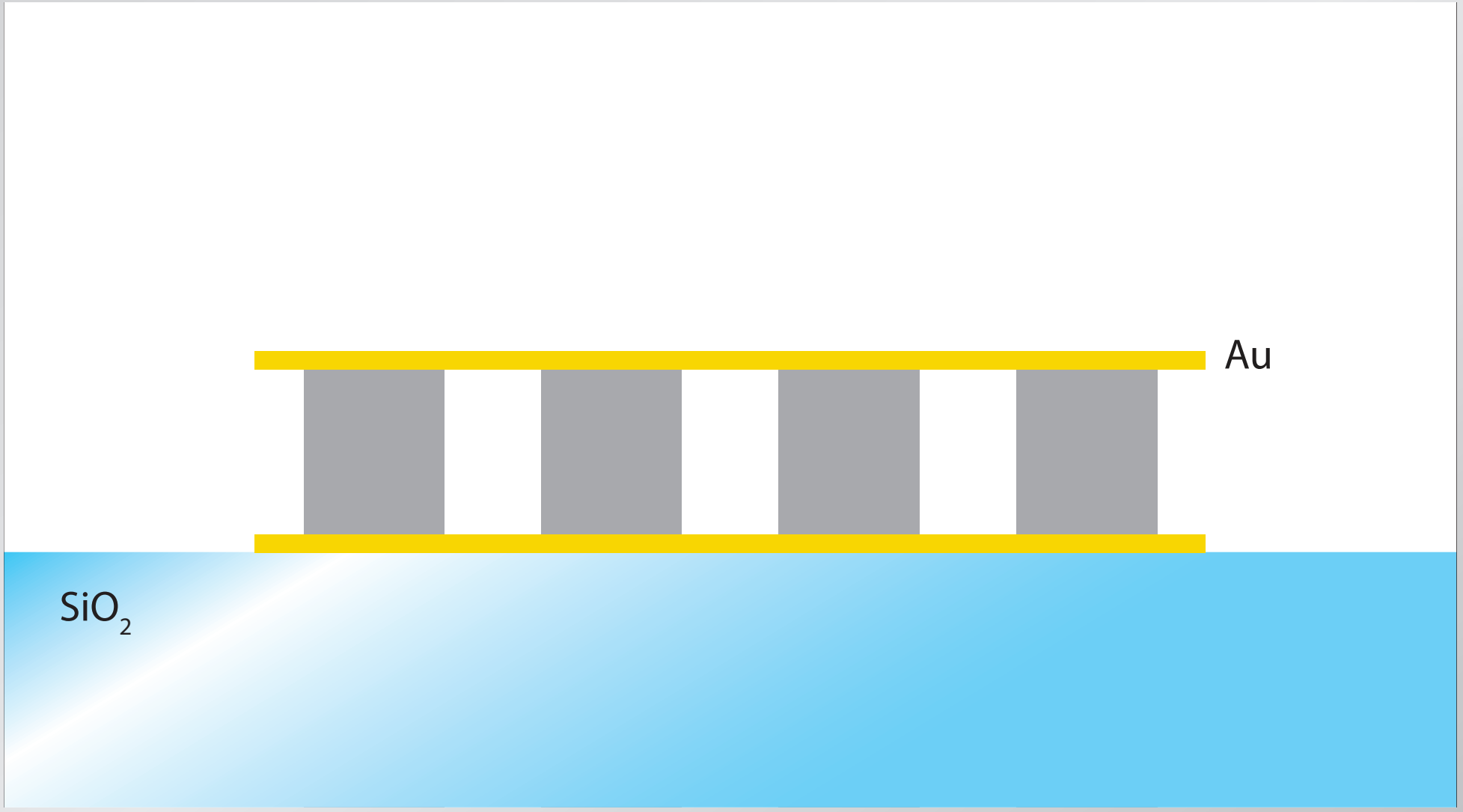
On-chip zero-index fabrication

On-chip zero-index fabrication

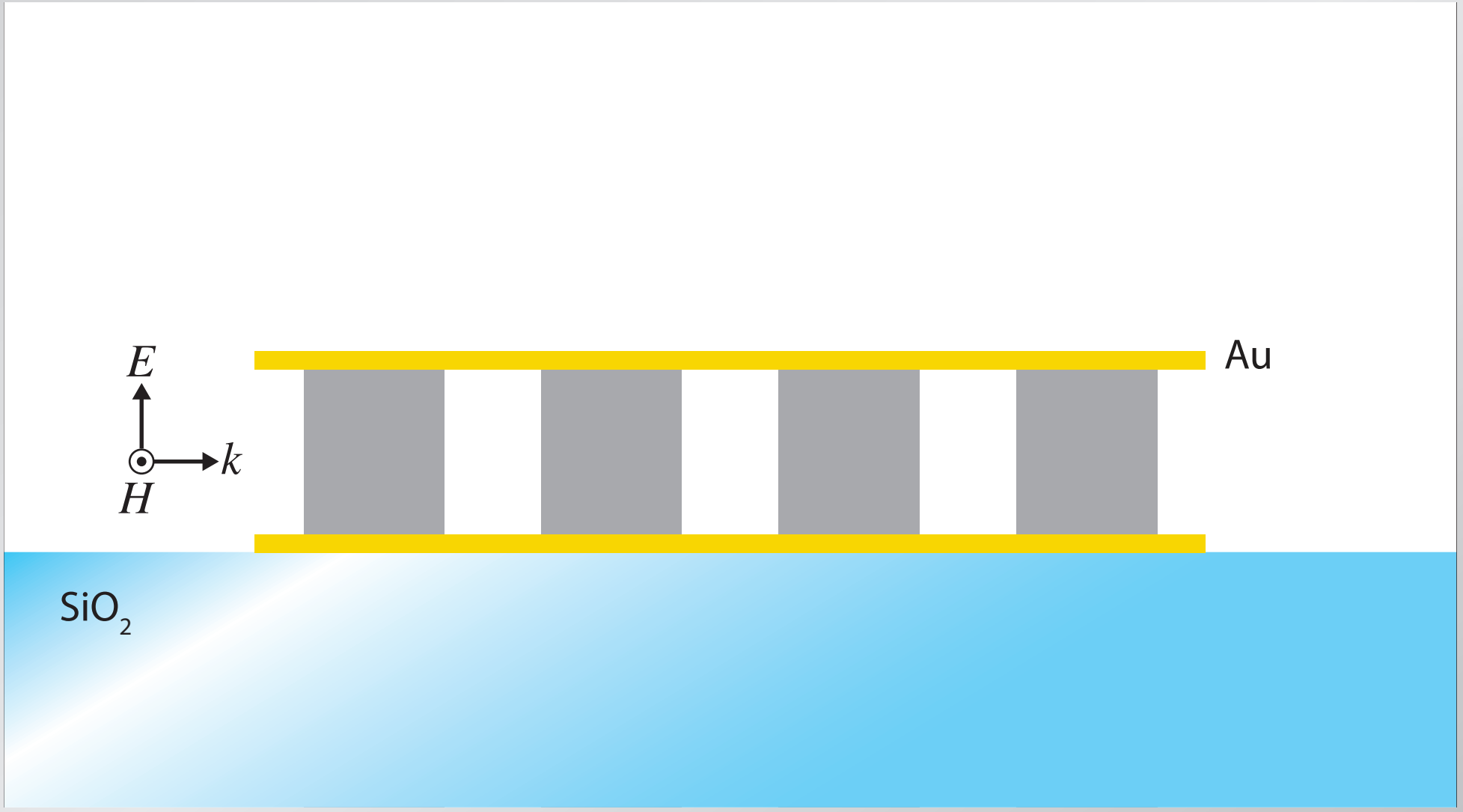


On-chip zero-index fabrication

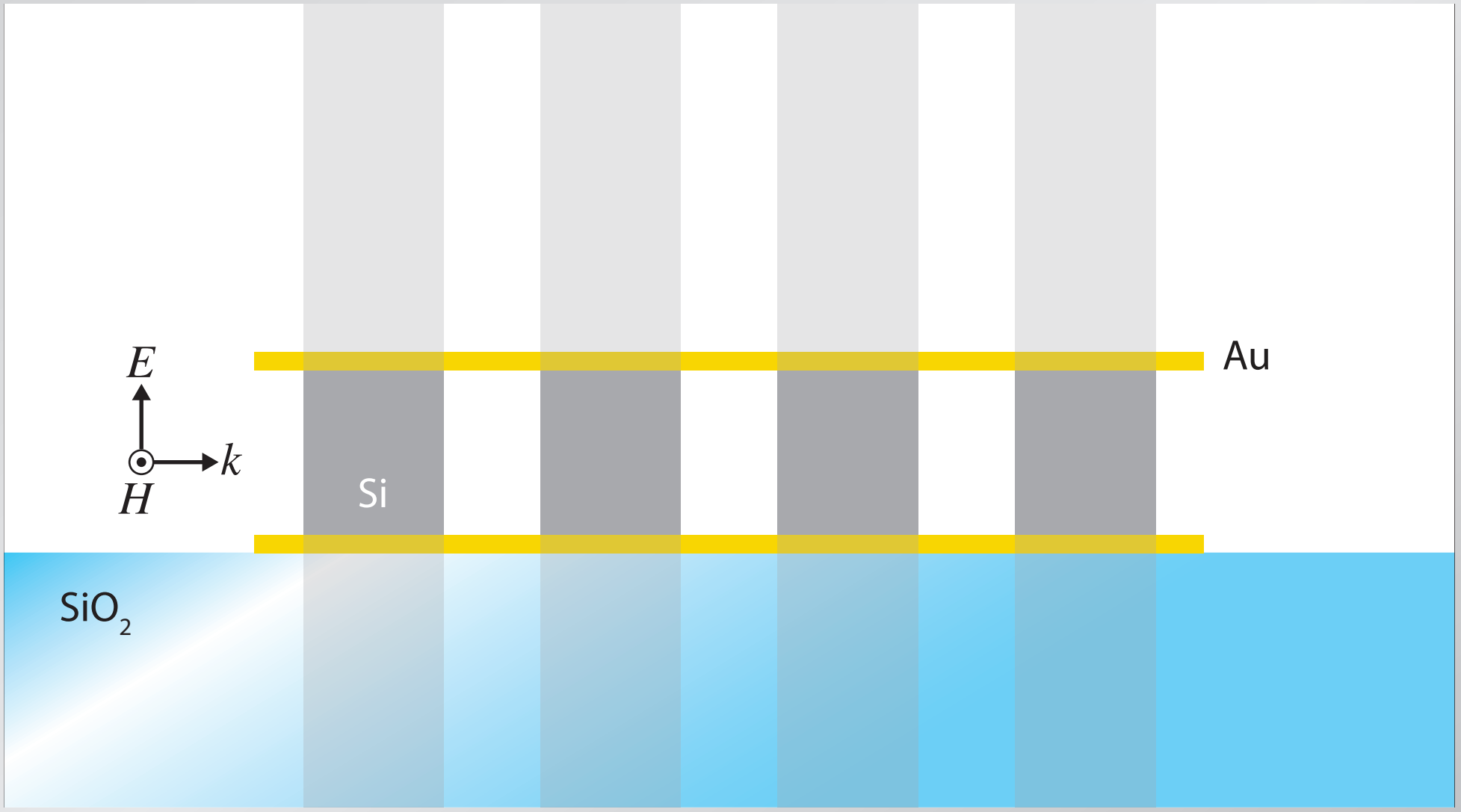
On-chip zero-index fabrication

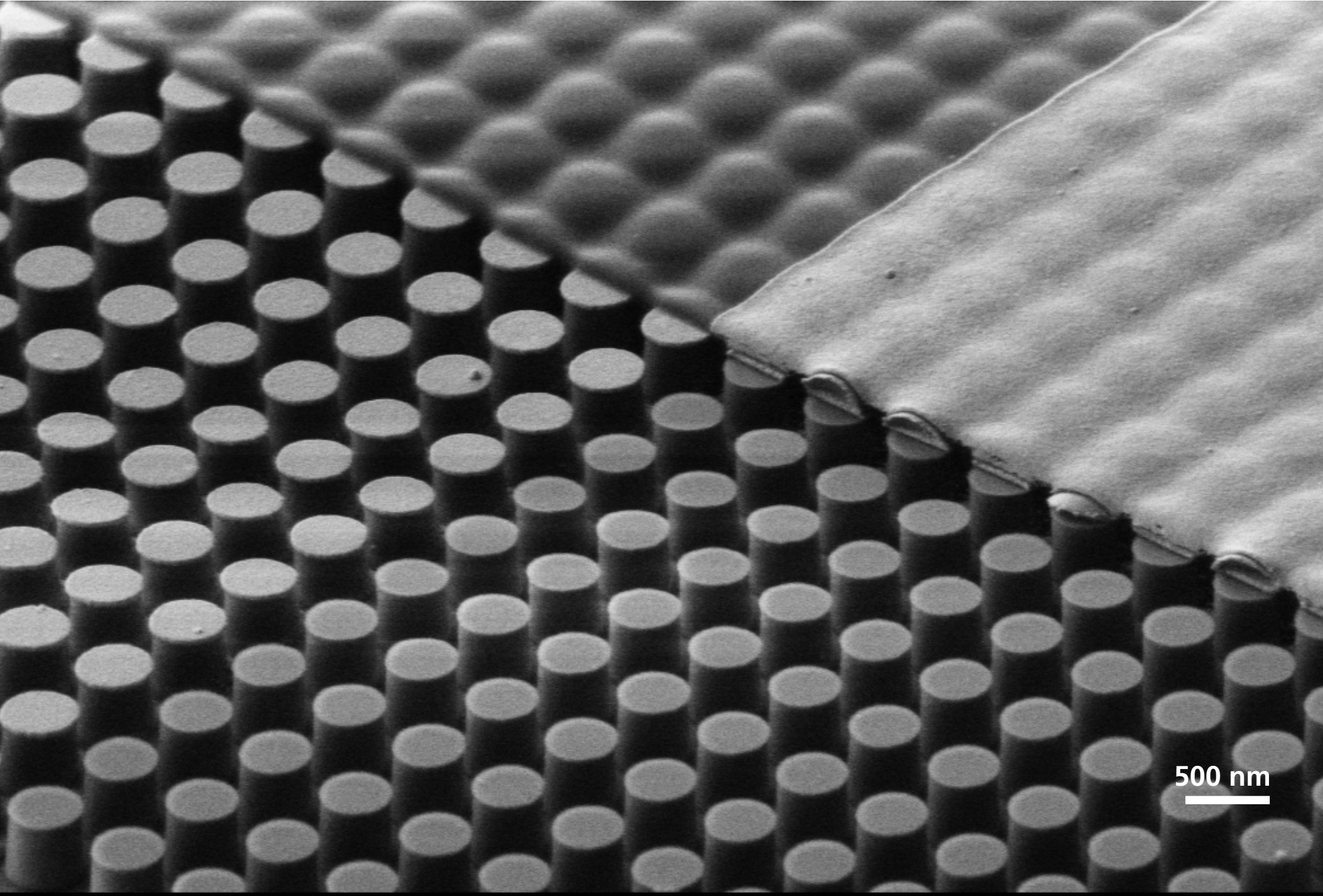


On-chip zero-index fabrication



On-chip zero-index fabrication

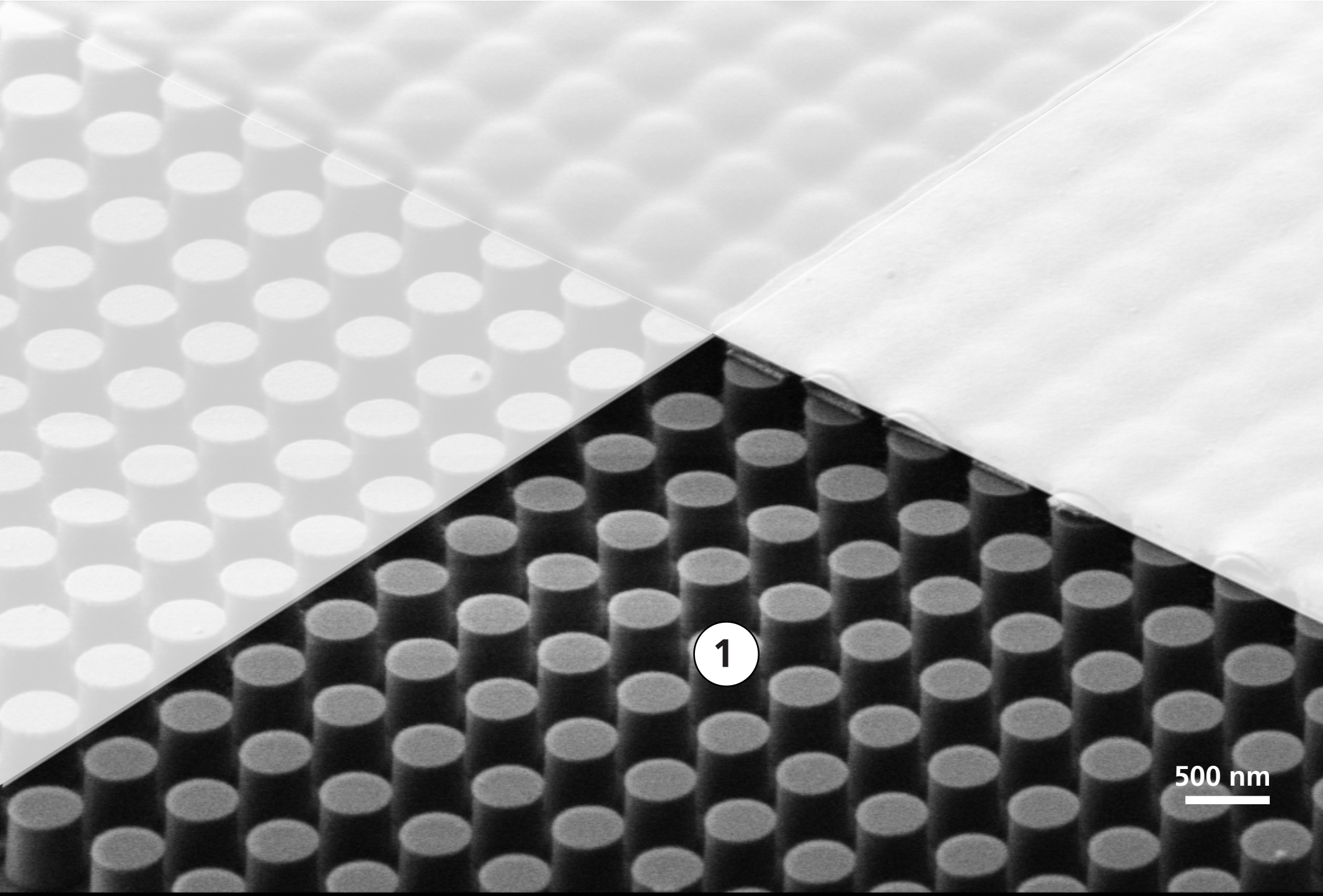




1 index

2 zero index

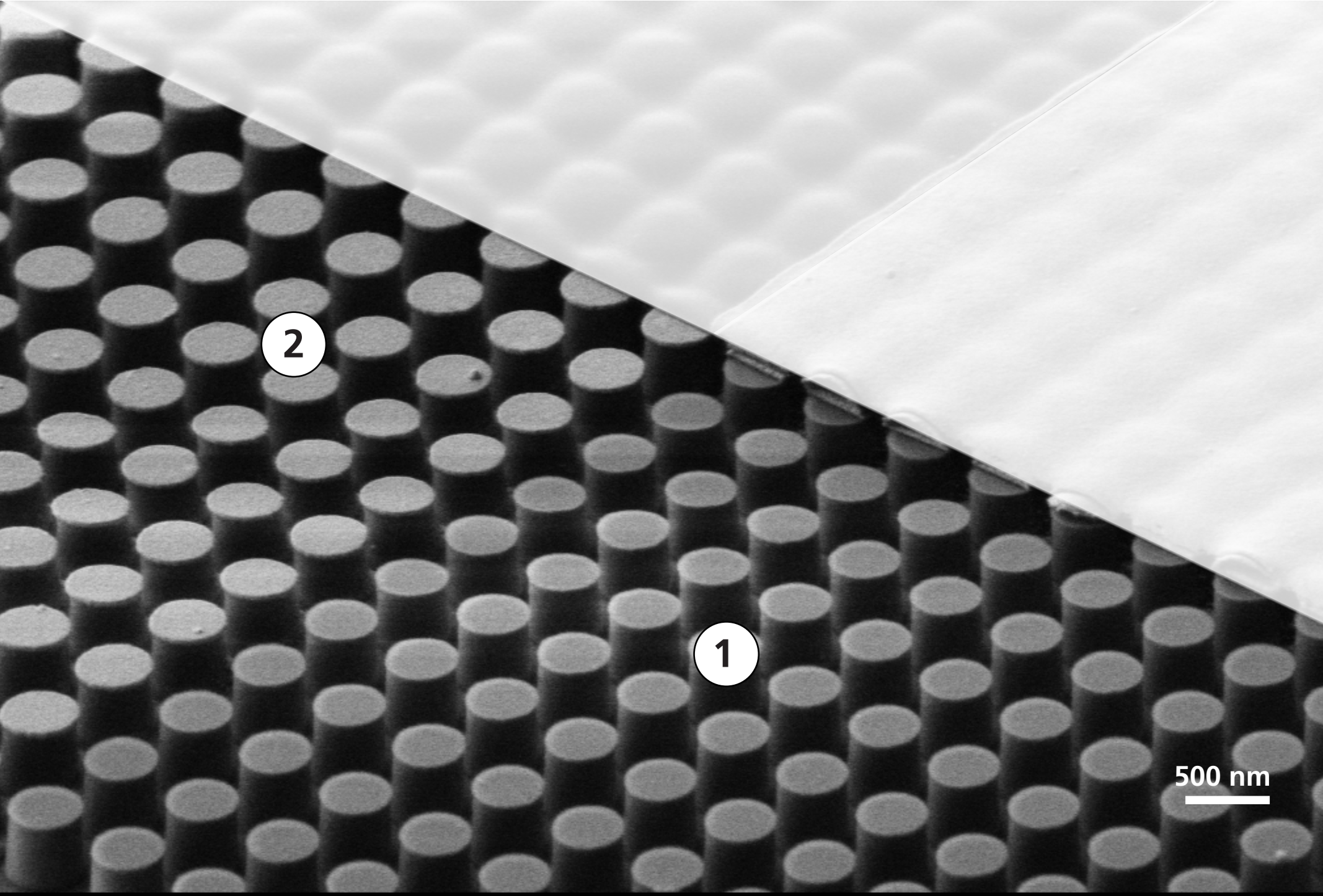
3 experiments



1 index

2 zero index

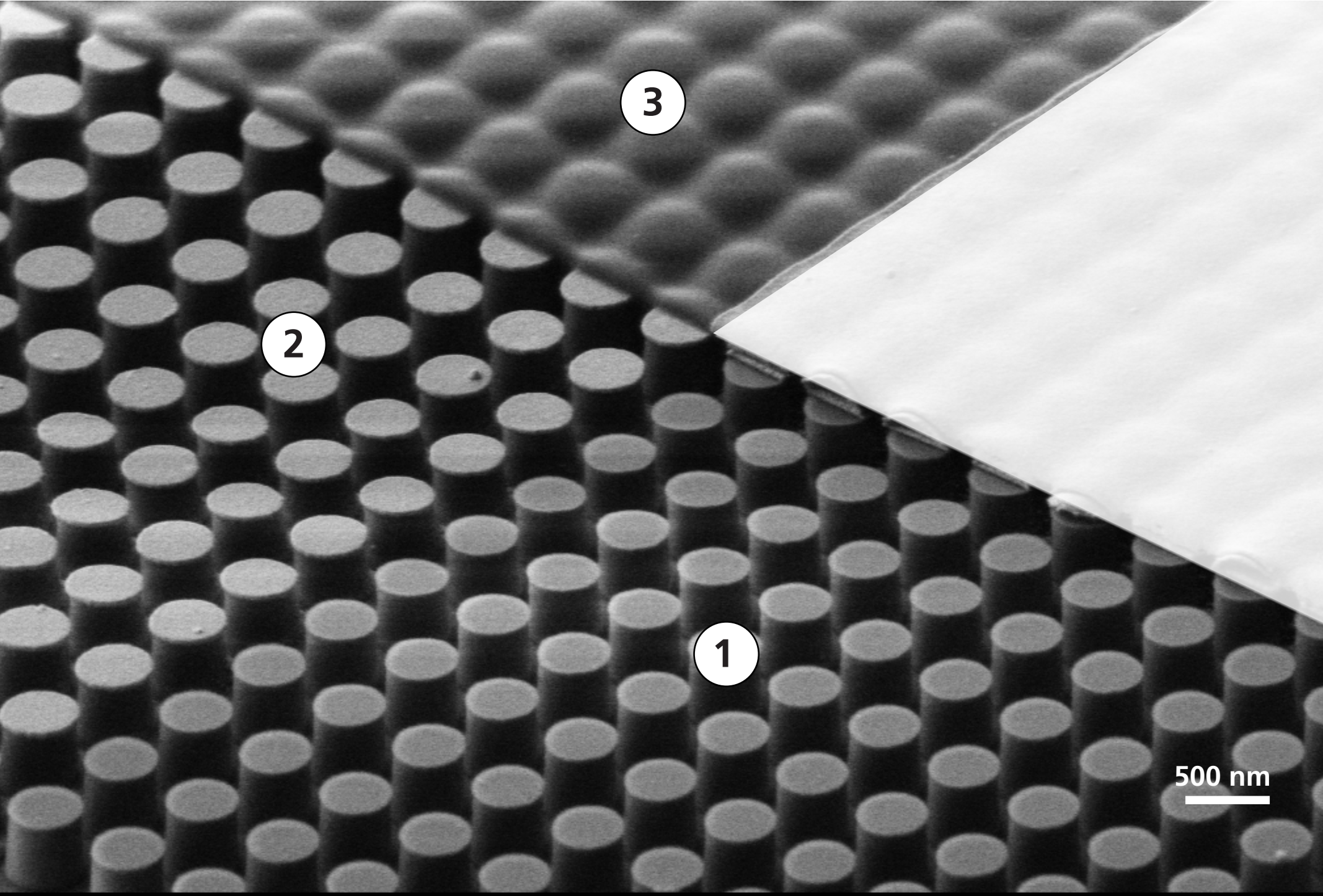
3 experiments



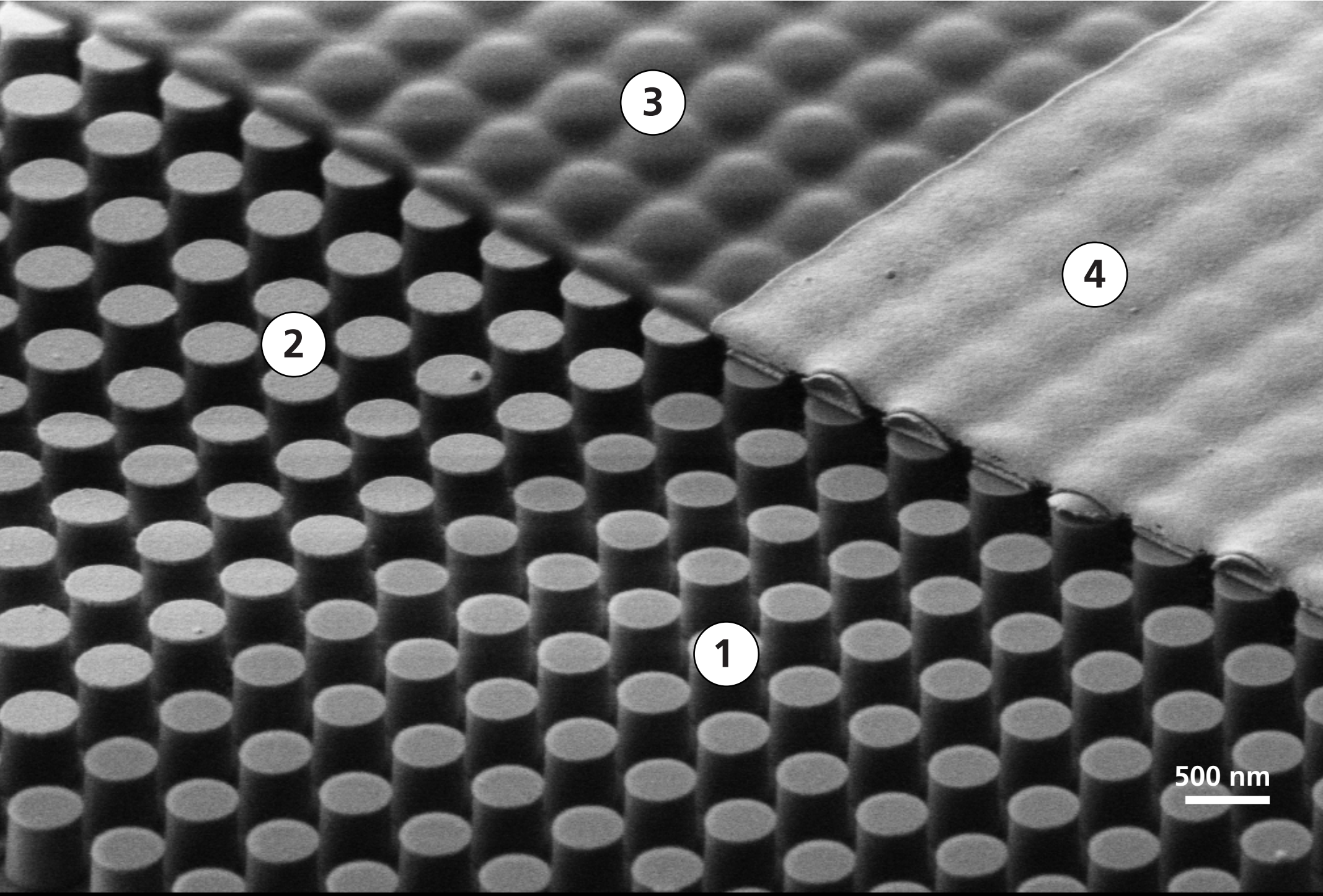
1 index

2 zero index

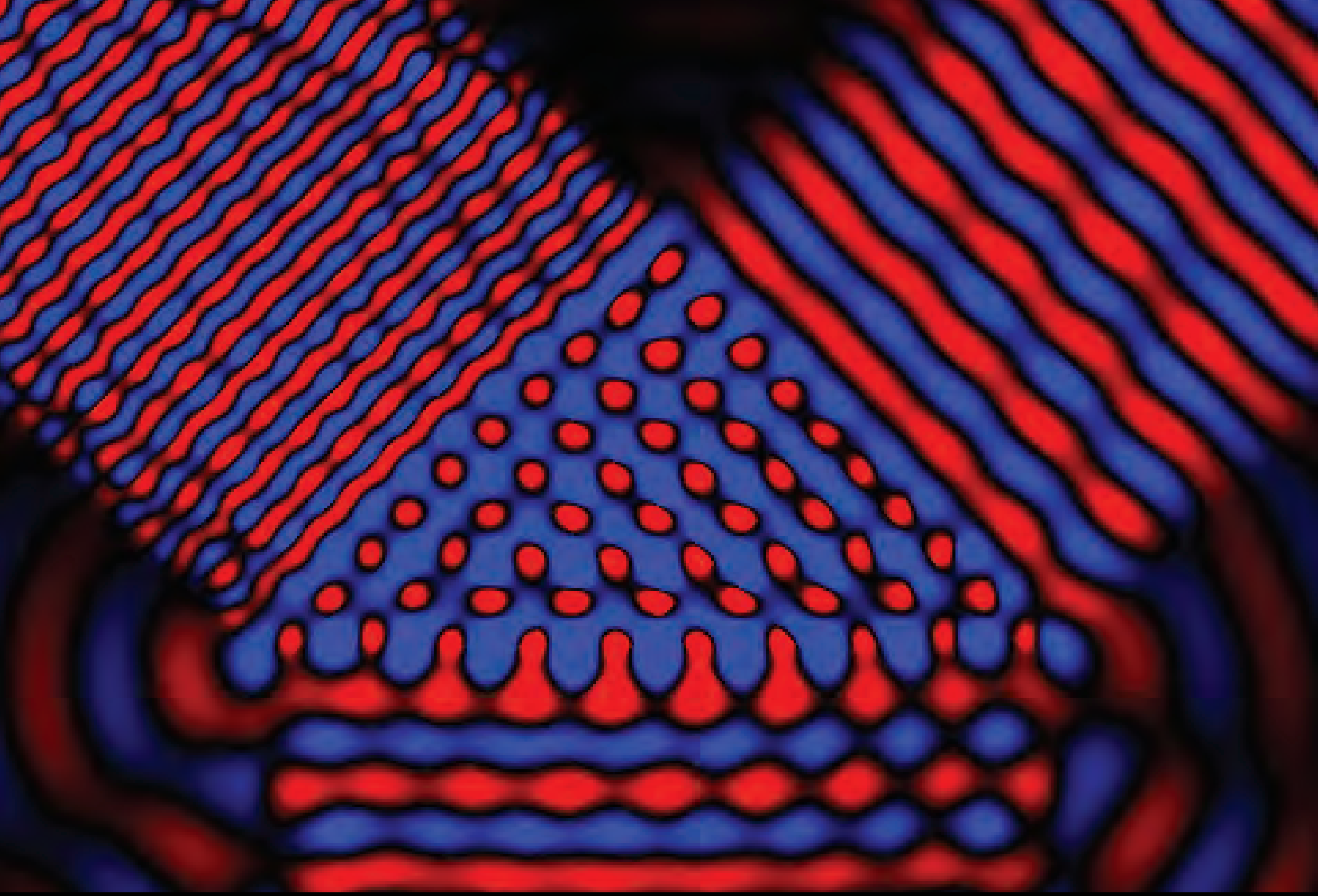
3 experiments



1 index 2 zero index 3 experiments



1 index 2 zero index 3 experiments



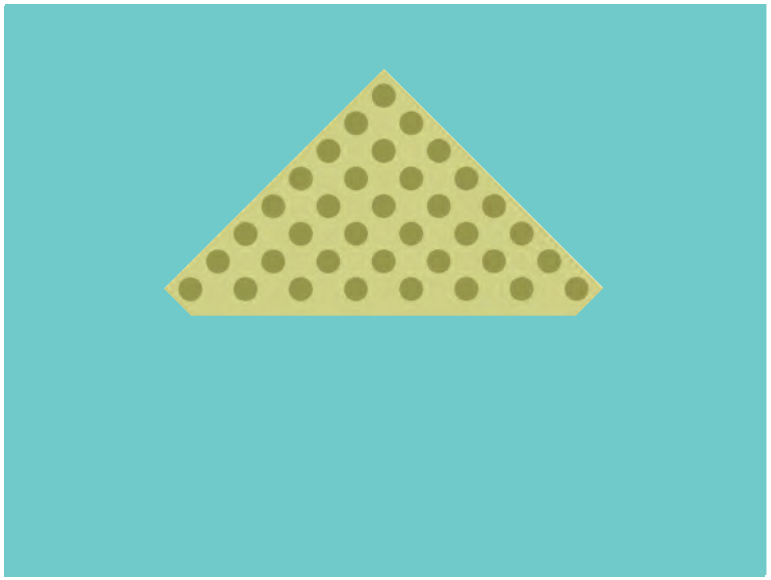
1 index

2 zero index

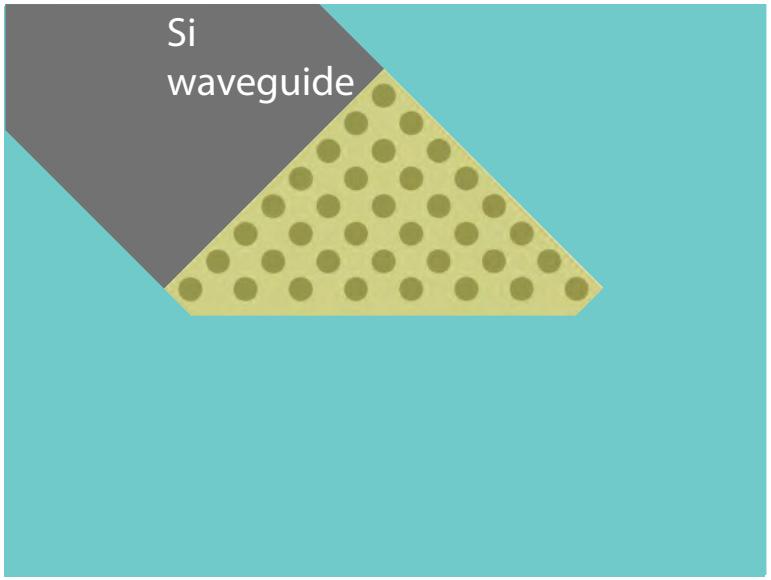
3 experiments

On-chip zero-index prism

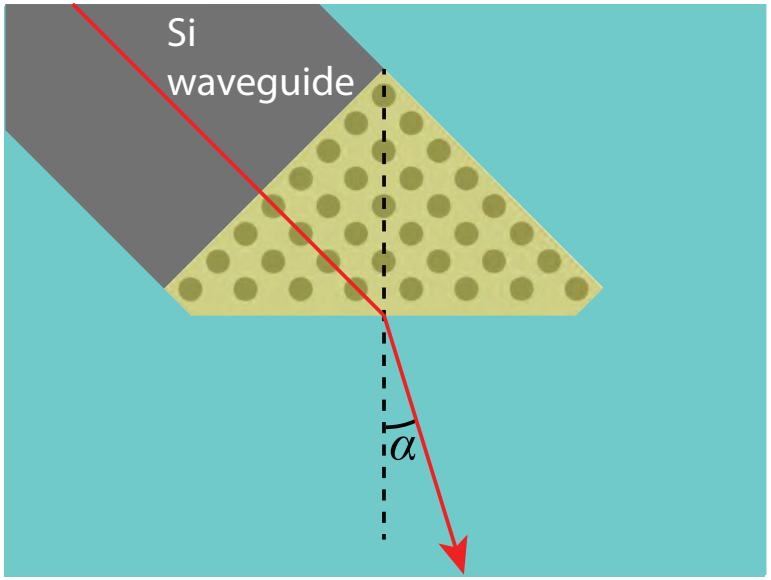
On-chip zero-index prism



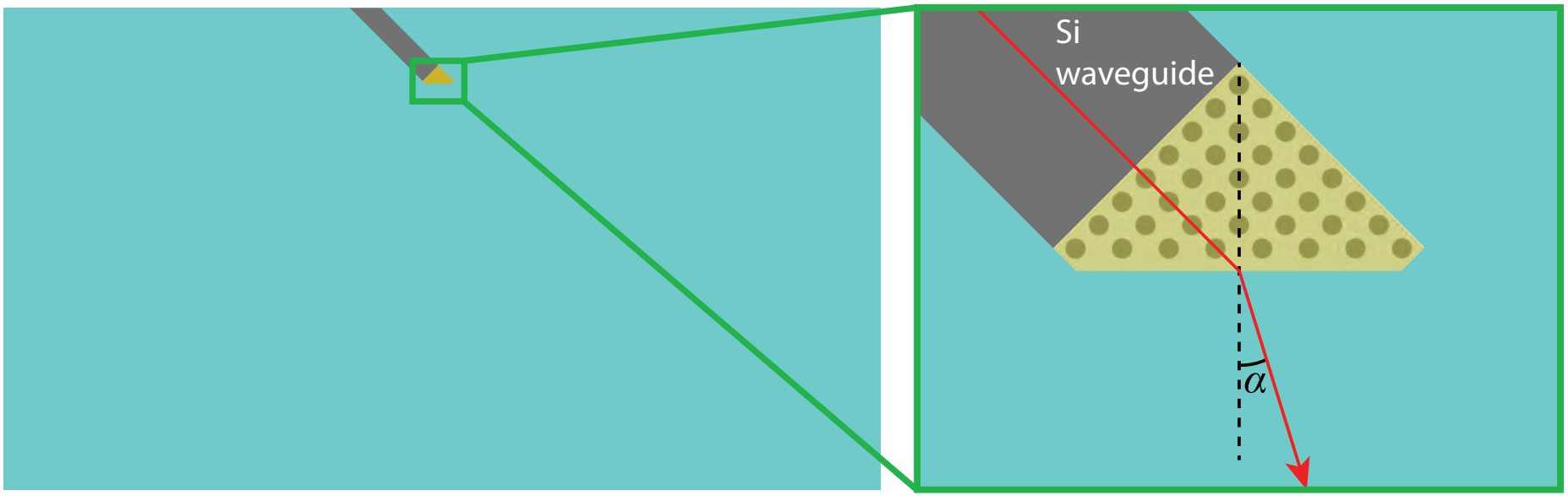
On-chip zero-index prism



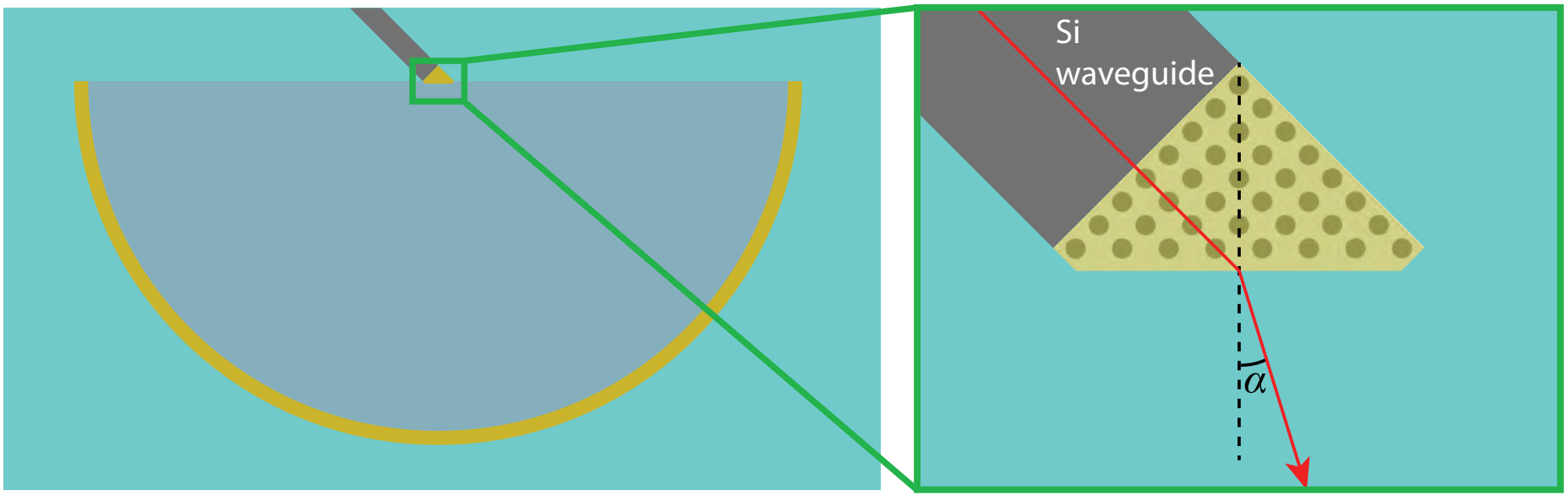
On-chip zero-index prism



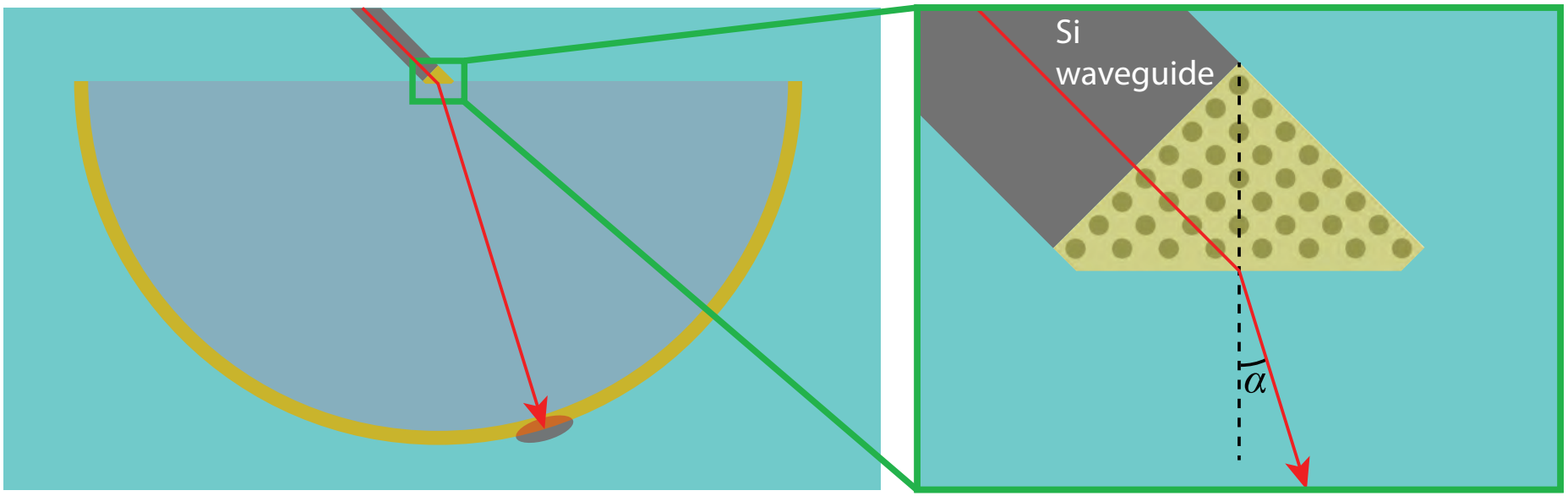
On-chip zero-index prism



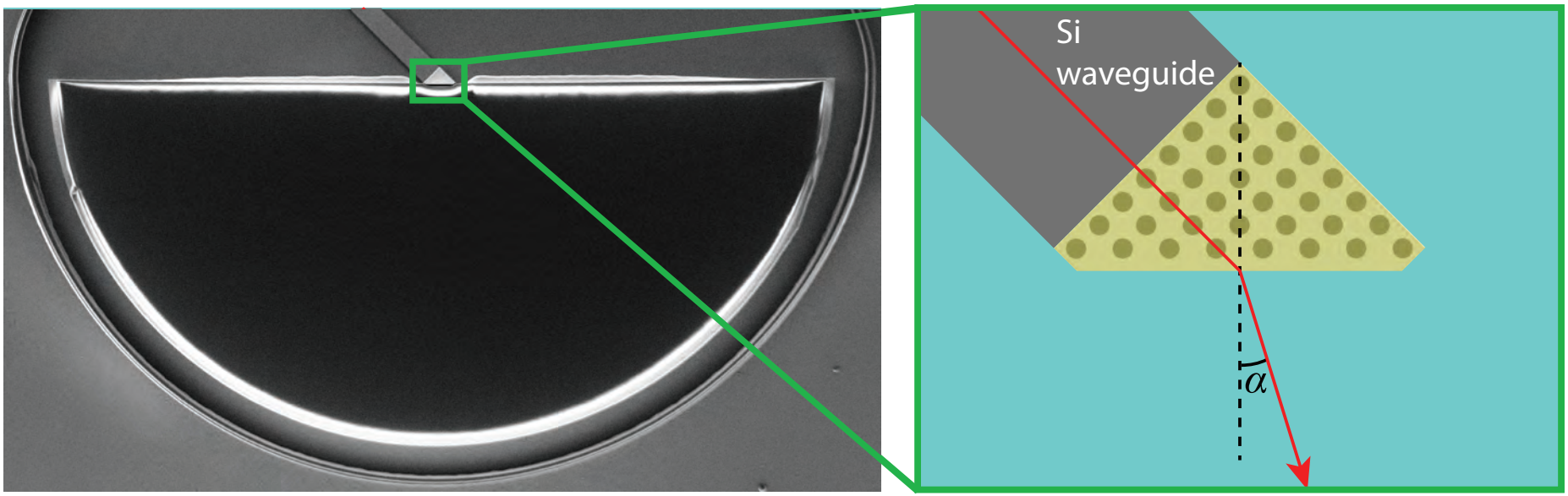
On-chip zero-index prism



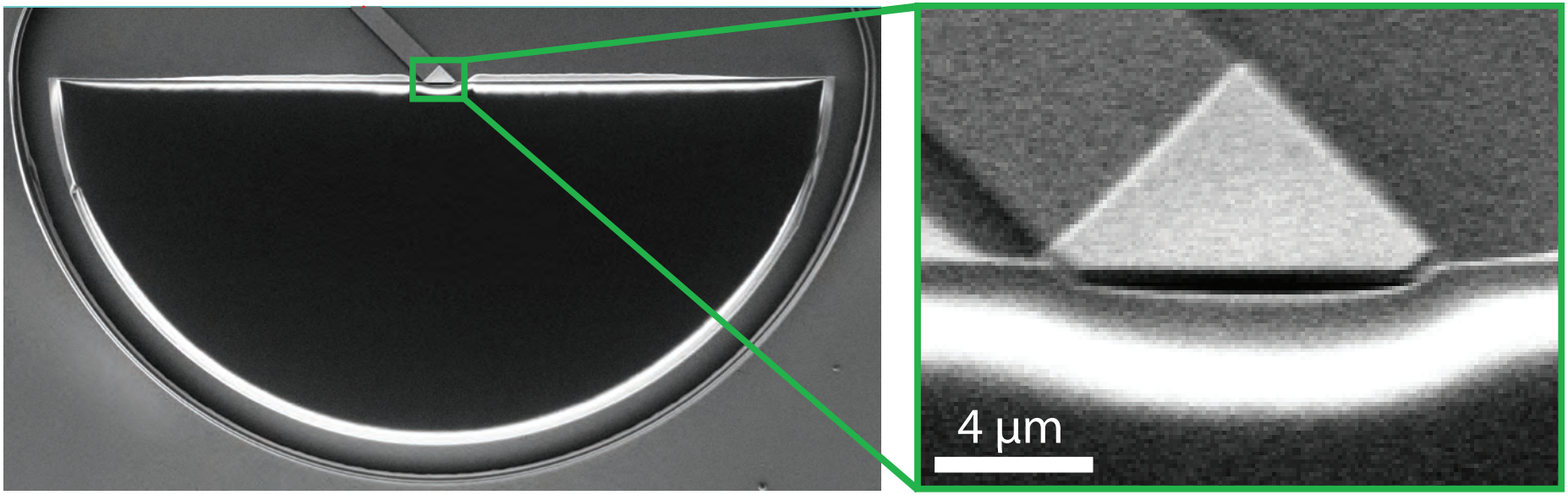
On-chip zero-index prism

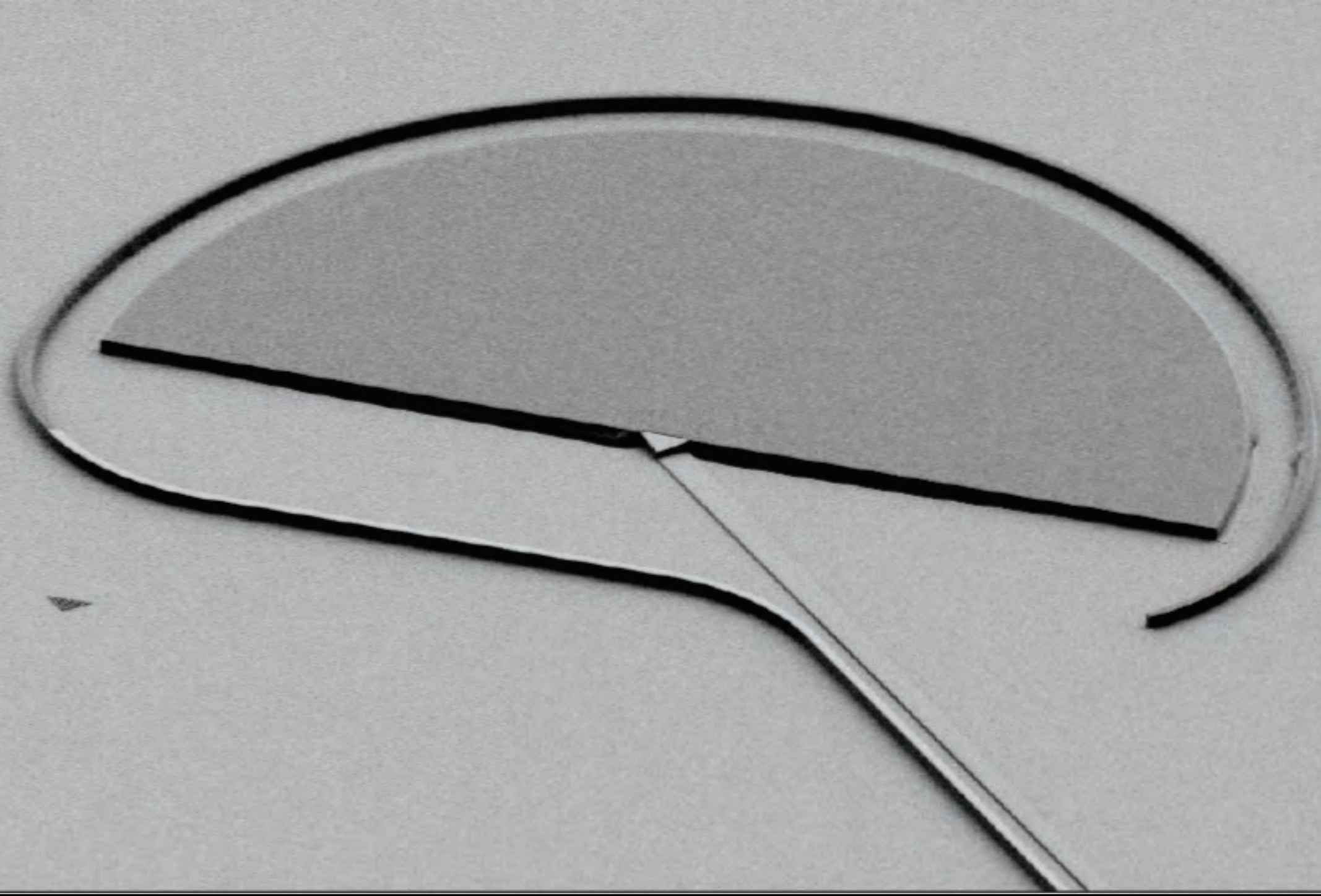


On-chip zero-index prism

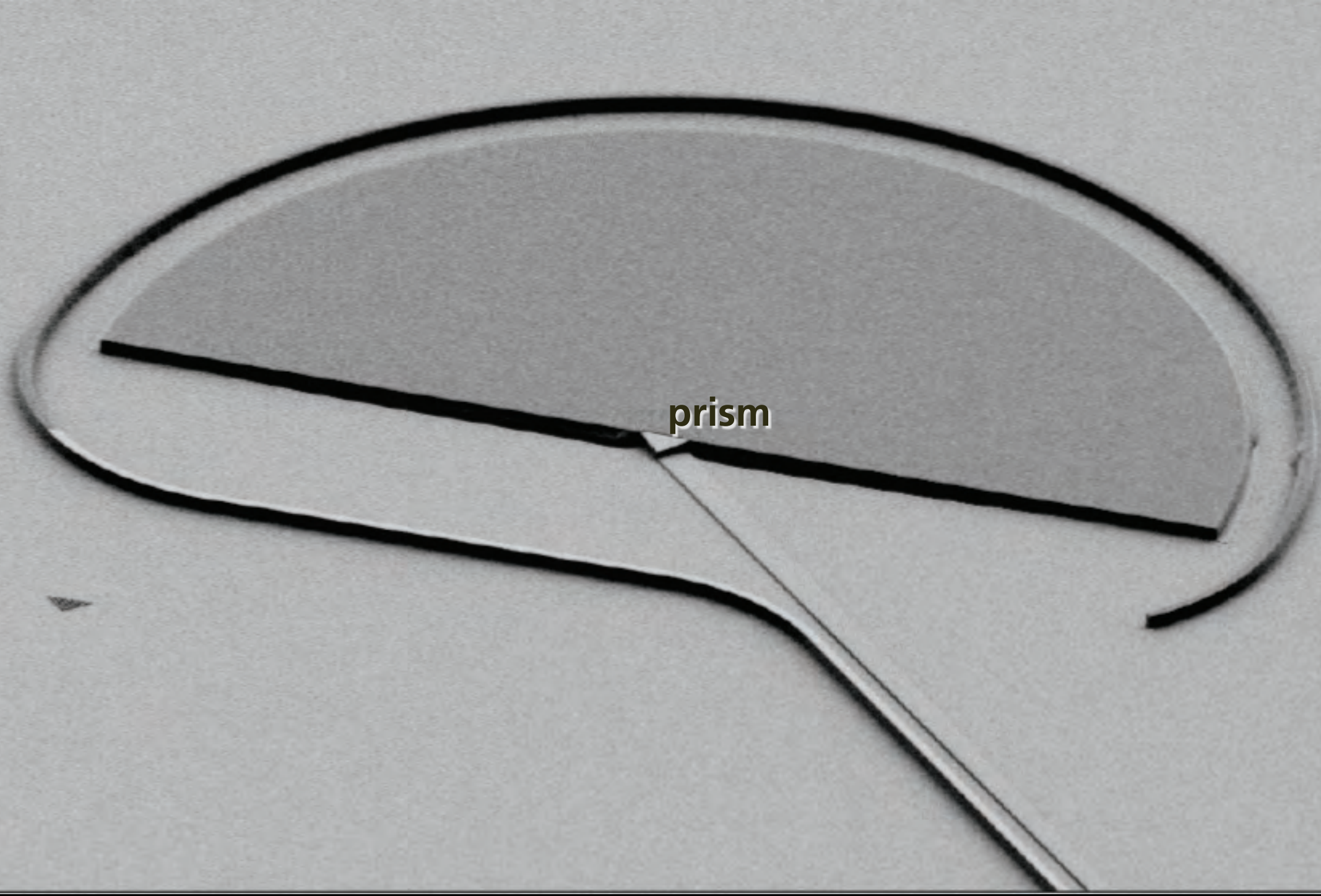


On-chip zero-index prism

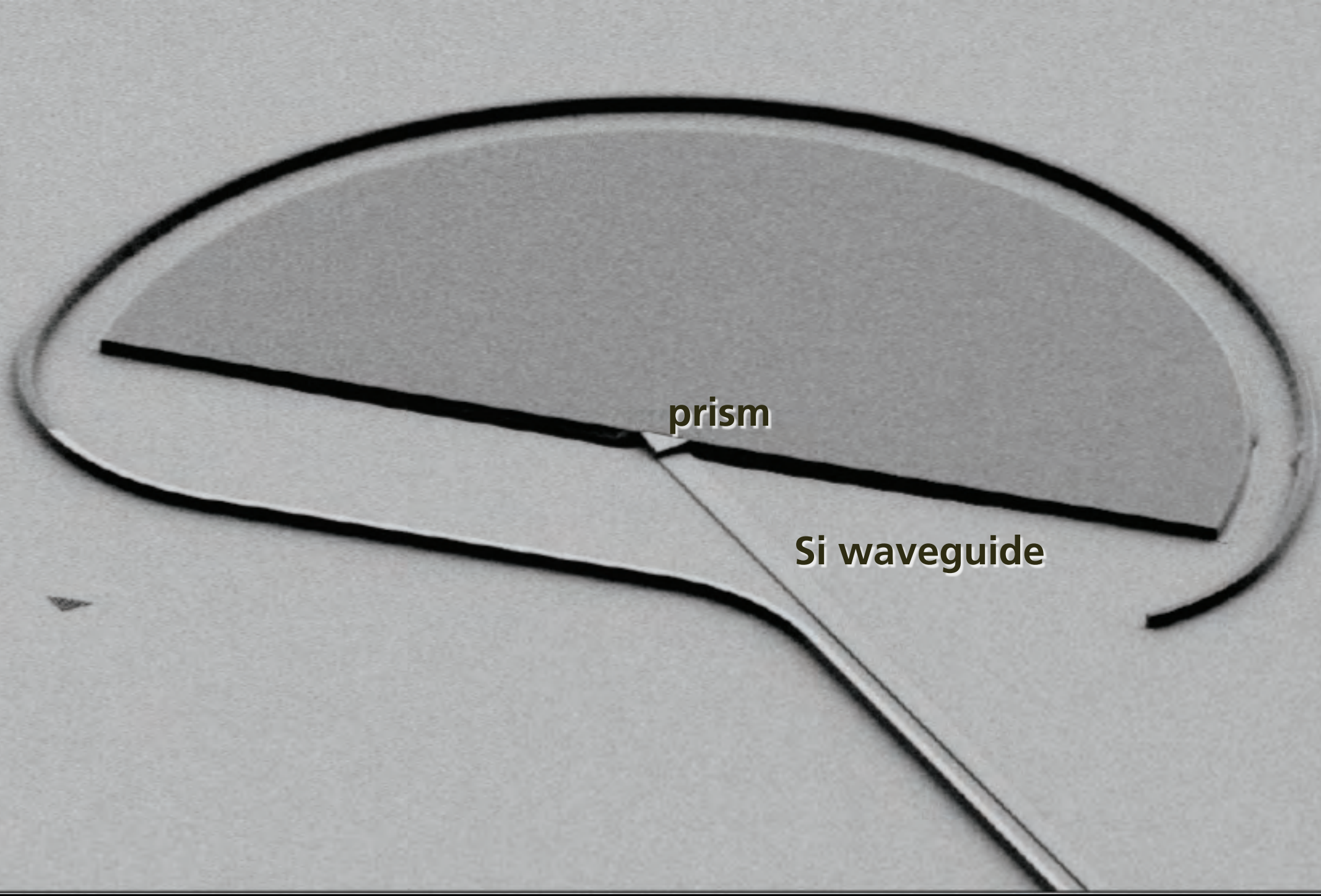


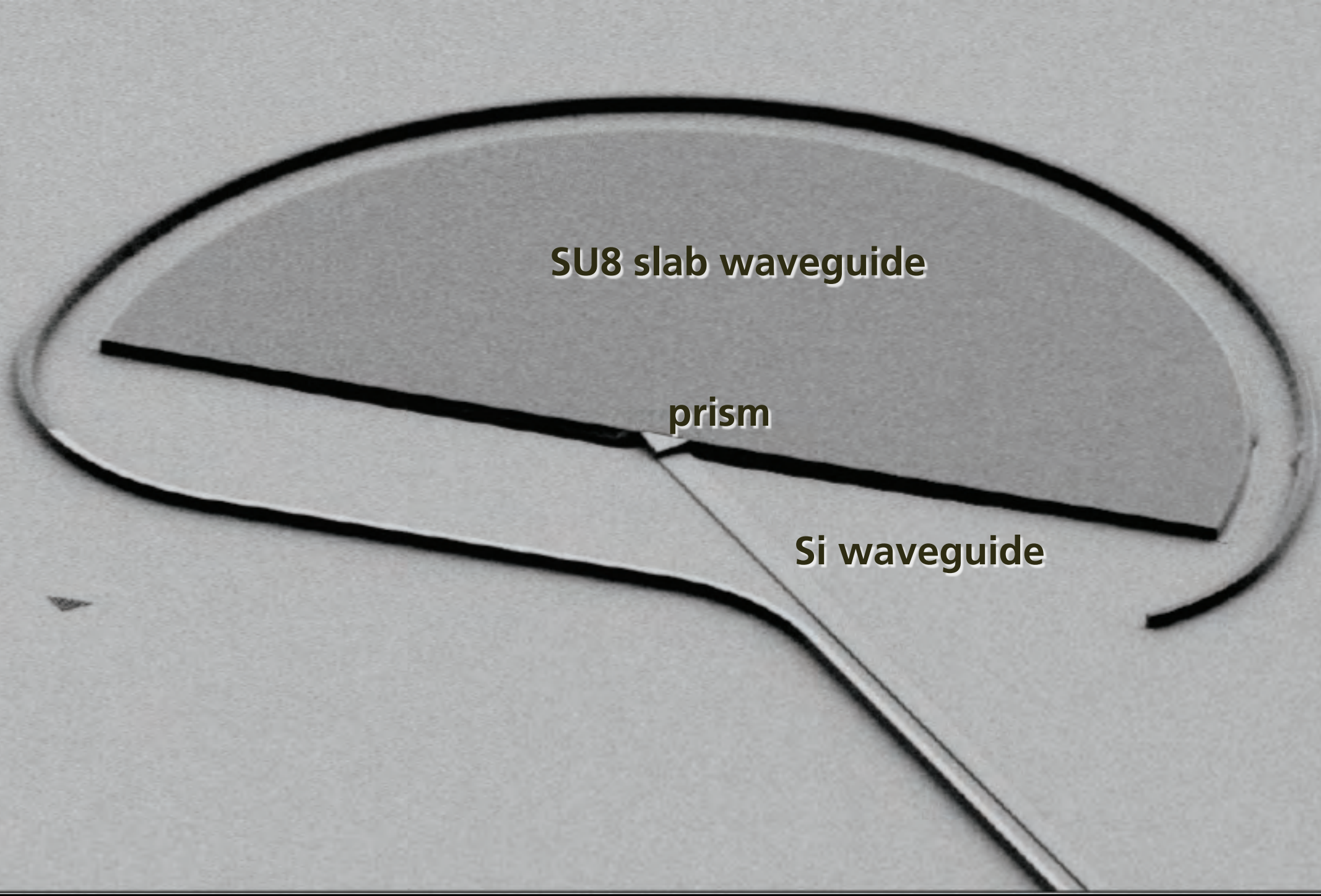


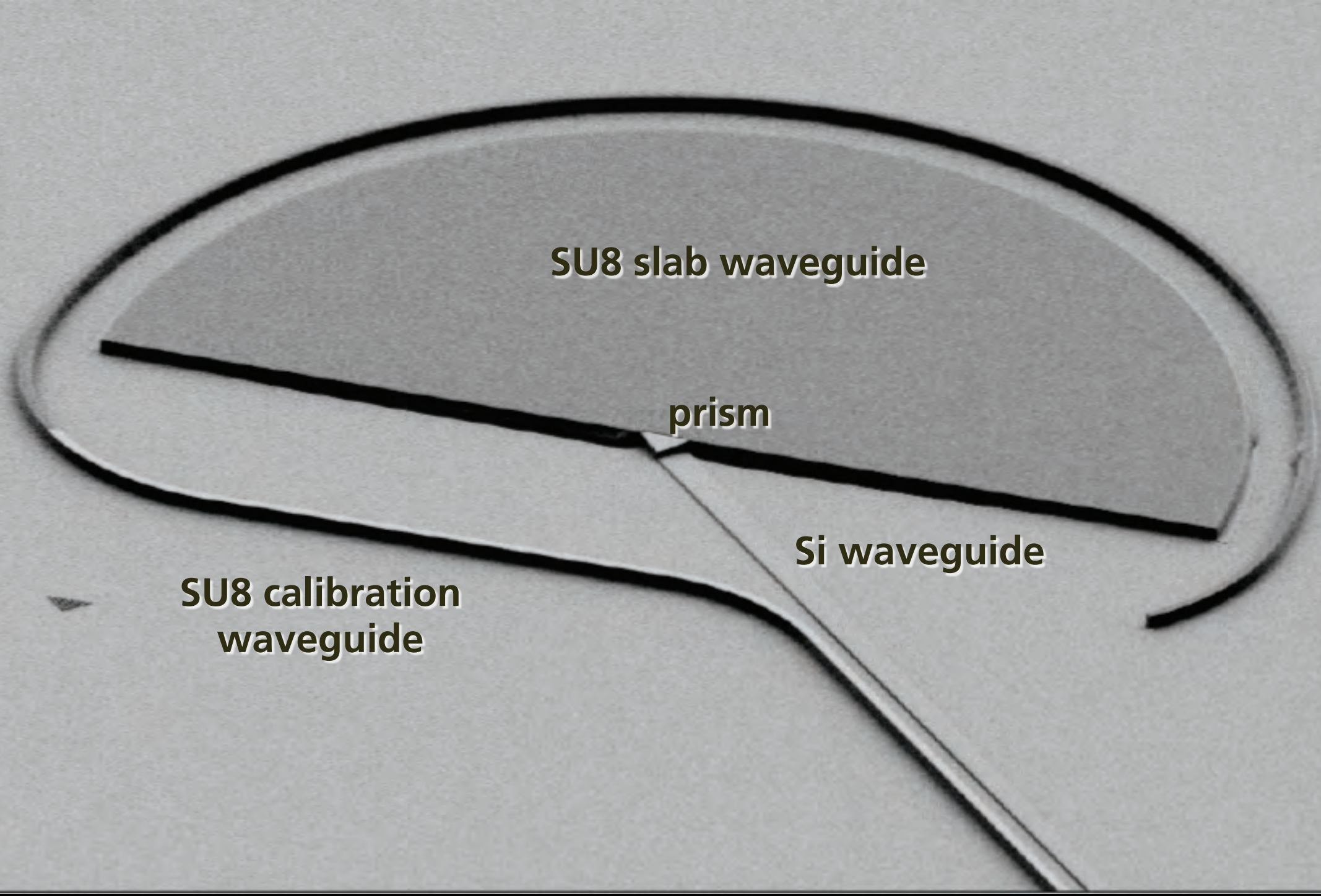
1 index 2 zero index 3 experiments

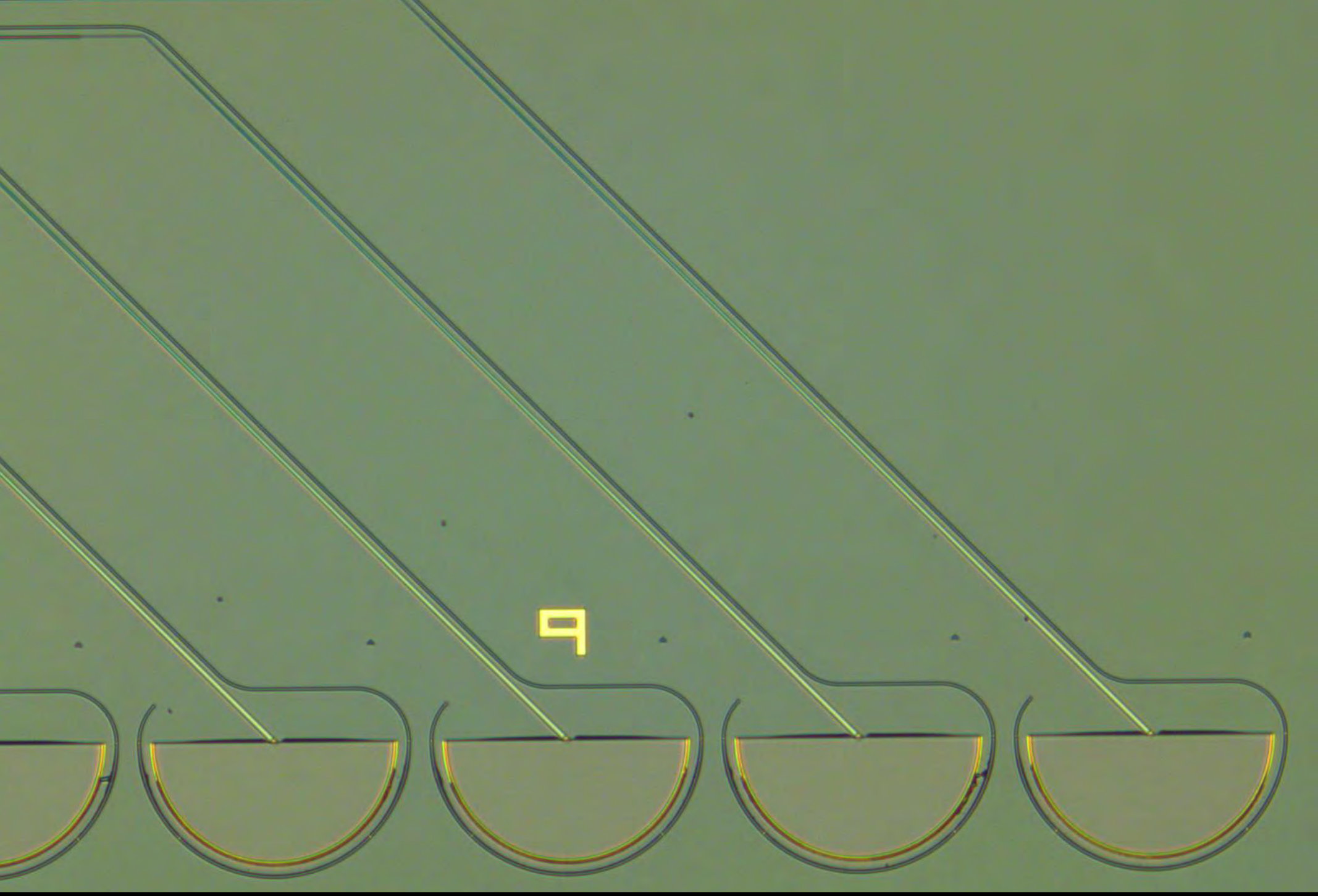


1 index 2 zero index 3 experiments


prism
Si waveguide


SU8 slab waveguide
prism
Si waveguide


SU8 slab waveguide
prism
Si waveguide
SU8 calibration
waveguide

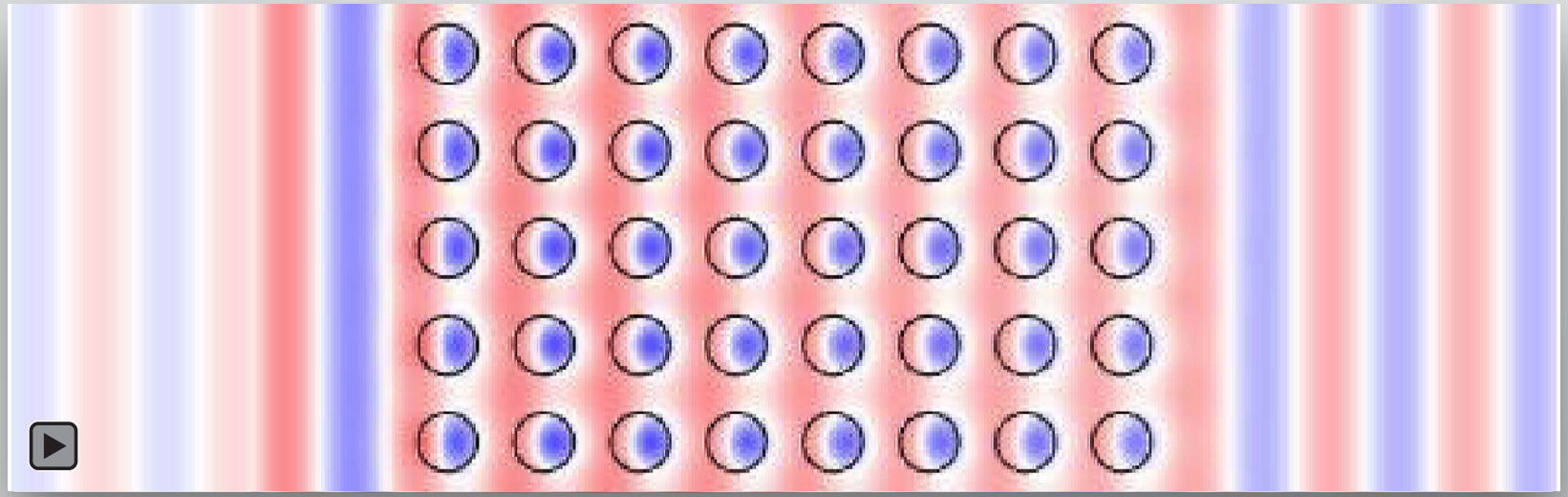


1 index

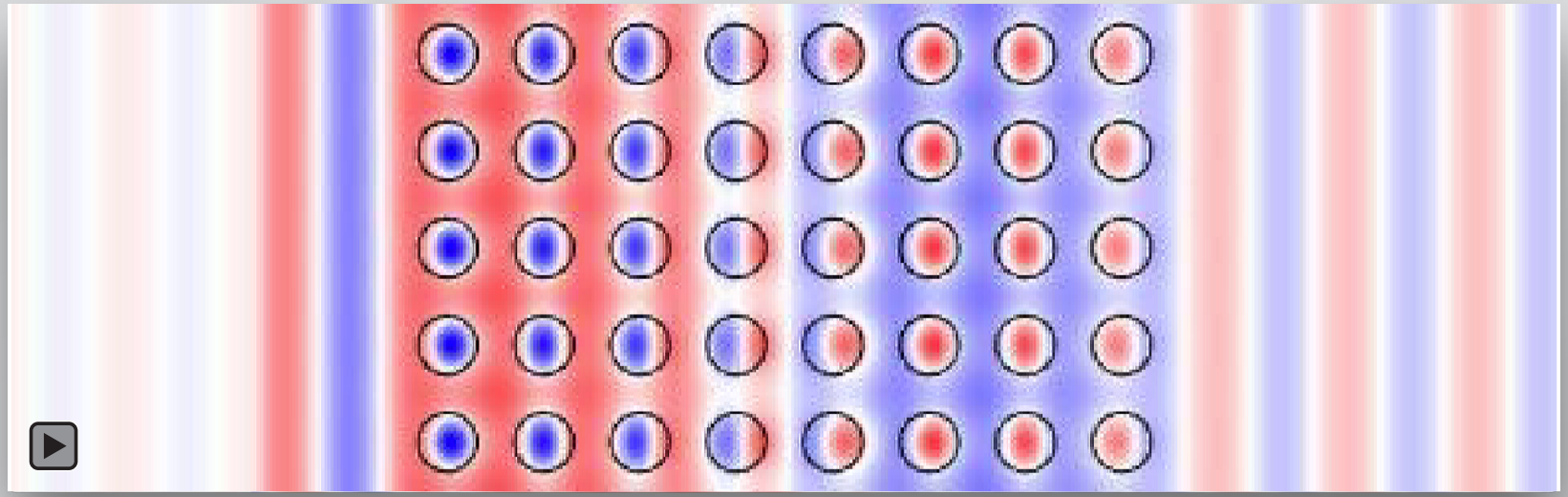
2 zero index

3 experiments

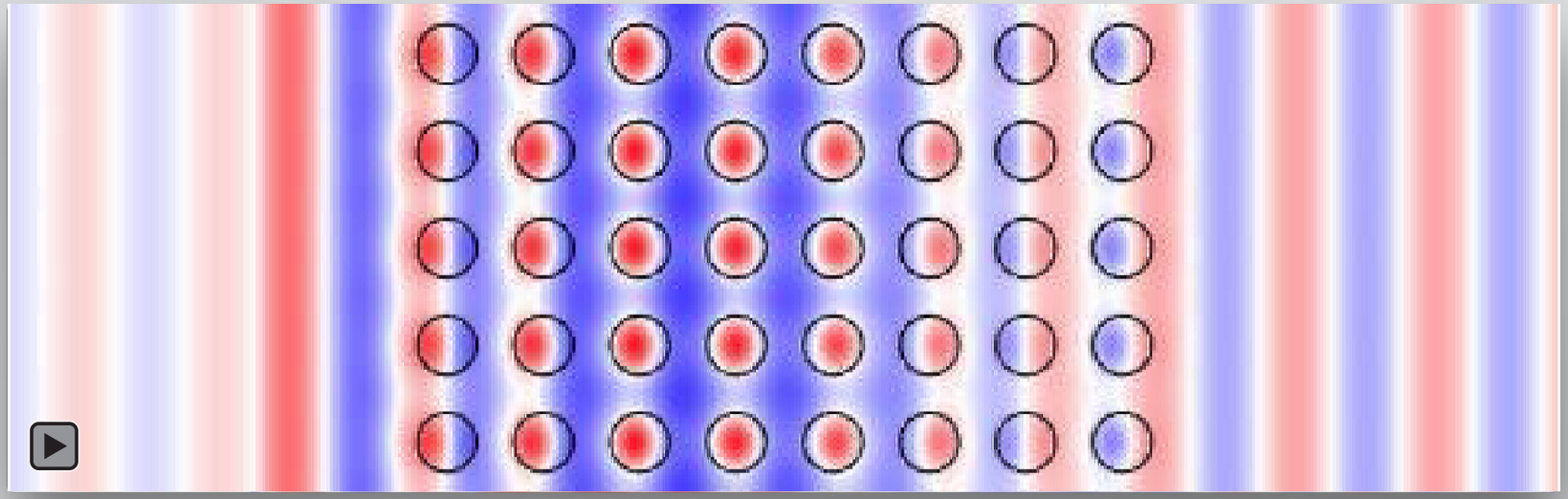
at design wavelength (1590 nm)



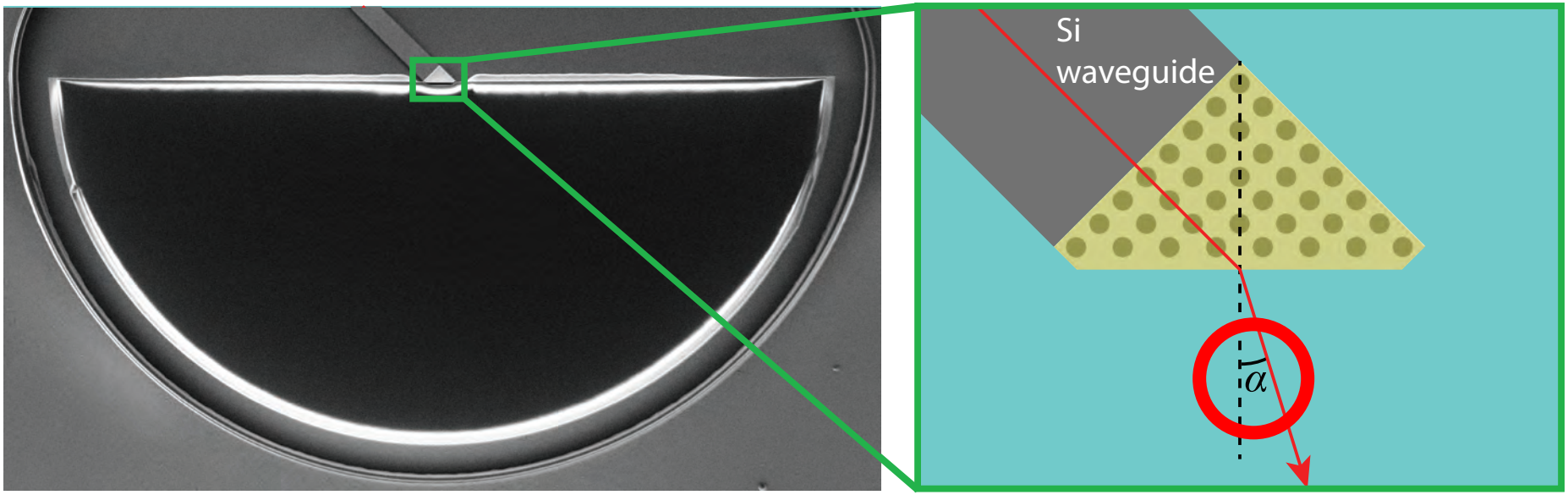
below design wavelength (1530 nm)

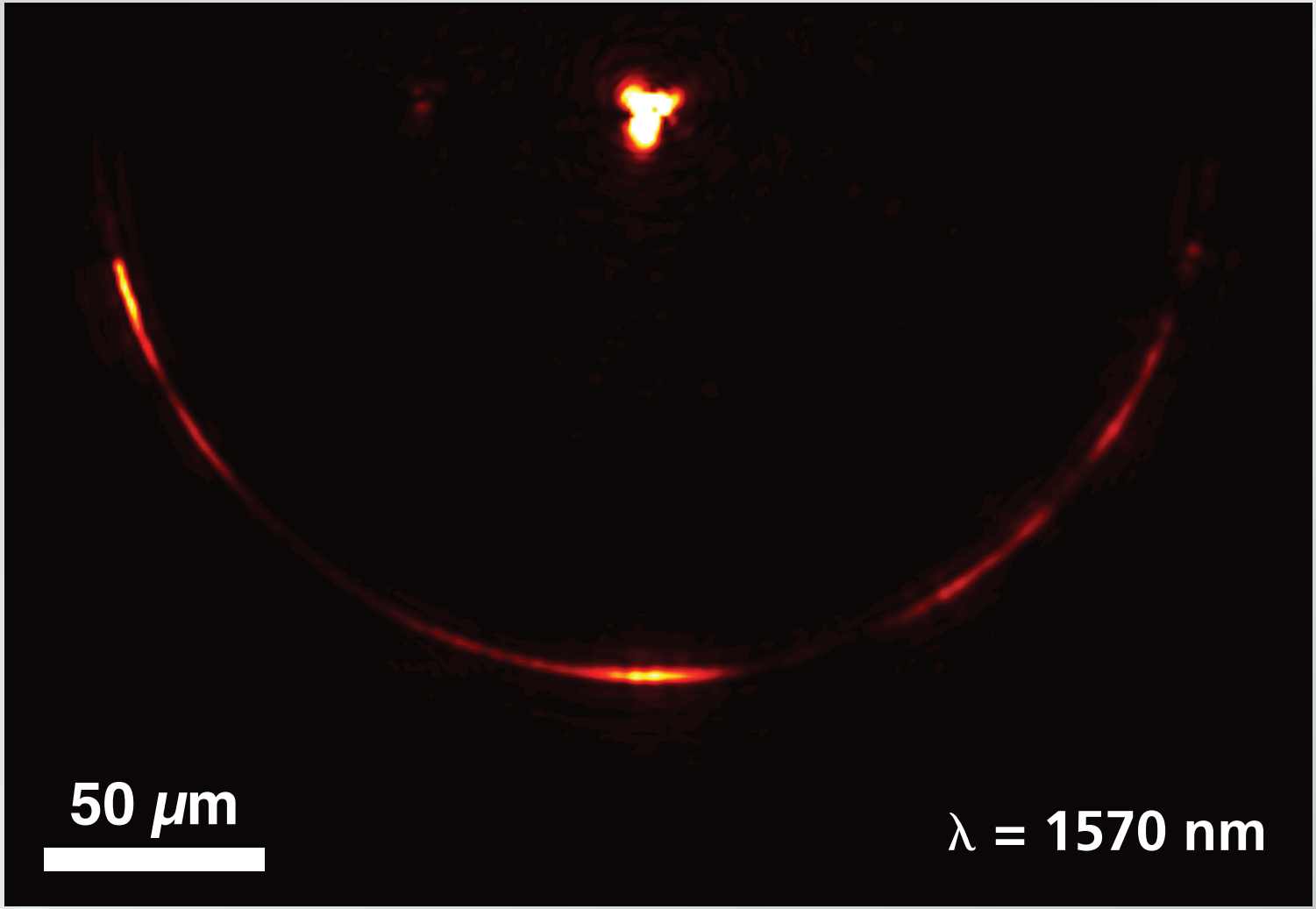


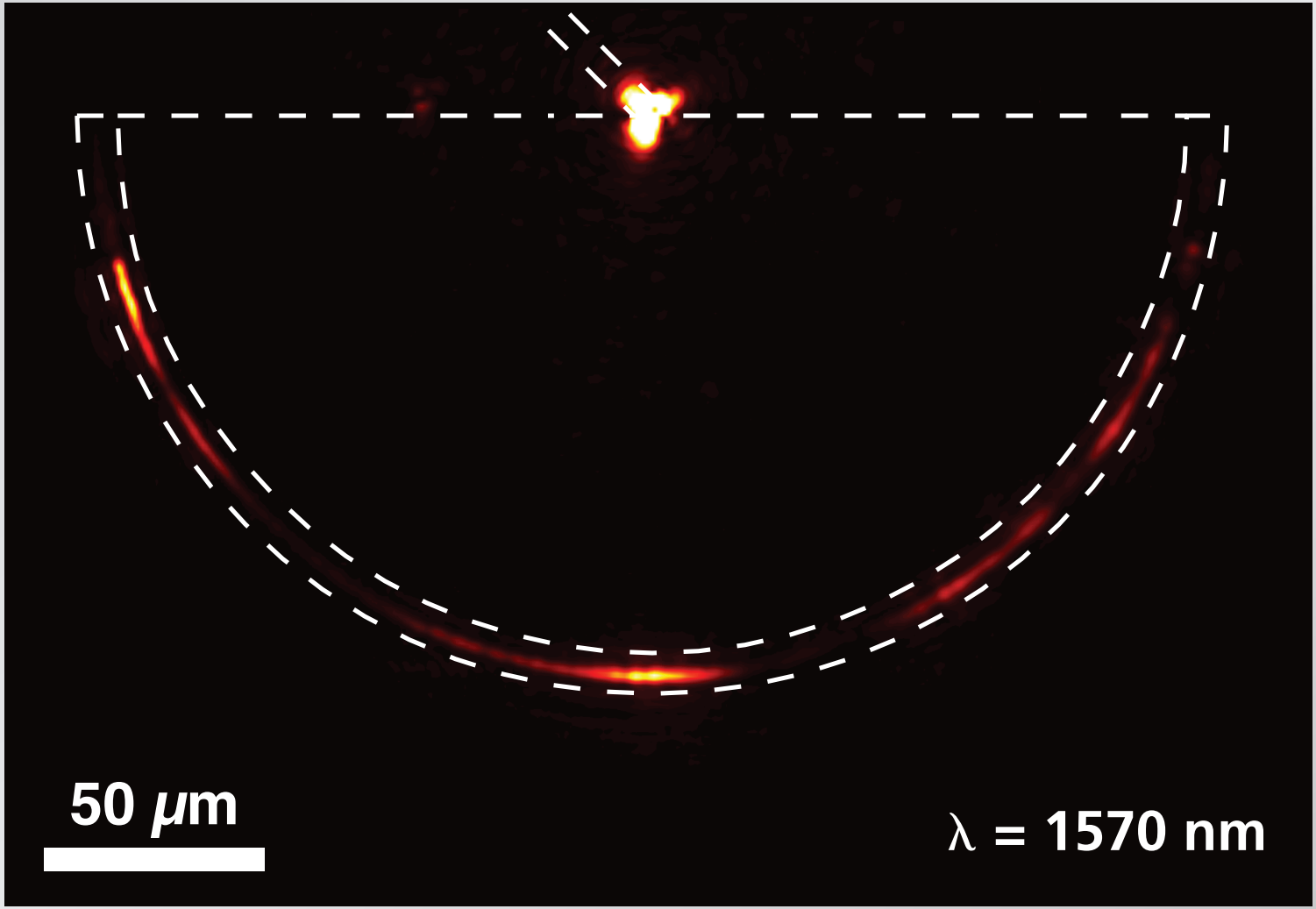
above design wavelength (1650 nm)

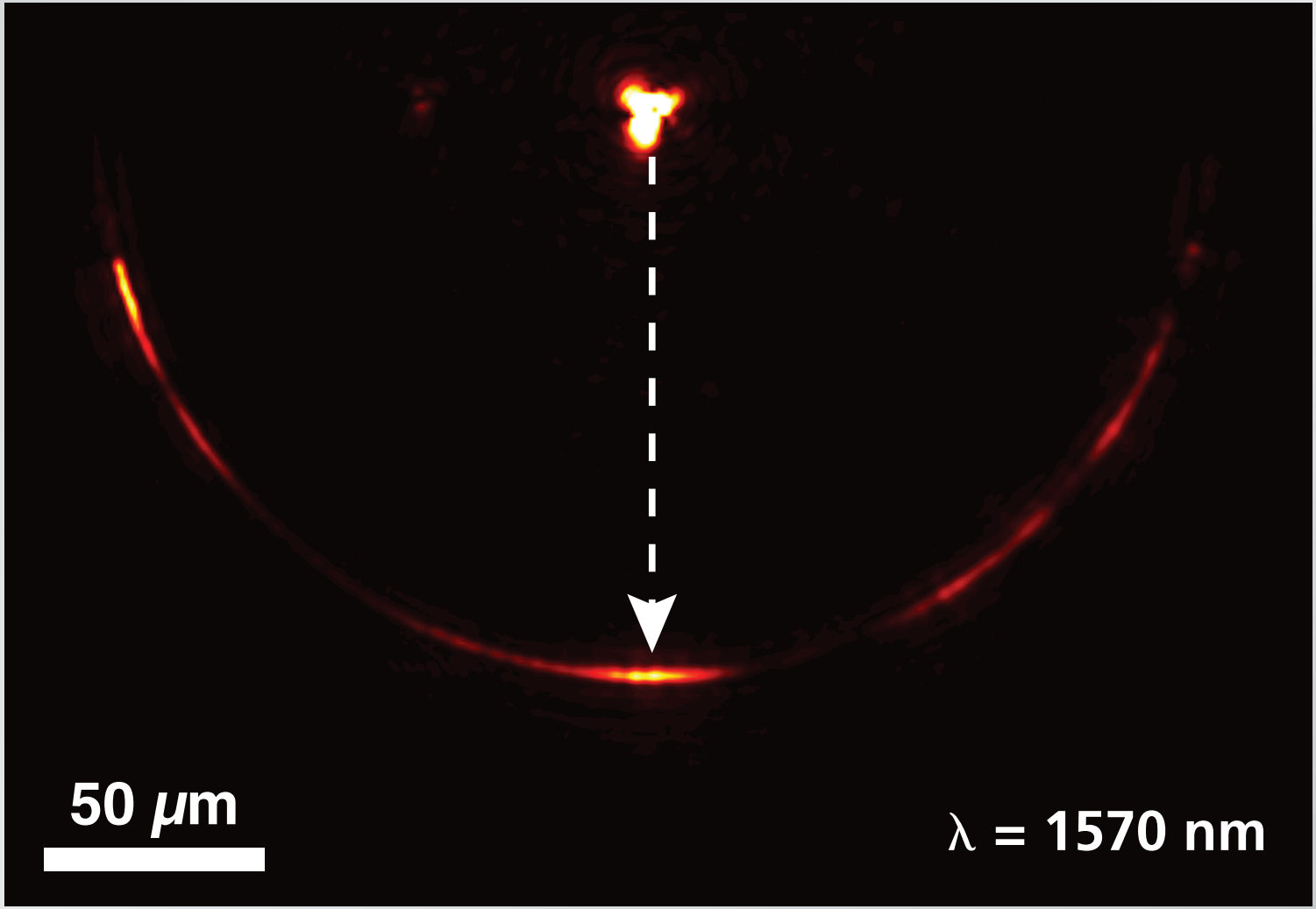


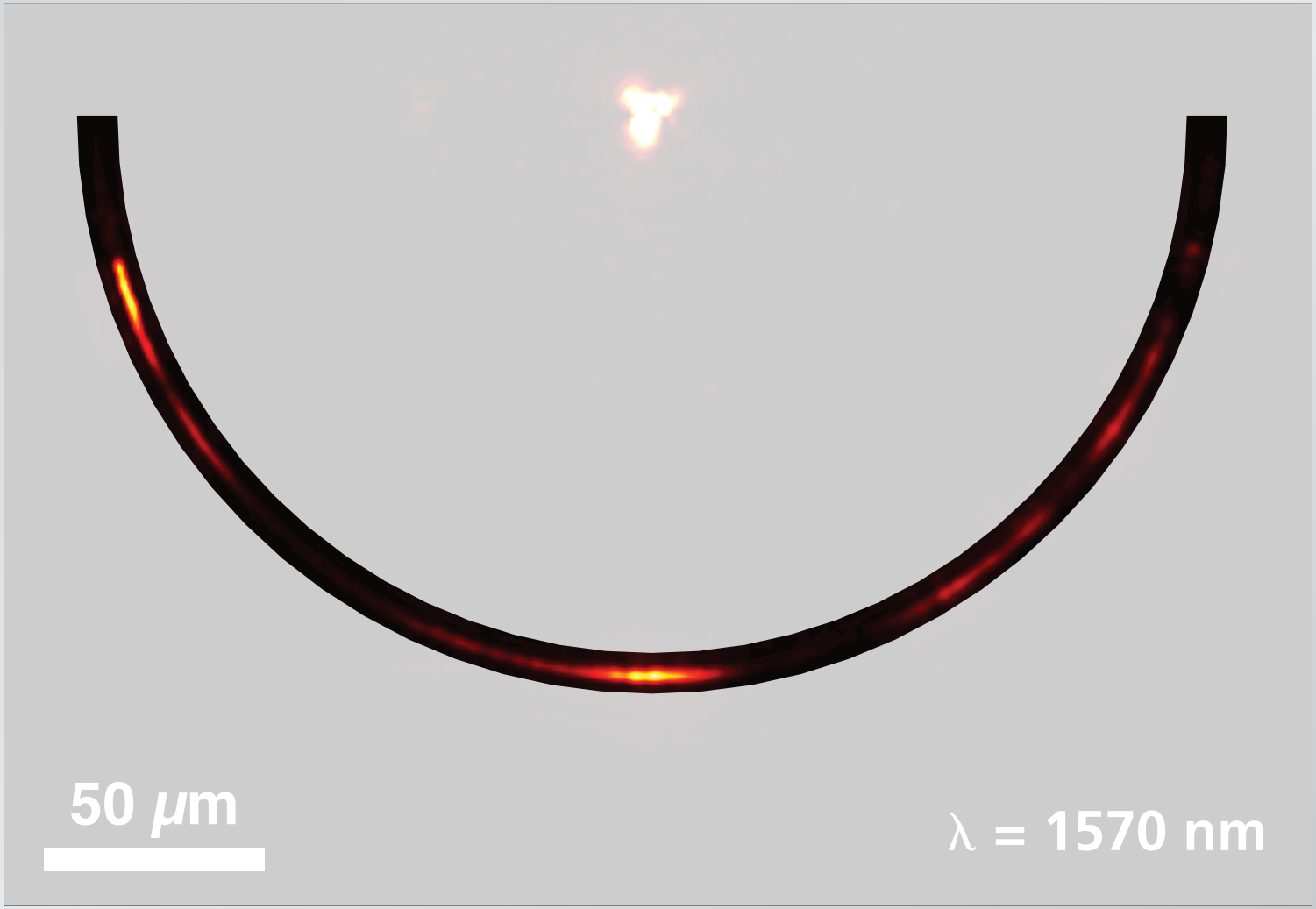
On-chip zero-index prism



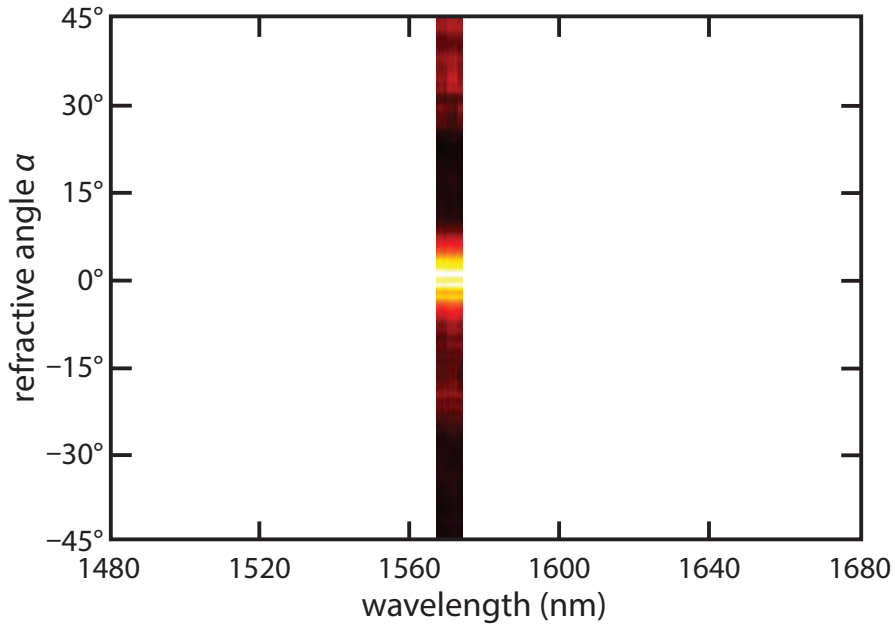

50 μm
λ = 1570 nm


50 μm
λ = 1570 nm

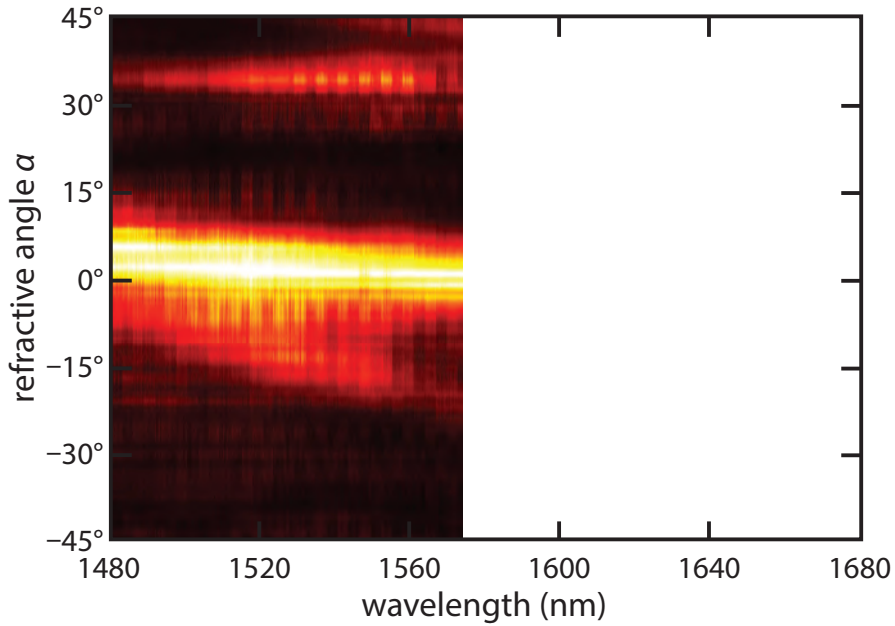

50 μm
λ = 1570 nm


50 μm
λ = 1570 nm

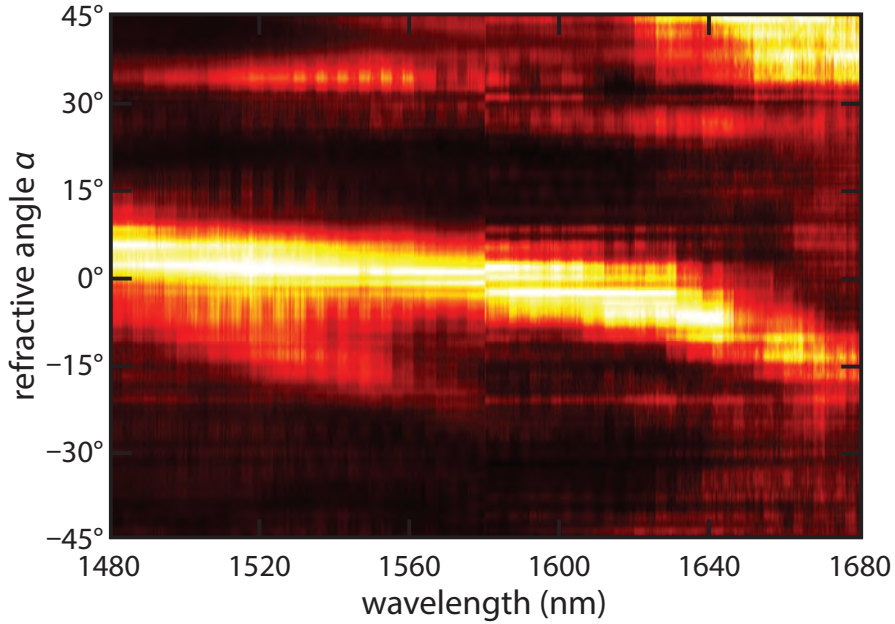
Wavelength dependence of refraction angle



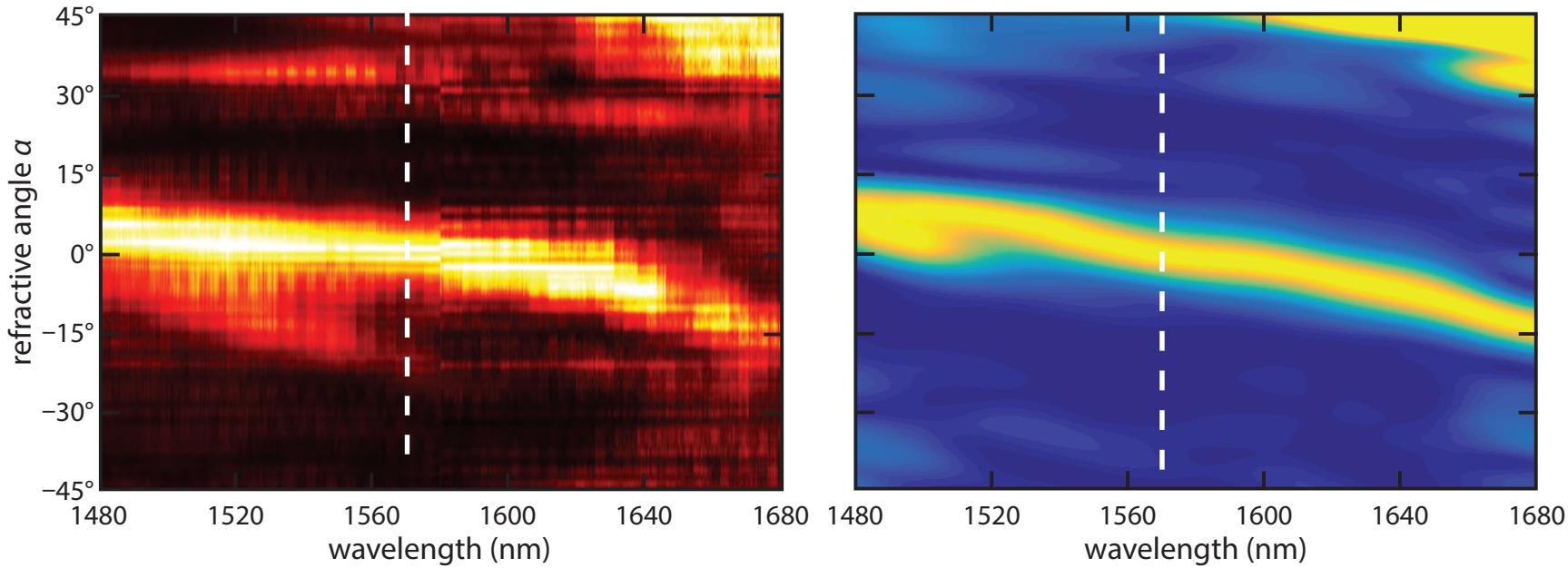
Wavelength dependence of refraction angle



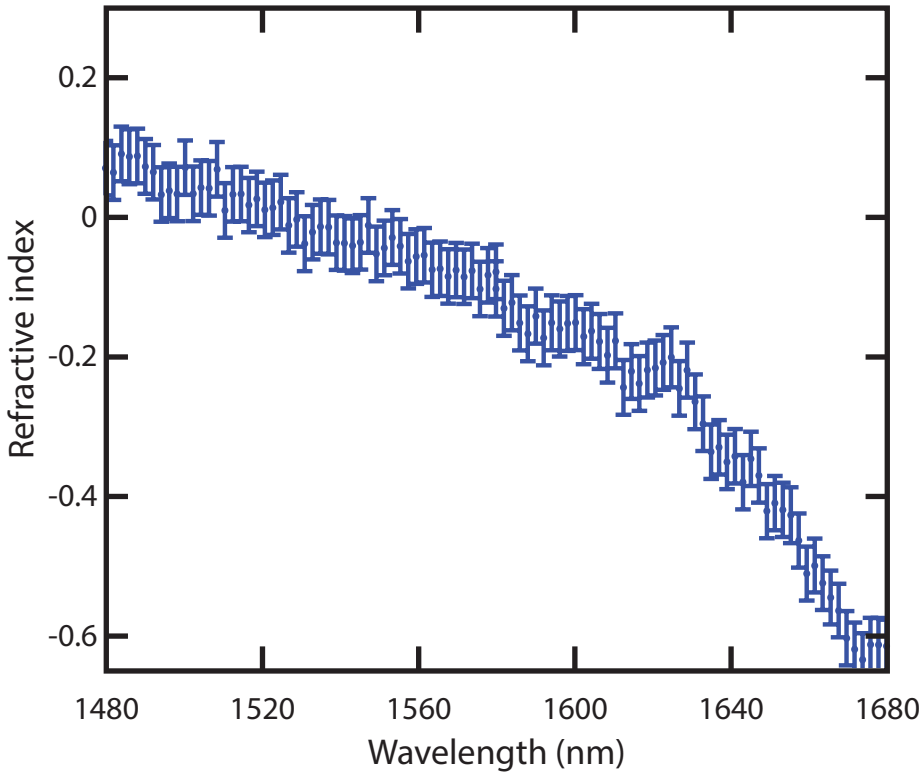
Wavelength dependence of refraction angle



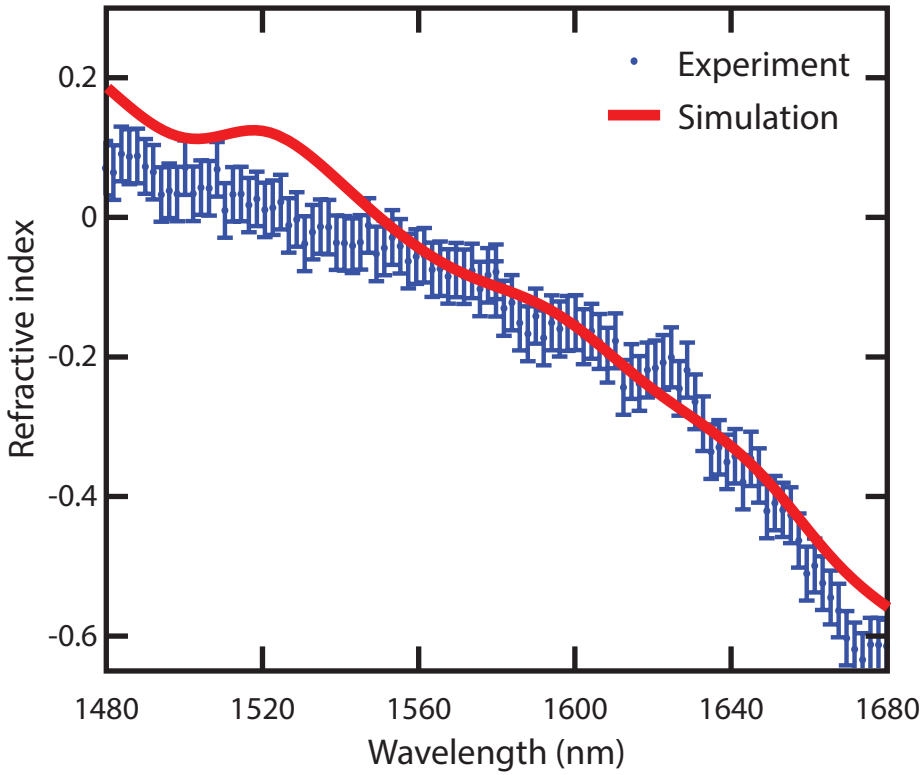
Wavelength dependence of refraction angle



Wavelength dependence of index

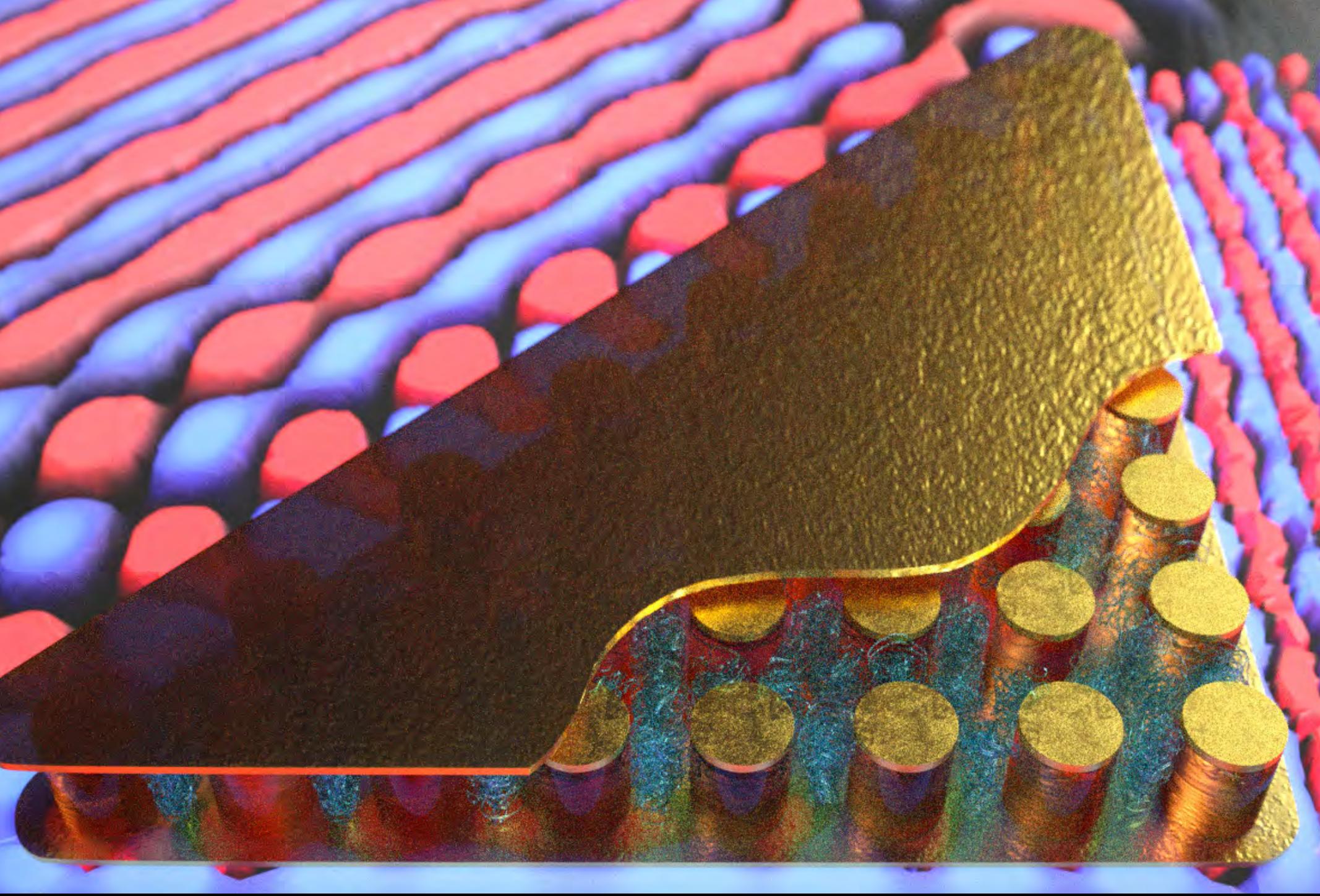


Wavelength dependence of index



More extreme optics

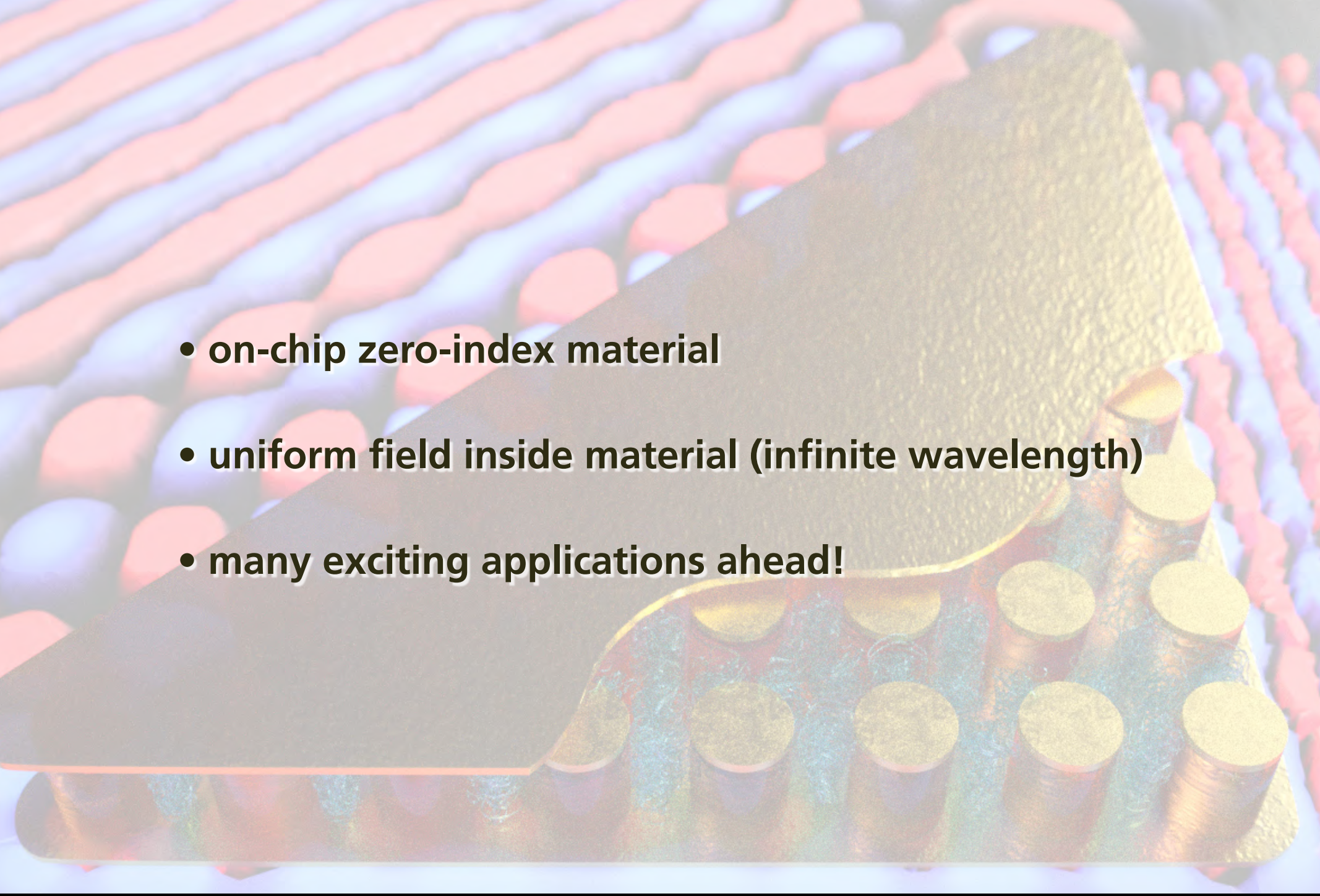
- suppressing losses
- beam steering & supercoupling
- nonlinear optics
- quantum optics



1 index

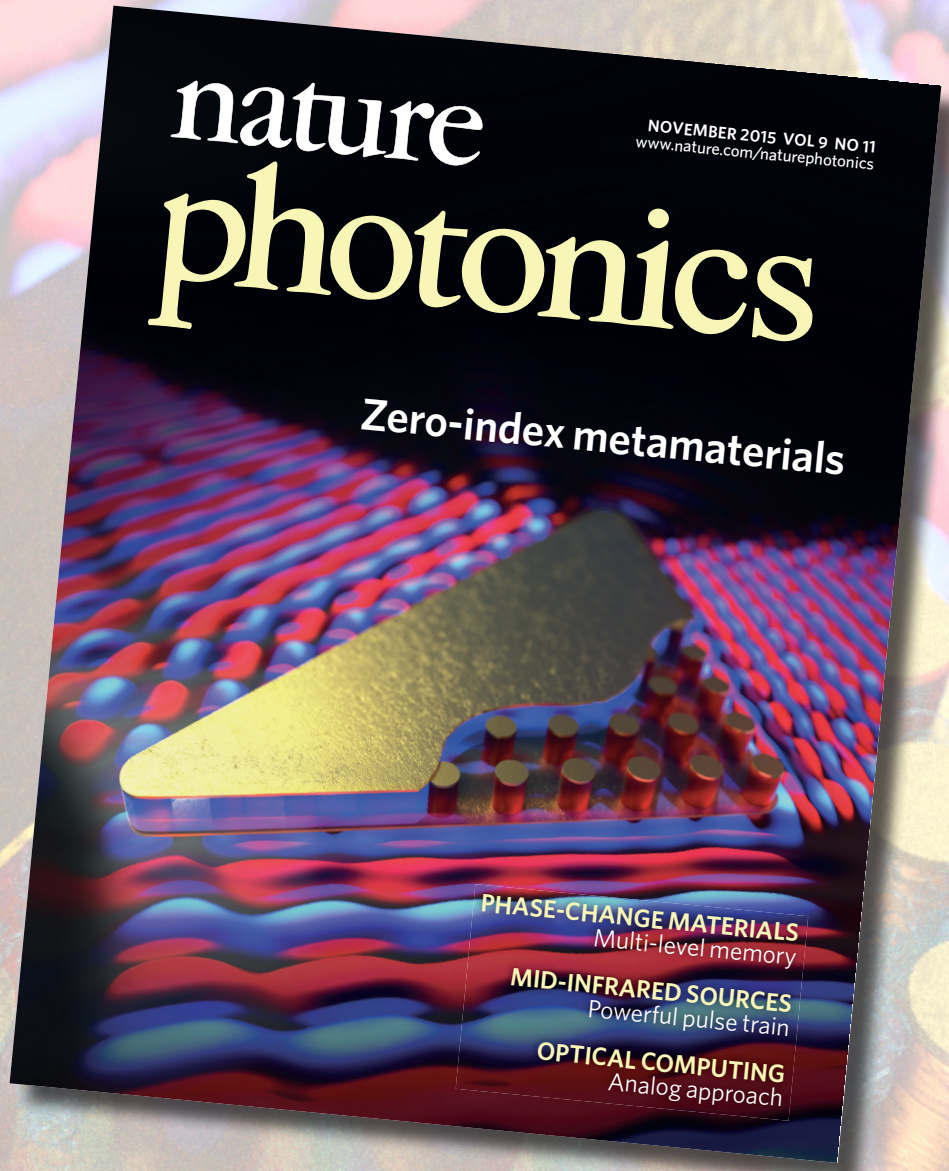
2 zero index

3 experiments



- on-chip zero-index material
- uniform field inside material (infinite wavelength)
- many exciting applications ahead!

More info: download paper!



The Team: Yang Li, Shota Kita,
Orad Reshef, Philip Muñoz, Daryl Vulis, Marko Lončar

Funding: National Science Foundation

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