Less is more:
Extreme optics with zero refractive index

NUS

Singapore, 24 August 2016

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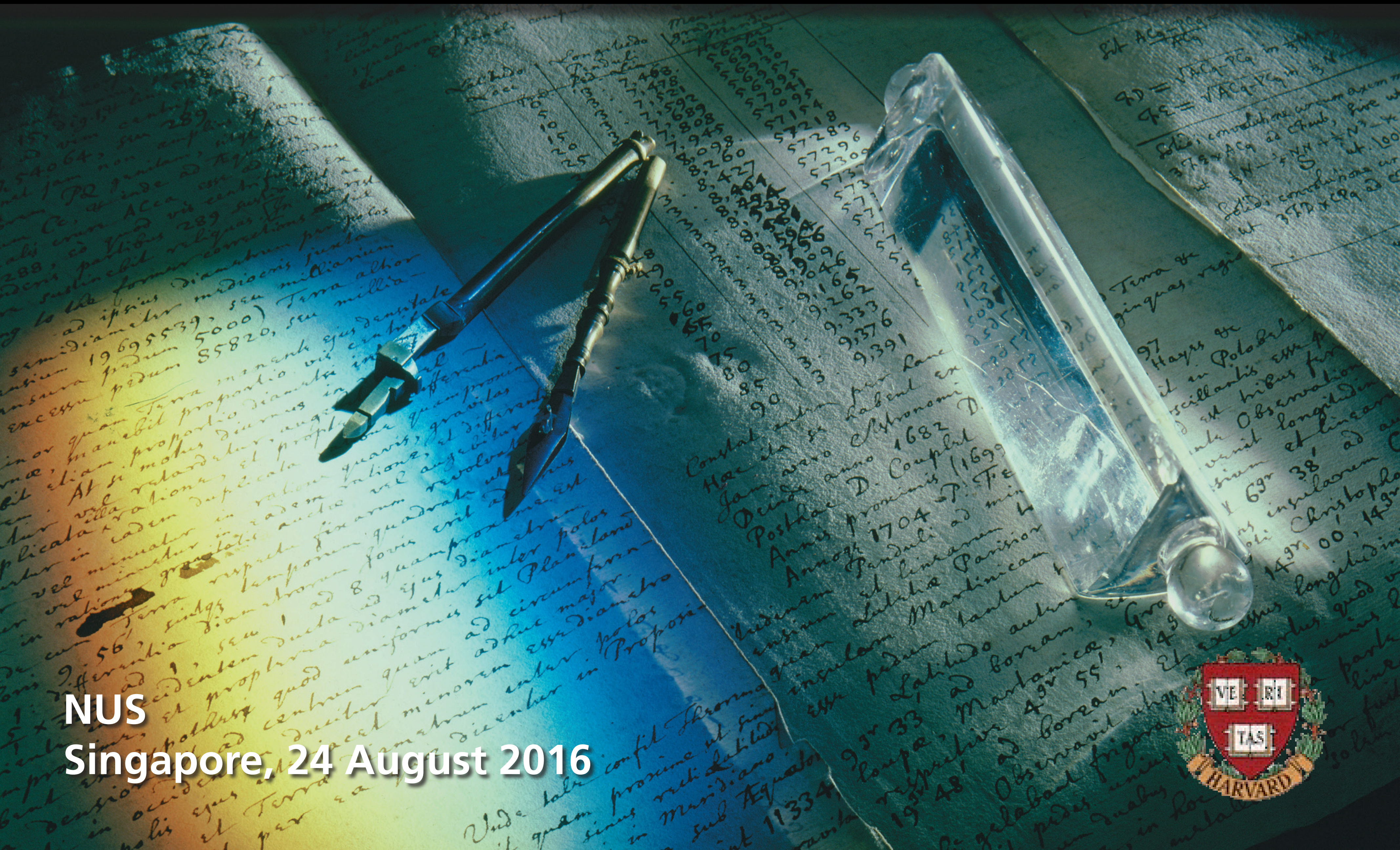

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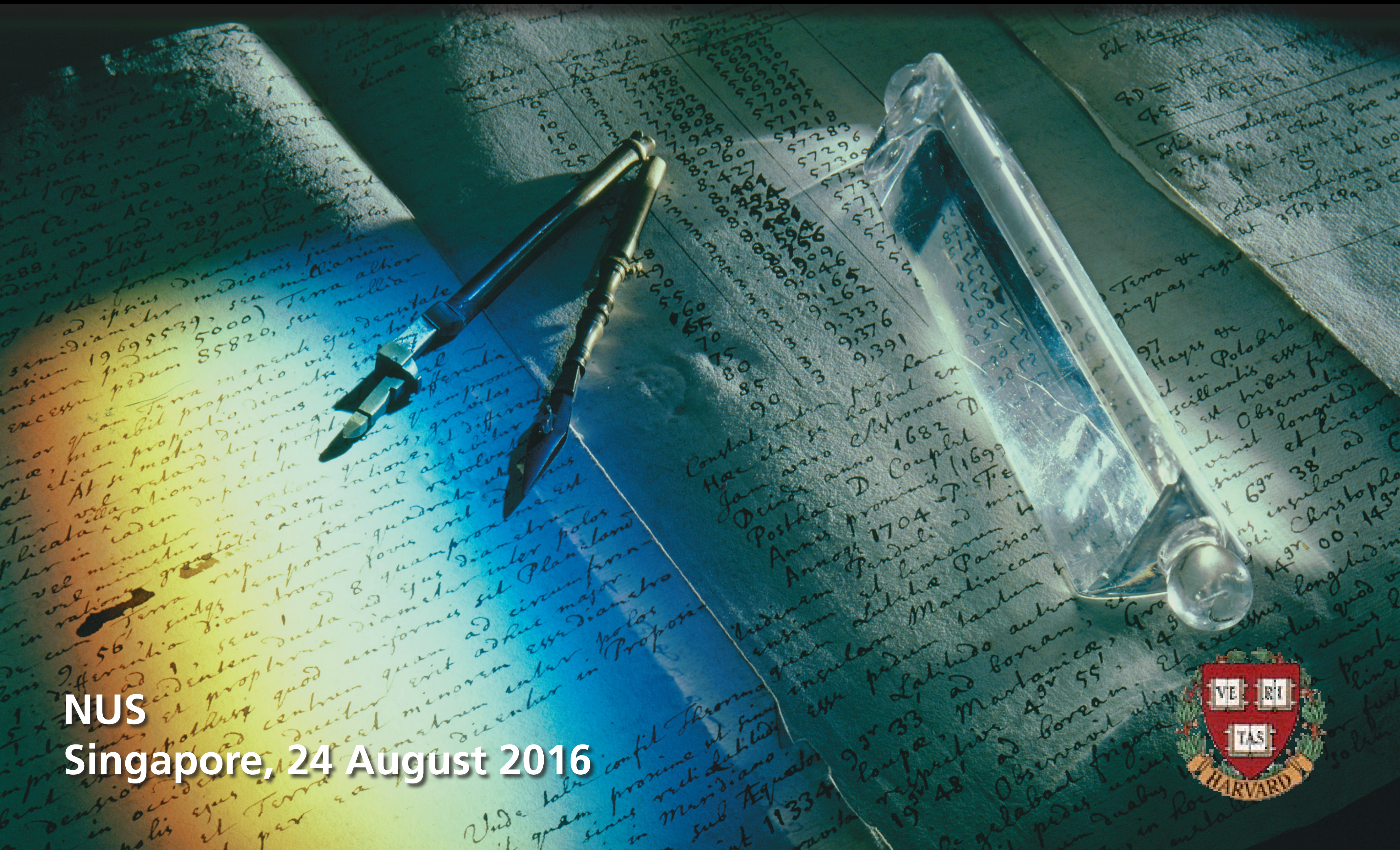

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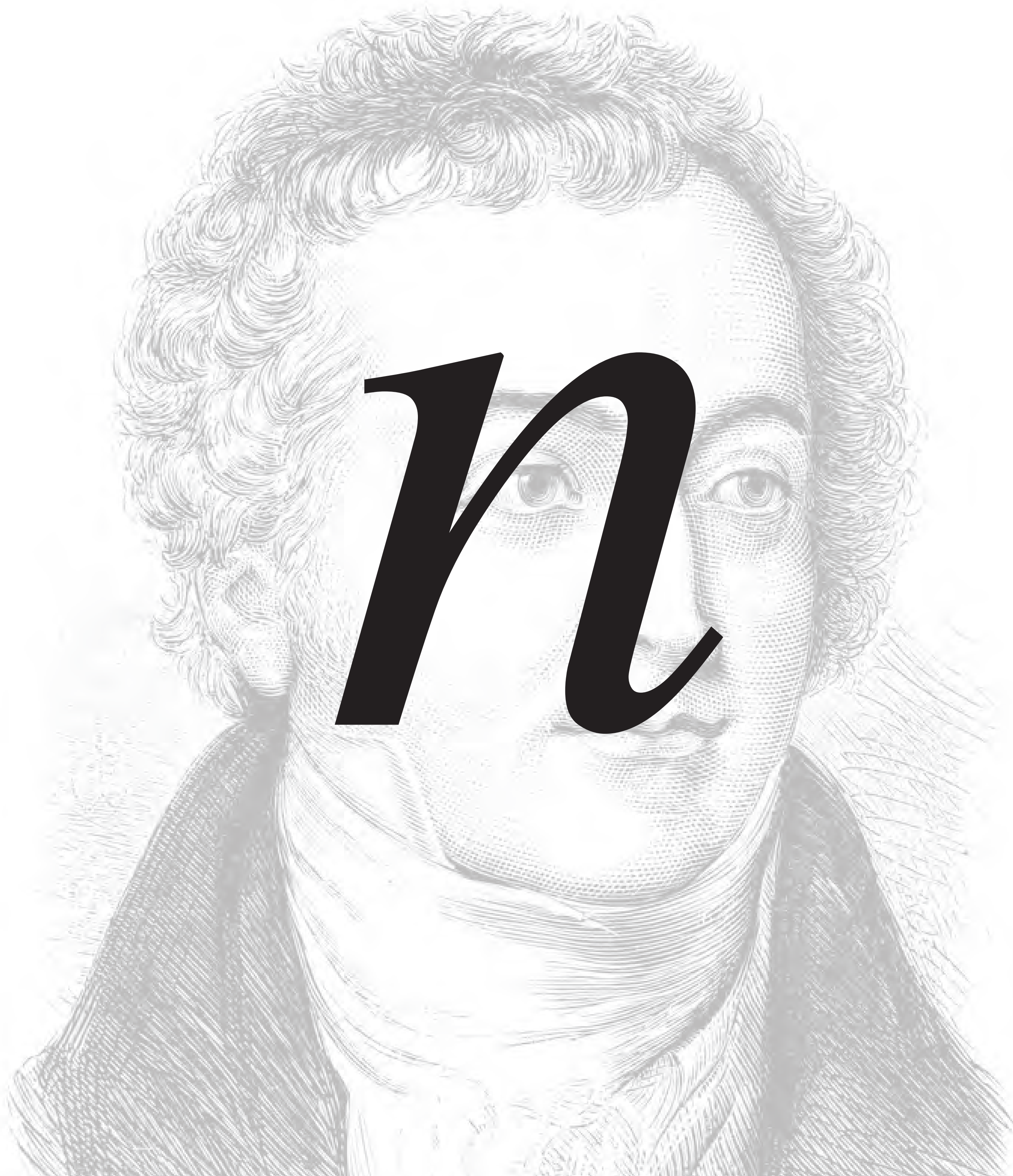
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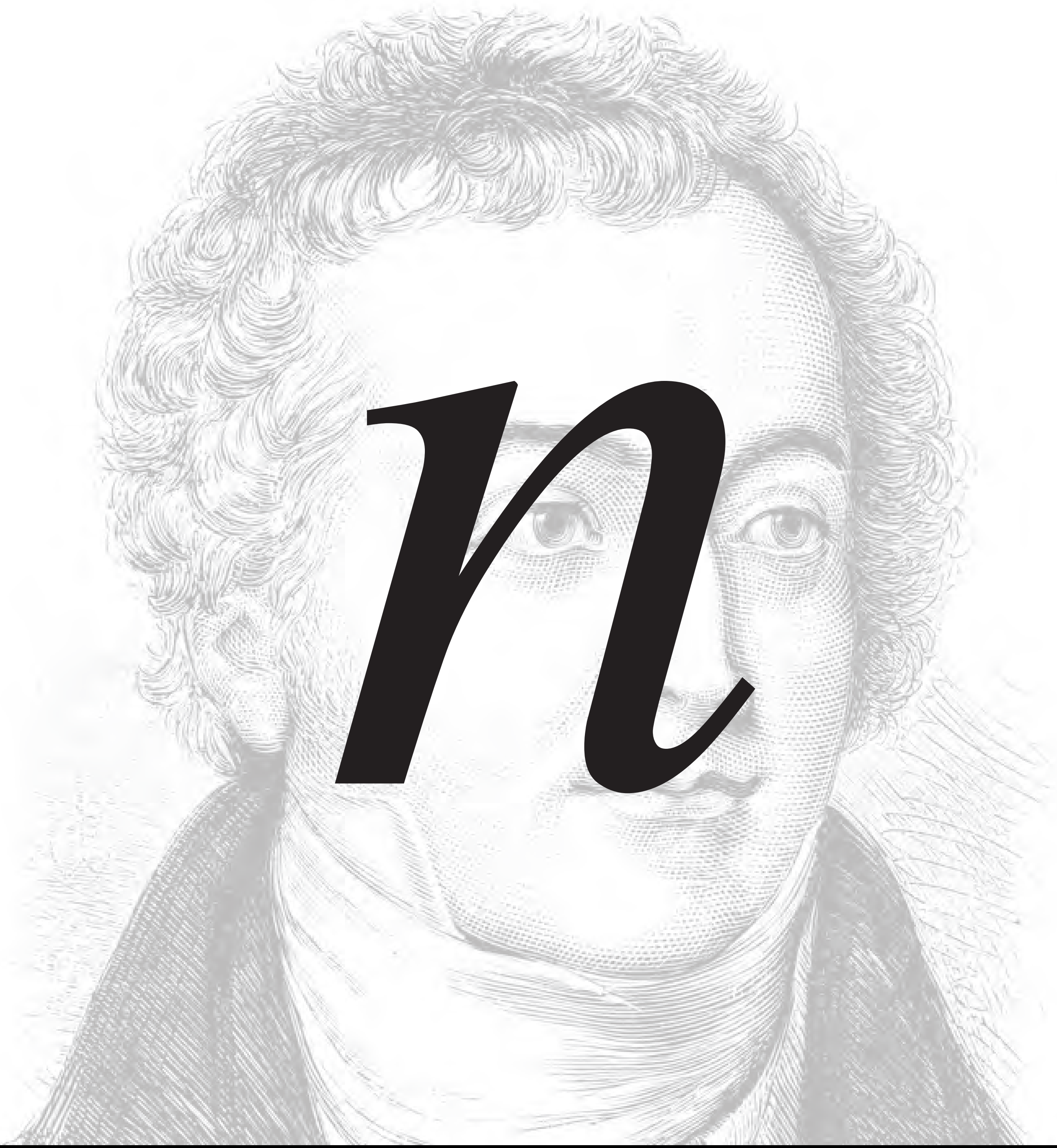
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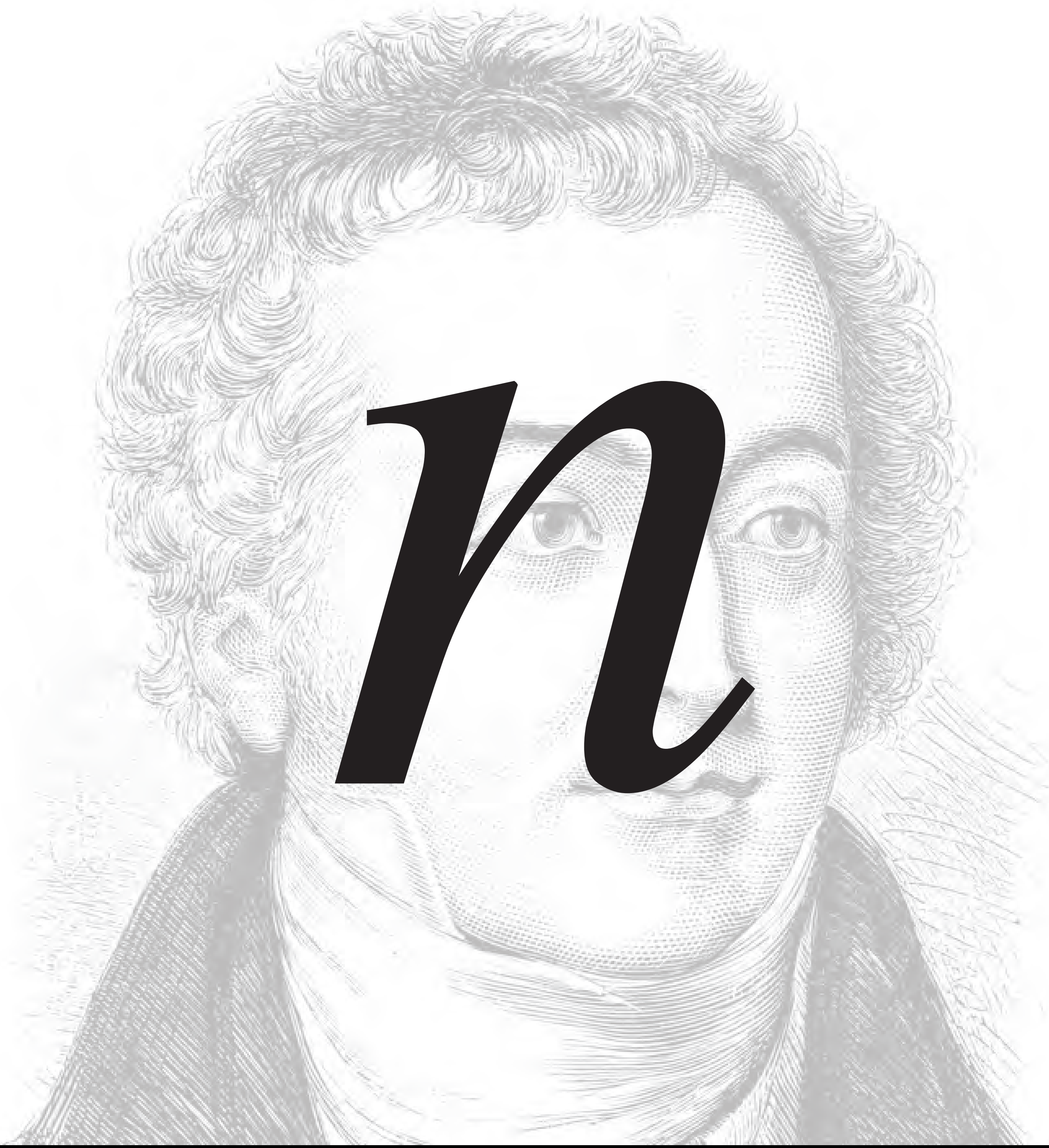
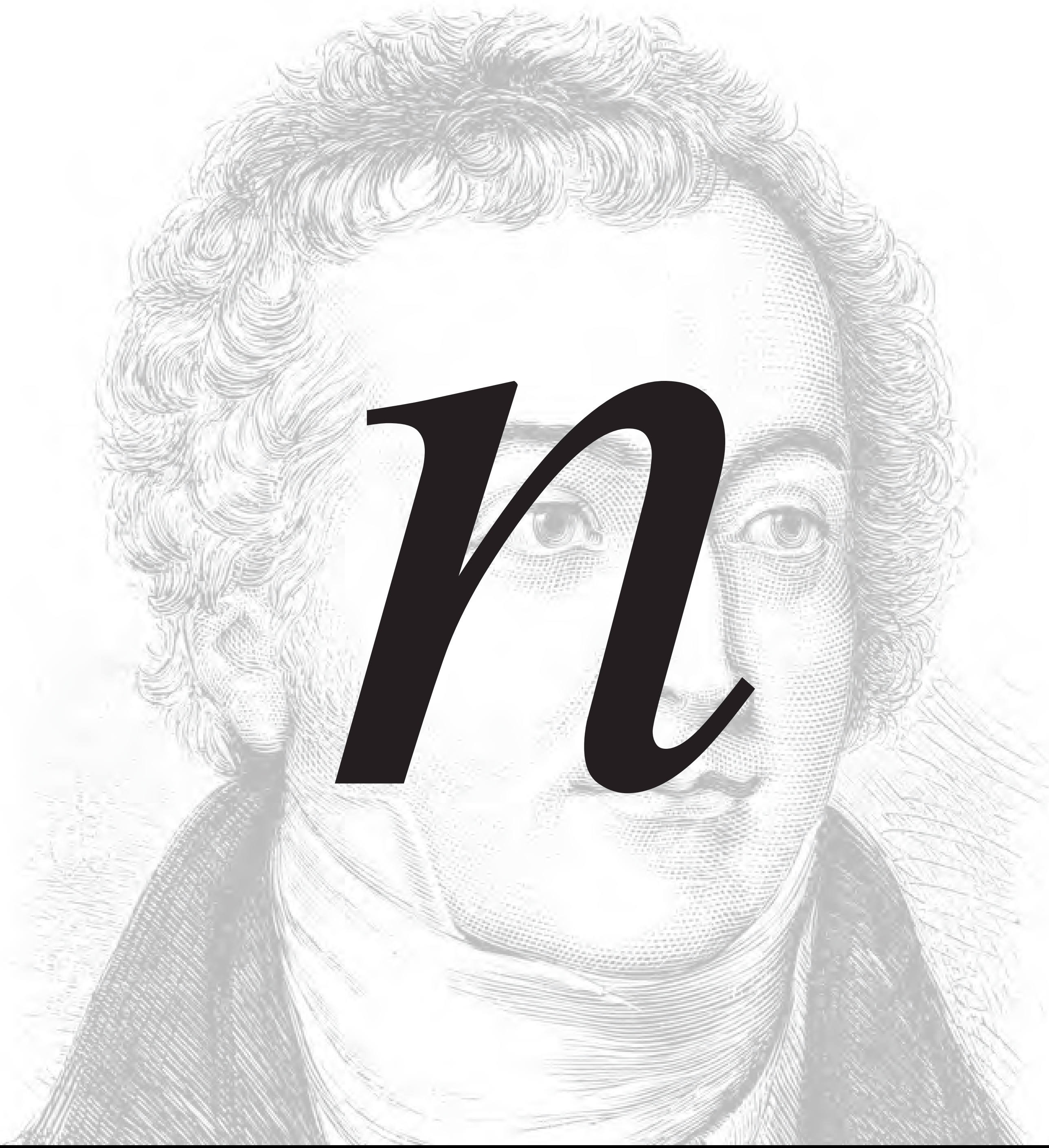


1 index

2 zero index



1 index

2 zero index

3 experiments

Propagation of EM wave

Propagation of EM wave

governed by wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

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and

$$n = \sqrt{\epsilon\mu} \, .$$

In dispersive media $n = n(\omega)$.

Index of refraction

$$n = \sqrt{\epsilon\mu}$$

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So $n(\omega)$ **determined by response of material to external fields**

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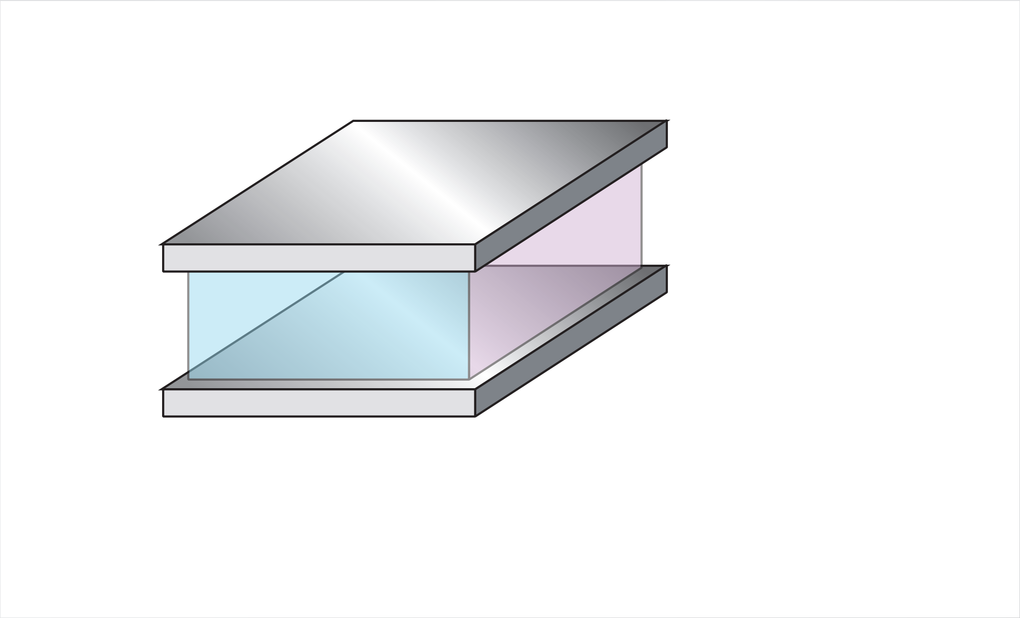

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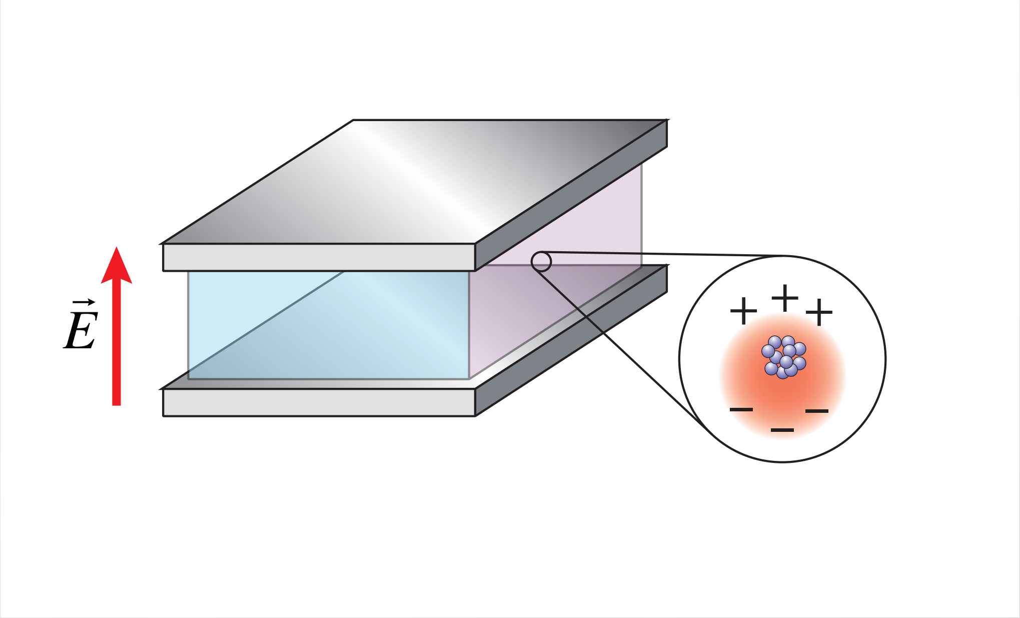

$\epsilon(\omega)$ measure of attenuation of electric field

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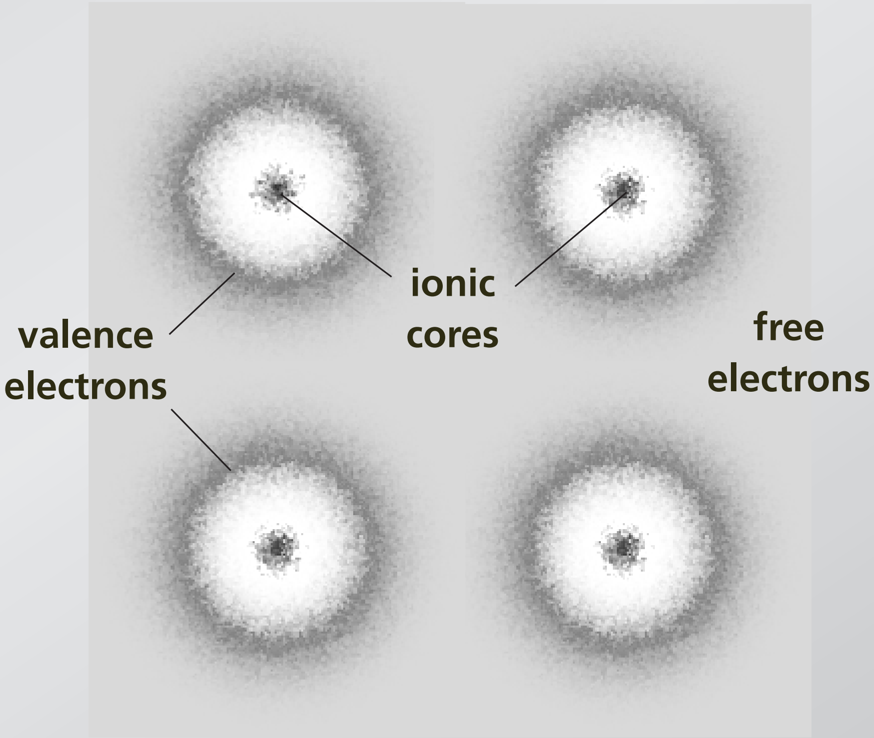
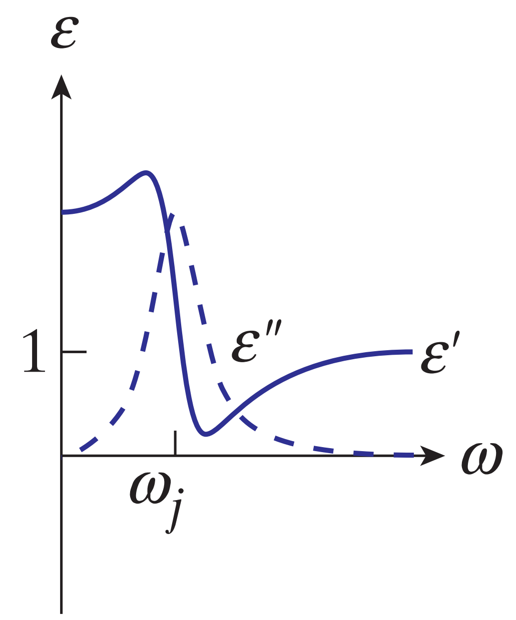


Dielectric constant

Lorentz oscillator



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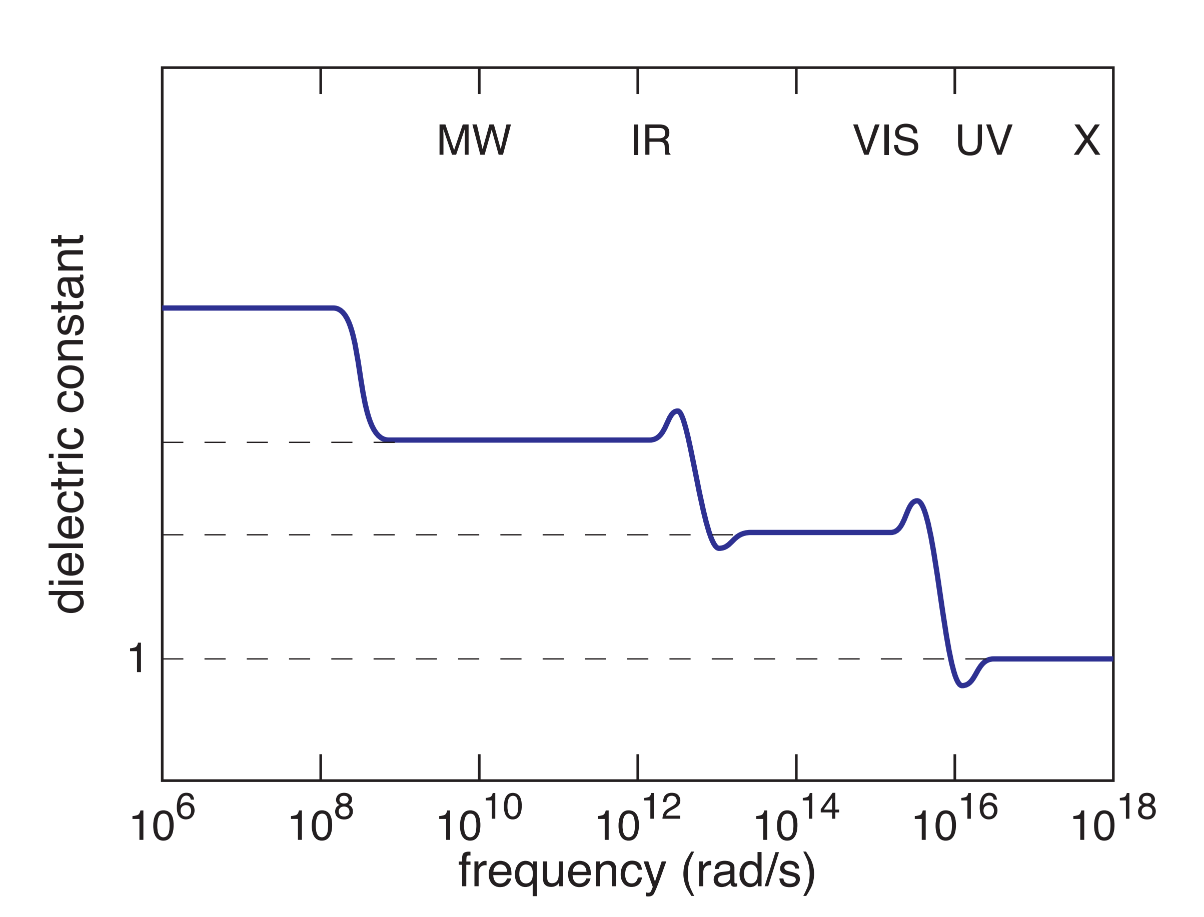

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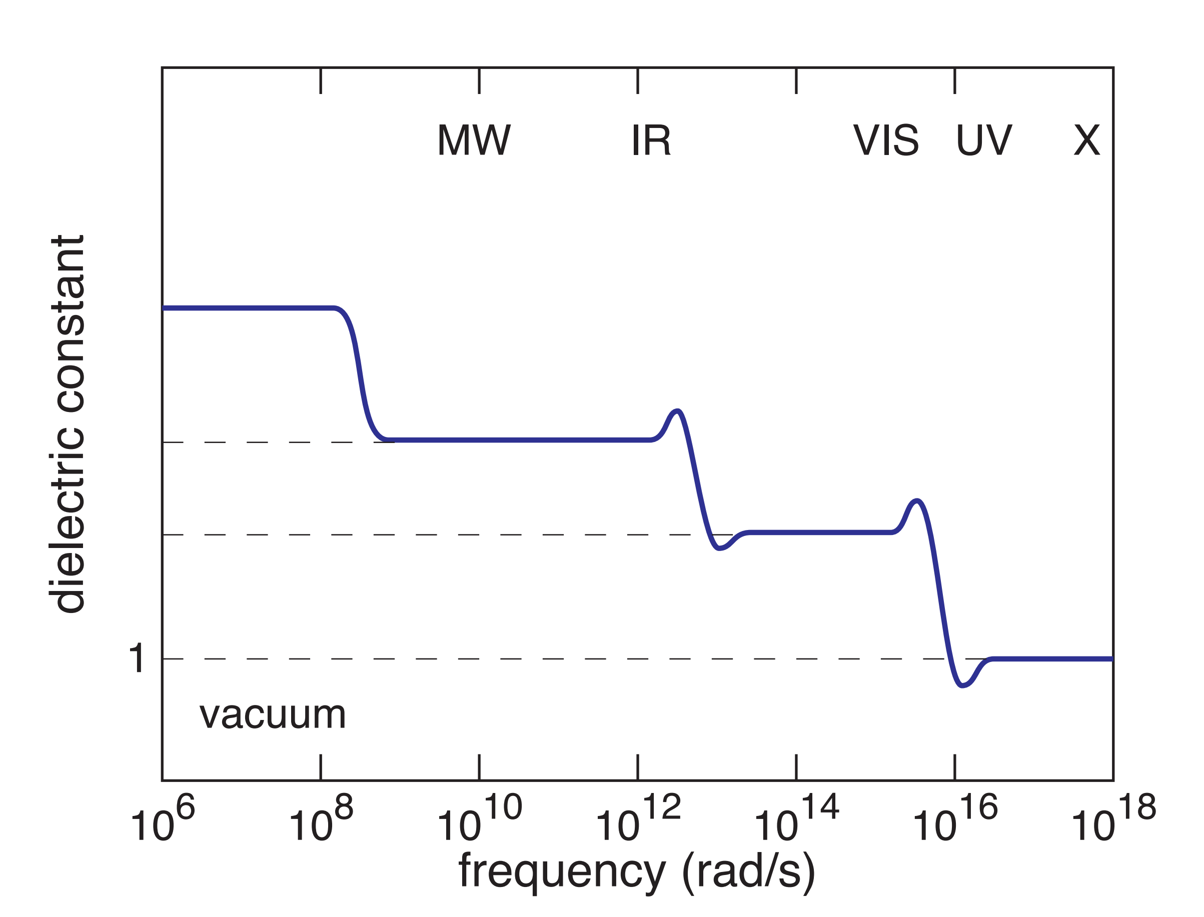

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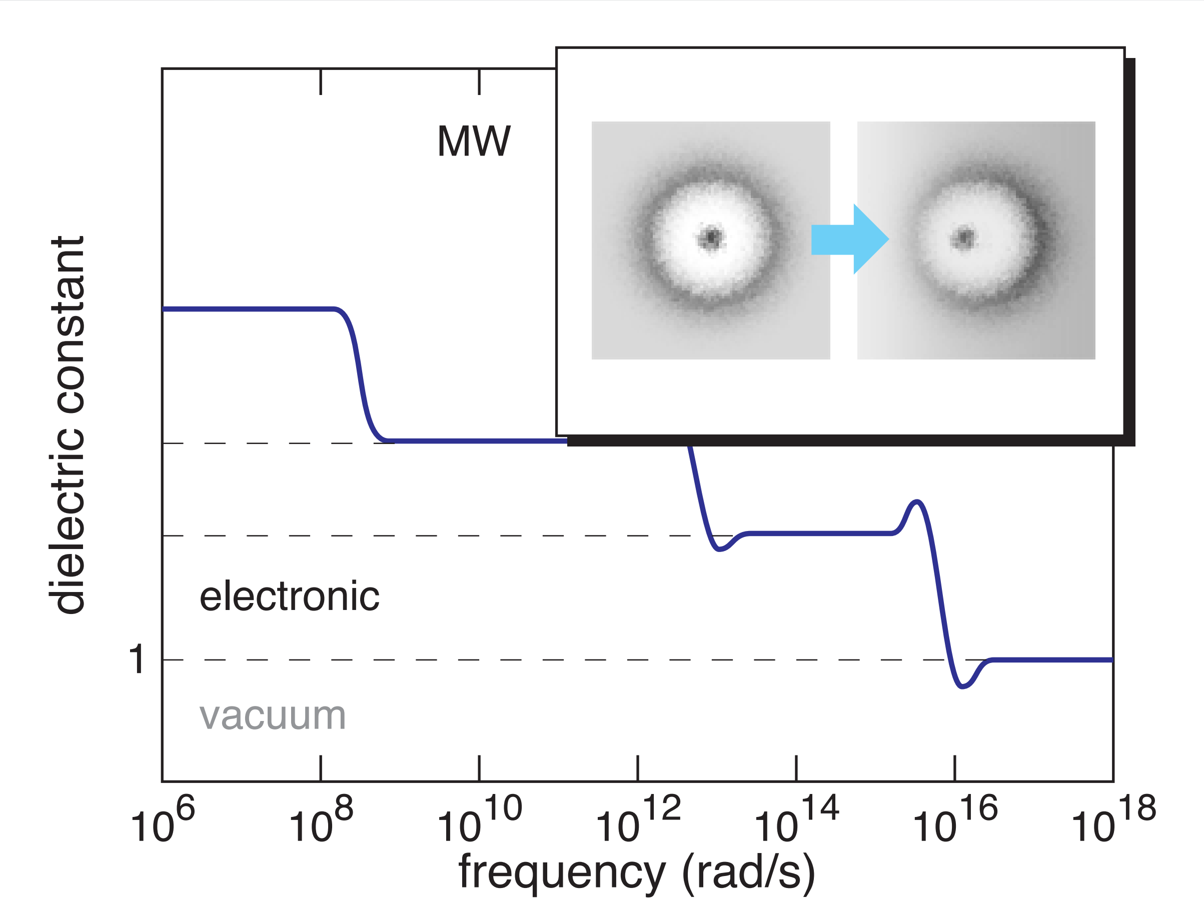

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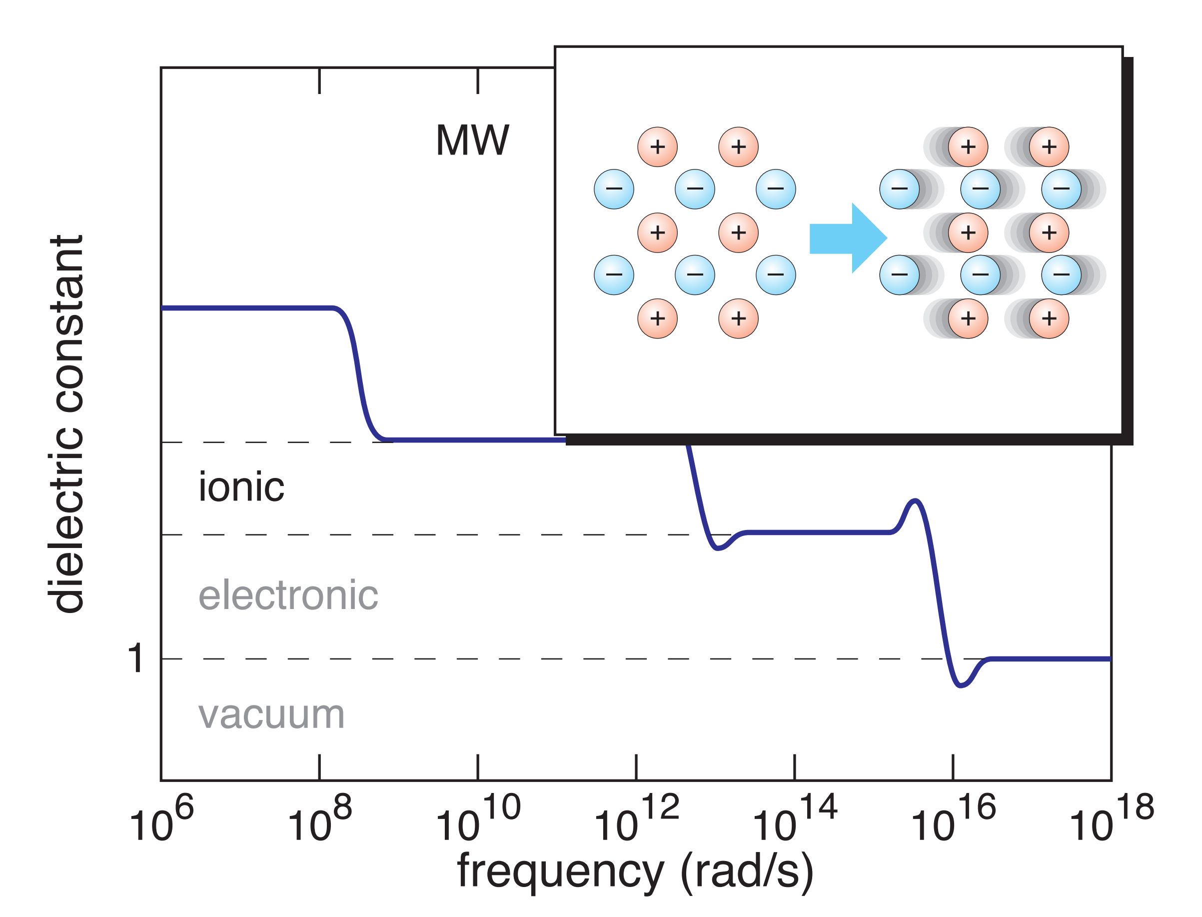

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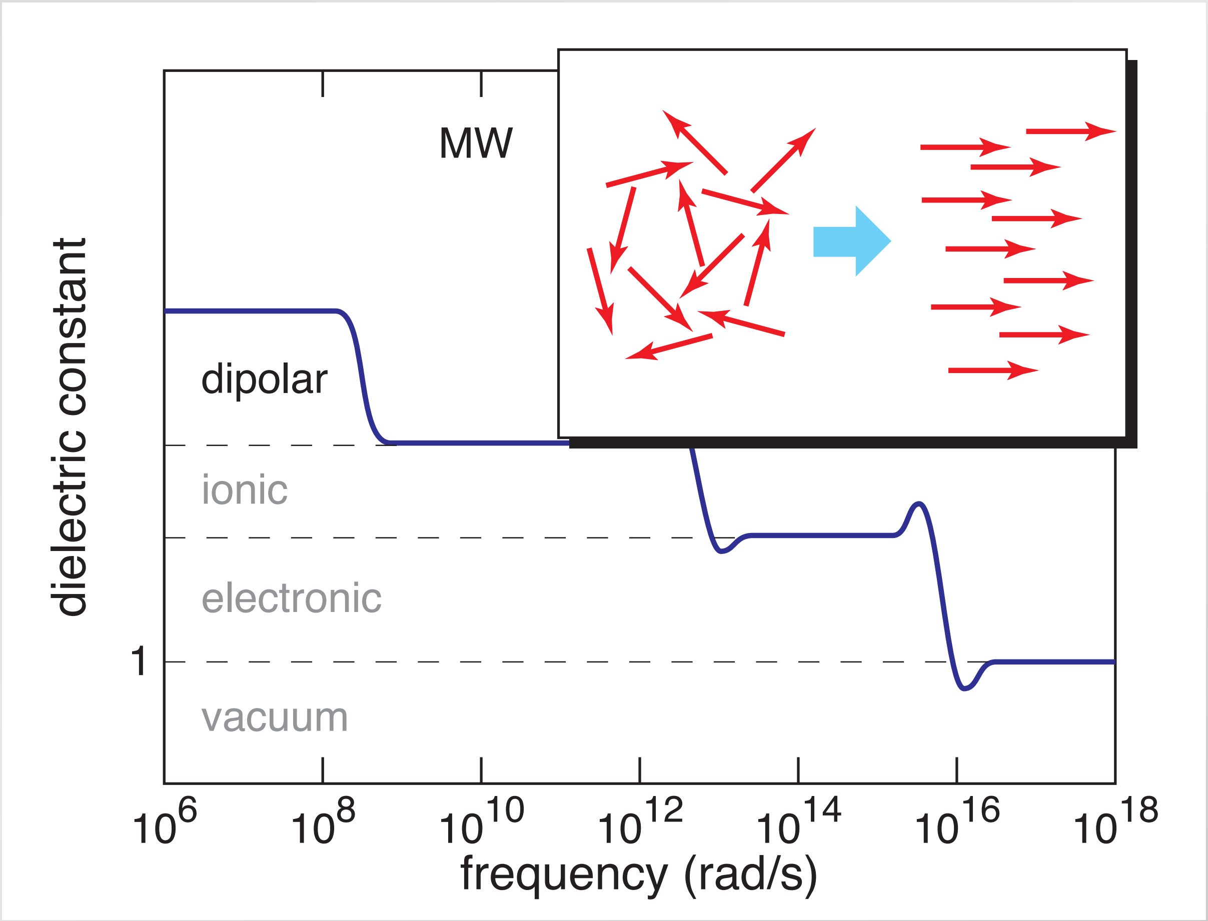

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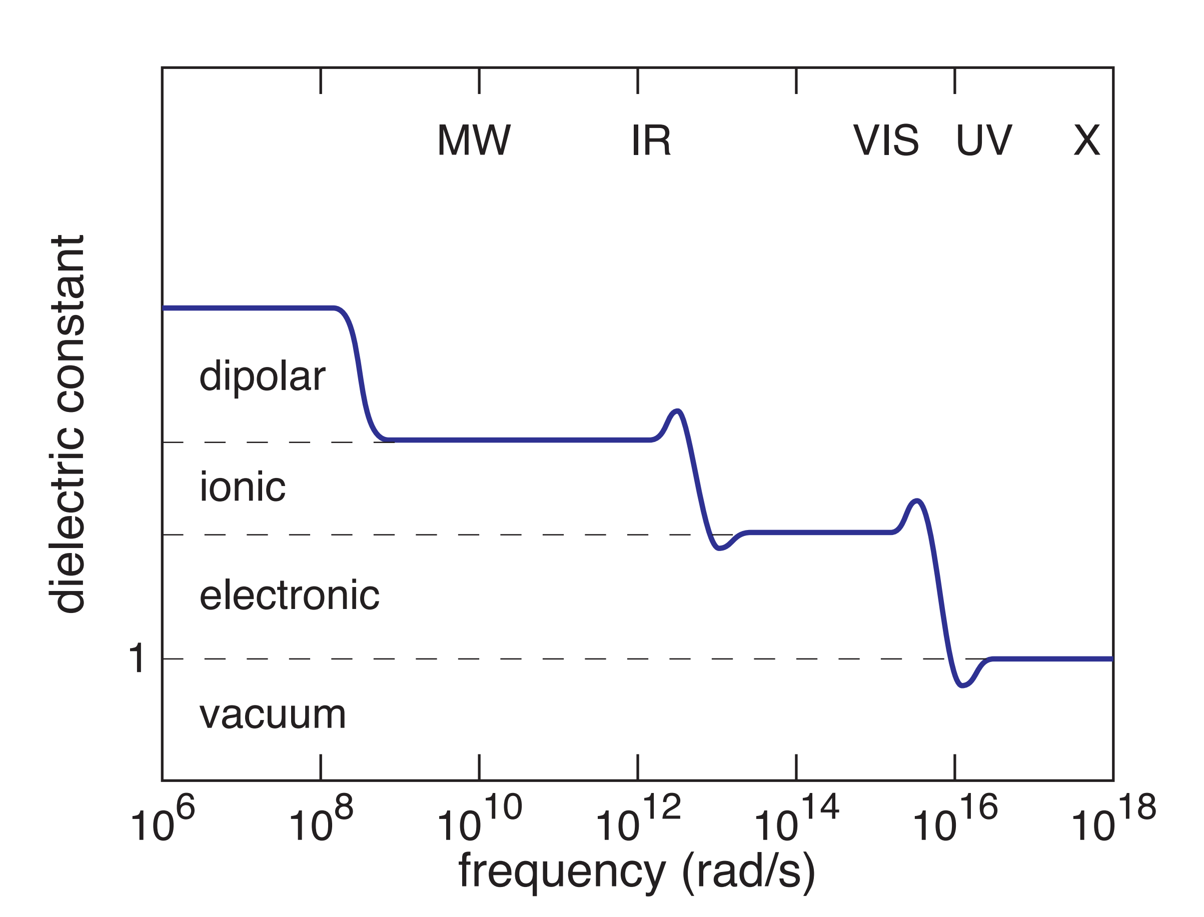
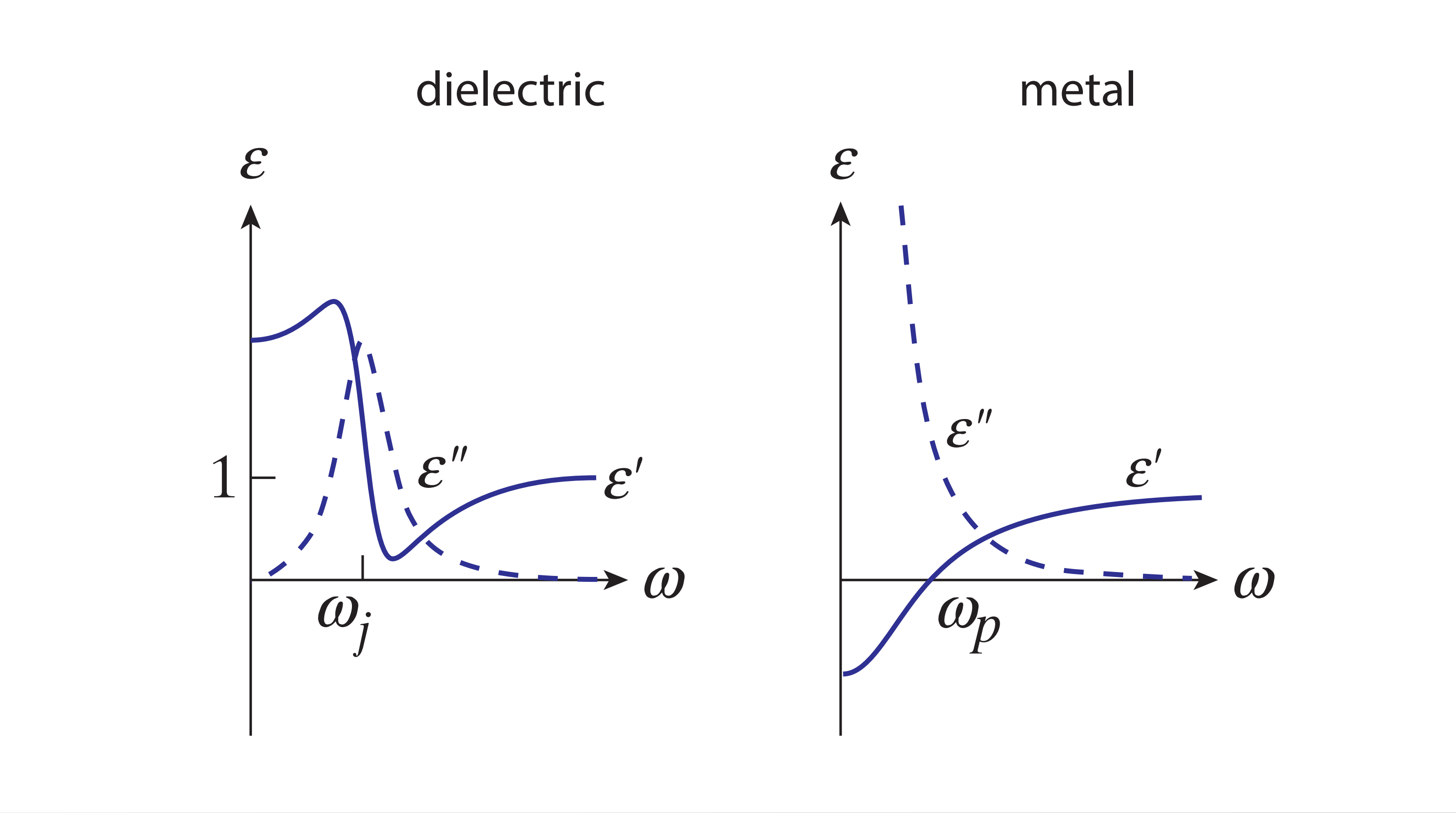
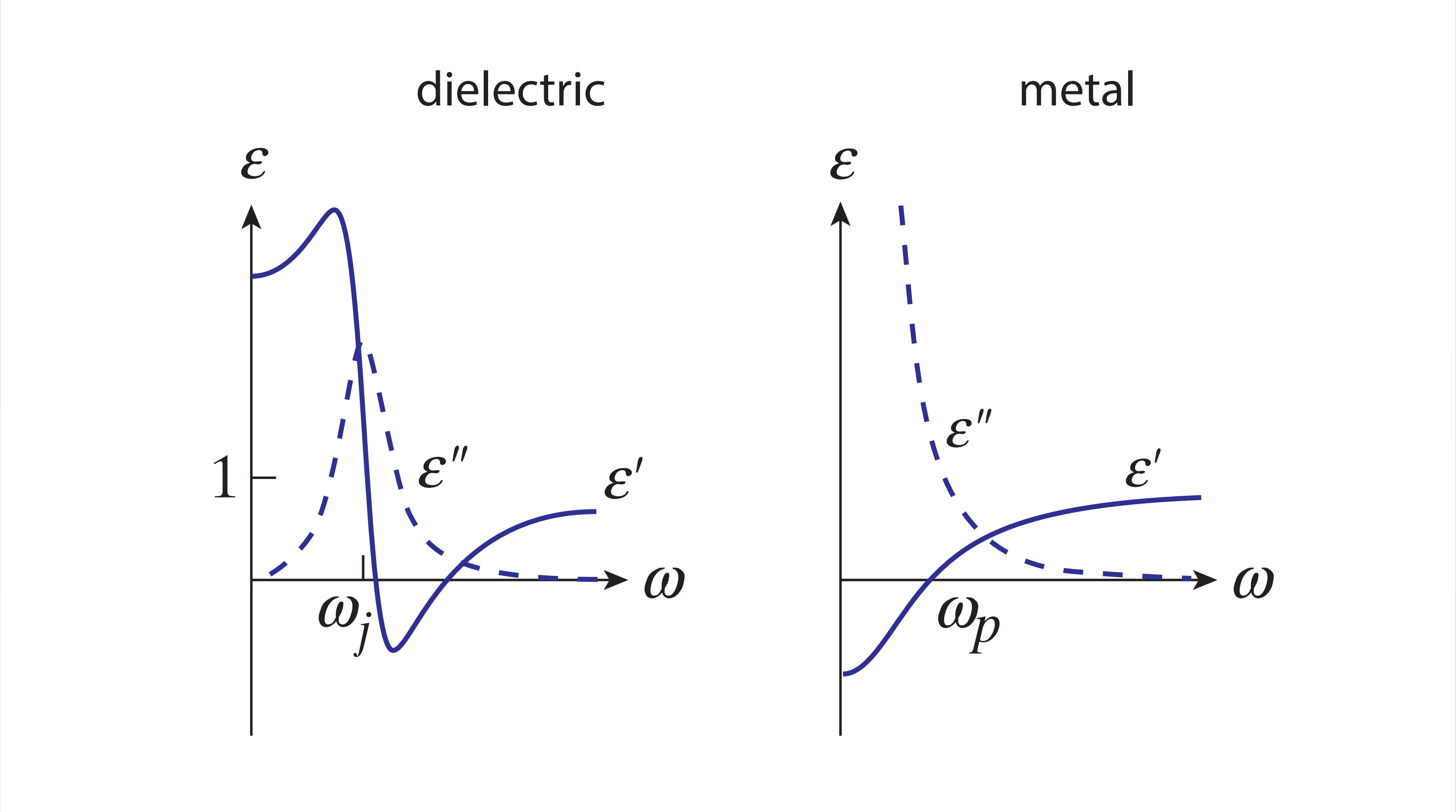


Lorentz and Drude models



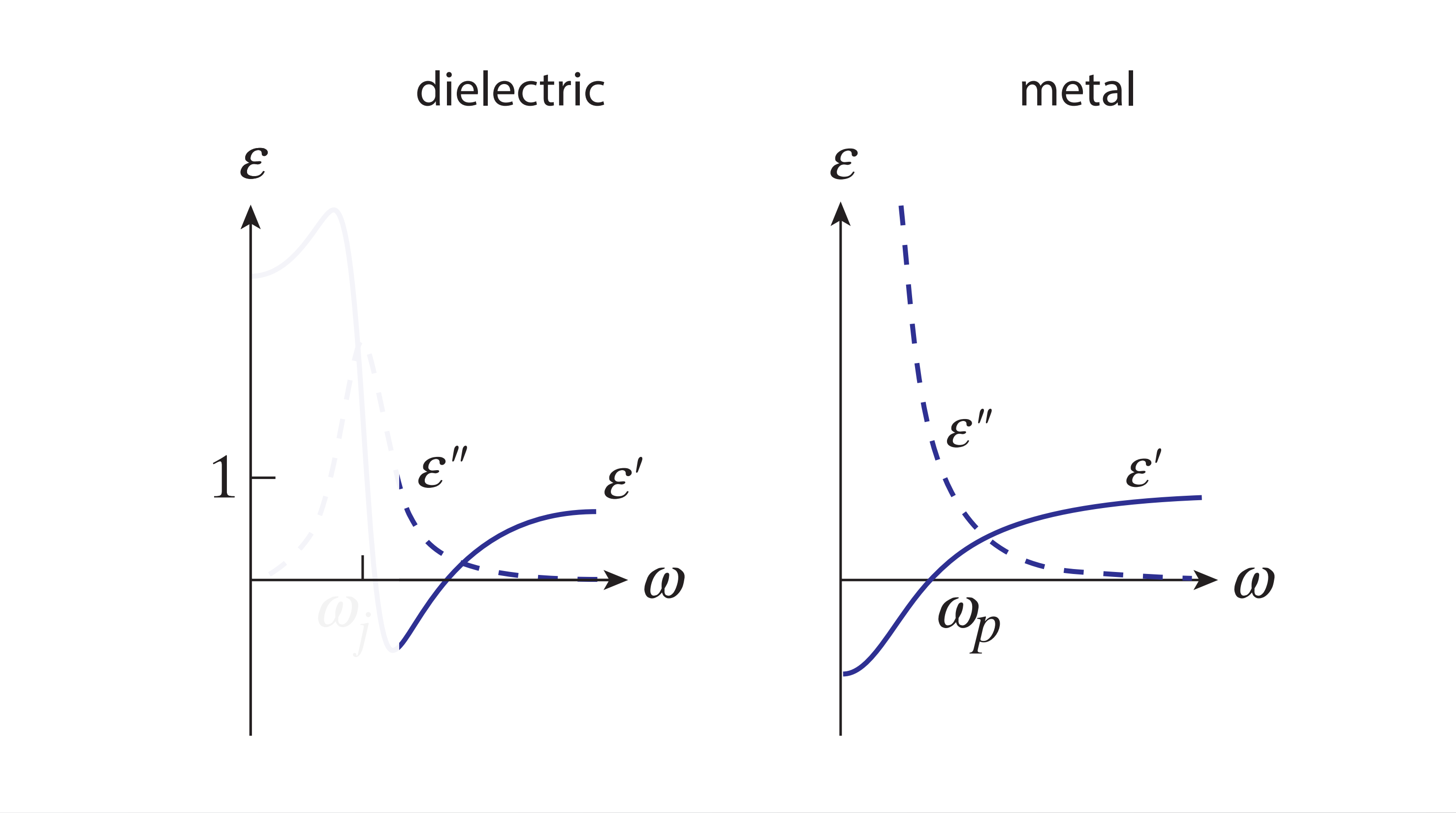
Lorentz and Drude models

for a strong (dielectric) resonance ε can become negative



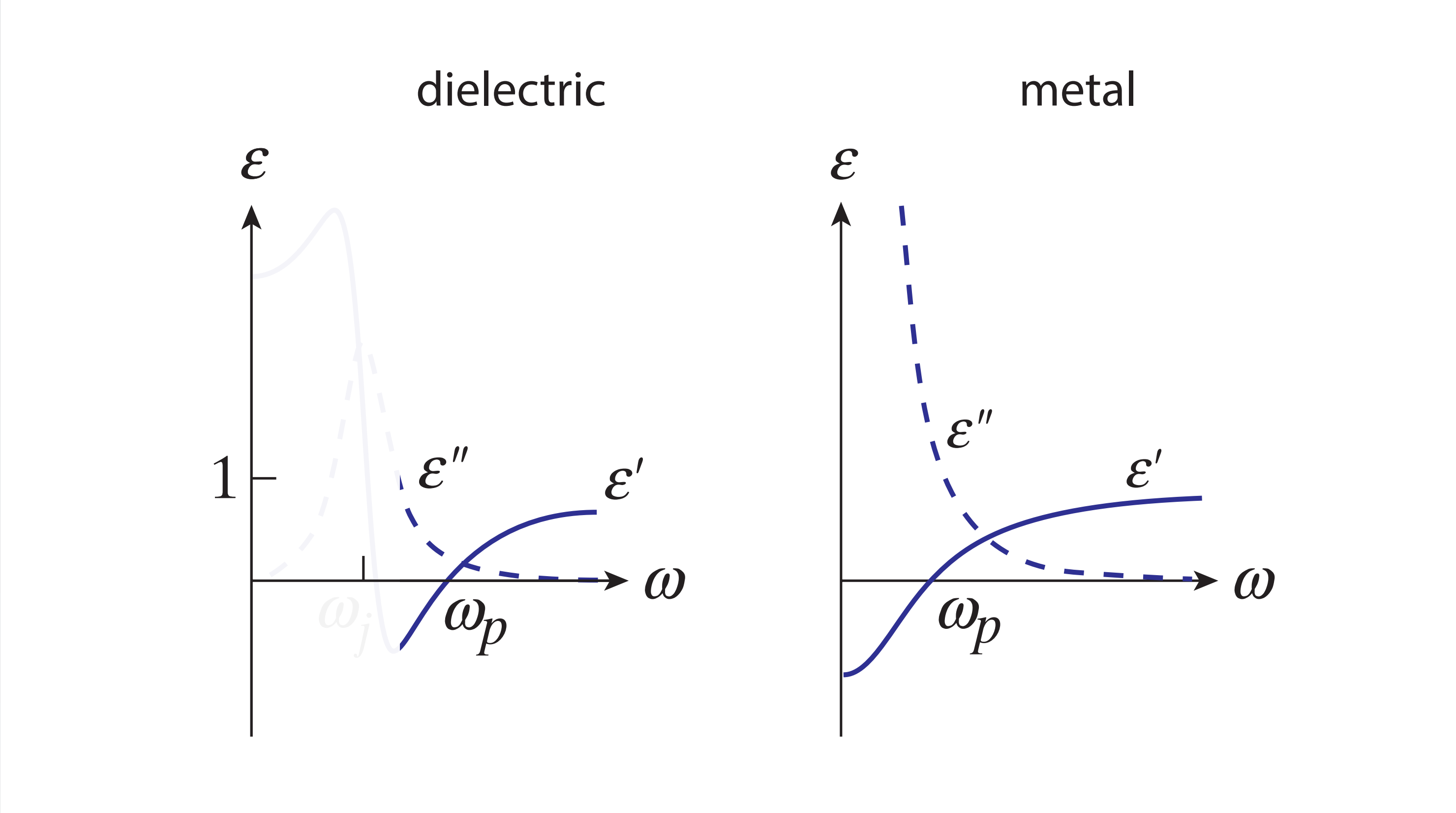
Lorentz and Drude models

valence electrons in dielectric then behave like a plasma



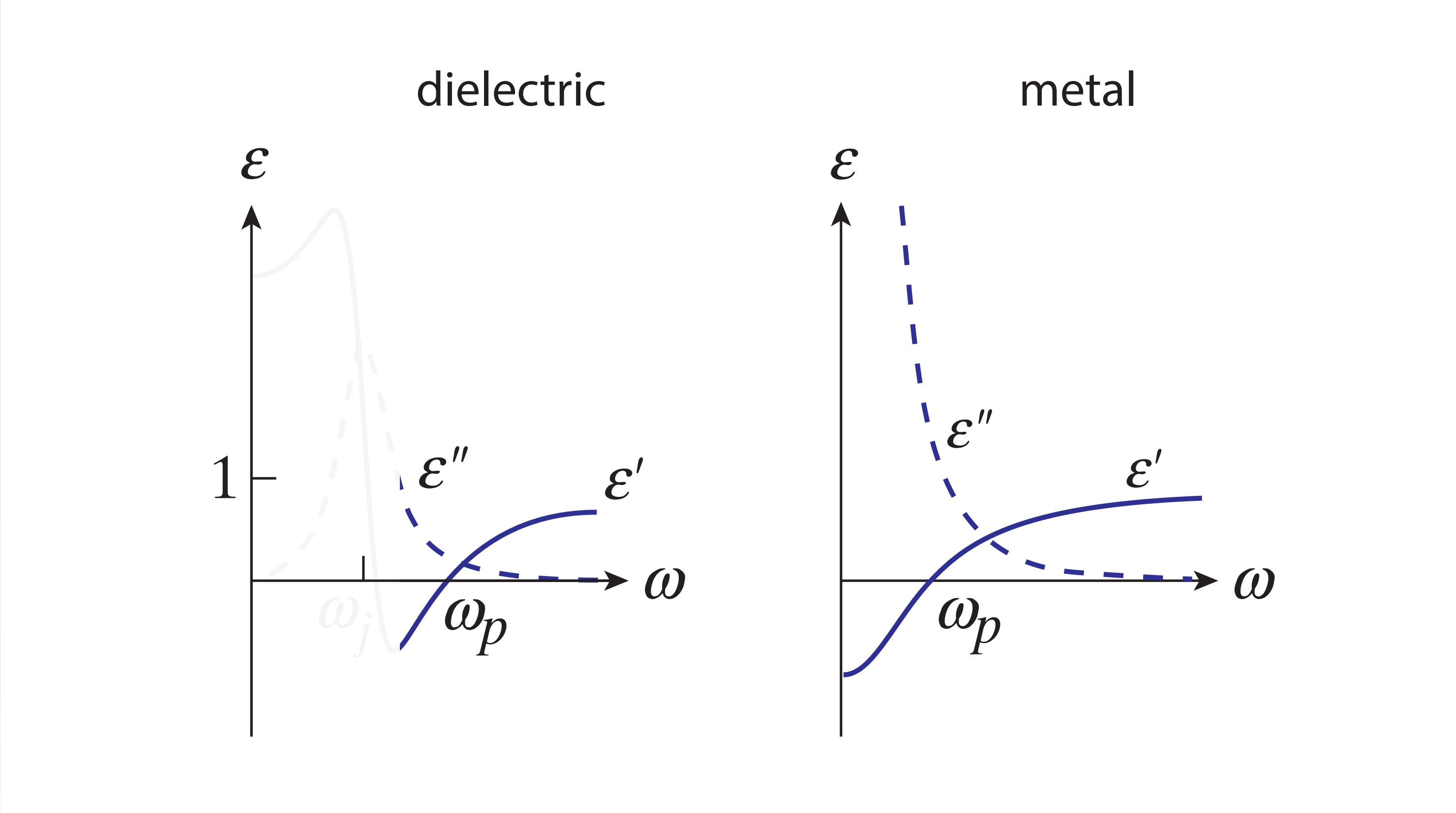
Lorentz and Drude models

with plasma frequency above the resonance



Lorentz and Drude models

(and far below the UV region)



Index also determined by magnetic response

$$n = \sqrt{\epsilon\mu}$$

1 index

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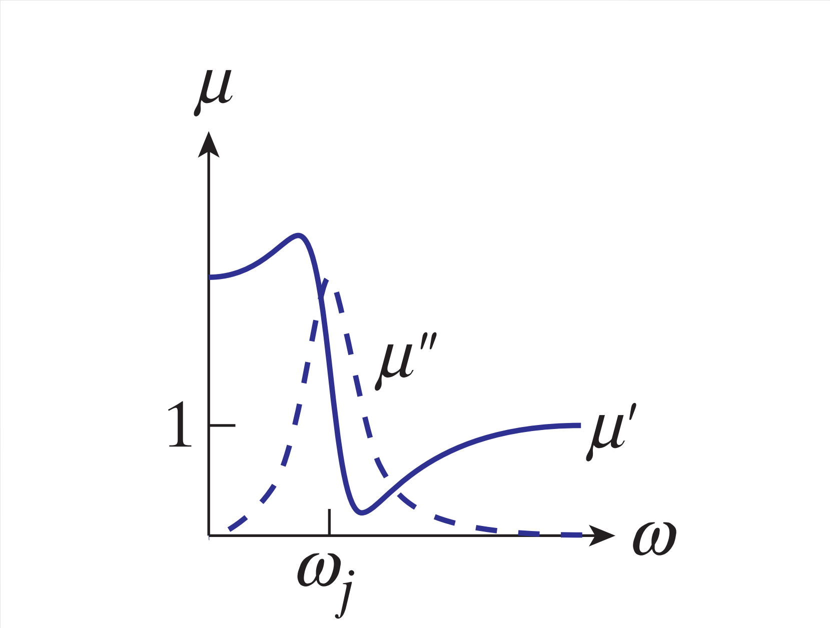
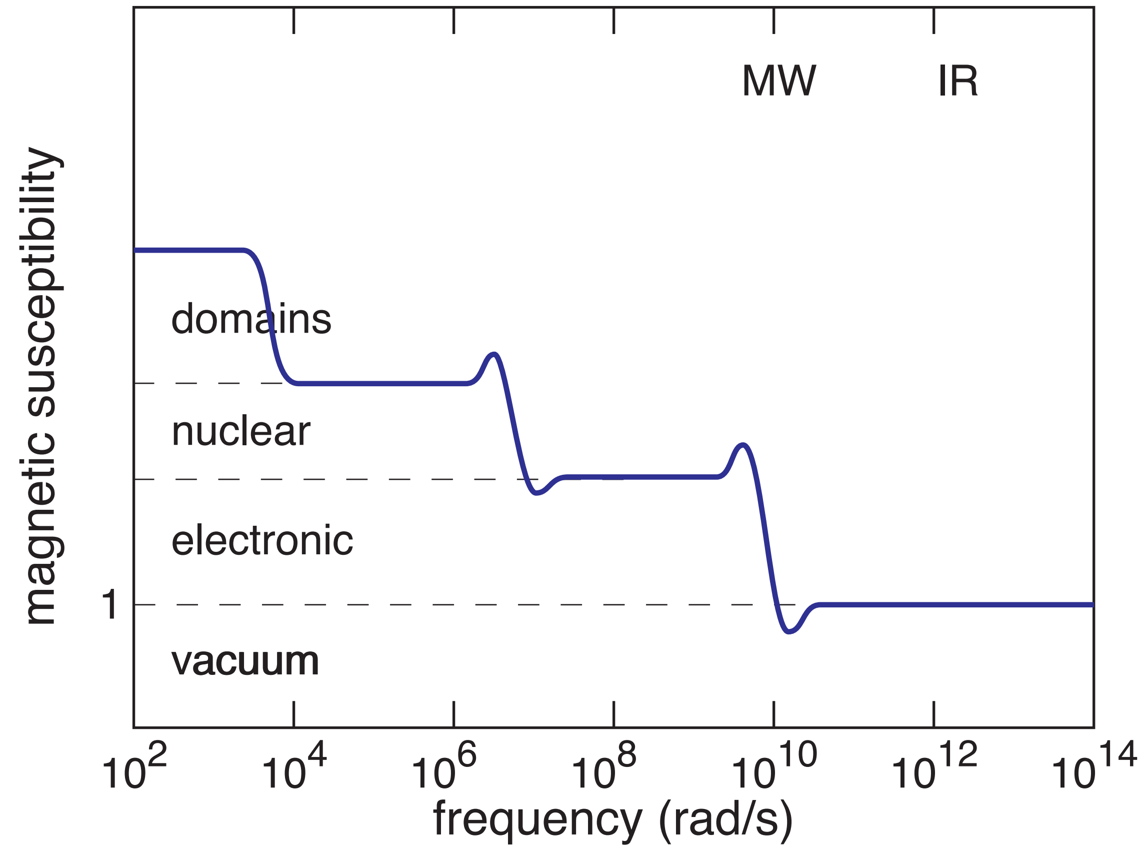
$$n = \sqrt{\epsilon\mu}$$

and magnetic response shows similar resonances

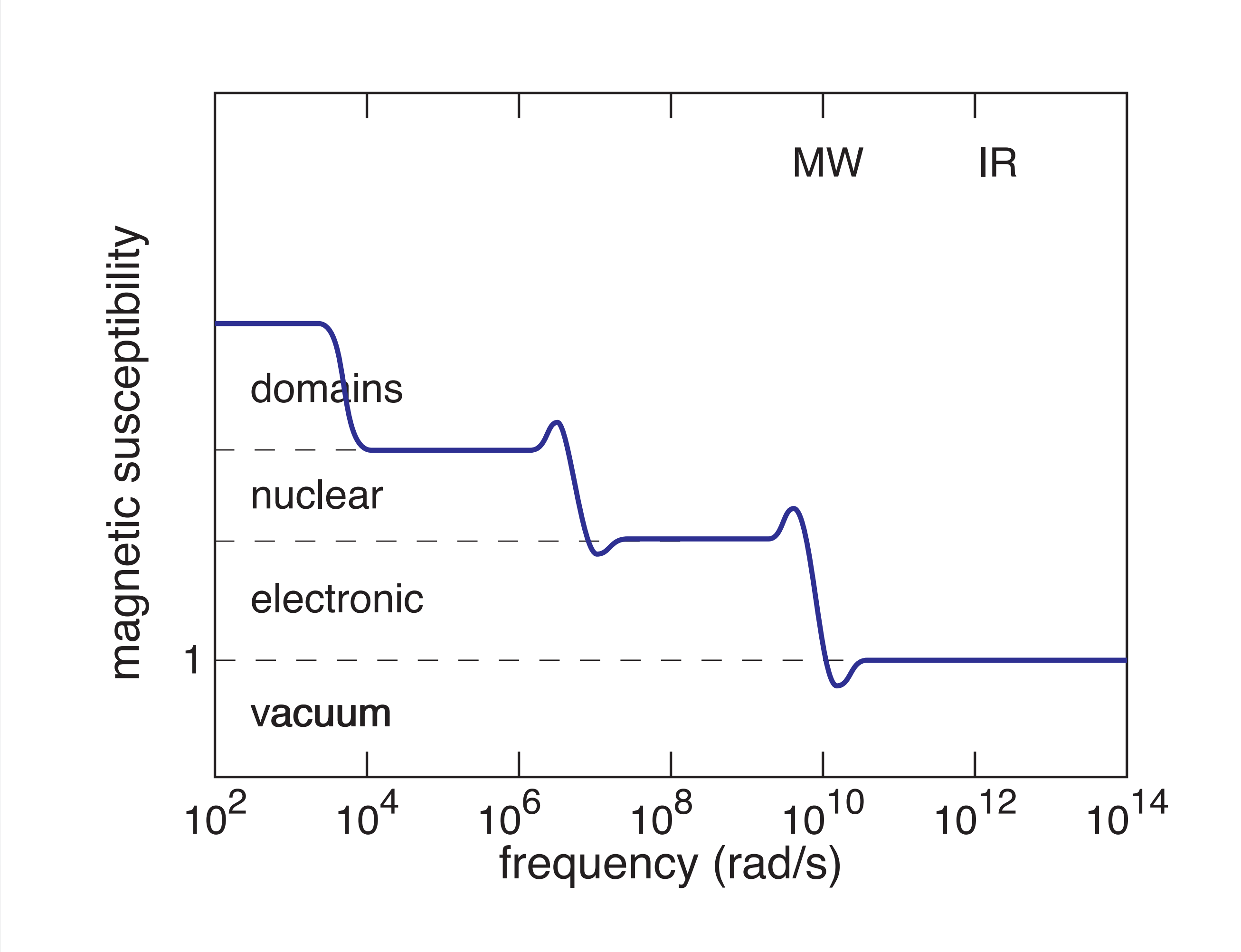


Magnetic response



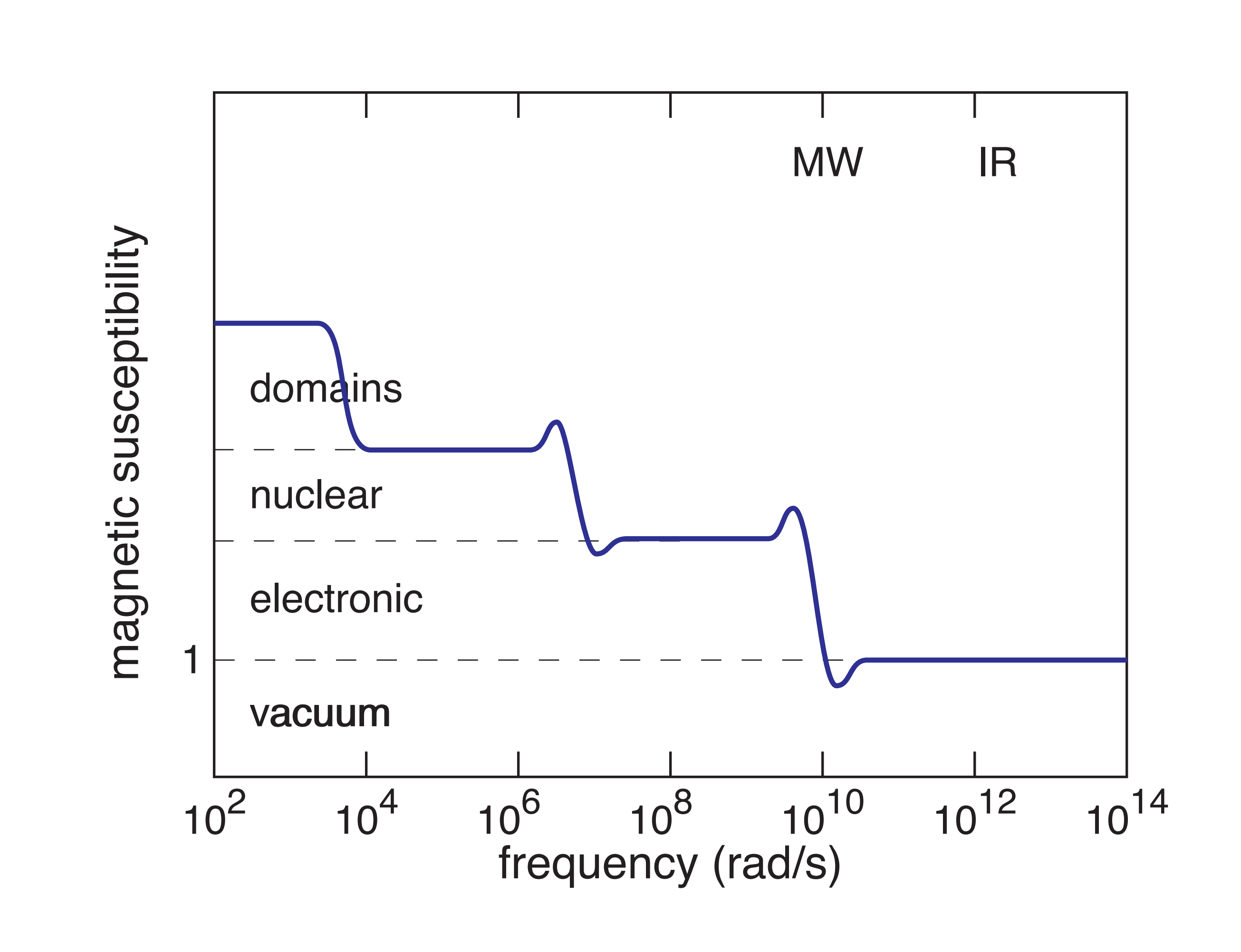
Magnetic response

but magnetic resonances occur below optical frequencies



Magnetic response

so, in optical regime, $\mu \approx 1$



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Both ϵ and μ are complex and their real parts can be negative.

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Both ϵ and μ are complex and their real parts can be negative.

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Write complex quantities as

$$\varepsilon = |\varepsilon| e^{i\theta} \qquad \mu = |\mu| e^{i\phi}$$

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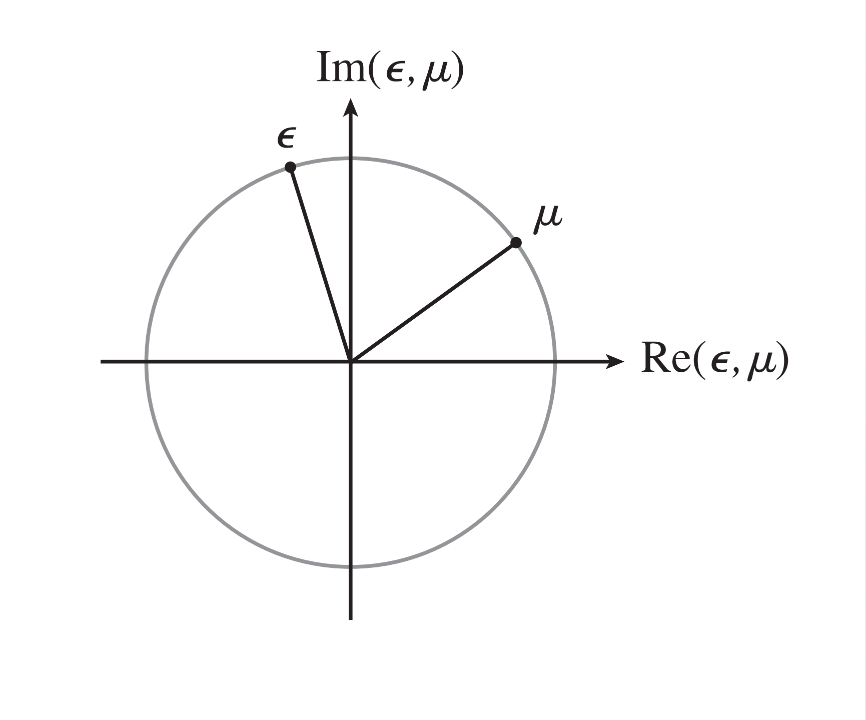
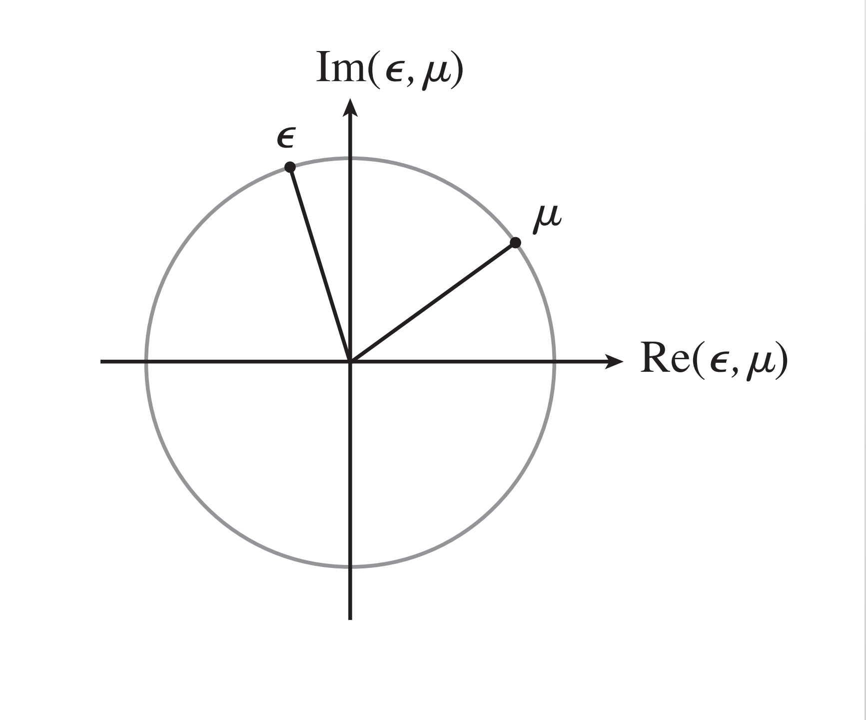


Write complex quantities as

$$\varepsilon = |\varepsilon| e^{i\theta} \qquad \mu = |\mu| e^{i\phi}$$

Index

$$n = \sqrt{|\varepsilon||\mu|}\, e^{i\frac{\theta+\phi}{2}}$$



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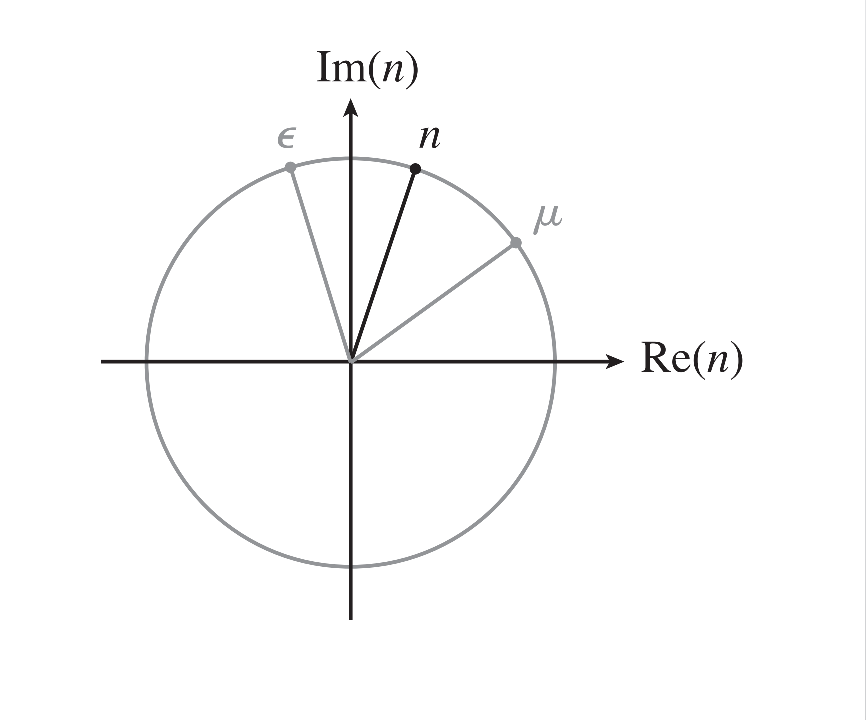

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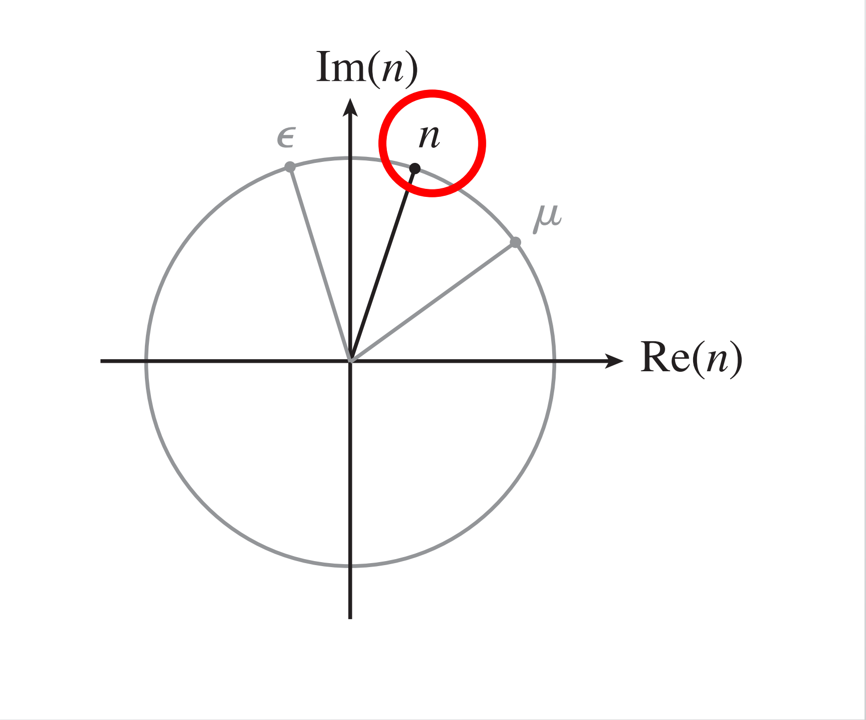
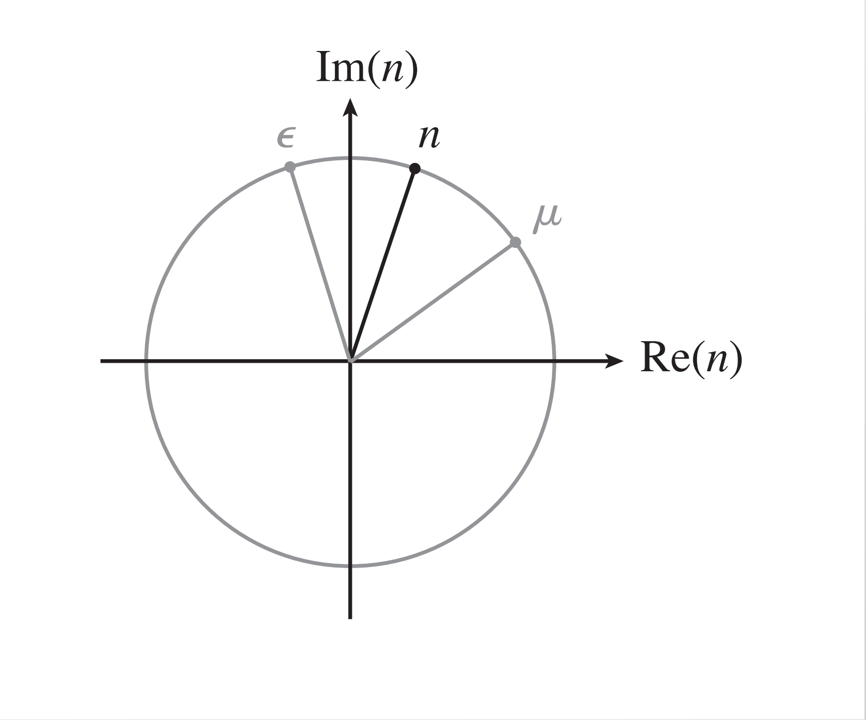


Q: Is this only possible value?

1. yes
2. no, there's one more
3. there are many more
4. it depends

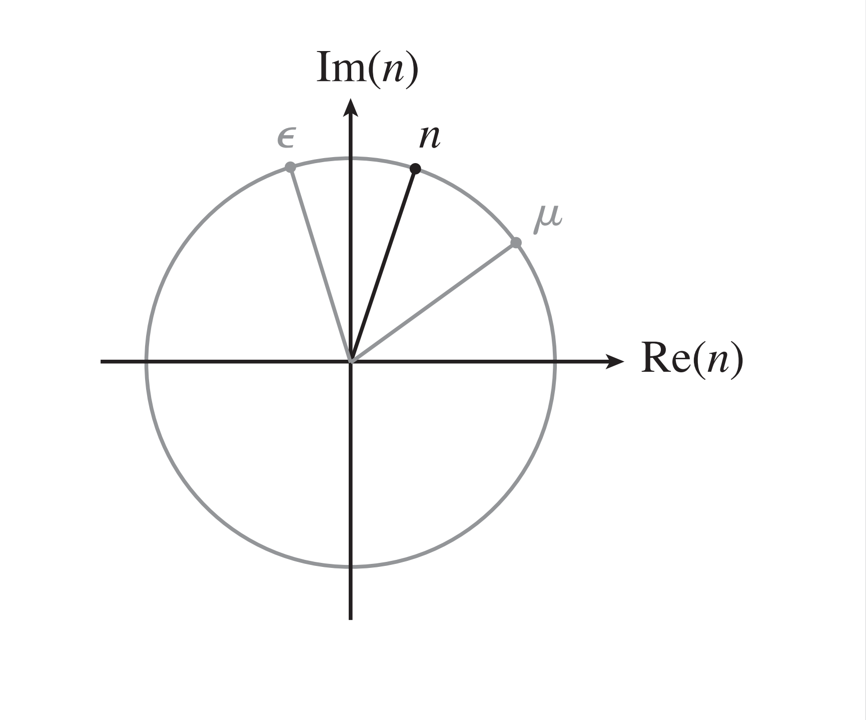
There *is* another root…



There *is* another root…

Can add 2π to exponent

$$e^{+i(\theta+\phi)} = e^{+i[\theta+\phi+2\pi]}$$



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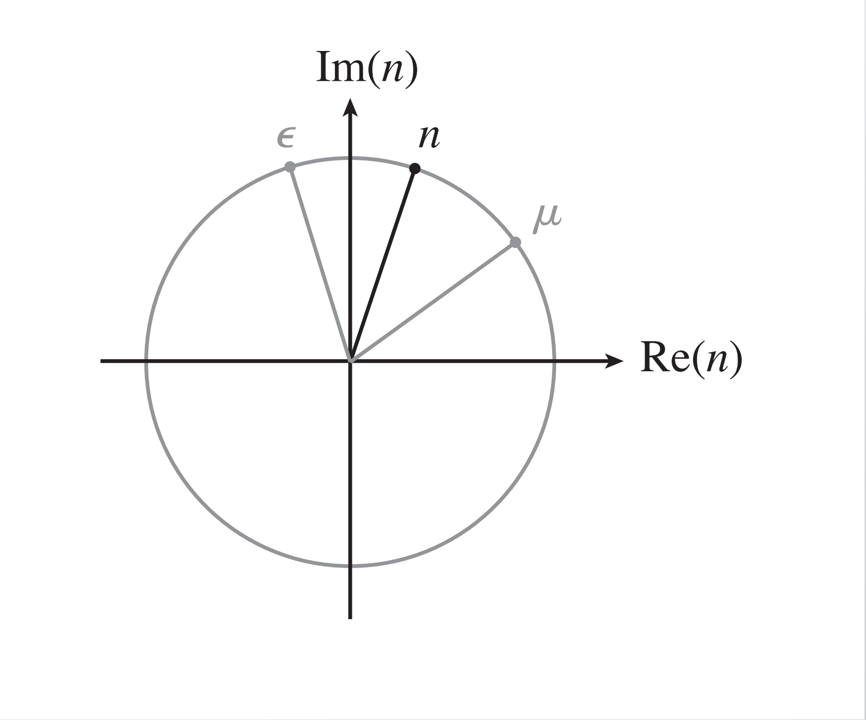

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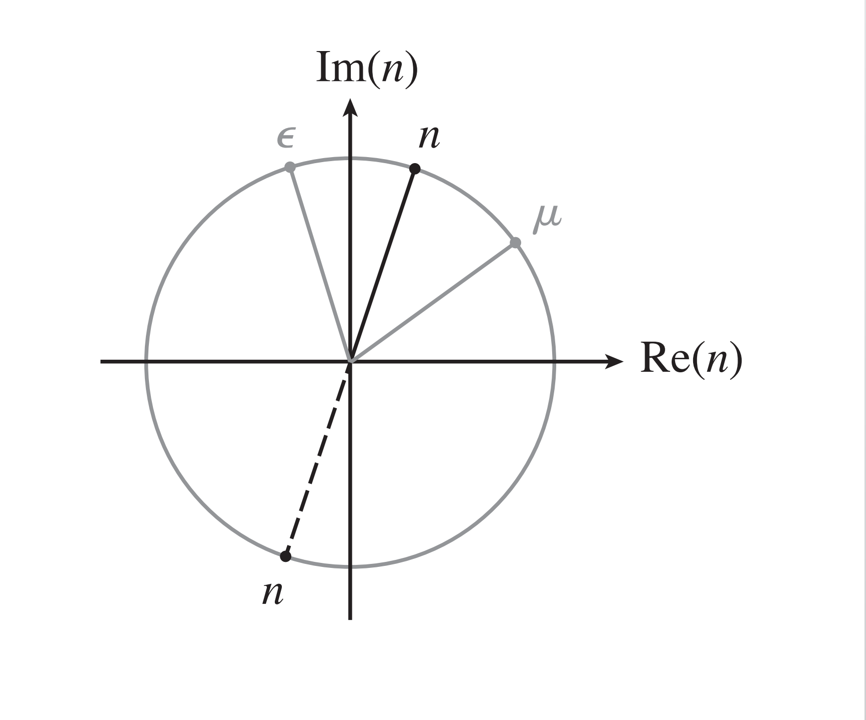

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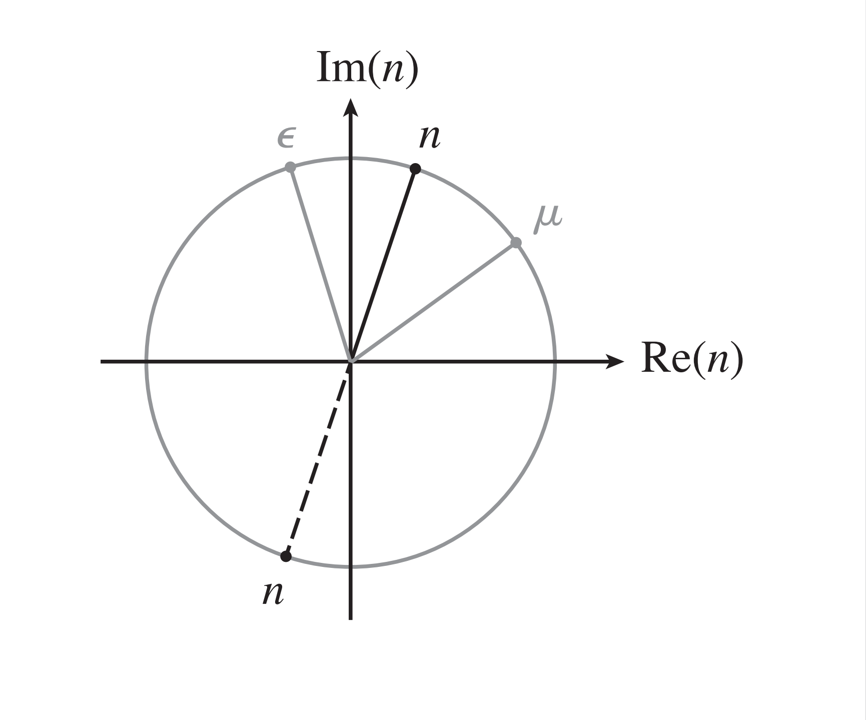

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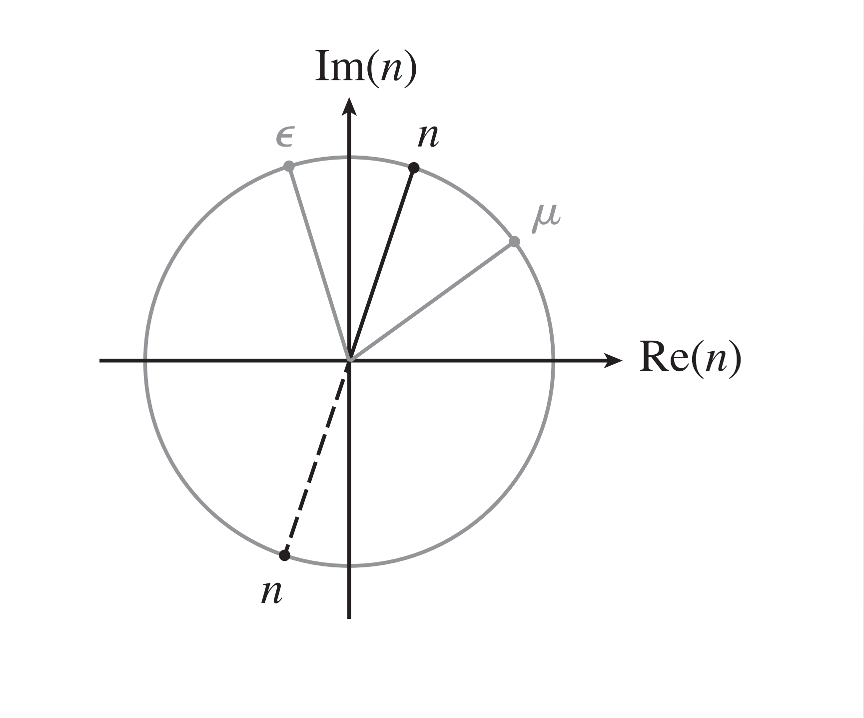

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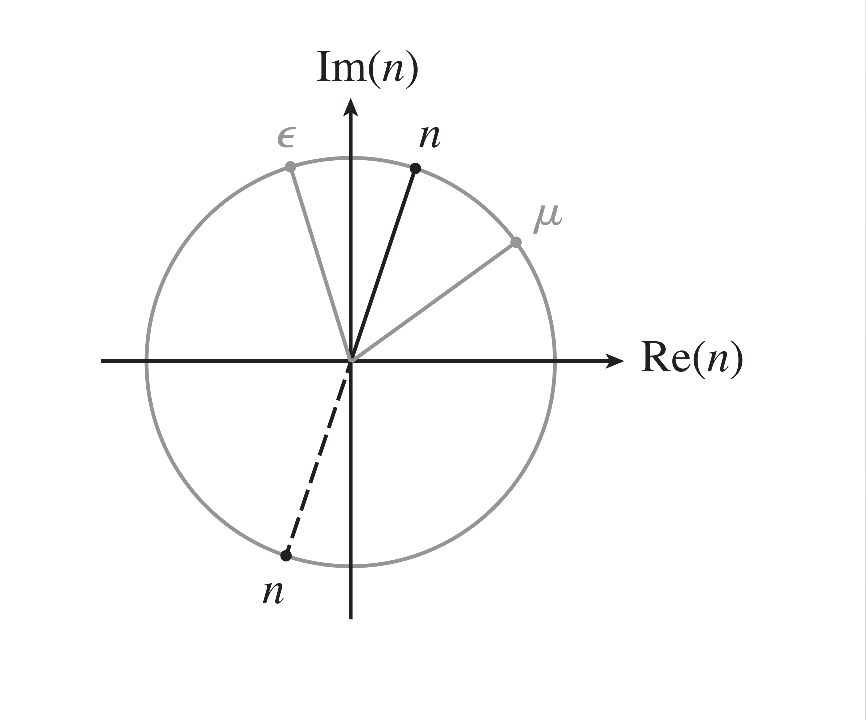

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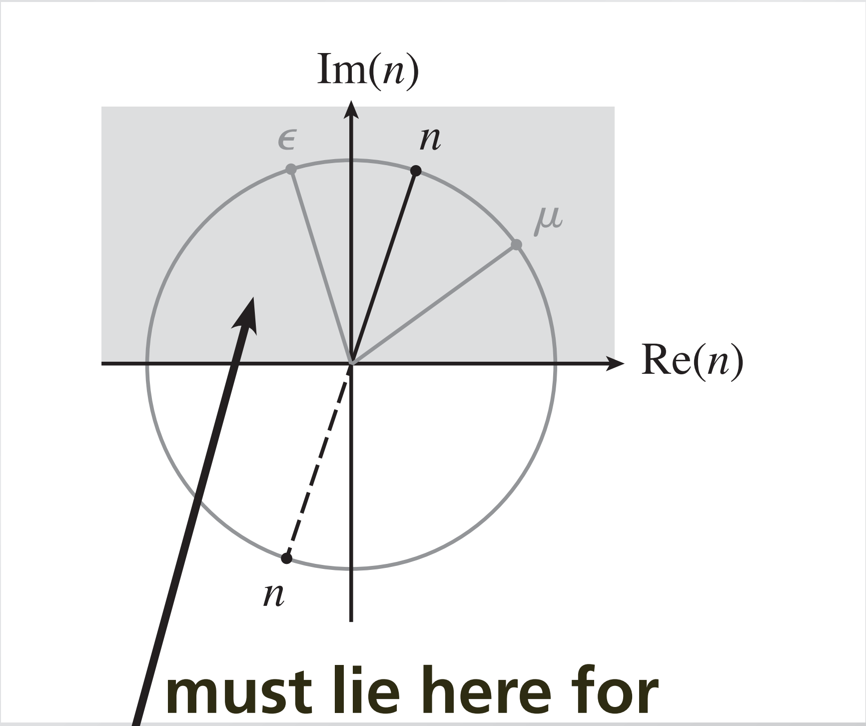

must lie here for passive material

$$k = \frac{2\pi n}{\lambda_o} = \frac{2\pi(n' + in'')}{\lambda_o} = k' + ik''$$

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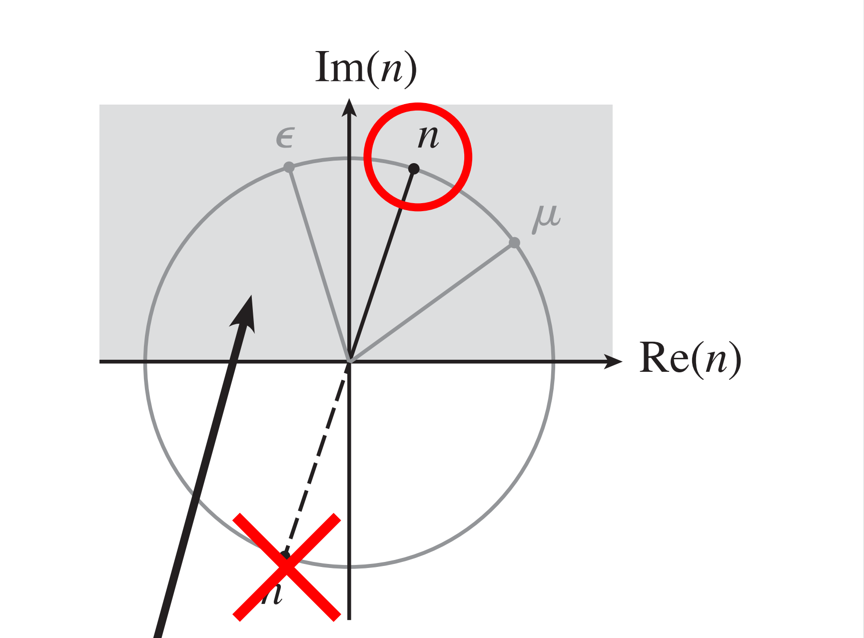

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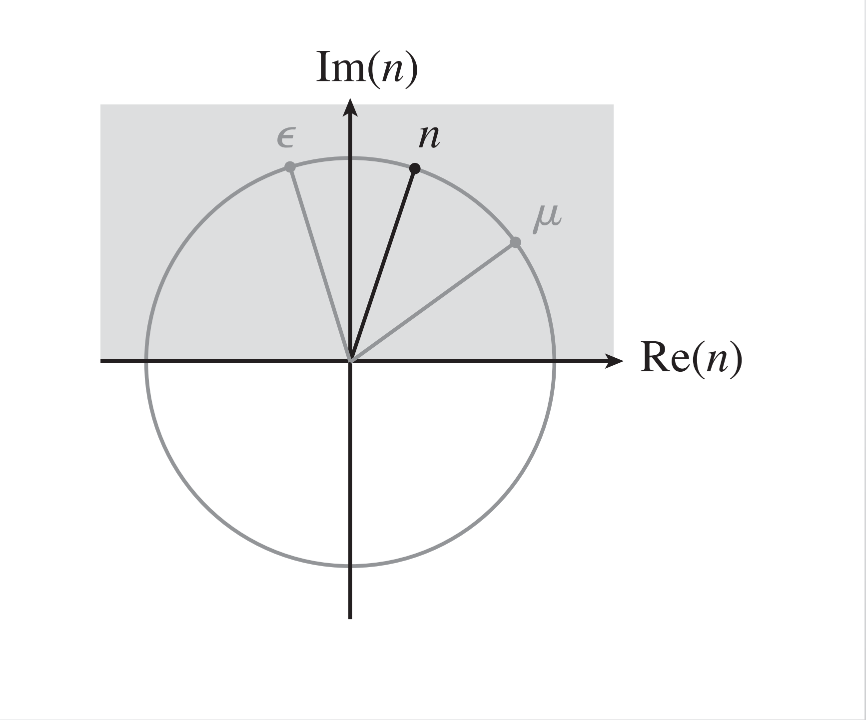
2. no, there's one more
3. there are many more
4. it depends



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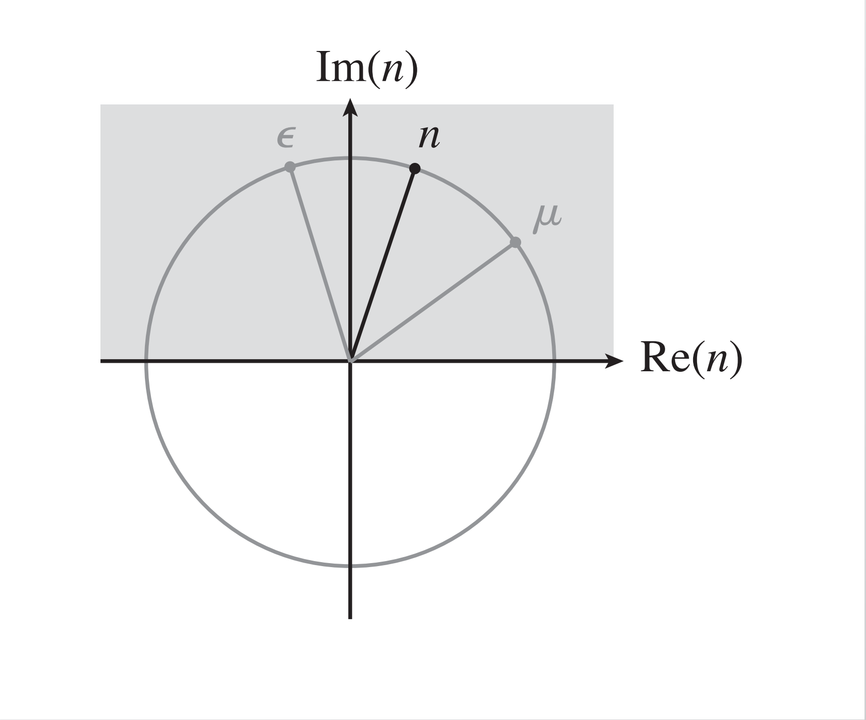

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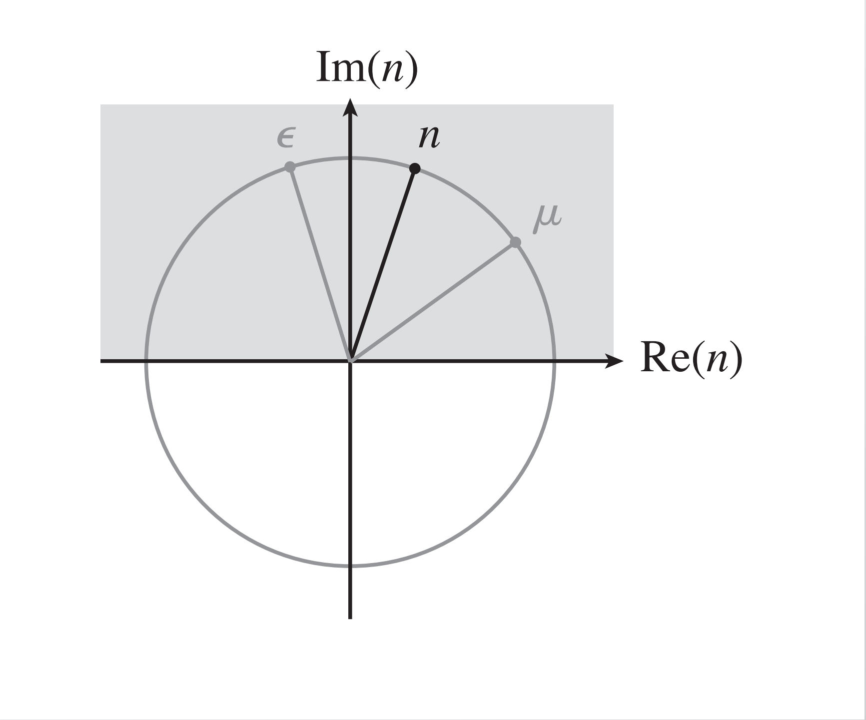

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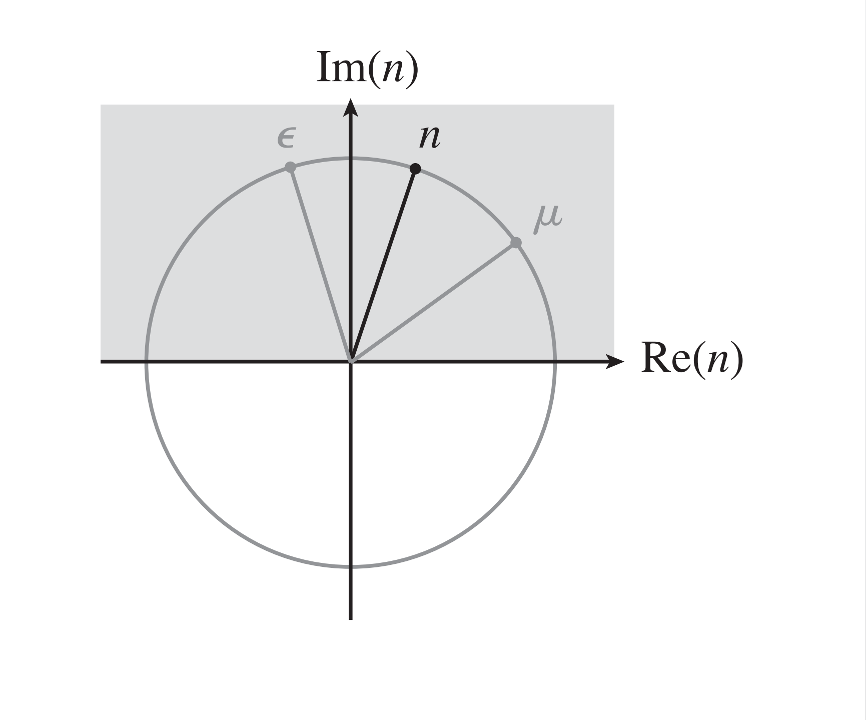
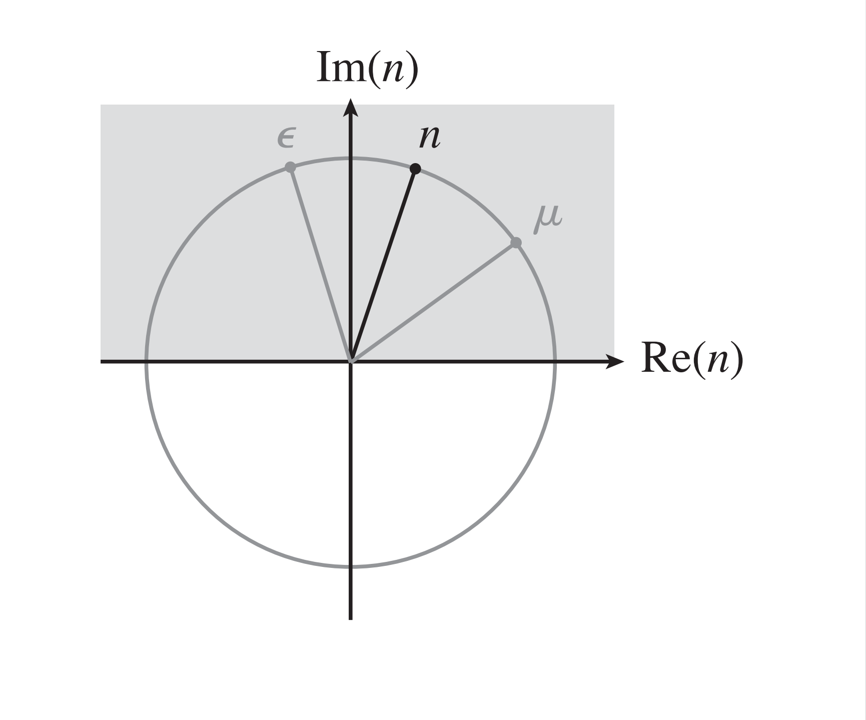


To find n (passive materials):

1. Draw line that bisects ϵ and μ

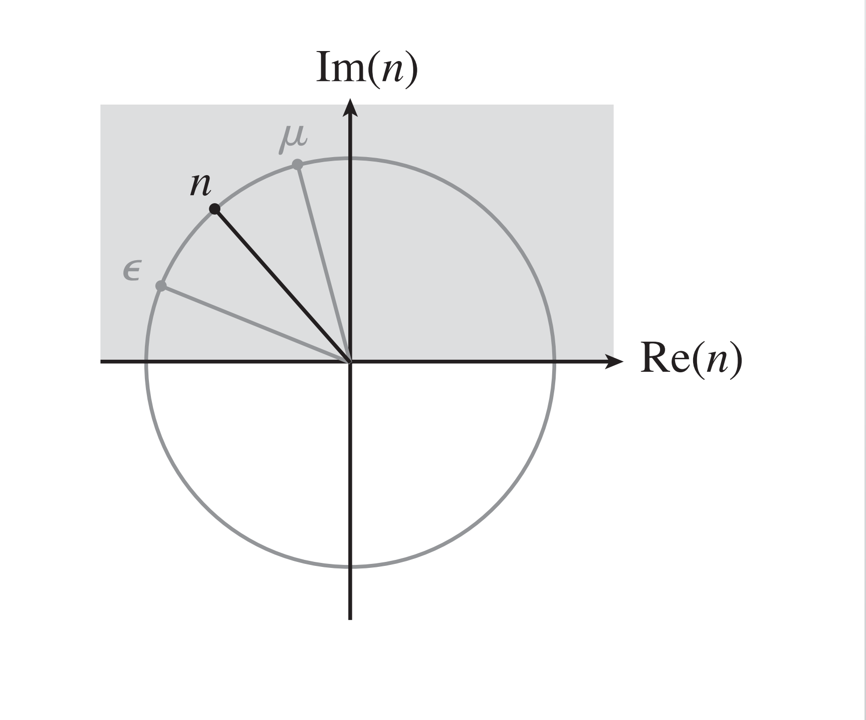
2. Choose upper branch



What happens when $\mathrm{Re}\,\epsilon$ and/or $\mathrm{Re}\,\mu$ is negative?

For certain values of ϵ and μ

we can get a *negative* $\mathrm{Re}(n)$!

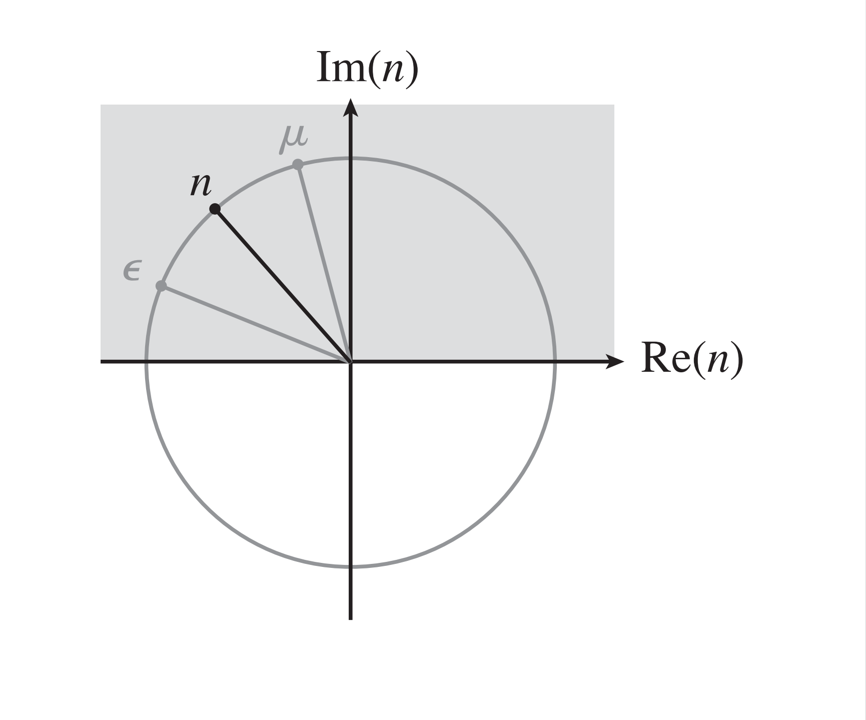


Q: **Must both** $\mathrm{Re}\,\epsilon < 0$ **and** $\mathrm{Re}\,\mu < 0$

to get a negative $\mathrm{Re}(n)$**?**

1. **yes**

2. **no**



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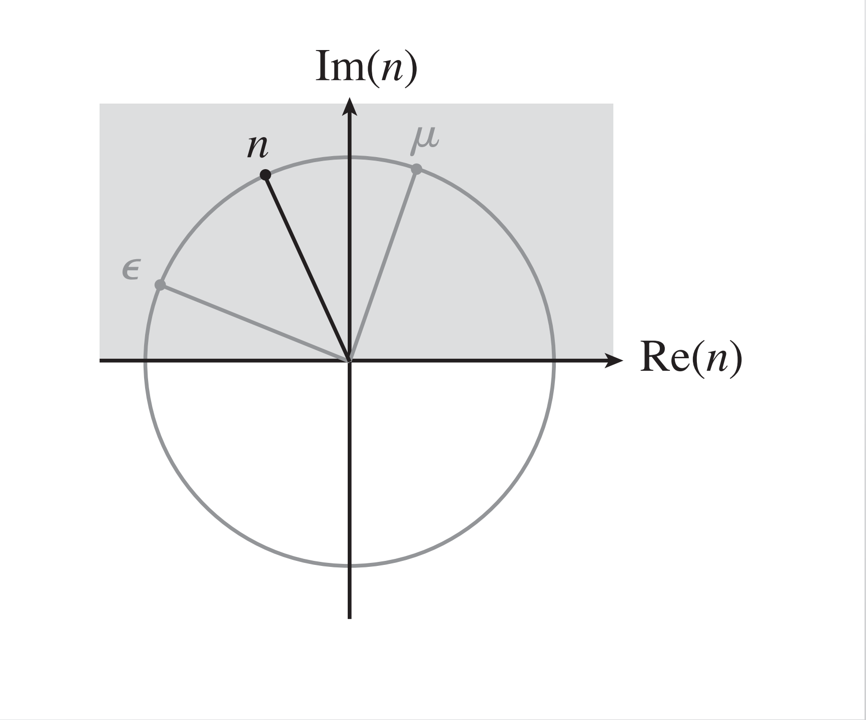
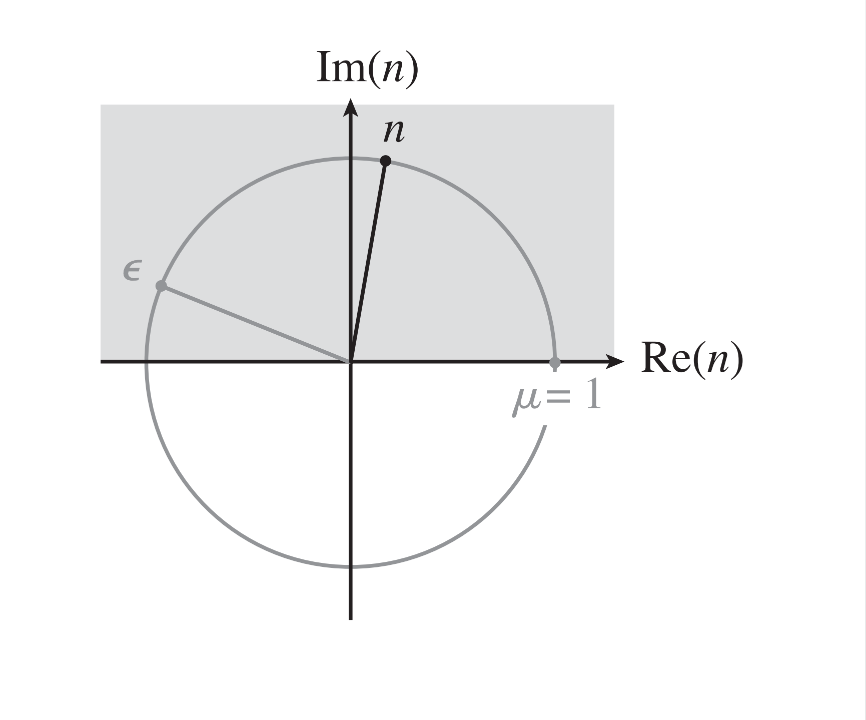


However, need magnetic response

to achieve $\mathrm{Re}(n) \leq 0$!



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Remember

$$k = \frac{2\pi n}{\lambda_o} = \frac{2\pi(n' + in'')}{\lambda_o} = k' + ik''$$

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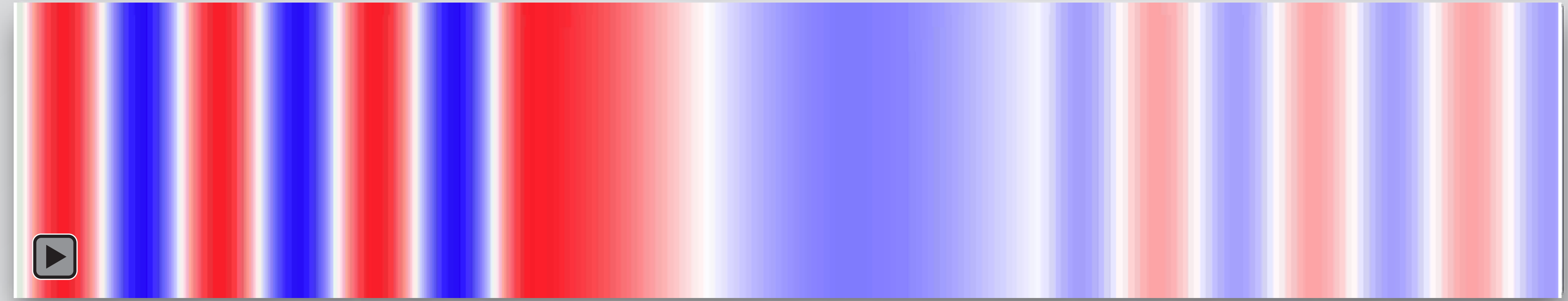
When $\mathrm{Re}(n) < 0$, $k' < 0$, and so phase velocity reversed!

What happens when $\mathrm{Re}(n) < 0$?



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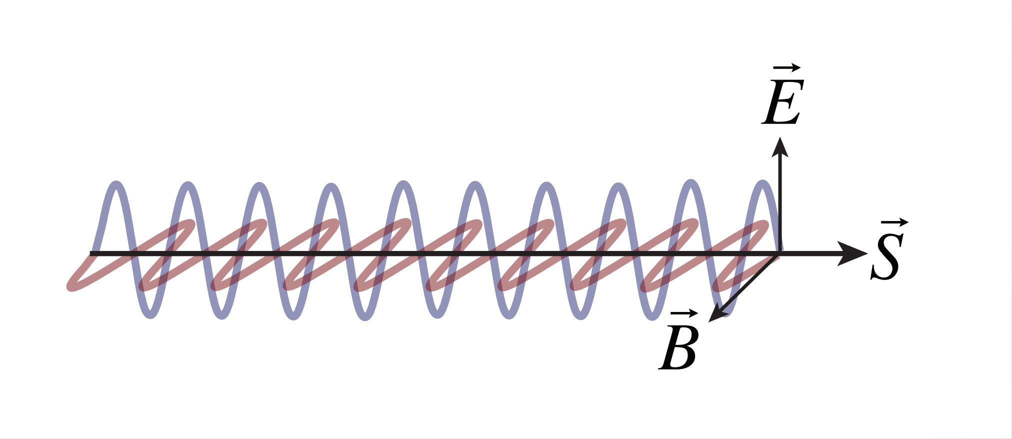

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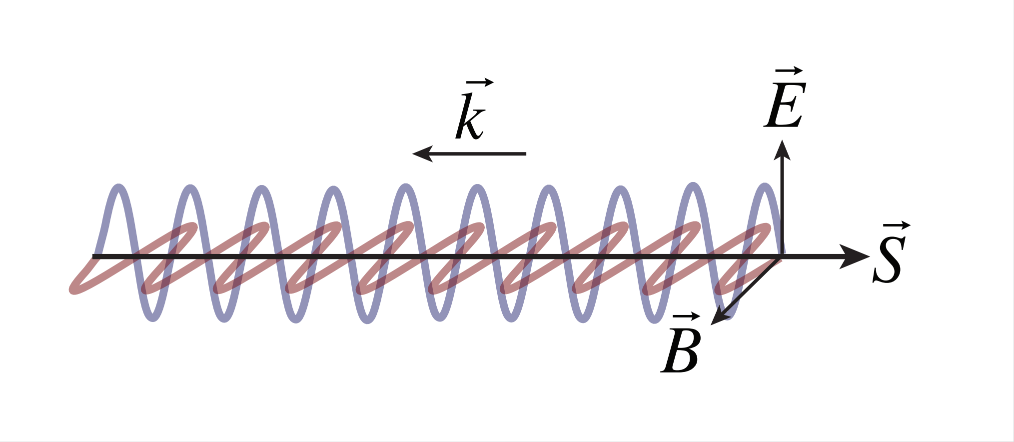

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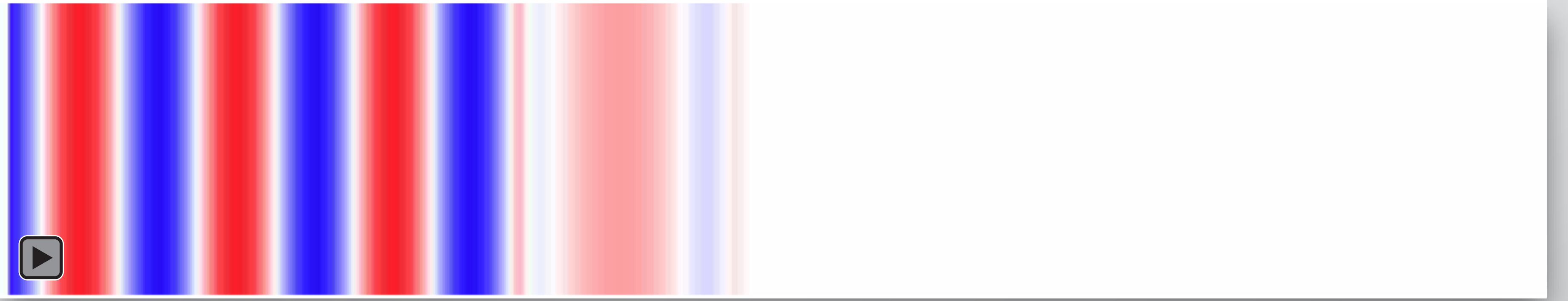
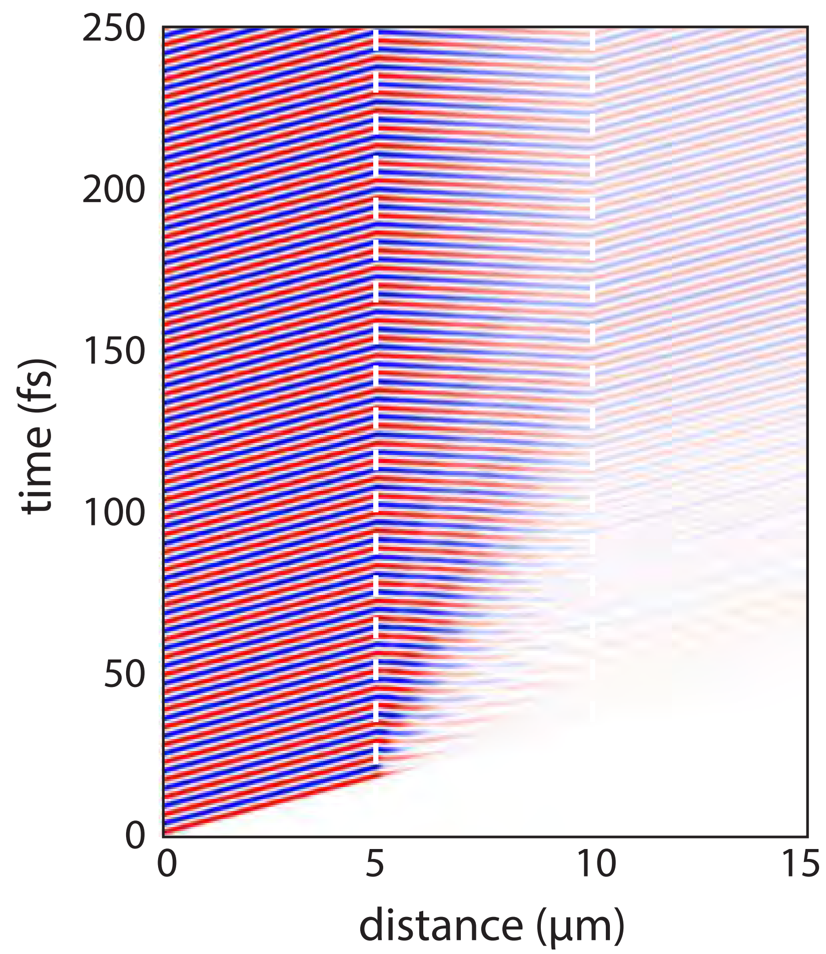
What about causality?

What about causality?

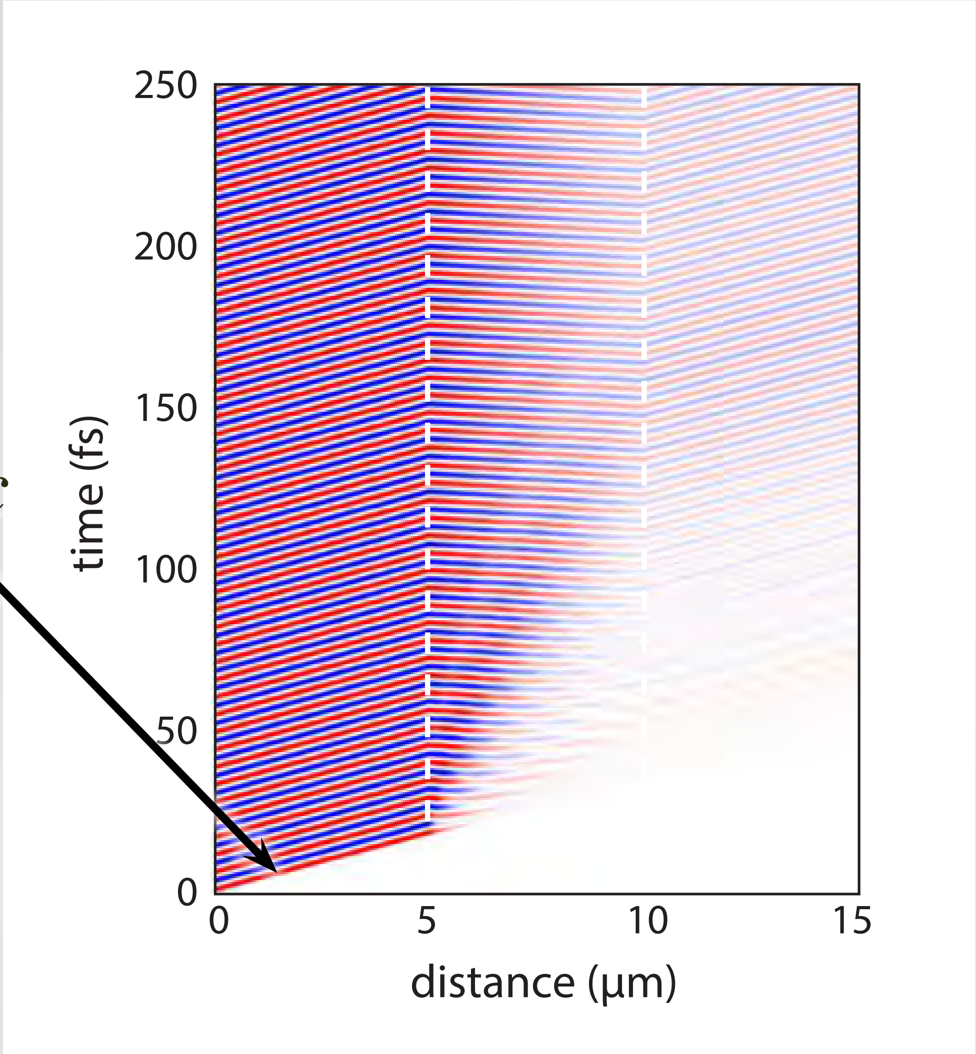


What about causality?



What about causality?

speed of light c



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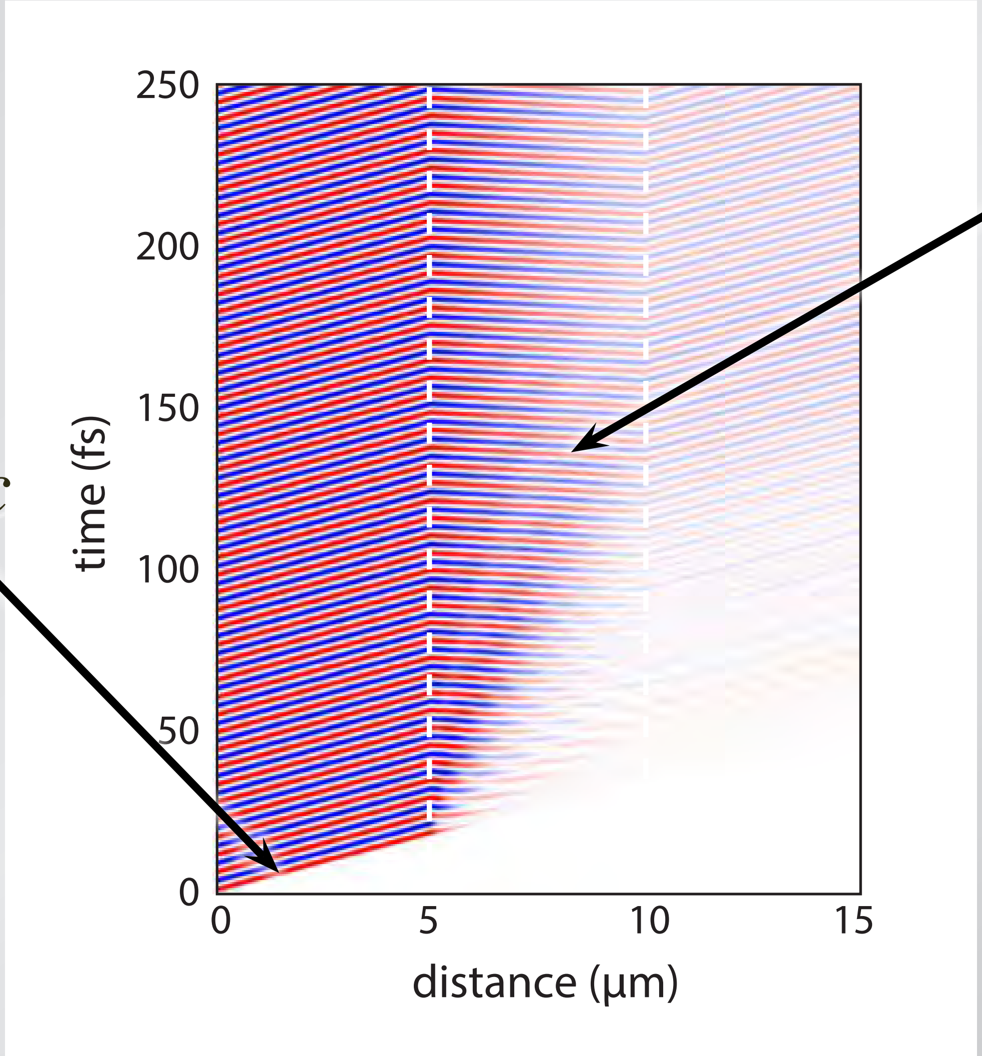


reverse phase propagation

speed of light c

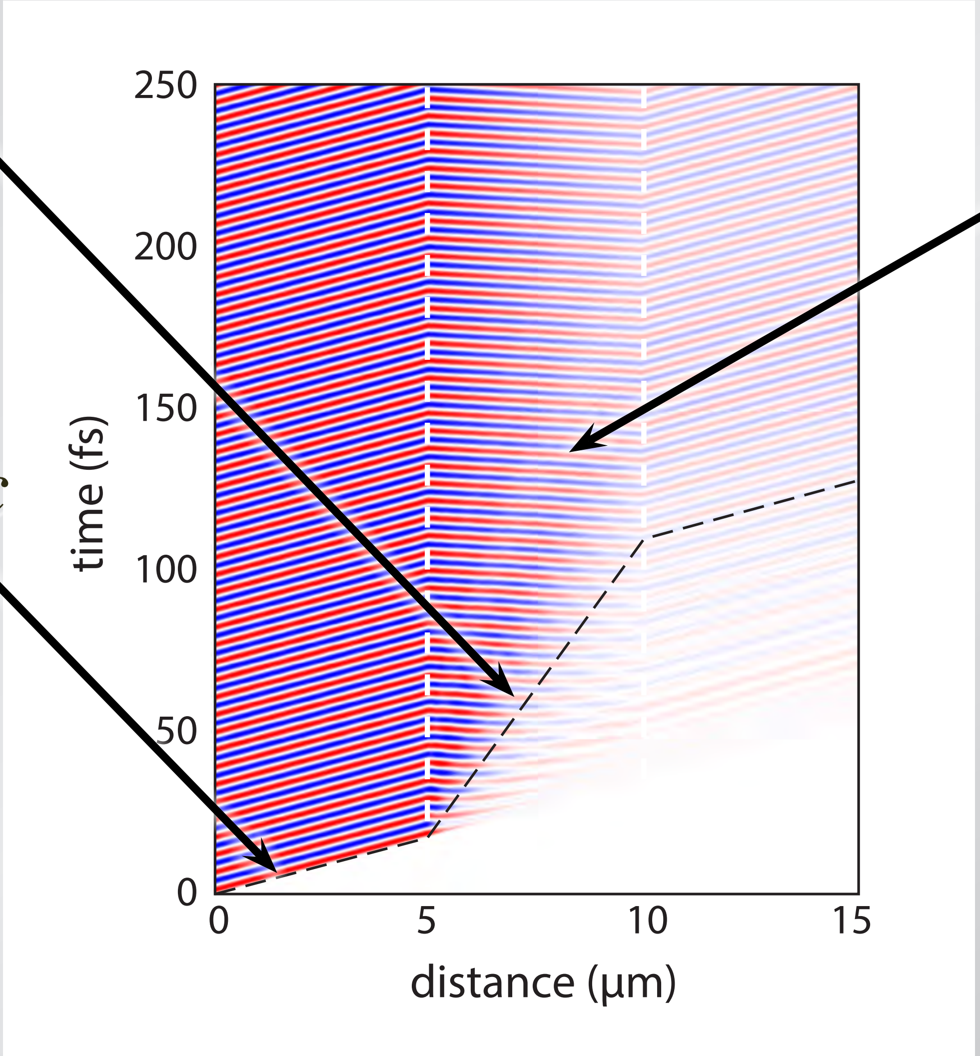
What about causality?

group velocity

$v_g < c$

reverse phase propagation

speed of light c



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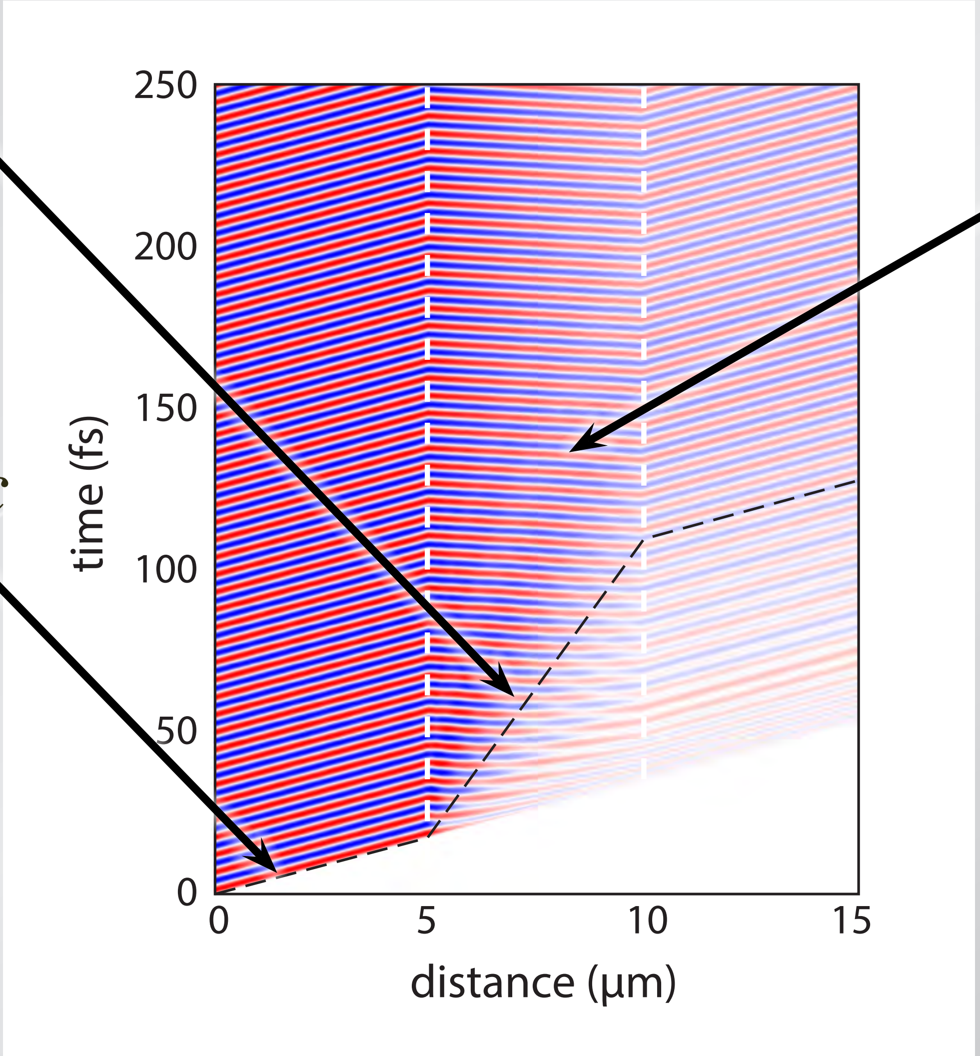


What about causality?

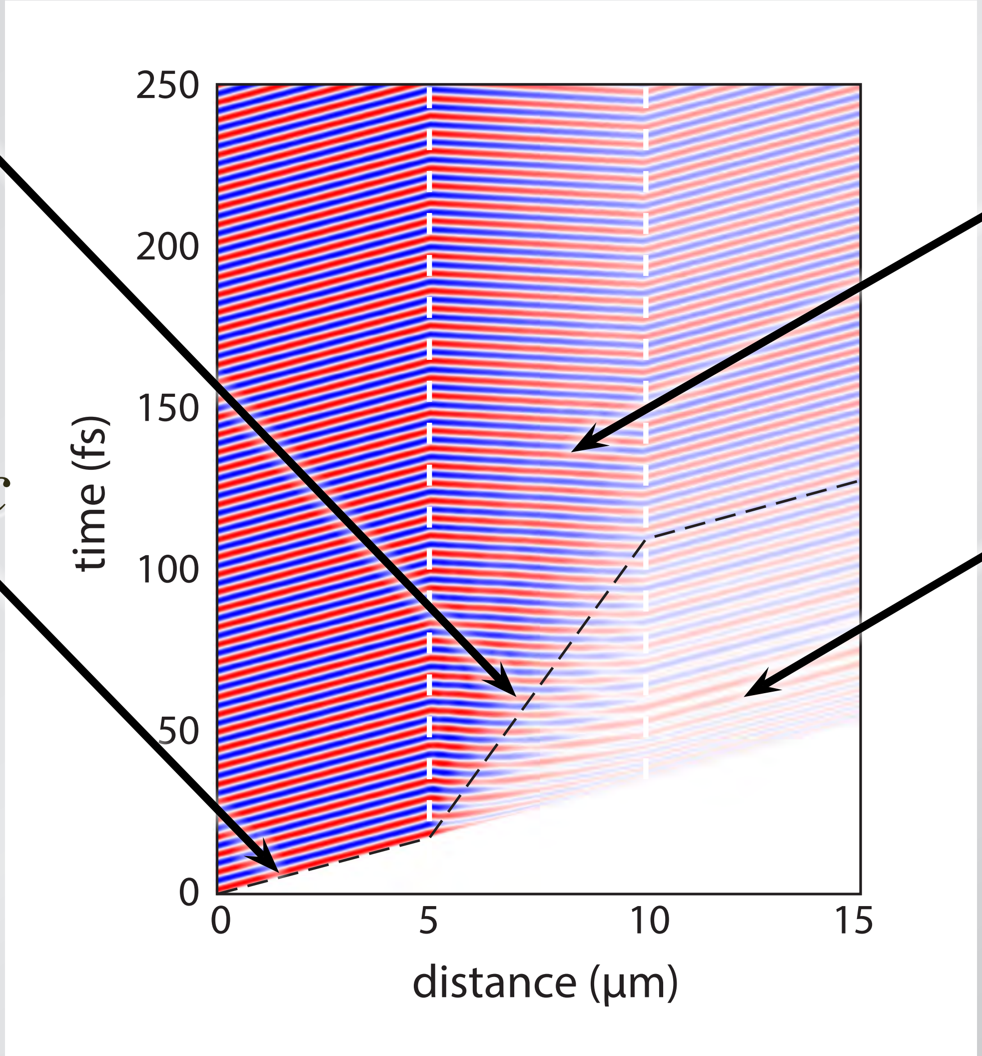
group velocity

$v_g < c$

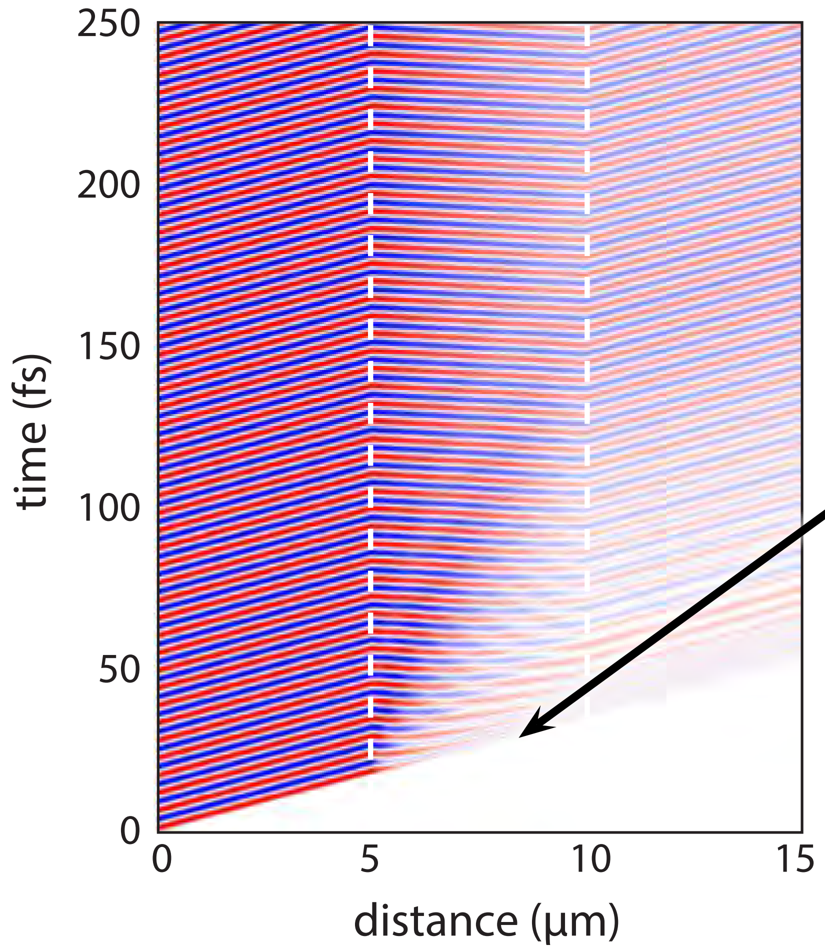
speed of light c

reverse phase propagation

high-frequency precursors

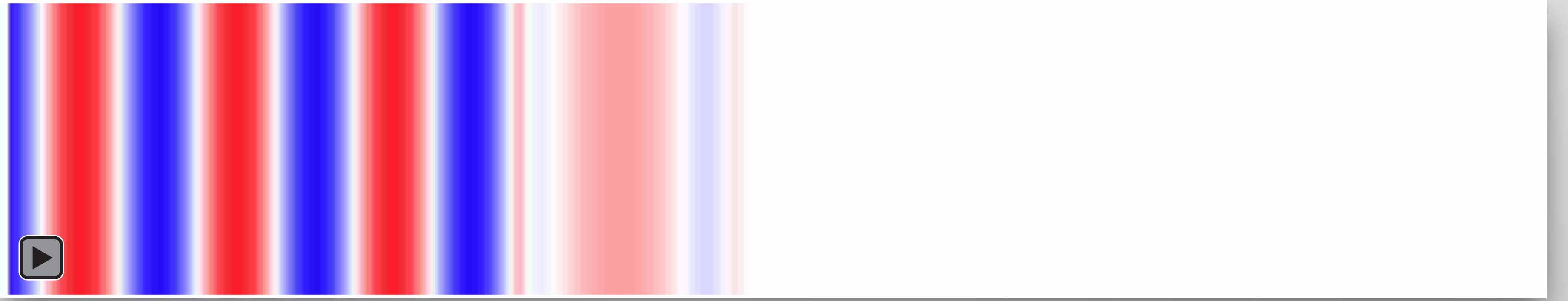


What about causality?

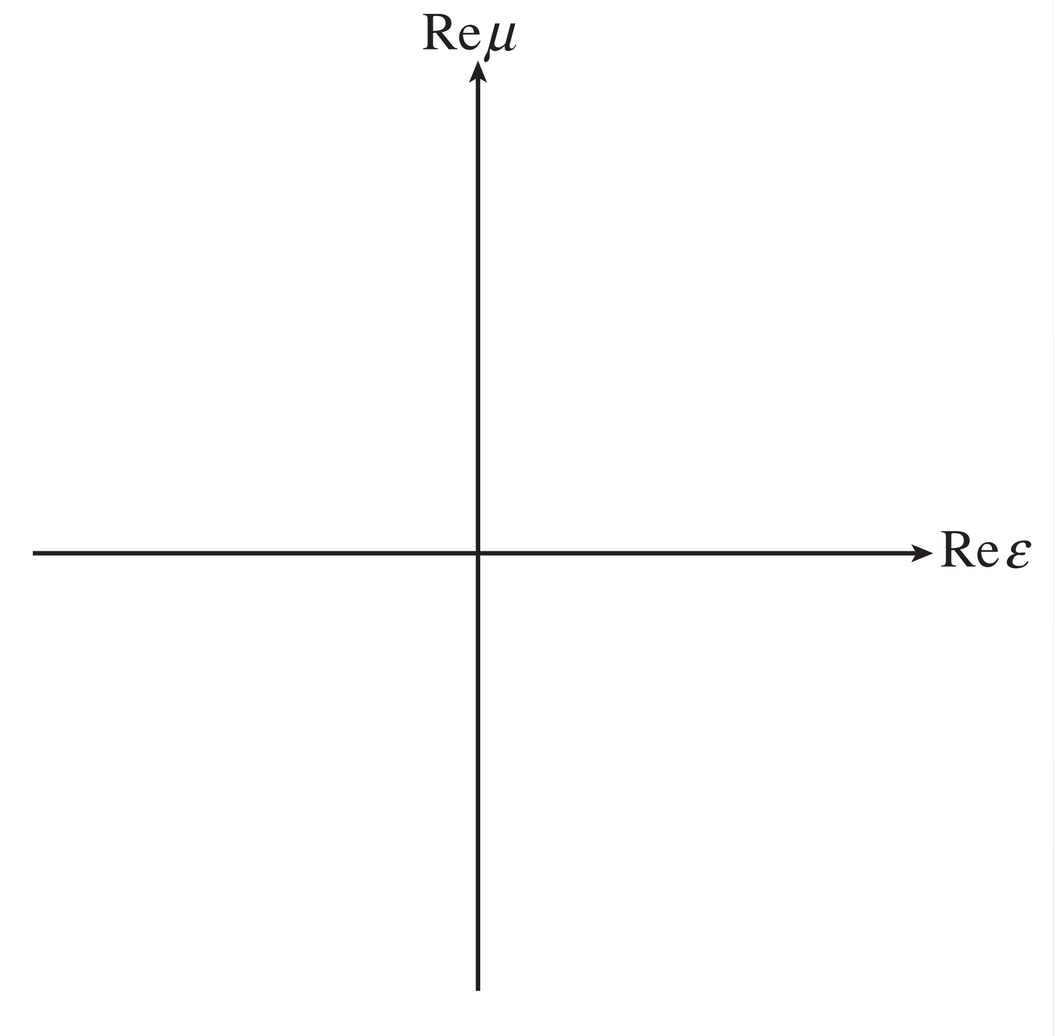


signal *always* travels at speed c!

What about causality?



classification of (non-lossy) materials



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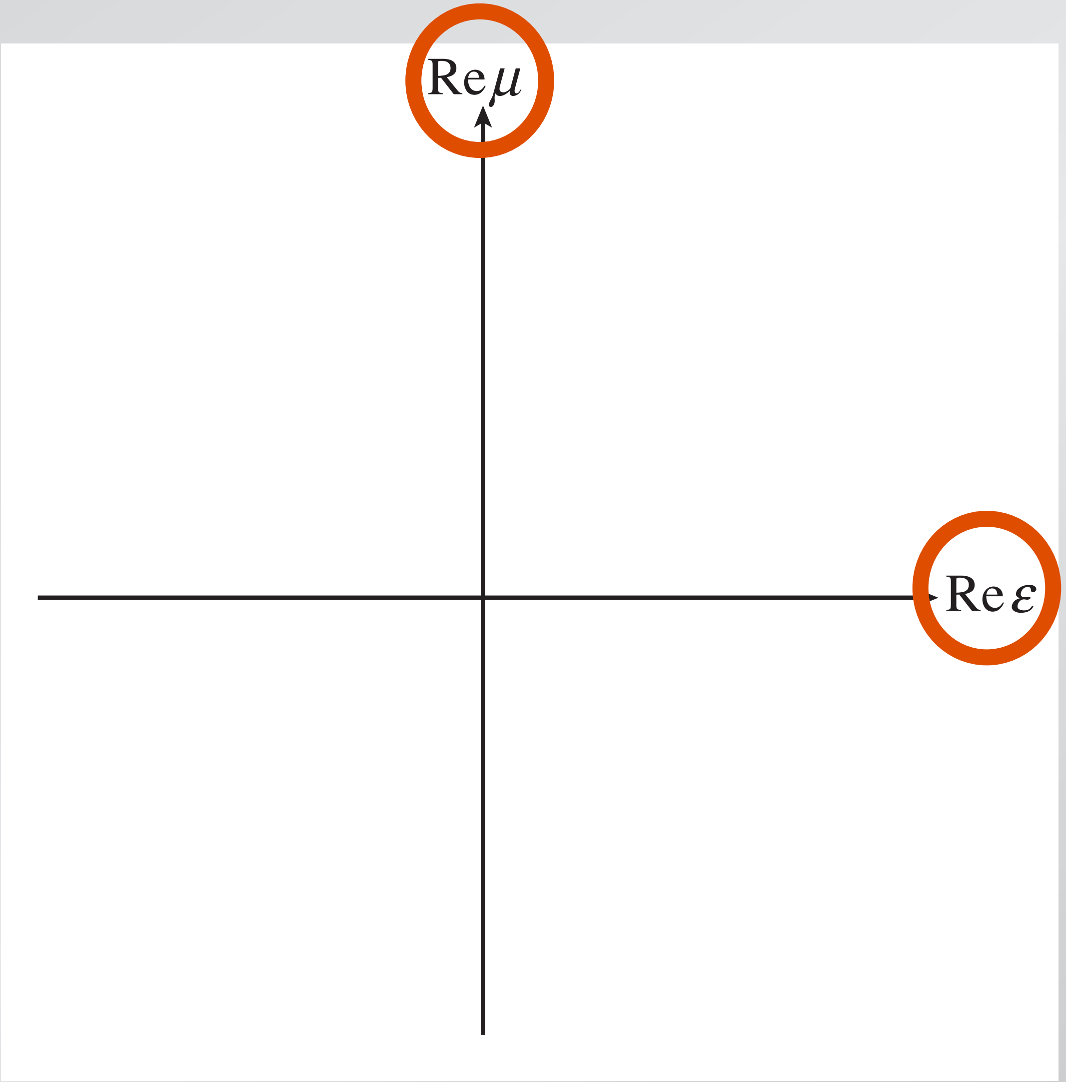

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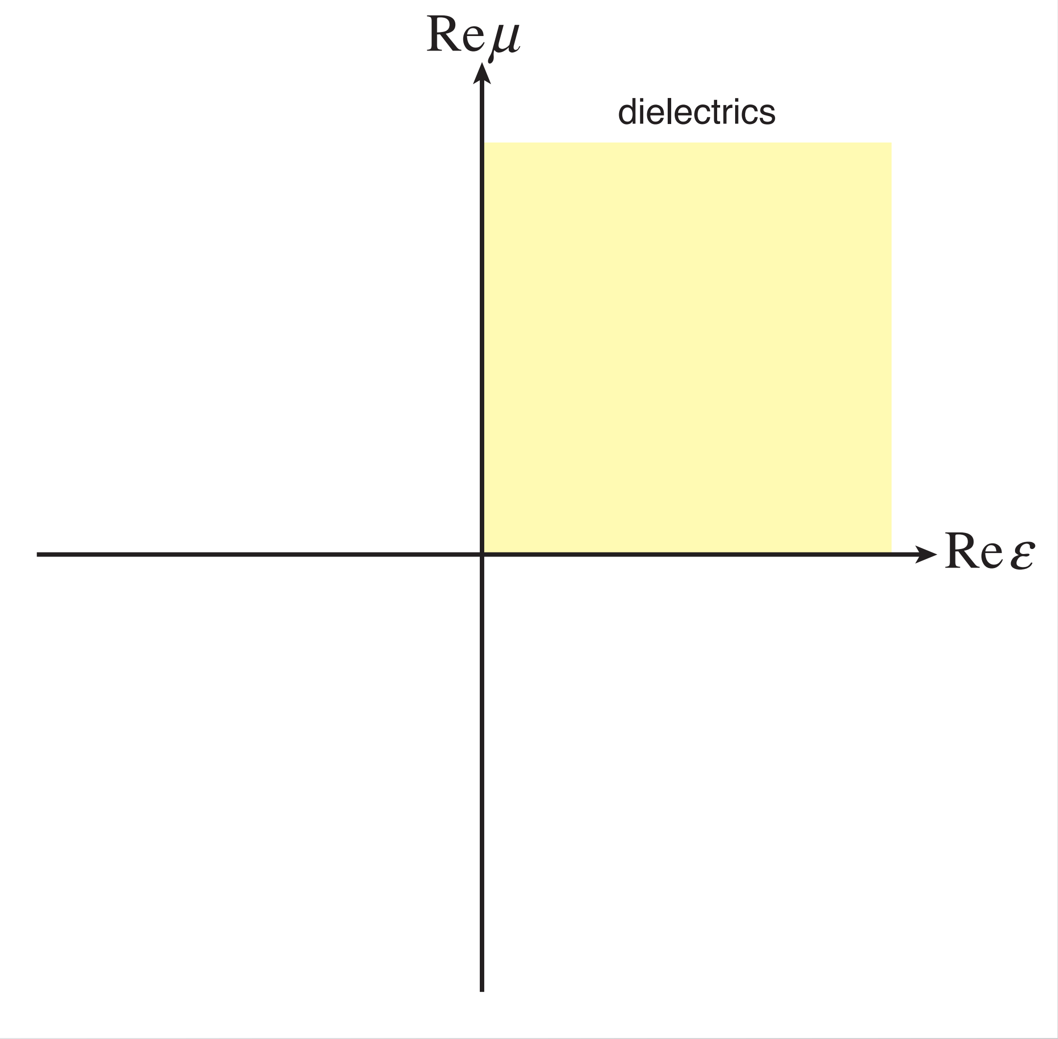

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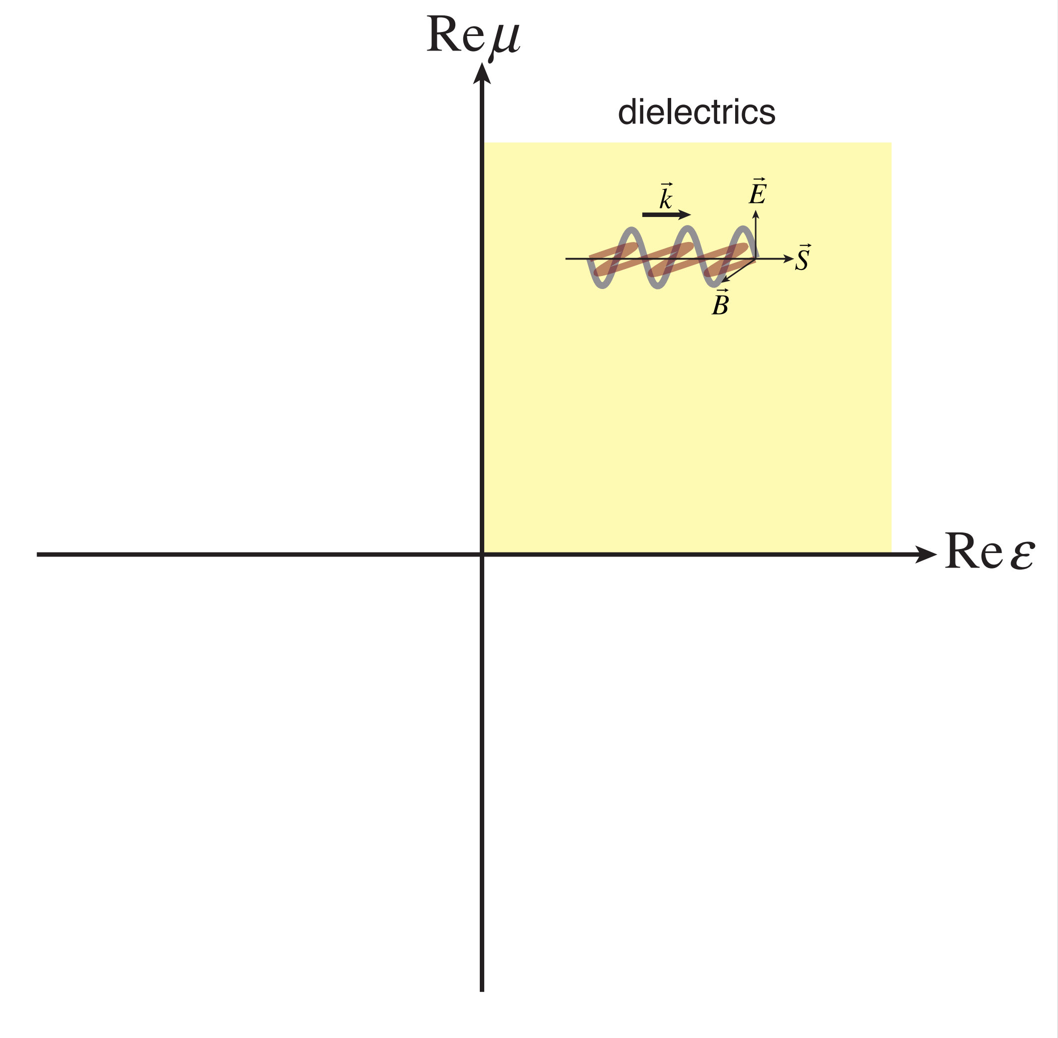

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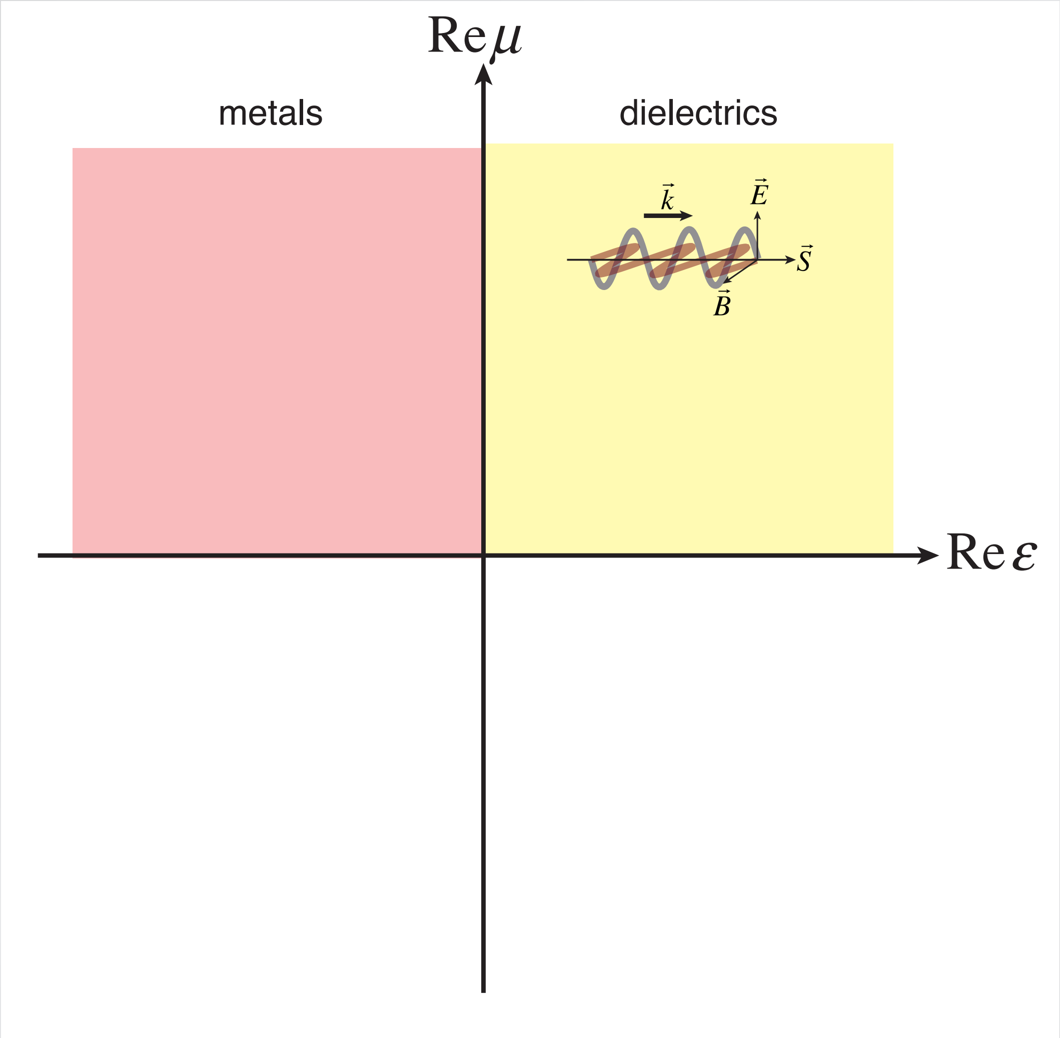

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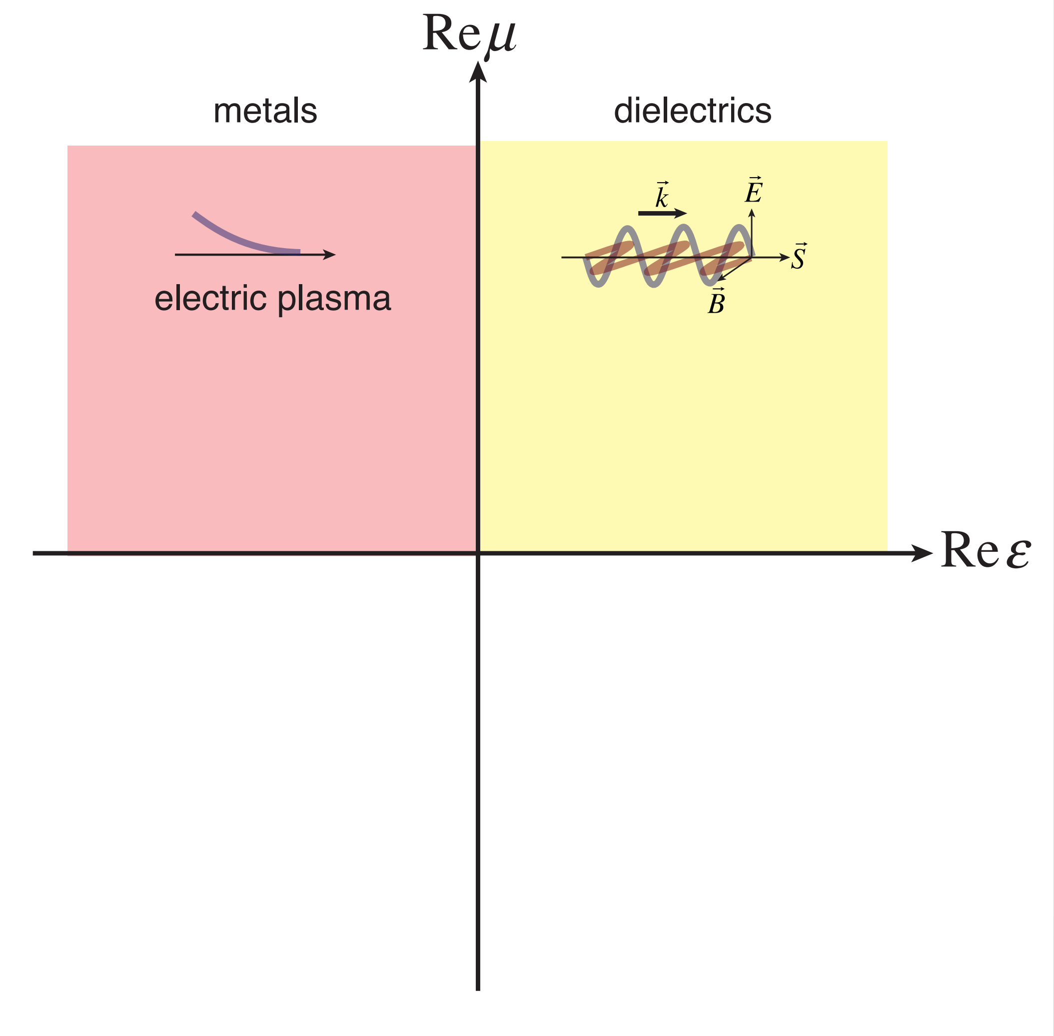

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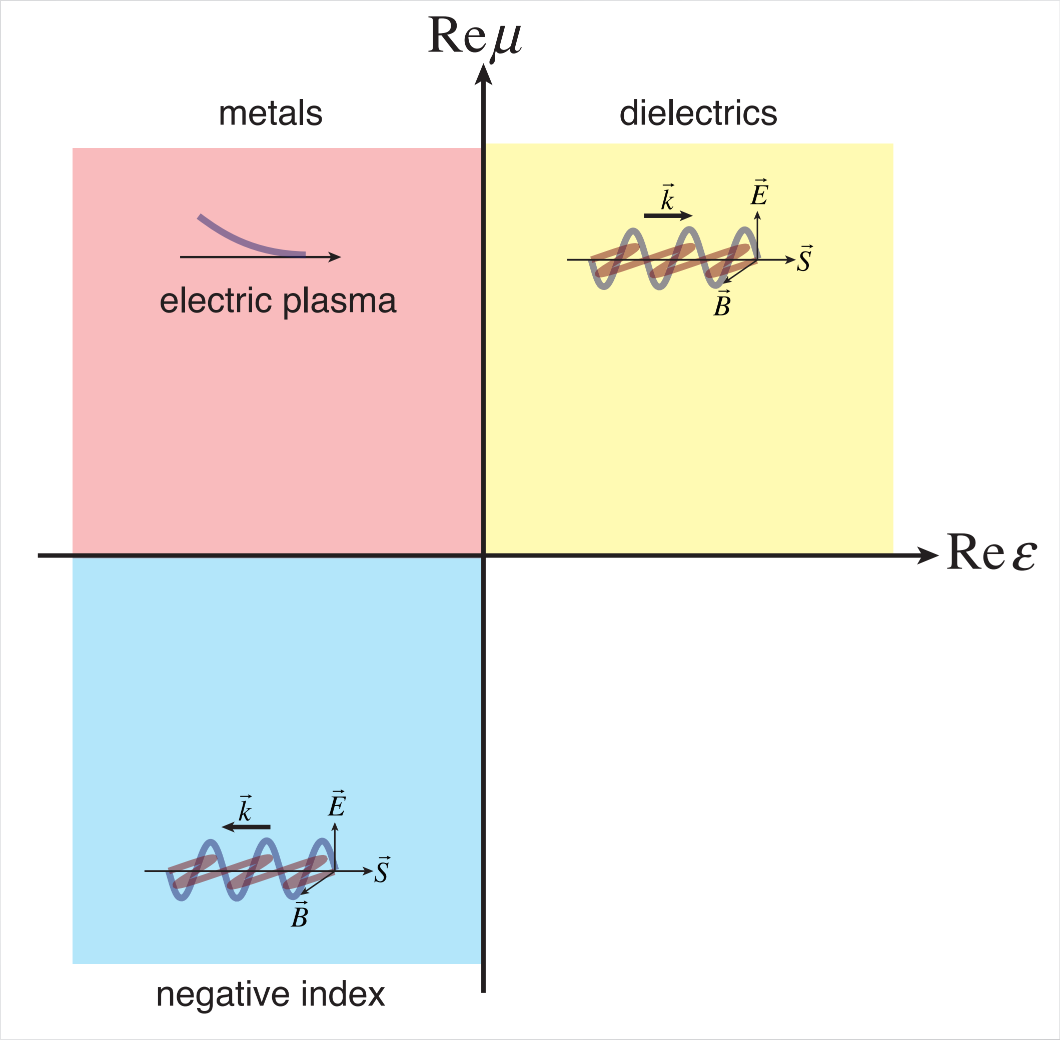

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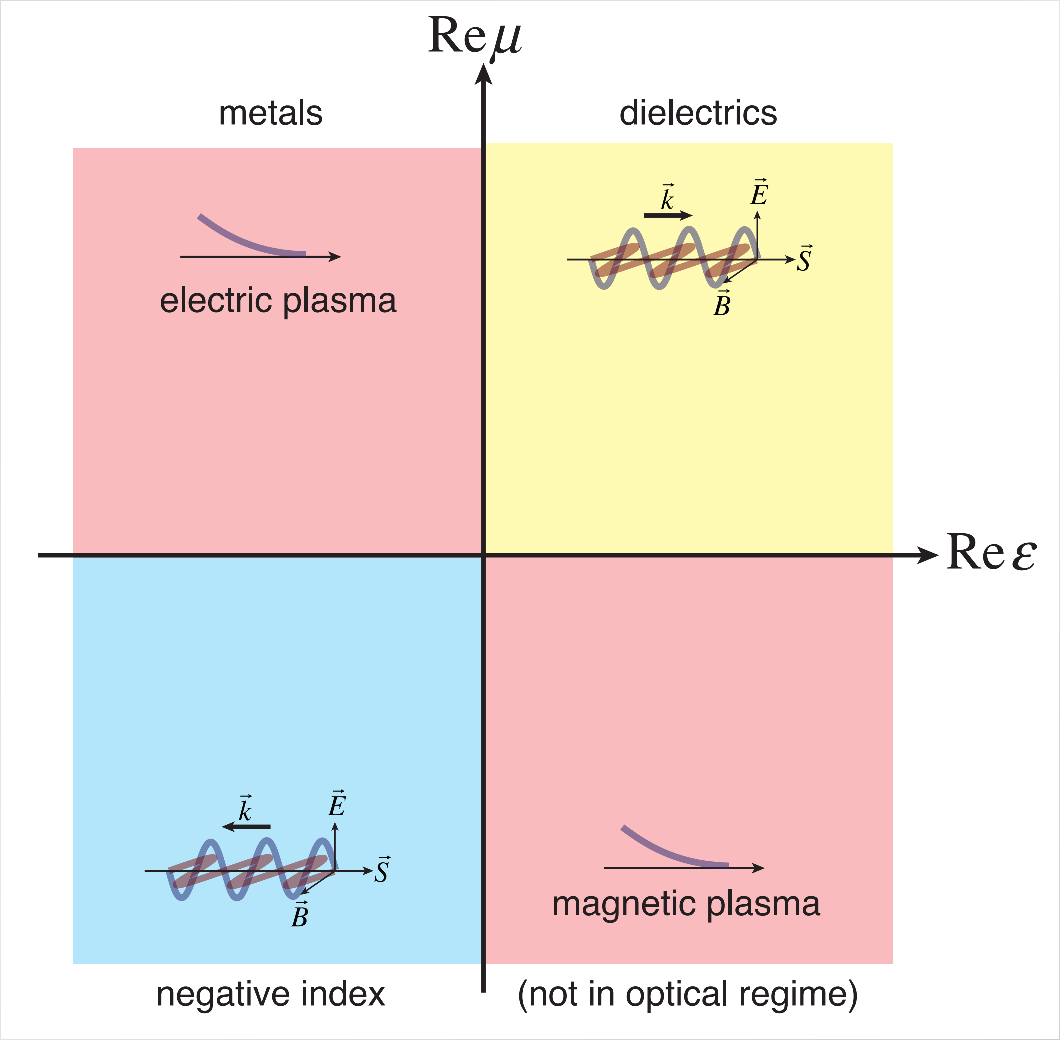

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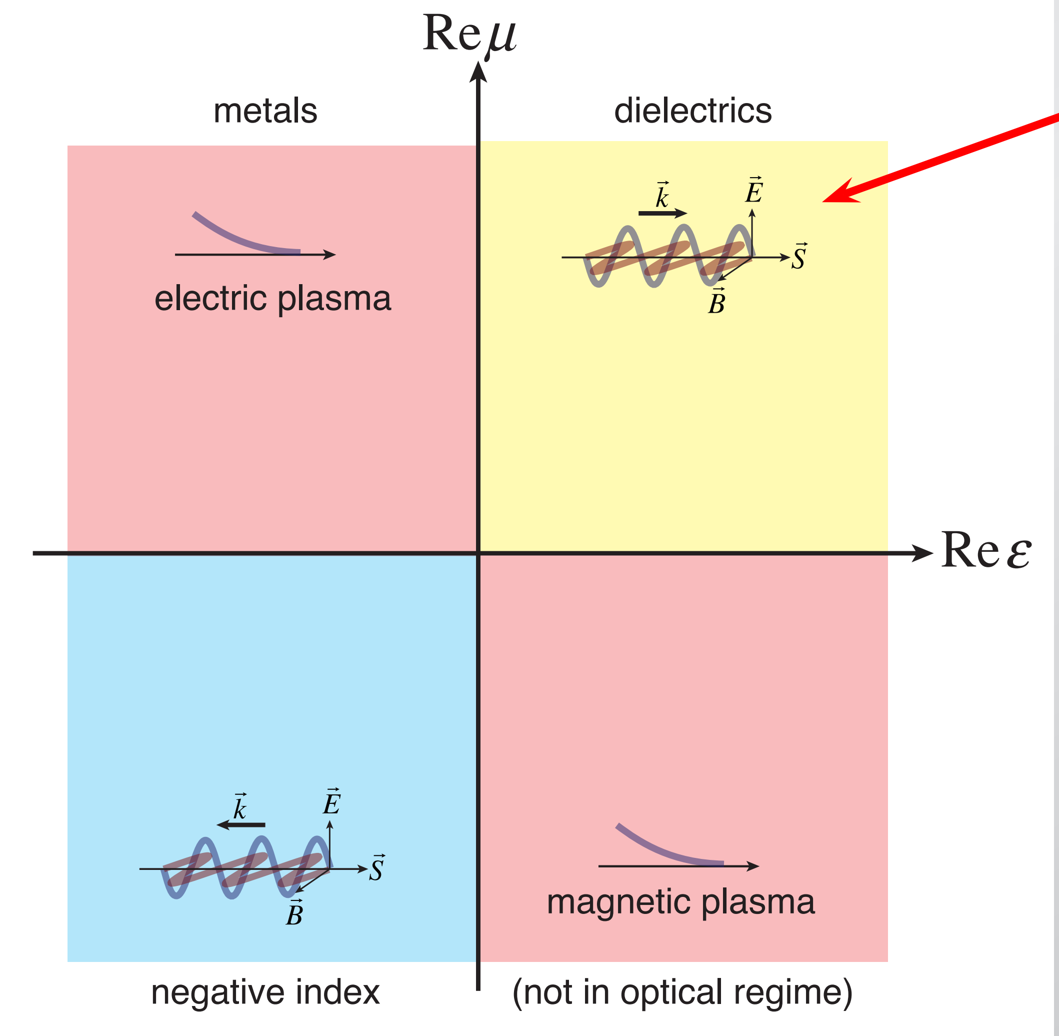
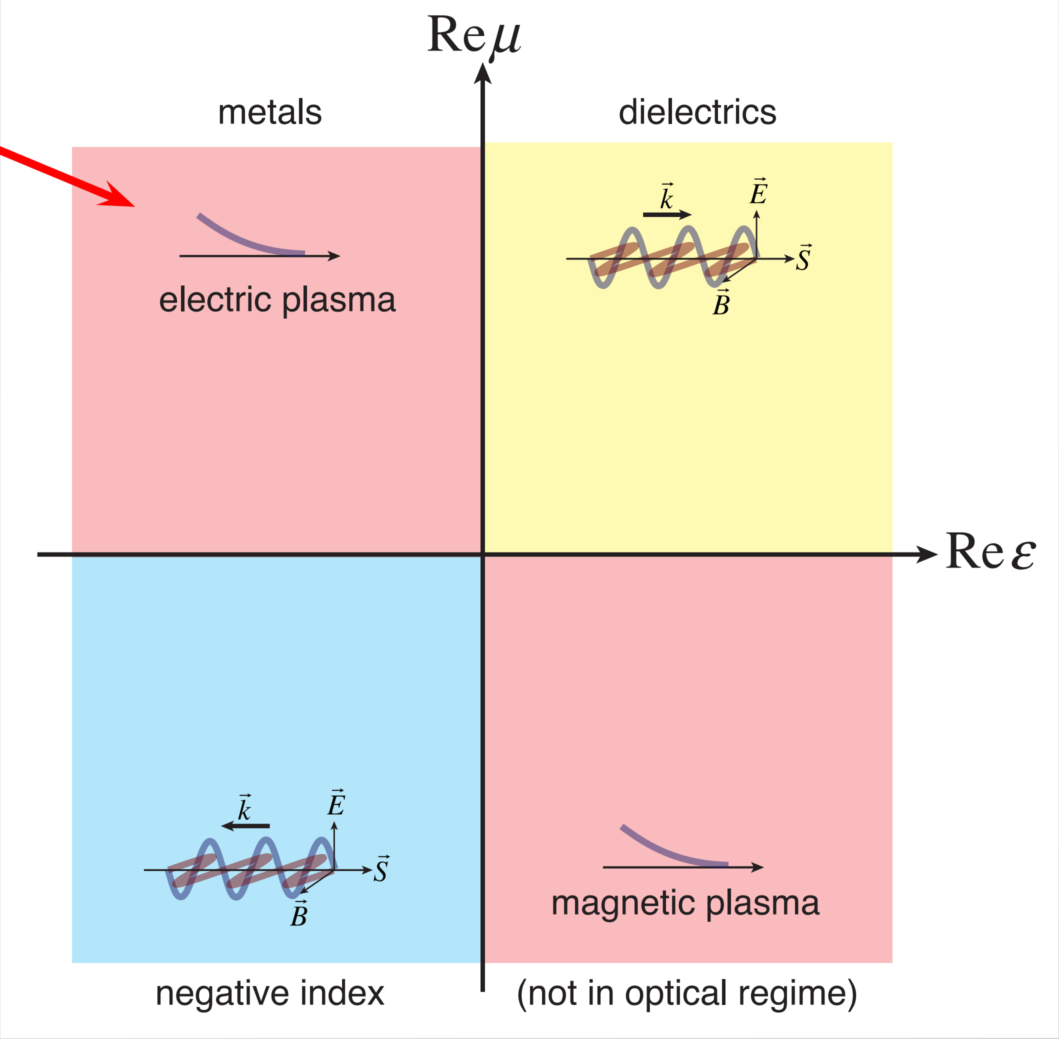


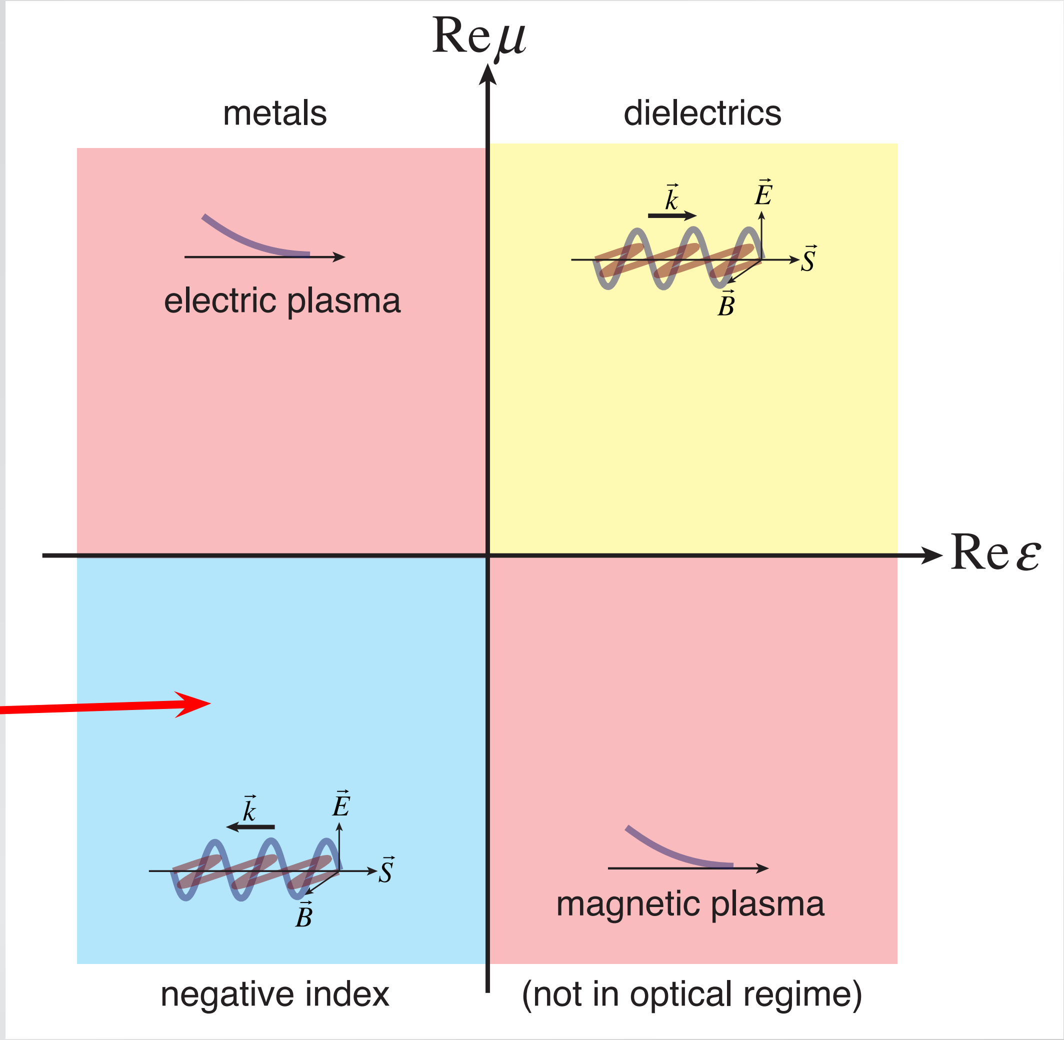
limited by diffraction

classification of (non-lossy) materials

no propagation

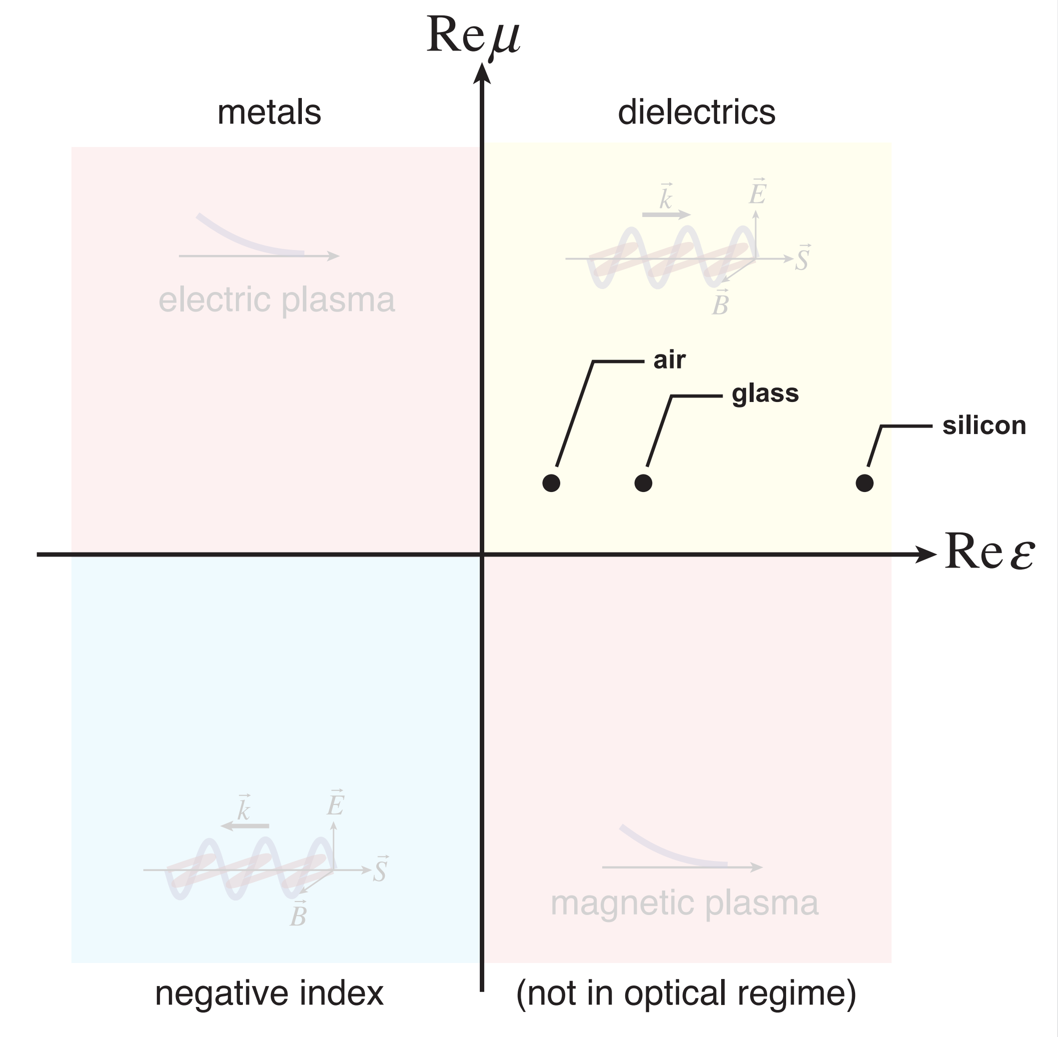


classification of (non-lossy) materials

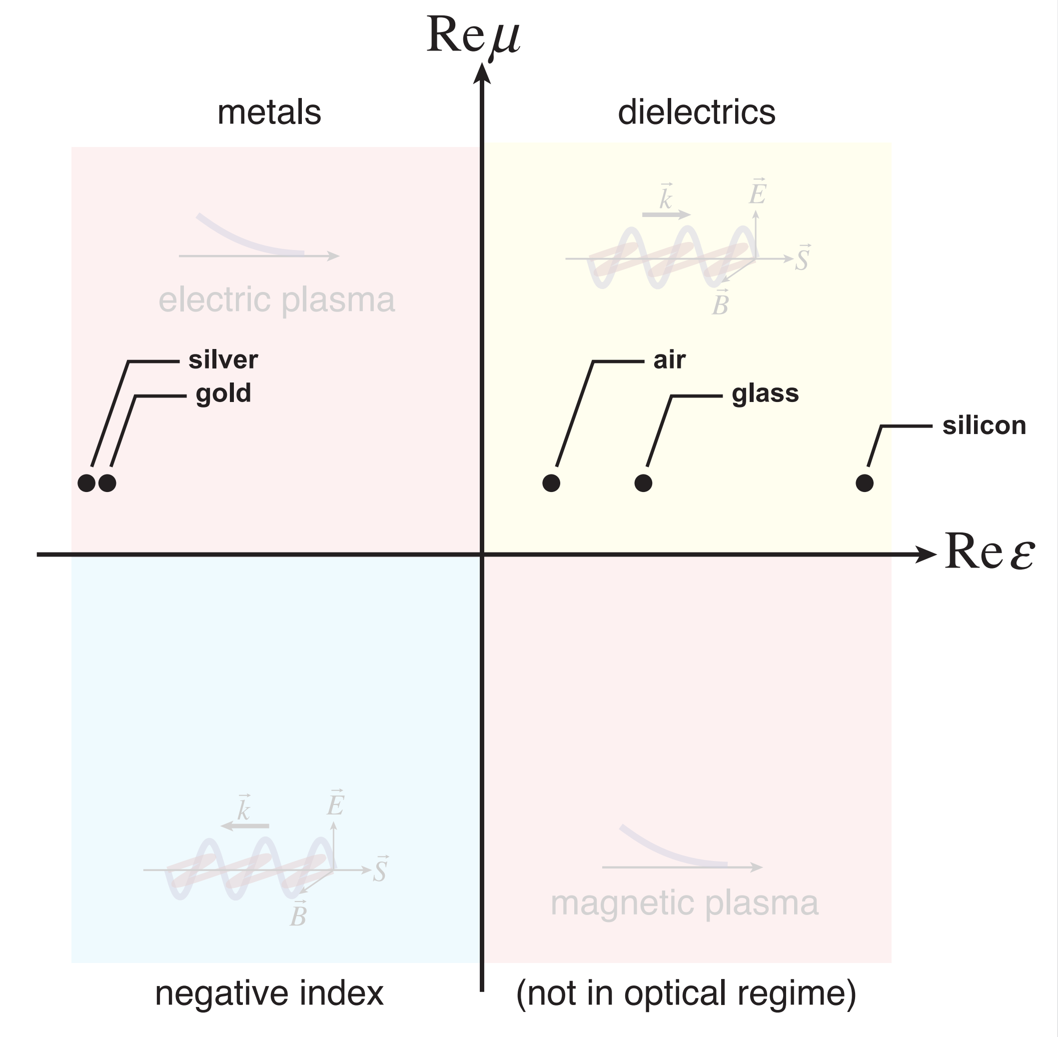


superlensing but…

common materials very limited

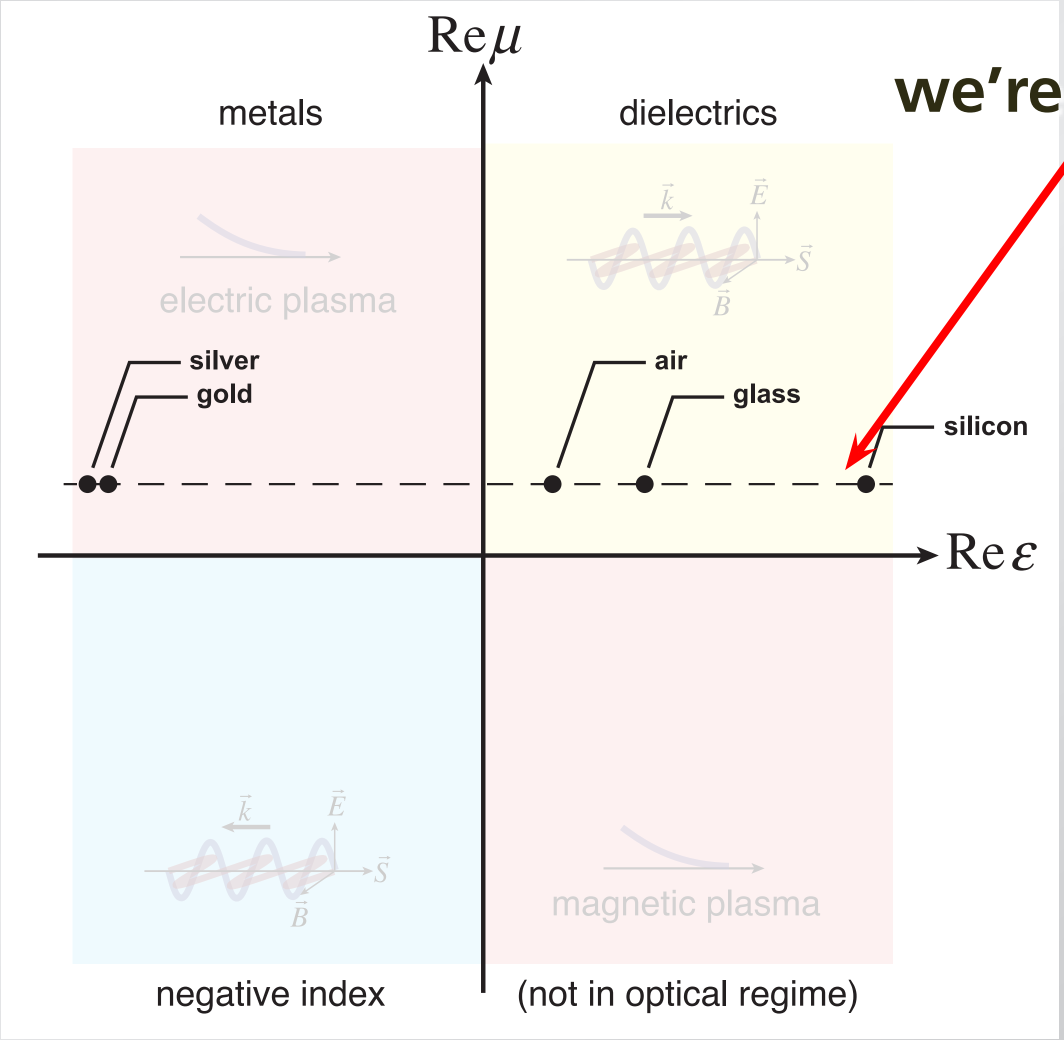


common materials very limited



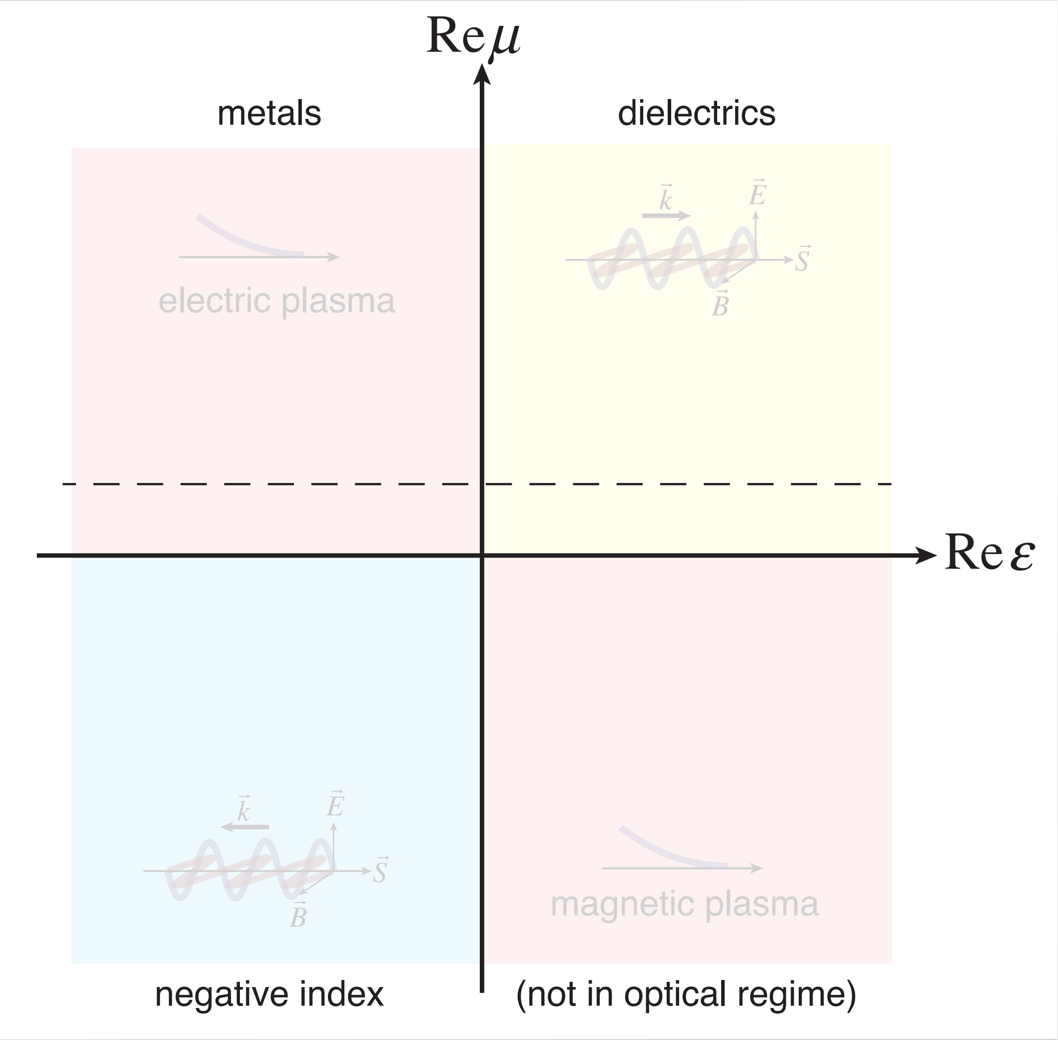
1 index

common materials very limited

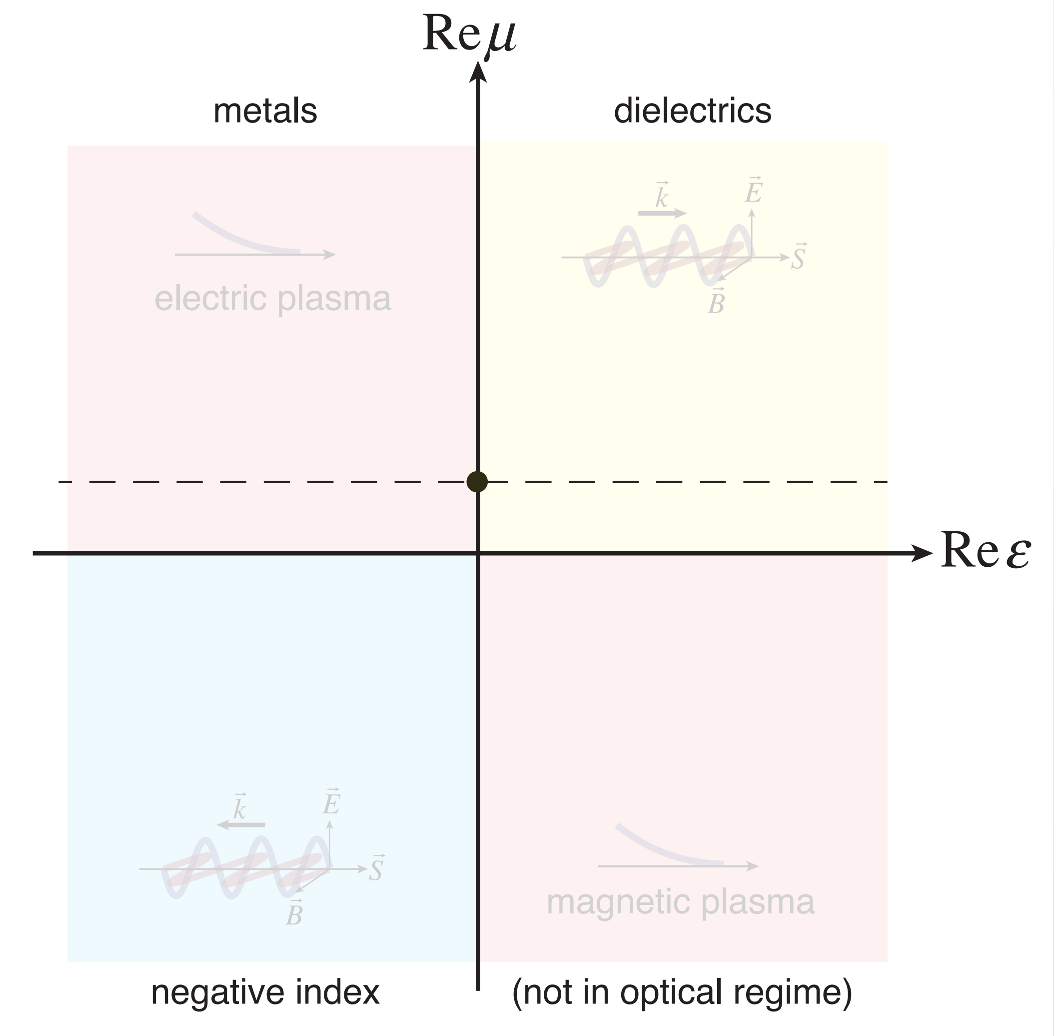


we’re stuck here!

What happens on the axes?



what if we let $\varepsilon = 0$?



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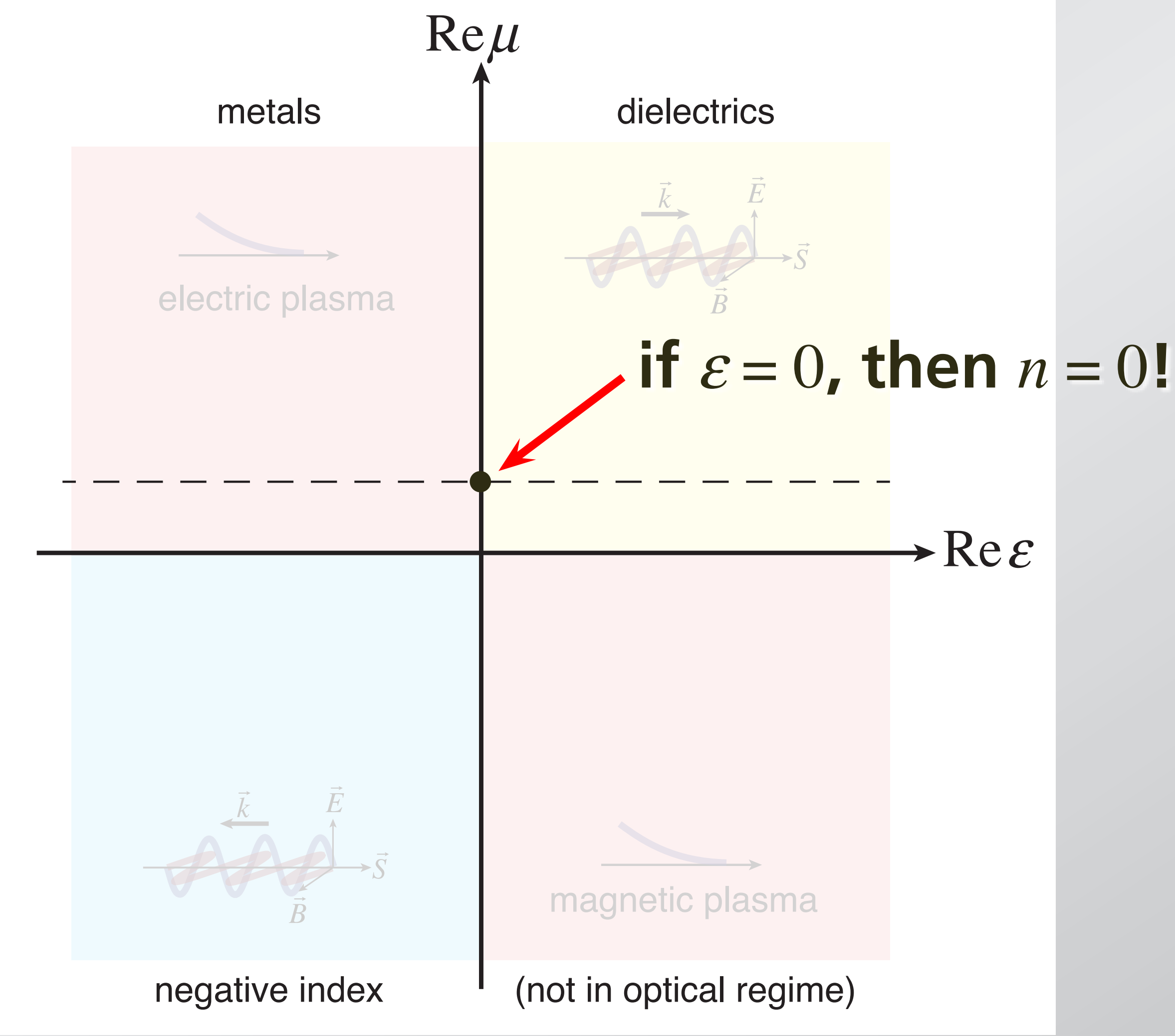

Q: If $n = 0$, which of the following is true?

1. the frequency goes to zero.
2. the phase velocity becomes infinite.
3. both of the above.
4. neither of the above.

1 index

2 zero index

wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

solution

$$\vec{E} = \vec{E}_o\, e^{i\,(kx - \omega t\,)}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}}\,c = \frac{1}{n}\,c$$

1 index

2 zero index

wave equation

$$\nabla^2 \vec{E} - \cancel{\frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}} = 0$$

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1 **index** 2 **zero index**

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$$\nabla^2 \vec{E} - \cancel{\frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}} = 0$$

solution

$$\vec{E} = \vec{E}_o\, e^{i\,(kx - \omega t\,)} \longrightarrow \vec{E} = \vec{E}_o\, e^{-i\omega t}$$

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1 index

2 zero index

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$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}}\, c = \frac{1}{n}\, c \longrightarrow \infty$$

1 index

2 zero index

Q: If $n = 0$, which of the following is true?

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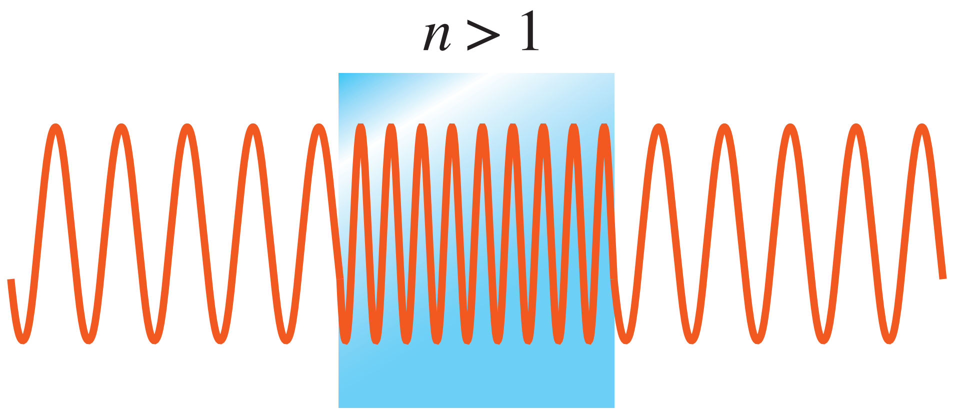
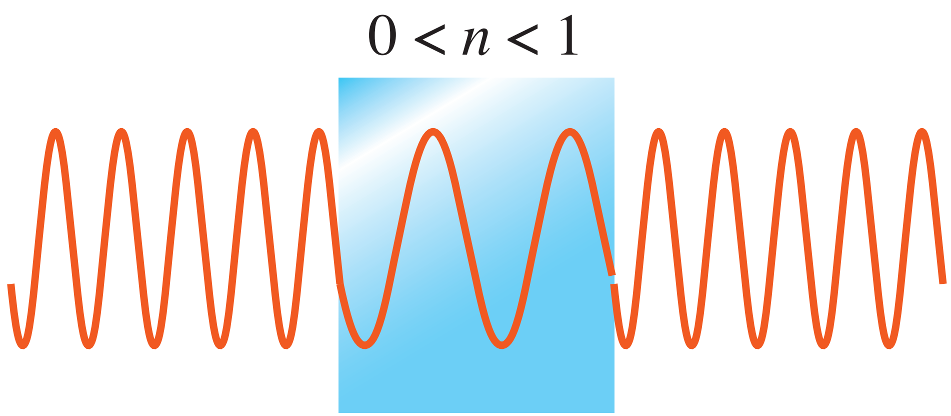
1 index

2 zero index

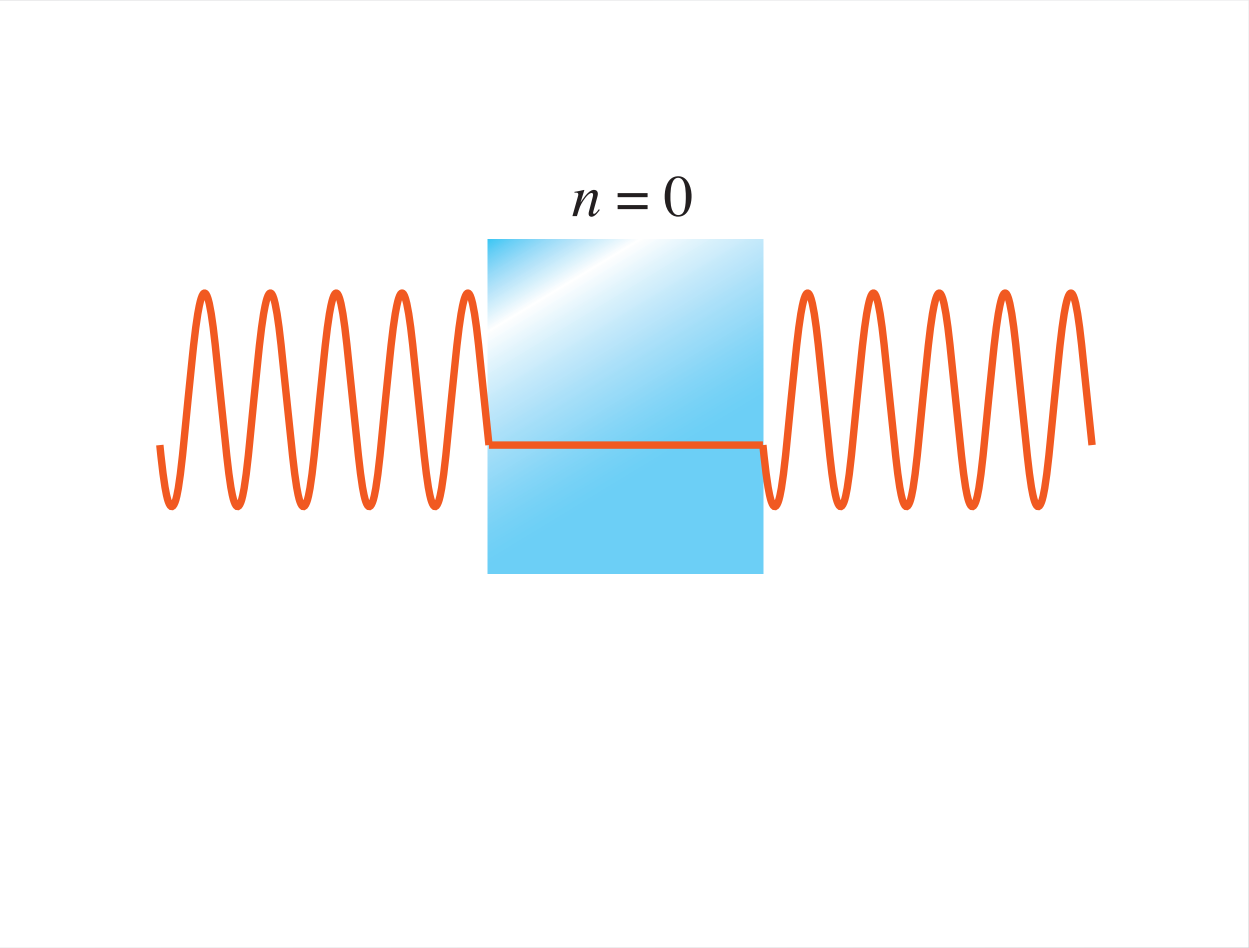


1 index 2 zero index



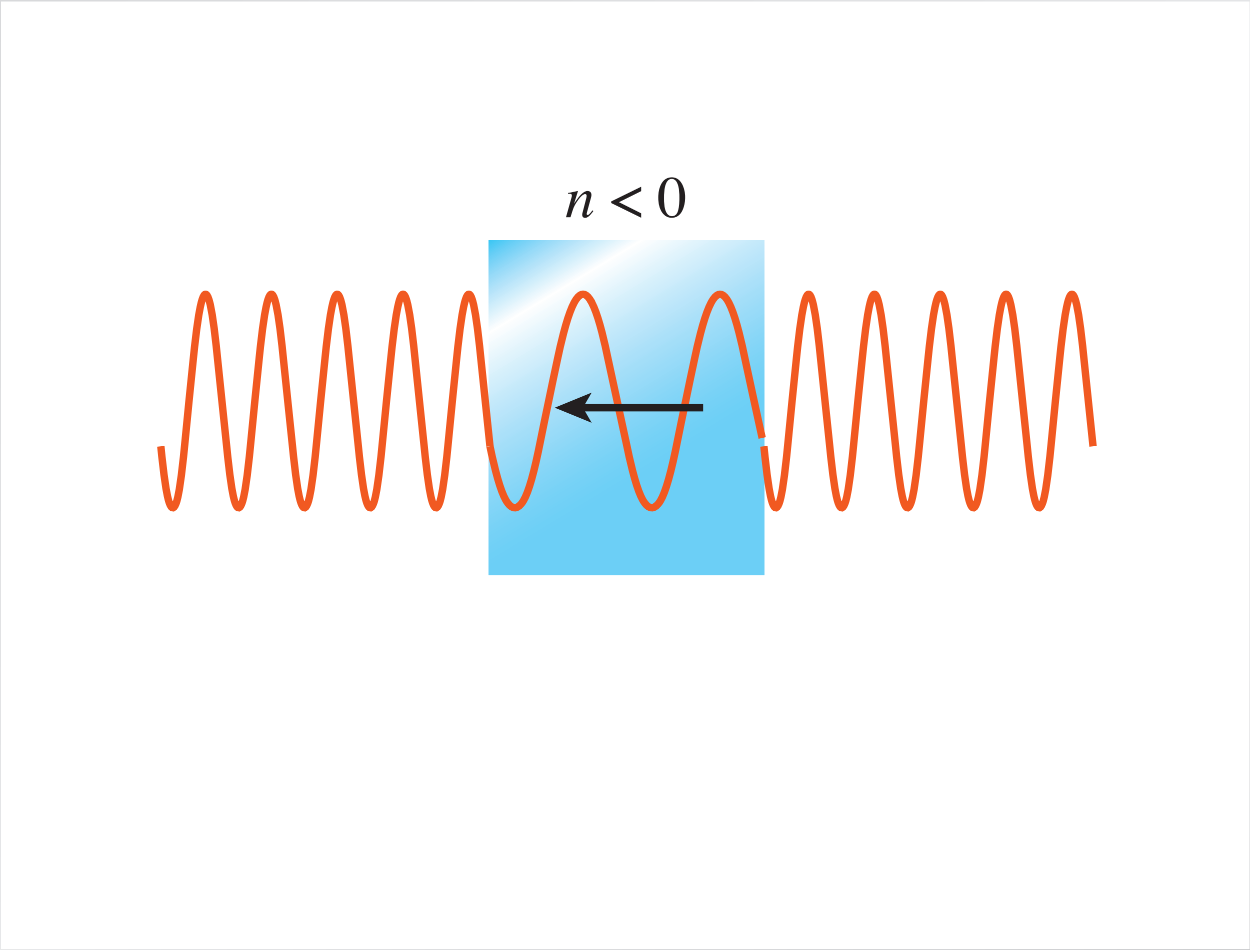
1 index

2 zero index



1 index

2 zero index



1 index

2 zero index

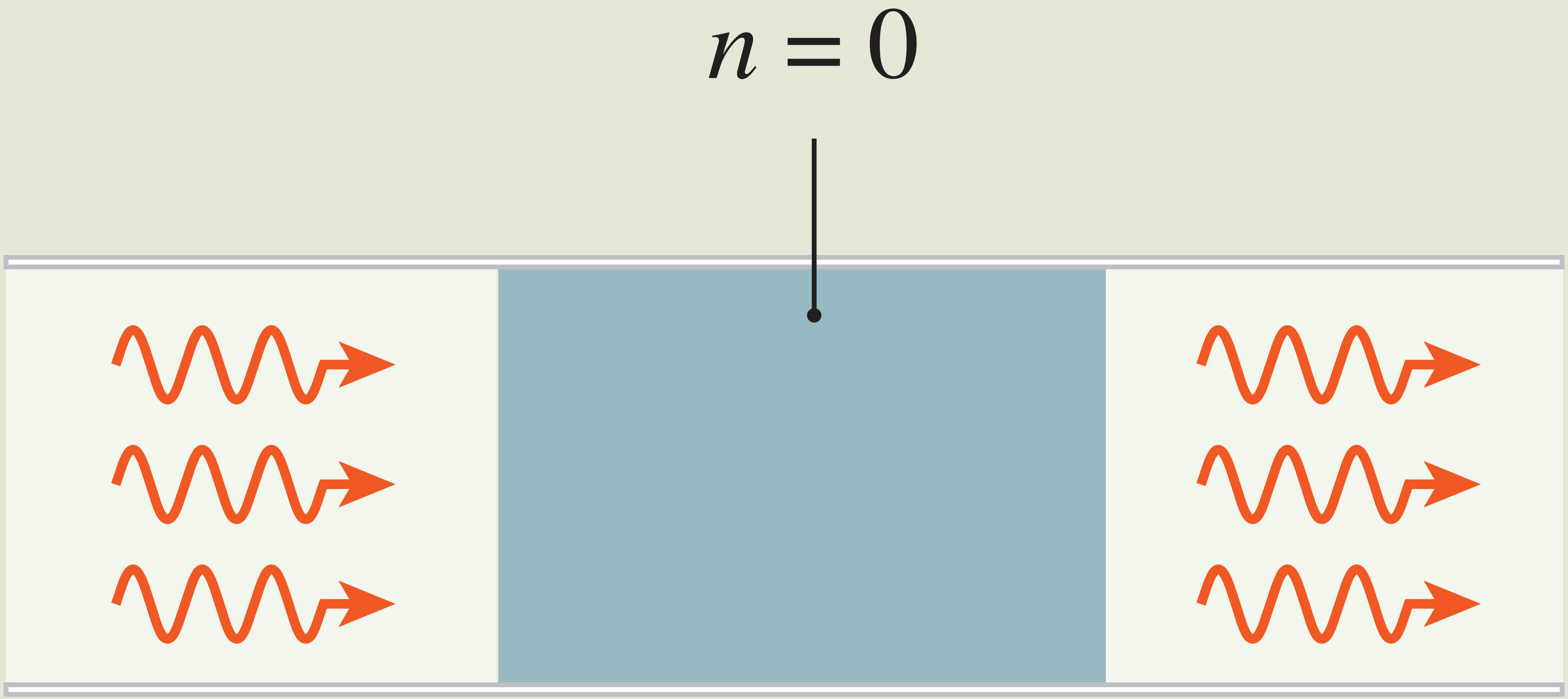
1 index

2 zero index

What can we do with uniform phase?

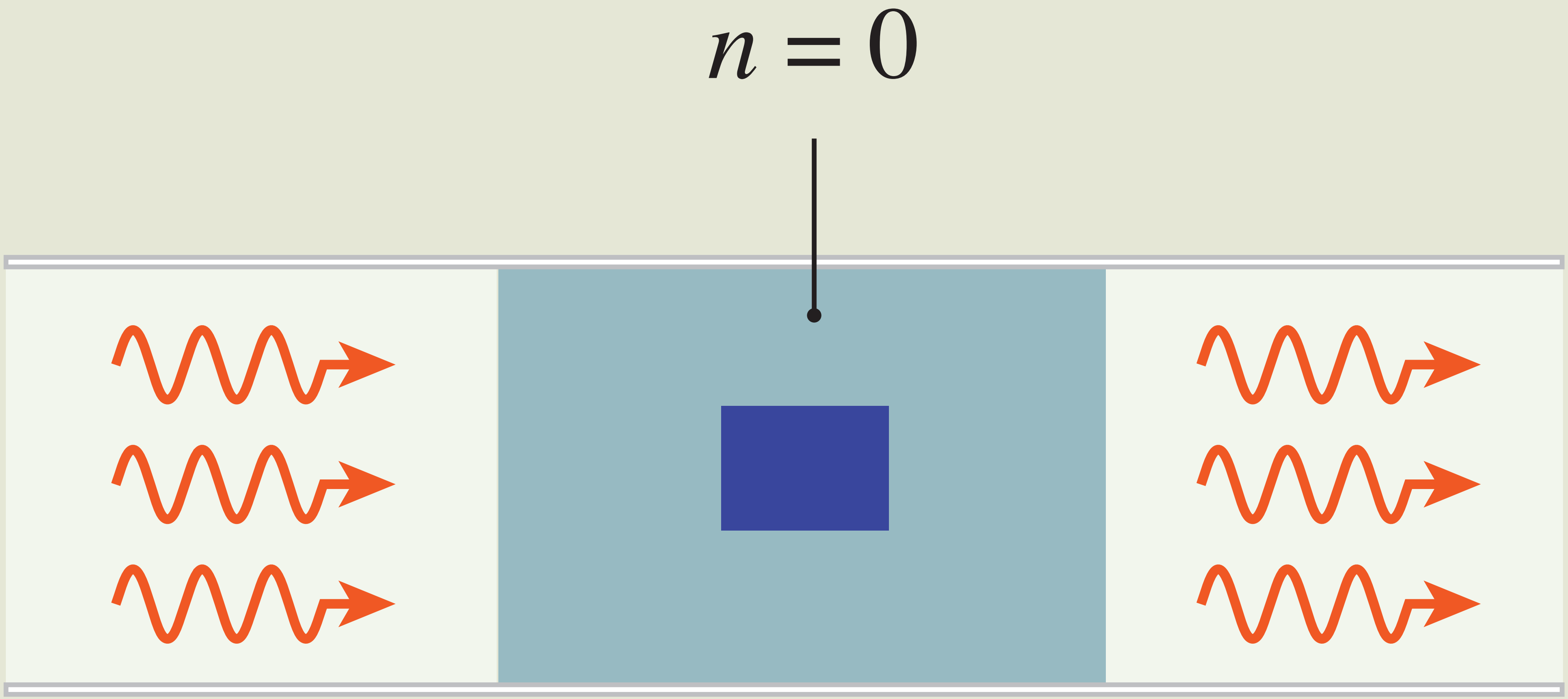
1 index

2 zero index

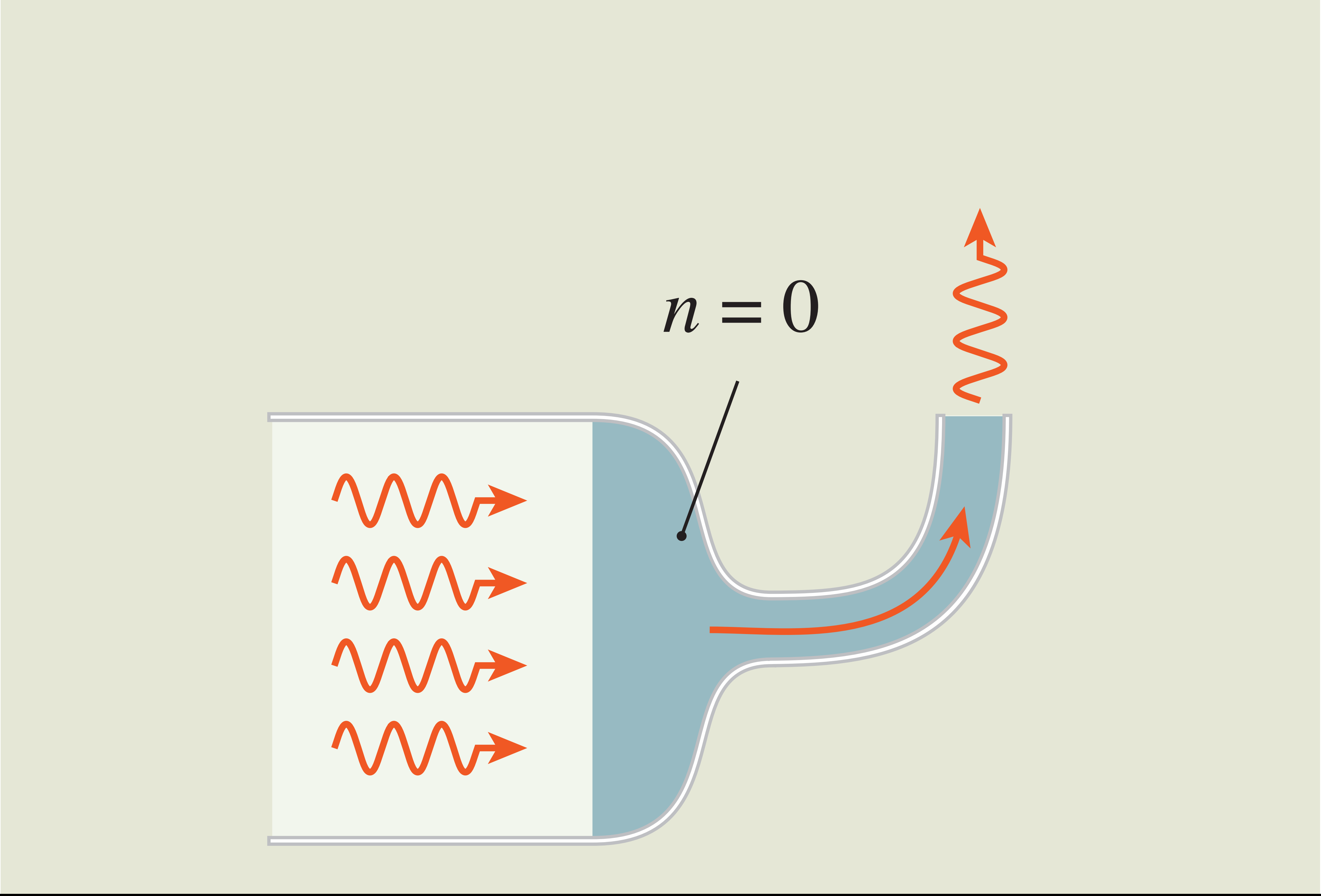


1 index

2 zero index



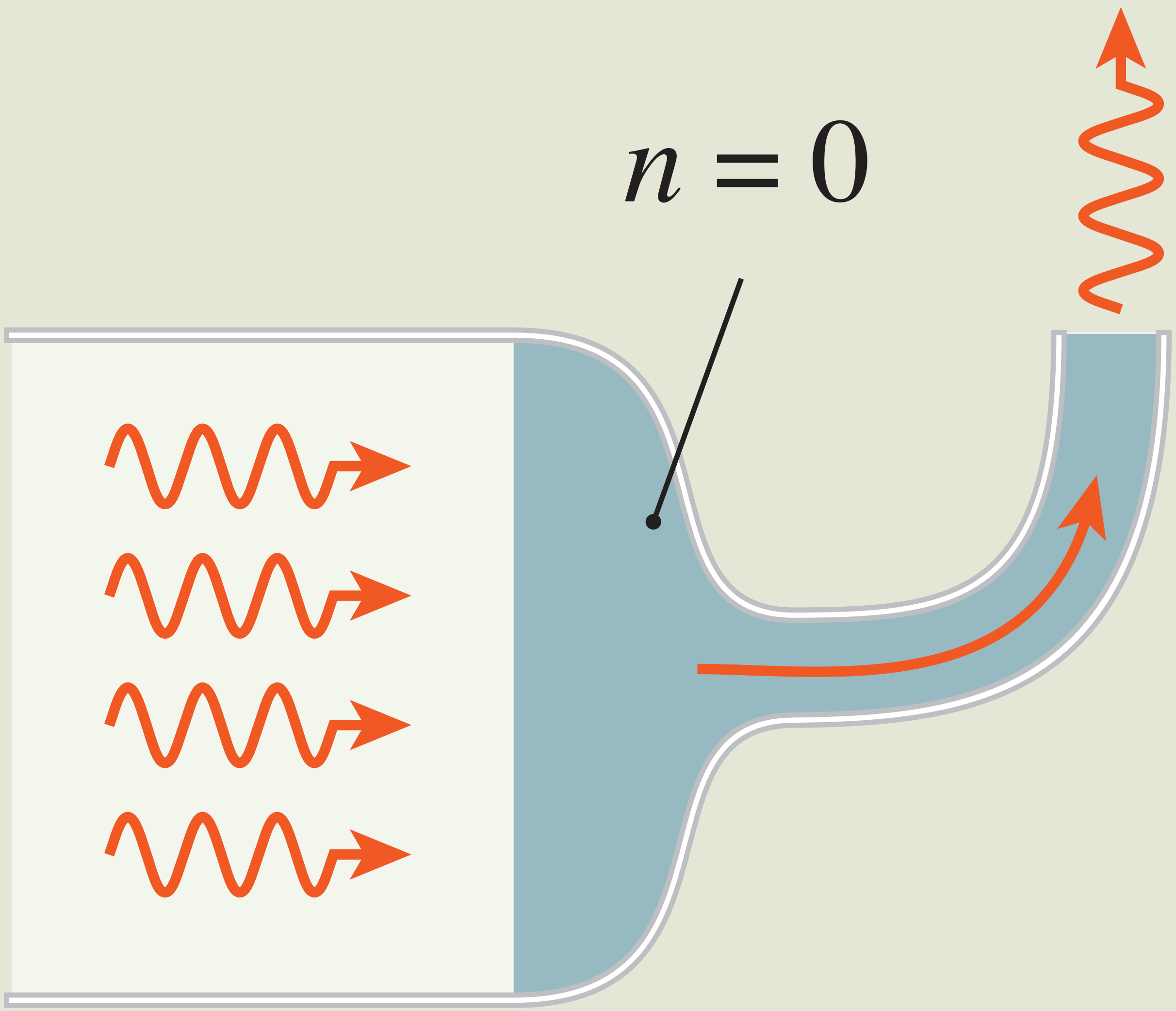
1 index 2 zero index



1 index

2 zero index

"tunneling with infinite decay length"



1 index 2 zero index

how?

$$n = \sqrt{\varepsilon\mu}$$

1 index

2 zero index

how?

$$n = \sqrt{\varepsilon\mu}$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

how?

$$n = \sqrt{\varepsilon\mu}$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

how?

$$\varepsilon \to 0 \qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

how?

$\varepsilon \to 0$ $\qquad$ $n = \sqrt{\varepsilon\mu} \;\longrightarrow 0$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \;\longrightarrow \infty$$

1 index $\qquad$ **2 zero index**

how?

$$\varepsilon \to 0 \qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1} \to 1$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to \infty$$

1 index 2 zero index

how?

$$\mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \;\to 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

how?

$$\mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \longrightarrow 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \longrightarrow 0$$

how?

$$\mu \rightarrow 0 \qquad n = \sqrt{\varepsilon\mu} \longrightarrow 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1} \longrightarrow -1$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \longrightarrow 0$$

1 index **2 zero index**

how?

$$\varepsilon, \mu \to 0 \qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

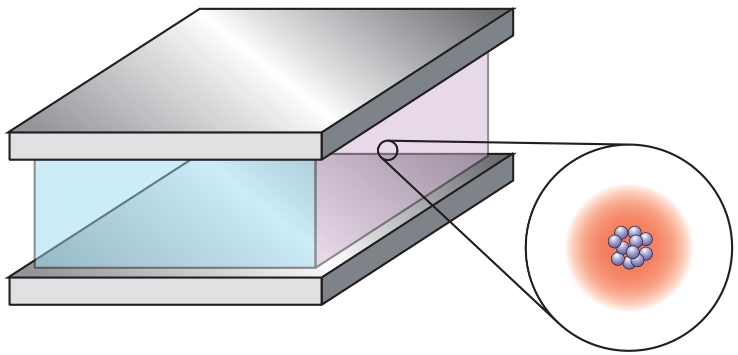
$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$ **finite!**

but $\mu \neq 1$ requires a magnetic response!

Engineering a magnetic response

Engineering a magnetic response

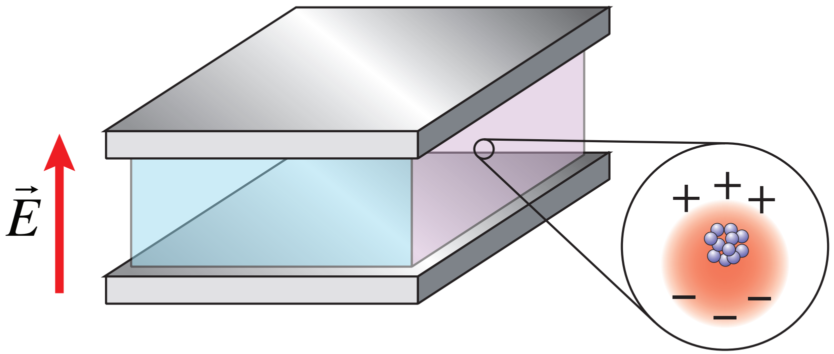
bulk material



properties derive from constituent atoms

Engineering a magnetic response

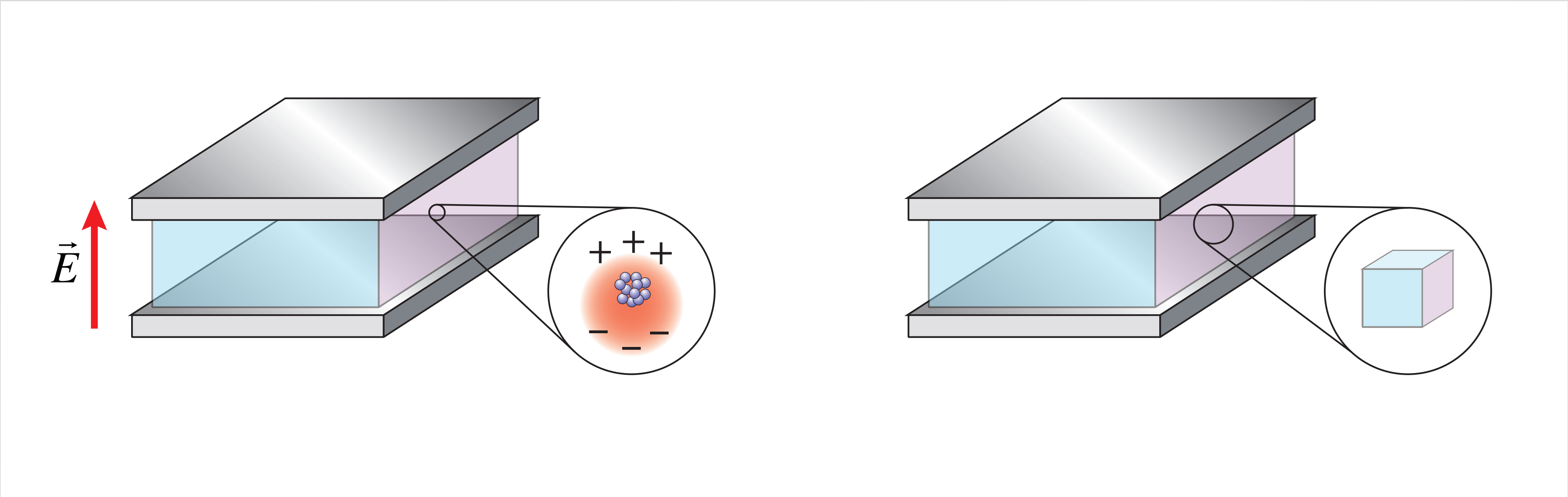
bulk material



properties derive from constituent atoms

Engineering a magnetic response

bulk material | **composite material**

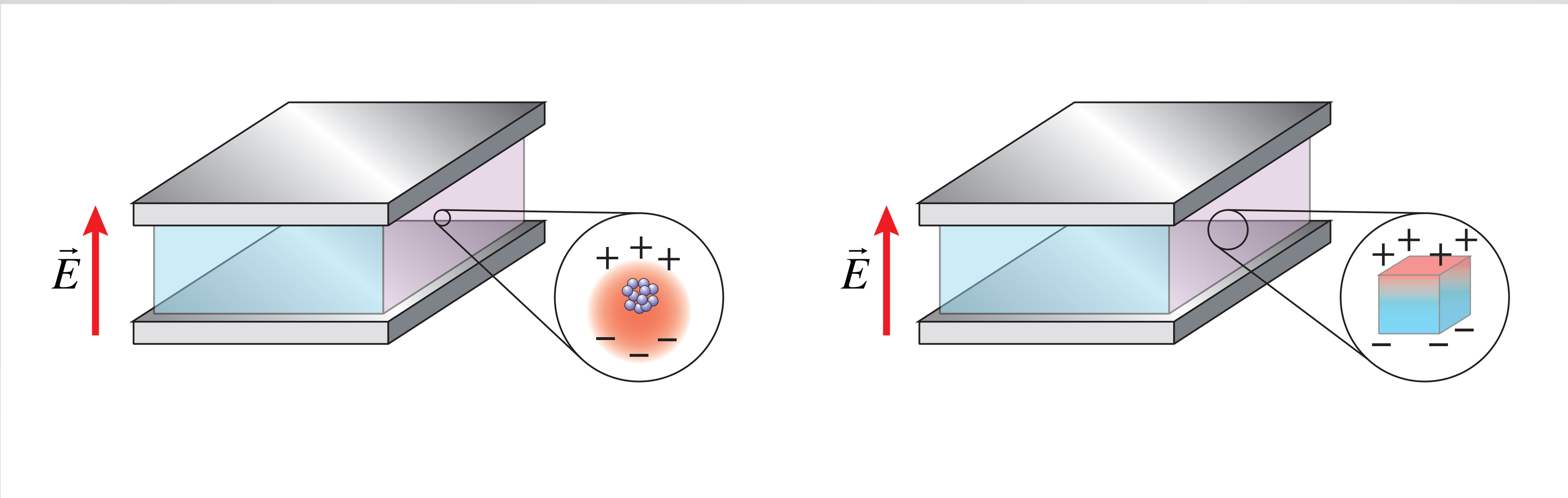


properties derive from constituent atoms

properties derive from constituent units

Engineering a magnetic response

bulk material | composite material



properties derive from constituent atoms

properties derive from constituent units

Engineering a magnetic response

use array of dielectric rods

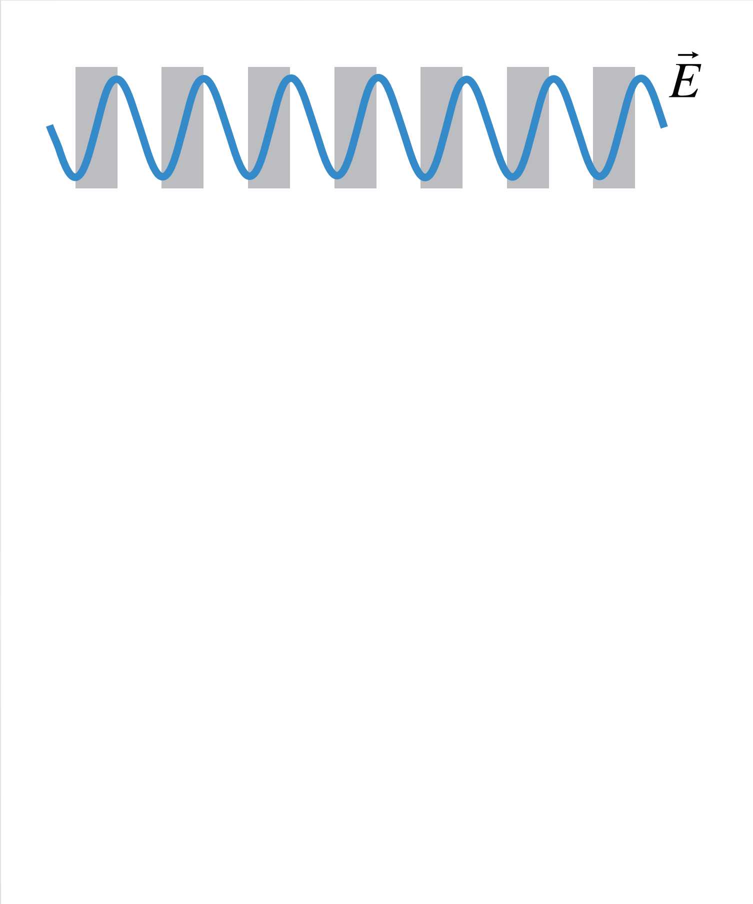
1 index

2 zero index

3 experiments

Engineering a magnetic response

incident electromagnetic wave ($\lambda_{\text{eff}} \approx a$)

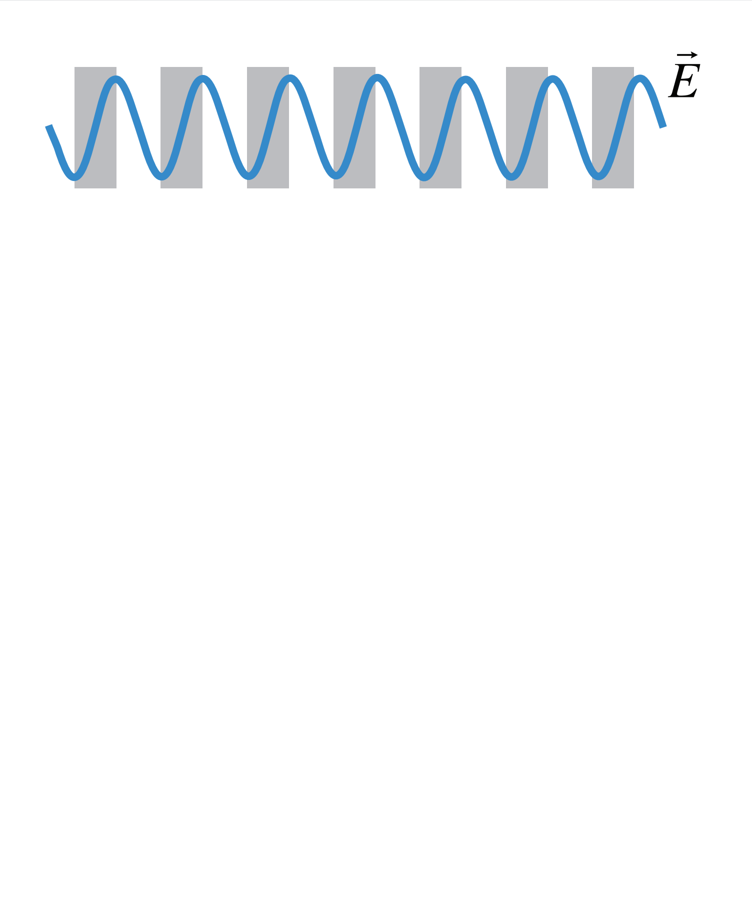


1 index

2 zero index

Engineering a magnetic response

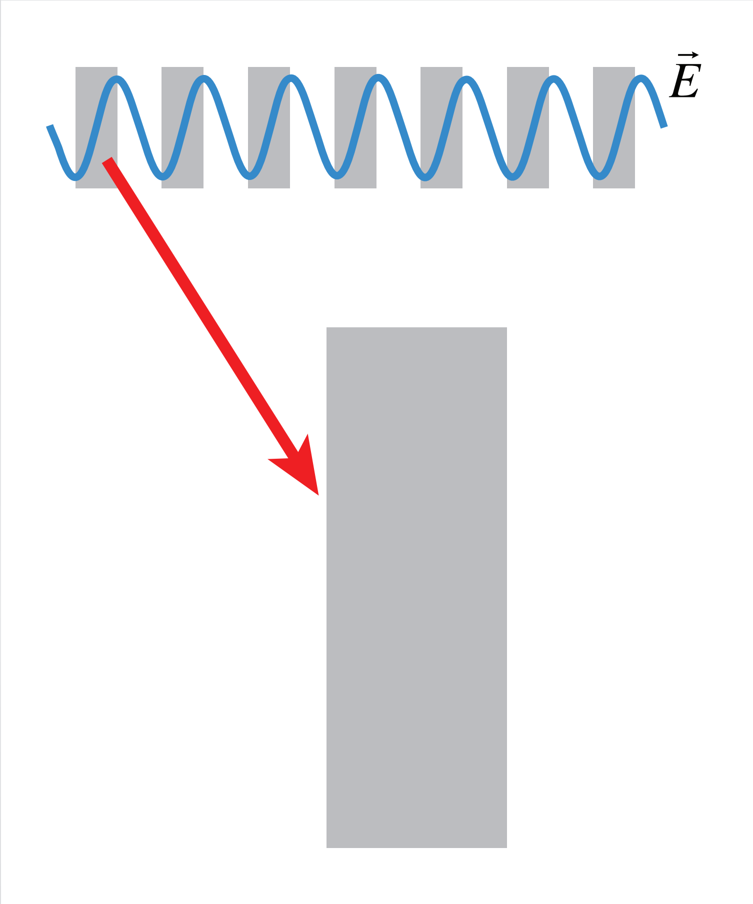
produces an electric response…



1 index 2 zero index 3 experiments

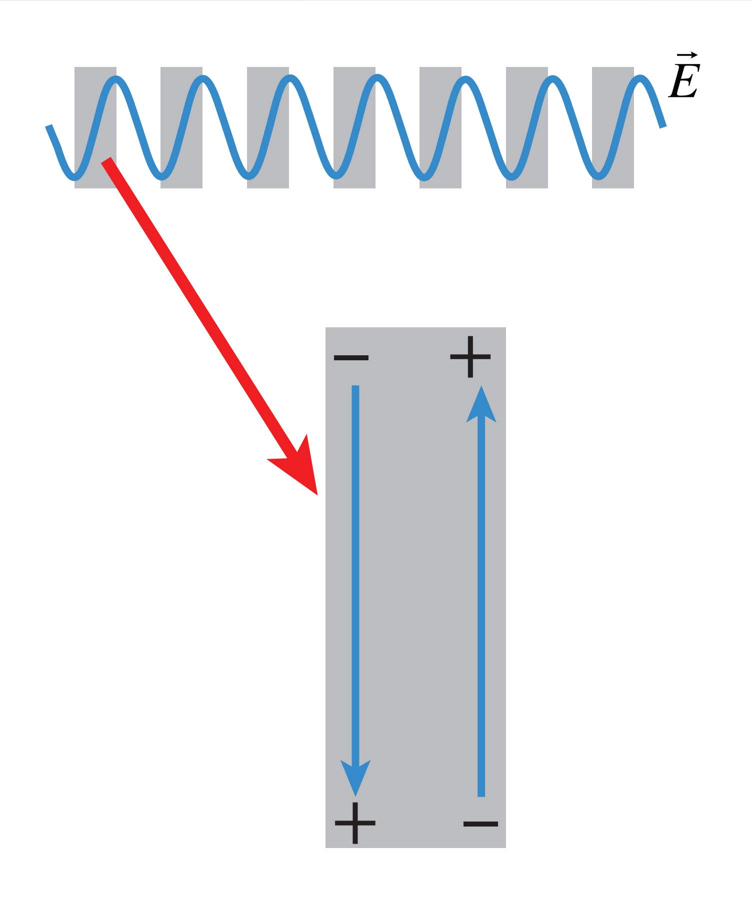
Engineering a magnetic response

… but different electric fields front and back…



Engineering a magnetic response

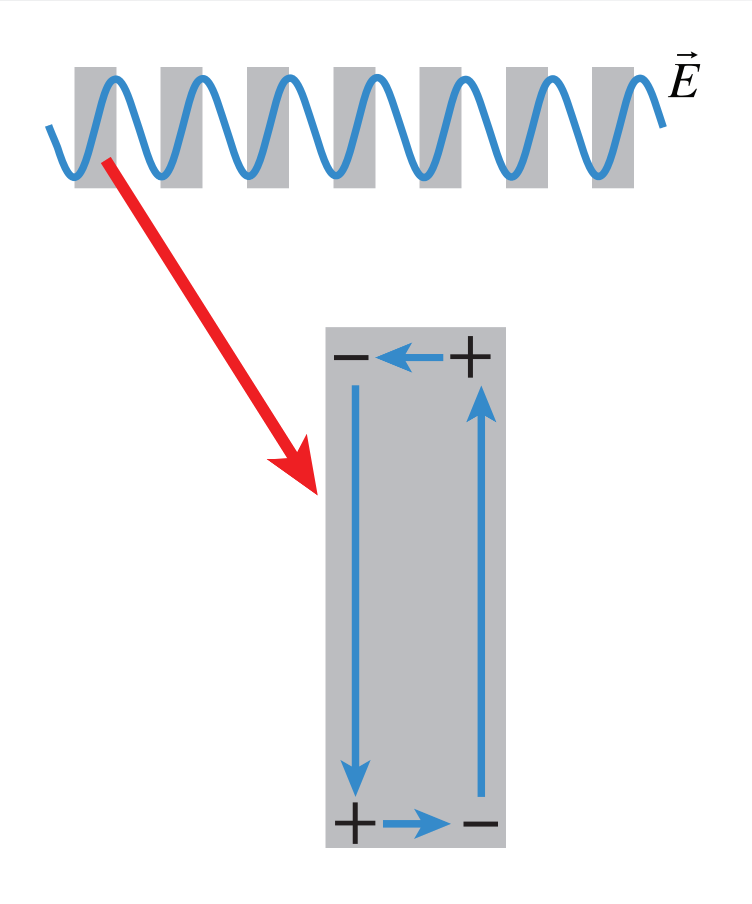
…induce different polarizations on opposite sides…



1 index   2 zero index   3 experiments

Engineering a magnetic response

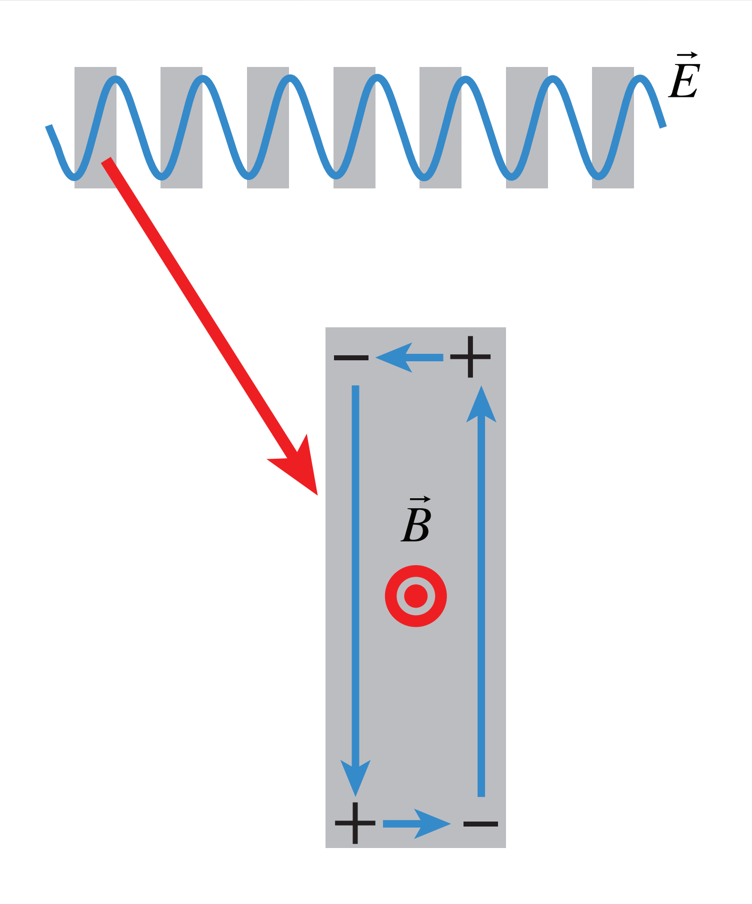
…causing a current loop…



1 index **2** zero index **3** experiments

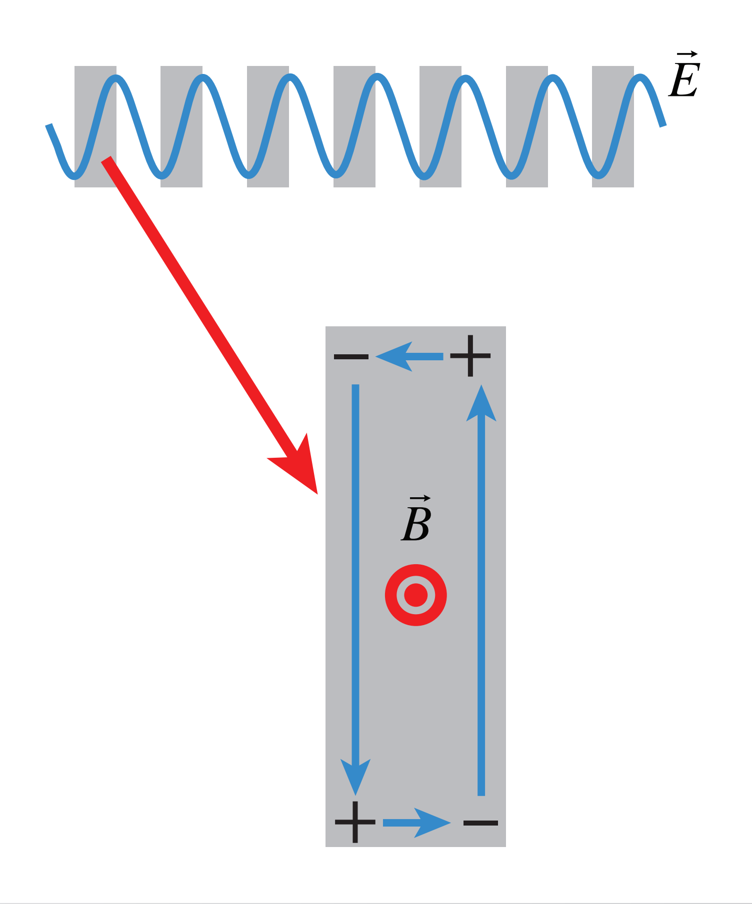
Engineering a magnetic response

…which, in turn, produces an induced magnetic field



Engineering a magnetic response

adjust design so electrical and magnetic resonances coincide



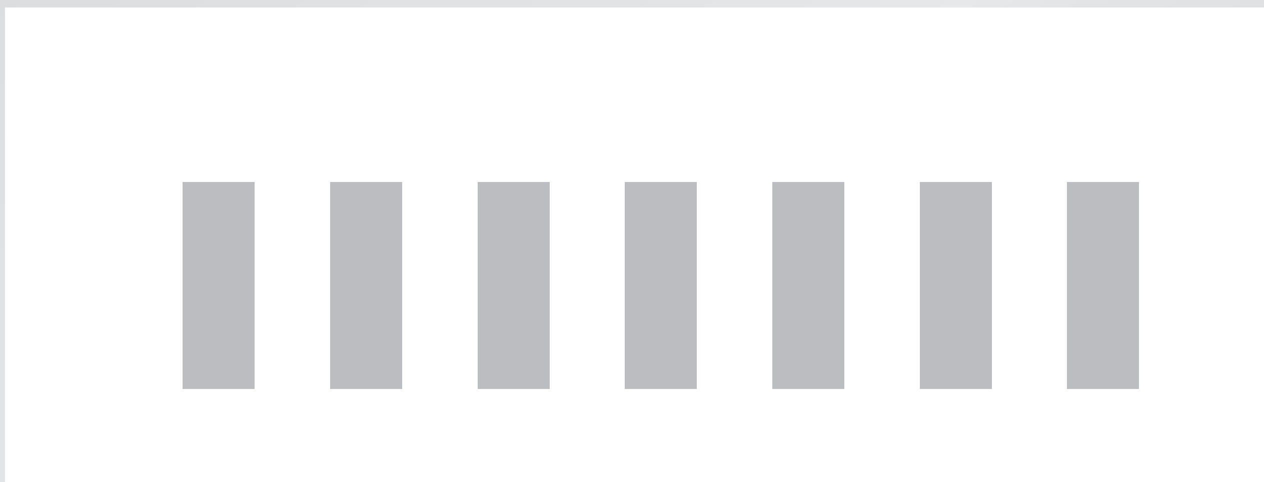
1 index

2 zero index

3 experiments

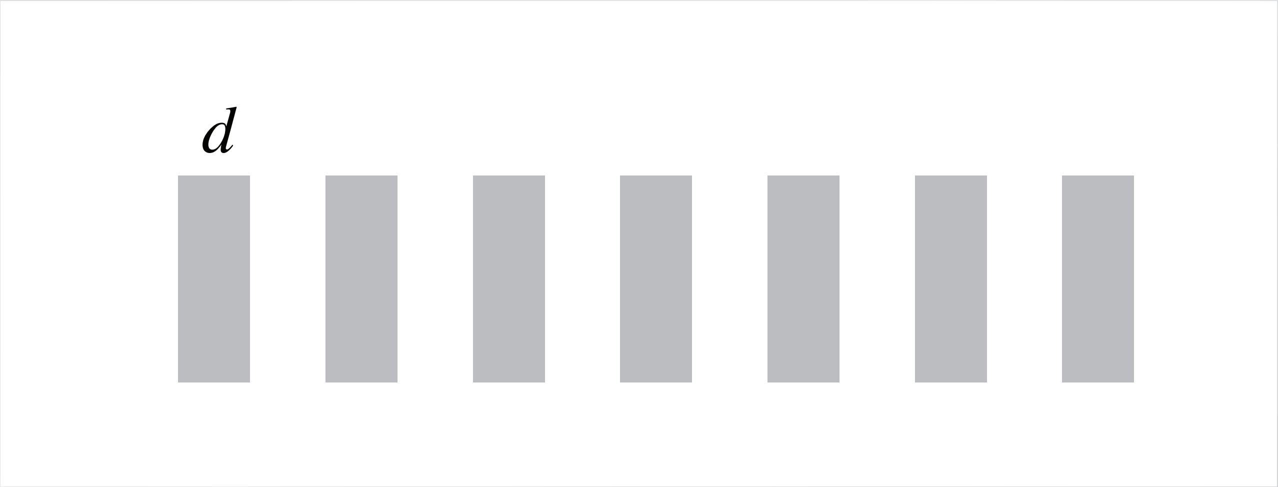
Engineering a magnetic response

adjustable parameters



Engineering a magnetic response

adjustable parameters



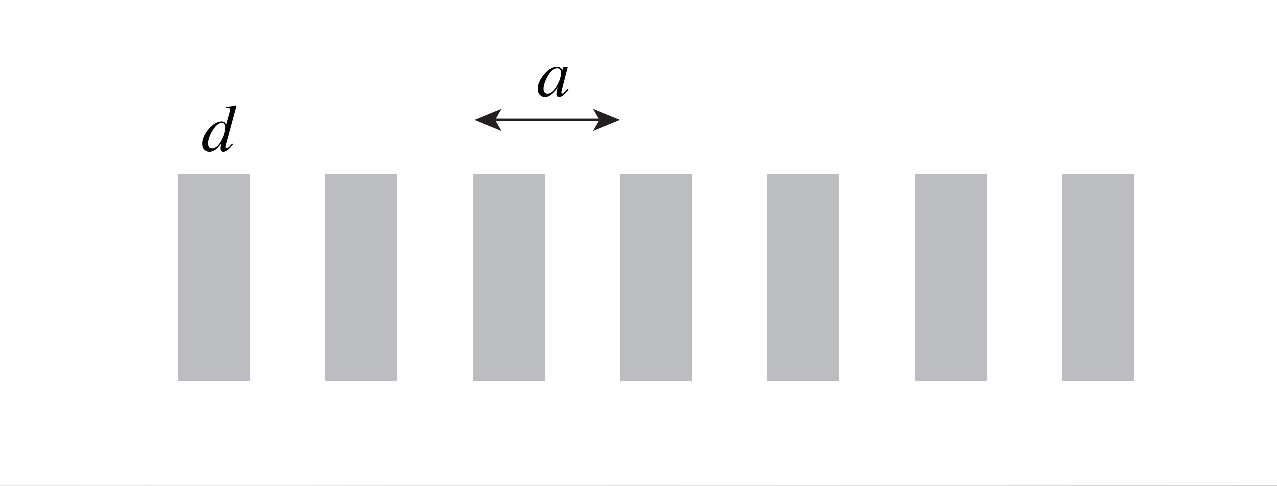
1 index

2 zero index

3 experiments

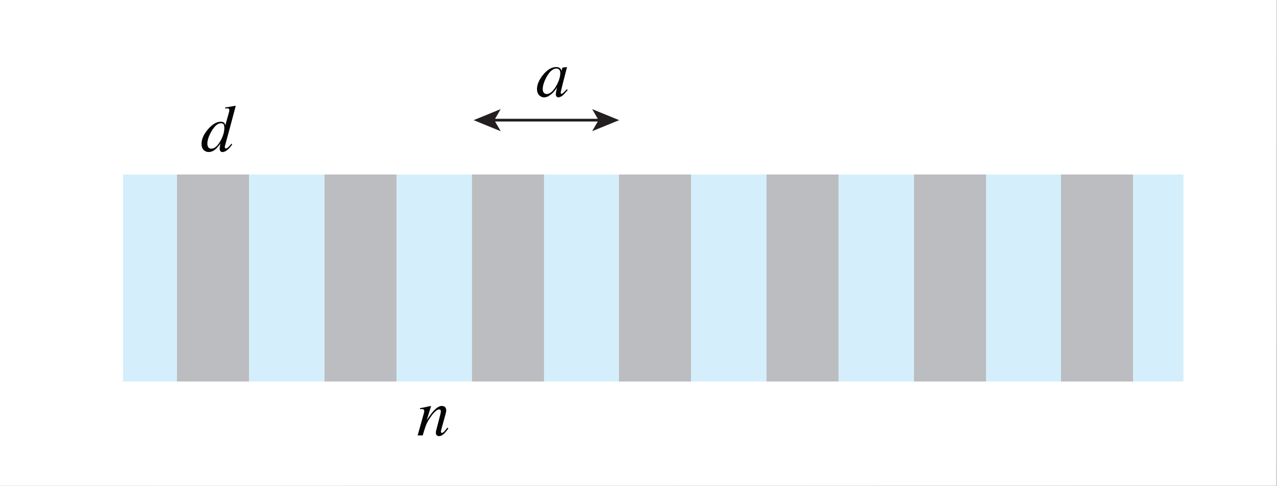
Engineering a magnetic response

adjustable parameters



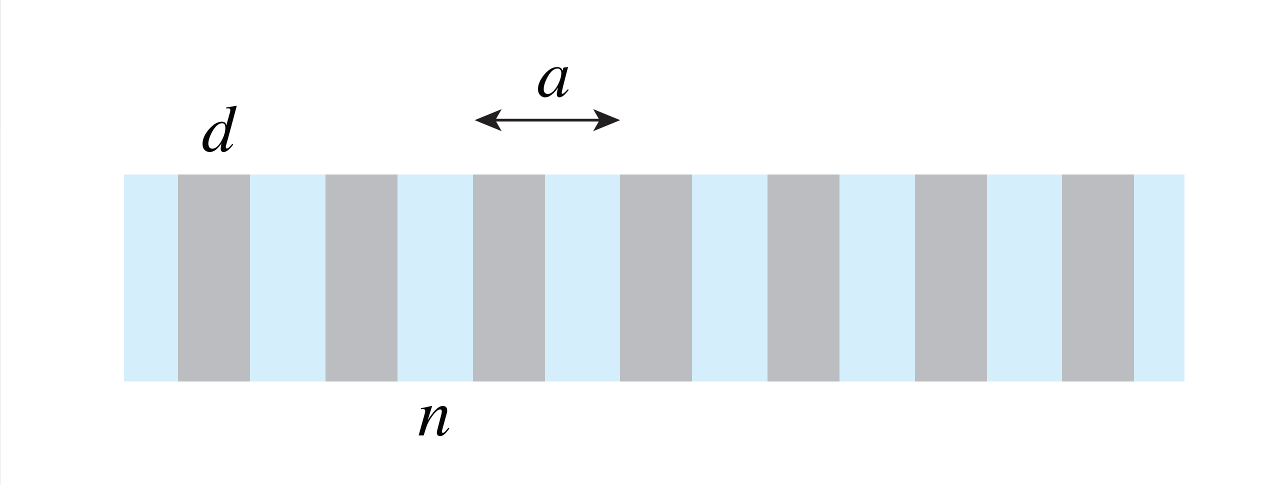
Engineering a magnetic response

adjustable parameters

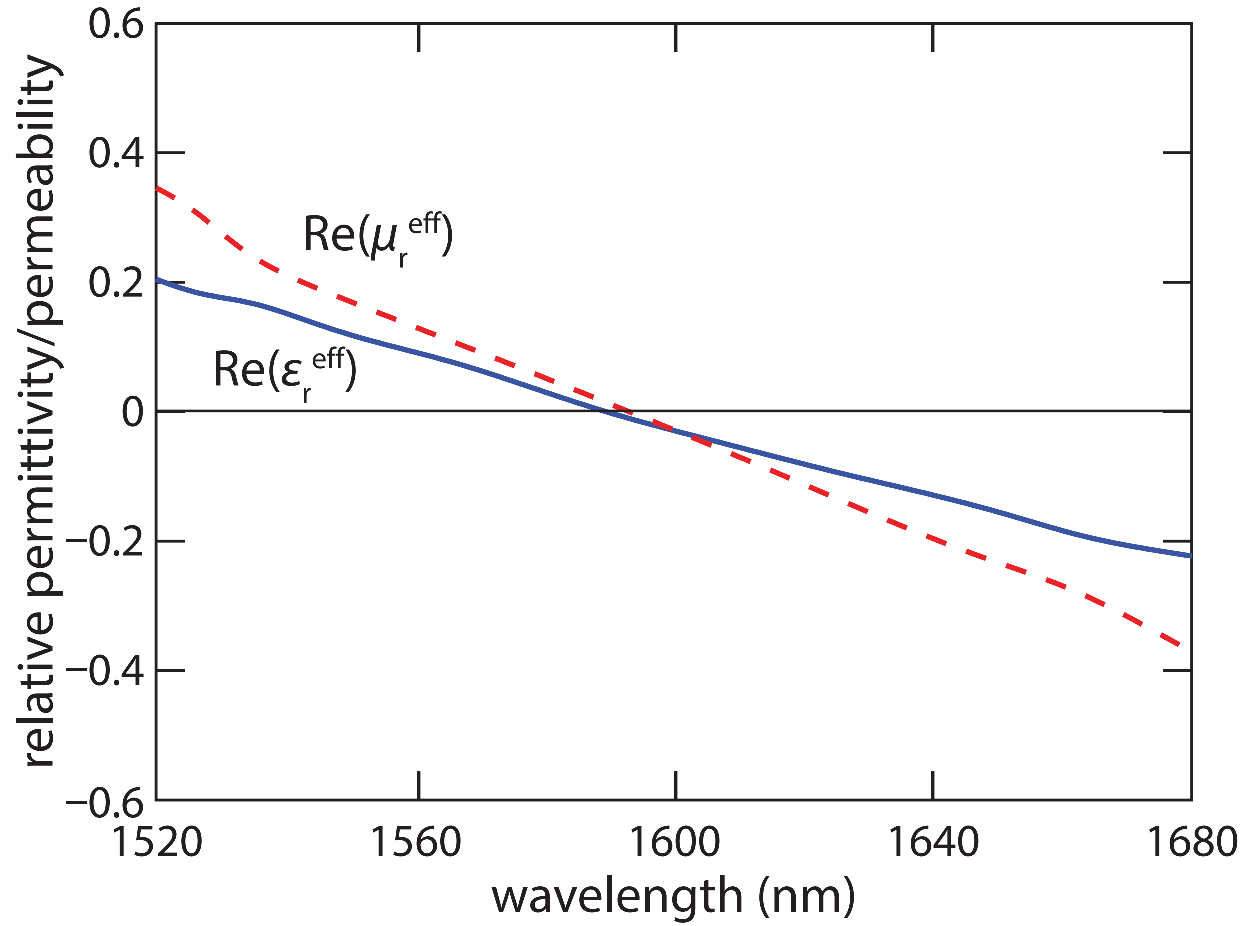


Engineering a magnetic response

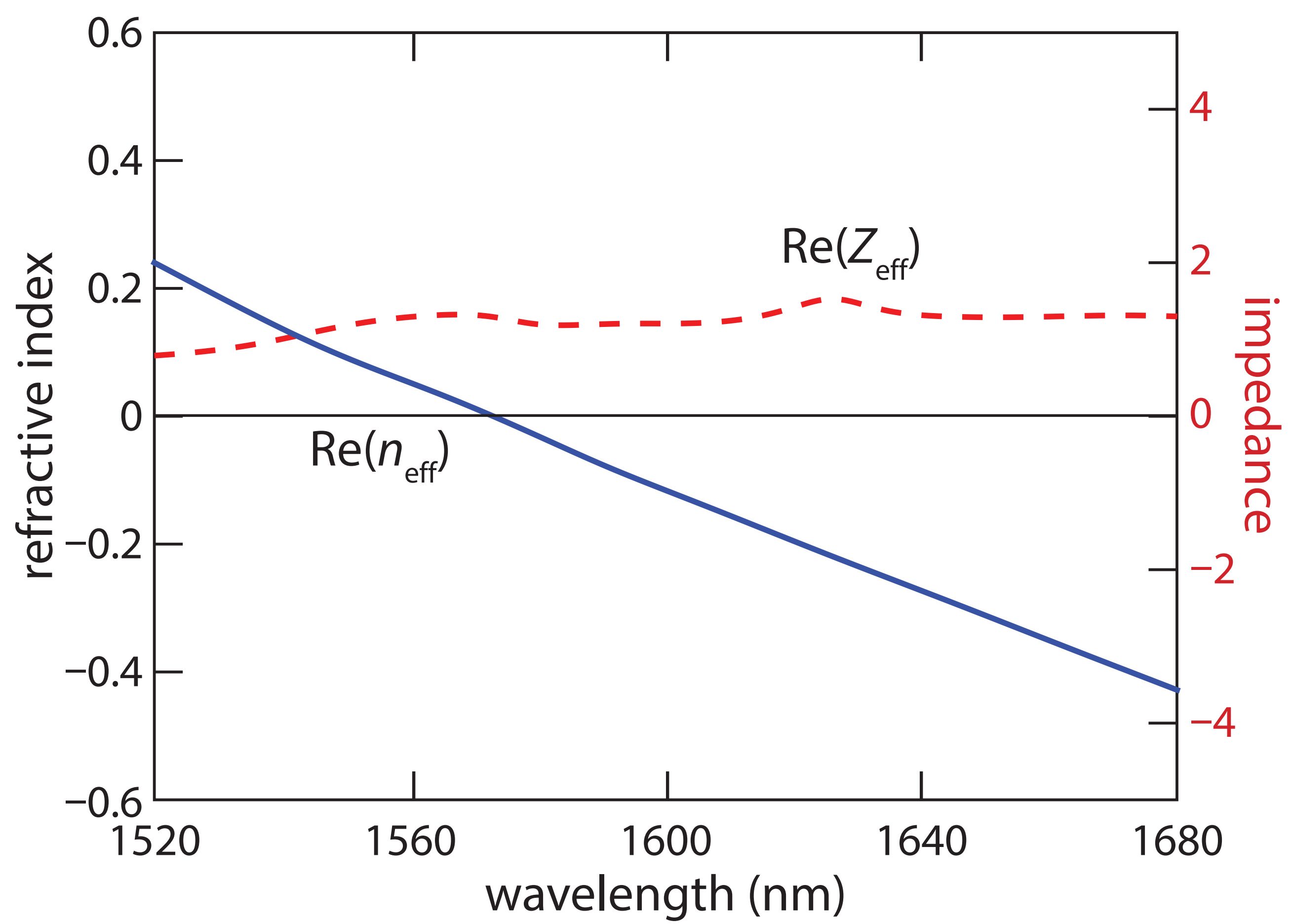
adjustable parameters



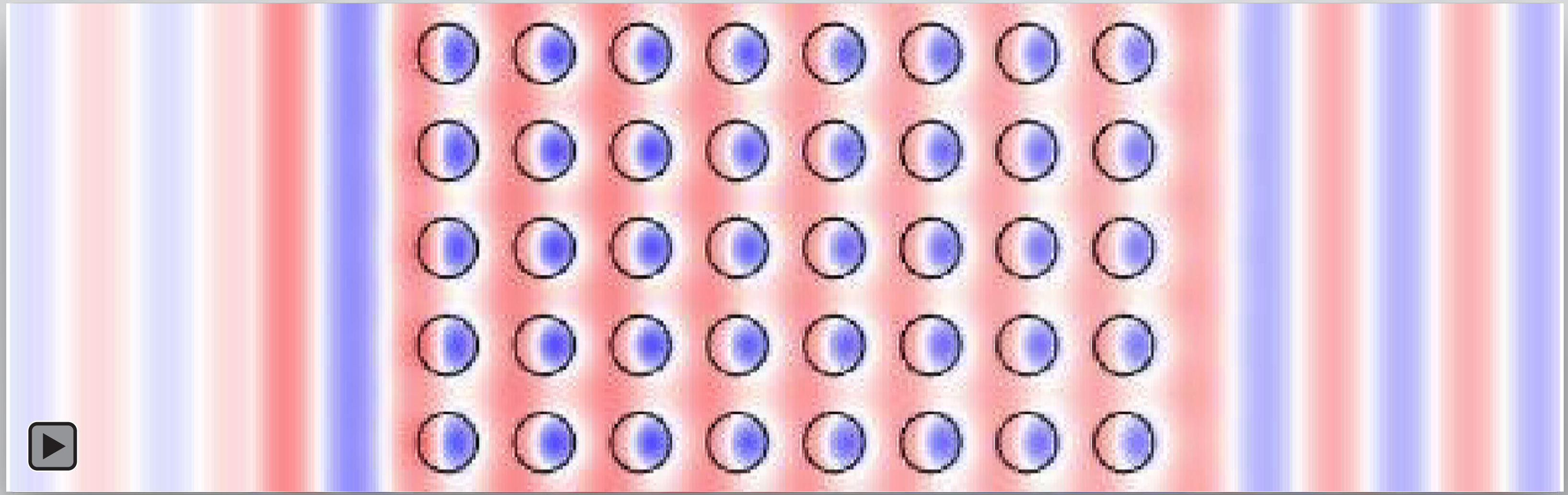
d = 422 nm, a = 690 nm, n = 1.57 (SU8)



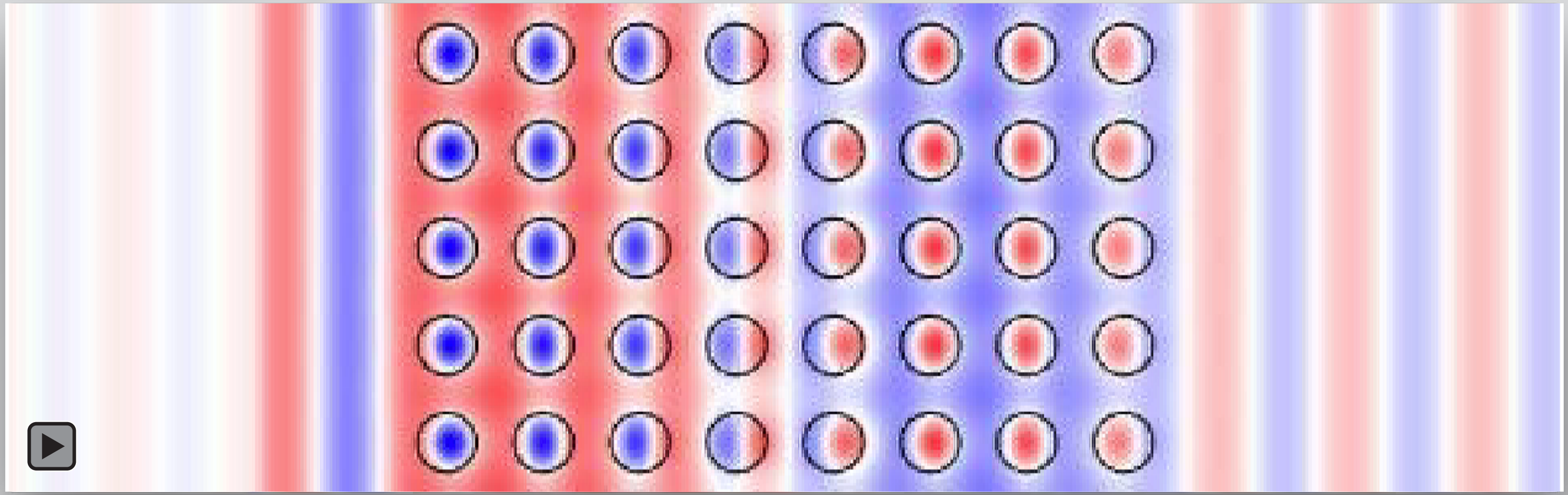
1 index 2 zero index 3 experiments


0.6
0.4
0.2
0
−0.2
−0.4
−0.6
refractive index
4
2
0
−2
−4
impedance
Re(Z_{eff})
Re(n_{eff})
1520
1560
1600
1640
1680
wavelength (nm)

at design wavelength (1590 nm)

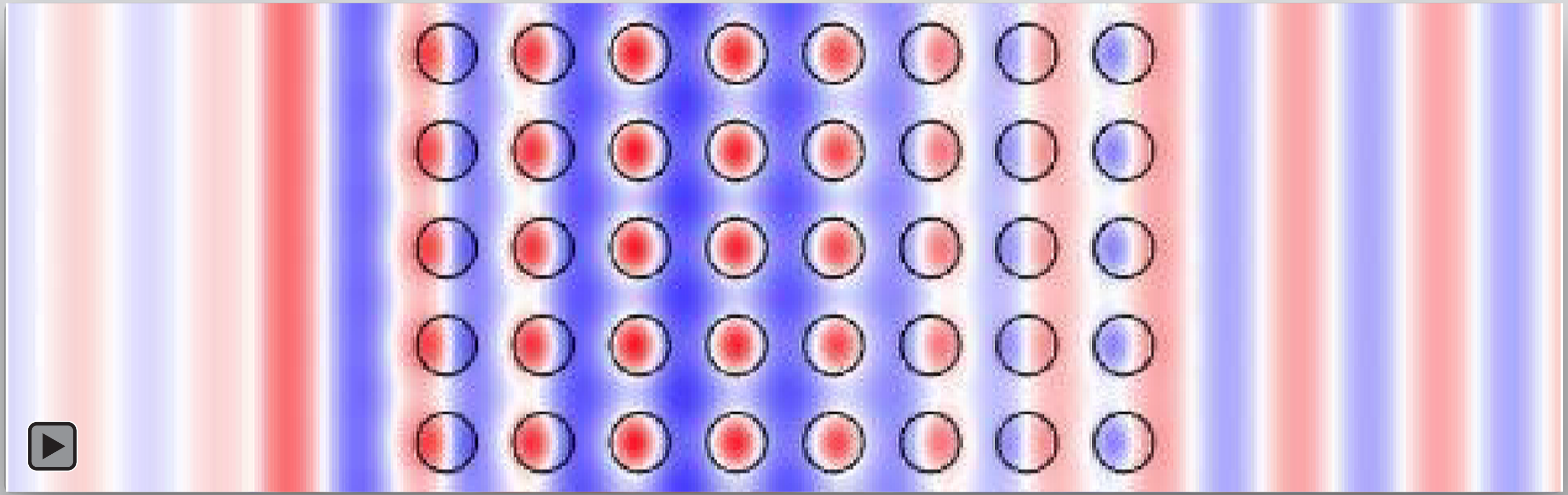


below design wavelength (1530 nm)



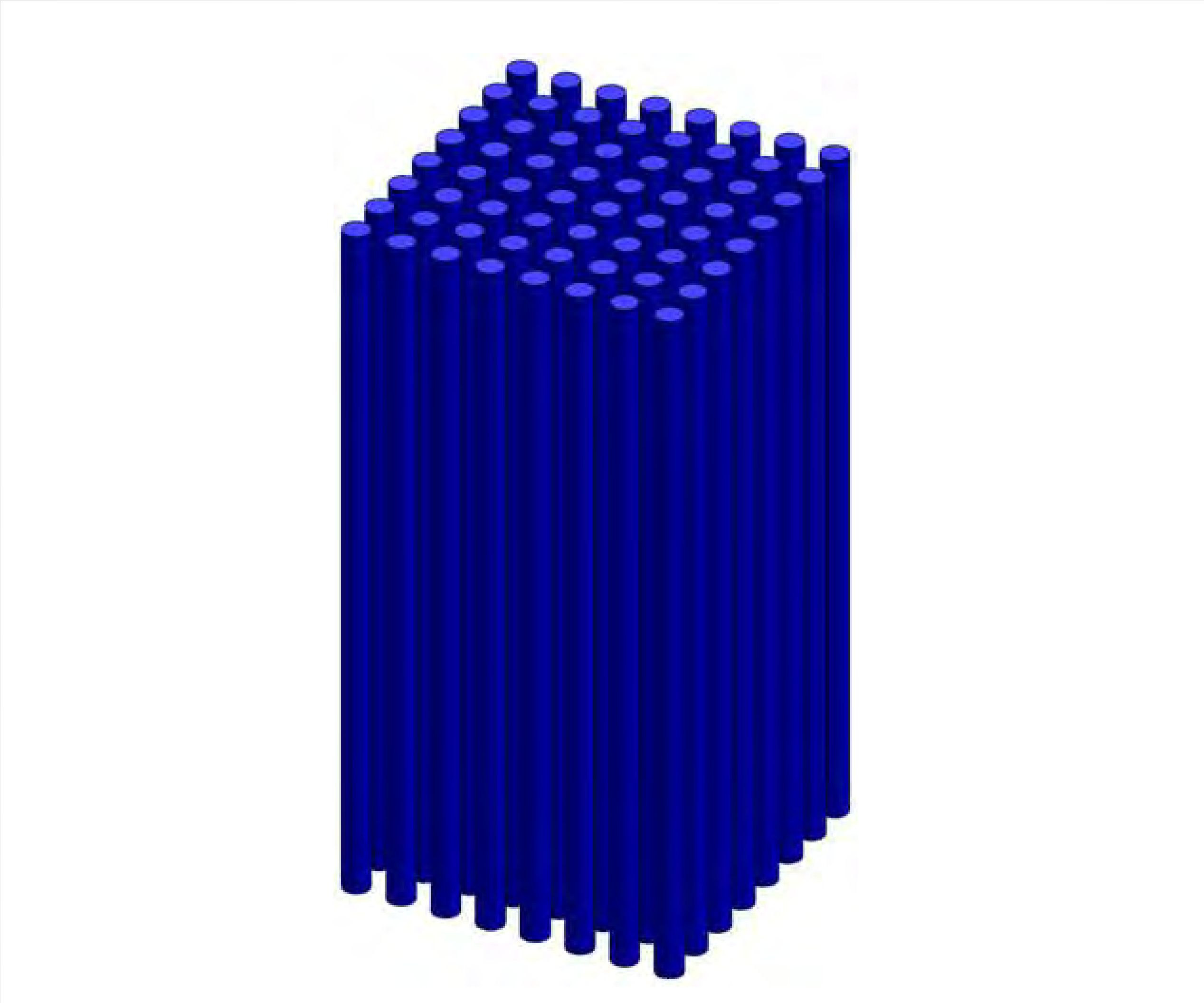
1 index   2 zero index   3 experiments

above design wavelength (1650 nm)



1 index   2 zero index   3 experiments

How to fabricate?



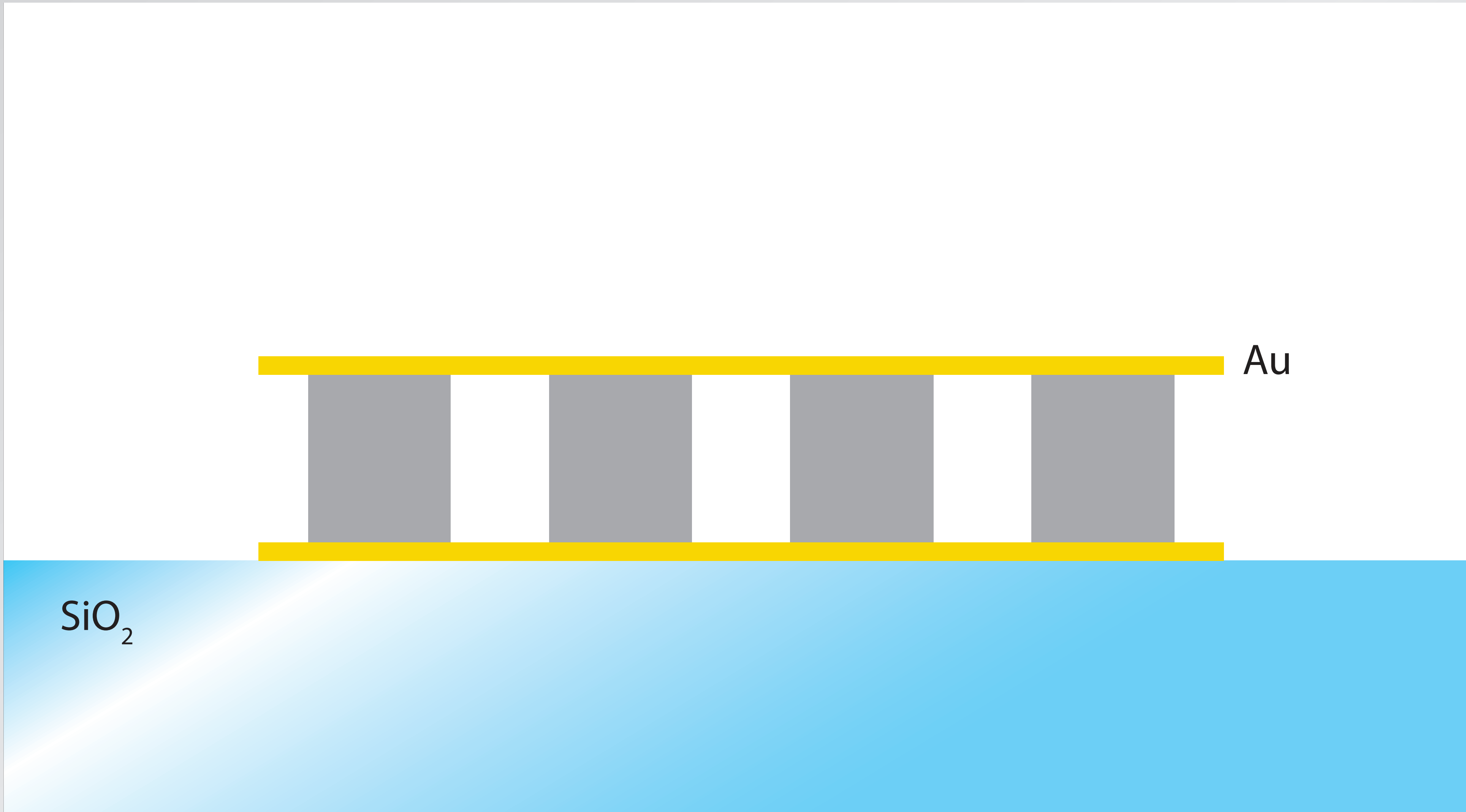
On-chip zero-index fabrication

1 index 2 zero index 3 experiments

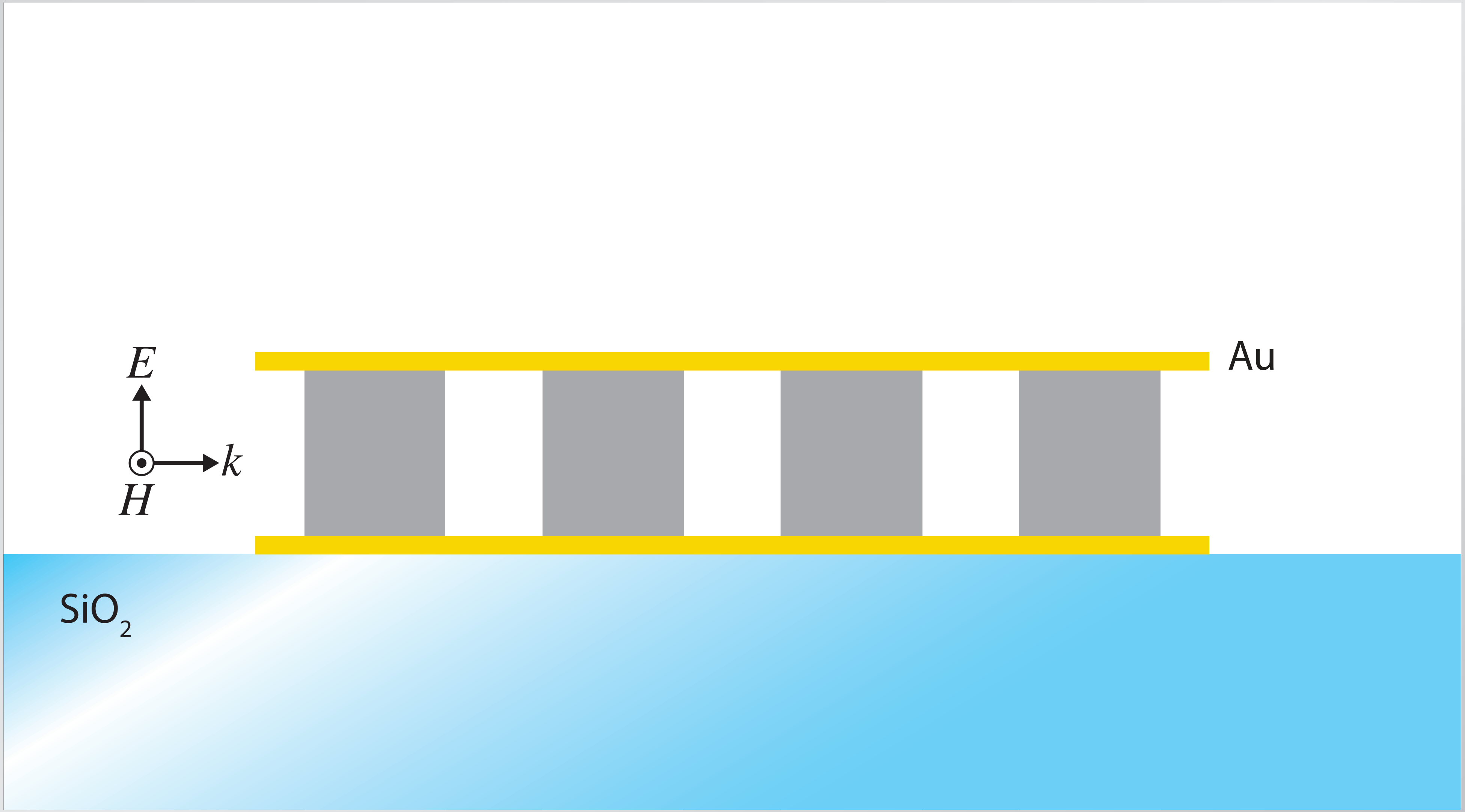
On-chip zero-index fabrication

On-chip zero-index fabrication

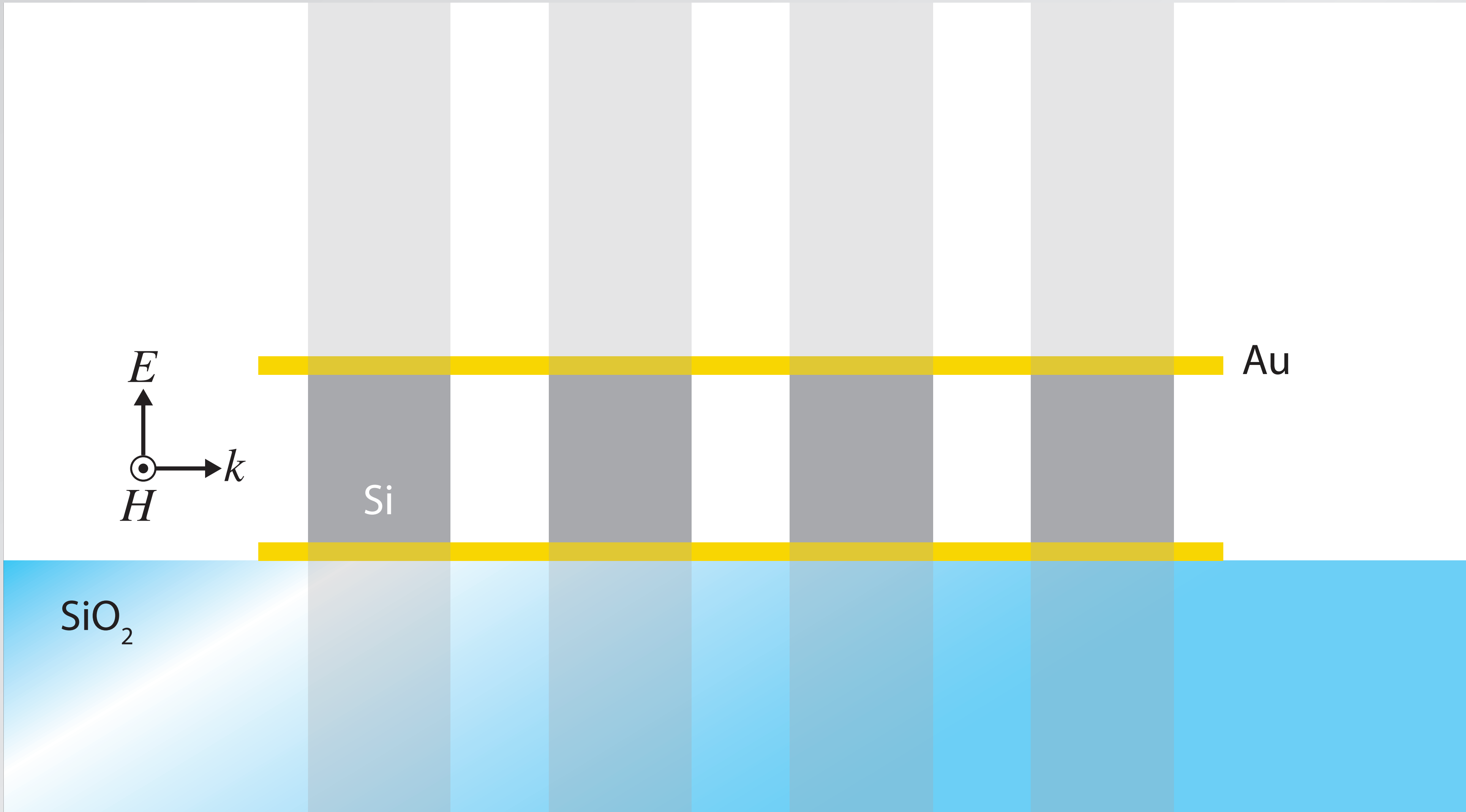
On-chip zero-index fabrication

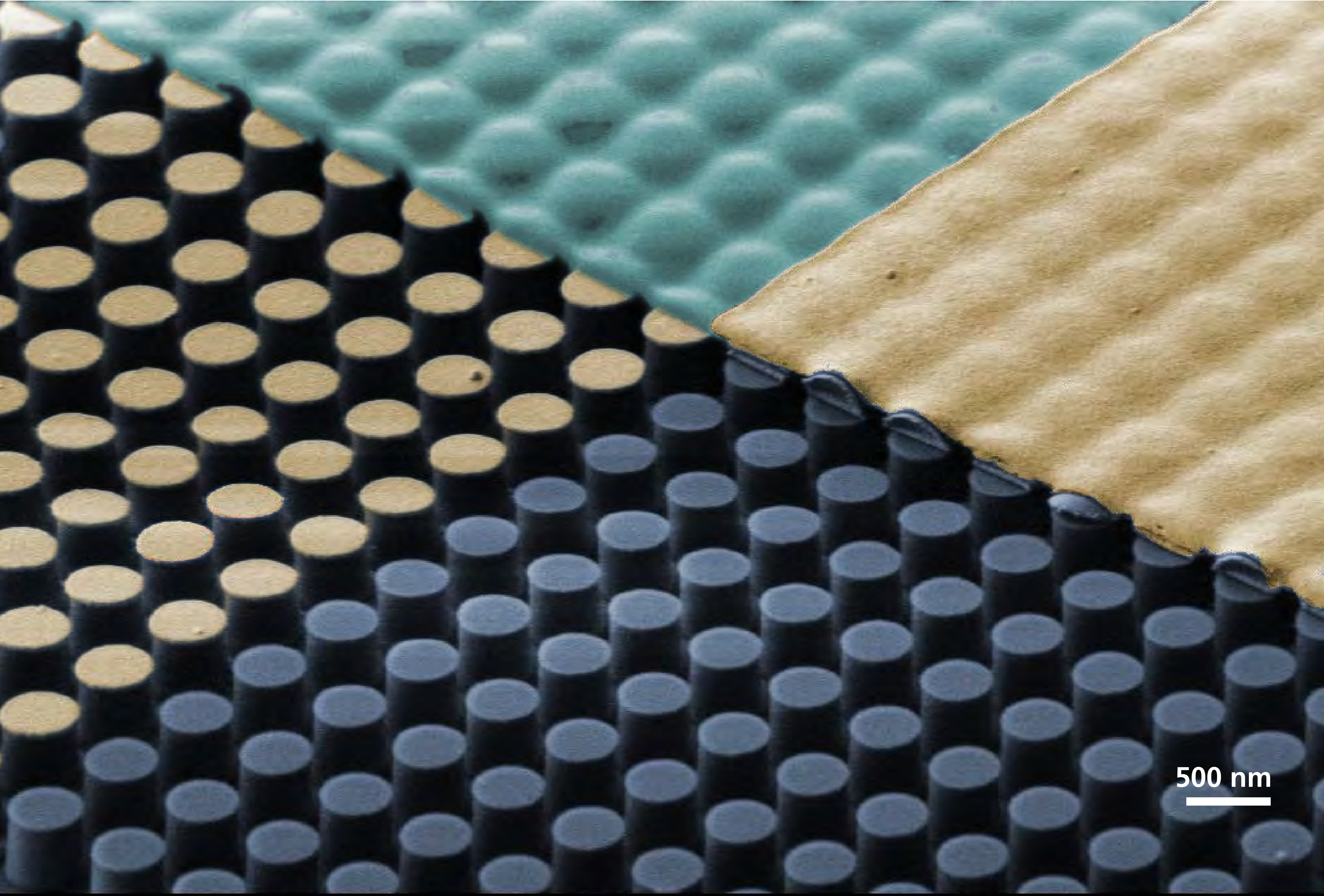


On-chip zero-index fabrication



On-chip zero-index fabrication

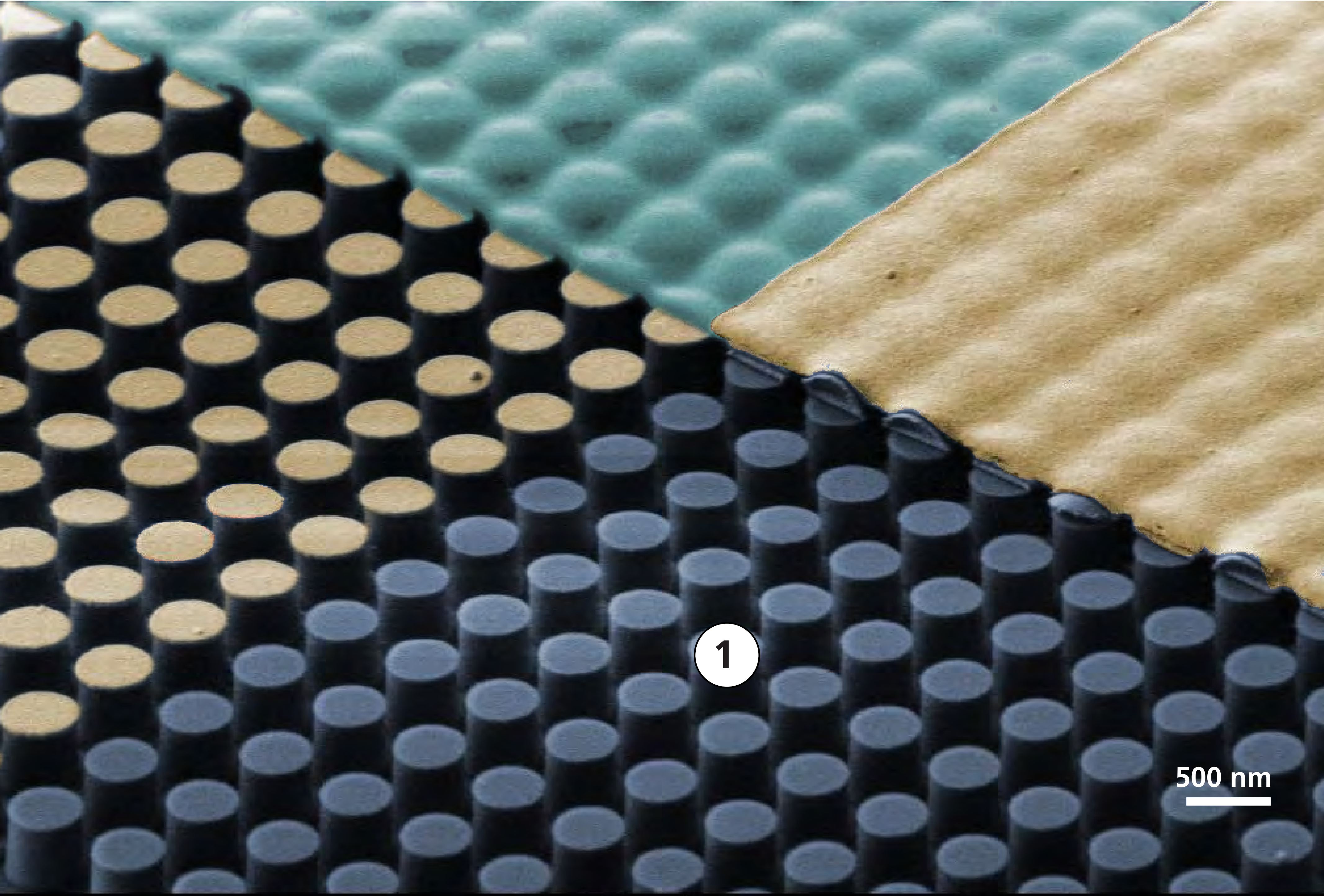




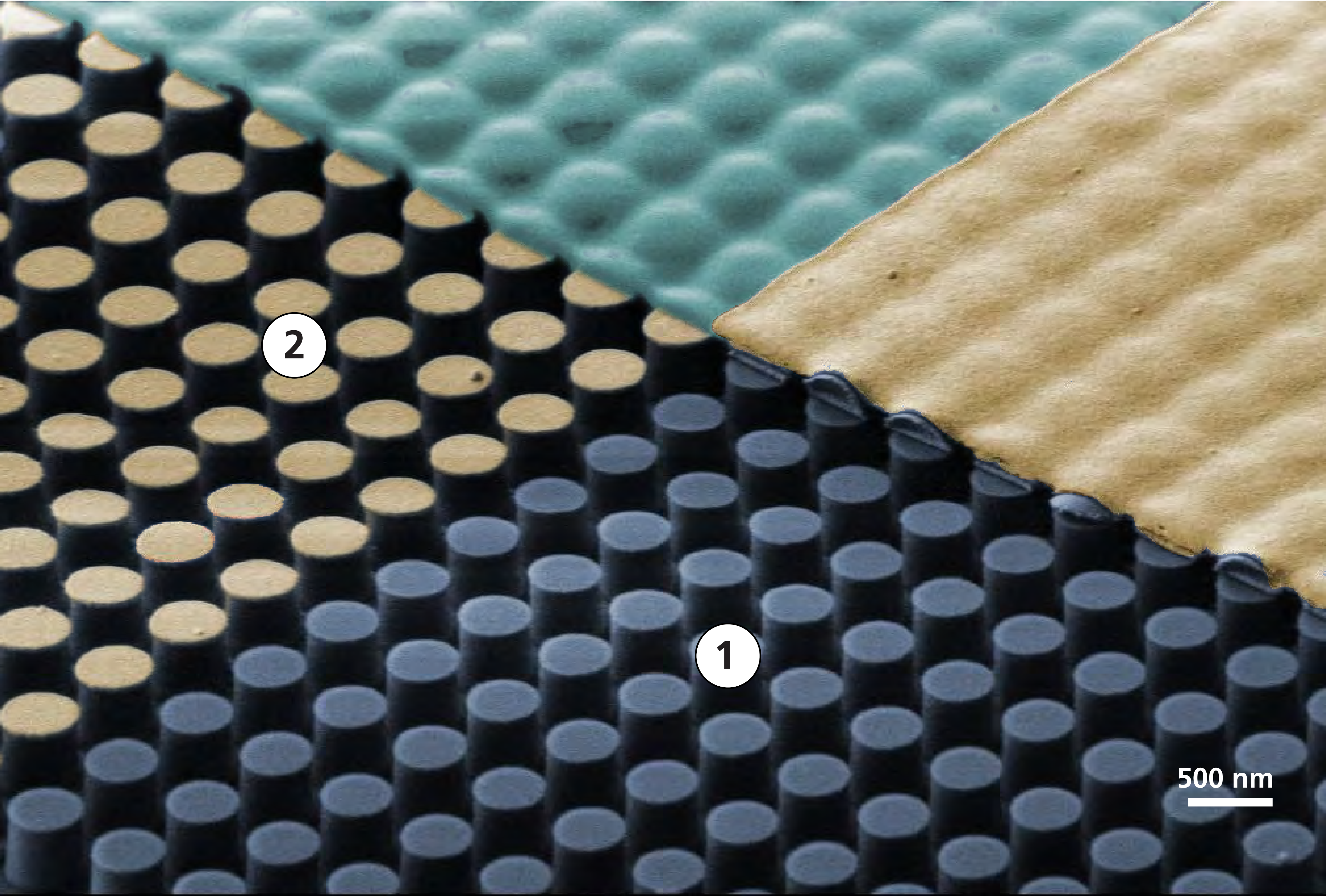
1 index

2 zero index

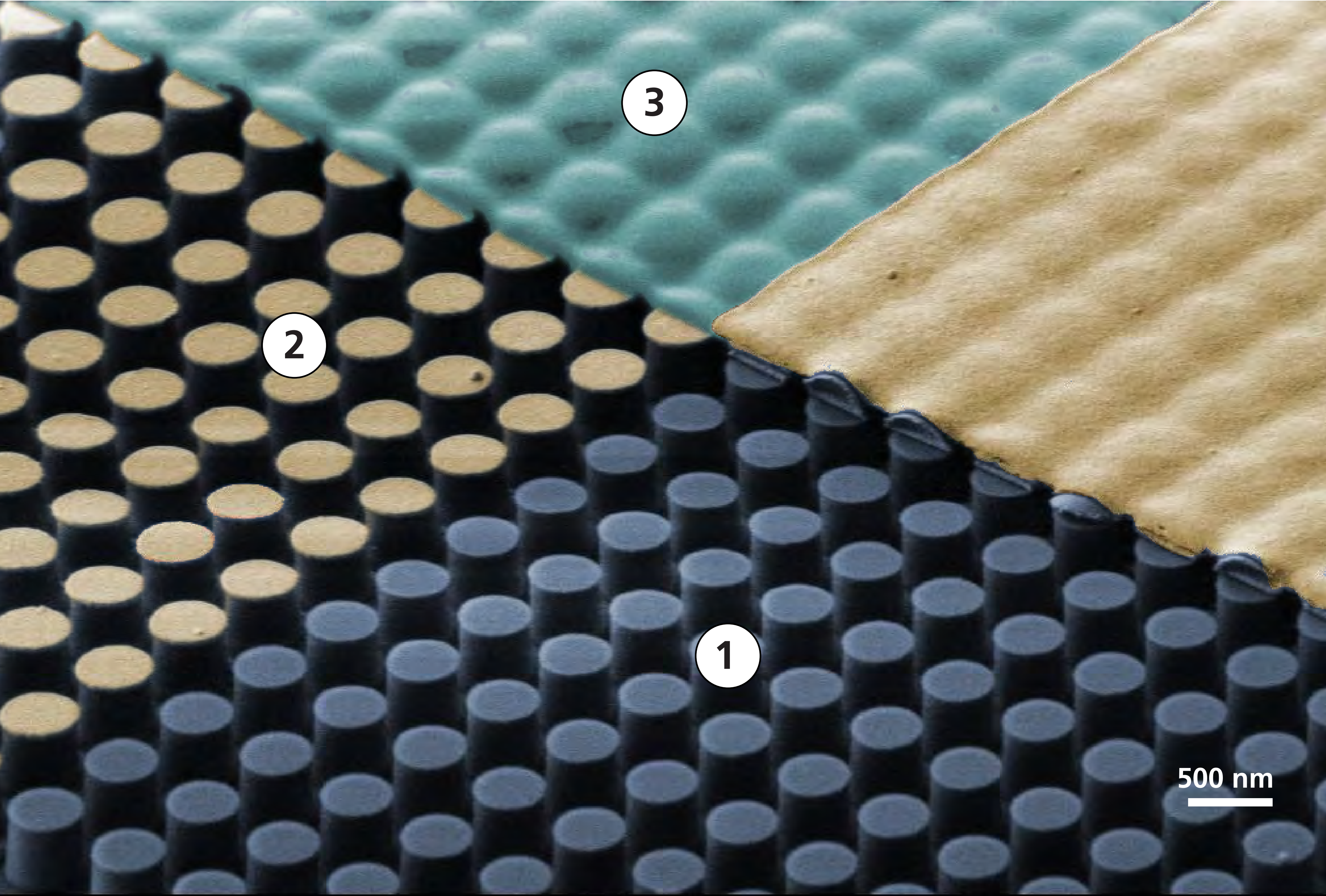
3 experiments



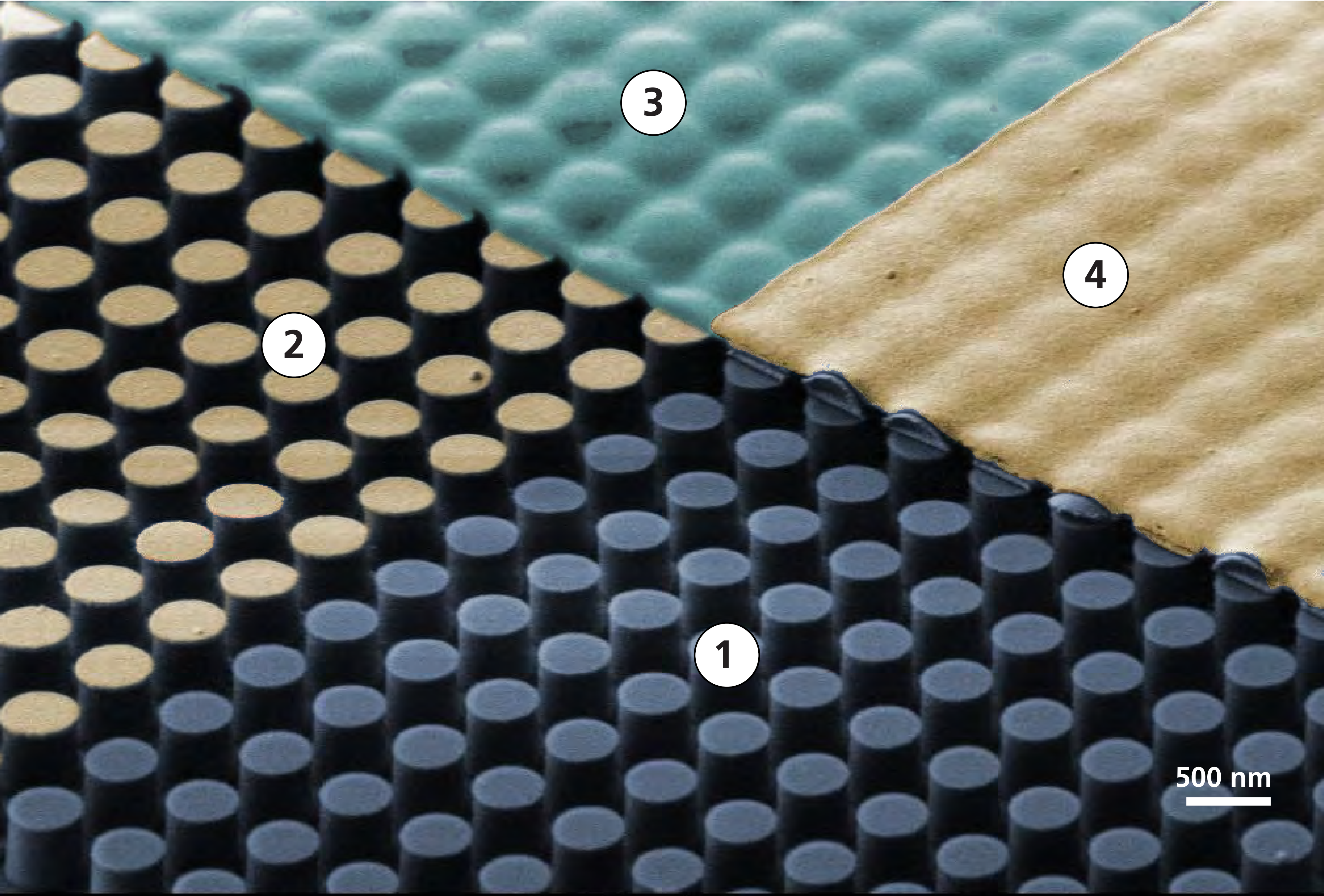
1 index   2 zero index   3 experiments



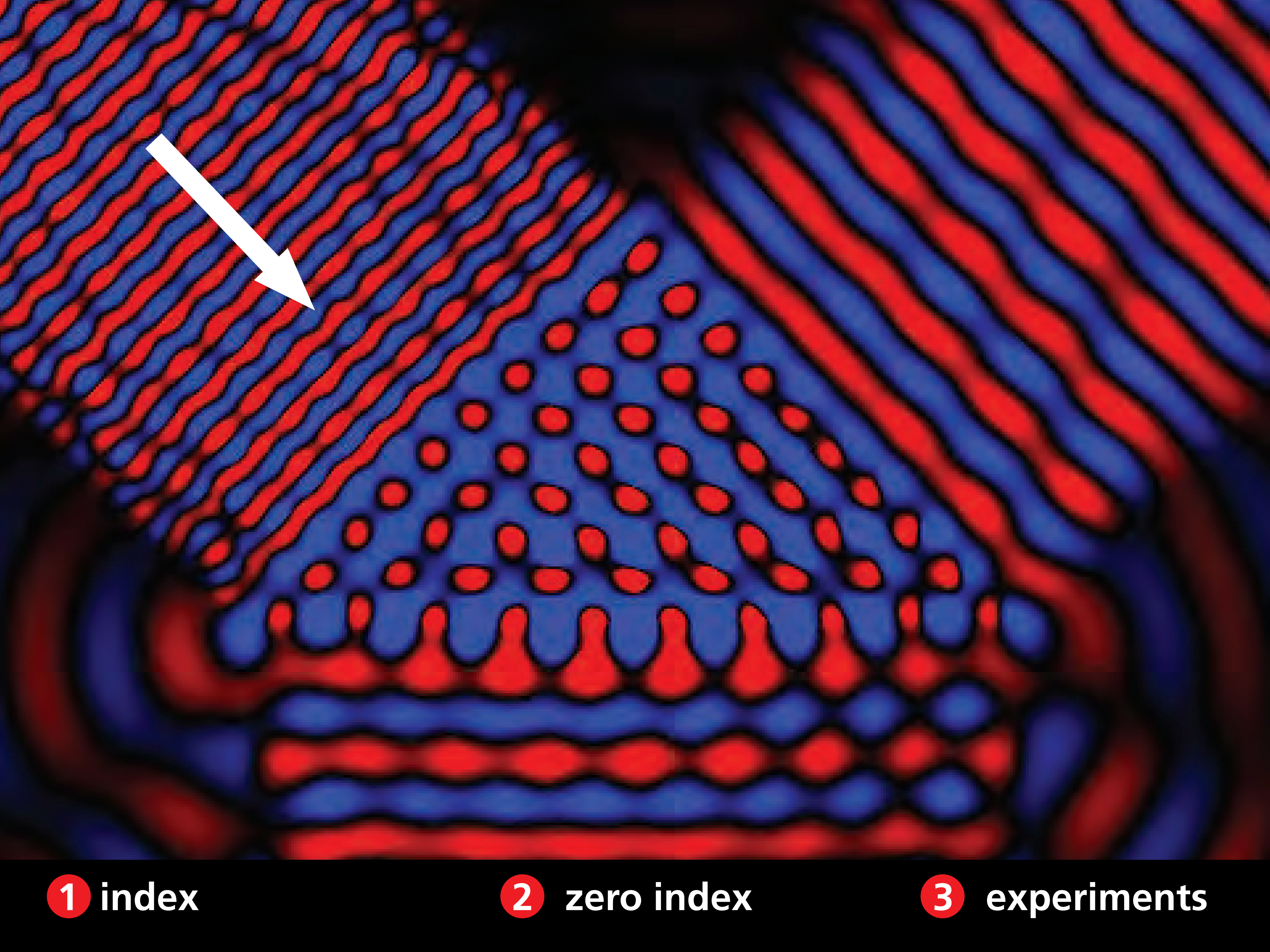
1 index 2 zero index 3 experiments



1 index 2 zero index 3 experiments

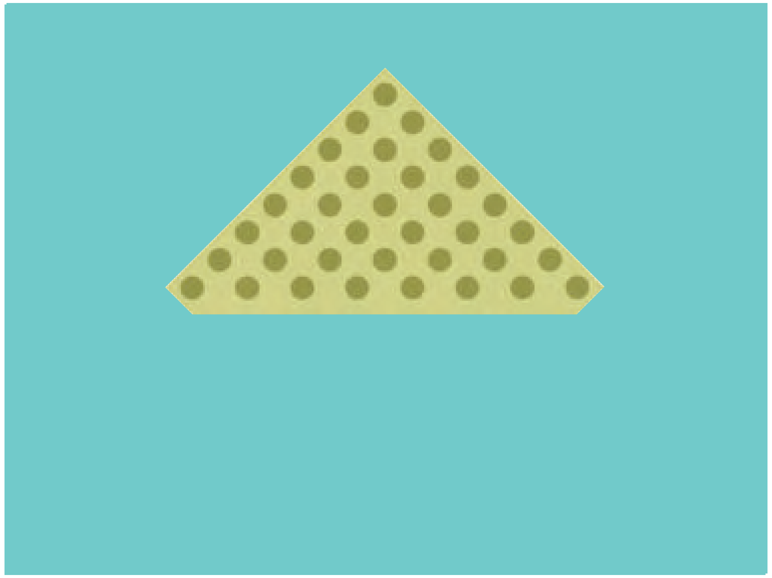


1 index 2 zero index 3 experiments


1 index
2 zero index
3 experiments

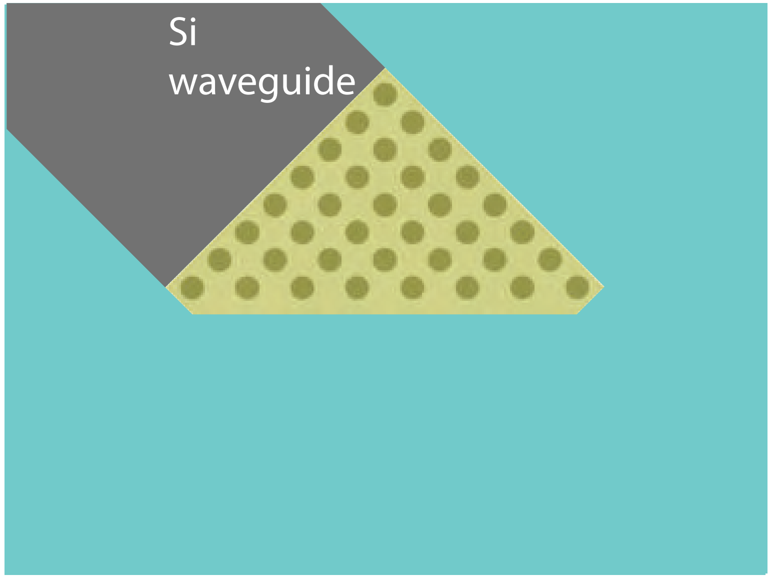
On-chip zero-index prism

On-chip zero-index prism



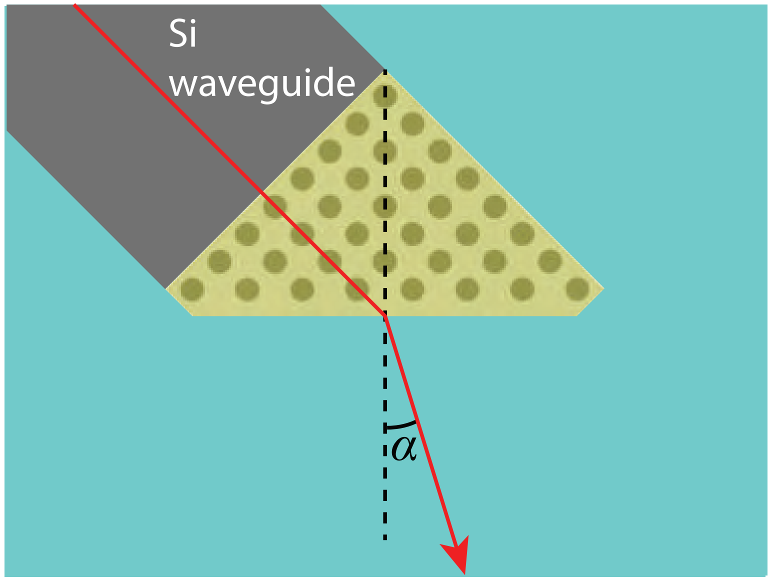
1 index 2 zero index 3 experiments

On-chip zero-index prism



1 index　　2 zero index　　3 experiments

On-chip zero-index prism

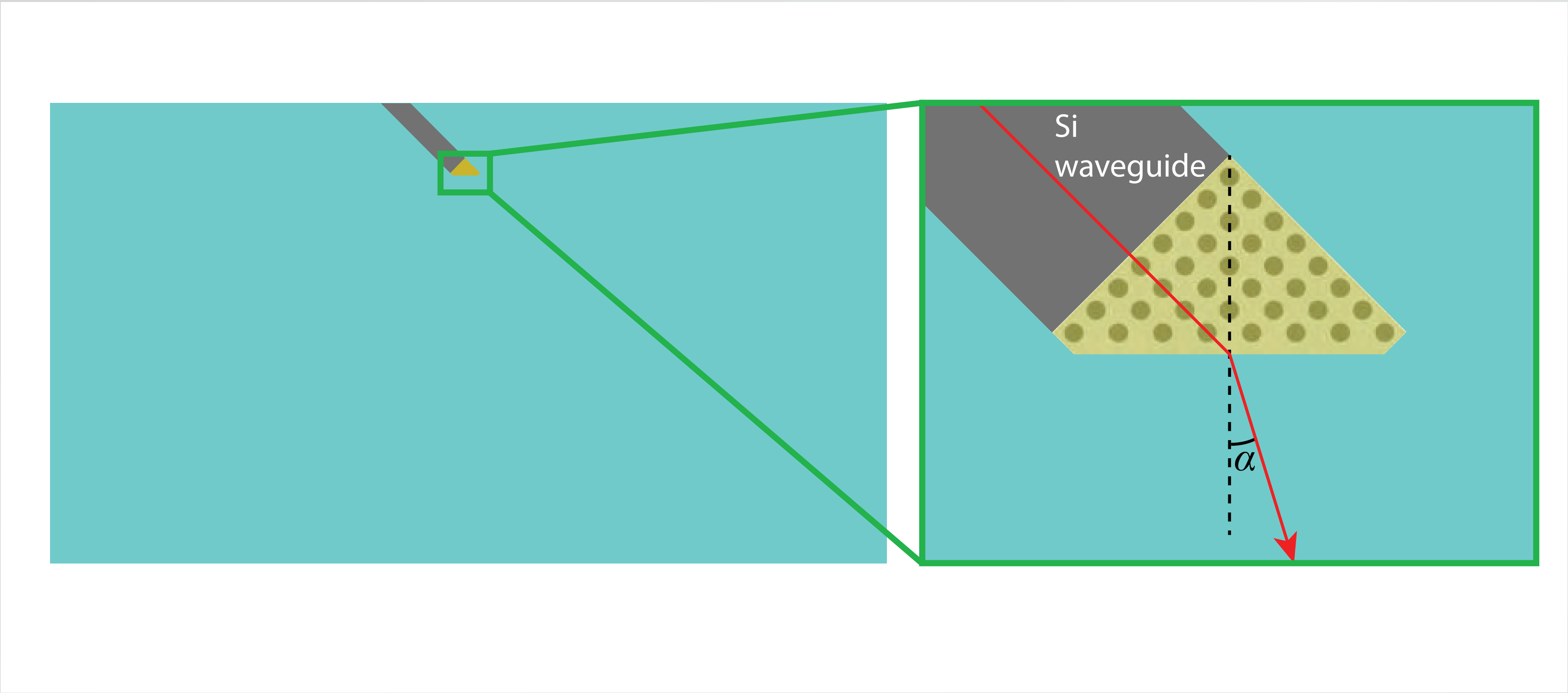


1 index

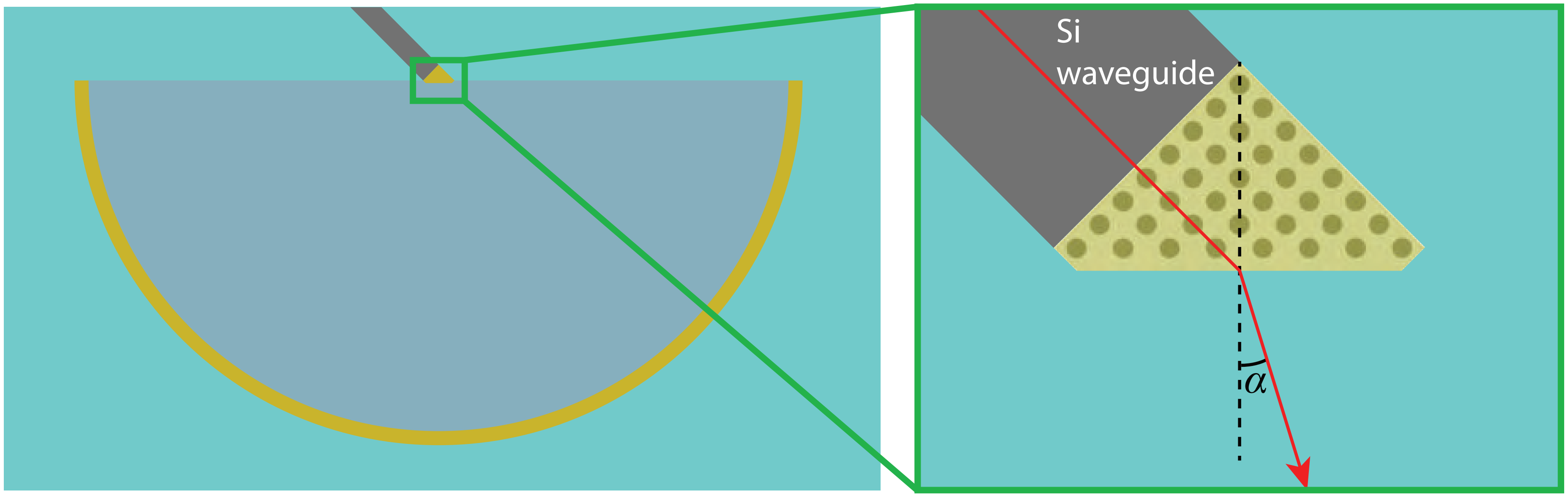
2 zero index

3 experiments

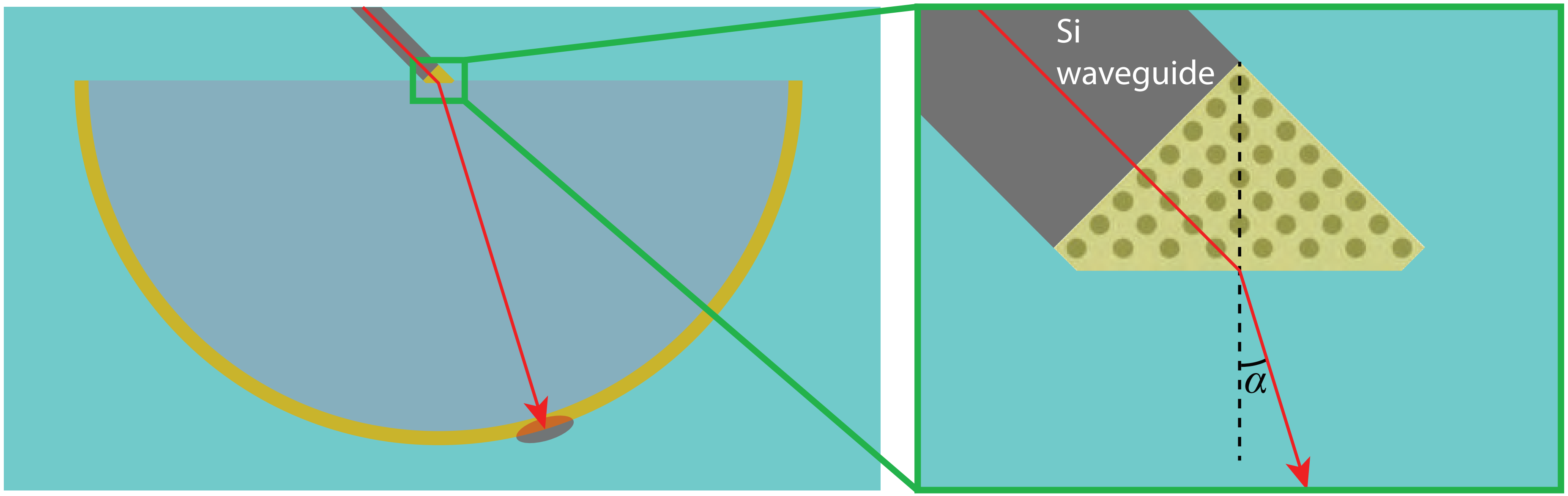
On-chip zero-index prism



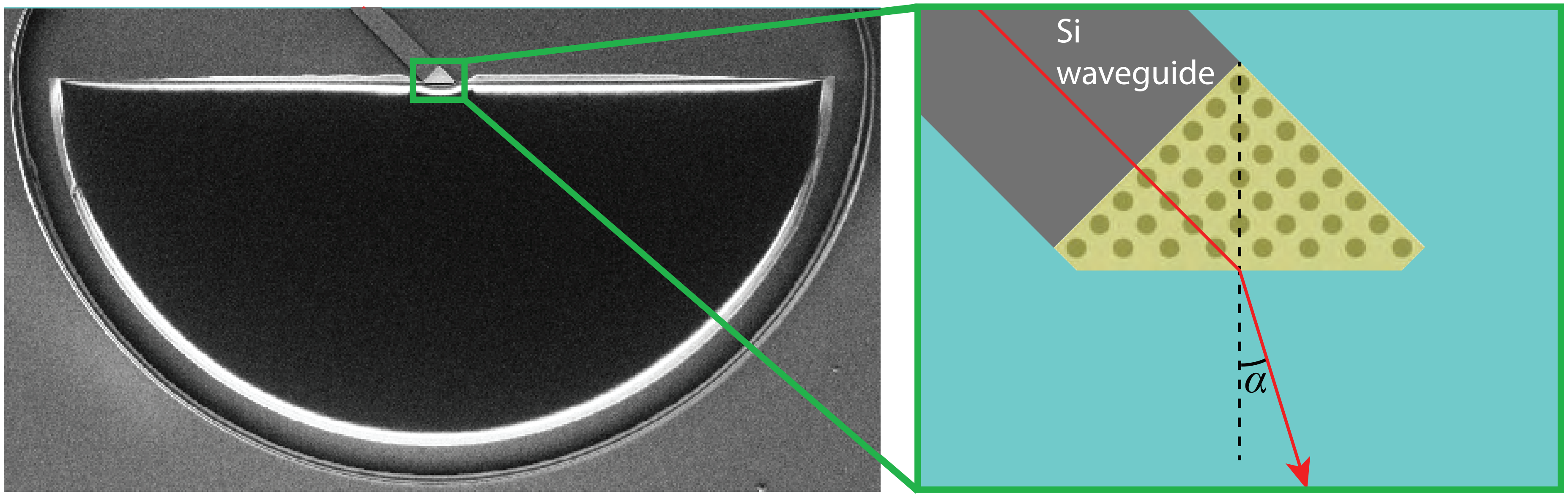
On-chip zero-index prism



On-chip zero-index prism



On-chip zero-index prism

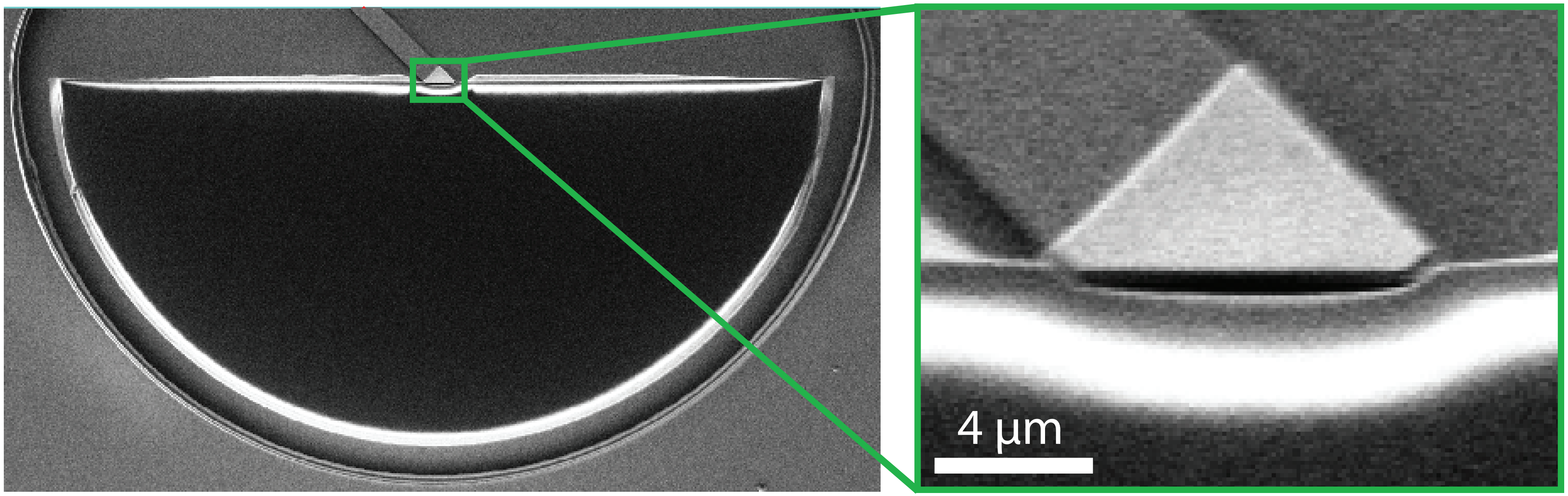


1 index

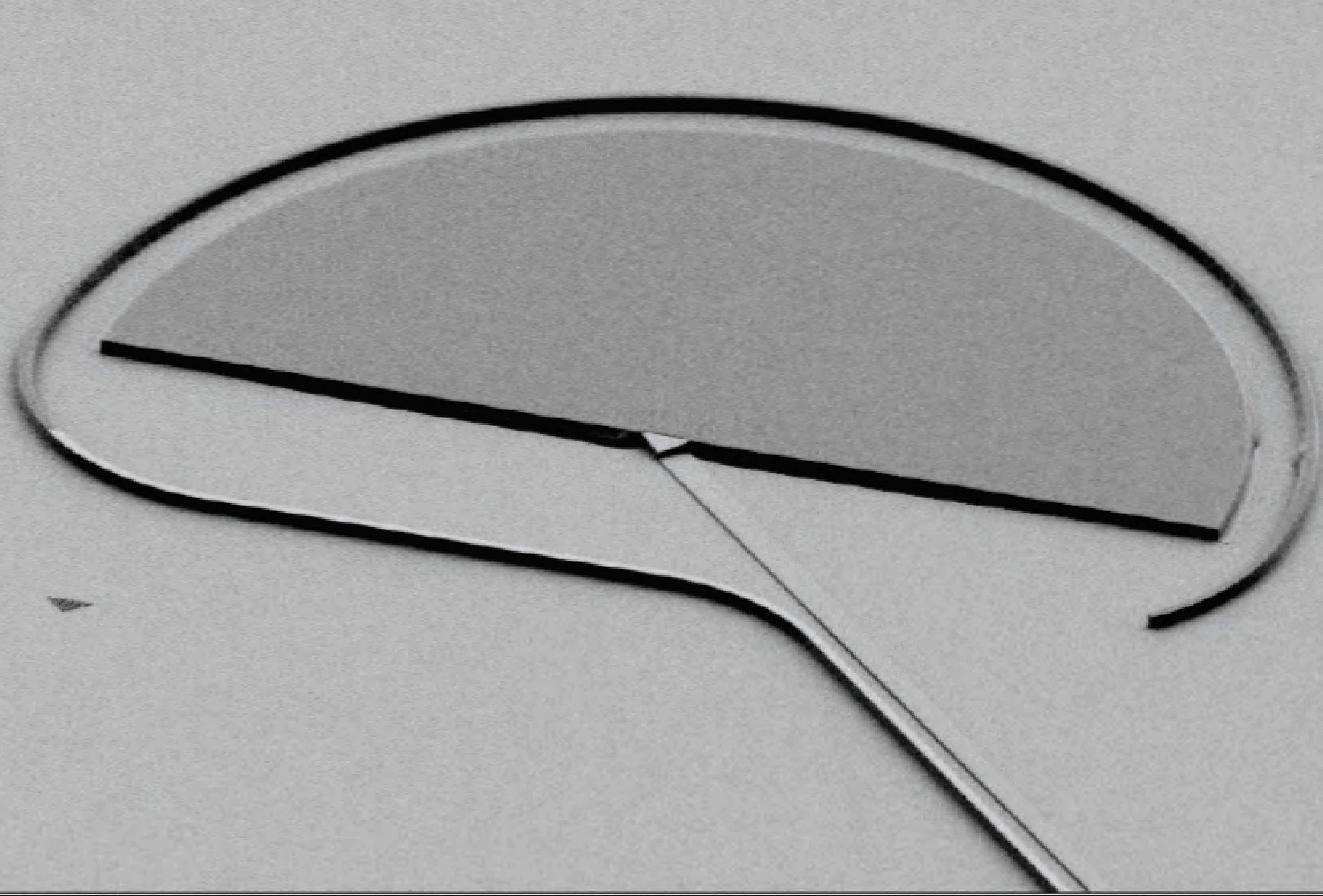
2 zero index

3 experiments

On-chip zero-index prism



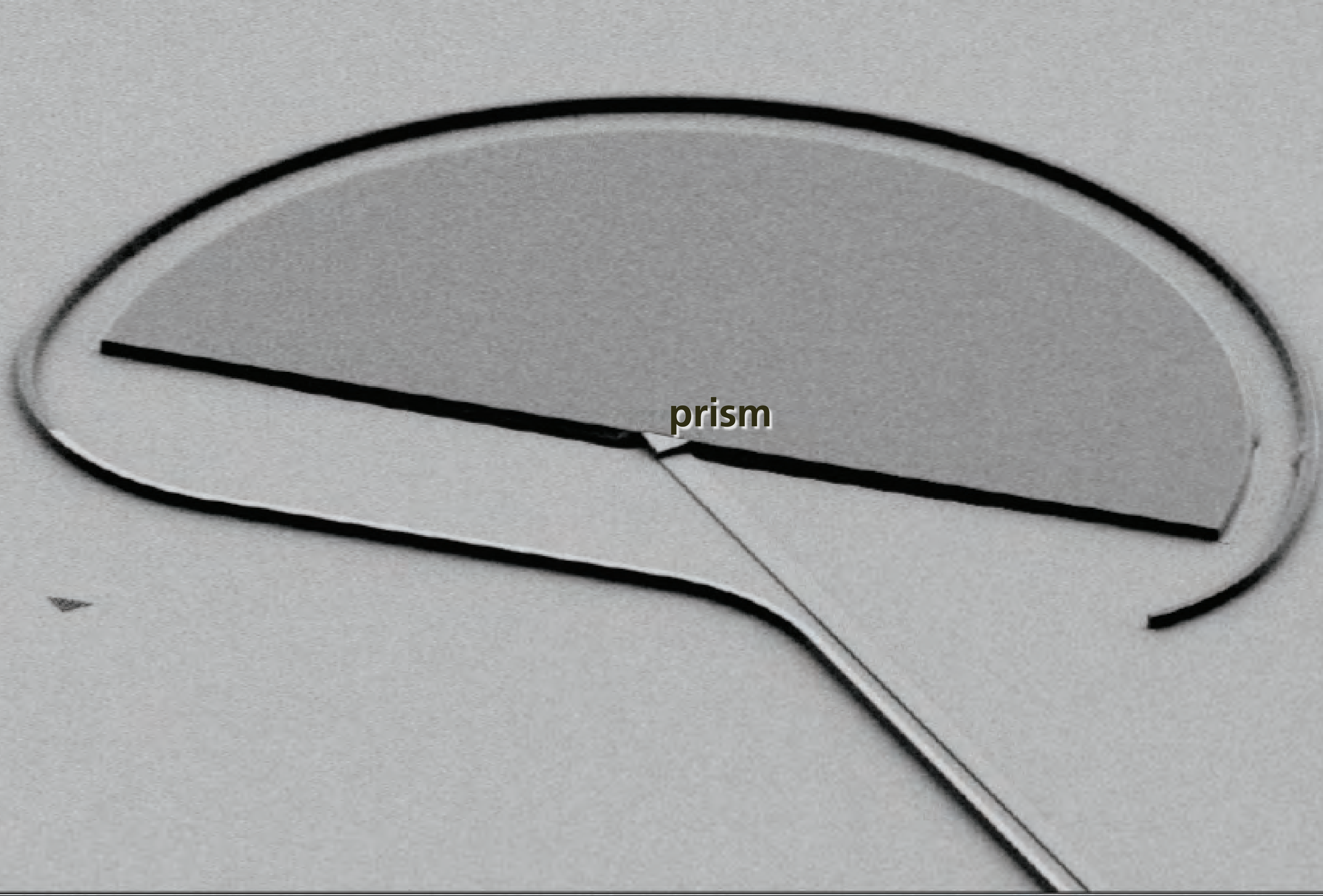
1 index 2 zero index 3 experiments

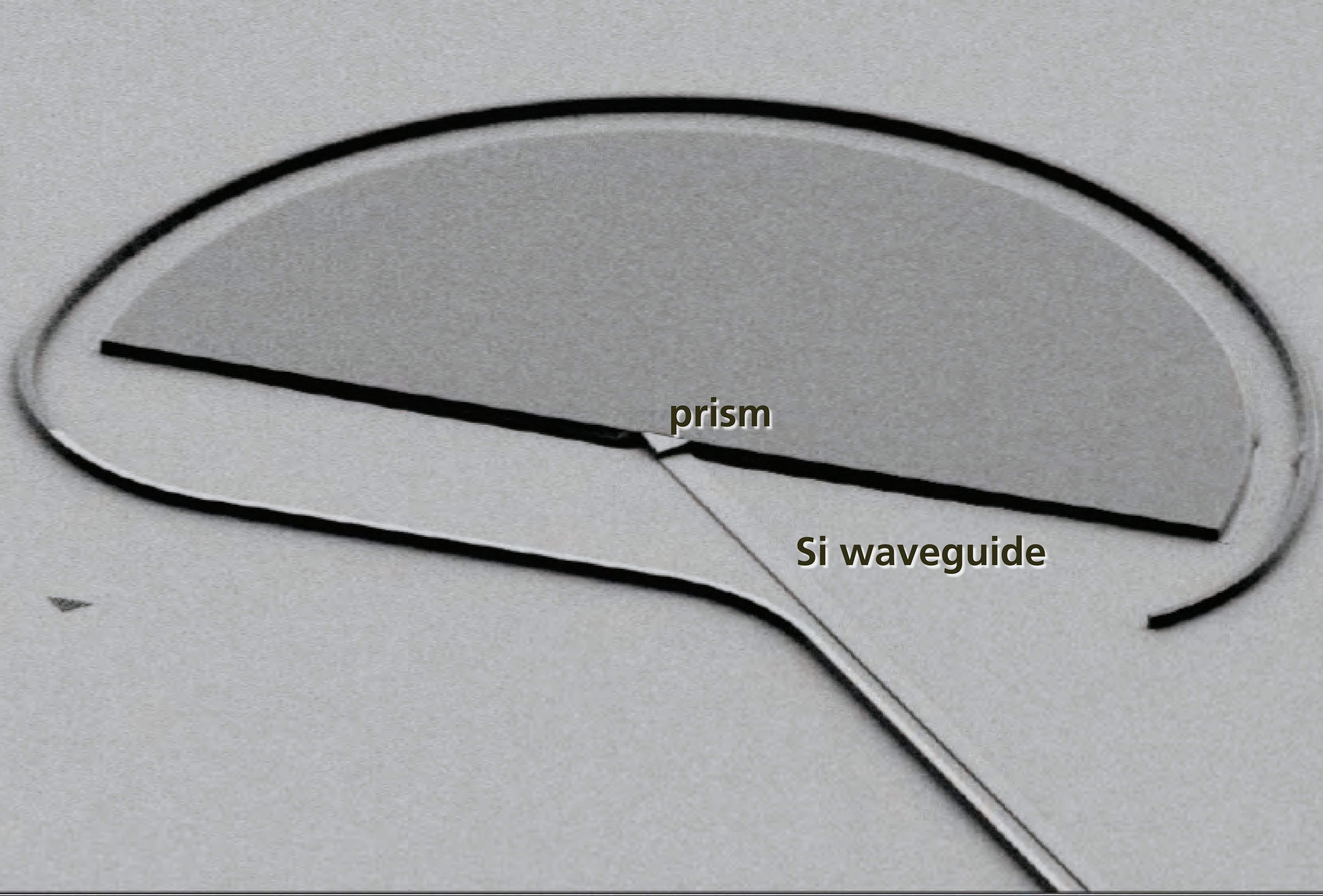


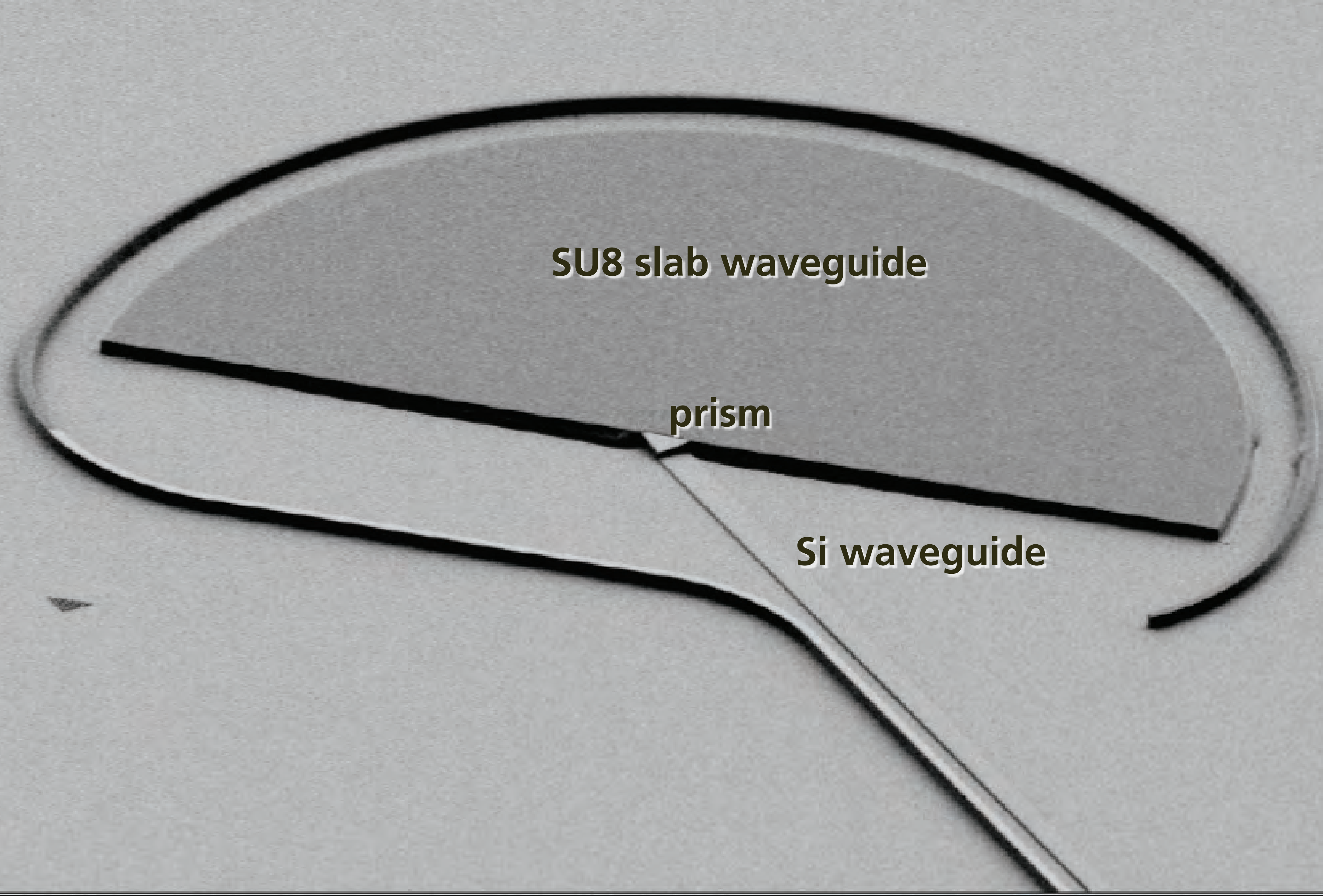
1 index

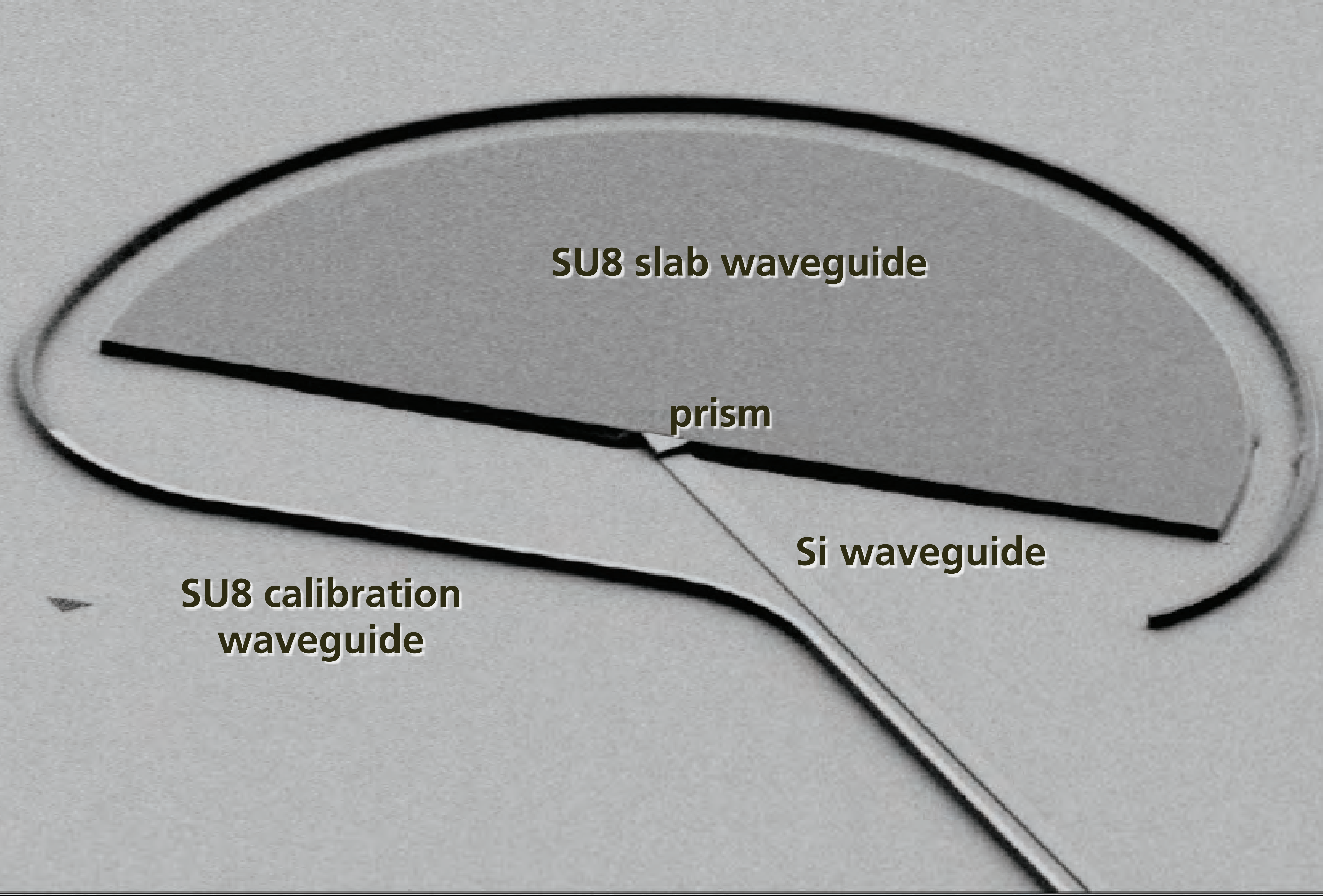
2 zero index

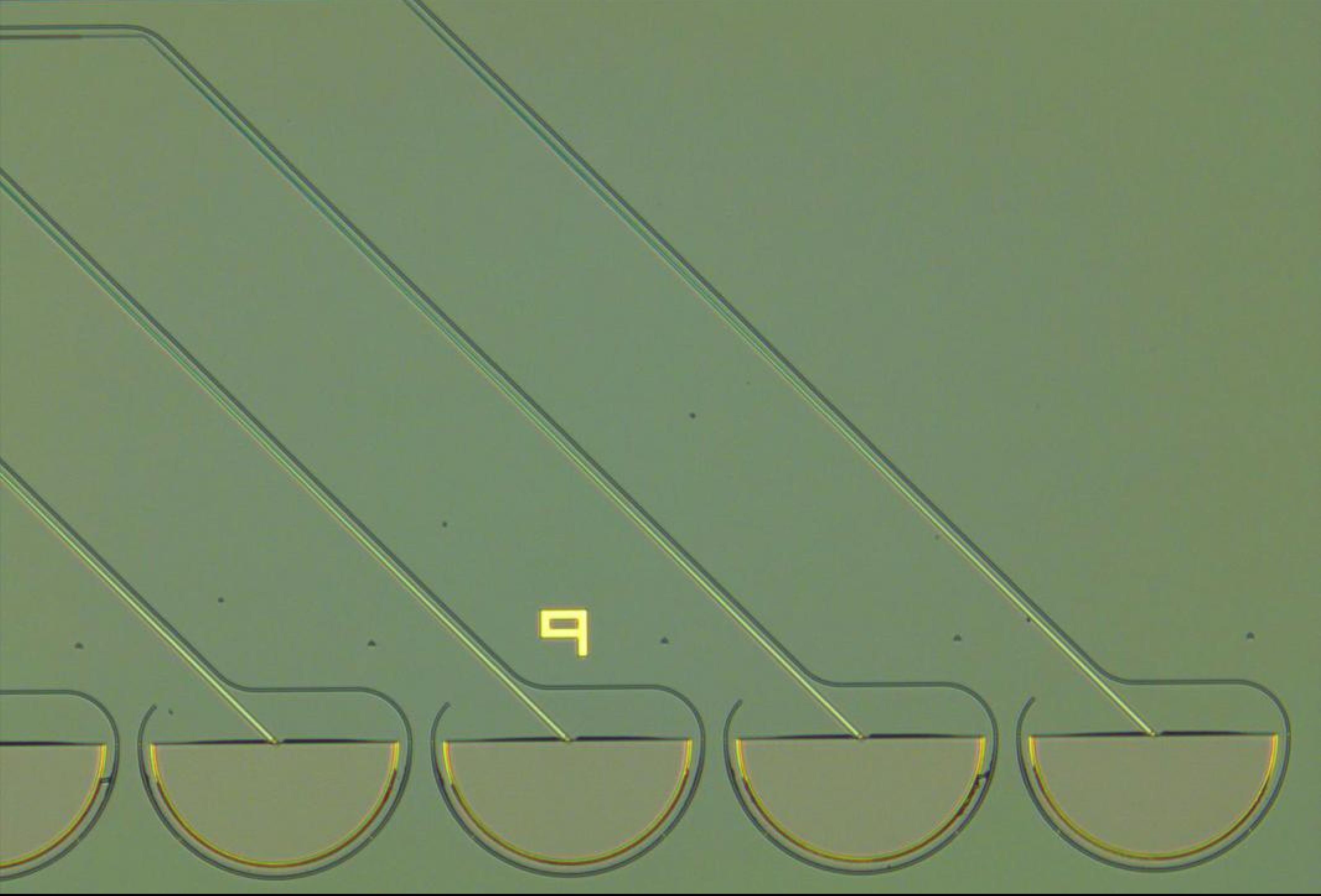
3 experiments


prism


prism
Si waveguide

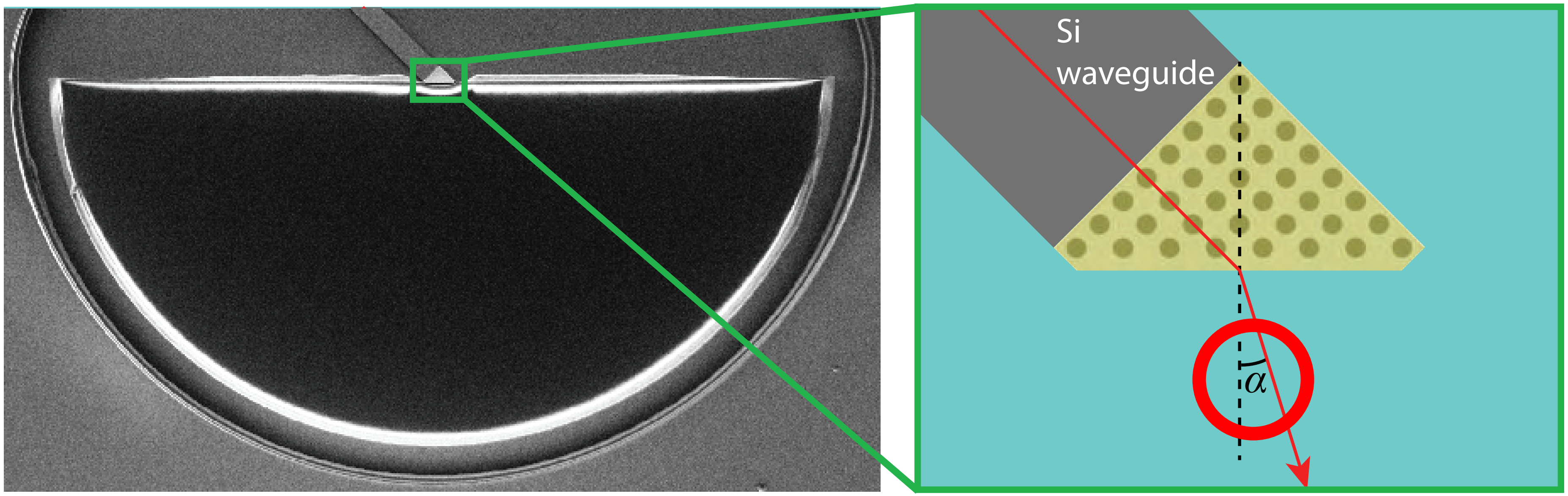

SU8 slab waveguide
prism
Si waveguide


SU8 slab waveguide
prism
Si waveguide
SU8 calibration waveguide



1 index　　2 zero index　　3 experiments

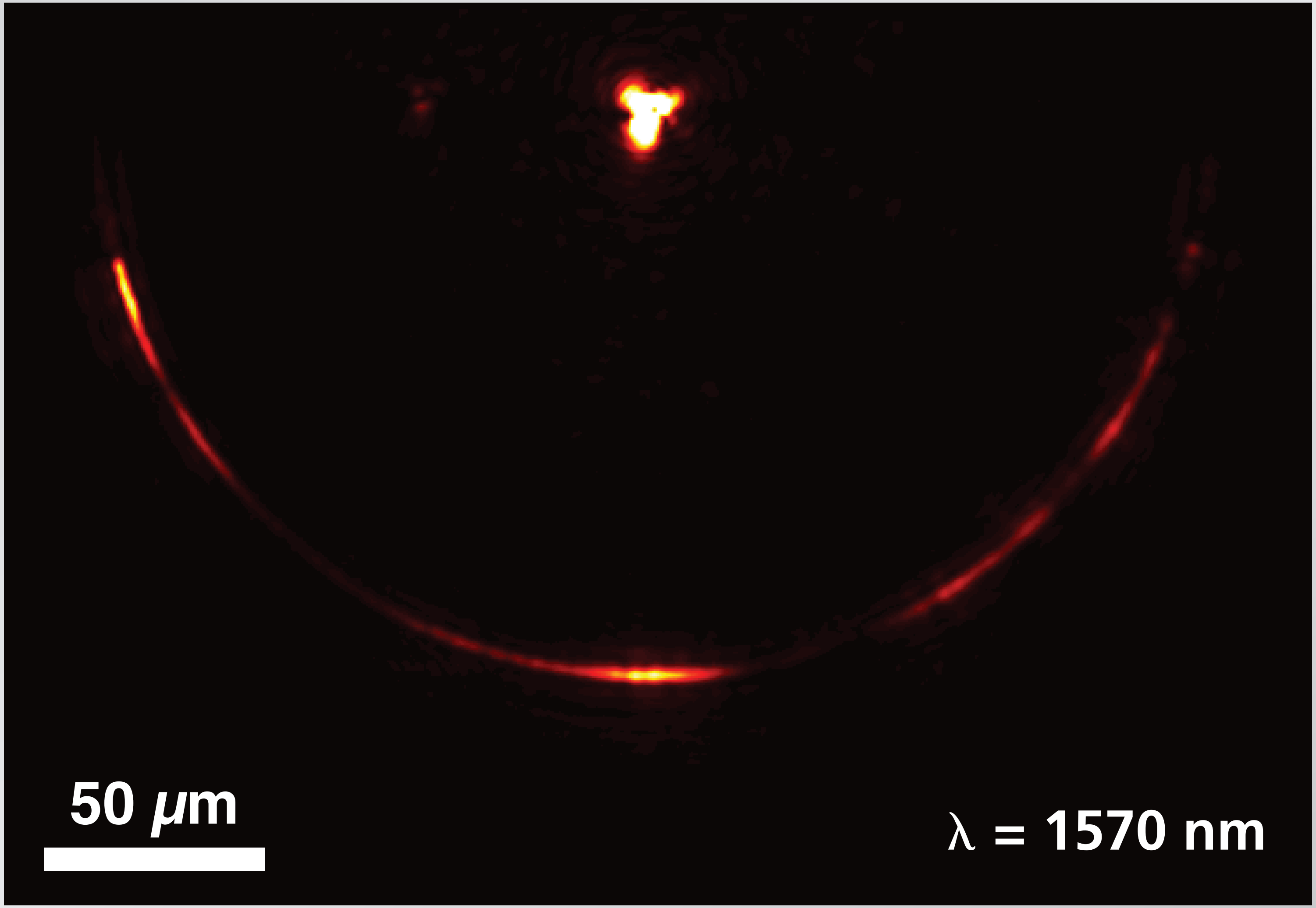
On-chip zero-index prism



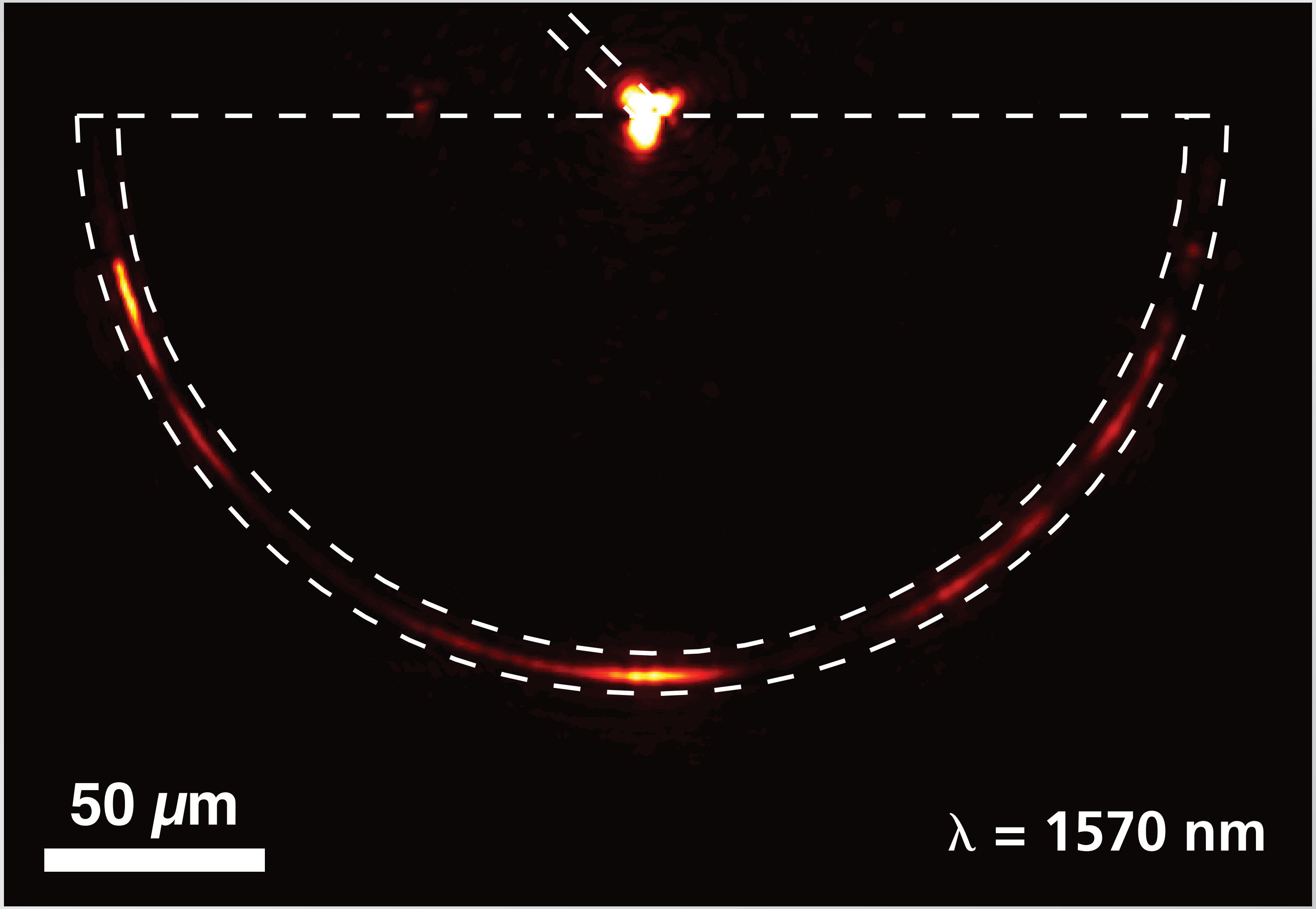
1 index

2 zero index

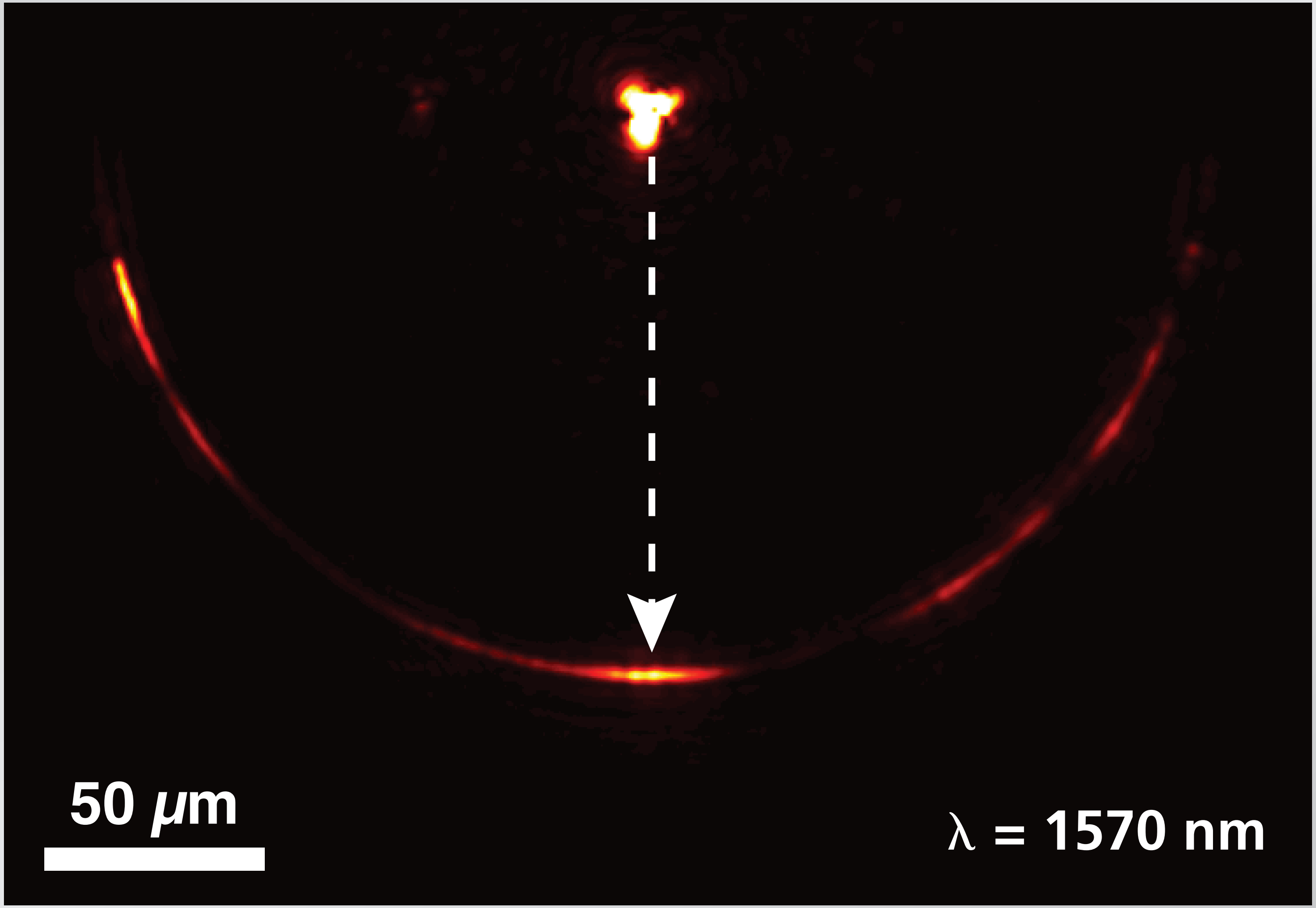
3 experiments



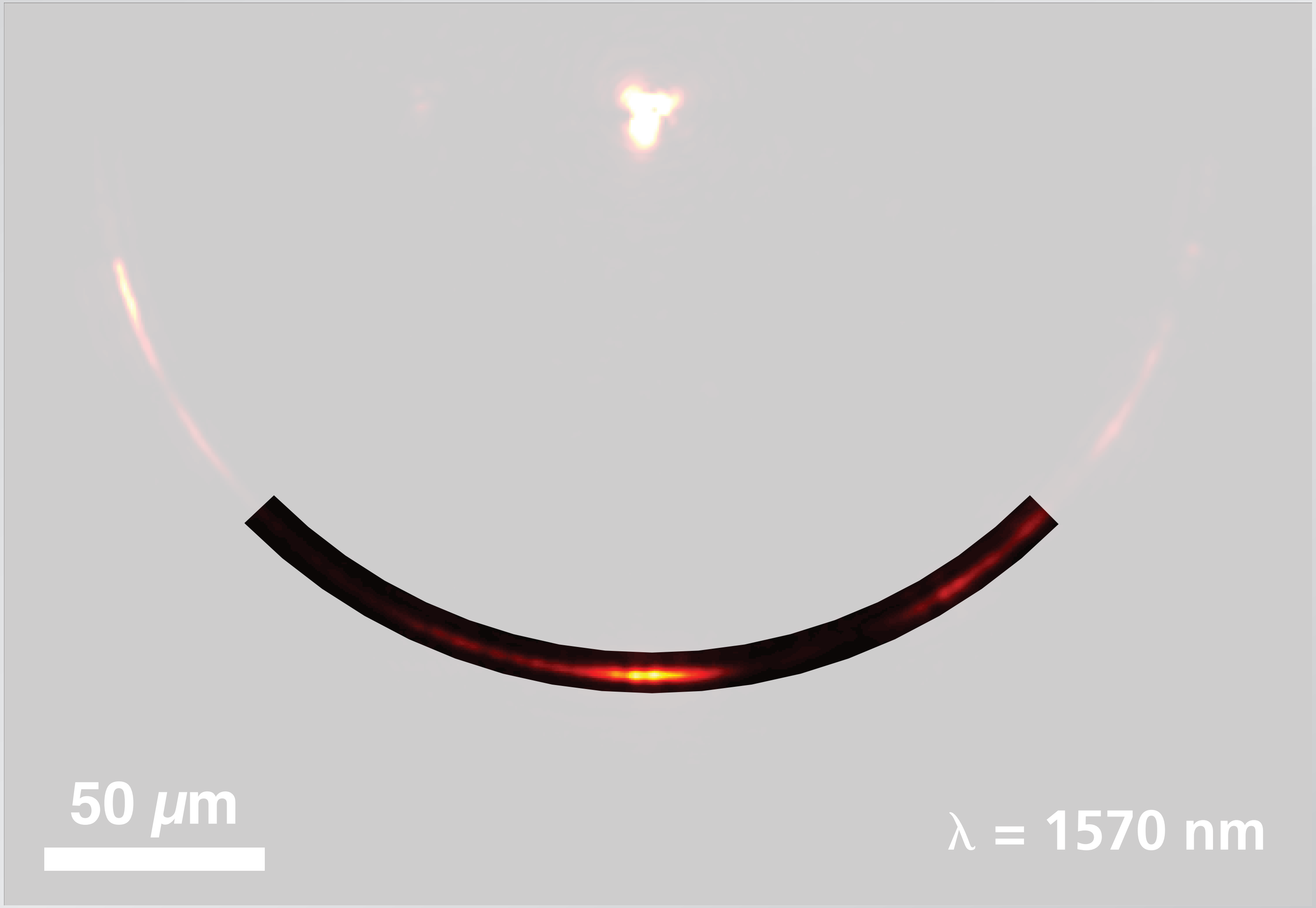
1 index 2 zero index 3 experiments



1 index 2 zero index 3 experiments



50 μm
λ = 1570 nm

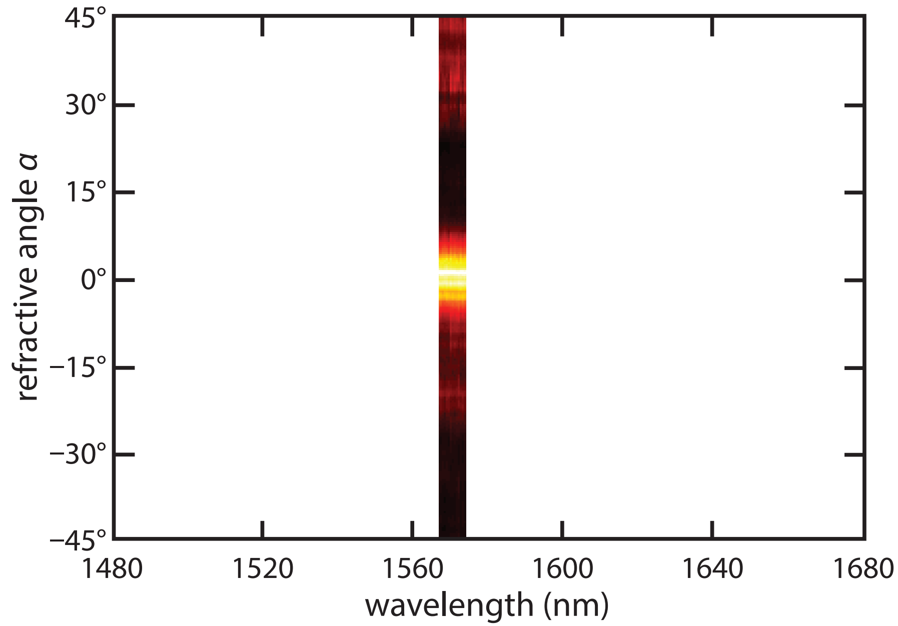


1 index

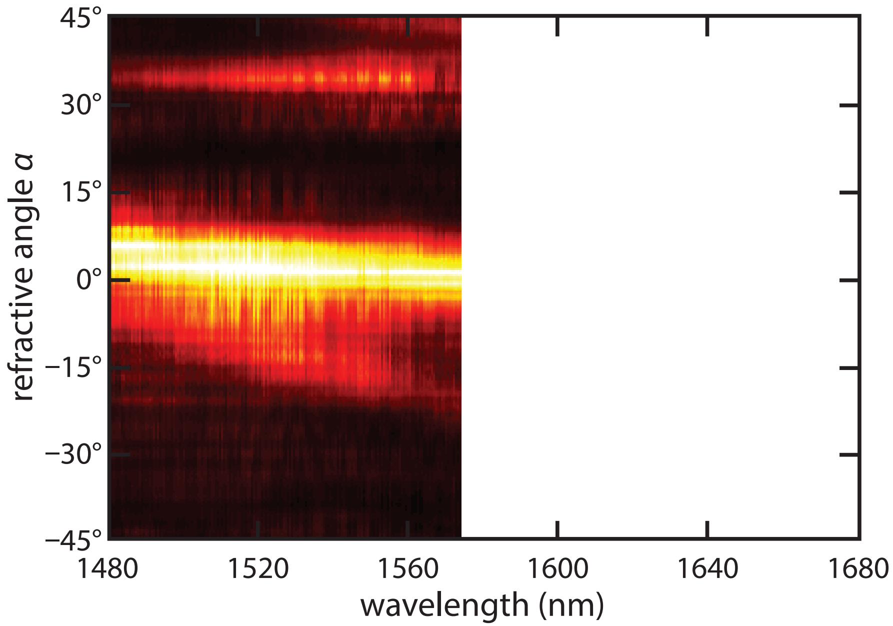
2 zero index

3 experiments

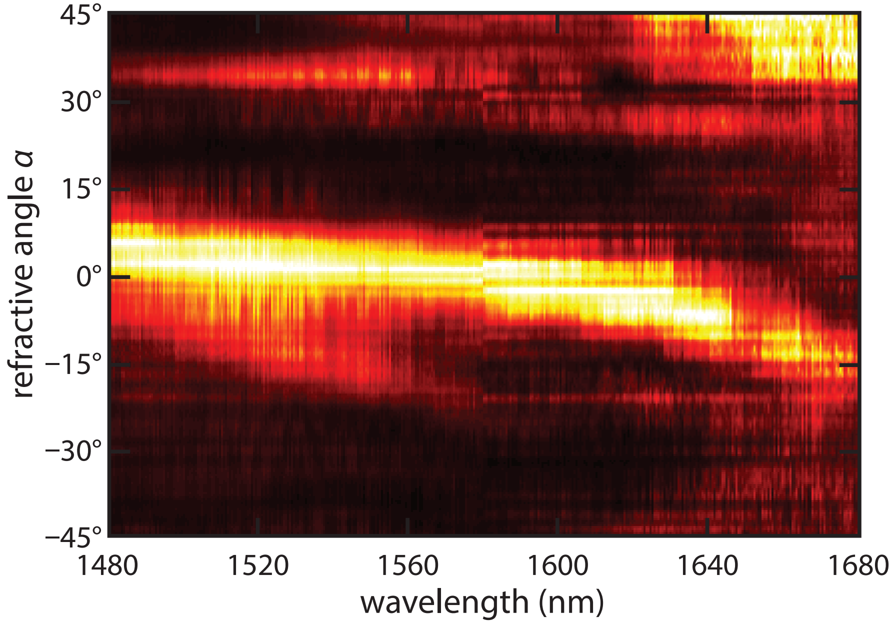
Wavelength dependence of refraction angle



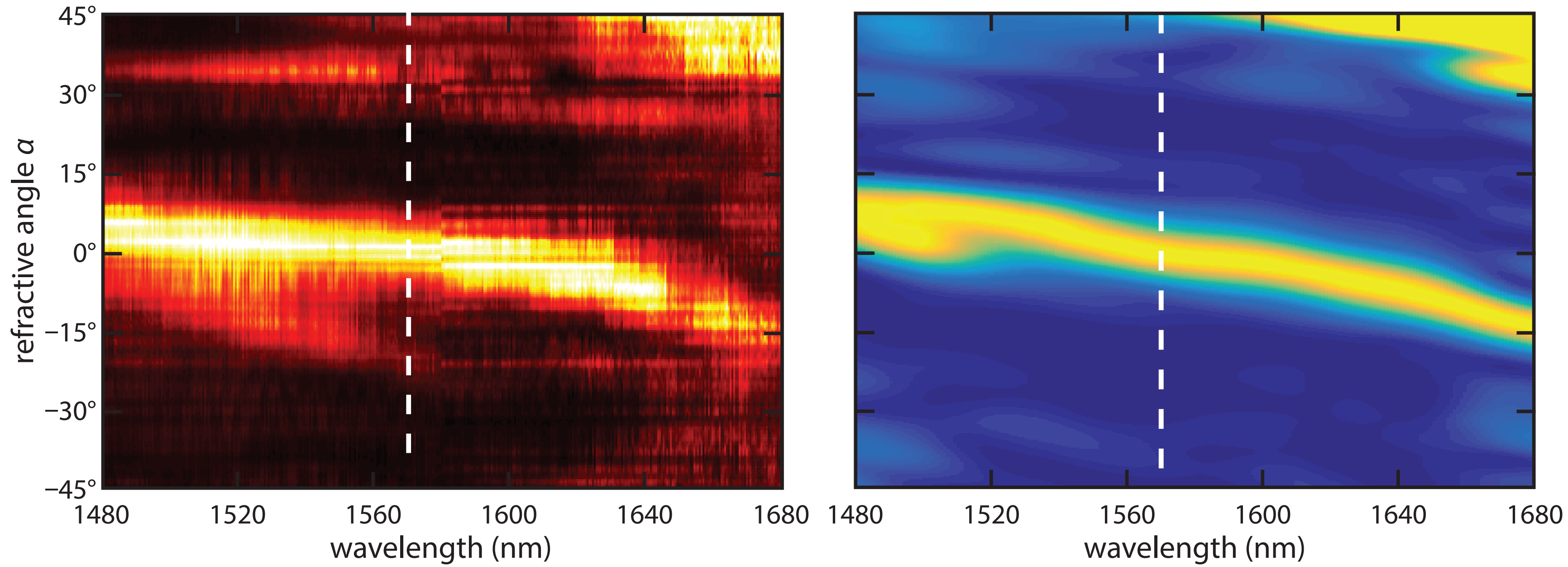
Wavelength dependence of refraction angle



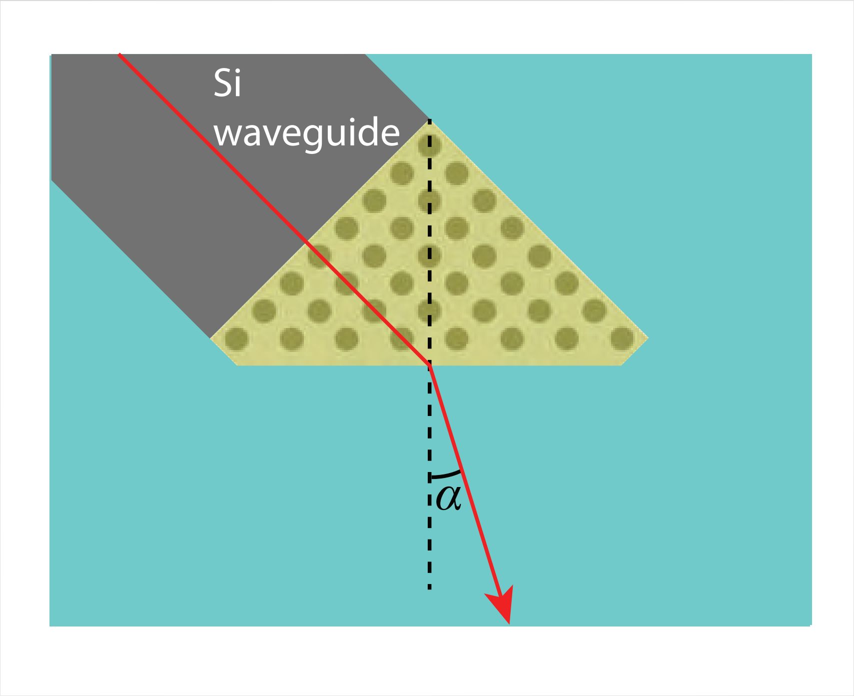
Wavelength dependence of refraction angle



Wavelength dependence of refraction angle

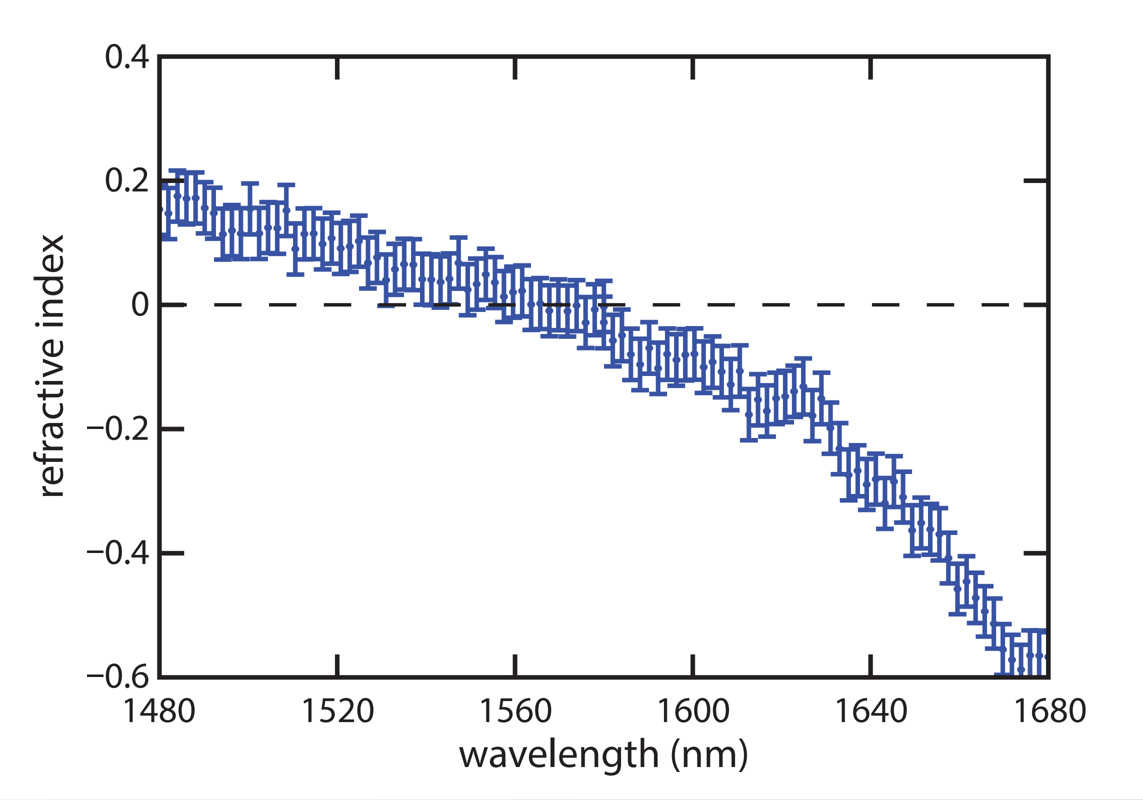


Wavelength dependence of index

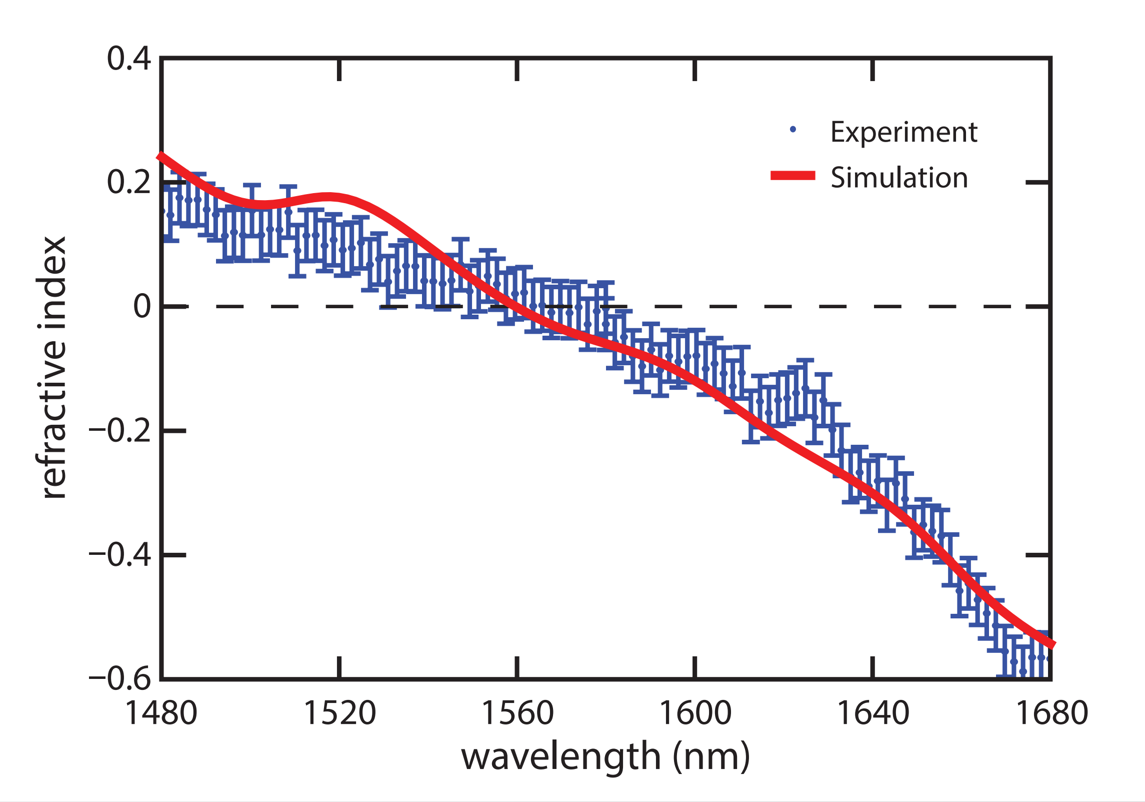


$$n_{\text{prism}} = n_{\text{slab}} \frac{\sin\alpha}{\sin 45°}$$

Wavelength dependence of index

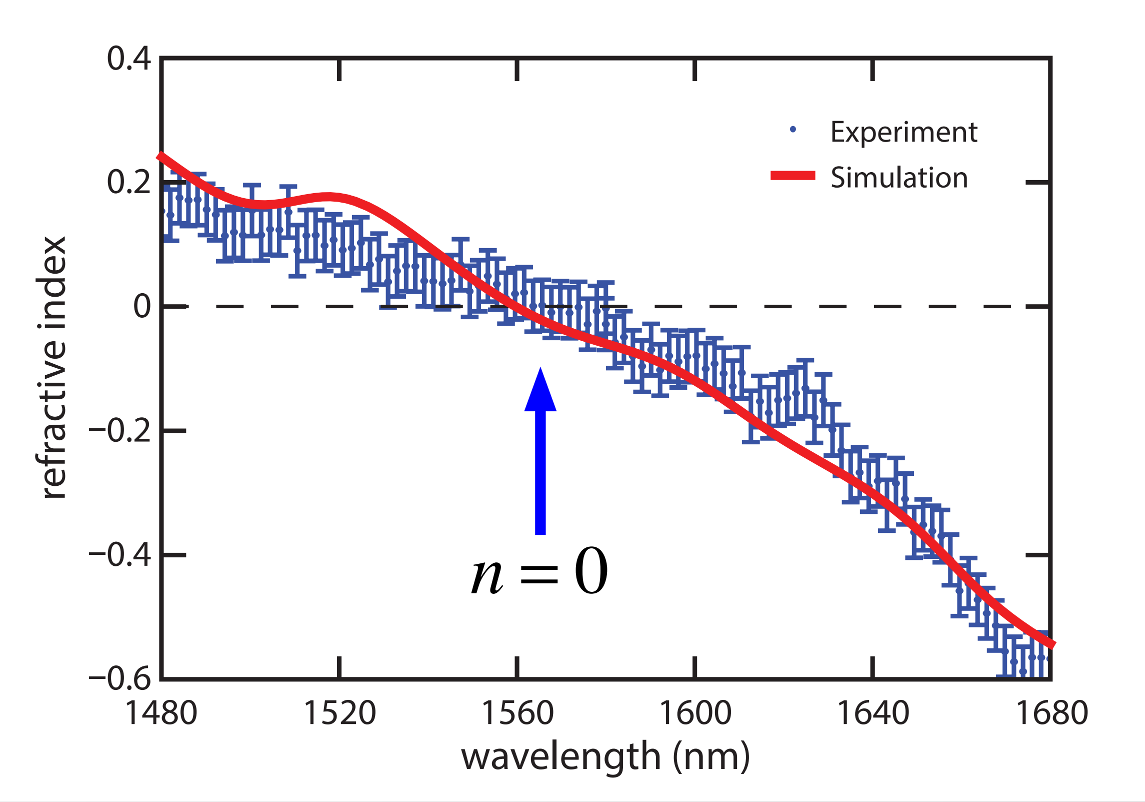


Wavelength dependence of index

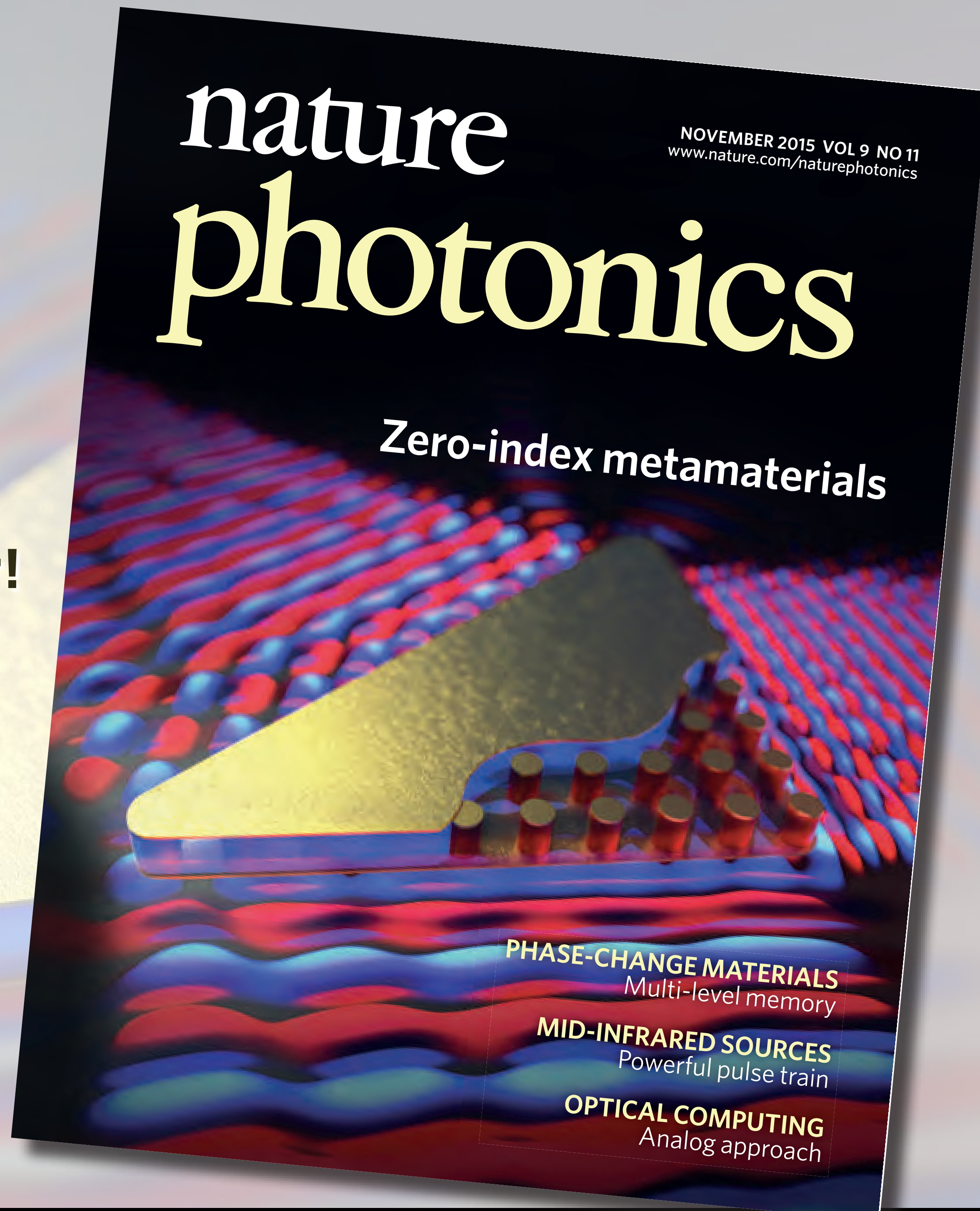


1 index
2 zero index
3 experiments

Wavelength dependence of index



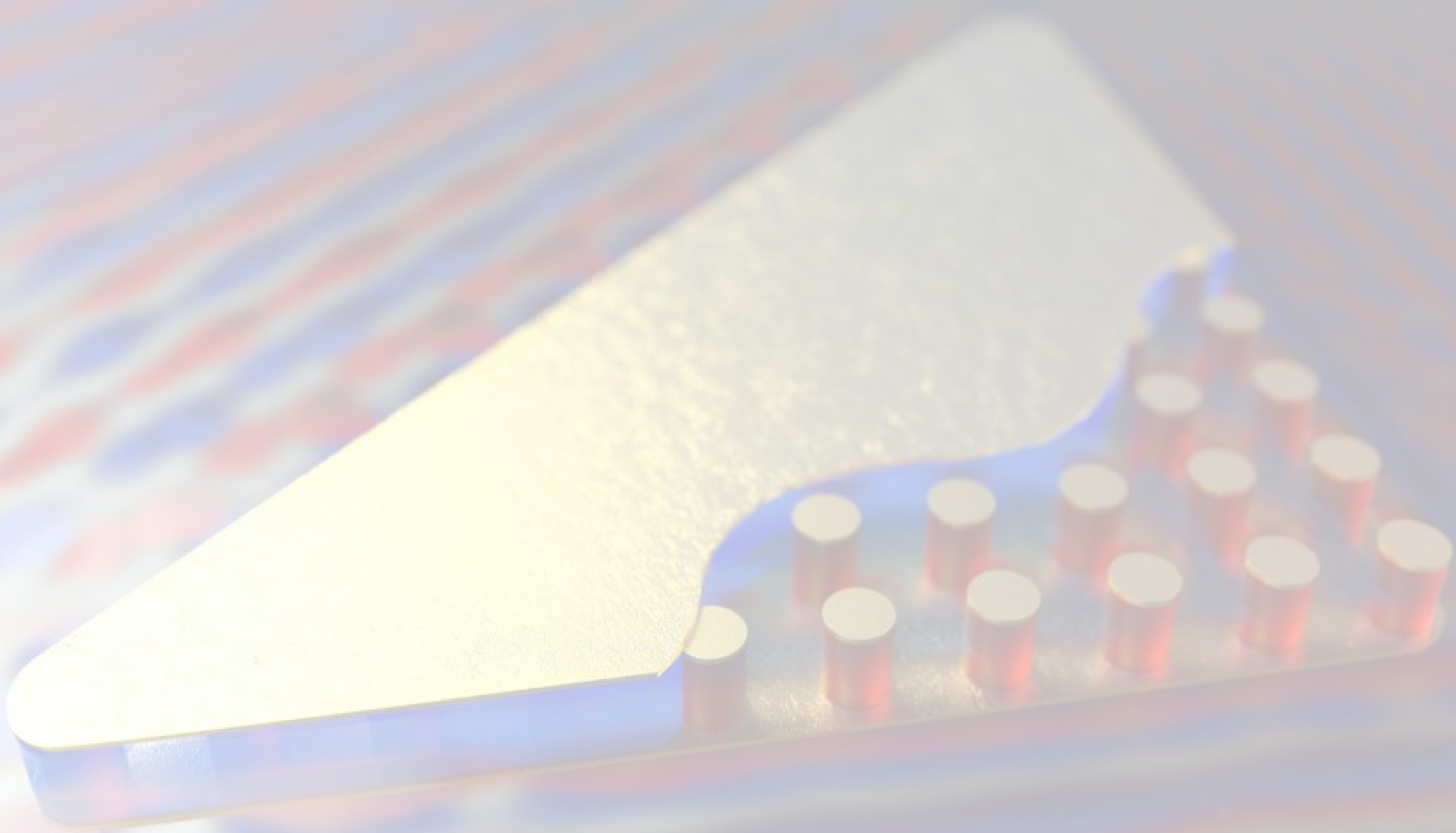
1 index
2 zero index
3 experiments



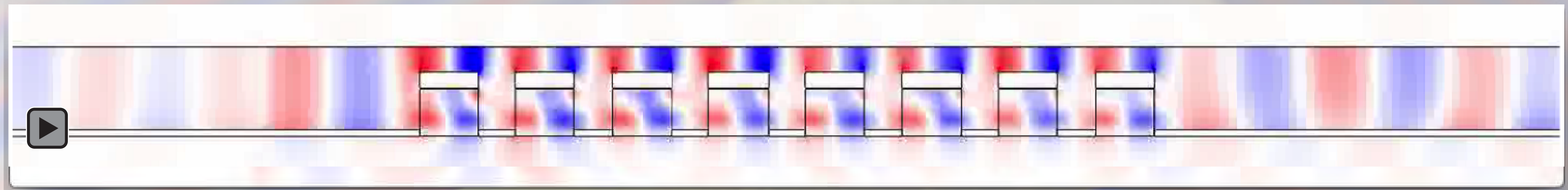
More info: download paper!

1 index 2 zero index 3 experiments

Where do we go from here?



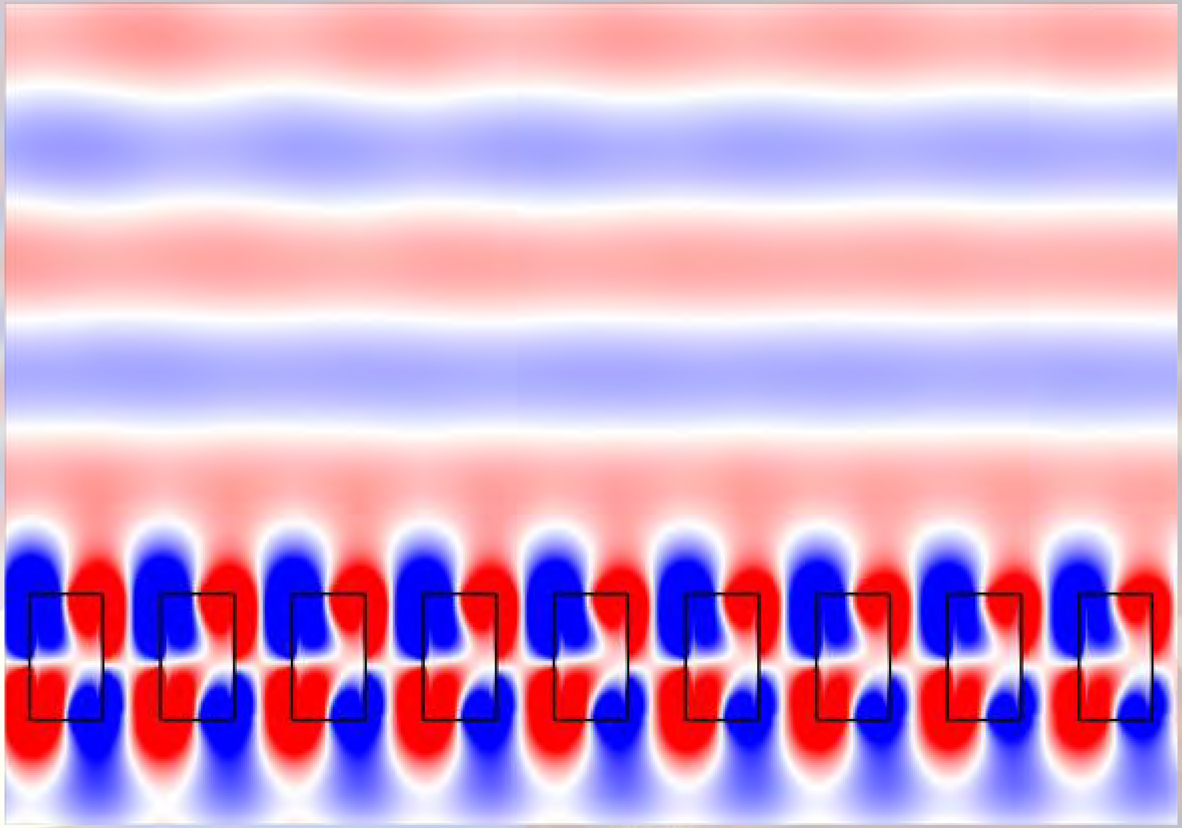
Where do we go from here?



Need to eliminate losses in metal mirrors

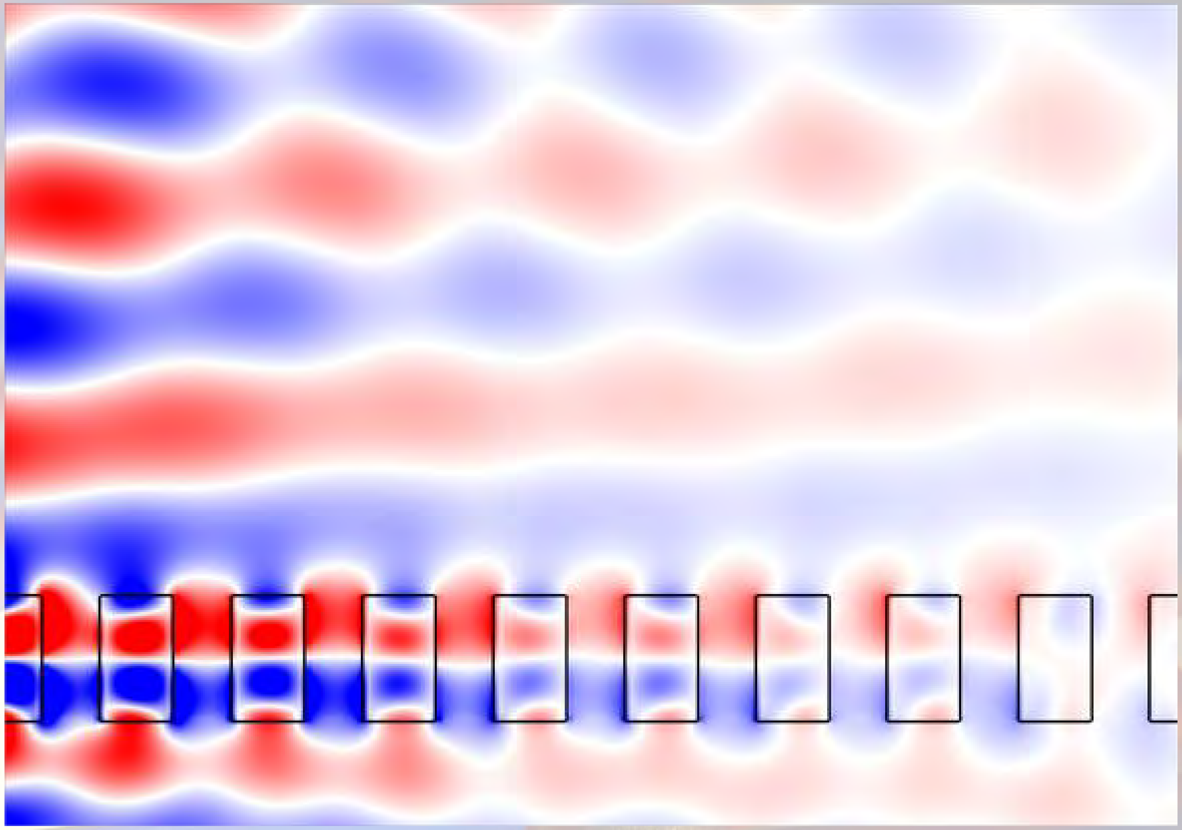
1 index 2 zero index 3 experiments

Where do we go from here?



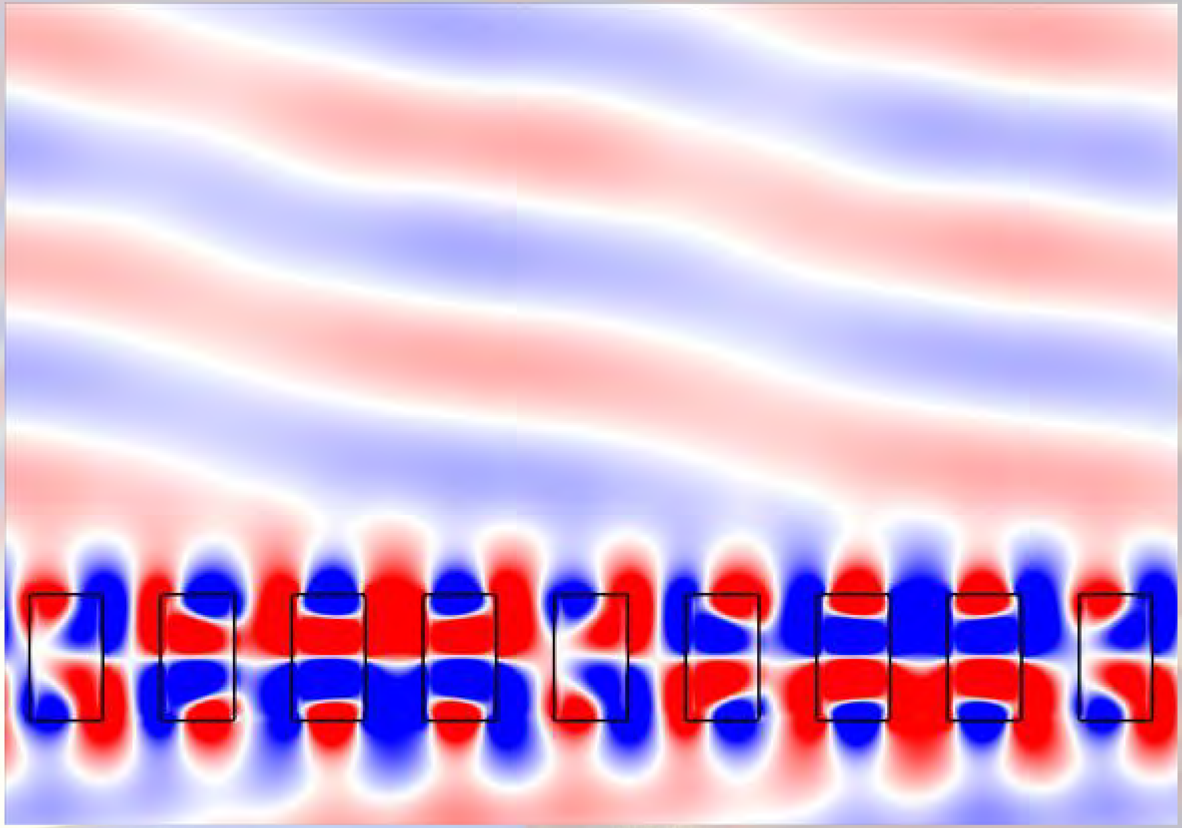
Removing mirrors causes radiative losses

Where do we go from here?



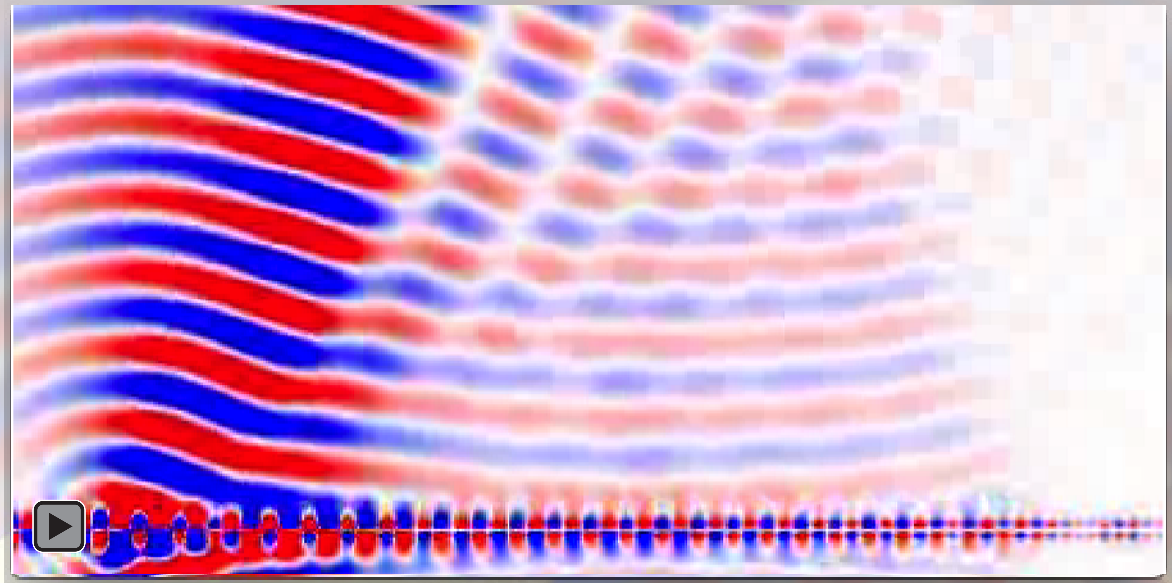
Radiative losses can be steered…

Where do we go from here?



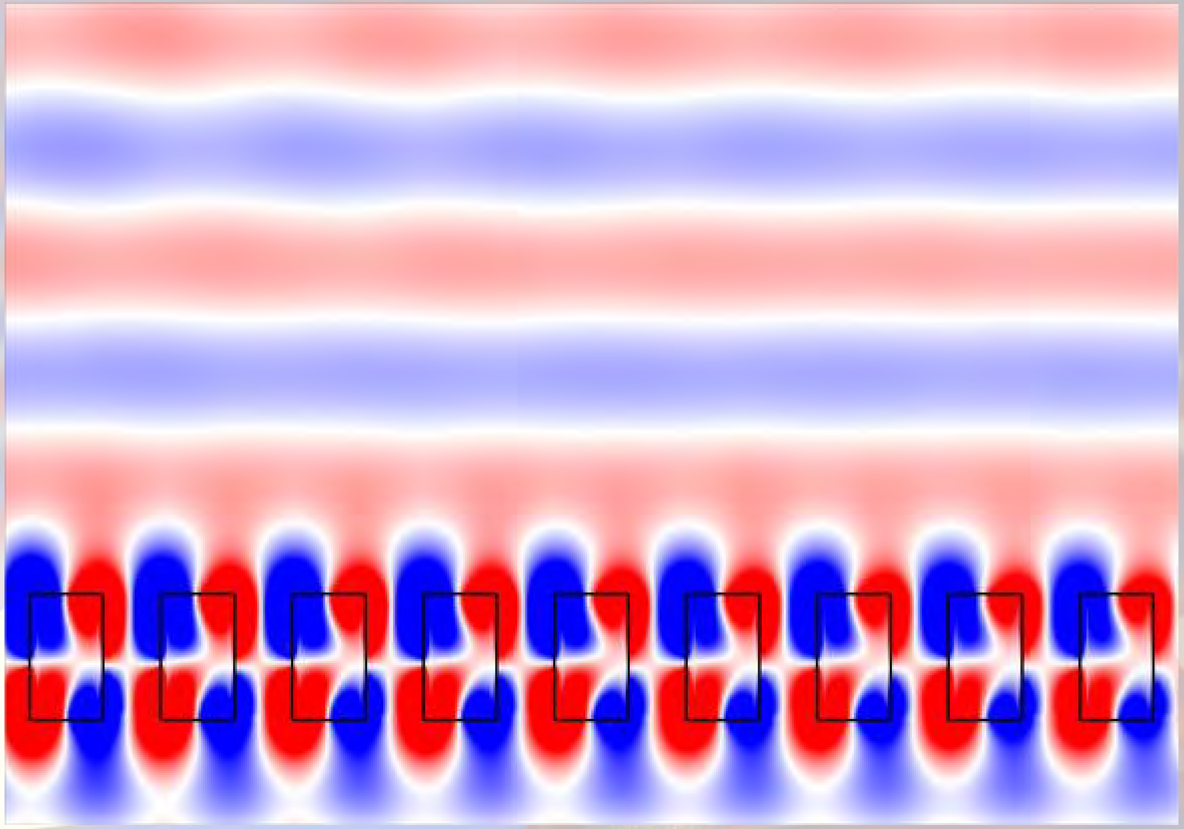
Radiative losses can be steered…

Where do we go from here?



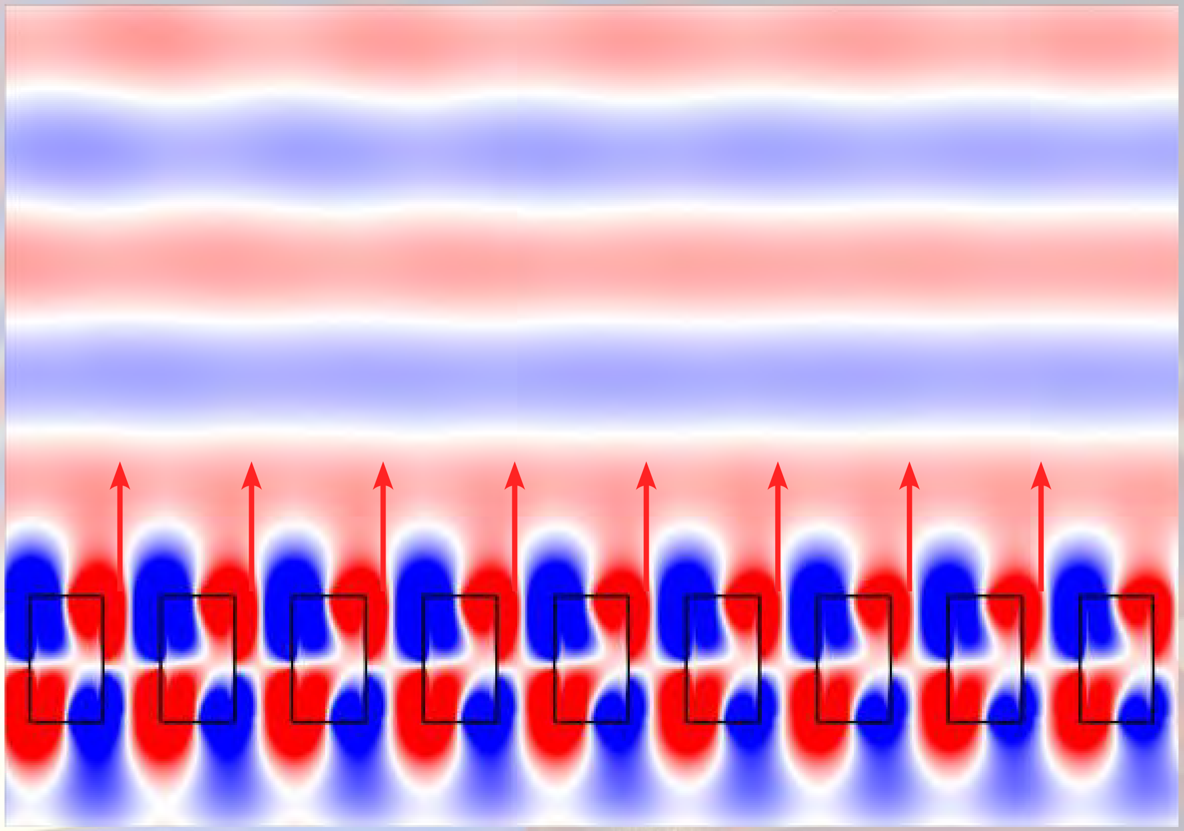
…or arranged to cause focusing…

Where do we go from here?



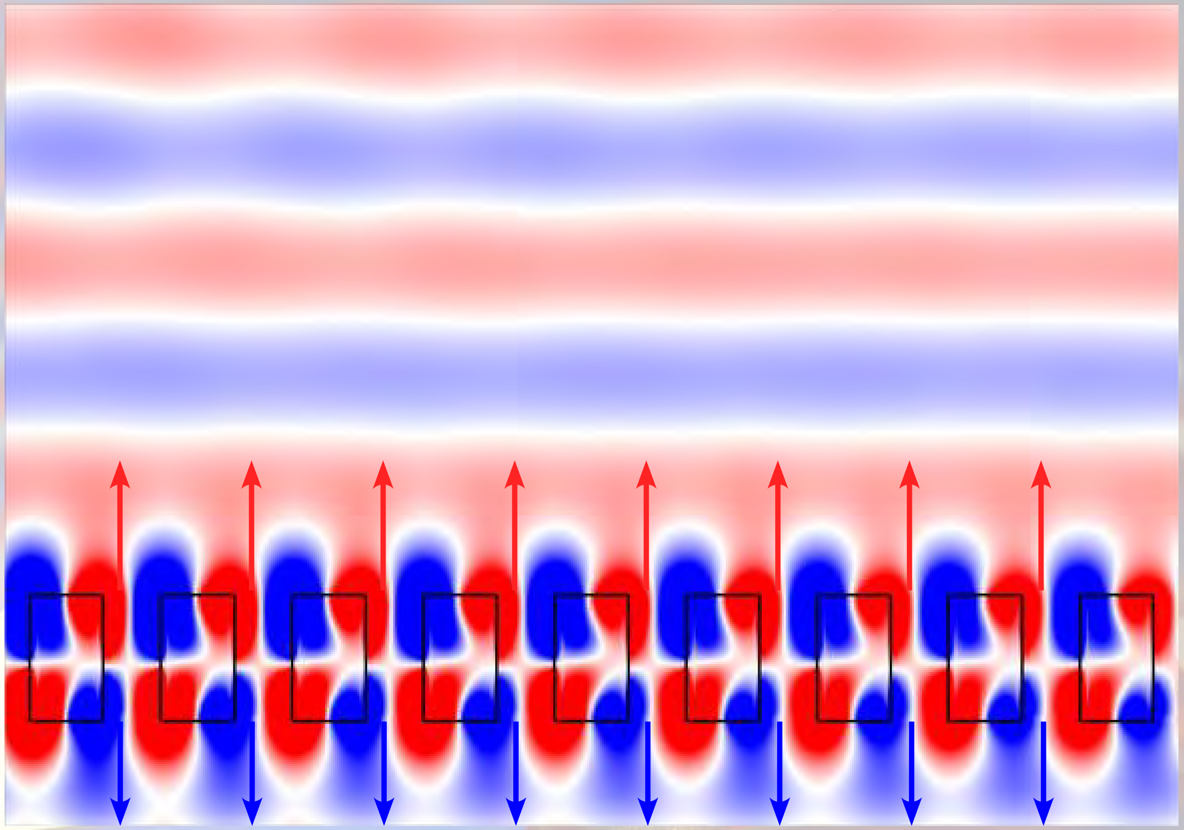
…or eliminated causing "bound in continuum" state

Where do we go from here?



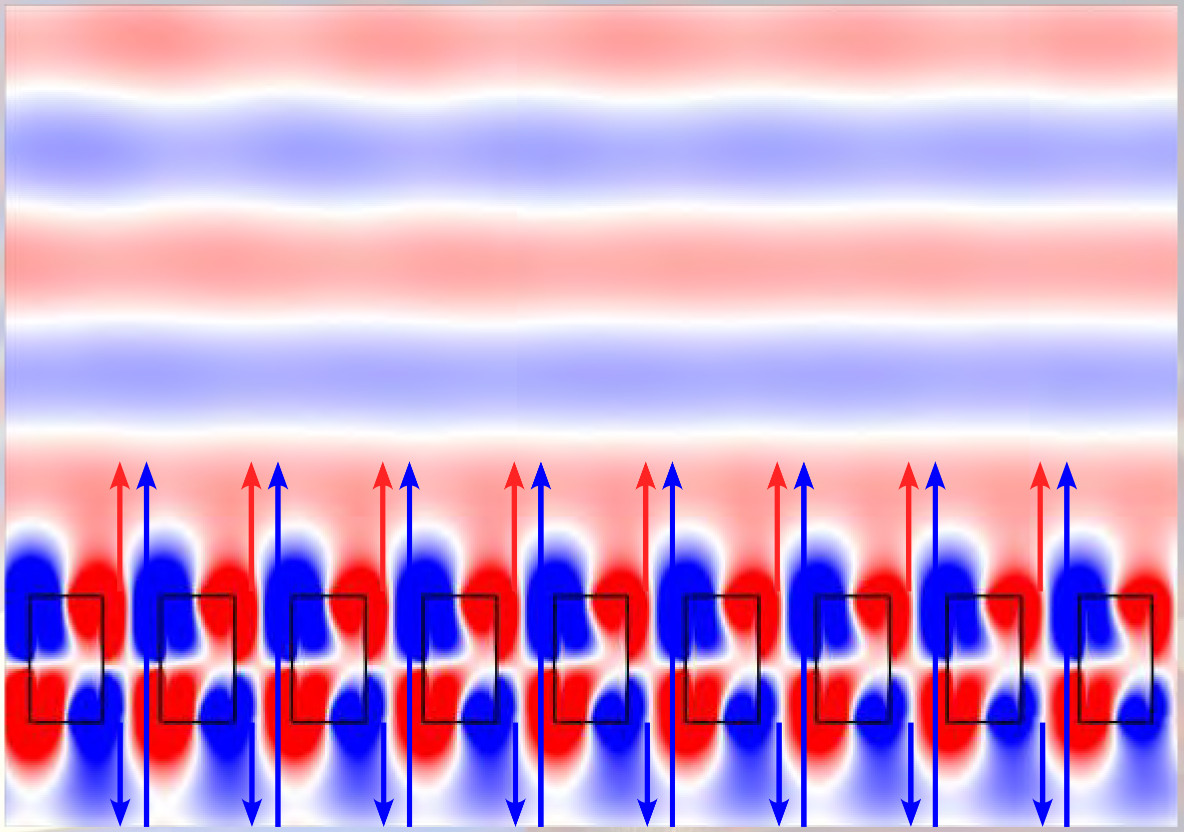
...or eliminated causing "bound in continuum" state

Where do we go from here?



...or eliminated causing “bound in continuum” state

Where do we go from here?



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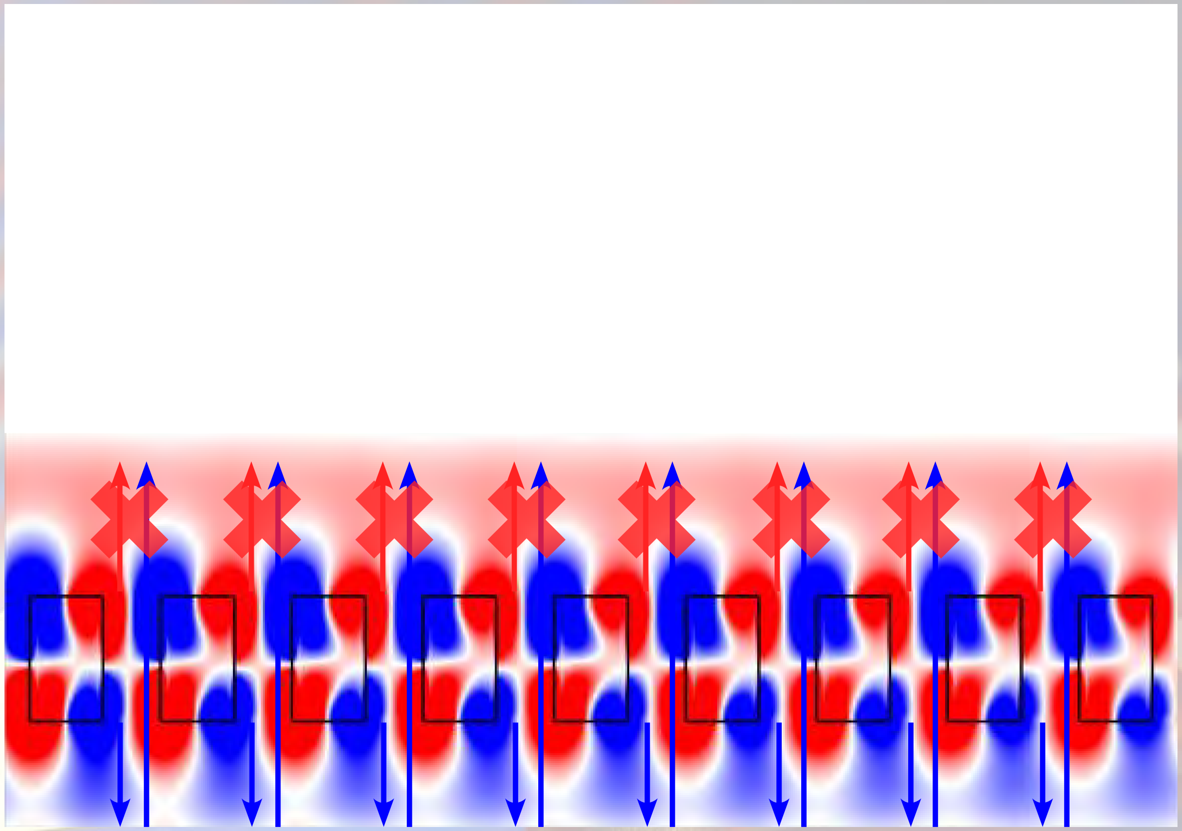
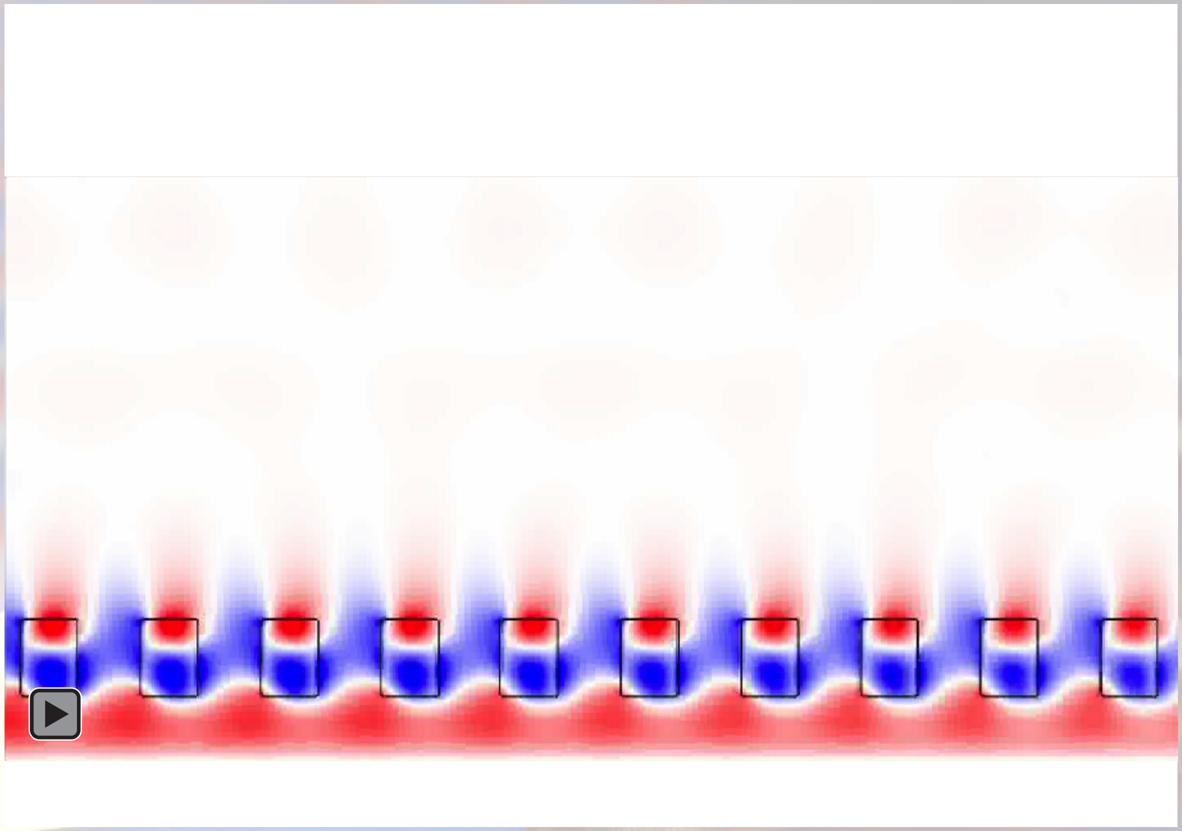
Where do we go from here?



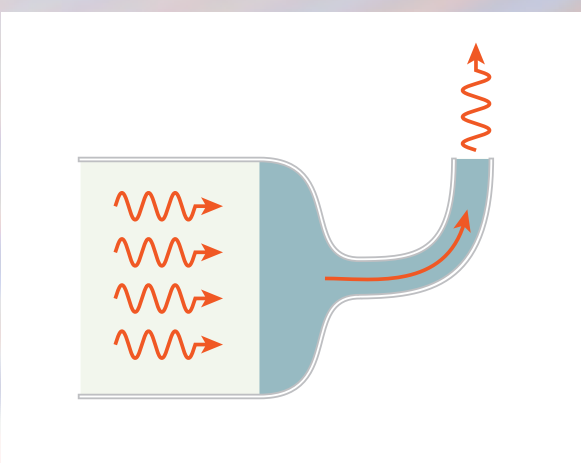
…or eliminated causing “bound in continuum” state

Where do we go from here?



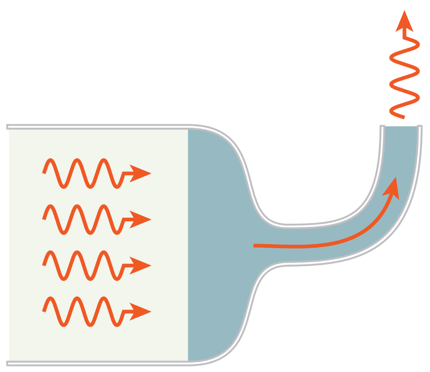
…or eliminated causing “bound in continuum” state

Exciting applications ahead



supercoupling

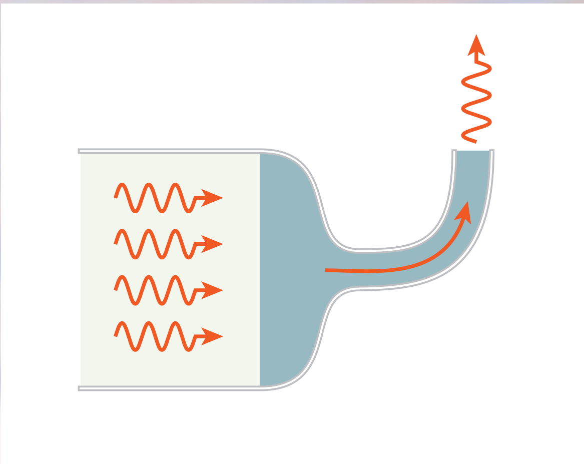
Exciting applications ahead



supercoupling

NLO

Exciting applications ahead

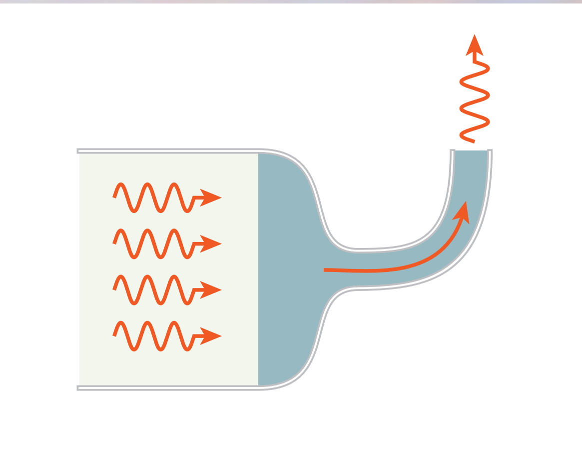


supercoupling

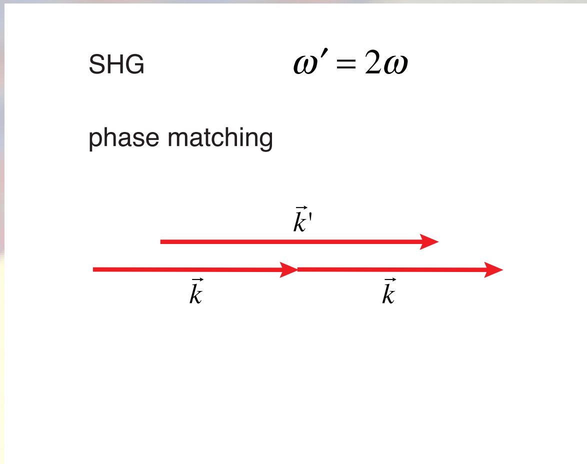
SHG $\omega' = 2\omega$

NLO

Exciting applications ahead

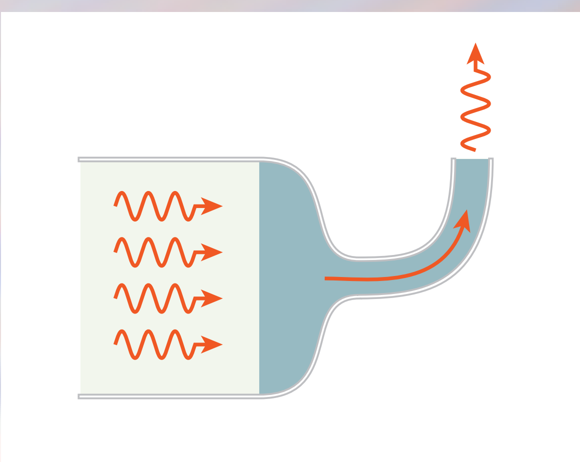


supercoupling

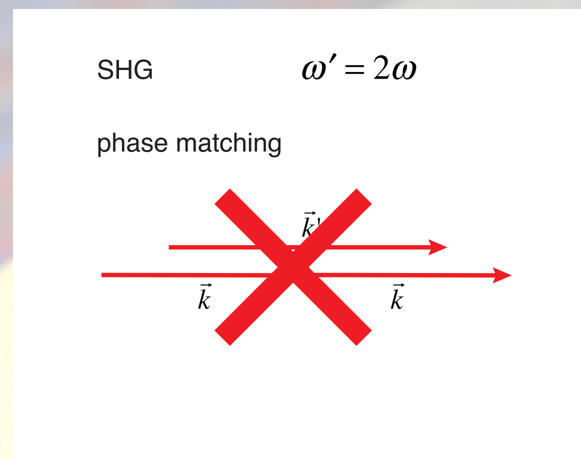


NLO

Exciting applications ahead

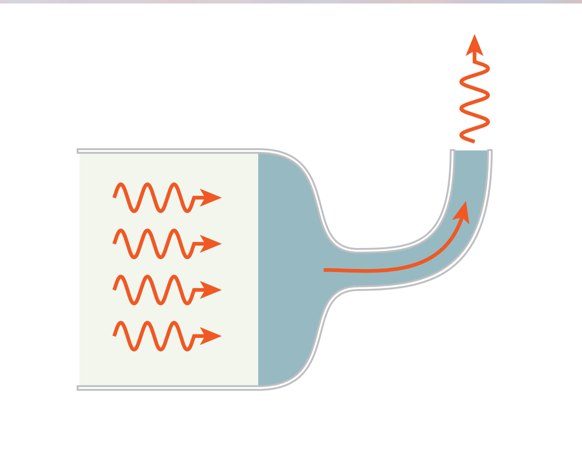


supercoupling

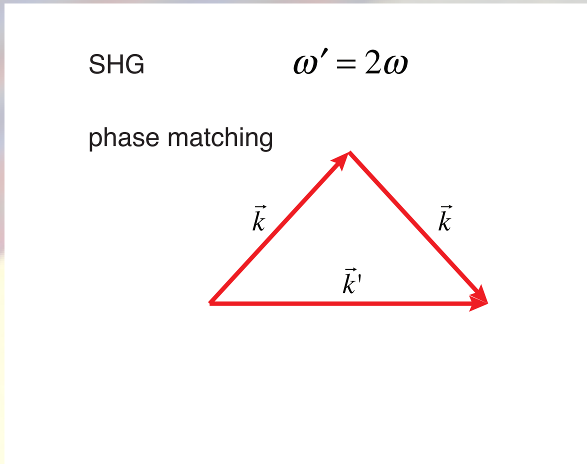


NLO

Exciting applications ahead

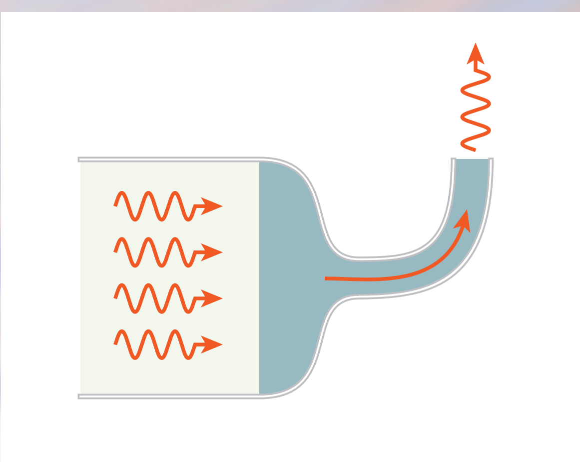


supercoupling



NLO

Exciting applications ahead



supercoupling

SHG $\omega' = 2\omega$

at zero index

$\vec{k} = 0$

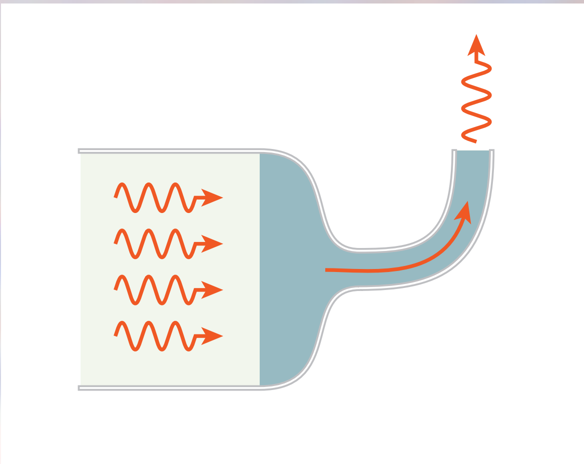
NLO

1 index

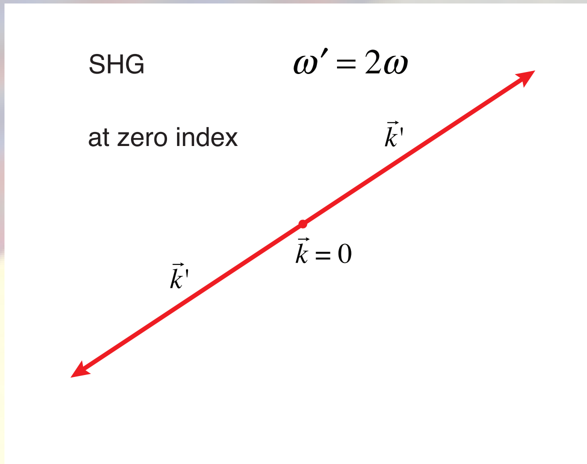
2 zero index

3 experiments

Exciting applications ahead

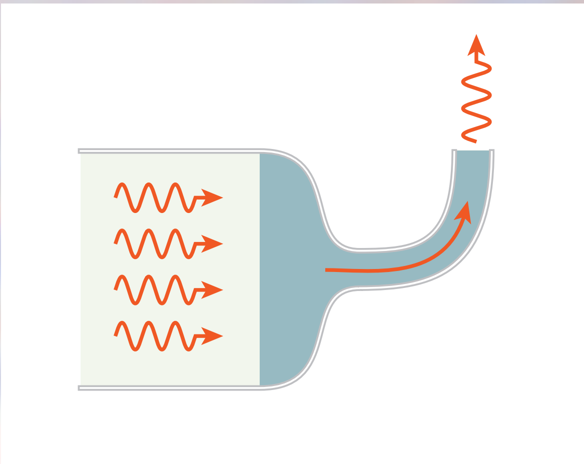


supercoupling

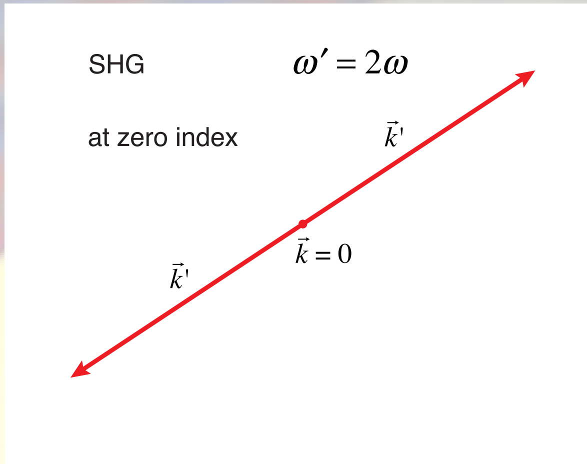


NLO

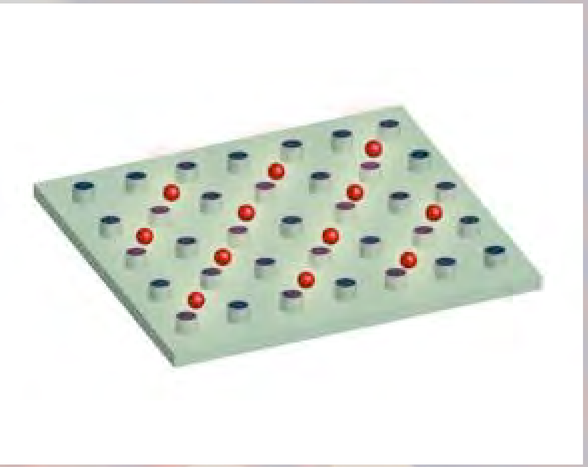
Exciting applications ahead



supercoupling



NLO



quantum optics

Yang Li, Shota Kita, Phil Muñoz, Orad Reshef,
Daryl Vulis, Mei Yin, Lysander Christakis, Zin Lin,
Cleaven Chia, Olivia Mello, Haoning Tang, Marko Lončar

National Science Foundation
Harvard Center for Nanoscale Systems

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