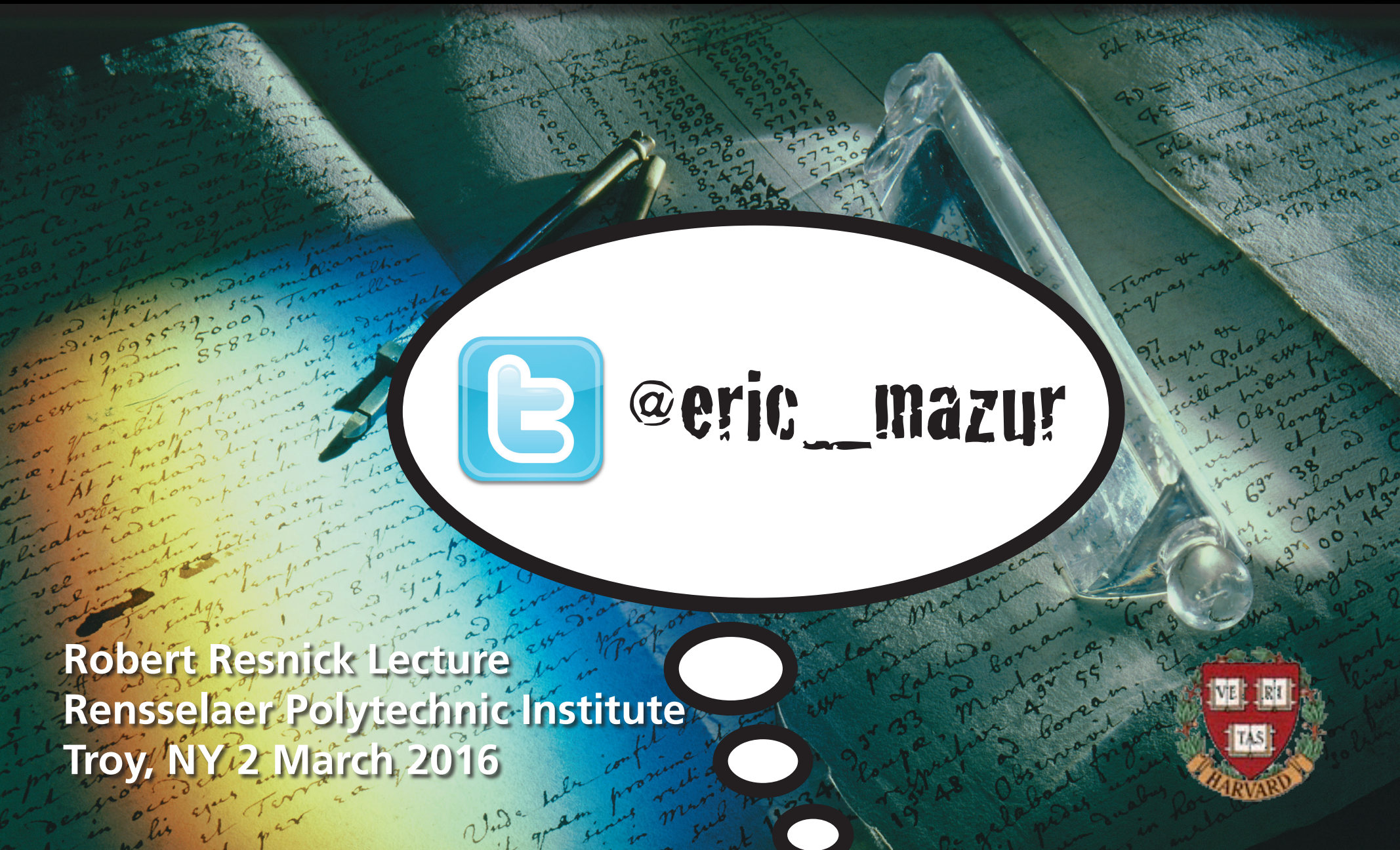
Less is more:
Extreme optics with zero refractive index



Robert Resnick Lecture
Rensselaer Polytechnic Institute
Troy, NY 2 March 2016

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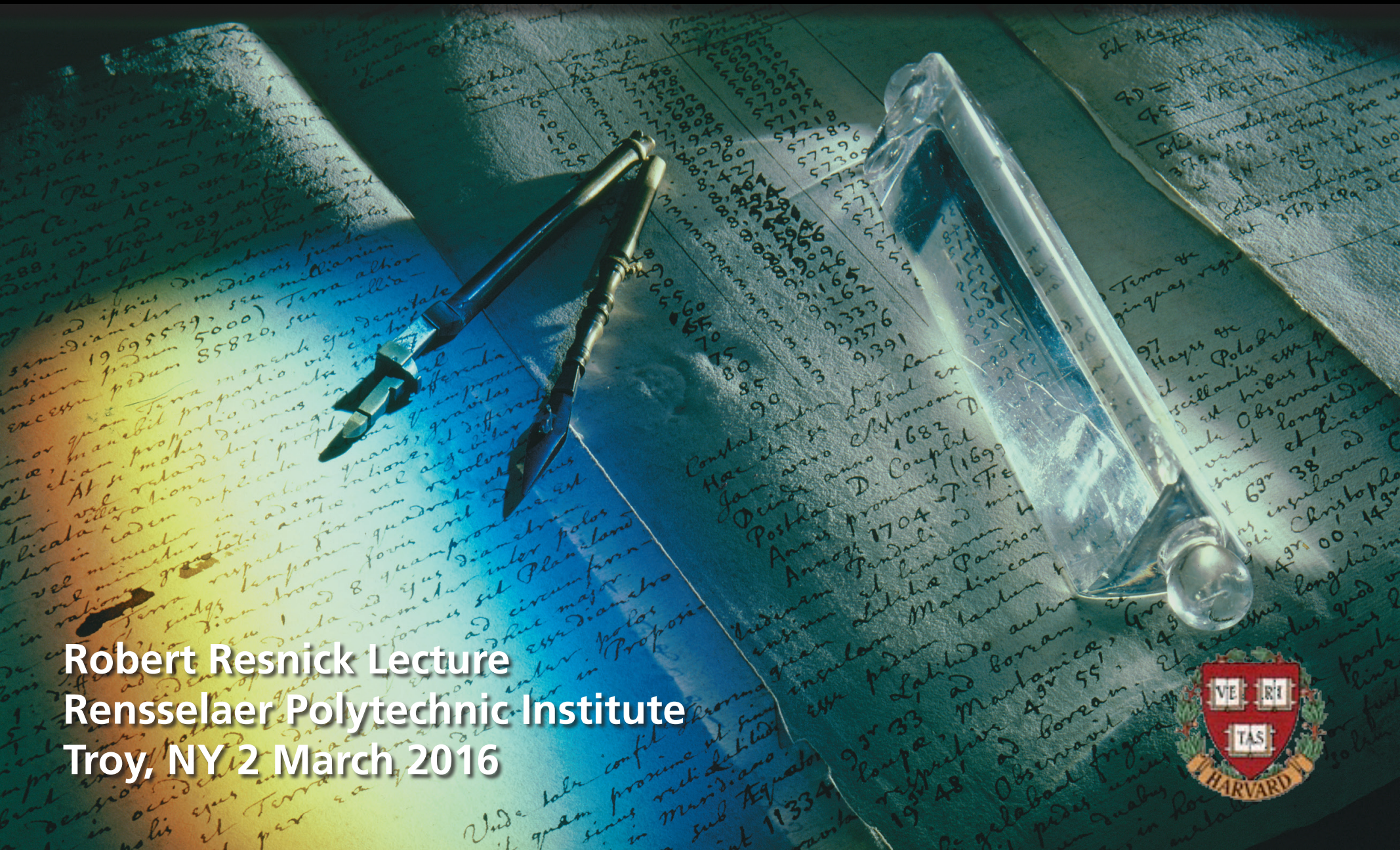

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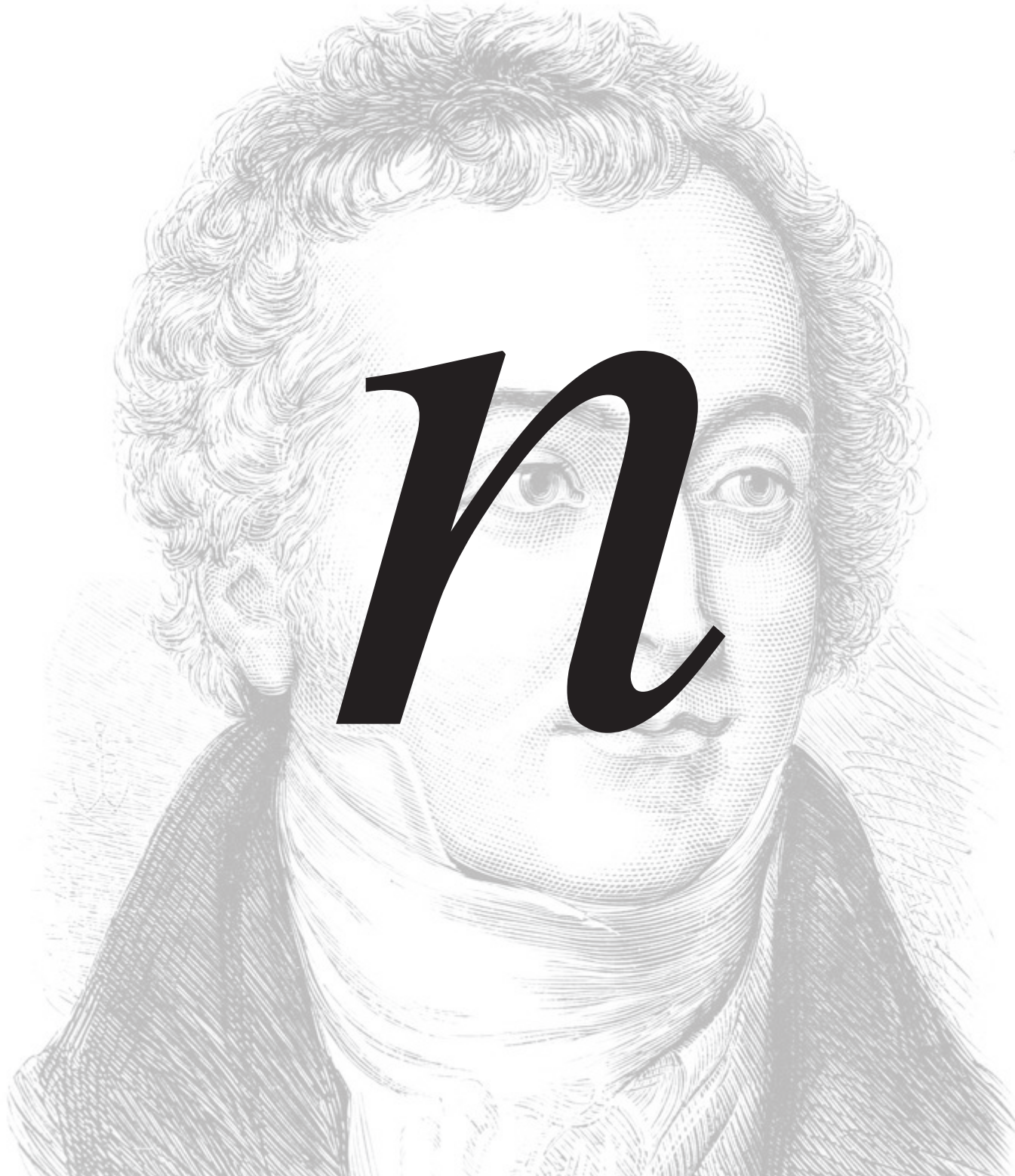
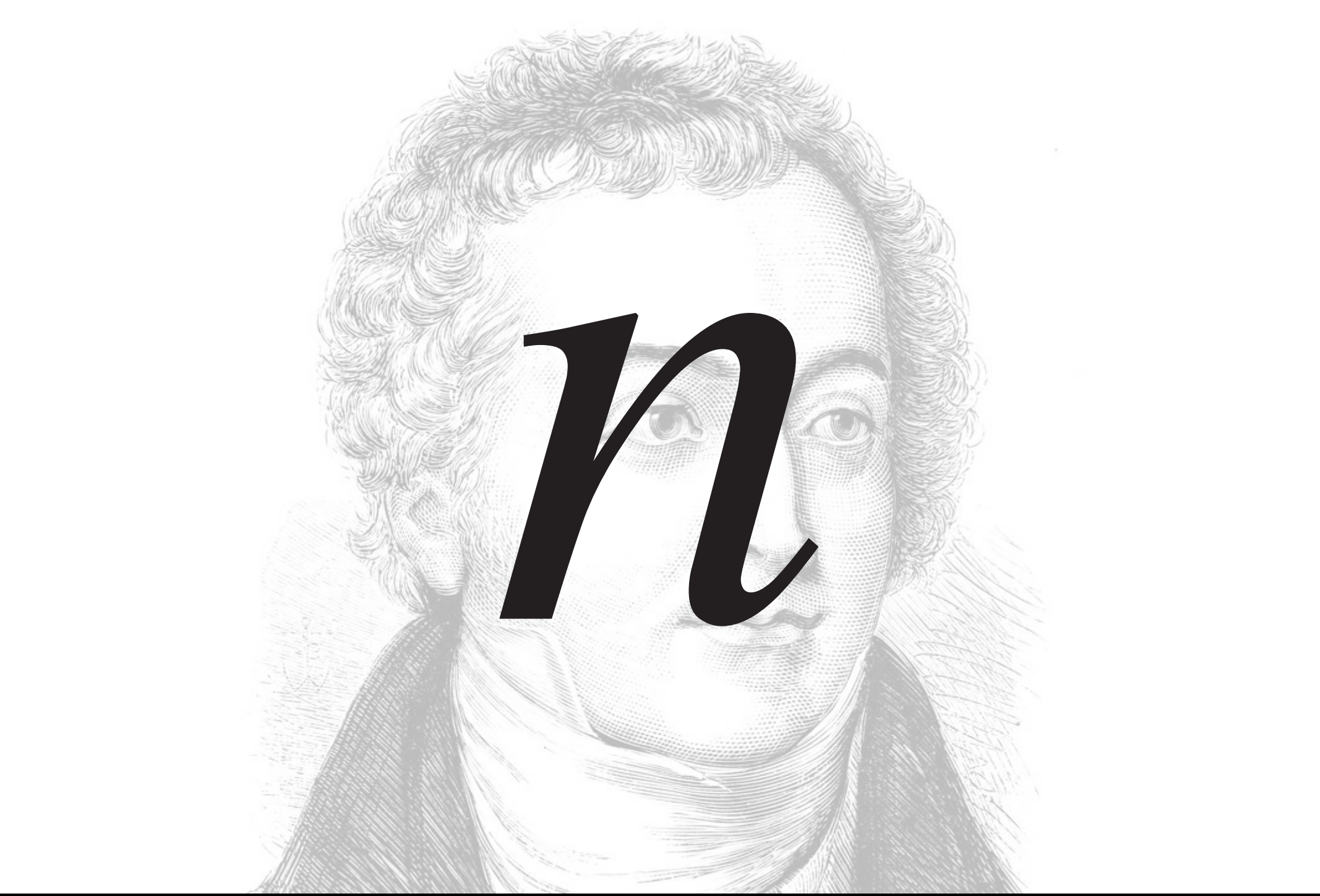
Reflecting a Century of Innovation





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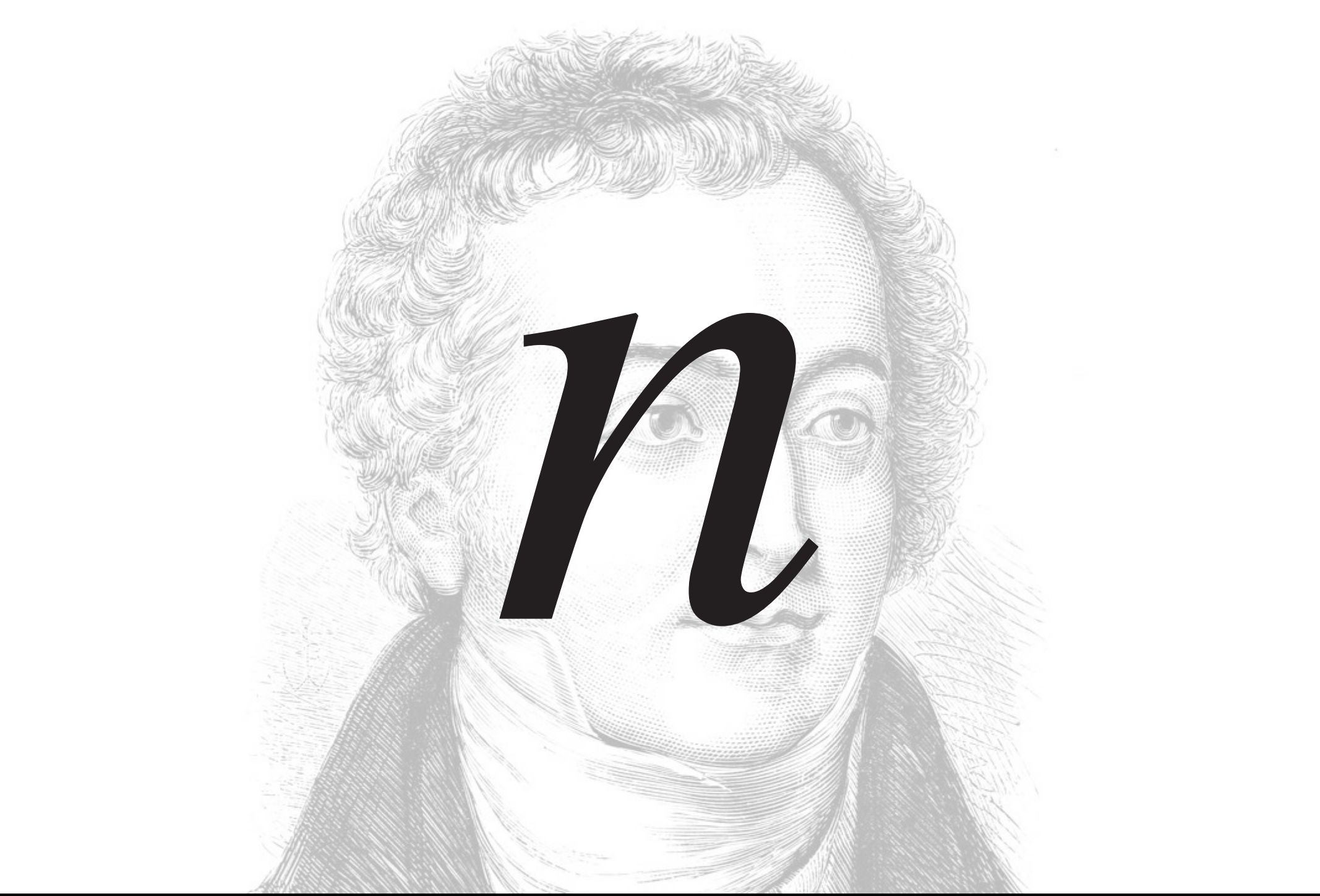
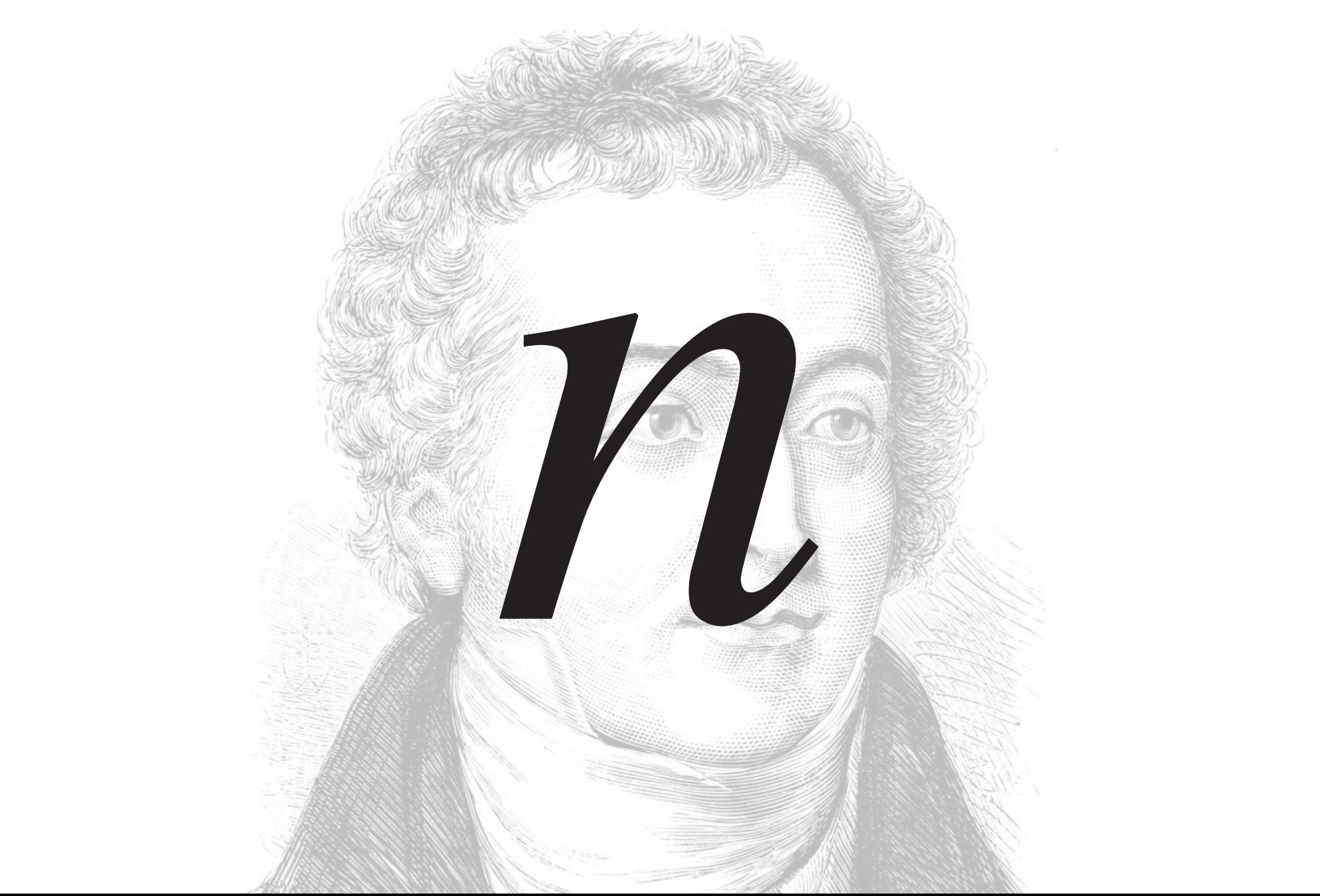


1 index

2 zero index



1 index

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3 experiments

Propagation of EM wave

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governed by wave equation

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In dispersive media $n = n(\omega)$.

Index of refraction

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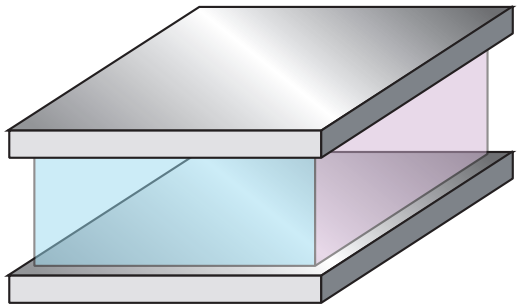

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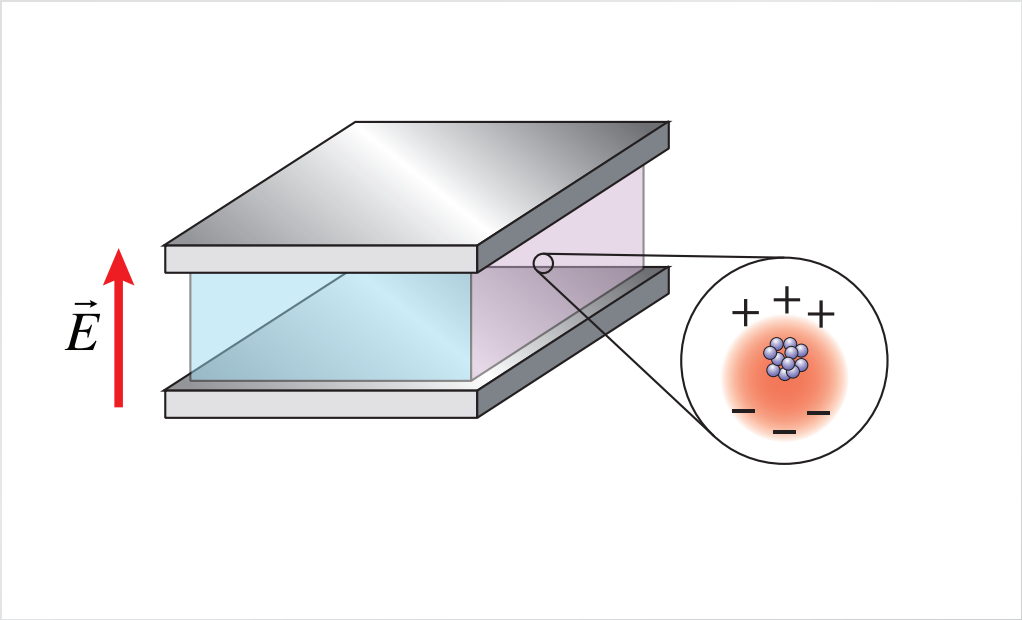

$\epsilon(\omega)$ measure of attenuation of electric field

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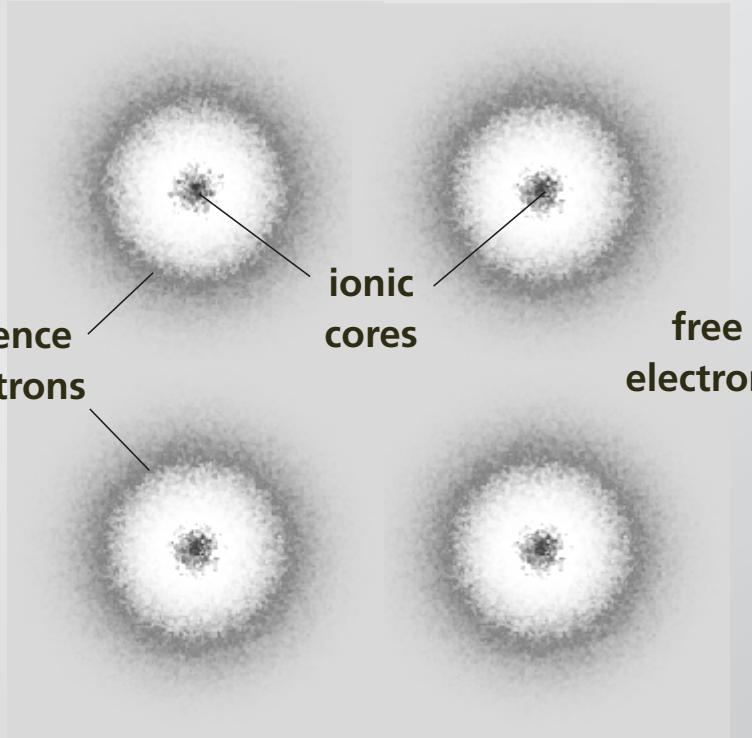
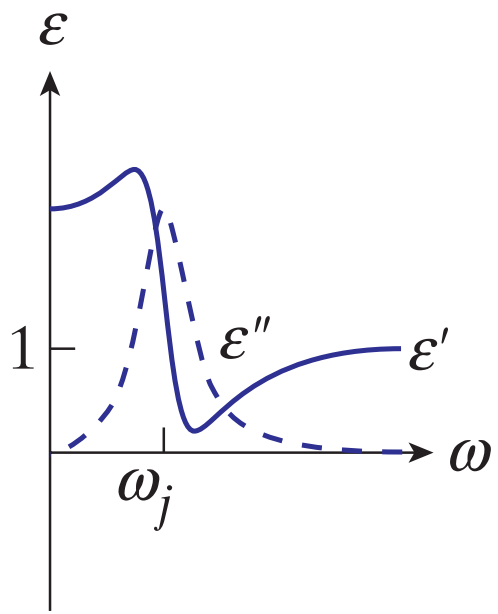


Dielectric constant

Lorentz oscillator



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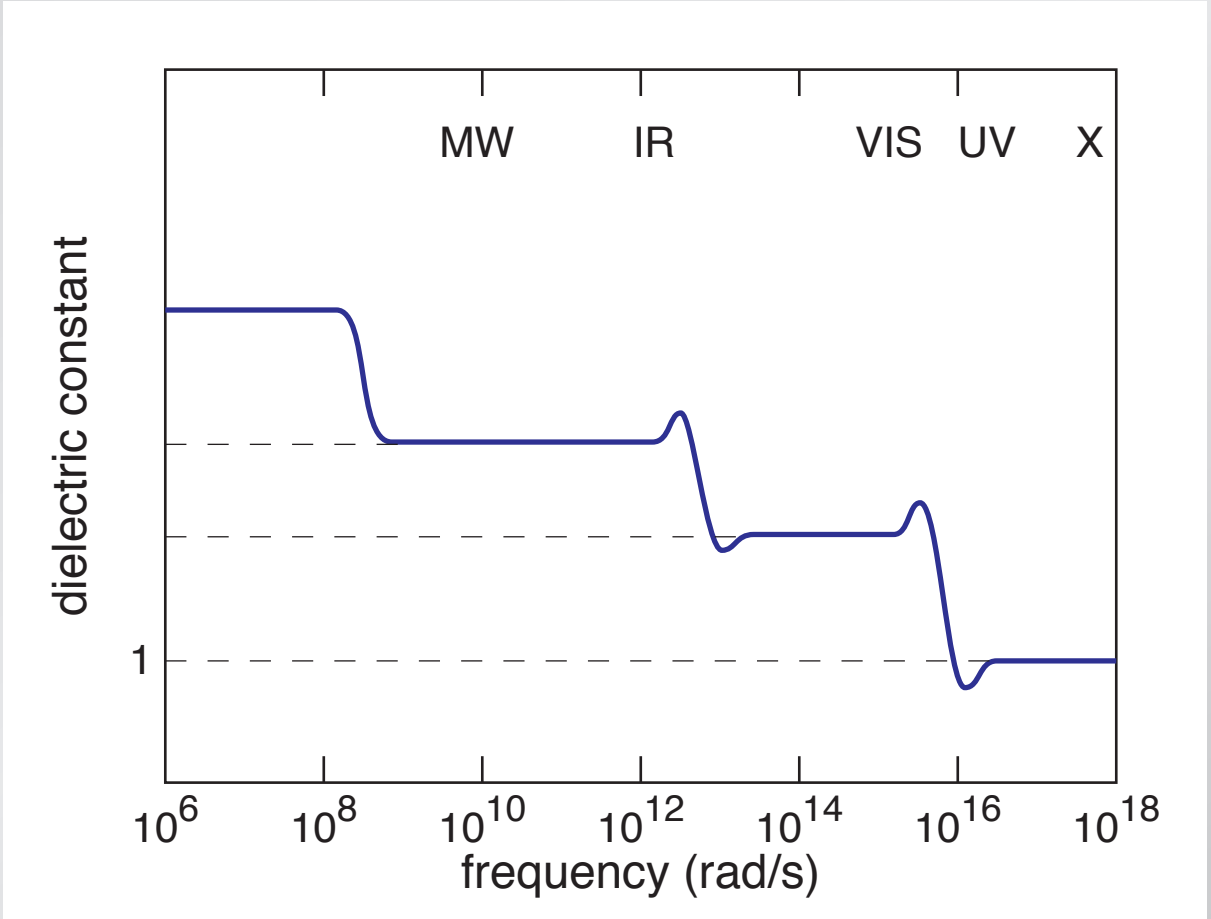

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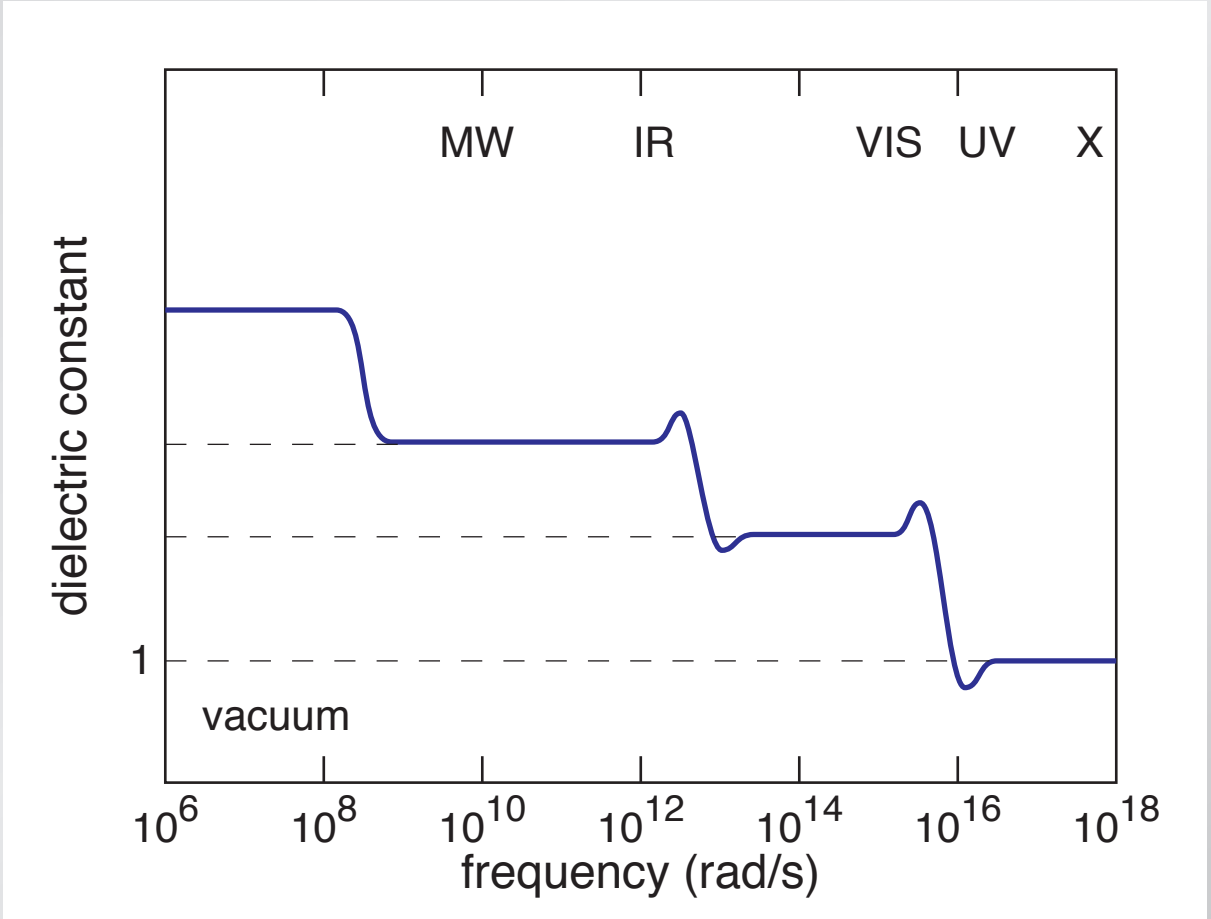

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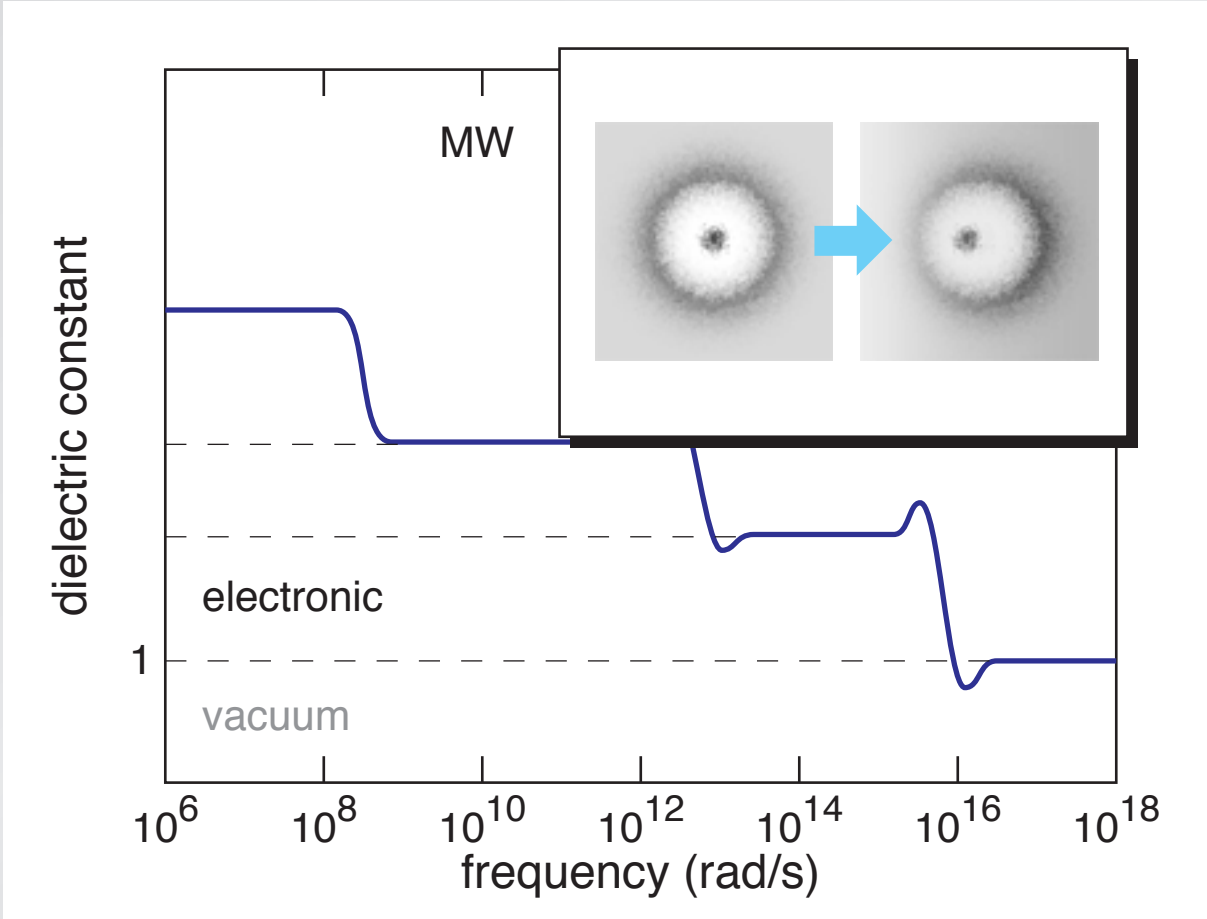

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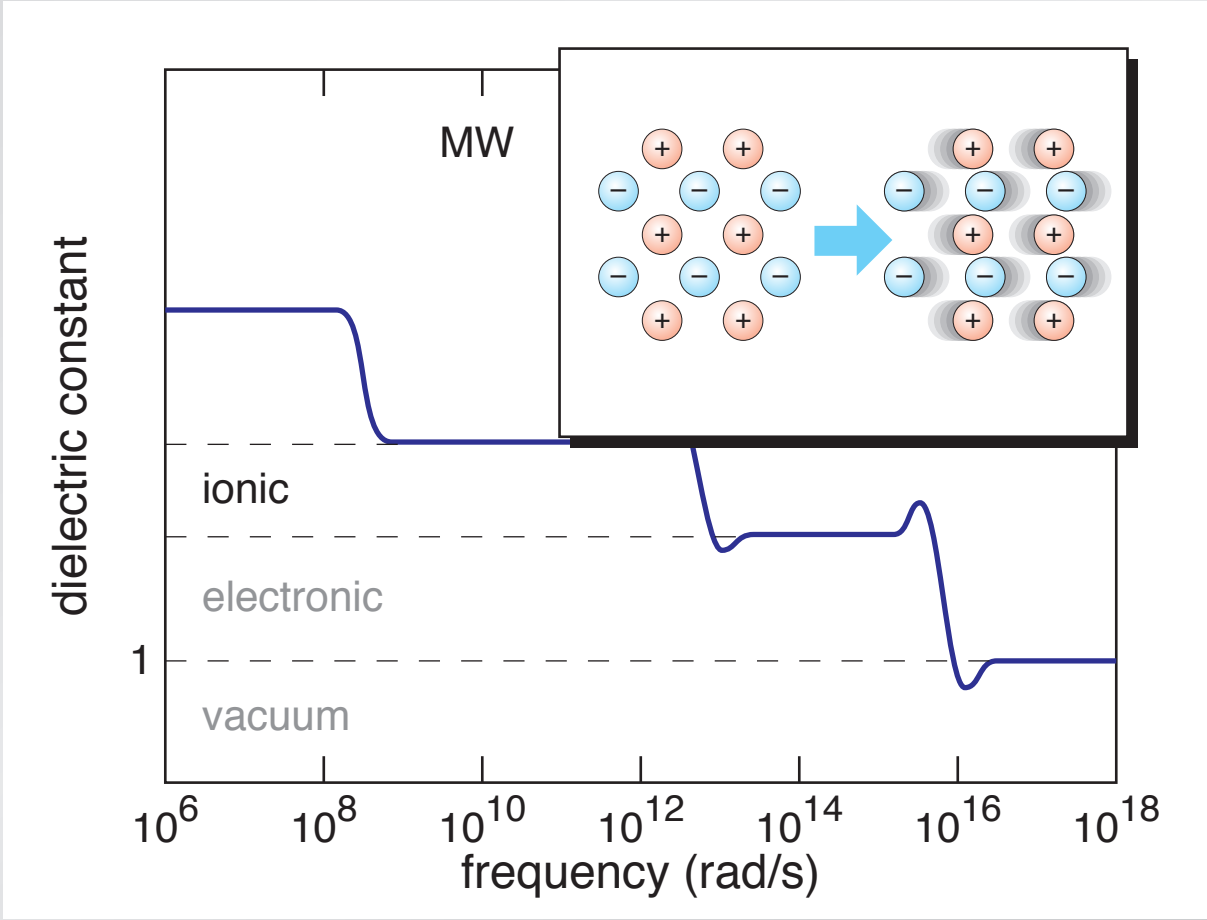

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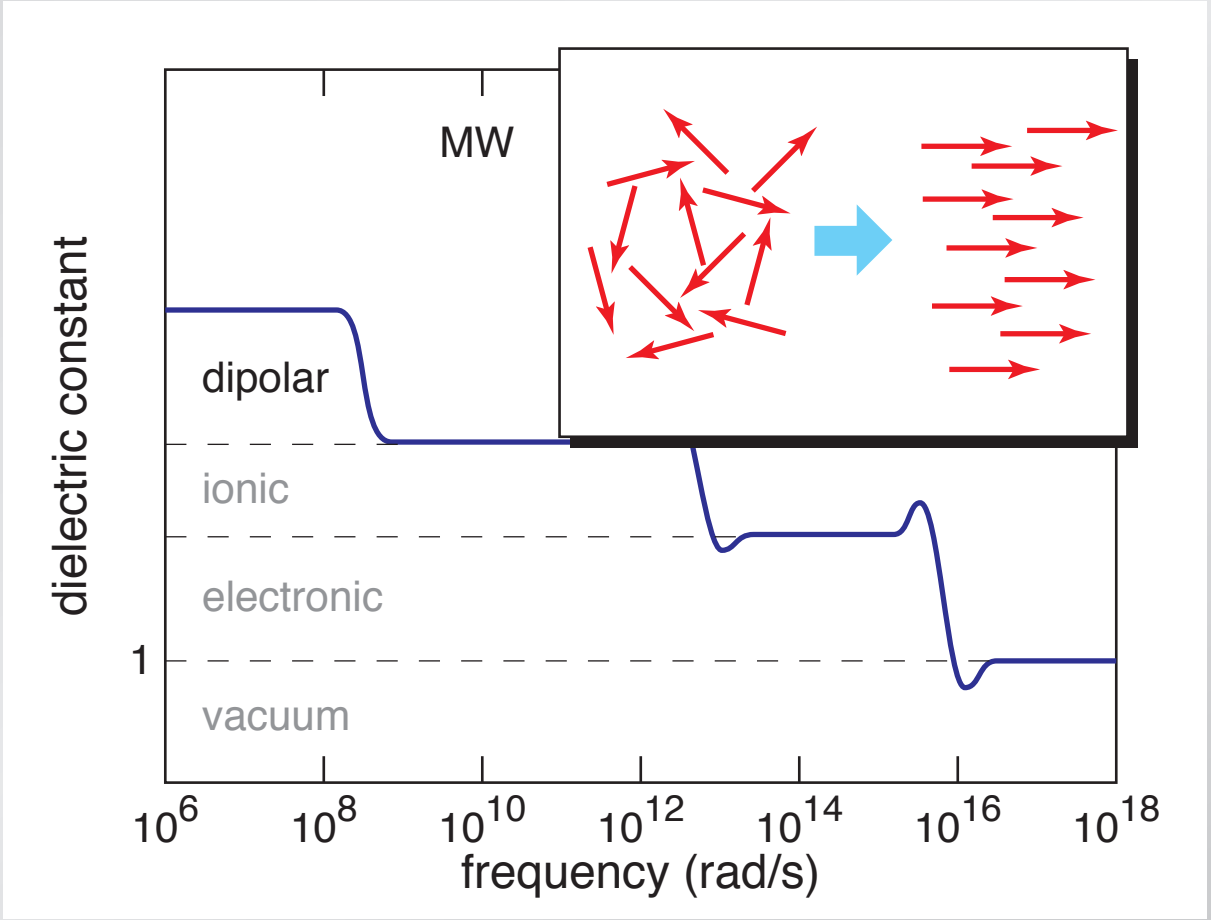

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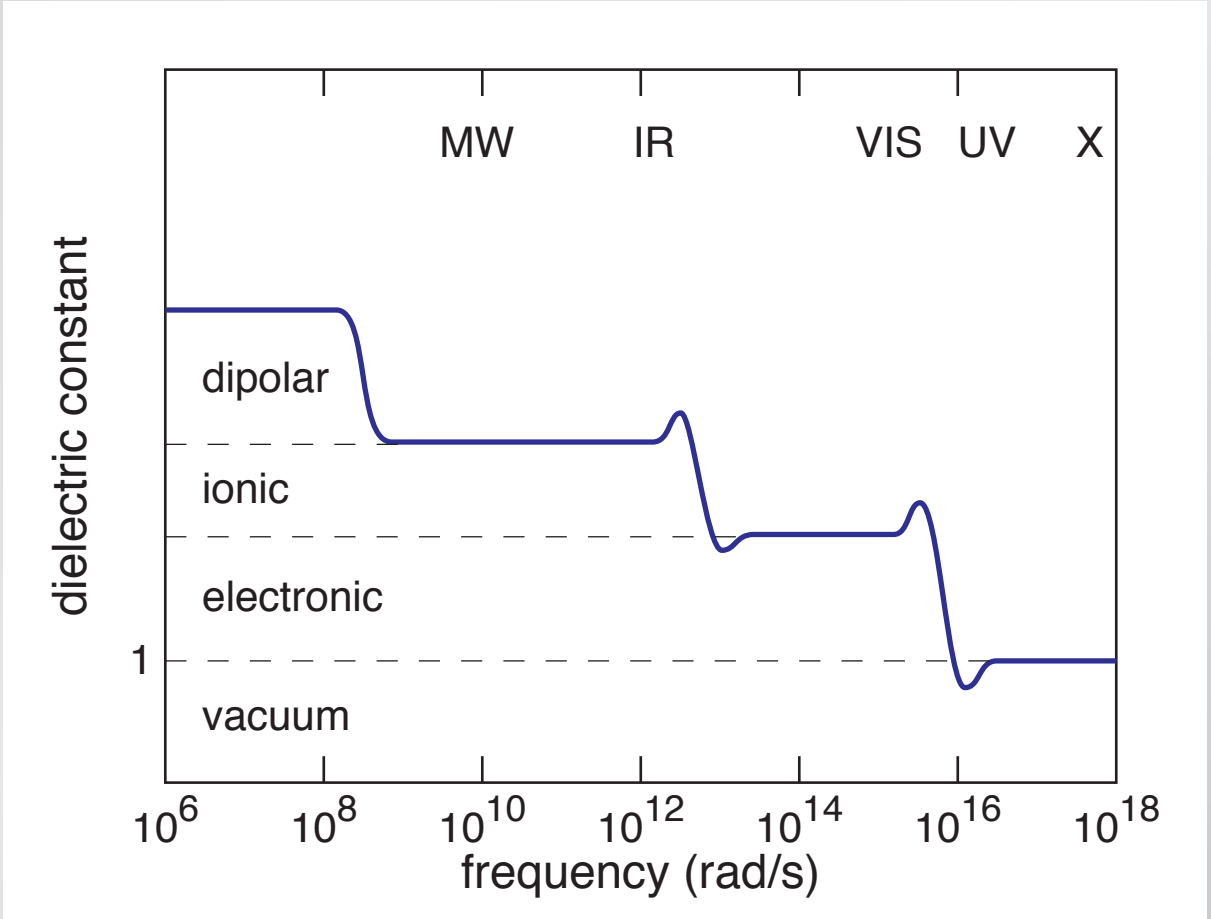
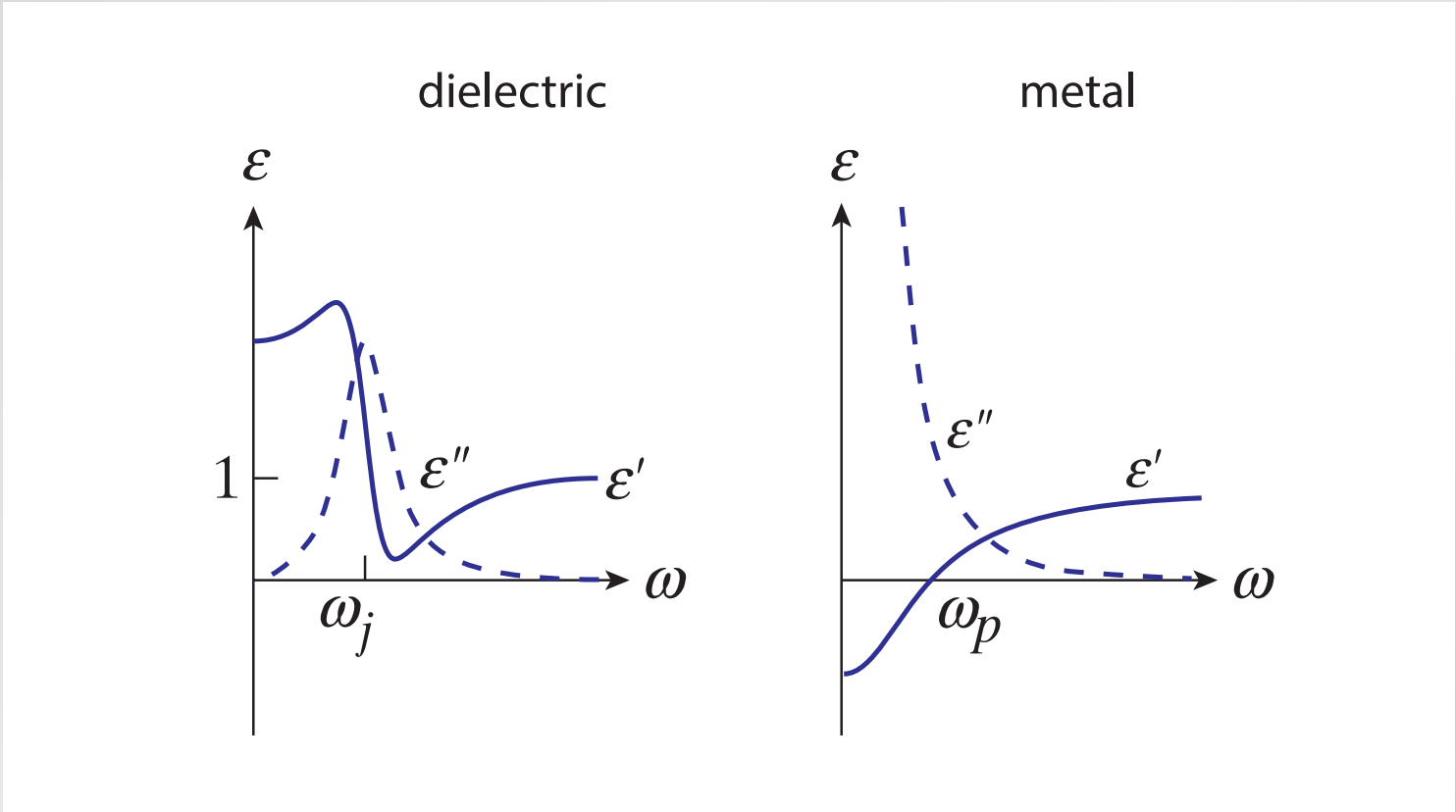
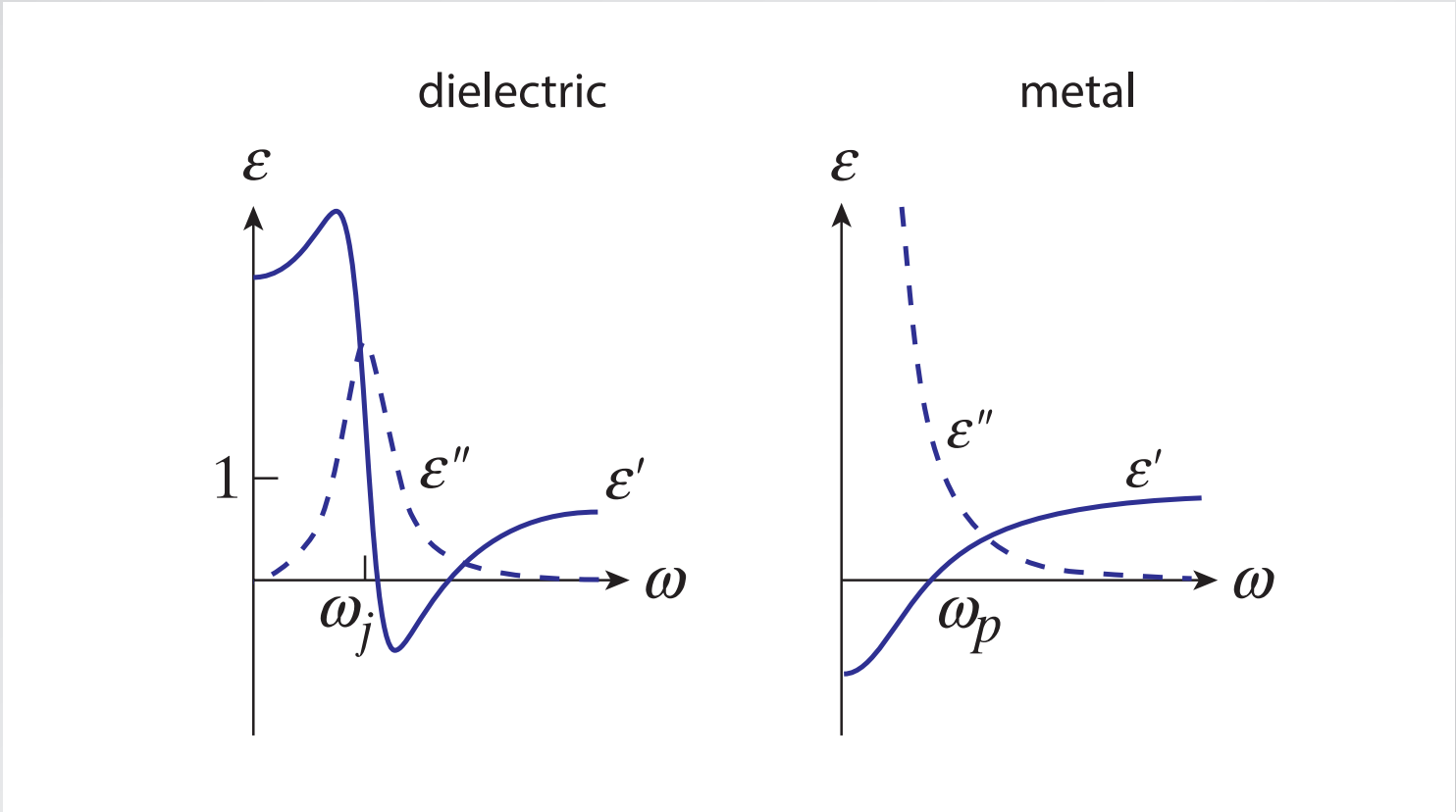


Lorentz and Drude models



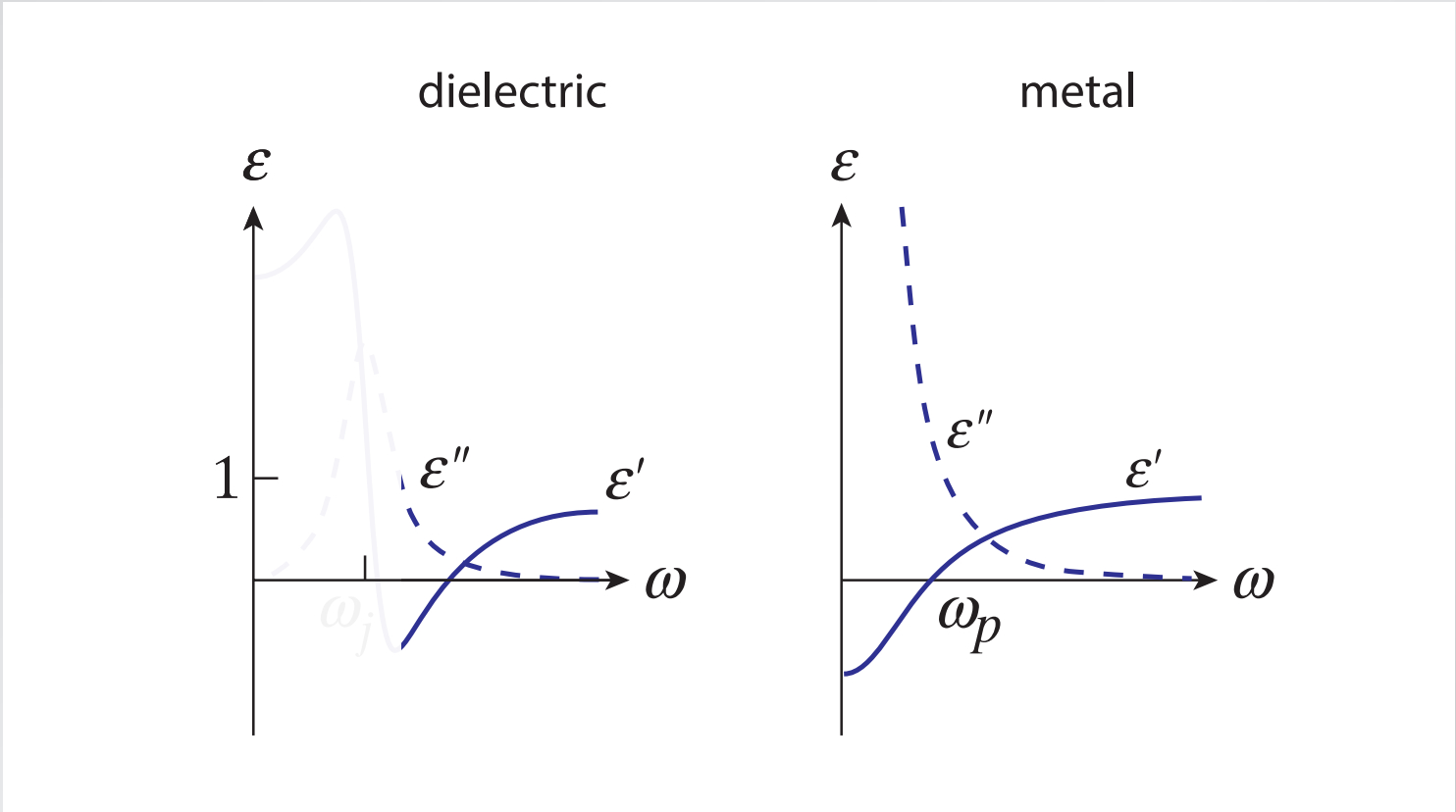
Lorentz and Drude models

for a strong (dielectric) resonance ε can become negative



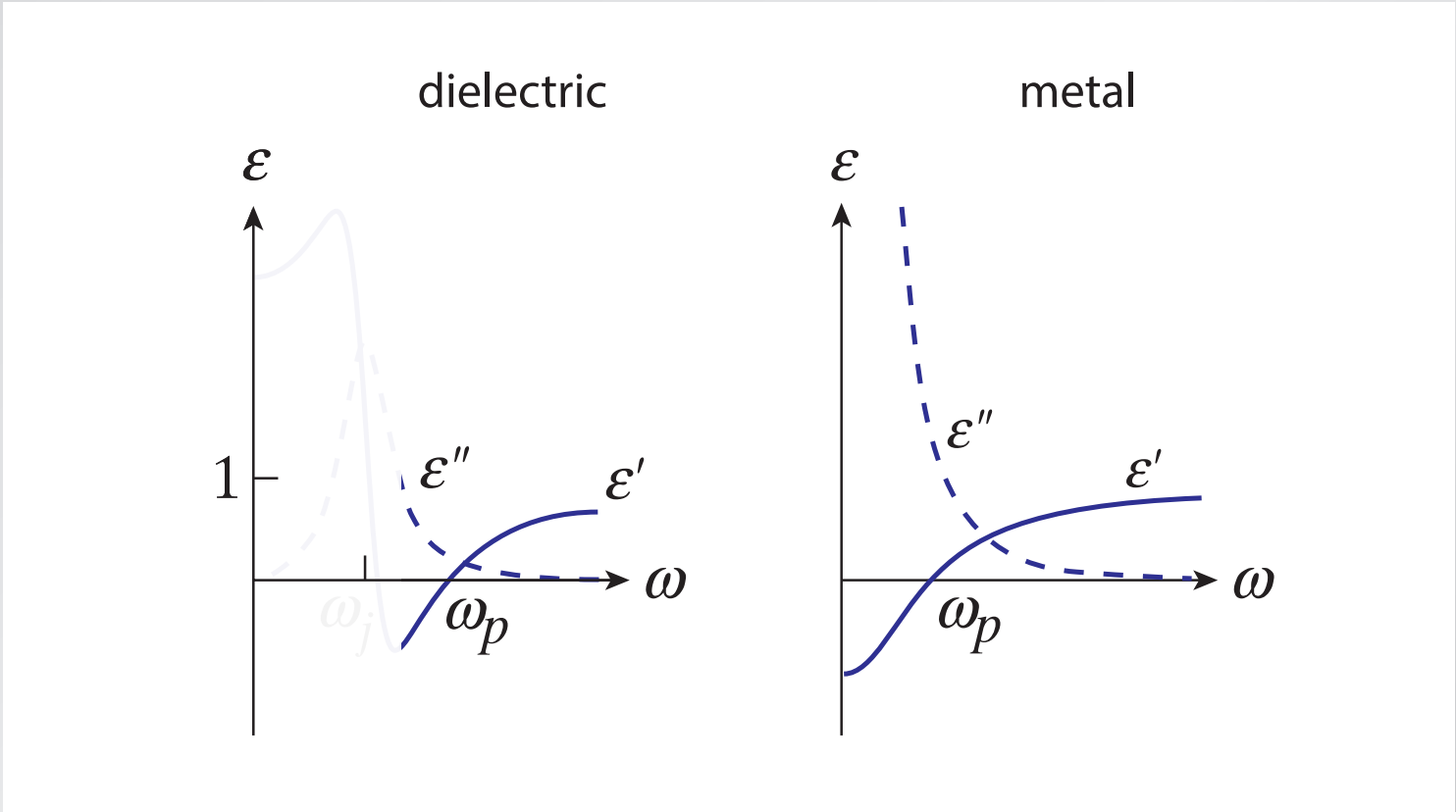
Lorentz and Drude models

valence electrons in dielectric then behave like a plasma



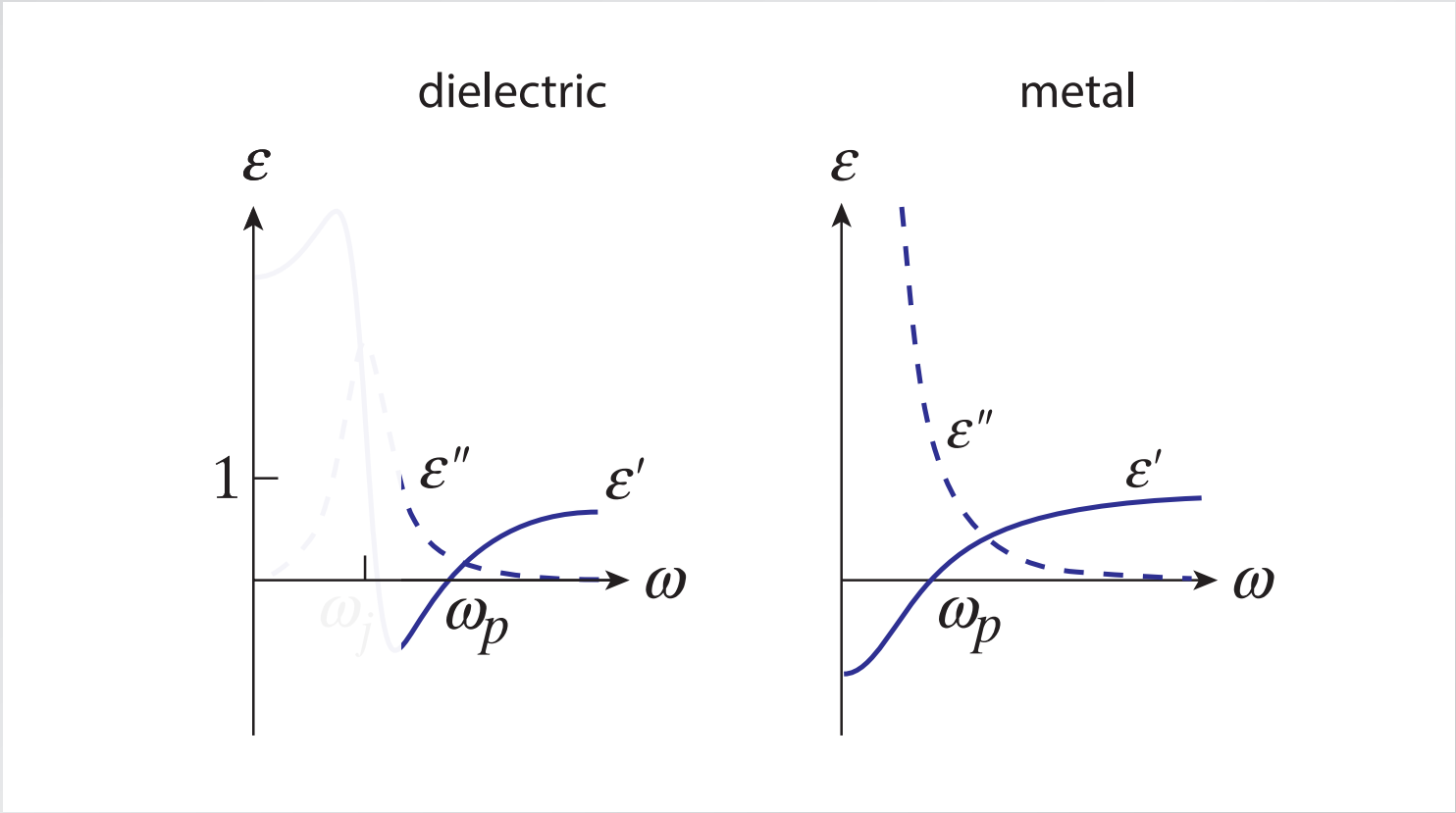
Lorentz and Drude models

with plasma frequency above the resonance



Lorentz and Drude models

(and far below the UV region)



Index also determined by magnetic response

$$n = \sqrt{\epsilon\mu}$$

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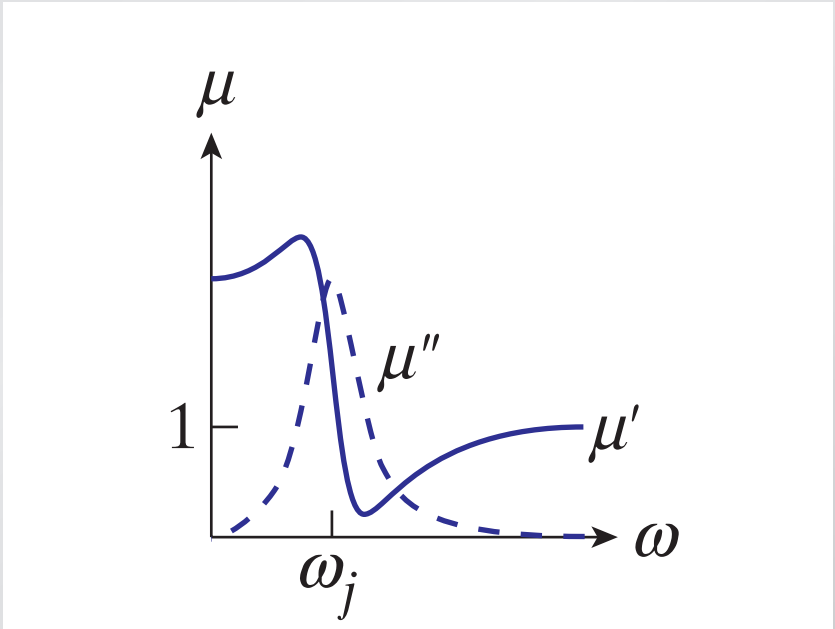
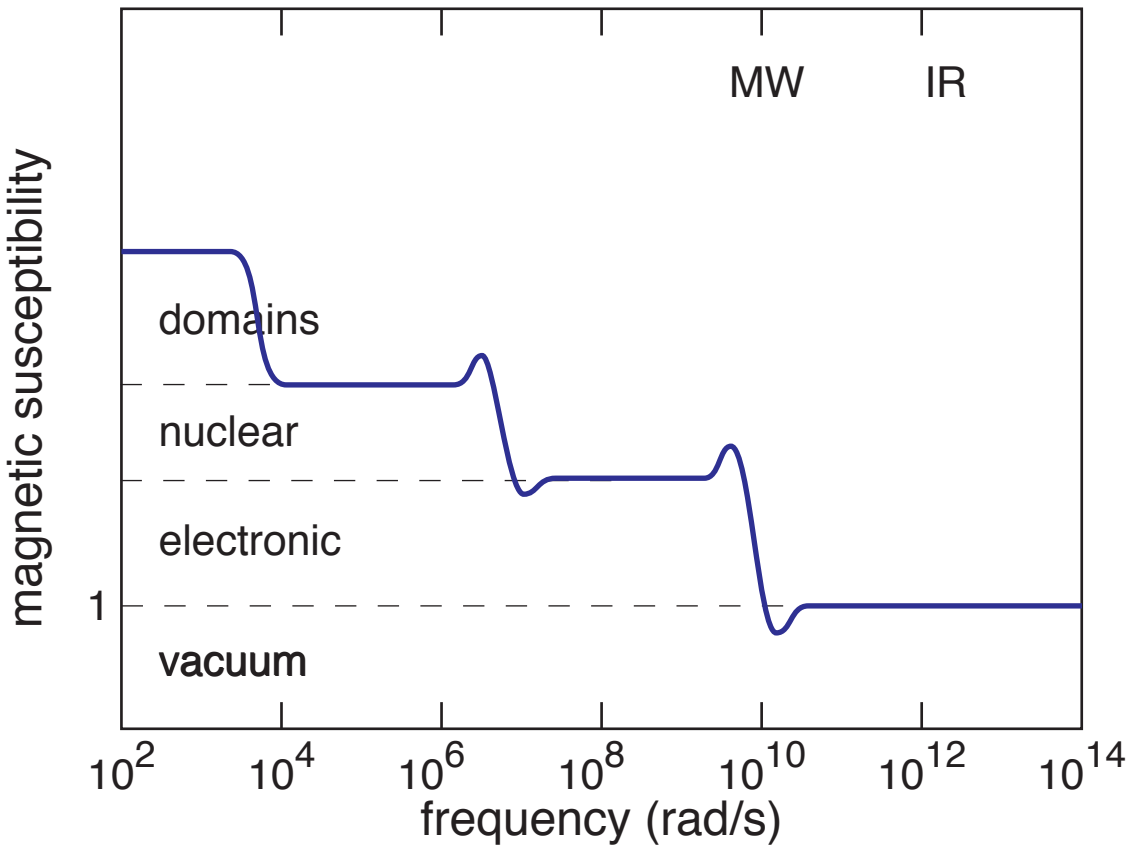
and magnetic response shows similar resonances

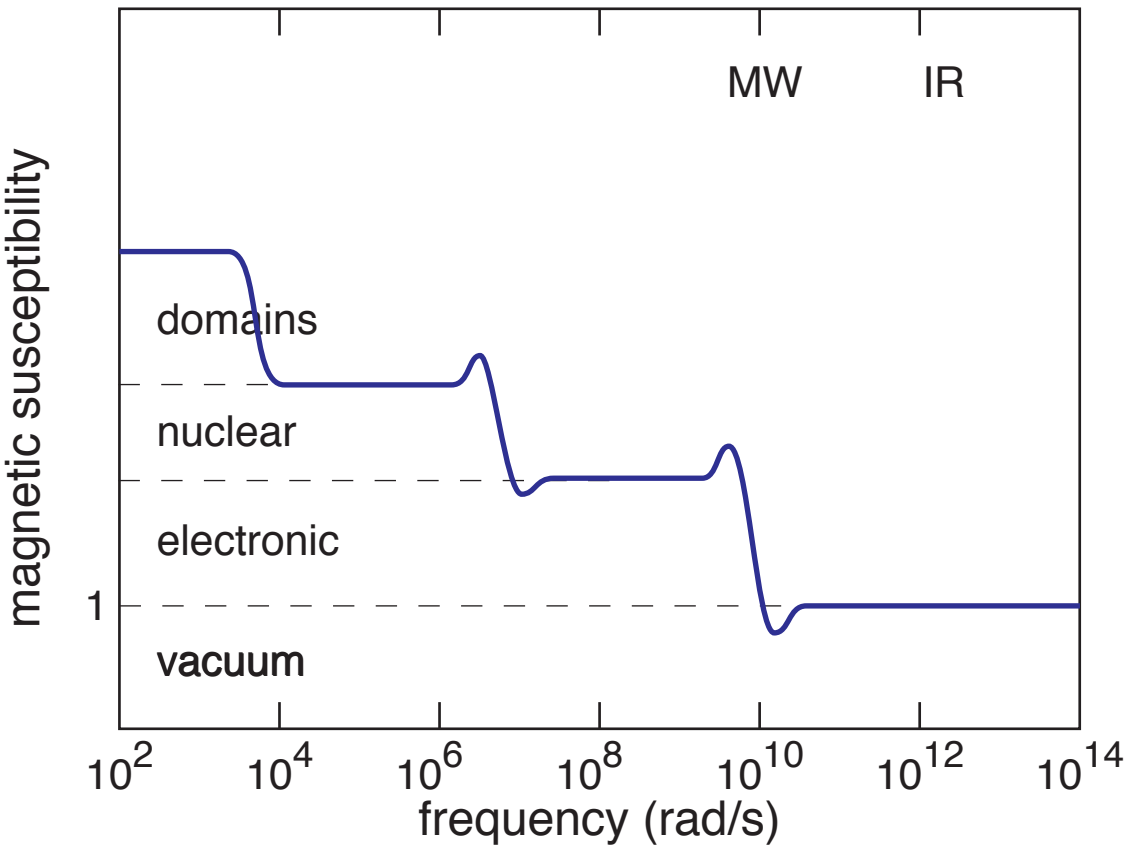


Magnetic response



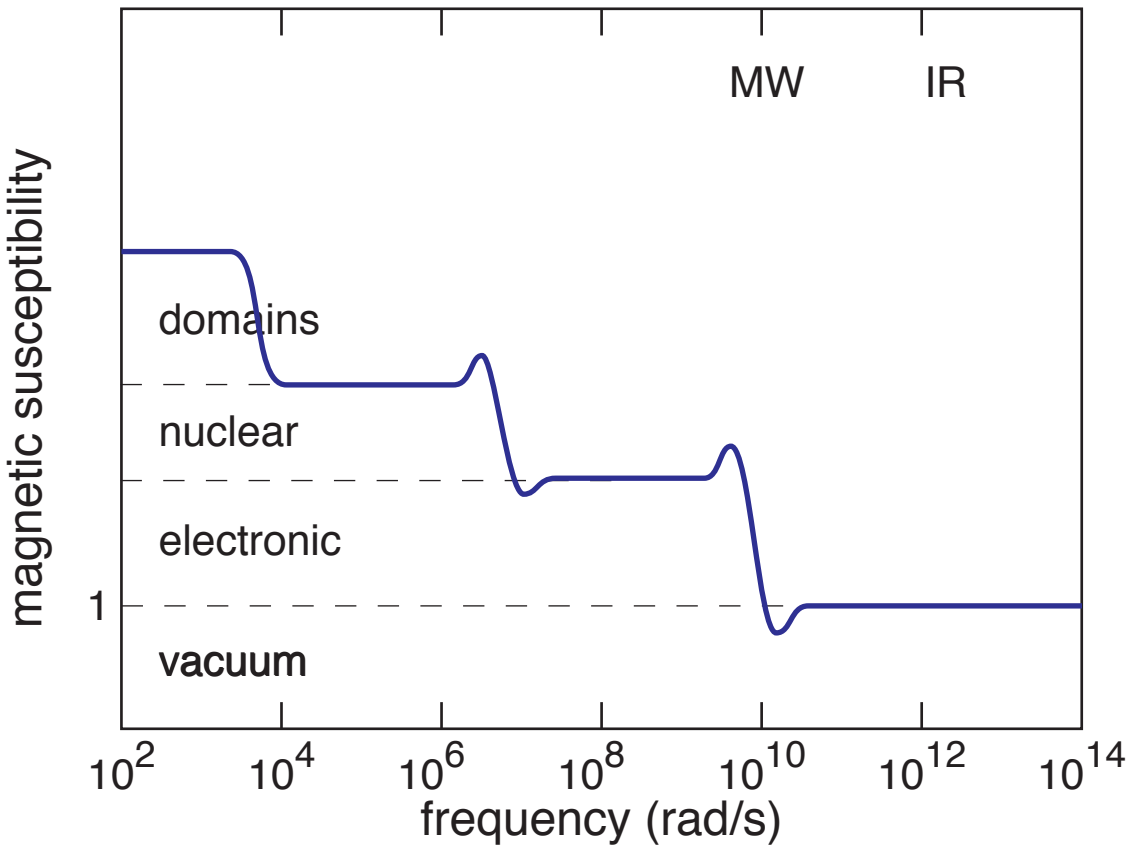
Magnetic response

but magnetic resonances occur below optical frequencies



Magnetic response

so, in optical regime, $\mu \approx 1$



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What happens when $\mathrm{Re}\epsilon$ and/or $\mathrm{Re}\mu$ is negative?

Write complex quantities as

$$\varepsilon = |\varepsilon| e^{i\theta} \qquad \mu = |\mu| e^{i\phi}$$

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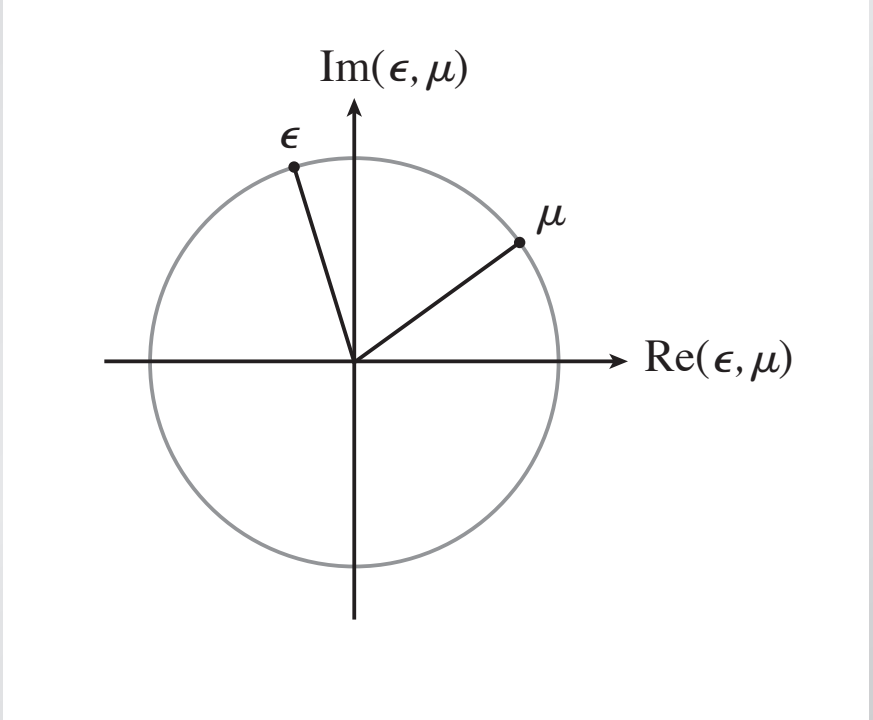

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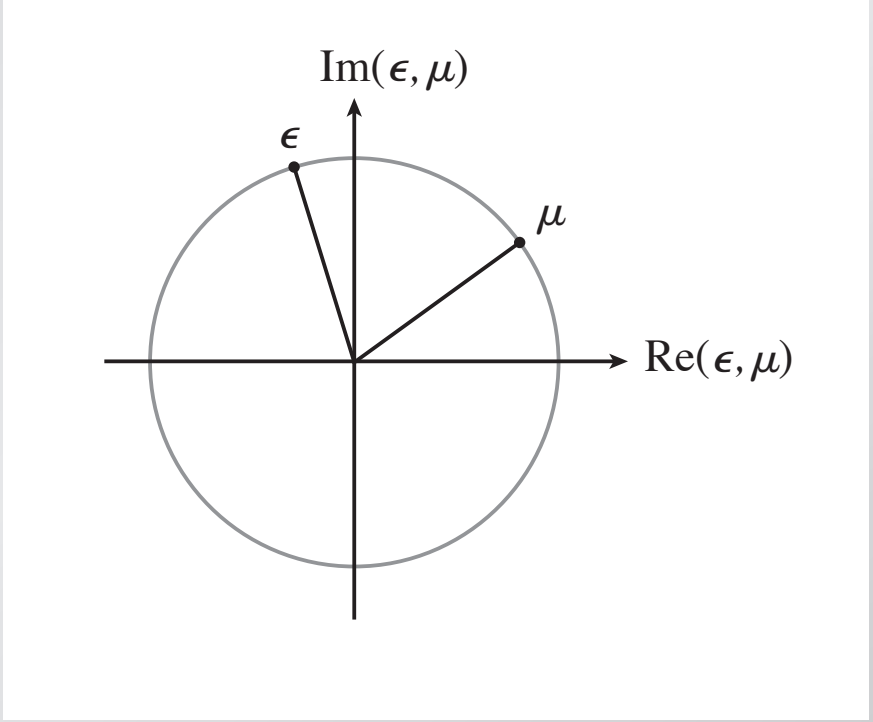

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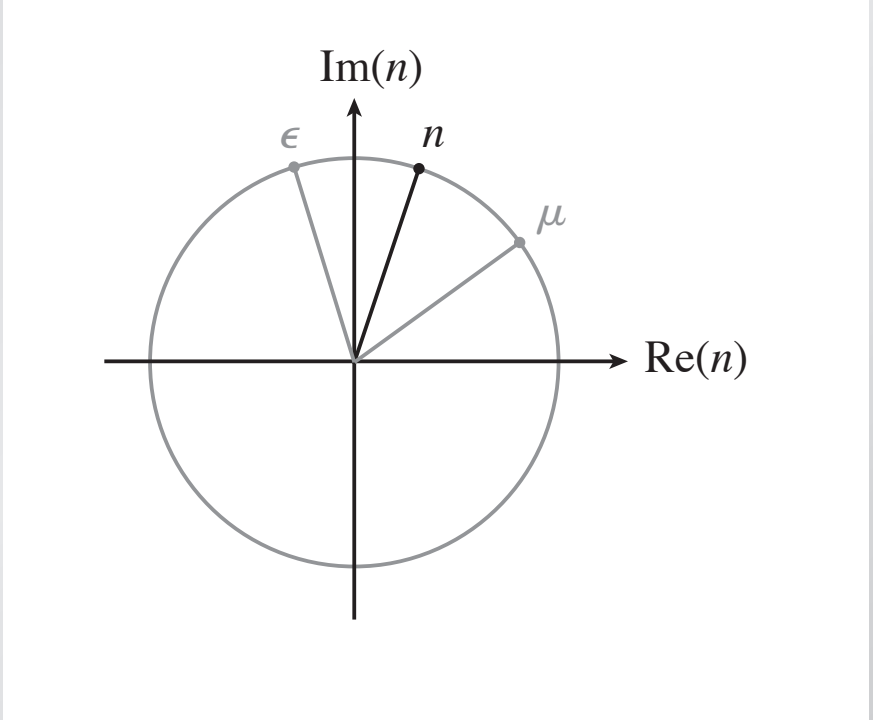

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Q: Is this only possible value?

1. yes
2. no, there's one more
3. there are many more
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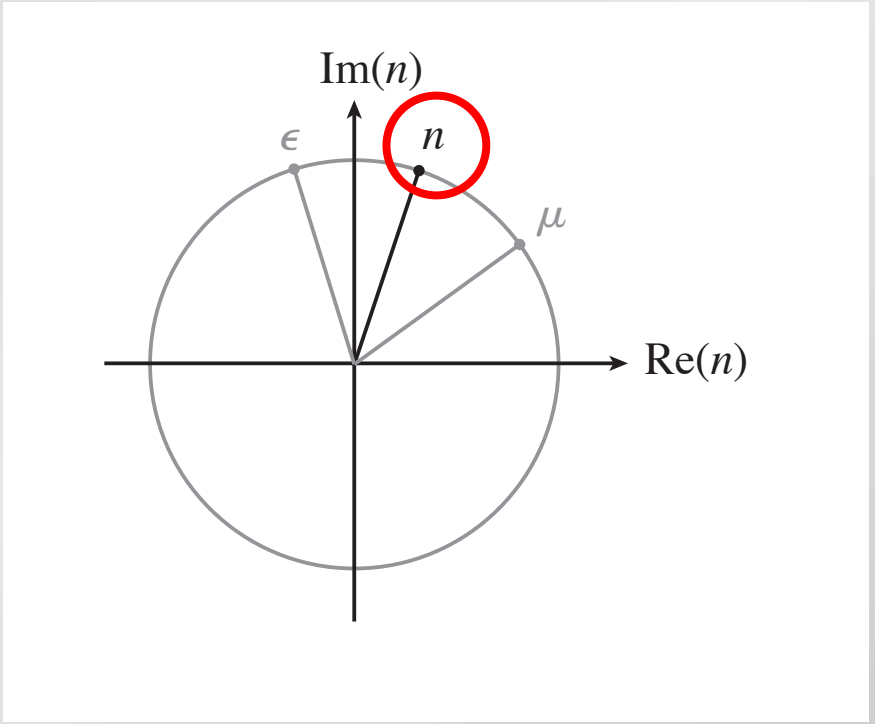

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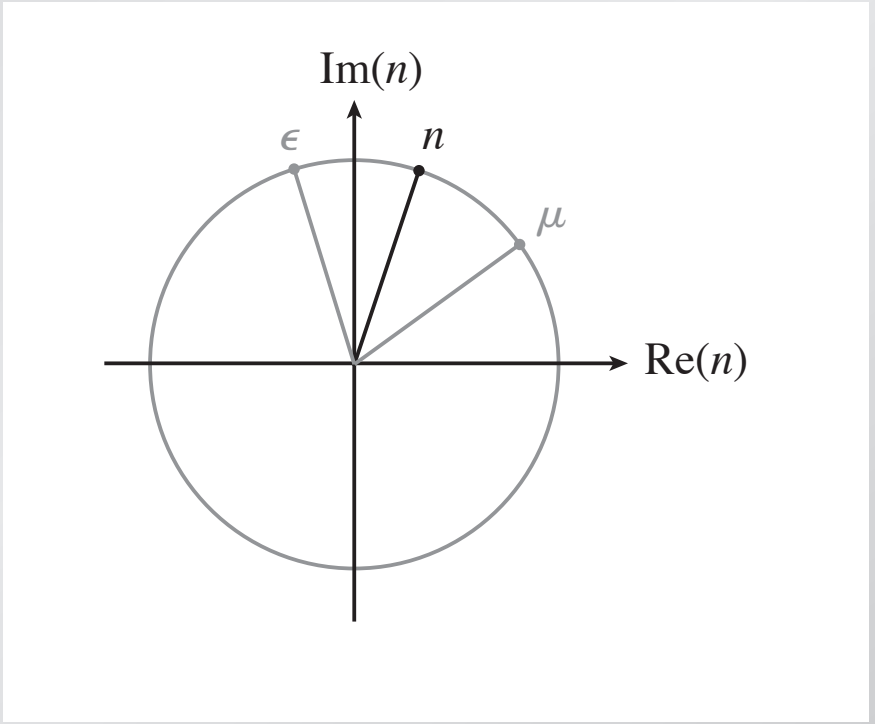

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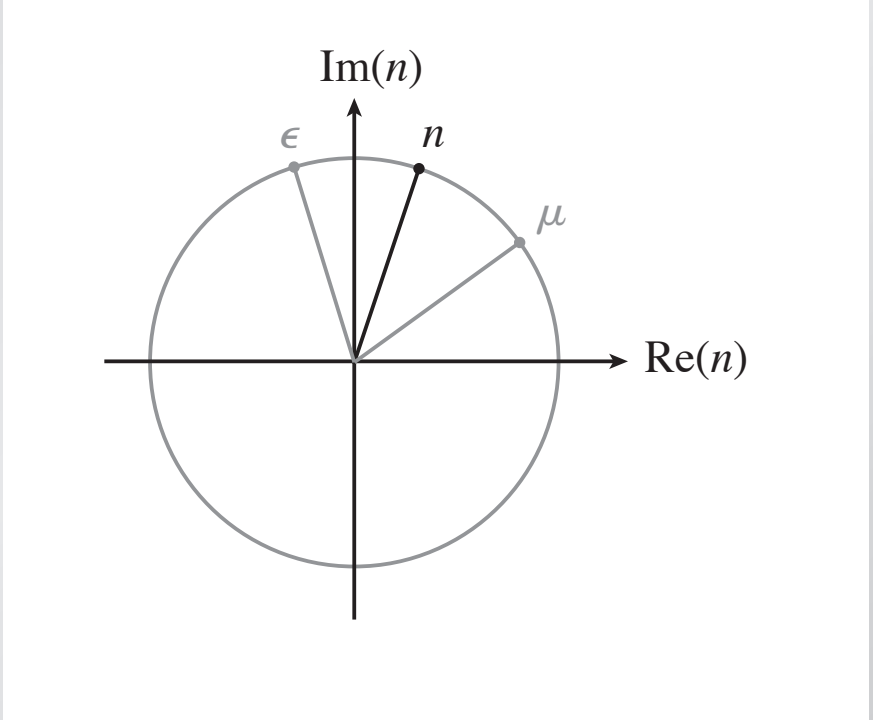
Can add 2π to exponent

$$e^{+i(\theta+\phi)} = e^{+i[\theta+\phi+2\pi]}$$



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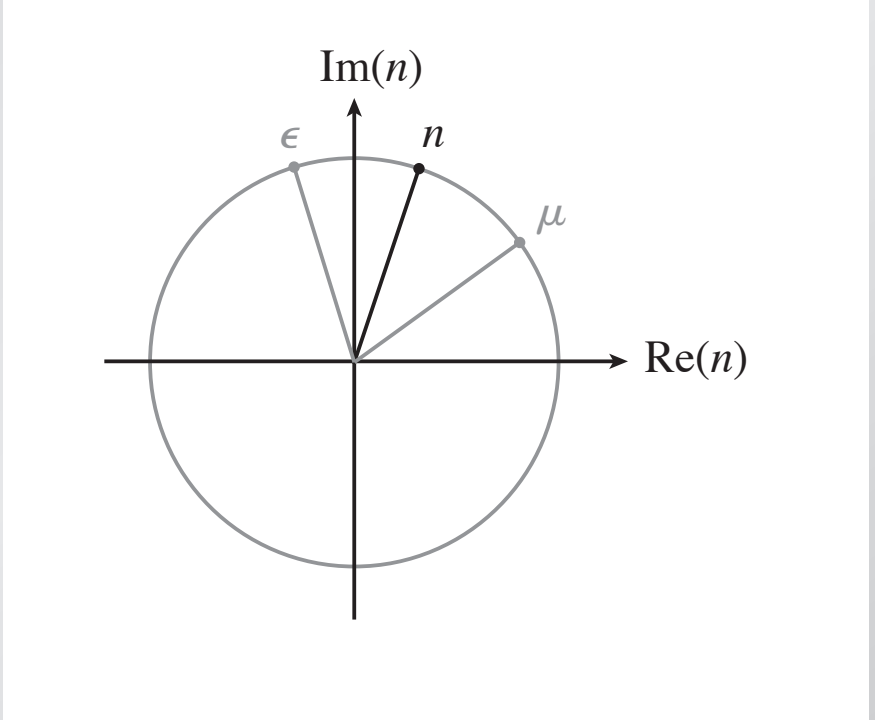

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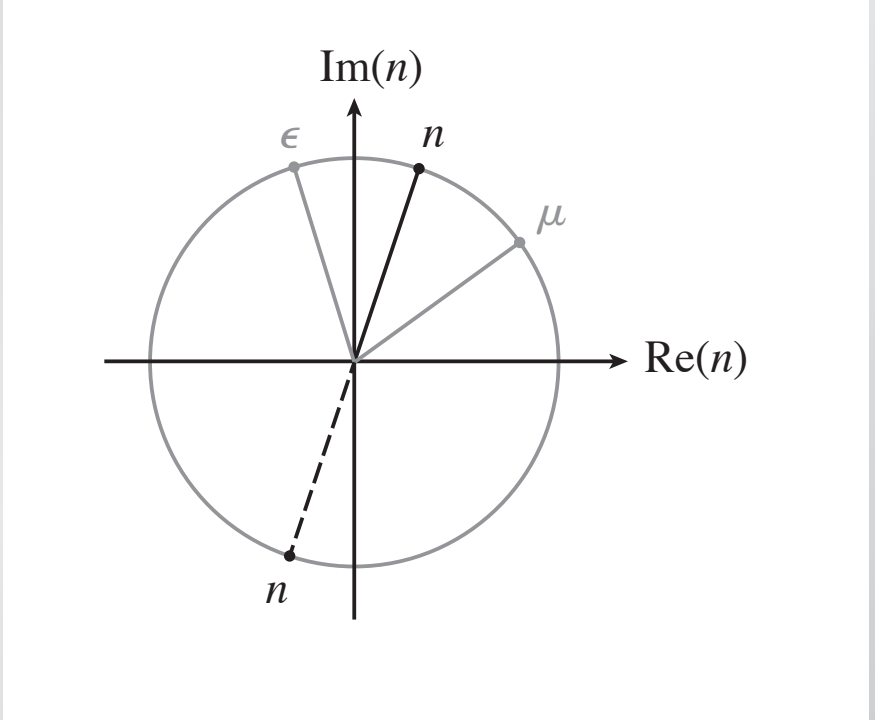

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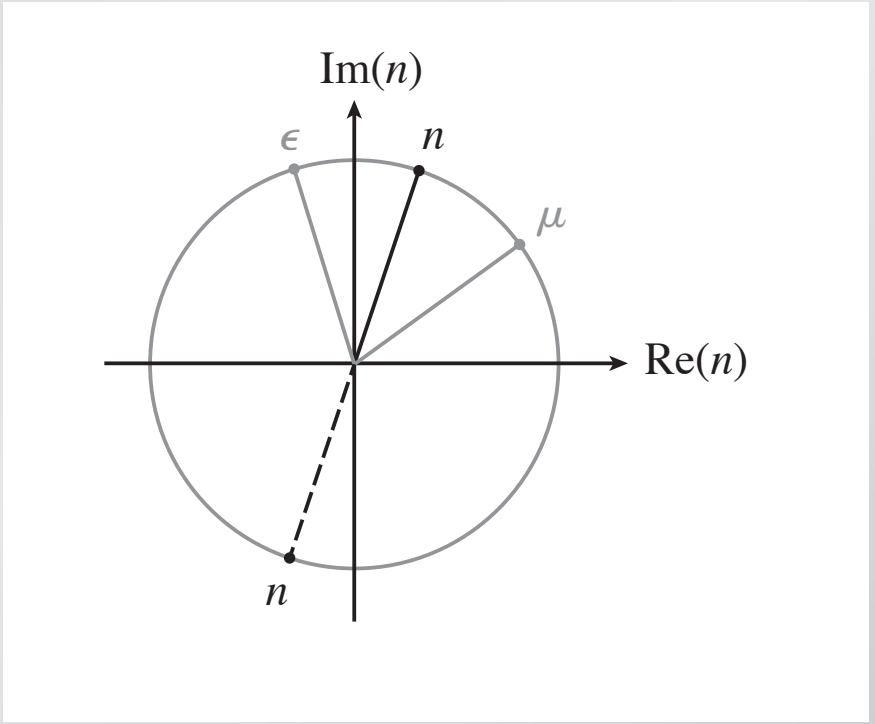

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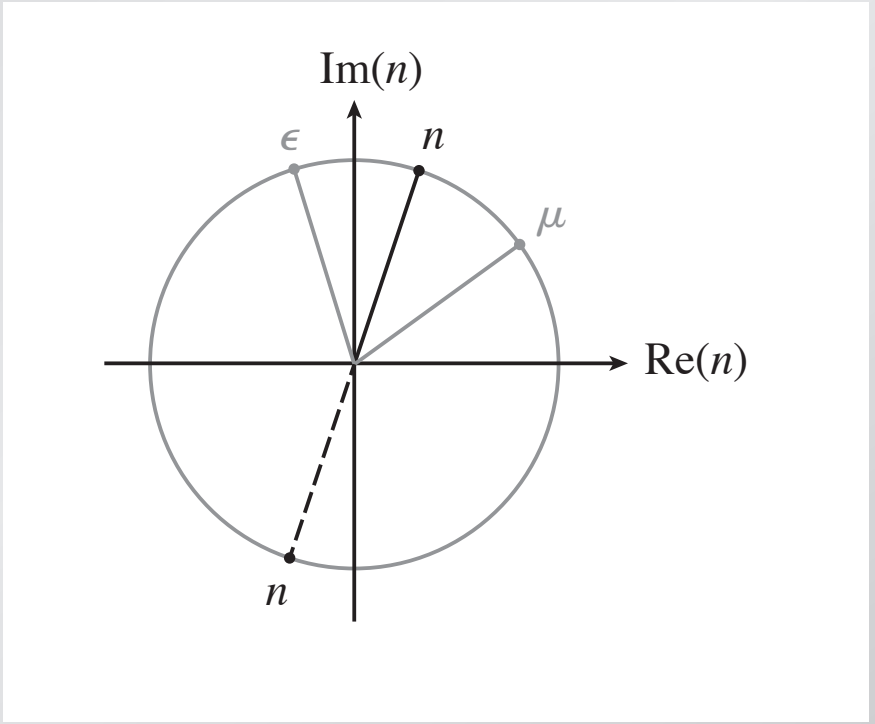

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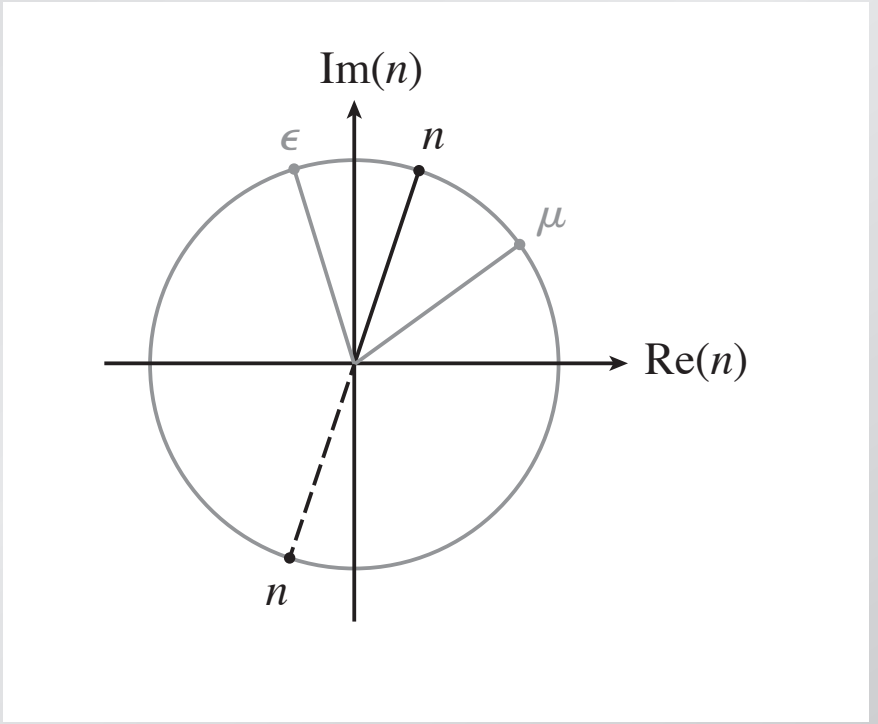

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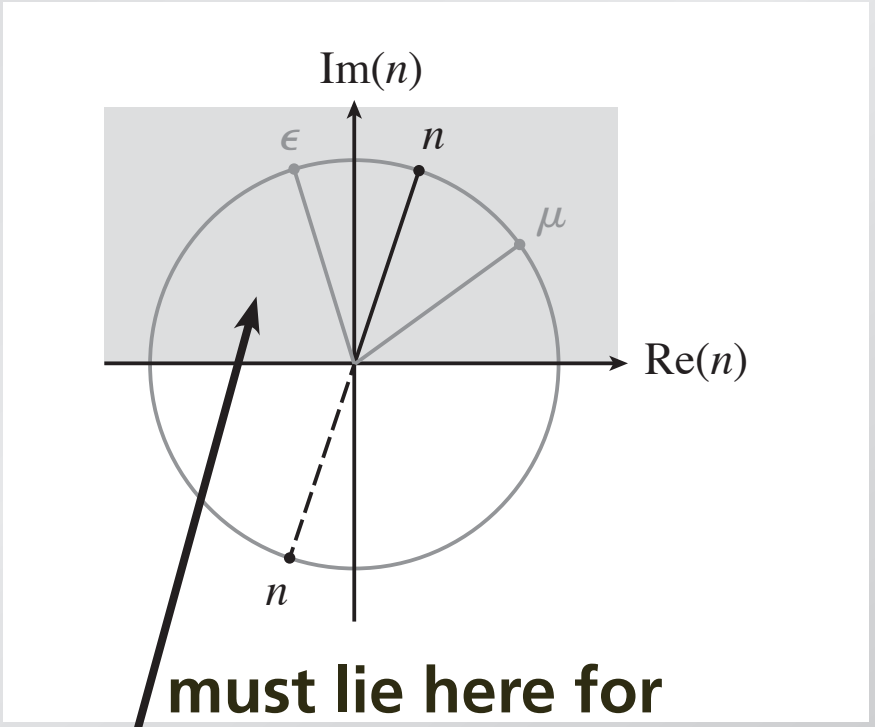

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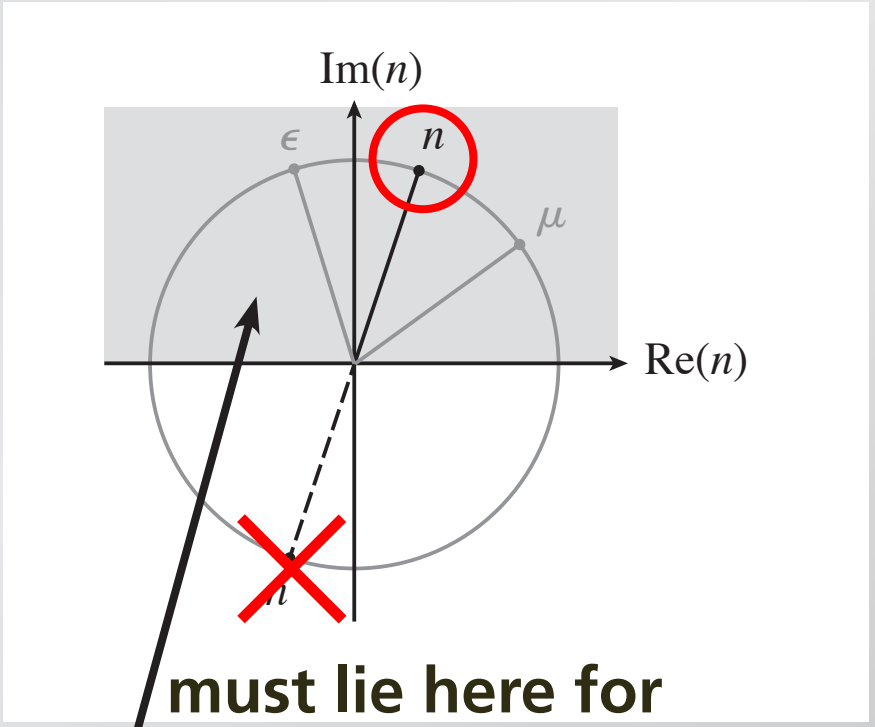

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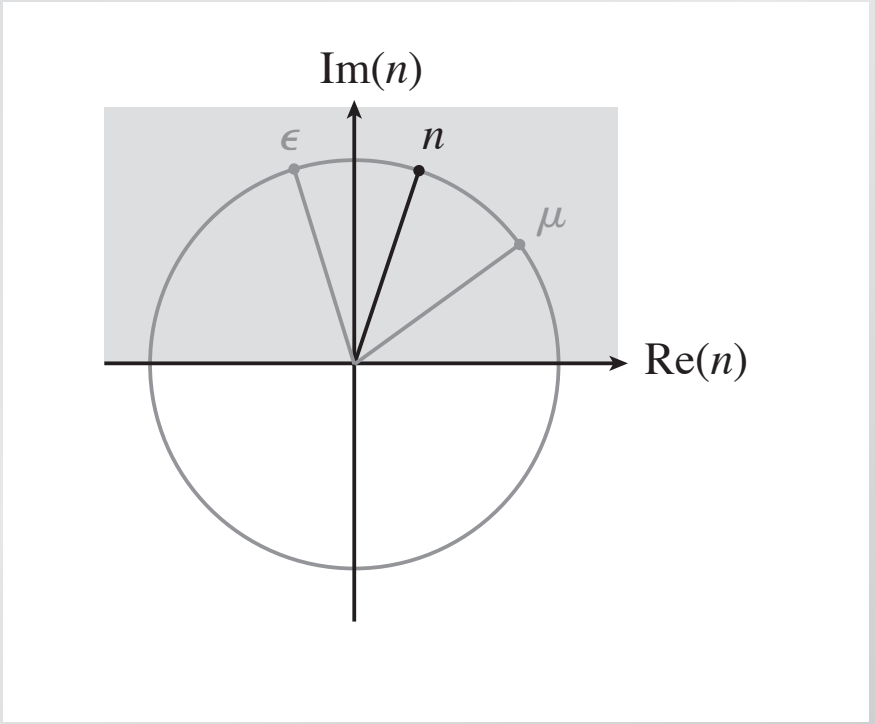

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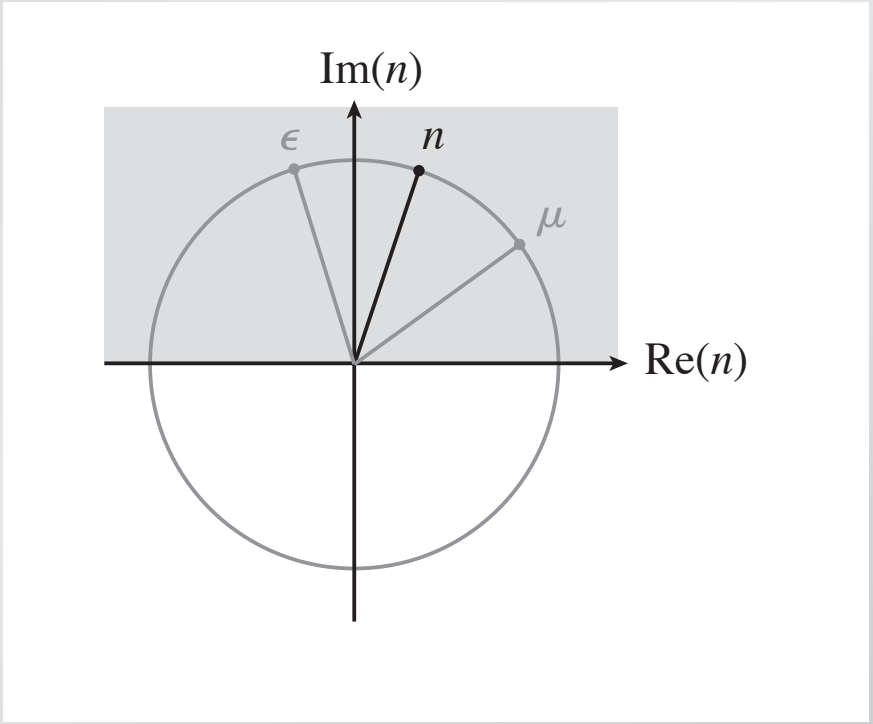

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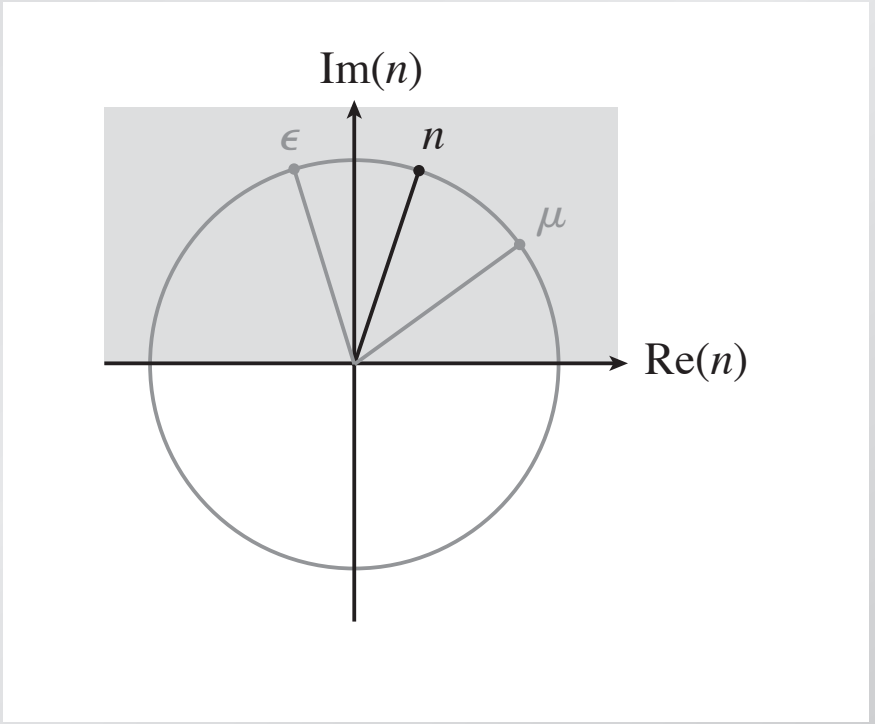
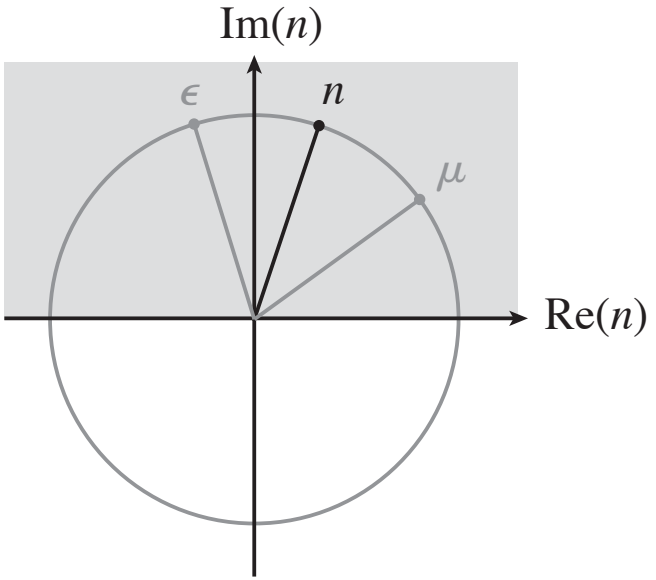
4. it depends ✔



Q: Is this the only possible value?

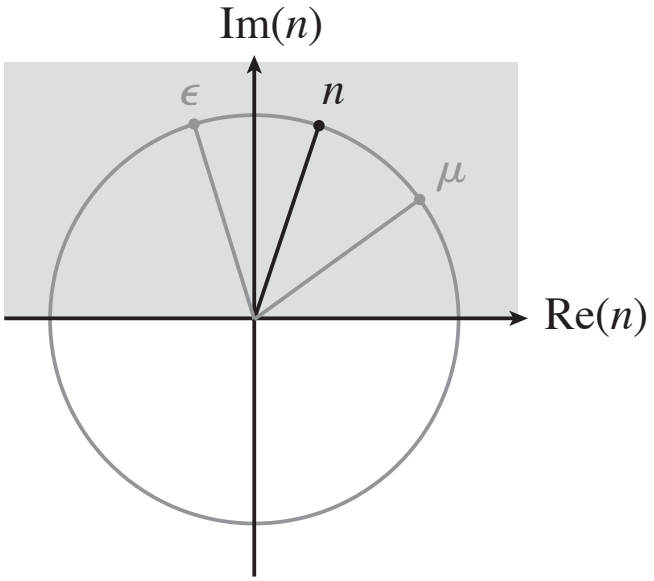
1. yes ✔
2. no, there's one more ✔
3. there are many more ✔
4. it depends ✔



To find n (passive materials):

1. Draw line that bisects ϵ and μ

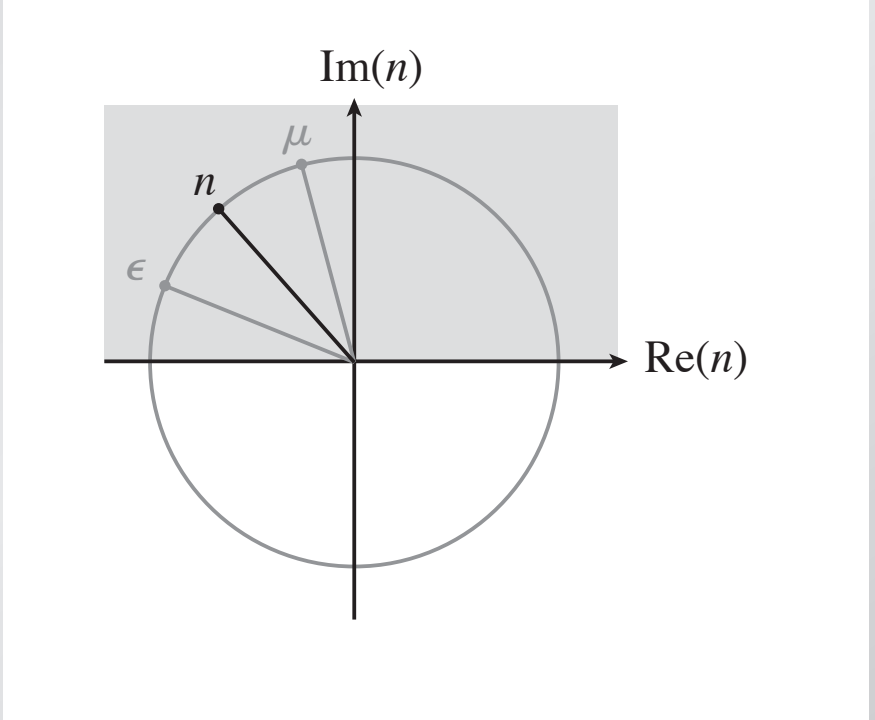
2. Choose upper branch



What happens when $\mathrm{Re}\epsilon$ and/or $\mathrm{Re}\mu$ is negative?

For certain values of ϵ and μ

we can get a *negative* $\mathrm{Re}(n)$!

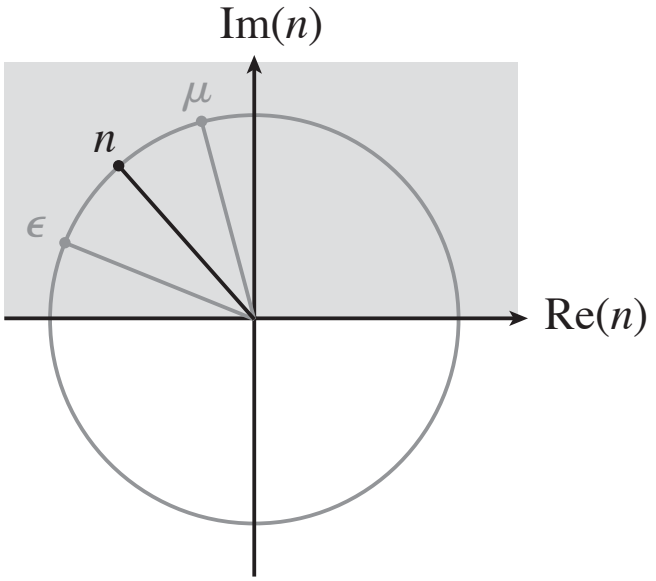


Q: Must both $\mathrm{Re}\epsilon < 0$ and $\mathrm{Re}\mu < 0$

to get a negative $\mathrm{Re}(n)$?

1. yes

2. no

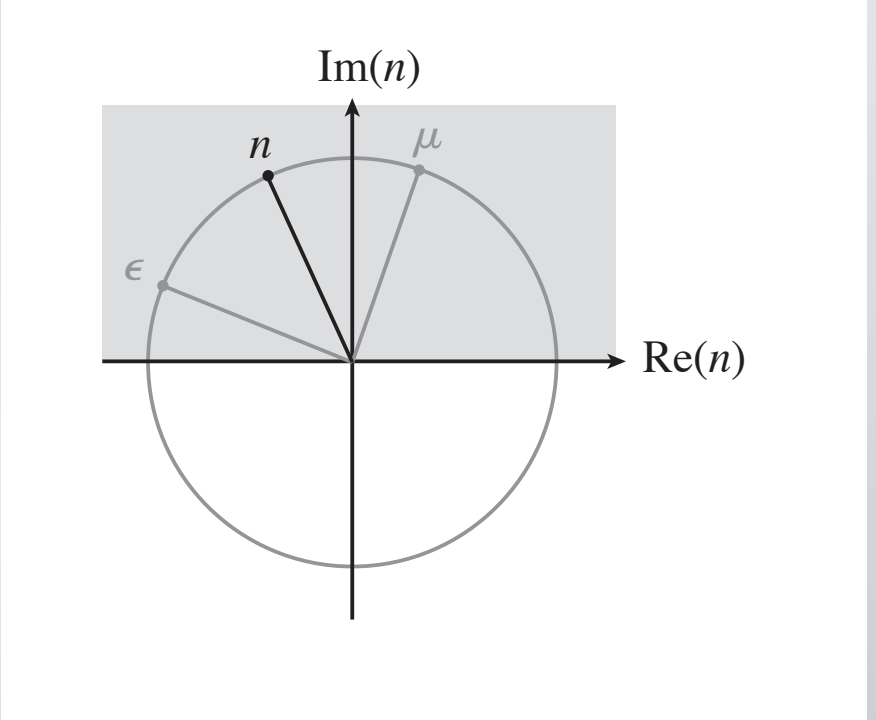


Q: Must both $\mathrm{Re}\epsilon < 0$ and $\mathrm{Re}\mu < 0$

to get a negative $\mathrm{Re}(n)$?

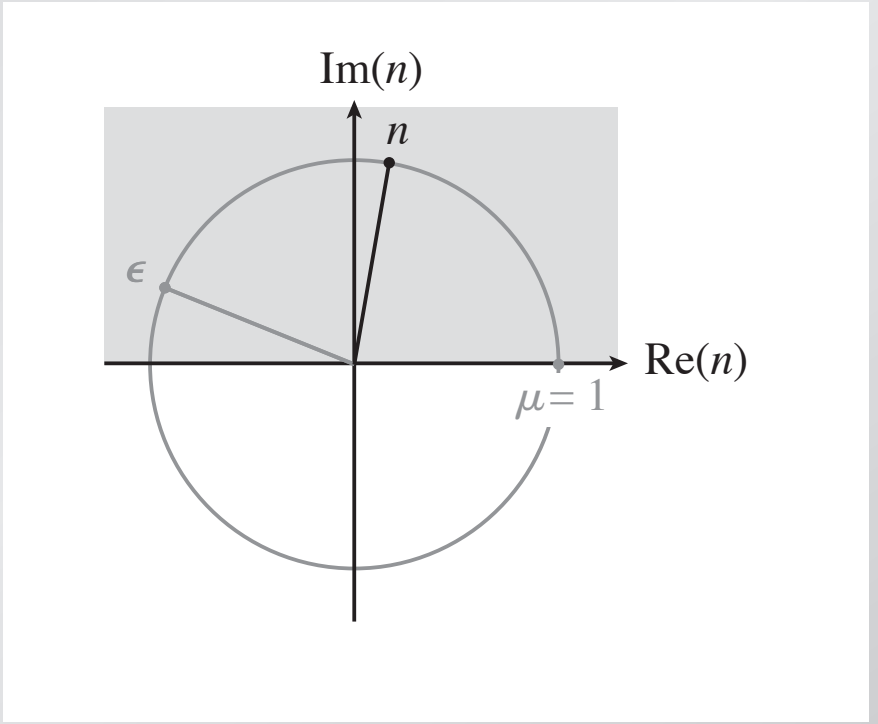
1. yes

2. no ✔



However, need magnetic response

to achieve $\mathrm{Re}(n) \leq 0$!



What happens when $\mathrm{Re}(n) < 0$?

What happens when $\mathrm{Re}(n) < 0$?

Remember

$$k = \frac{2\pi n}{\lambda_o} = \frac{2\pi(n' + in'')}{\lambda_o} = k' + ik''$$

$$E = E_o e^{i(kx-\omega t)} = E_o e^{-k''x} e^{i(k'x-\omega t)}$$

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When $\mathrm{Re}(n) < 0$, $k' < 0$, and so phase velocity reversed!

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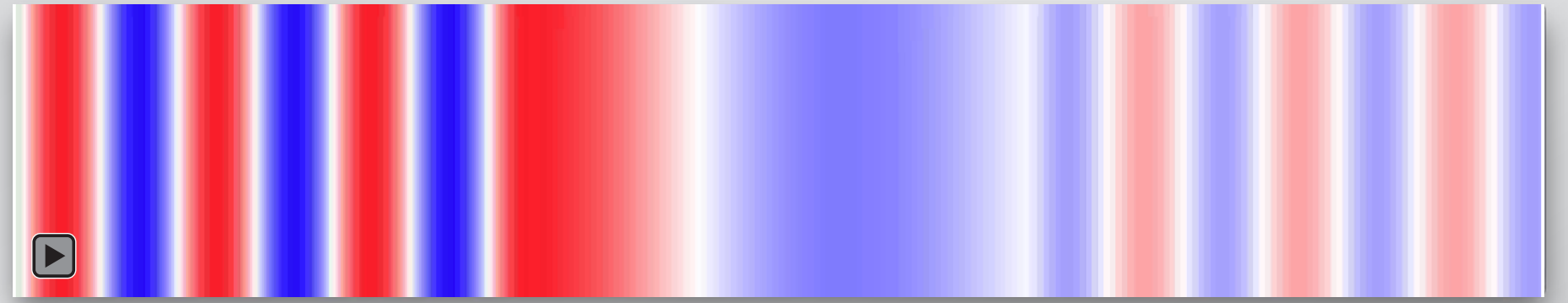

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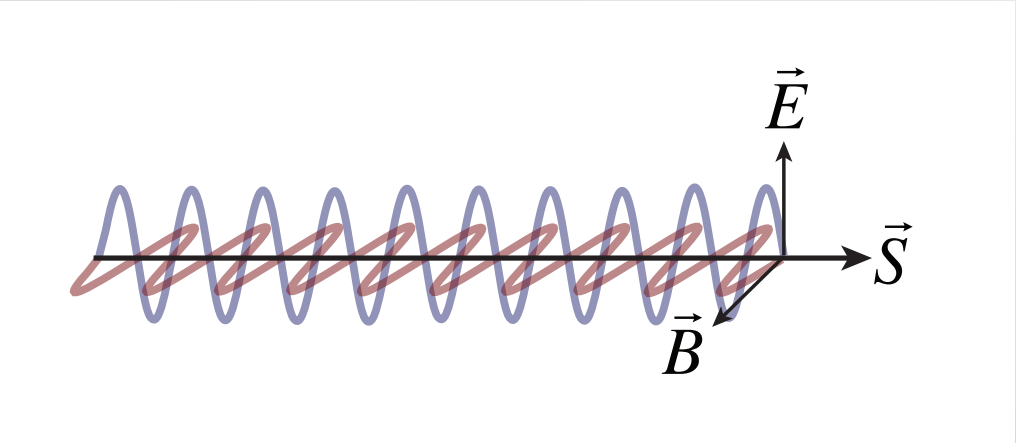

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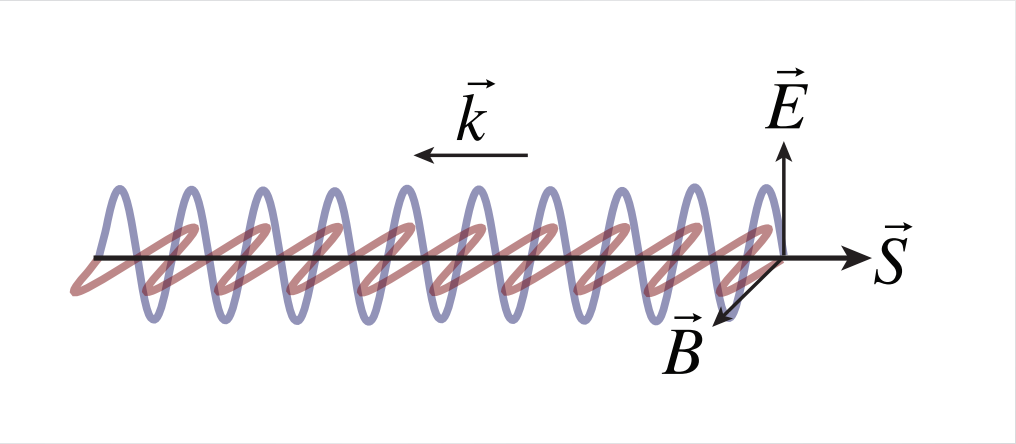

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What about causality?

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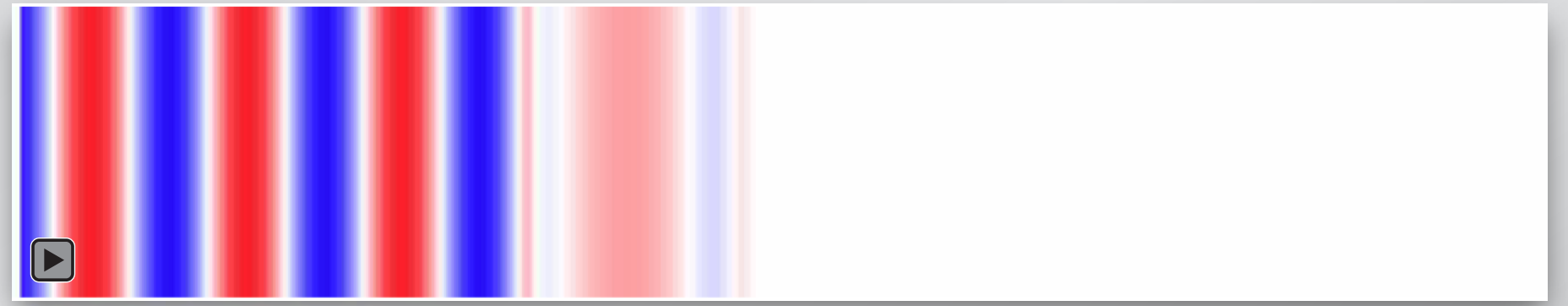

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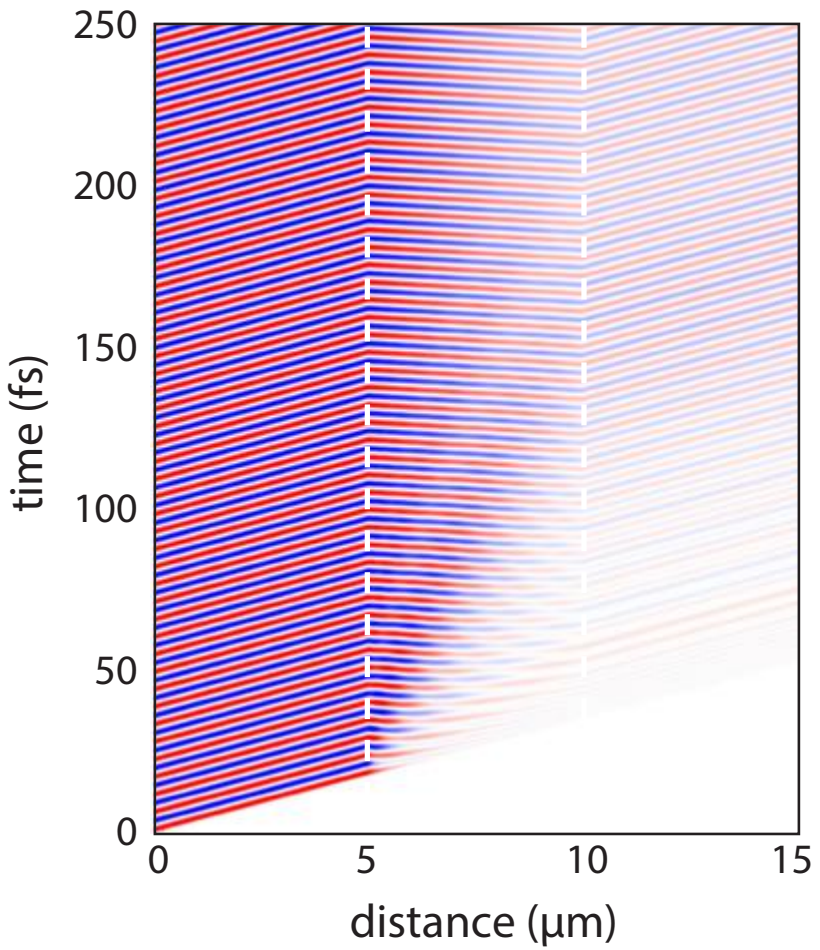
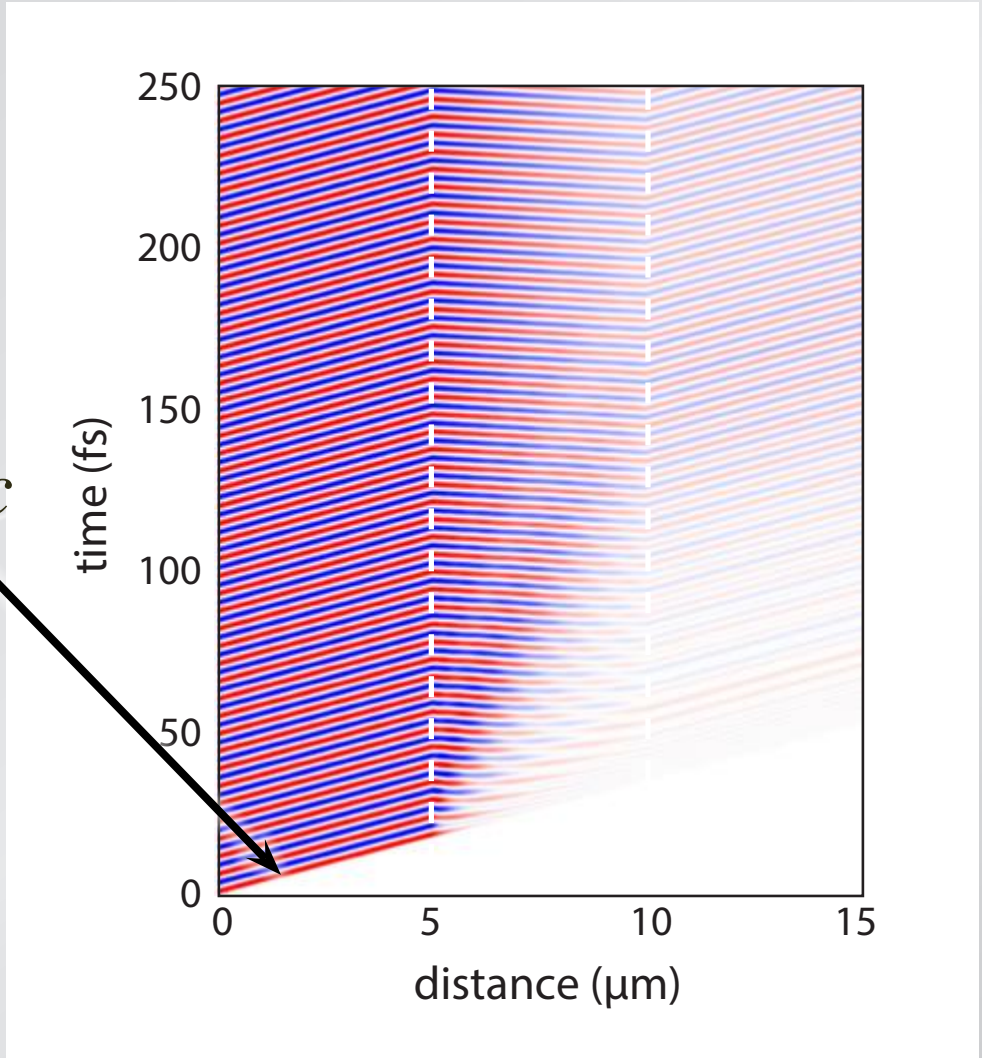


What about causality?

speed of light c



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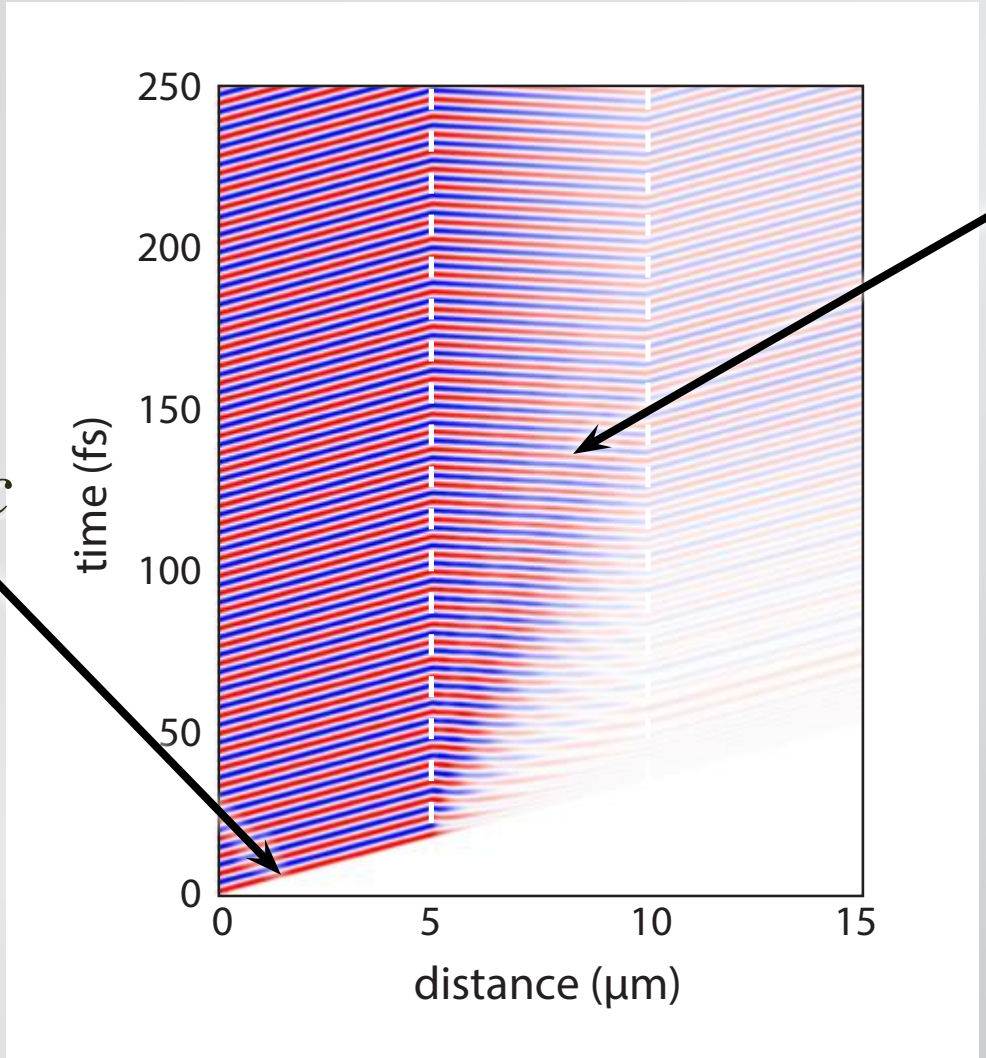


reverse phase propagation

speed of light c

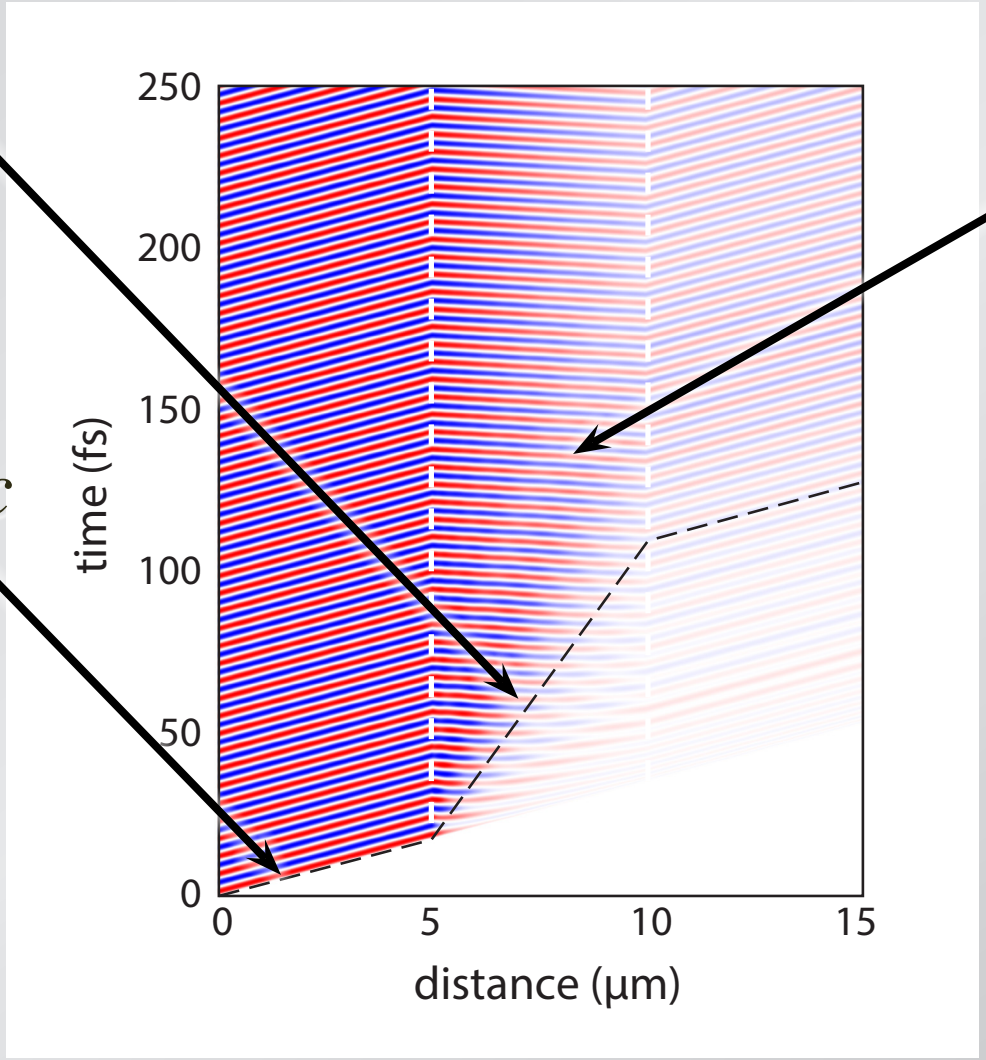
What about causality?

group velocity

$$v_g < c$$

speed of light c

reverse phase propagation



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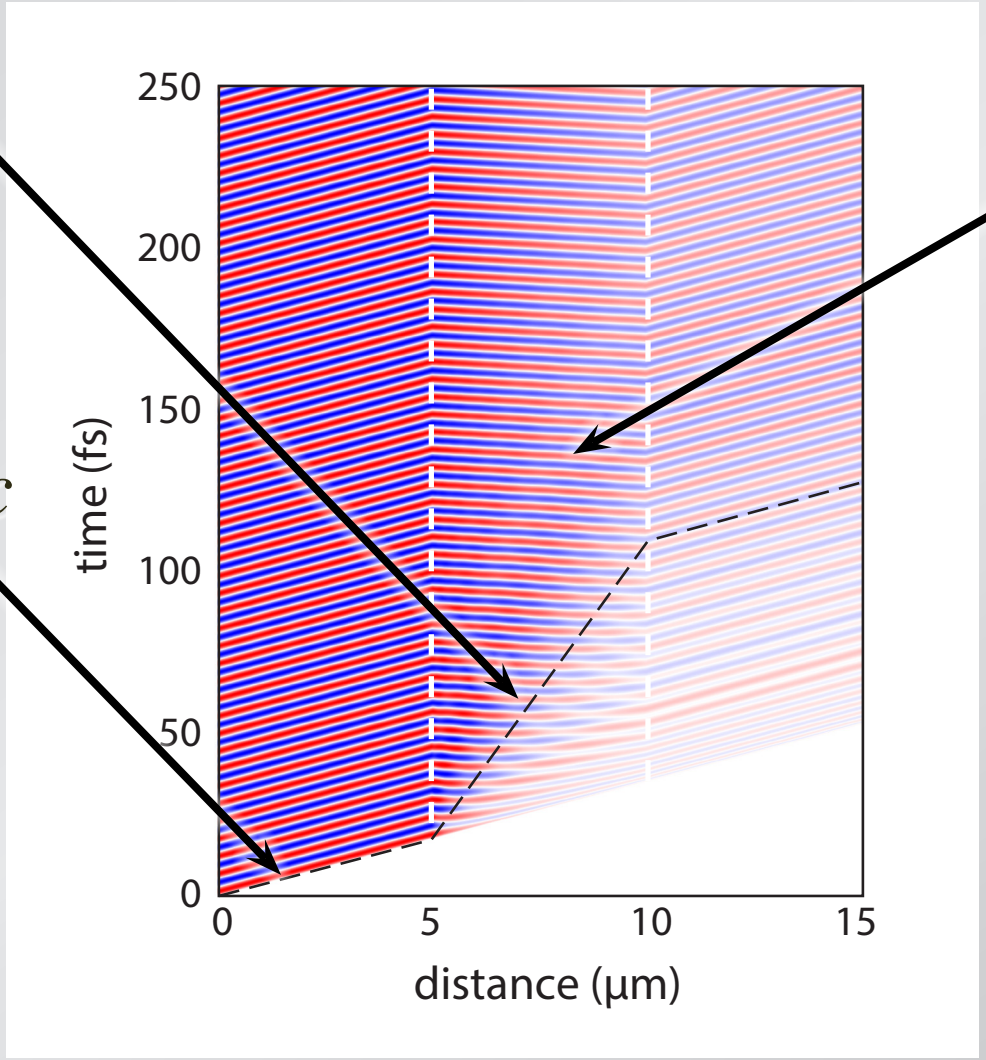

reverse phase propagation

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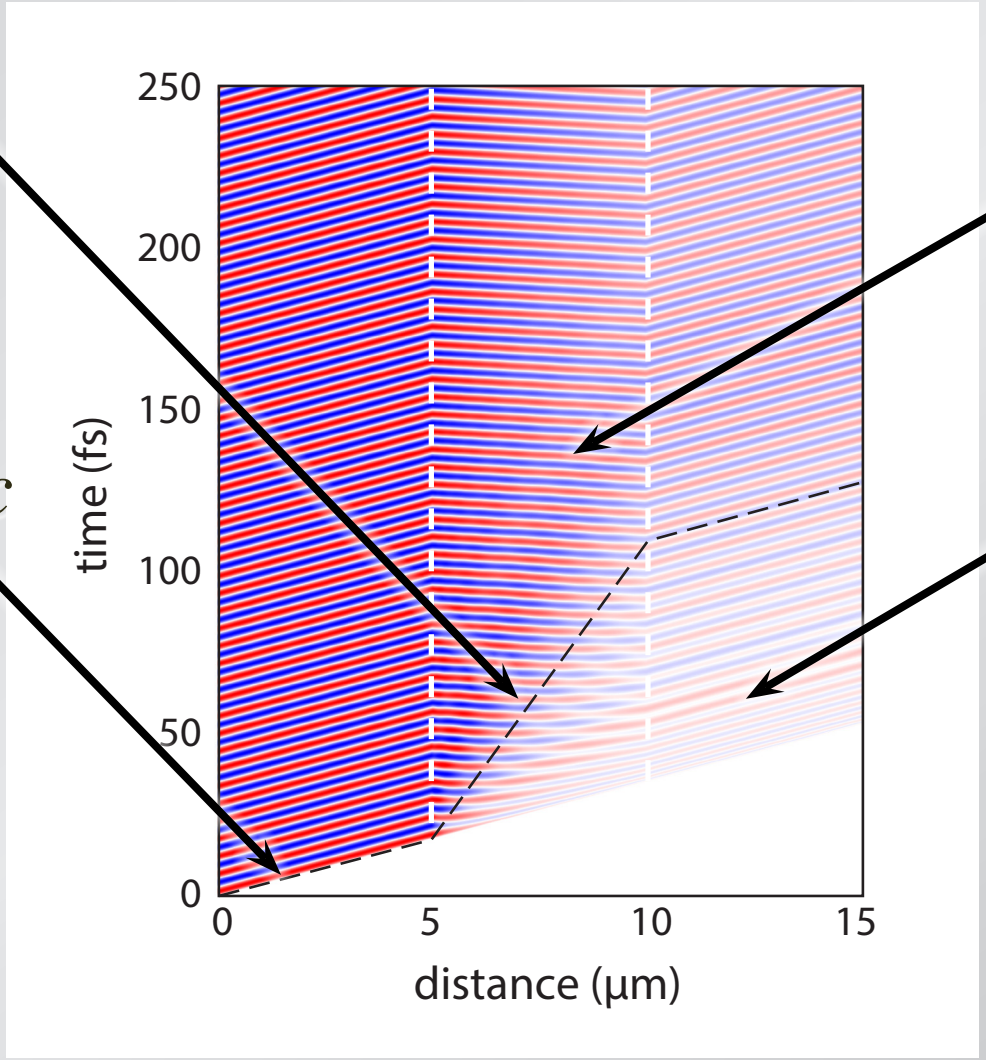
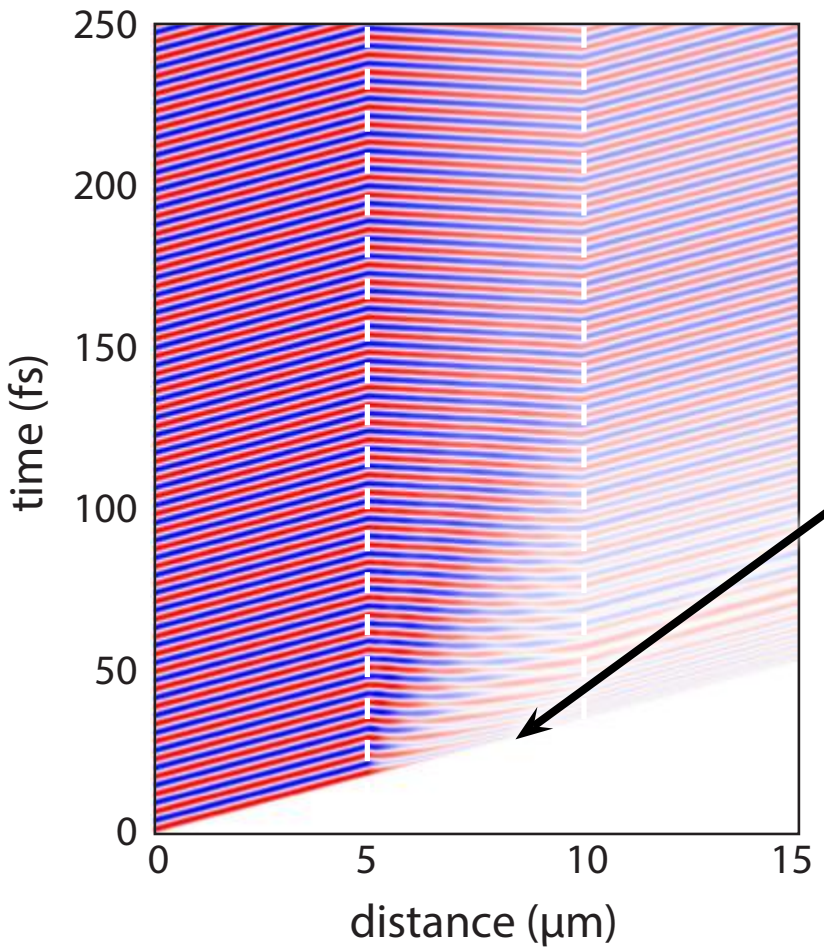


reverse phase propagation

high-frequency precursors

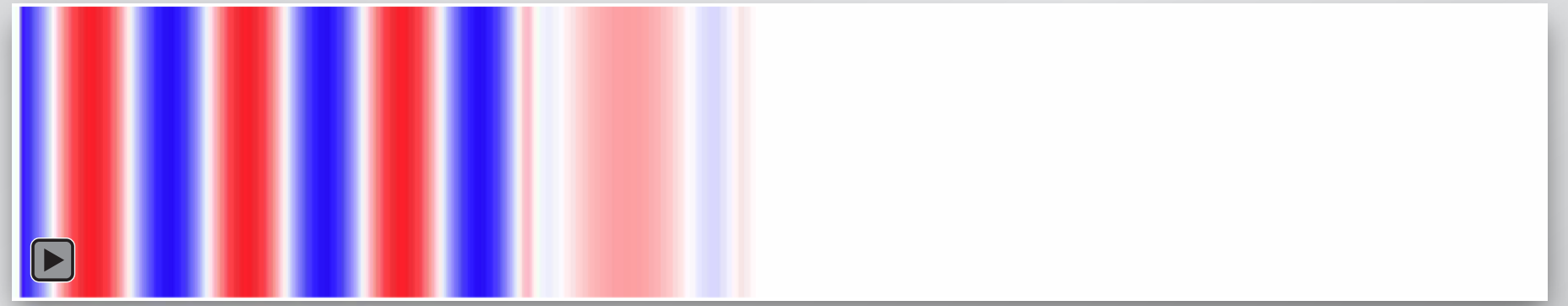
What about causality?



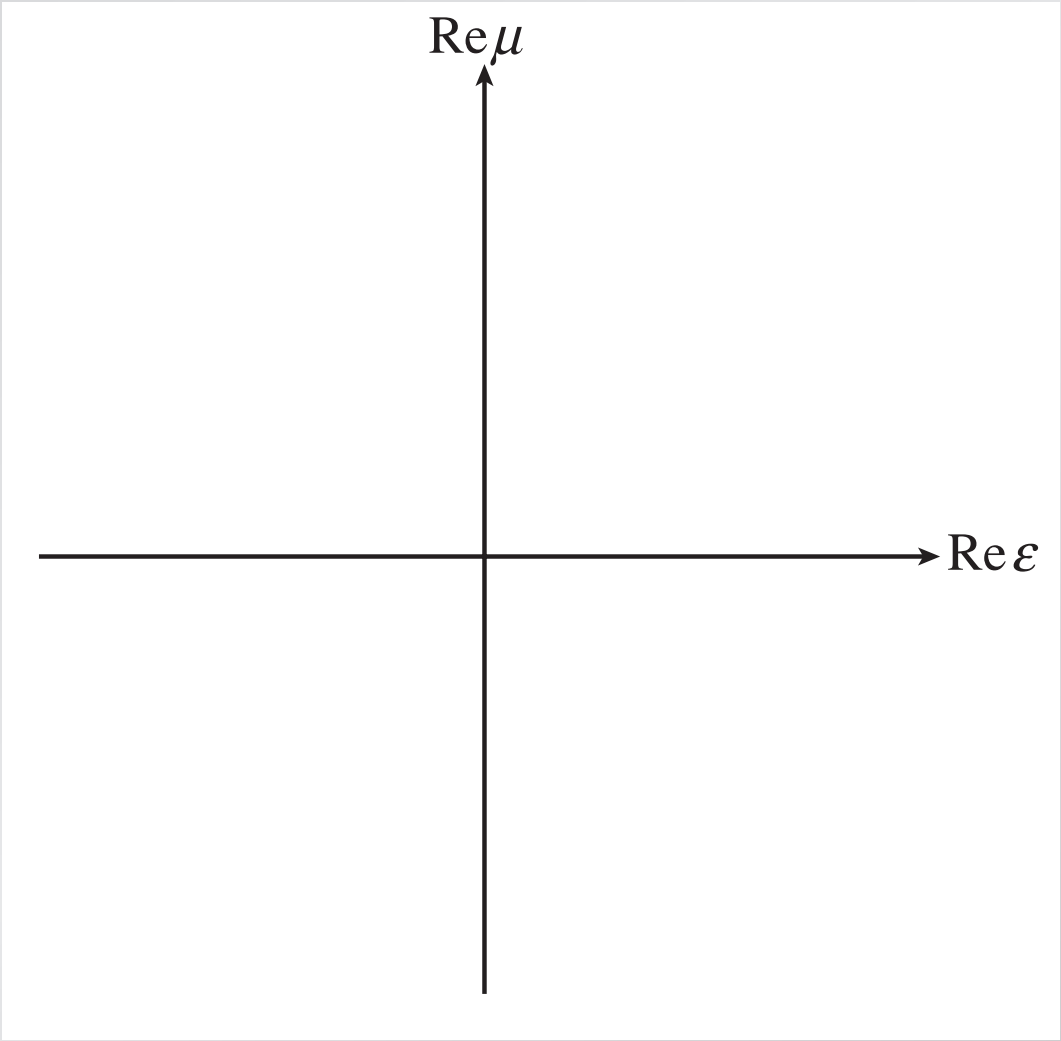
signal *always*

travels at speed c!

What about causality?



classification of (non-lossy) materials



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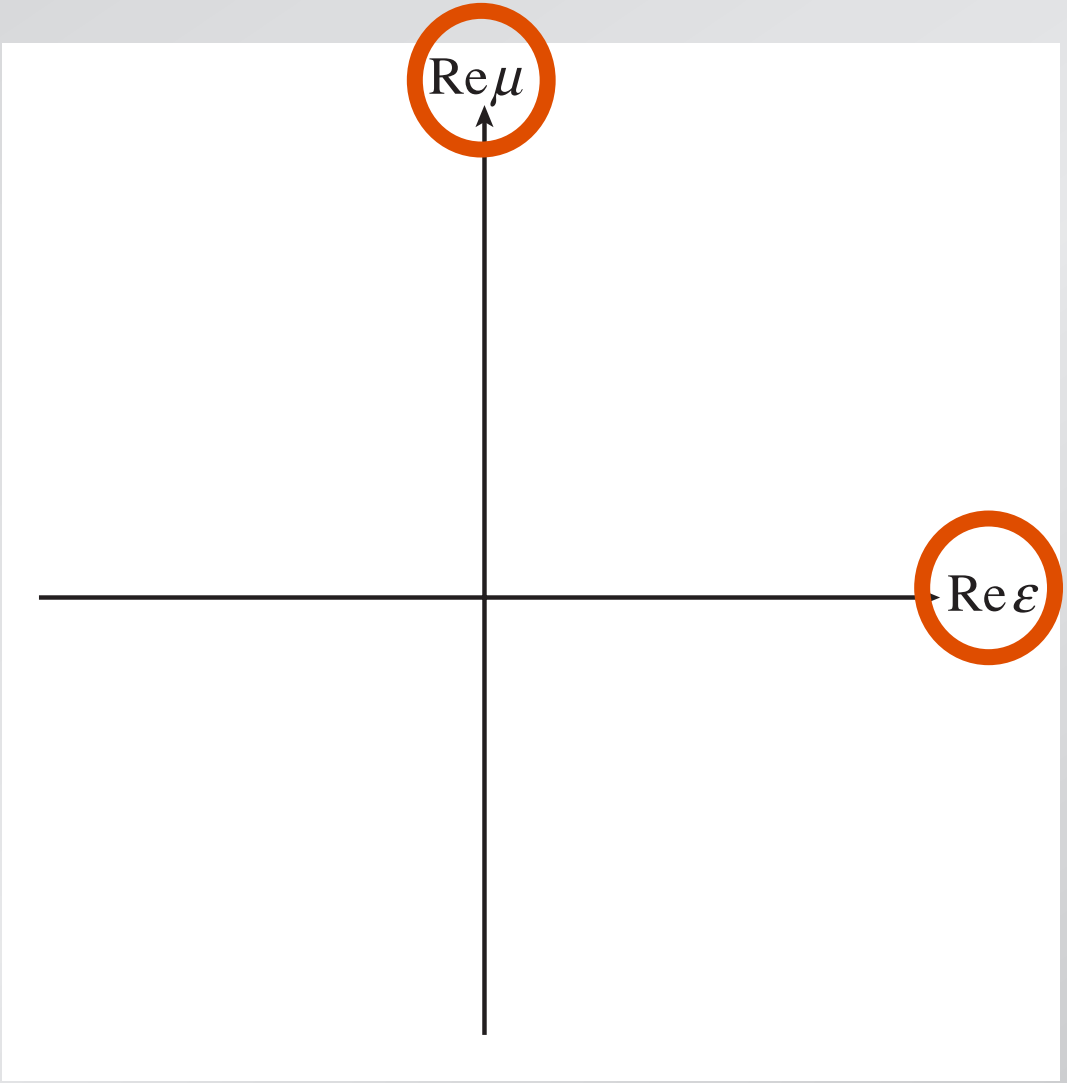

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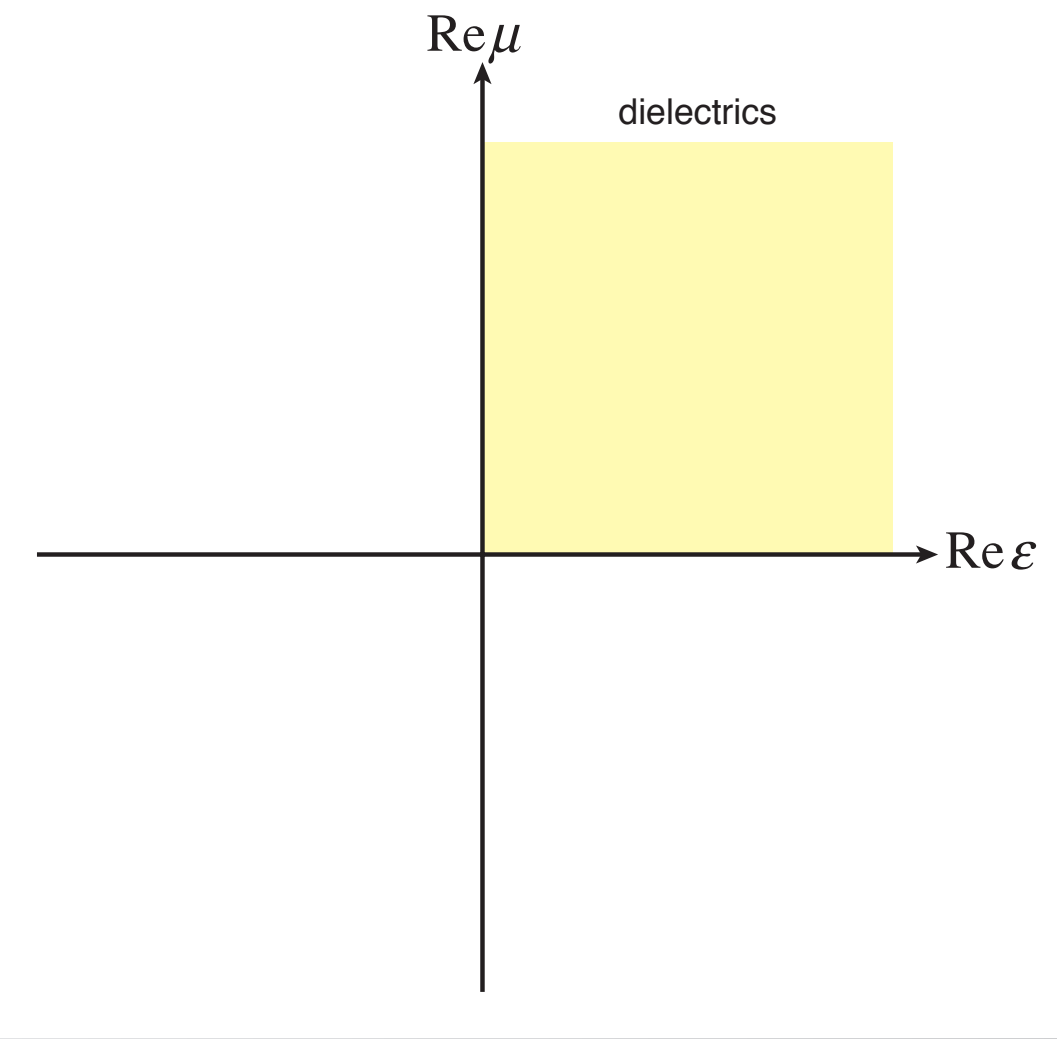

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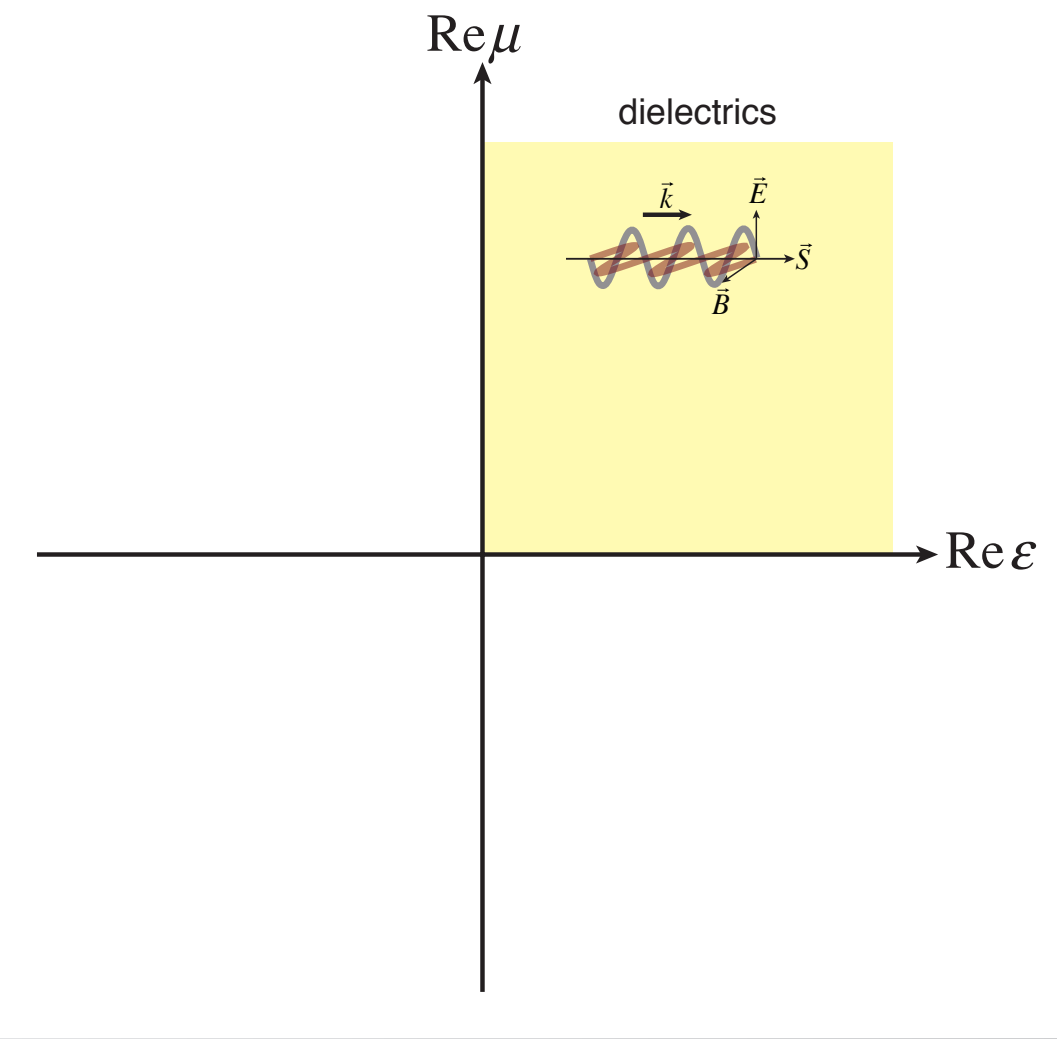

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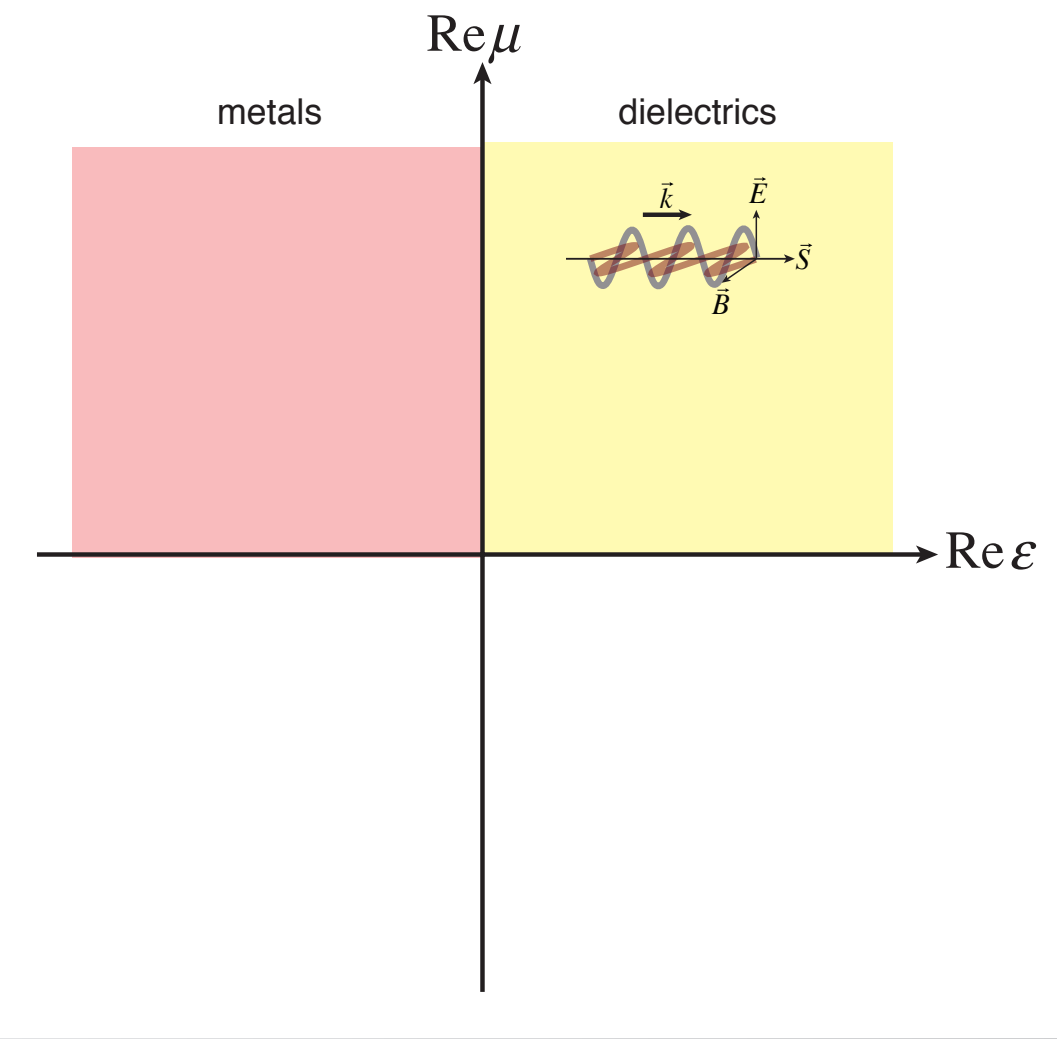

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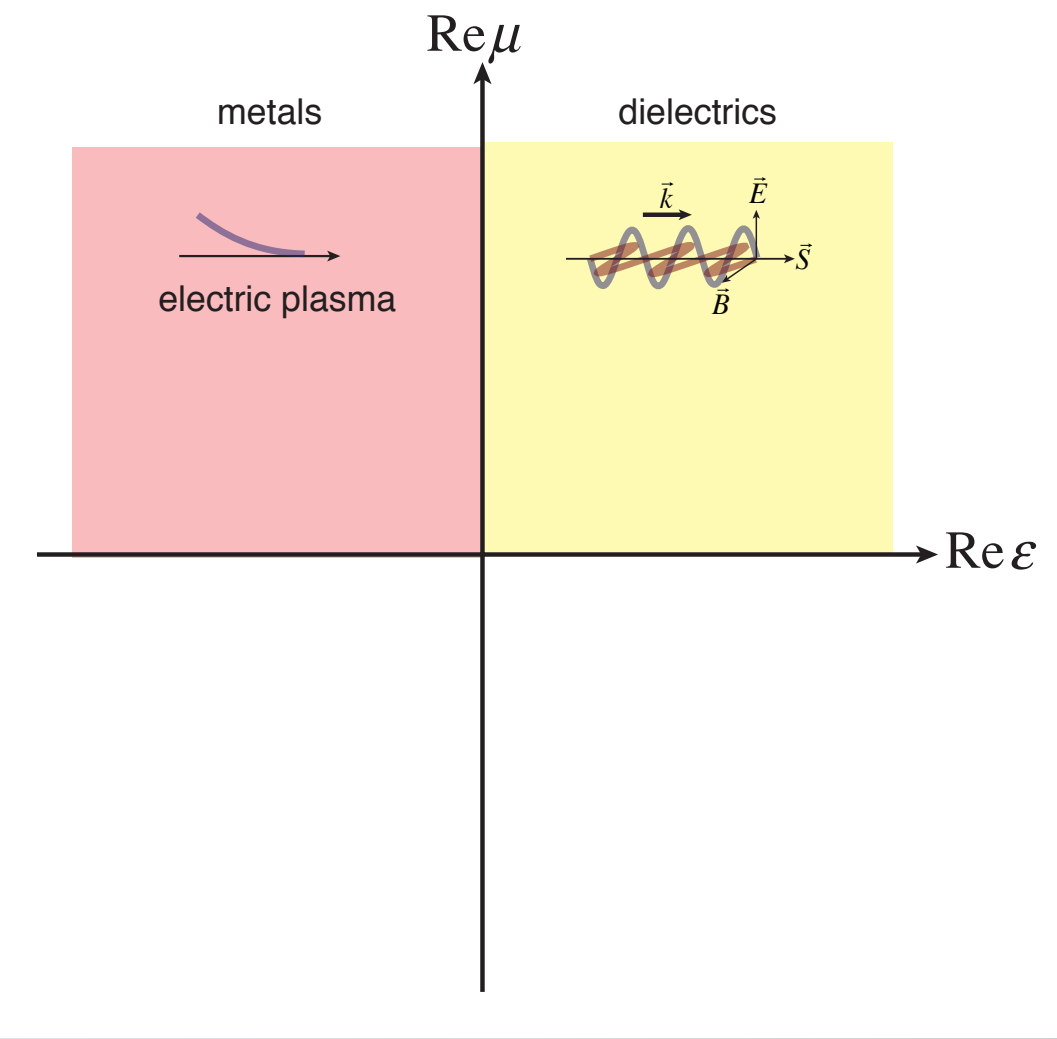

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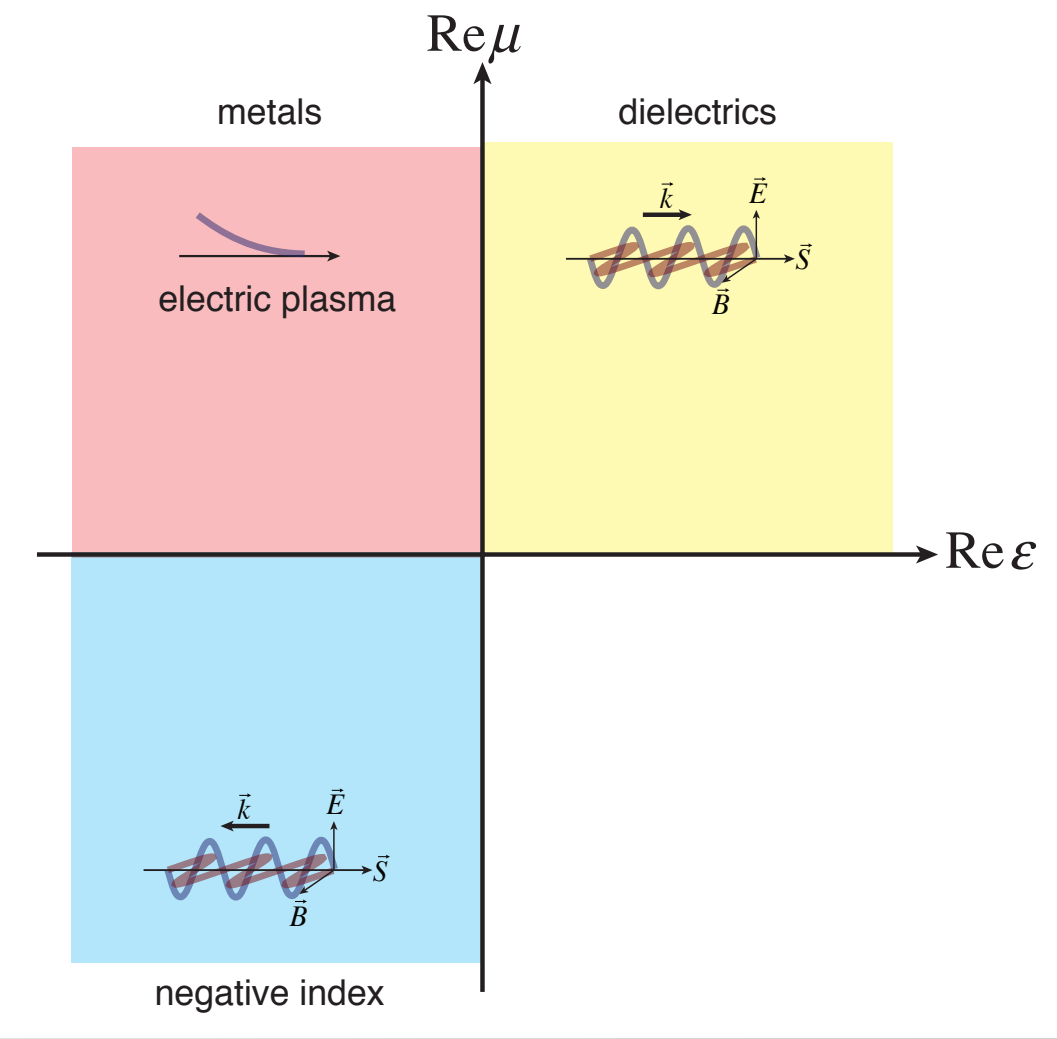

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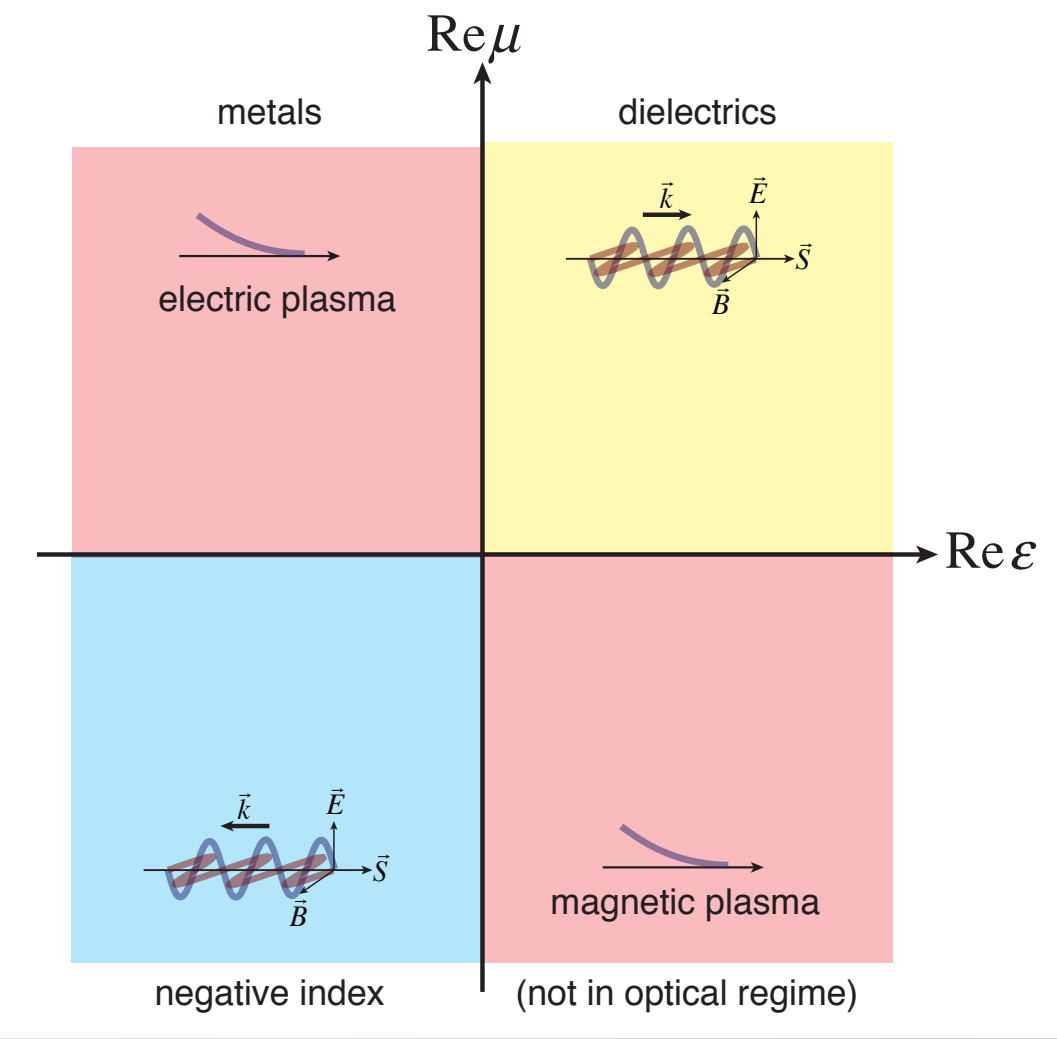

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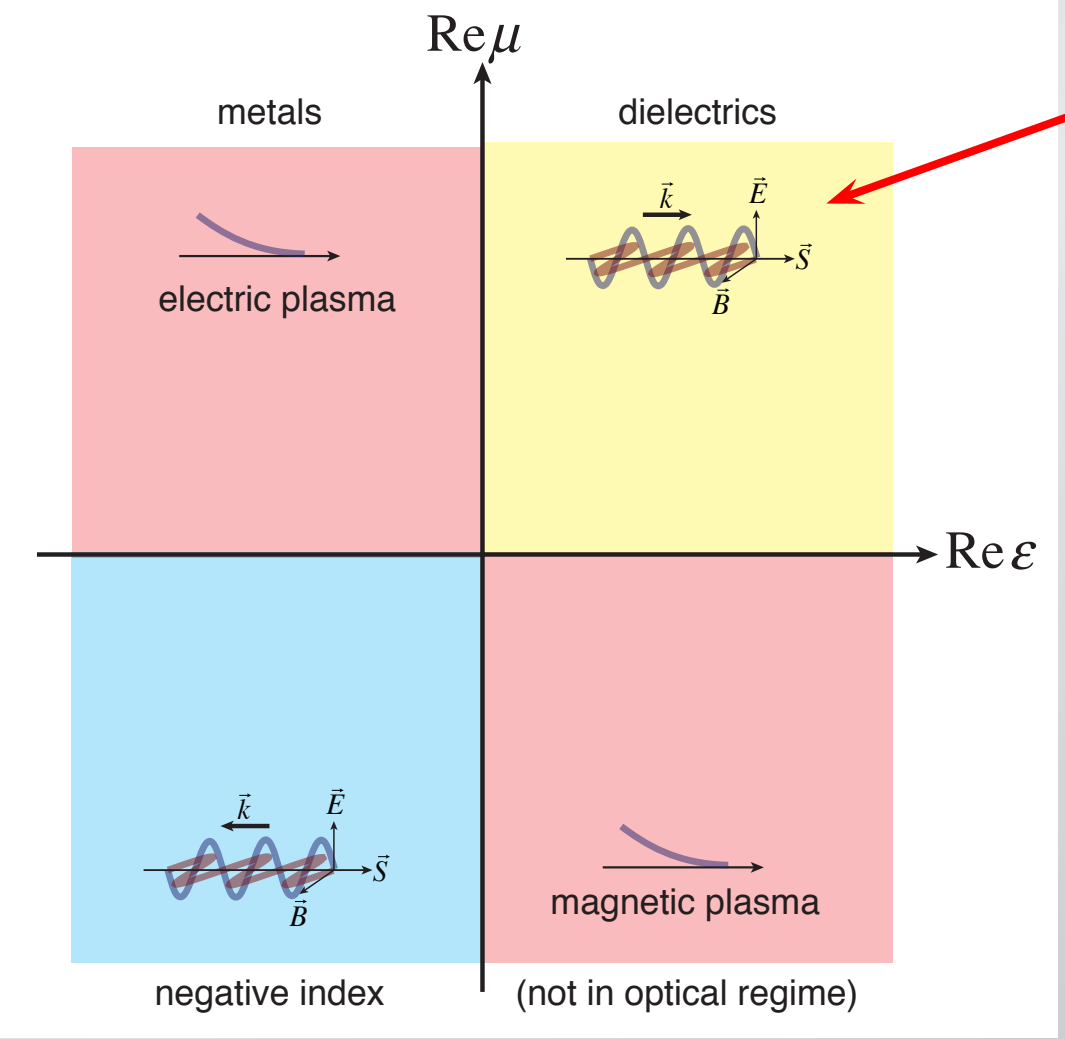
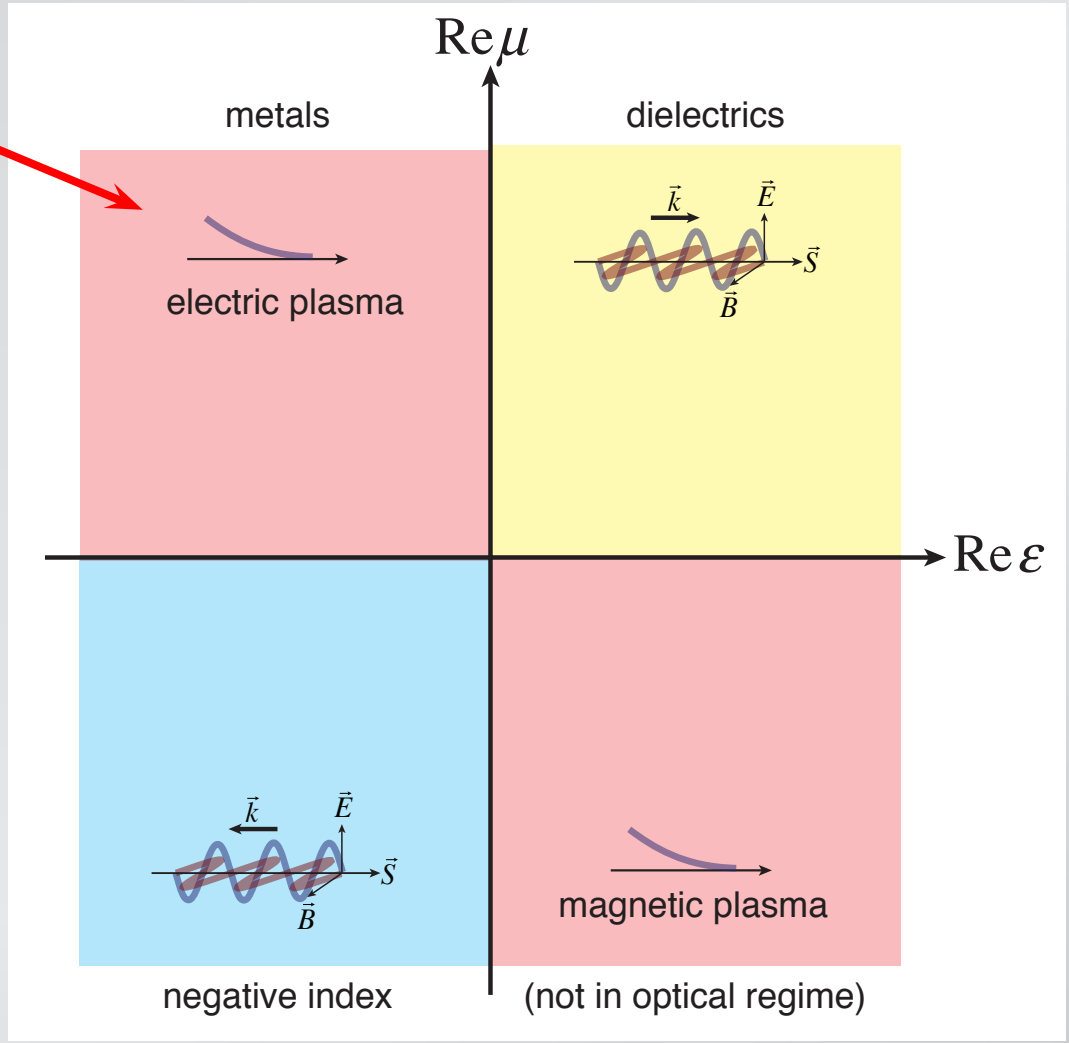


limited by diffraction

classification of (non-lossy) materials

no propagation



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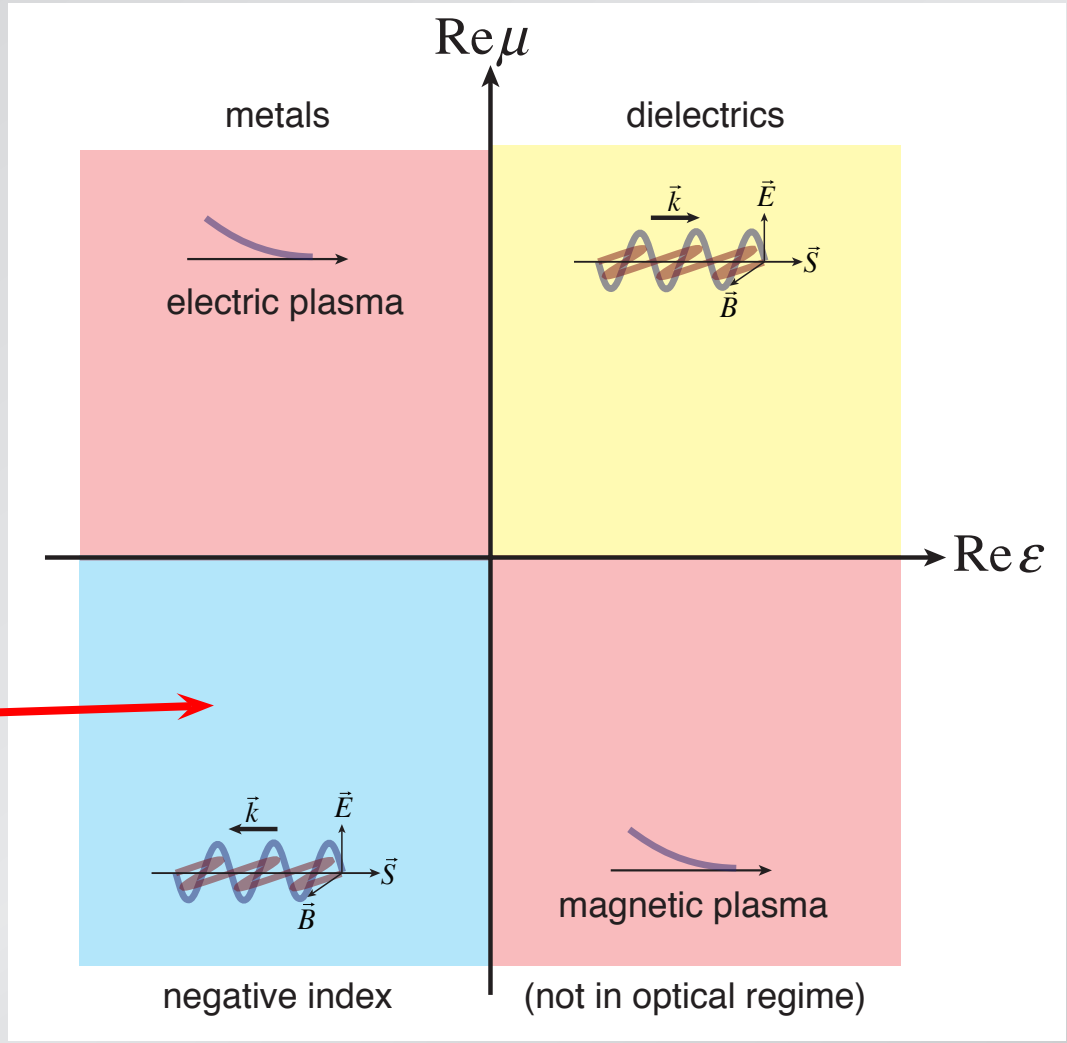
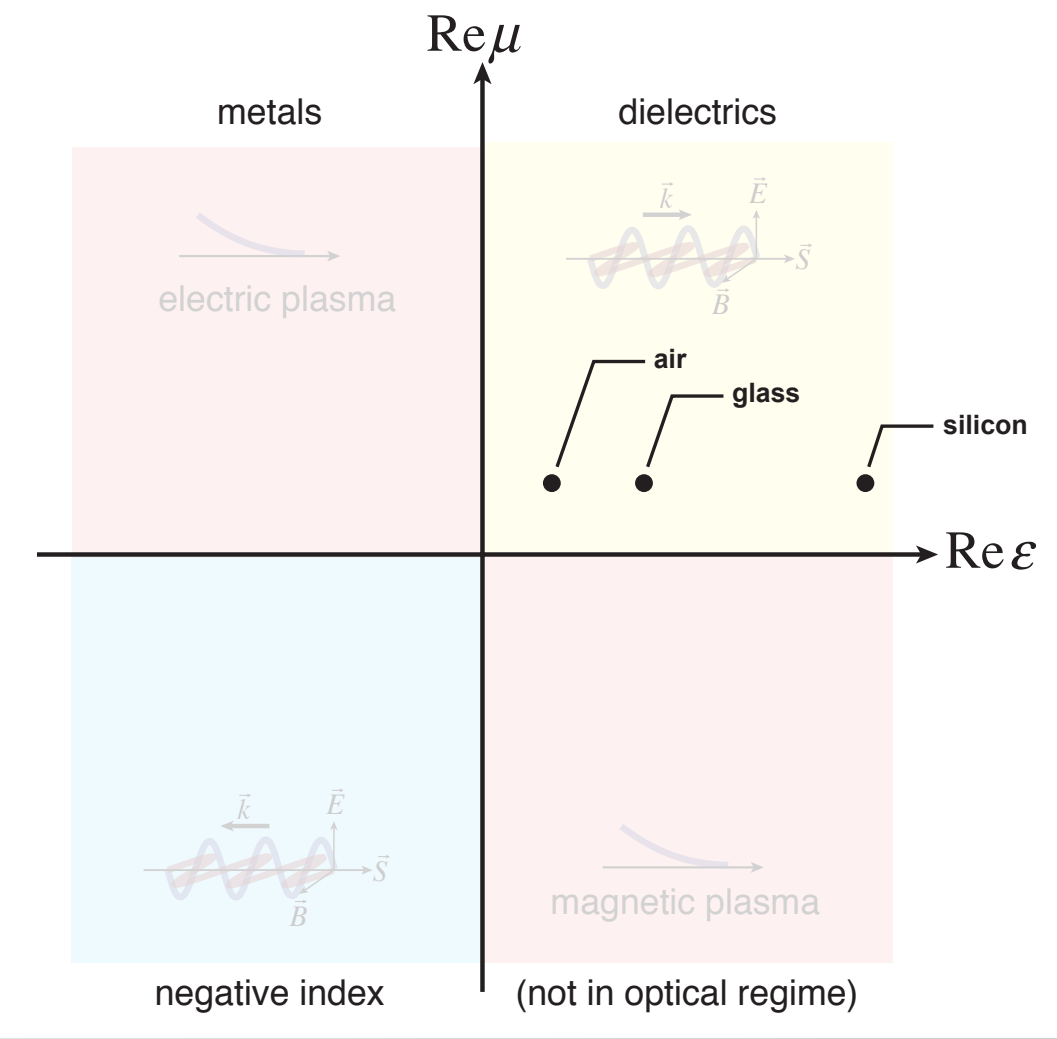
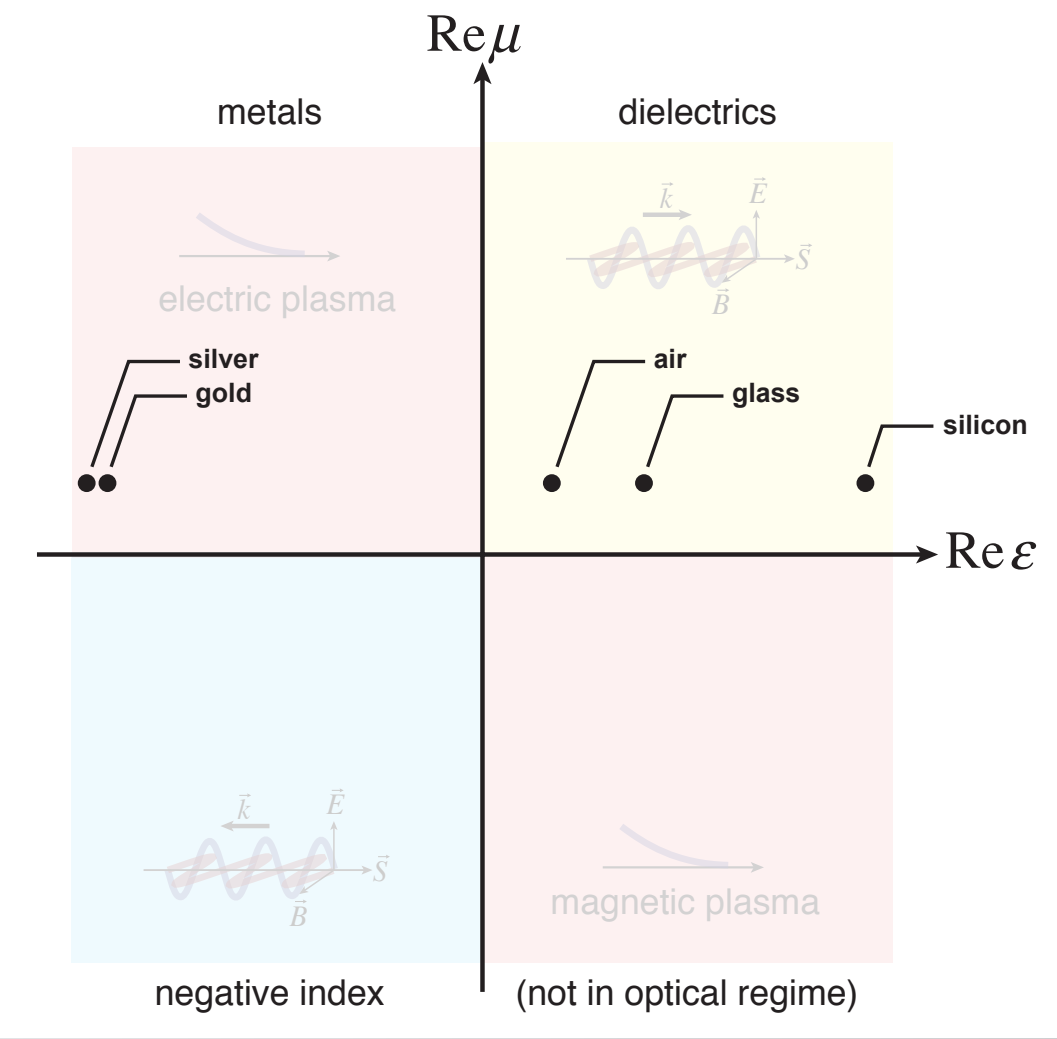


superlensing but…

common materials very limited



common materials very limited



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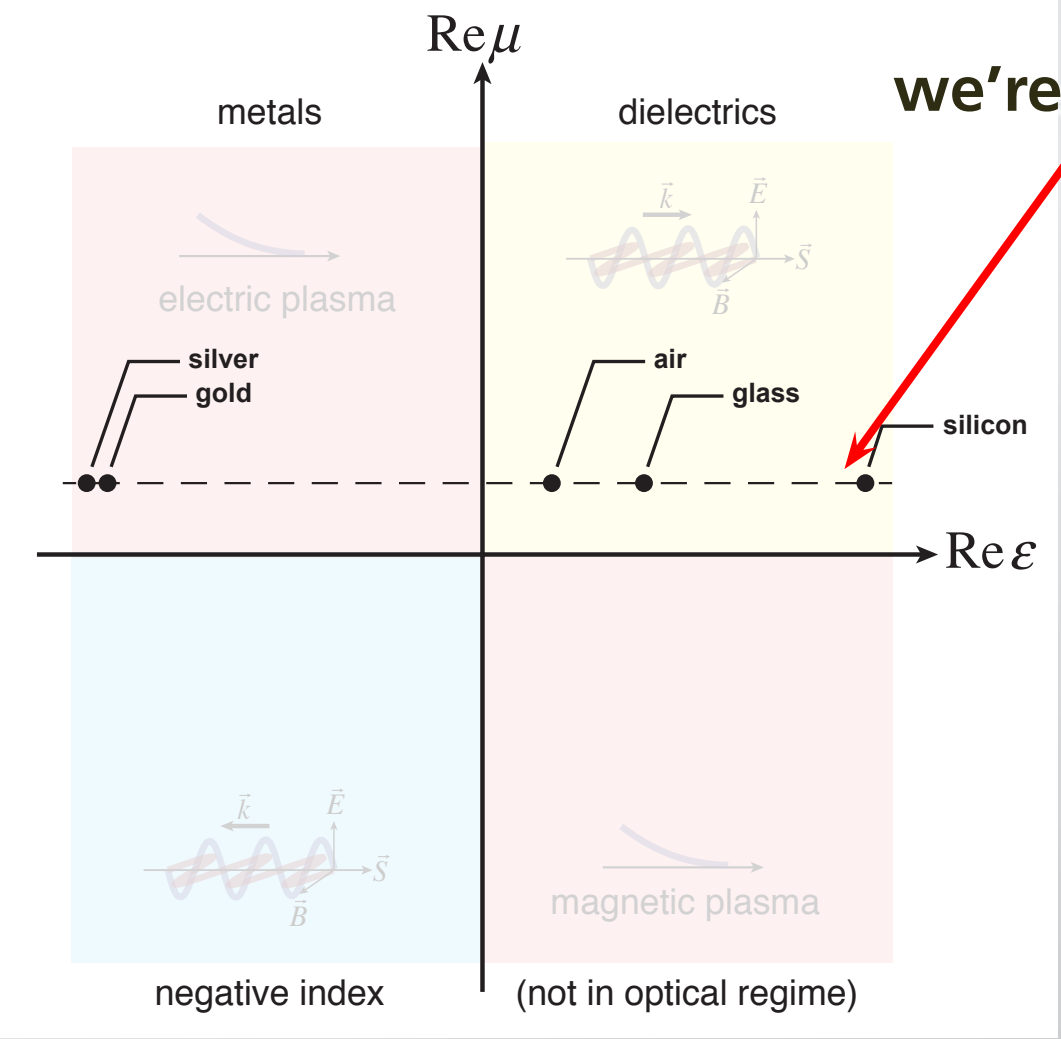
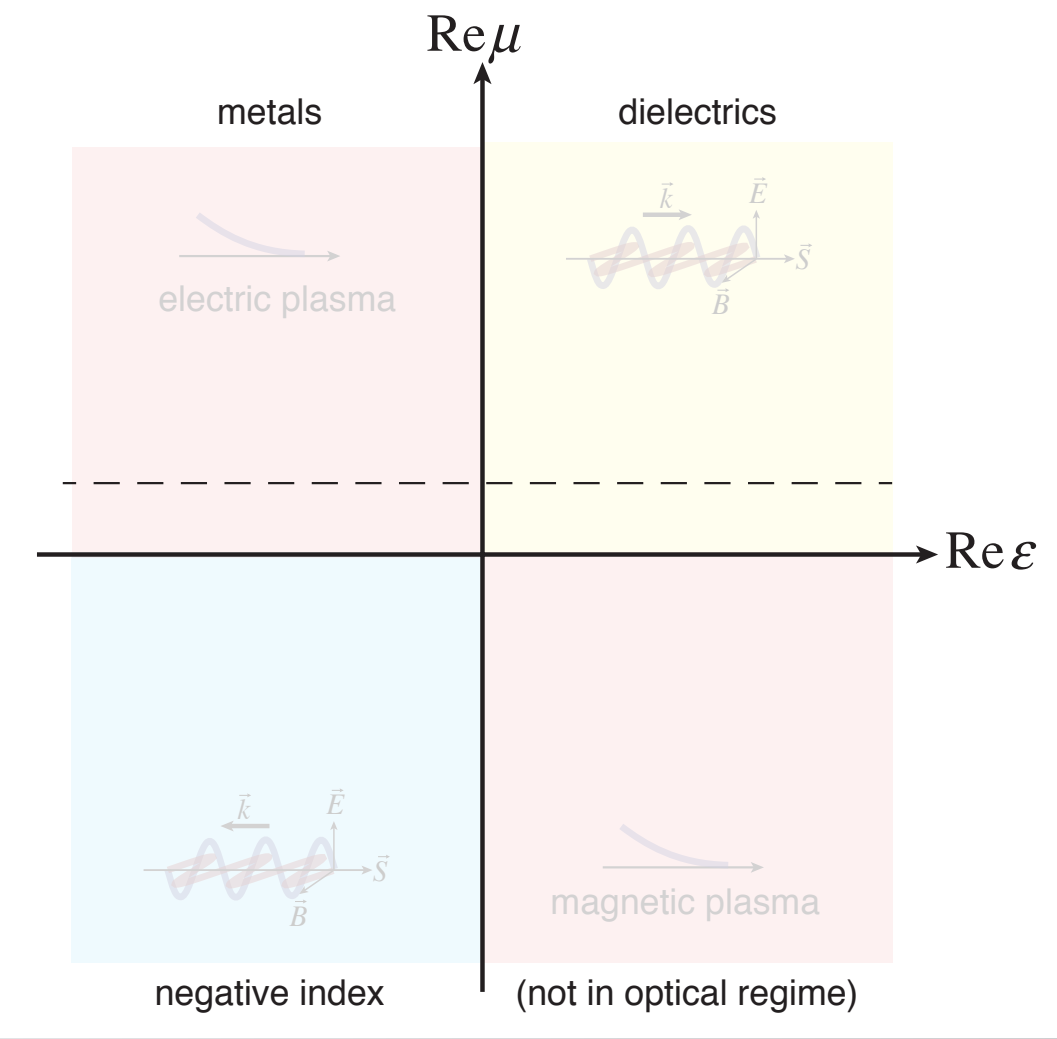
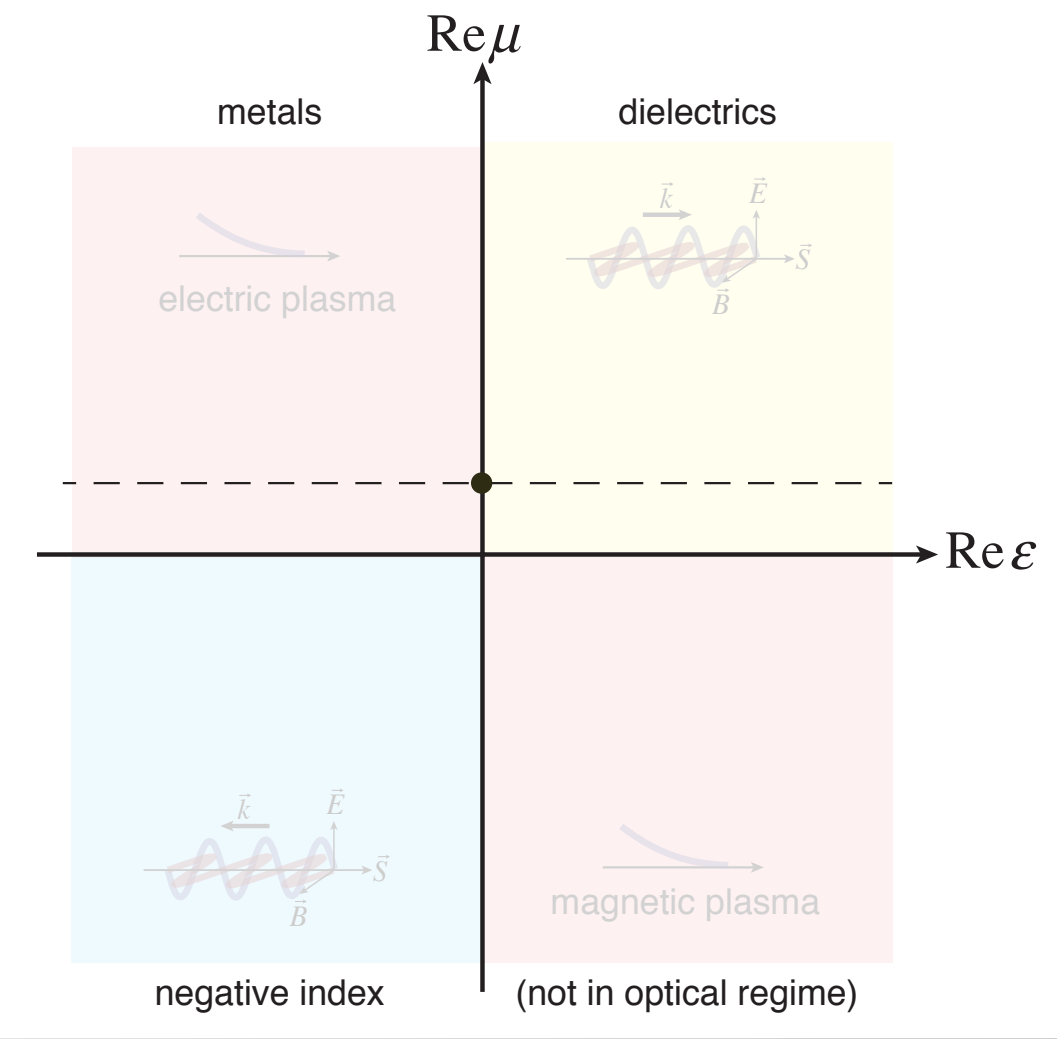


What happens on the axes?



what if we let $\varepsilon = 0$?



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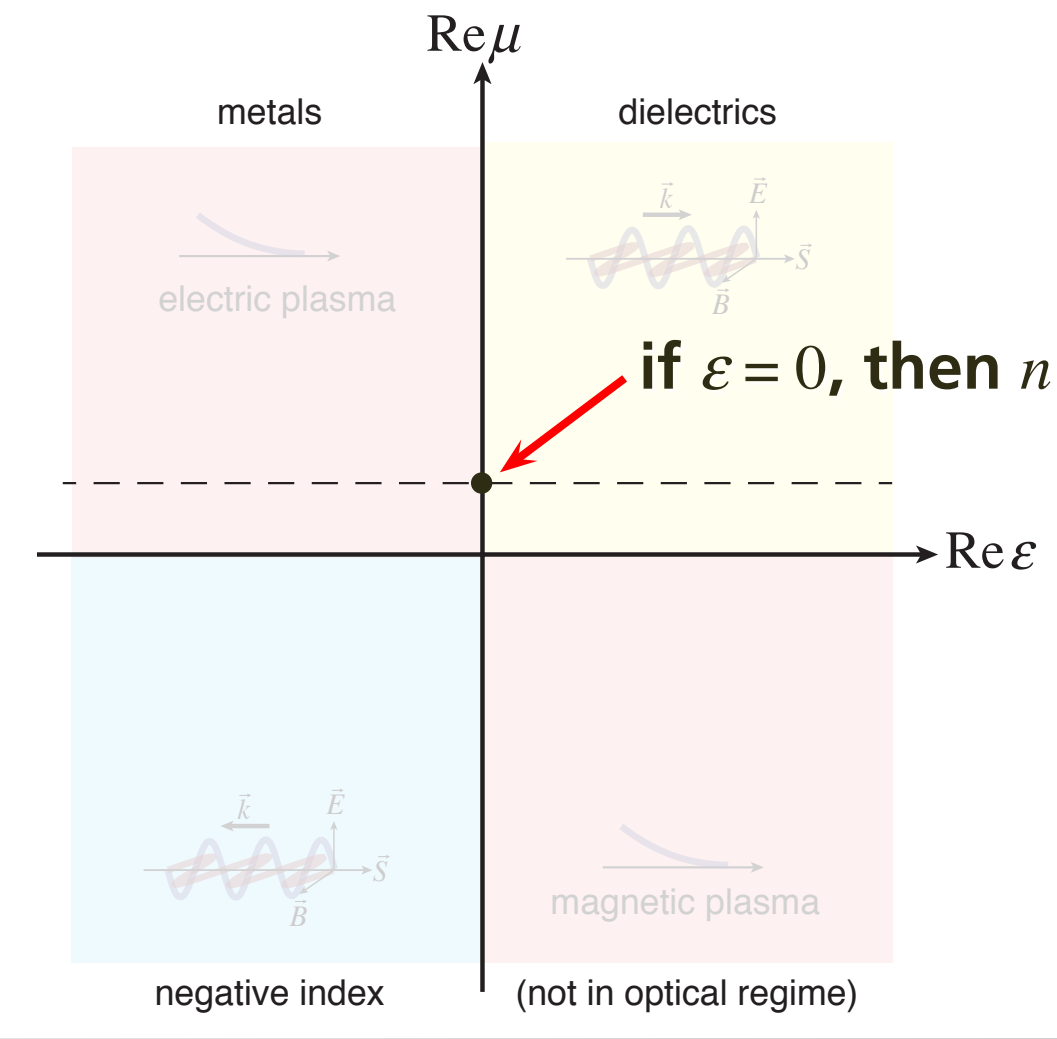

Q: If $n = 0$, which of the following is true?

1. the frequency goes to zero.
2. the phase velocity becomes infinite.
3. both of the above.
4. neither of the above.

1 index 2 zero index

wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

solution

$$\vec{E} = \vec{E}_o \, e^{i(kx - \omega t)}$$

where

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}} c = \frac{1}{n} c$$

wave equation

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1 index

2 zero index

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1 index

2 zero index

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where

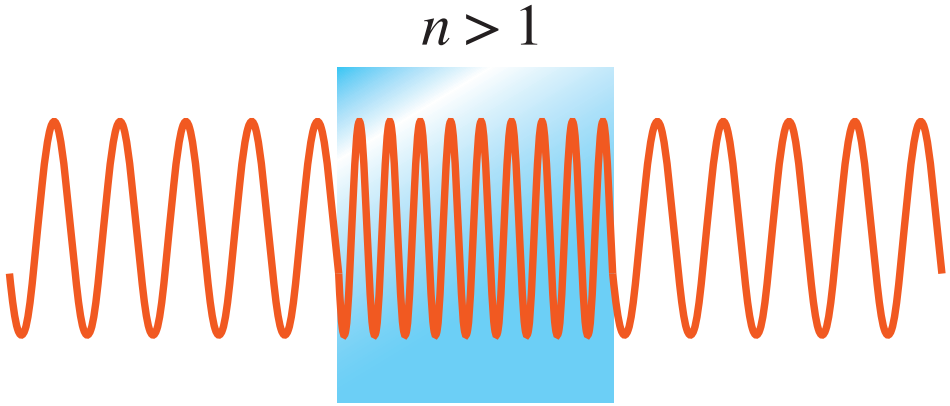
$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon \mu}} c = \frac{1}{n} c \longrightarrow \infty$$

1 index

2 zero index

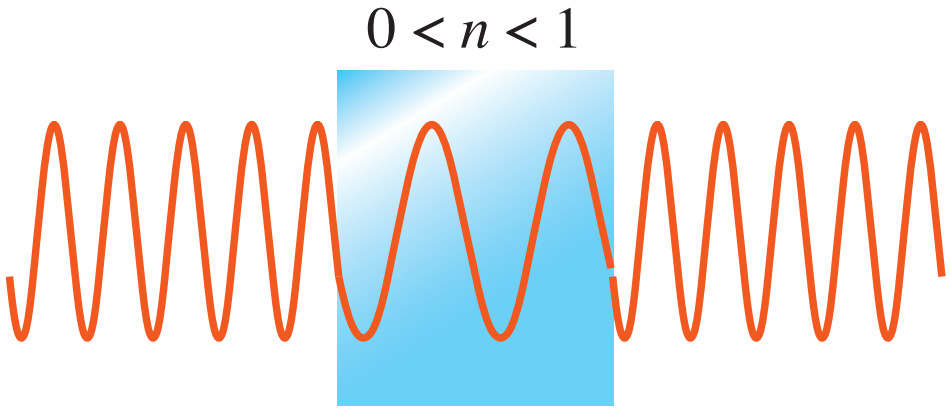
Q: If $n = 0$, which of the following is true?

1. the frequency goes to zero.
2. **the phase velocity becomes infinite.** ✔
3. both of the above.
4. neither of the above.



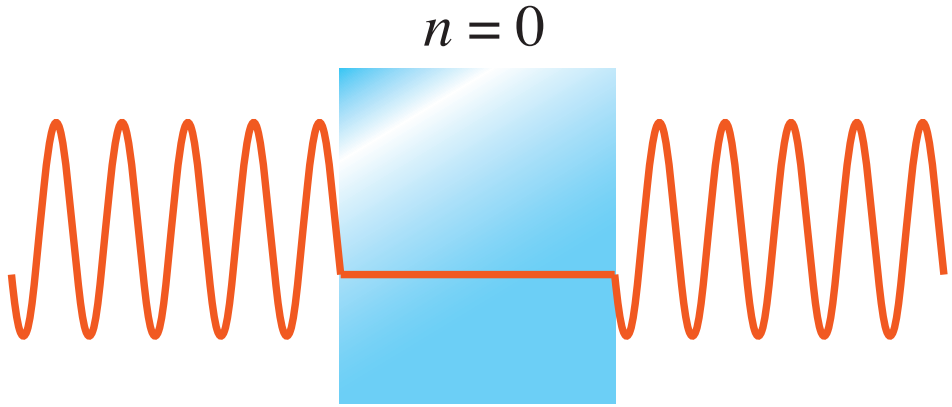
1 index

2 zero index



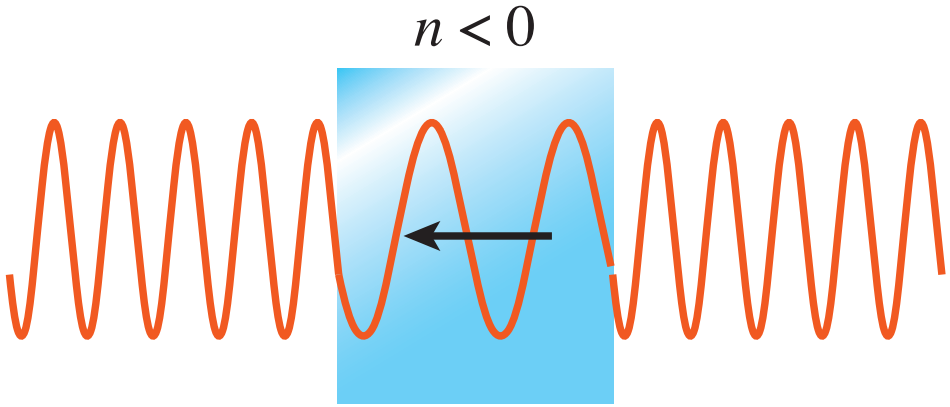
1 index

2 zero index



1 index

2 zero index



1 index

2 zero index

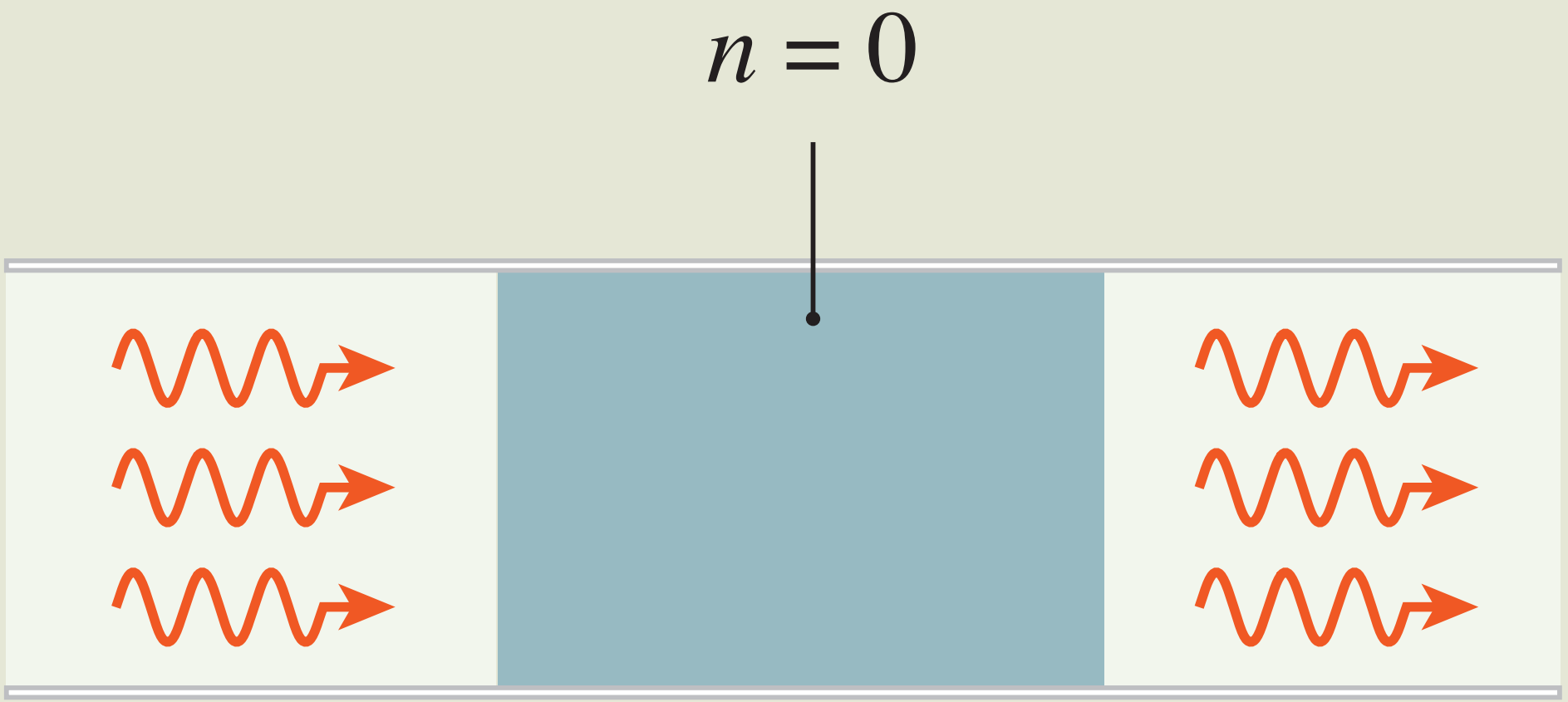
1 index

2 zero index

What can we do with uniform phase?

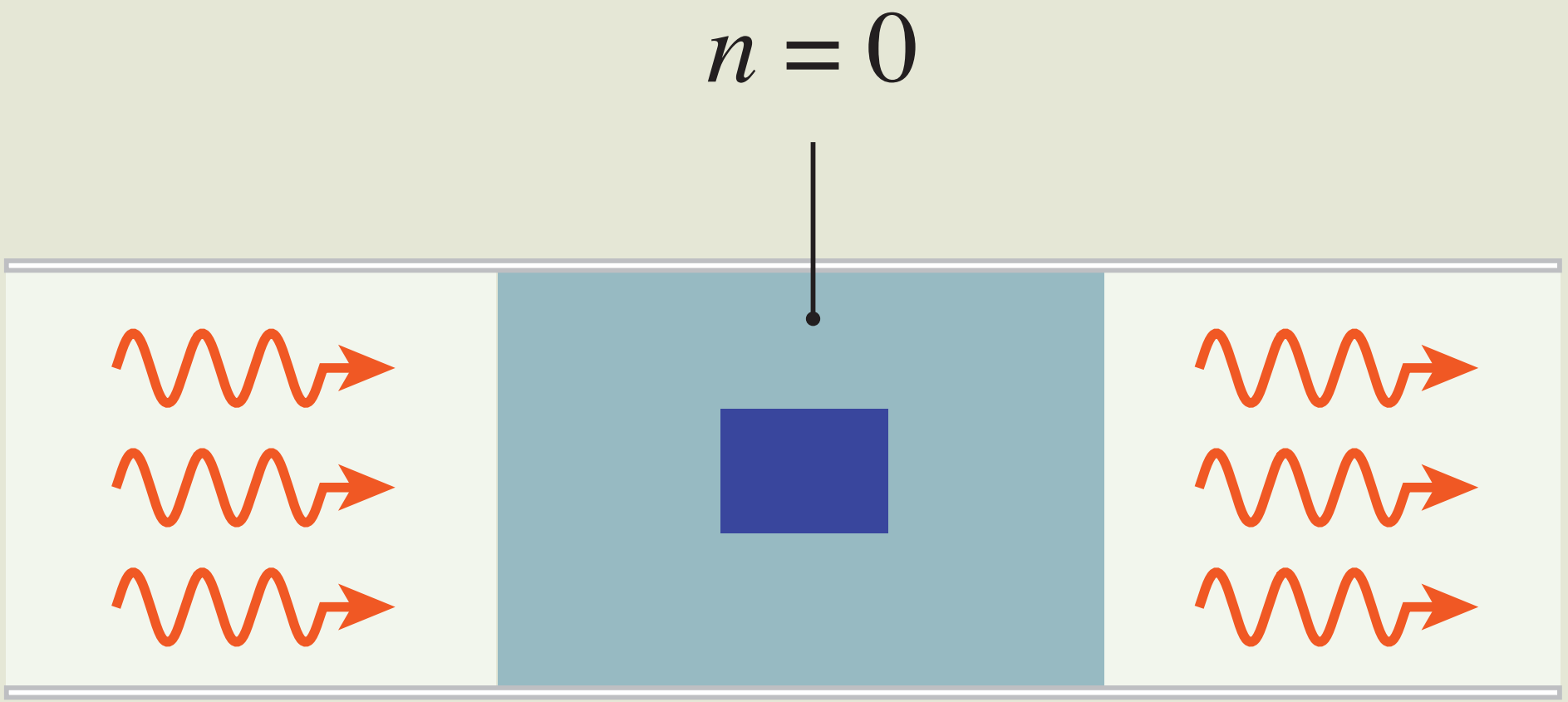
1 index

2 zero index



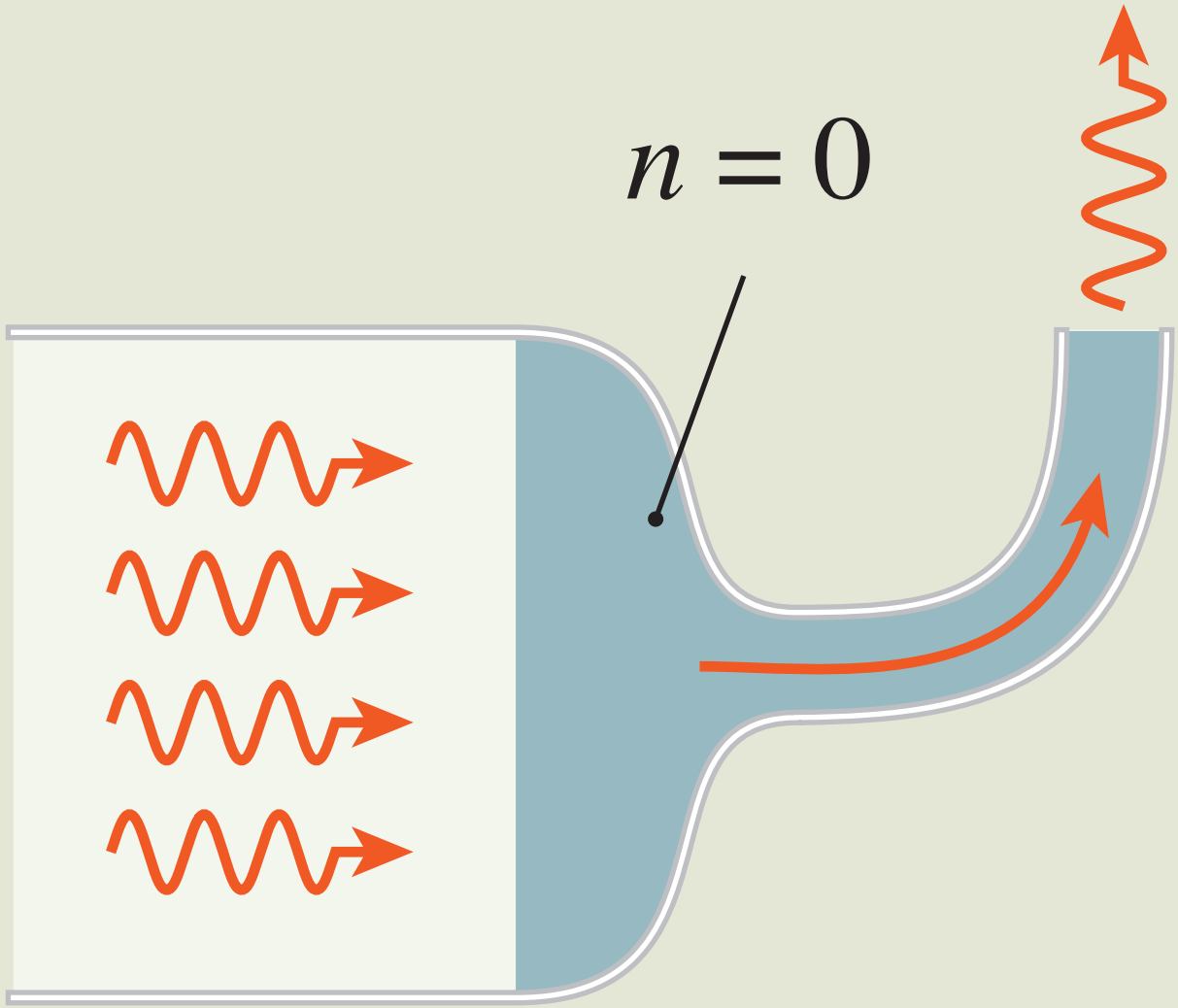
1 index

2 zero index



1 index

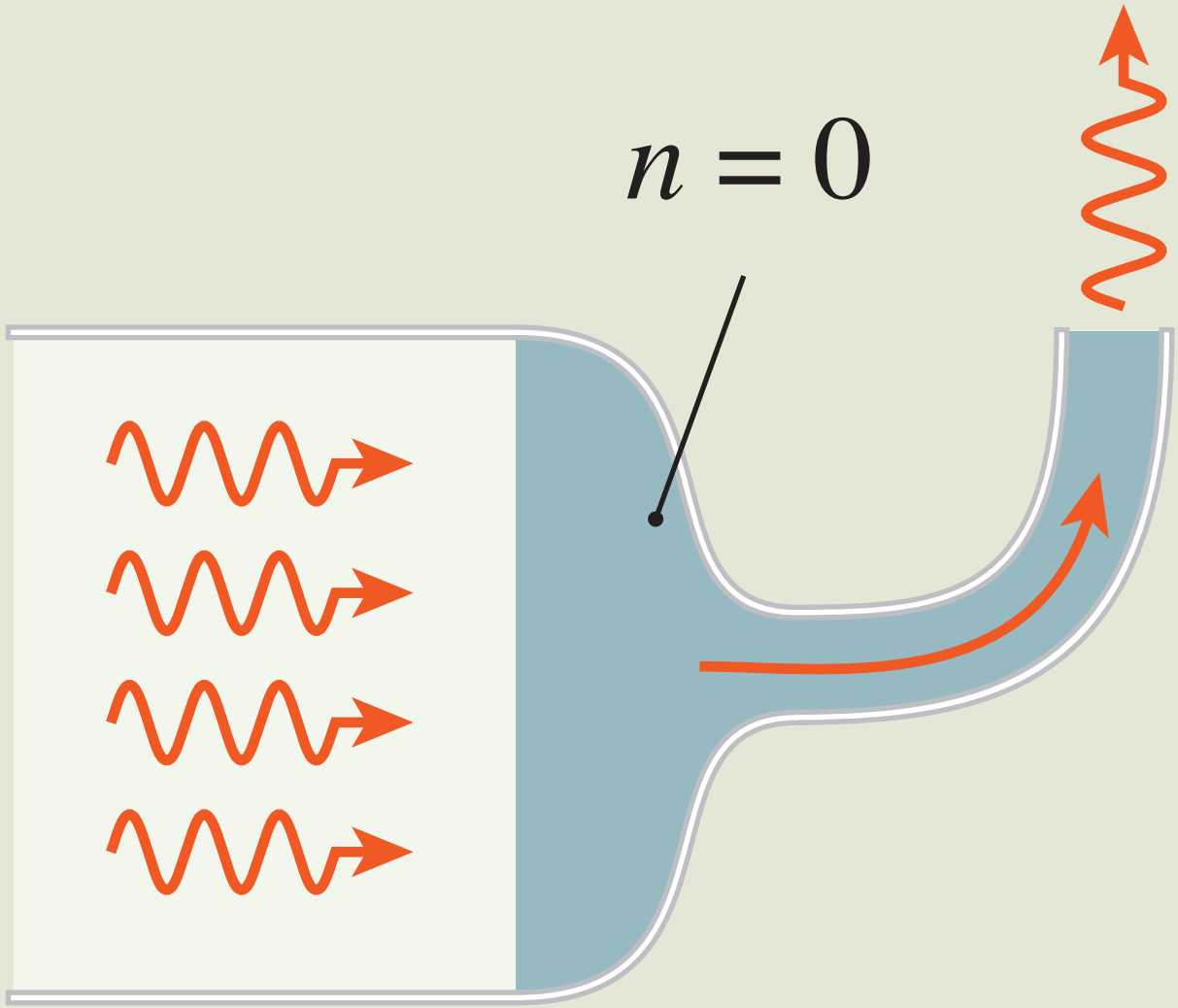
2 zero index



1 index

2 zero index

"tunneling with infinite decay length"



how?

$$n = \sqrt{\varepsilon\mu}$$

1 index

2 zero index

how?

$$n = \sqrt{\varepsilon\mu}$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

1 index

2 zero index

how?

$$n = \sqrt{\varepsilon\mu}$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

1 index

2 zero index

how?

$$\varepsilon \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

how?

$$\varepsilon \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to \infty$$

1 index 2 zero index

how?

$$\varepsilon \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1} \to 1$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to \infty$$

1 index

2 zero index

how?

$$\mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

how?

$$\mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to 0$$

1 index 2 zero index

how?

$$\mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1} \to -1$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \to 0$$

1 index

2 zero index

how?

$$\varepsilon, \mu \to 0 \qquad\qquad n = \sqrt{\varepsilon\mu} \to 0$$

but ϵ and μ also determine reflectivity

$$R = \frac{Z-1}{Z+1}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

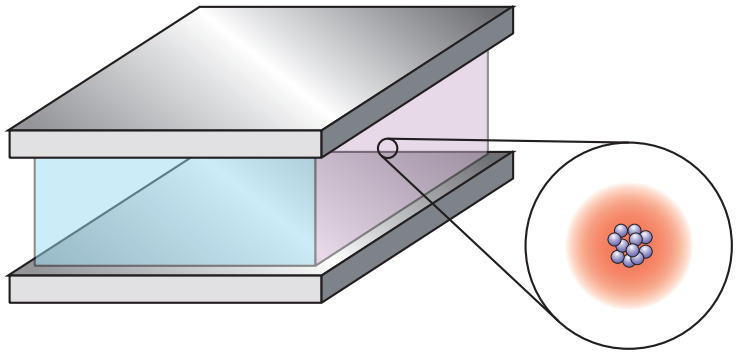
finite!

but $\mu \neq 1$ requires a magnetic response!

Engineering a magnetic response

Engineering a magnetic response

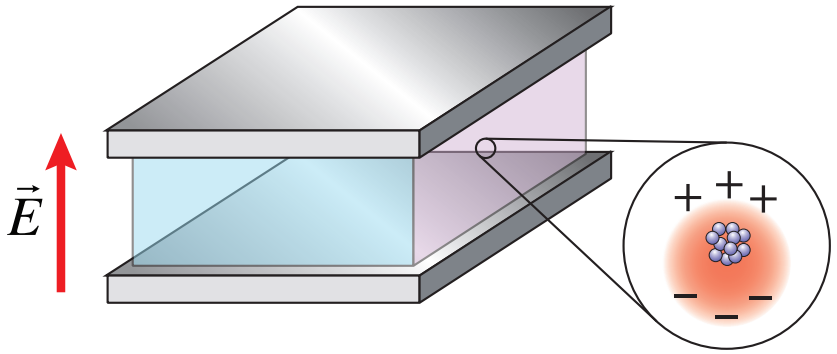
bulk material



properties derive from
constituent atoms

Engineering a magnetic response

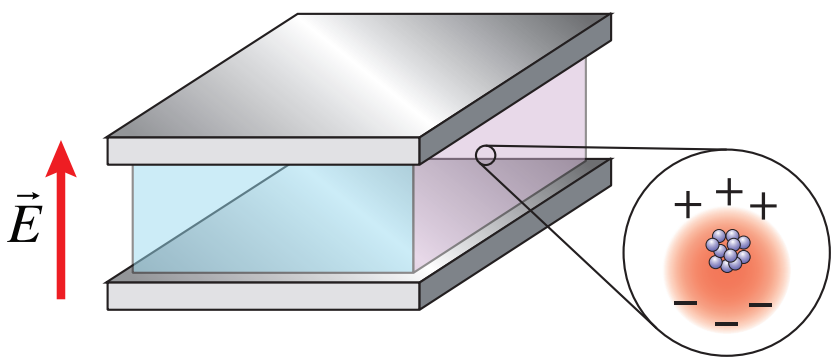
bulk material



properties derive from
constituent atoms

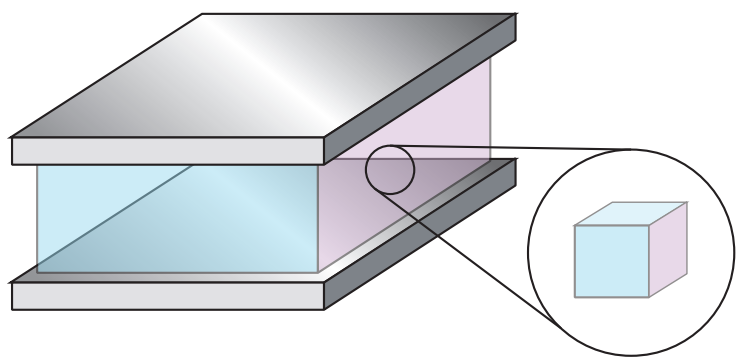
Engineering a magnetic response

bulk material



properties derive from constituent atoms

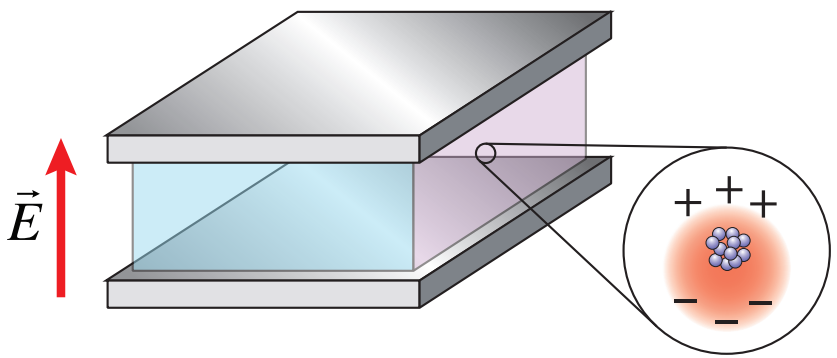
composite material



properties derive from constituent units

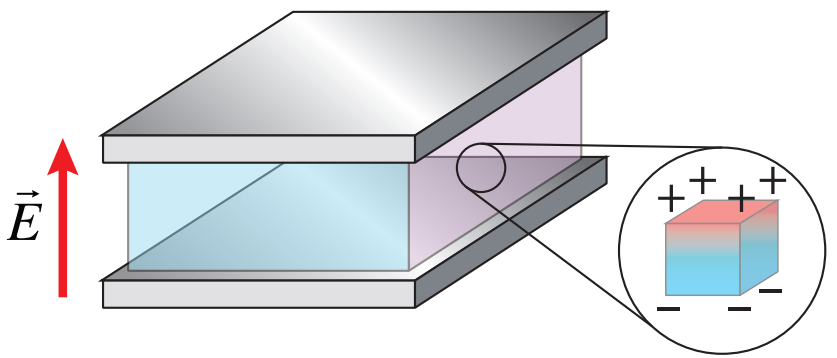
Engineering a magnetic response

bulk material



properties derive from constituent atoms

composite material



properties derive from constituent units

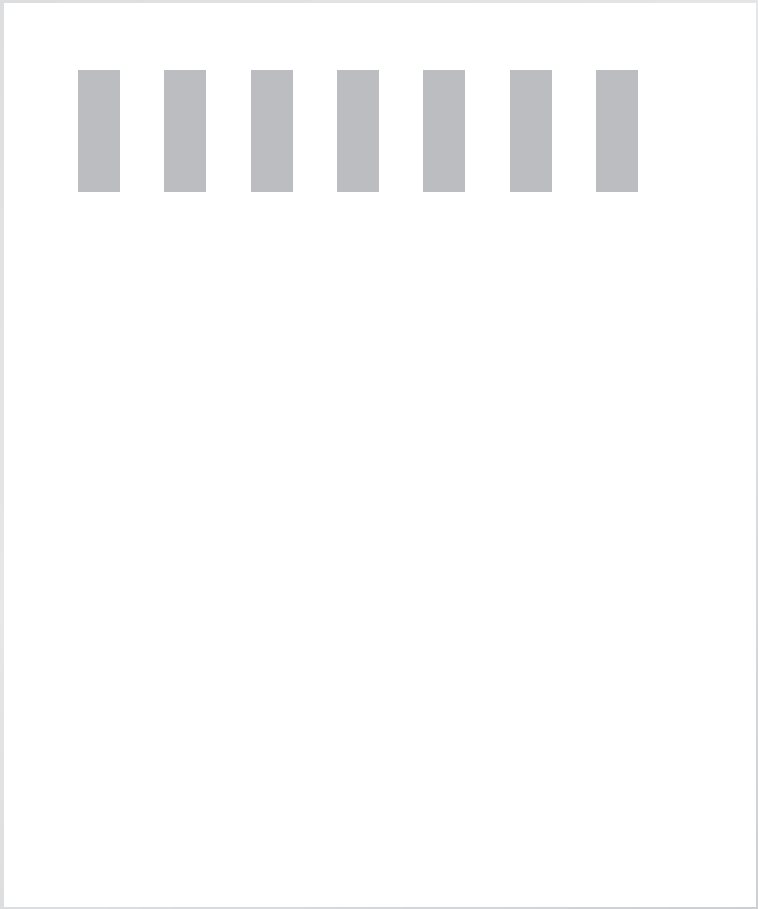
1 index

2 zero index

3 experiments

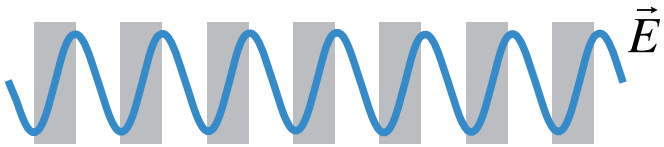
Engineering a magnetic response

use array of dielectric rods



Engineering a magnetic response

incident electromagnetic wave ($\lambda_{\text{eff}} \approx a$)

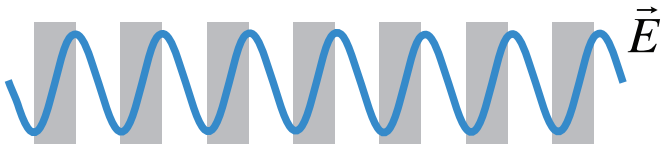


1 index

2 zero index

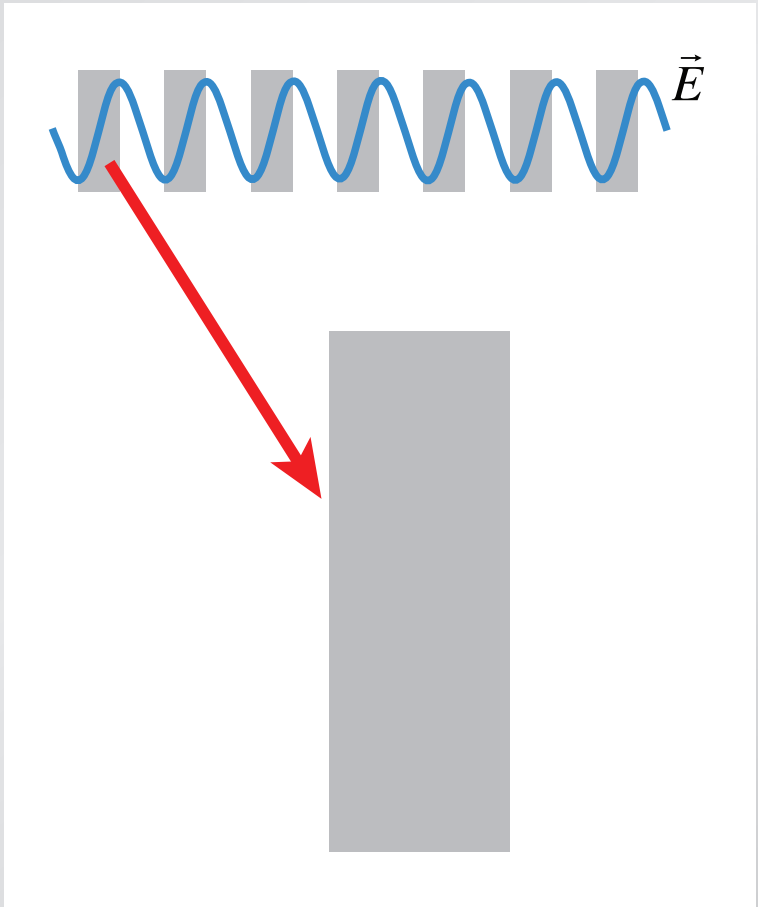
Engineering a magnetic response

produces an electric response…



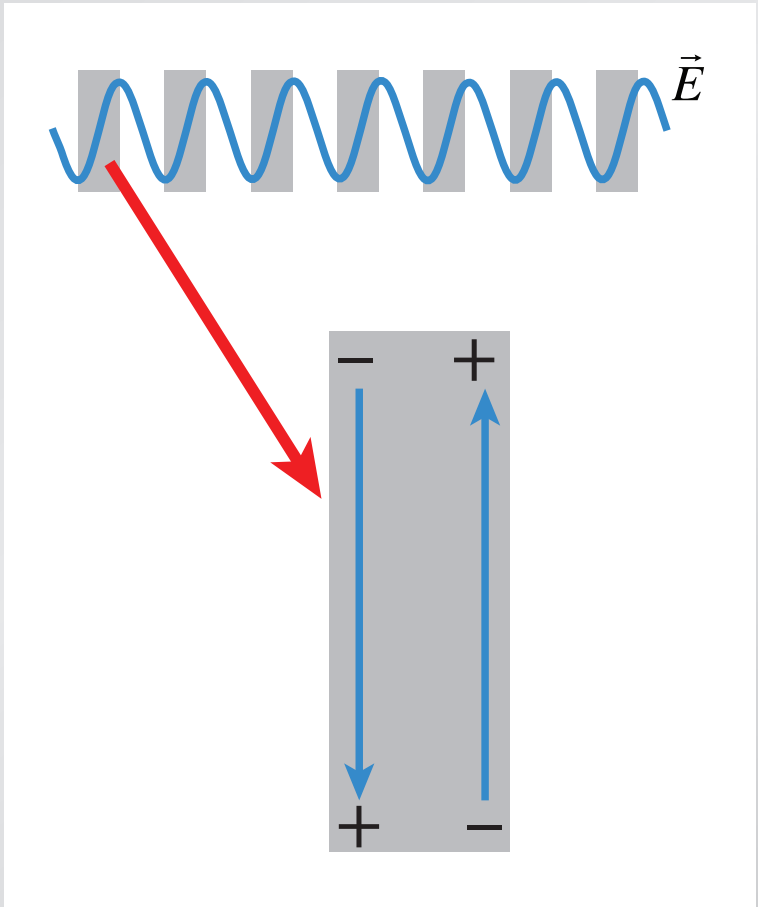
Engineering a magnetic response

… but different electric fields front and back…



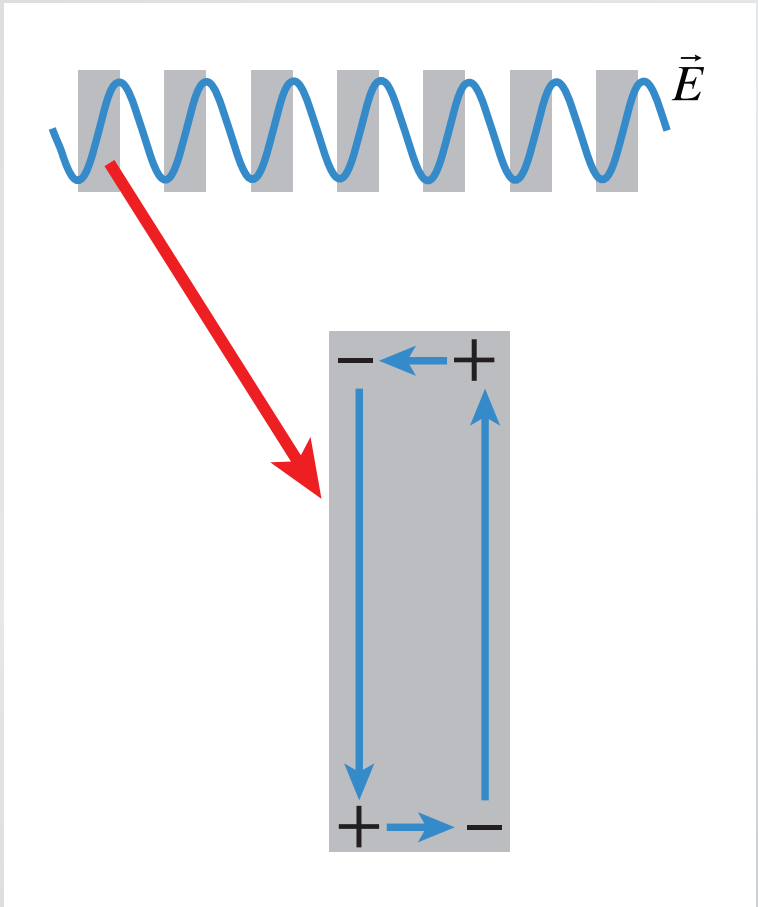
Engineering a magnetic response

…induce different polarizations on opposite sides…



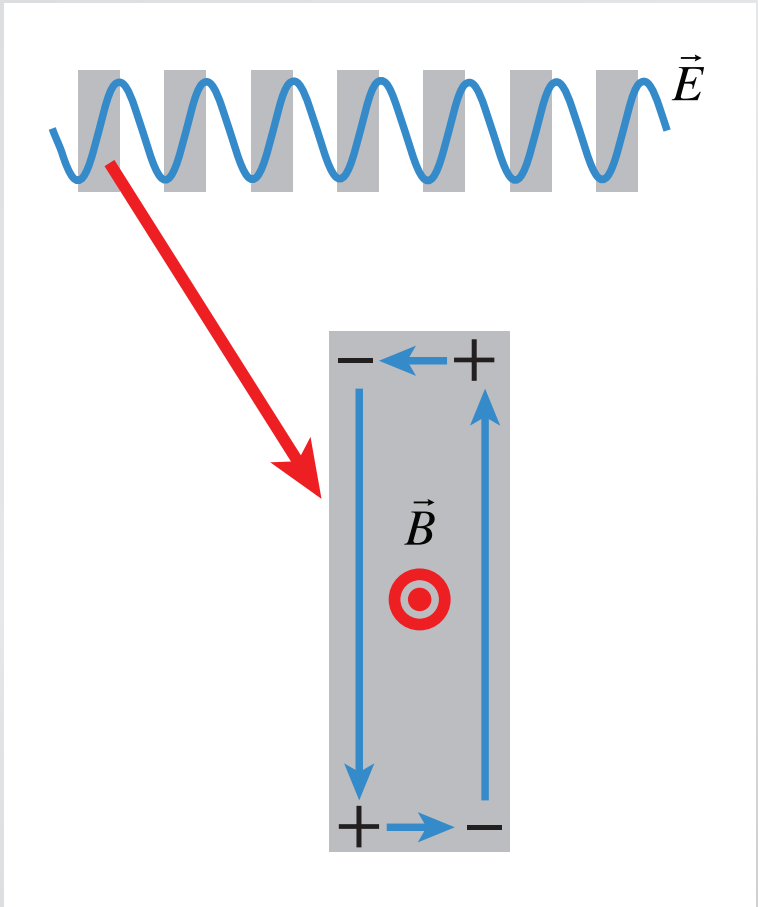
Engineering a magnetic response

…causing a current loop…



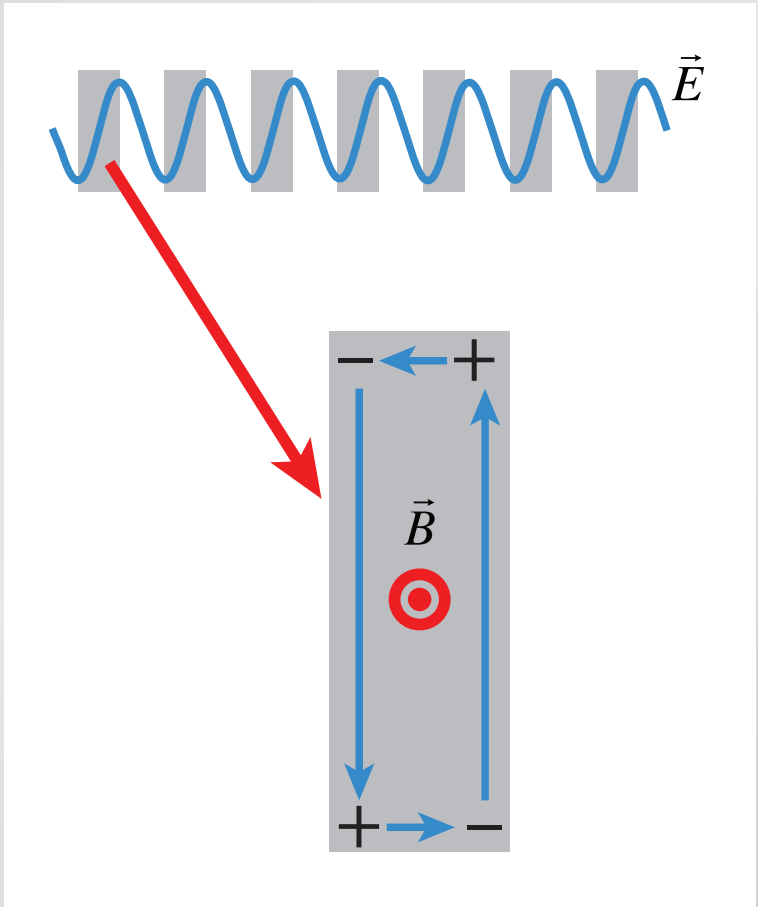
Engineering a magnetic response

…which, in turn, produces an induced magnetic field



Engineering a magnetic response

adjust design so electrical and magnetic resonances coincide



Engineering a magnetic response

adjustable parameters

1 index 2 zero index 3 experiments

Engineering a magnetic response

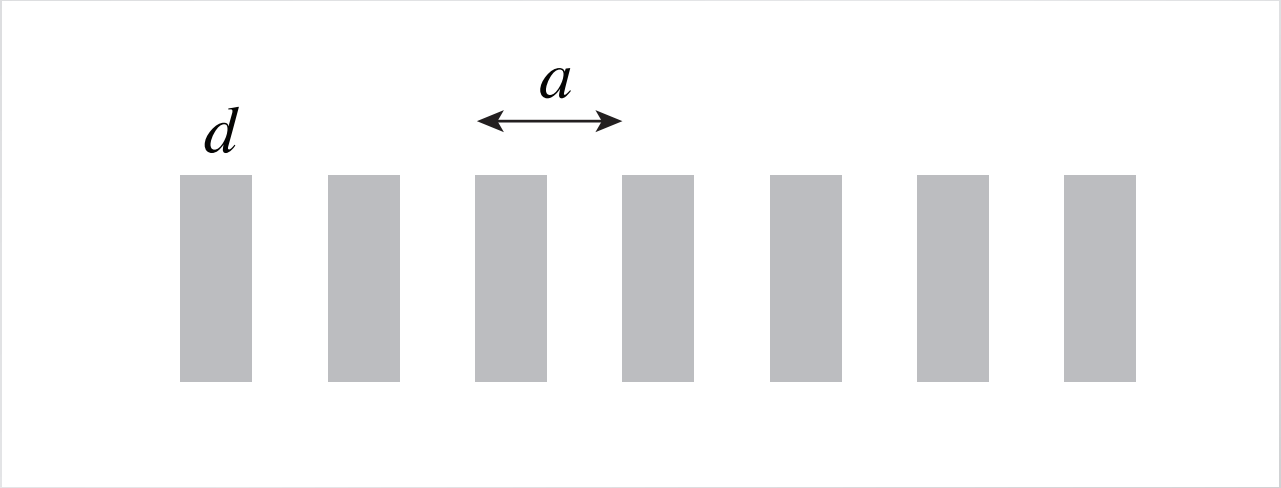
adjustable parameters

Engineering a magnetic response

adjustable parameters

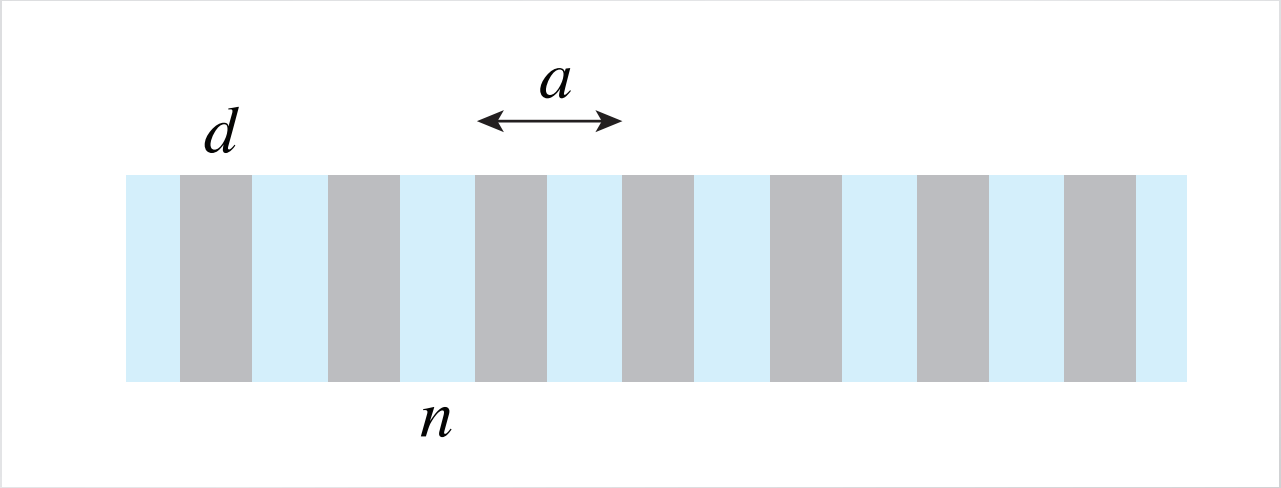
Engineering a magnetic response

adjustable parameters



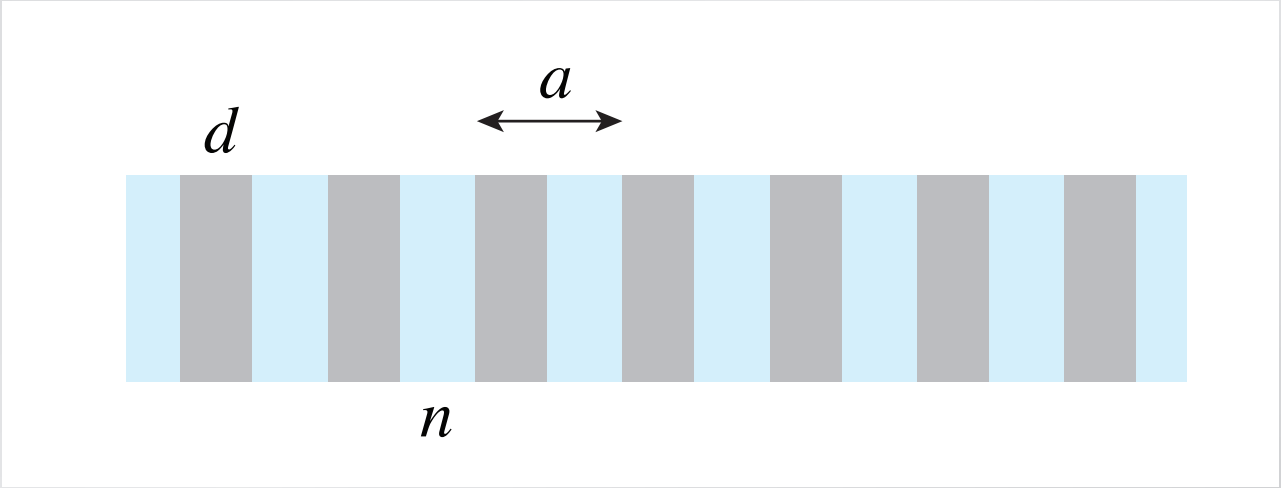
Engineering a magnetic response

adjustable parameters

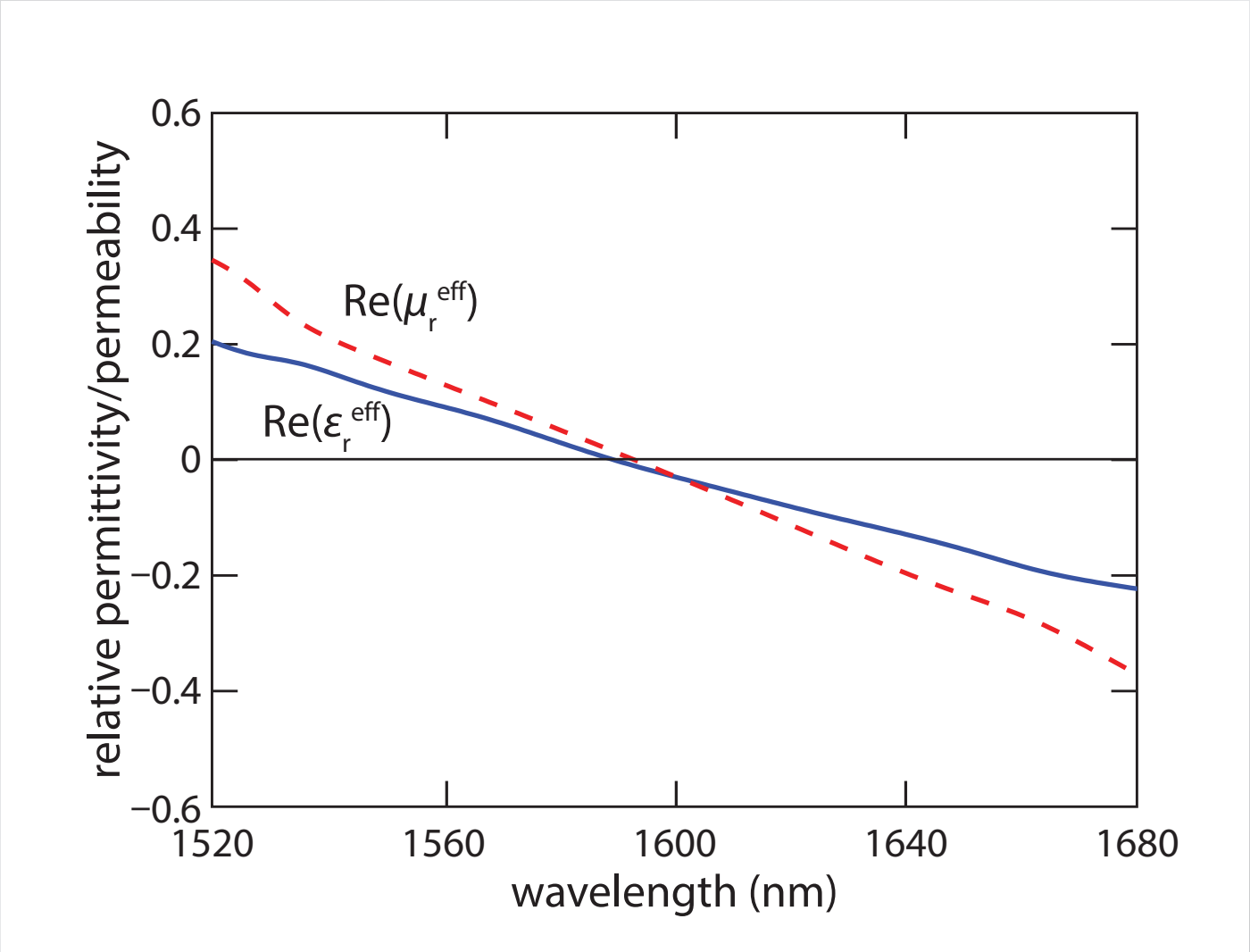


Engineering a magnetic response

adjustable parameters



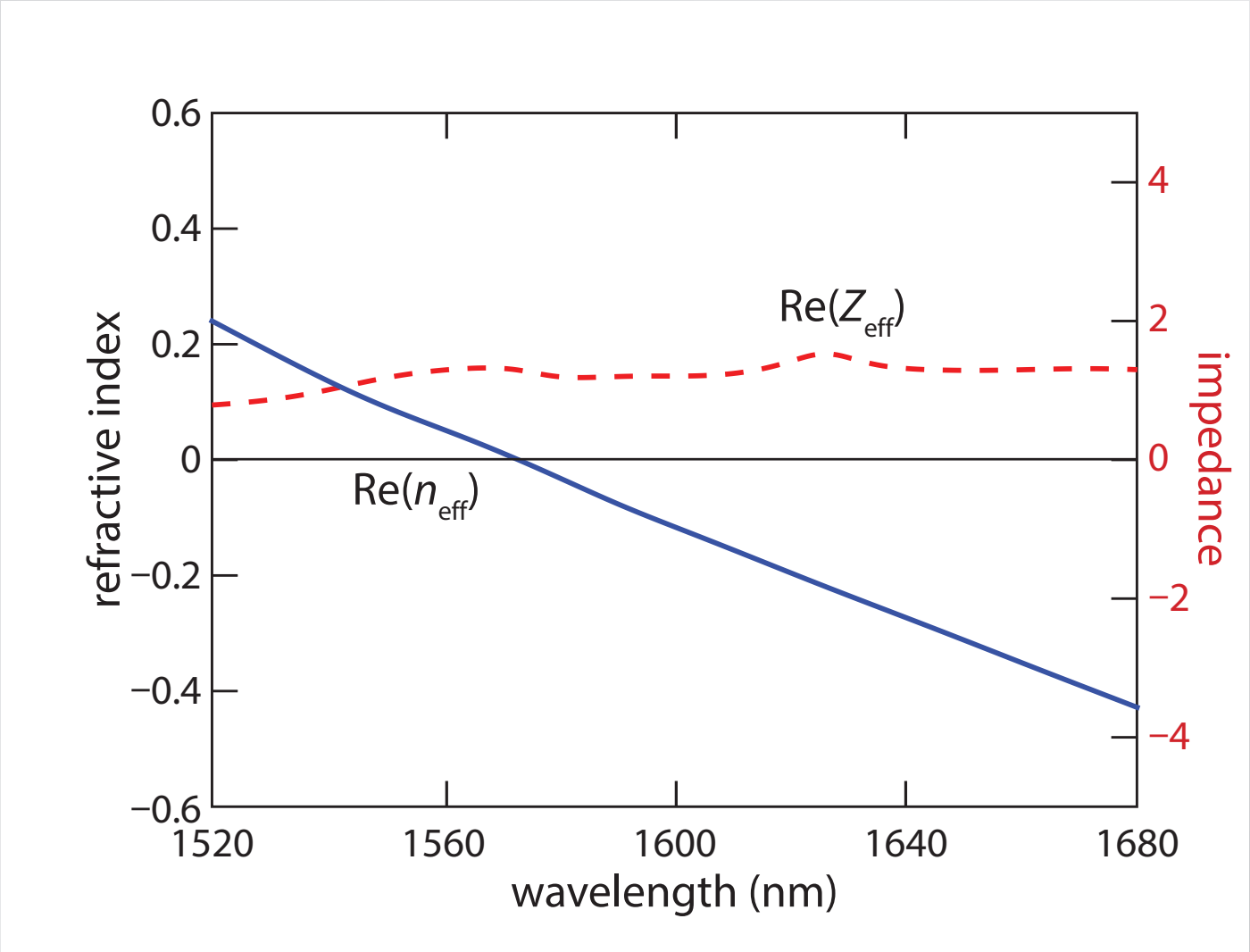
d = 422 nm, a = 690 nm, n = 1.57 (SU8)



1 index

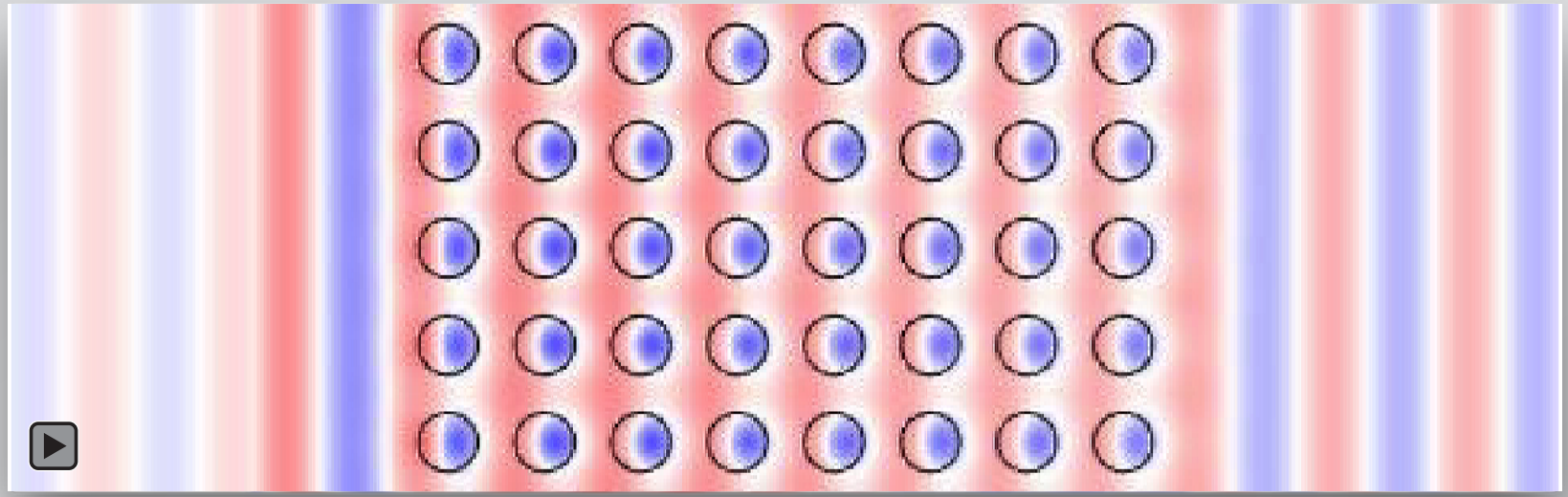
2 zero index

3 experiments

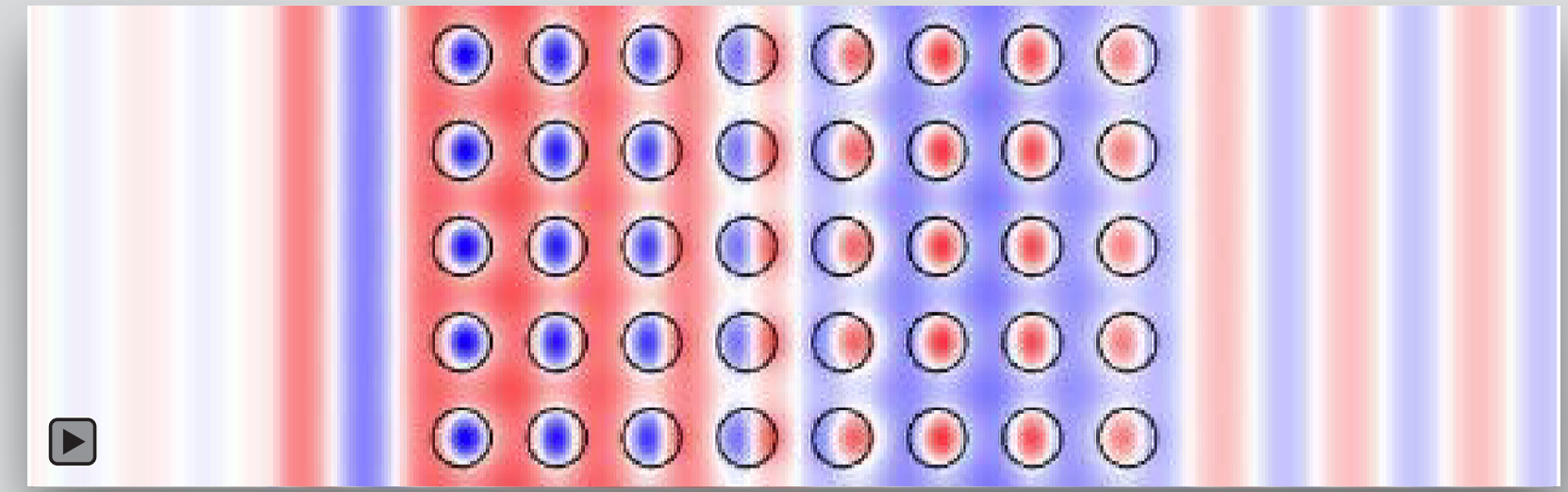


1 index 2 zero index 3 experiments

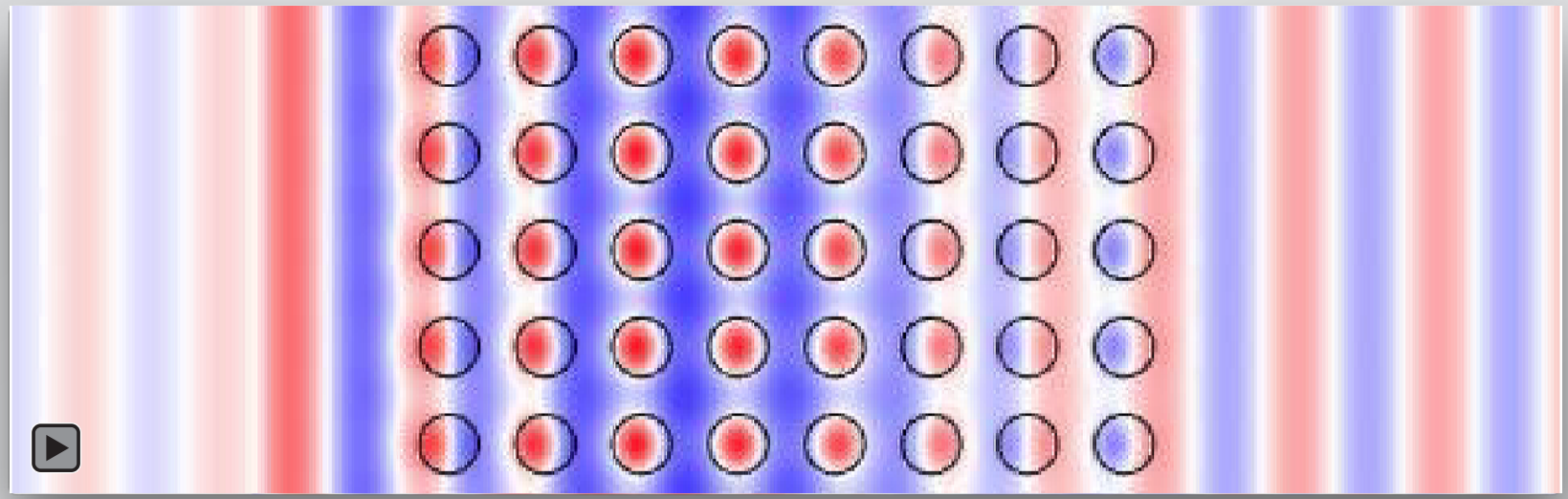
at design wavelength (1590 nm)



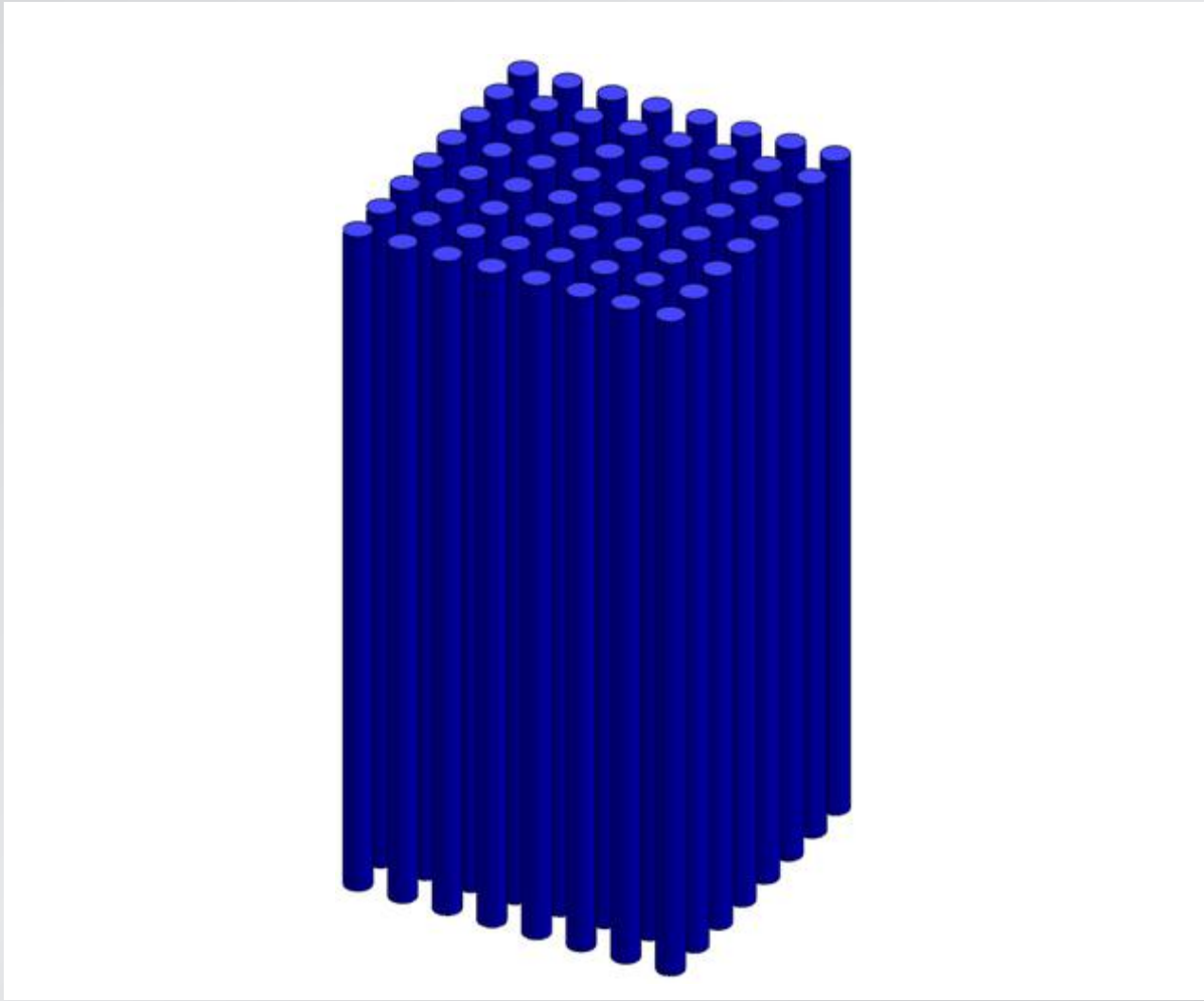
below design wavelength (1530 nm)



above design wavelength (1650 nm)



How to fabricate?



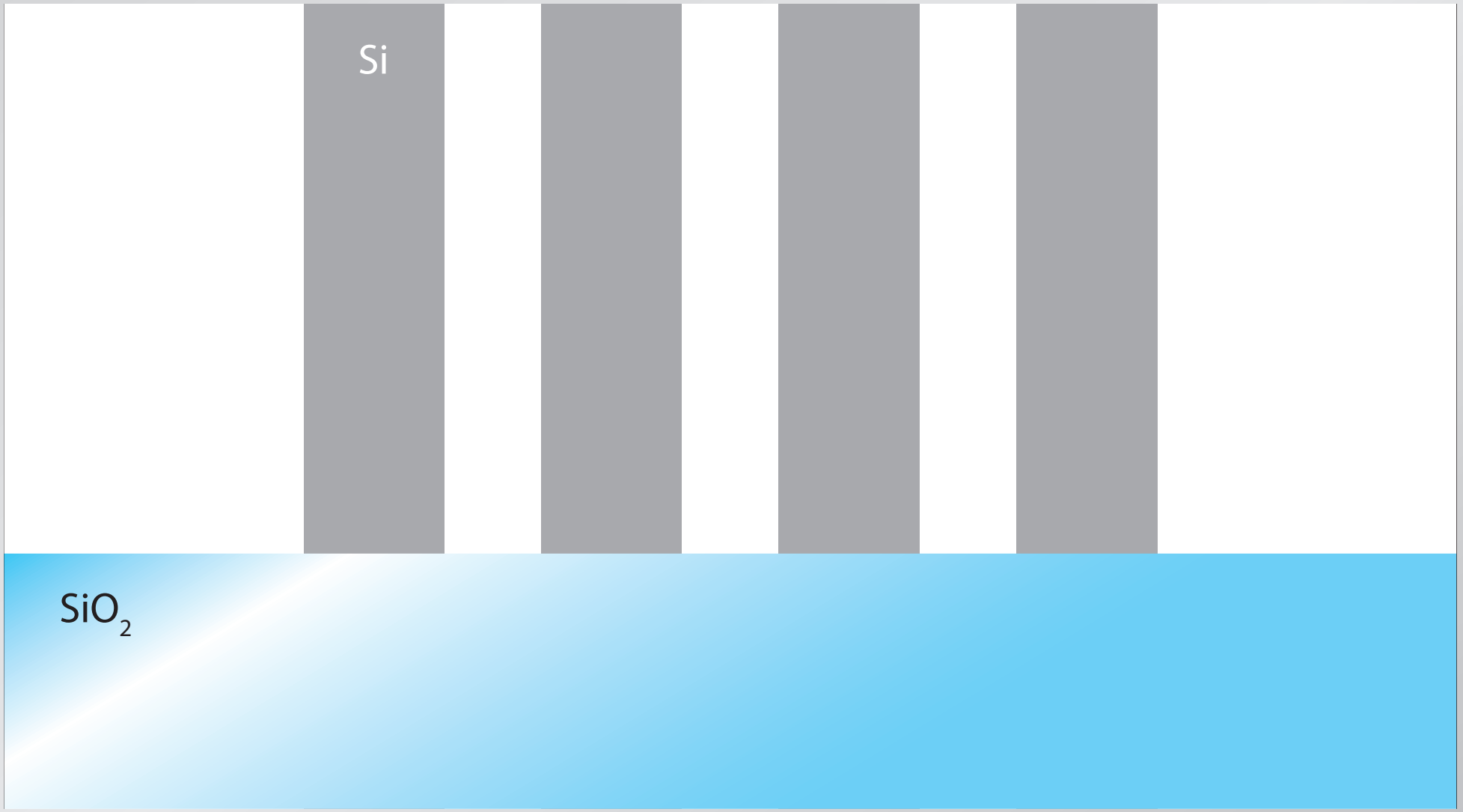
1 index

2 zero index

3 experiments

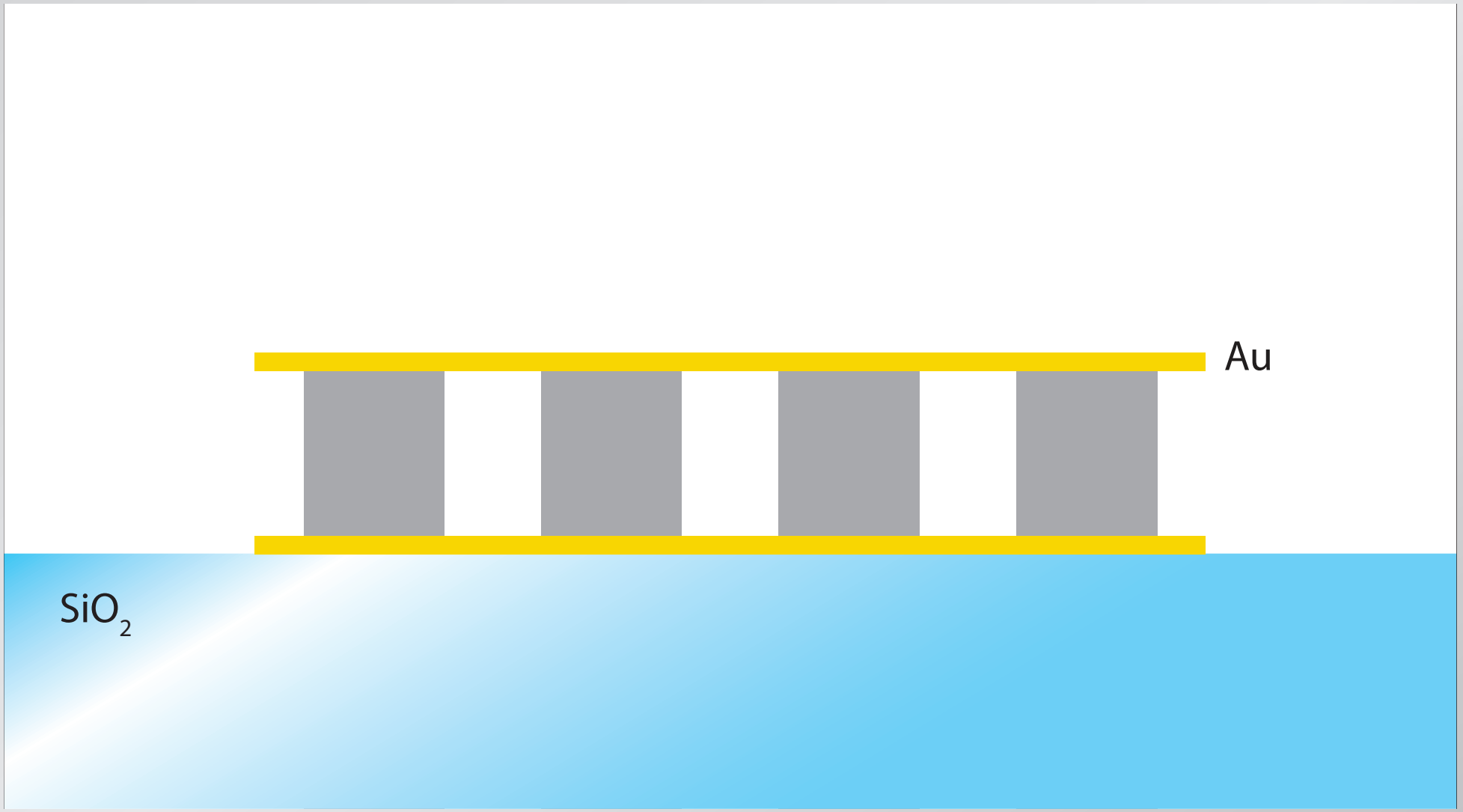
On-chip zero-index fabrication

On-chip zero-index fabrication

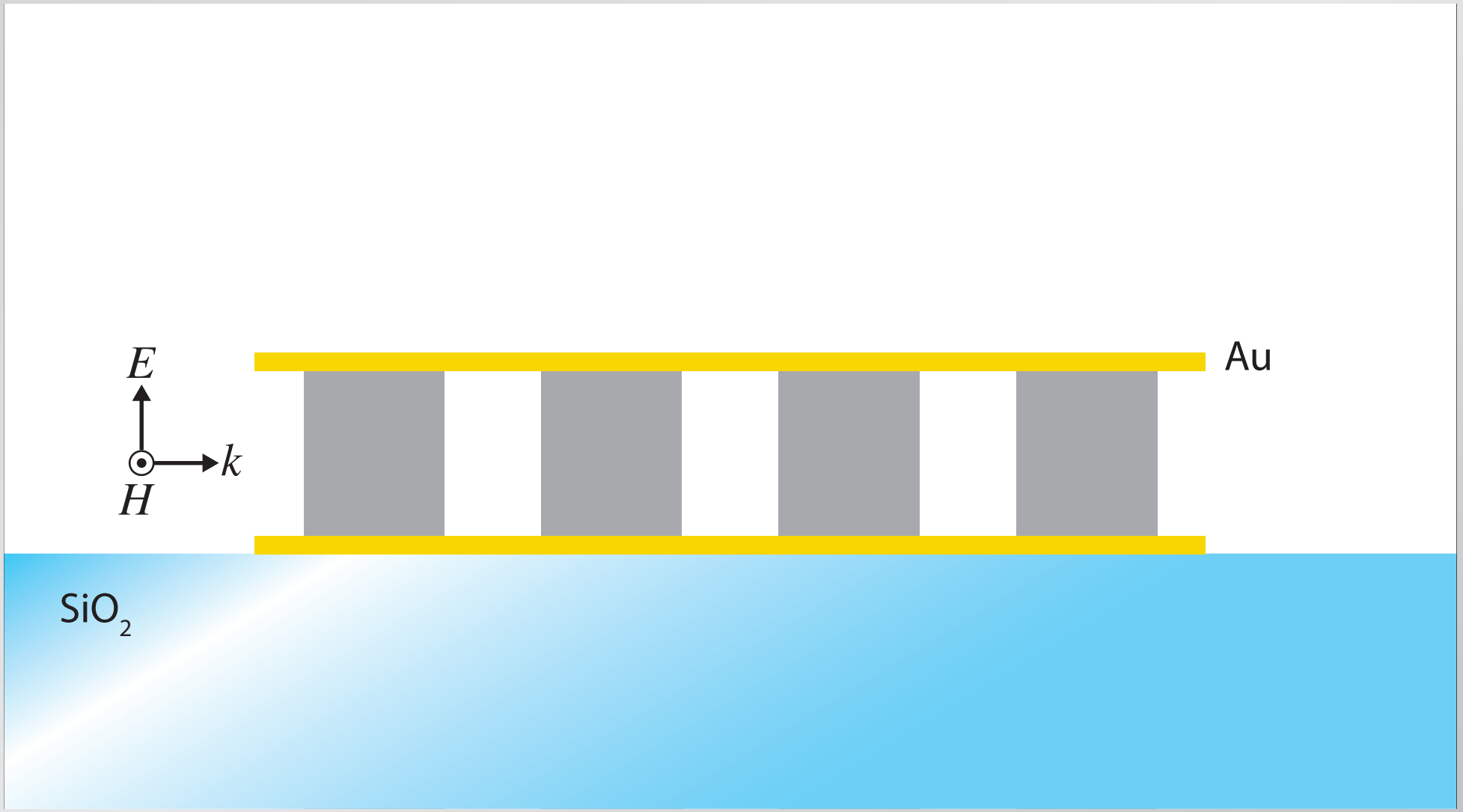


On-chip zero-index fabrication

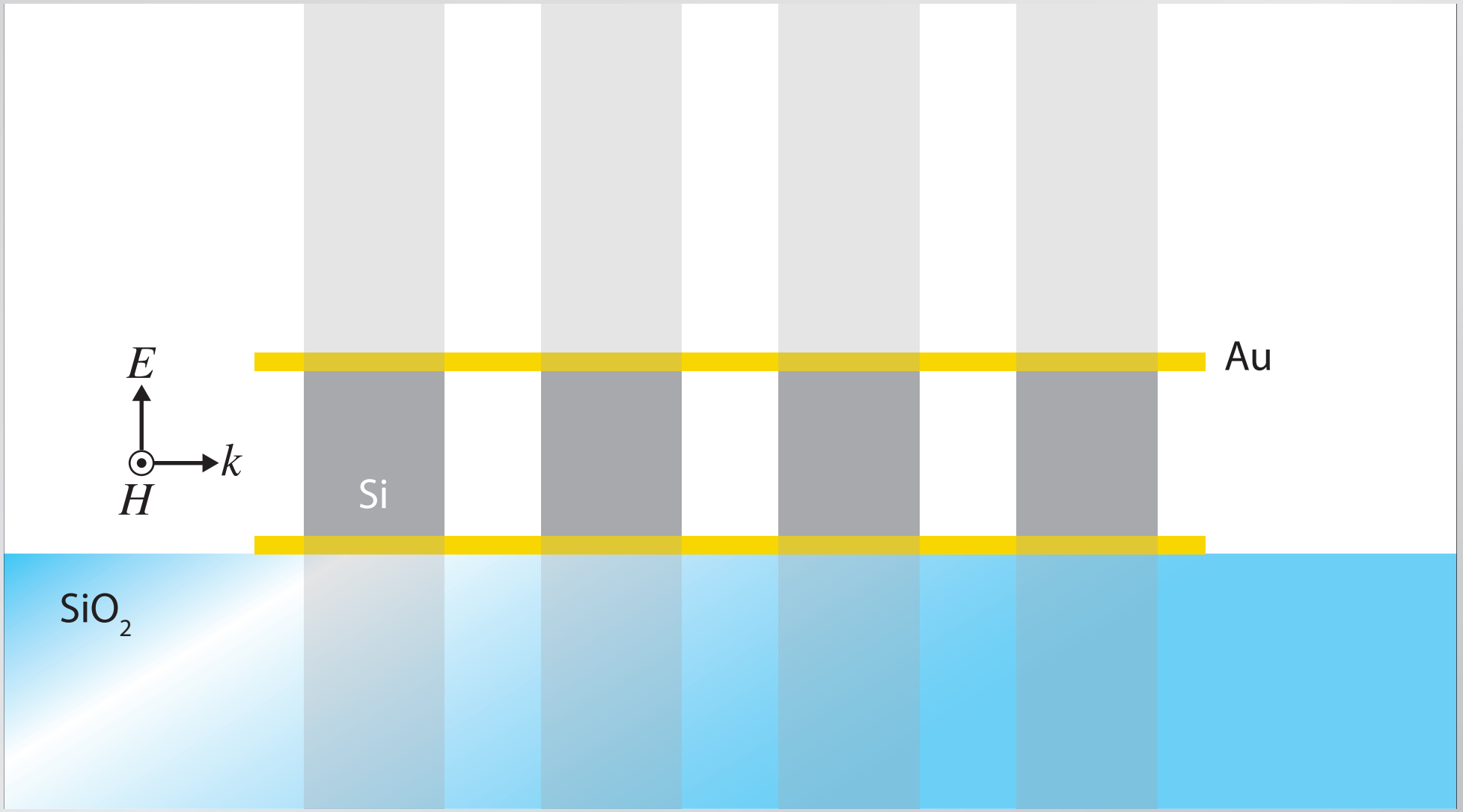
On-chip zero-index fabrication



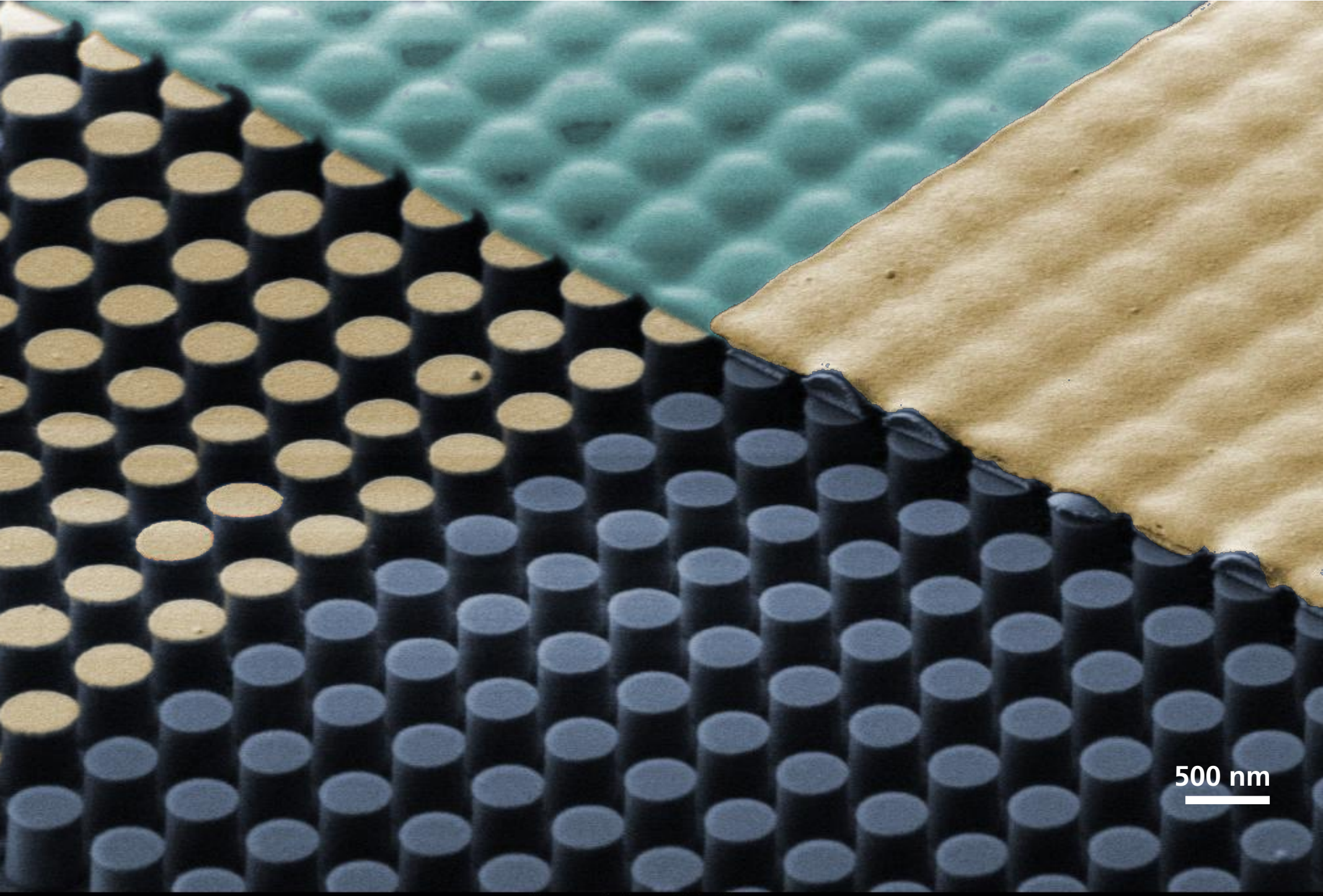
On-chip zero-index fabrication



On-chip zero-index fabrication



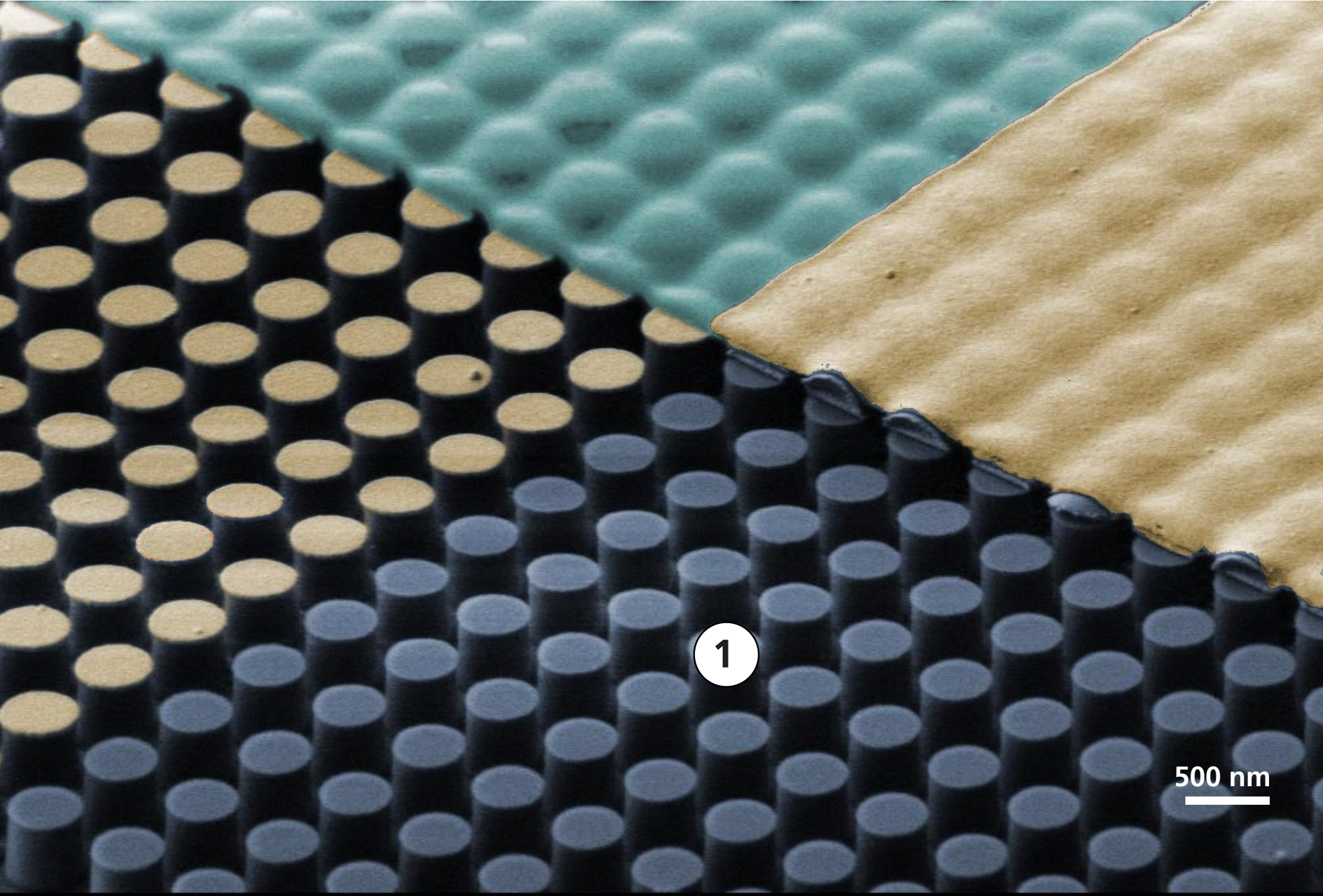
1 index 2 zero index 3 experiments



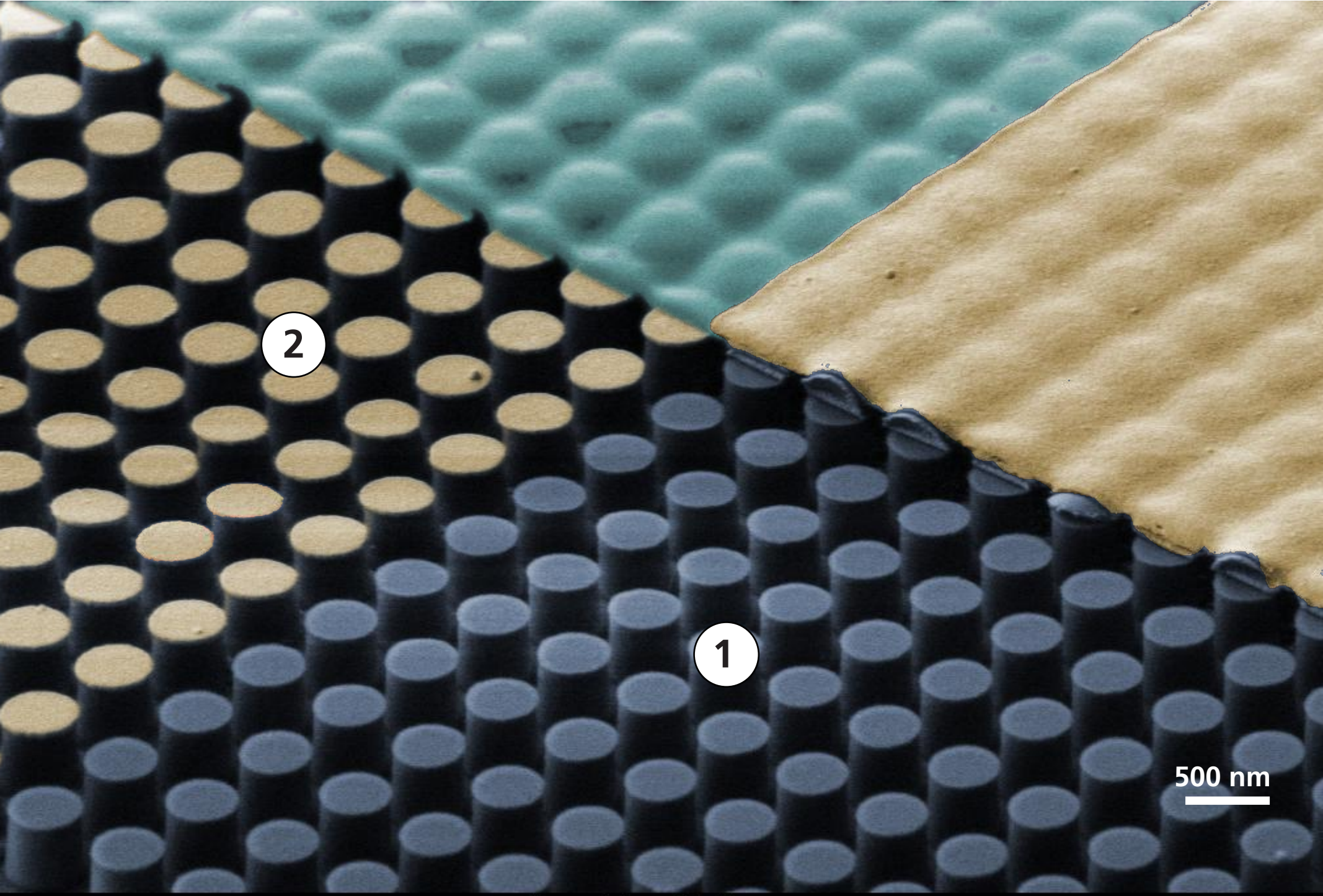
1 index

2 zero index

3 experiments



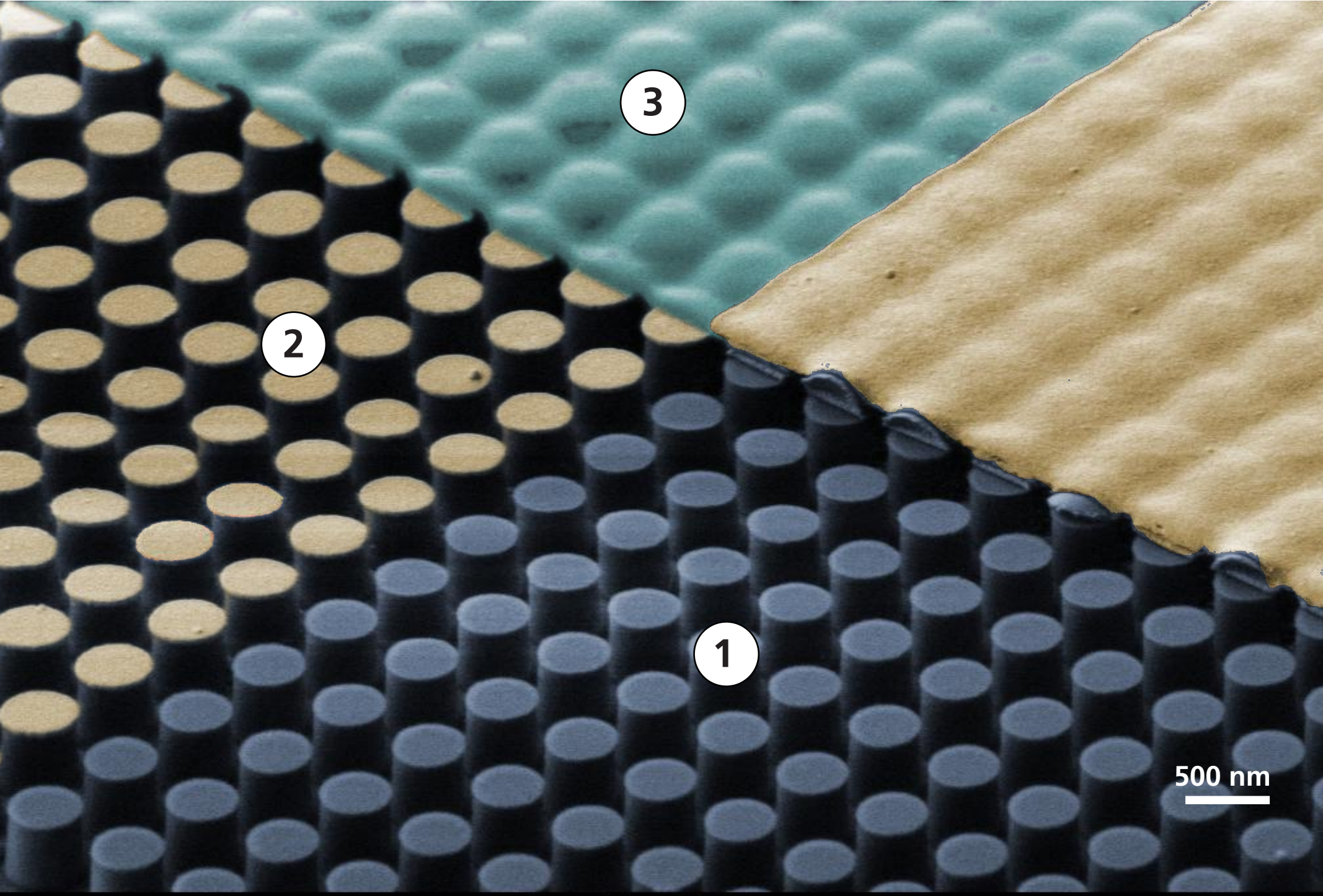
1 index 2 zero index 3 experiments



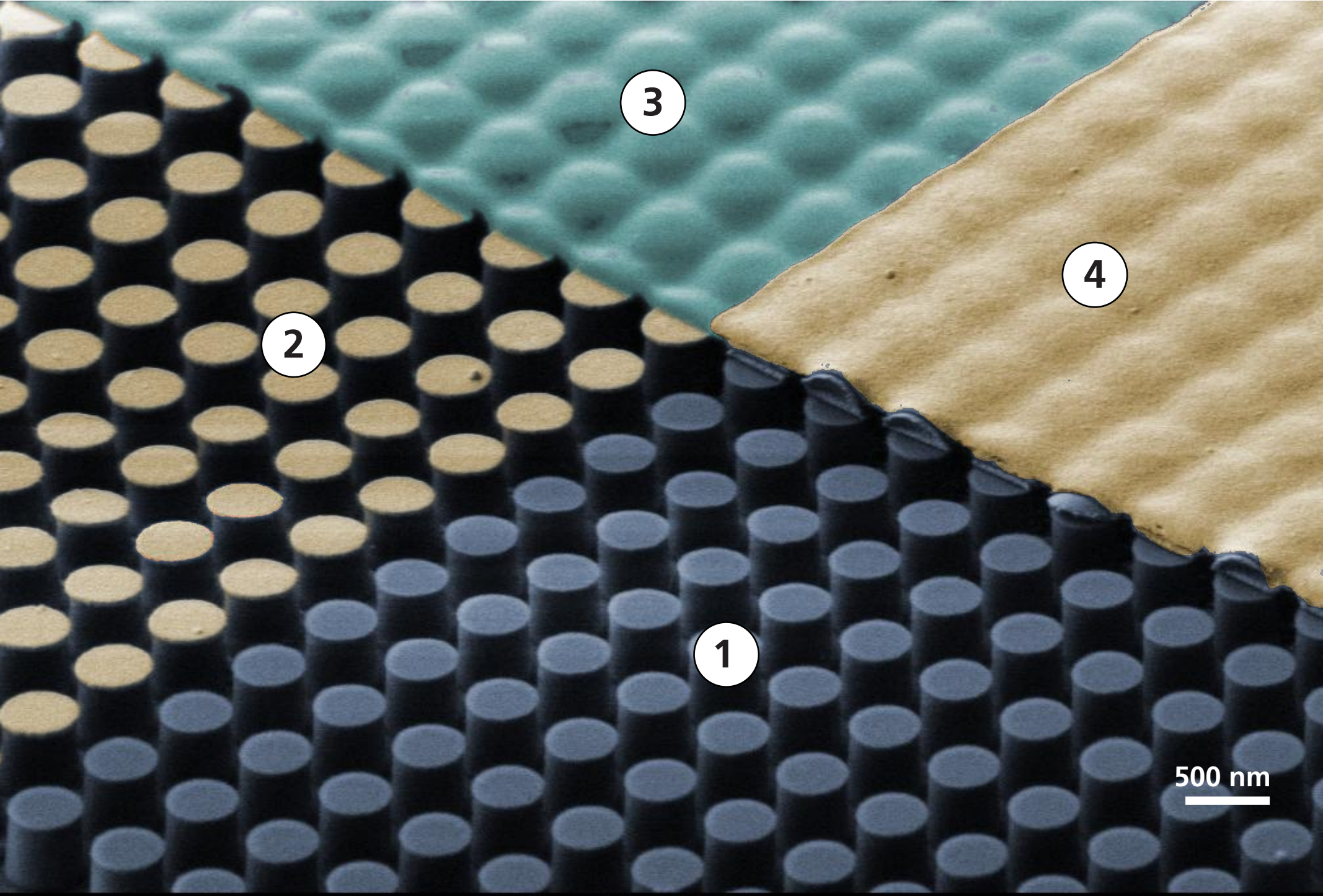
1 index

2 zero index

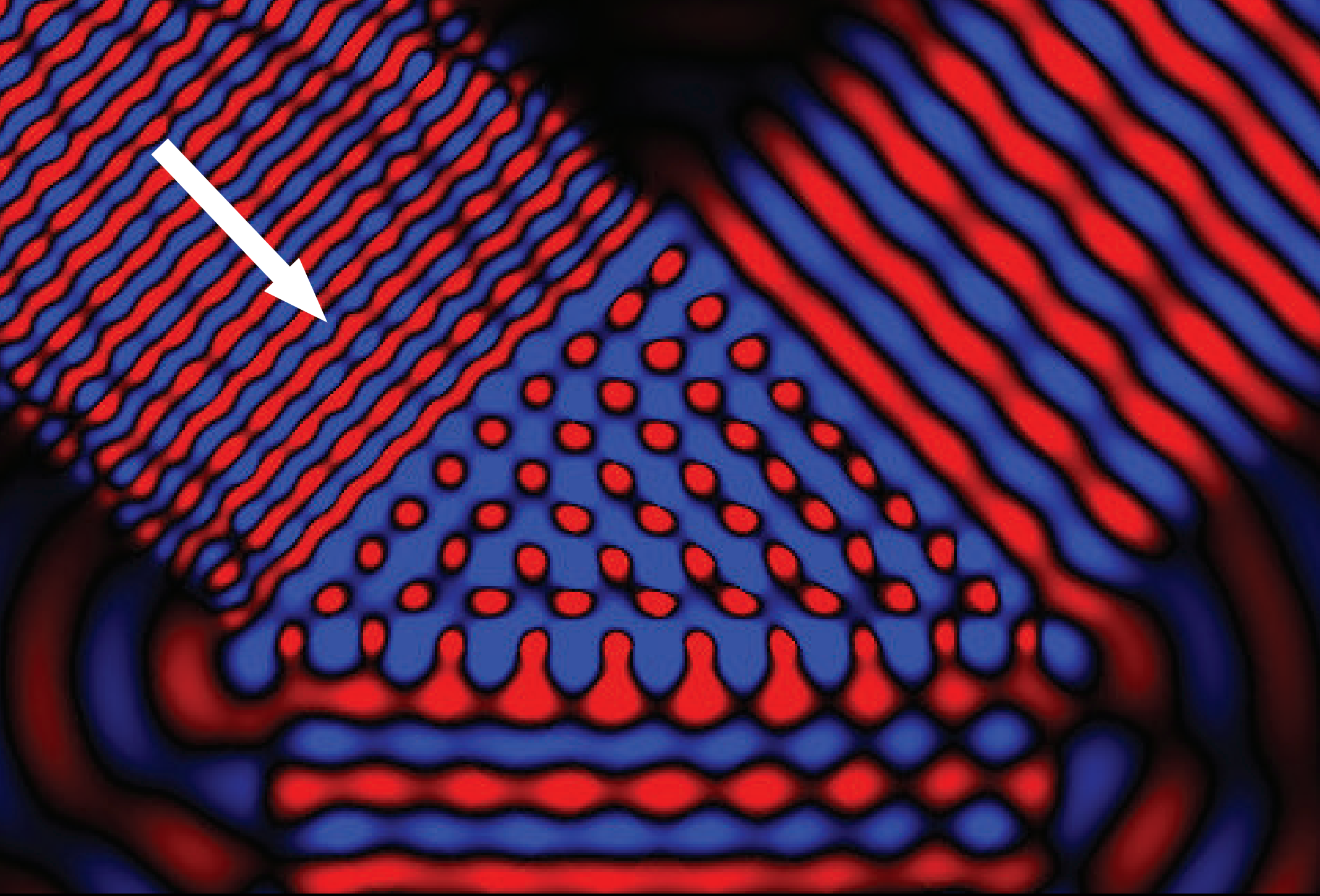
3 experiments



1 index 2 zero index 3 experiments

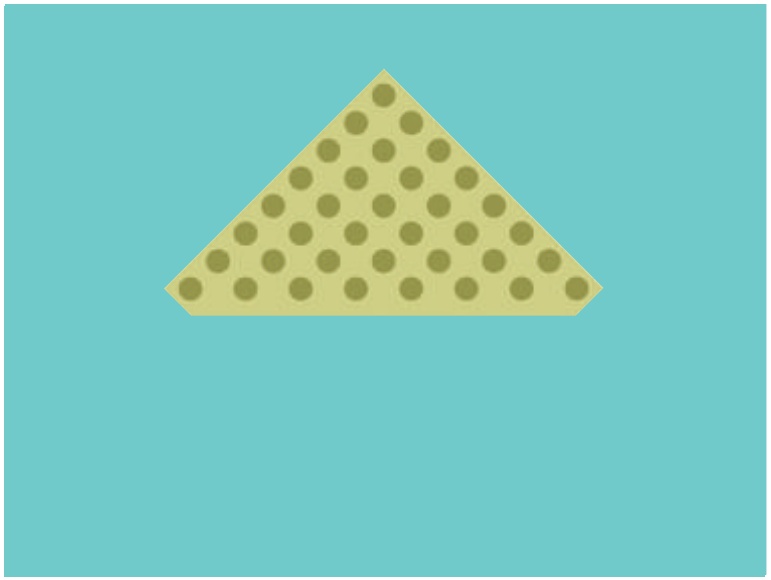


1 index 2 zero index 3 experiments

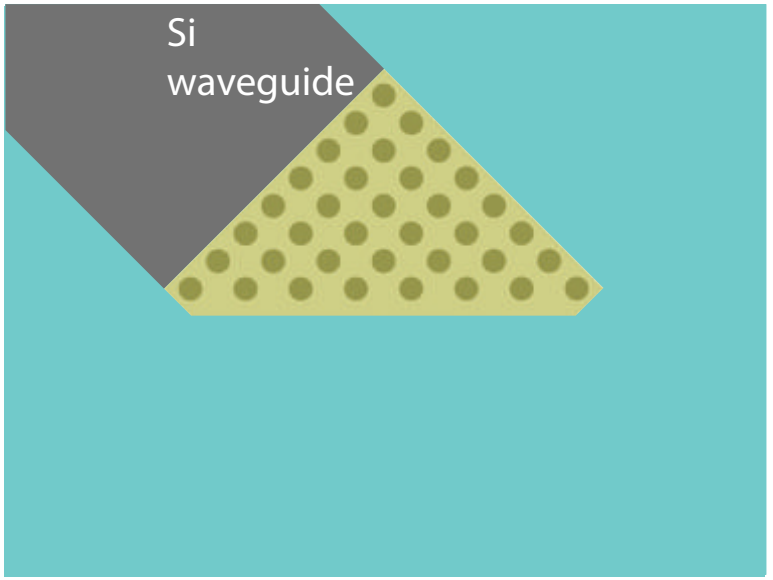

1 index
2 zero index
3 experiments

On-chip zero-index prism

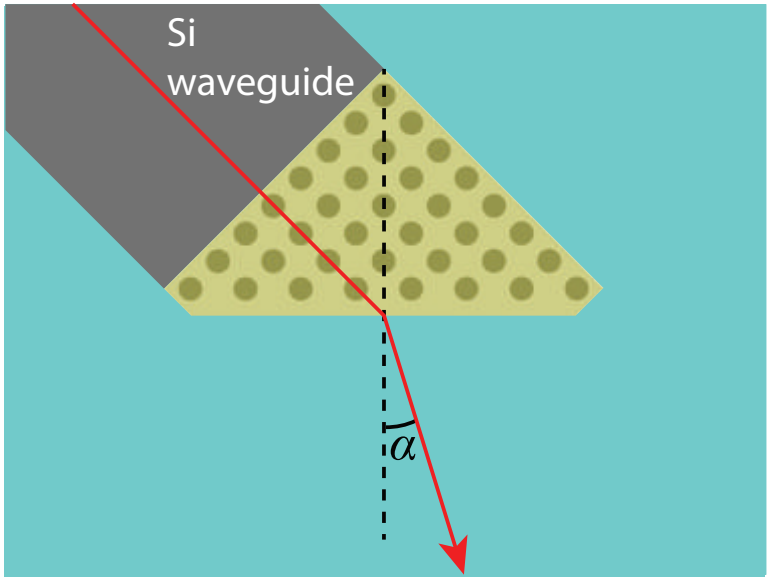
On-chip zero-index prism



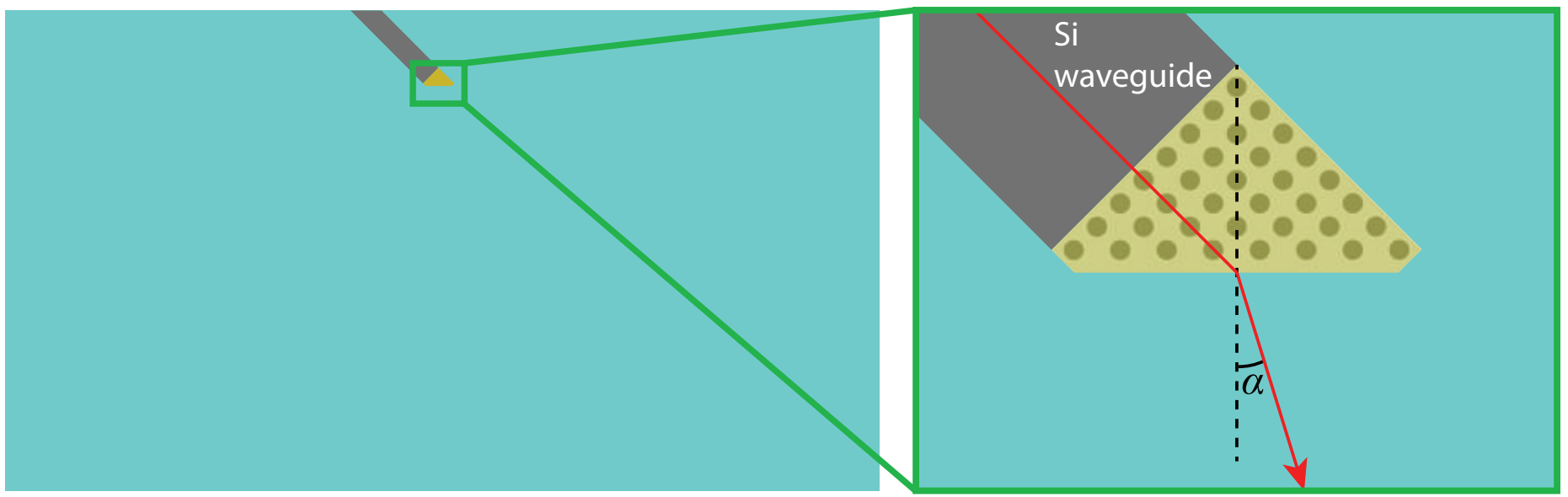
On-chip zero-index prism



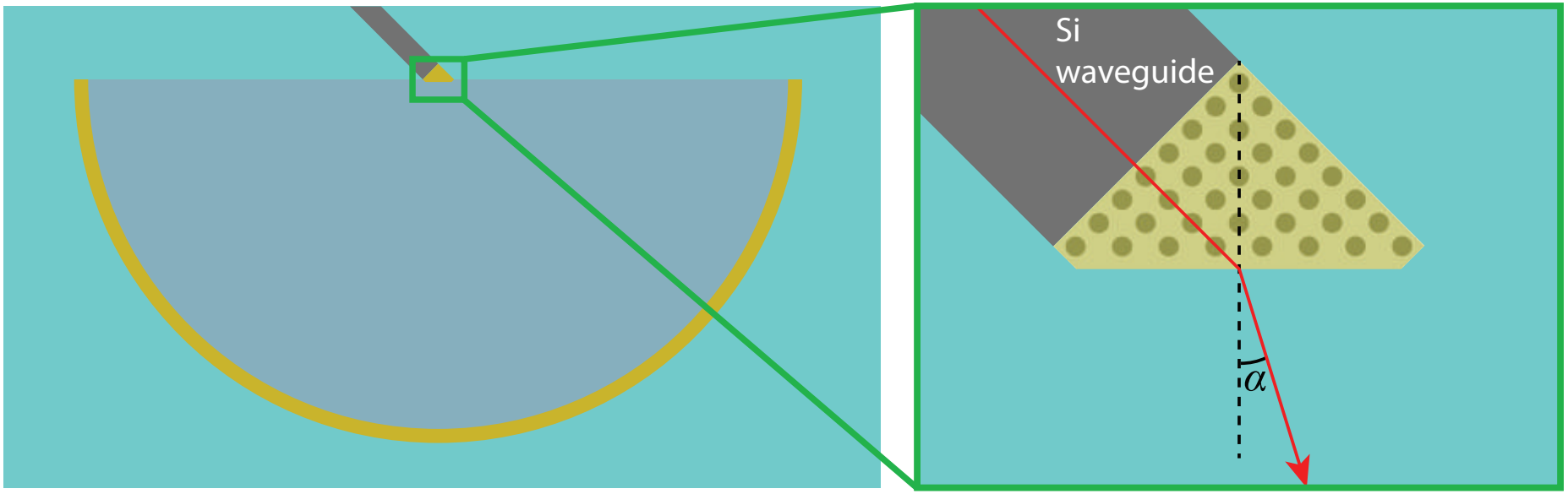
On-chip zero-index prism



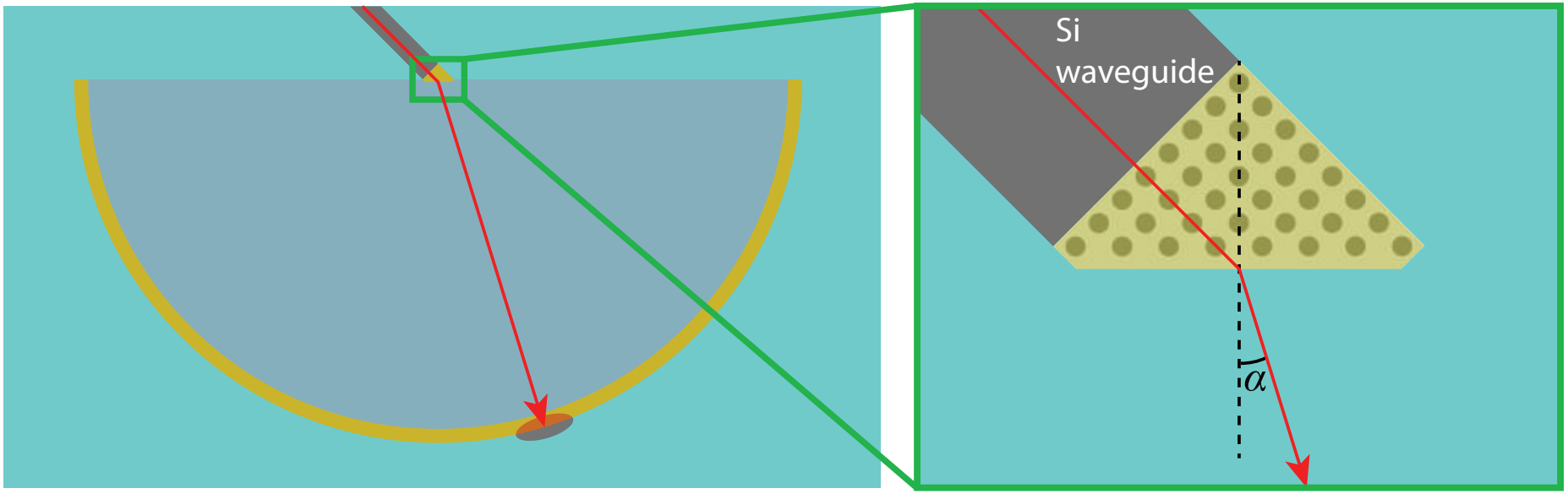
On-chip zero-index prism



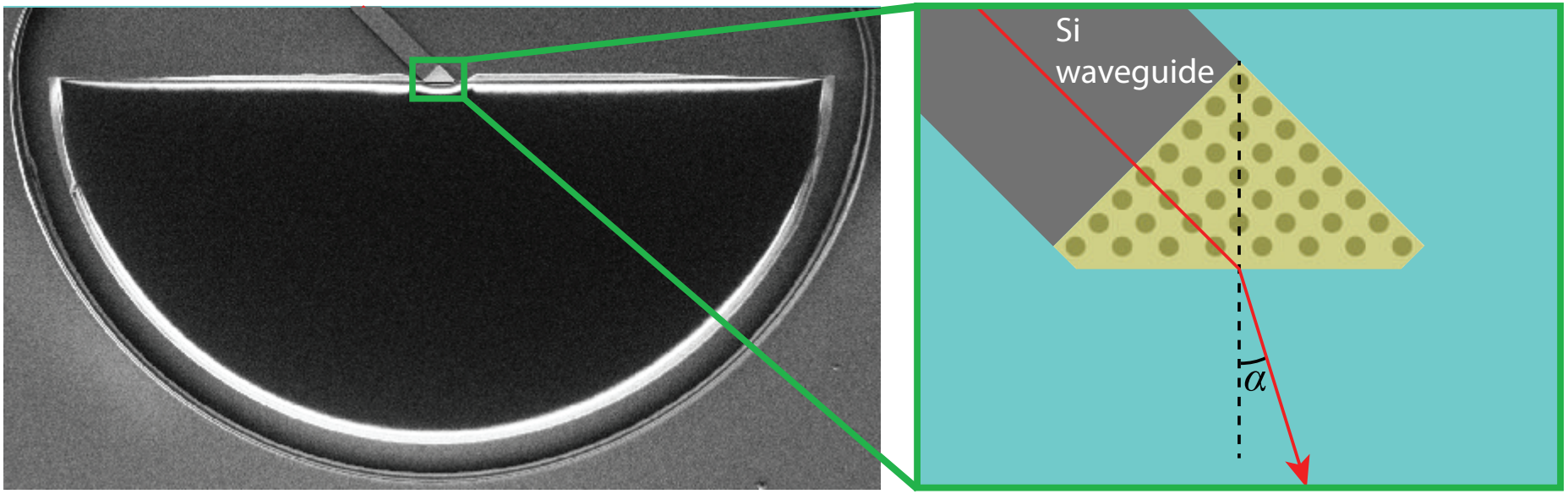
On-chip zero-index prism



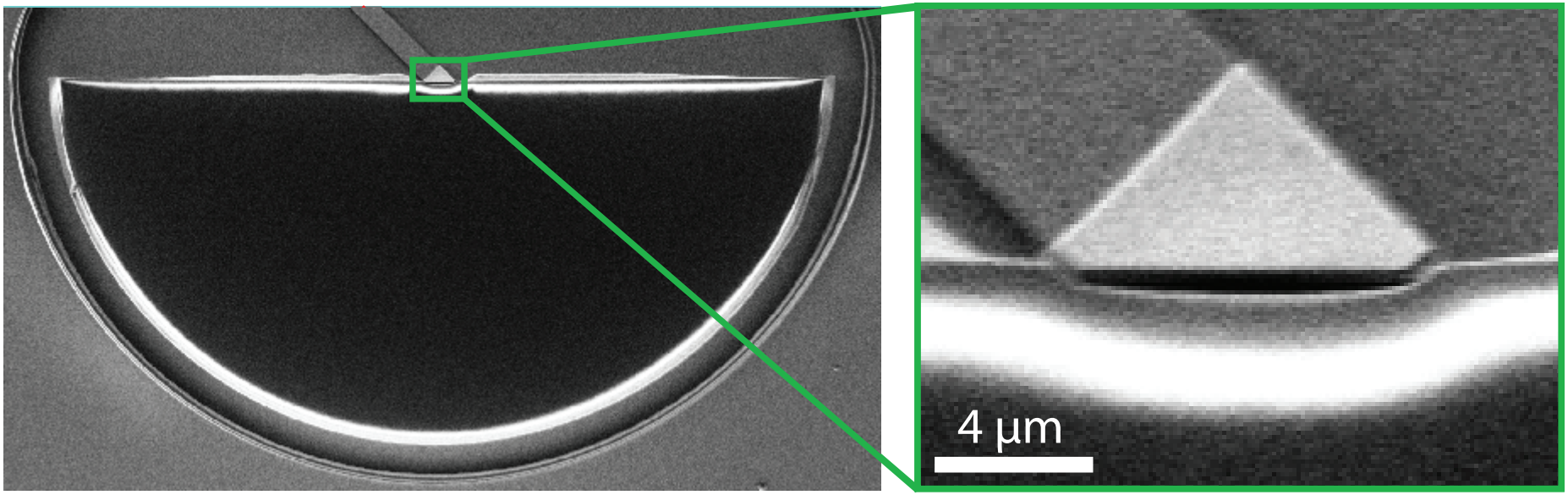
On-chip zero-index prism

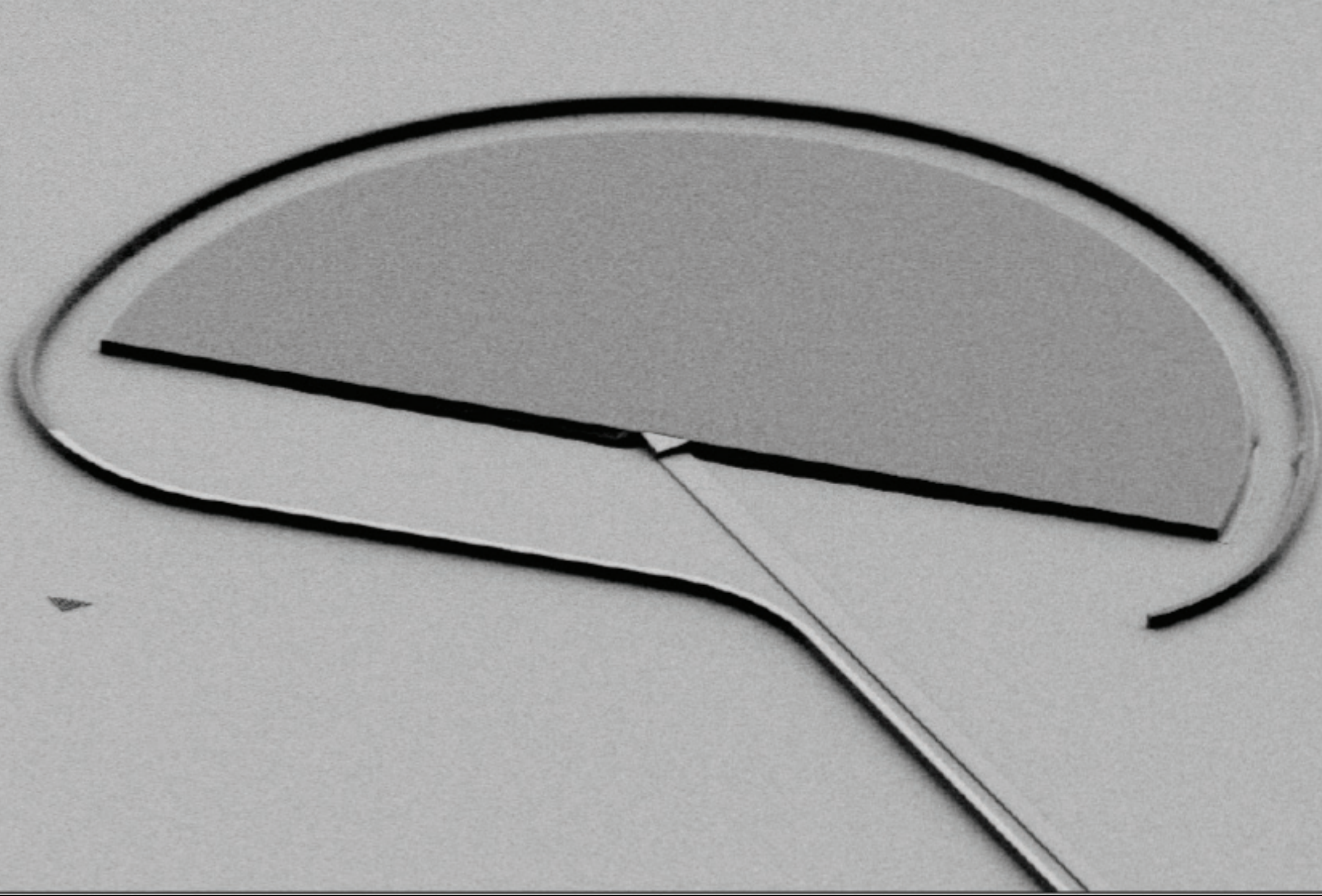


On-chip zero-index prism

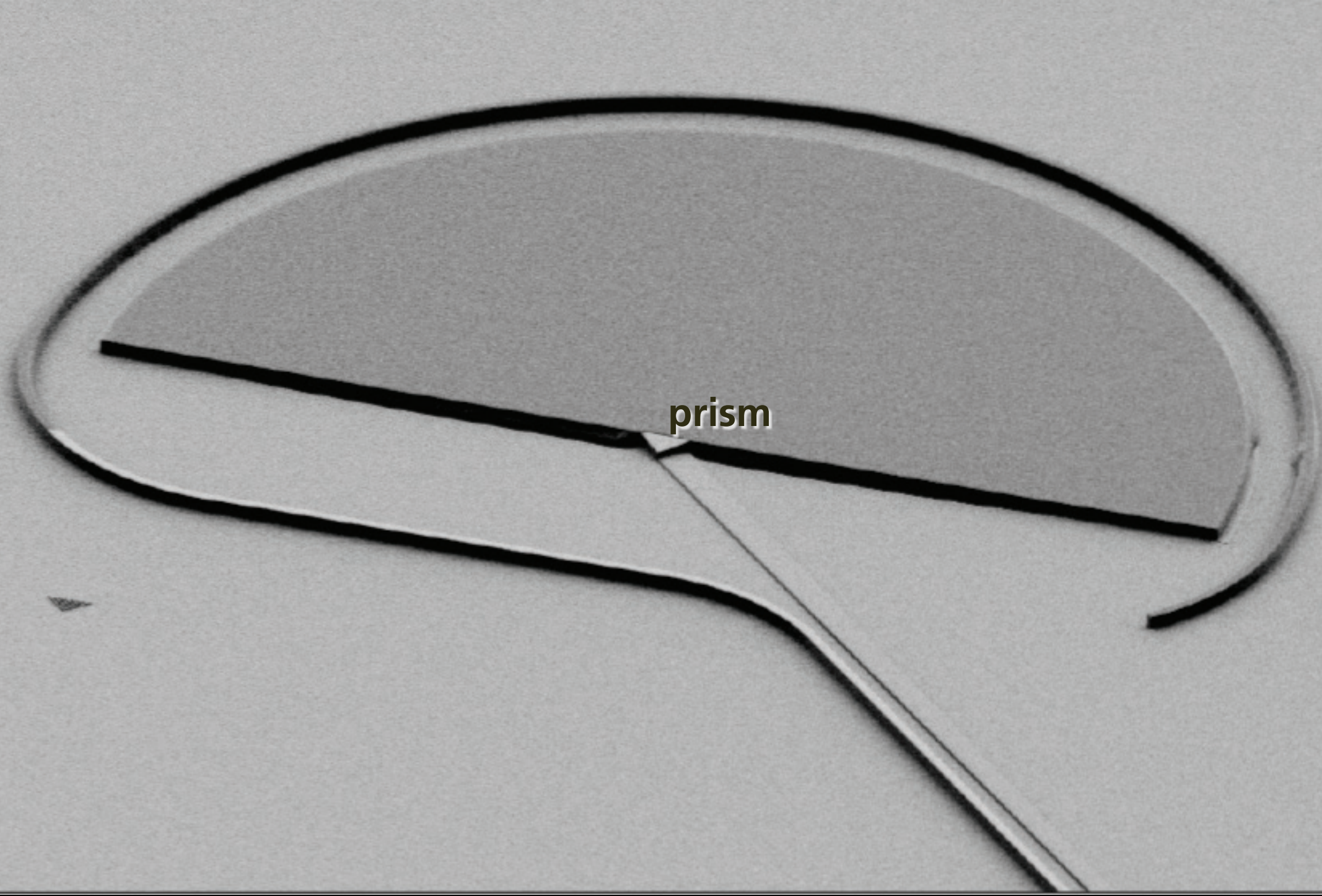


On-chip zero-index prism





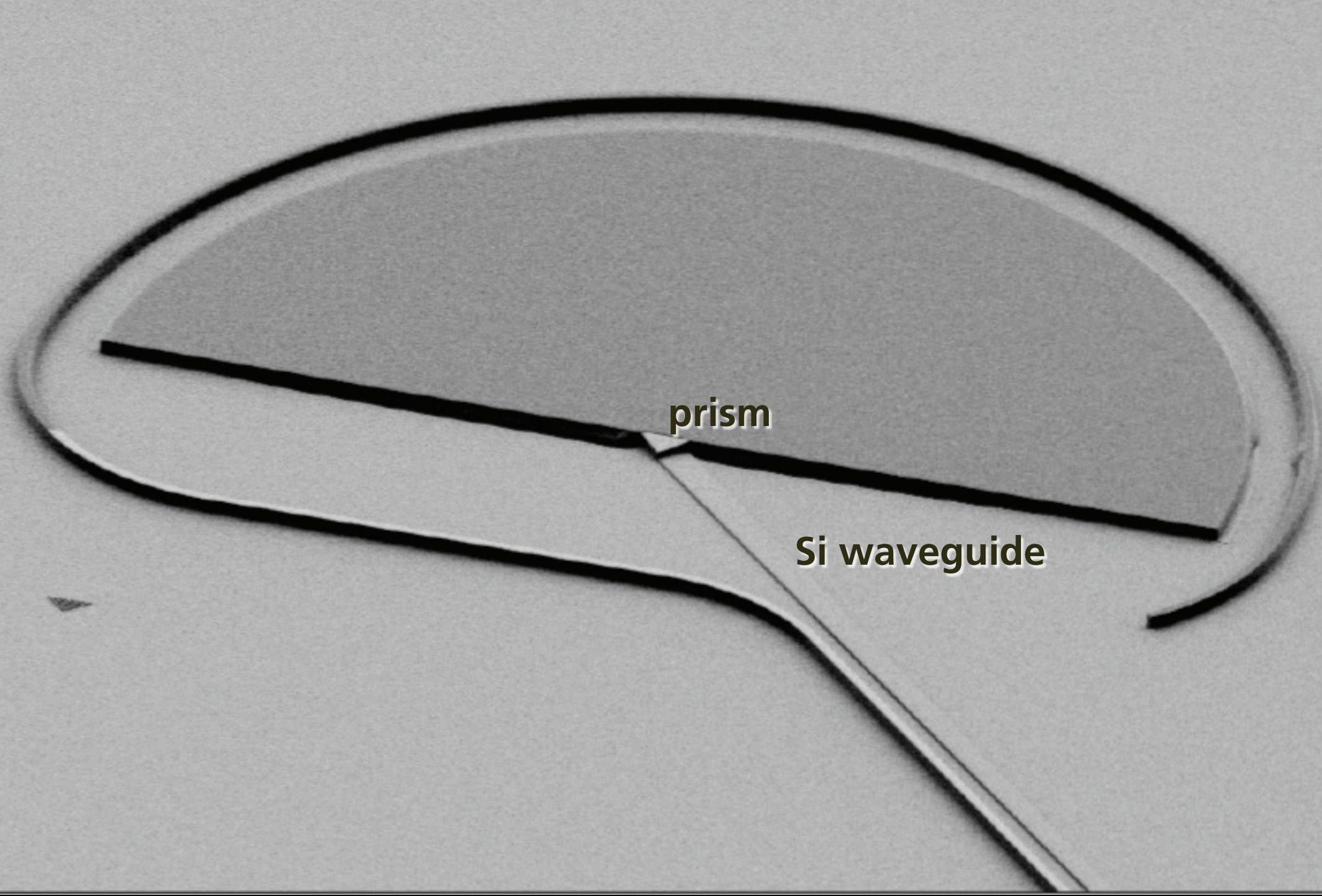
1 index 2 zero index 3 experiments



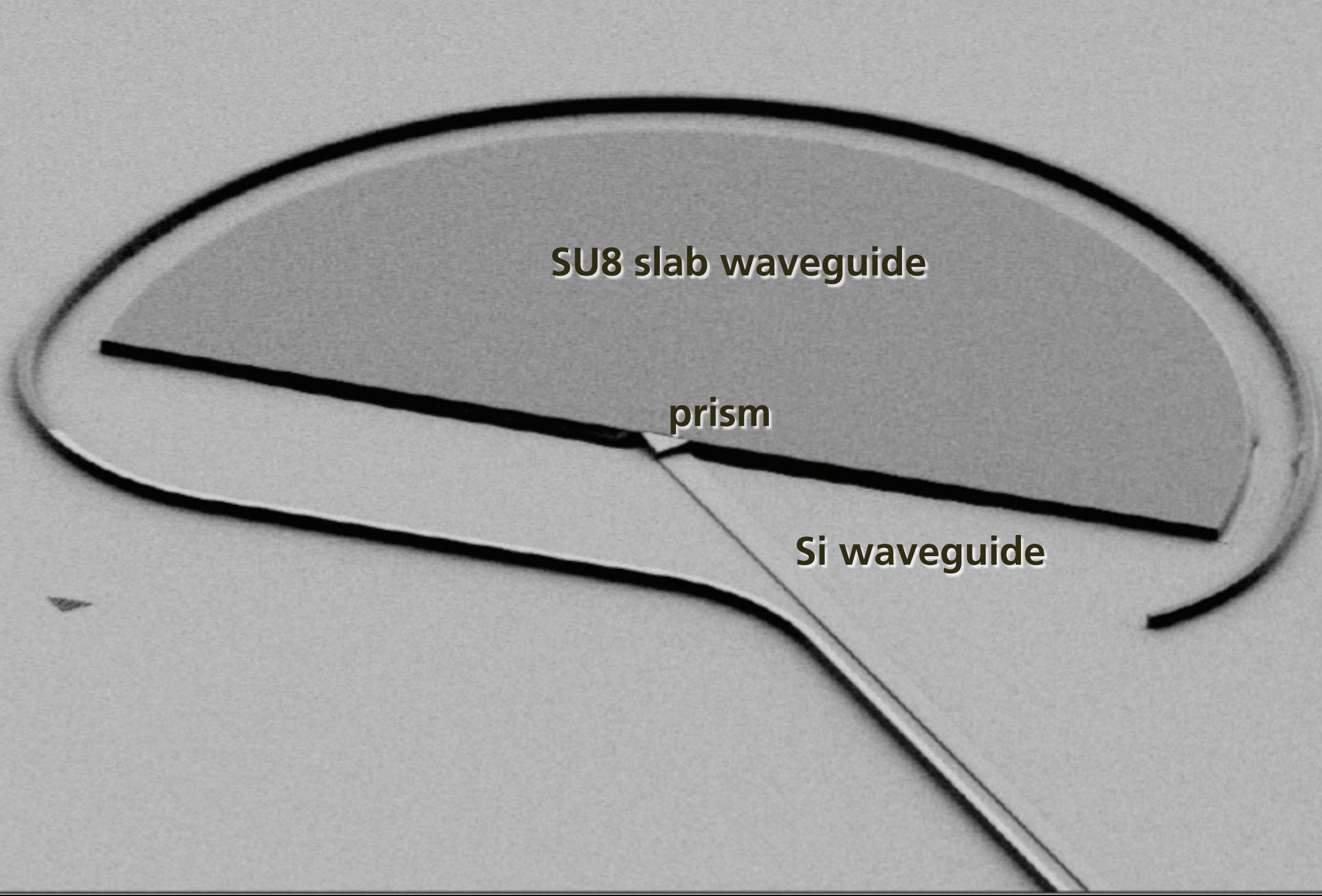
1 index

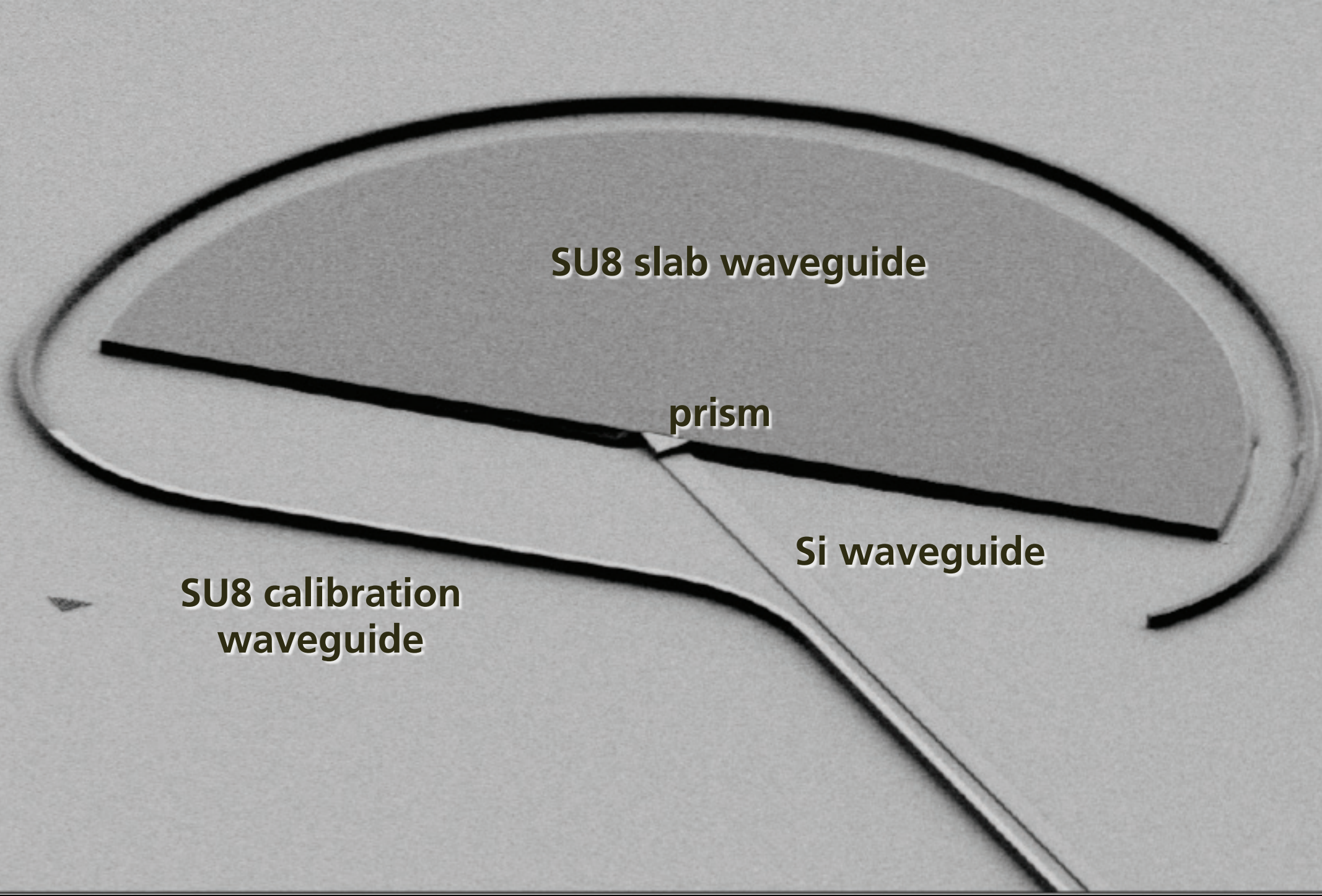
2 zero index

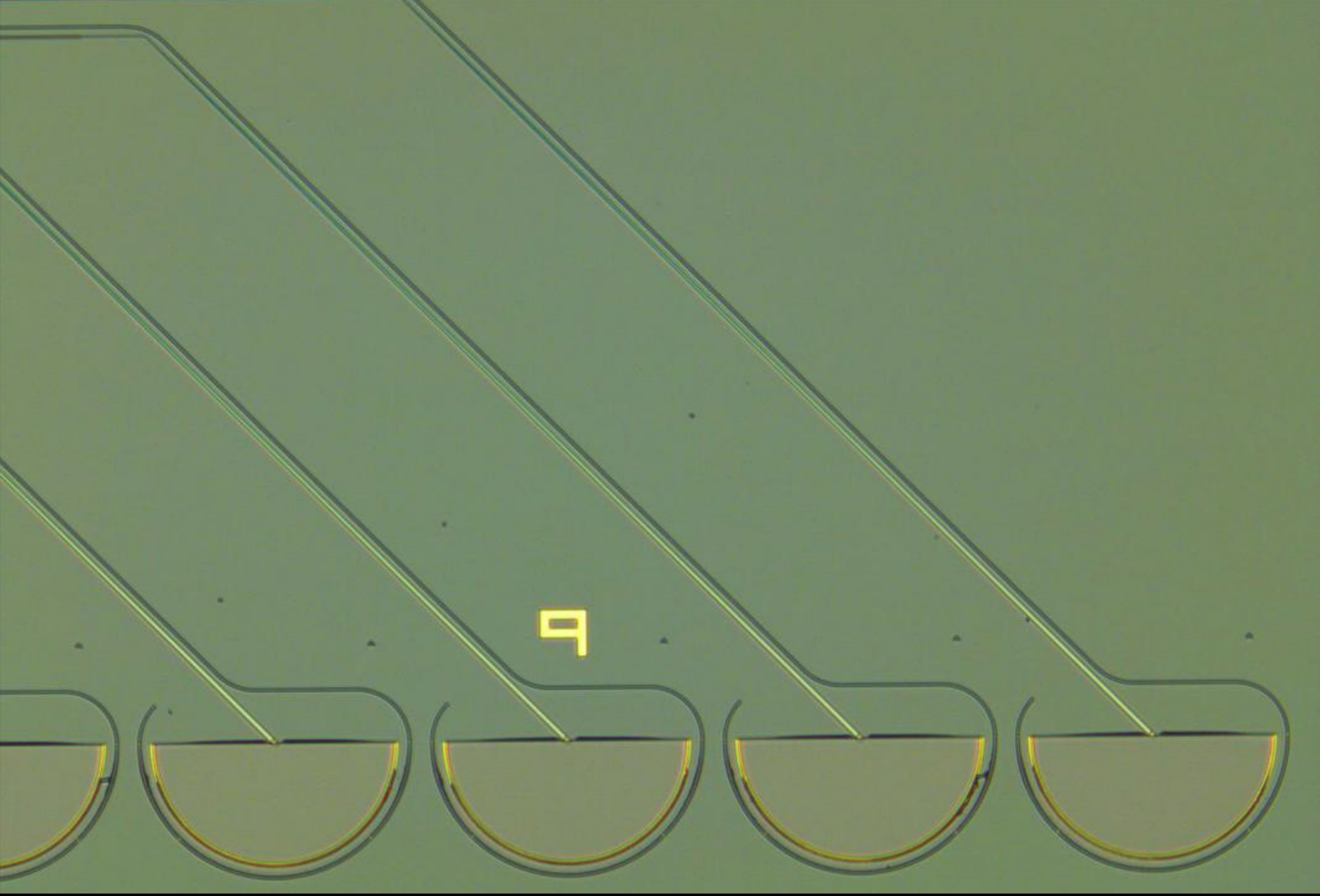
3 experiments



1 index 2 zero index 3 experiments

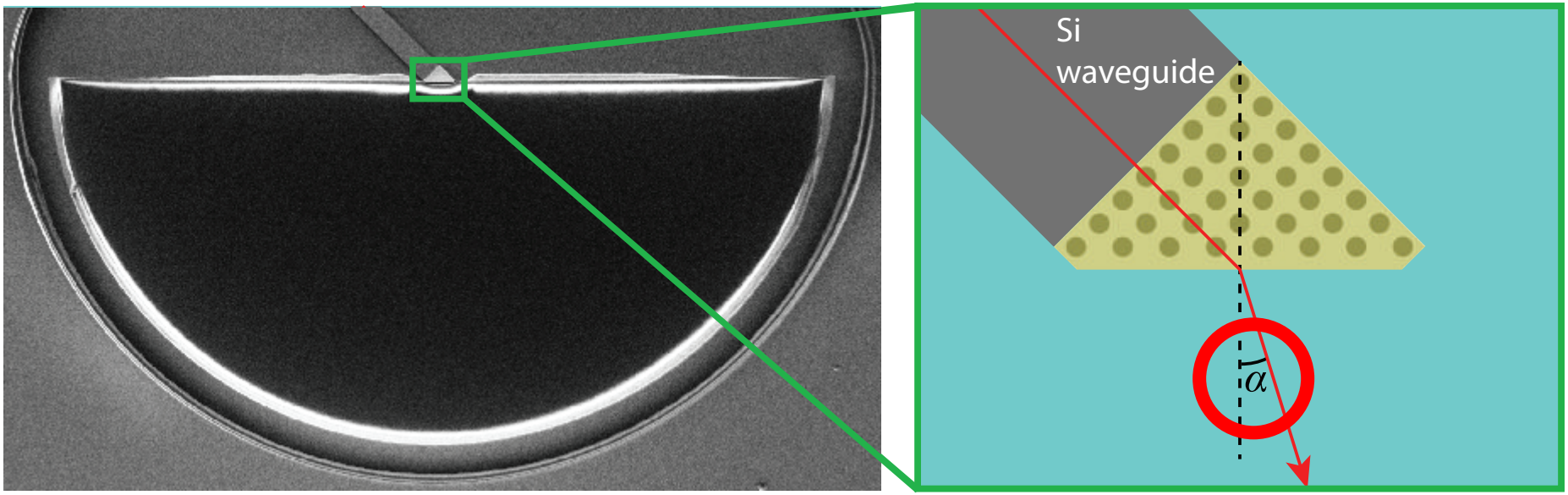

SU8 slab waveguide
prism
Si waveguide

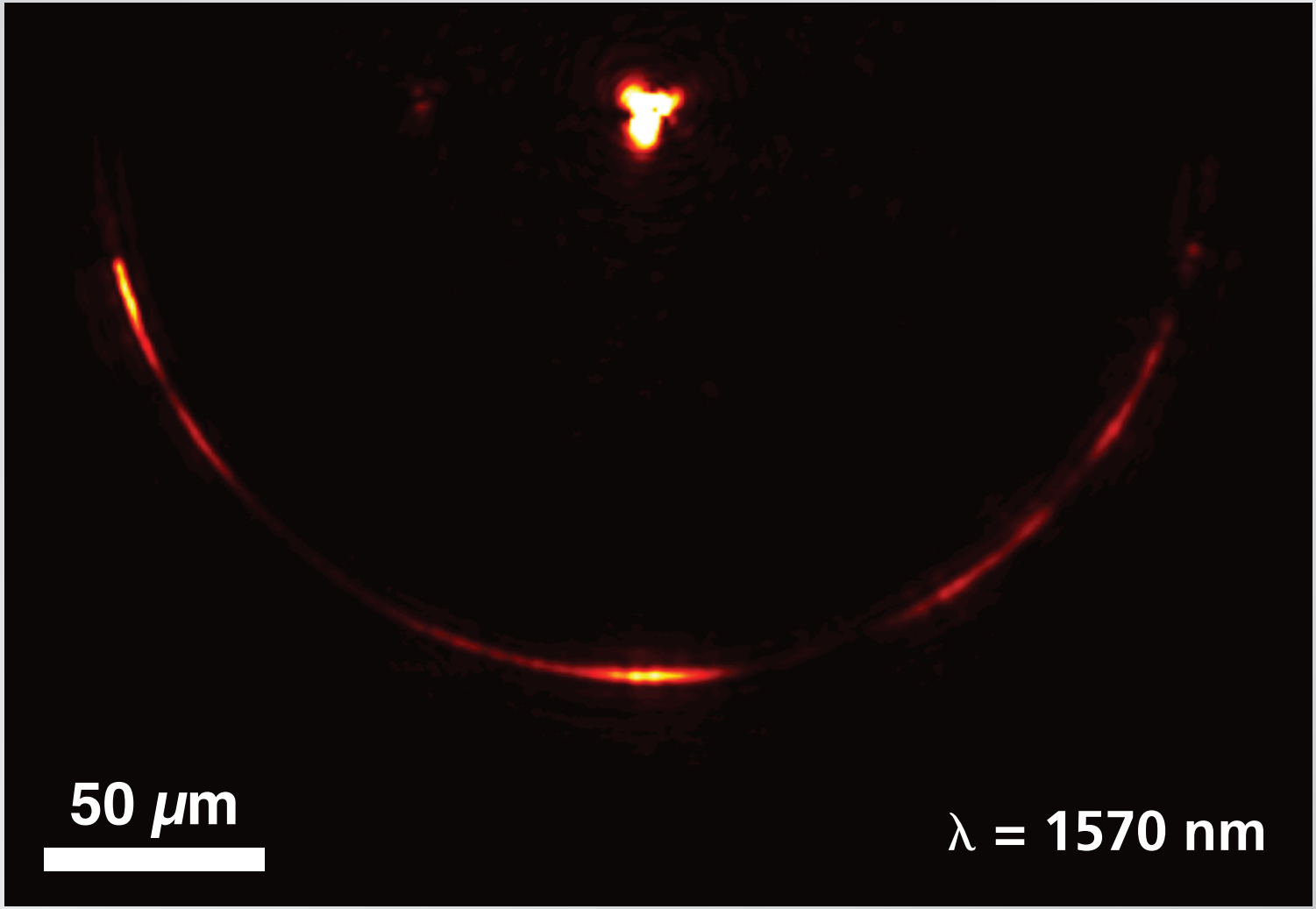

SU8 slab waveguide
prism
Si waveguide
SU8 calibration
waveguide



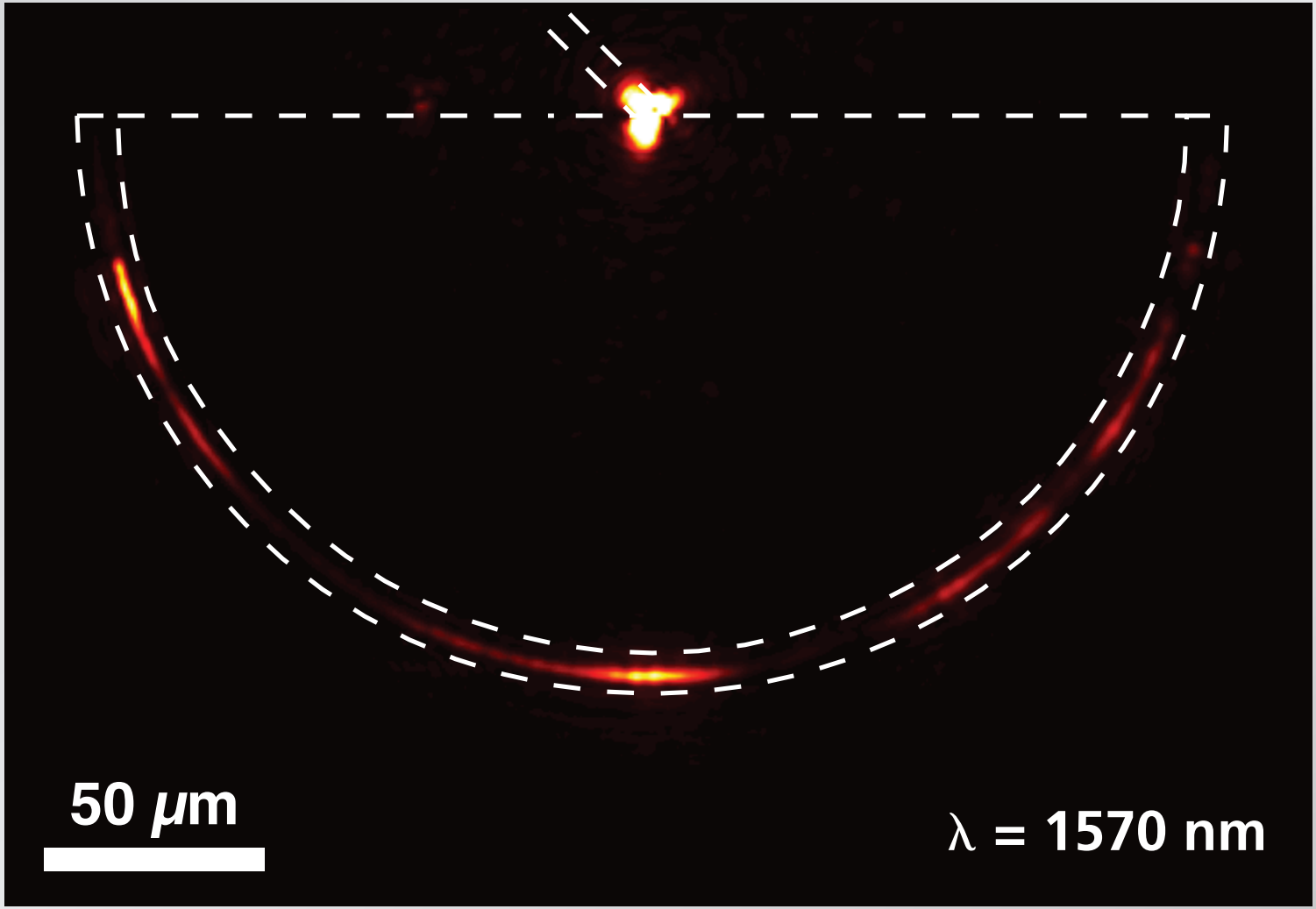
1 index 2 zero index 3 experiments

On-chip zero-index prism

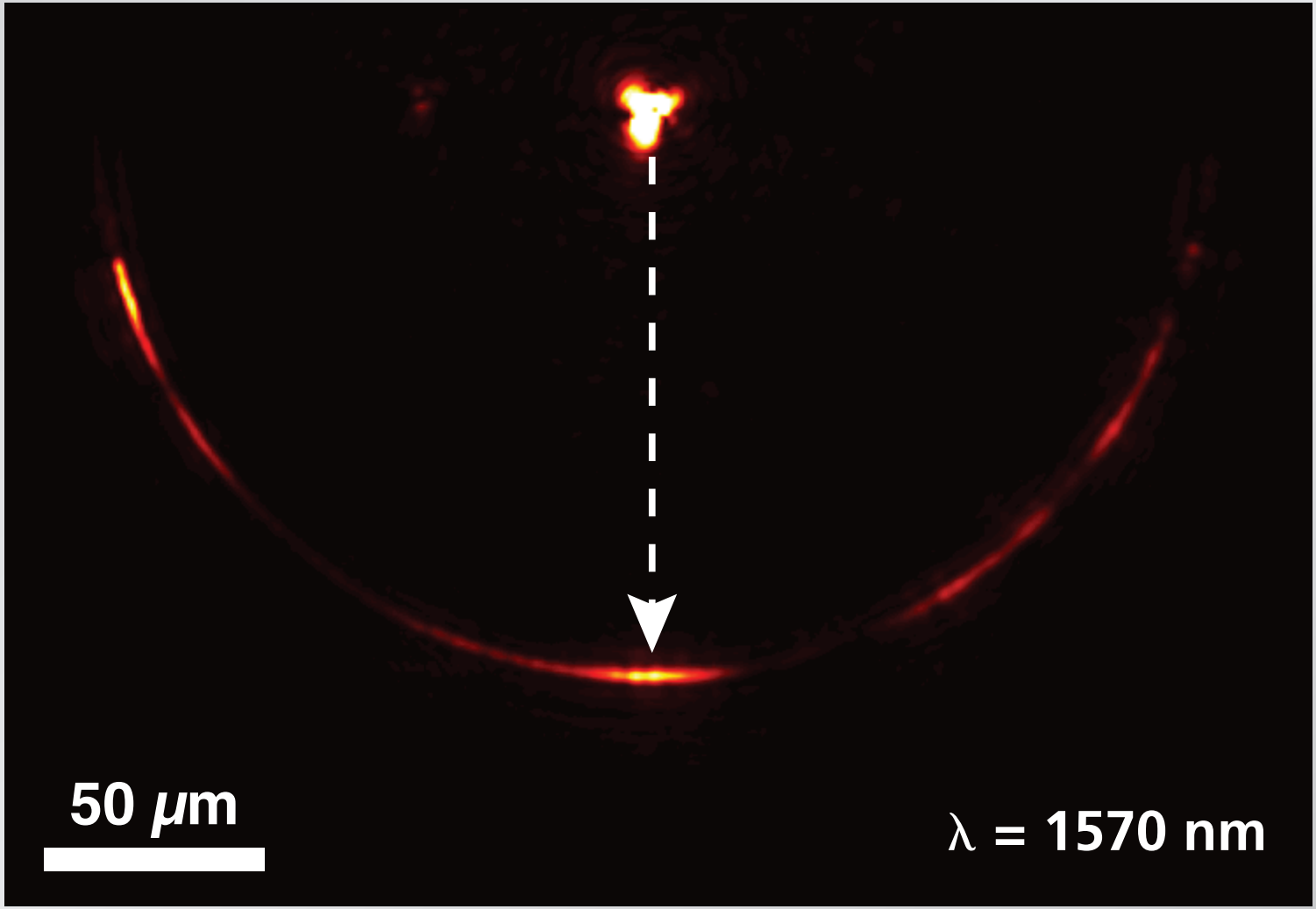




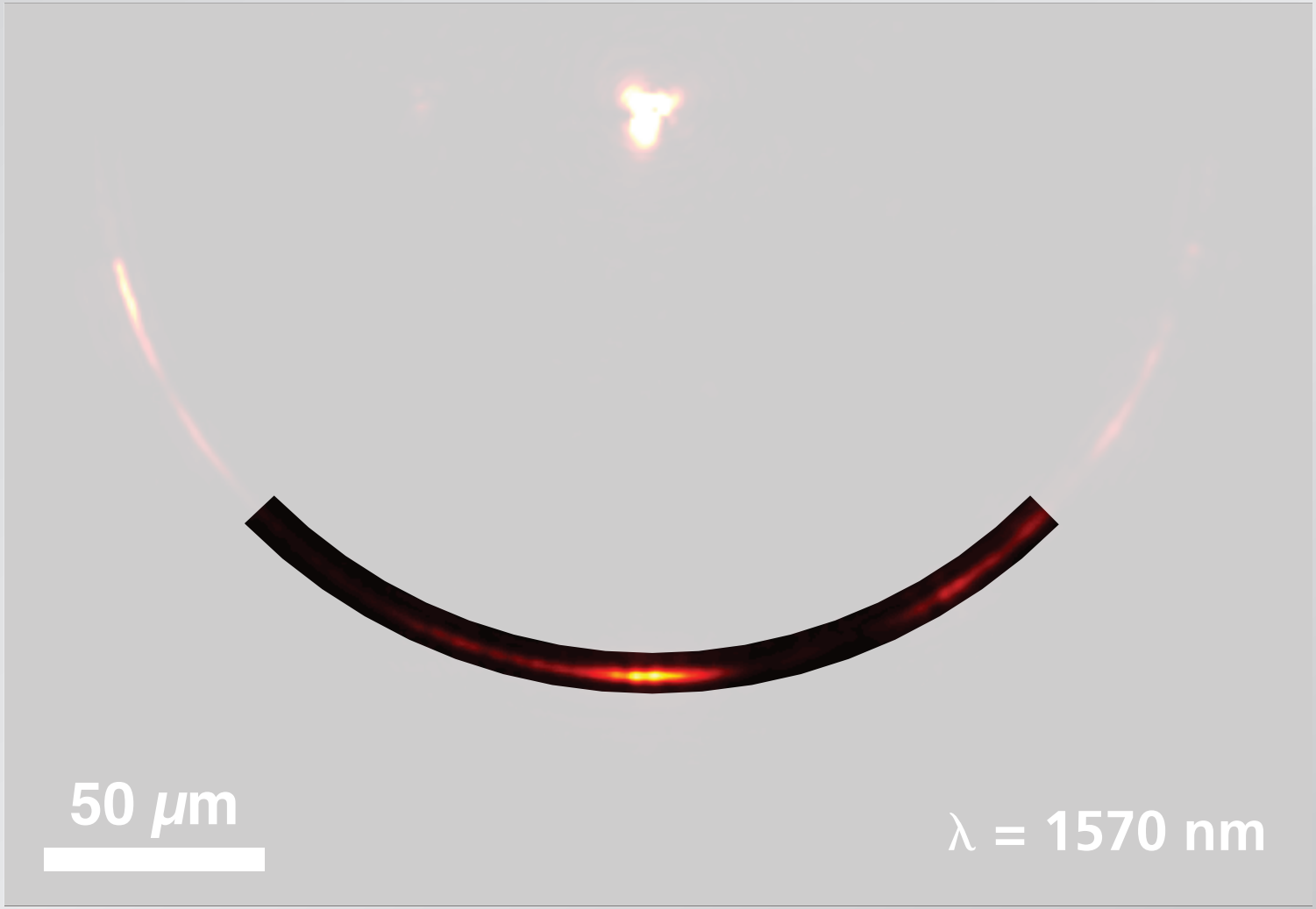
1 index 2 zero index 3 experiments



1 index 2 zero index 3 experiments

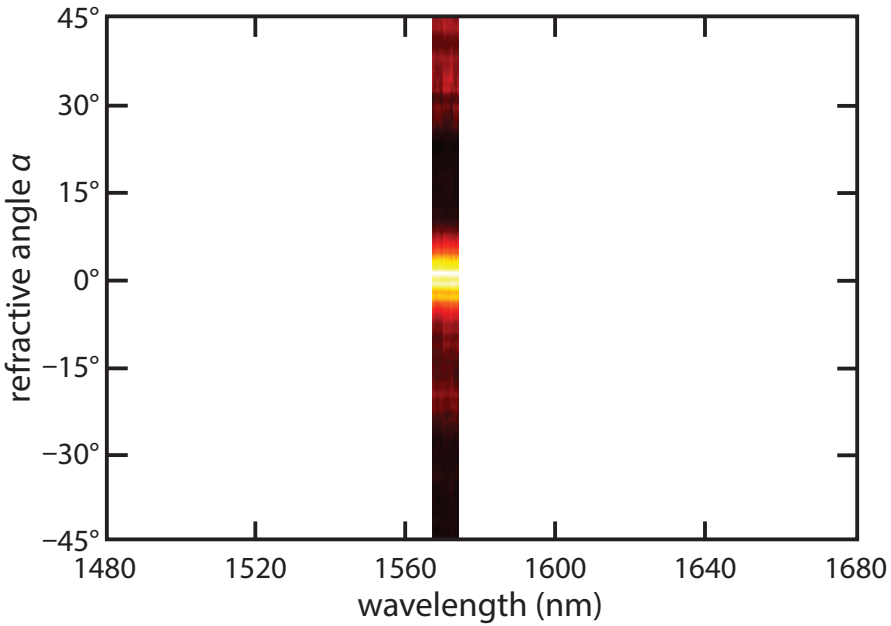


1 index 2 zero index 3 experiments

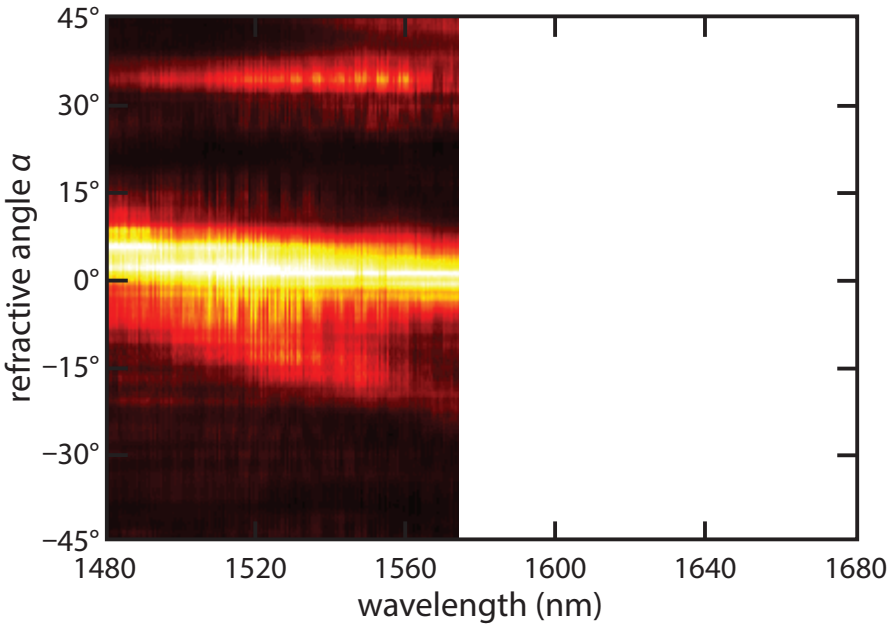


1 index 2 zero index 3 experiments

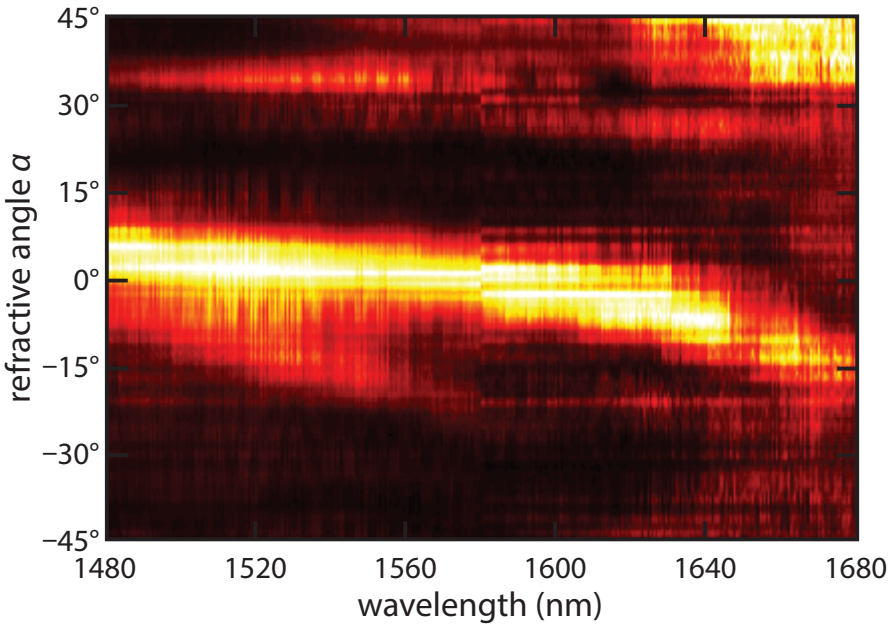
Wavelength dependence of refraction angle



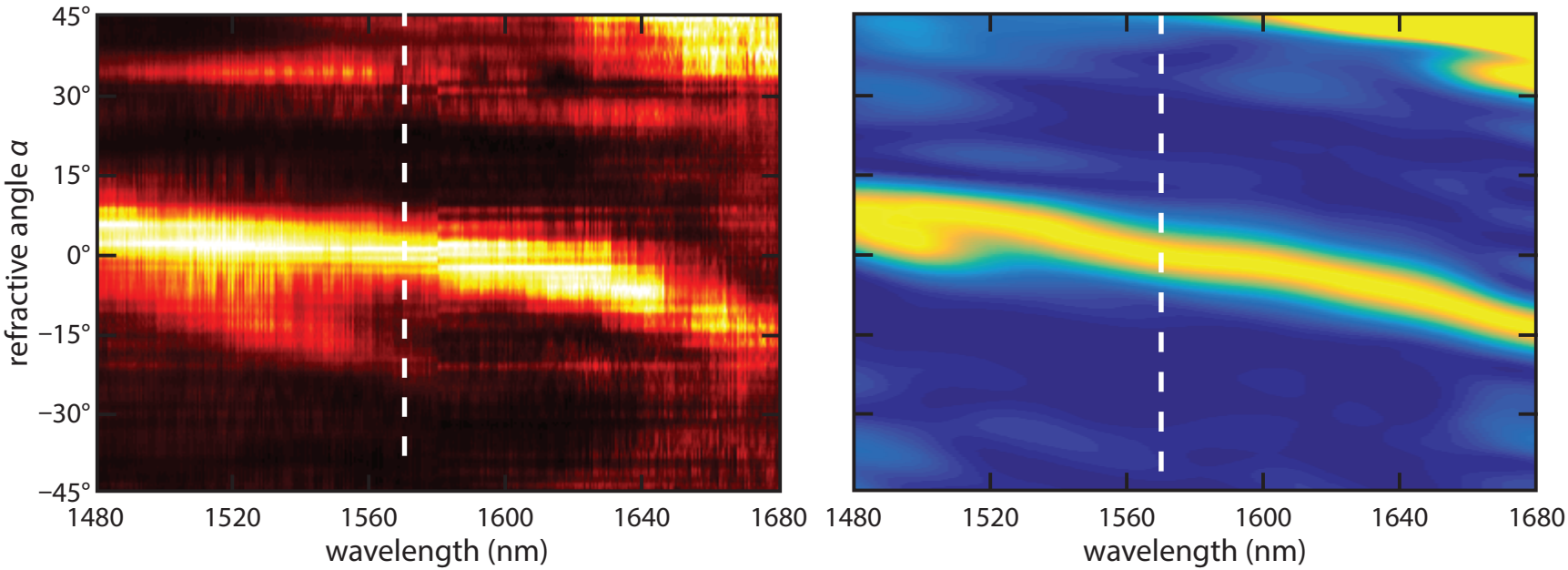
Wavelength dependence of refraction angle



Wavelength dependence of refraction angle



Wavelength dependence of refraction angle

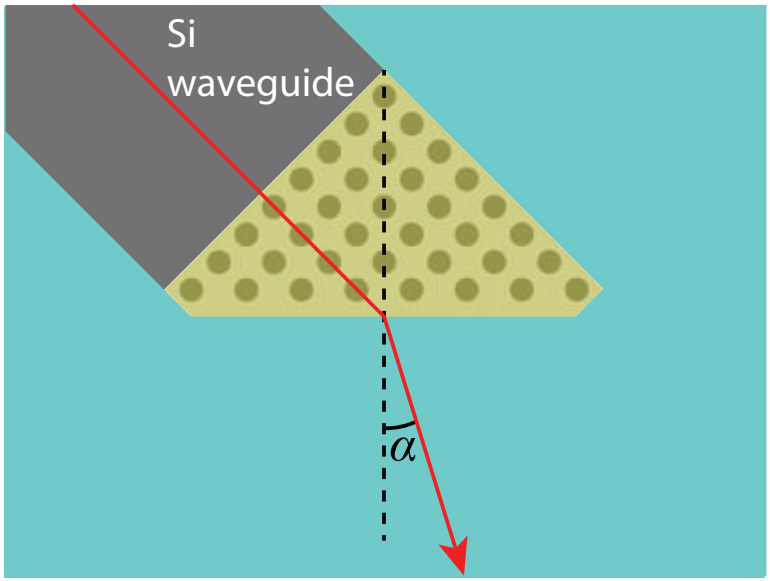


1 index

2 zero index

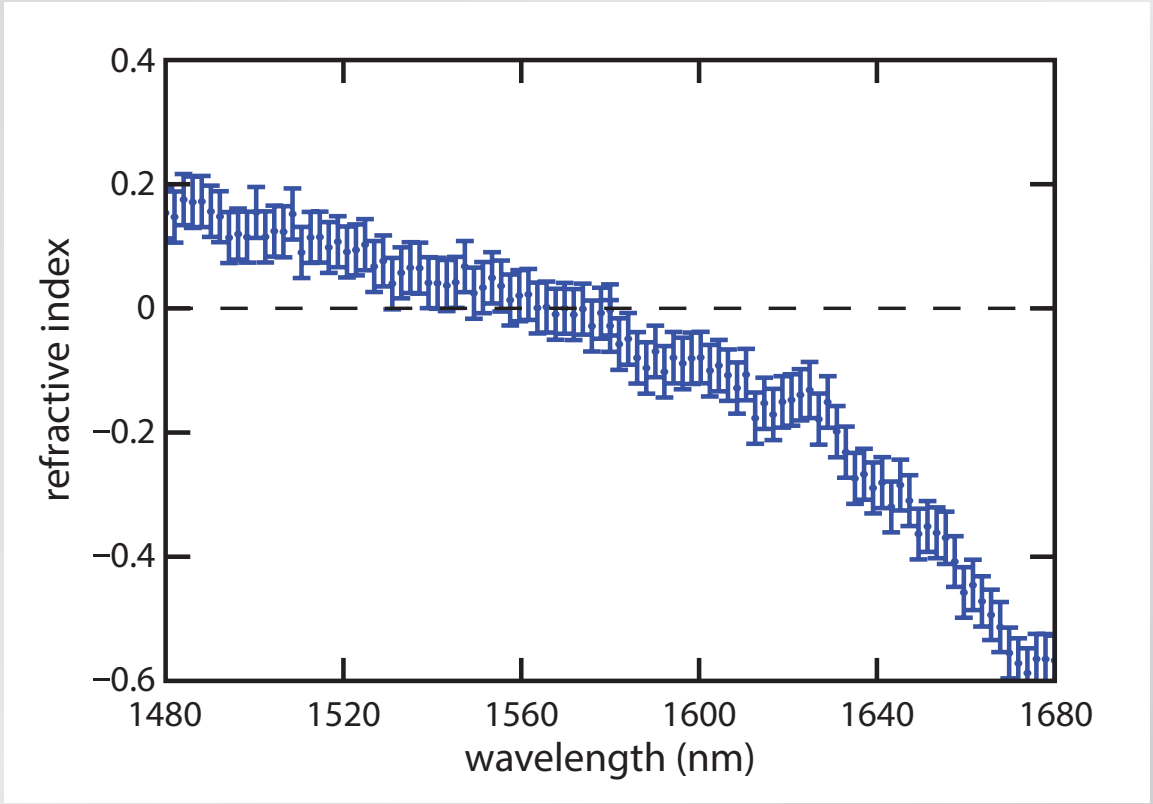
3 experiments

Wavelength dependence of index

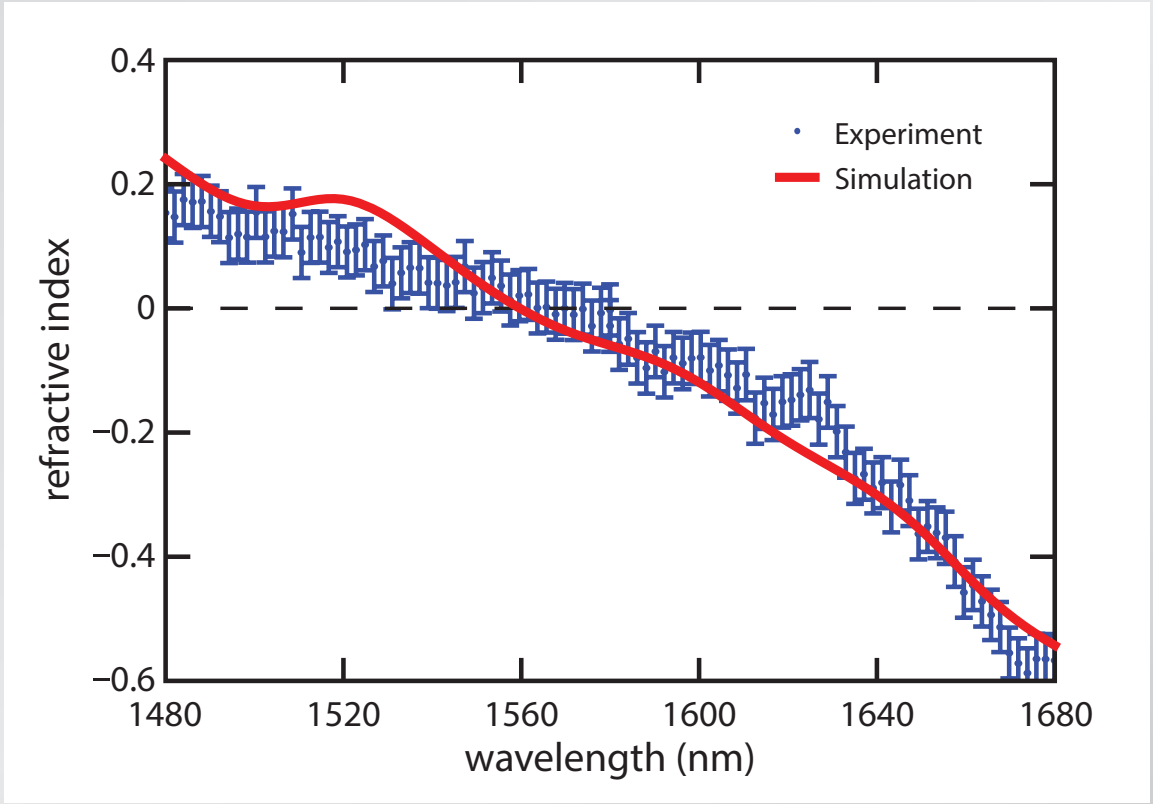


$$n_{\text{prism}} = n_{\text{slab}} \frac{\sin\alpha}{\sin 45°}$$

Wavelength dependence of index



Wavelength dependence of index

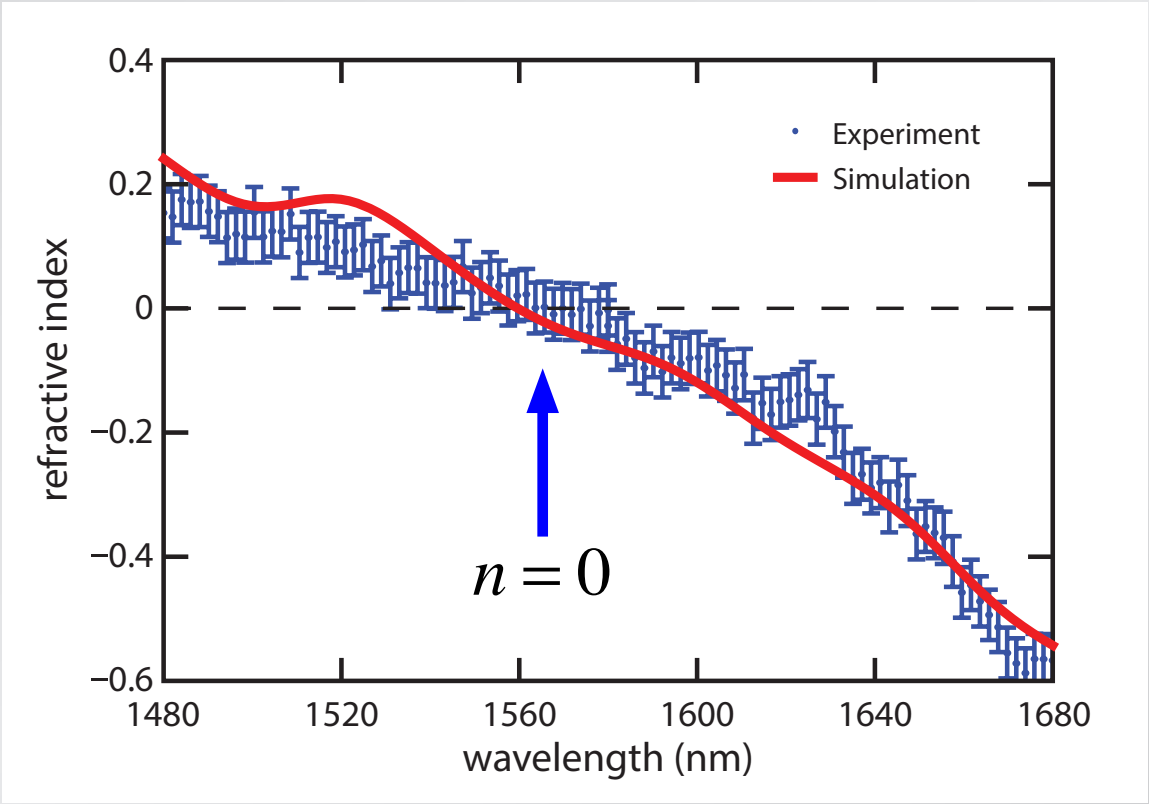


1 index

2 zero index

3 experiments

Wavelength dependence of index



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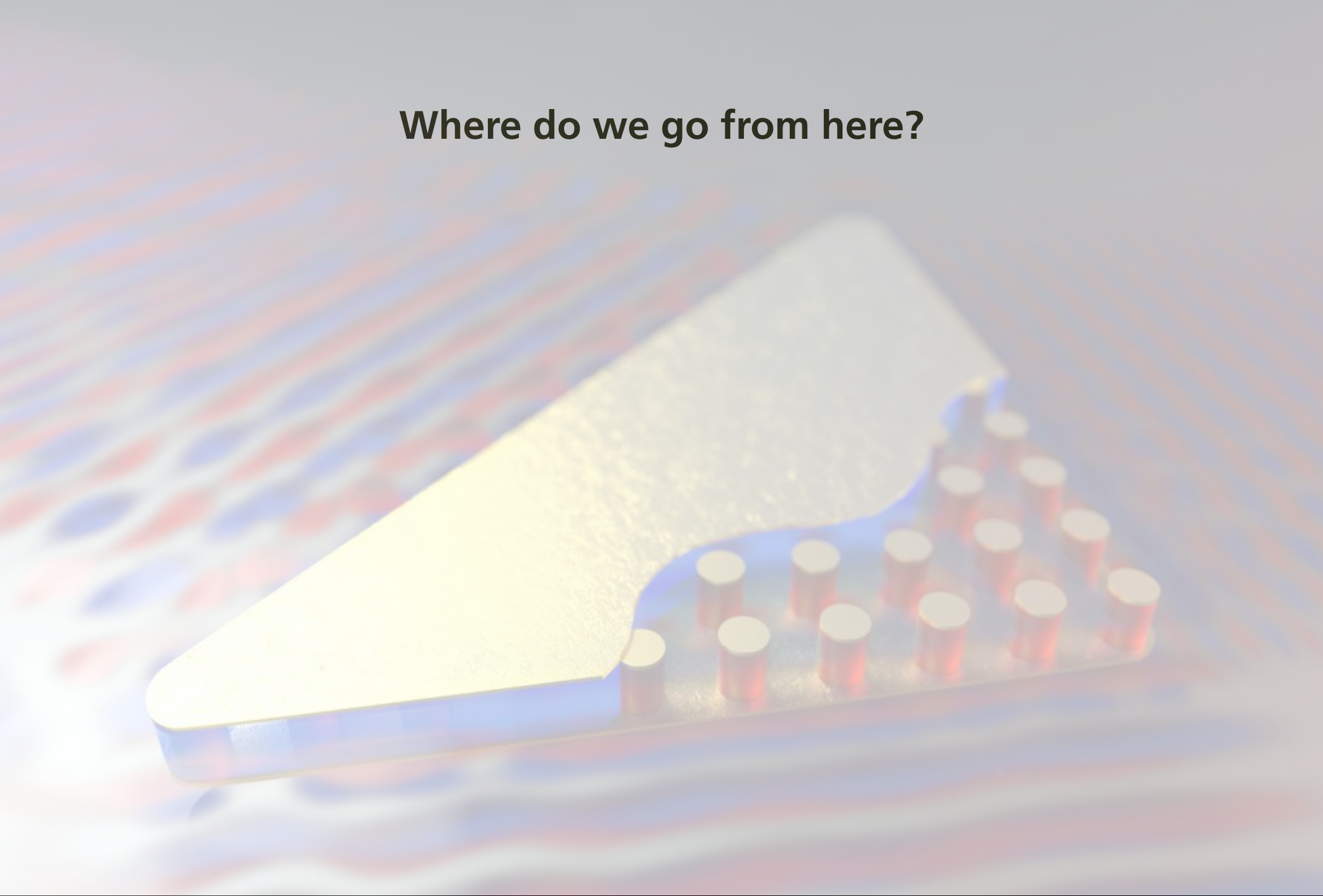
1 index
2 zero index
3 experiments

Where do we go from here?



1 index

2 zero index

3 experiments

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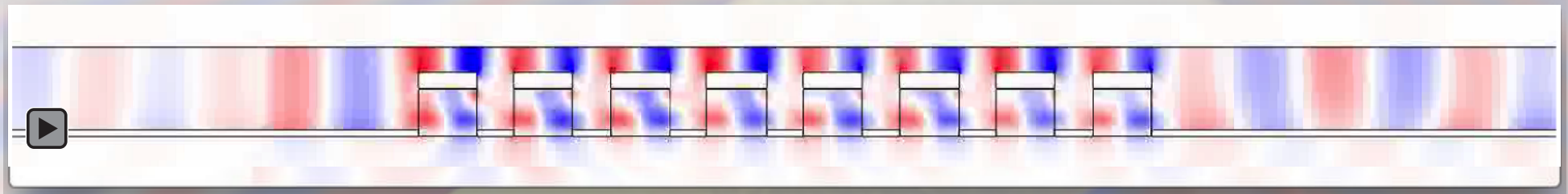


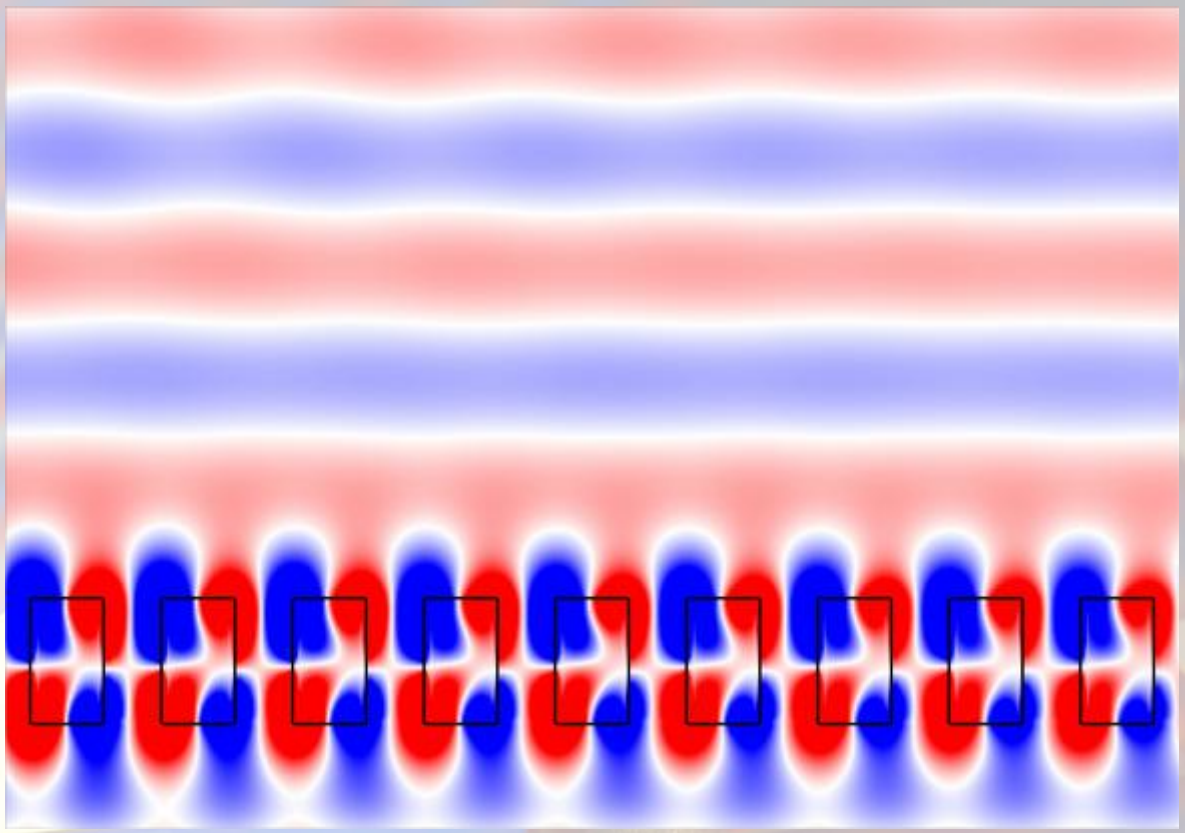
Need to eliminate losses in metal mirrors

1 index

2 zero index

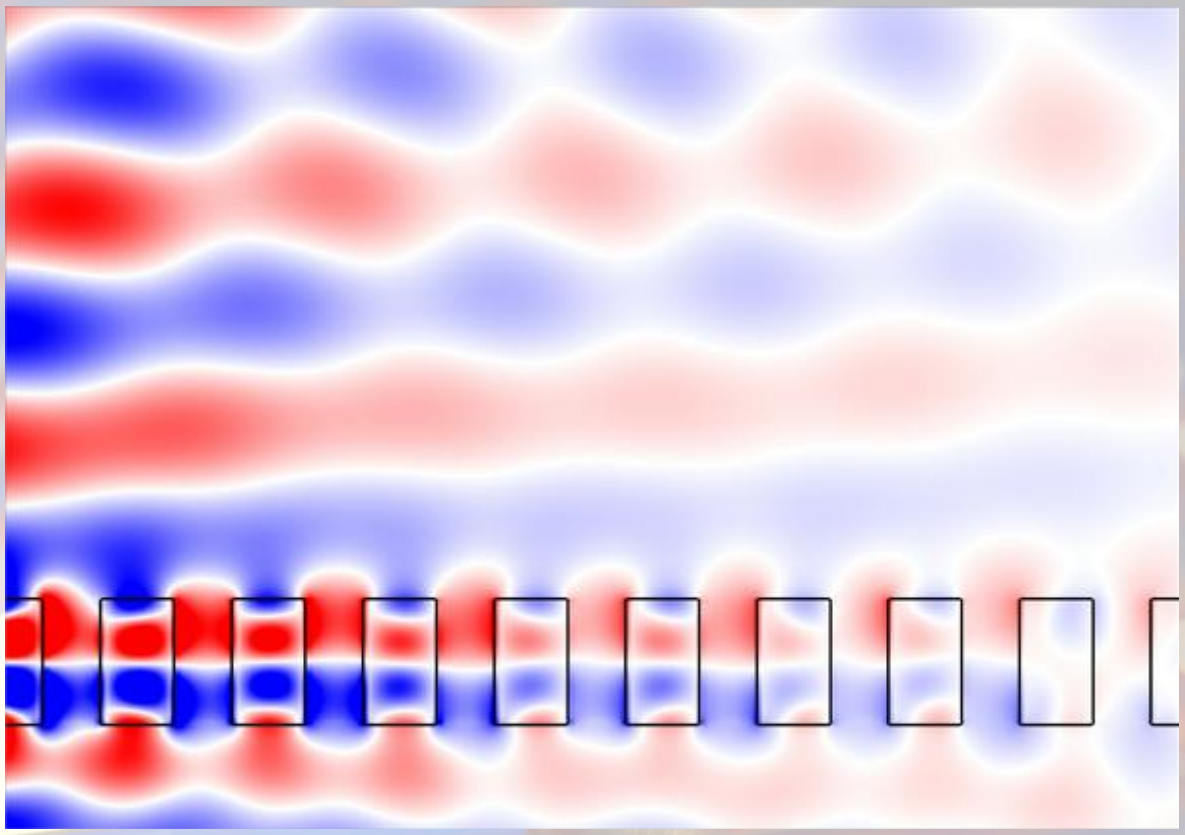
3 experiments

Where do we go from here?



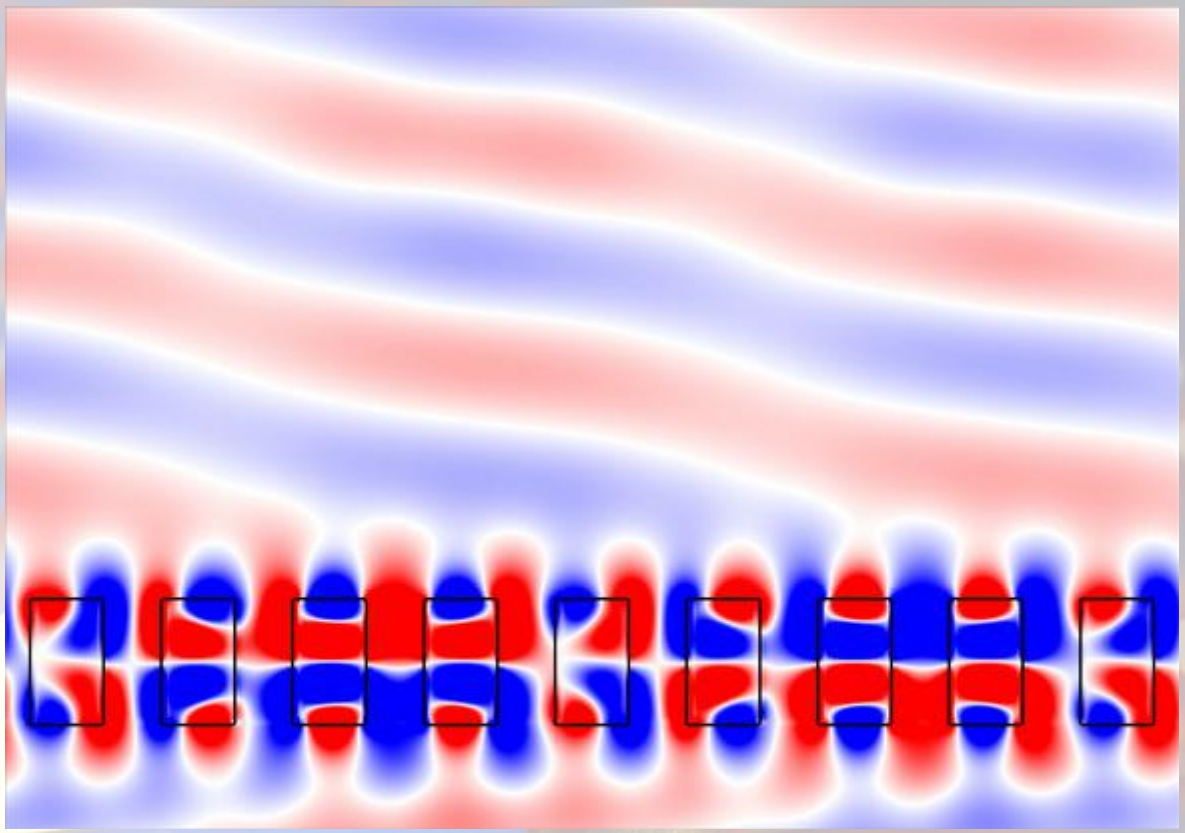
Removing mirrors causes radiative losses

Where do we go from here?



Radiative losses can be steered…

Where do we go from here?



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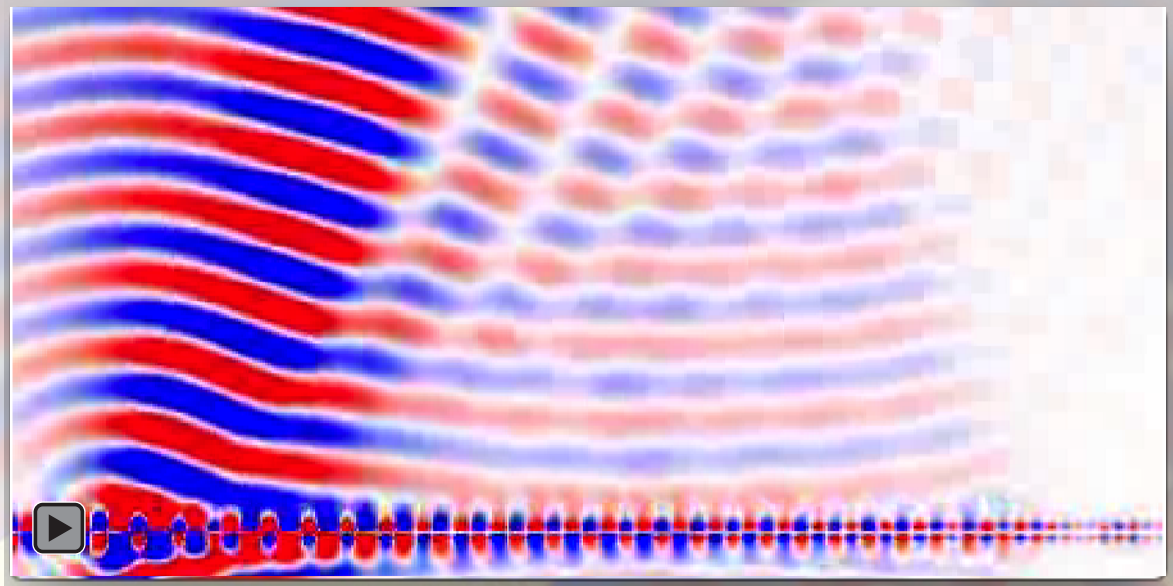
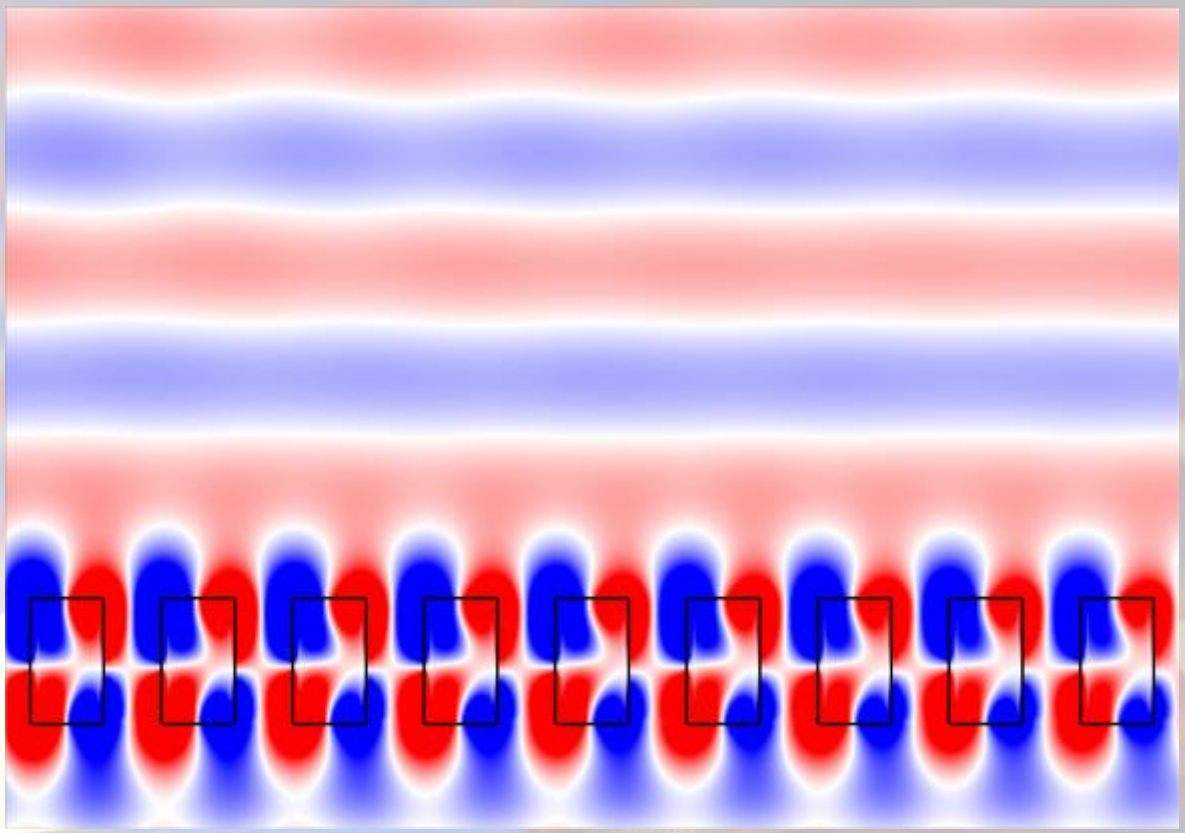


…or arranged to cause focusing…

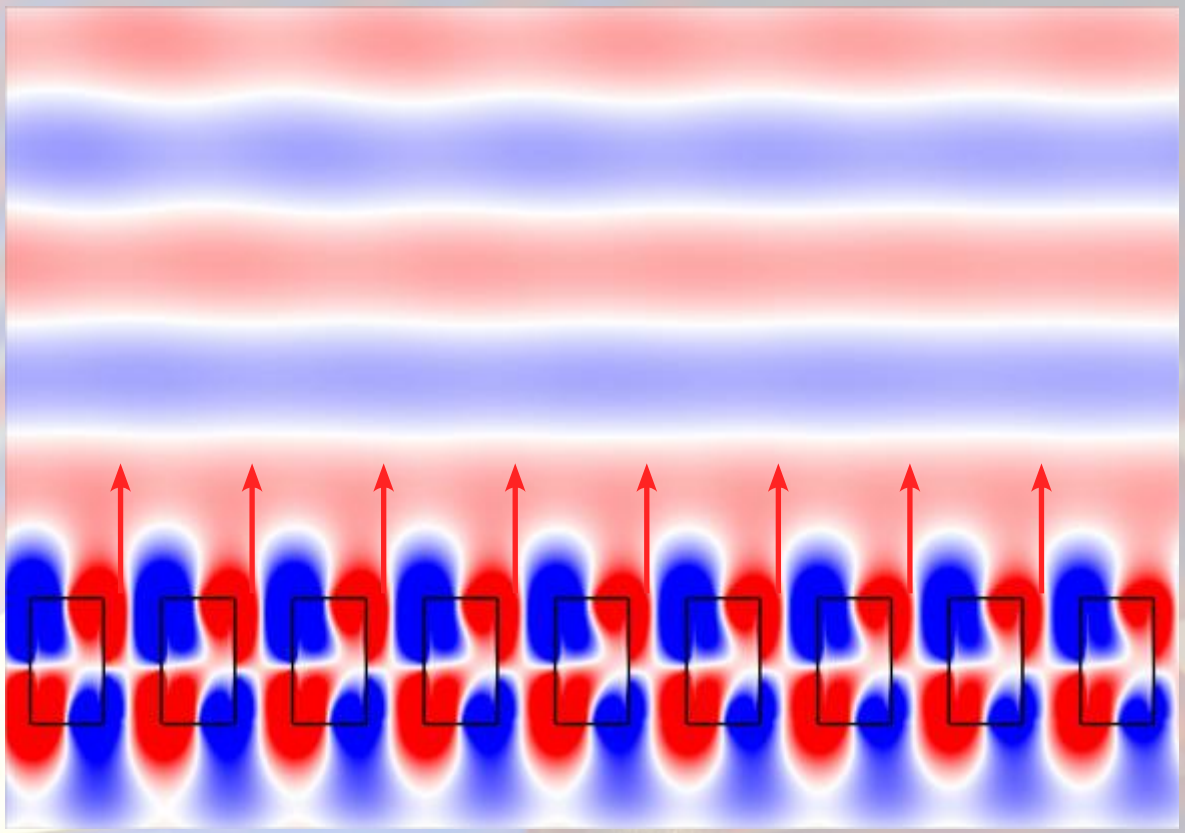
1 index 2 zero index 3 experiments

Where do we go from here?



…or eliminated causing “bound in continuum” state

Where do we go from here?



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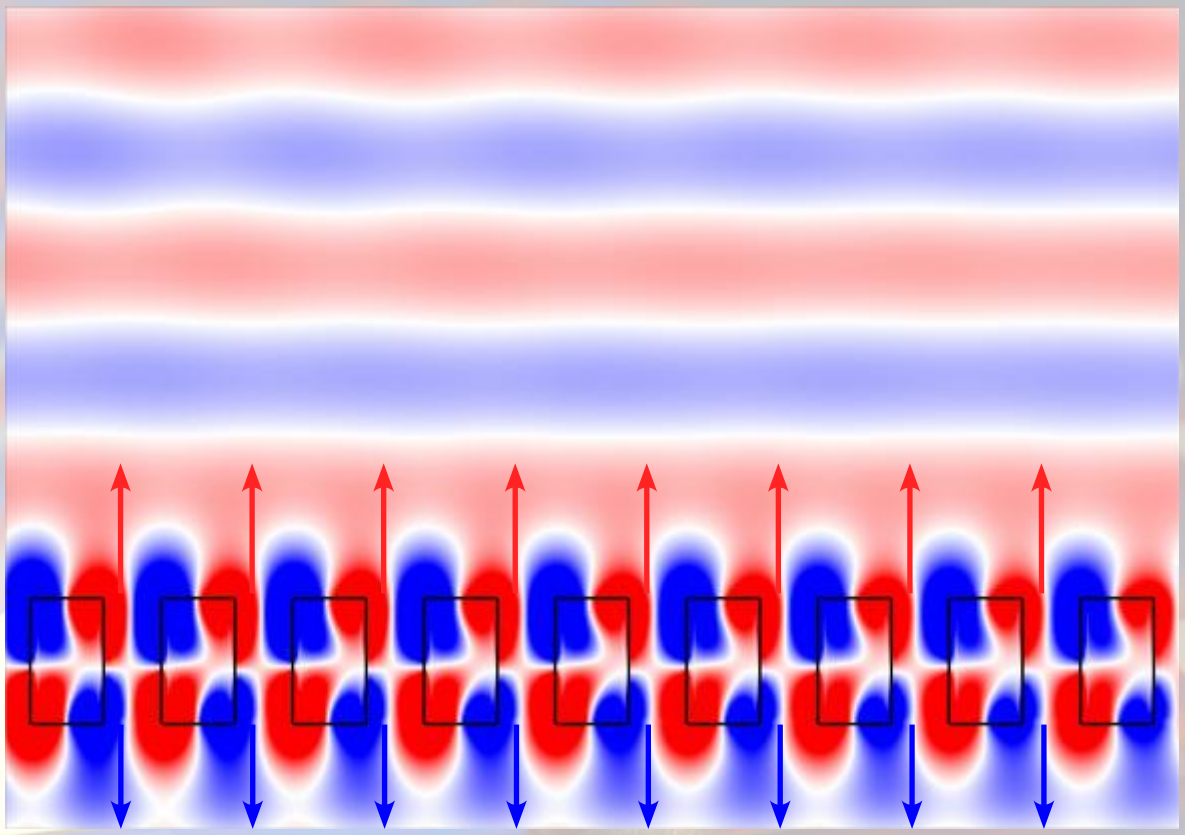

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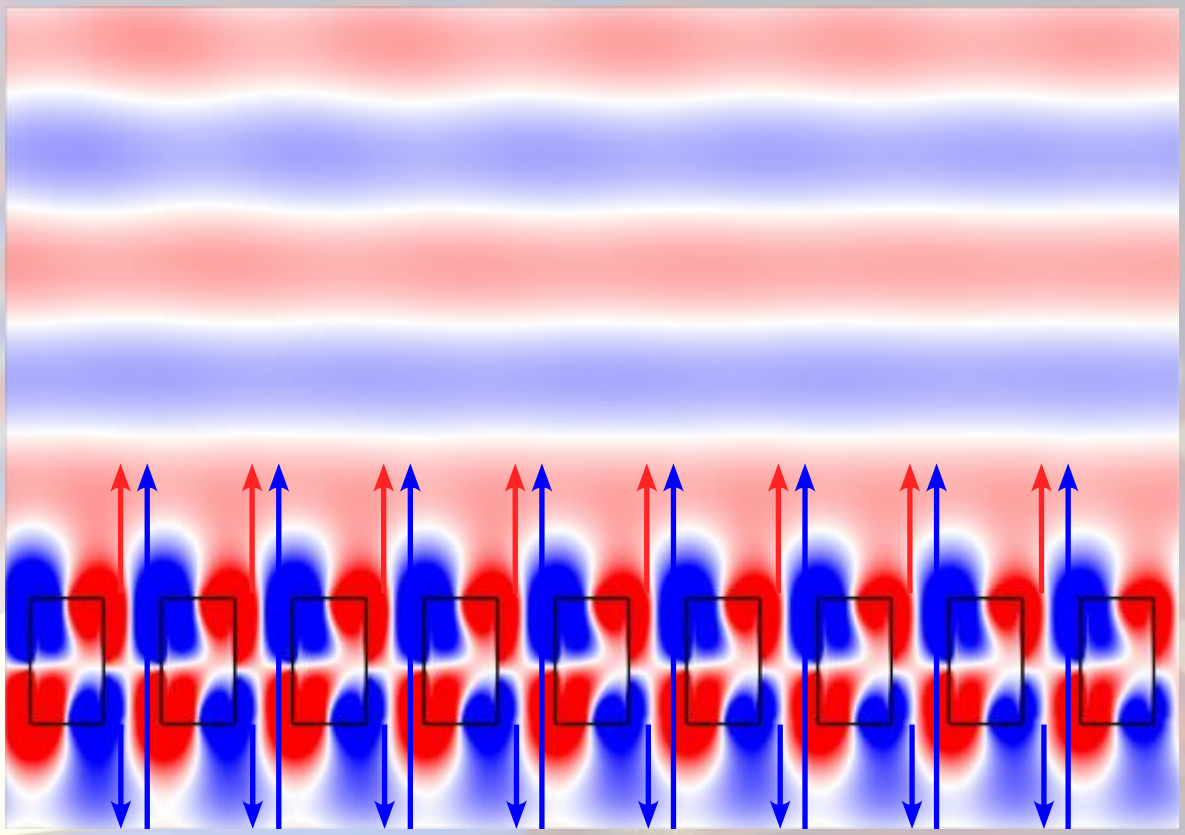

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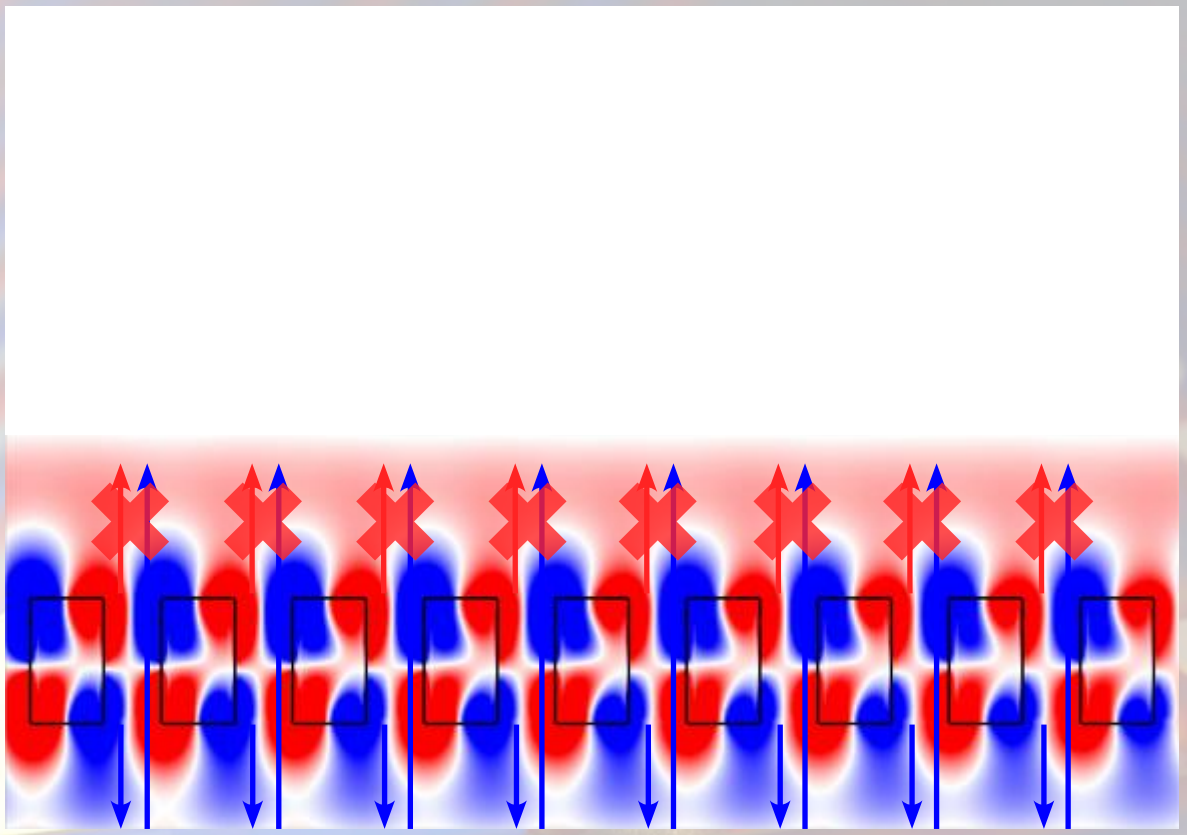

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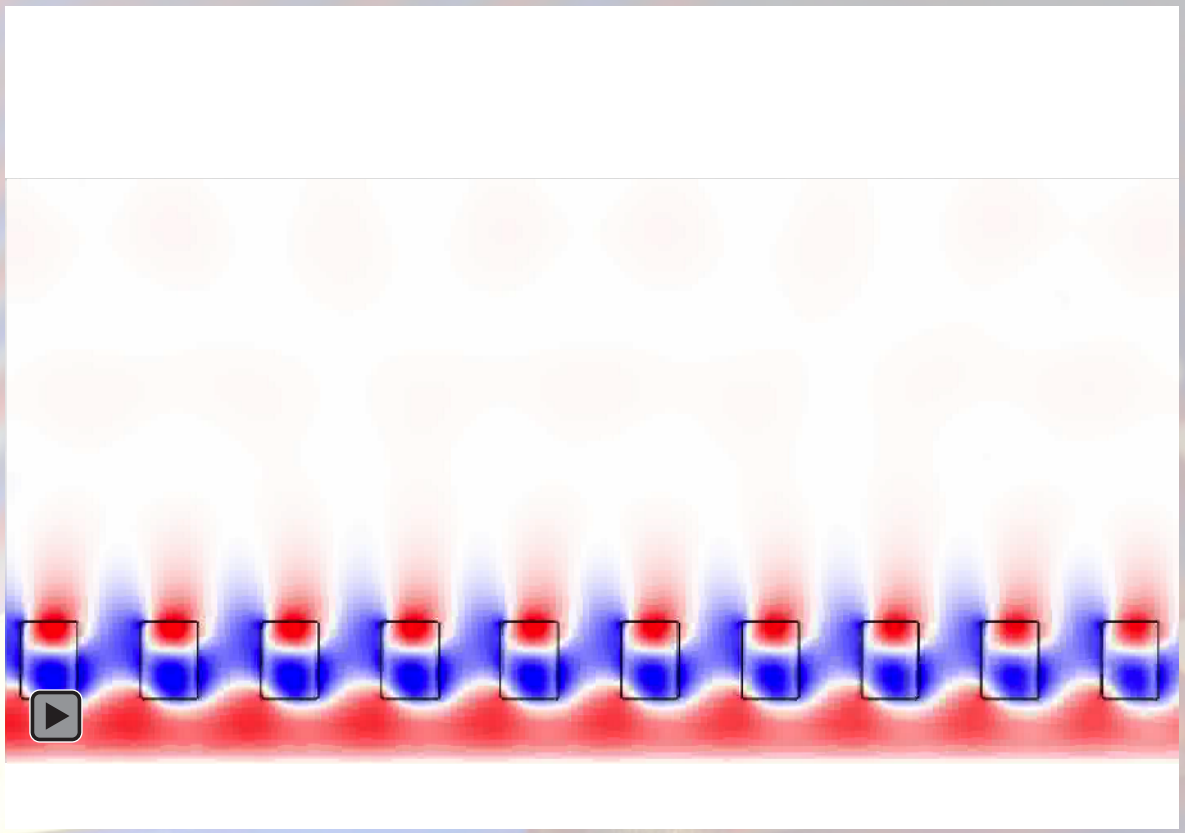
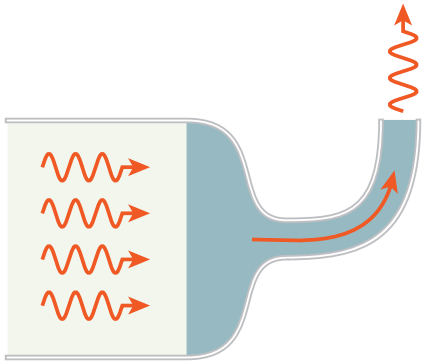
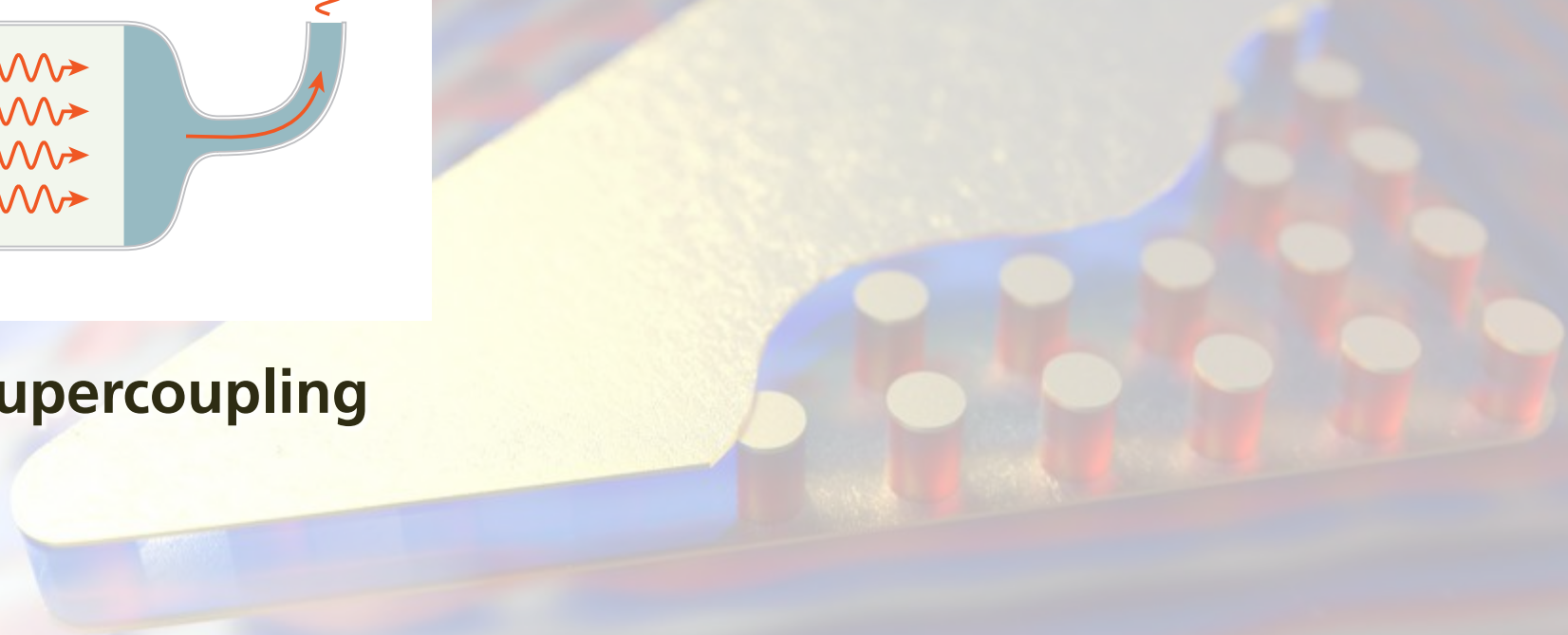


…or eliminated causing “bound in continuum” state

Exciting applications ahead



supercoupling

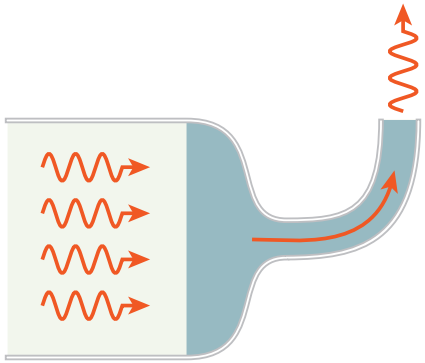


1 index

2 zero index

3 experiments

Exciting applications ahead



supercoupling

NLO

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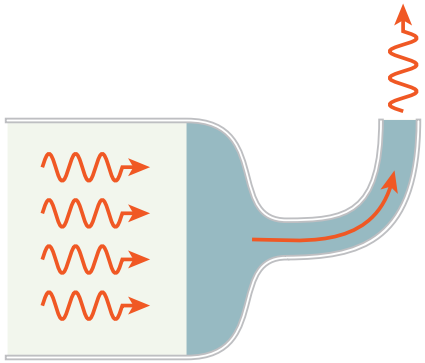
2 zero index

3 experiments

Exciting applications ahead



SHG $\omega' = 2\omega$

supercoupling

NLO

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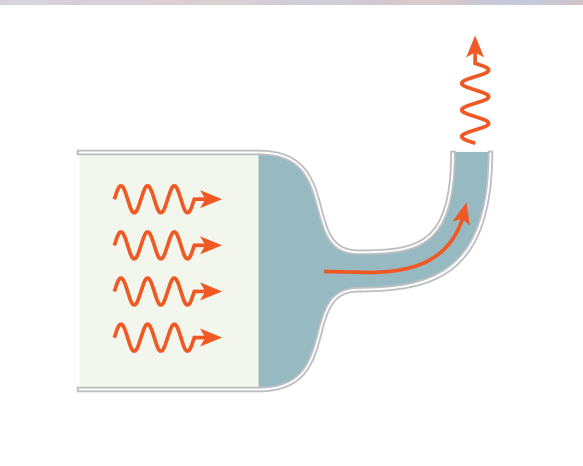
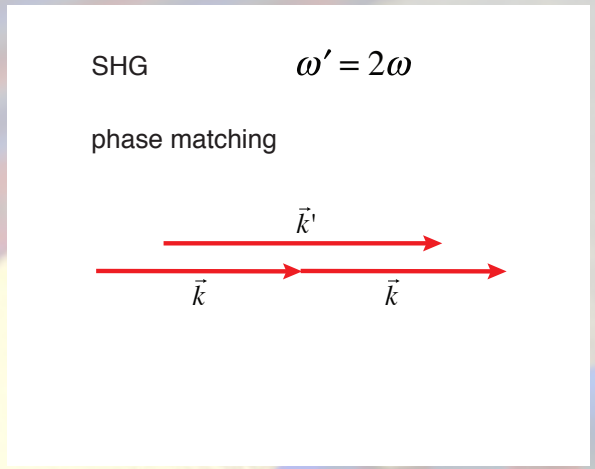


supercoupling



NLO

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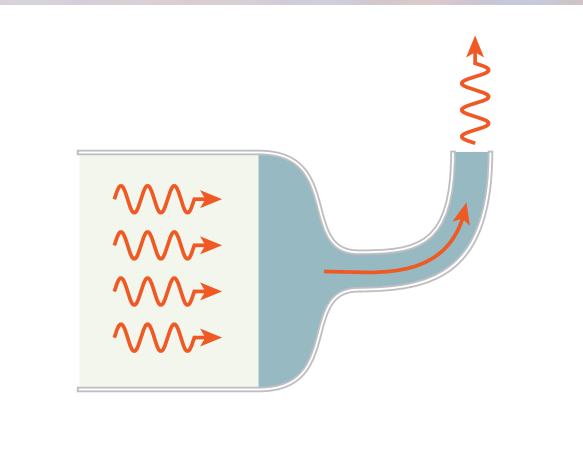
Exciting applications ahead



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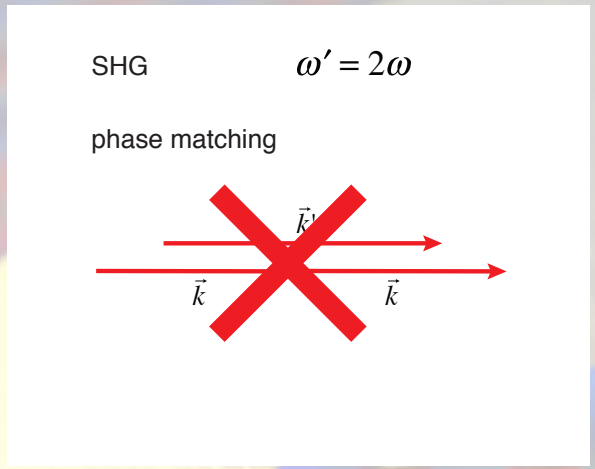


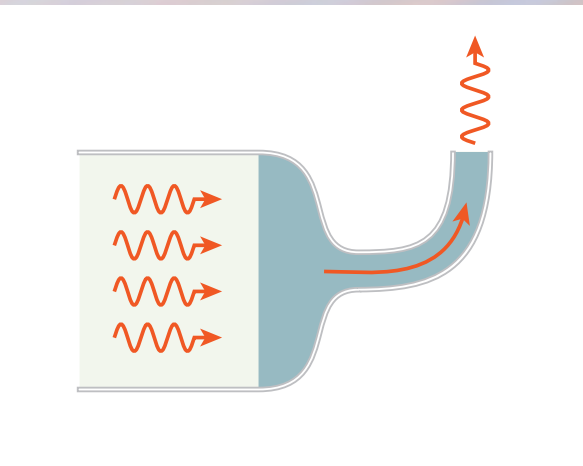
NLO

1 index

2 zero index

3 experiments

Exciting applications ahead



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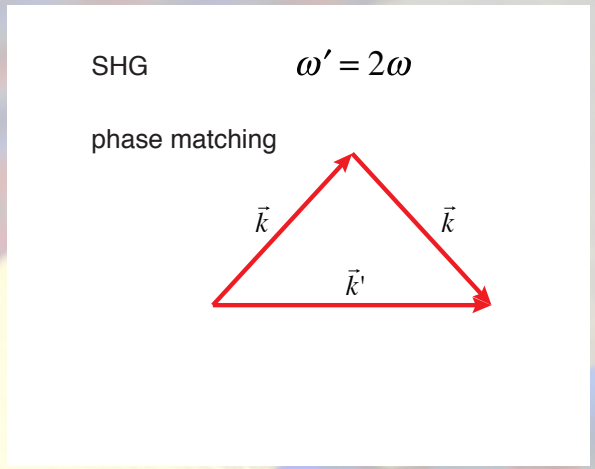


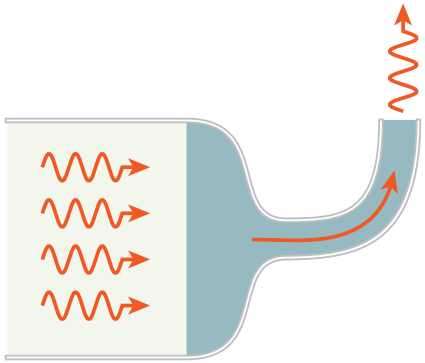
NLO

1 index

2 zero index

3 experiments

Exciting applications ahead



supercoupling

SHG $\omega' = 2\omega$

at zero index

$\dot{\vec{k}} = 0$

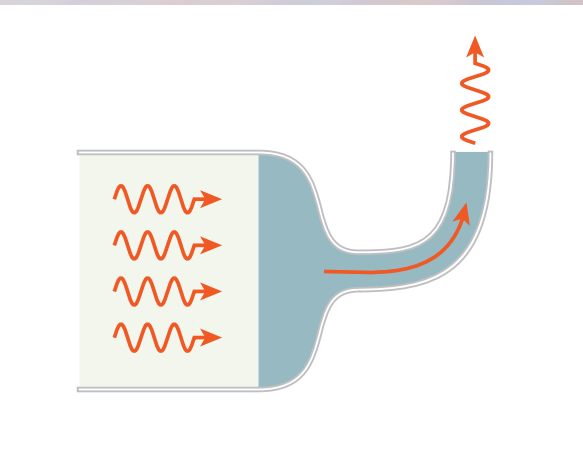
NLO

1 index

2 zero index

3 experiments

Exciting applications ahead



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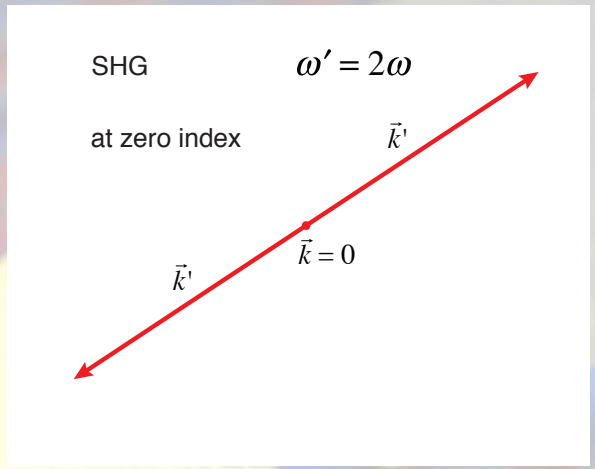

NLO

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3 experiments

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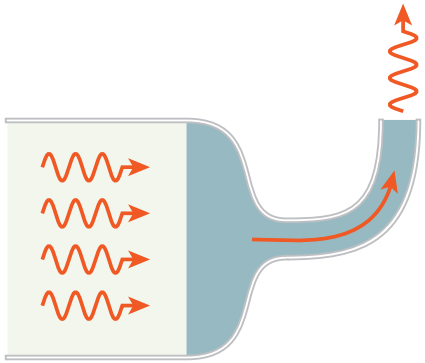

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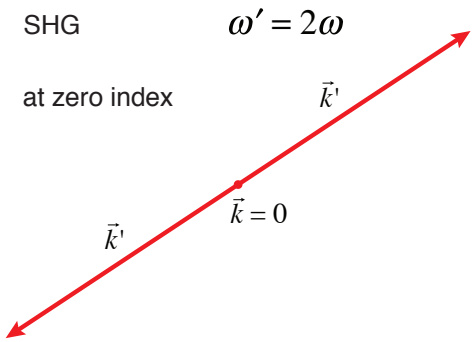
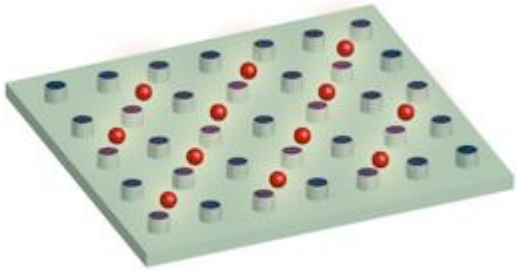


NLO



quantum optics

1 index

2 zero index

3 experiments

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