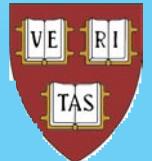
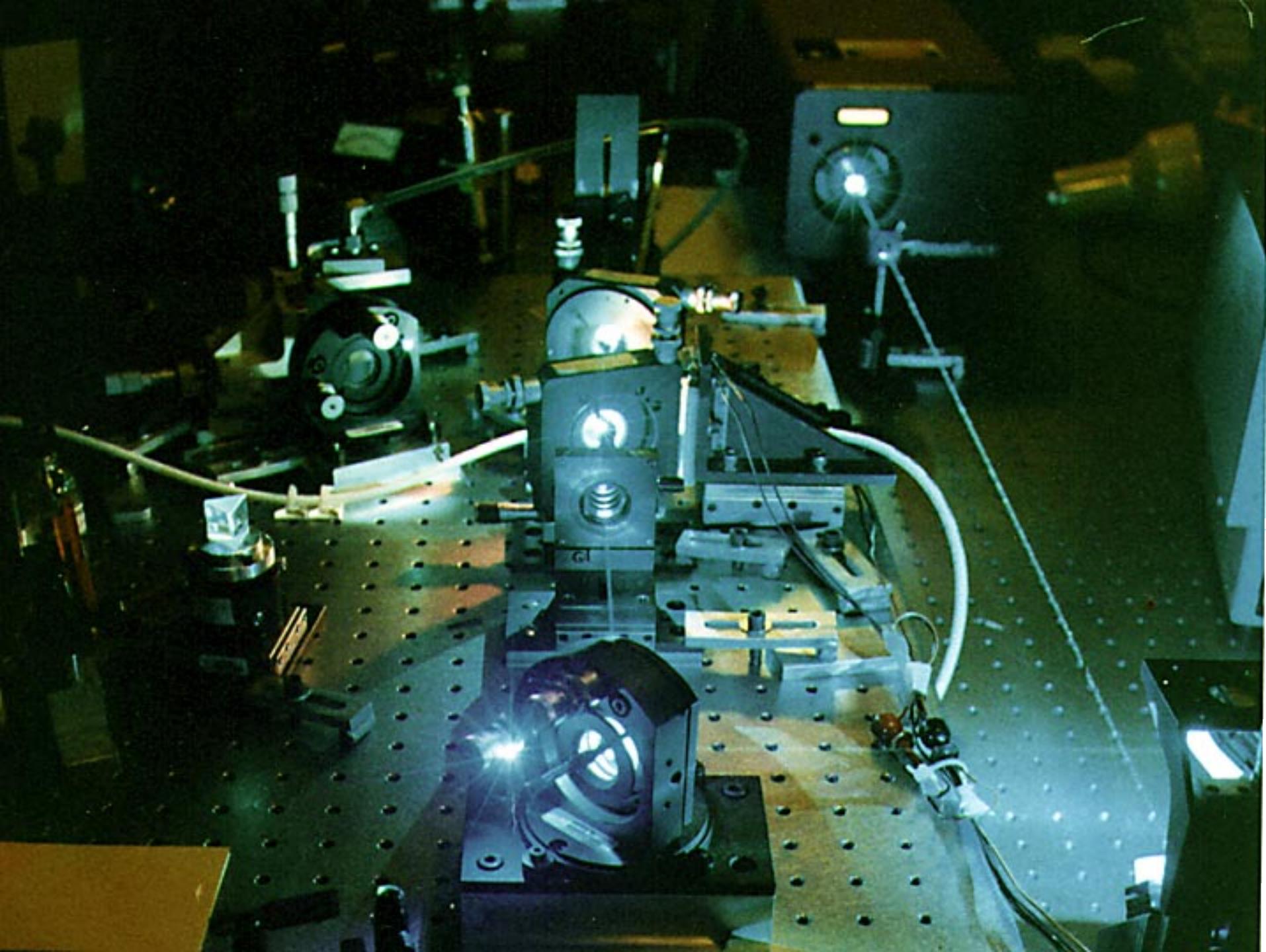


**Creation,
Characterization and Applications
of Ultrashort Laser Pulses**

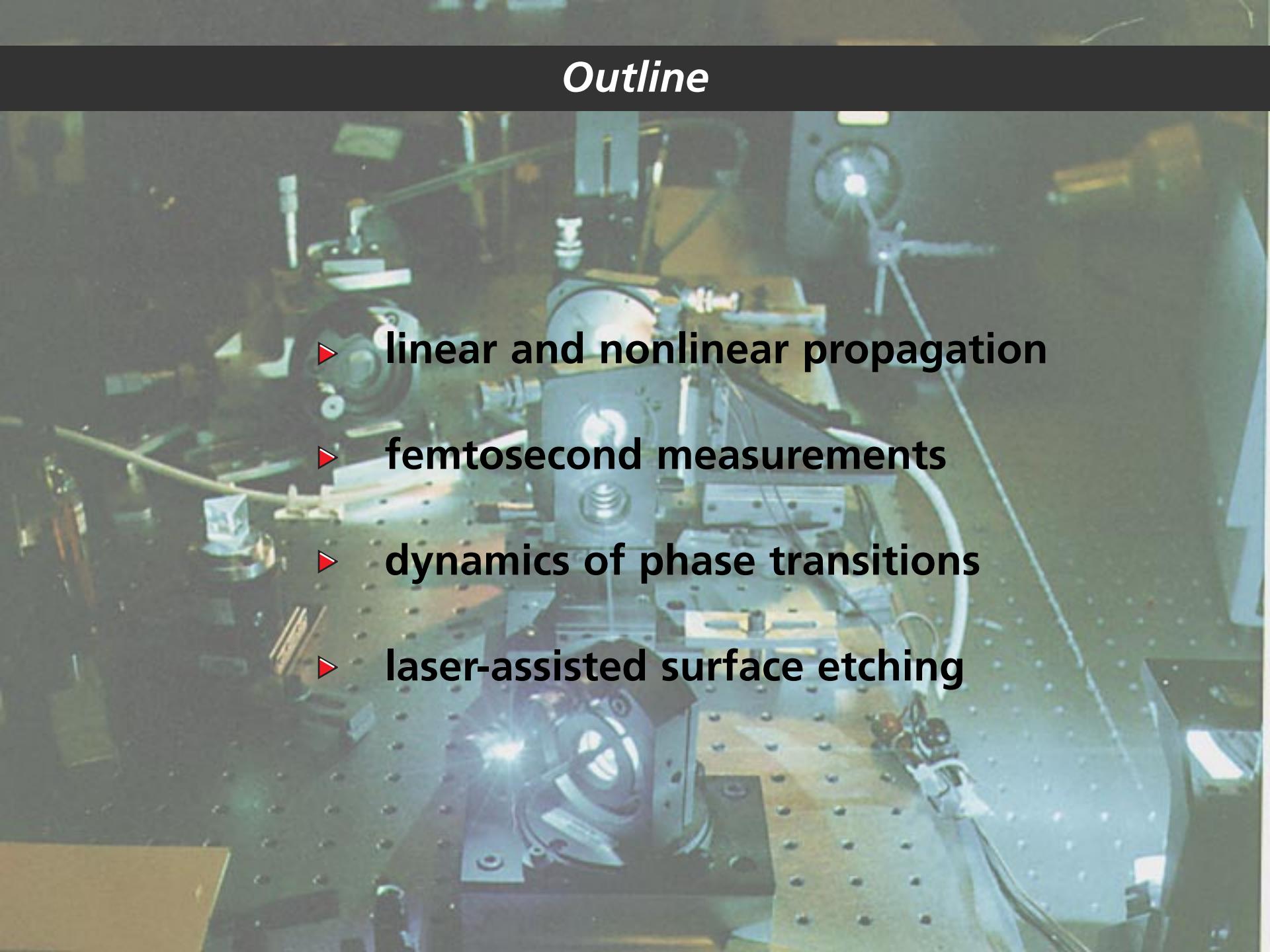
**Eric Mazur
Harvard University**

**NATO Advanced Summer Institute, Erice, Italy
Spectroscopy of systems with confined spatial structures
20 June 2001**





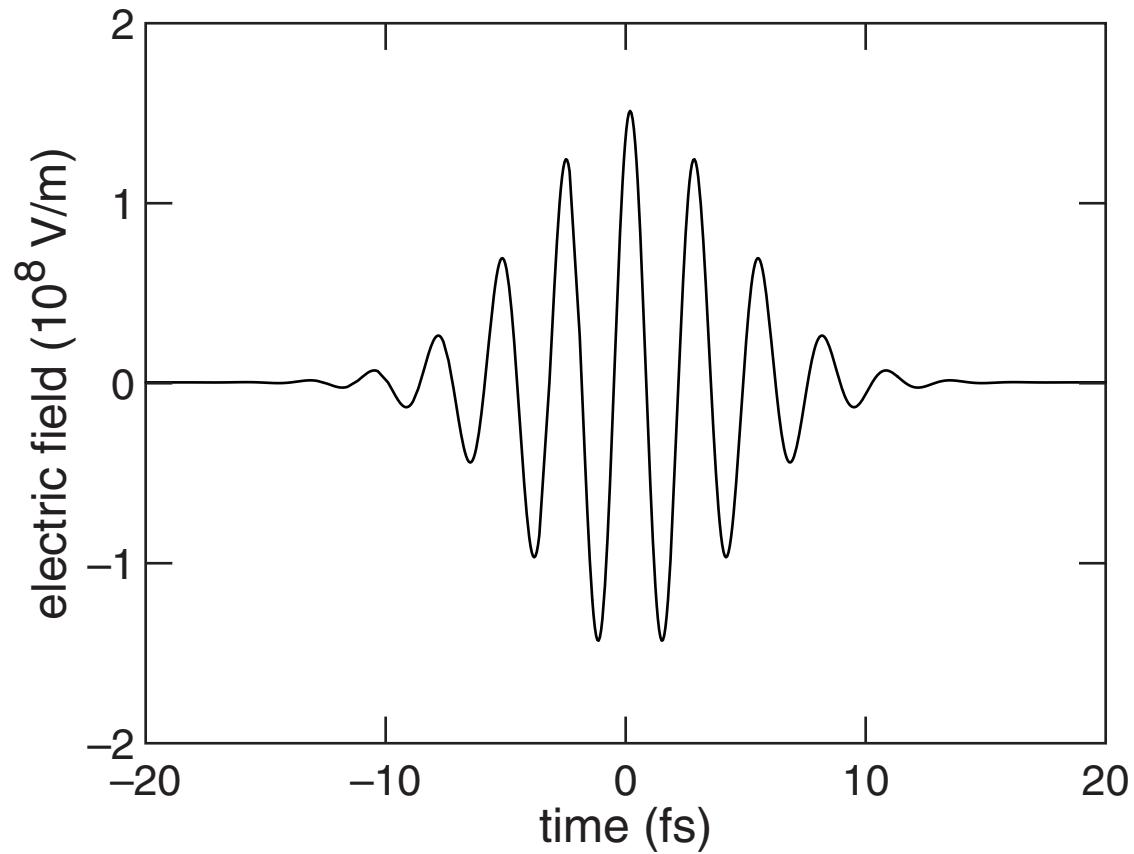
Outline

- 
- ▶ linear and nonlinear propagation
 - ▶ femtosecond measurements
 - ▶ dynamics of phase transitions
 - ▶ laser-assisted surface etching

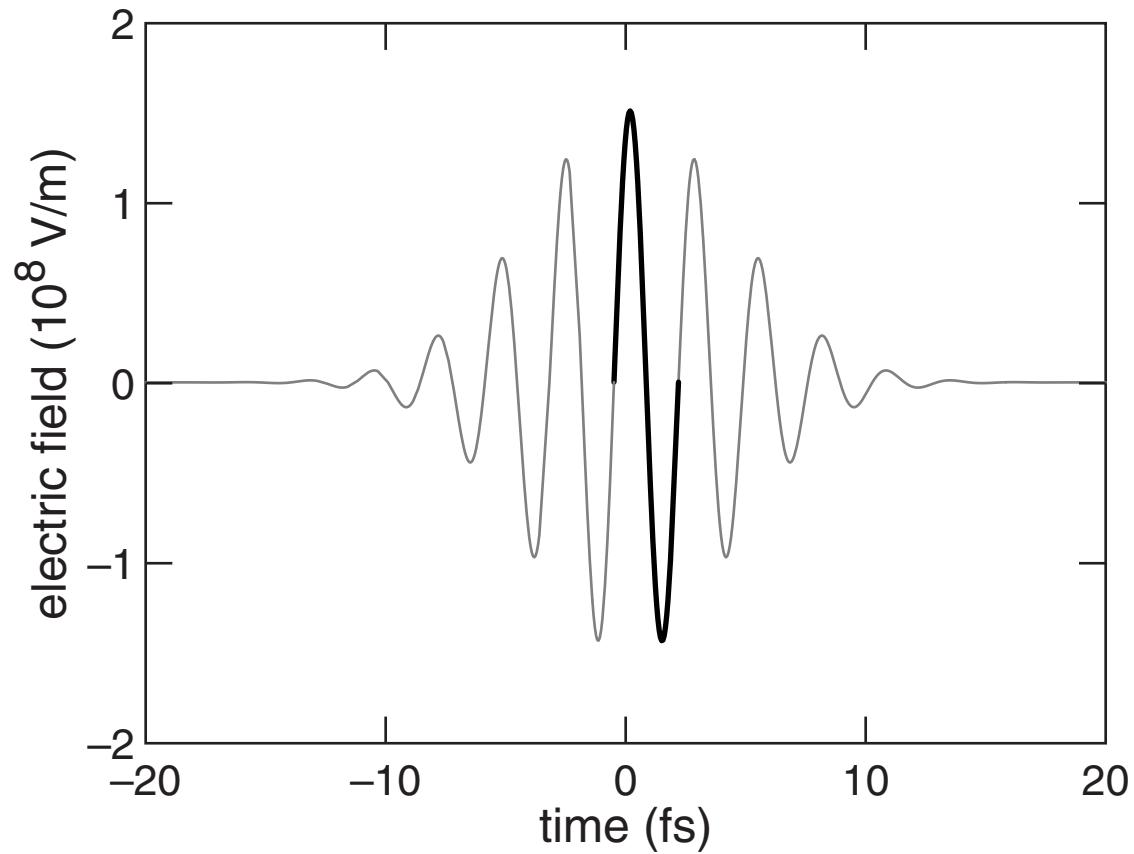
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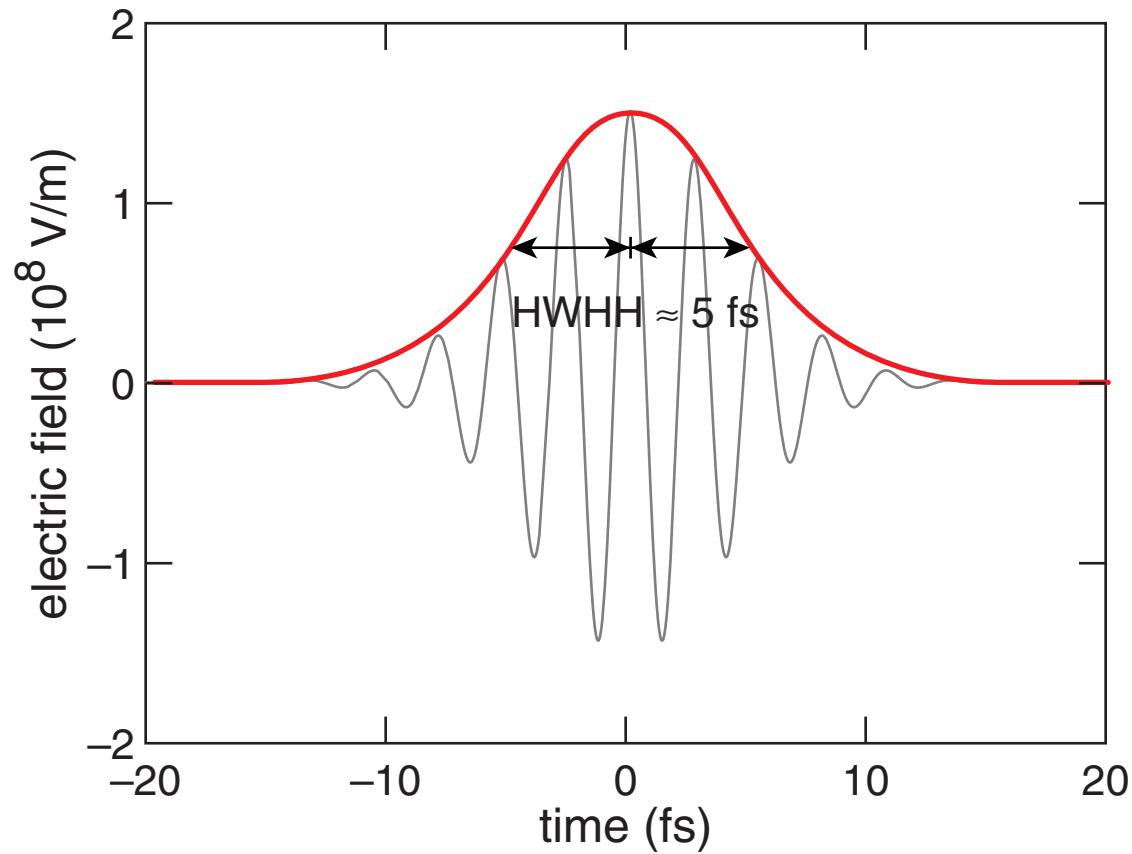
Introduction



Introduction



Introduction



Introduction

- ▶ **time resolution**
- ▶ **high intensity**
- ▶ **nonlinear optics**
- ▶ **new physics**

Propagation of EM waves through medium

Governed by wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

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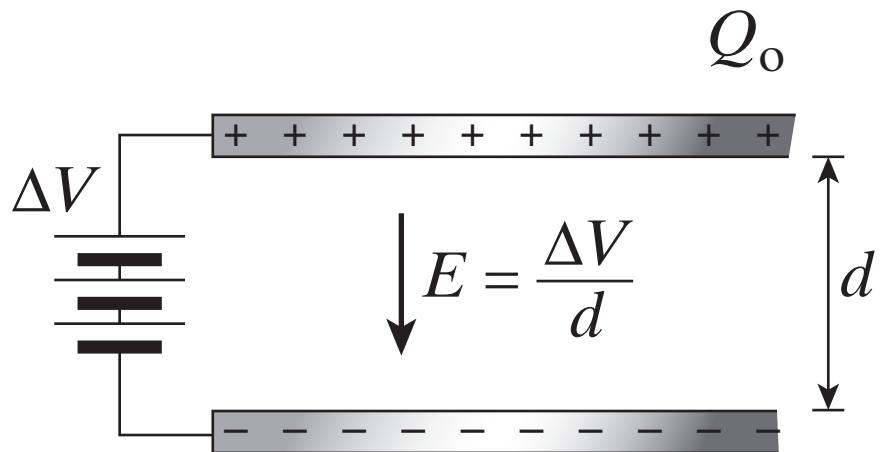
In non-ferromagnetic media $\mu \approx 1$, and so $n \approx \sqrt{\epsilon}$.

In dispersive media $n = n(\omega)$.

Propagation of EM waves through medium

Dielectric constant measures increase in capacitance

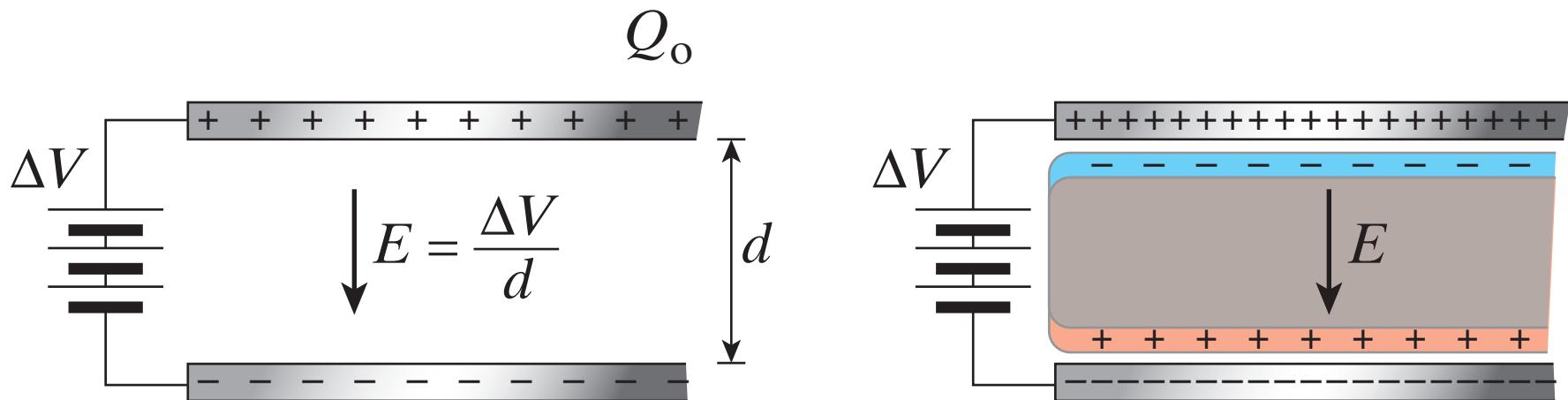
$$\epsilon = \frac{C_d}{C_o}$$



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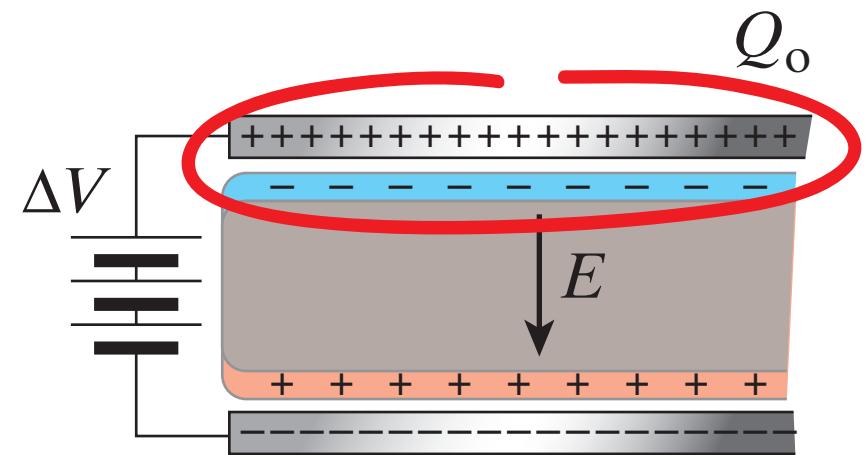
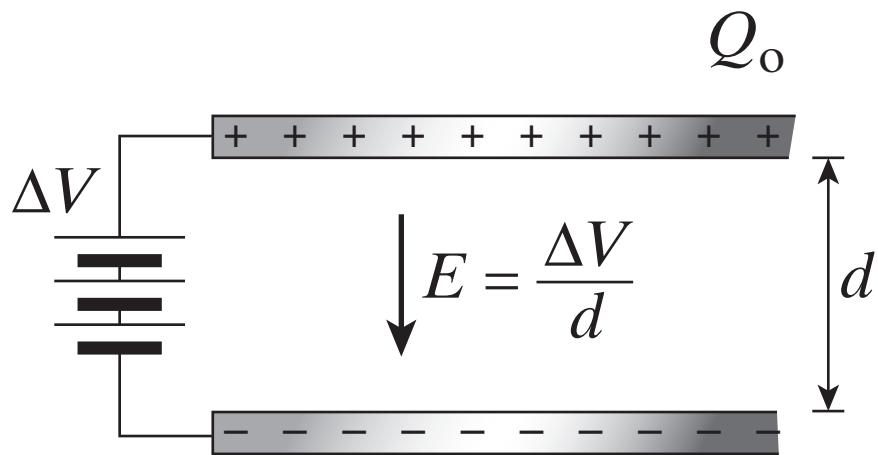
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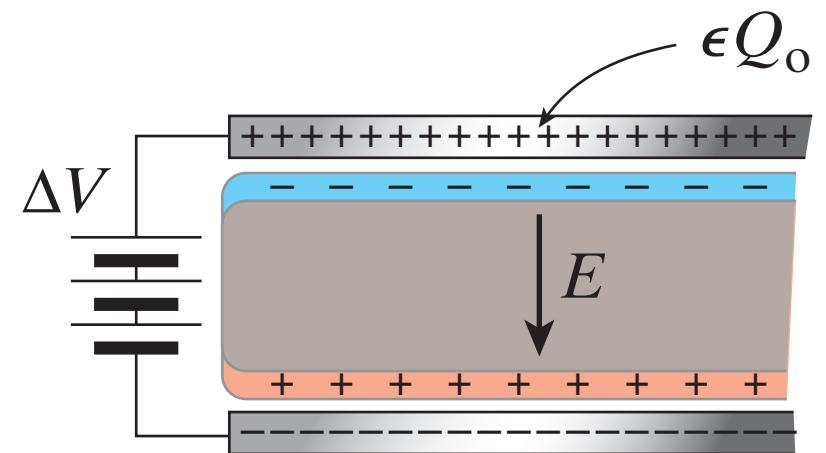
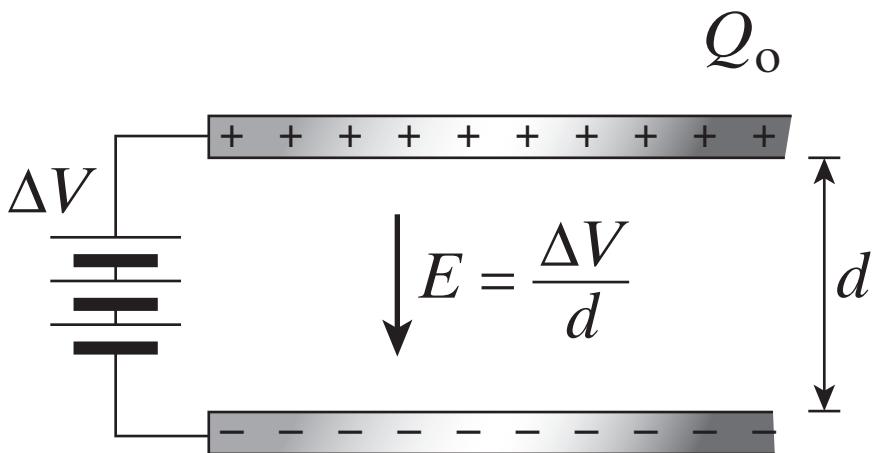
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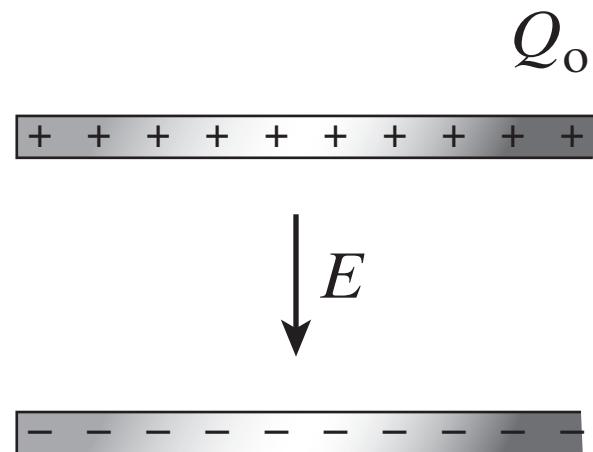
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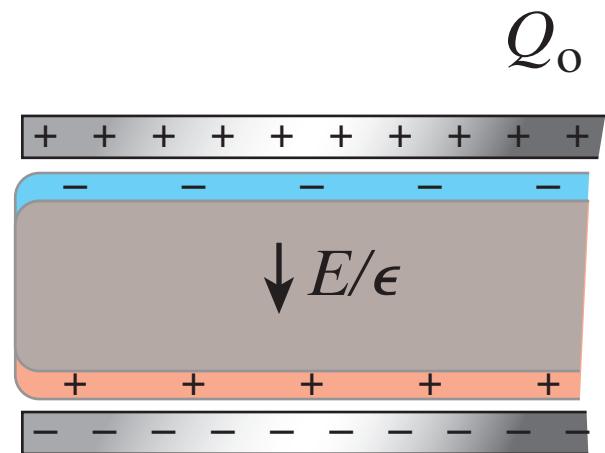
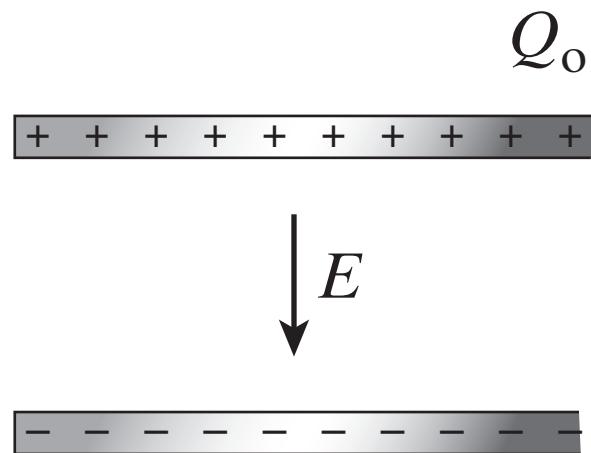
Propagation of EM waves through medium

Alternatively, ϵ is measure of the attenuation of the field



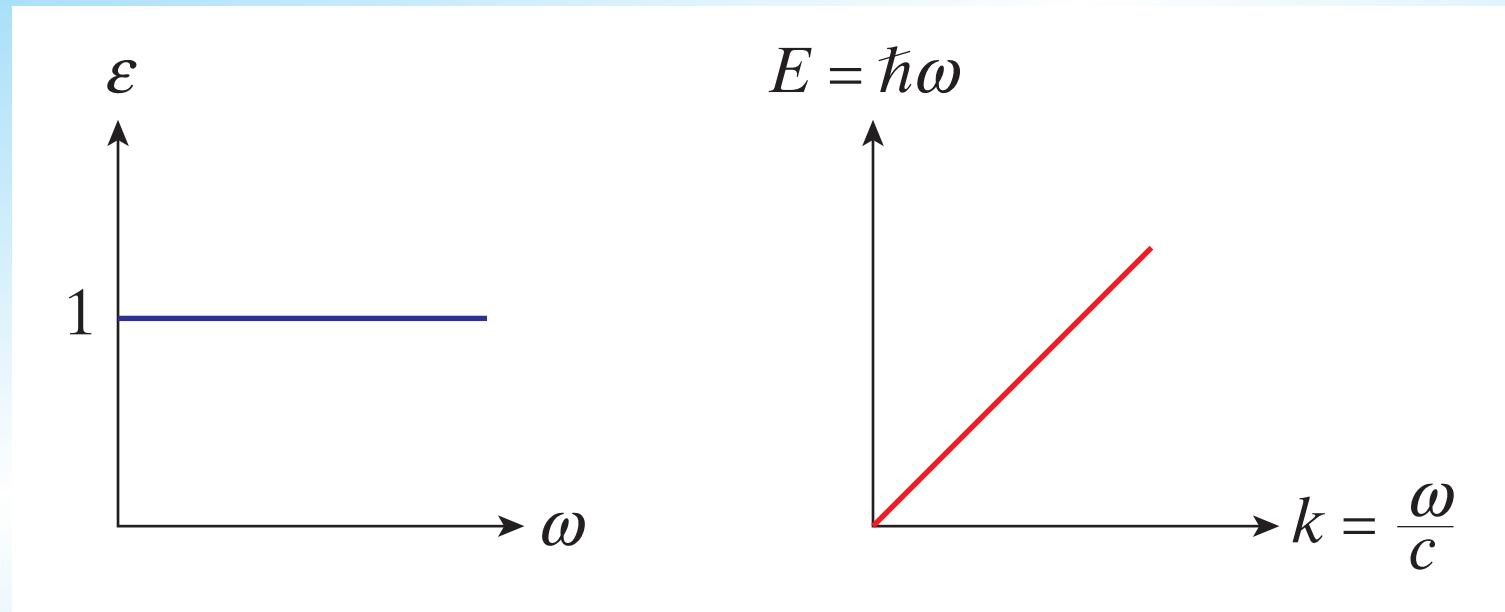
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Propagation of EM waves through medium

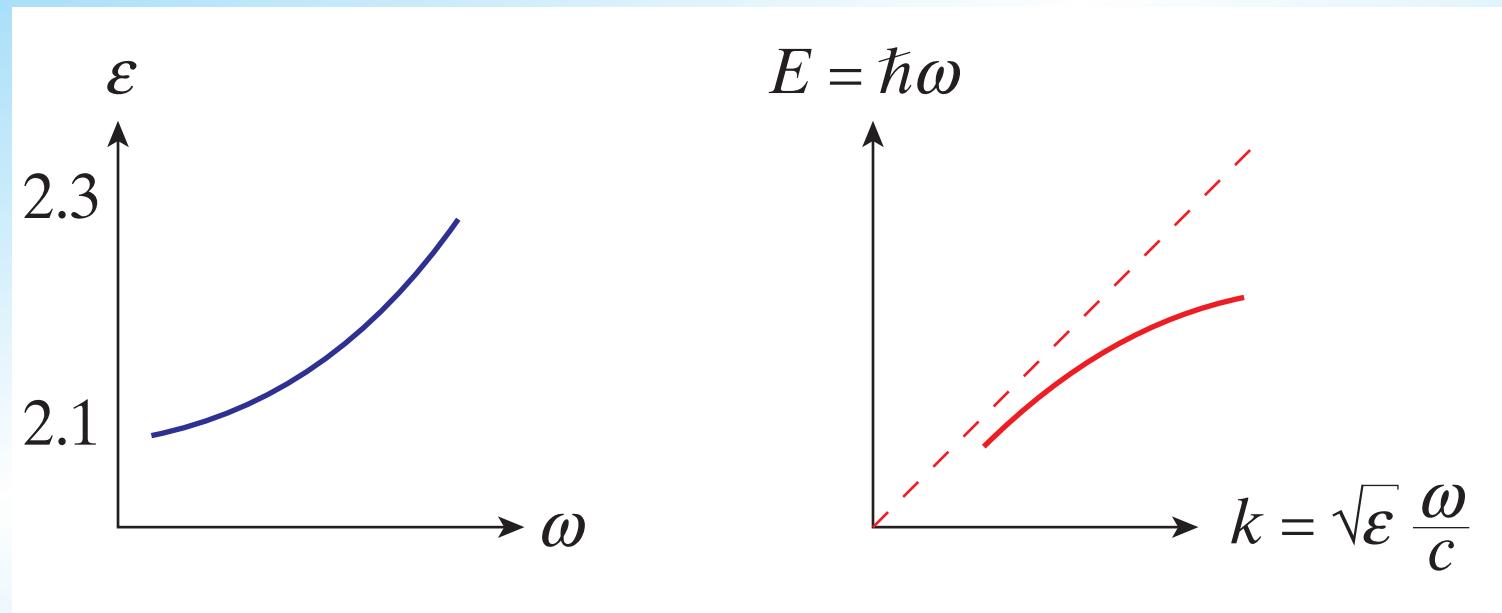
In vacuum: $f\lambda = \frac{\omega}{k} = c \Rightarrow \omega = c k$



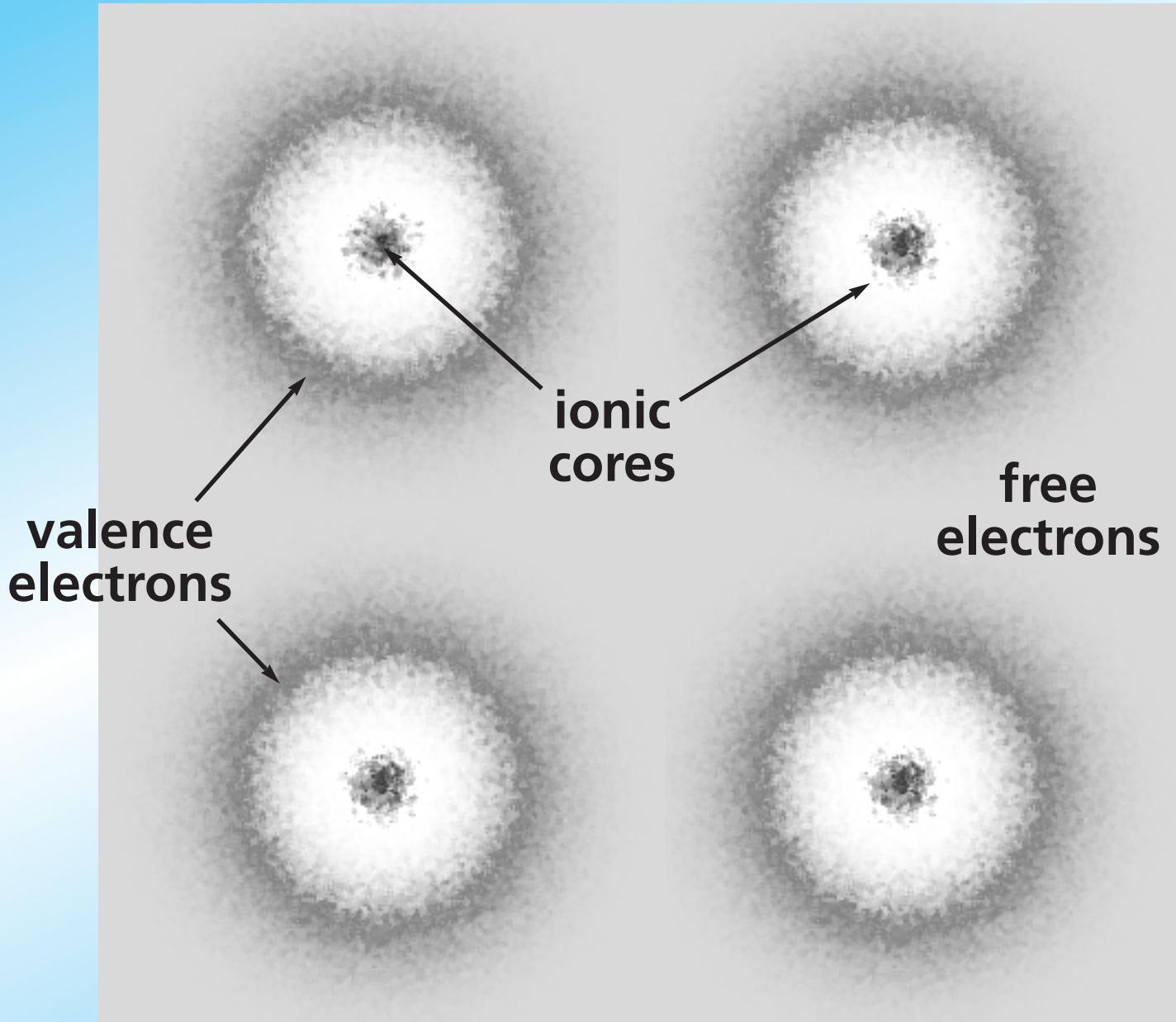
Propagation of EM waves through medium

In medium:

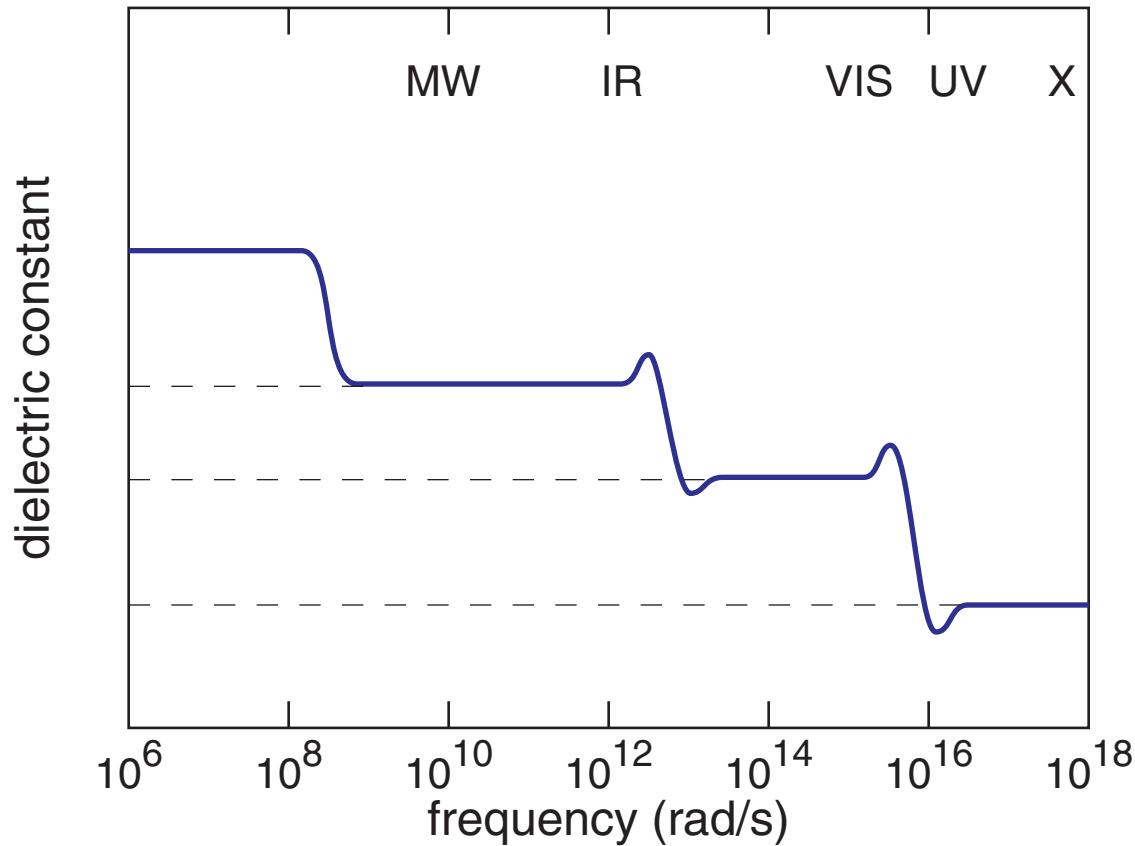
$$v = \frac{c}{\sqrt{\epsilon(\omega)}} = \frac{c}{n} \quad \Rightarrow \quad \omega = \frac{c}{\sqrt{\epsilon}} k$$



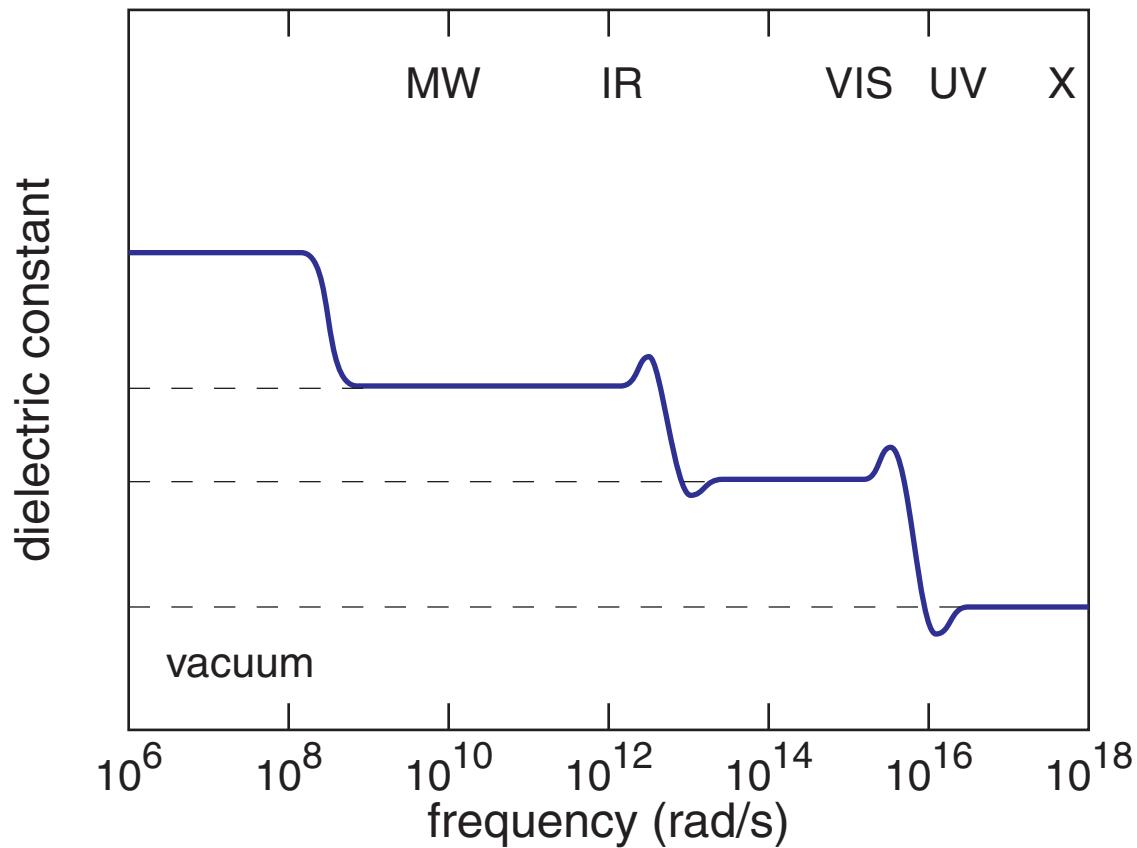
Which charges participate?



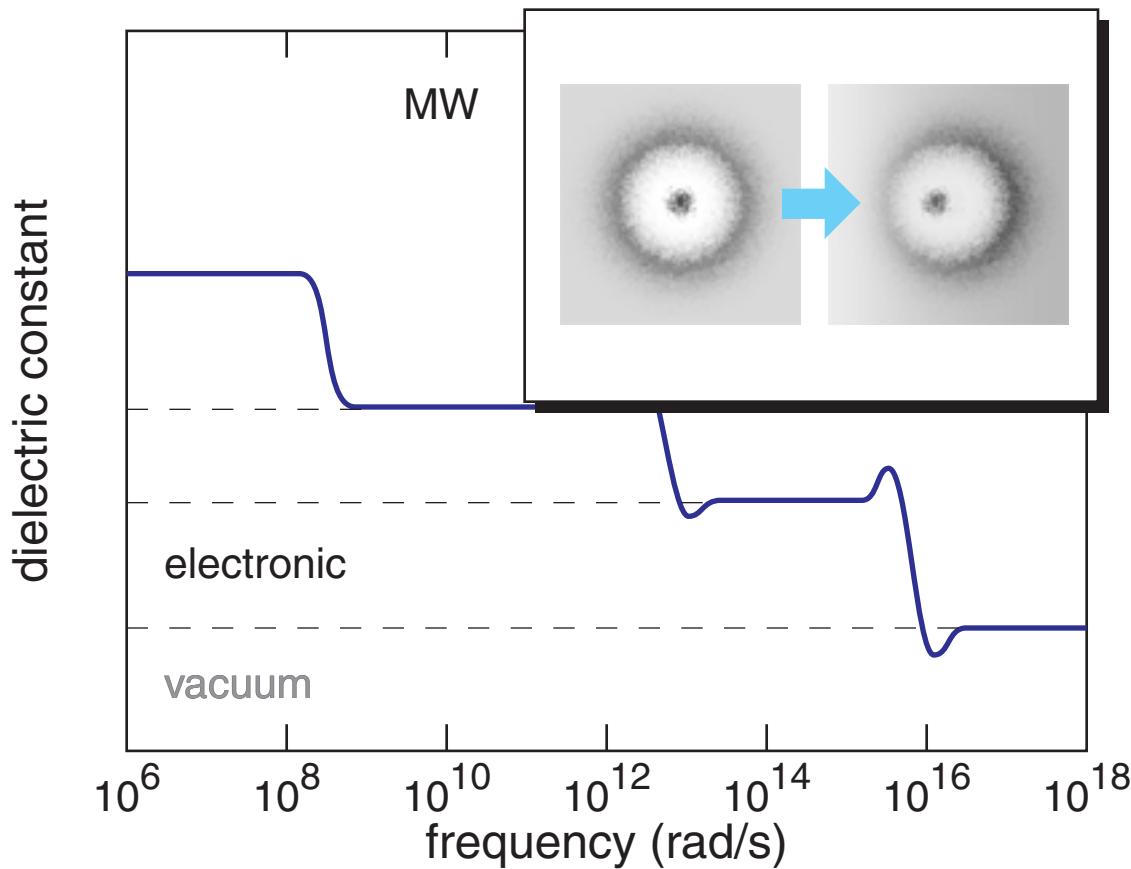
Dielectric function



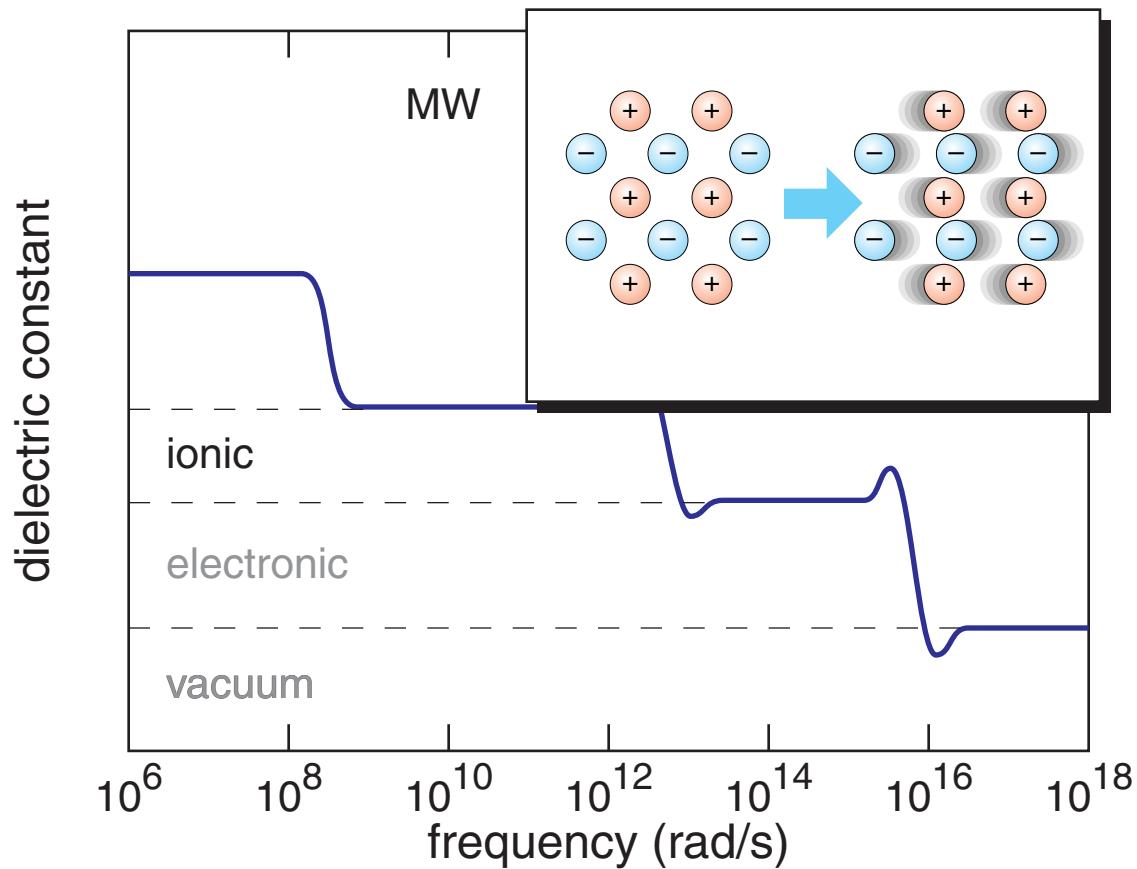
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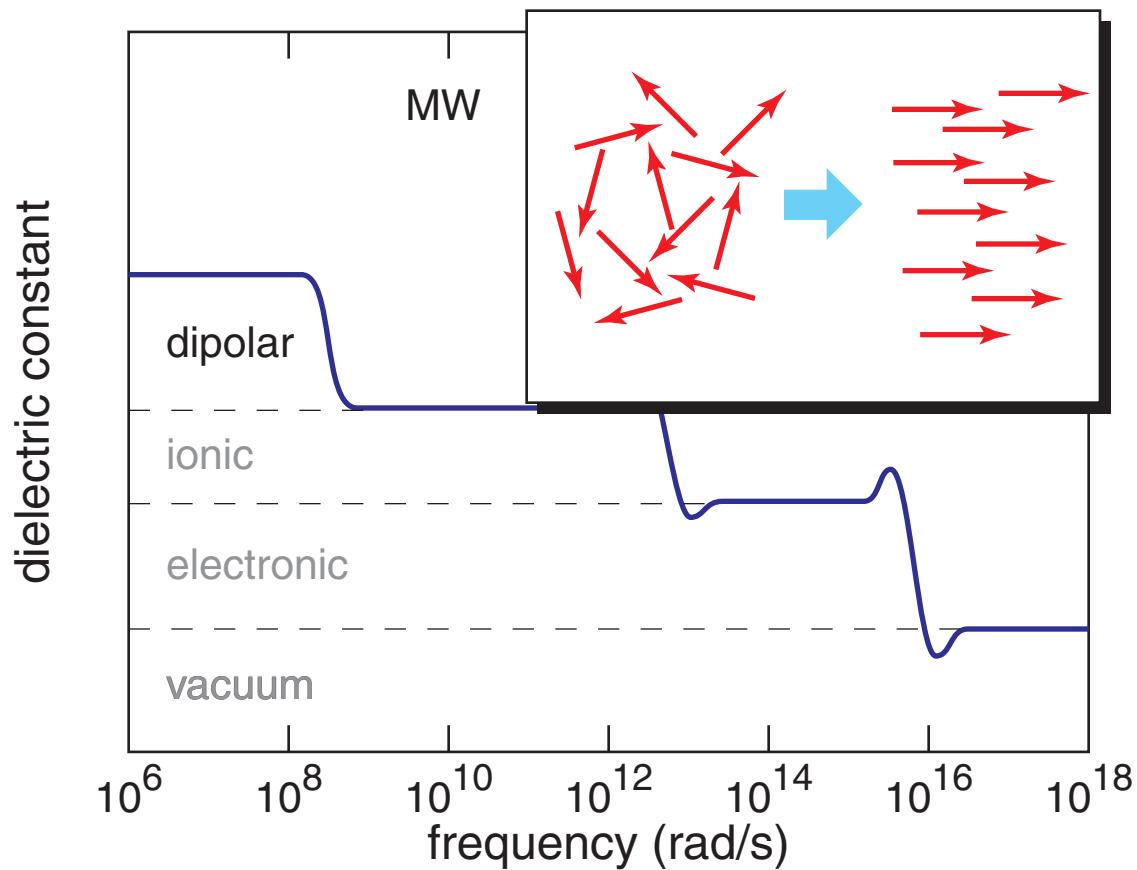
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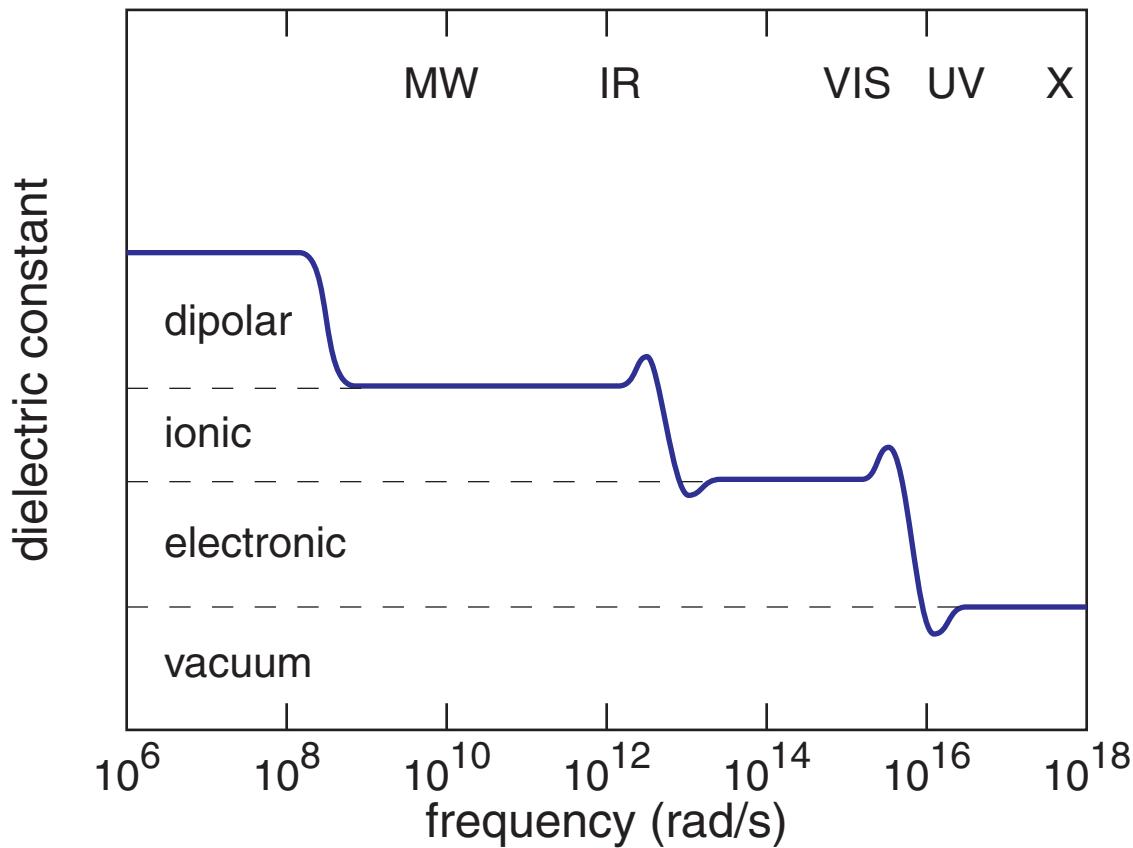
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Bound electrons

Electron on a string:

$$F_{binding} = - m_e \omega_o^2 x$$

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$$m \frac{d^2x}{dt^2} + m\gamma \frac{dx}{dt} + m\omega_o^2 x = - eE$$

Bound electrons

Steady state: electron oscillates at driving frequency

$$x(t) = x_o e^{-i\omega t} \quad x_o = - \frac{e}{m} \frac{1}{(\omega_o^2 - \omega^2) - i\gamma\omega} E_o$$

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$$P(t) = \left(\frac{Ne^2}{m} \right) \sum_j \frac{f_j}{(\omega_j^2 - \omega^2) - i\gamma_j\omega} E(t) \equiv \epsilon_o \chi_e E(t)$$

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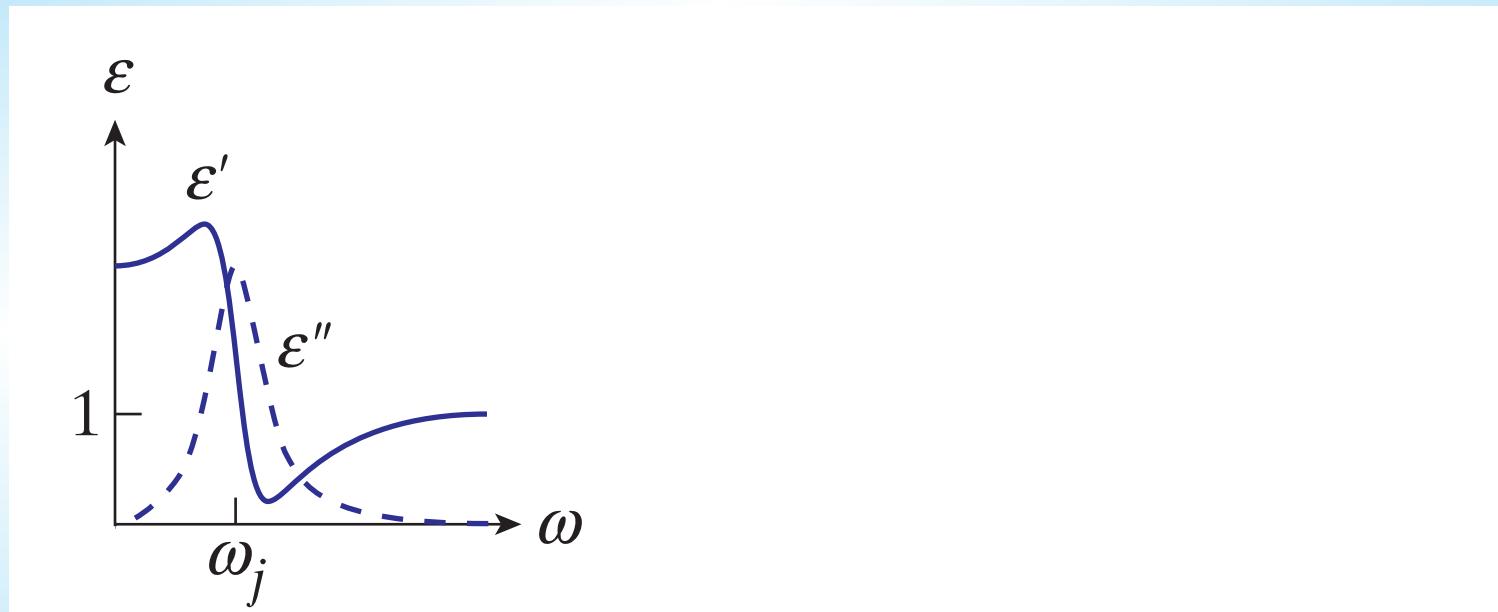
Dielectric function

$$\epsilon(\omega) \equiv 1 + \chi_e = 1 + \frac{Ne^2}{\epsilon_0 m} \sum_j \frac{f_j}{(\omega_j^2 - \omega^2) - i\gamma_j \omega}$$

Bound electrons

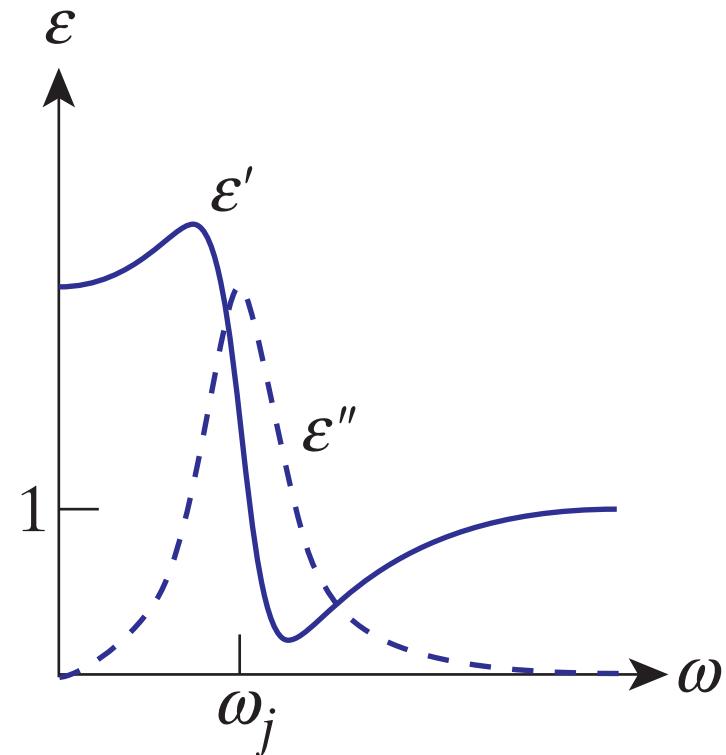
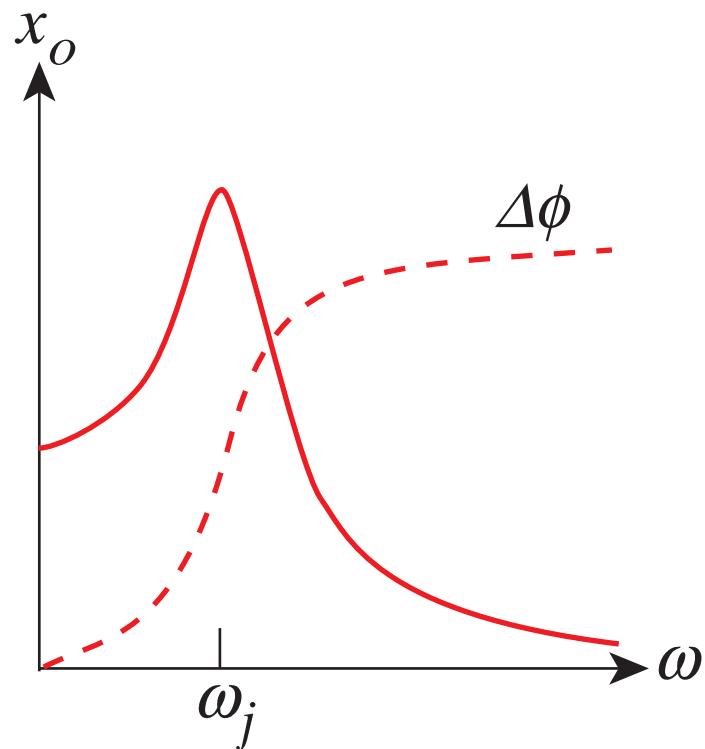
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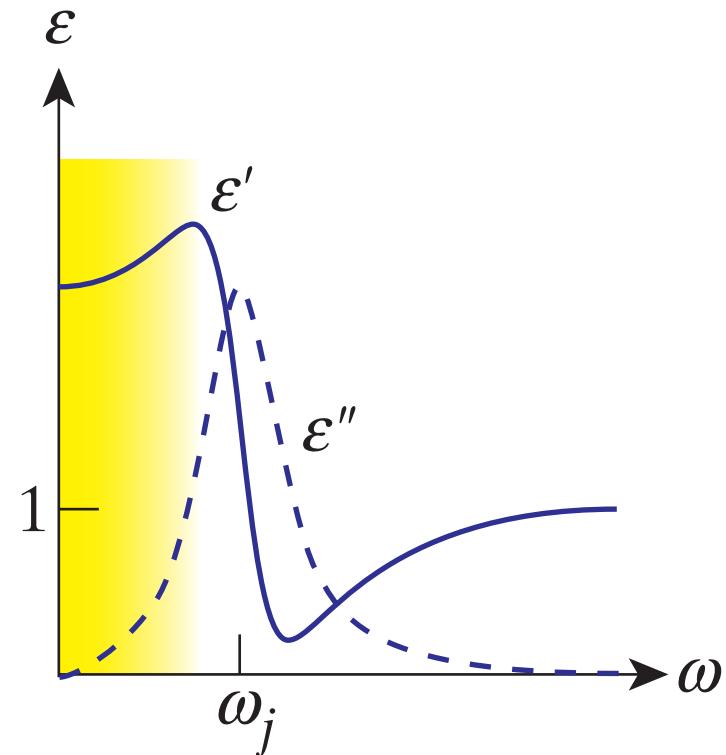
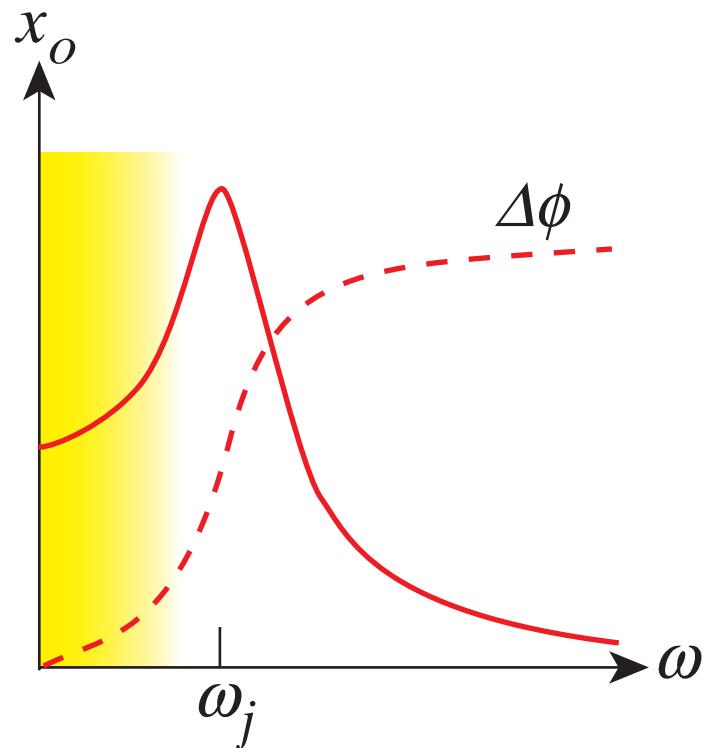
Bound electrons

amplitude of bound charge oscillation



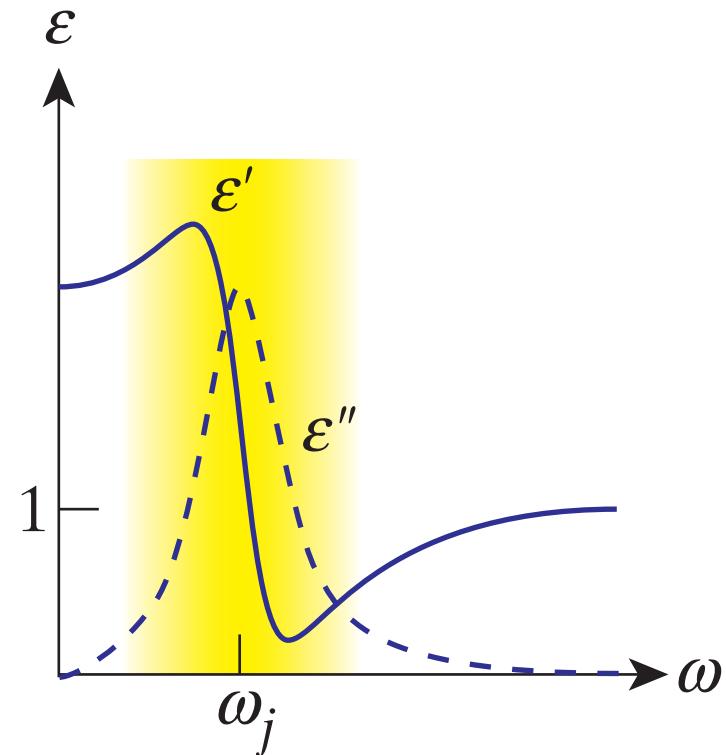
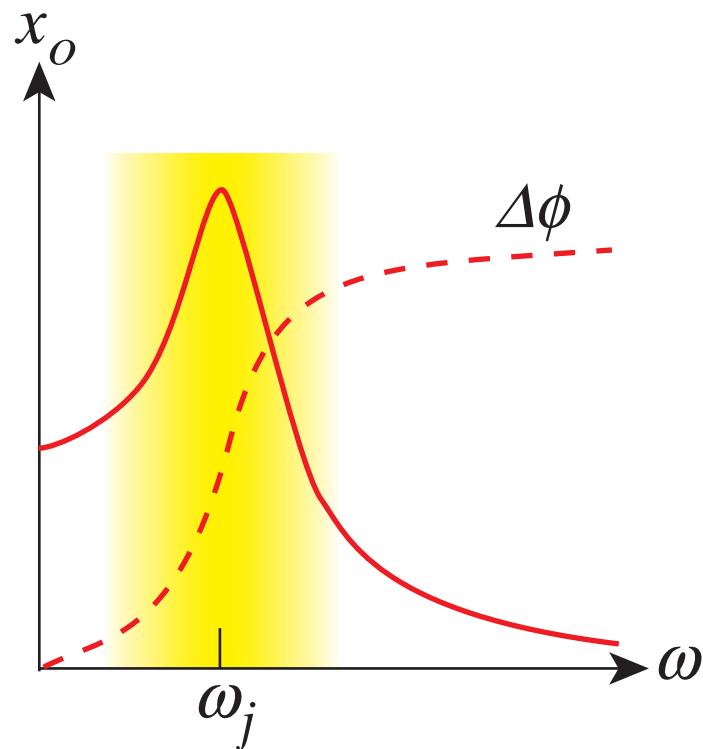
Bound electrons

Below resonance: bound charges keep up with driving field \Rightarrow field attenuated, wave propagates more slowly



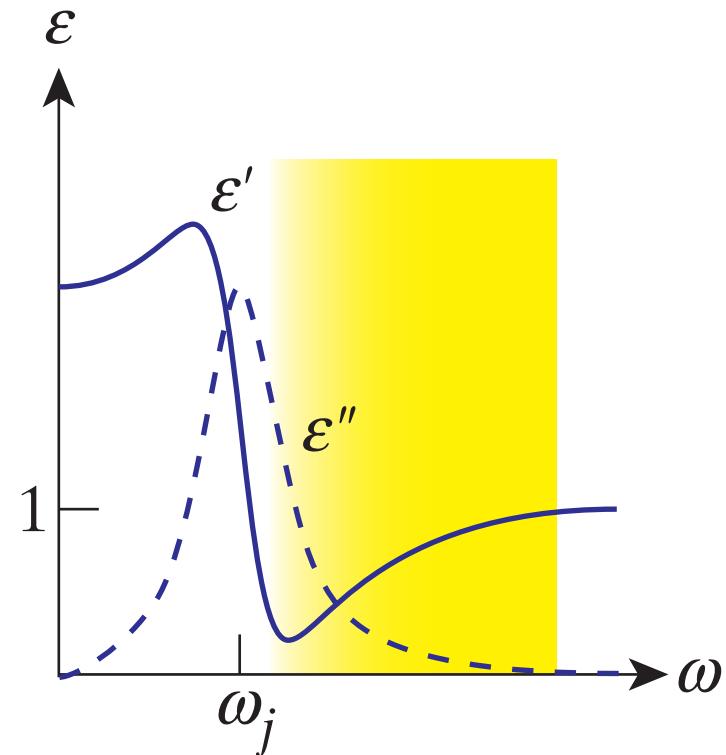
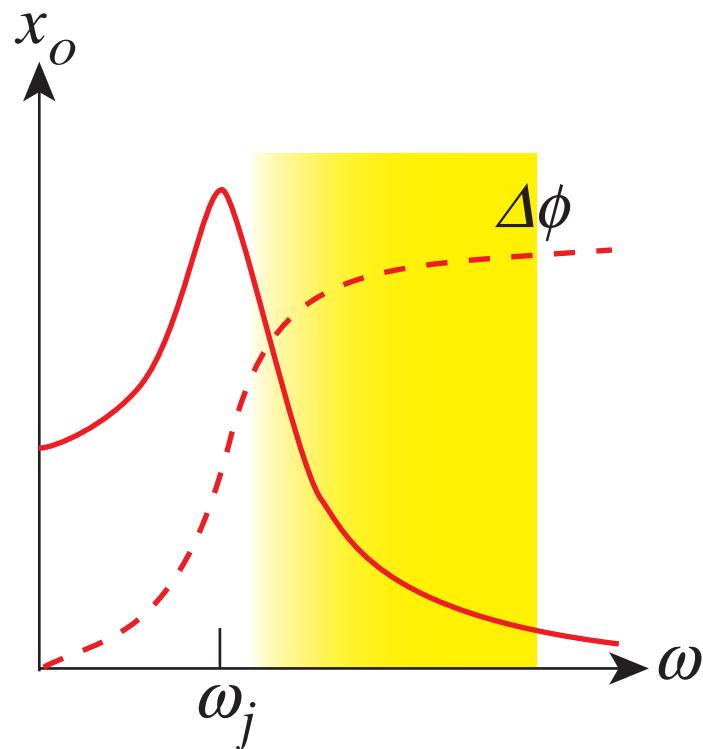
Bound electrons

At resonance: energy transfer from wave to bound charges \Rightarrow wave attenuates (absorption)



Bound electrons

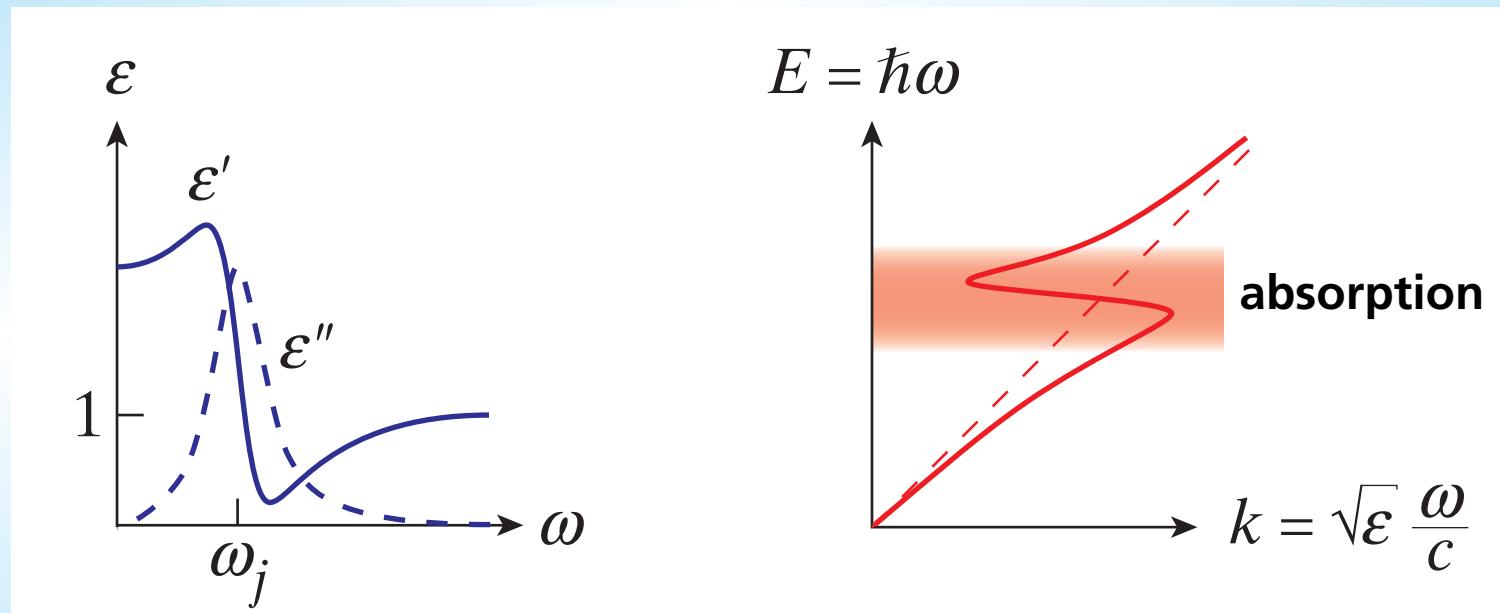
Above resonance: bound charges cannot keep up with driving field \Rightarrow dielectric like a vacuum



Bound electrons

Dielectric function

$$\epsilon(\omega) \equiv 1 + \chi_e = 1 + \frac{Ne^2}{\epsilon_0 m} \sum_j \frac{f_j}{(\omega_j^2 - \omega^2) - i\gamma_j \omega}$$



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Low frequency ($\omega \ll 1$) \Rightarrow current generated

$$J = -Ne \frac{dx}{dt} = \frac{Ne^2}{m} \frac{1}{\gamma - i\omega} E \approx \frac{Ne^2}{m\gamma} E \equiv \sigma E$$

Free electrons

$\omega \gg \gamma$: **σ complex** \Rightarrow J out of phase with E

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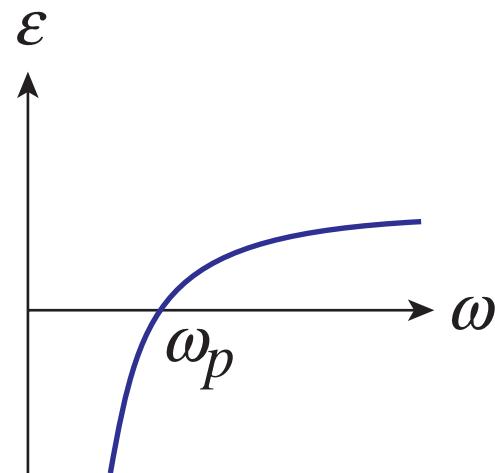
Dielectric function

$$\epsilon(\omega) \equiv 1 + \chi_e = 1 - \frac{Ne^2}{m\epsilon_0} \frac{1}{\omega^2 + i\gamma\omega} = \epsilon'(\omega) + i\epsilon''(\omega)$$

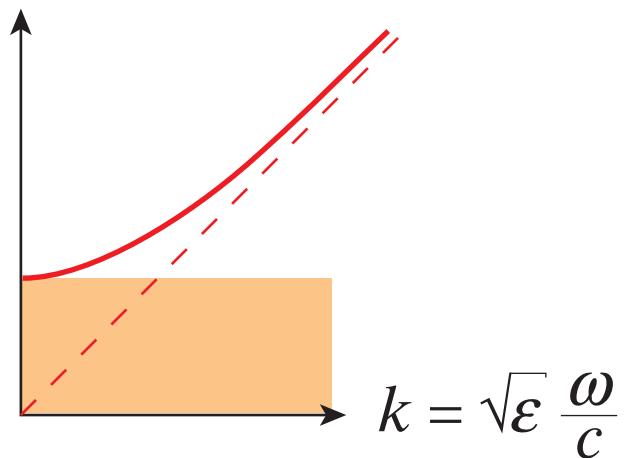
Plasma

$$\gamma \approx 0 \quad \Rightarrow \quad \epsilon'' = 0$$

$$\epsilon'(\omega) = 1 - \frac{Ne^2}{m\epsilon_0} \frac{1}{\omega^2} \equiv 1 - \frac{\omega_p^2}{\omega^2}$$



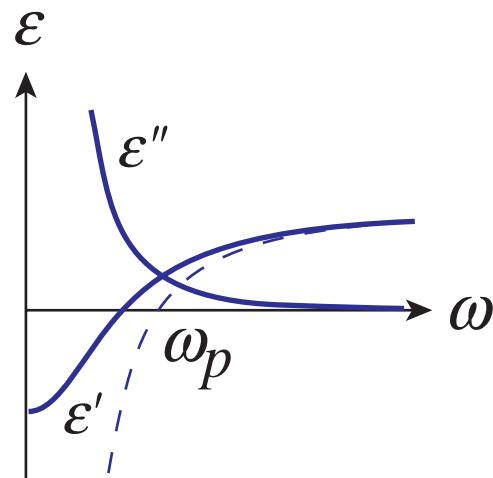
$$E = \hbar\omega$$



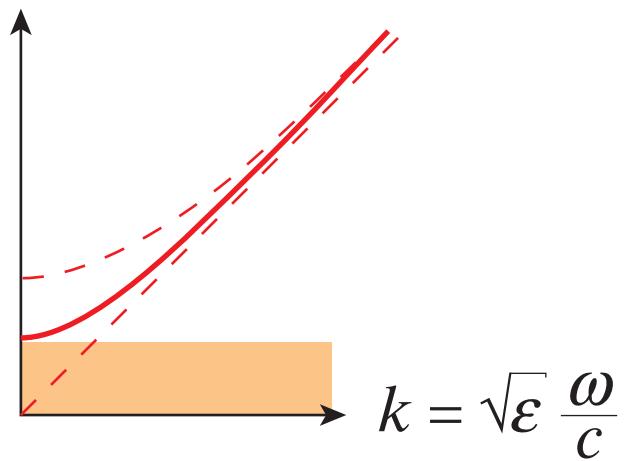
Plasma

Add damping

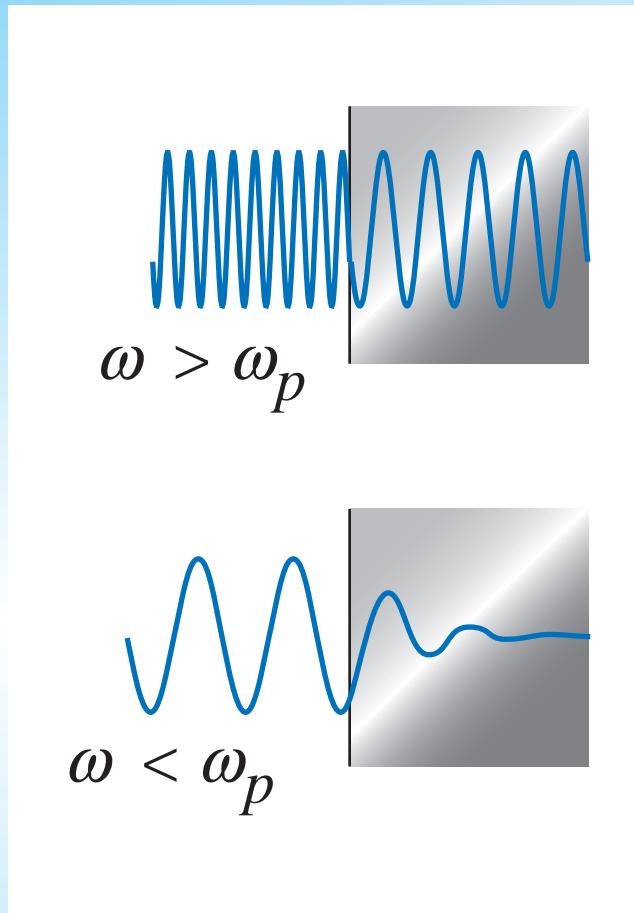
$$\gamma \lesssim \omega_p$$



$$E = \hbar\omega$$

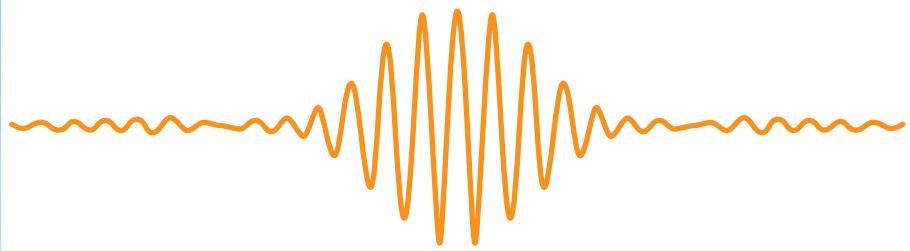


Plasma acts like a high-pass filter:

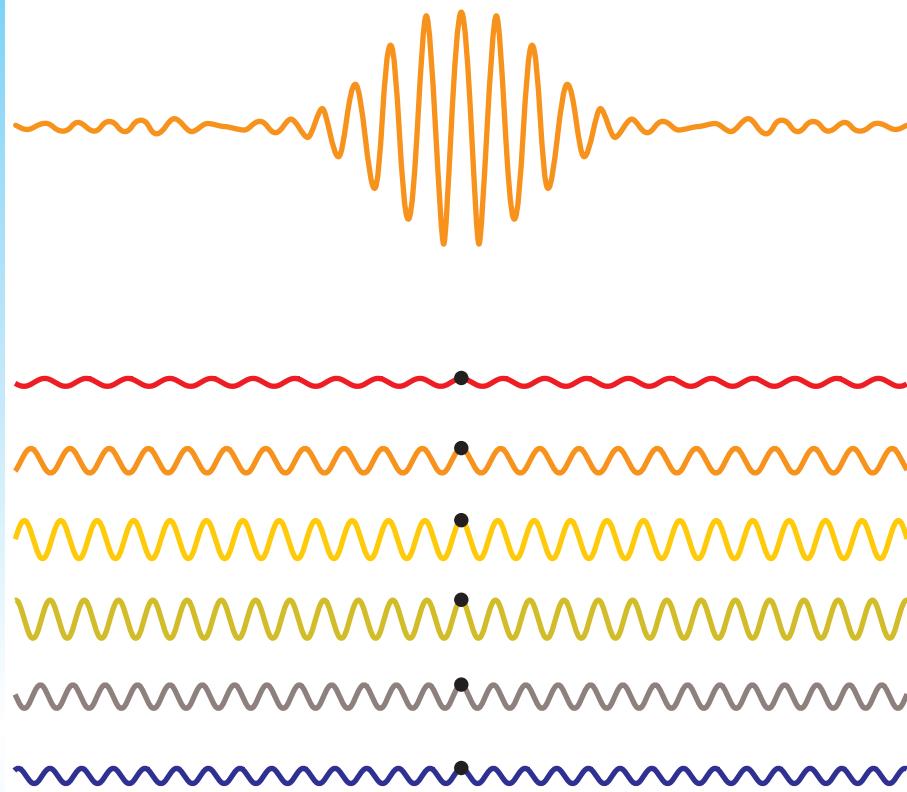


$\log N$ (cm ⁻³)	ω_p (rad s ⁻¹)	λ_p
22	6×10^{15}	330 nm
18	6×10^{13}	33 μm
14	6×10^{11}	3.3 mm
10	6×10^9	0.33 m

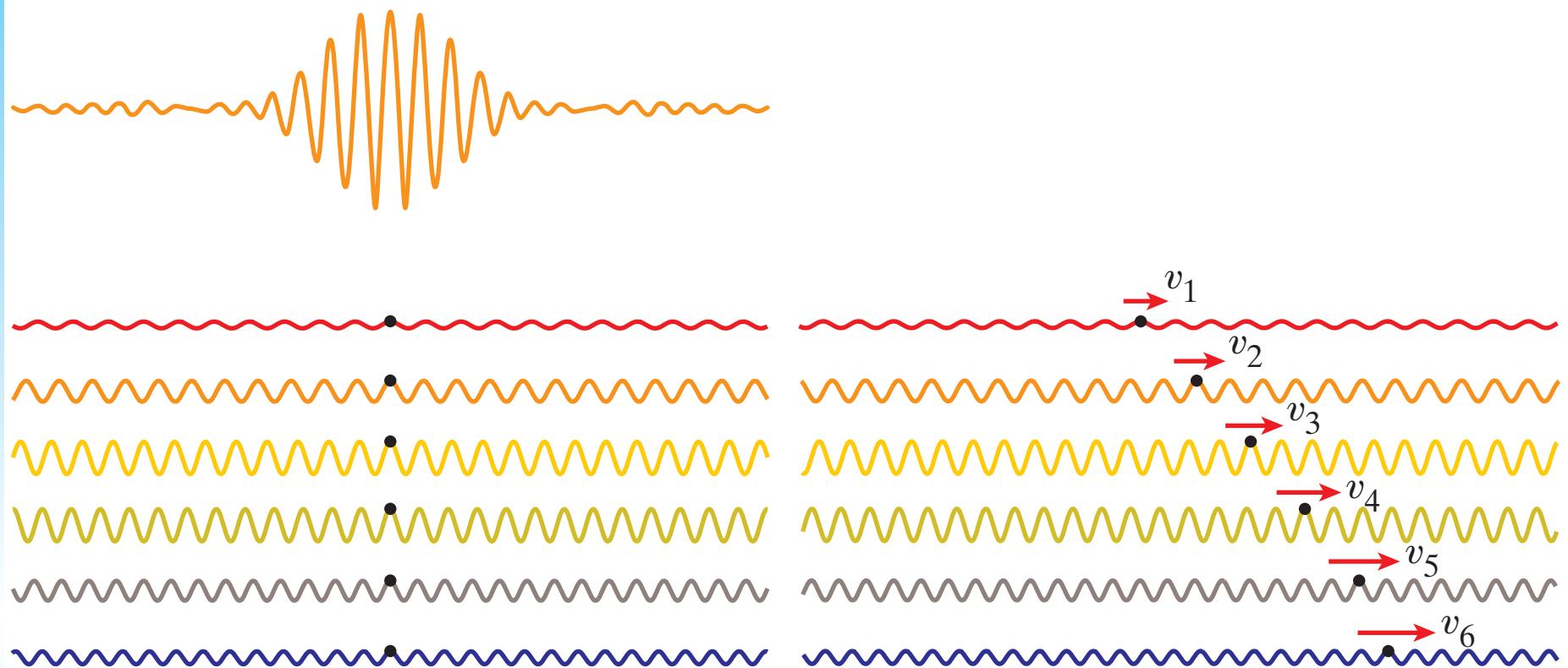
Pulse dispersion



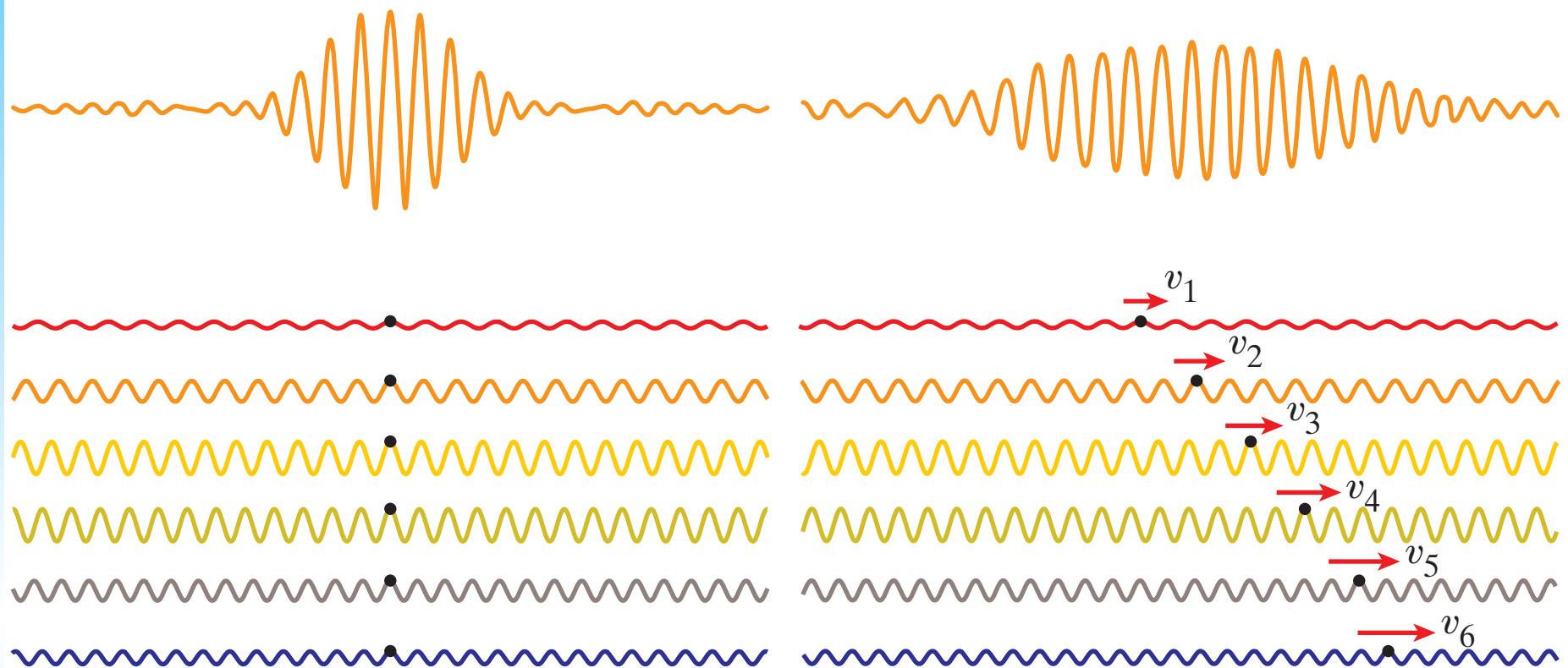
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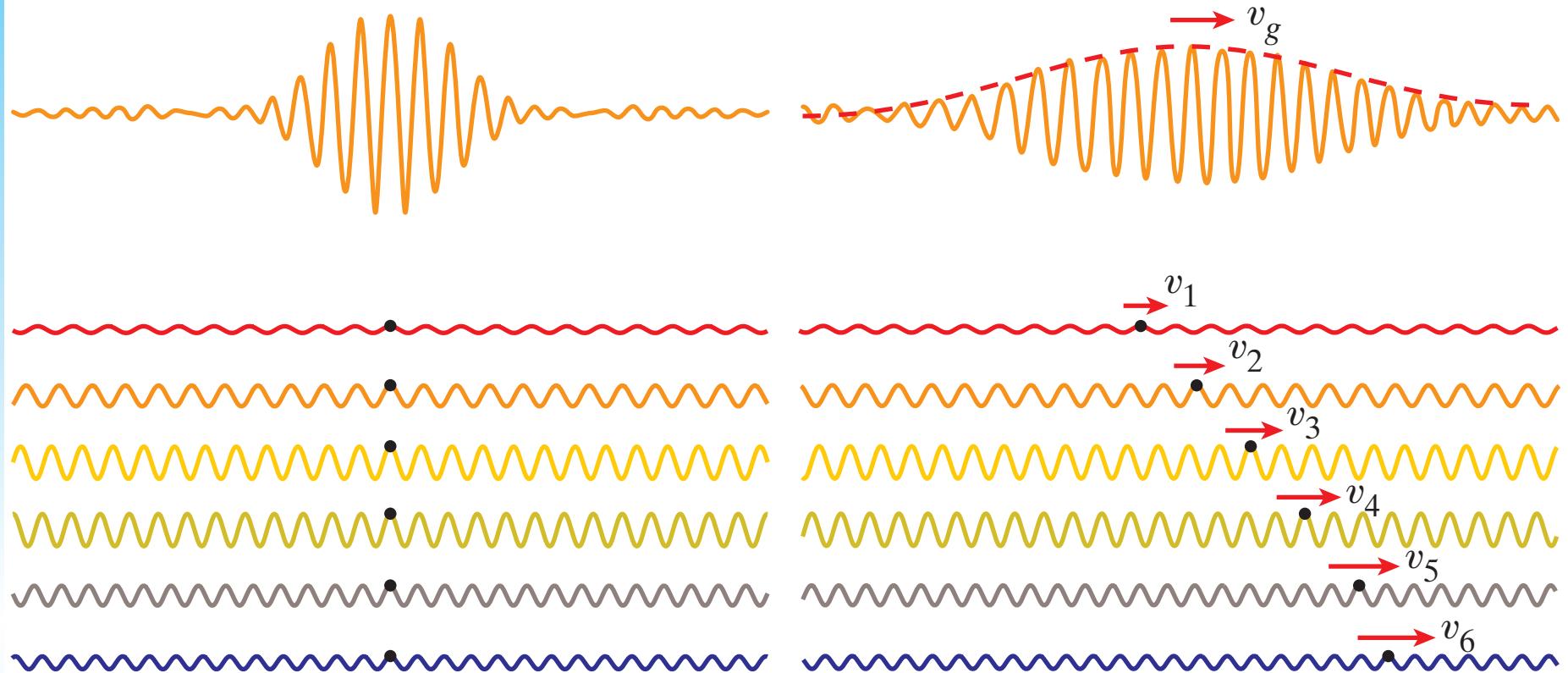
Pulse dispersion



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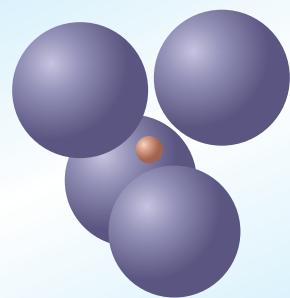
Pulse dispersion



Nonlinear optics

Linear response

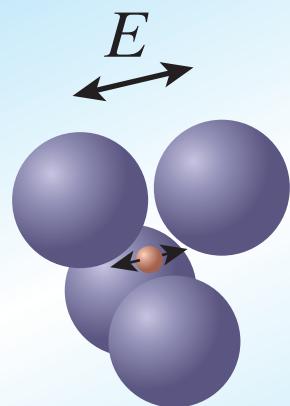
$$P(t) = \epsilon_0 \chi_e E(t)$$



Nonlinear optics

Linear response

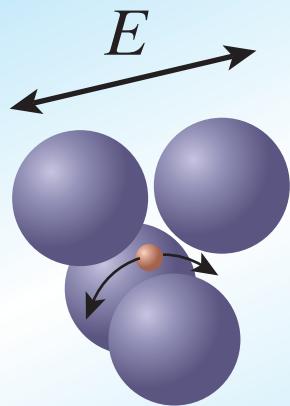
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Nonlinear optics

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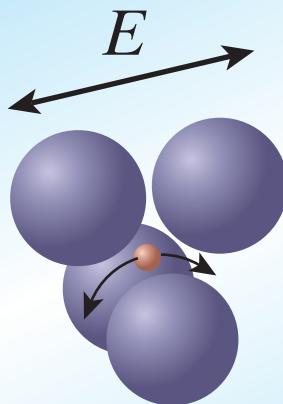
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Nonlinear optics

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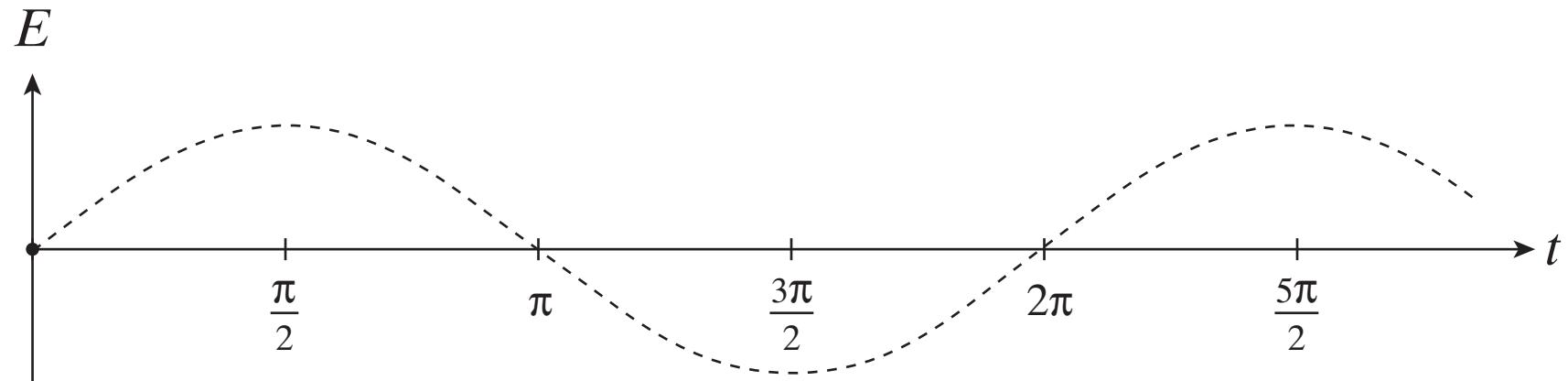


Nonlinear polarization:

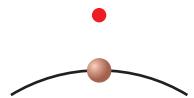
$$P = \chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \dots$$

$$P = P^{(1)} + P^{(2)} + P^{(3)} + \dots$$

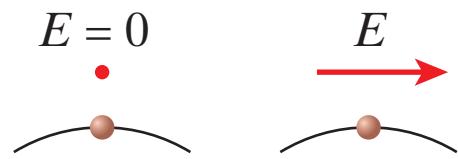
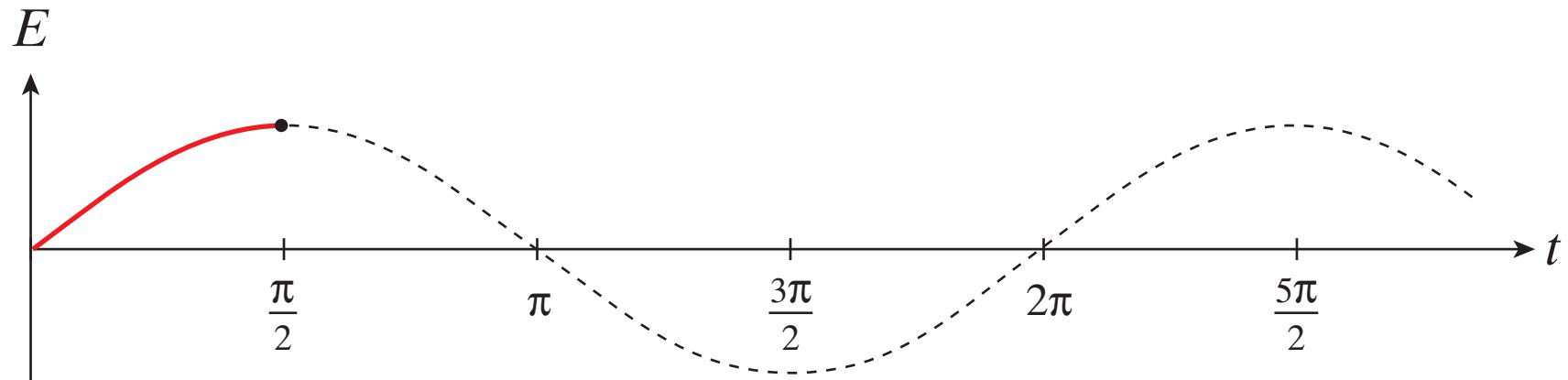
Nonlinear optics



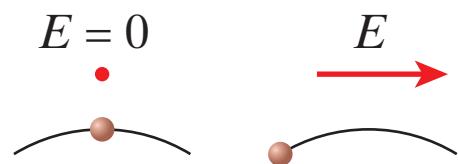
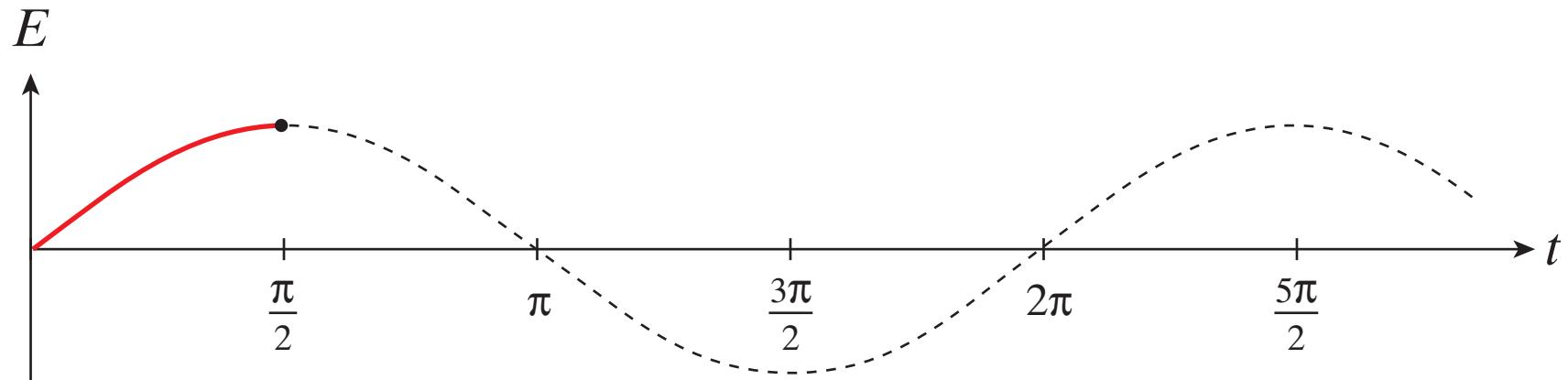
$$E = 0$$



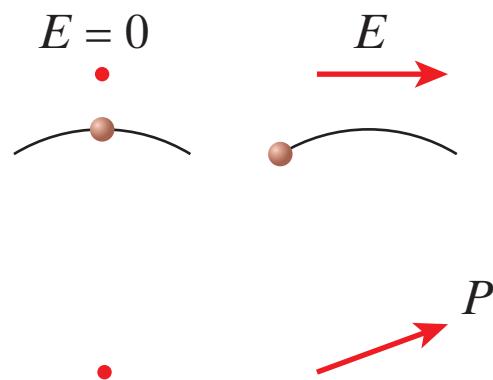
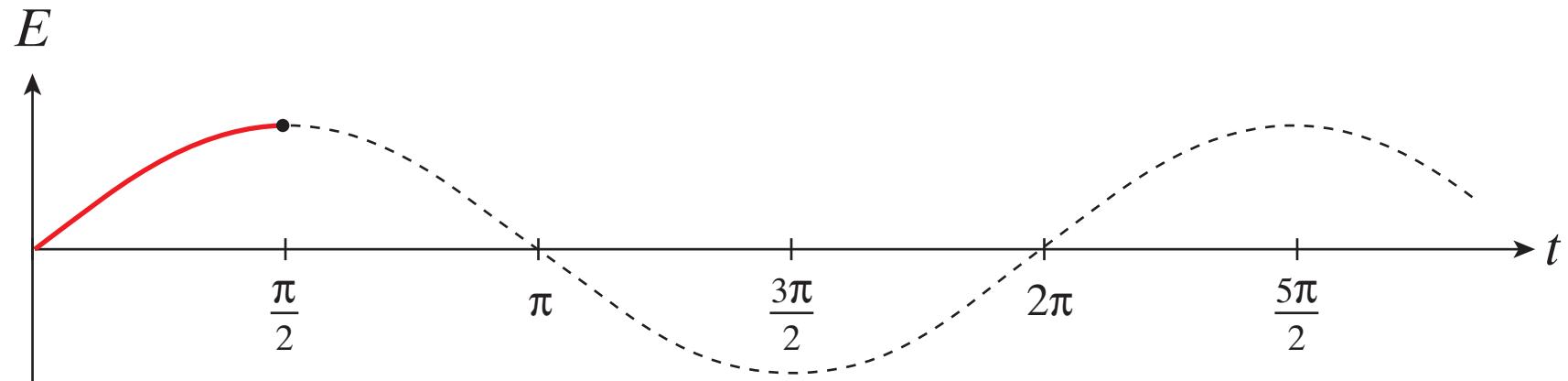
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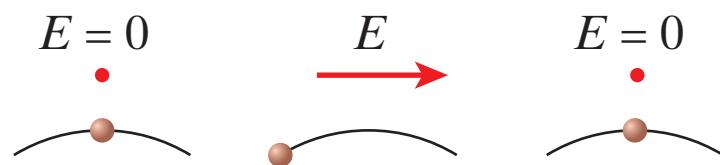
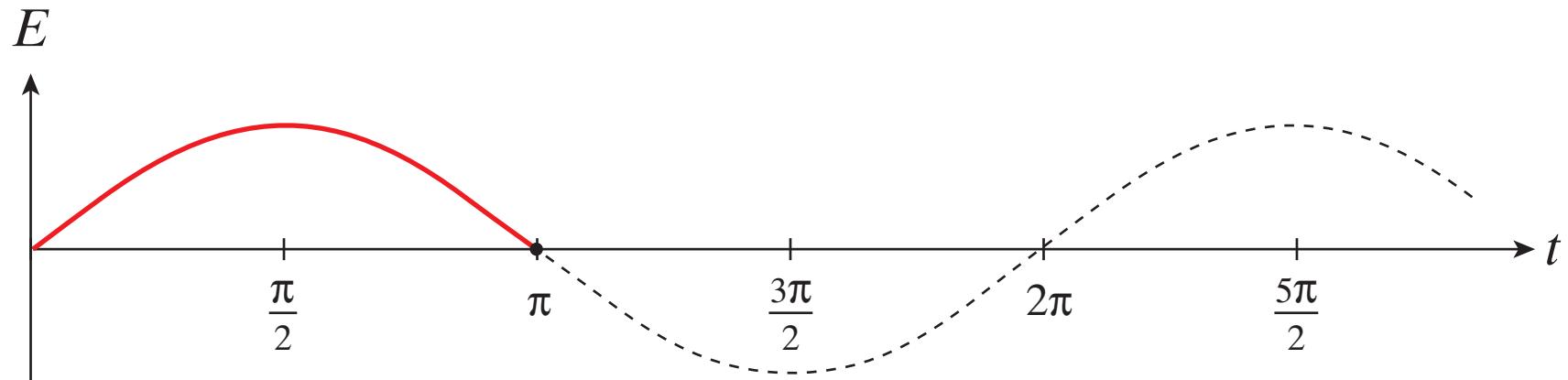
Nonlinear optics



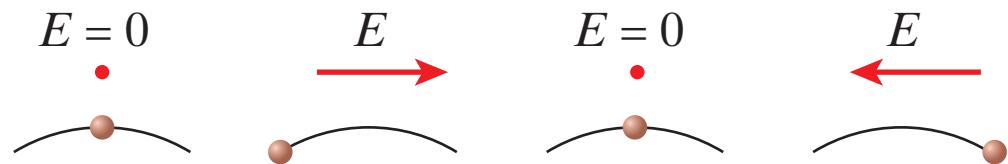
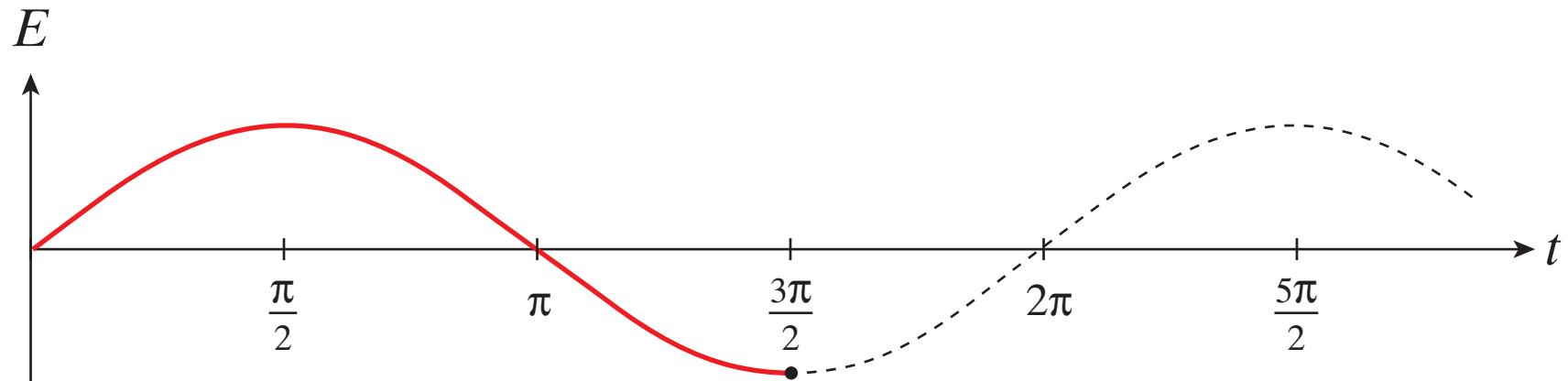
Nonlinear optics



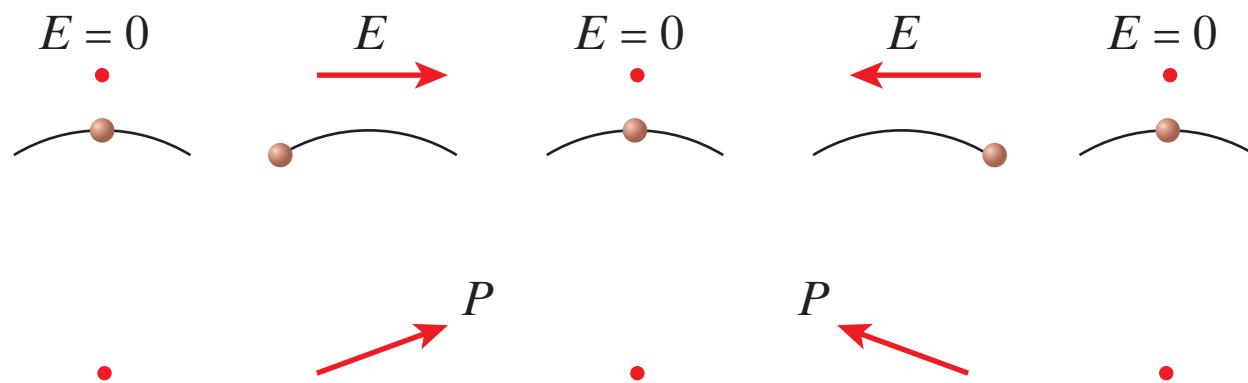
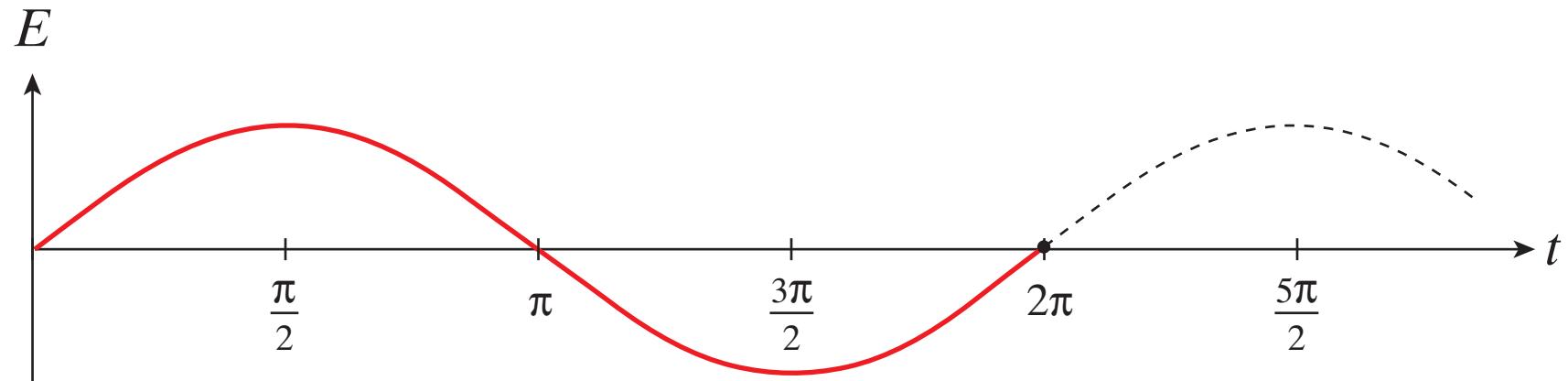
Nonlinear optics



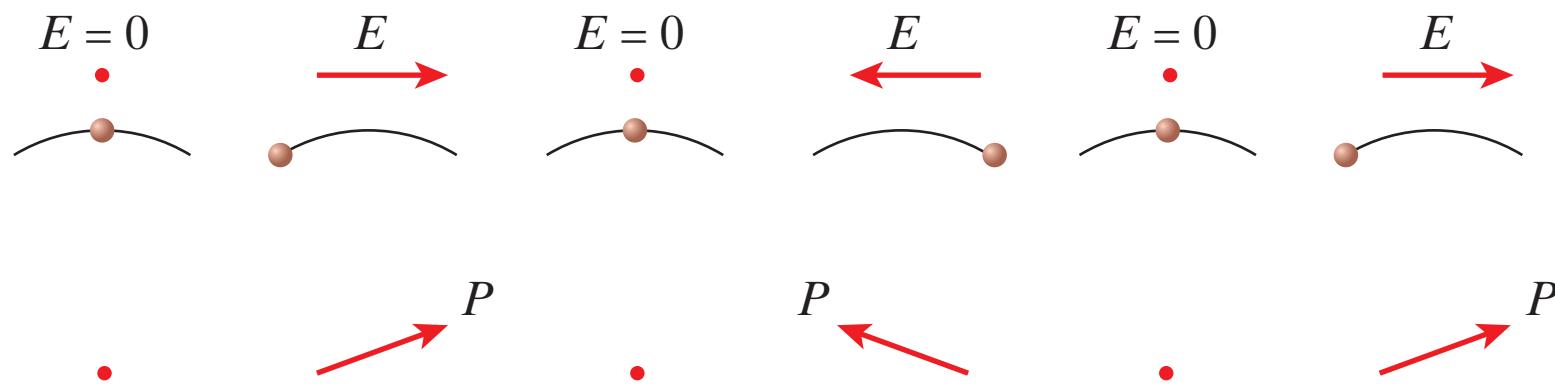
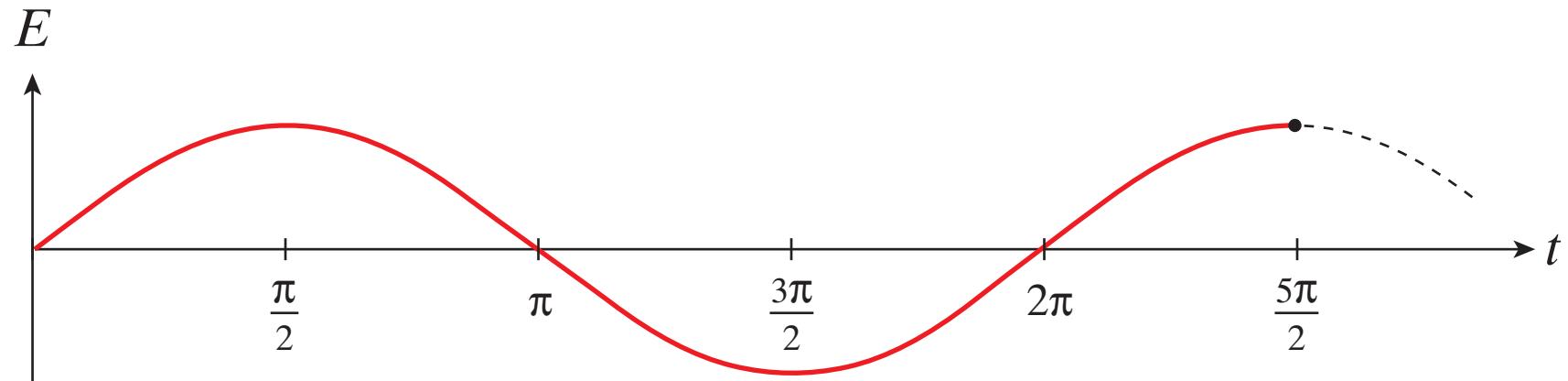
Nonlinear optics



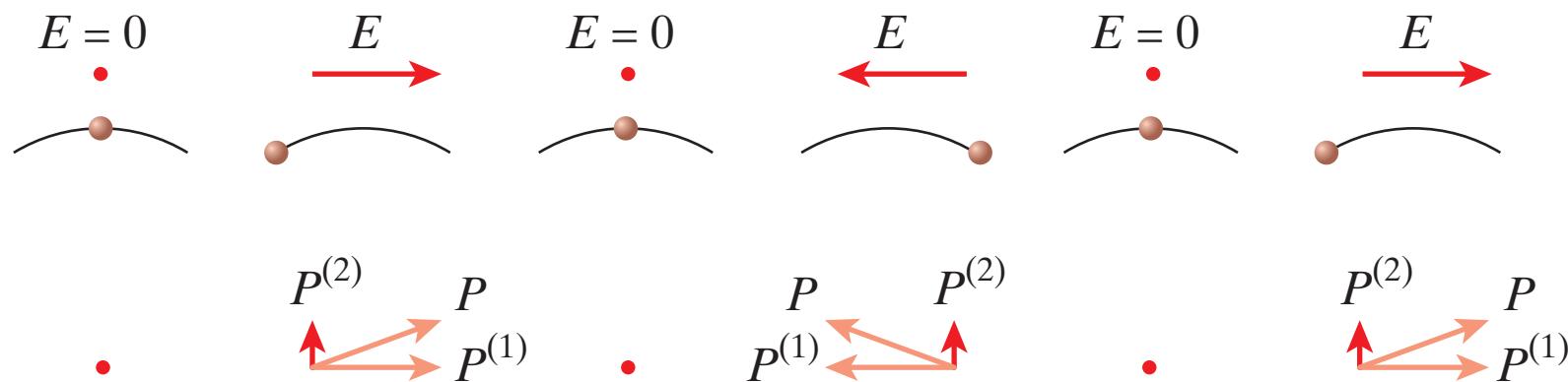
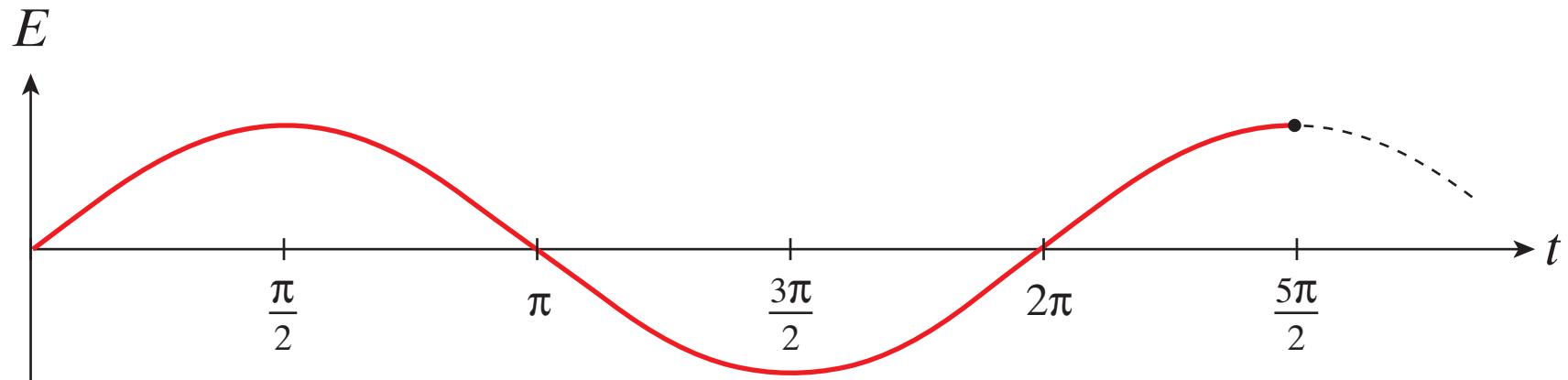
Nonlinear optics



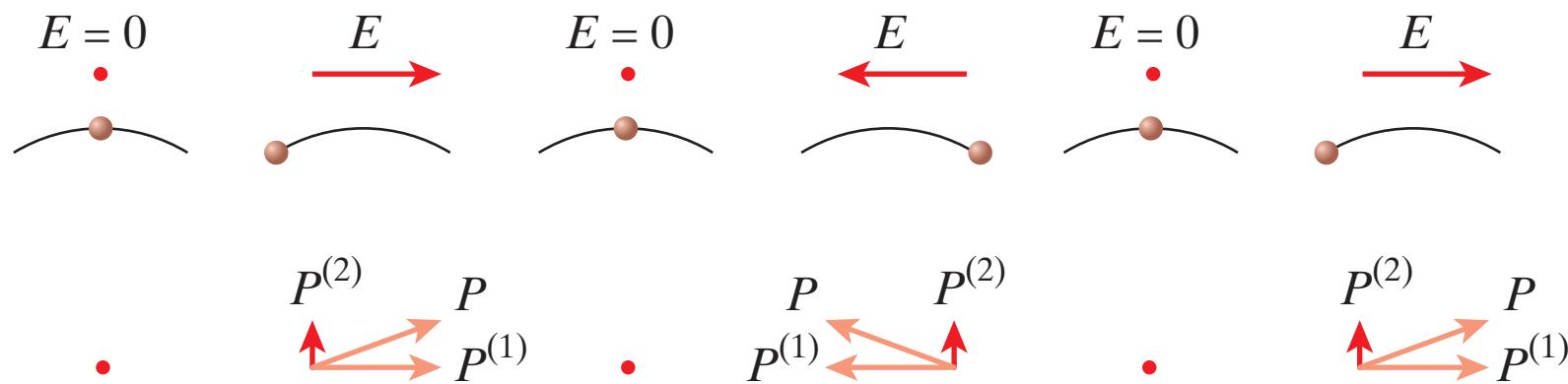
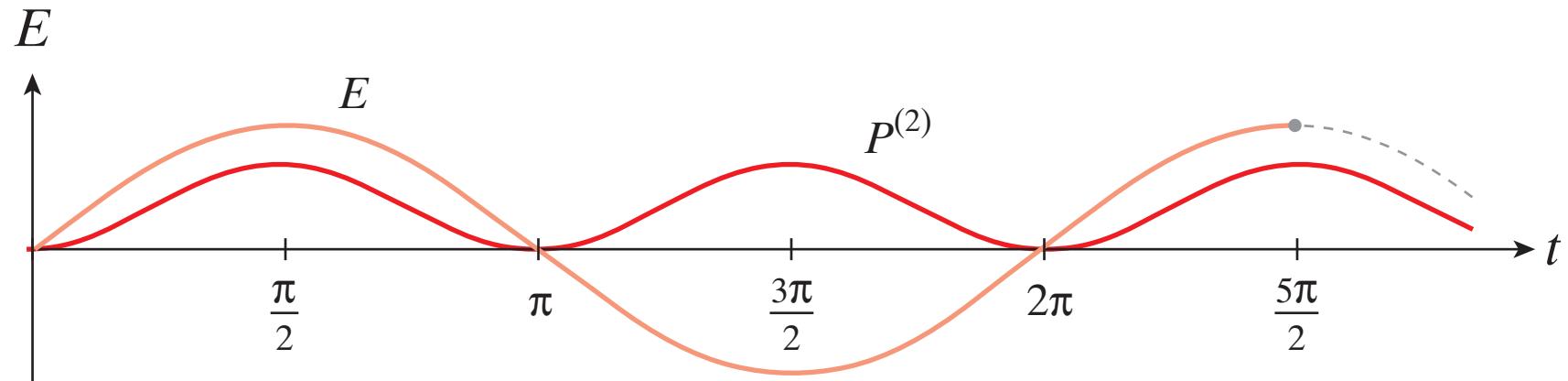
Nonlinear optics



Nonlinear optics



Nonlinear optics



In medium with inversion symmetry

$$\vec{P}^{(2)} = \vec{\chi}^{(2)} : \vec{E} \vec{E} \quad \Rightarrow \quad -\vec{P}^{(2)} = \vec{\chi}^{(2)} : (-\vec{E})(-\vec{E})$$

In medium with inversion symmetry

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and so

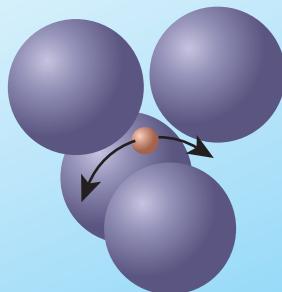
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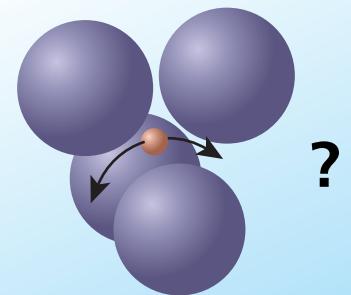
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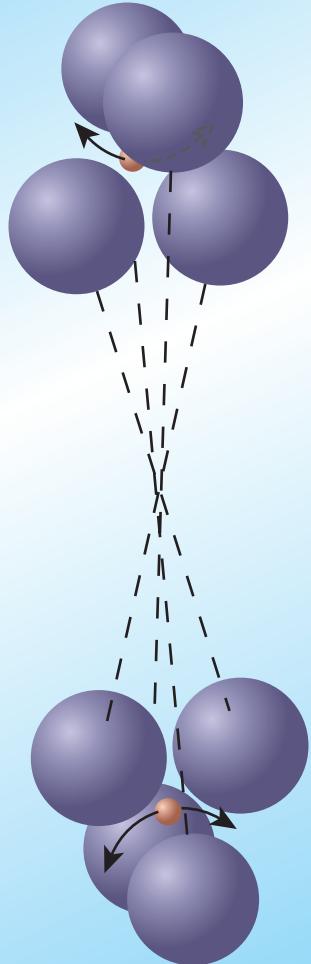
... but ...

Nonlinear optics

How to reconcile $\chi^{(2)} = -\chi^{(2)} = 0$ **with**



Nonlinear optics



Nonlinear optics

Nonlinear polarization:

$$P = \chi^{(1)}E + \chi^{(3)}E^3 + \dots$$

Nonlinear optics

Nonlinear polarization:

$$P = \chi^{(1)}E + \chi^{(3)}E^3 + \dots$$

Third order polarization

$$P^{(3)}(t) = \chi^{(3)}E(t)E^*(t)E(t) = \chi^{(3)}I(t)E(t)$$

Nonlinear optics

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and so $P = P^{(1)} + P^{(3)} = (\chi^{(1)} + \chi^{(3)}I)E \equiv \chi_{eff}E$

Nonlinear optics

Nonlinear polarization:

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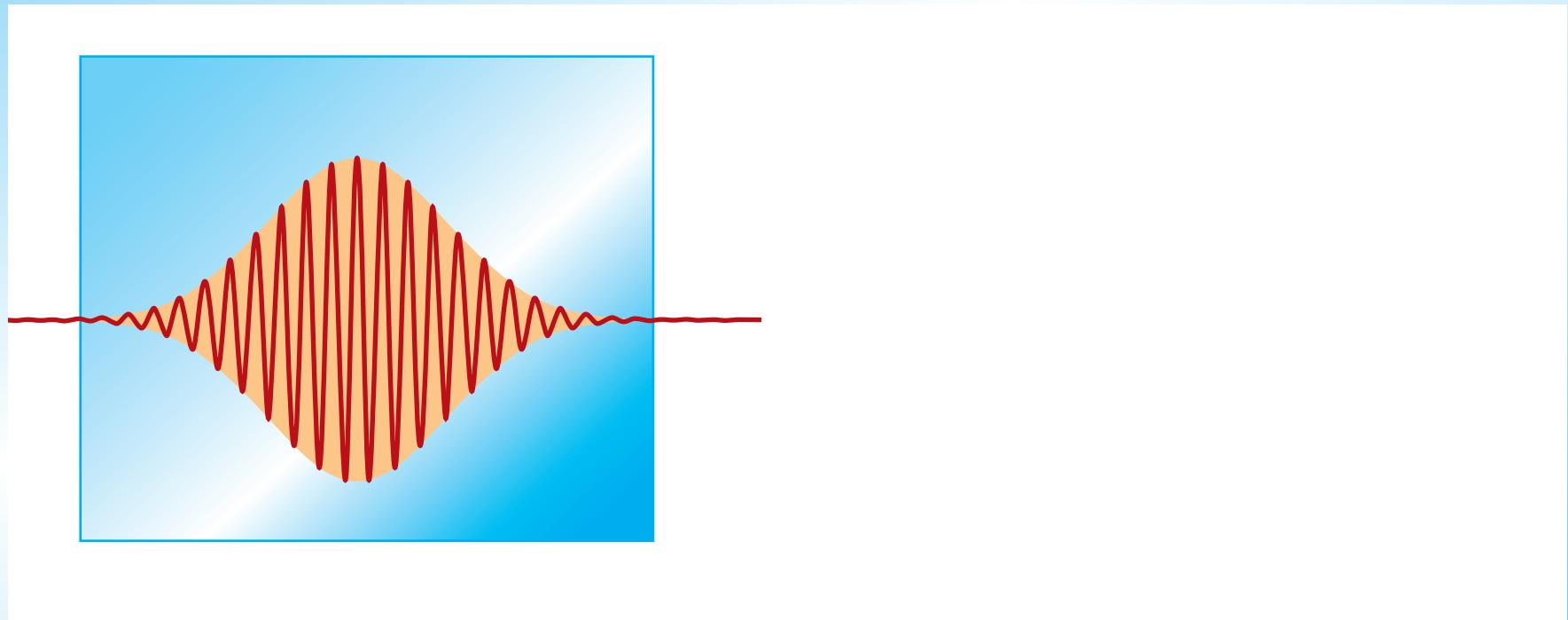
and so $P = P^{(1)} + P^{(3)} = (\chi^{(1)} + \chi^{(3)}I)E \equiv \chi_{eff}E$

$$n = \sqrt{\epsilon} = \sqrt{1 + \chi_{eff}} \approx \sqrt{1 + \chi^{(1)}} + \frac{1}{2} \frac{\chi^{(3)}I}{\sqrt{1 + \chi^{(1)}}} = n_o + n_2 I$$

Nonlinear optics

Intensity dependent index of refraction:

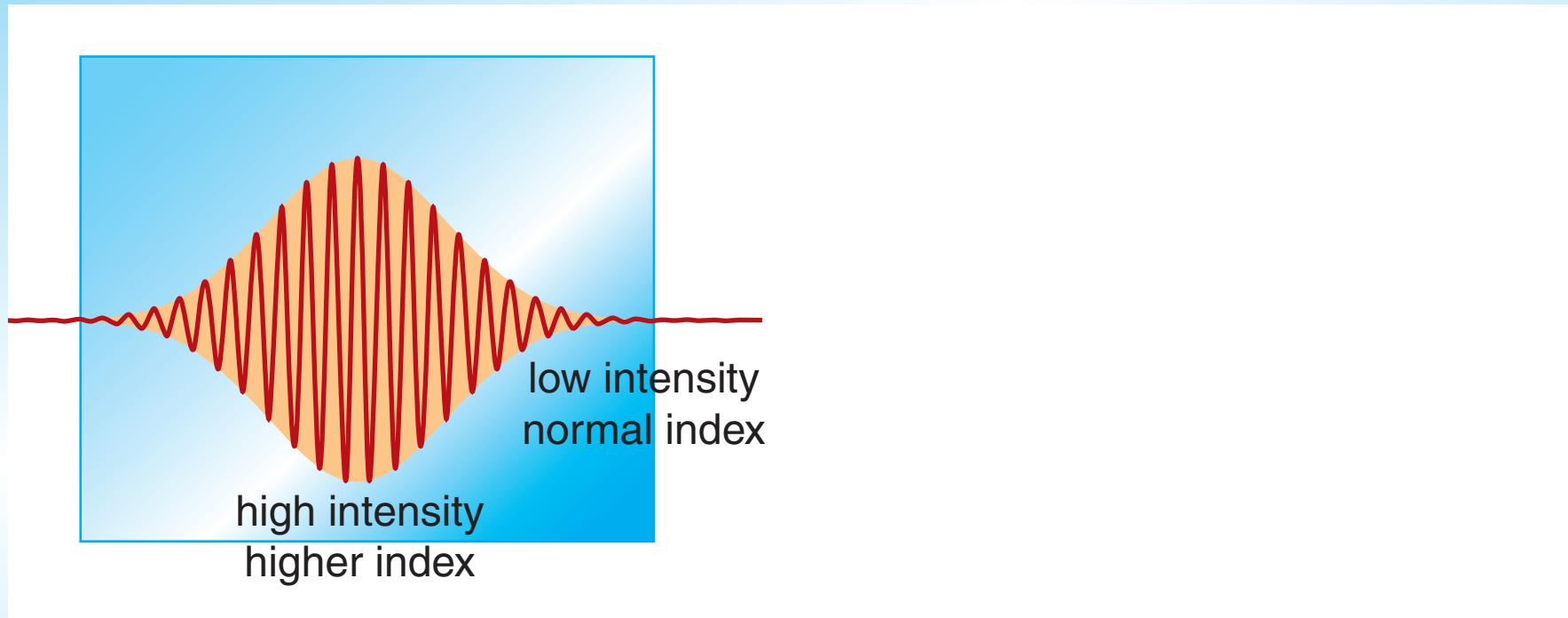
$$n = n_o + n_2 I$$



Nonlinear optics

Intensity dependent index of refraction:

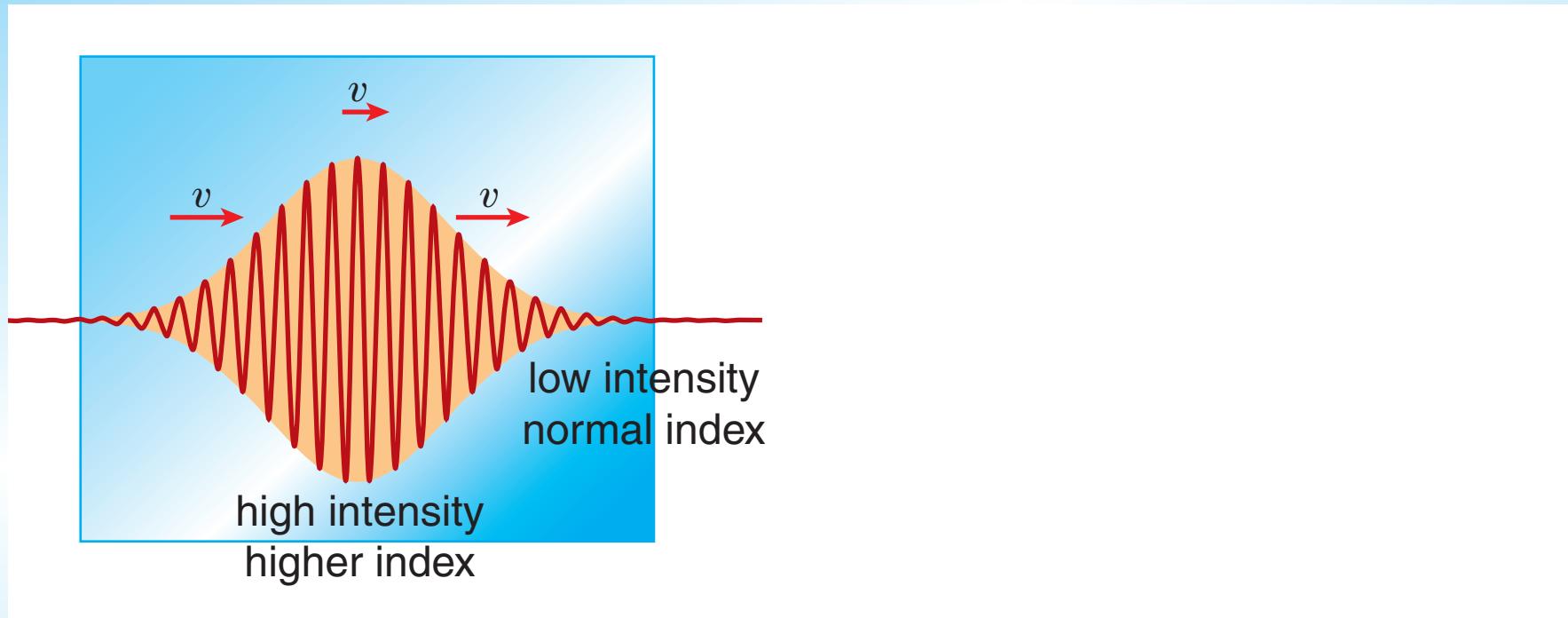
$$n = n_o + n_2 I$$



Nonlinear optics

Intensity dependent index of refraction:

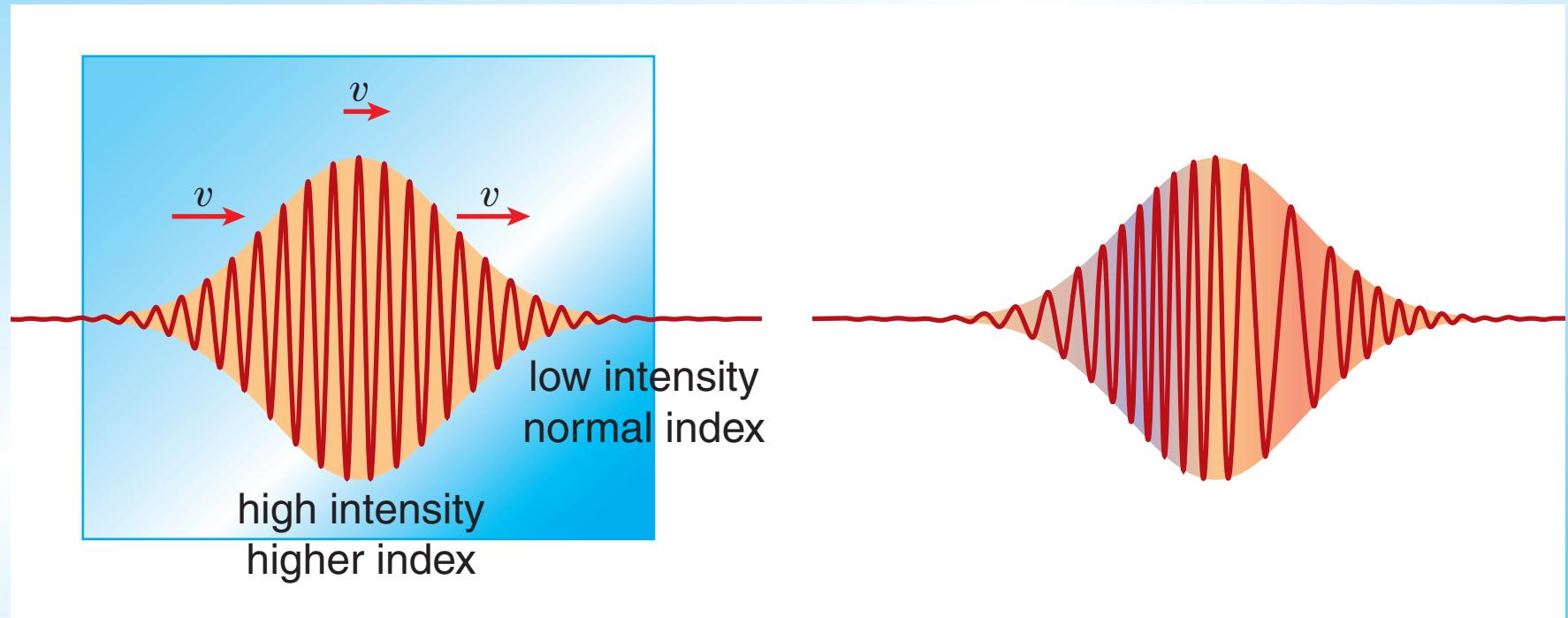
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Nonlinear optics

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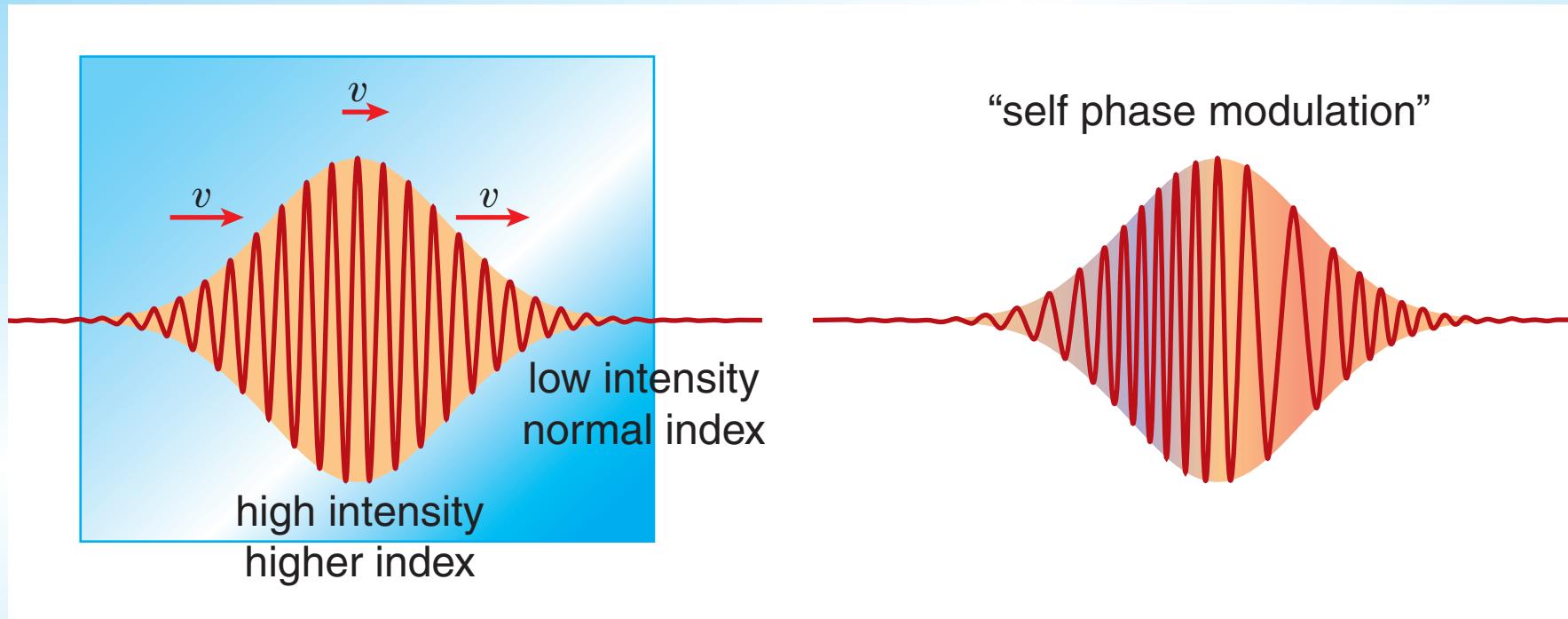
$$n = n_o + n_2 I$$



Nonlinear optics

Intensity dependent index of refraction:

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Nonlinear optics

Phase:

$$\frac{\phi}{2\pi} = \frac{nL}{\lambda}$$

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Nonlinear optics

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Frequency change:

$$\Delta\omega = -\frac{d\phi}{dt} = \frac{-2\pi}{\lambda} L n_2 \frac{dI}{dt}$$

Nonlinear optics

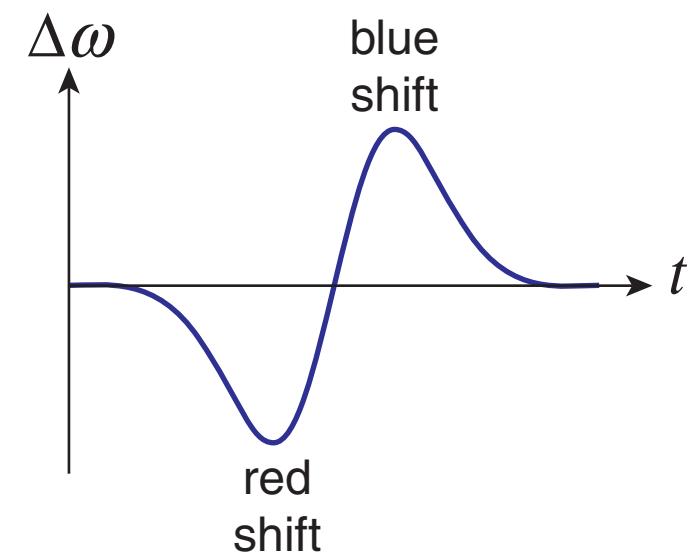
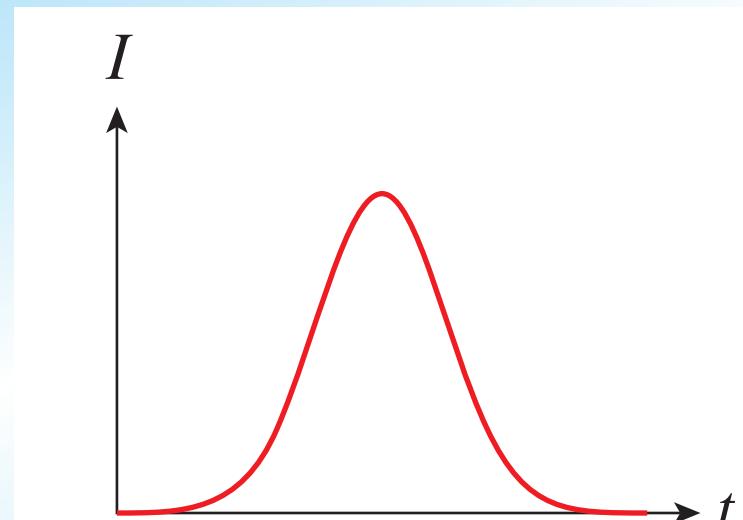
Phase:

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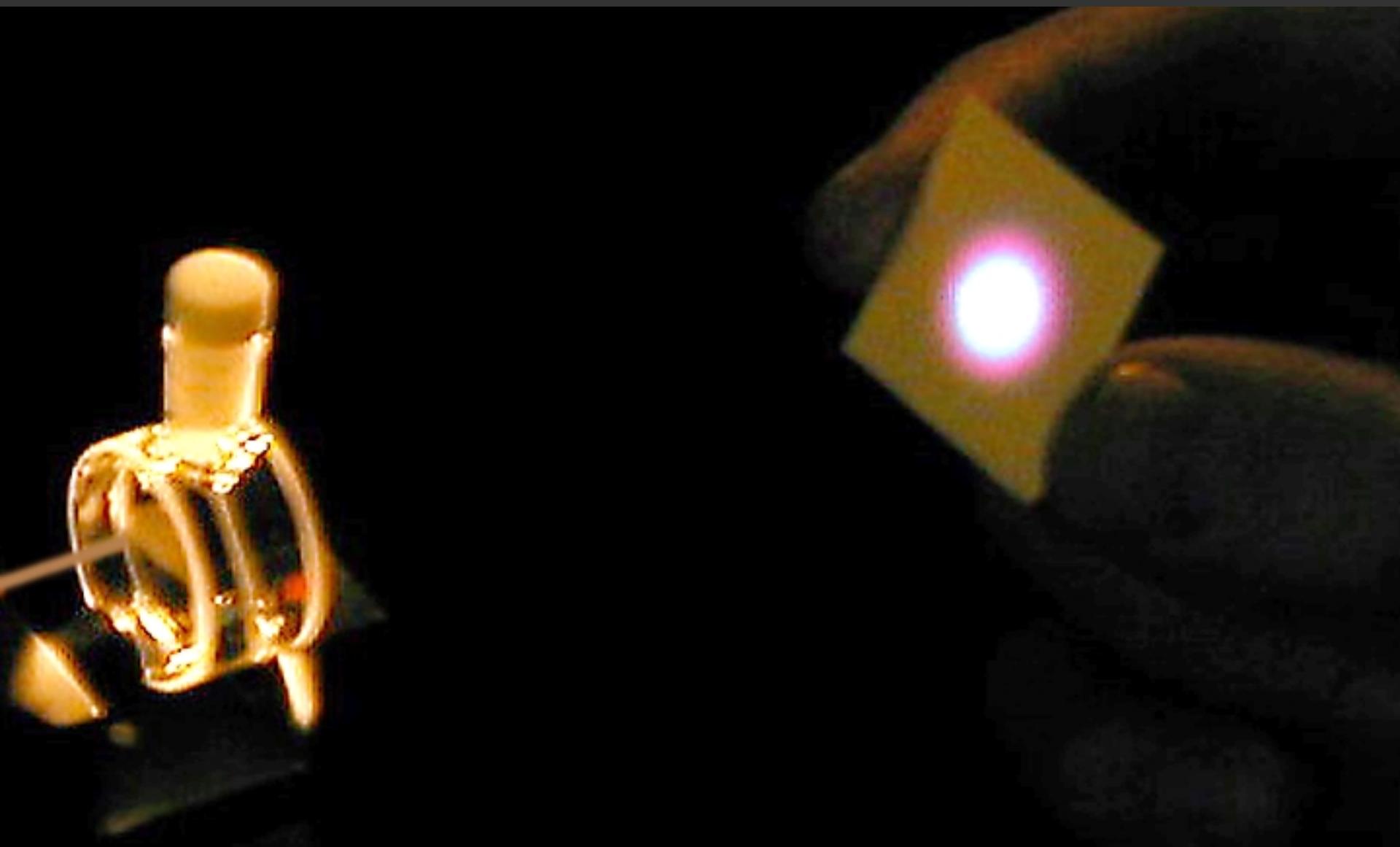
$$\phi = \frac{2\pi}{\lambda} L(n_o + n_2 I)$$

Frequency change:

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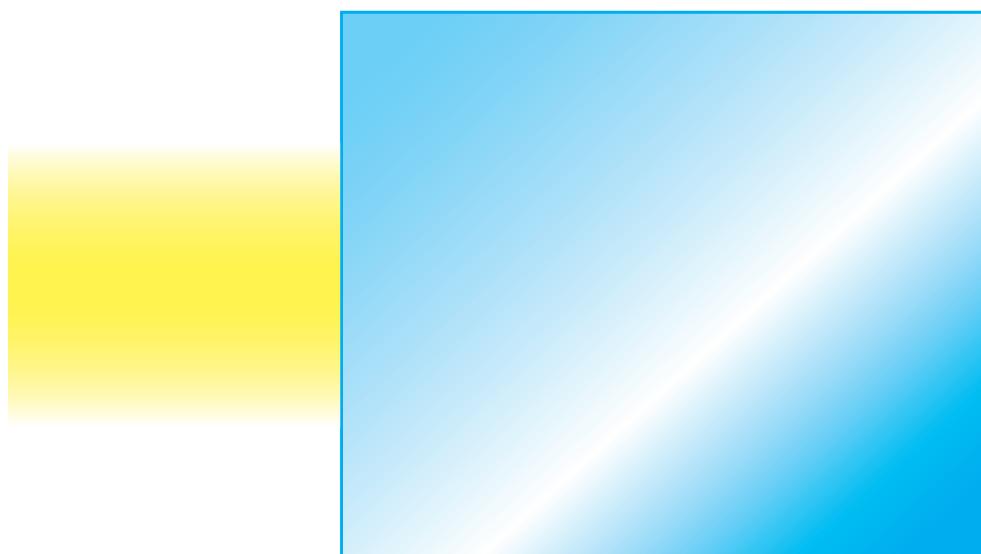


Nonlinear optics



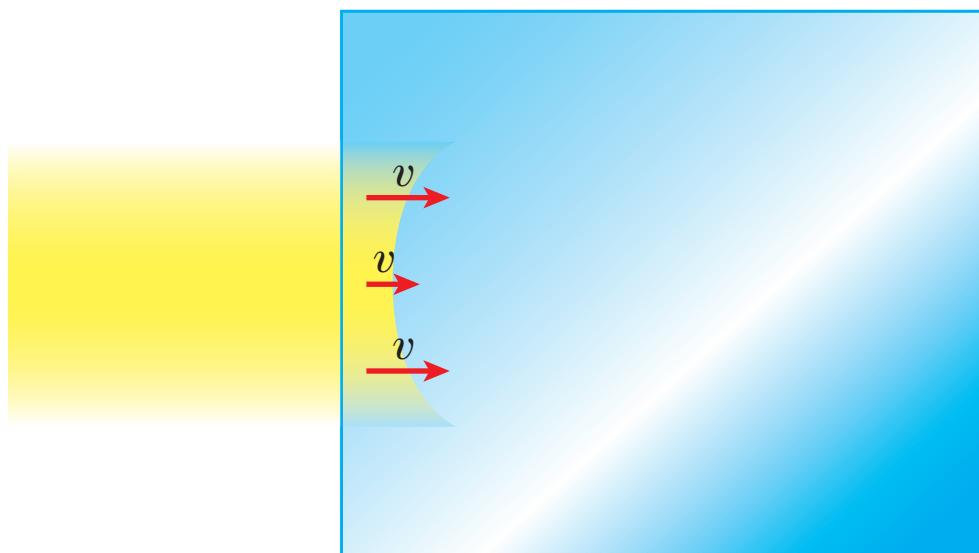
Nonlinear optics

Spatial intensity profile...



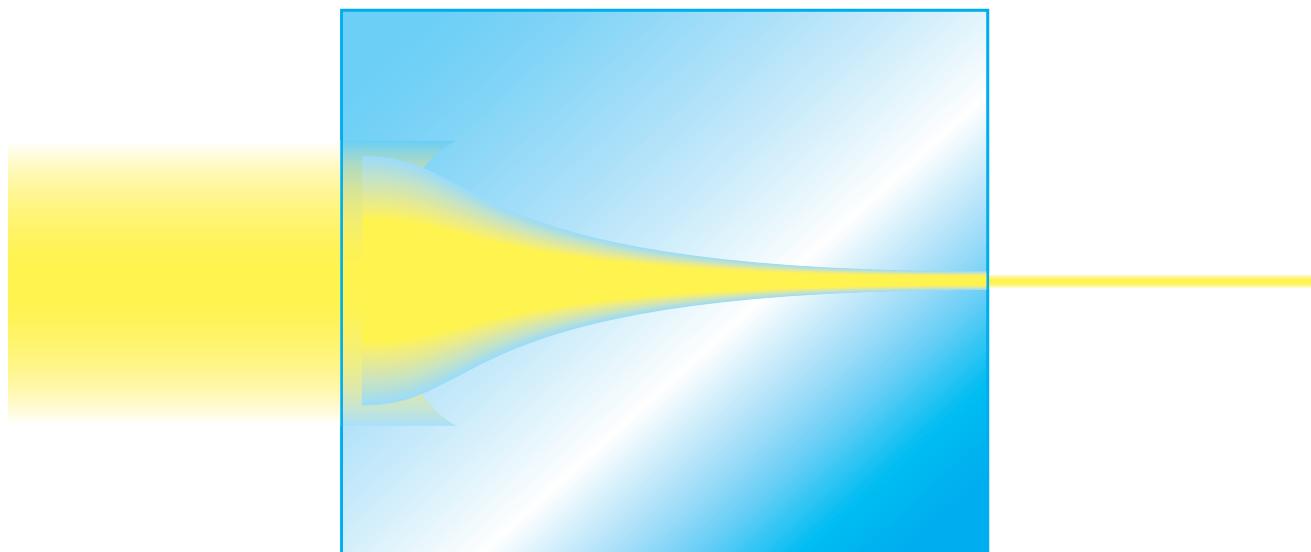
Nonlinear optics

Spatial intensity profile...

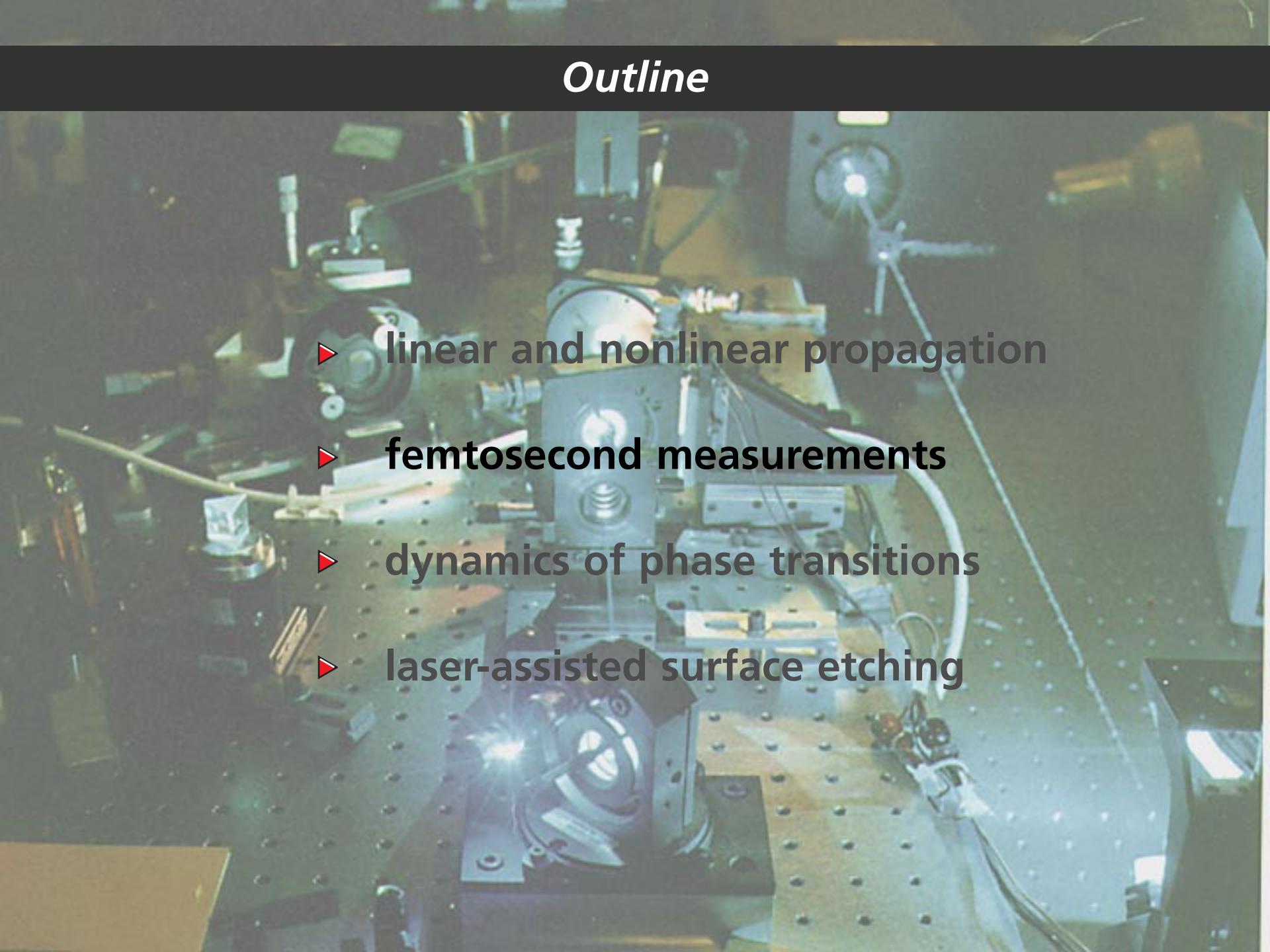


Nonlinear optics

...causes self-focusing

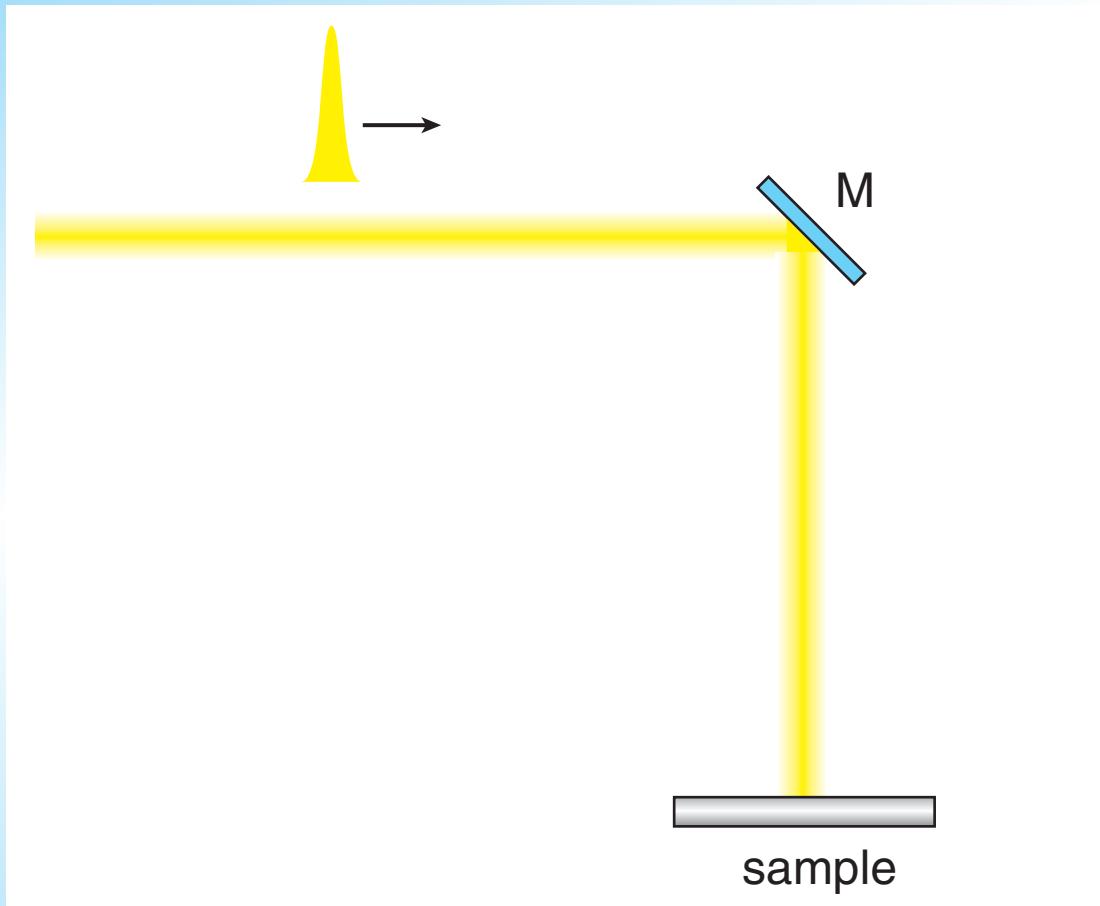


Outline

- 
- ▶ linear and nonlinear propagation
 - ▶ femtosecond measurements
 - ▶ dynamics of phase transitions
 - ▶ laser-assisted surface etching

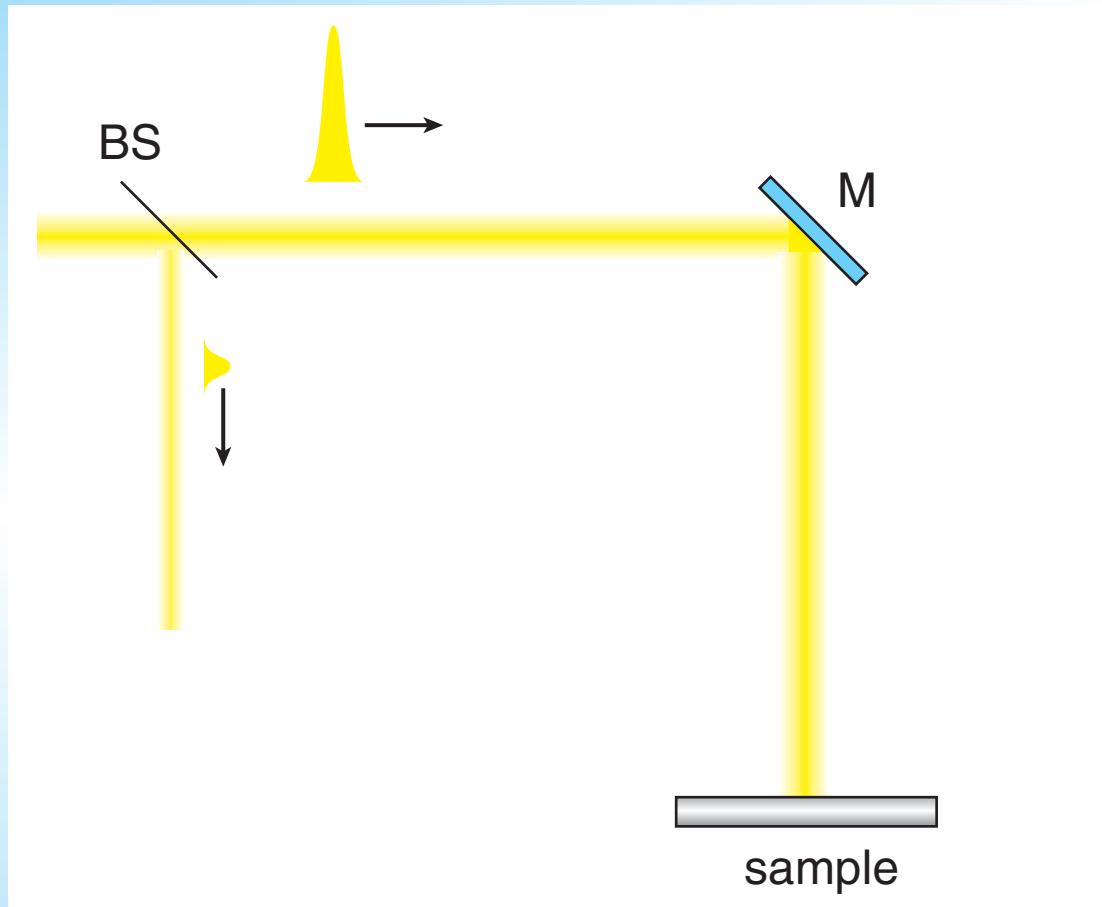
Introduction

How to measure on the femtosecond time scale?



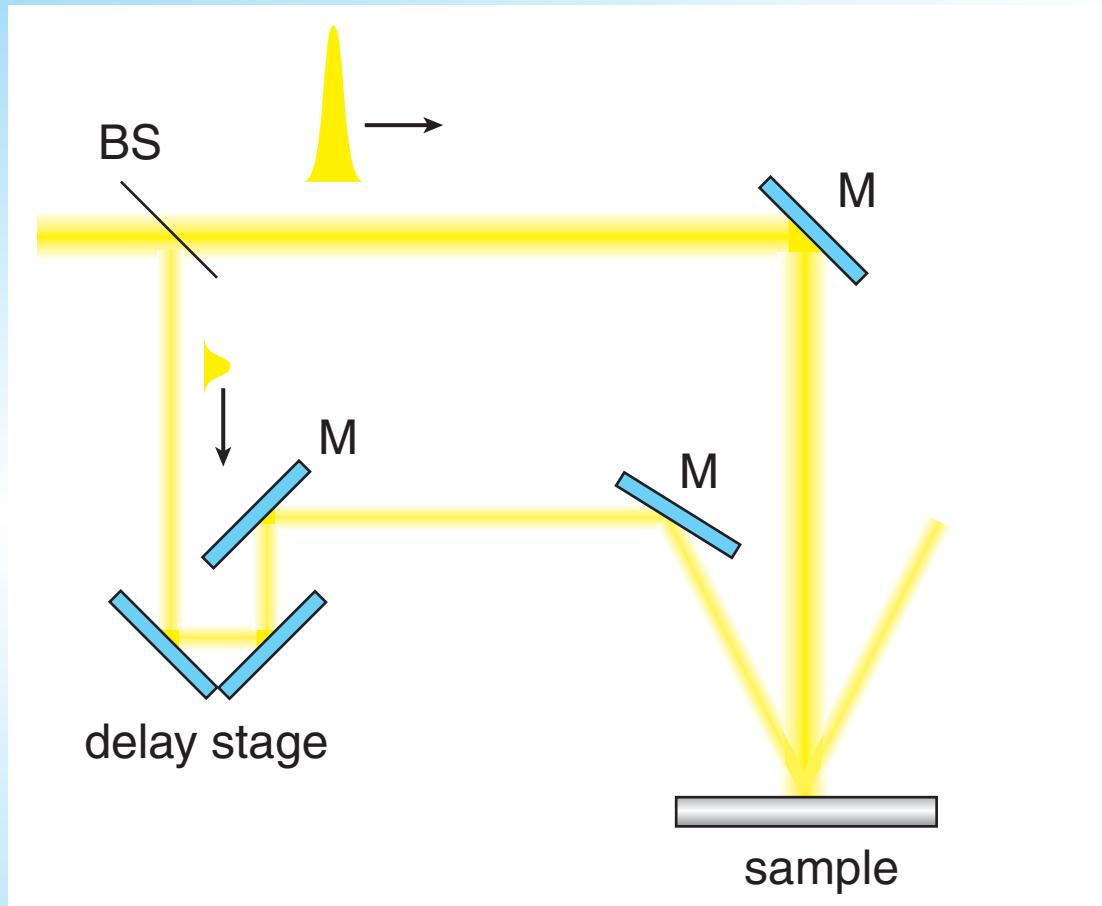
Introduction

Use pump-probe technique



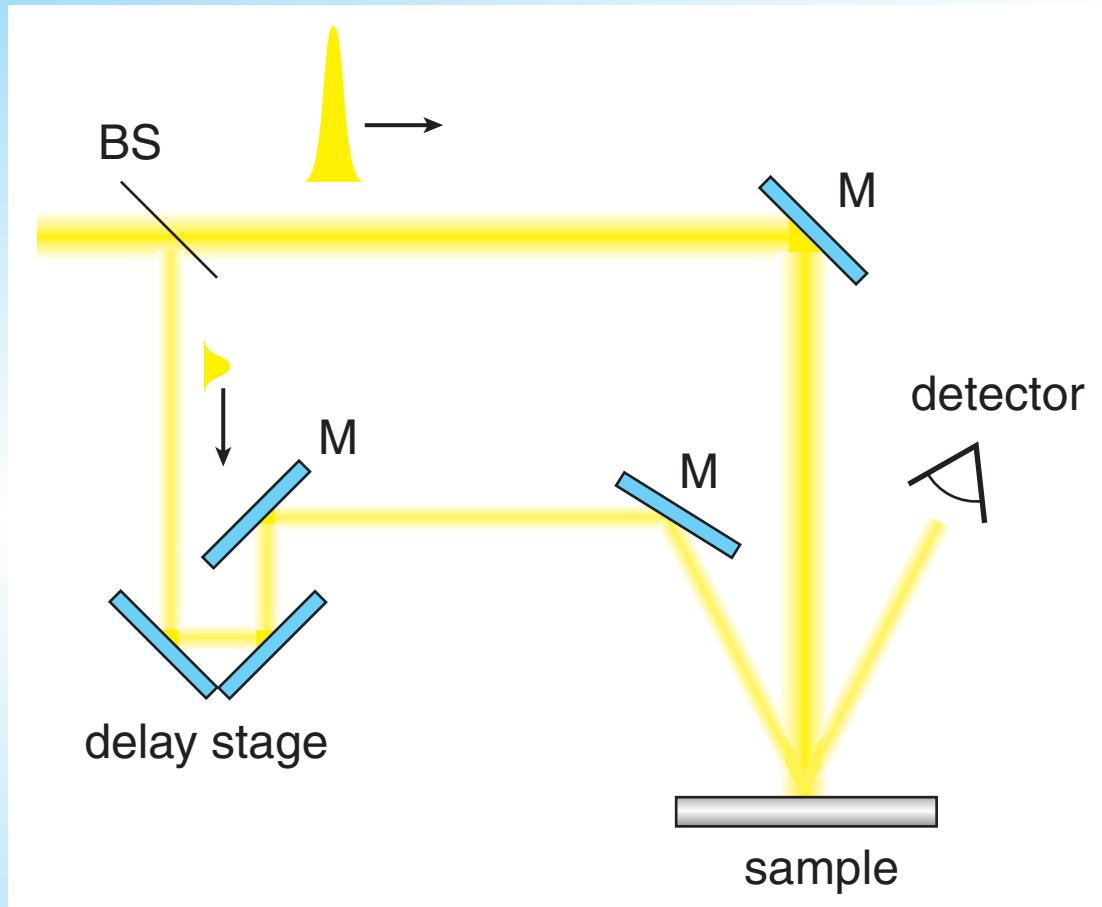
Introduction

Use pump-probe technique



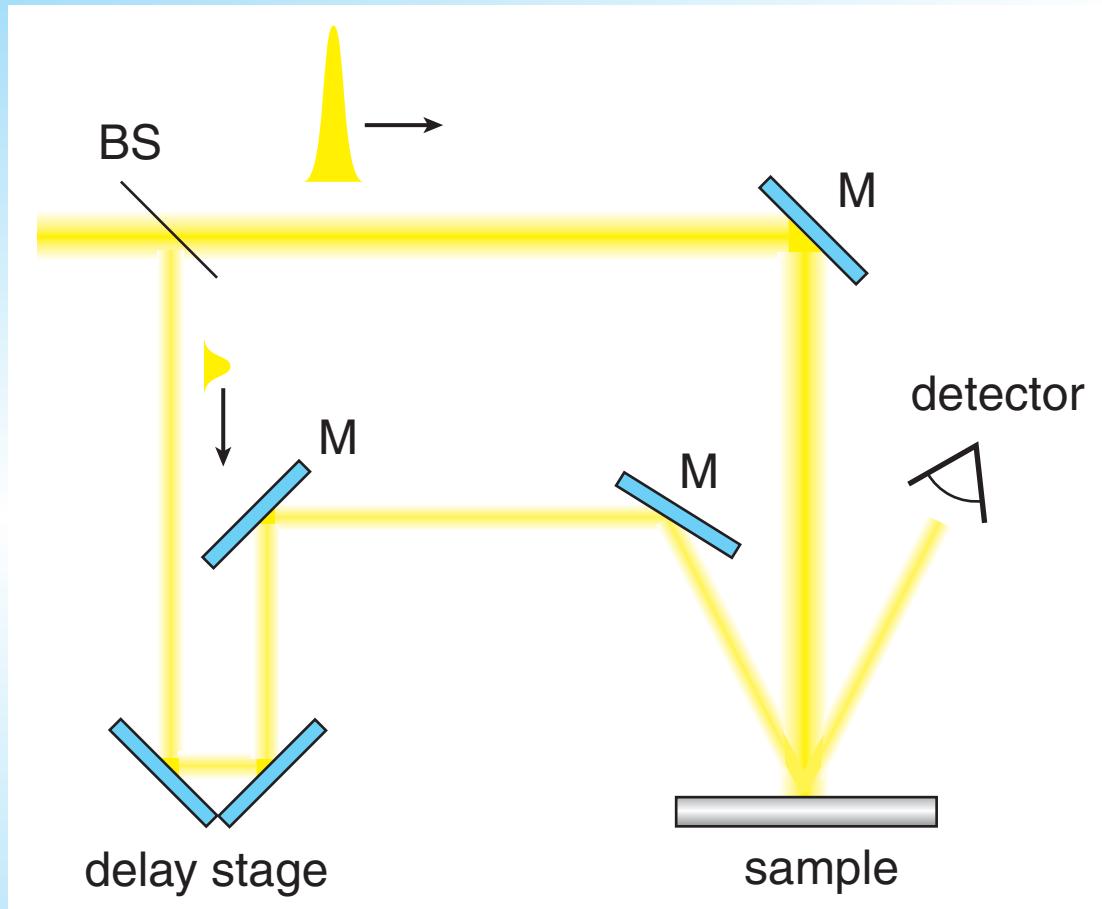
Introduction

Use pump-probe technique



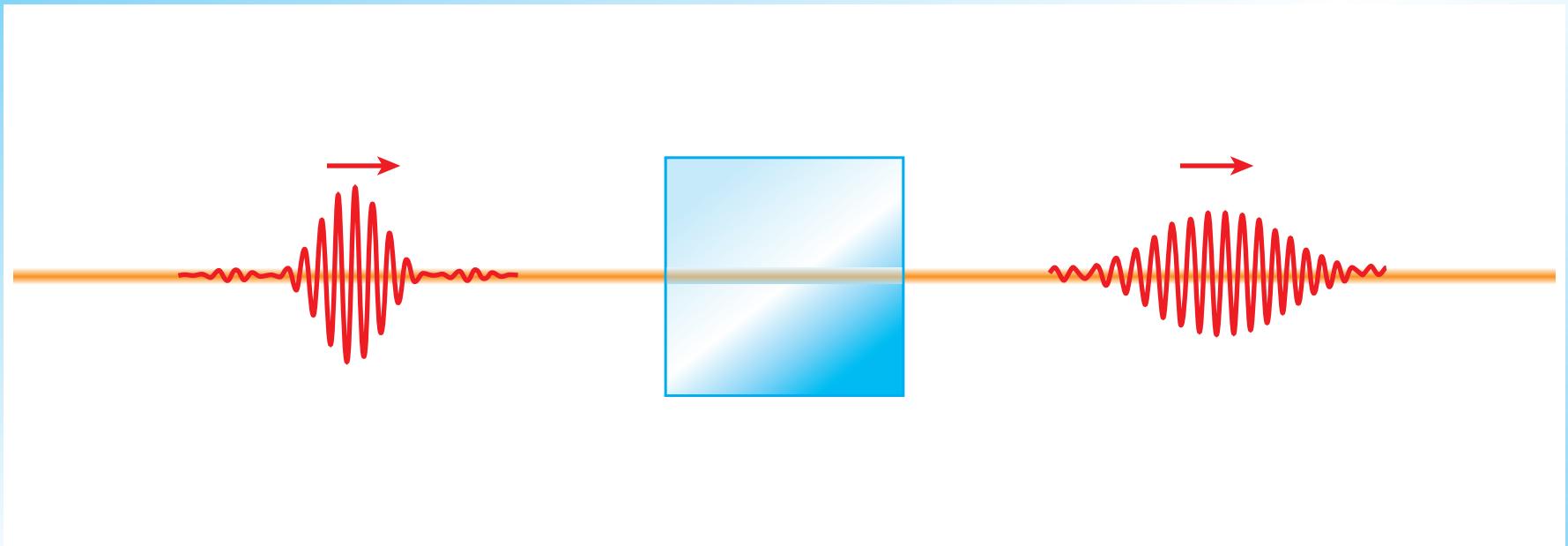
Introduction

Vary delay to get time resolution



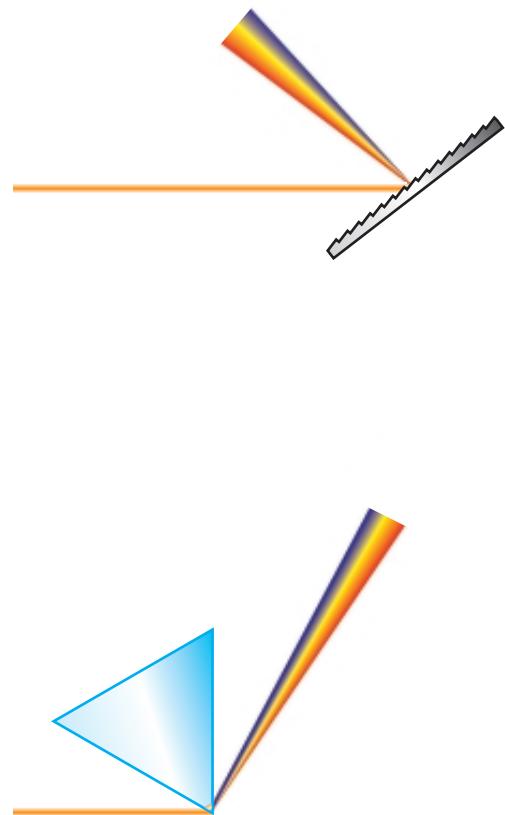
Dispersion compensation

Dispersion stretches the pulse

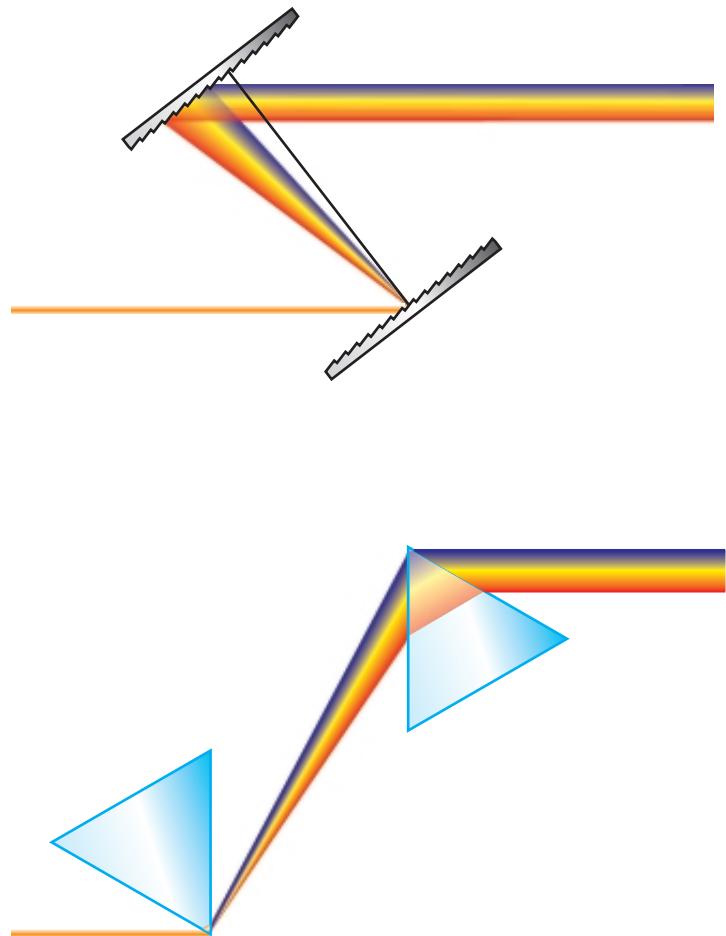


Compensate by rearranging spectral components!

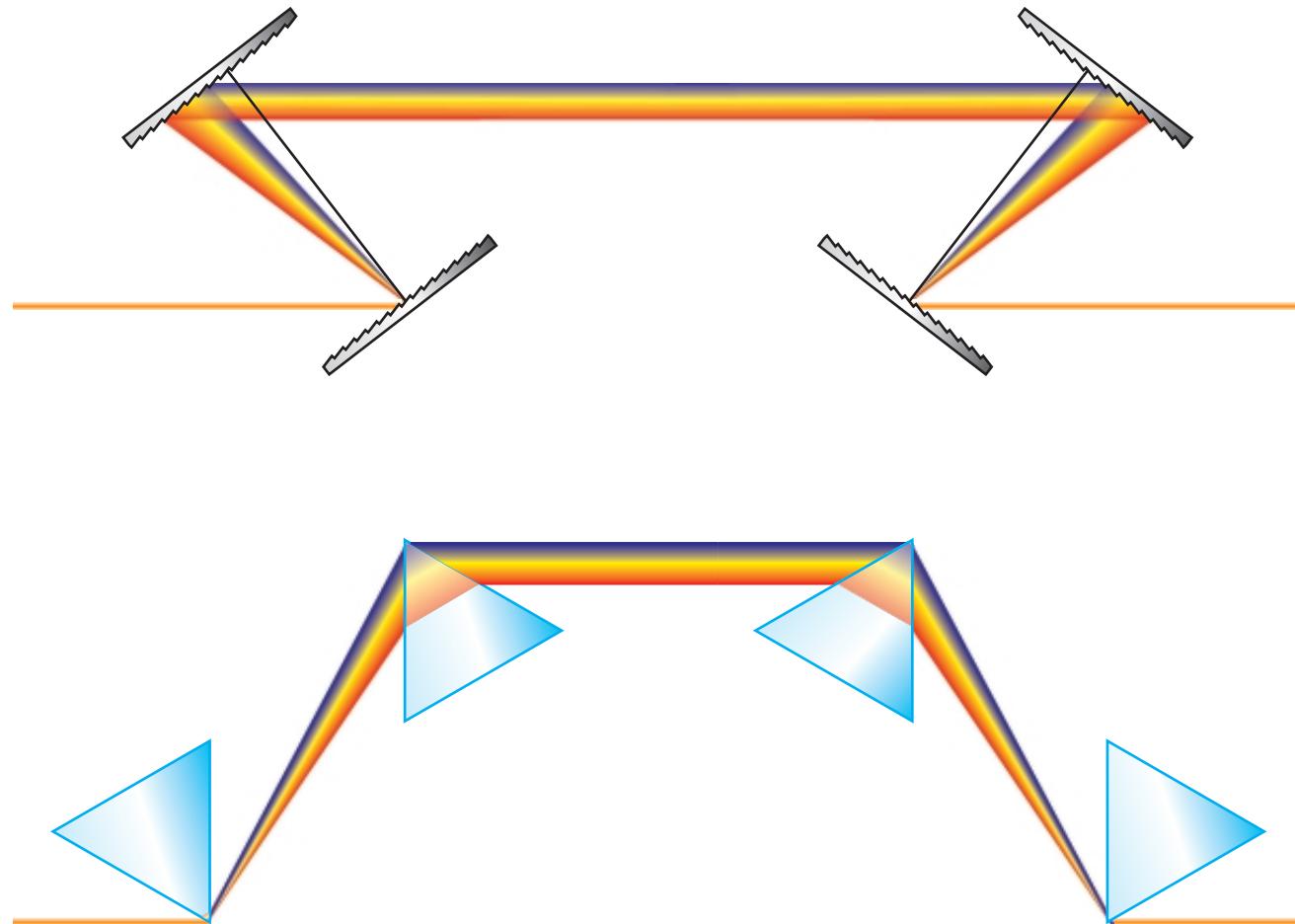
Dispersion compensation



Dispersion compensation



Dispersion compensation

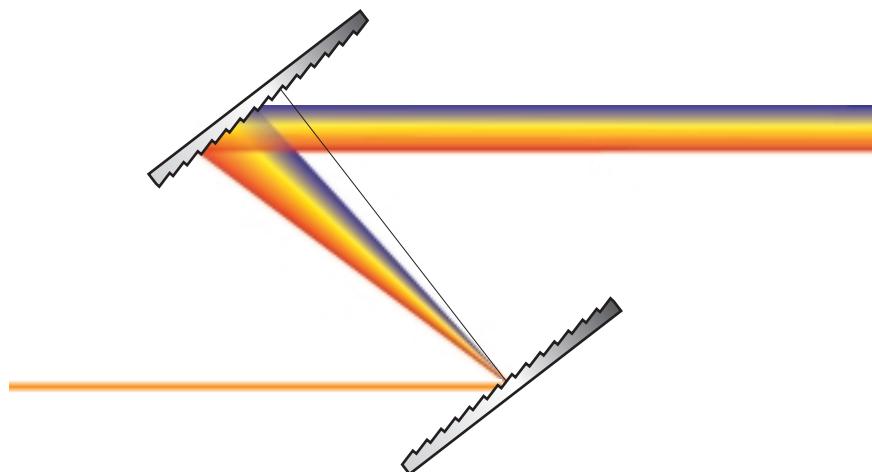


Dispersion compensation

How do these arrangements work?

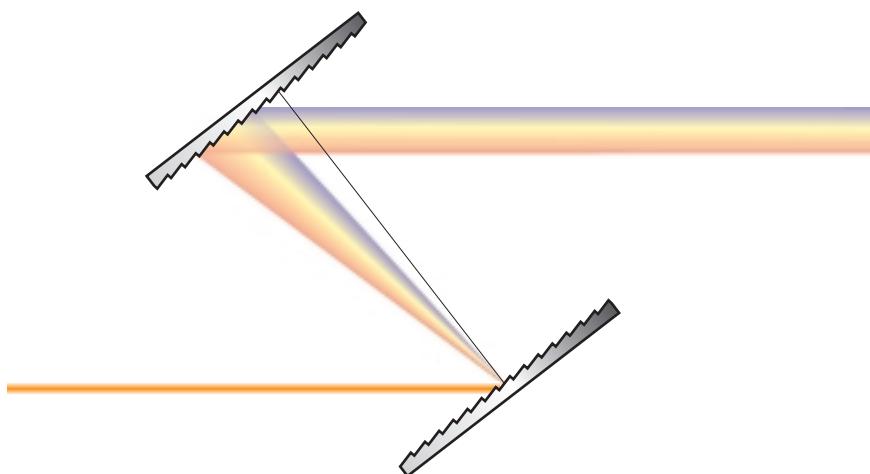
Dispersion compensation

Does path length difference compensate?



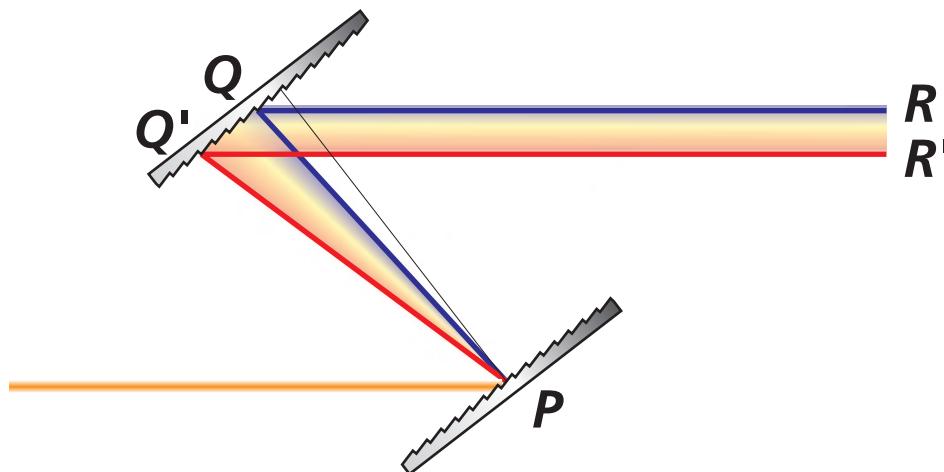
Dispersion compensation

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Dispersion compensation

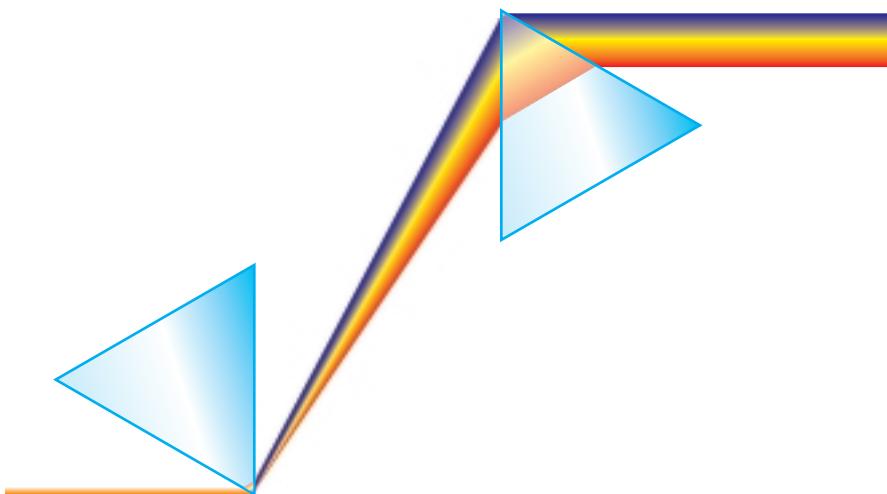
Does path length difference compensate?



Grating gives low frequency longer path length...

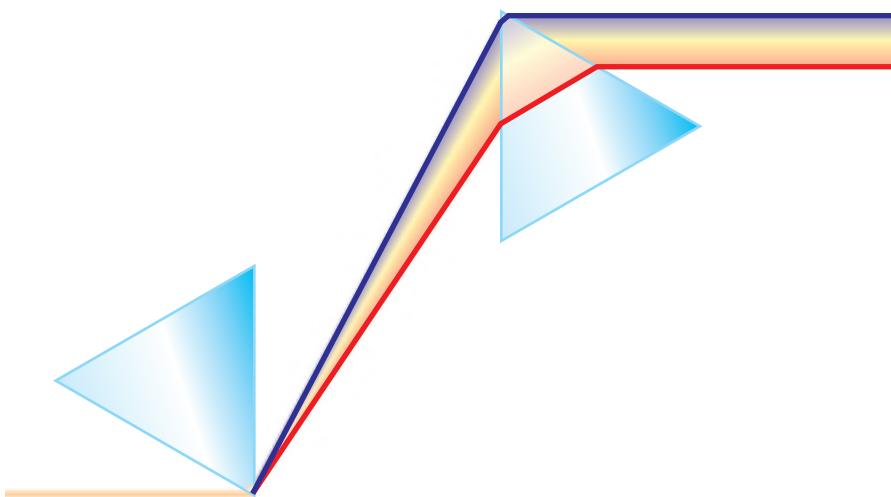
Dispersion compensation

Does path length difference compensate?



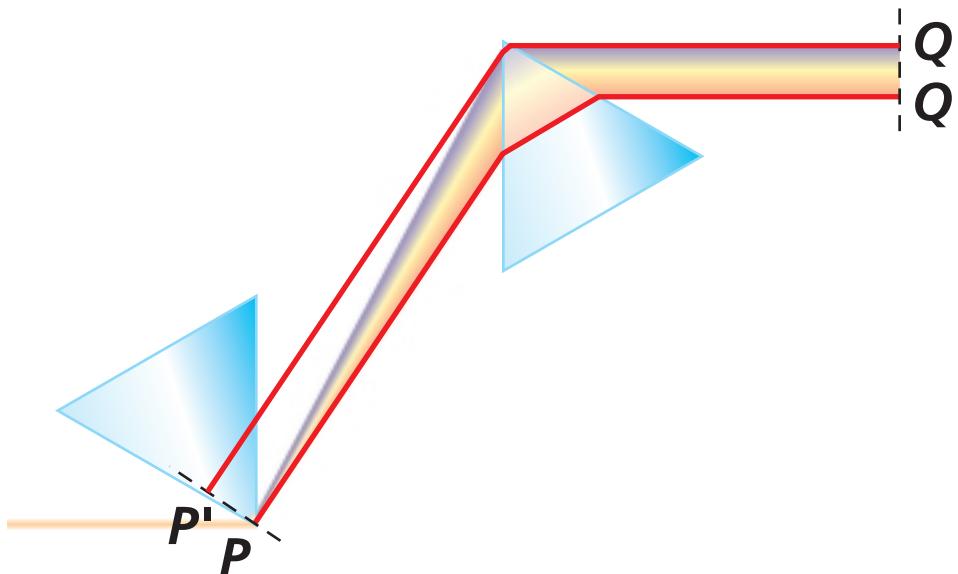
Dispersion compensation

Does path length difference compensate?



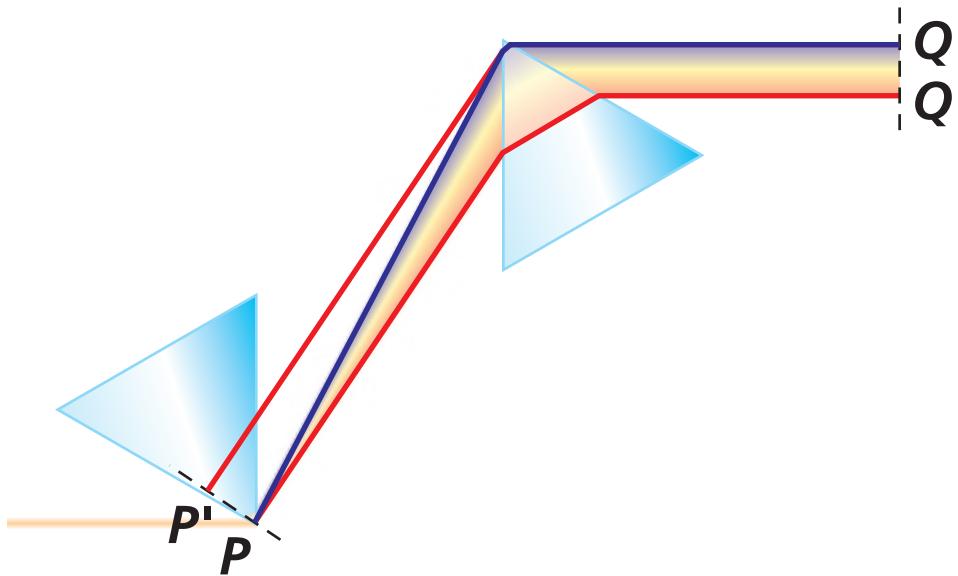
Dispersion compensation

Does path length difference compensate?



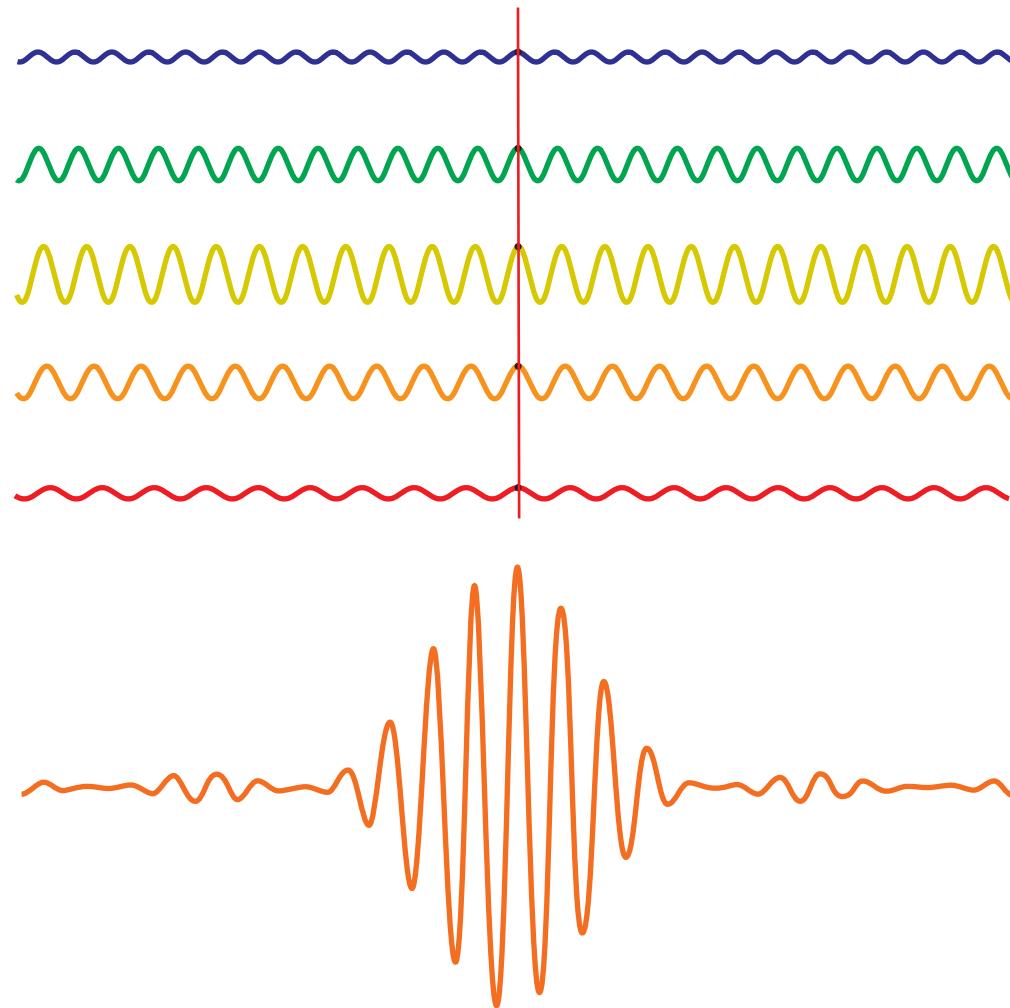
Dispersion compensation

Does path length difference compensate?

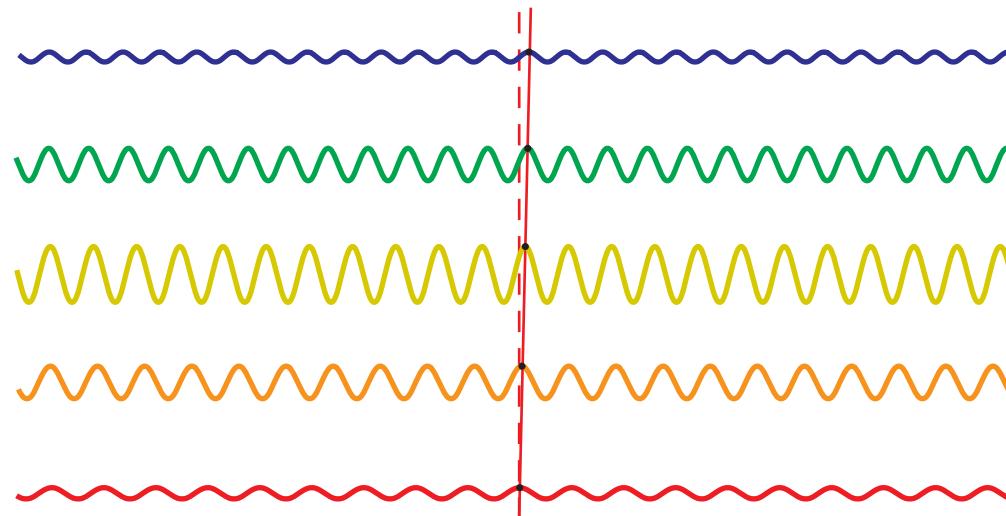


...so prism gives low frequency *shorter* path length...

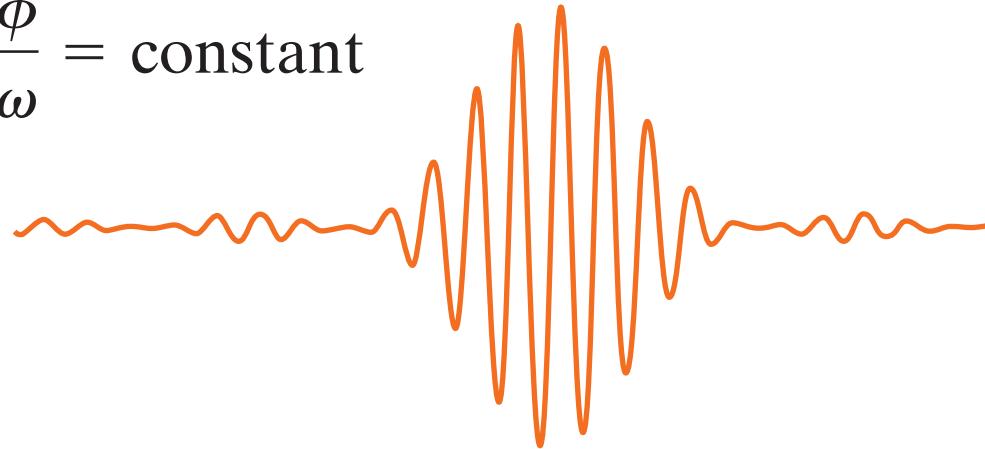
Dispersion compensation



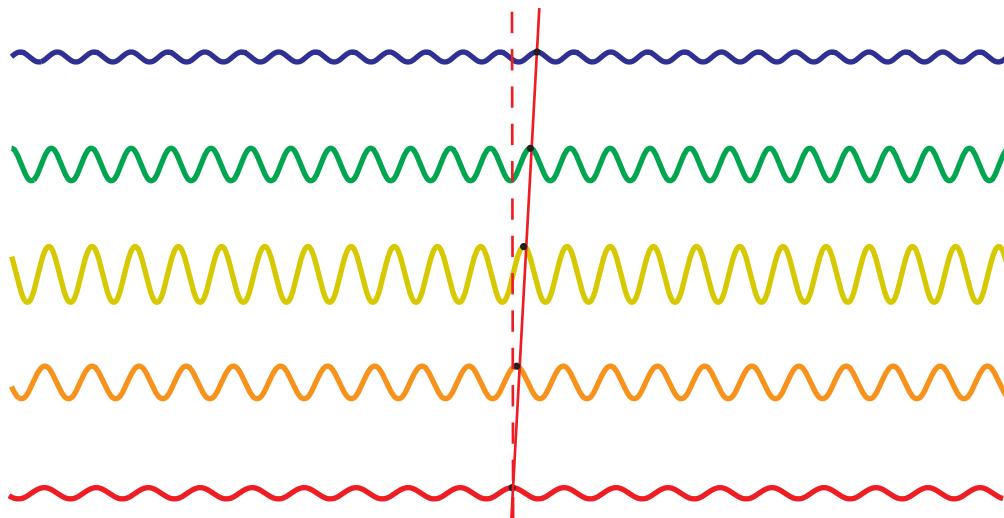
Dispersion compensation



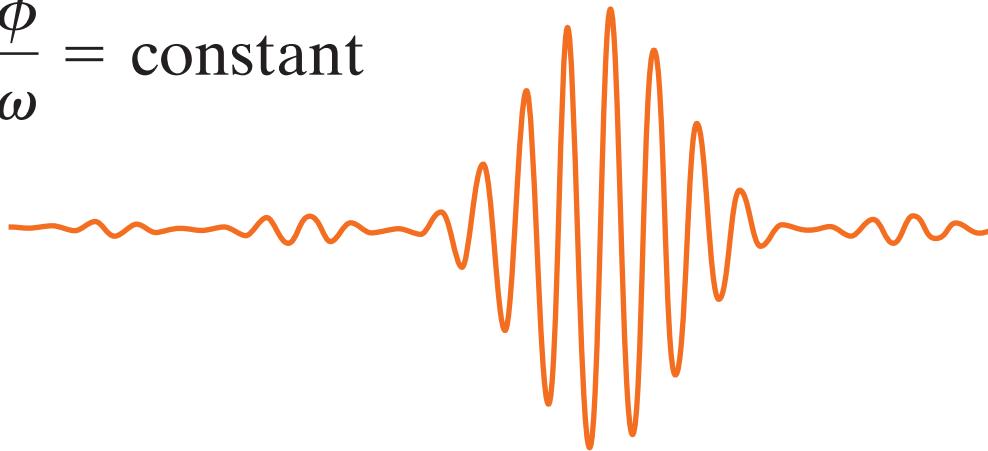
$$\frac{d\phi}{d\omega} = \text{constant}$$



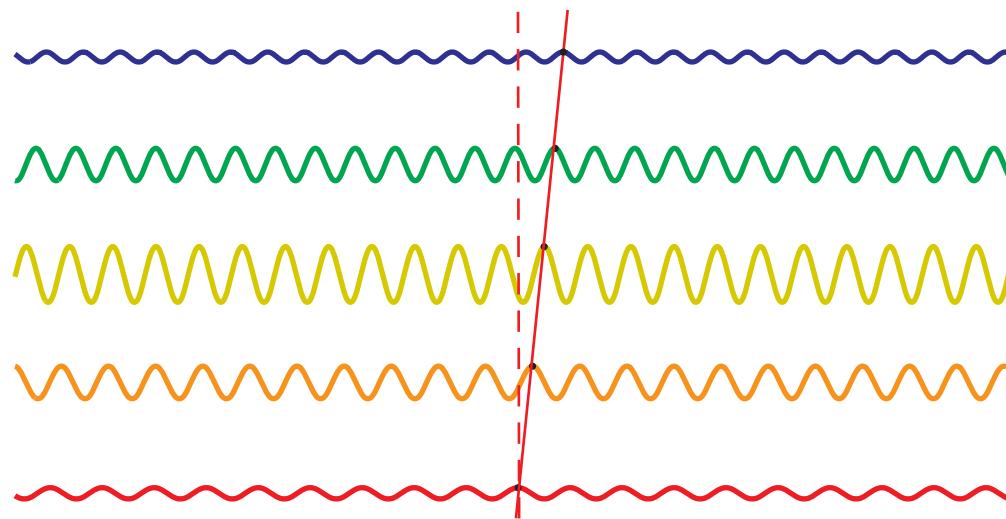
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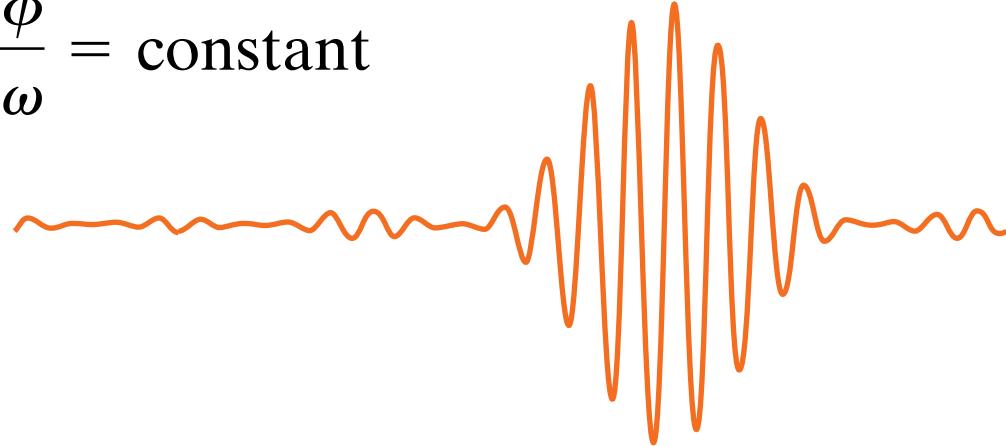
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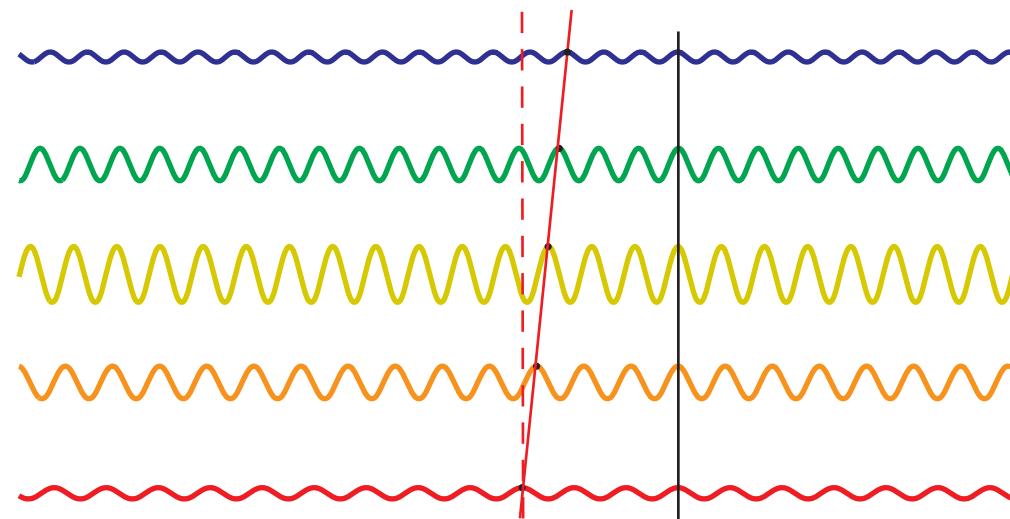
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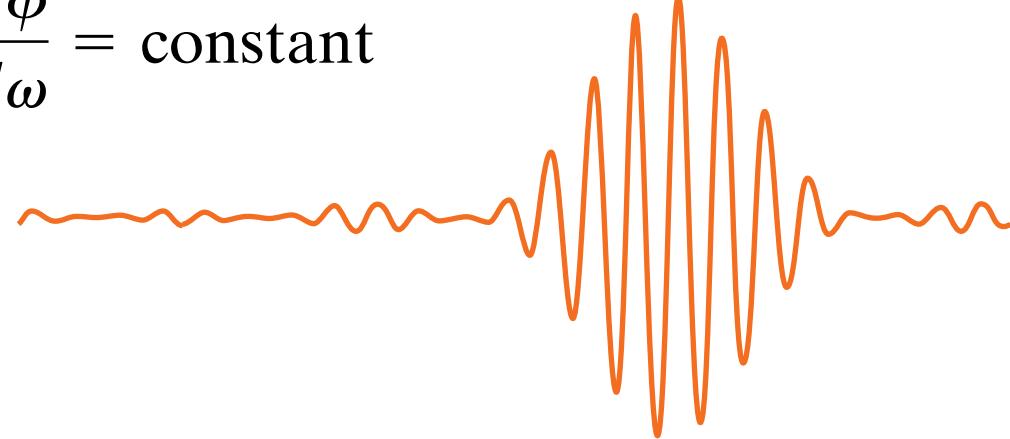
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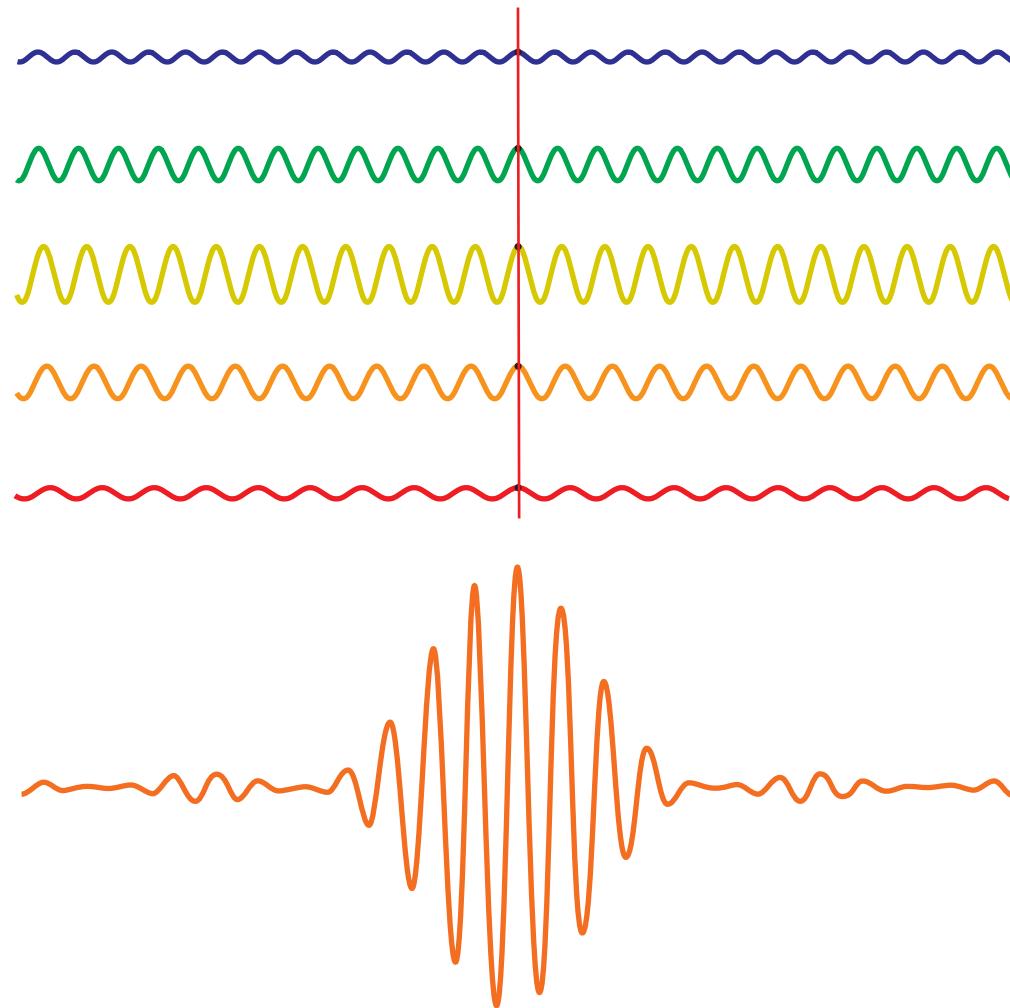
Dispersion compensation



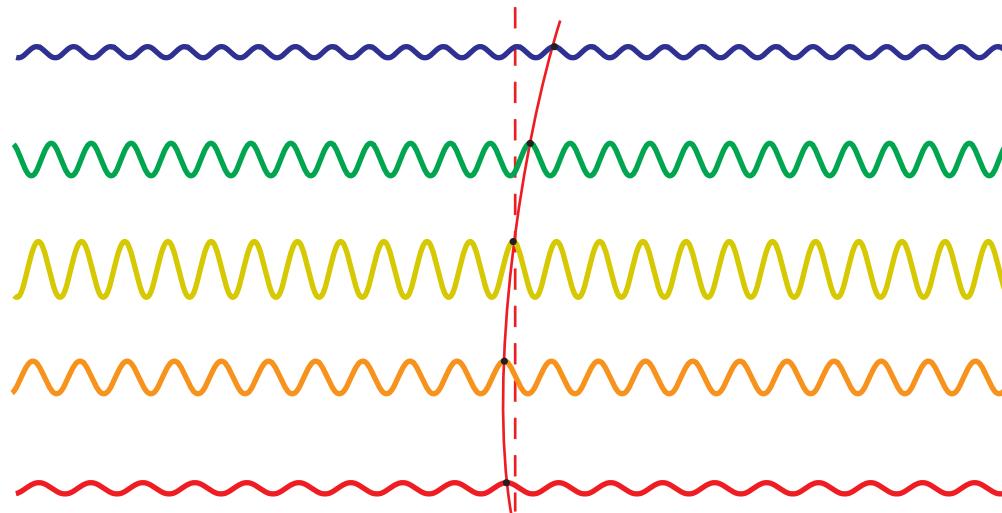
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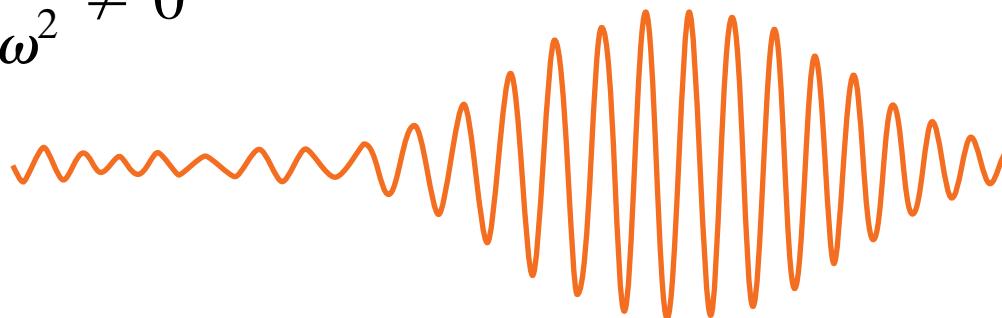
Dispersion compensation



Dispersion compensation



$$\frac{d^2\phi}{d\omega^2} \neq 0$$

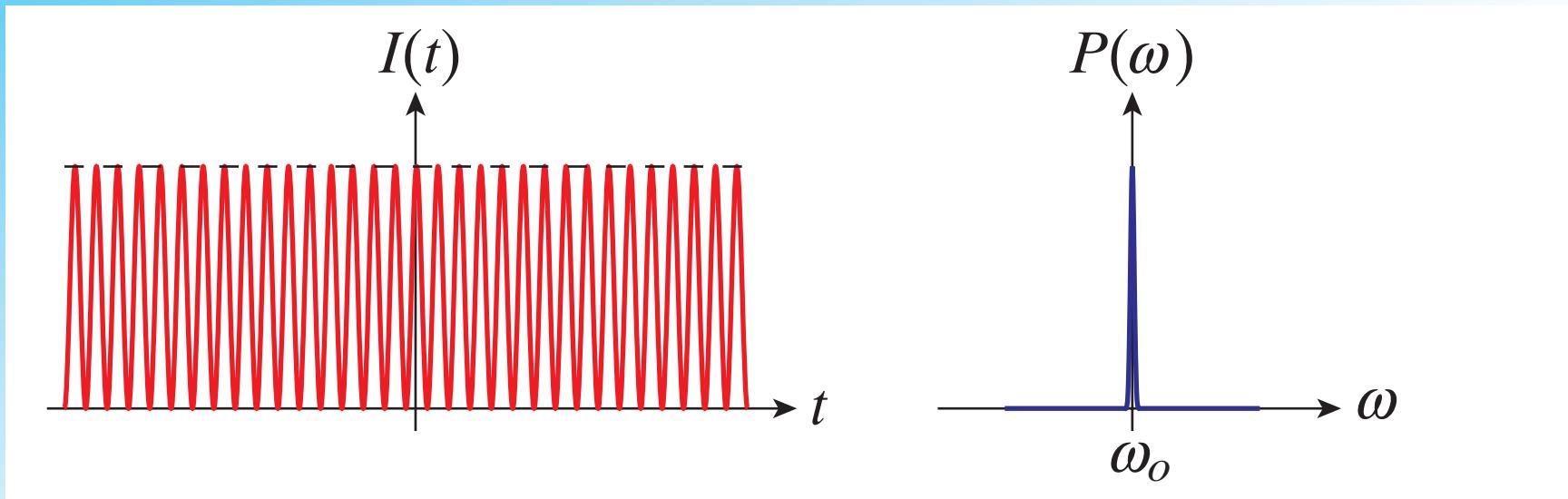


Dispersion compensation

So not path length but $\frac{d^2\phi}{d\omega^2}$ matters!

	$\frac{dl_{eff}}{d\omega}$	$\frac{d^2\phi}{d\omega^2}$
dispersion	+	+
gratings	-	-
prisms	+	-

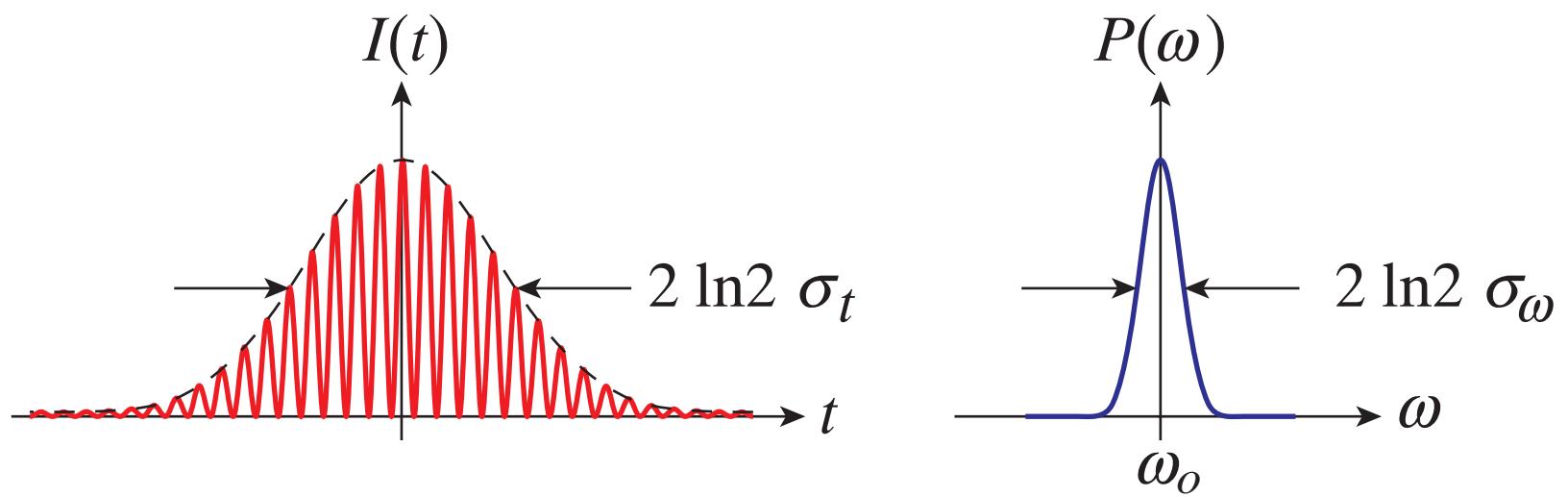
Representation of pulses



Spectrum of sinusoidal intensity is a delta function

$$I(t) = \cos^2(\omega_o t) \quad \Rightarrow \quad P(\omega) = \delta(\omega - \omega_o)$$

Representation of pulses



Modulate amplitude

$$I(t) = \exp\left[-\frac{t^2}{\sigma_t^2}\right] \cos^2(\omega_o t)$$

Representation of pulses

Fourier relations:

$$E(t) = \exp\left[-\frac{t^2}{2\sigma_t^2} - i\omega_o t\right]$$

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$$= \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{\sigma_t^2(\omega - \omega_o)^2}{2}\right] \int_{-\infty}^{\infty} \exp\left[-\left[\frac{t}{\sqrt{2}\sigma_t} - i\frac{(\omega - \omega_o)\sigma_t}{\sqrt{2}}\right]^2\right] dt =$$

Representation of pulses

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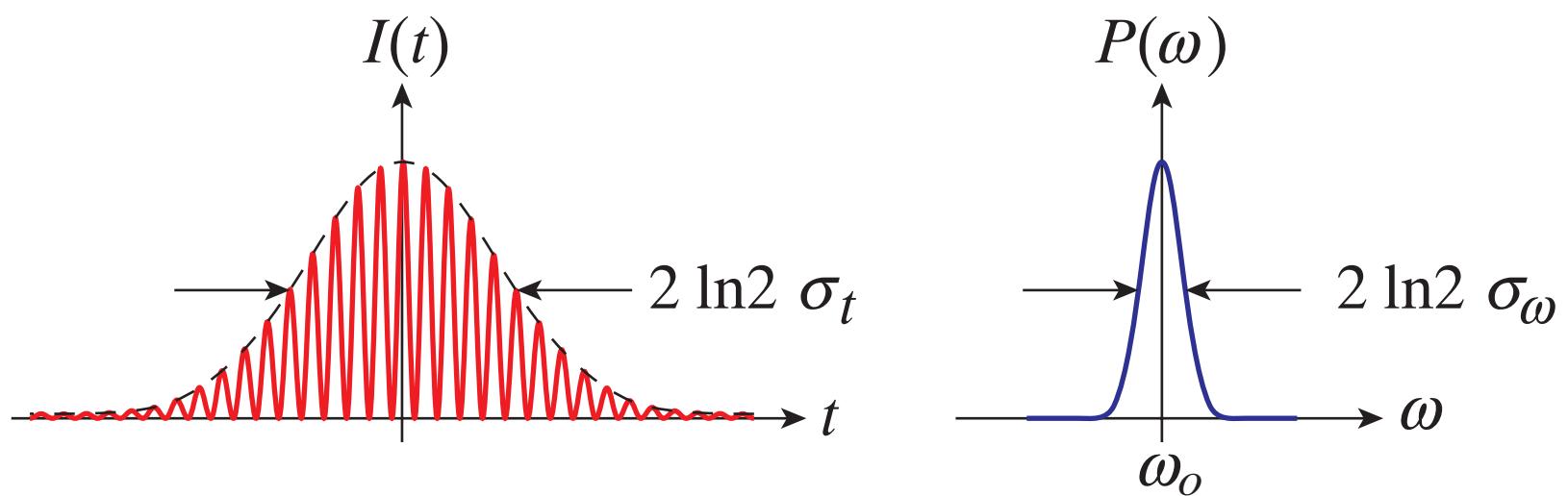
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$$= \sigma_t \exp\left[-\frac{\sigma_t^2(\omega - \omega_o)^2}{2}\right] \equiv \sigma_t \exp\left[-\frac{(\omega - \omega_o)^2}{2\sigma_\omega^2}\right]$$

Representation of pulses

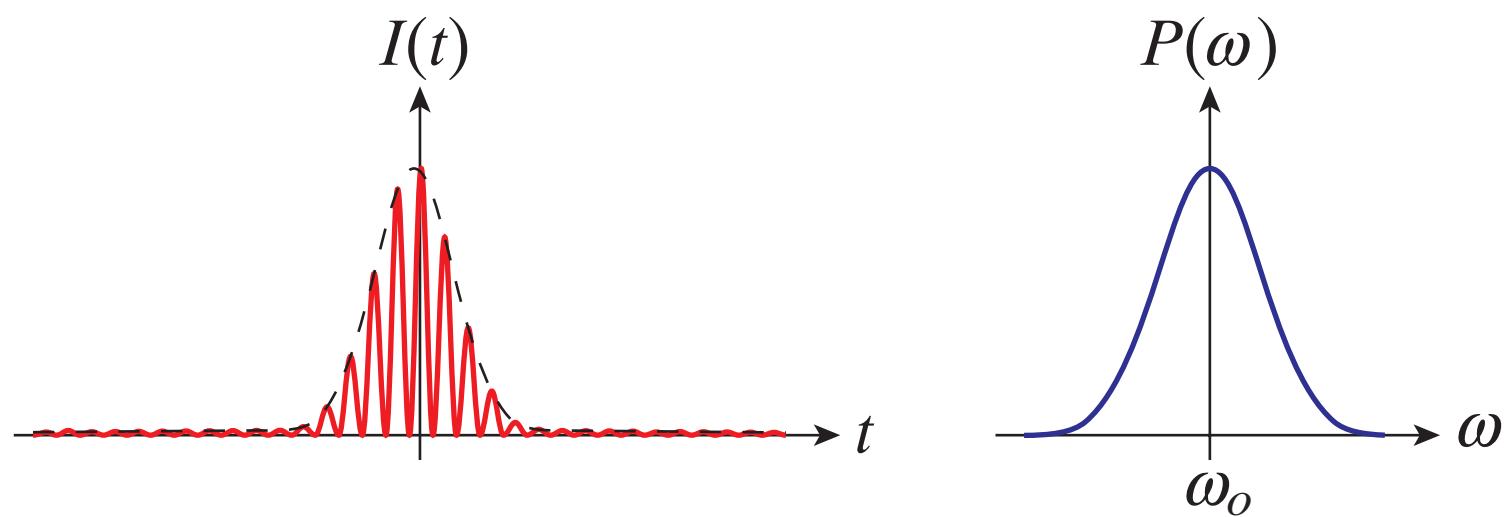


Pulse duration-bandwidth product: $\sigma_t \sigma_\omega = 1$

$$I(t) = [\operatorname{Re} E(t)]^2 \propto \exp\left[-\frac{t^2}{\sigma_t^2}\right] \cos^2(\omega_o t)$$

$$P(\omega) = E(\omega)E^*(\omega) = \exp\left[-\frac{(\omega - \omega_o)^2}{\sigma_\omega^2}\right]$$

Representation of pulses



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Joint time-frequency representation

Wigner representation:

$$\begin{aligned} W(t, \omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} E\left(\omega + \frac{\omega'}{2}\right) E^*\left(\omega - \frac{\omega'}{2}\right) e^{-i\omega't} d\omega' = \\ &= \int_{-\infty}^{\infty} E\left(t + \frac{t'}{2}\right) E^*\left(t - \frac{t'}{2}\right) e^{-i\omega t'} dt' \end{aligned}$$

Joint time-frequency representation

Wigner representation:

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$$= \int_{-\infty}^{\infty} E\left(t + \frac{t'}{2}\right) E^*\left(t - \frac{t'}{2}\right) e^{-i\omega t'} dt'$$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} W(t, \omega) d\omega = |E(t)|^2 = I(t)$$

Joint time-frequency representation

Wigner representation:

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$$= \int_{-\infty}^{\infty} E\left(t + \frac{t'}{2}\right) E^*\left(t - \frac{t'}{2}\right) e^{-i\omega t'} dt'$$

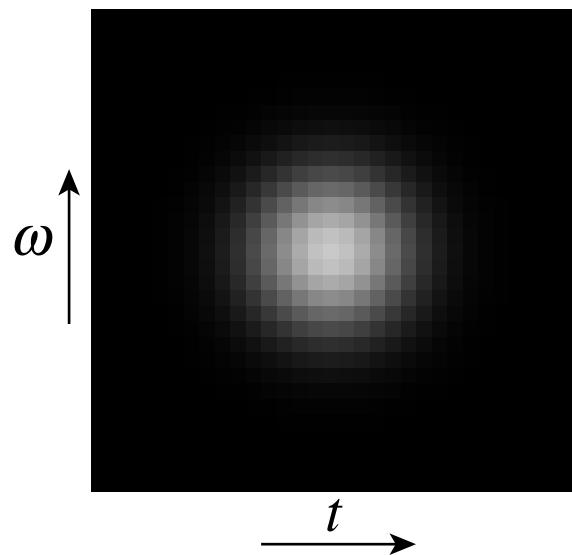
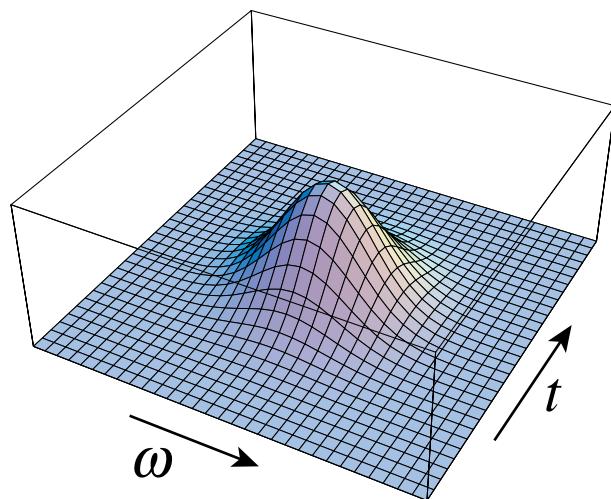
$$\frac{1}{2\pi} \int_{-\infty}^{\infty} W(t, \omega) d\omega = |E(t)|^2 = I(t)$$

$$\int_{-\infty}^{\infty} W(t, \omega) dt = |E(\omega)|^2 = I(\omega)$$

Joint time-frequency representation

Energy:

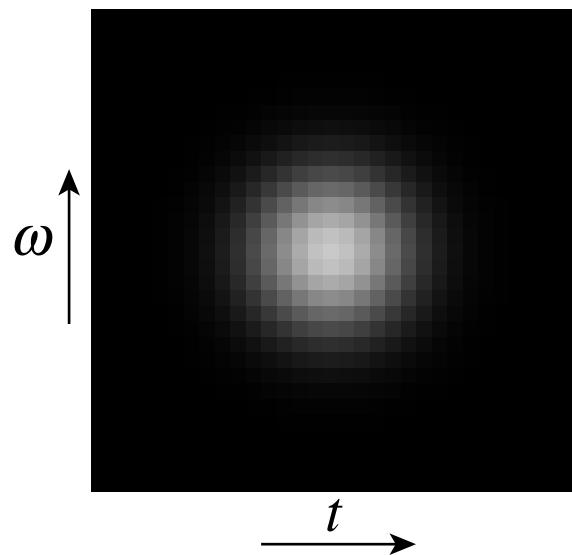
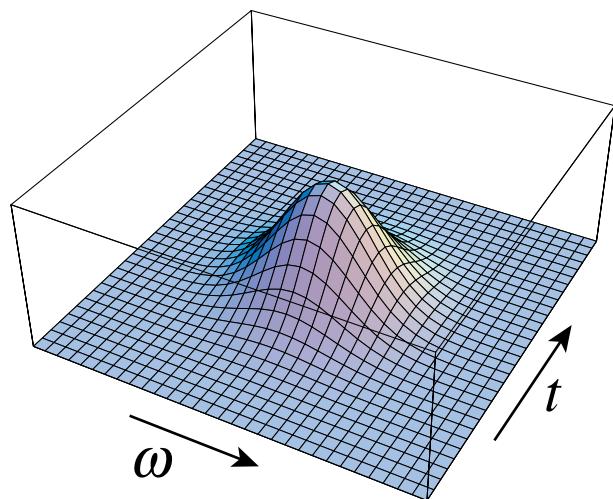
$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(t, \omega) dt d\omega$$



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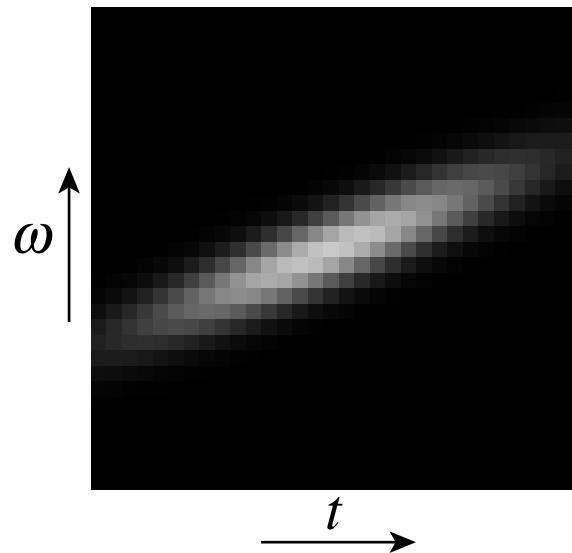
$W(t, \omega)$ must be nonzero in phase-space area larger than π

Joint time-frequency representation

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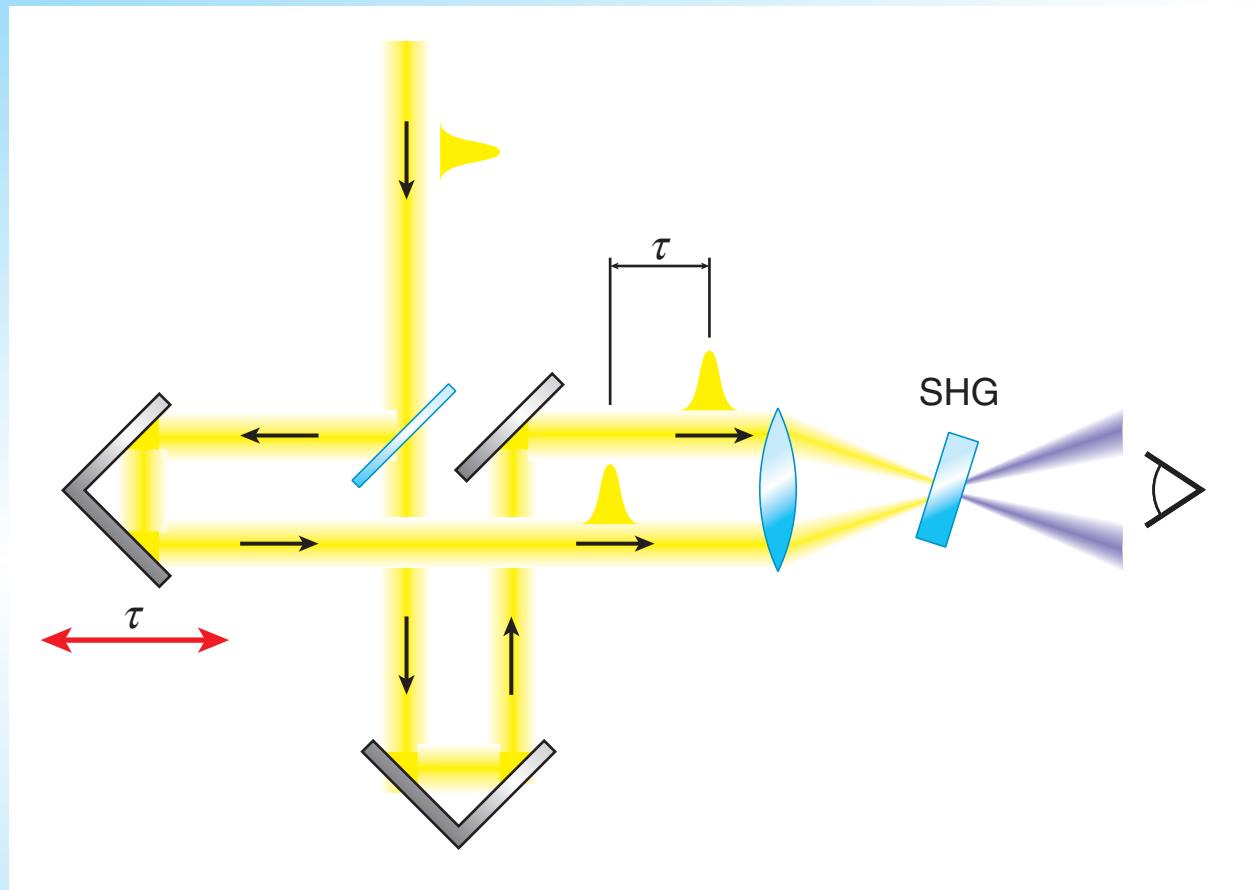
chirped pulse



$W(t, \omega)$ must be nonzero in phase-space area larger than π

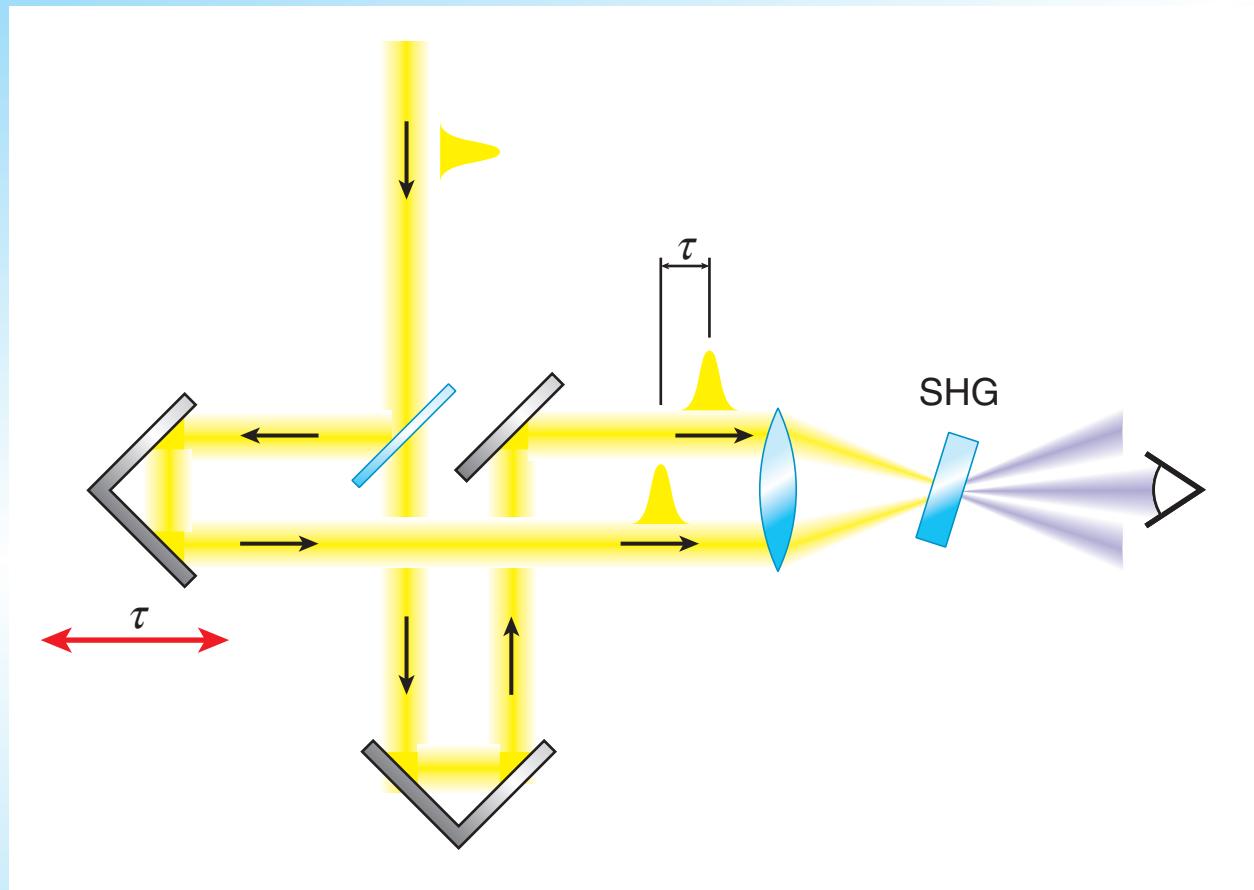
Temporal characterization

Use pulse to measure itself...



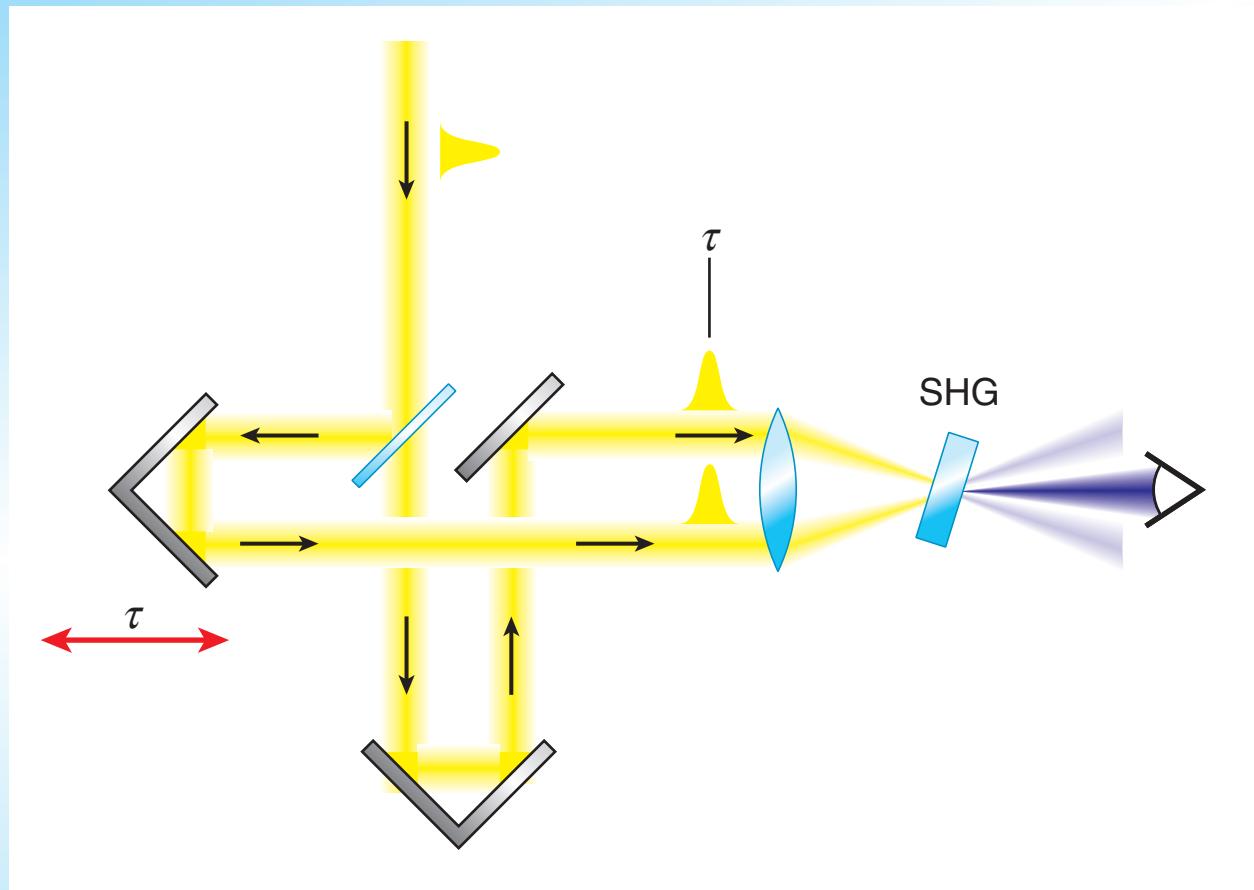
Temporal characterization

Use pulse to measure itself...



Temporal characterization

Use pulse to measure itself...



Temporal characterization

Electric field at SHG crystal

$$E_{tot}(t,\tau) = \frac{1}{\sqrt{2}}E_1(t) + \frac{1}{\sqrt{2}}E_2(t + \tau)$$

Temporal characterization

Electric field at SHG crystal

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Second harmonic field

$$E_{2\omega} \propto \chi^{(2)} E_{tot}^2$$

Temporal characterization

Electric field at SHG crystal

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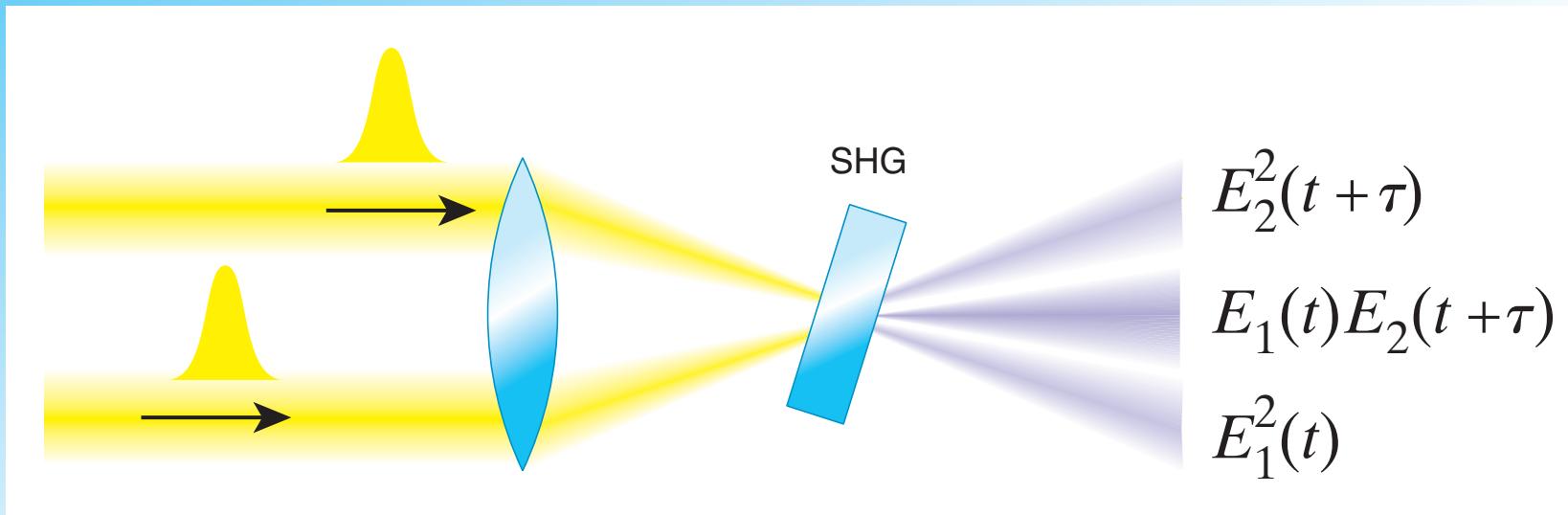
Second harmonic field

$$E_{2\omega} \propto \chi^{(2)} E_{tot}^2$$

Second harmonic intensity

$$I_{2\omega}(t, \tau) \propto |\chi^{(2)}|^2 |E_{tot}^2|^2 = |\chi^{(2)}|^2 |E_1^2(t) + 2E_1(t)E_2(t+\tau) + E_2^2(t+\tau)|^2$$

Temporal characterization



Second harmonic intensity

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detector selects middle term

Temporal characterization

Integrated detector signal yields intensity autocorrelation

$$A(\tau) = \int I_{2\omega}(t, \tau) dt \propto \int |\chi^{(2)}|^2 4 |E_1(t)|^2 |E_2(t + \tau)|^2 dt$$

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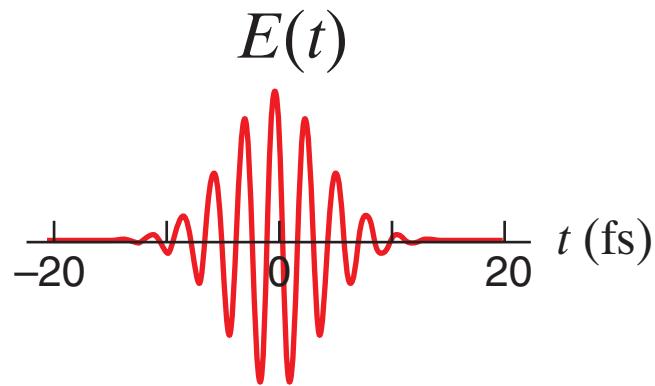
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Temporal characterization

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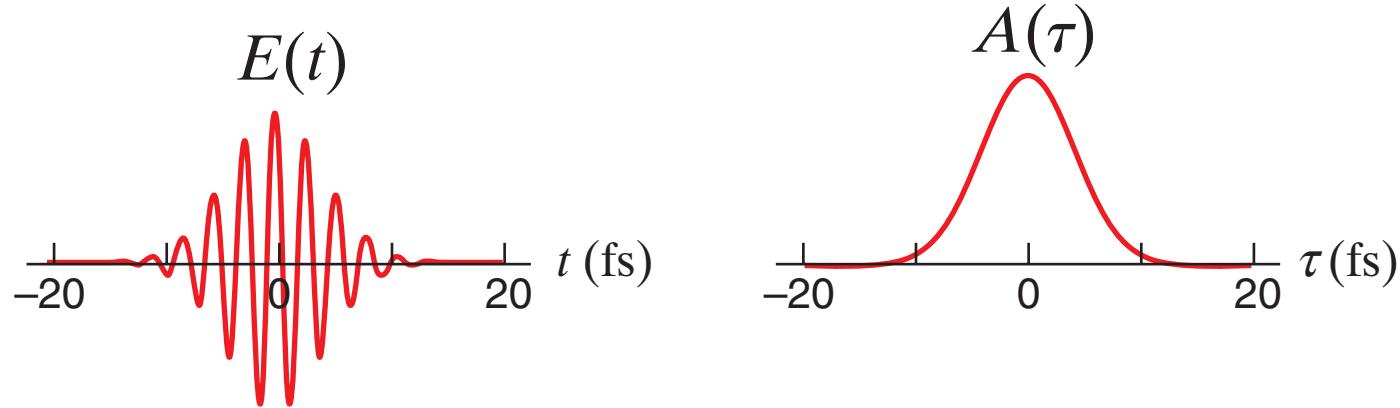


Temporal characterization

Integrated detector signal yields intensity autocorrelation

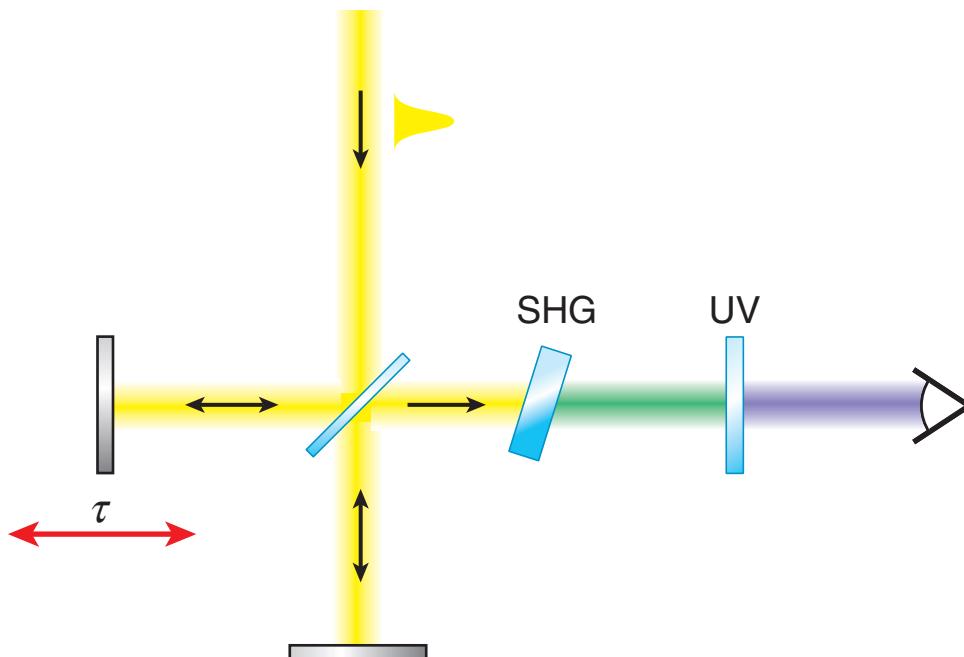
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Temporal characterization

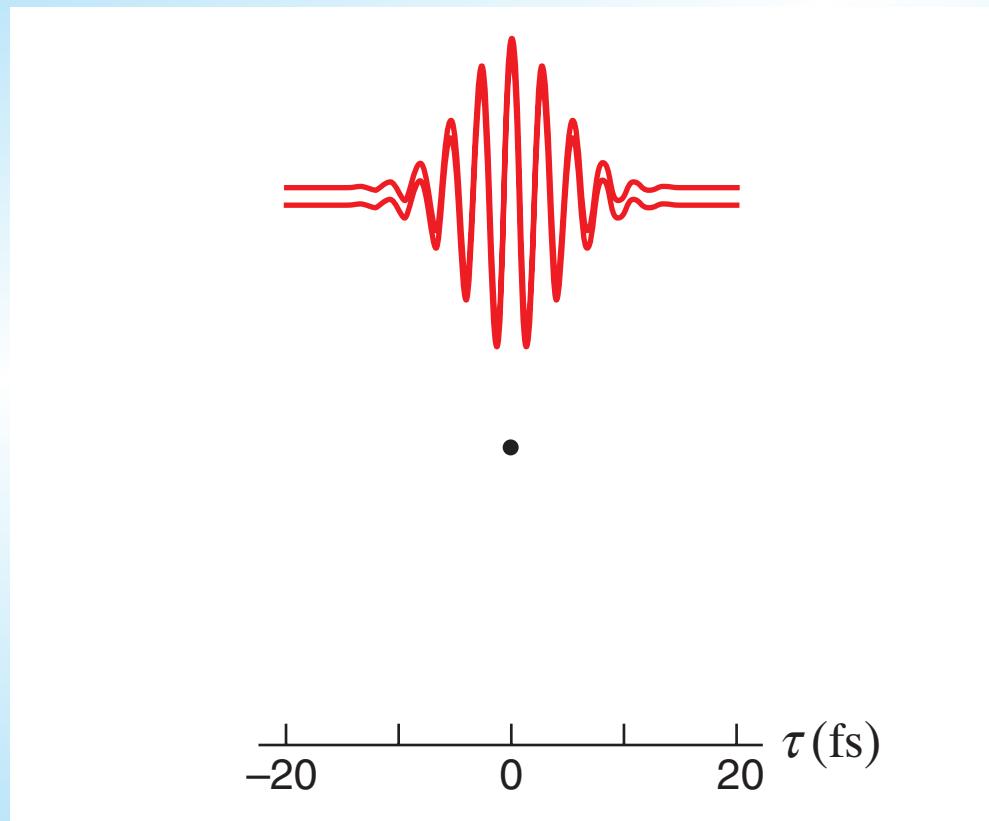
Alternative colinear geometry



Temporal characterization

All terms now contribute:

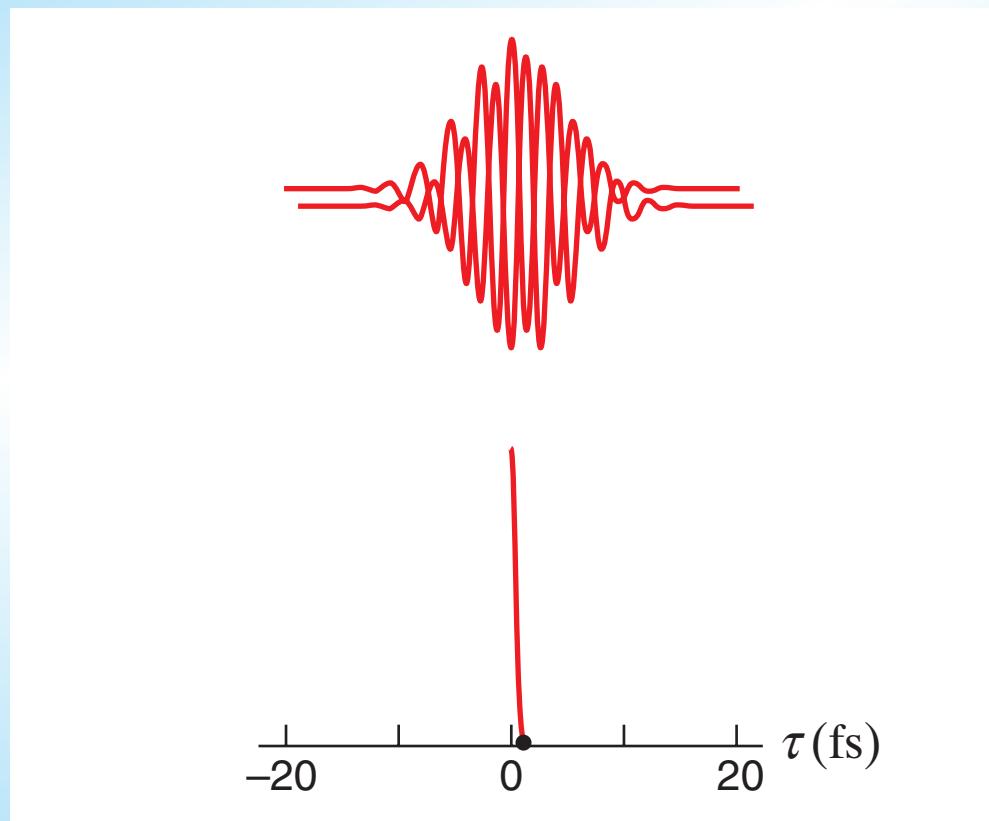
$$I_{2\omega}(t, \tau) \propto |\chi^{(2)}|^2 |E_{tot}^2|^2 = |\chi^{(2)}|^2 |E_1^2(t) + 2E_1(t)E_2(t+\tau) + E_2^2(t+\tau)|^2$$



Temporal characterization

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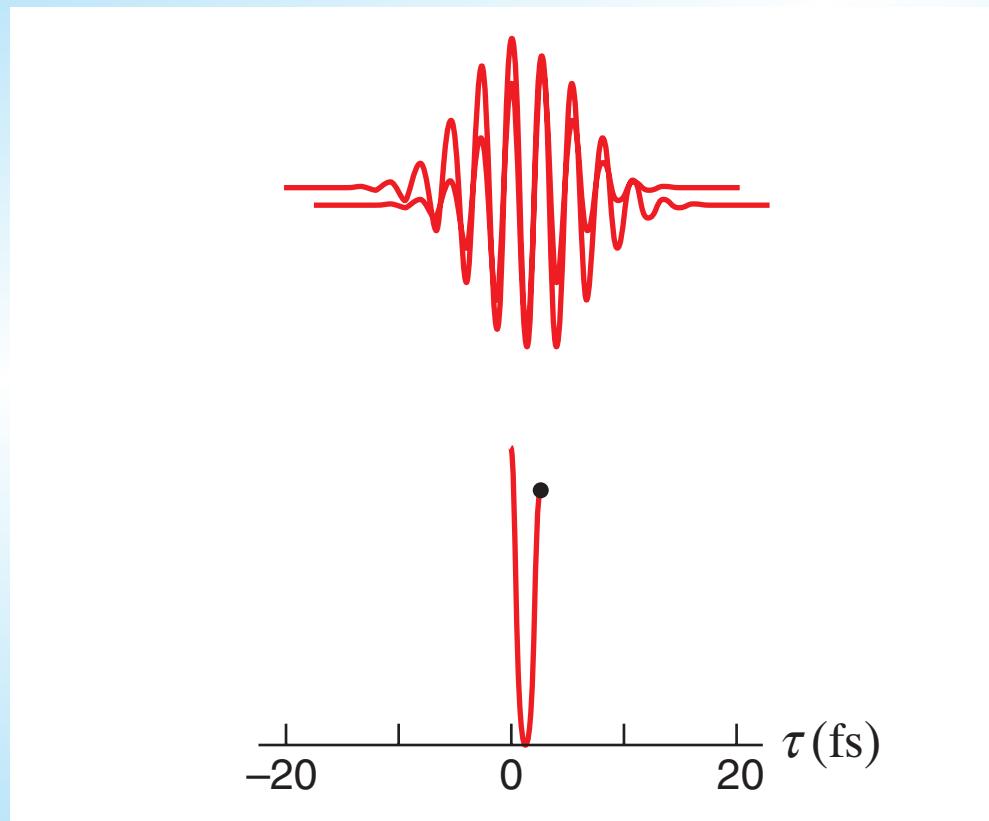
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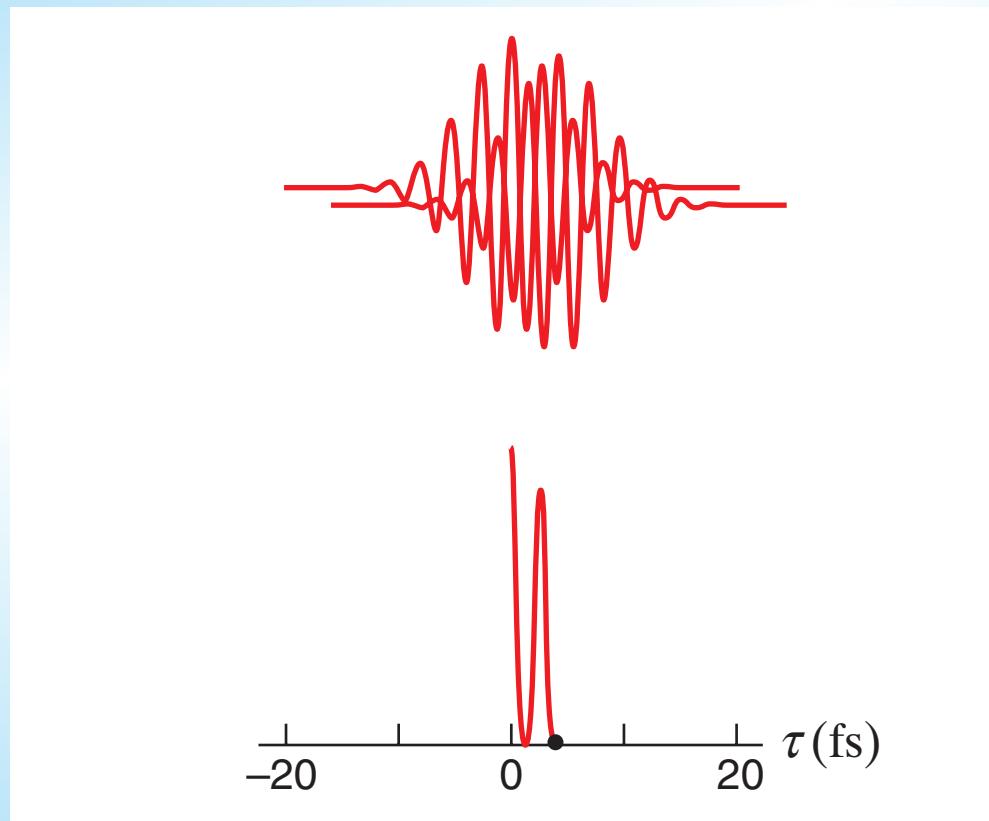
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Temporal characterization

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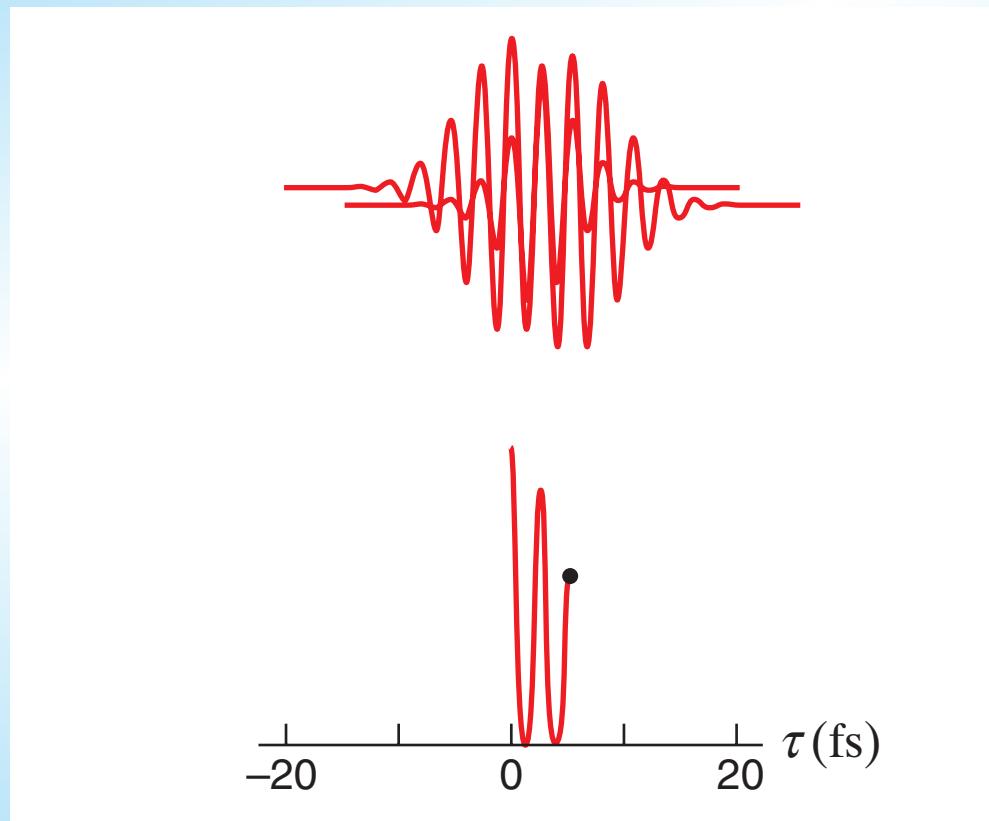
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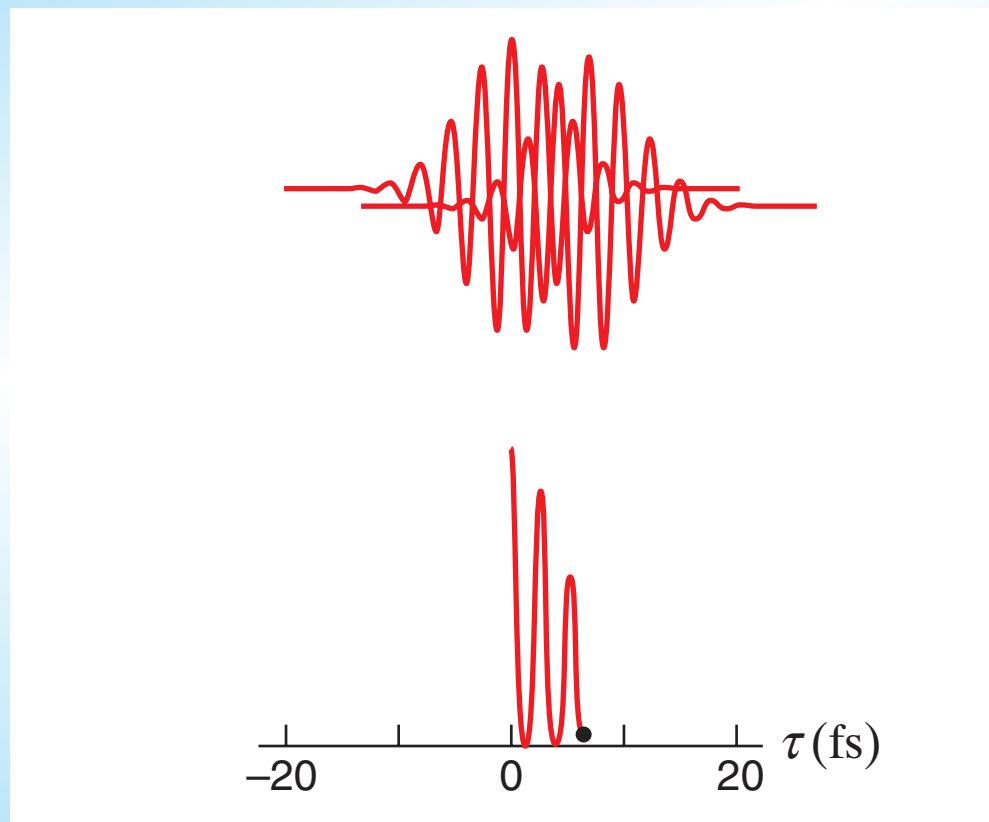
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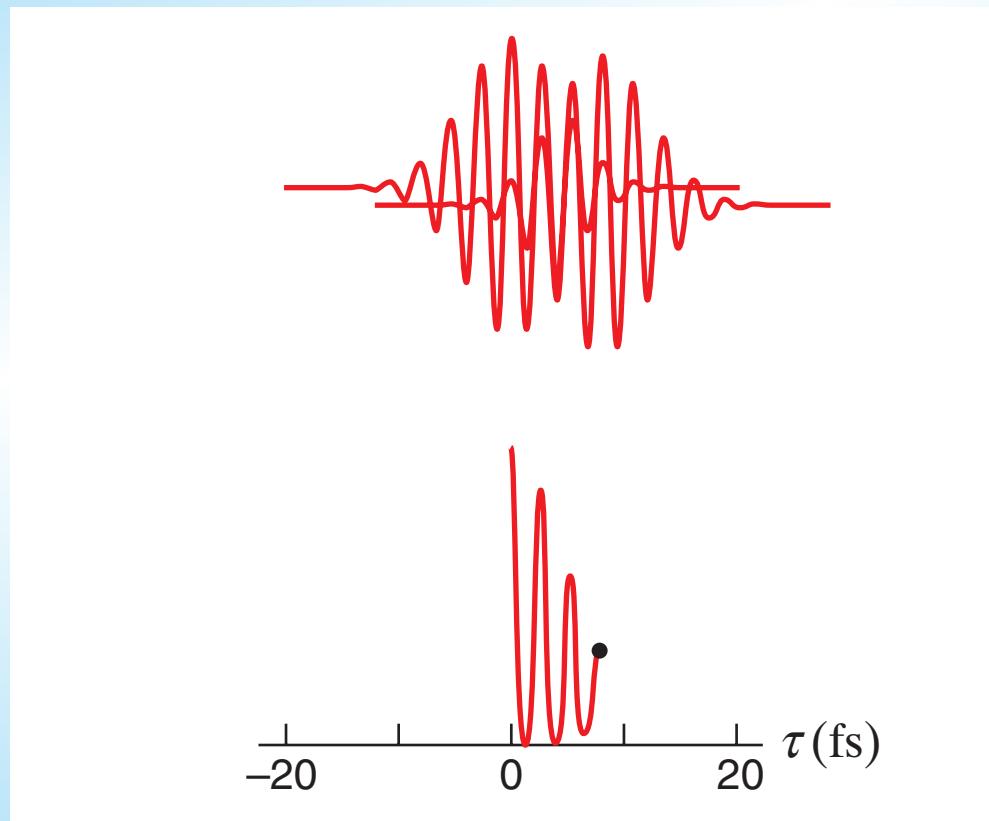
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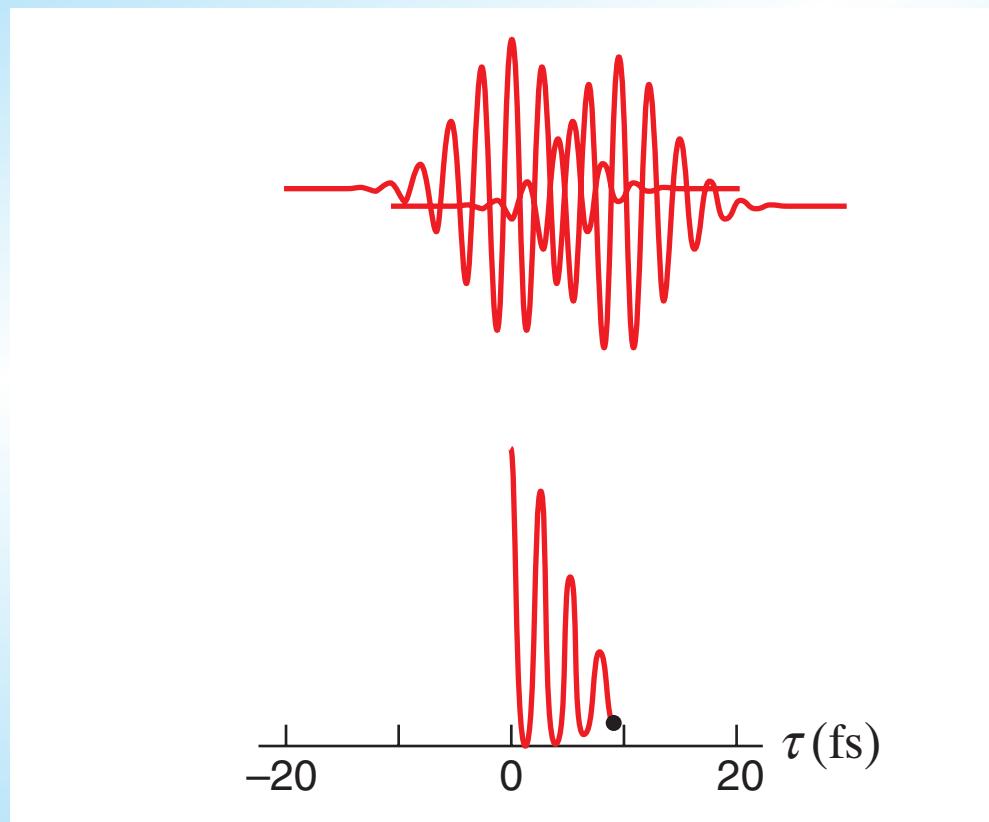
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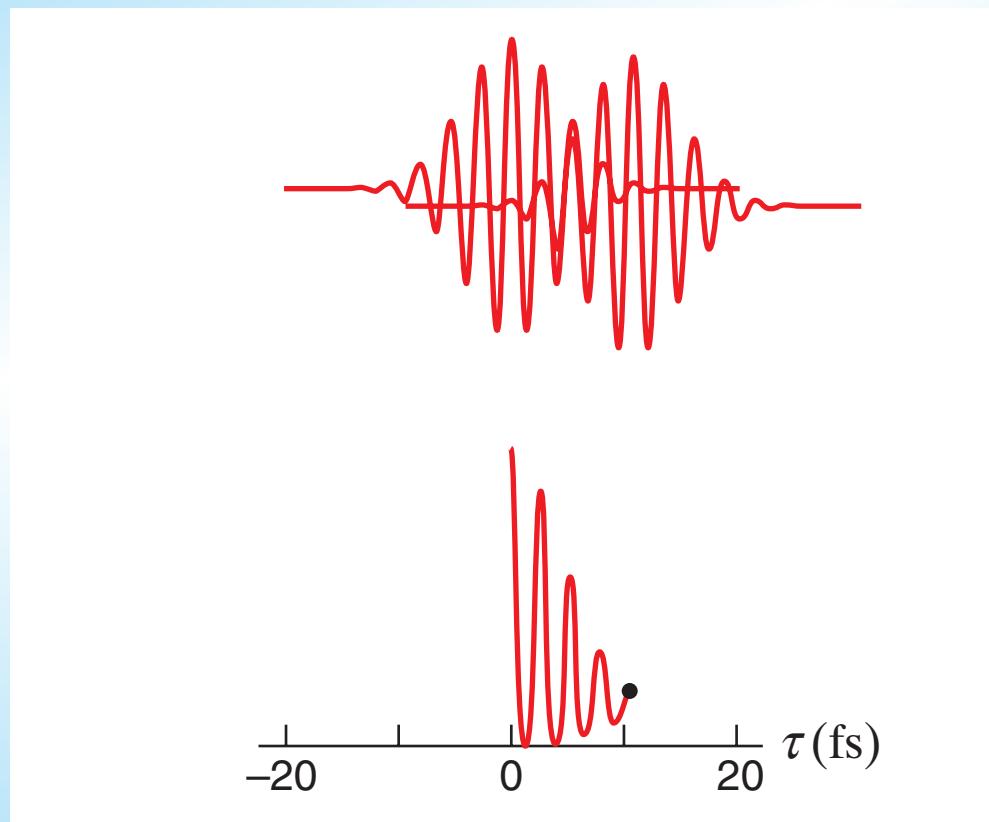
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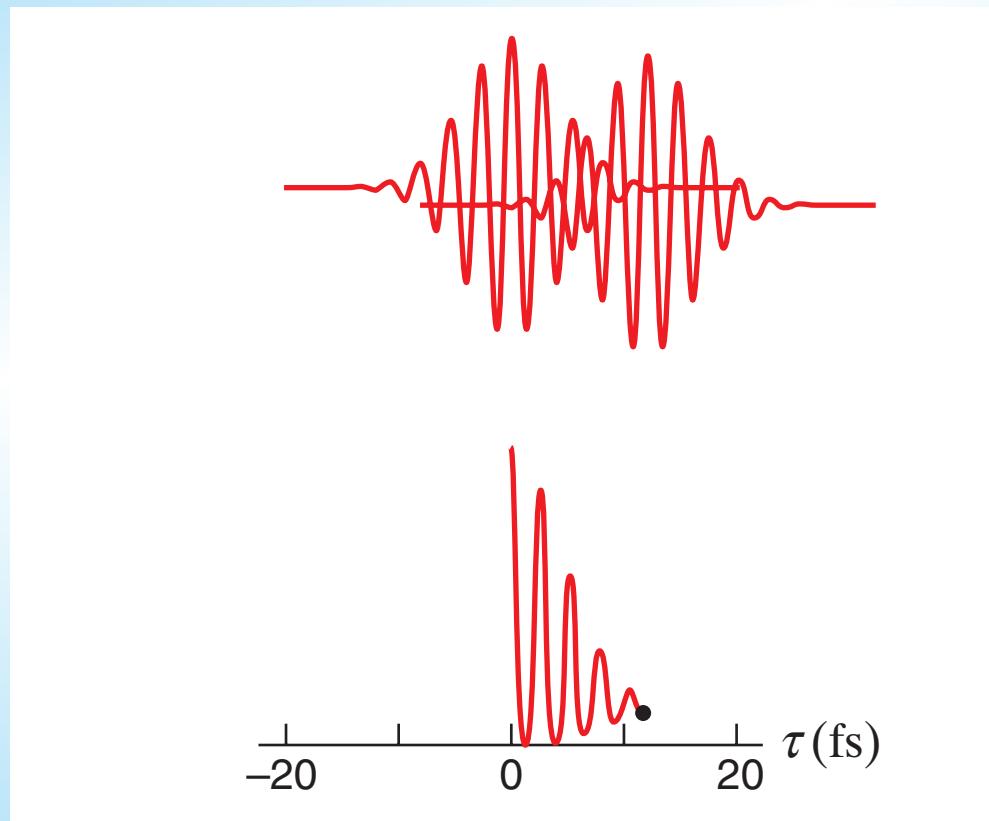
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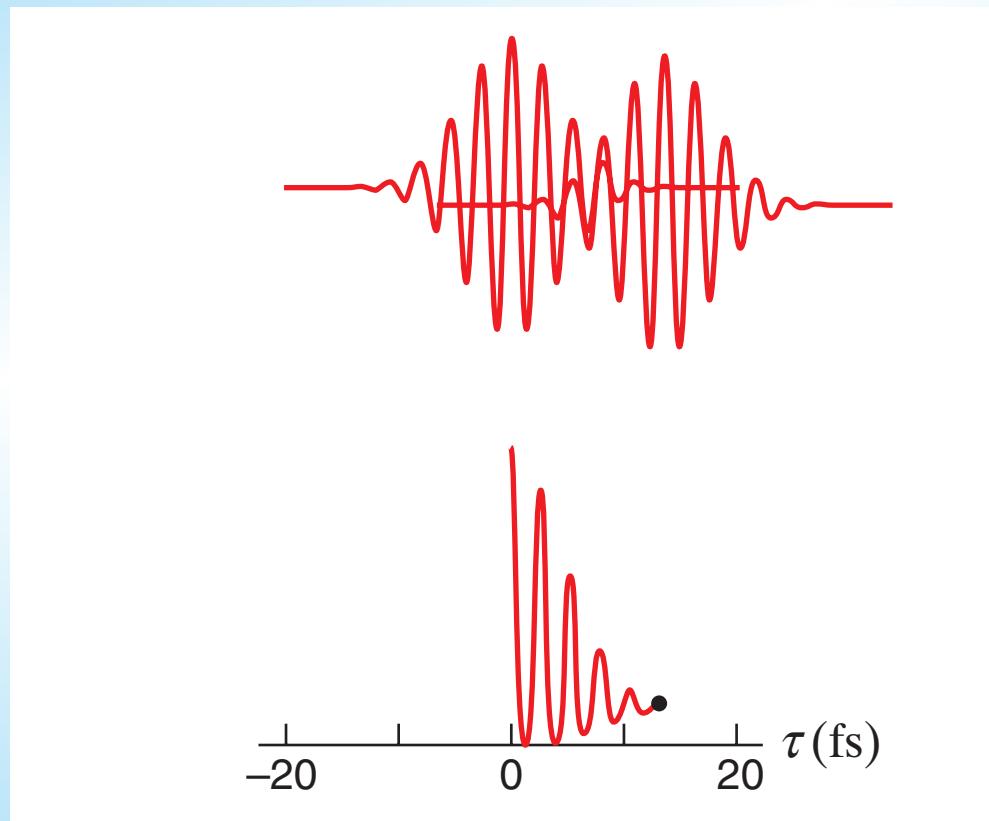
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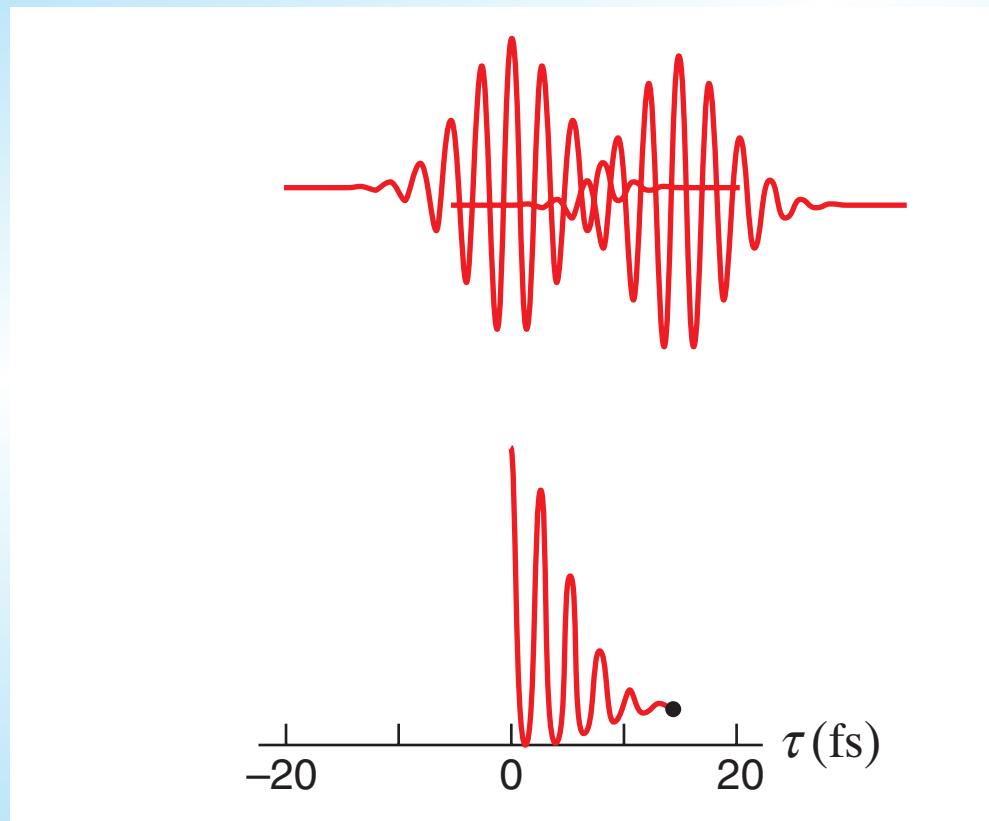
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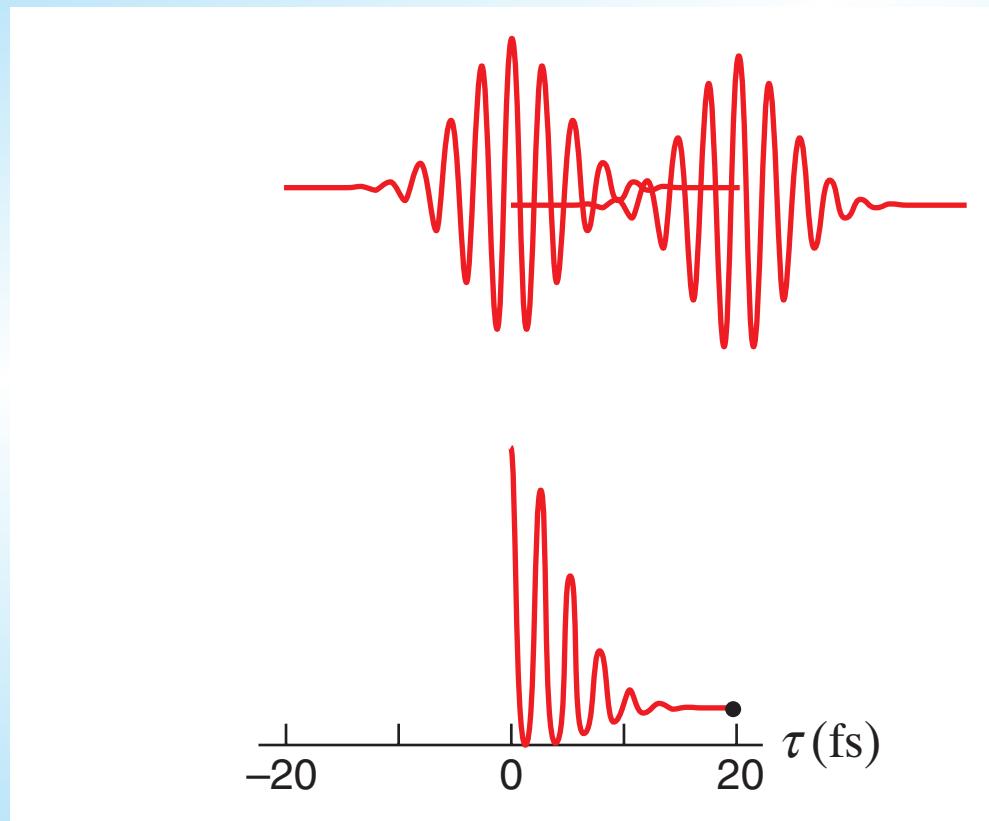
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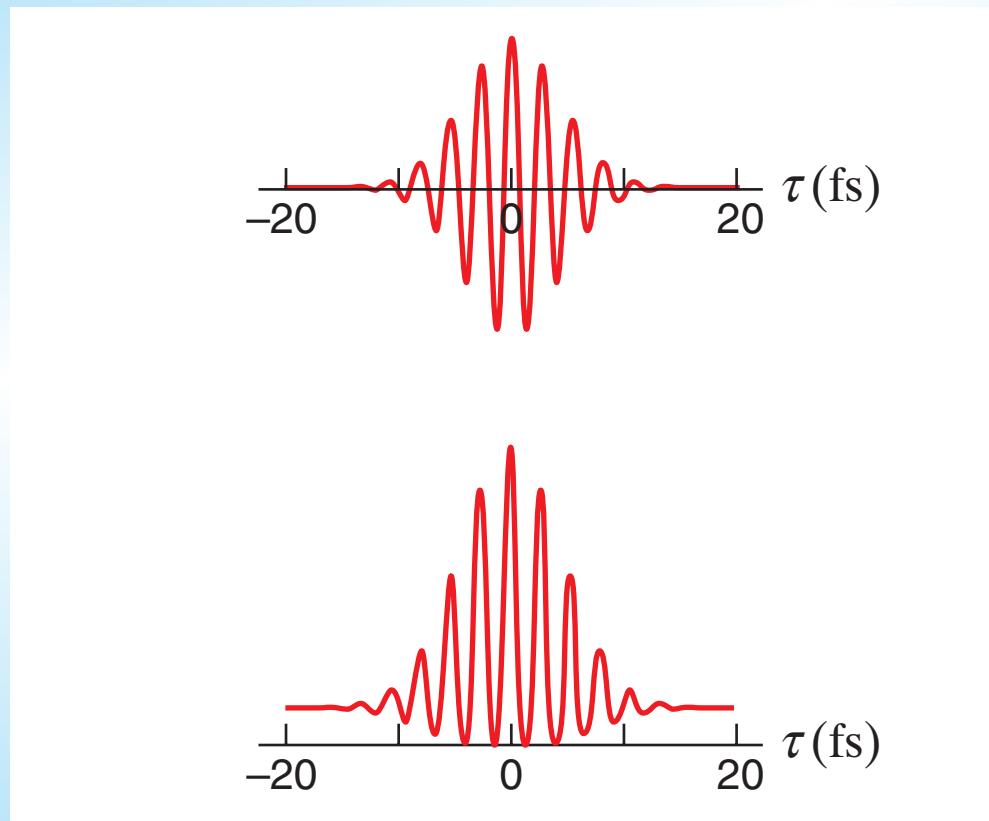
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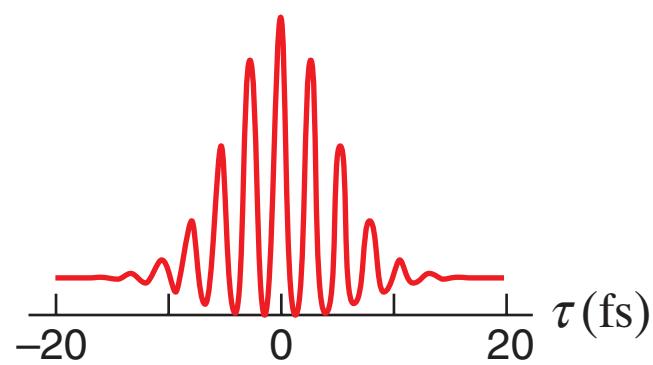
Temporal characterization

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at $\tau = 0$:

$$I_{2\omega}(t, \tau) \propto 16E^4(t)$$



Temporal characterization

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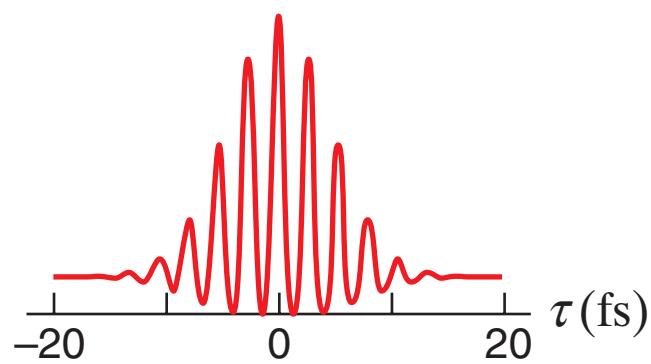
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as $\tau \rightarrow \pm\infty$:

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Temporal characterization

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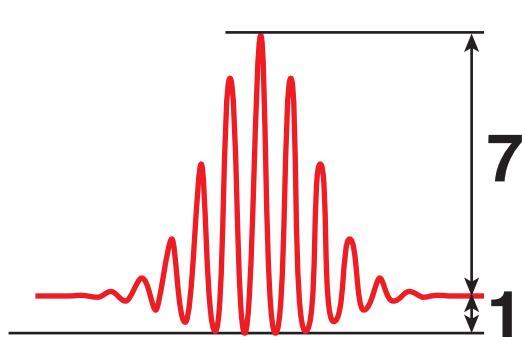
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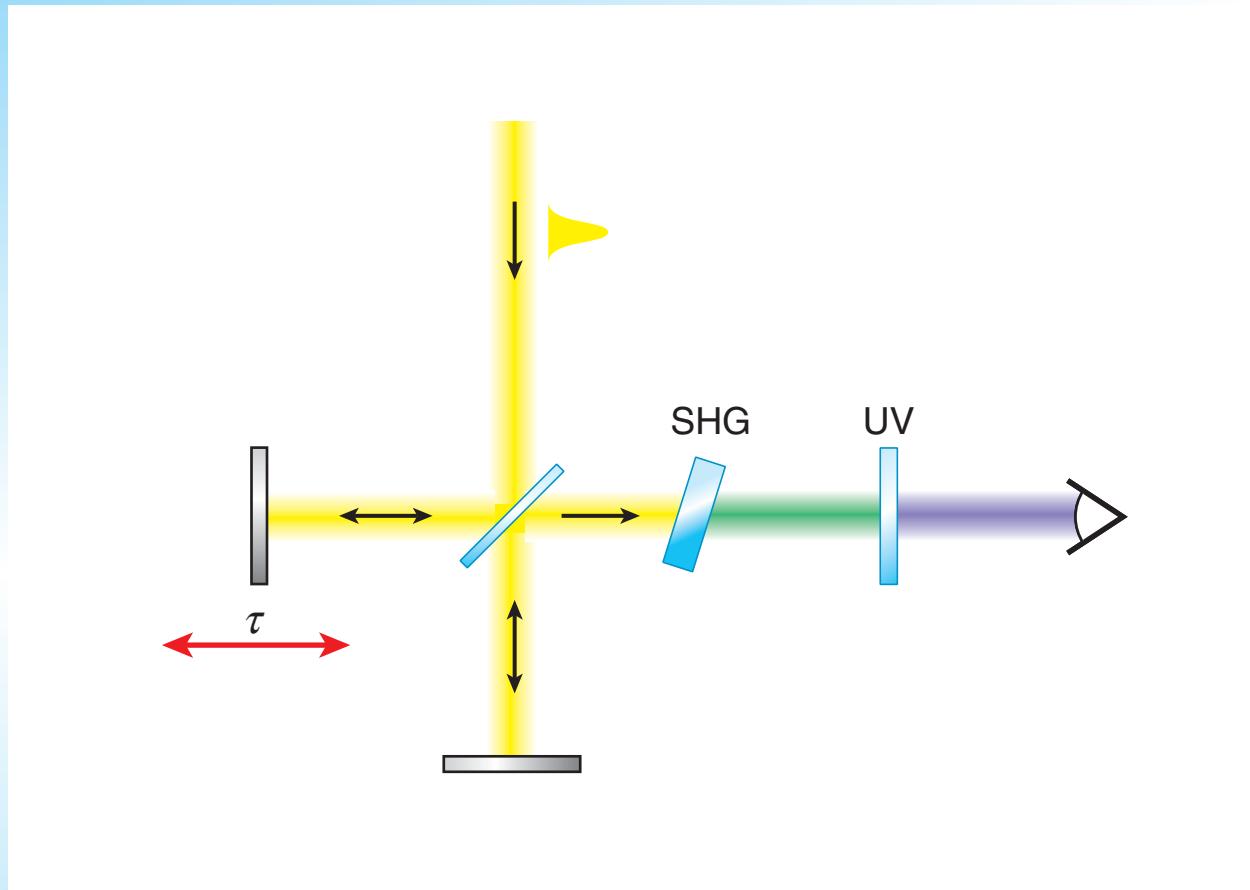
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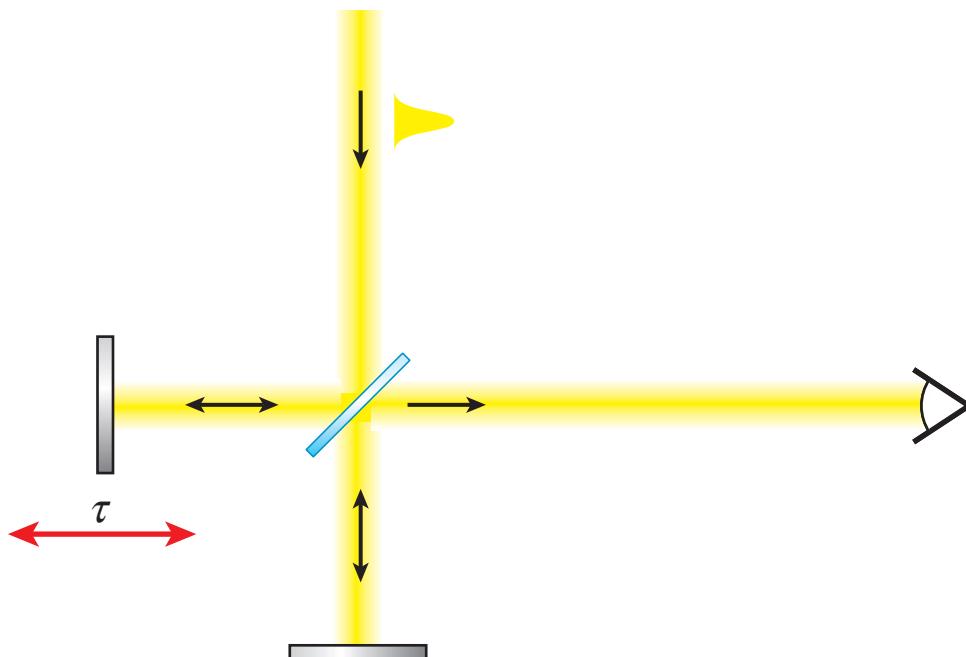
Temporal characterization

Do we really need the second-harmonic crystal...?



Temporal characterization

Would this work?



Temporal characterization

Intensity at detector

$$I_\omega(t, \tau) \propto |E_1(t) + E_2(t + \tau)|^2$$

Temporal characterization

Intensity at detector

$$I_\omega(t, \tau) \propto |E_1(t) + E_2(t + \tau)|^2$$

Detected signal

$$S_\omega(\tau) = \int I_\omega(t, \tau) dt$$

Temporal characterization

Intensity at detector

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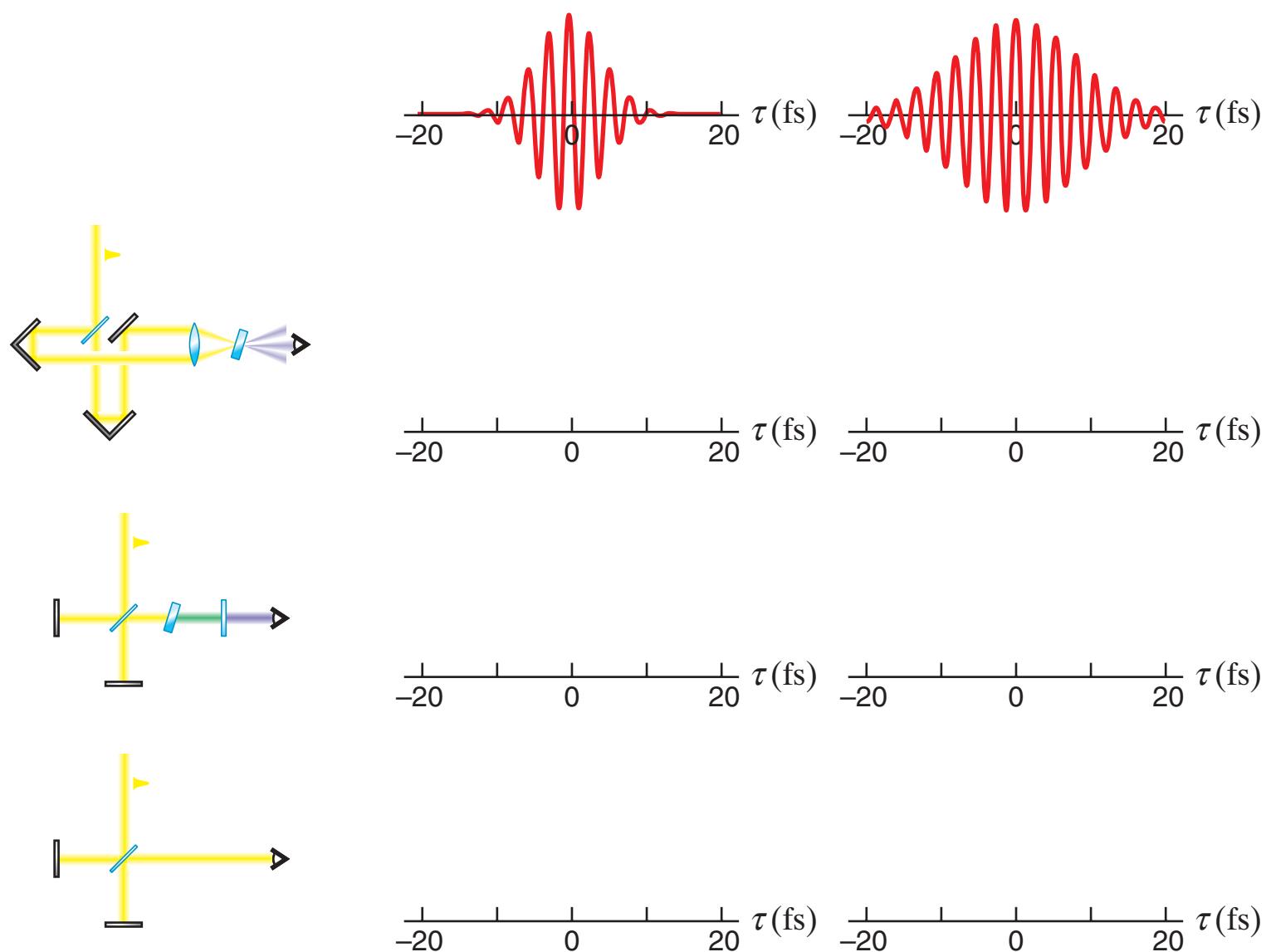
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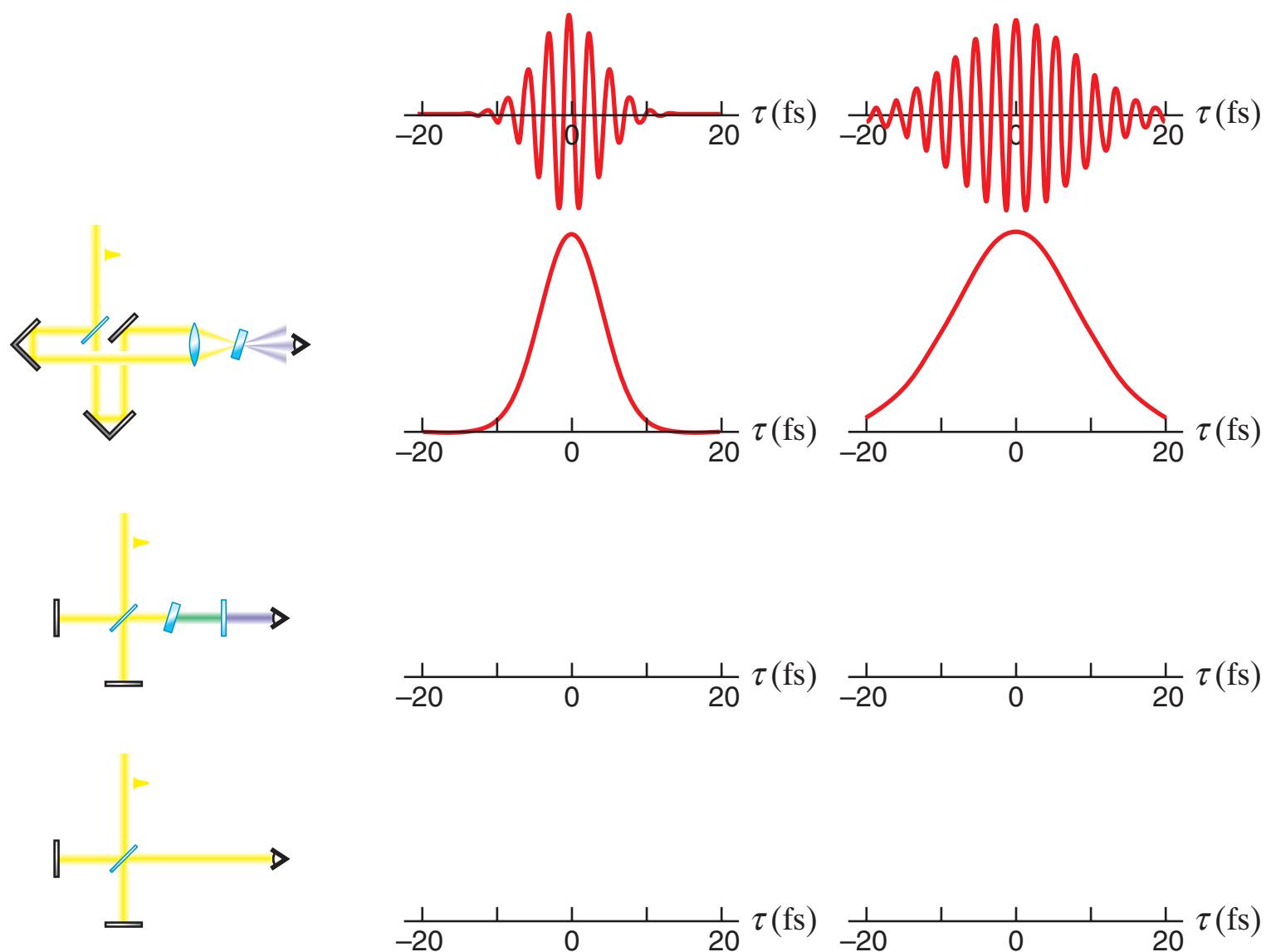
so

$$S_\omega(\tau) \propto \int \{|E_1(t)|^2 + |E_2(t + \tau)|^2 + E_1(t)E_2^*(t + \tau) + E_1^*(t)E_2(t + \tau)\} dt$$

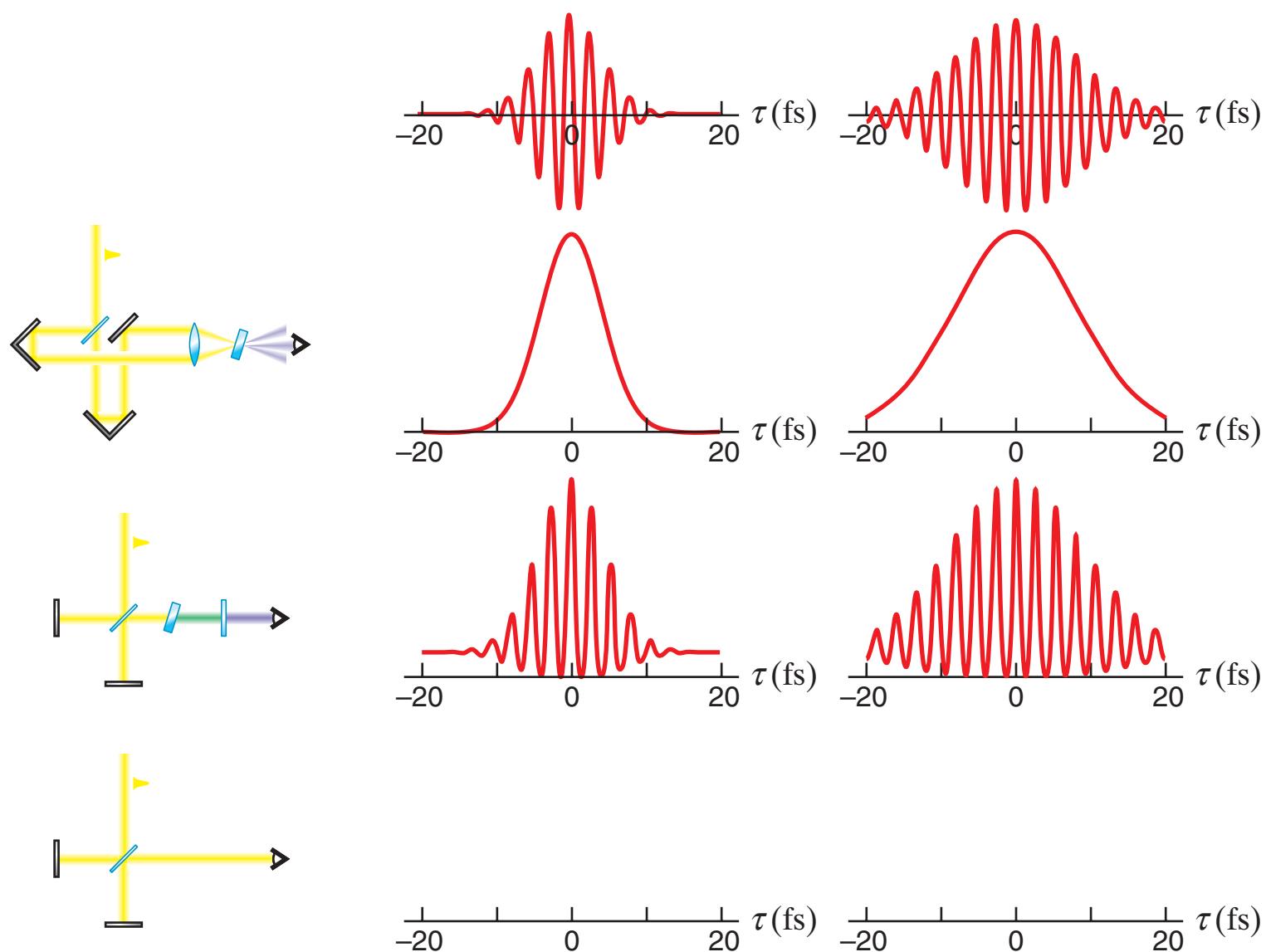
Temporal characterization



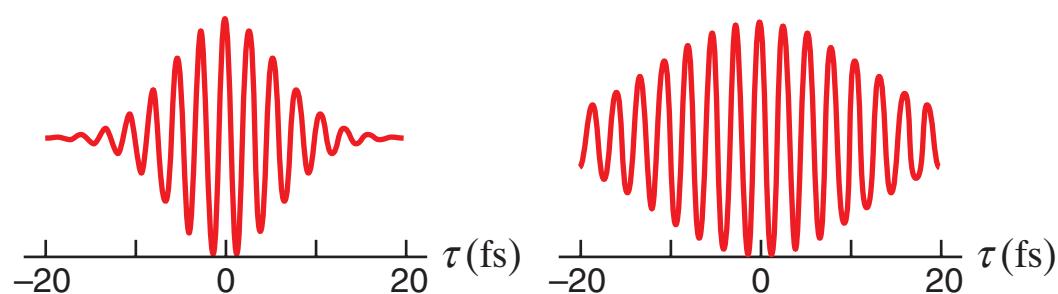
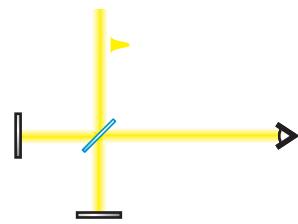
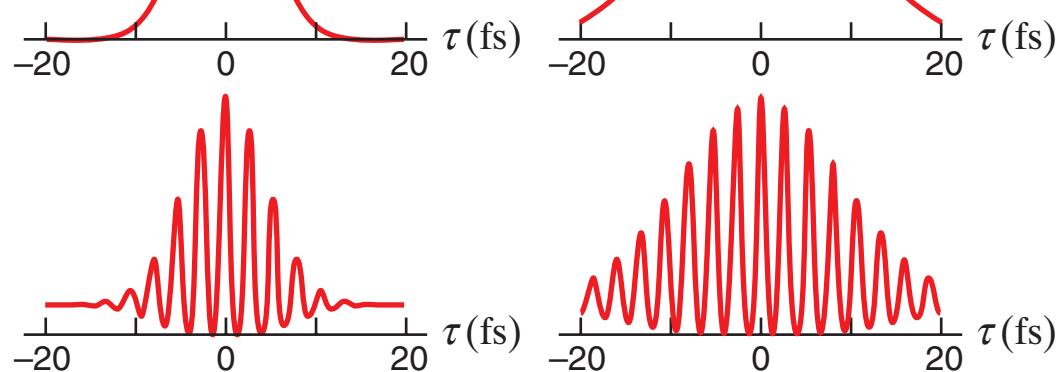
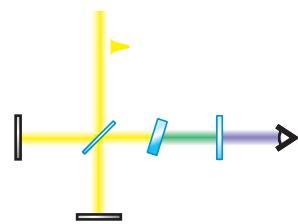
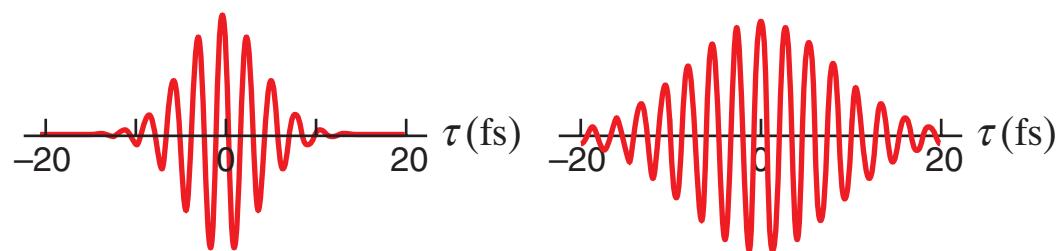
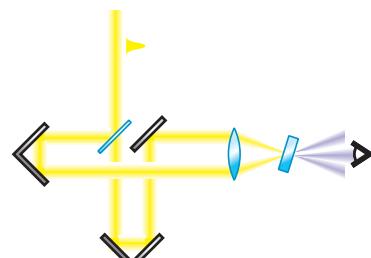
Temporal characterization



Temporal characterization



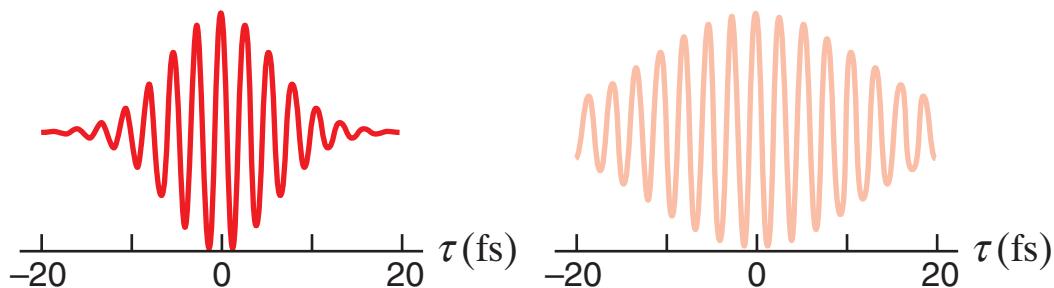
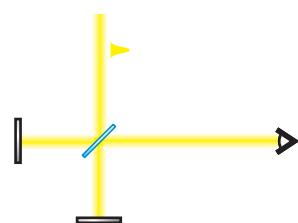
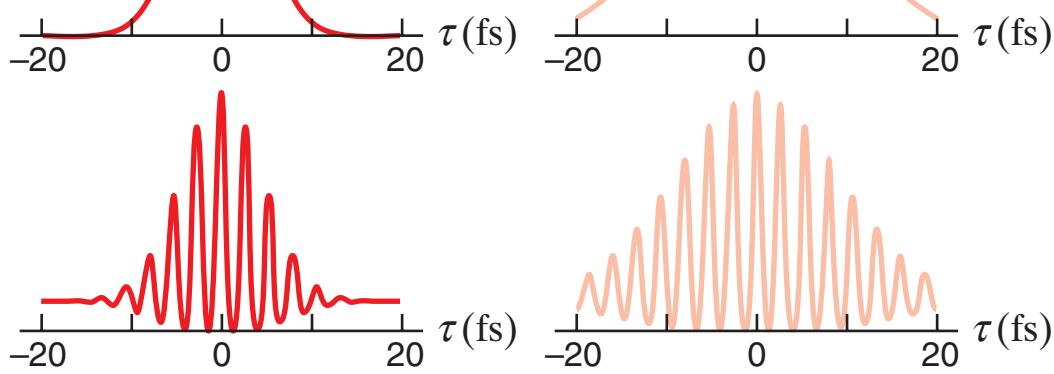
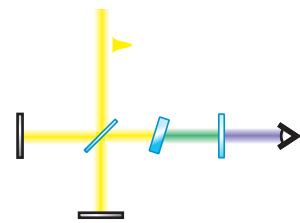
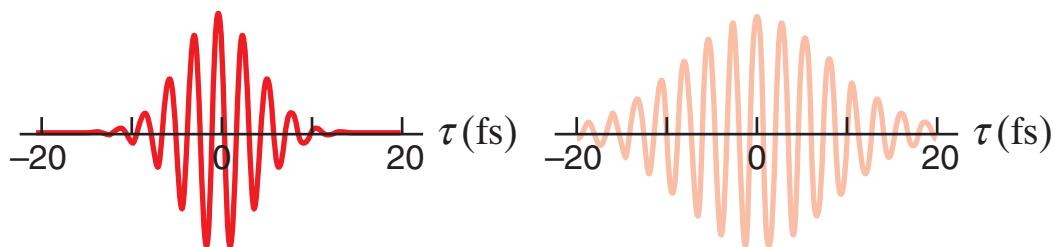
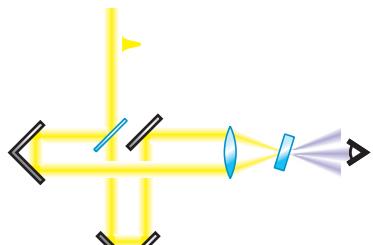
Temporal characterization



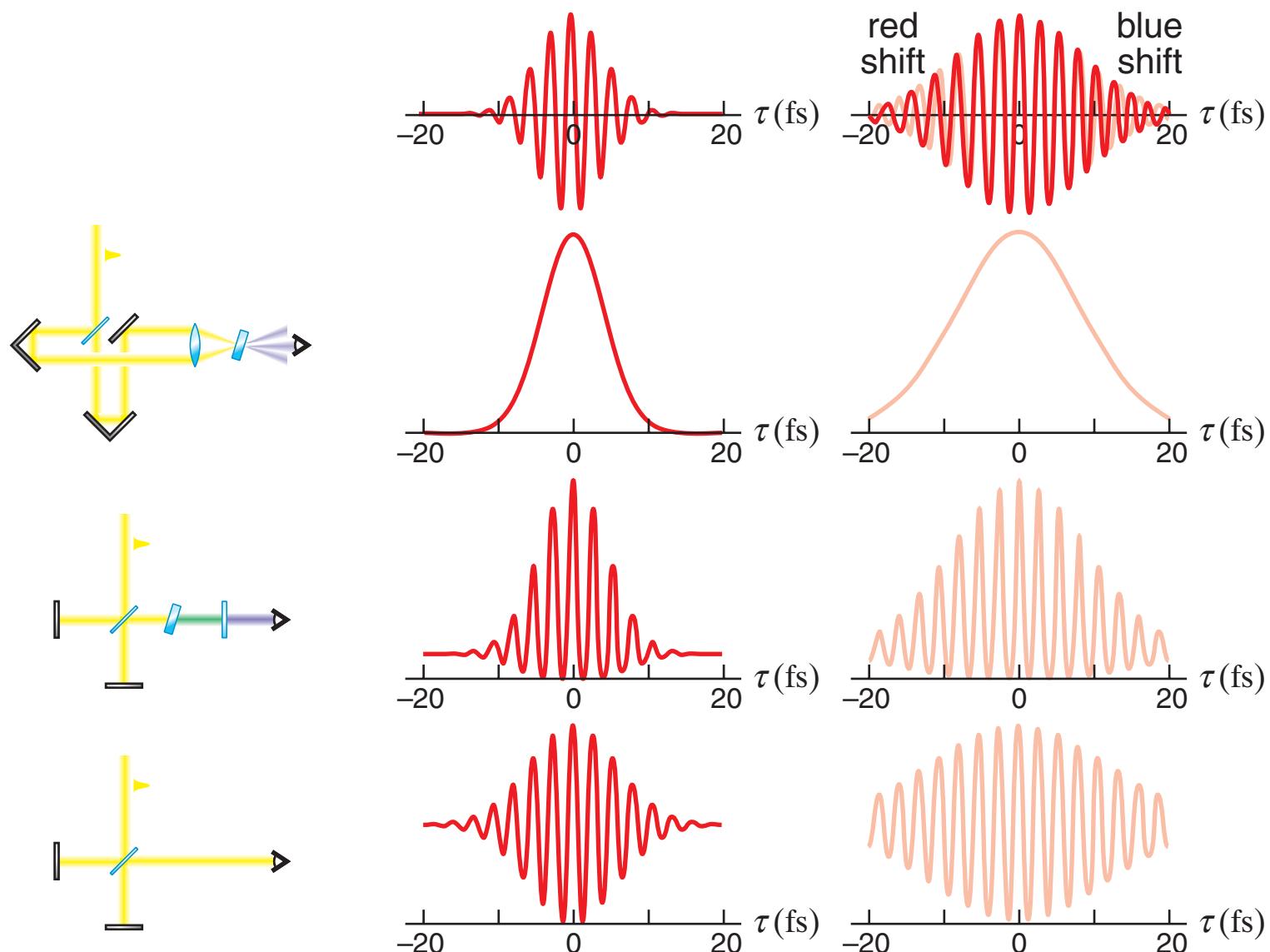
Temporal characterization

But what about dispersion?

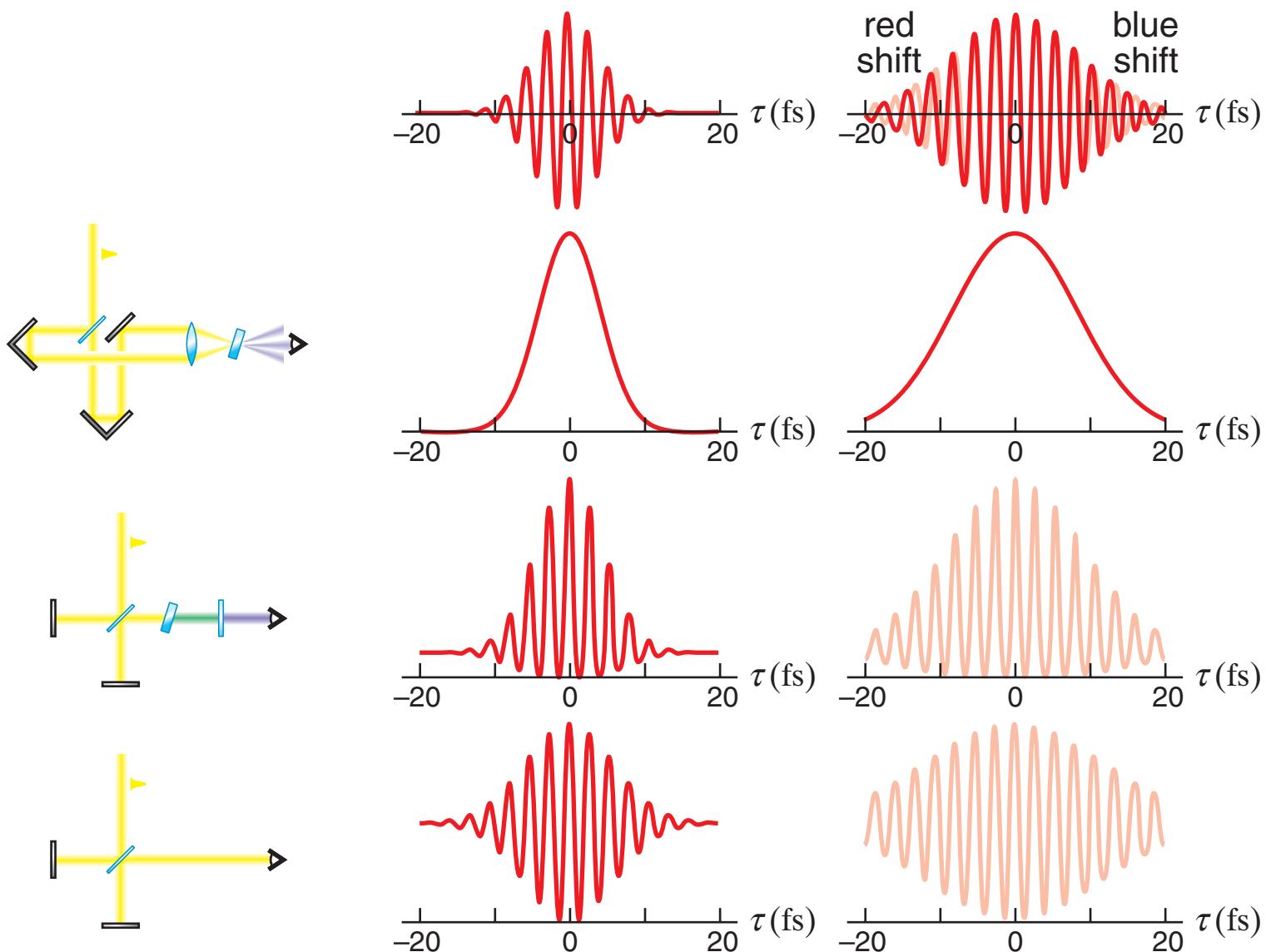
Temporal characterization



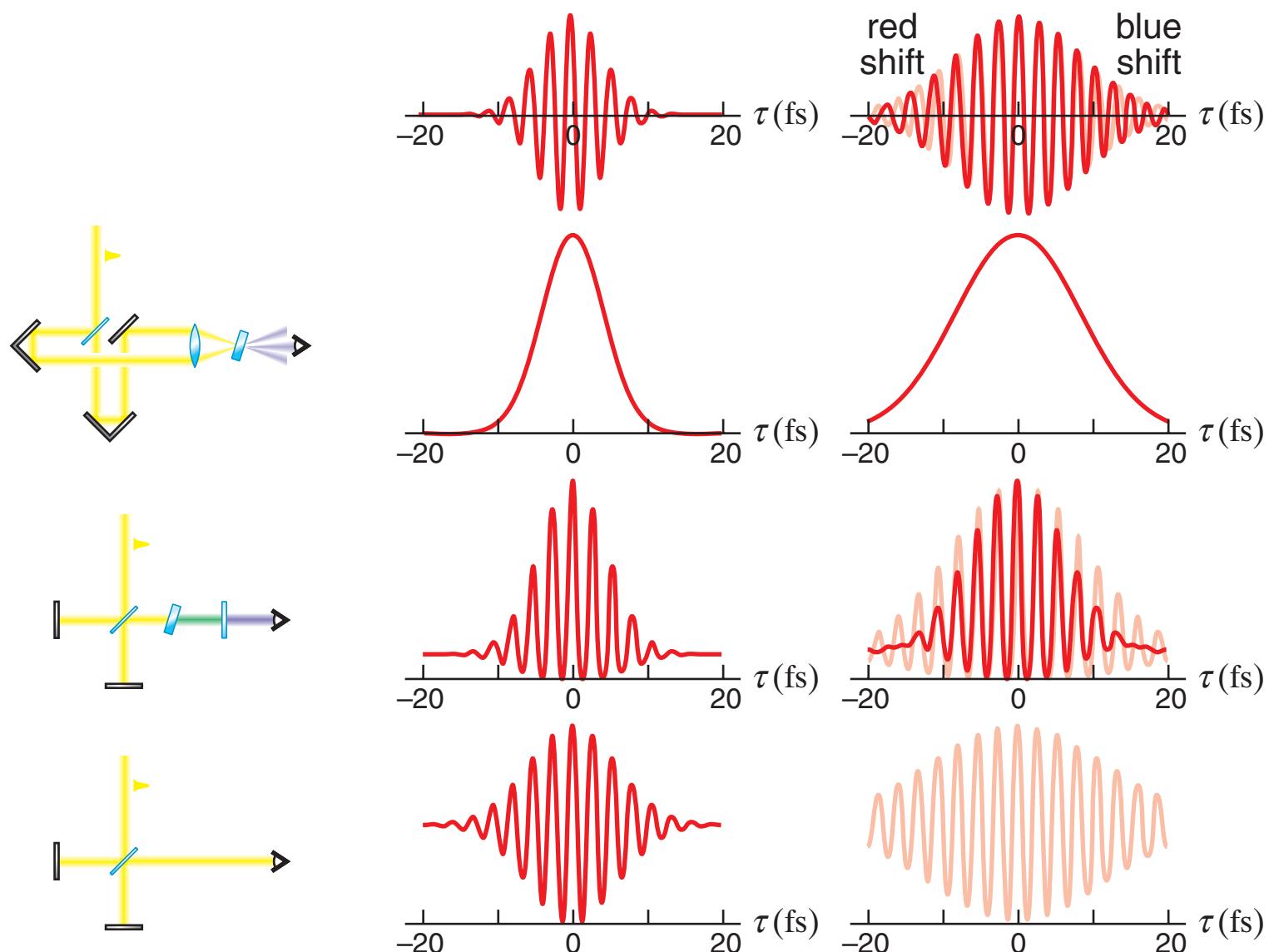
Temporal characterization



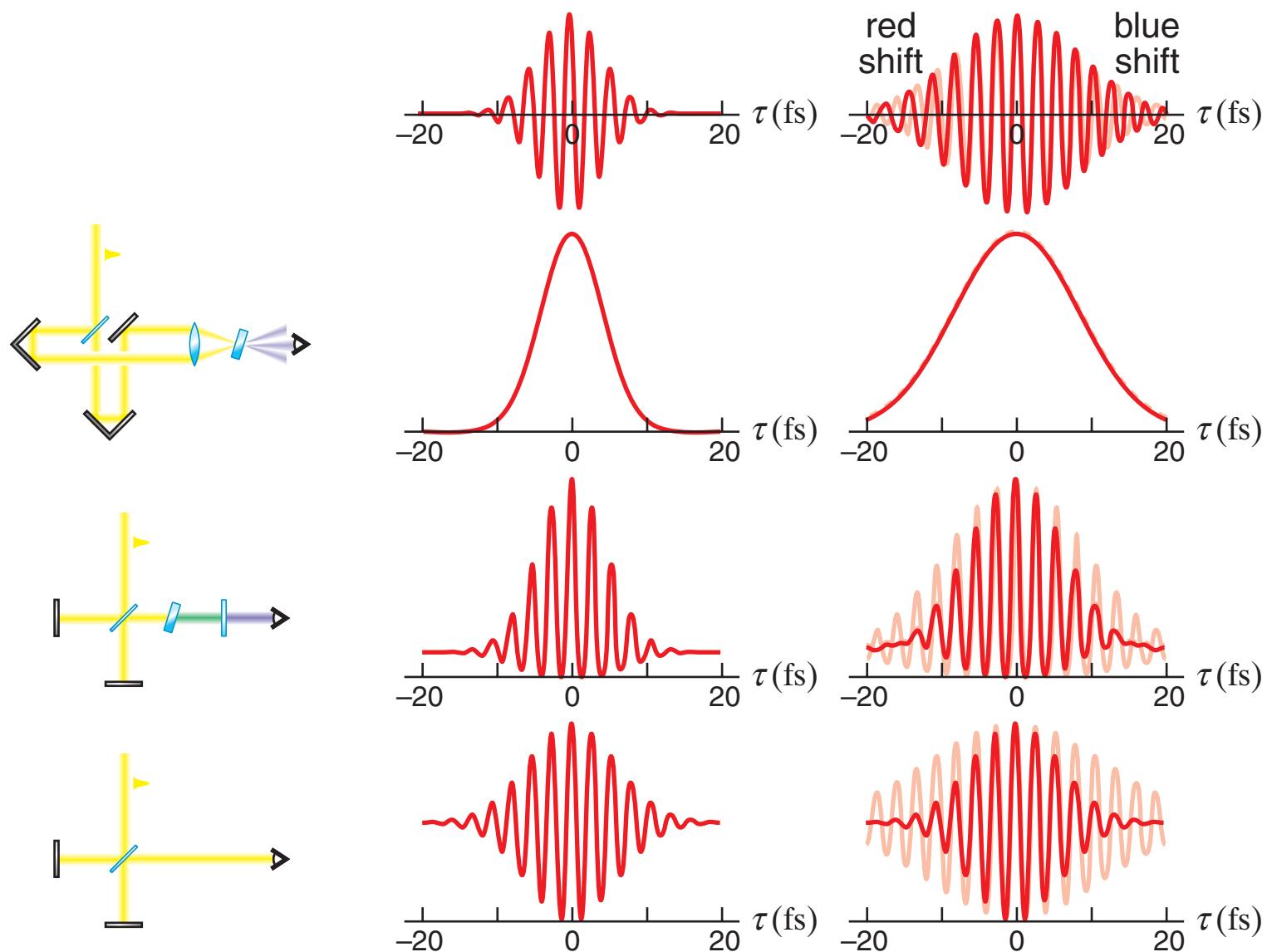
Temporal characterization



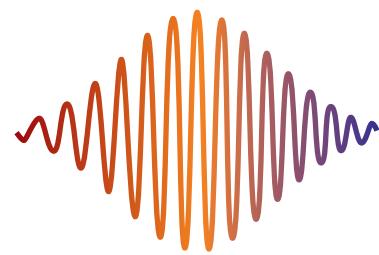
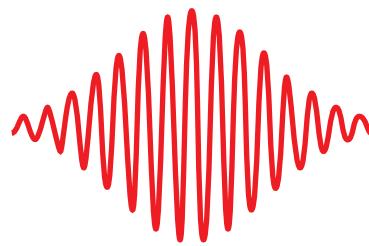
Temporal characterization



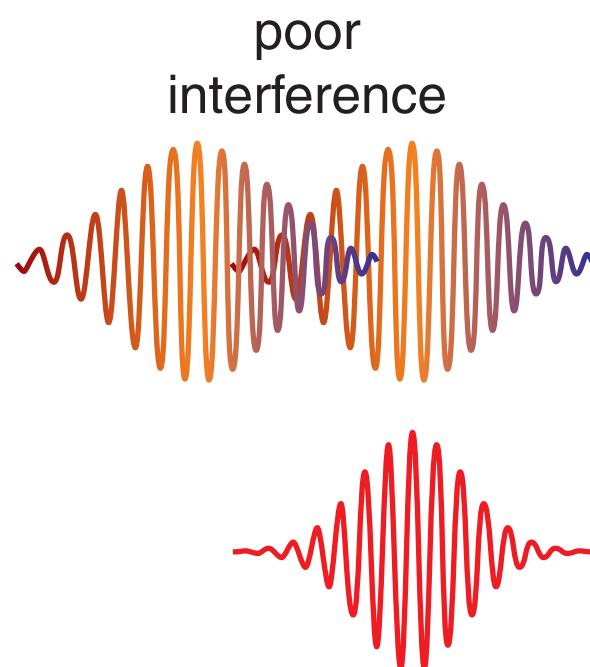
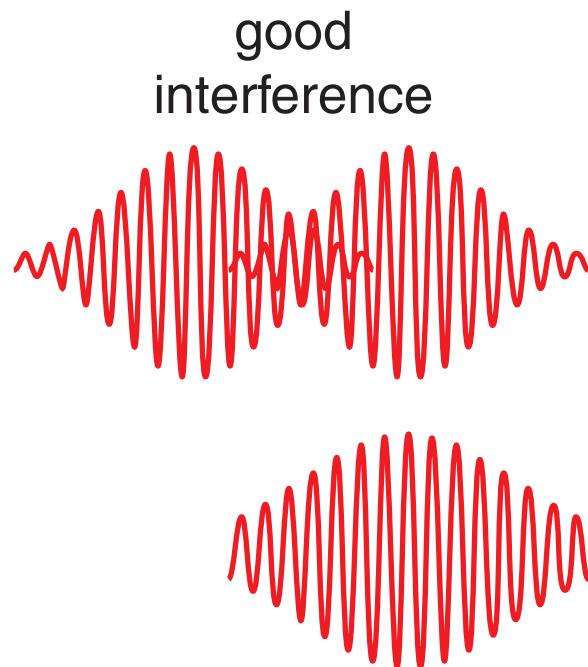
Temporal characterization



Temporal characterization



Temporal characterization



Temporal characterization

Let $E_{disp}(\omega) = E_{orig}(\omega)e^{-i\phi(\omega)}$.

Temporal characterization

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$$f_1(t) \otimes f_2(t) \equiv \int f_1(t+\tau) {f_2}^*(t) dt = \mathcal{F}^{-1}\{f_1(\omega) {f_2}^*(\omega)\}$$

Temporal characterization

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Interference term in linear autocorrelation:

$$\int E_{disp}(t + \tau) {E_{disp}}^*(t) dt = \mathcal{F}^{-1}\{E_{disp}(\omega) {E_{disp}}^*(\omega)\} =$$

Temporal characterization

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Interference term in linear autocorrelation:

$$\begin{aligned} \int E_{disp}(t + \tau) {E_{disp}^*(t)} dt &= \mathcal{F}^{-1}\{E_{disp}(\omega) {E_{disp}^*(\omega)}\} = \\ &= \mathcal{F}^{-1}\{E_{orig}(\omega) e^{i\phi(\omega)} {E_{orig}^*(\omega)} e^{-i\phi(\omega)}\} = \end{aligned}$$

Temporal characterization

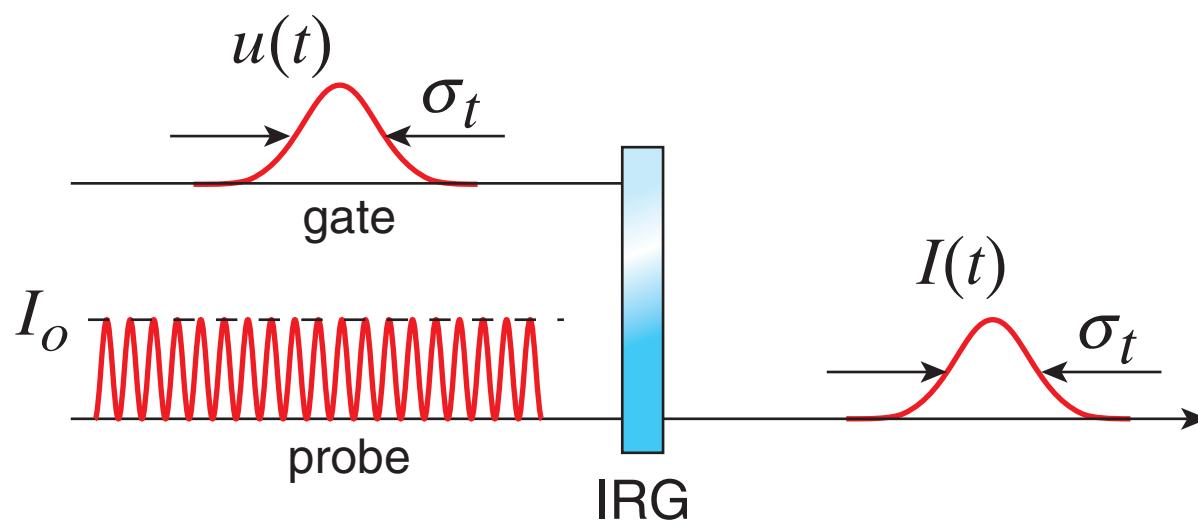
Let $E_{disp}(\omega) = E_{orig}(\omega)e^{-i\phi(\omega)}$. Convolution theorem

$$f_1(t) \otimes f_2(t) \equiv \int f_1(t + \tau) {f_2}^*(t) dt = \mathcal{F}^{-1}\{f_1(\omega) {f_2}^*(\omega)\}$$

Interference term in linear autocorrelation:

$$\begin{aligned} \int E_{disp}(t + \tau) {E_{disp}}^*(t) dt &= \mathcal{F}^{-1}\{E_{disp}(\omega) {E_{disp}}^*(\omega)\} = \\ &= \mathcal{F}^{-1}\{E_{orig}(\omega) e^{i\phi(\omega)} {E_{orig}}^*(\omega) e^{-i\phi(\omega)}\} = \\ &= \mathcal{F}^{-1}\{E_{orig}(\omega) {E_{orig}}^*(\omega)\} = \int E_{orig}(t + \tau) {E_{orig}}^*(t) dt \end{aligned}$$

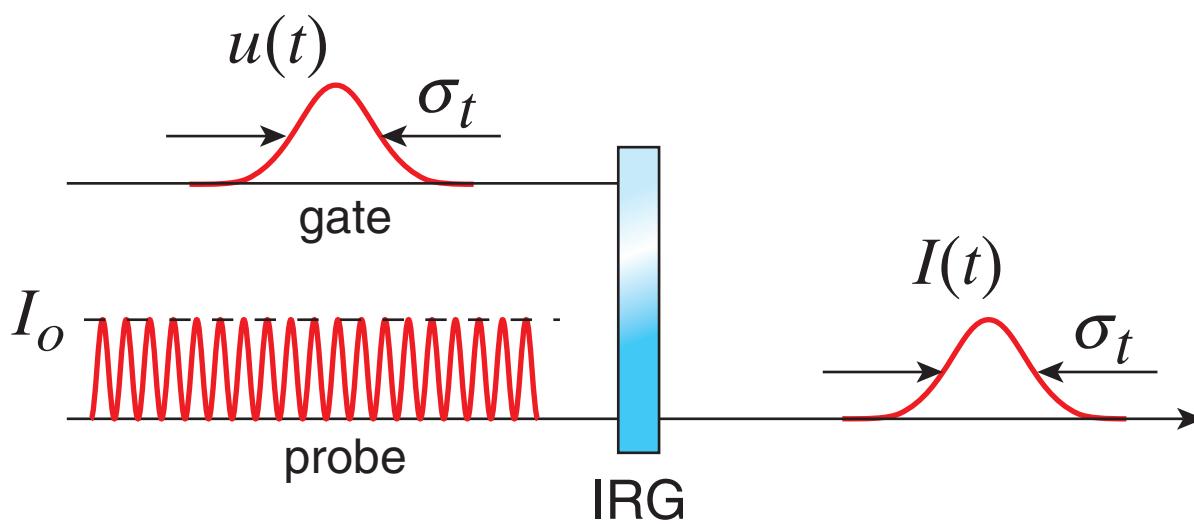
Joint time-frequency measurements



IRG (“instantaneous response gate”): device whose transmittance of a weak probe pulse is proportional to the intensity envelope of the pump (“gate”)

$$T(t) = u(t)$$

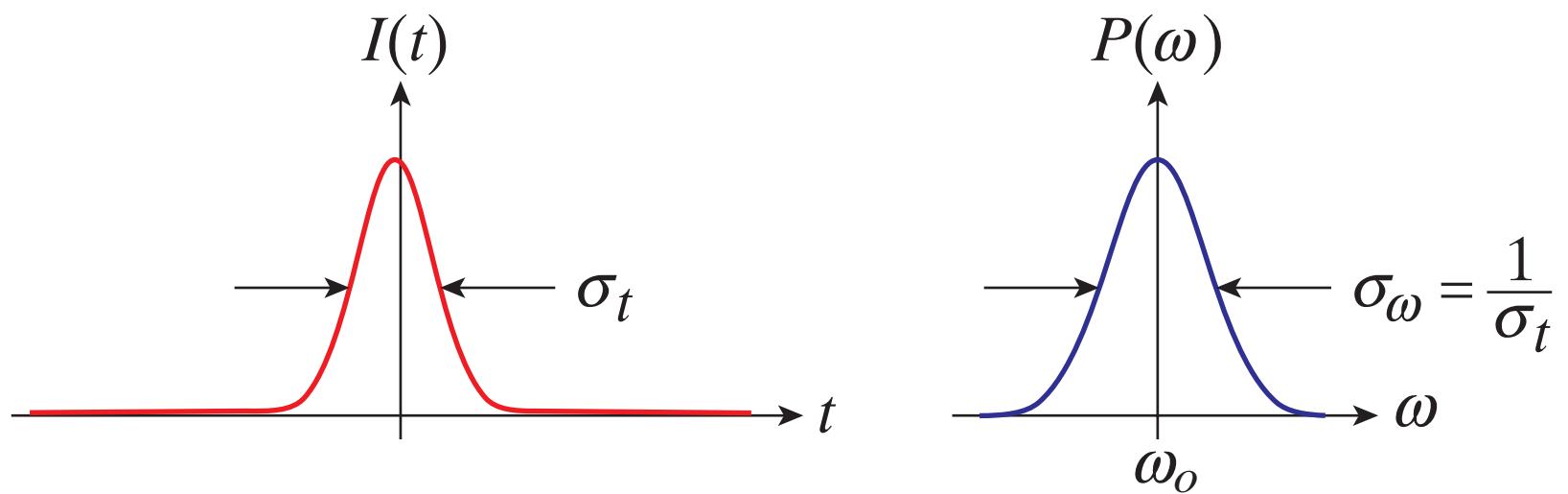
Joint time-frequency measurements



Transmitted intensity

$$I(t) = I_o T(t) = I_o u(t) = I_o \exp\left[-\frac{t^2}{\sigma_t^2}\right]$$

Joint time-frequency measurements

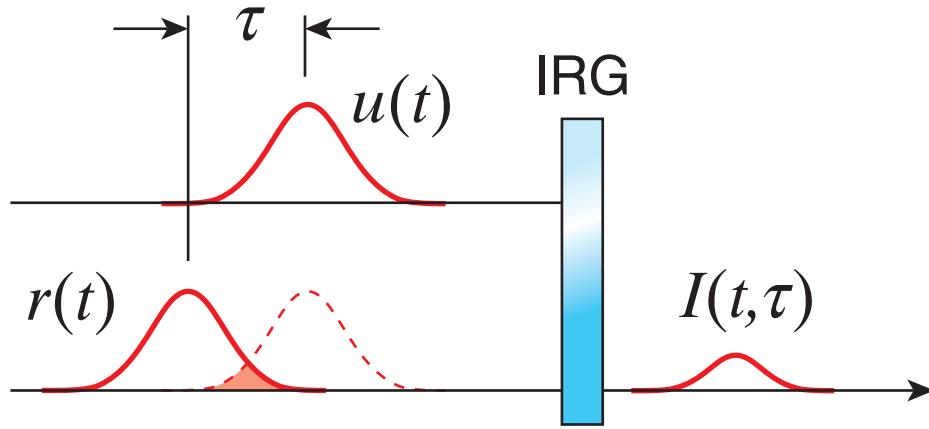


Transmitted intensity

$$I(t) = I_o T(t) = I_o u(t) = I_o \exp\left[-\frac{t^2}{\sigma_t^2}\right]$$

$$\sigma_t \sigma_\omega = 1$$

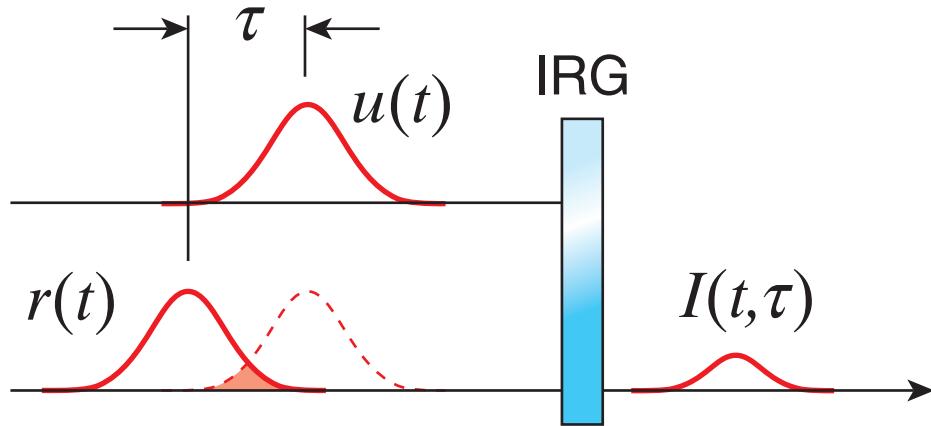
Joint time-frequency measurements



Transmitted intensity

$$\begin{aligned} I(t, \tau) &= u(t)r(t+\tau) = \exp\left[-\frac{t^2}{\sigma^2}\right] \exp\left[-\left(\frac{t+\tau}{\sigma}\right)^2\right] = \\ &= \exp\left[-\frac{2t^2+2t\tau+\tau^2}{\sigma^2}\right] = \exp\left[-\frac{2t^2+2t\tau+\tau^2/2}{\sigma^2}\right] \exp\left[-\frac{\tau^2}{2\sigma^2}\right] = \end{aligned}$$

Joint time-frequency measurements

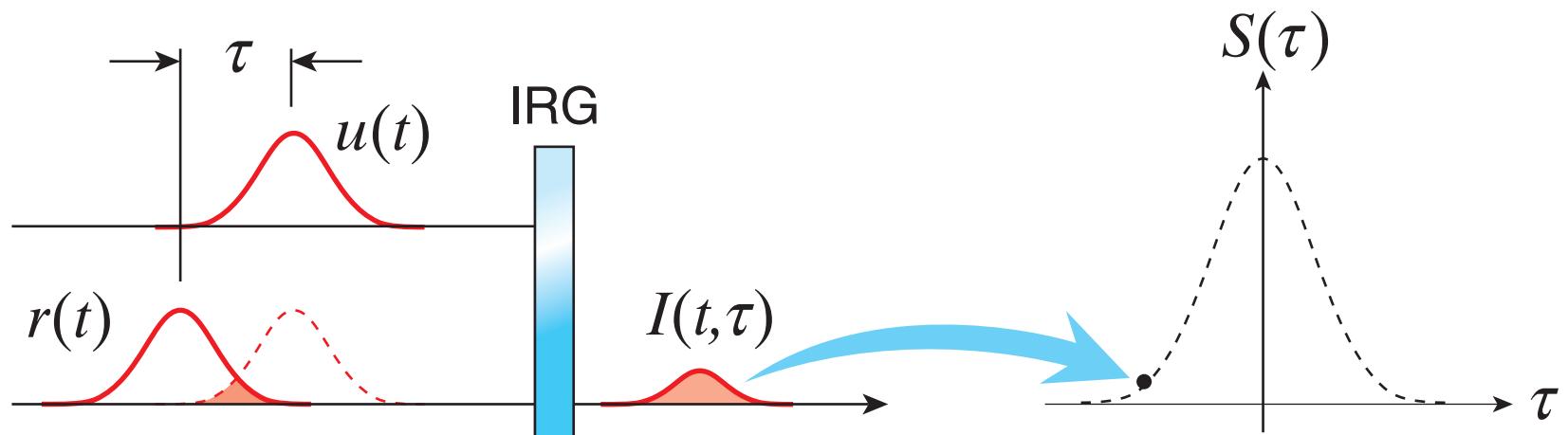


Transmitted intensity

$$I(t, \tau) = \exp\left[-\frac{\tau^2}{2\sigma^2}\right] \exp\left[-\left(\frac{t + \tau/2}{\sigma/\sqrt{2}}\right)^2\right]$$

so $I(t, \tau)$ narrowed by $\sqrt{2}$

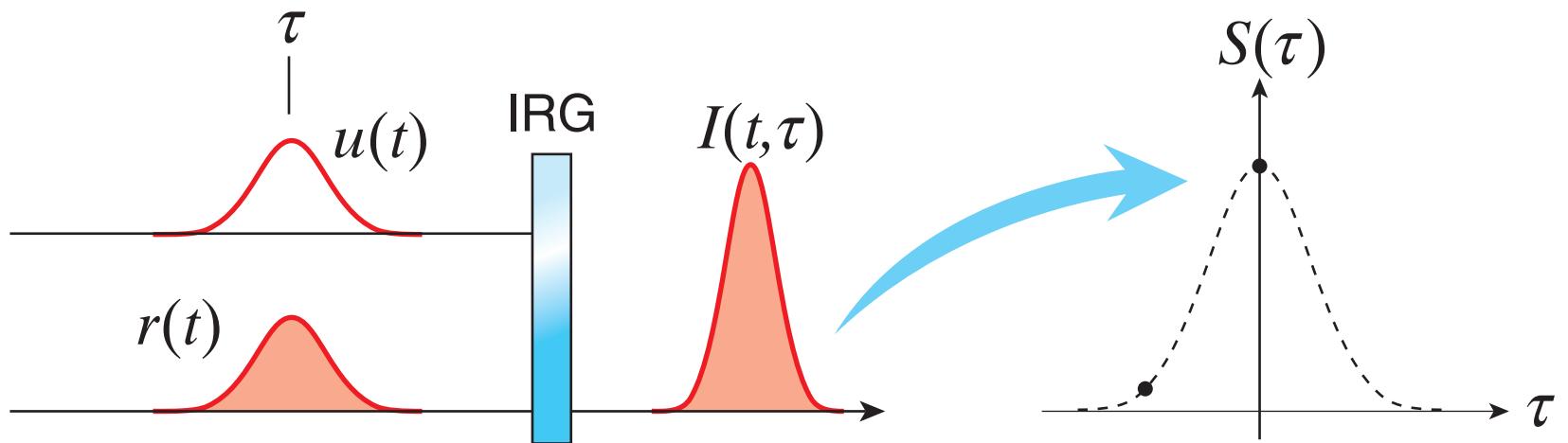
Joint time-frequency measurements



...but detector integrates $I(t, \tau)$:

$$S(\tau) = \int_{-\infty}^{\infty} I(t, \tau) dt = \int_{-\infty}^{\infty} \exp\left[-\frac{\tau^2}{2\sigma^2}\right] \exp\left[-2\left(\frac{t + \tau/2}{\sigma^2}\right)^2\right] dt$$

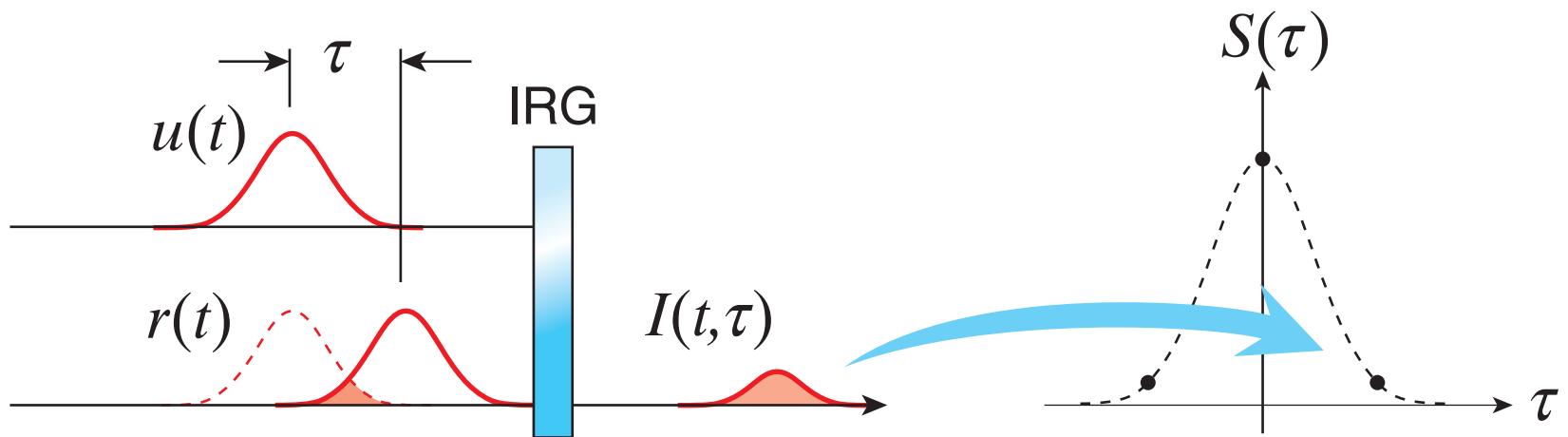
Joint time-frequency measurements



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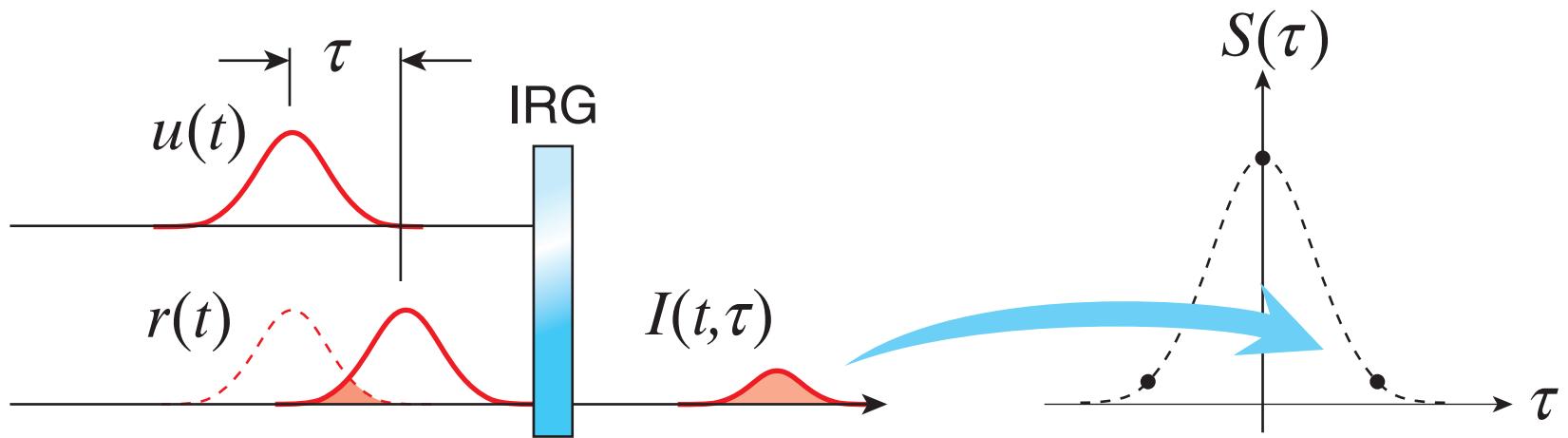
Joint time-frequency measurements



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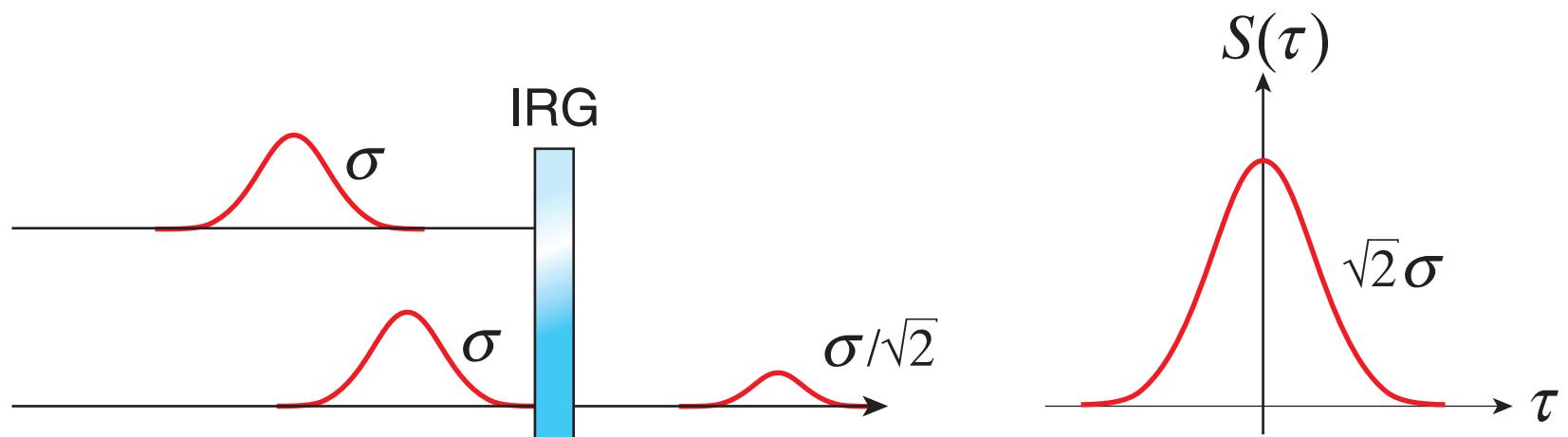
Joint time-frequency measurements



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$$\begin{aligned} S(\tau) &= \int_{-\infty}^{\infty} I(t, \tau) dt = \int_{-\infty}^{\infty} \exp\left[-\frac{\tau^2}{2\sigma^2}\right] \exp\left[-2\left(\frac{t+\tau/2}{\sigma}\right)^2\right] dt \\ &= \exp\left[-\frac{\tau^2}{2\sigma^2}\right] \int_{-\infty}^{\infty} \exp\left[-\frac{2u^2}{\sigma^2}\right] du = \sqrt{\frac{\pi}{2}} \sigma \exp\left[-\frac{\tau^2}{(\sigma\sqrt{2})^2}\right] \end{aligned}$$

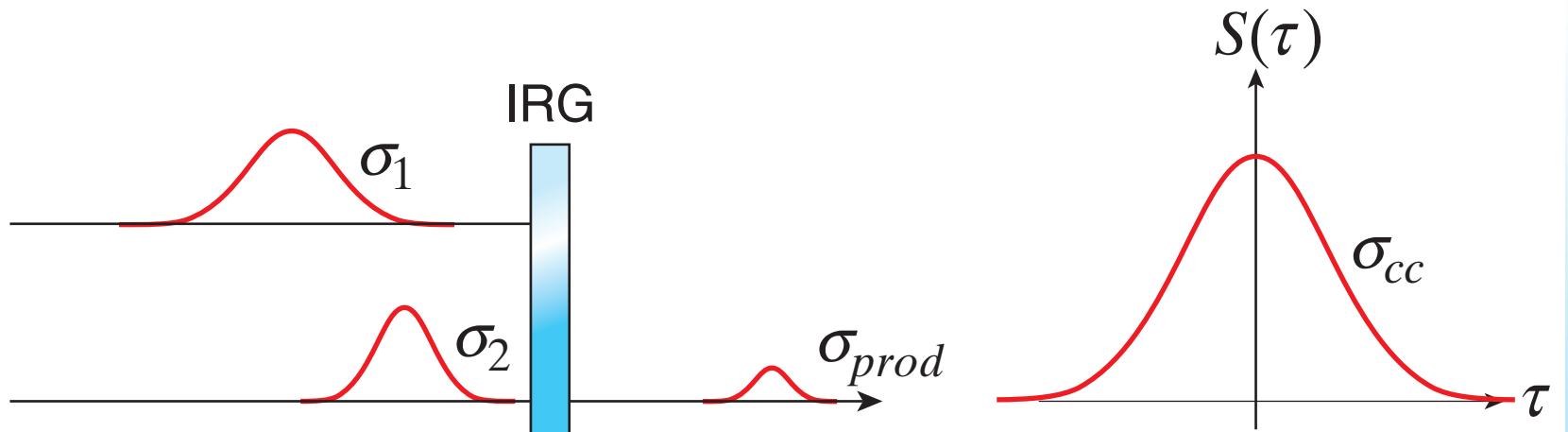
Joint time-frequency measurements



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Joint time-frequency measurements

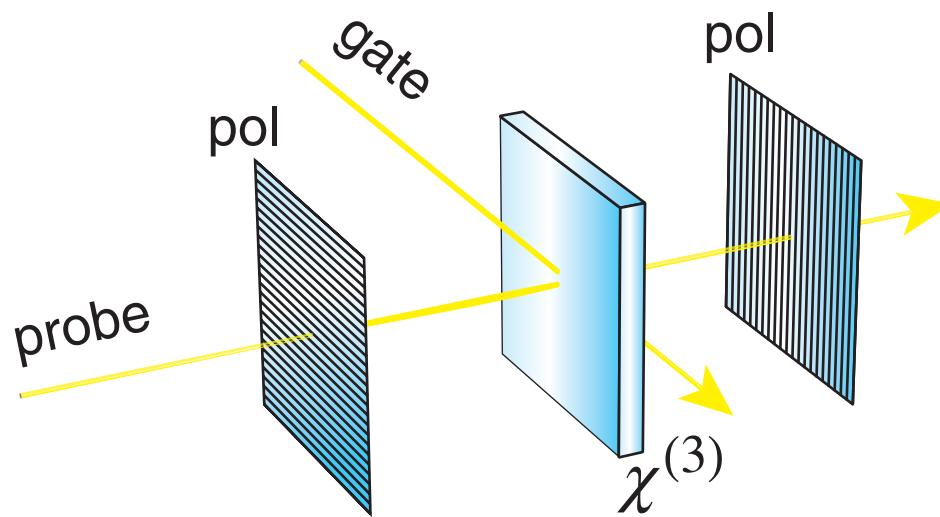


If gate and probe unequal:

$$\sigma_{prod}^2 = \frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 + \sigma_2^2} \quad (\text{narrower than both})$$

$$\sigma_{cc}^2 = \sigma_1^2 + \sigma_2^2 \quad (\text{wider than both})$$

Joint time-frequency measurements

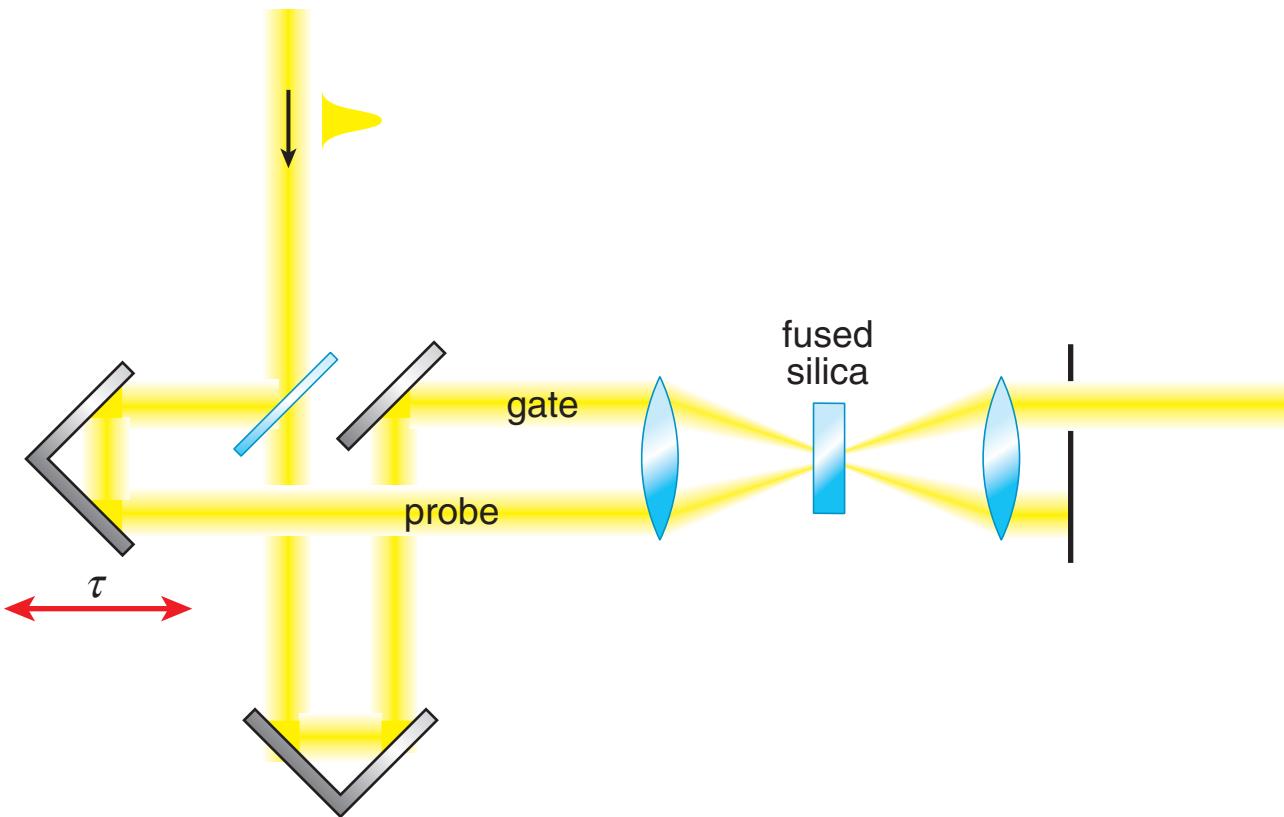


Transmitted field:

$$E_{trans}(t, \tau) \propto \chi^{(3)} E_{probe}(t) |E_{gate}(t + \tau)|^2$$

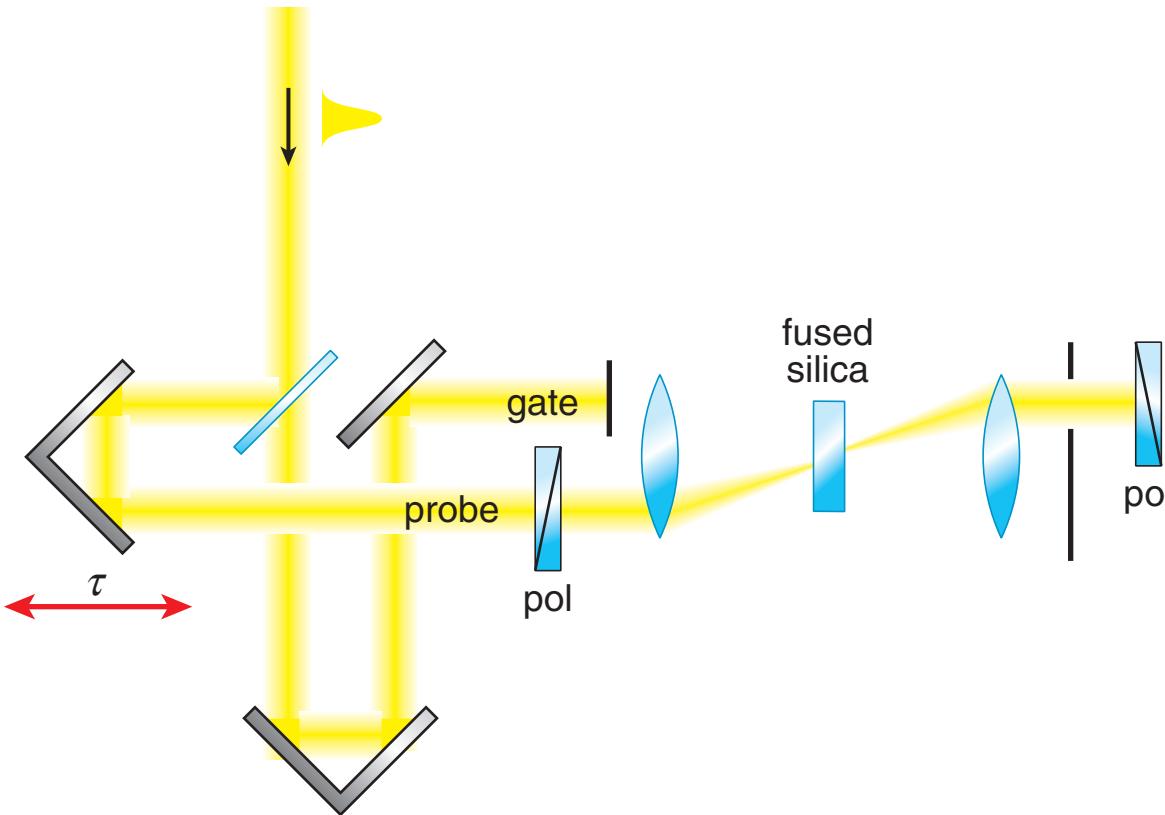
Joint time-frequency measurements

FROG: frequency-resolved optical gating



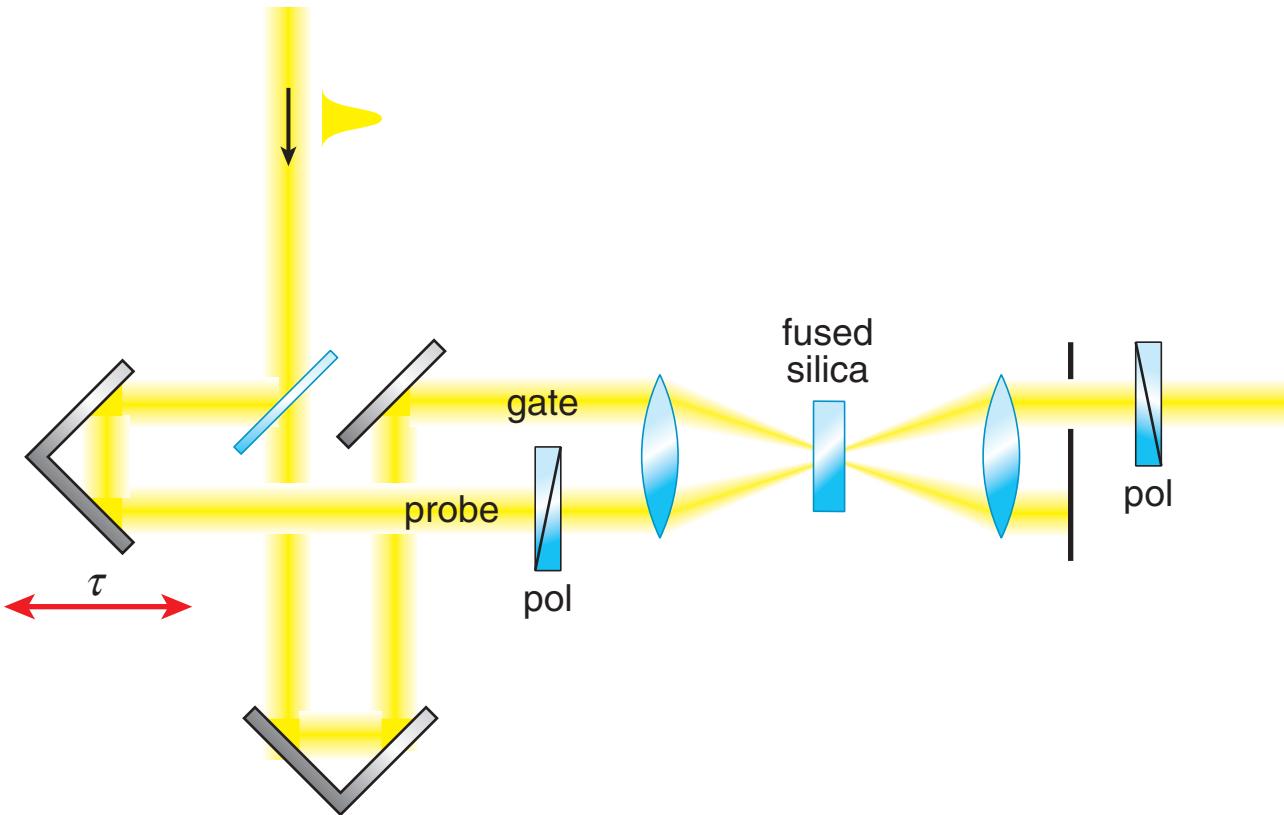
Joint time-frequency measurements

FROG: frequency-resolved optical gating



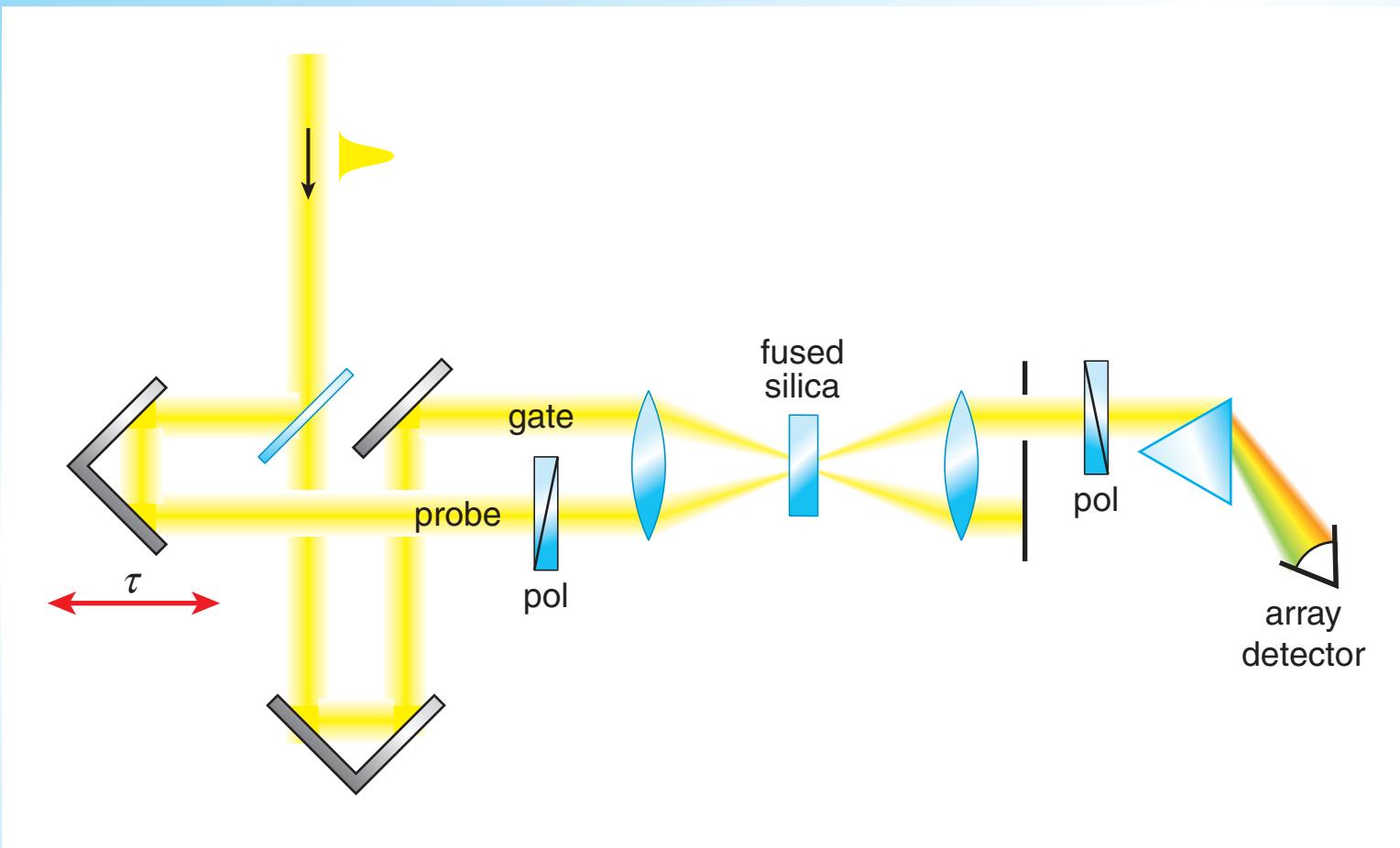
Joint time-frequency measurements

FROG: frequency-resolved optical gating

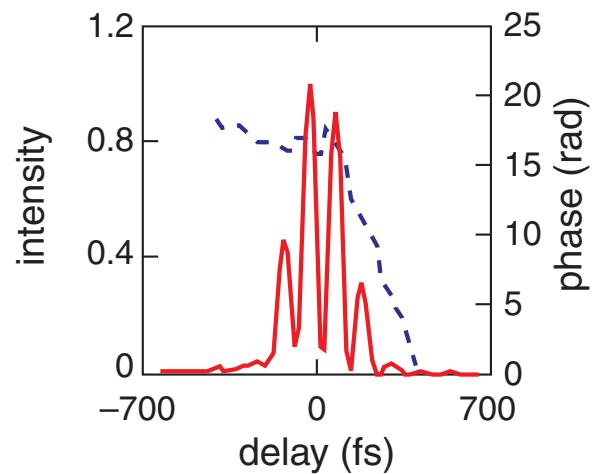
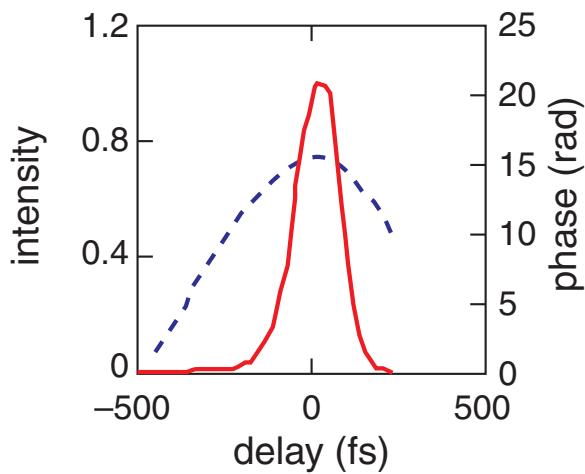
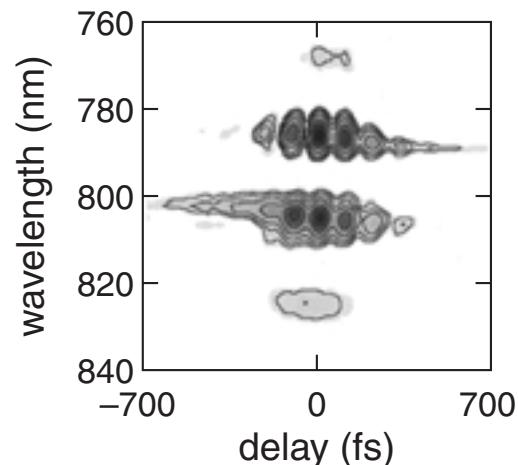
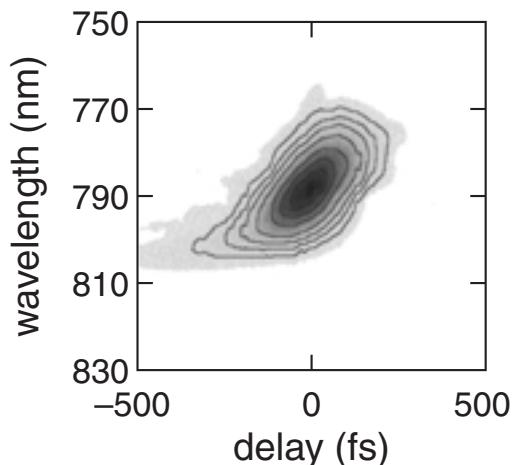


Joint time-frequency measurements

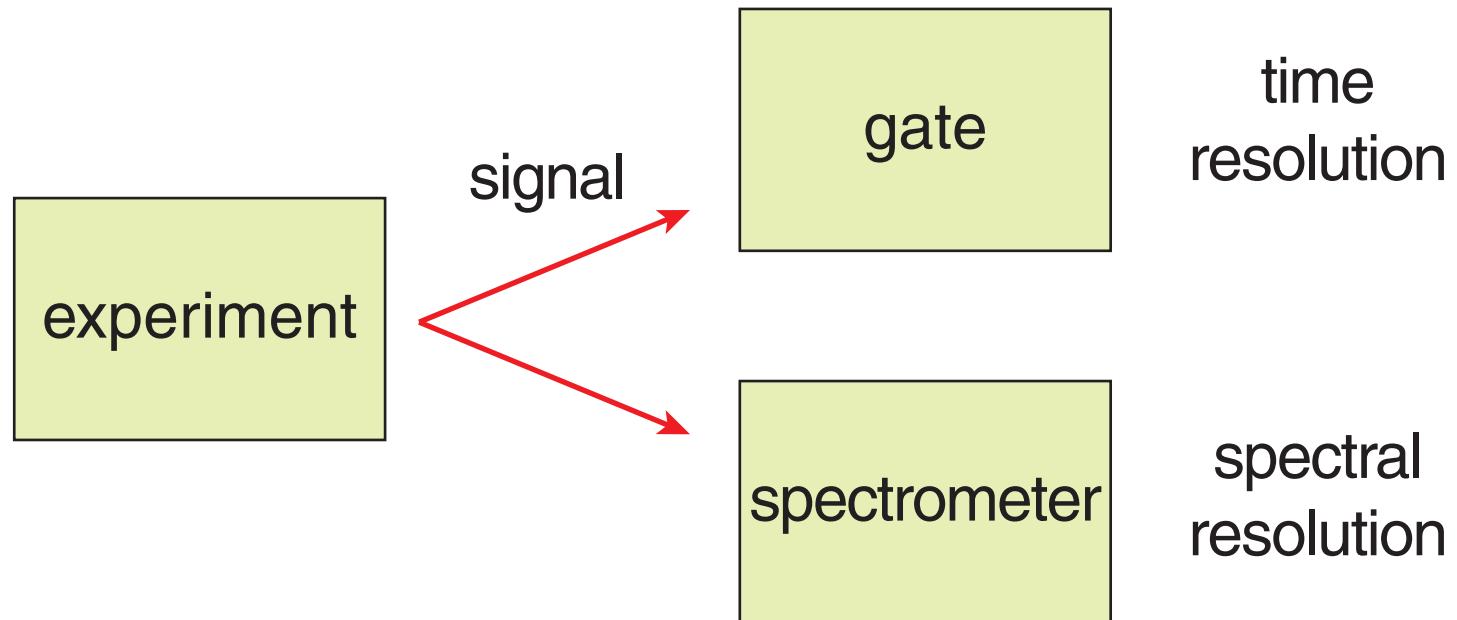
FROG: frequency-resolved optical gating



Joint time-frequency measurements



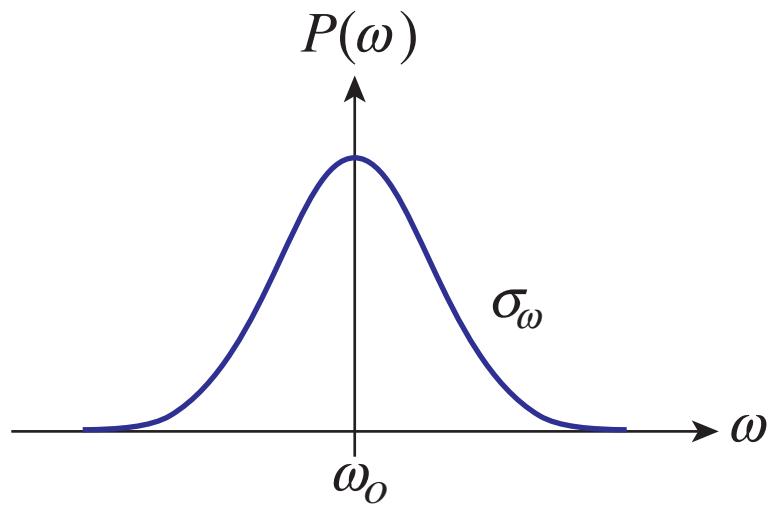
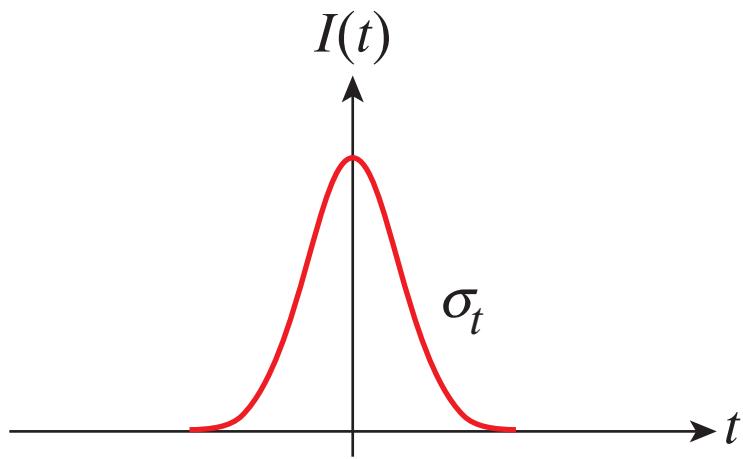
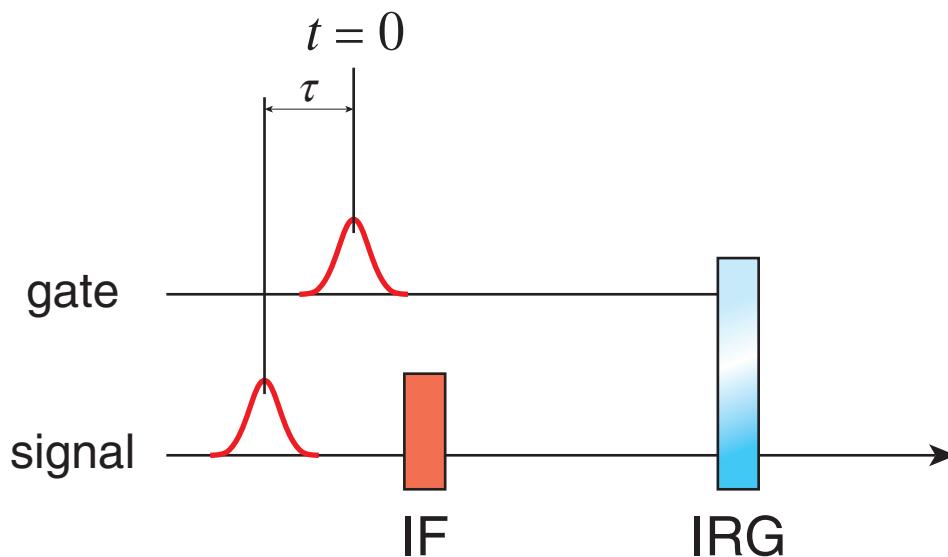
Joint time-frequency measurements



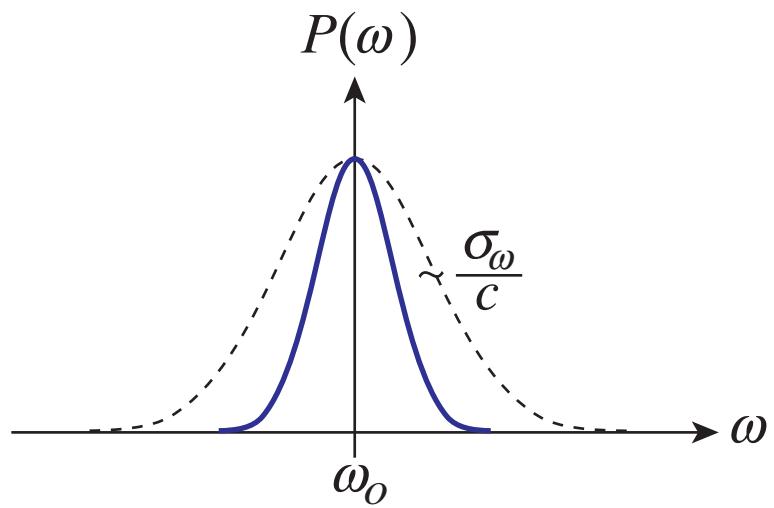
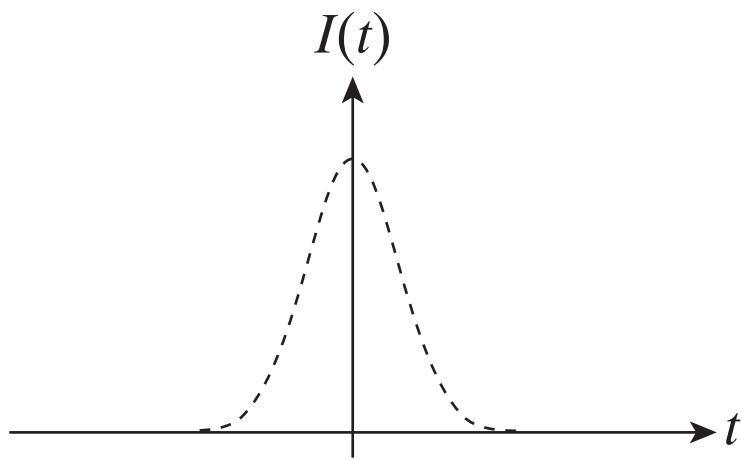
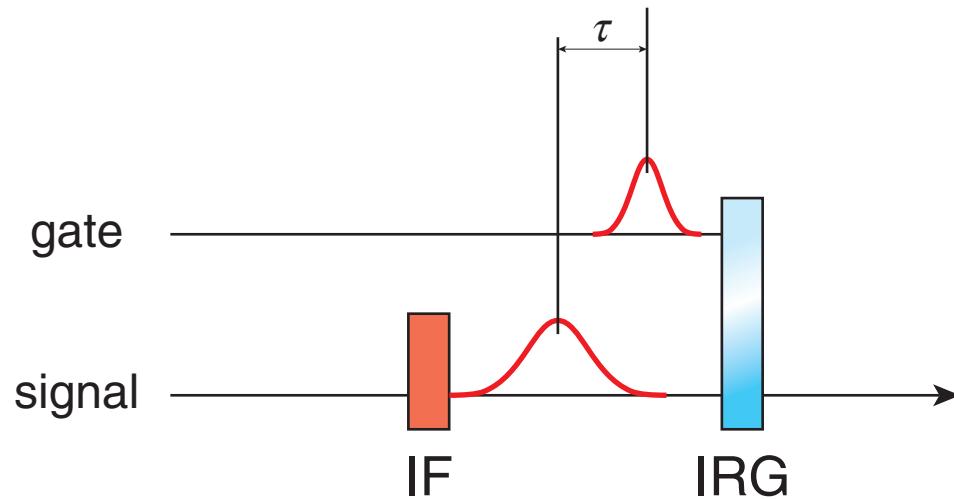
Joint time-frequency measurements

What are the resolution limits?

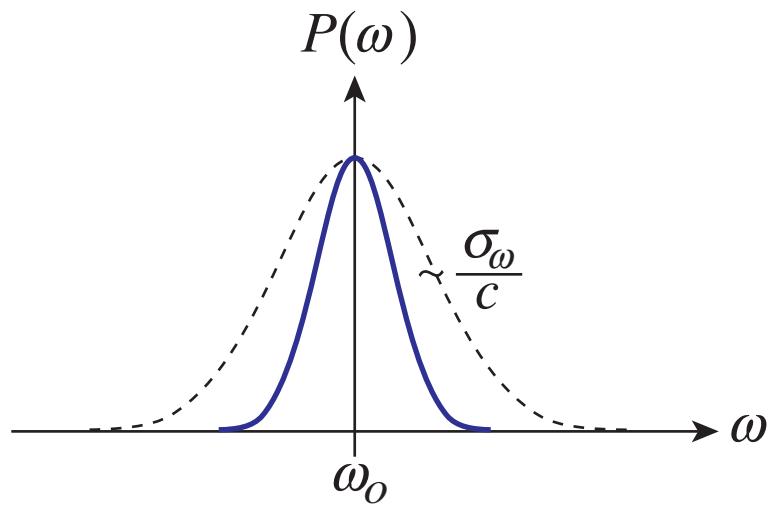
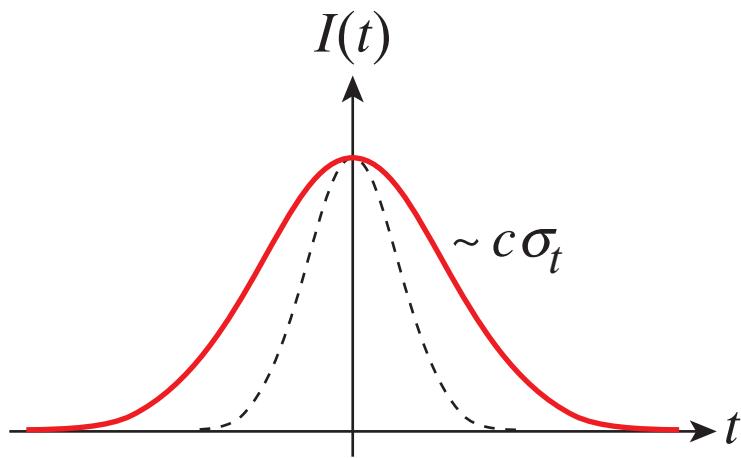
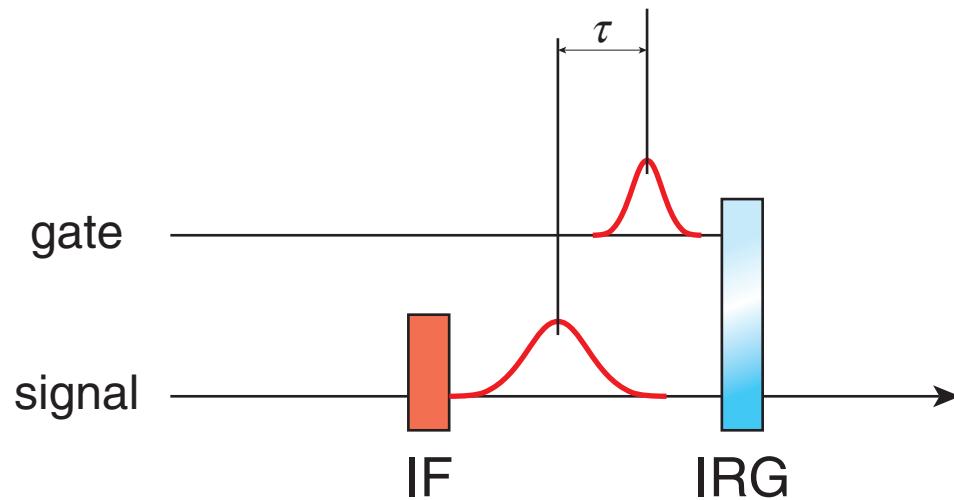
Experiment 1



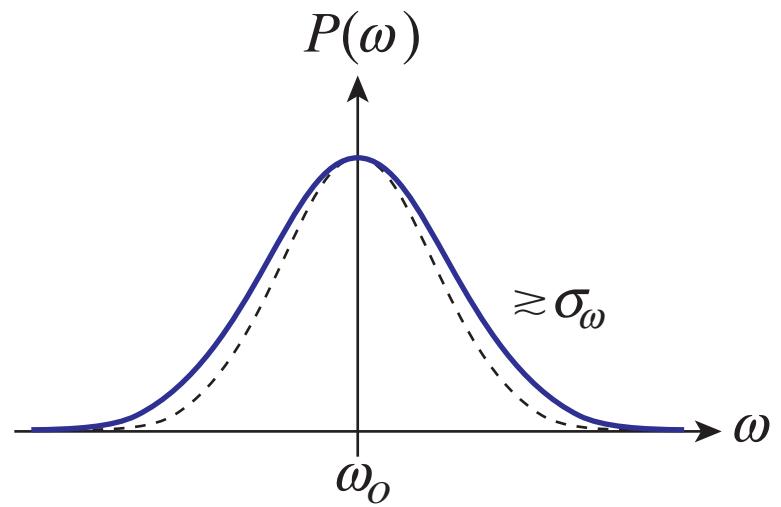
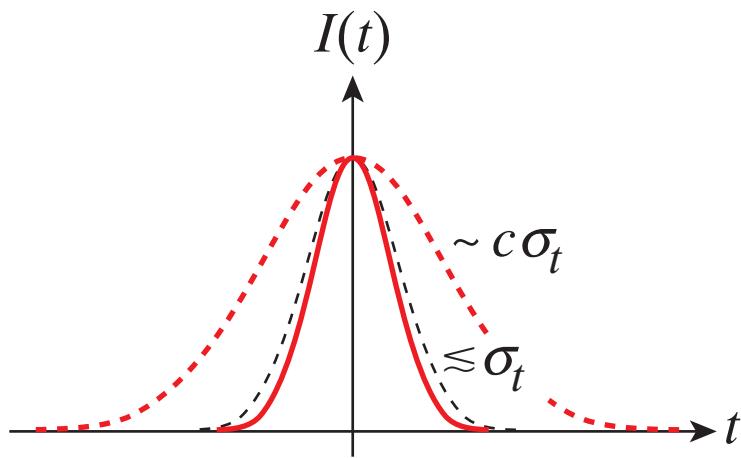
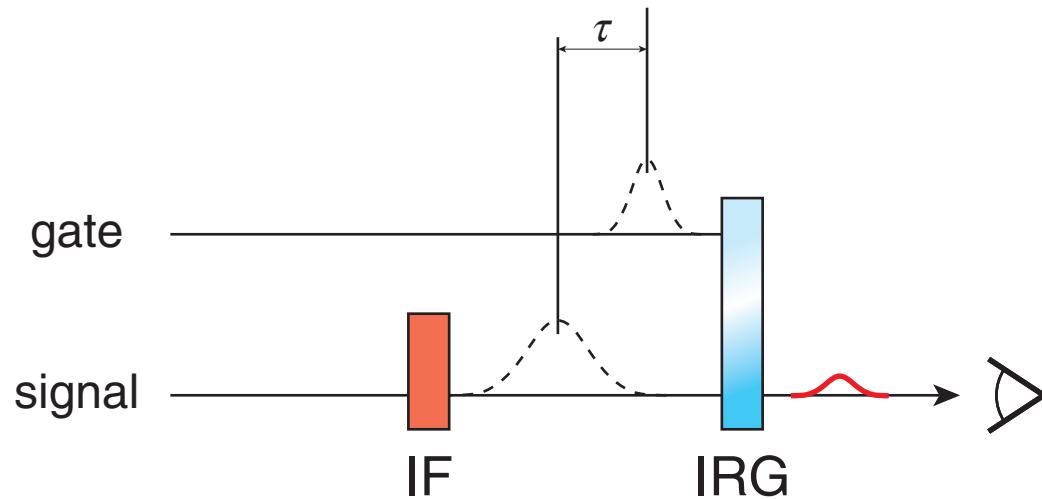
Experiment 1



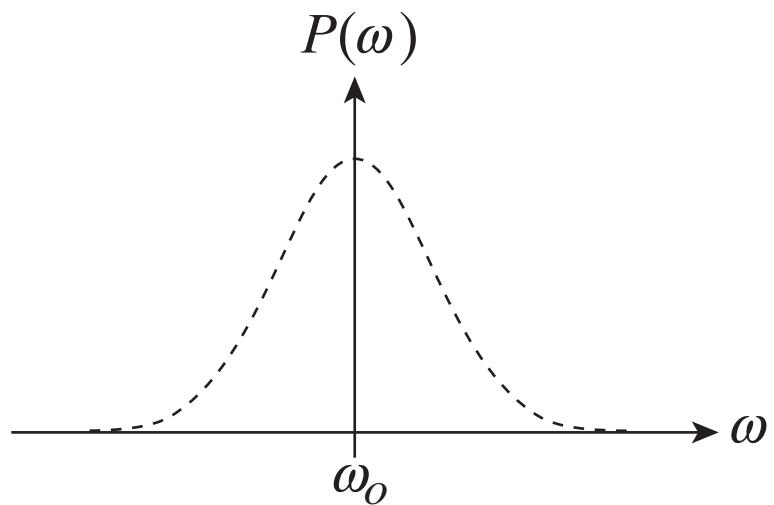
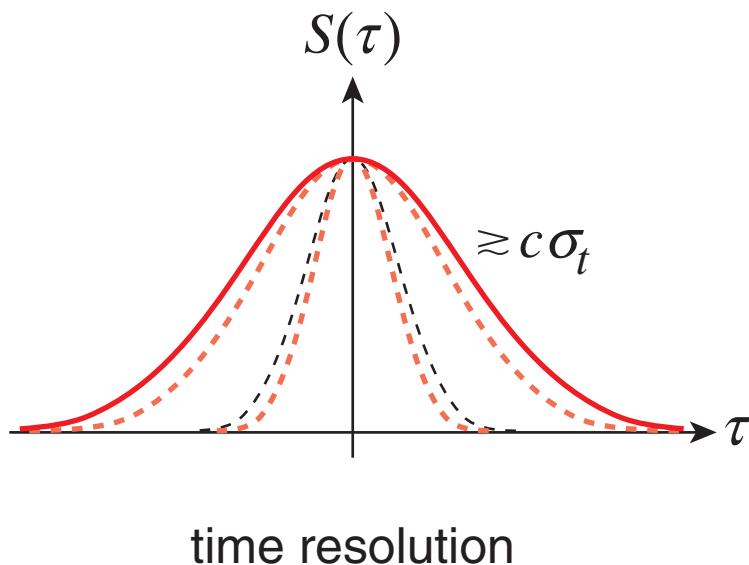
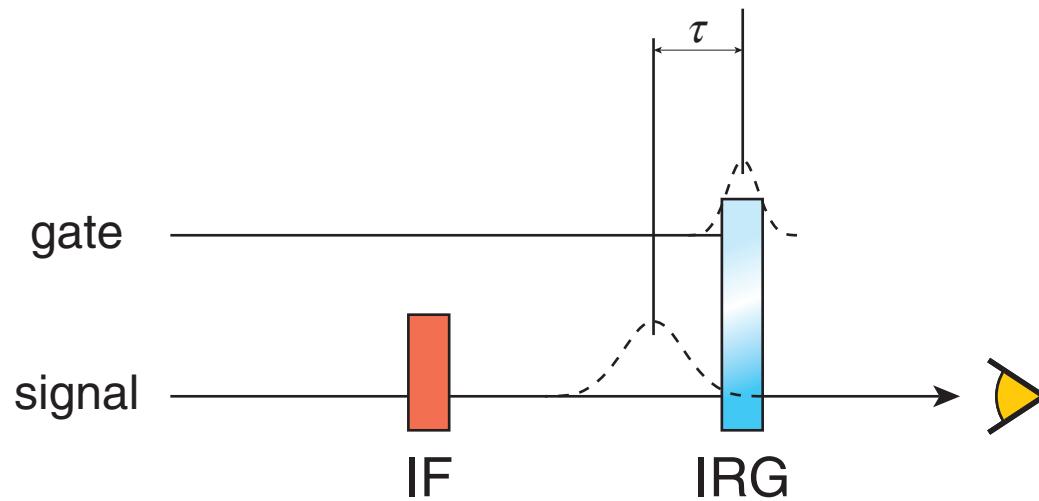
Experiment 1



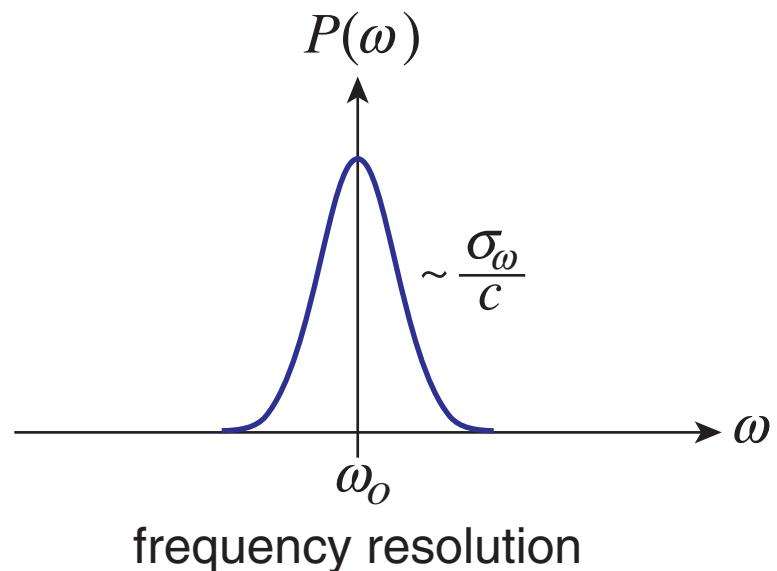
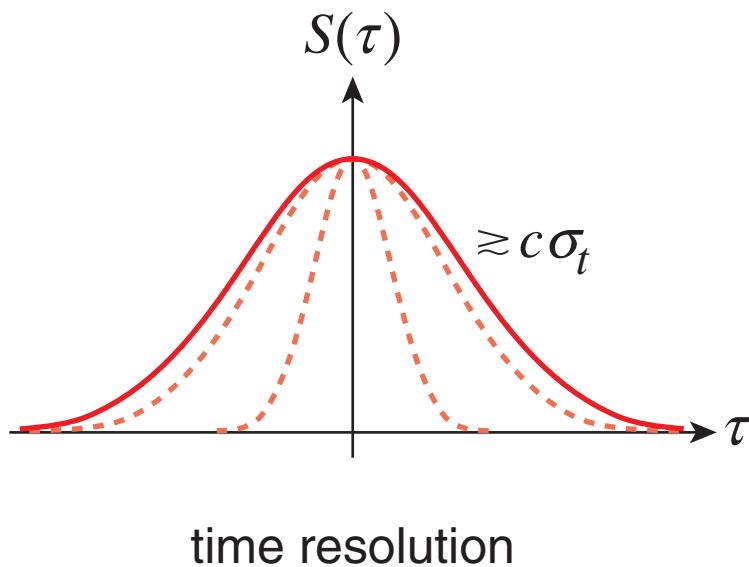
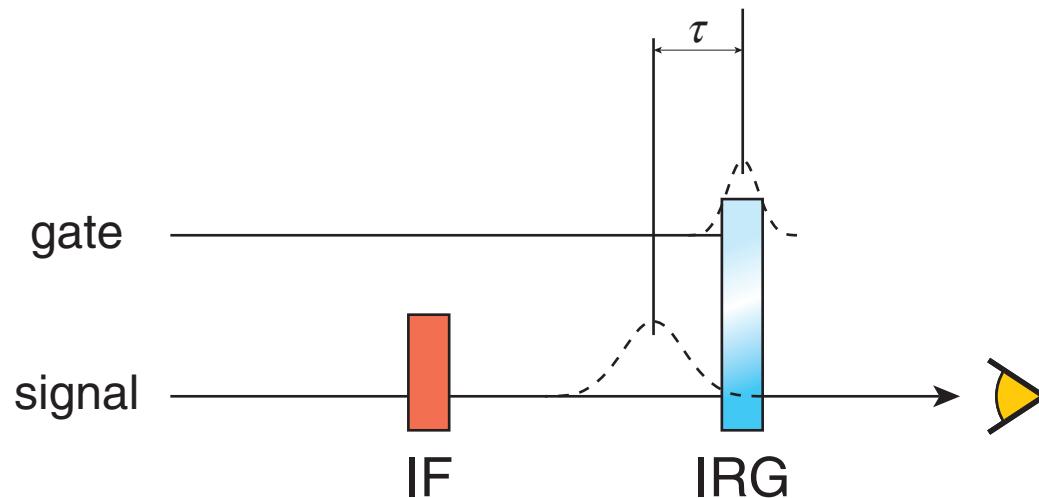
Experiment 1



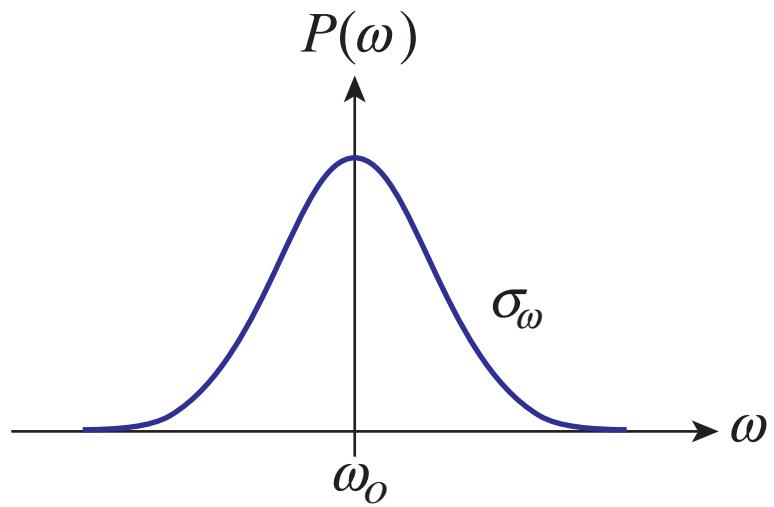
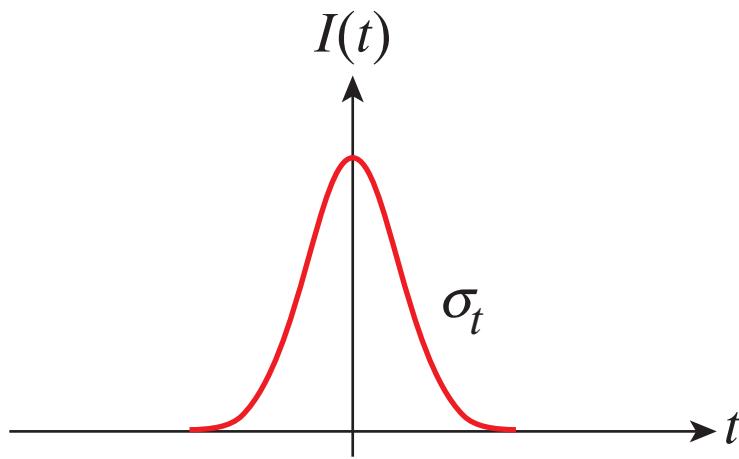
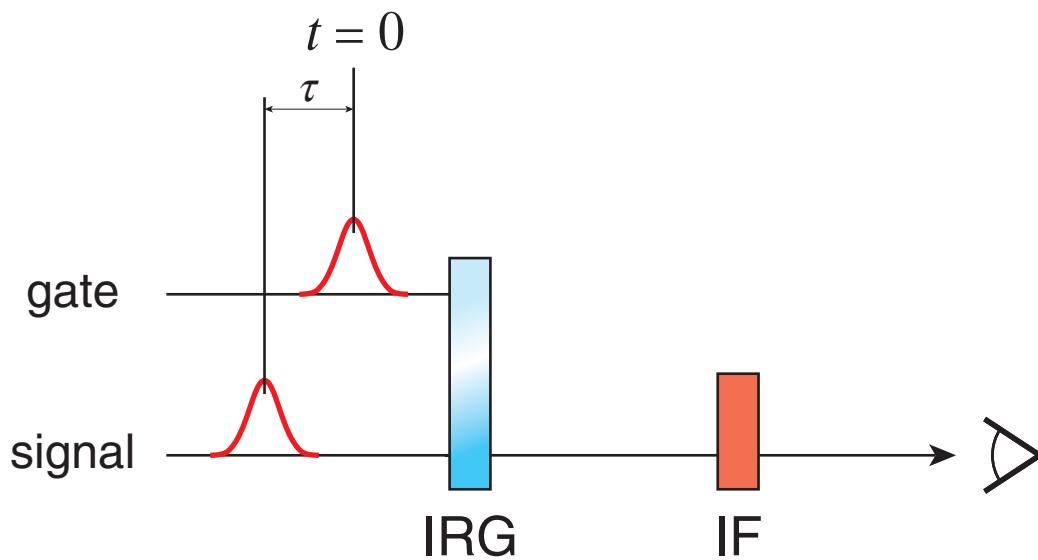
Experiment 1



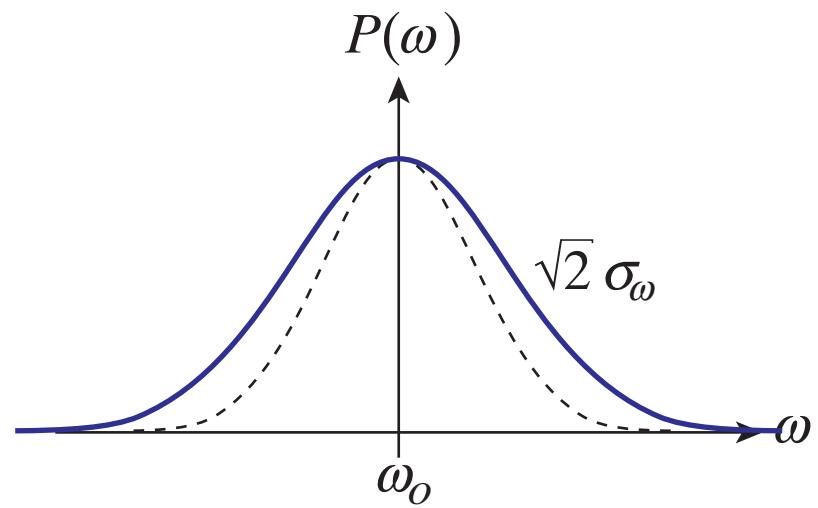
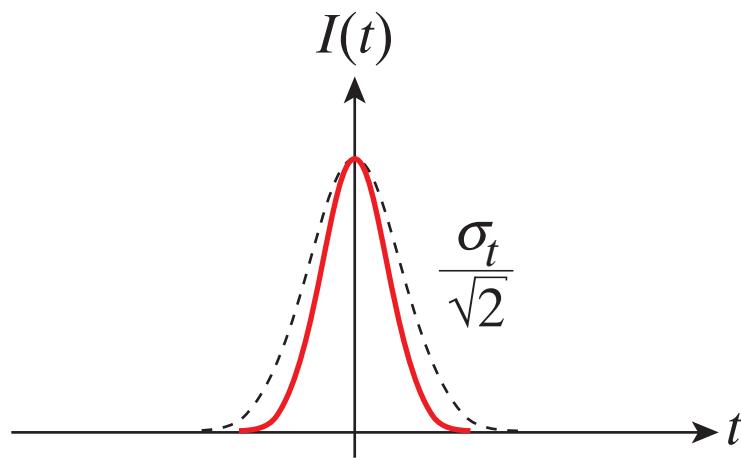
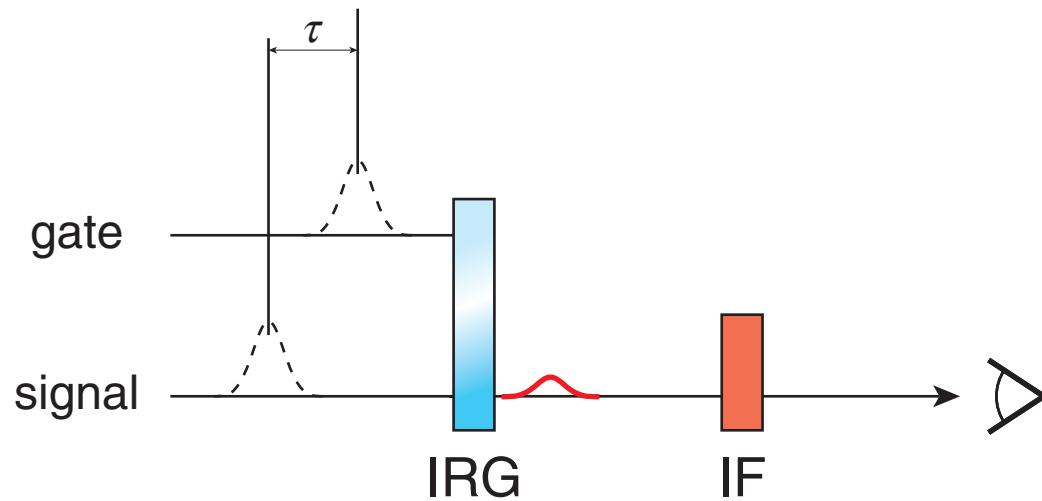
Experiment 1



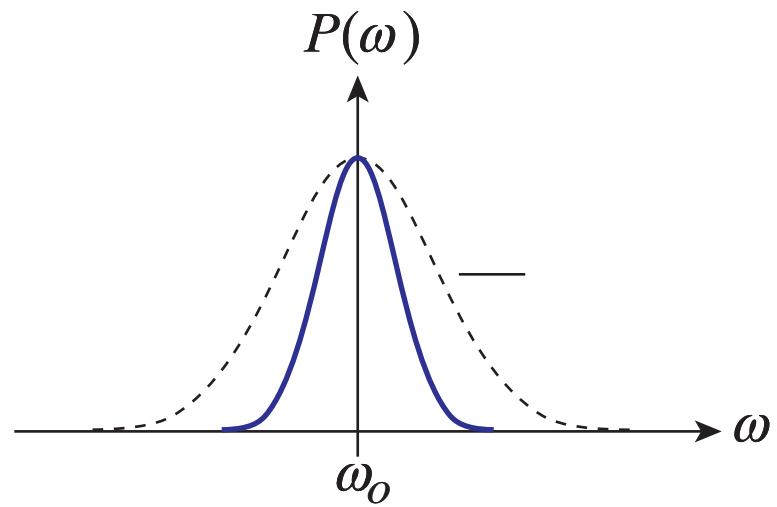
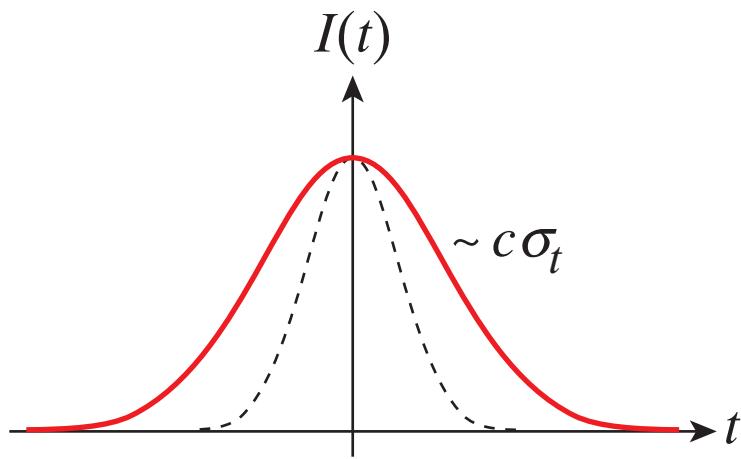
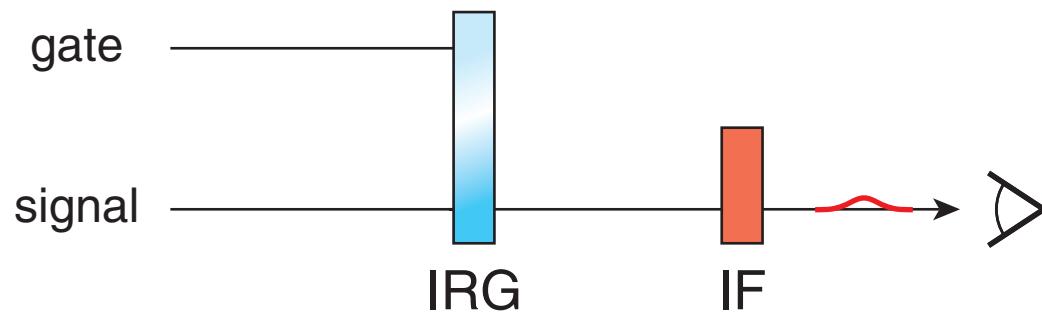
Experiment 2



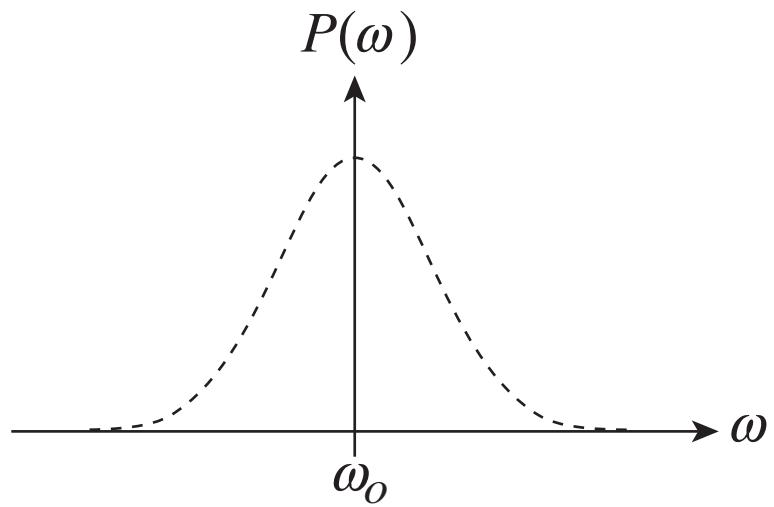
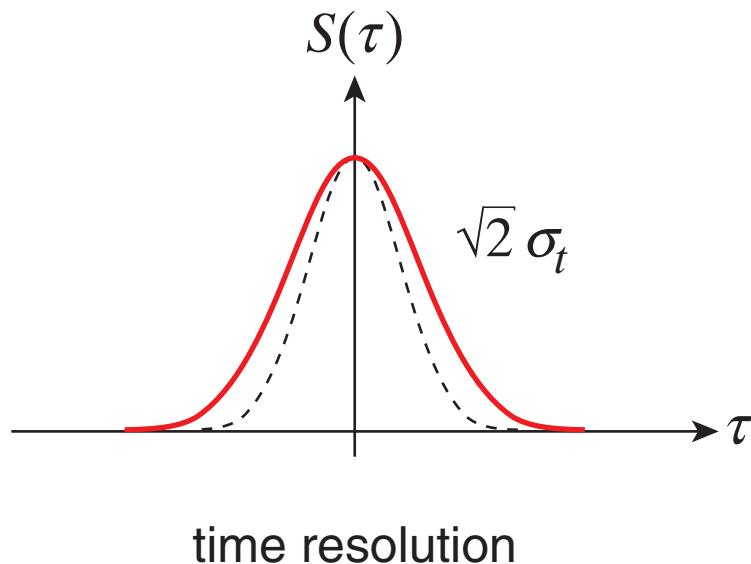
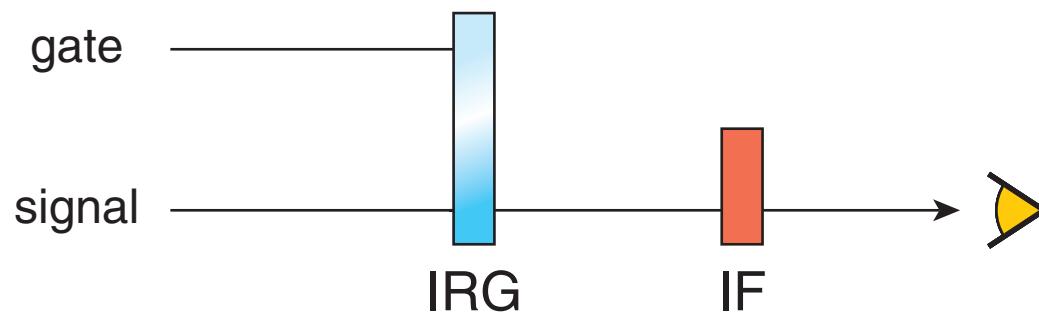
Experiment 2



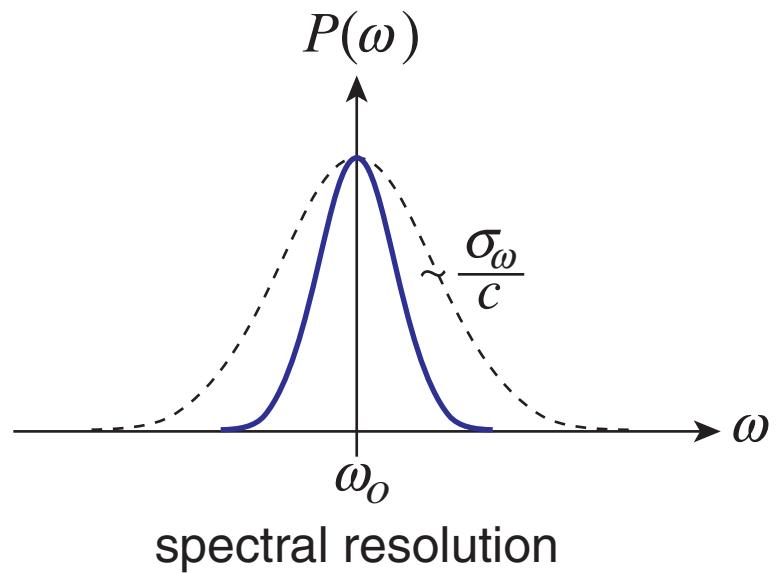
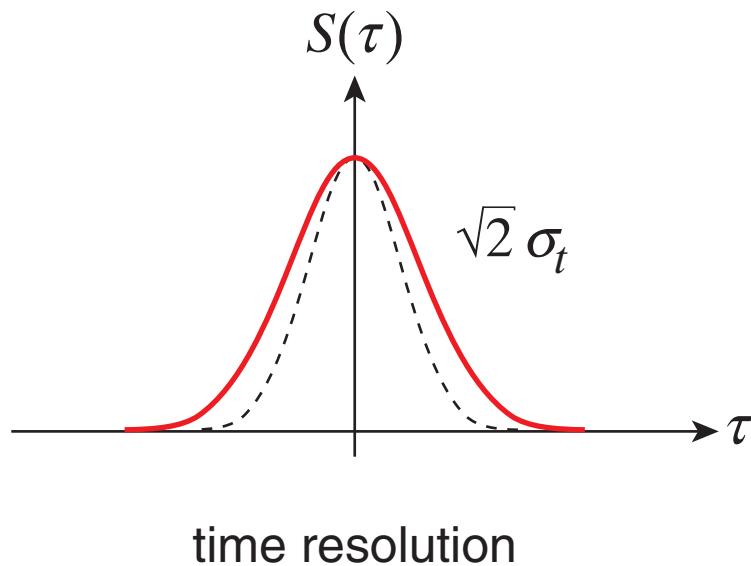
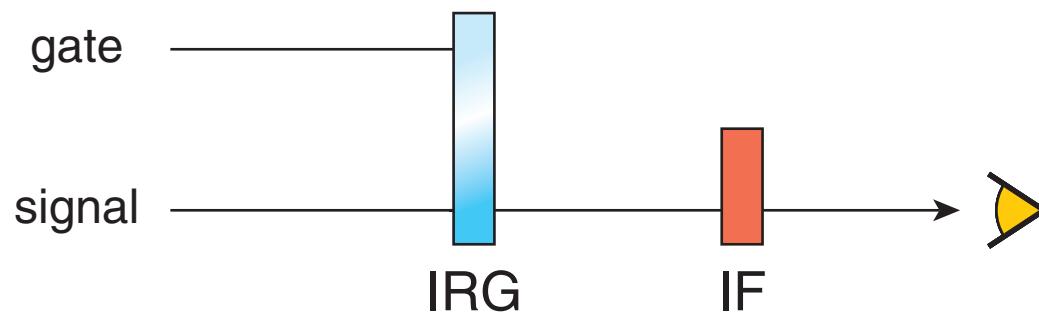
Experiment 2



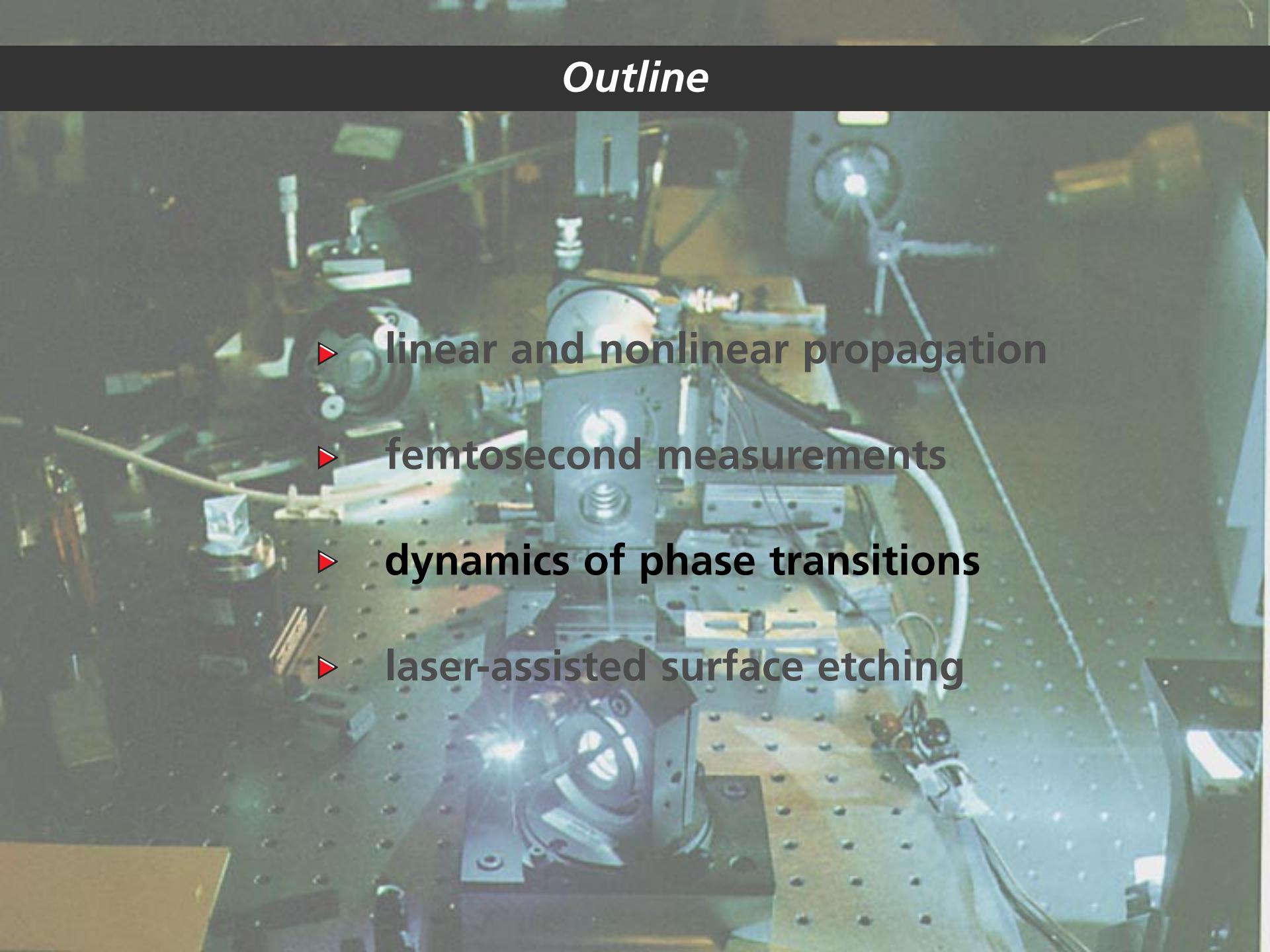
Experiment 2



Experiment 2

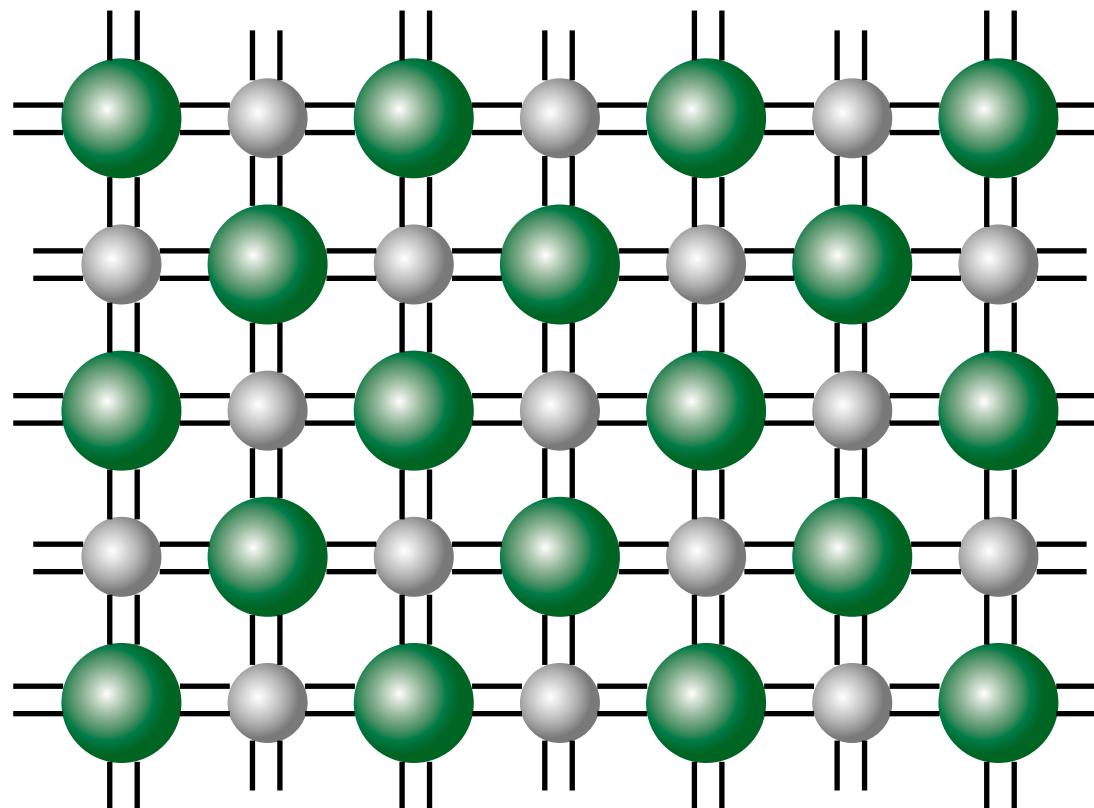


Outline

- 
- ▶ linear and nonlinear propagation
 - ▶ femtosecond measurements
 - ▶ dynamics of phase transitions
 - ▶ laser-assisted surface etching

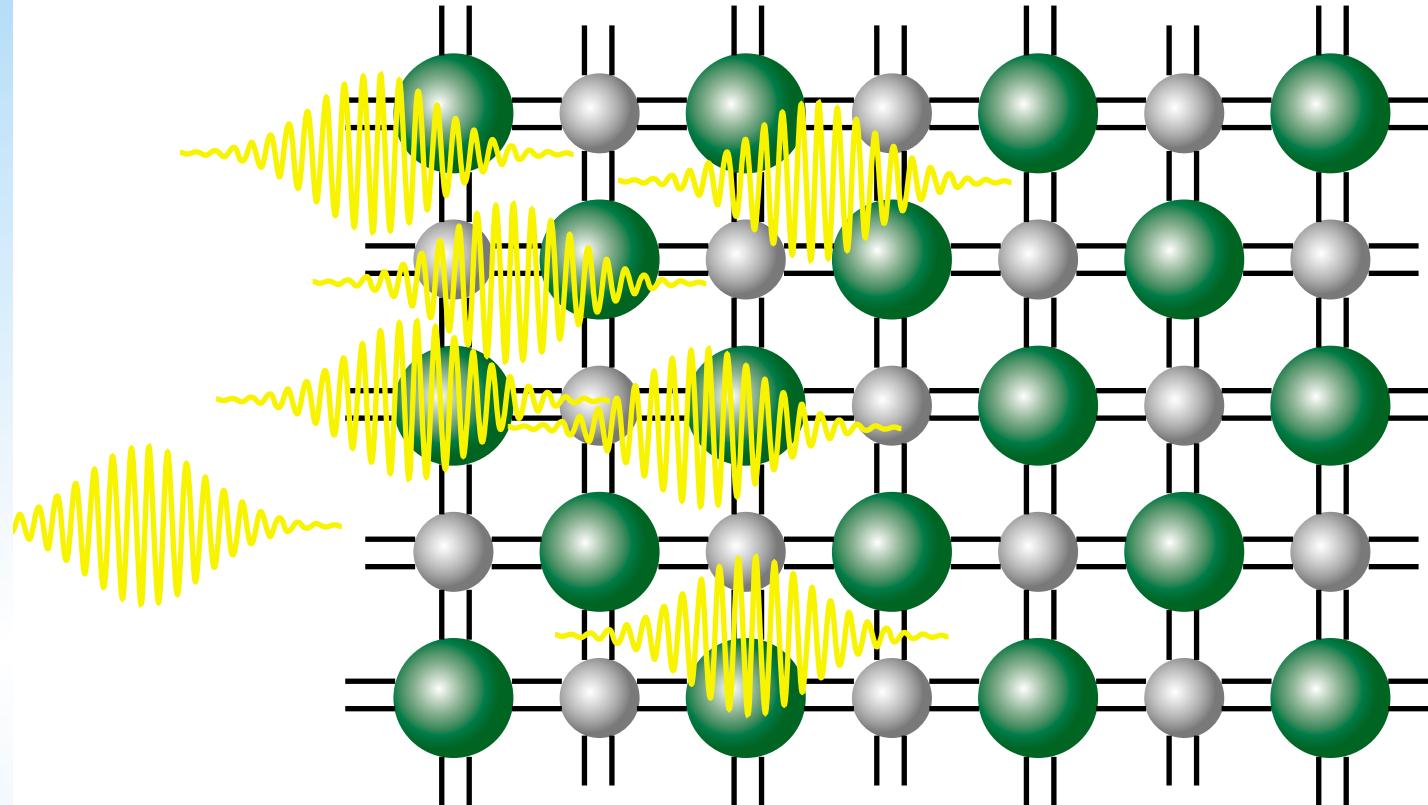
Introduction

how do femtosecond laser pulses alter a solid?



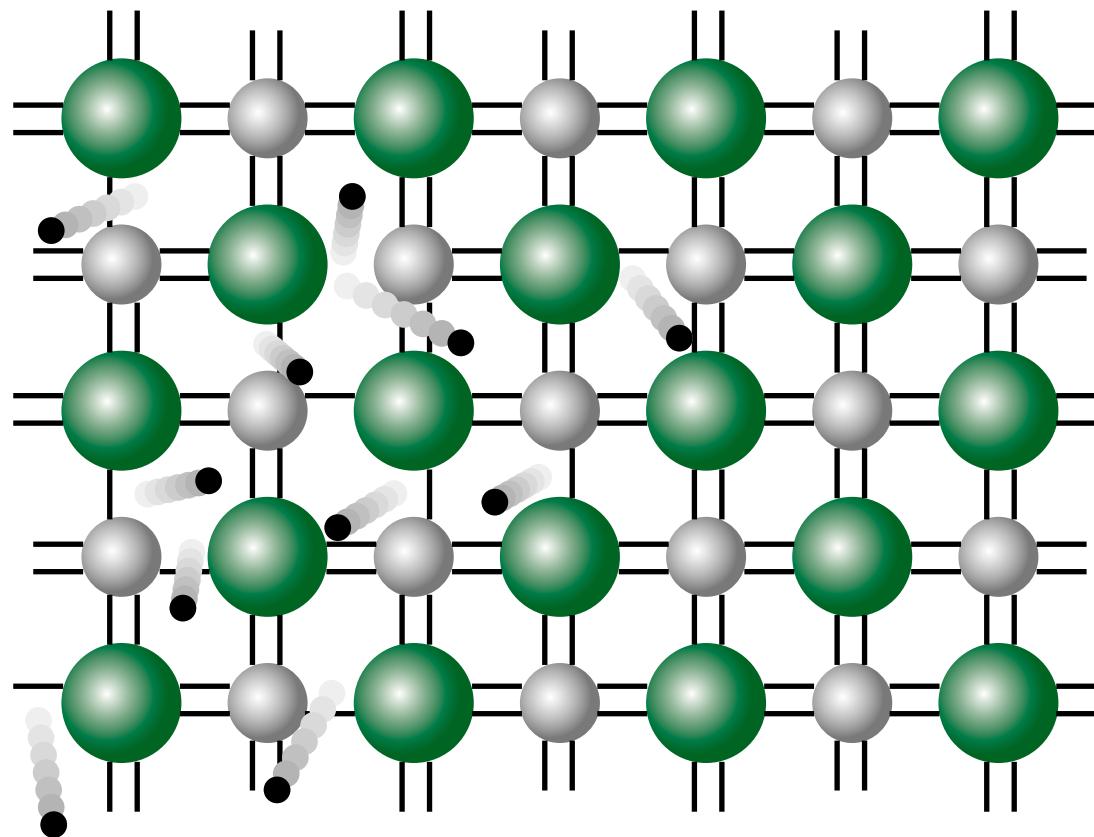
Introduction

photons excite valence electrons...



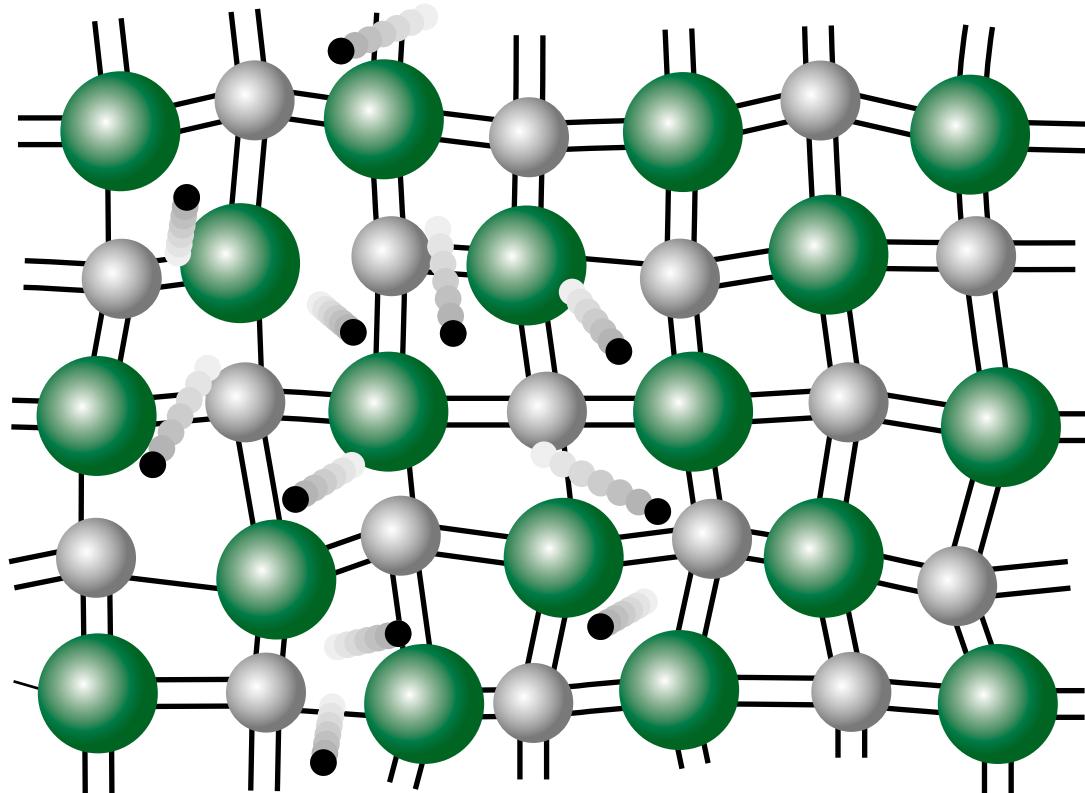
Introduction

...and create free electrons...



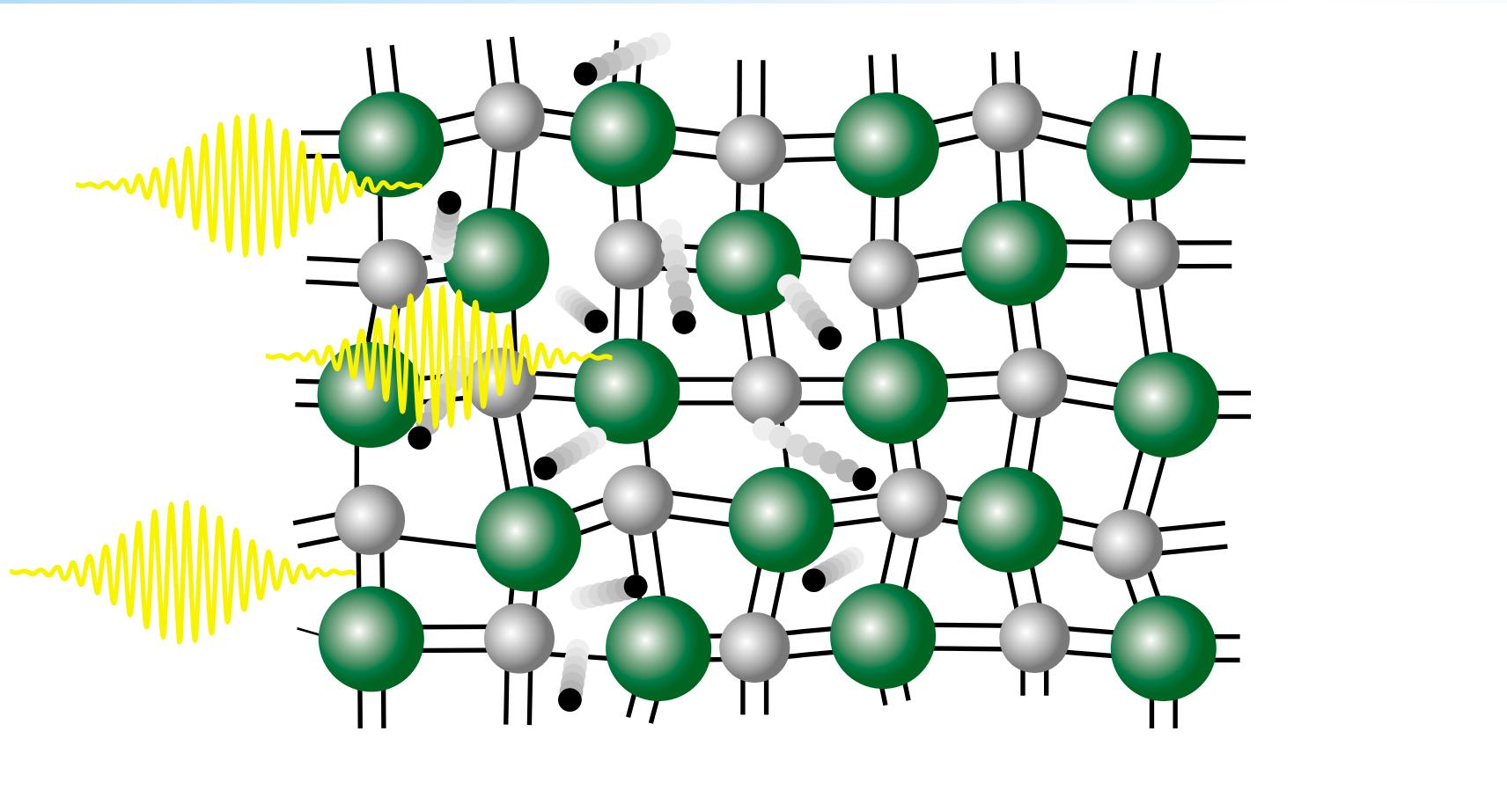
Introduction

...causing electronic and structural changes...



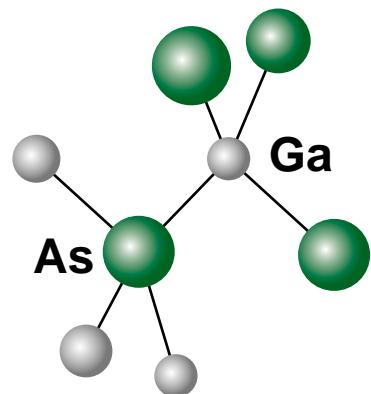
Introduction

...which we measure with another pulse



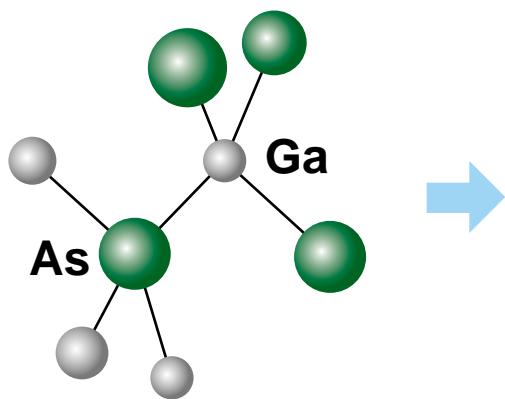
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structure

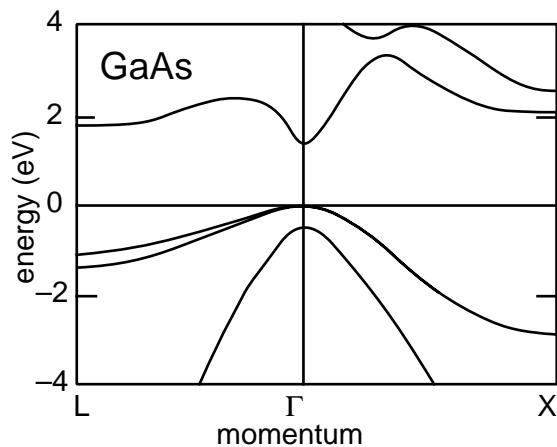


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structure

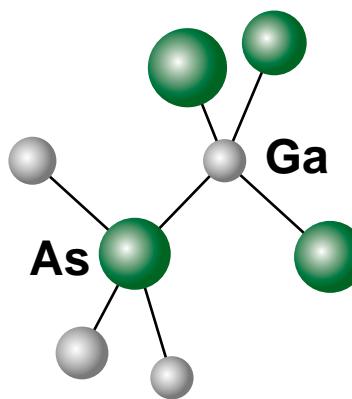


bandstructure

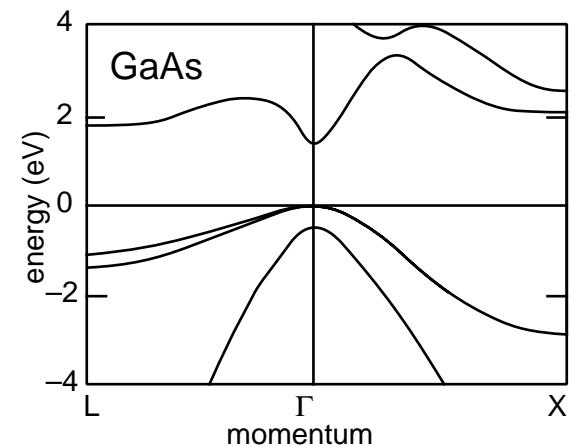


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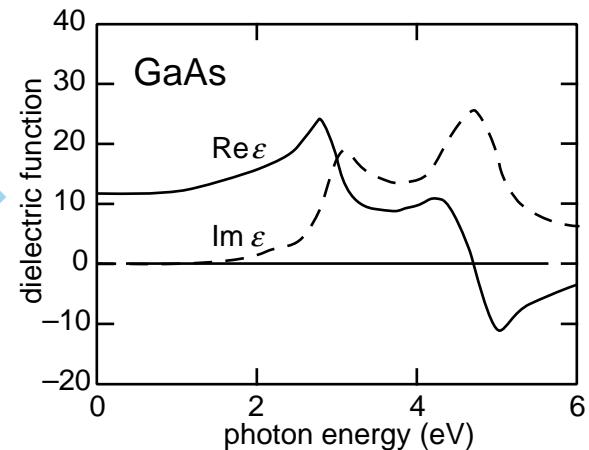
structure



bandstructure

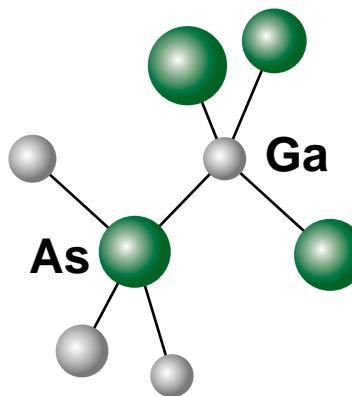


dielectric function

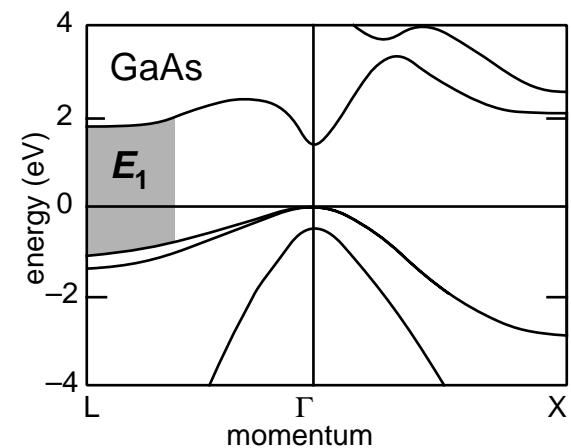


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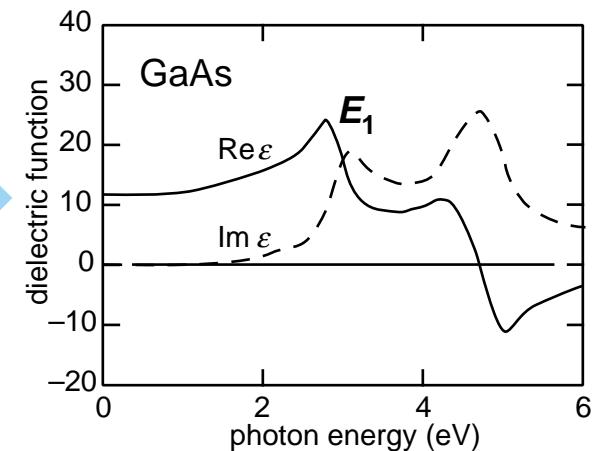
structure



bandstructure

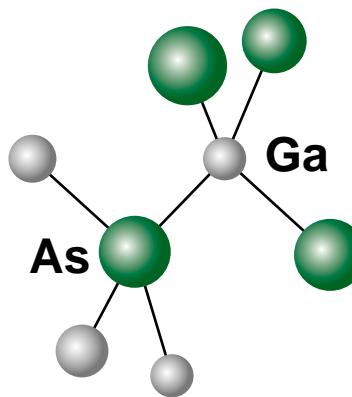


dielectric function

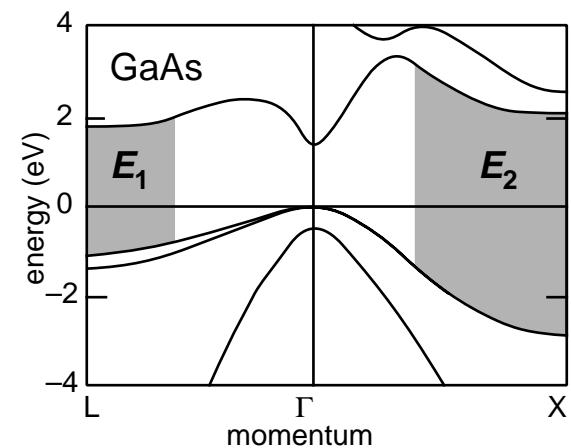


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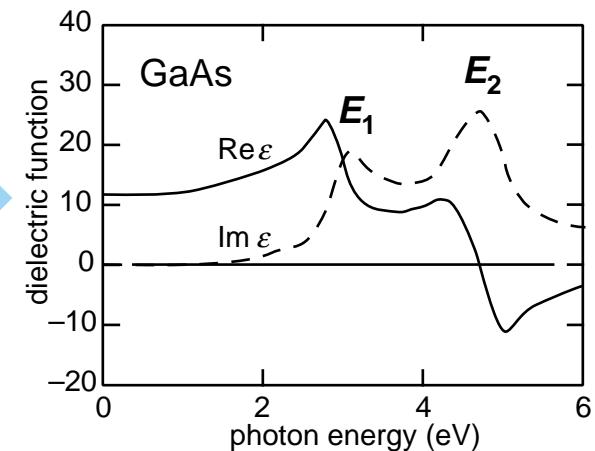
structure



bandstructure

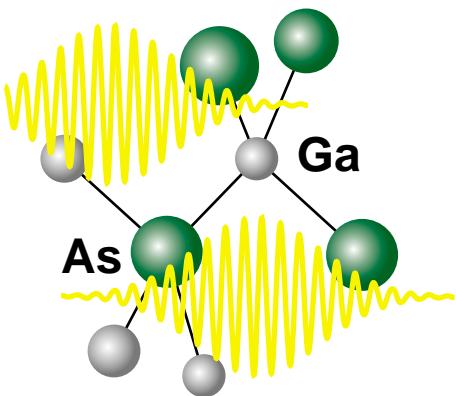


dielectric function



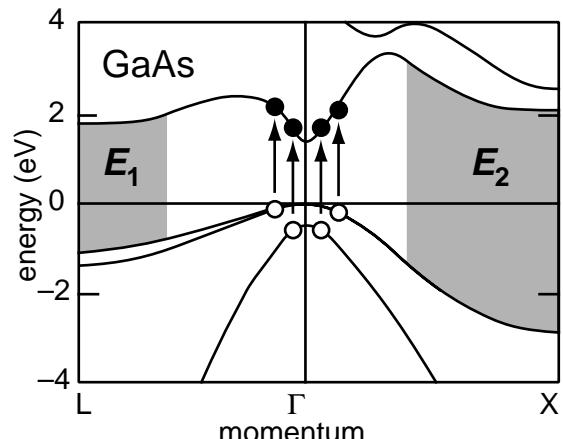
Introduction

structure



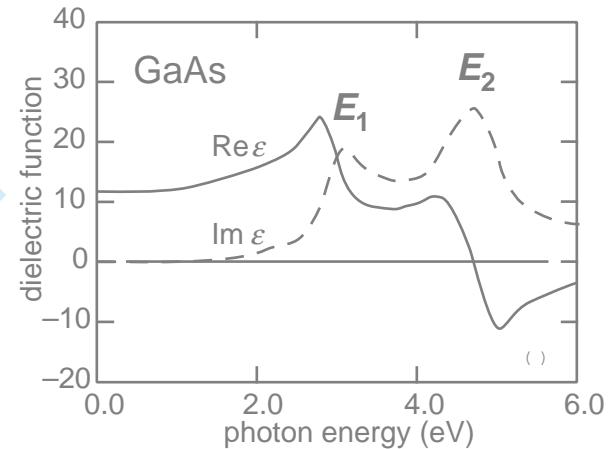
$70 \text{ fs}, 10^{16} \text{ W/m}^2$

bandstructure



$10^{20} - 10^{22} \text{ cm}^{-3}$

dielectric function



Introduction

structure

bandstructure

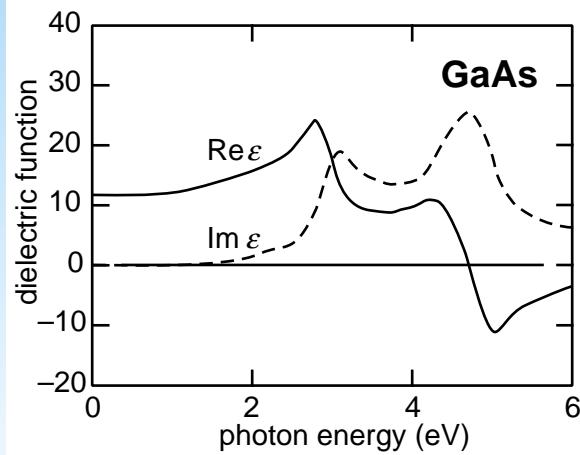
dielectric function

?

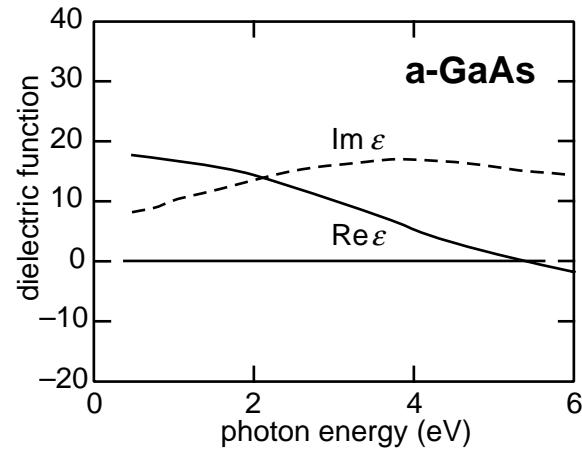
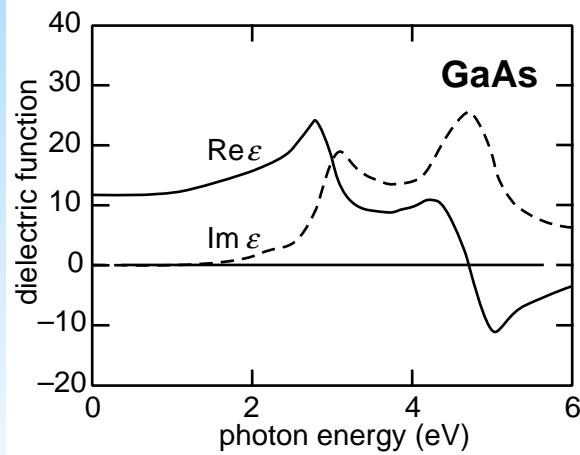
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?

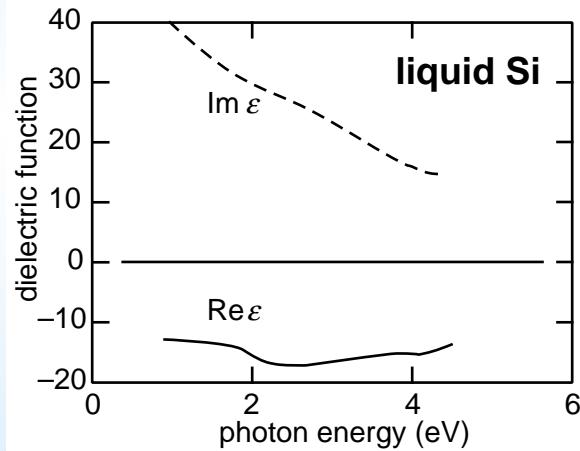
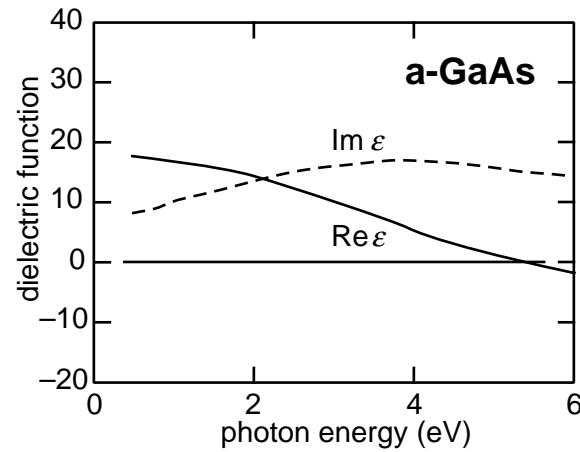
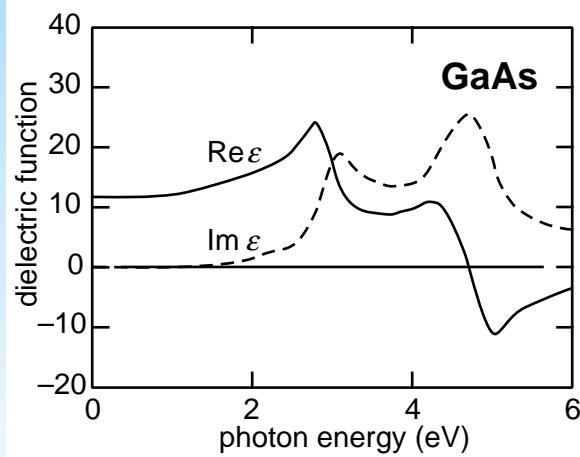
Introduction



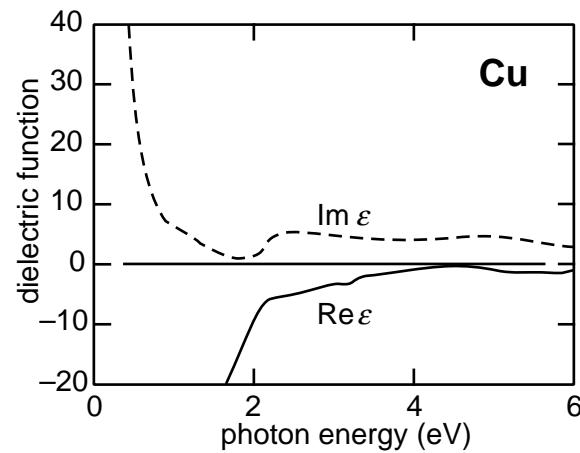
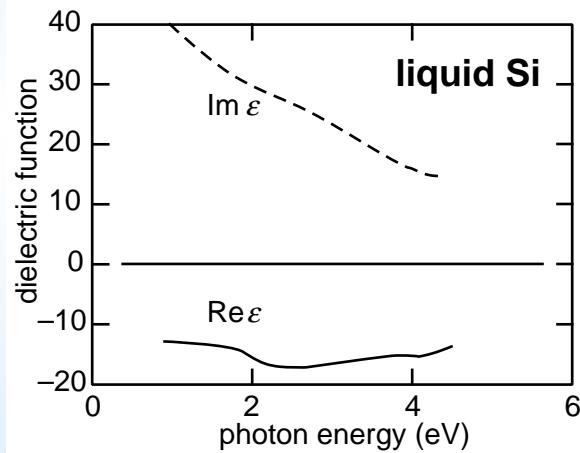
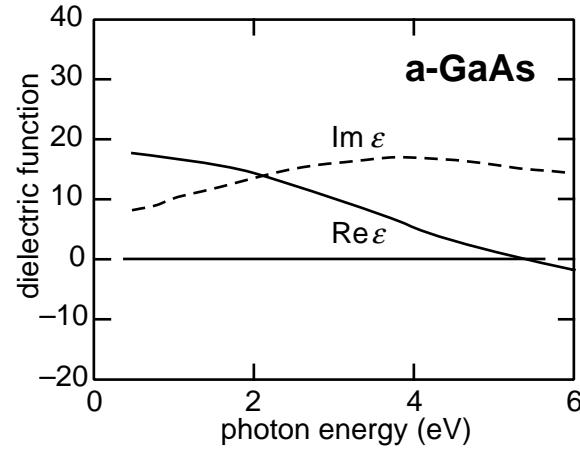
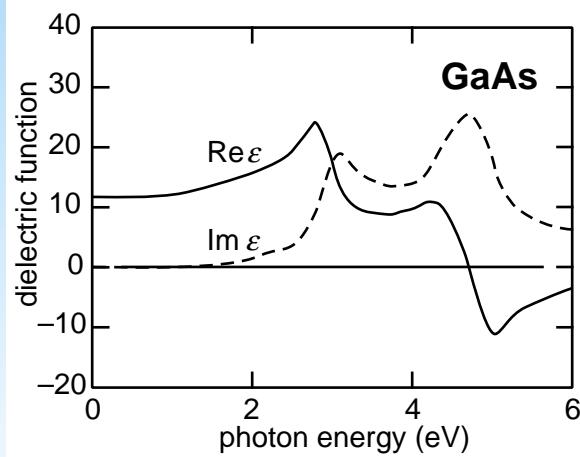
Introduction

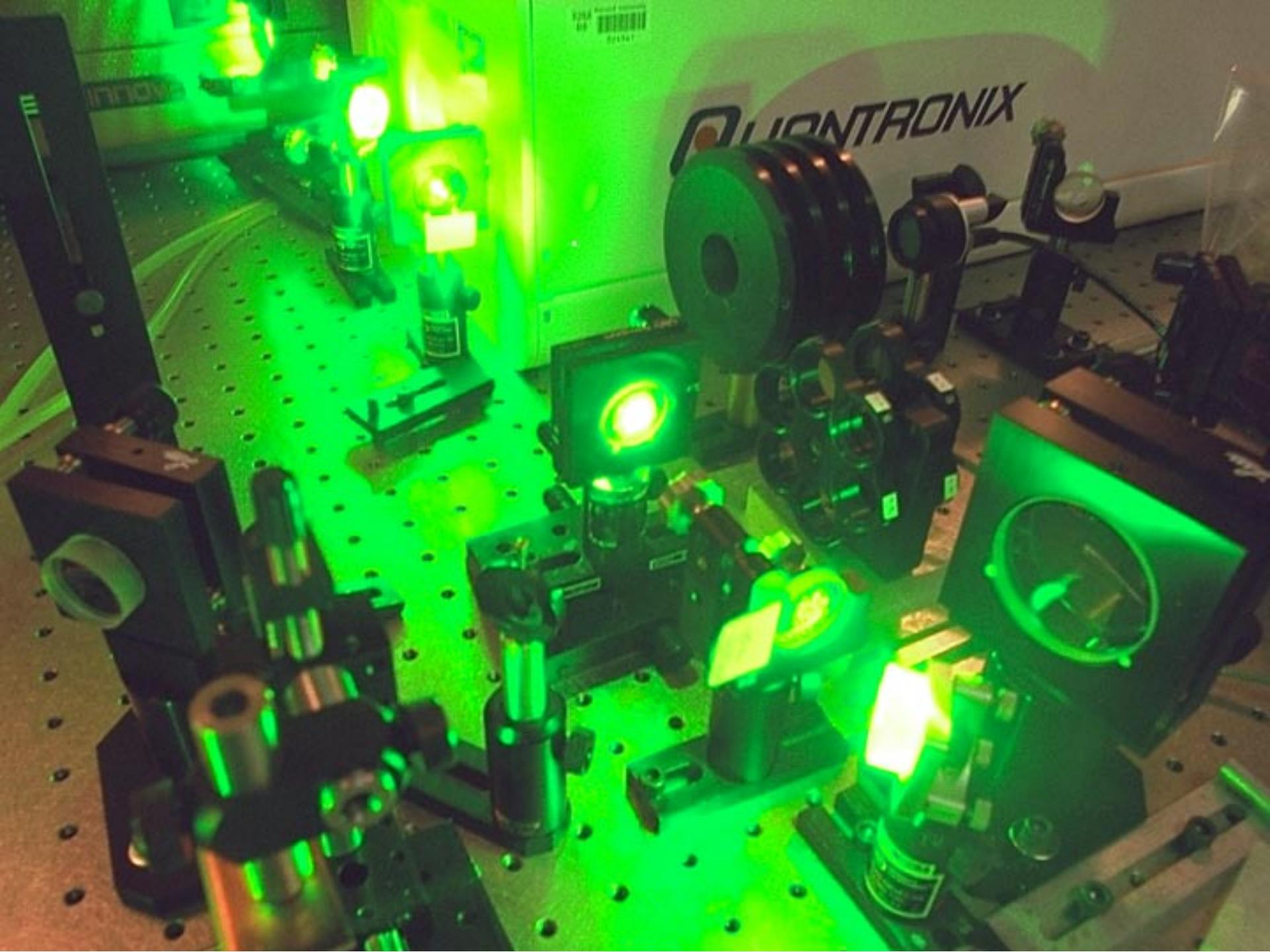


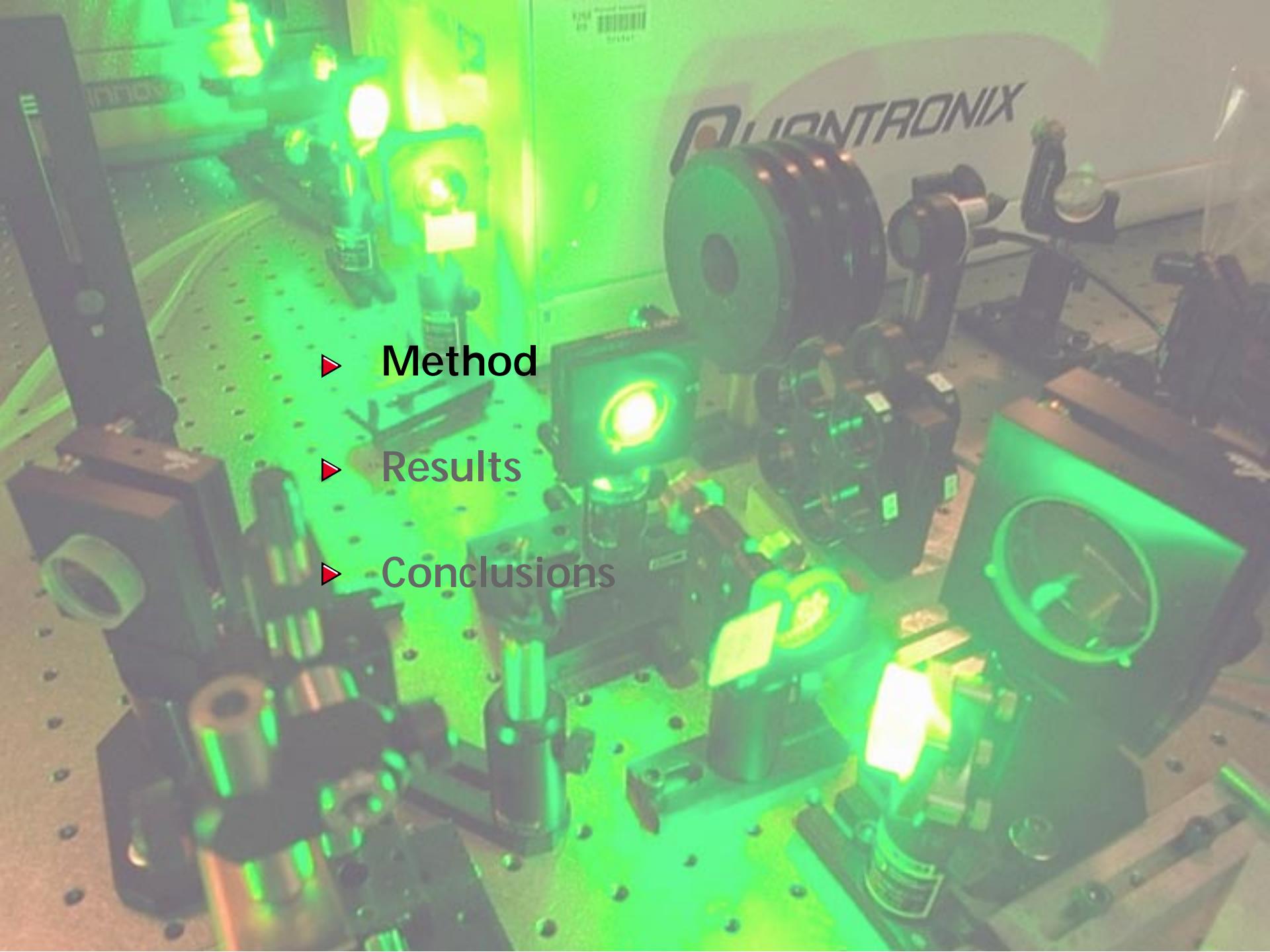
Introduction



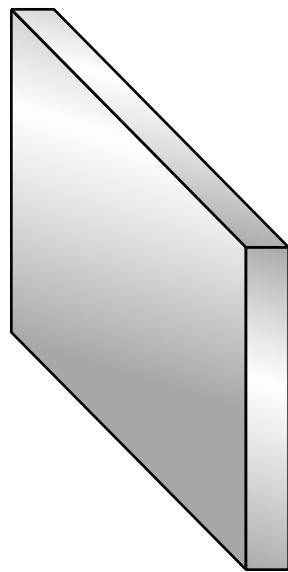
Introduction





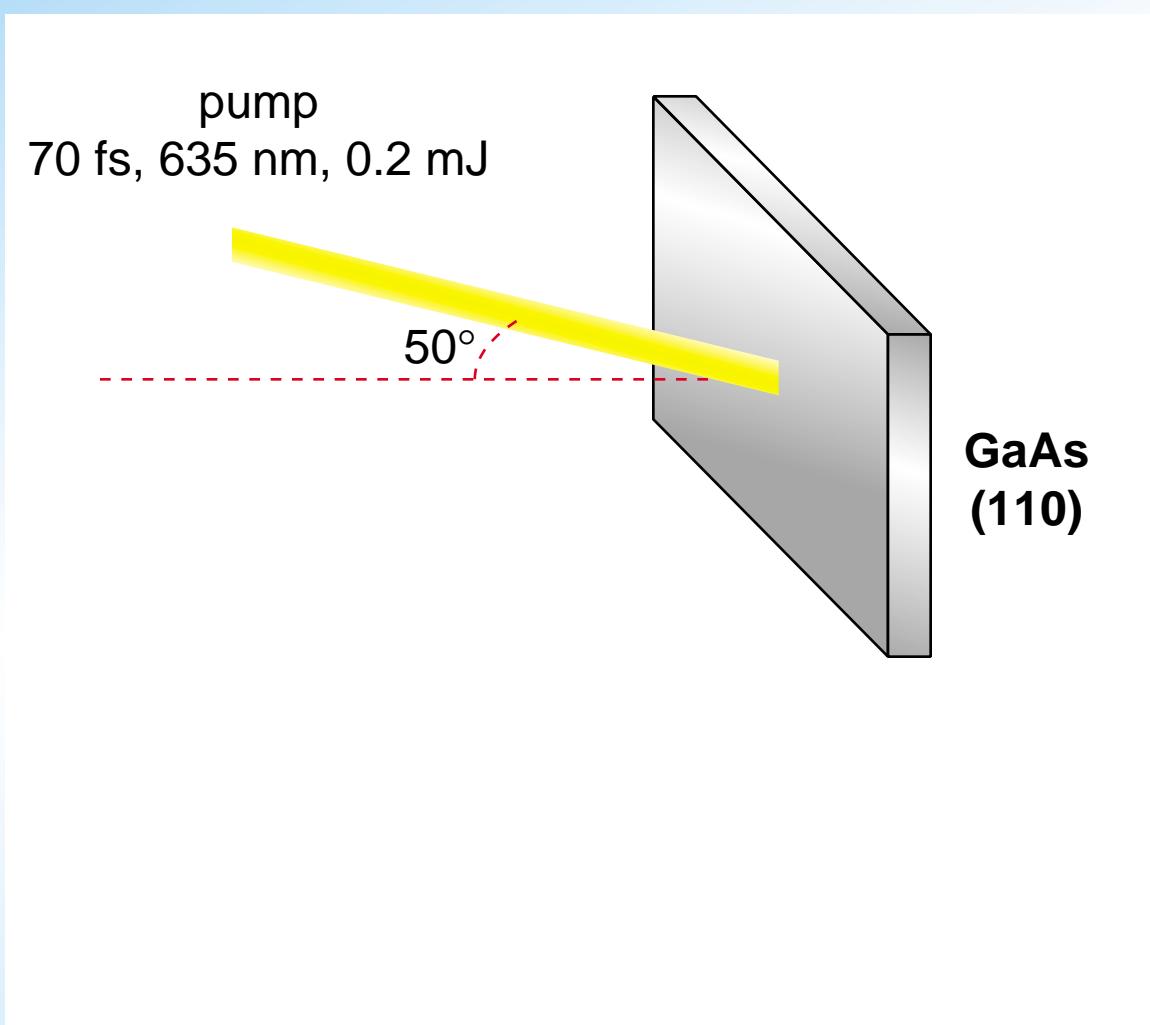
- 
- ▶ Method
 - ▶ Results
 - ▶ Conclusions

Method

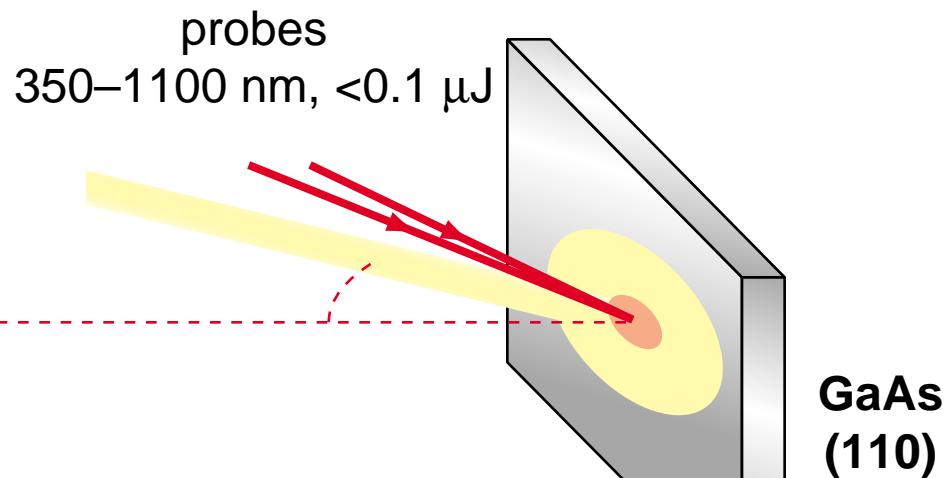


**GaAs
(110)**

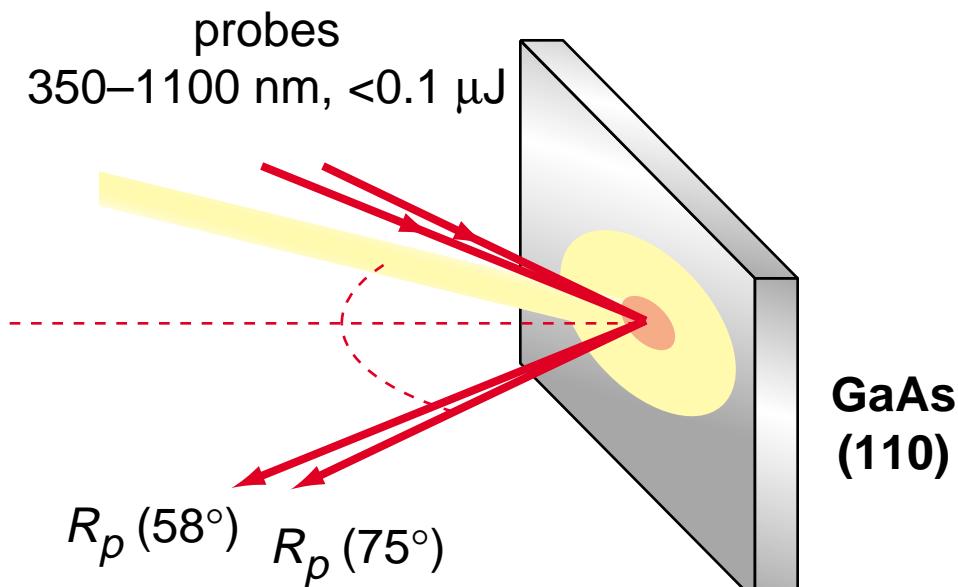
Method



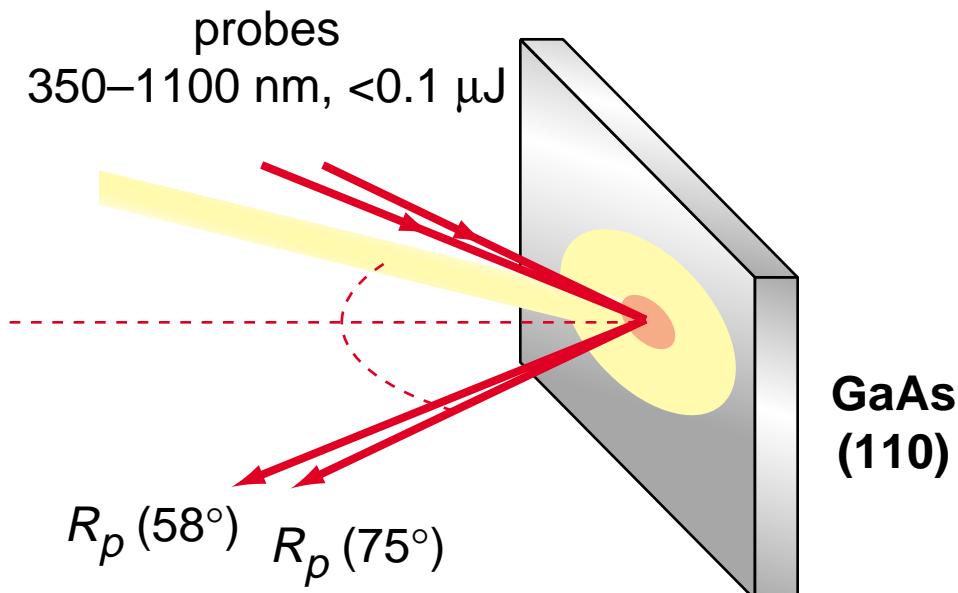
Method



Method

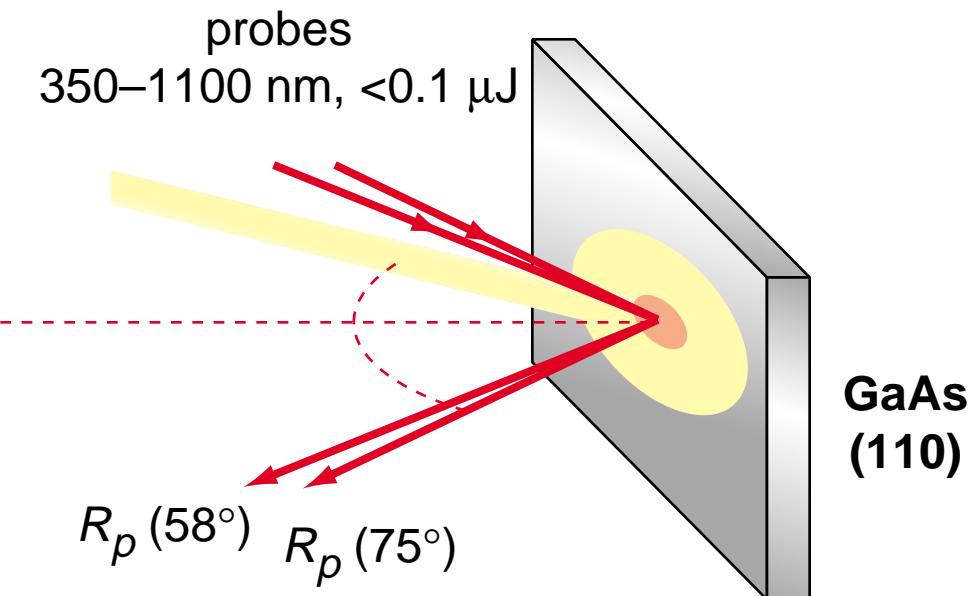


Method



Fresnel
equations

Method



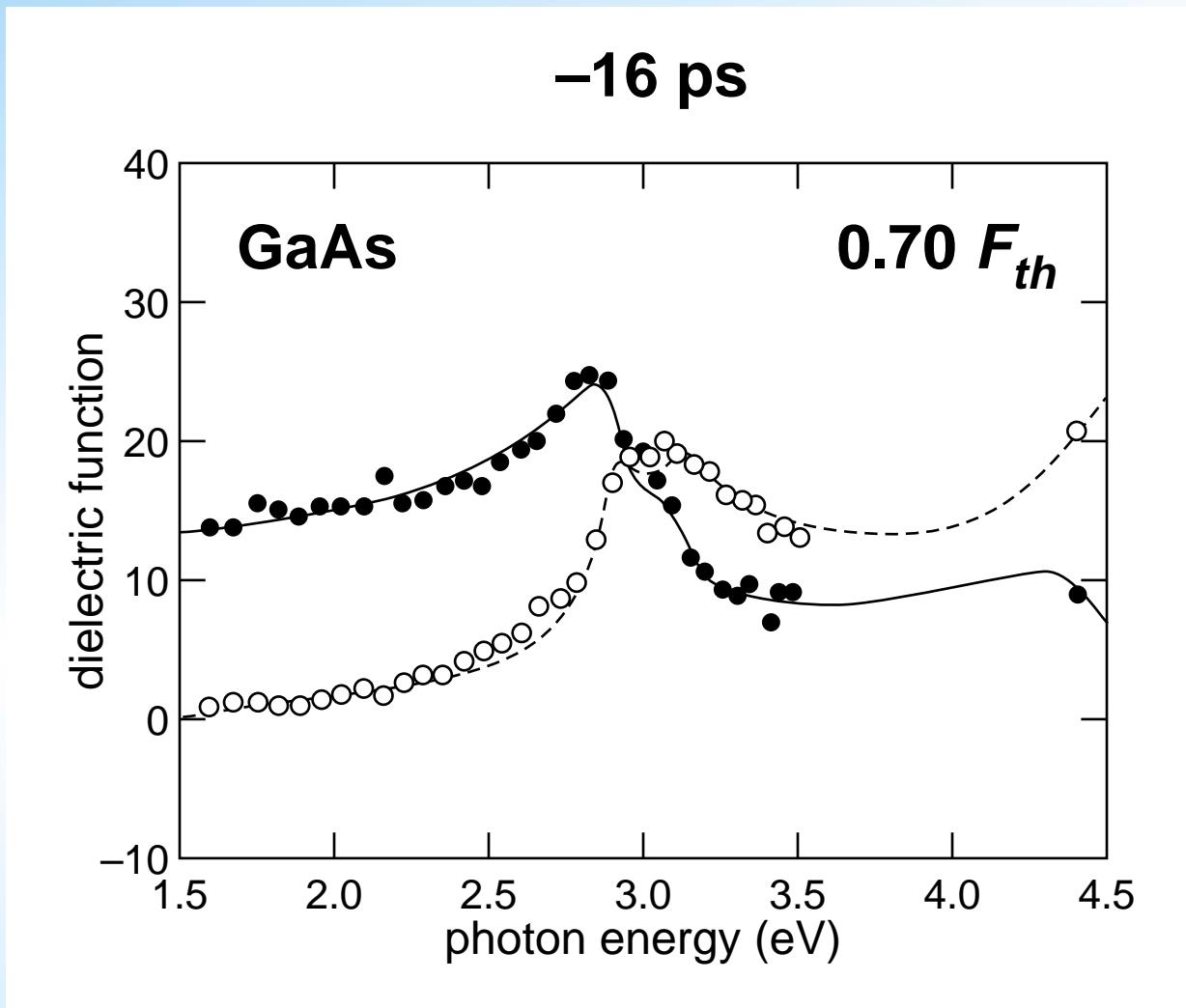
Fresnel
equations



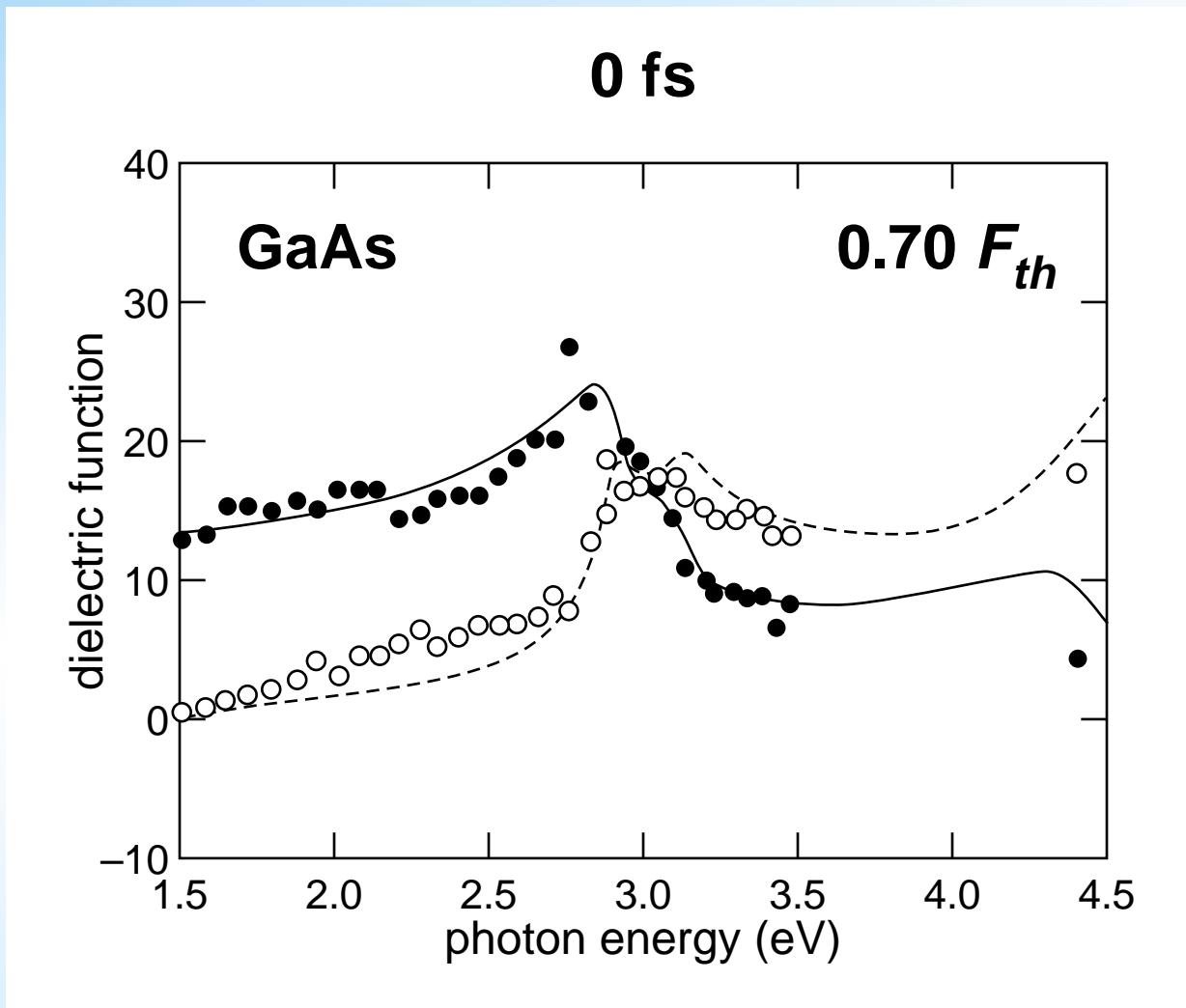
$\text{Re } \varepsilon(\omega)$
 $\text{Im } \varepsilon(\omega)$

-
- ▶ Method
 - ▶ Results: c-GaAs
 - ▶ Conclusions

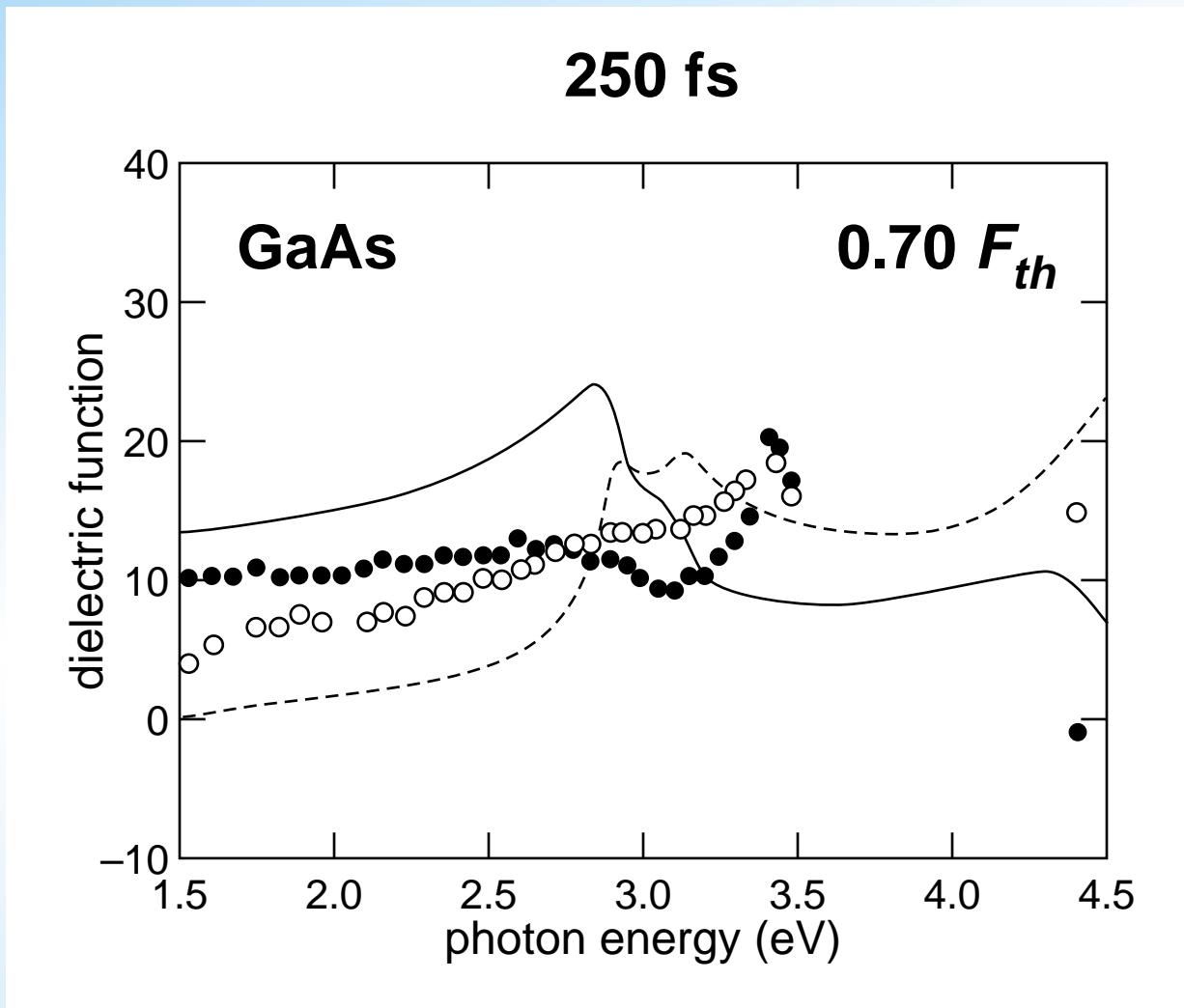
Results



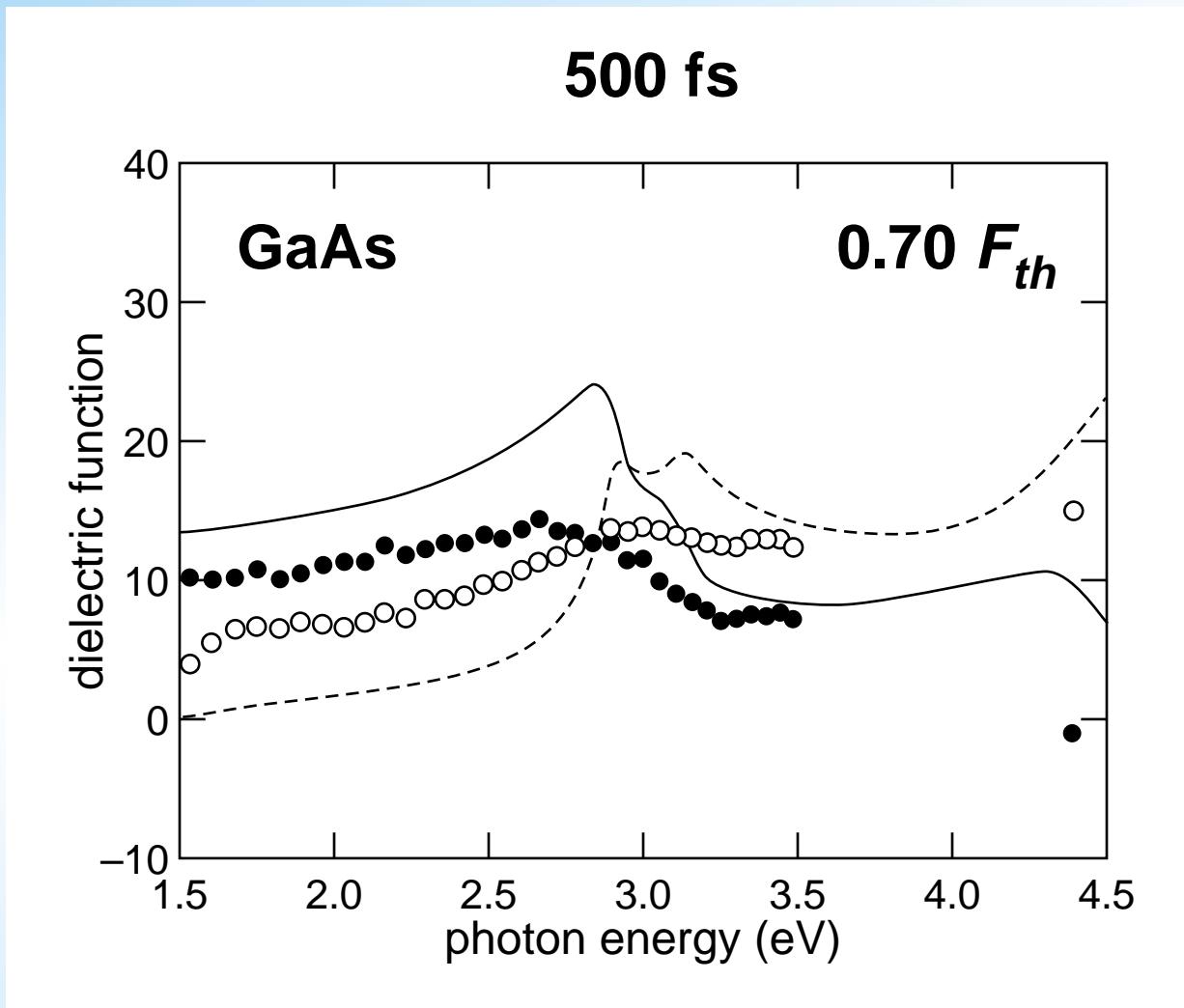
Results



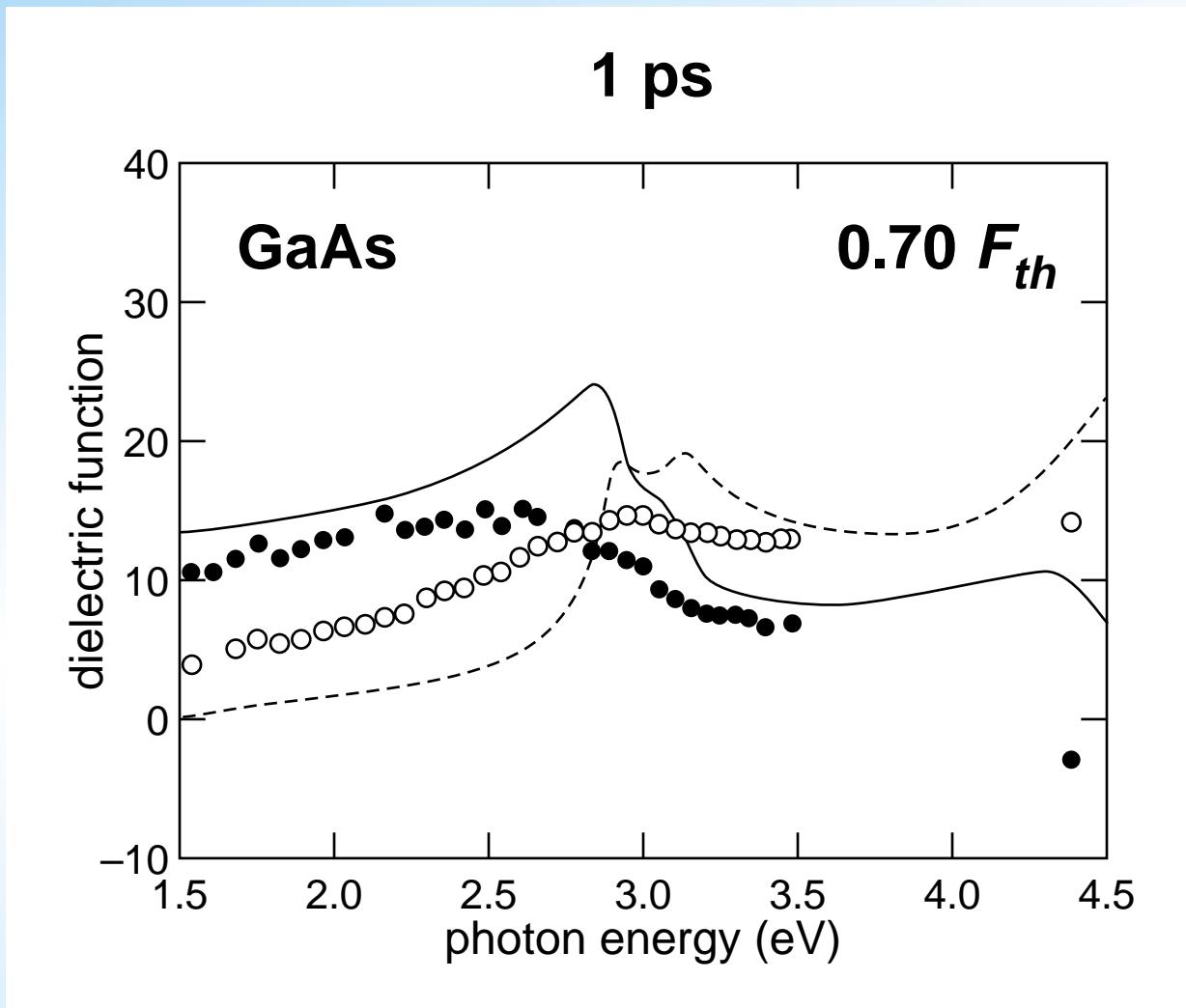
Results



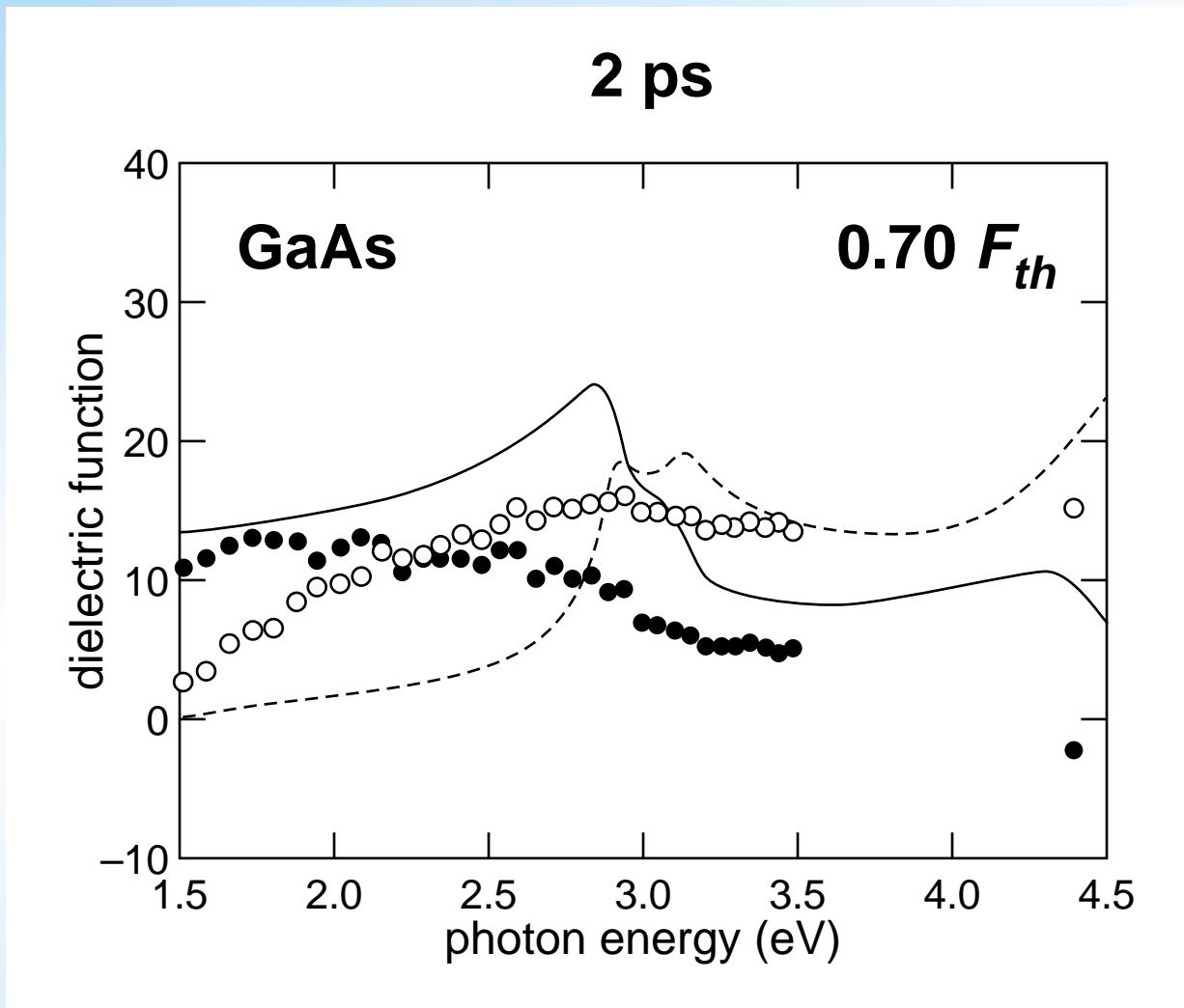
Results



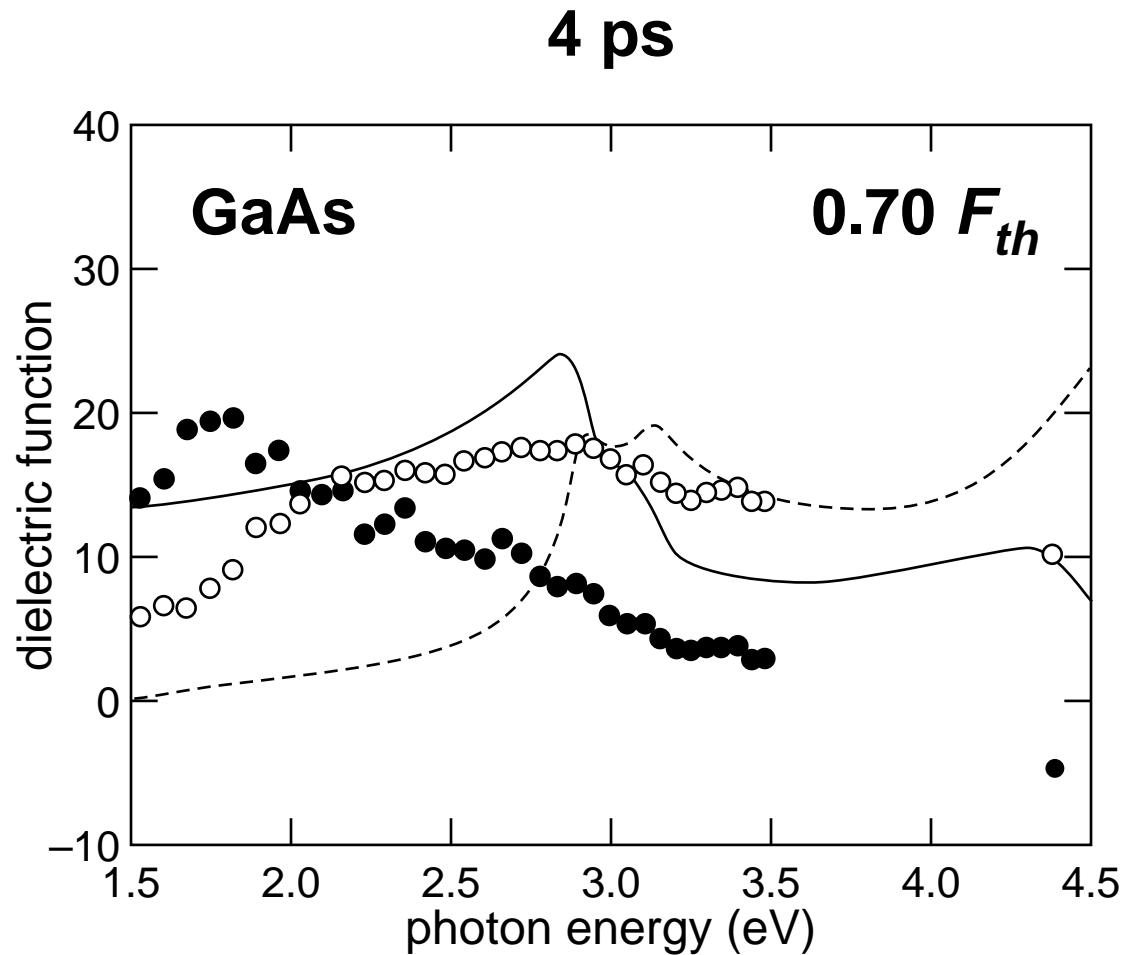
Results



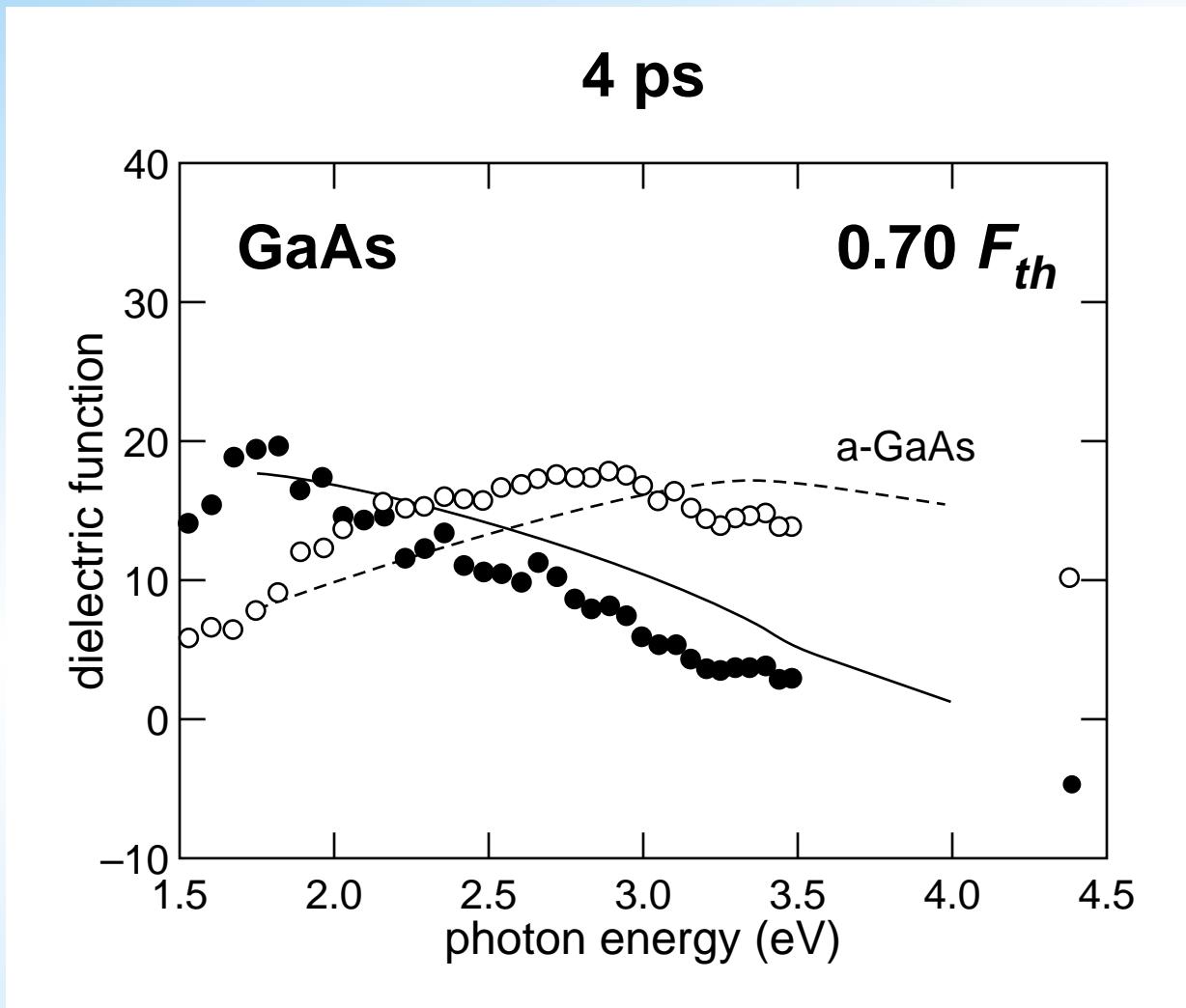
Results



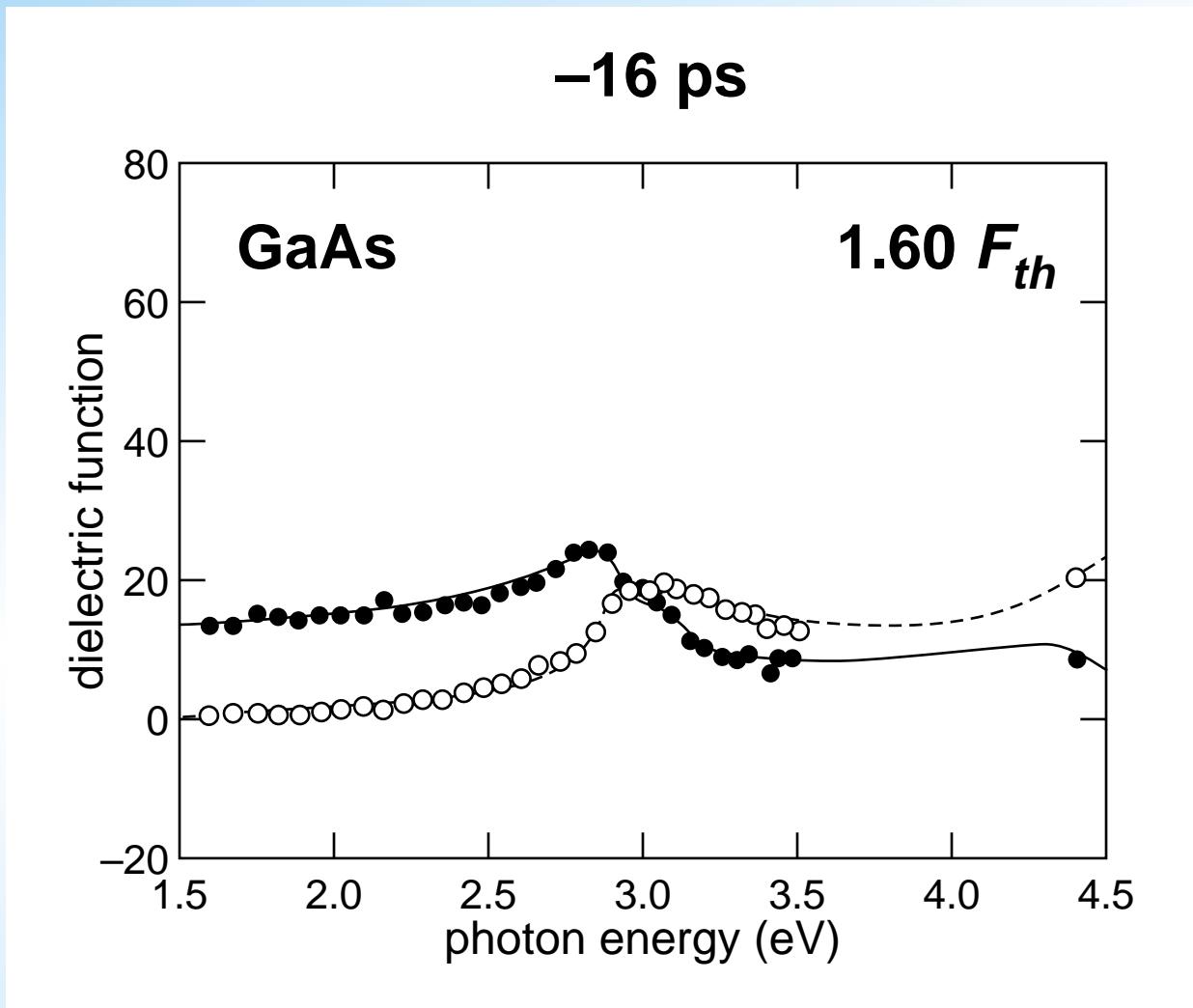
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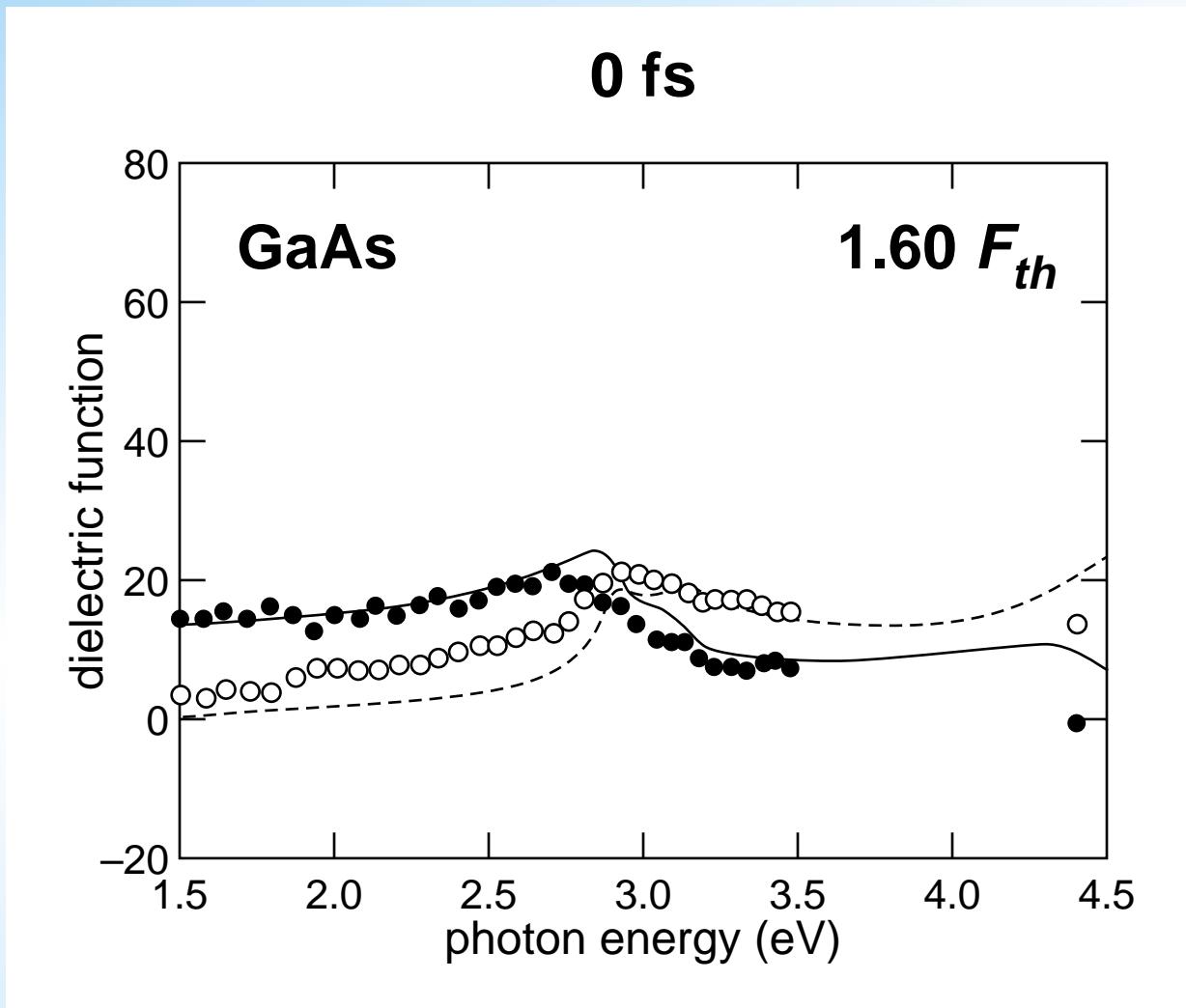
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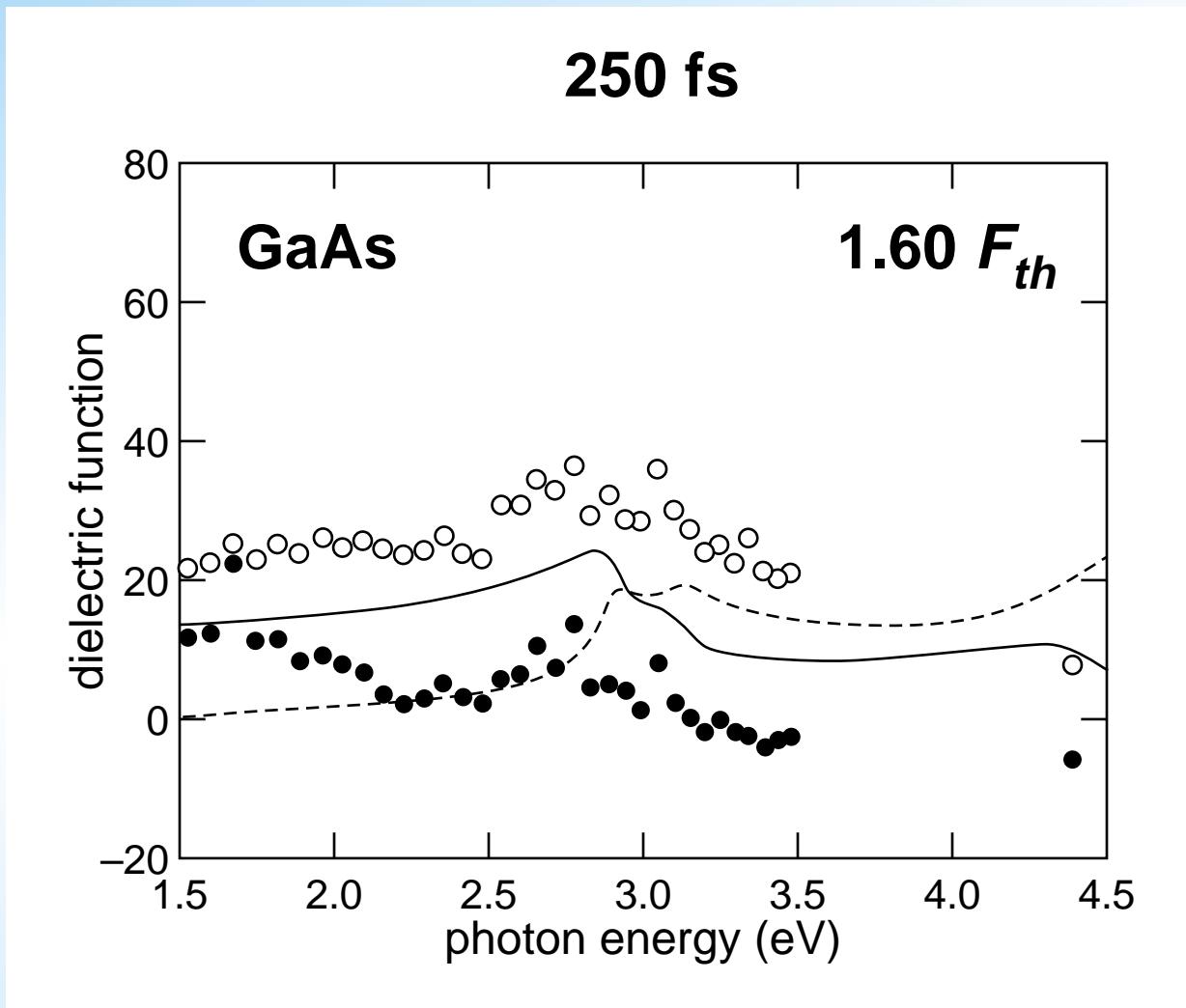
Results



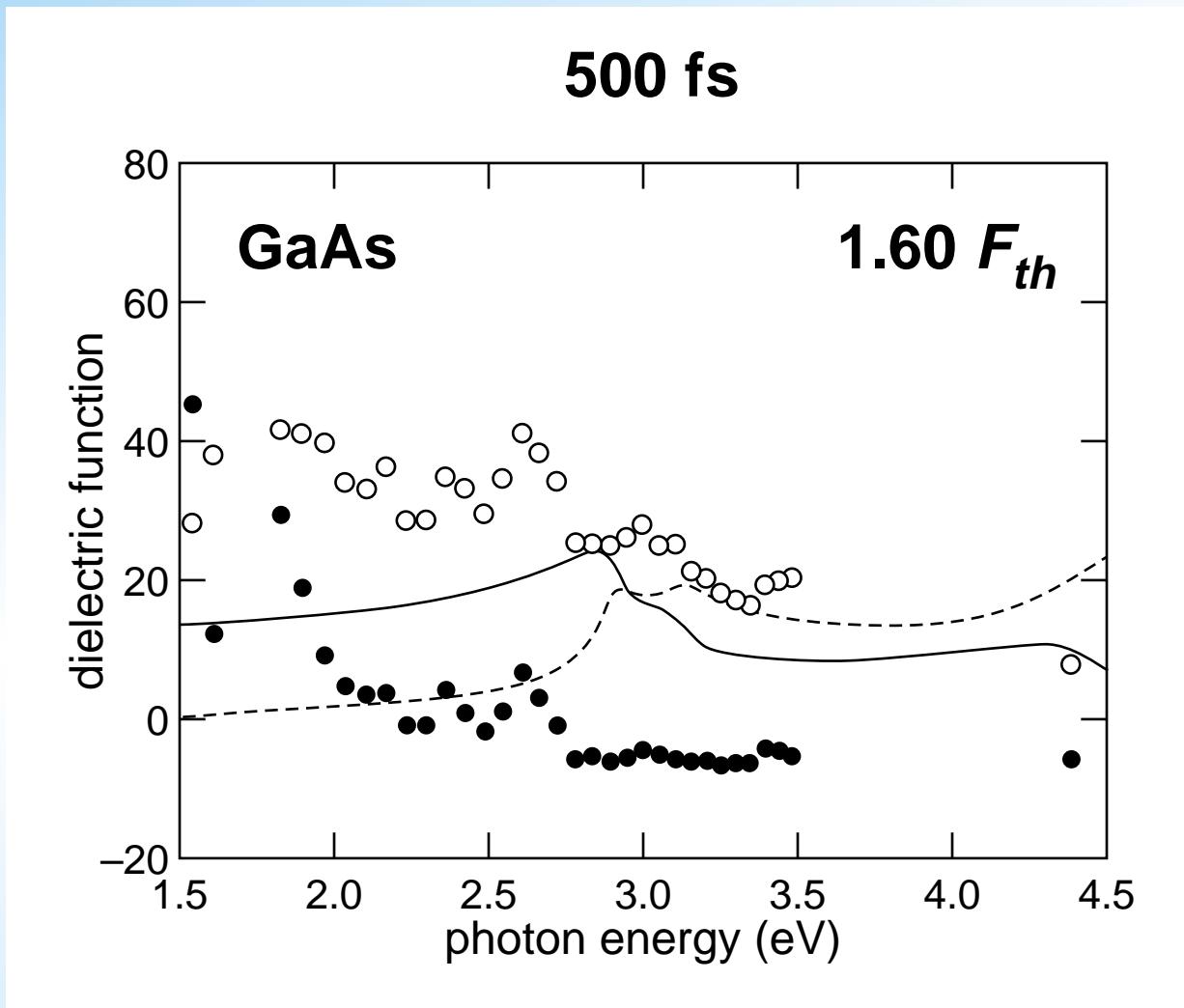
Results



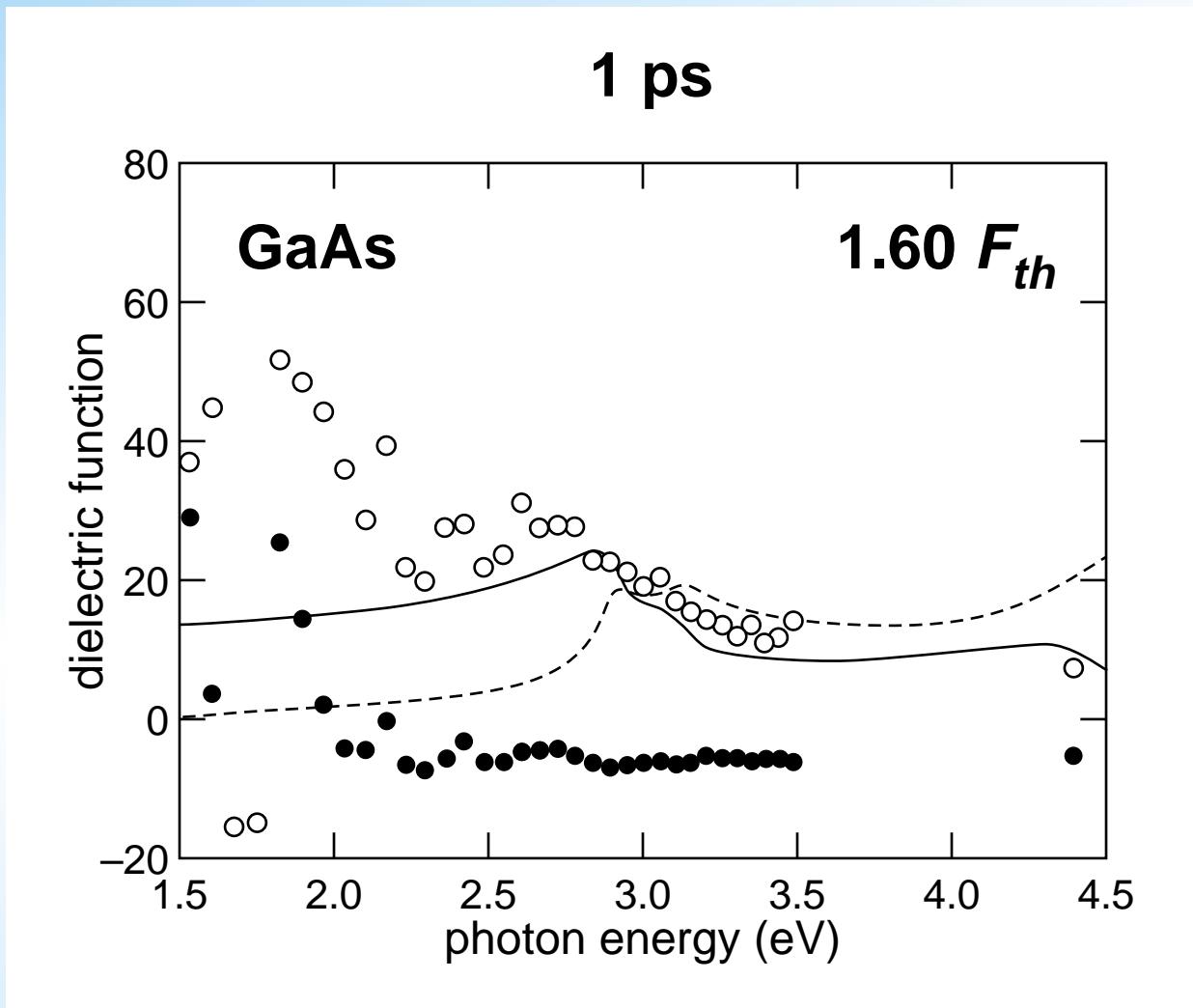
Results



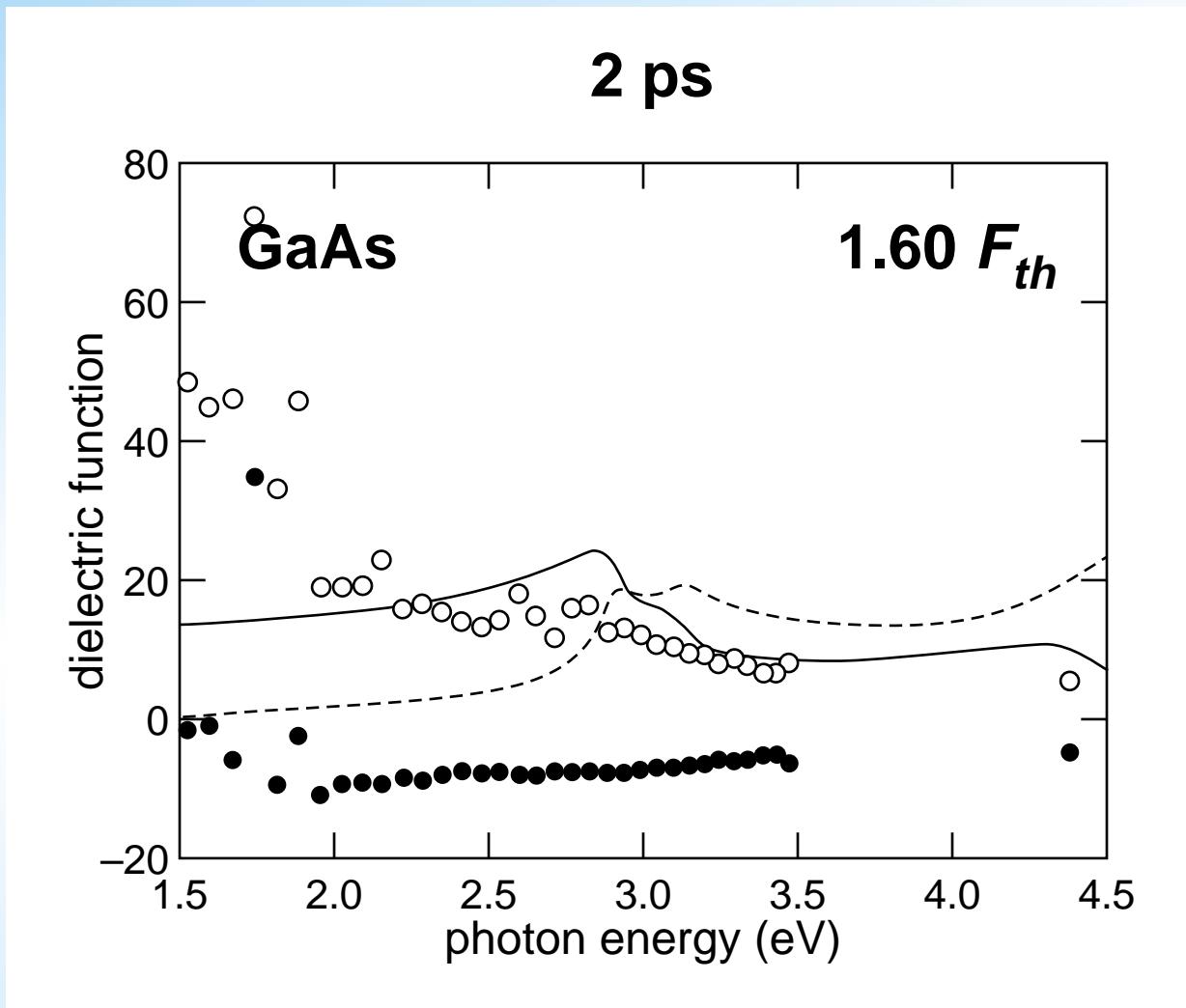
Results



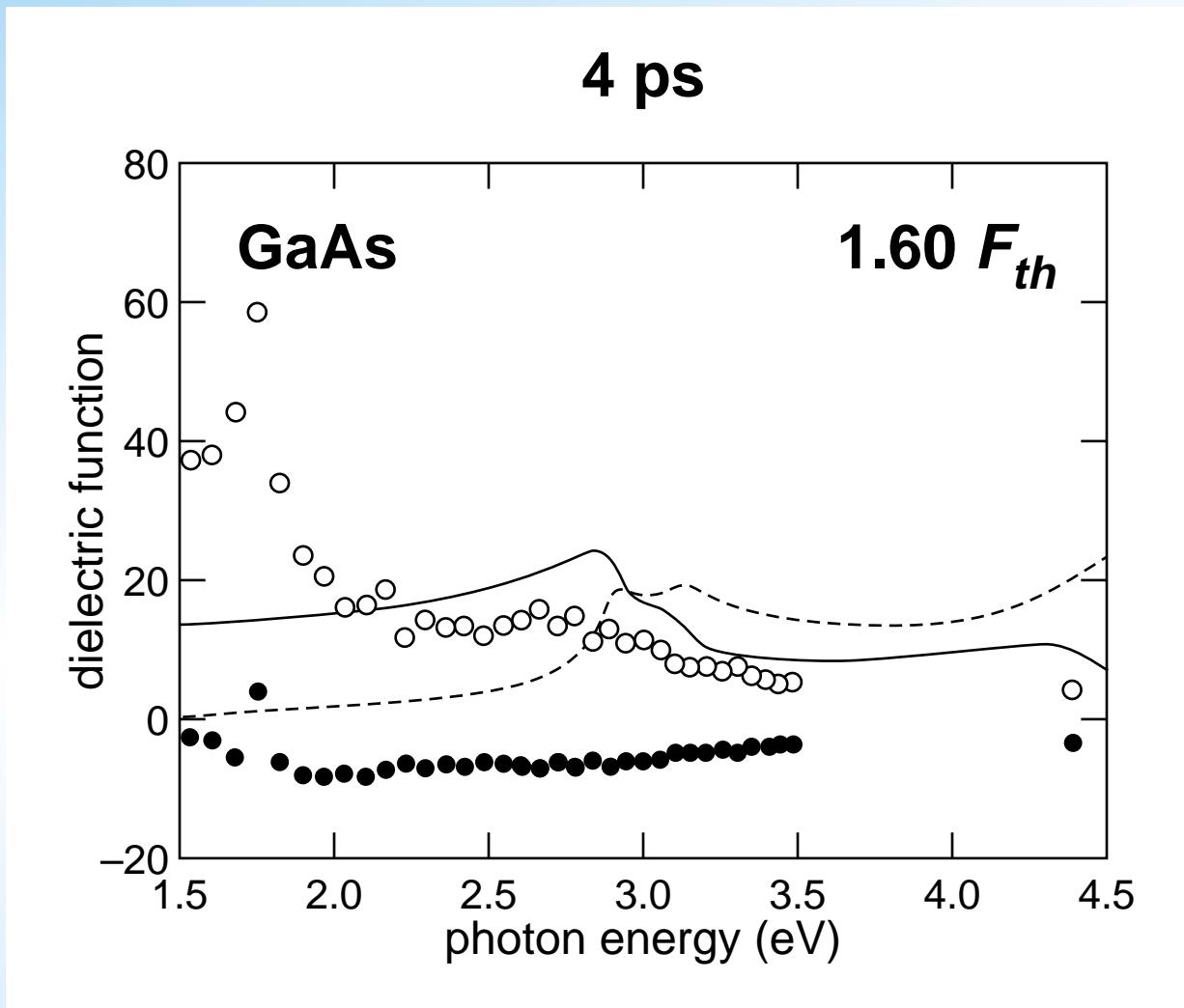
Results



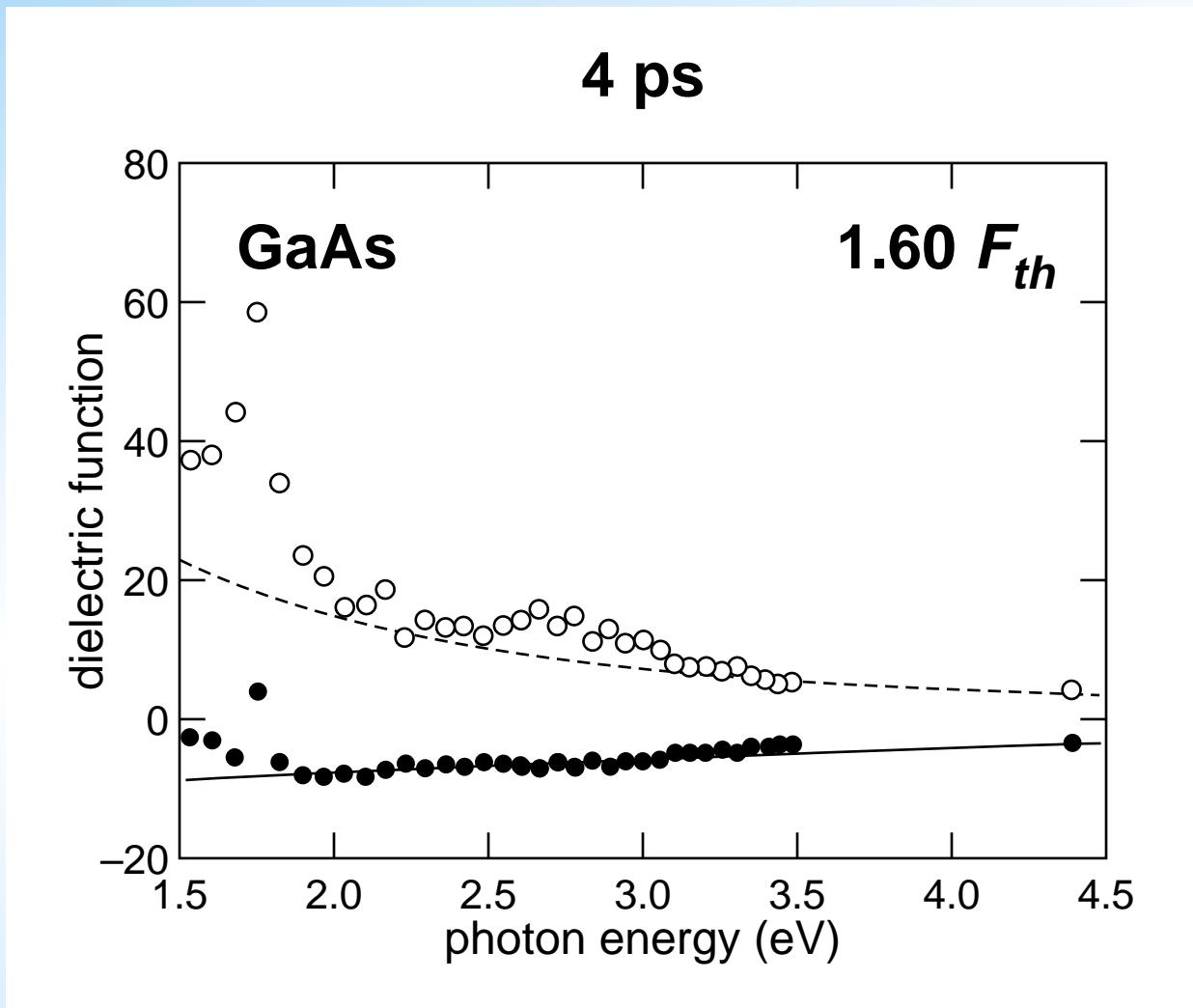
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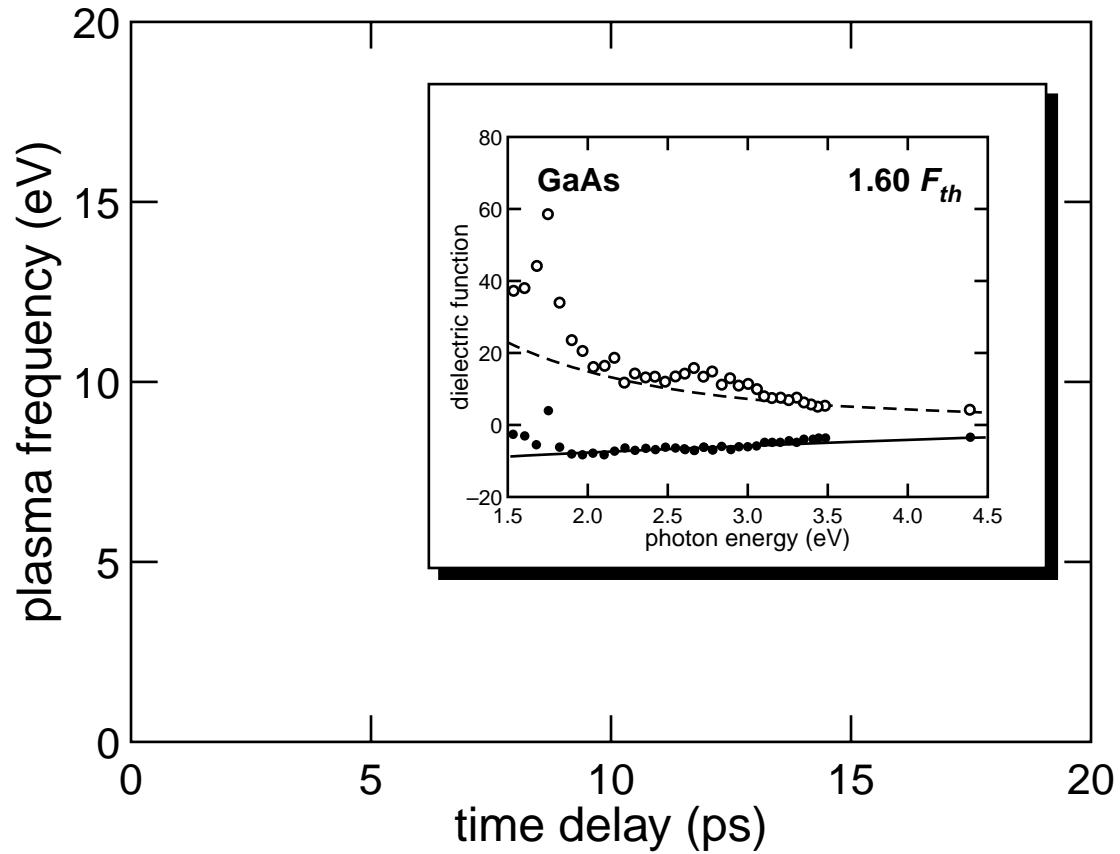
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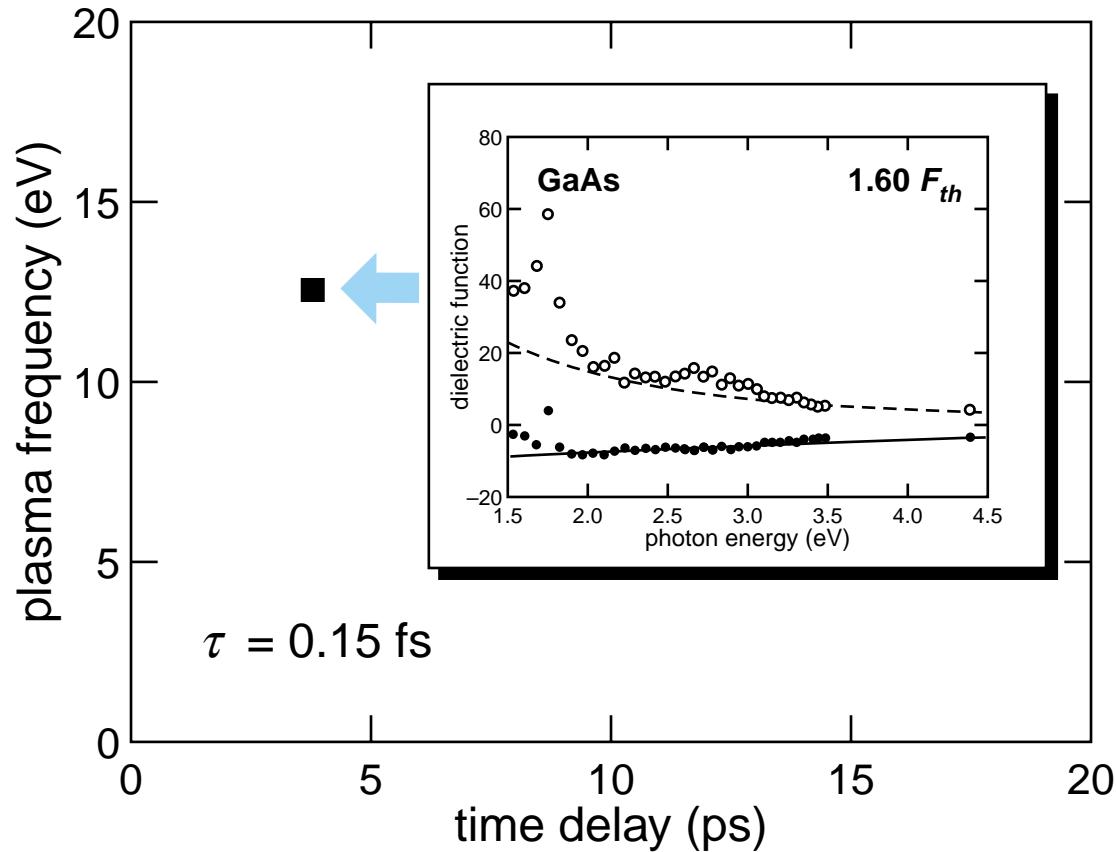
Results



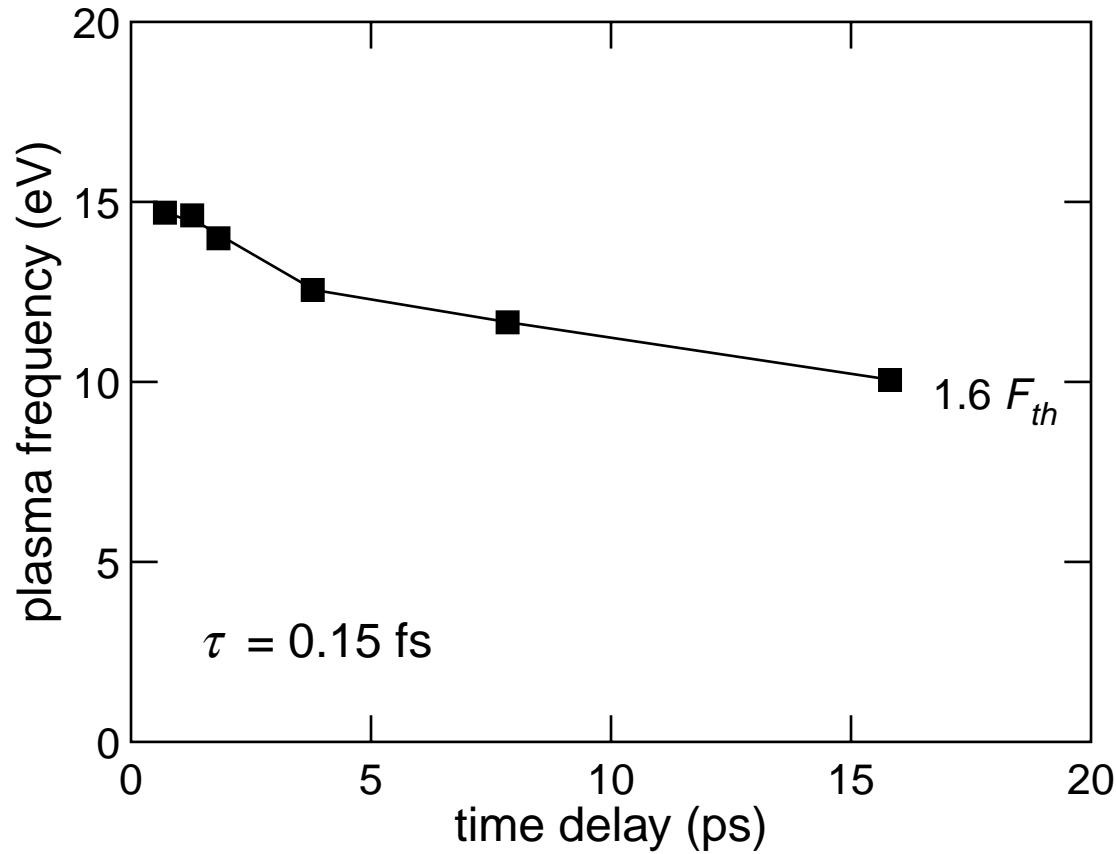
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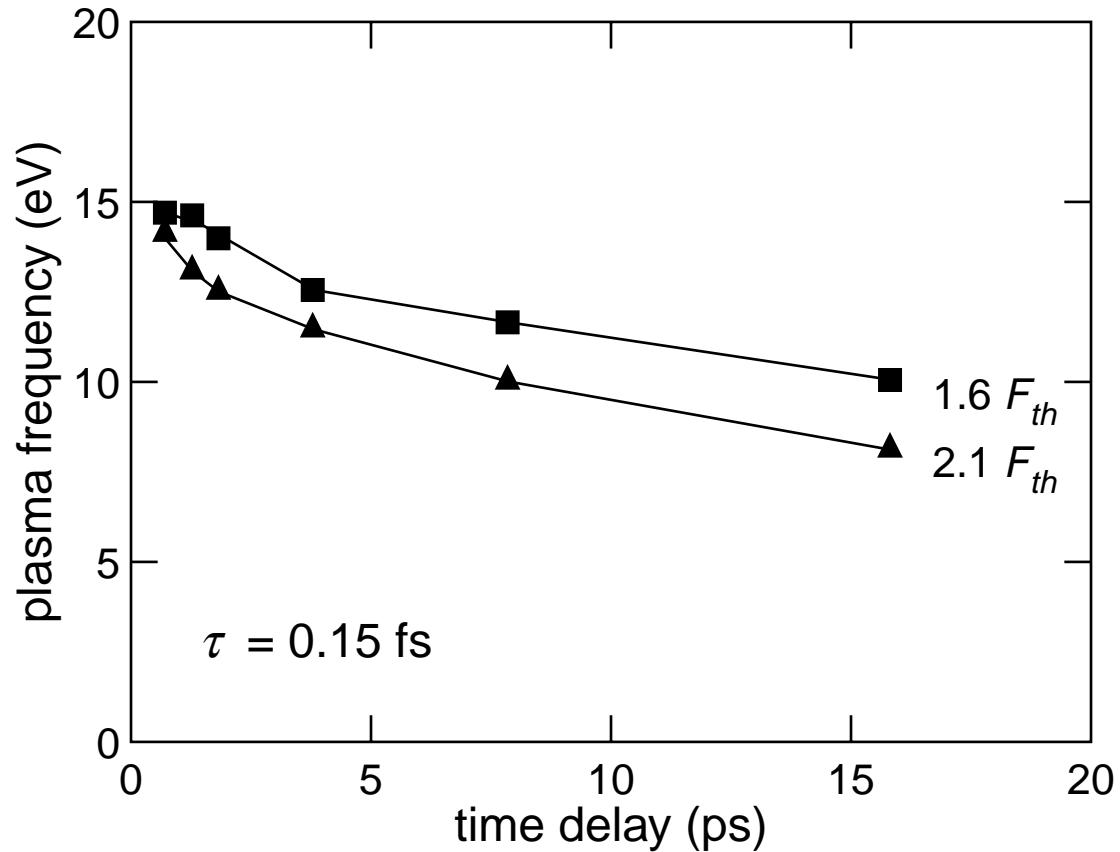
Results



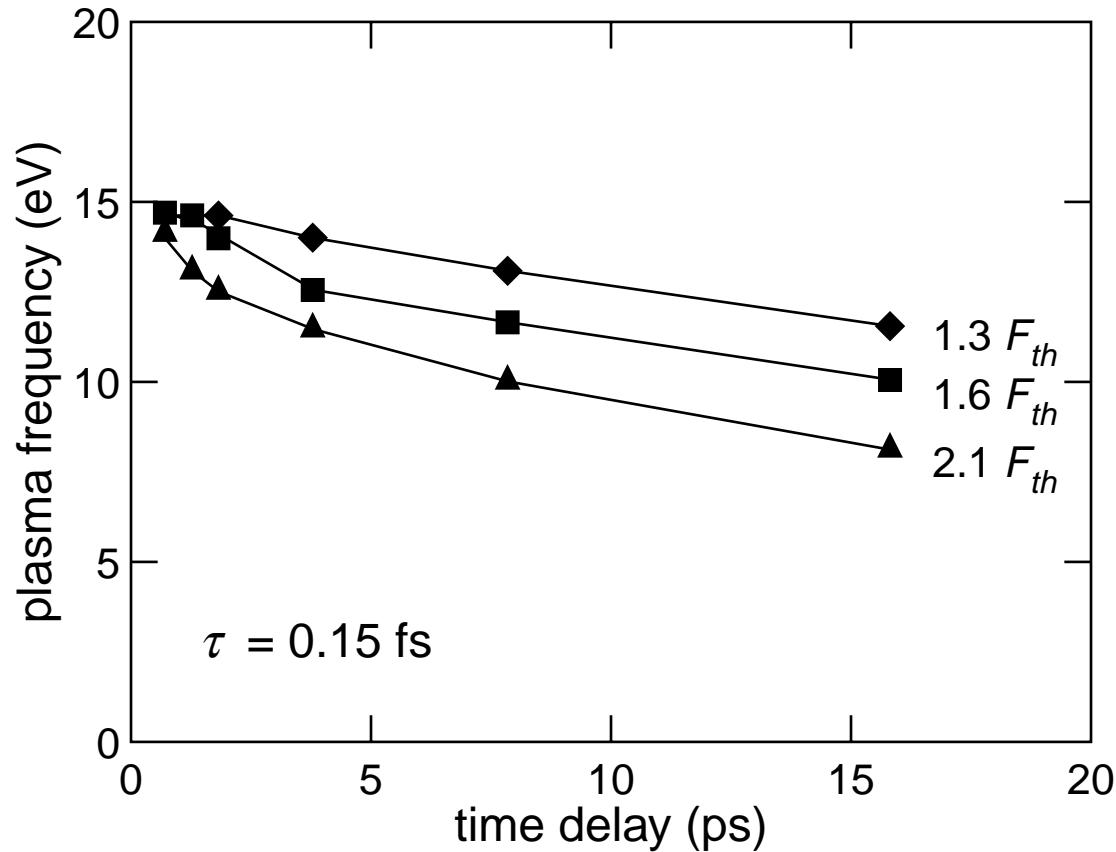
Results



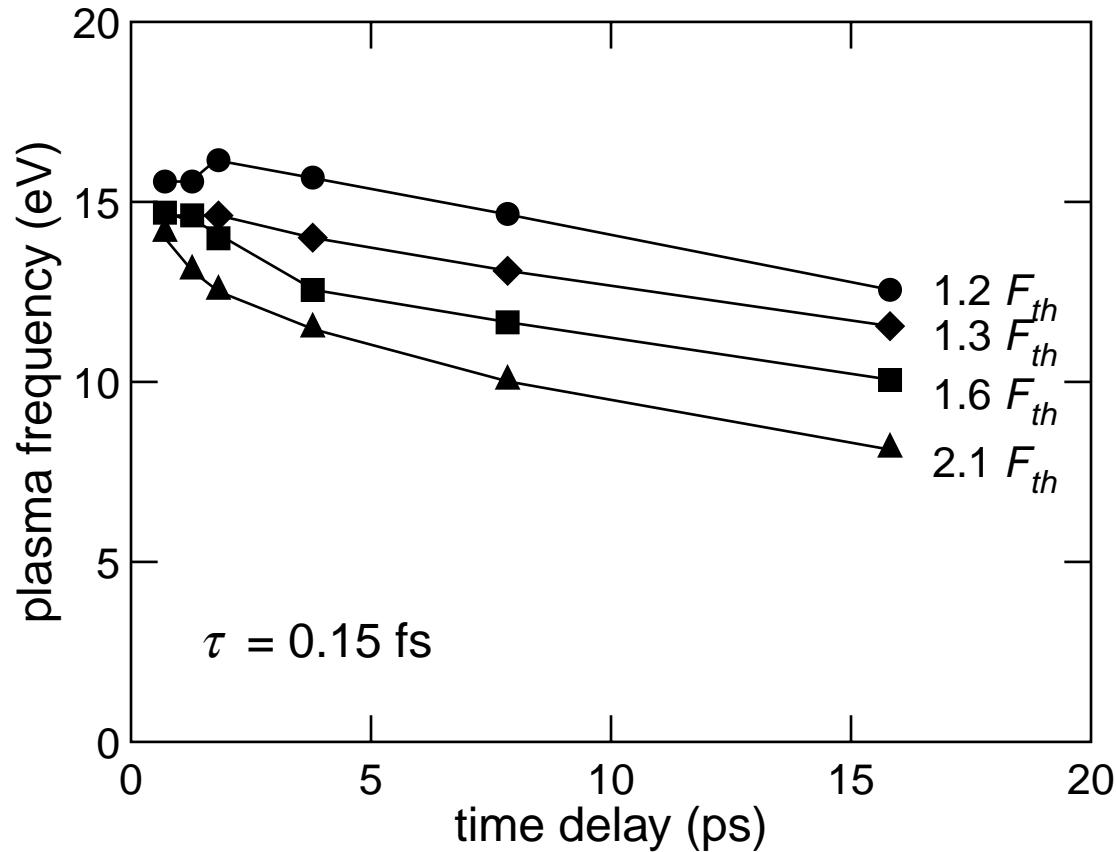
Results



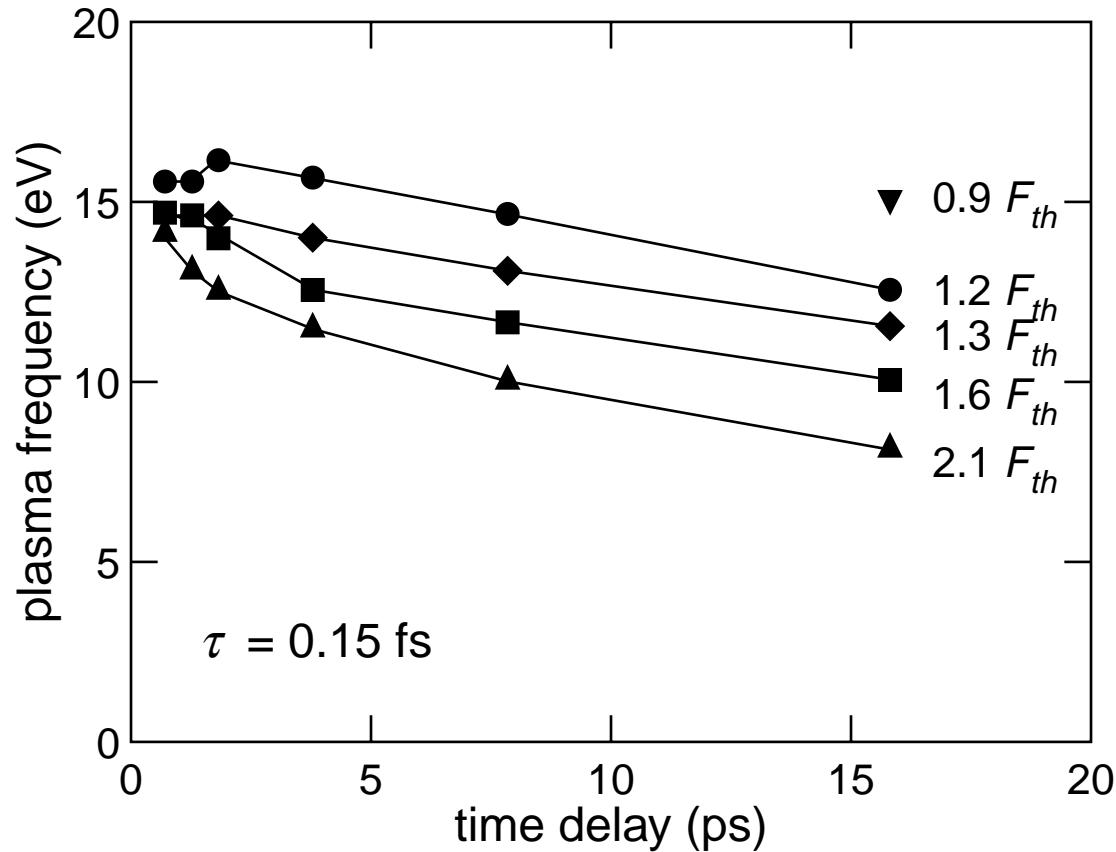
Results



Results



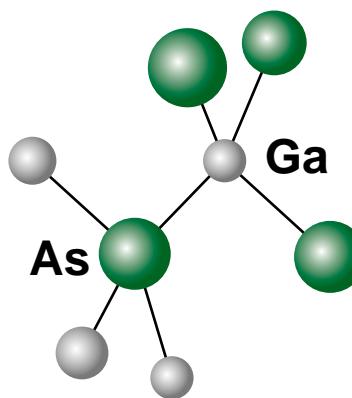
Results



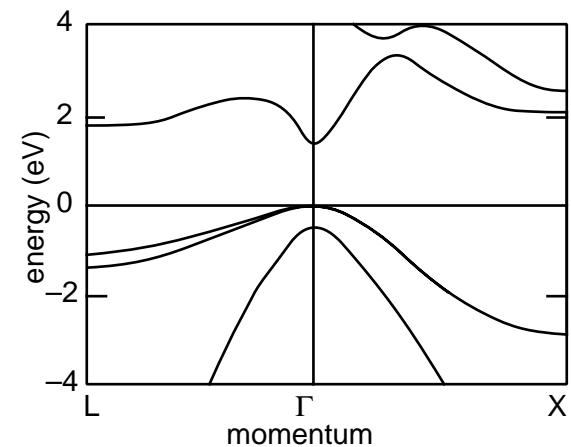
Discussion

short time scale

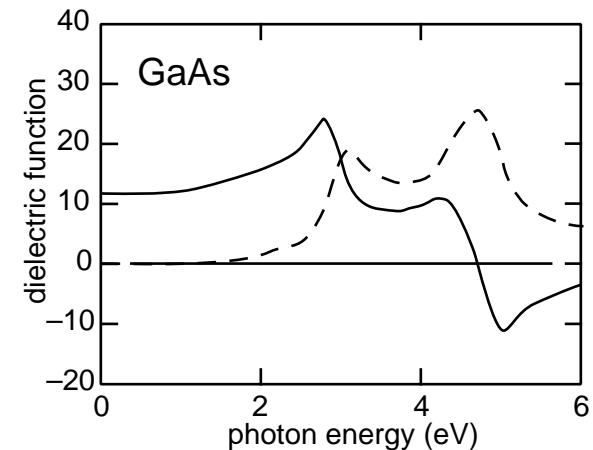
structure



bandstructure



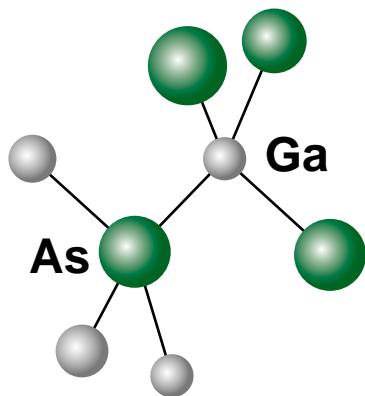
dielectric function



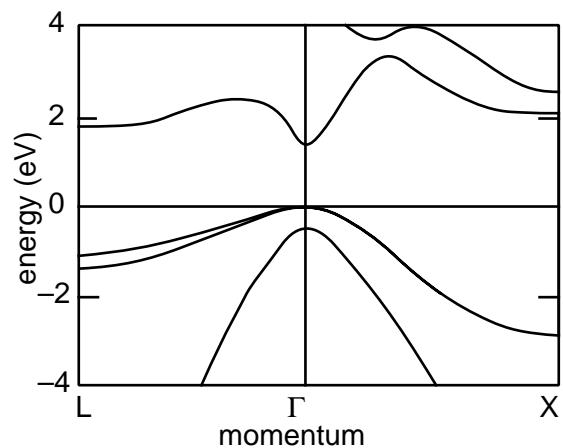
Discussion

short time scale

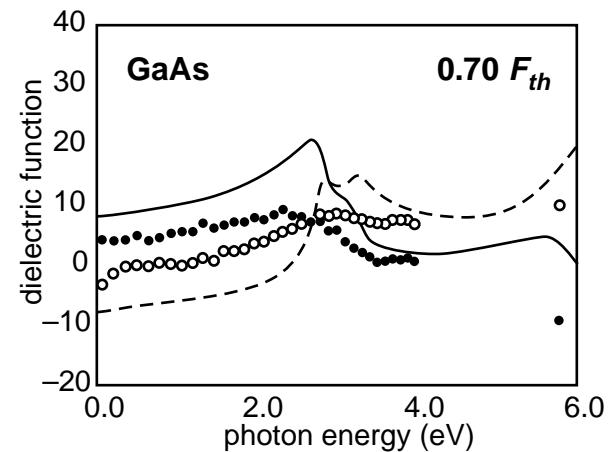
structure



bandstructure



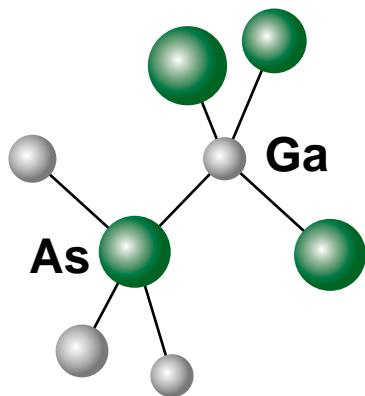
dielectric function



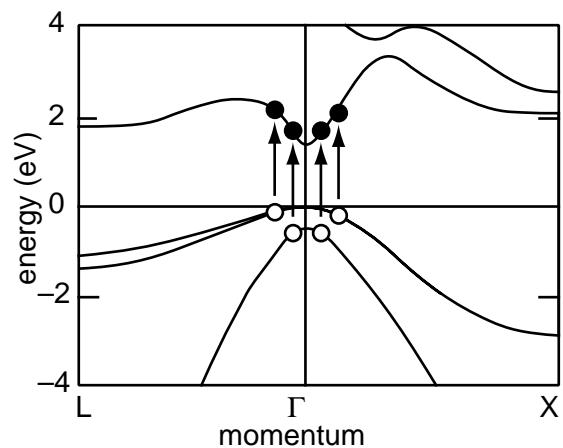
Discussion

short time scale

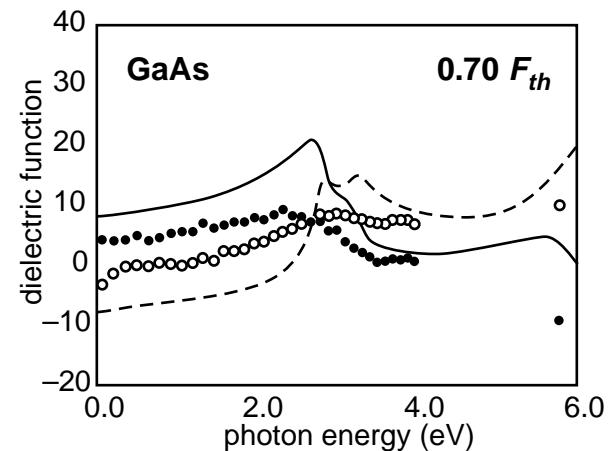
structure



bandstructure



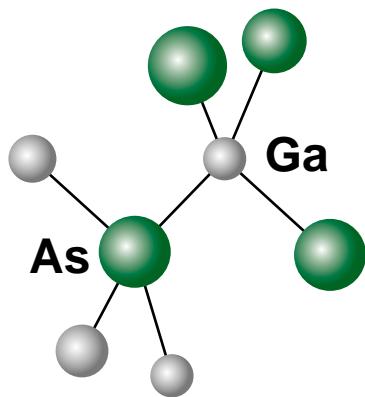
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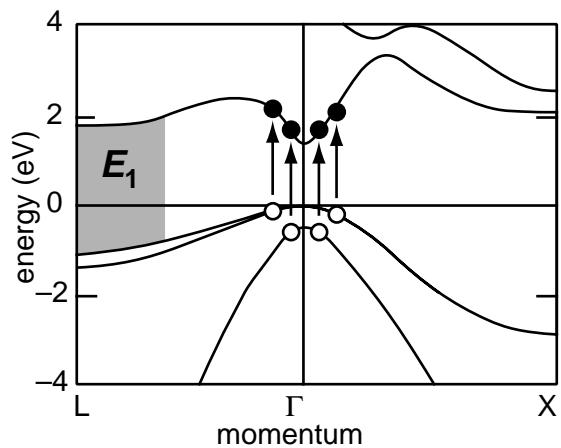
Discussion

short time scale

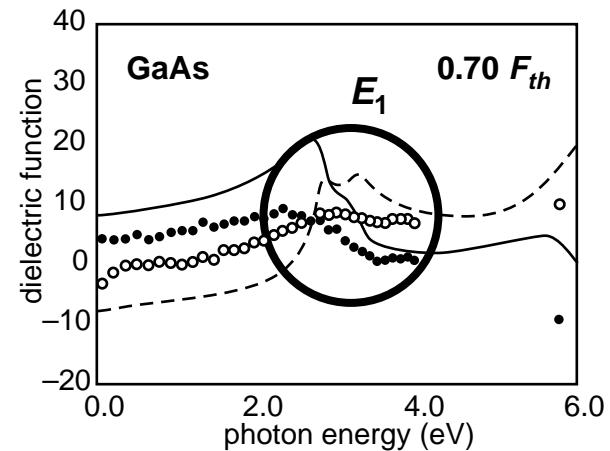
structure



bandstructure



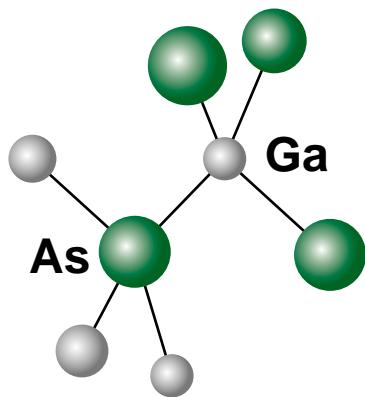
dielectric function



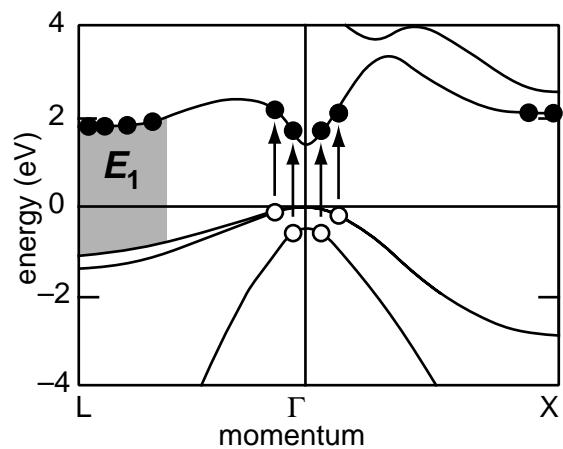
Discussion

short time scale

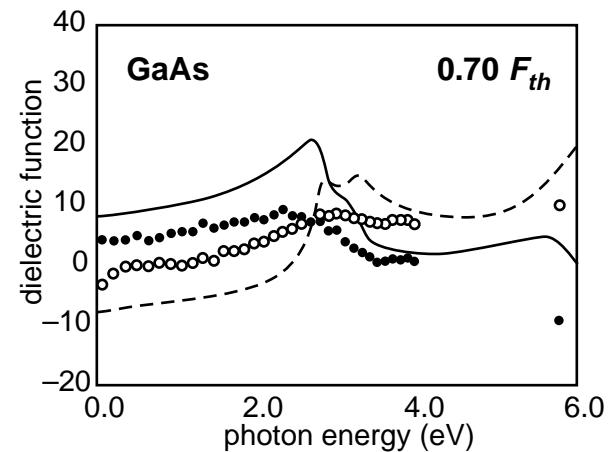
structure



bandstructure



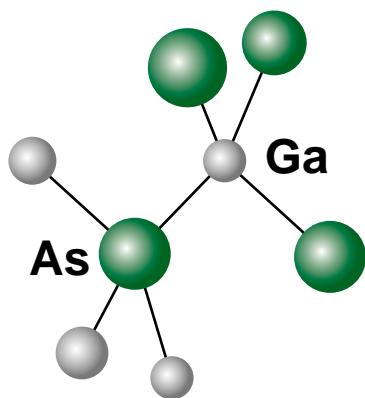
dielectric function



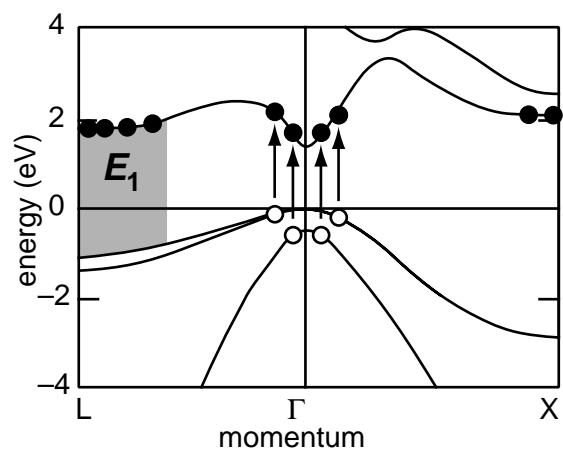
Discussion

short time scale

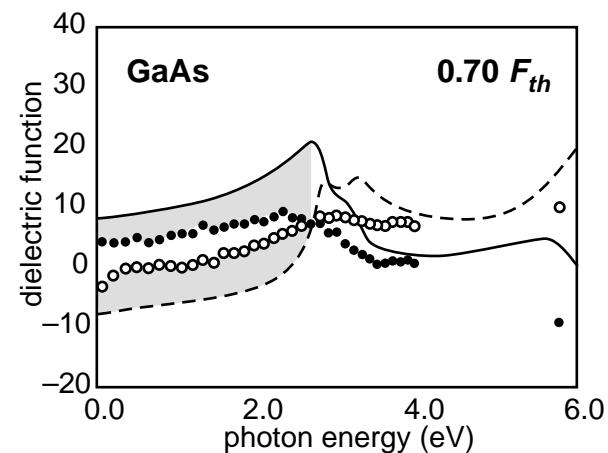
structure



bandstructure



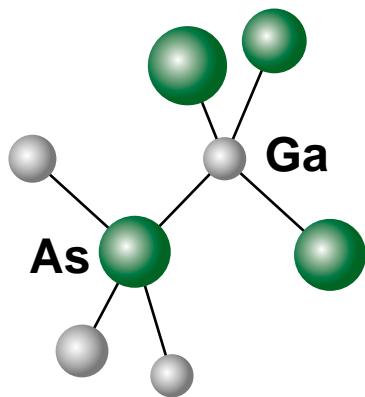
dielectric function



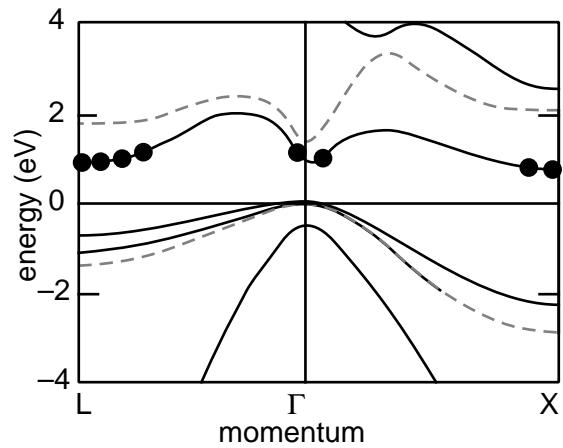
Discussion

short time scale

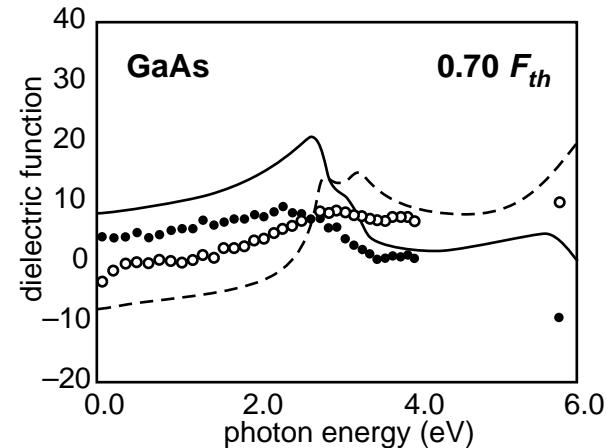
structure



bandstructure



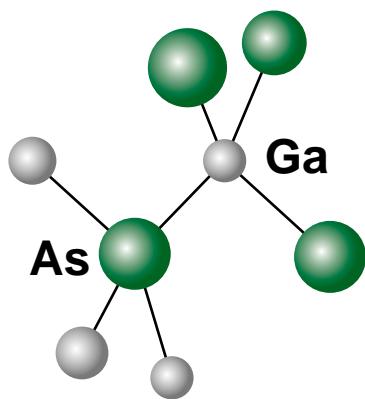
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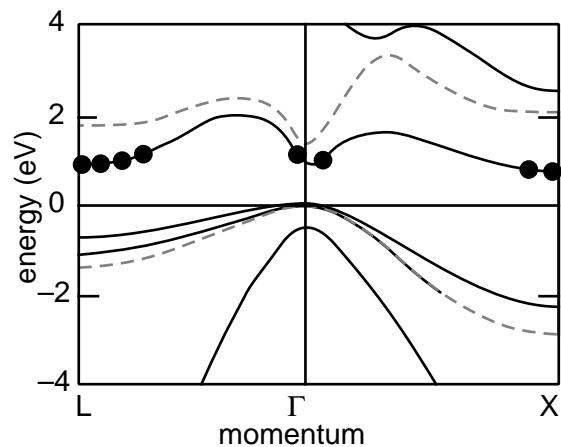
Discussion

short time scale

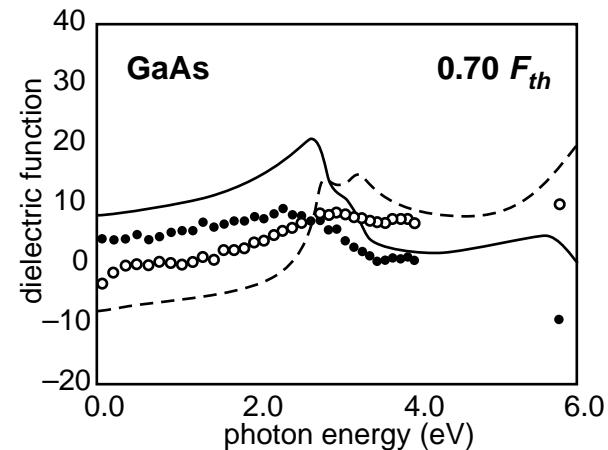
structure



bandstructure



dielectric function

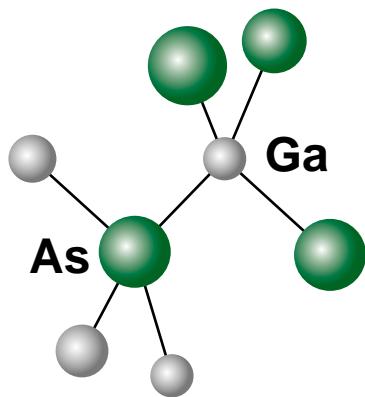


D.H. Kim, et al., Sol. State Comm. 89, 119 (1994)

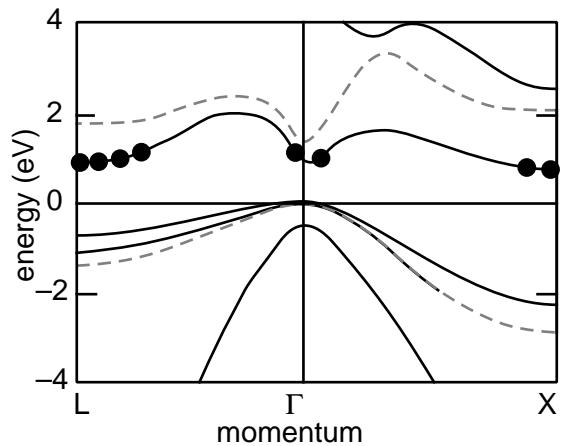
Discussion

short time scale

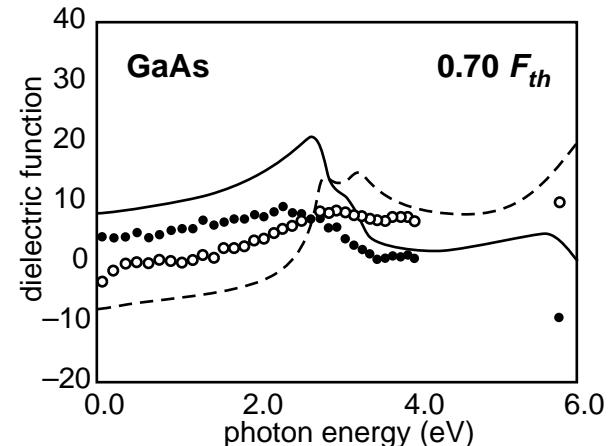
structure



bandstructure



dielectric function

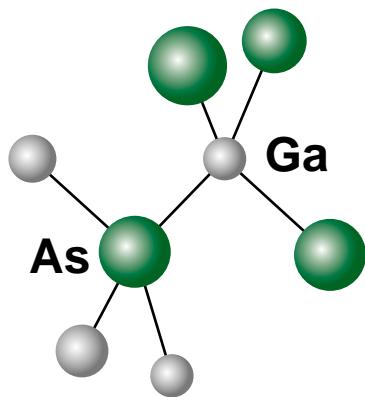


electronic effects dominate at short time scales...

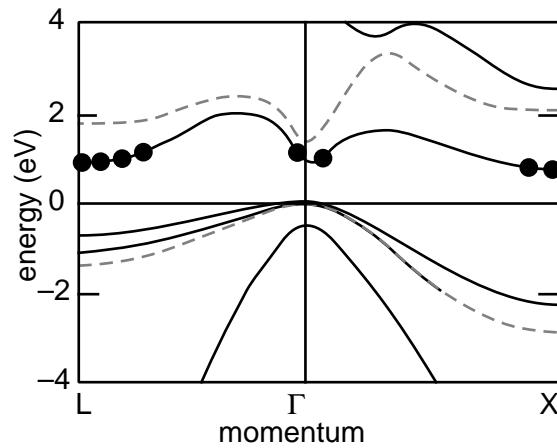
Discussion

short time scale

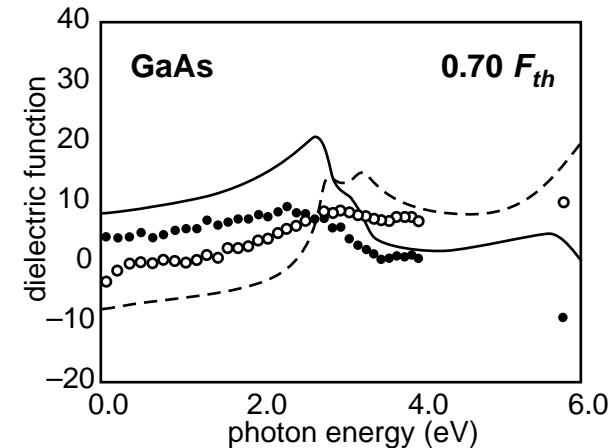
structure



bandstructure



dielectric function

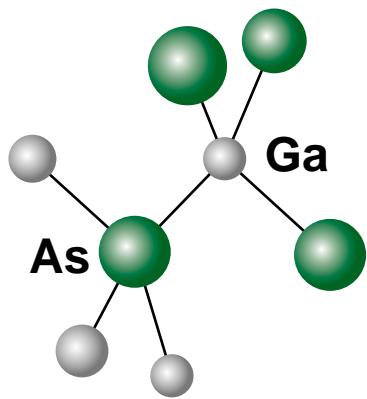


...but they are not as simple as we used to think

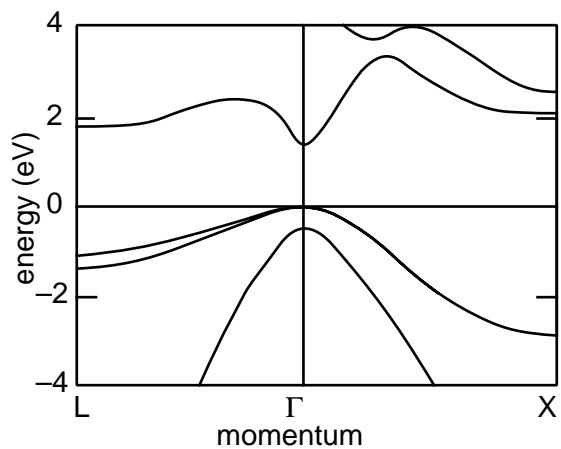
Discussion

long time scale, high fluence

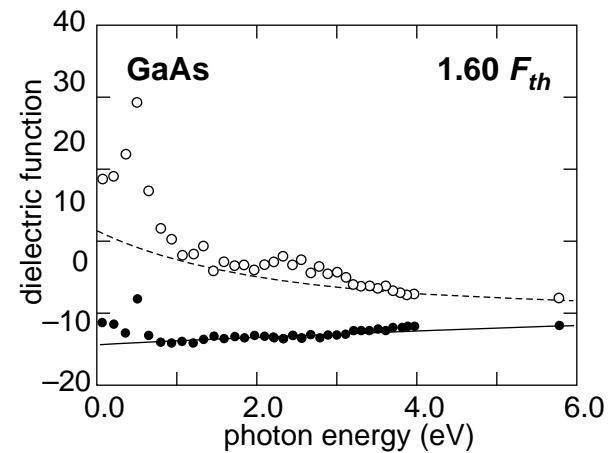
structure



bandstructure



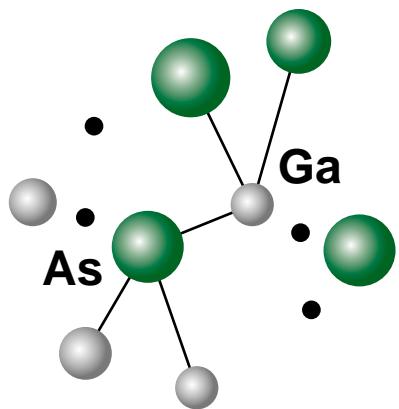
dielectric function



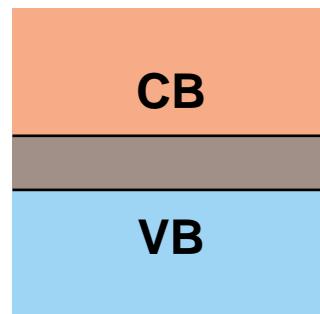
Discussion

long time scale, high fluence

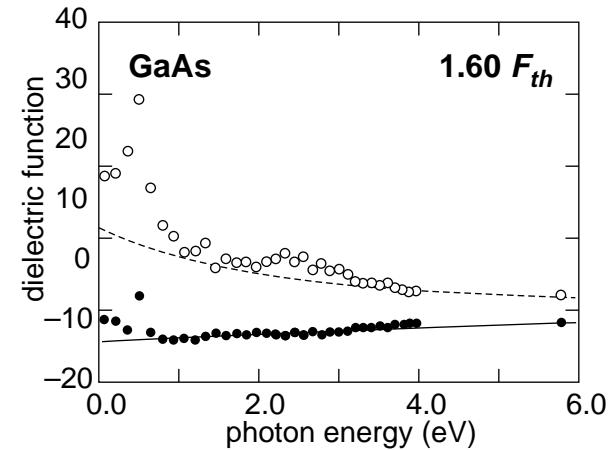
structure



bandstructure



dielectric function

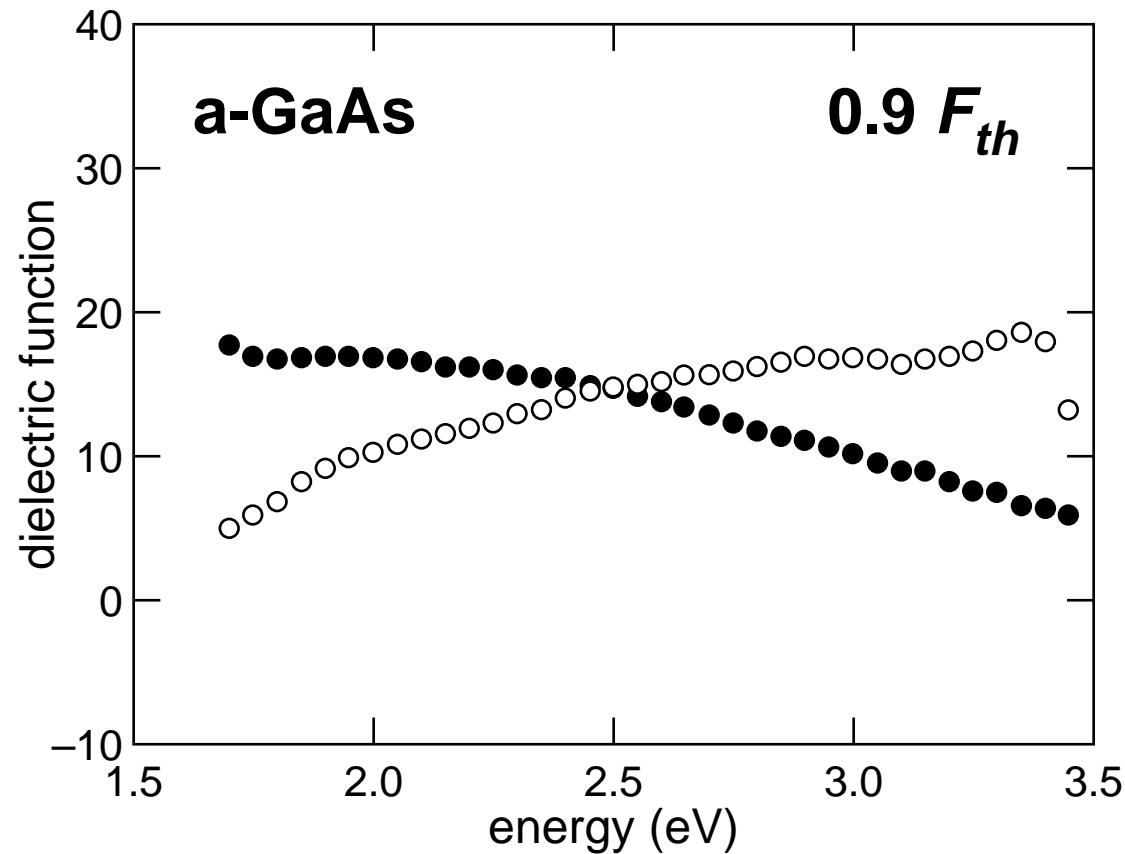


gradual drop in gap → not electronic effect

-
- ▶ Method
 - ▶ Results: a-GaAs
 - ▶ Conclusions

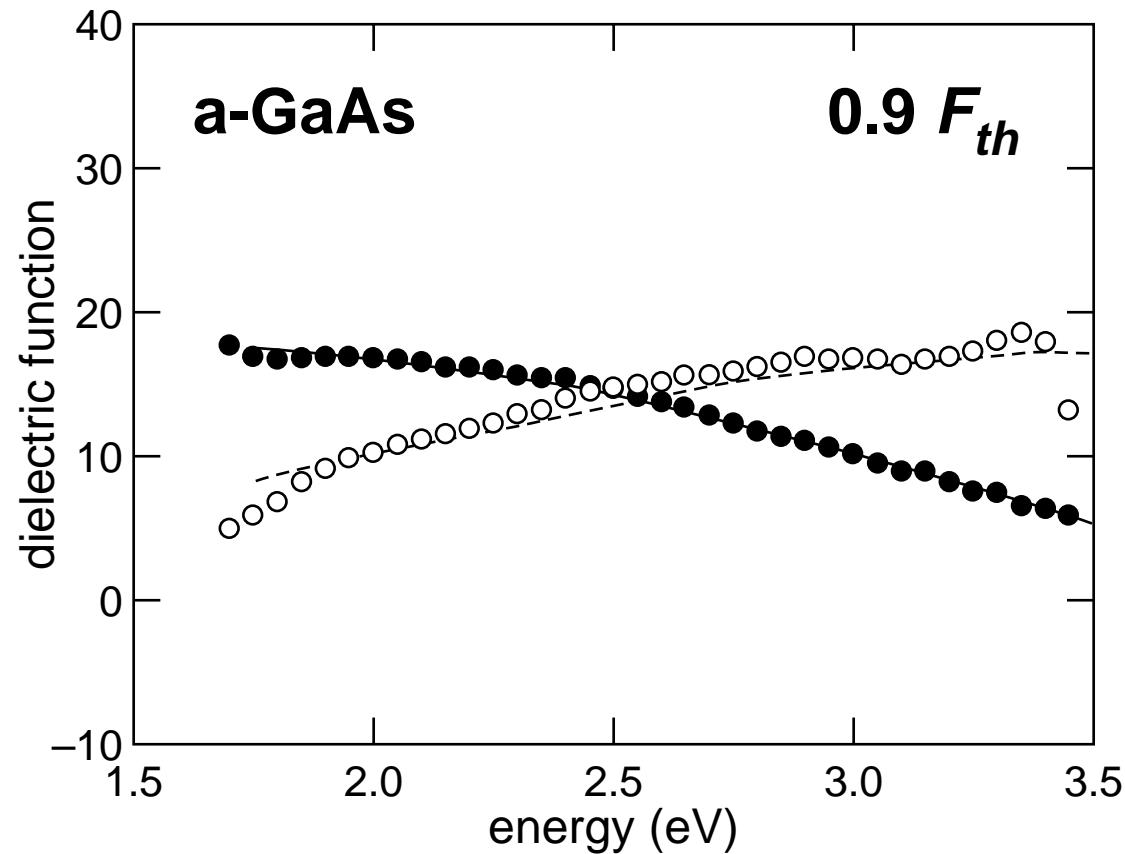
Results

-16 ps

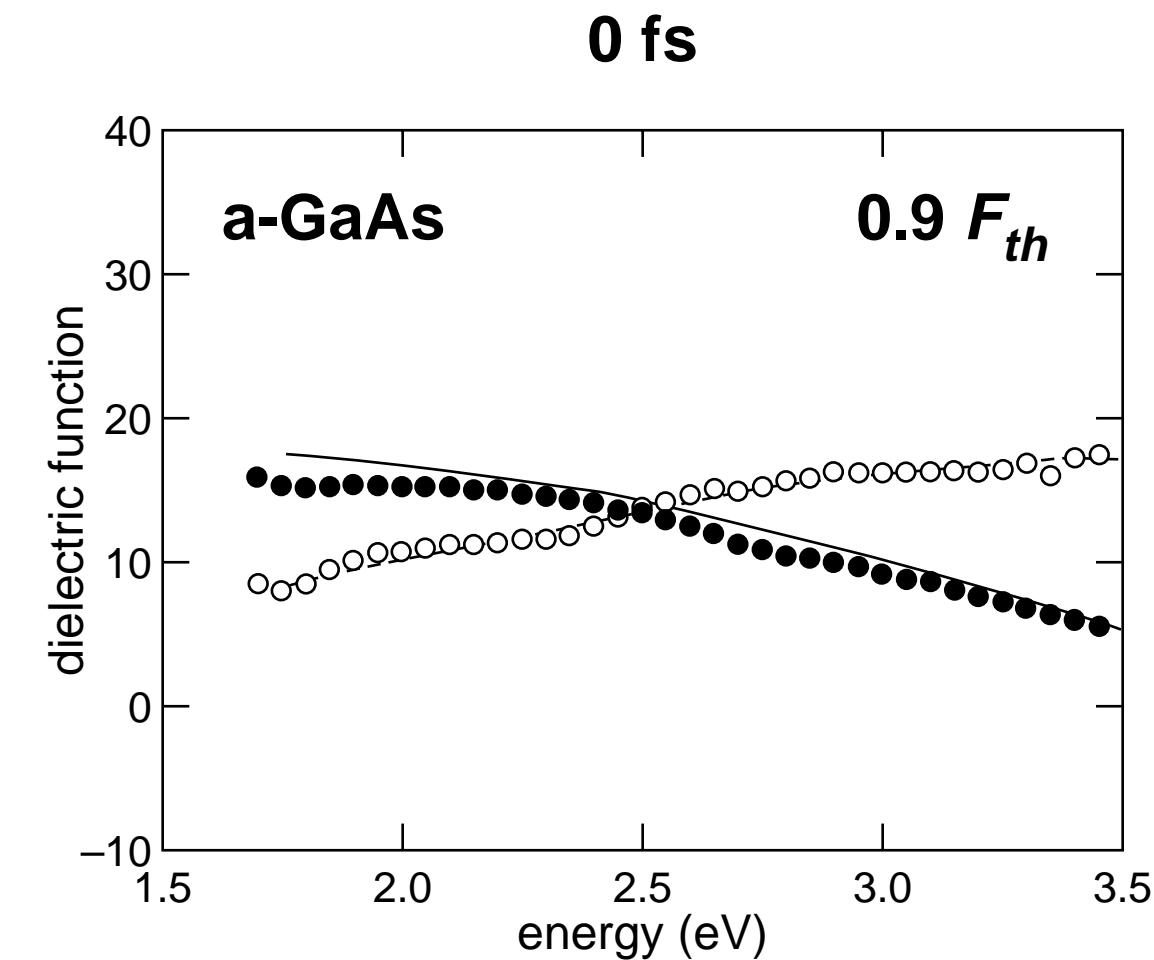


Results

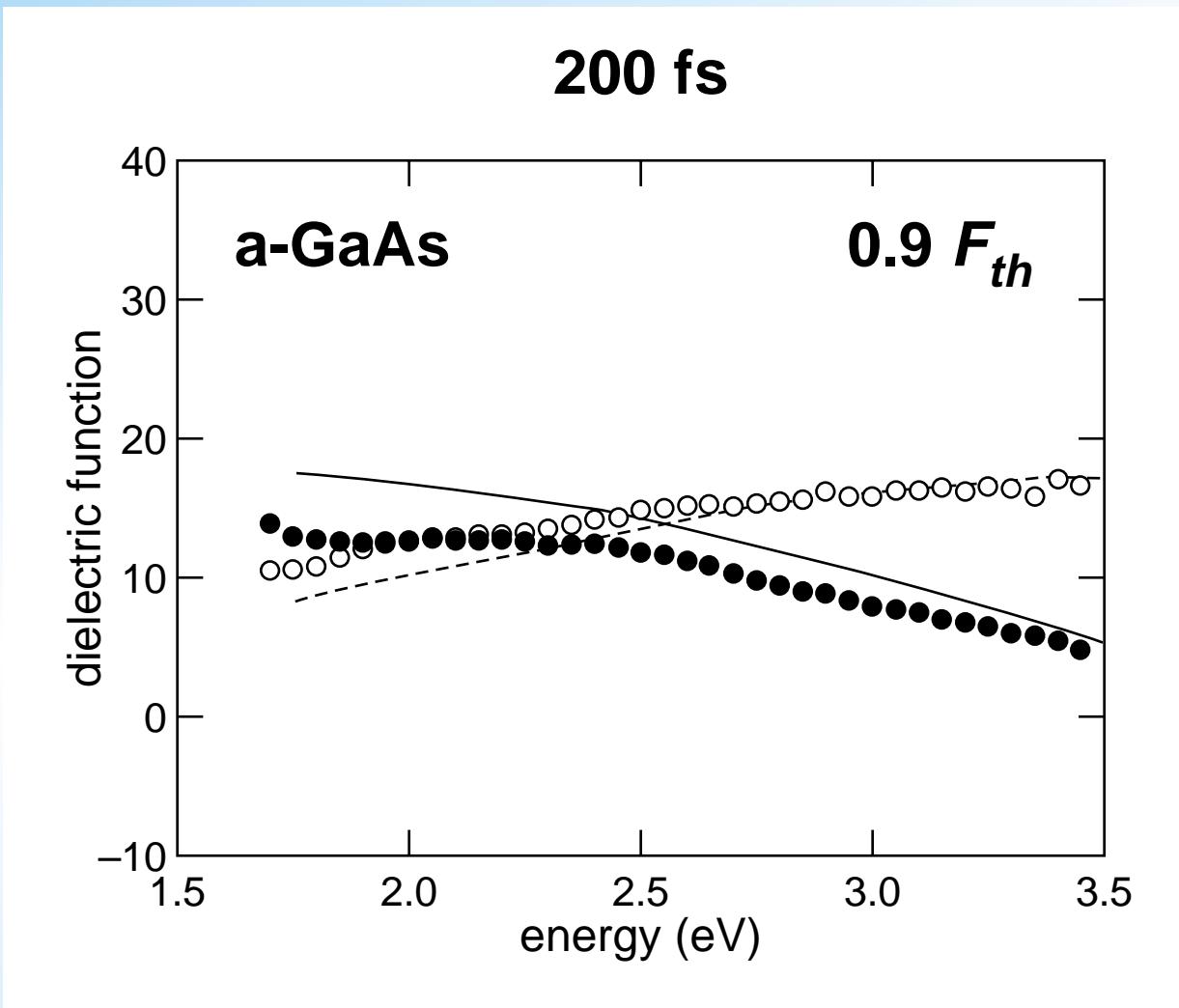
-16 ps



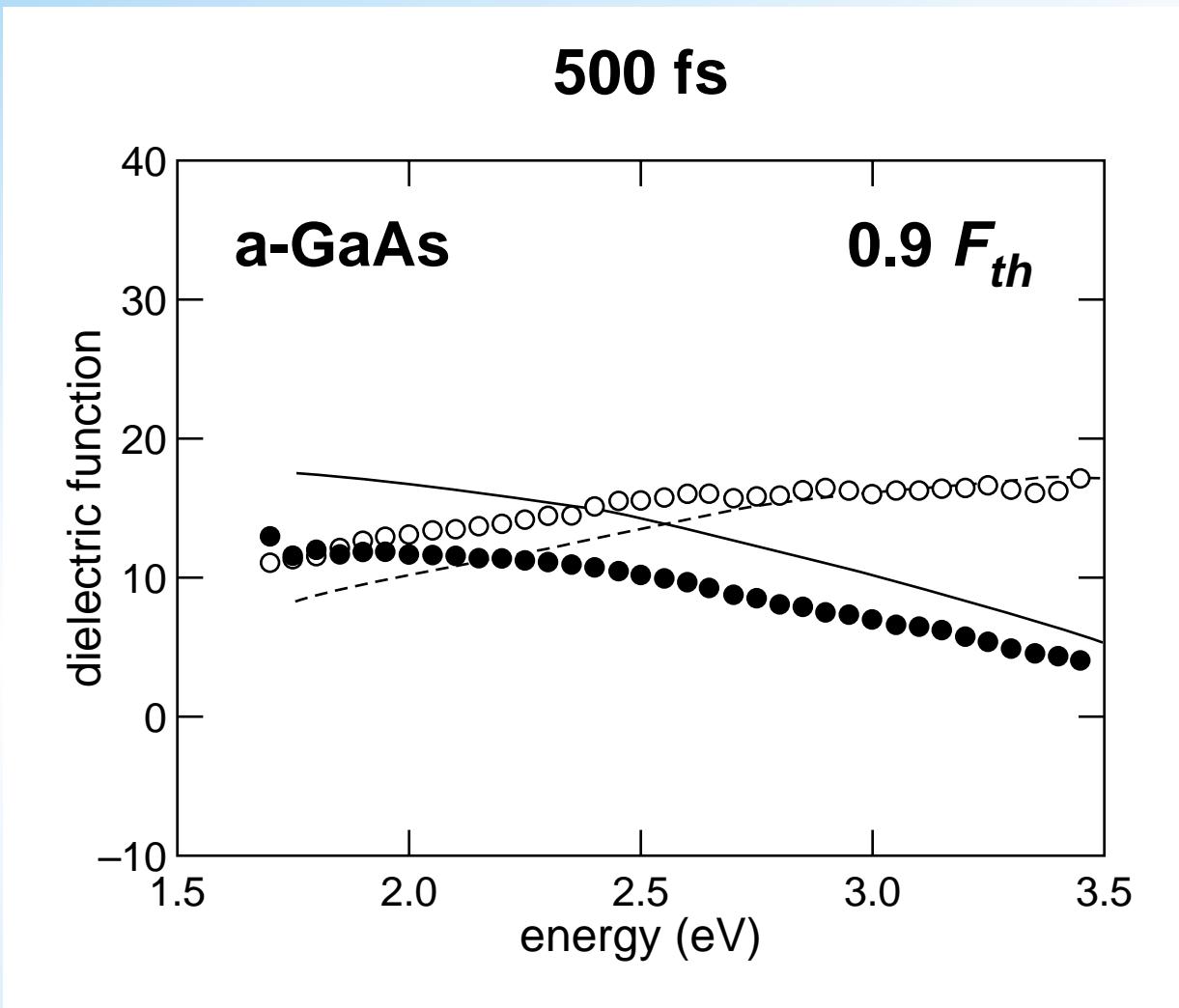
Results



Results

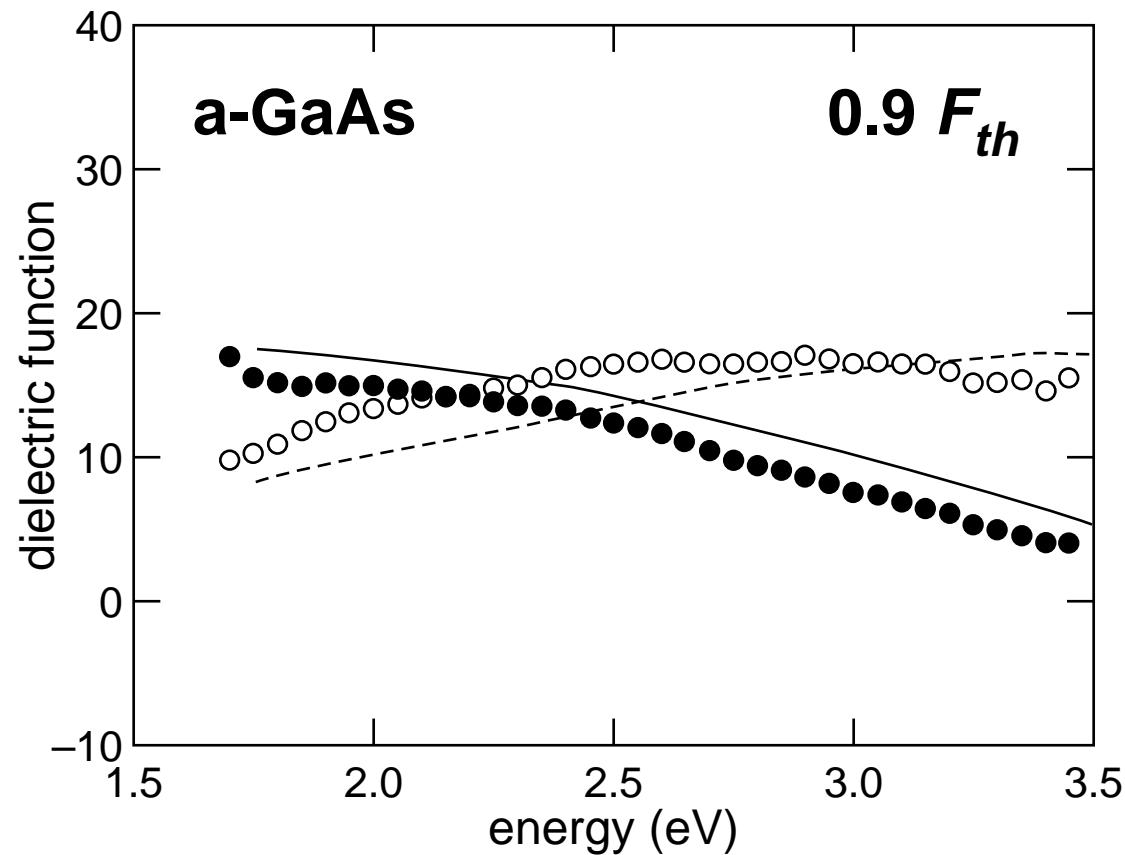


Results

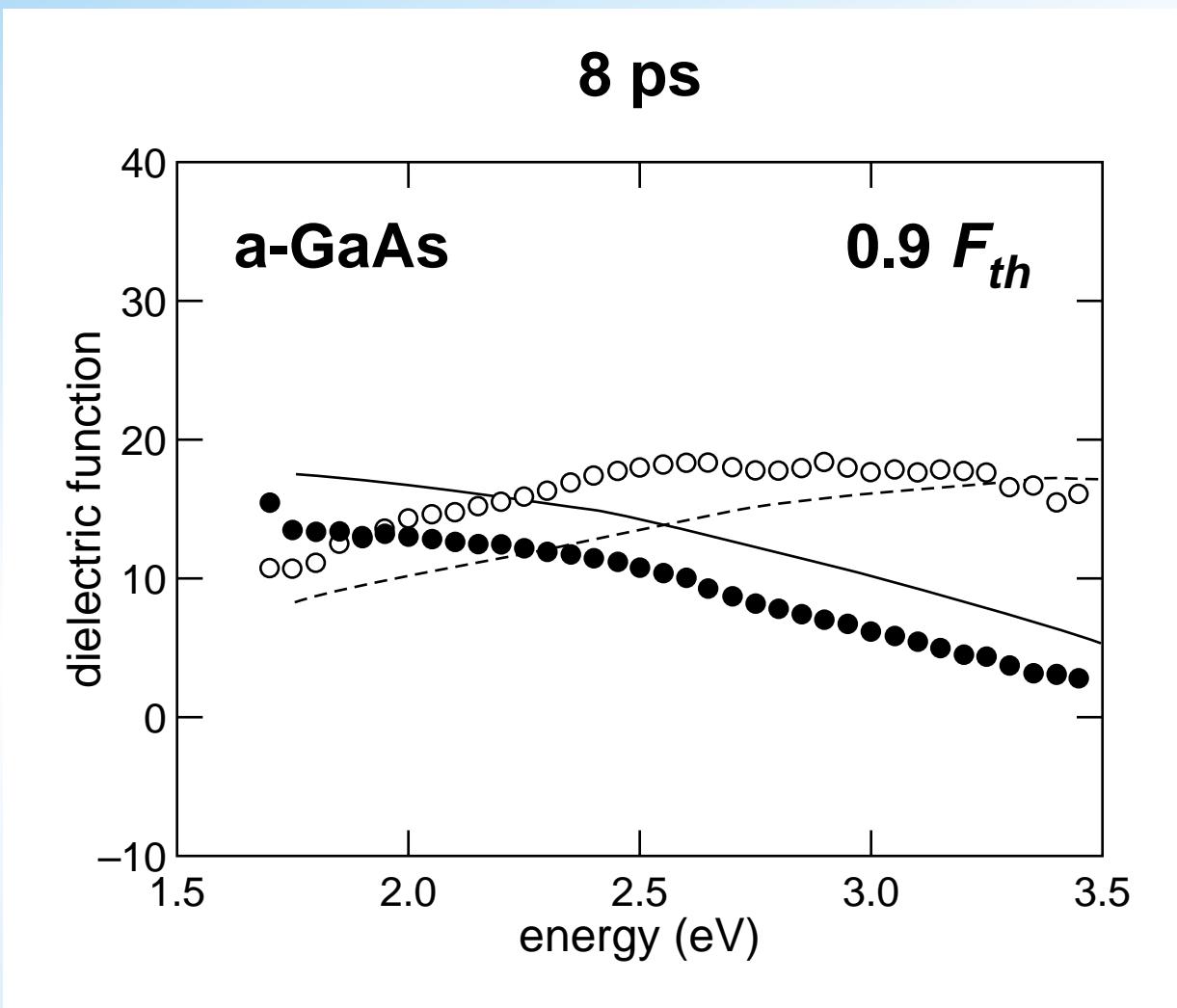


Results

2 ps

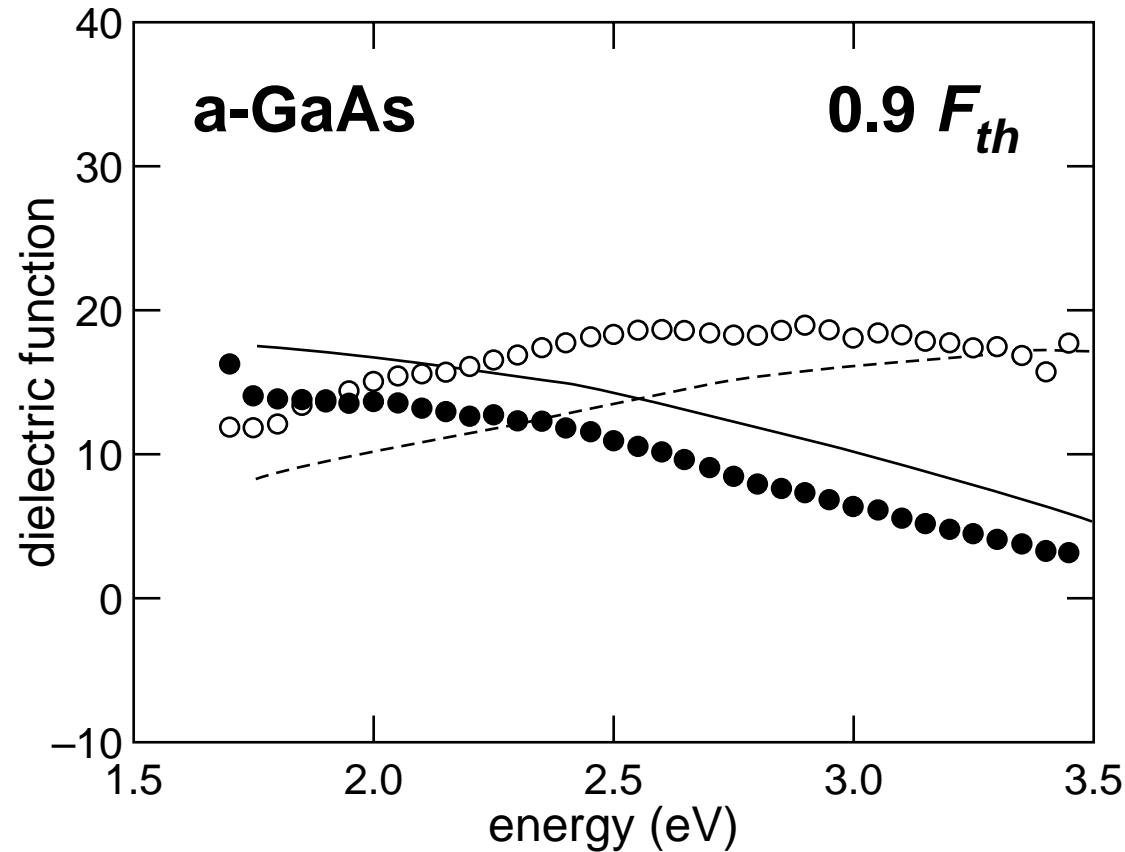


Results



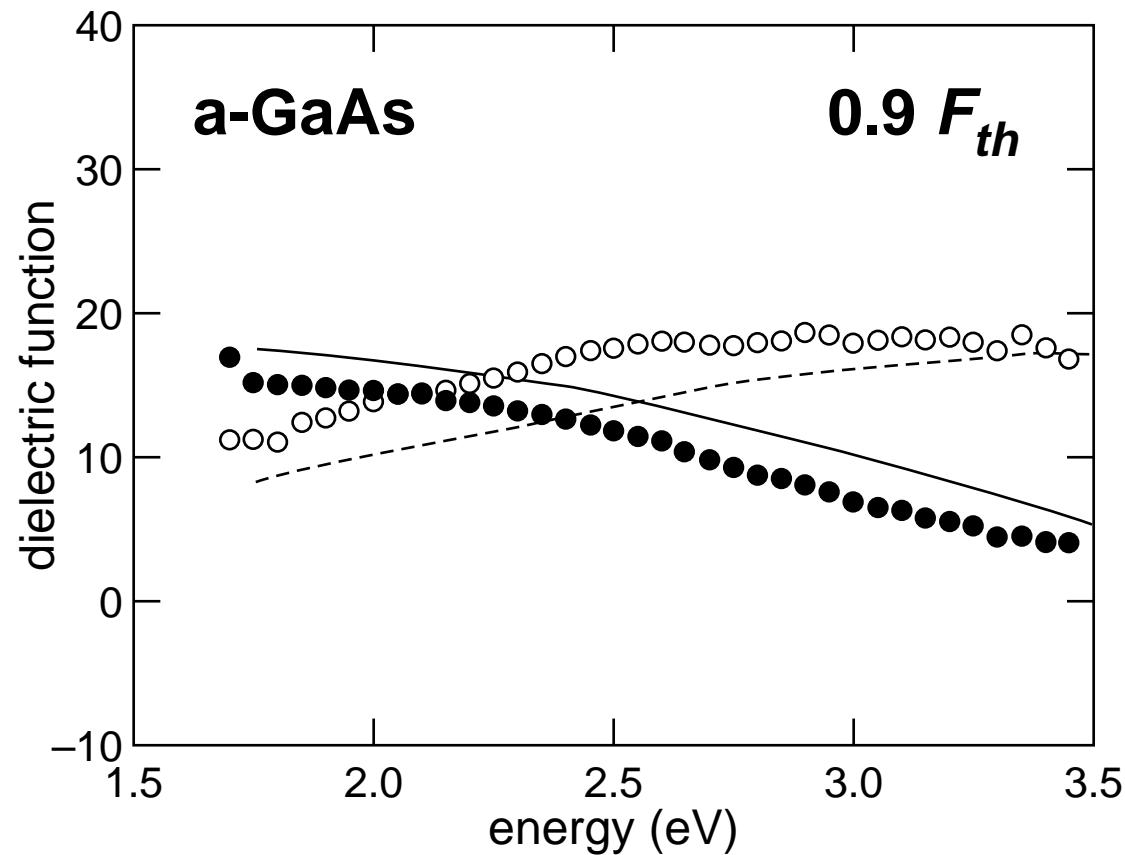
Results

16 ps



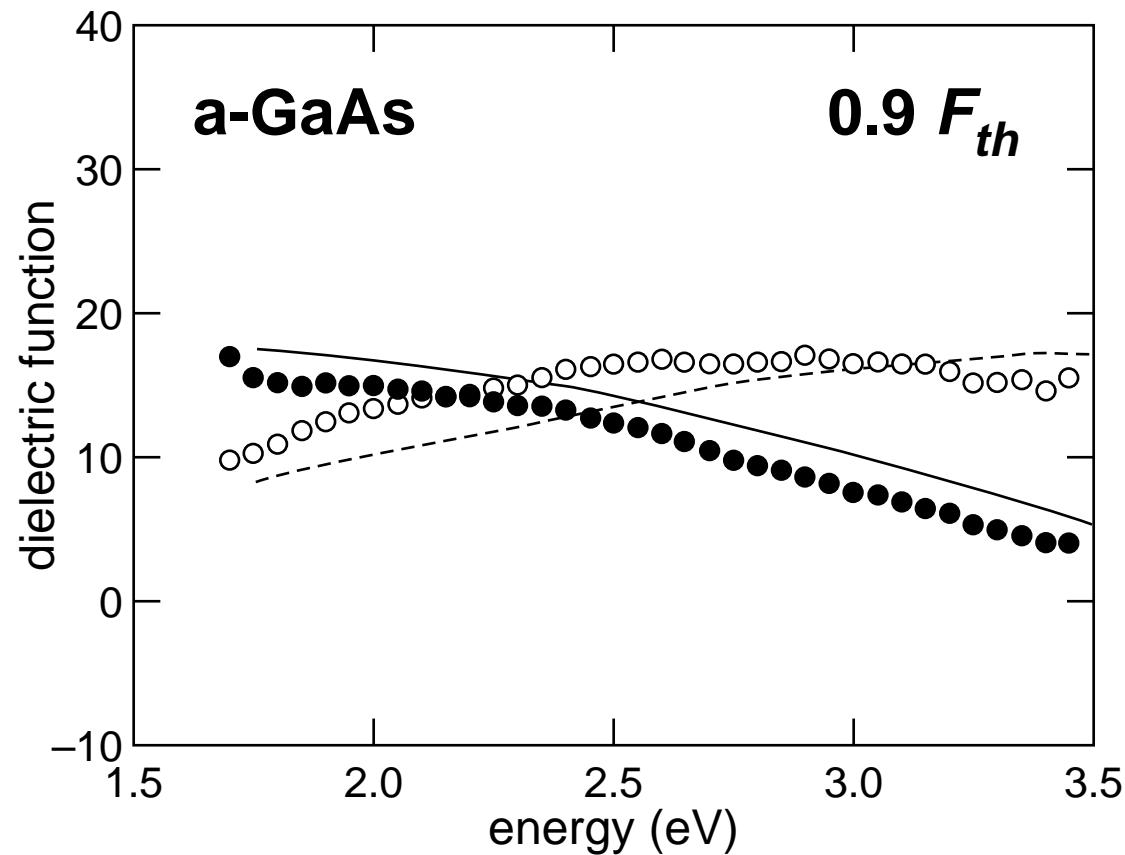
Results

400 ps



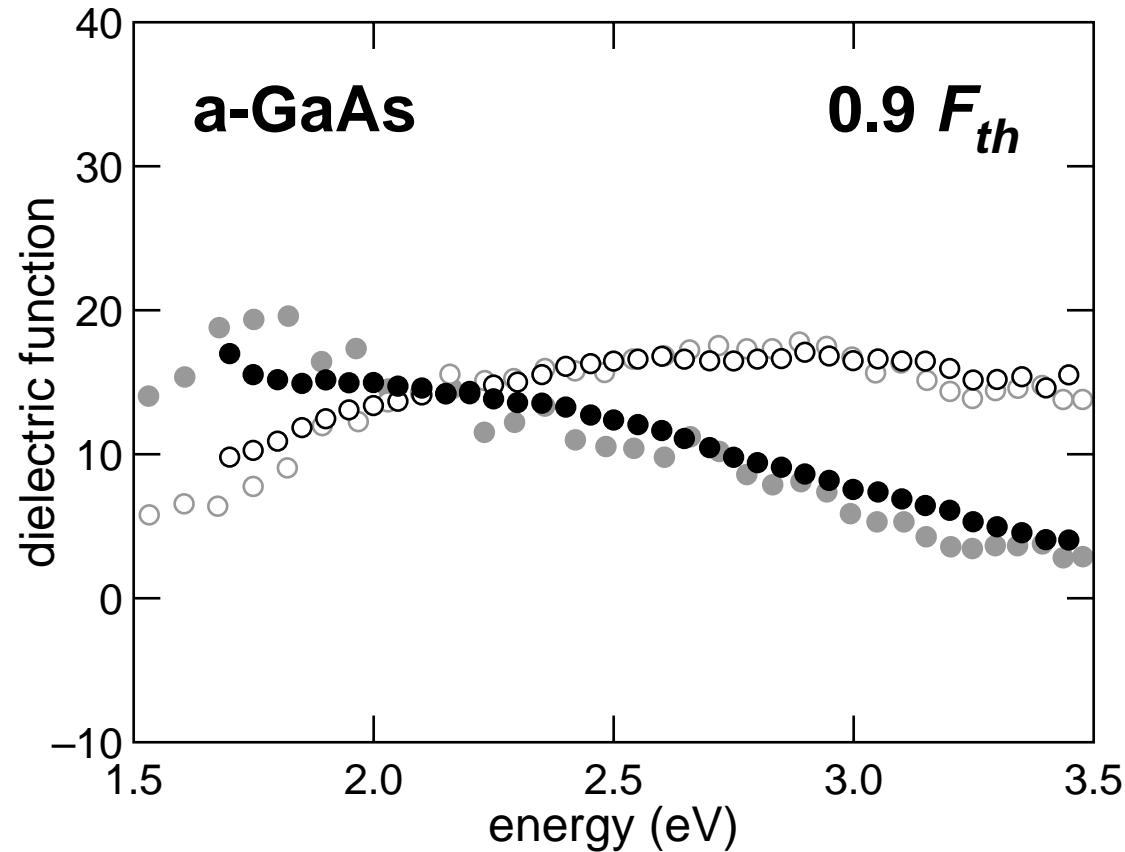
Results

2 ps



Results

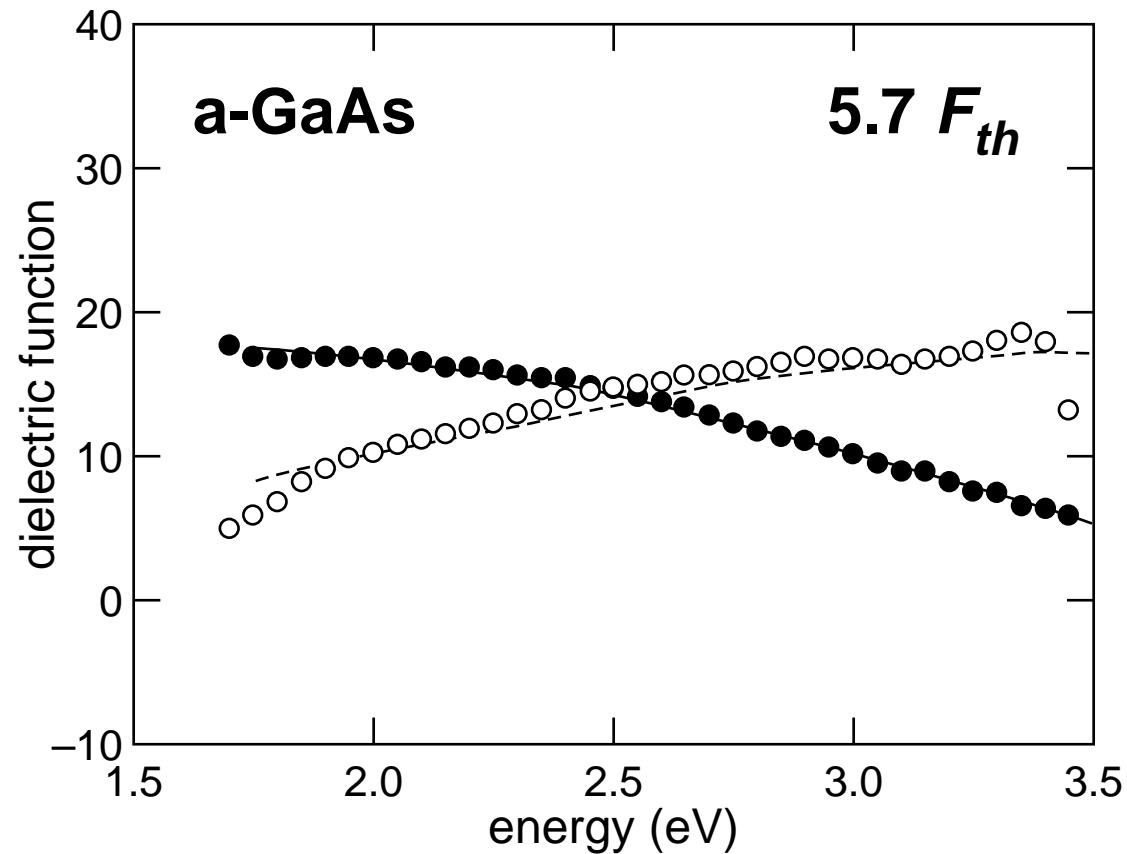
2 ps



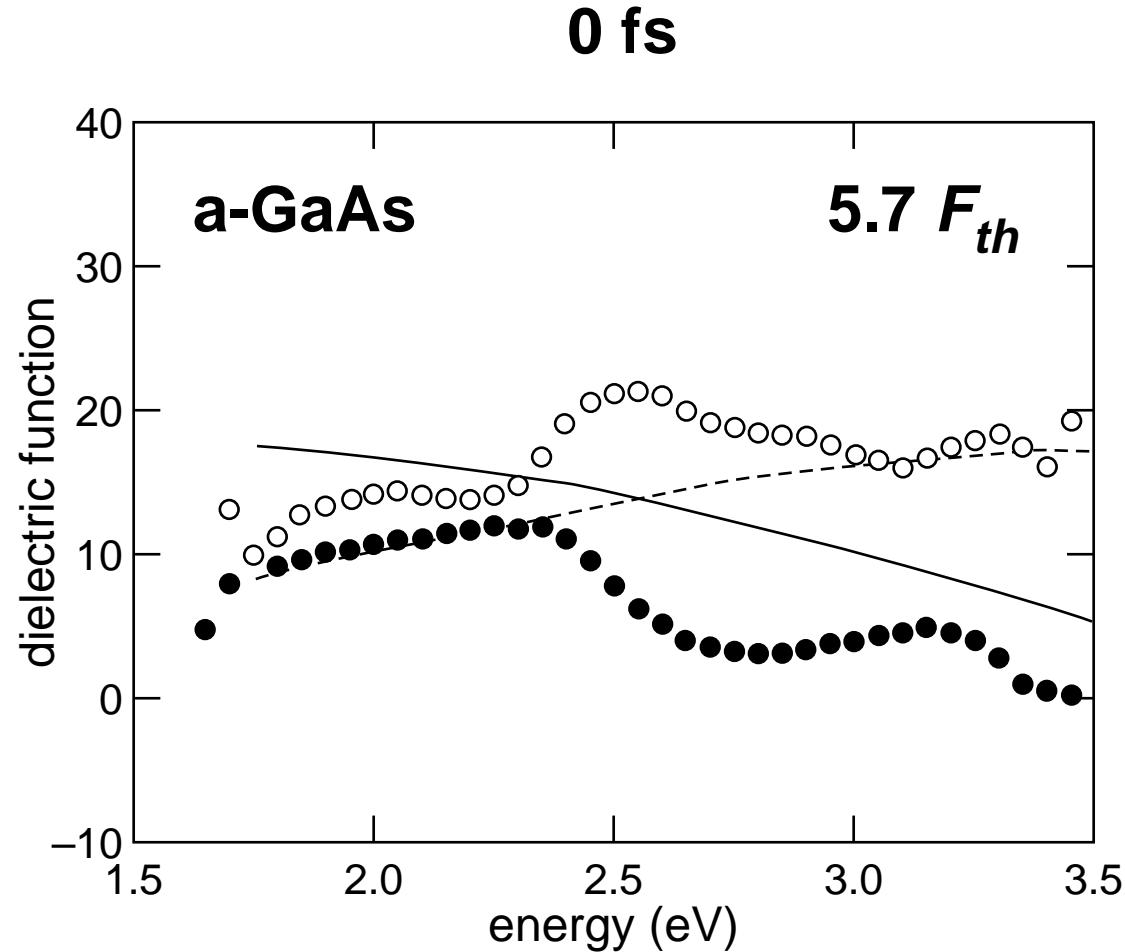
matches disordered c-GaAs

Results

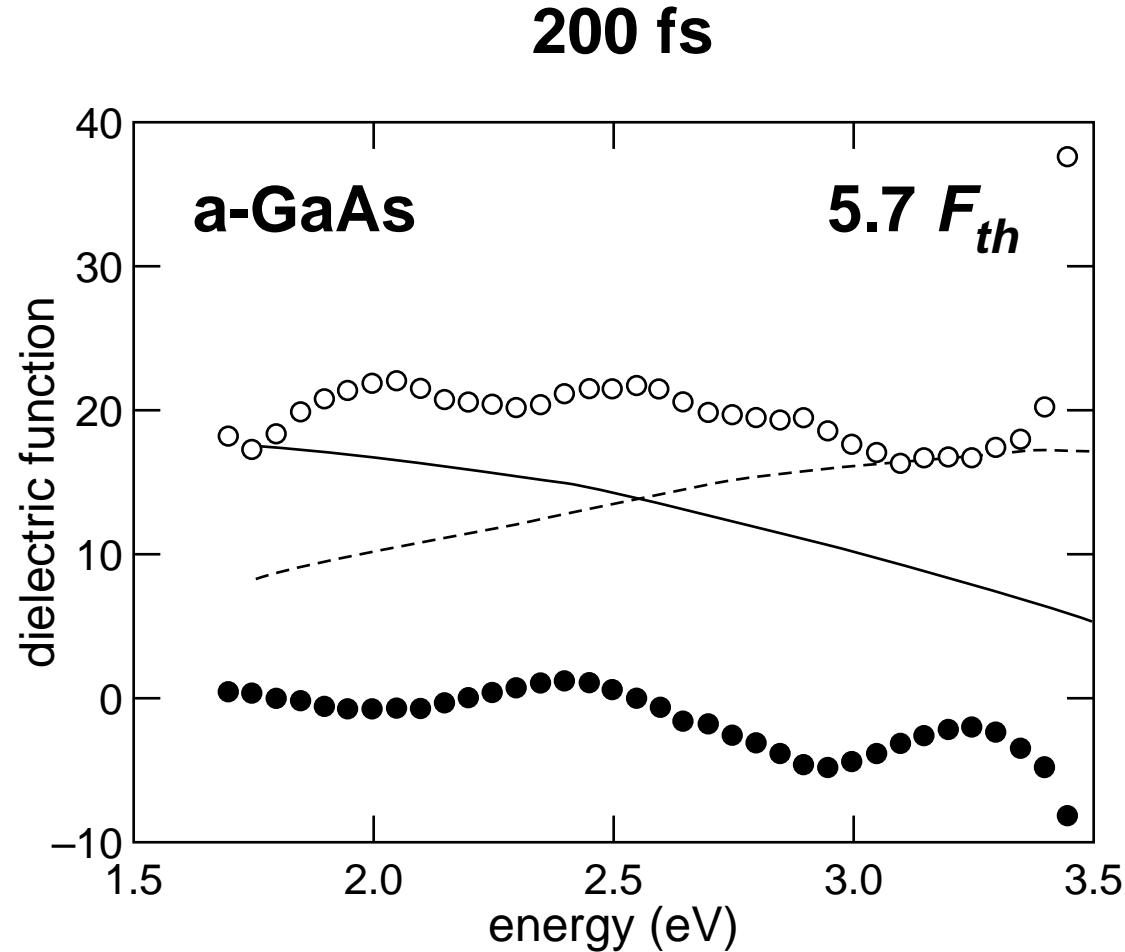
-16 ps



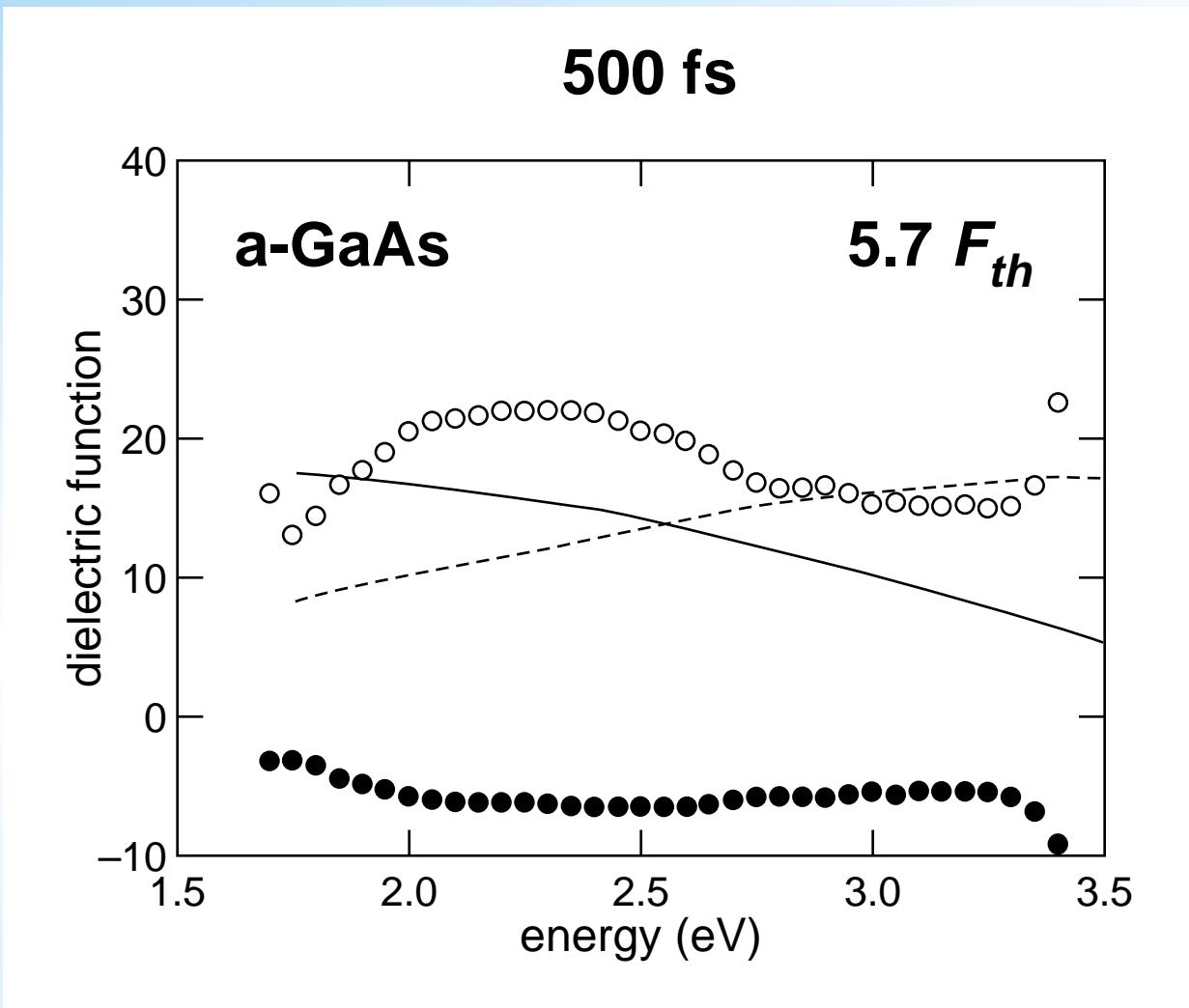
Results



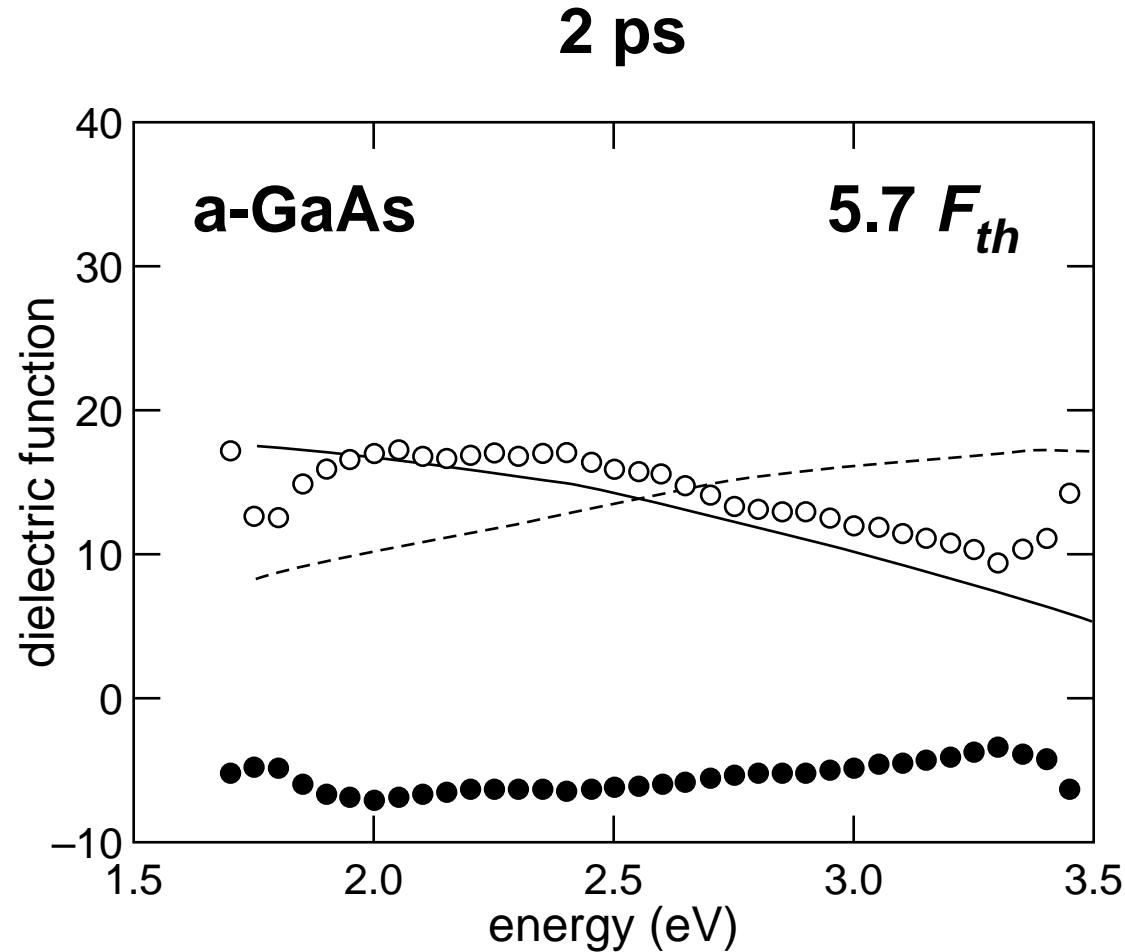
Results



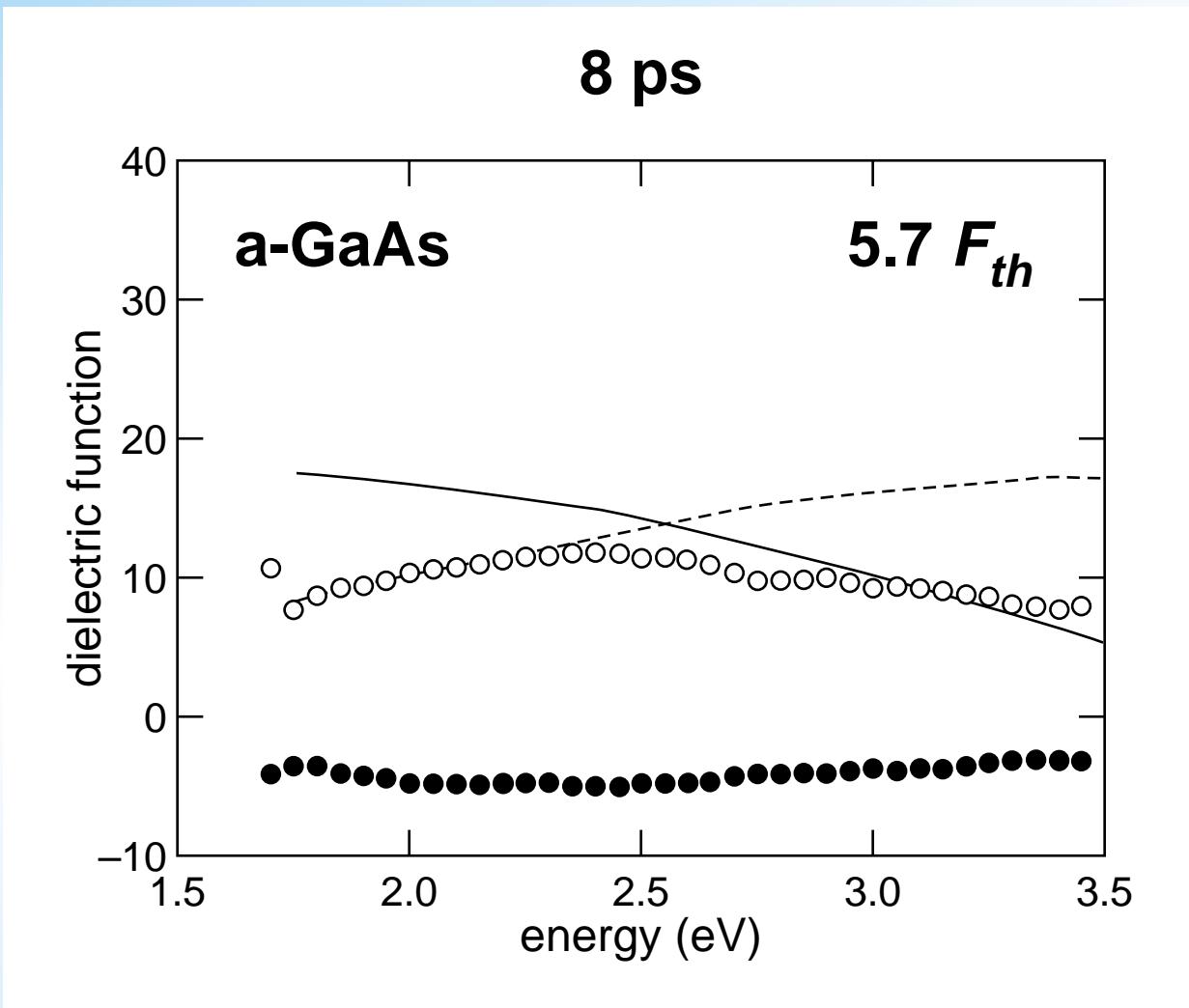
Results



Results

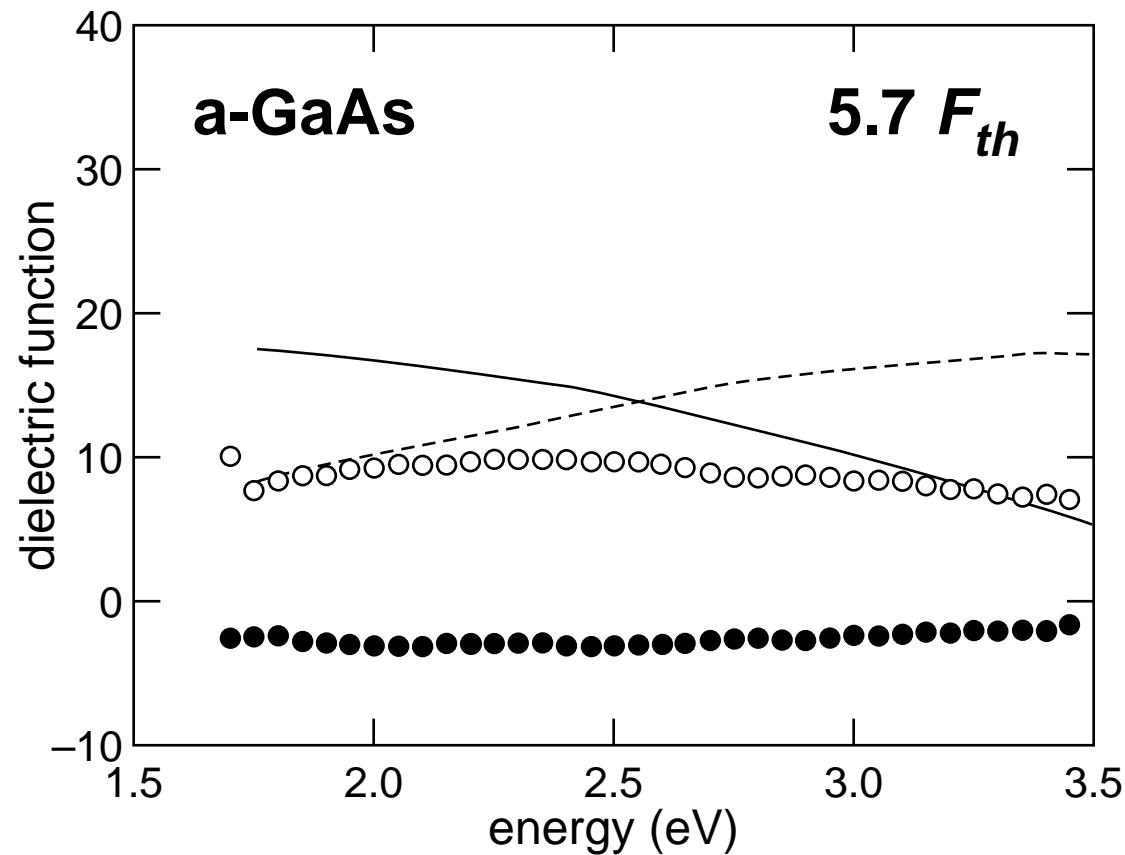


Results



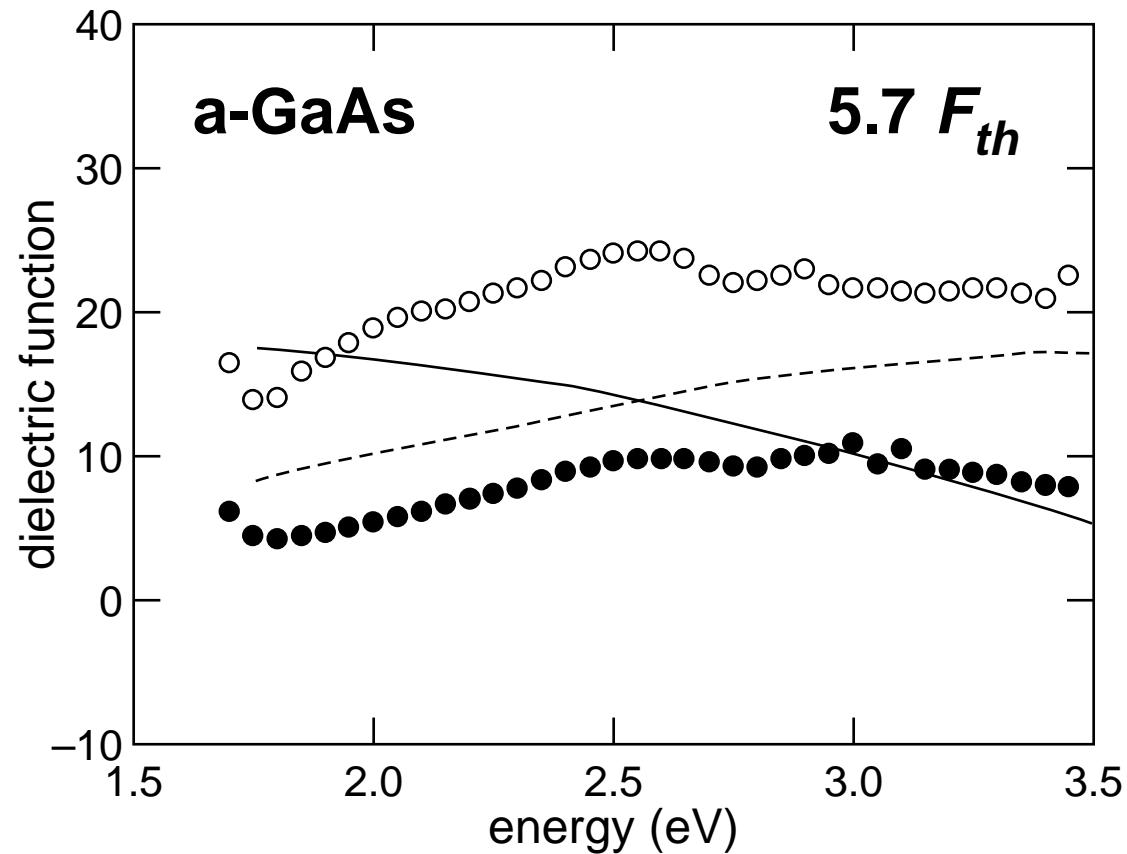
Results

16 ps

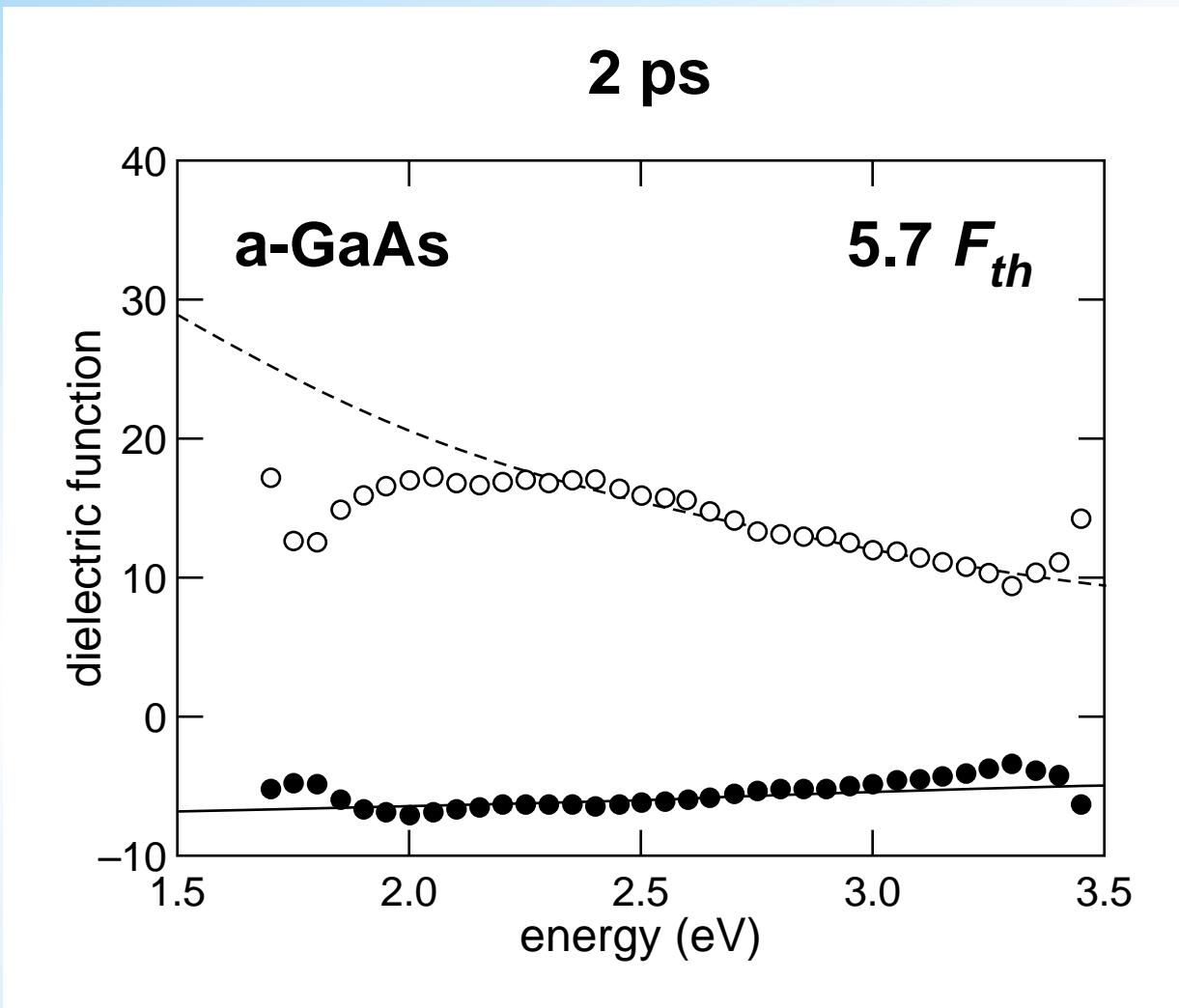


Results

400 ps



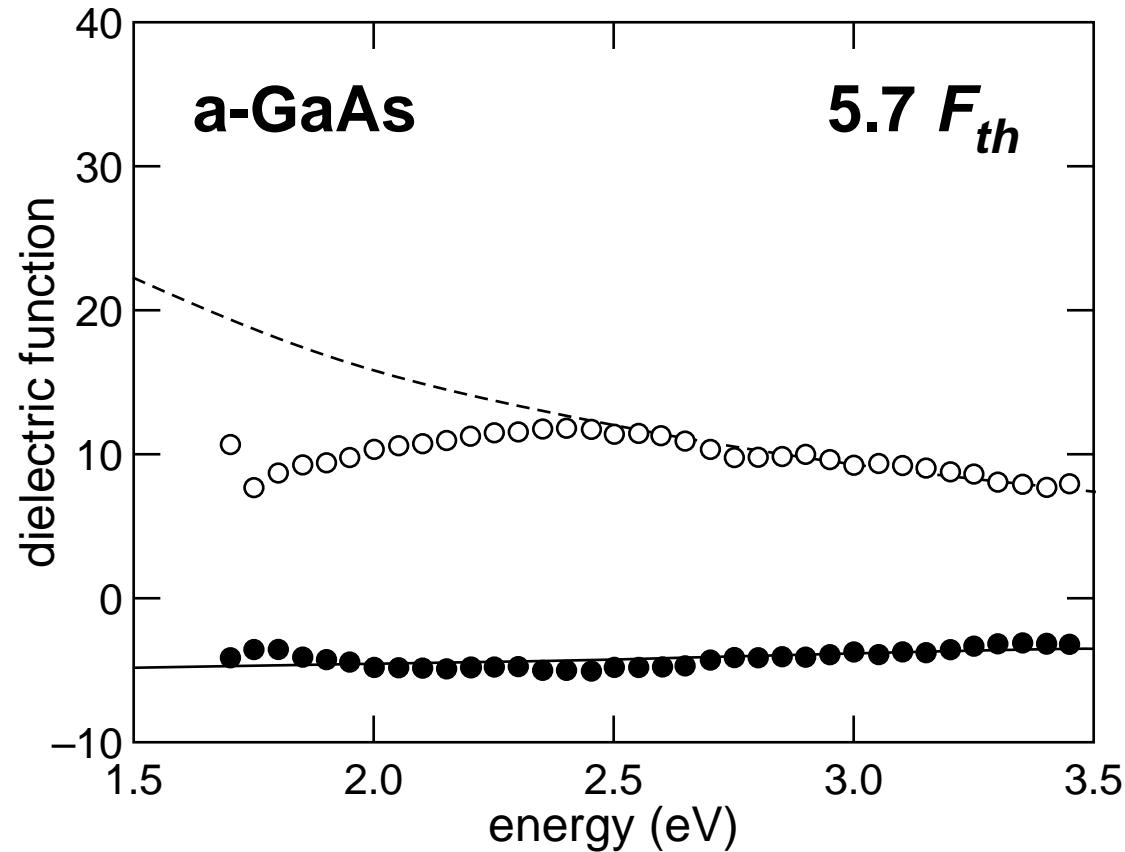
Results



Drude-like after 2 ps

Results

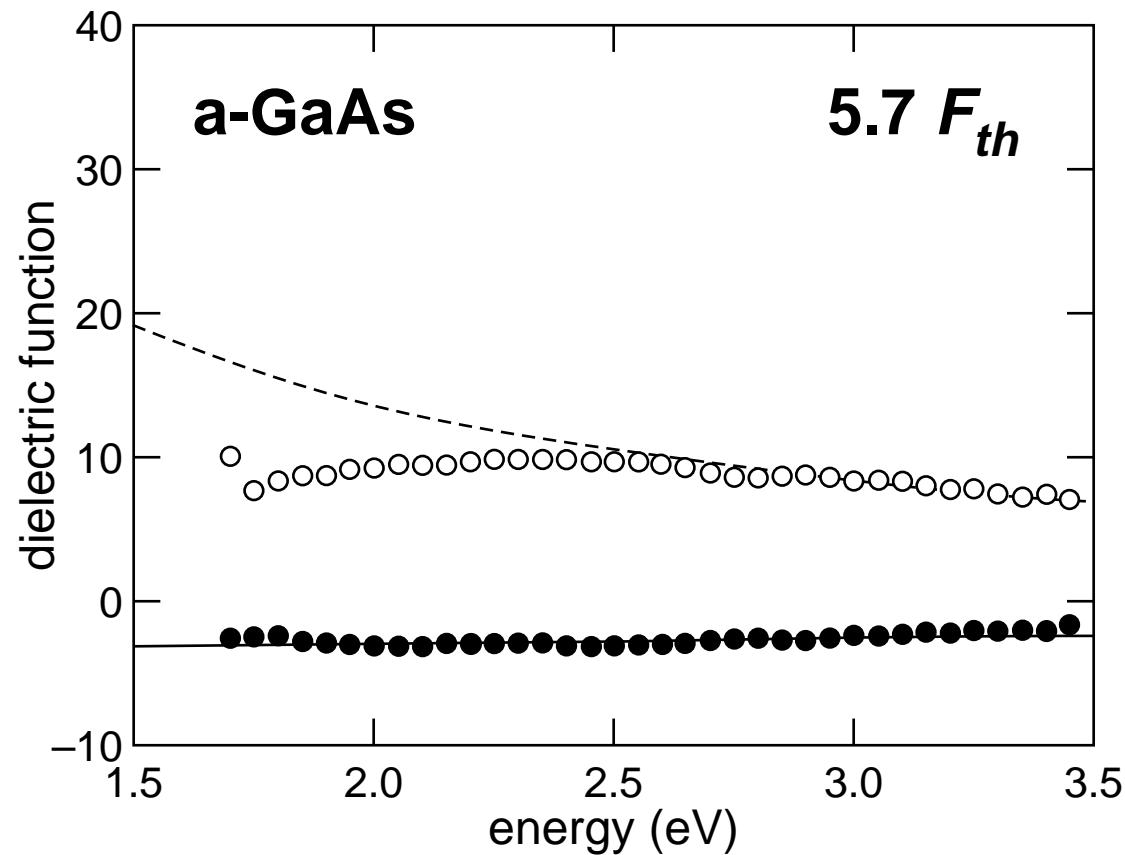
8 ps



plasma frequency decreases

Results

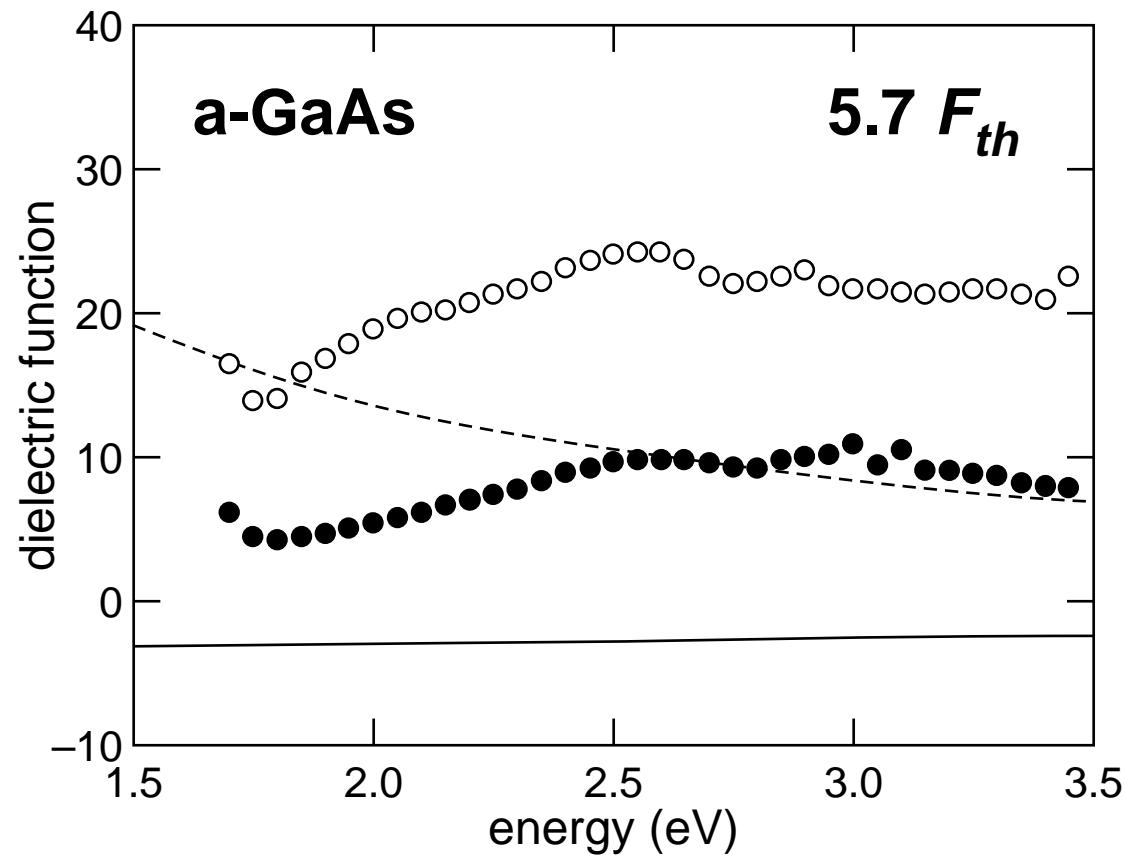
16 ps



plasma frequency decreases

Results

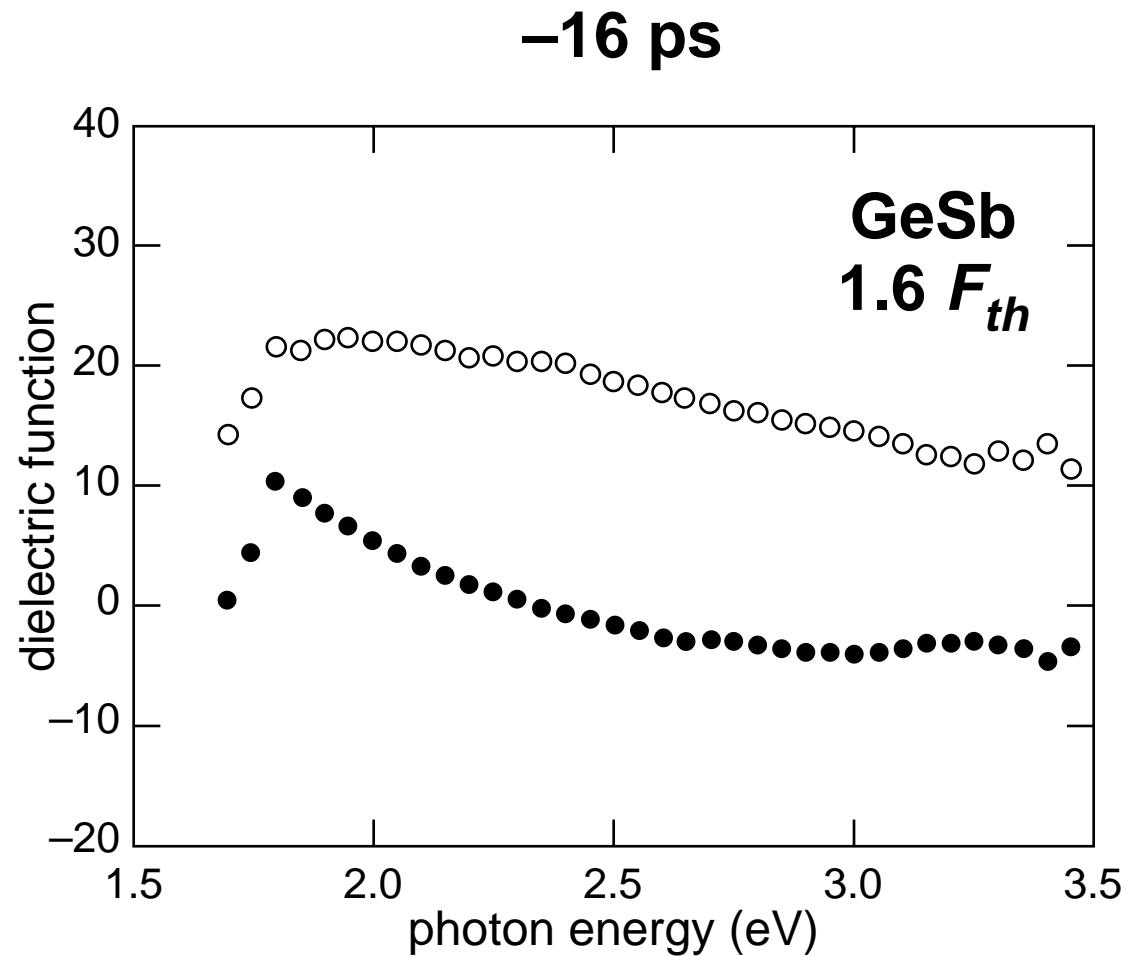
400 ps



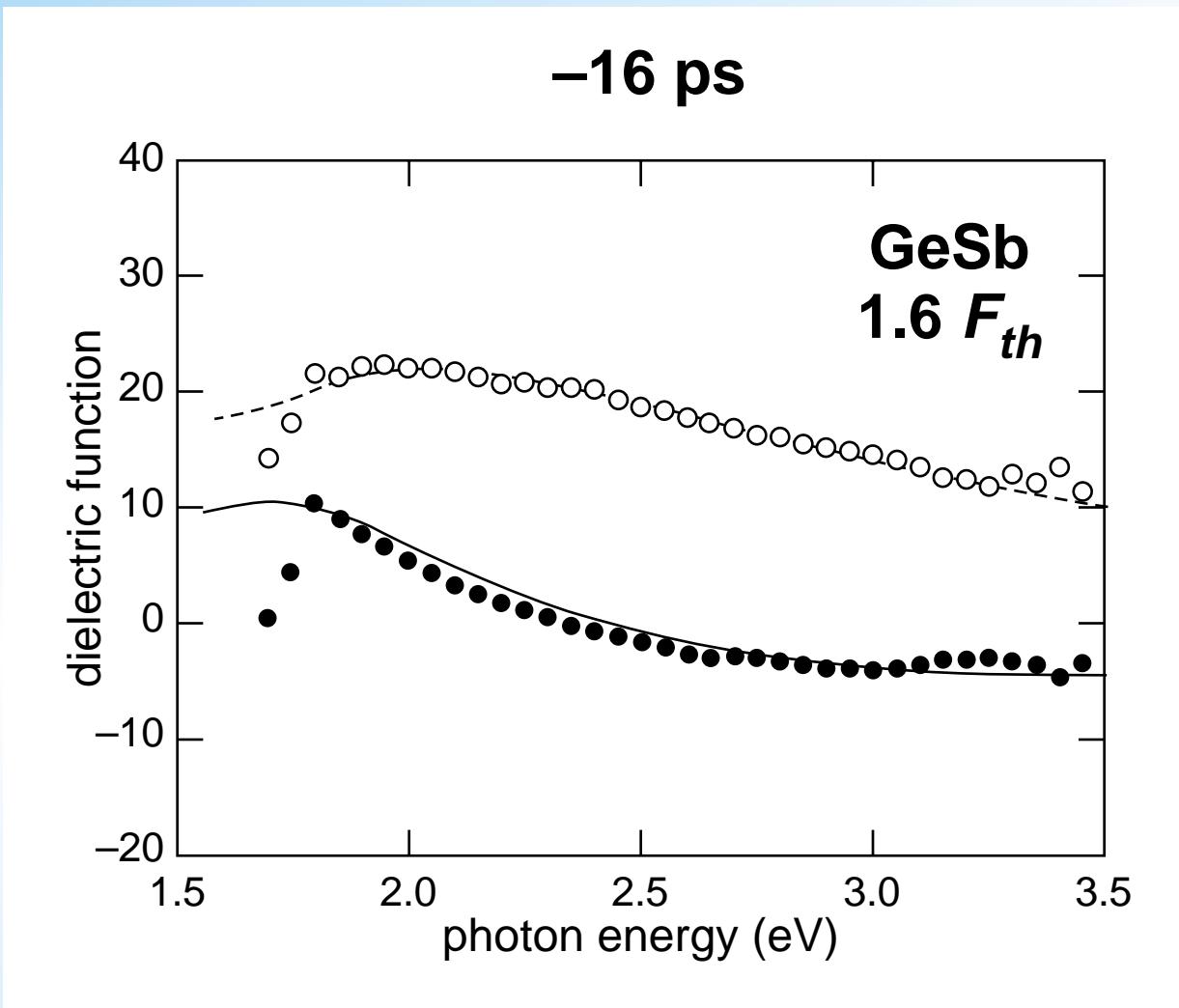
resolidification

-
- ▶ Method
 - ▶ Results: GeSb
 - ▶ Conclusions

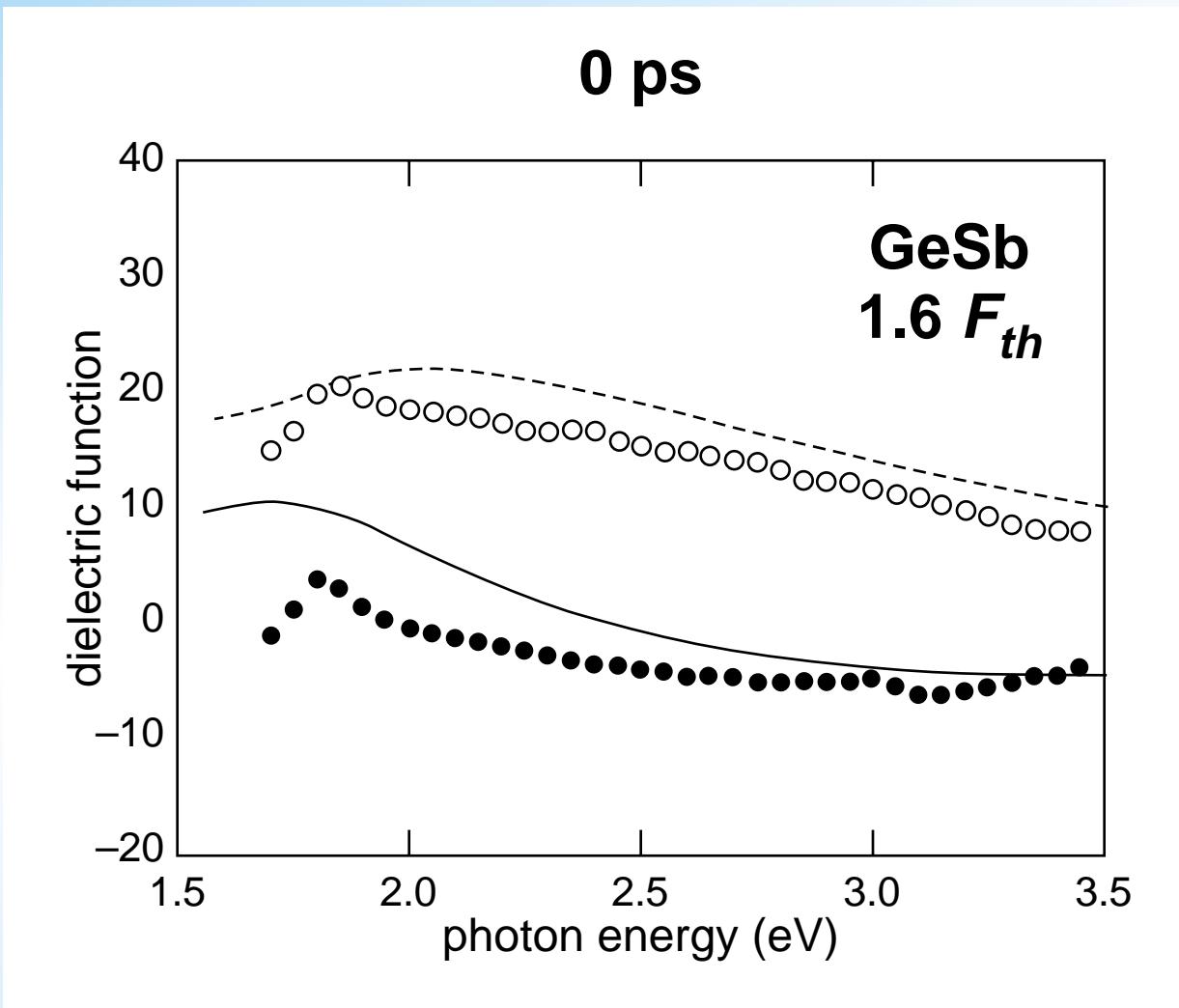
Results



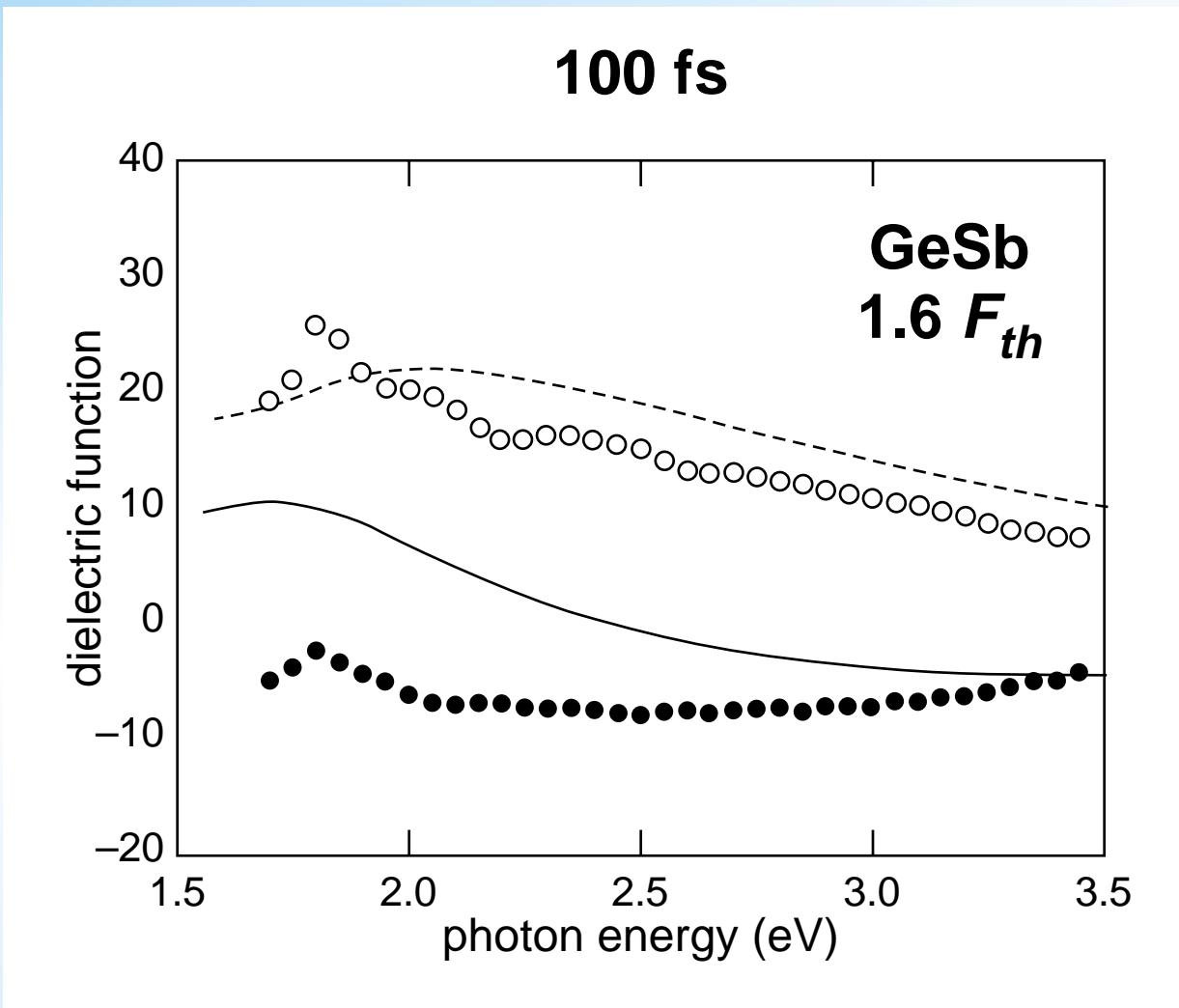
Results



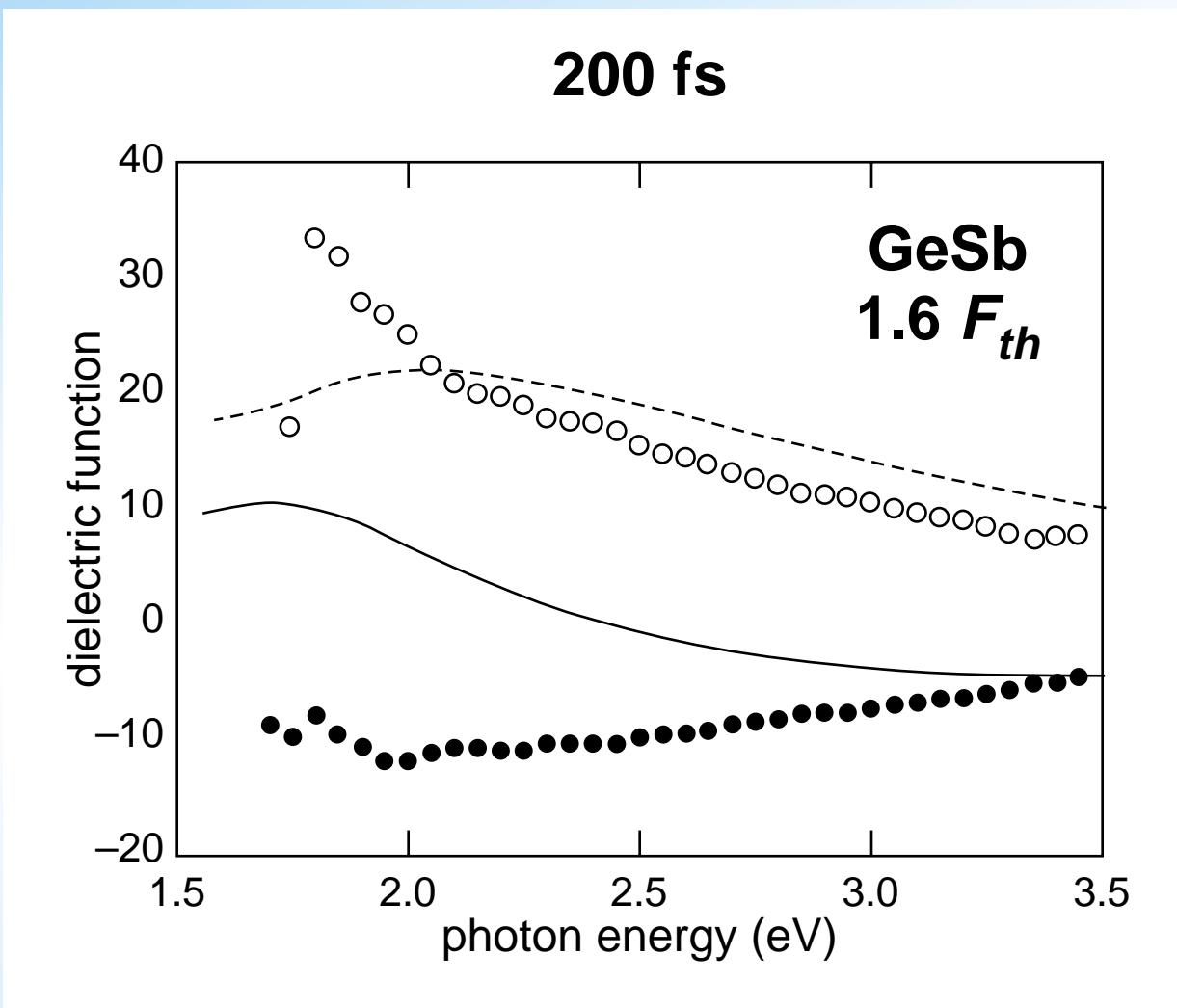
Results



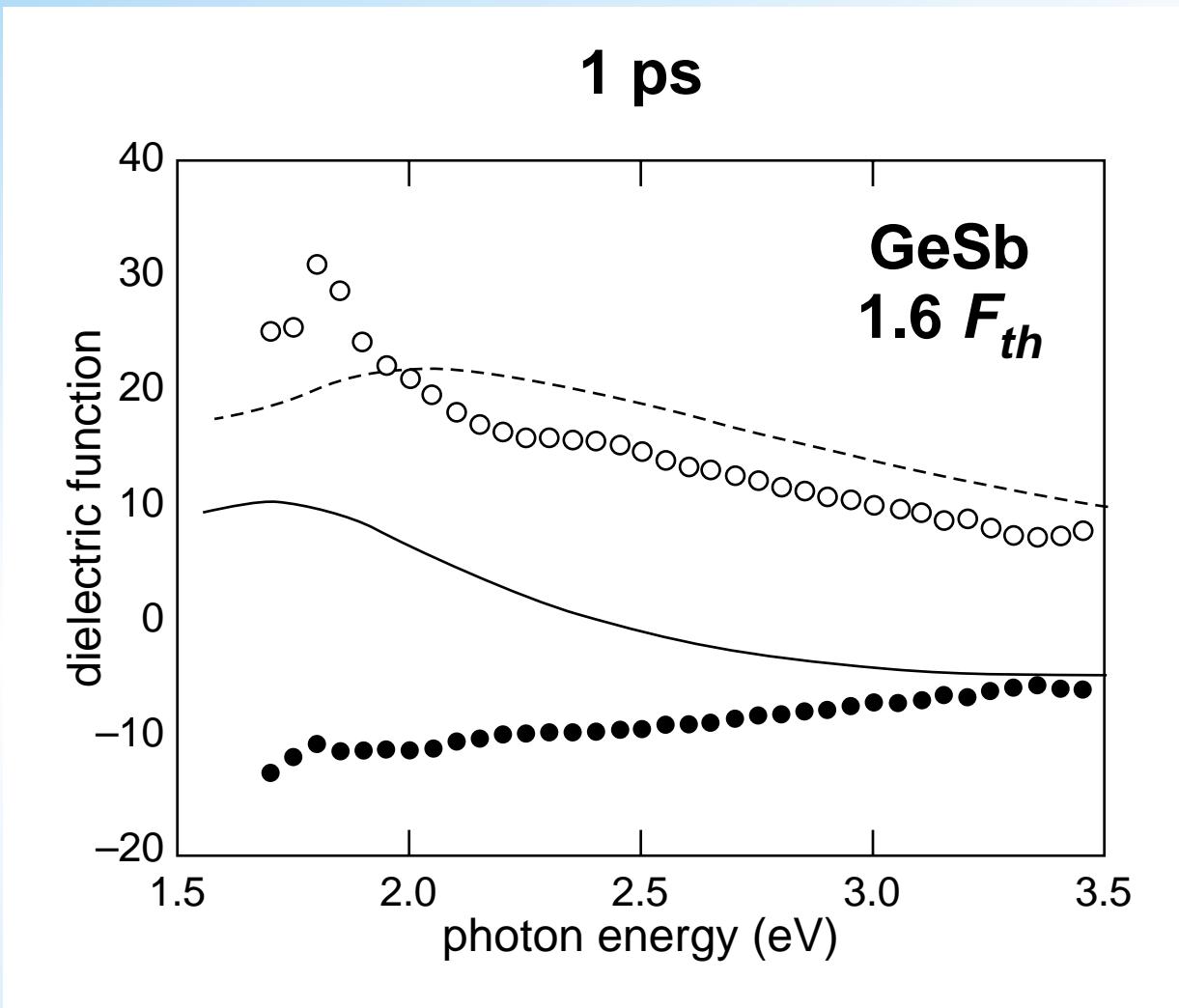
Results



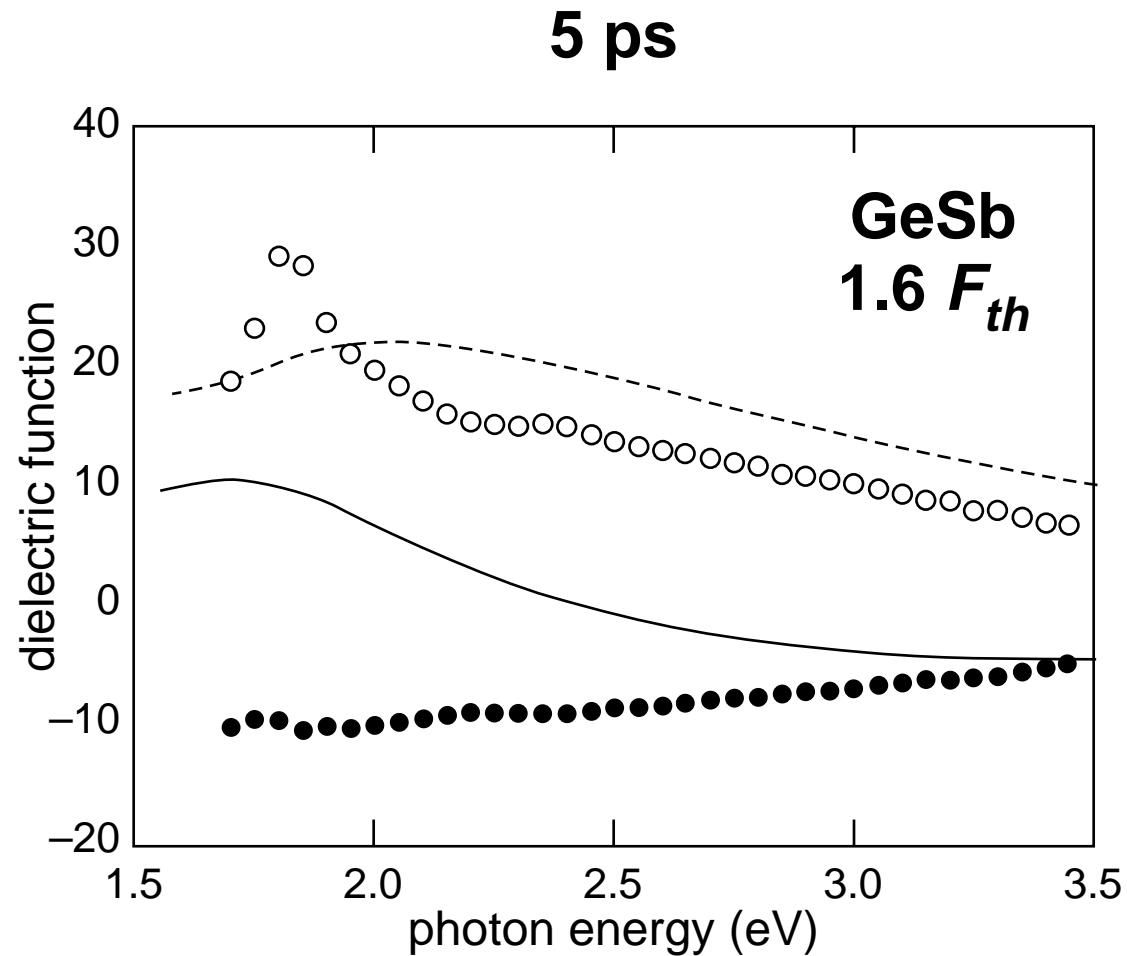
Results



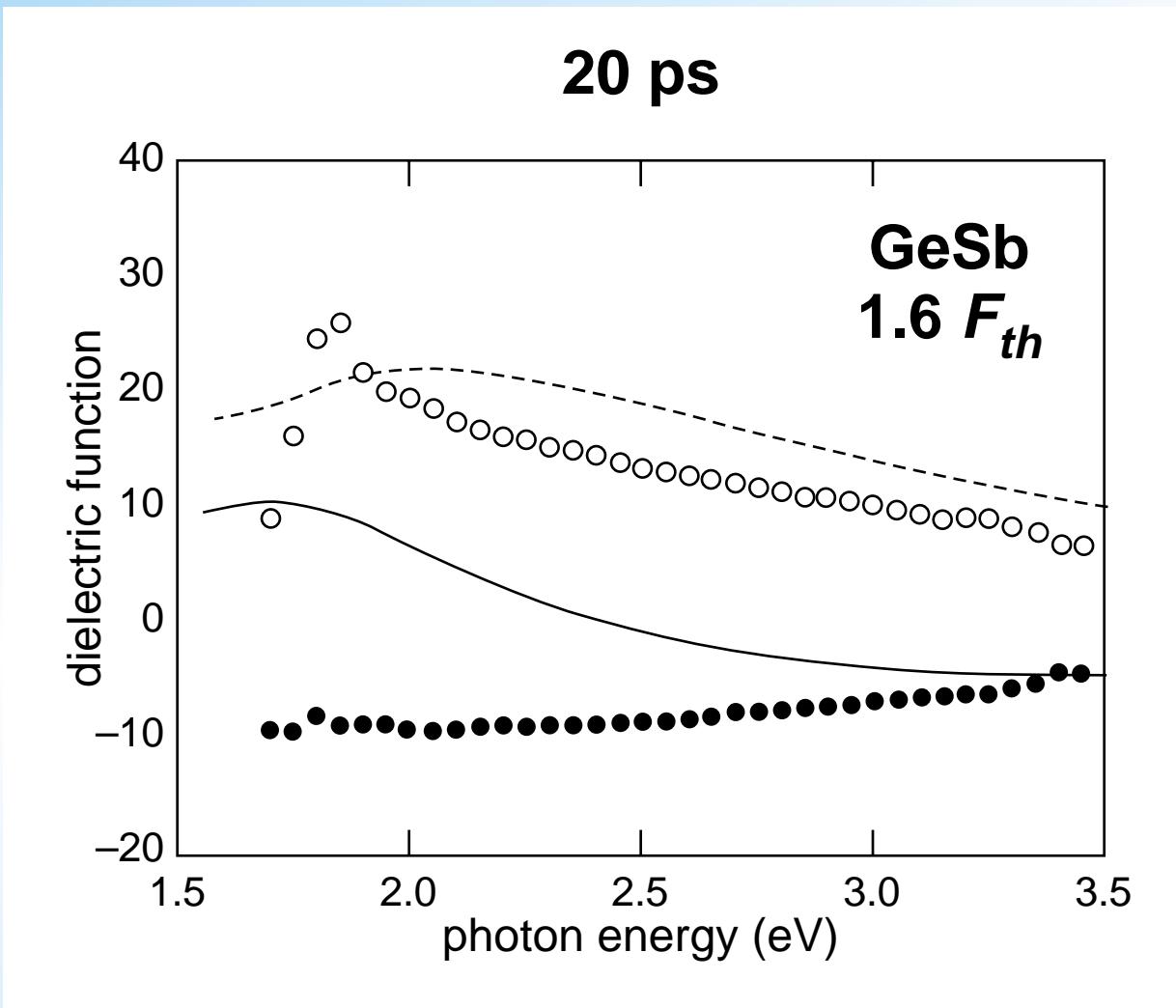
Results



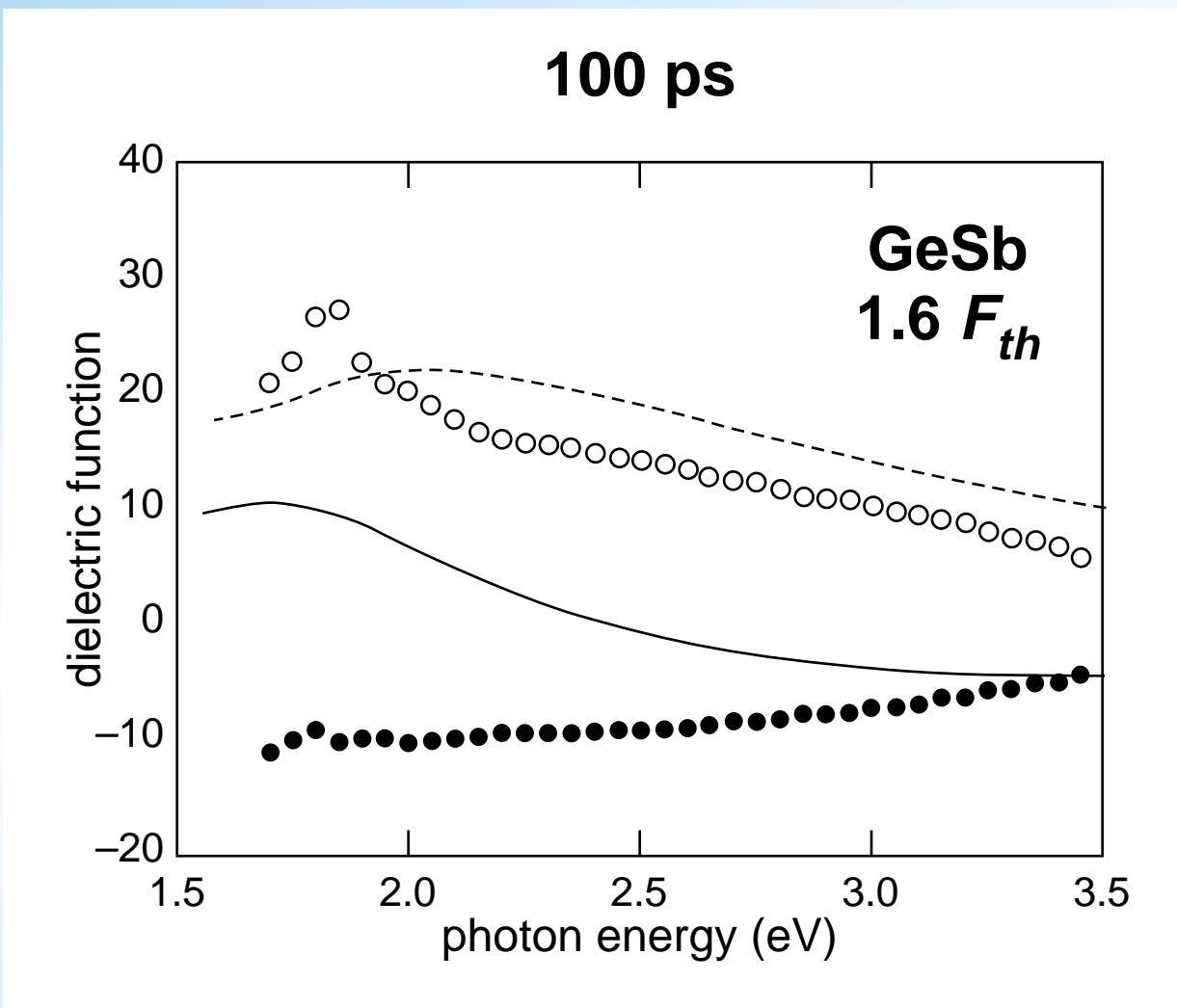
Results



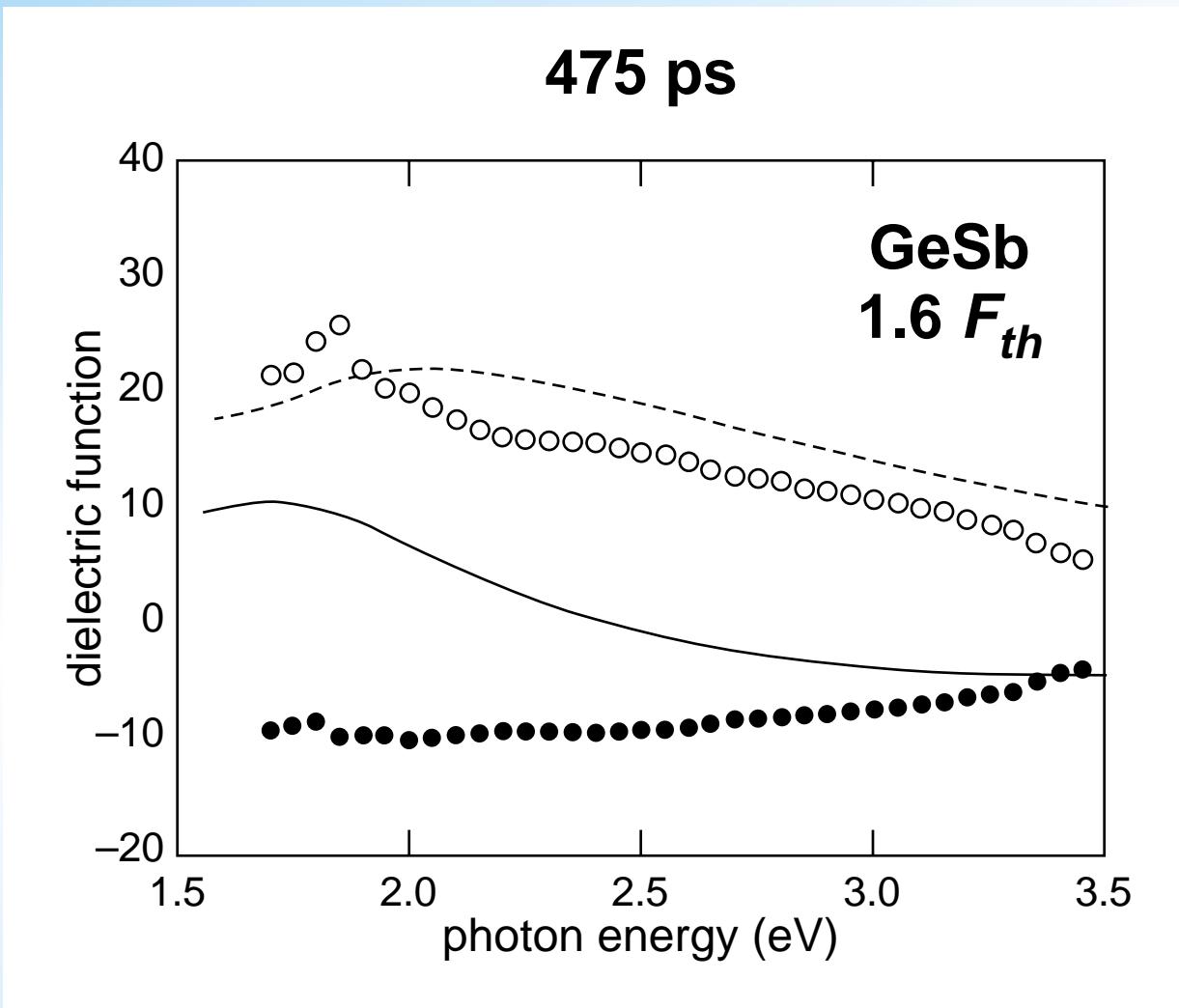
Results



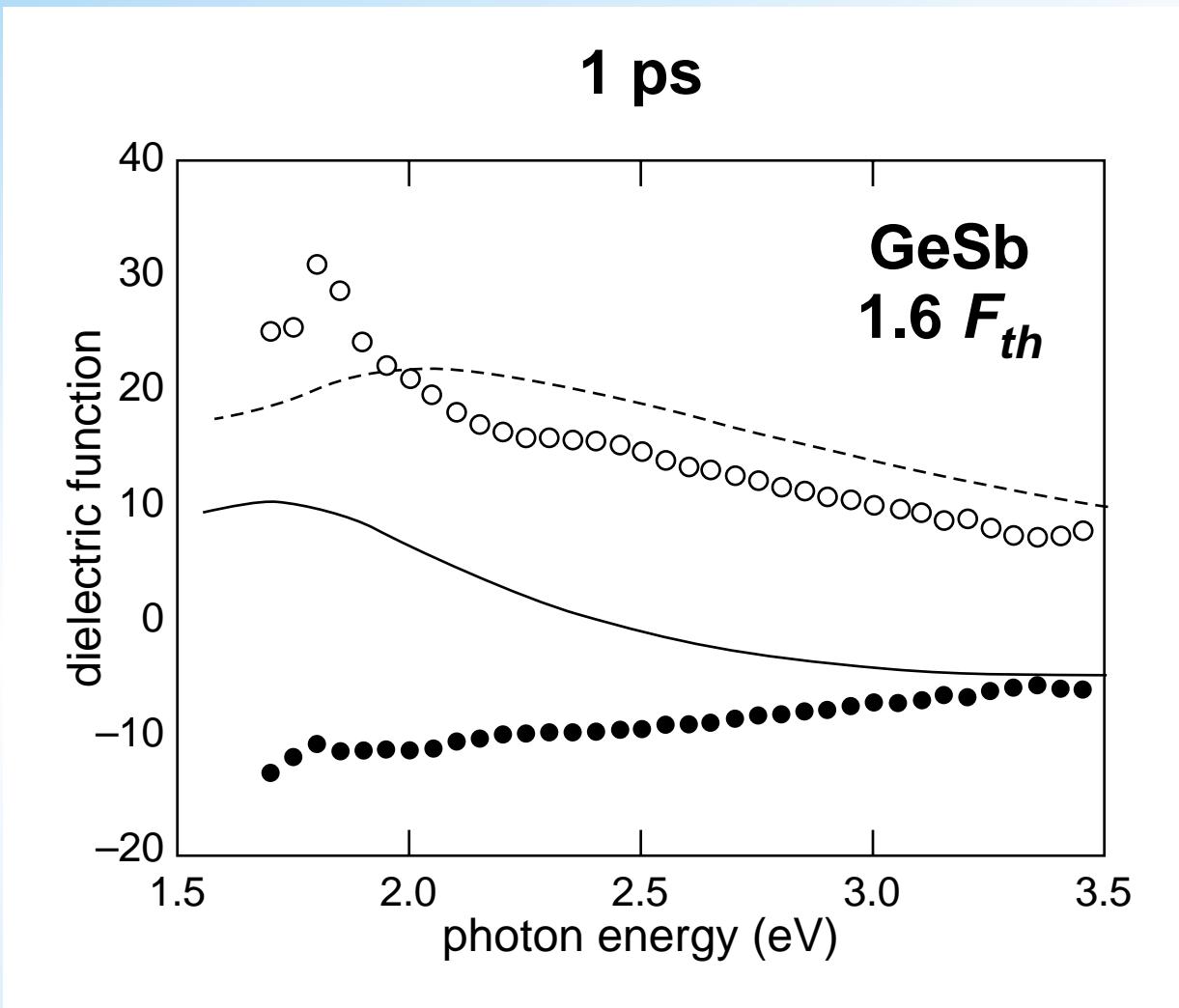
Results



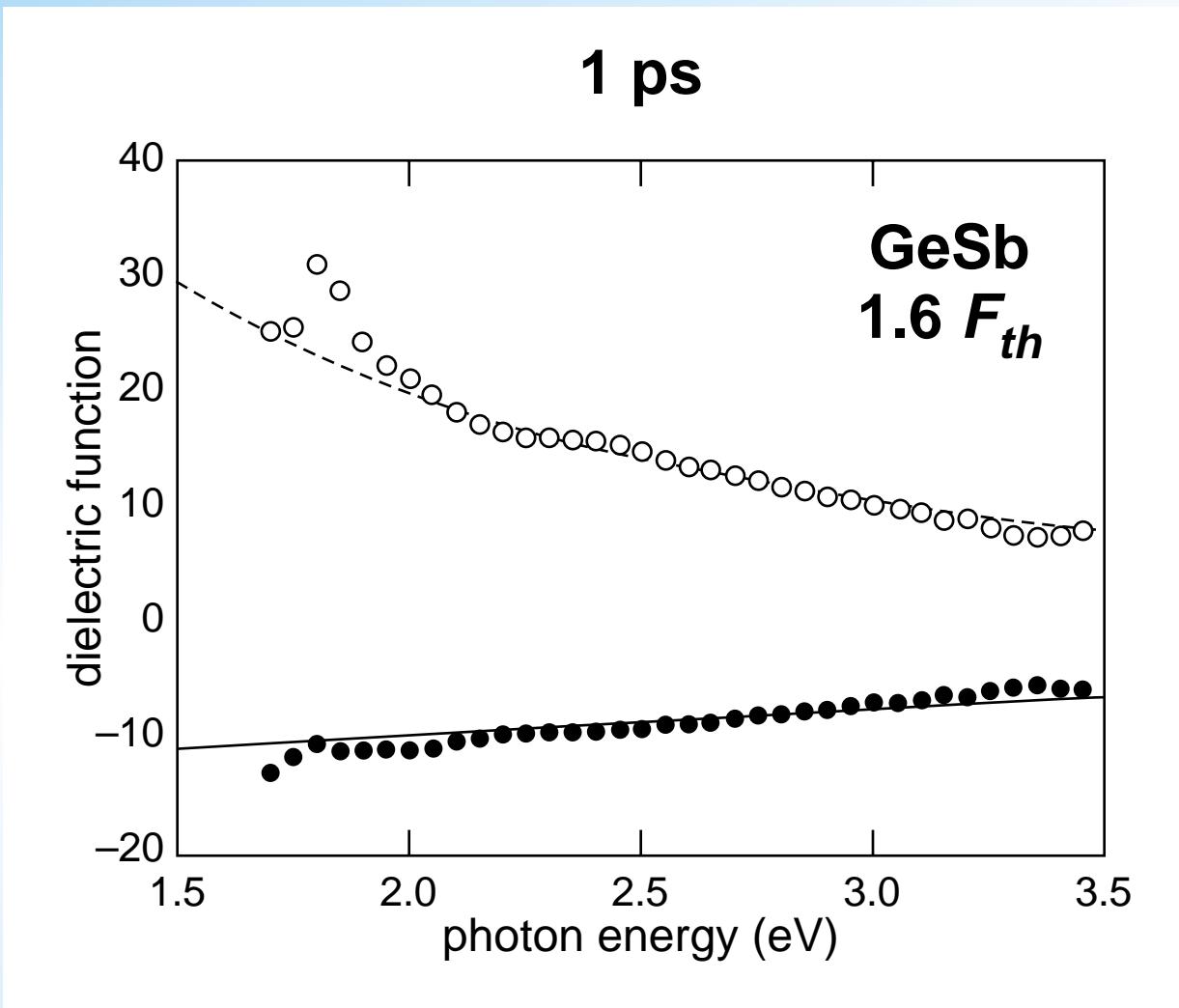
Results



Results

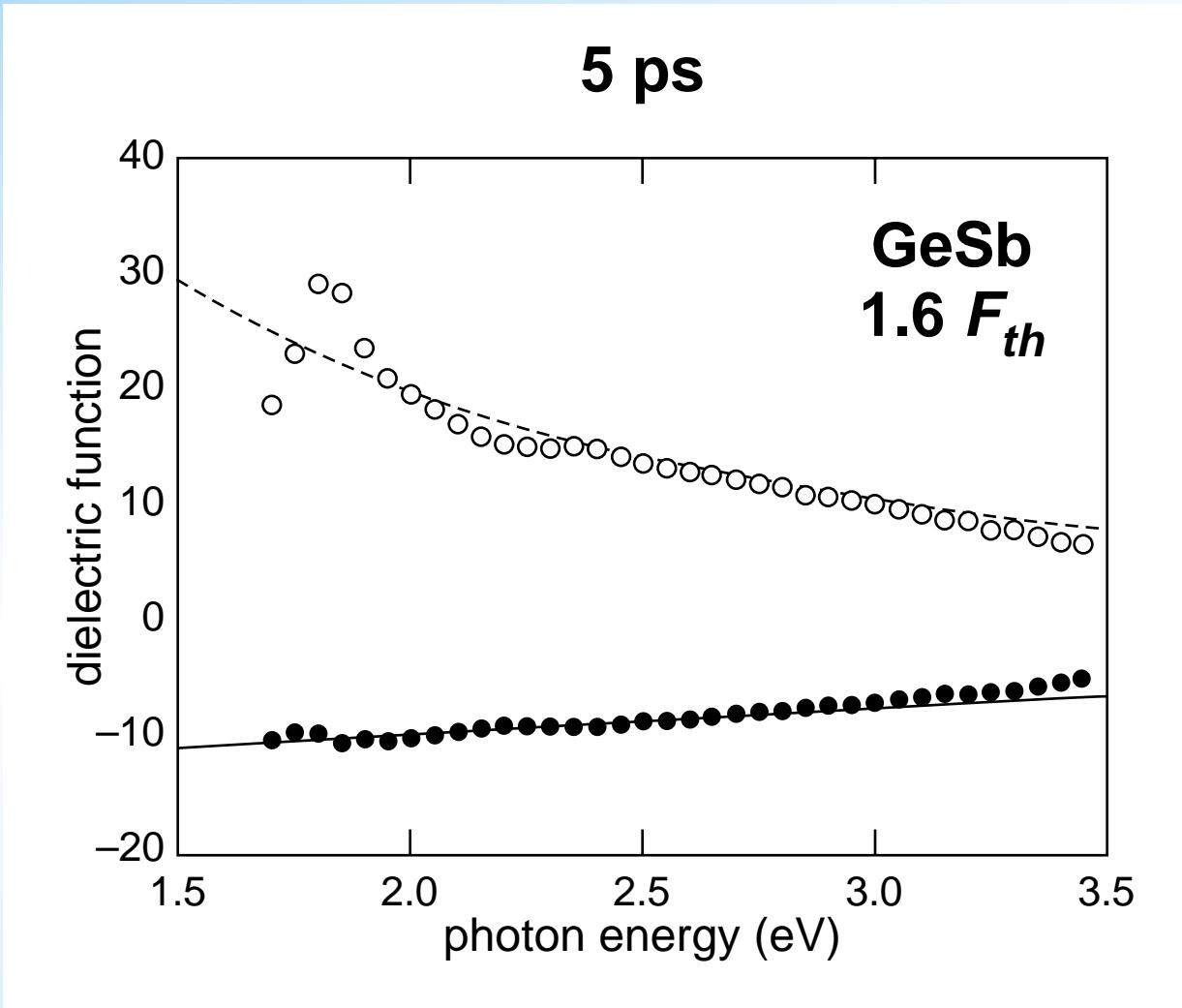


Results



Drude-like after 1 ps

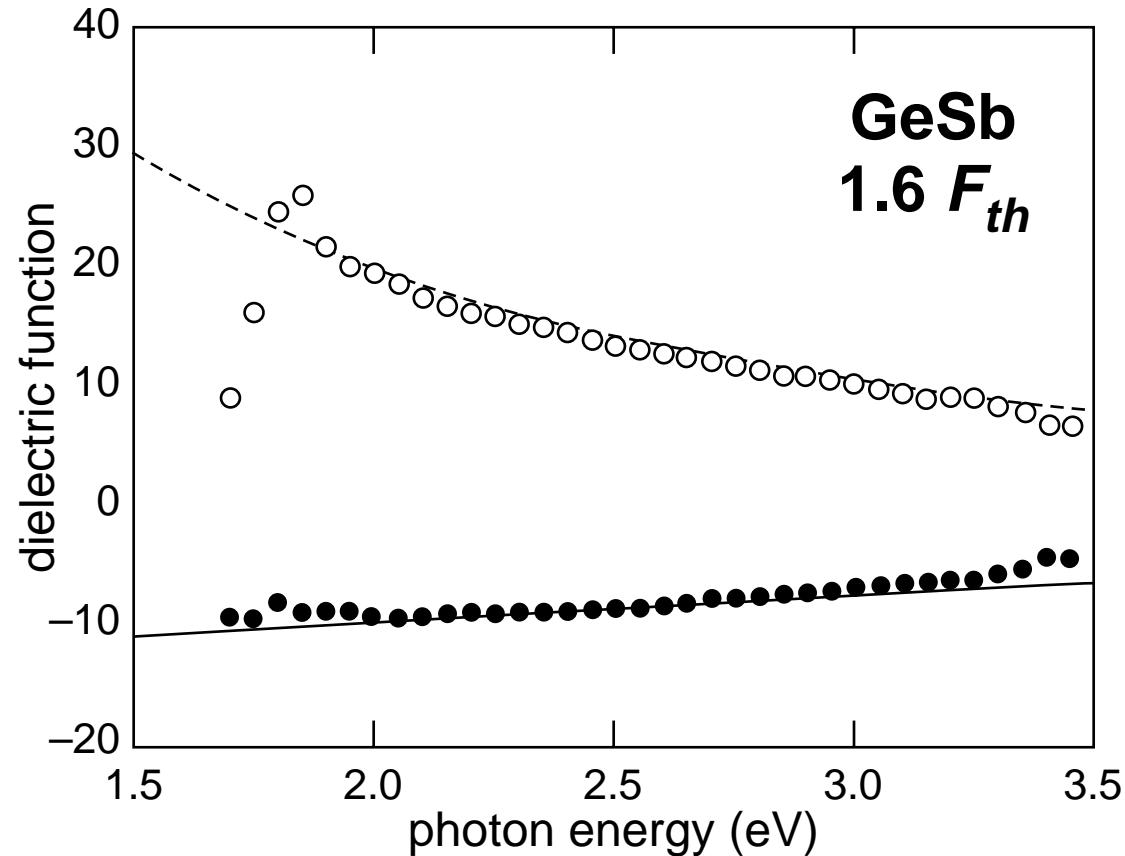
Results



Drude-like after 1 ps

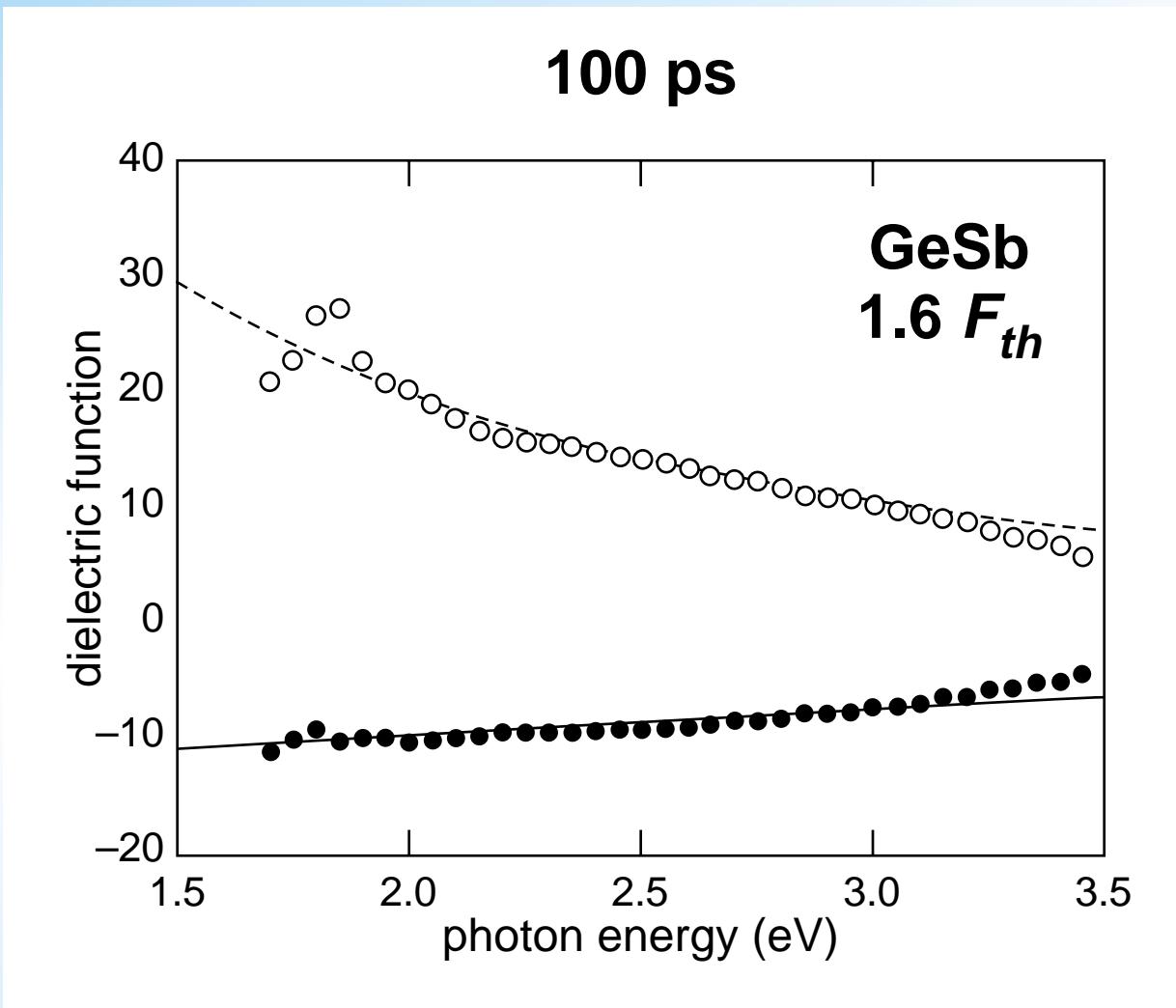
Results

20 ps



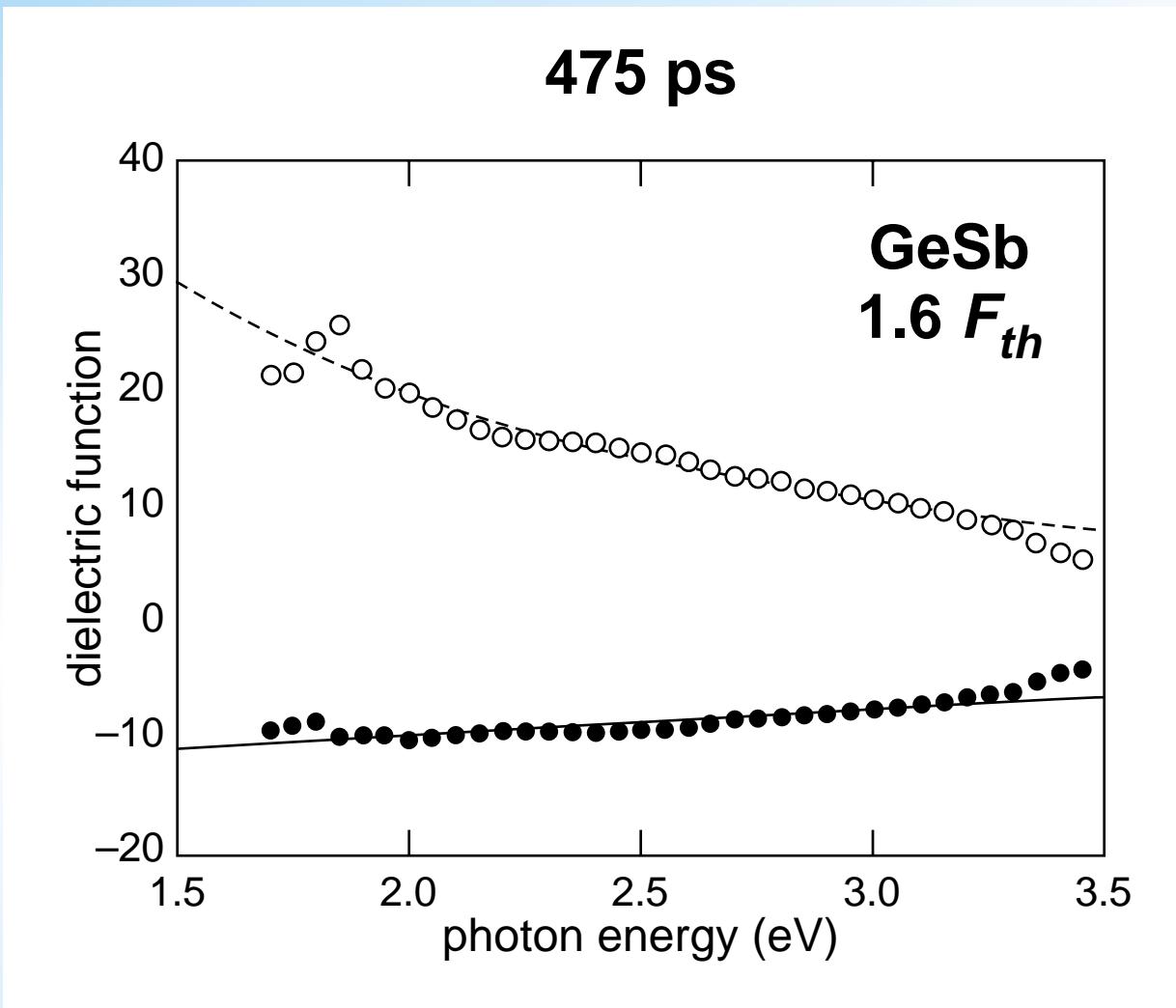
Drude-like after 1 ps

Results



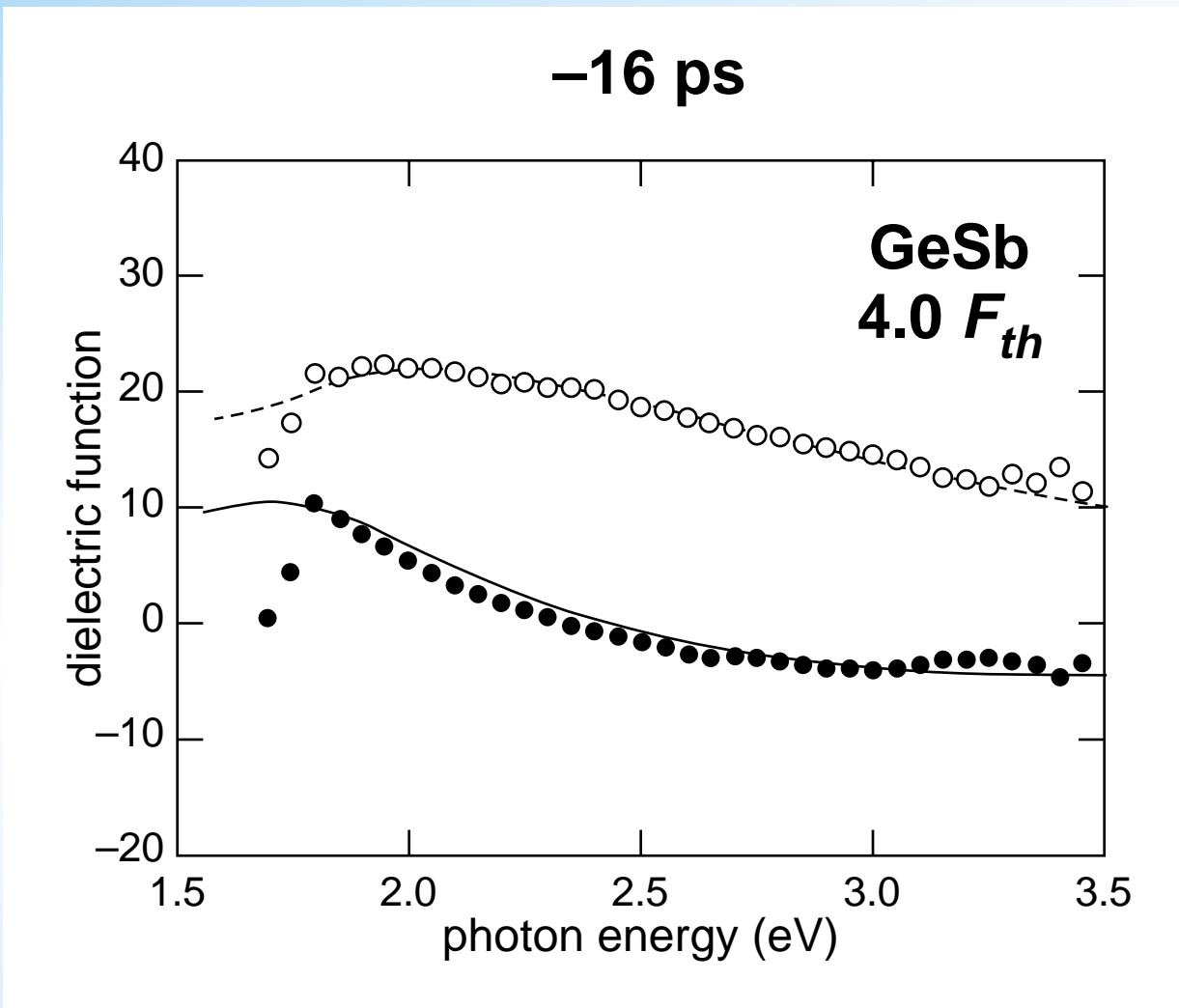
Drude-like after 1 ps

Results

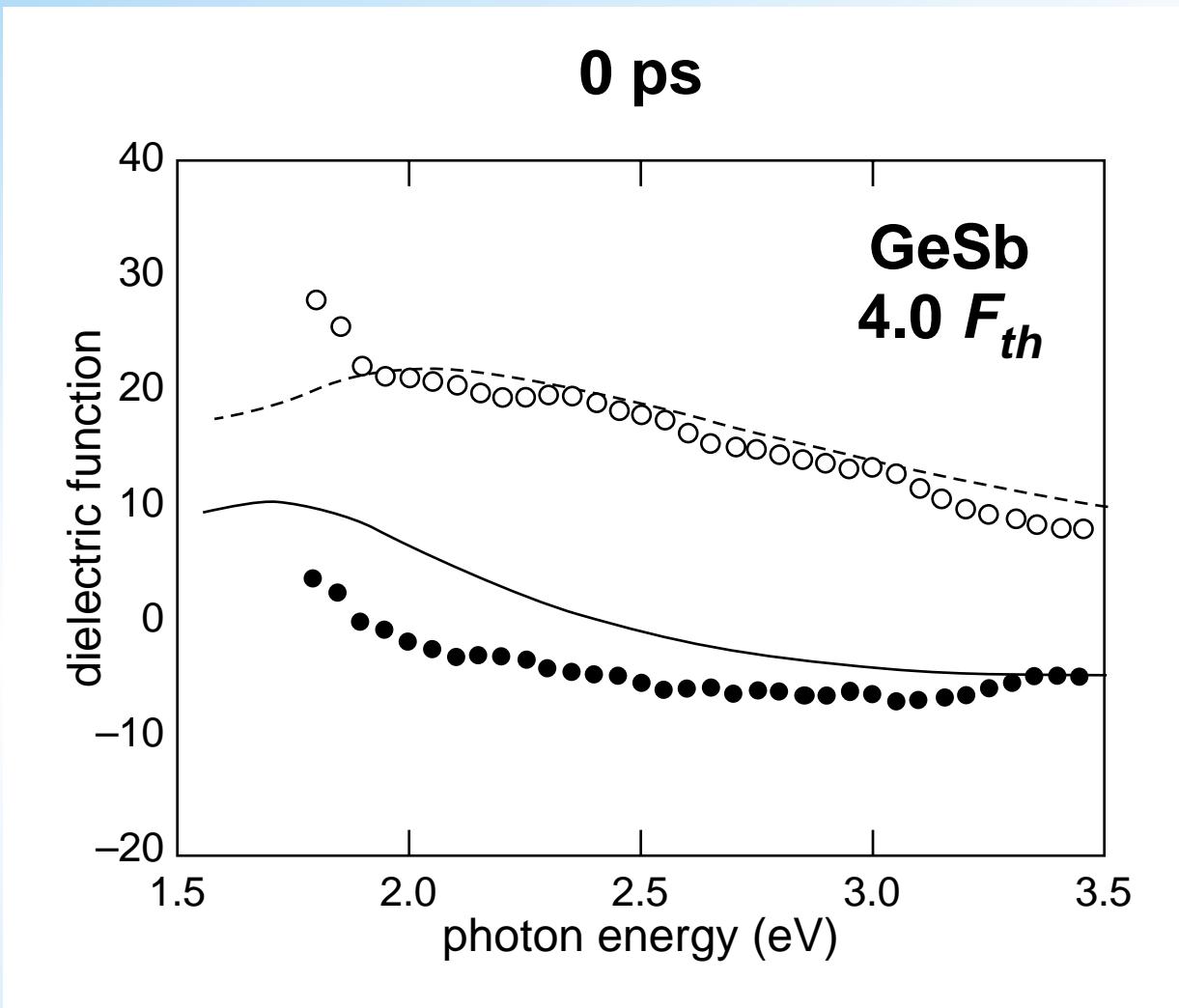


Drude-like after 1 ps

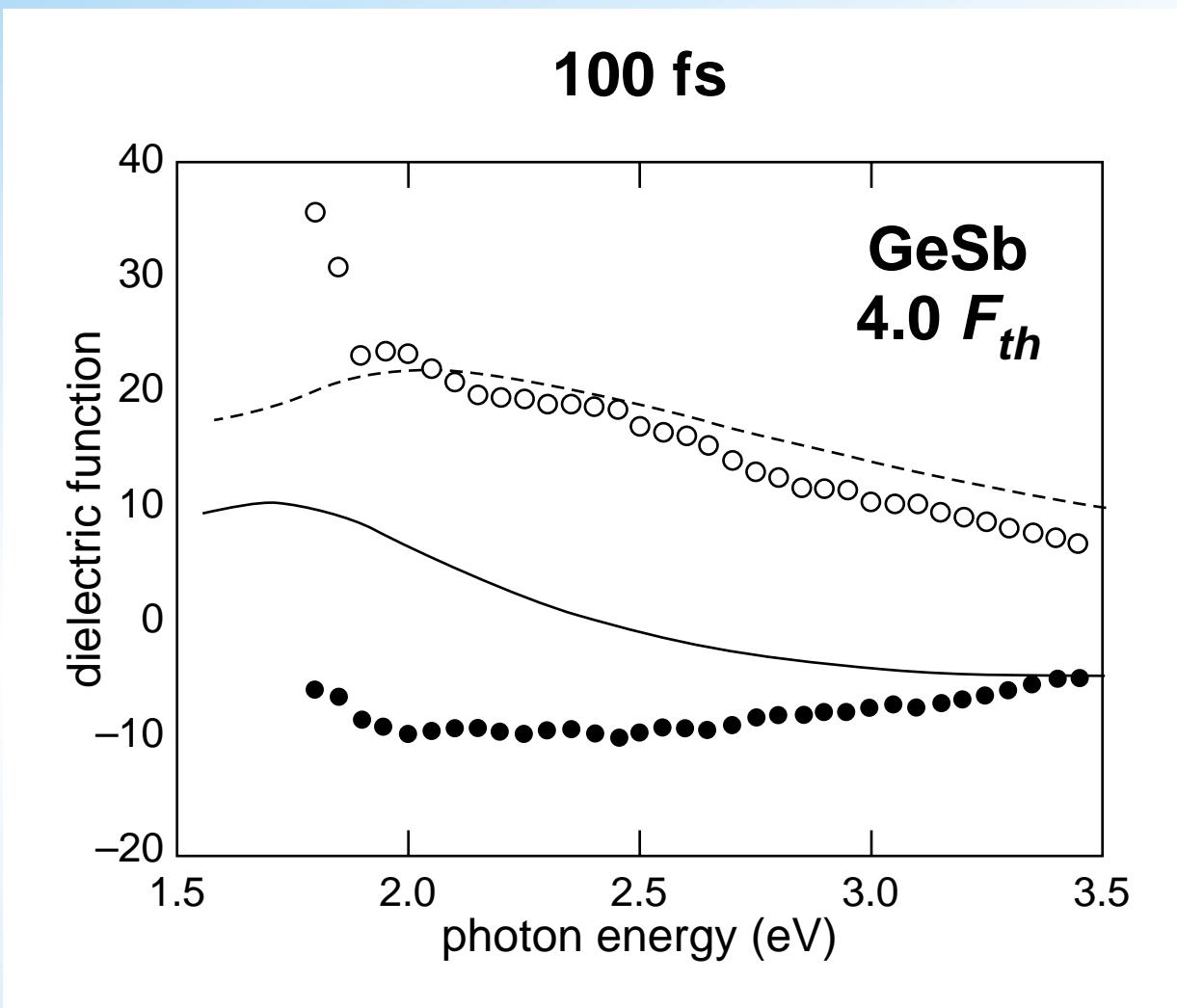
Results



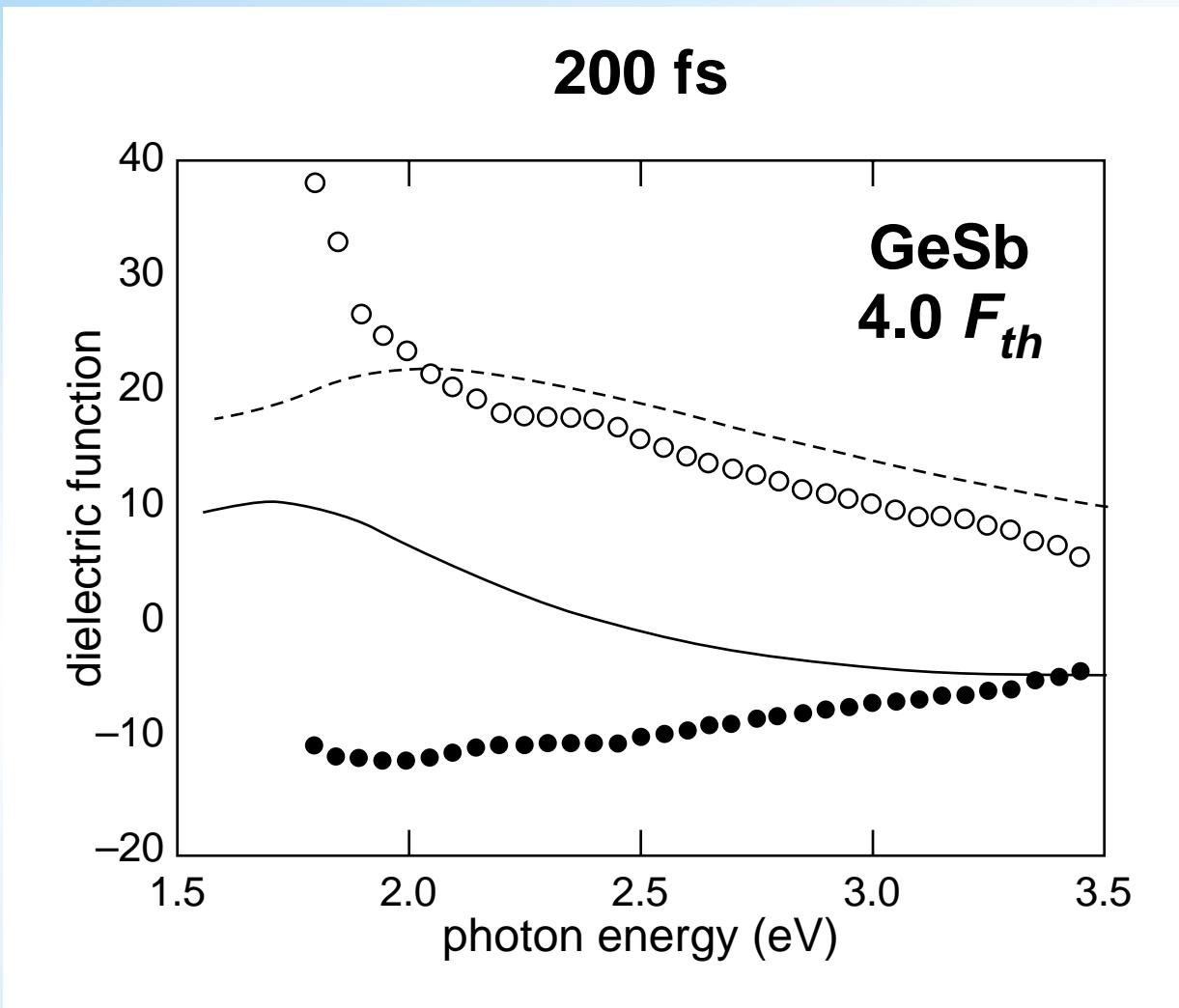
Results



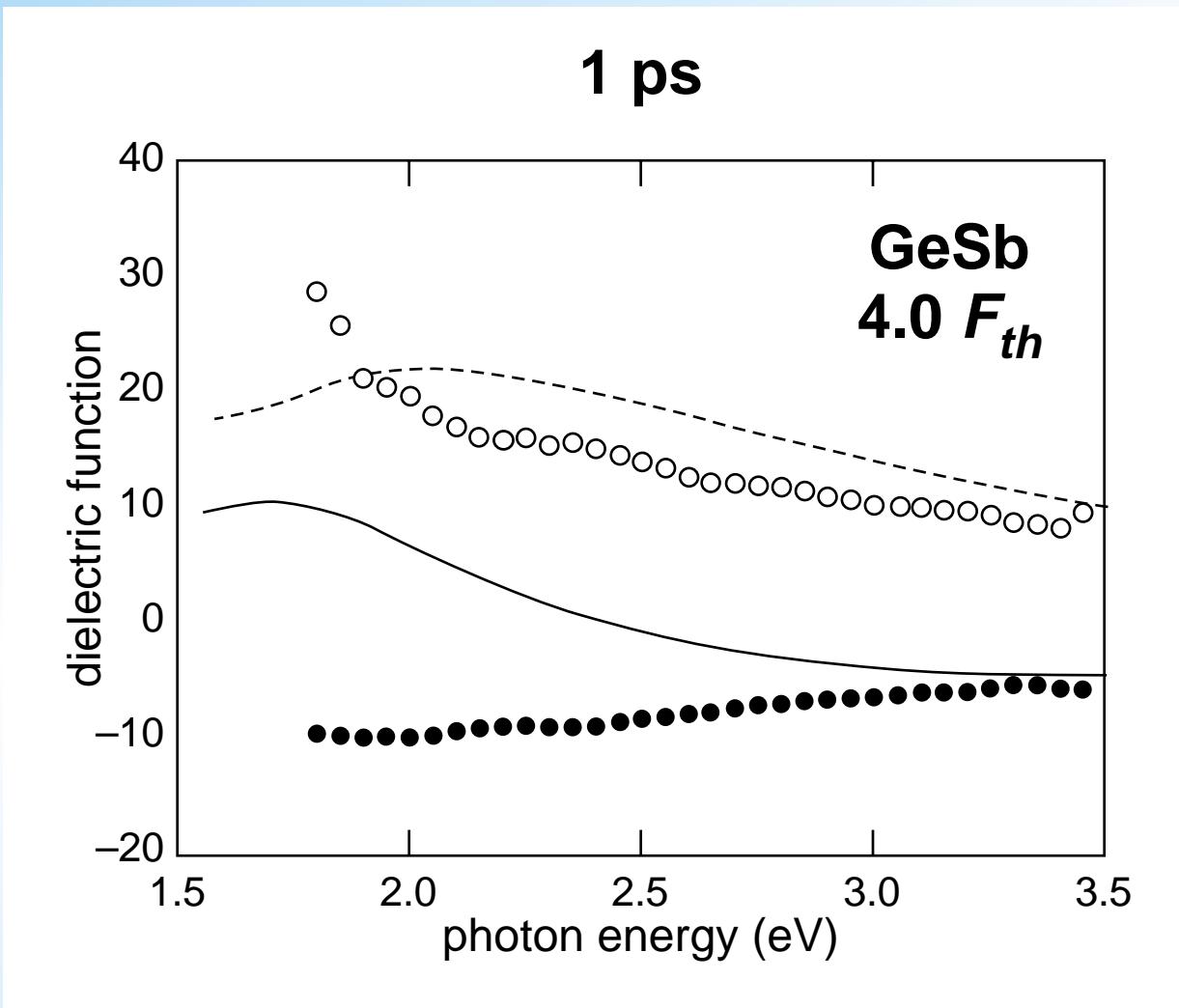
Results



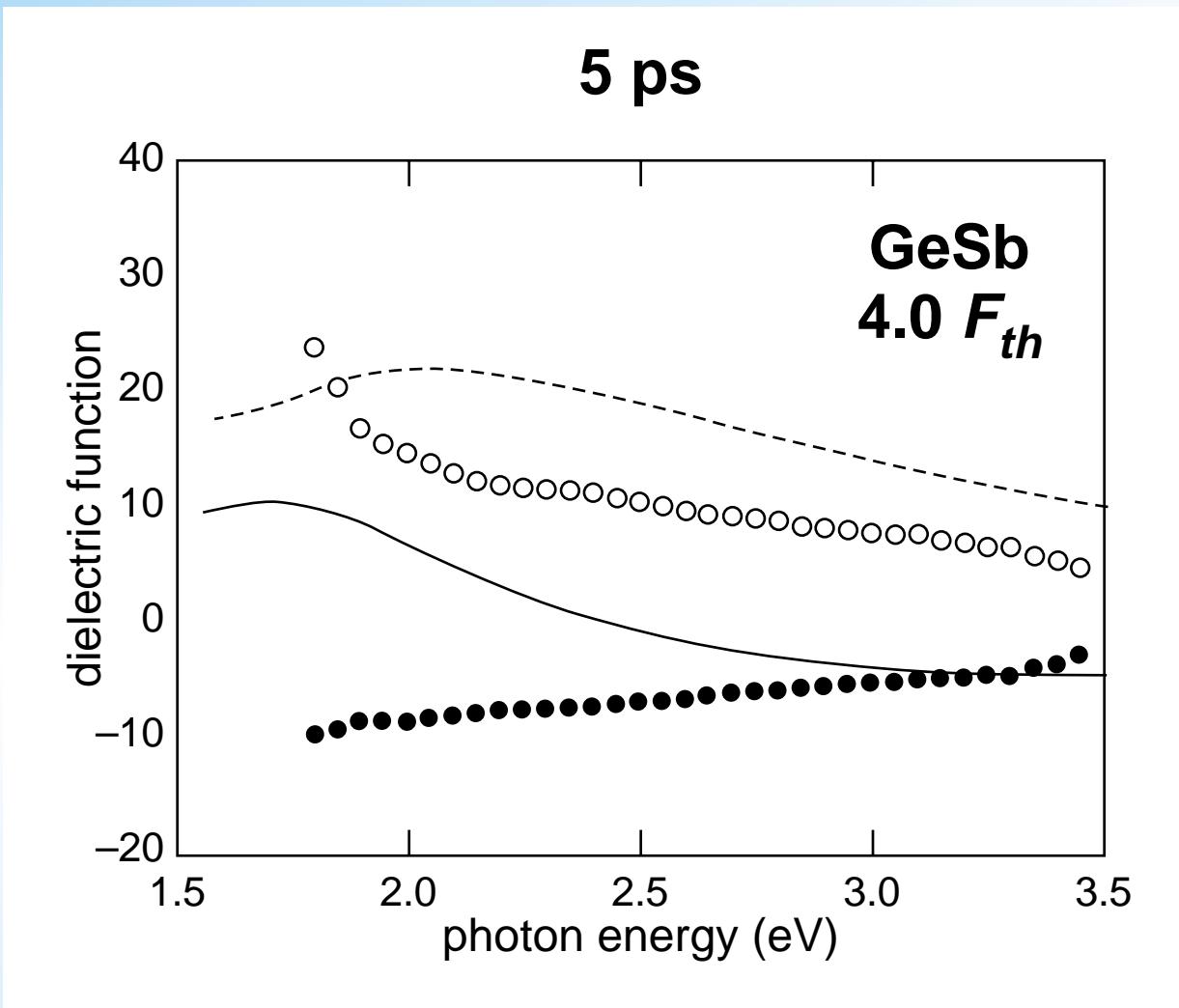
Results



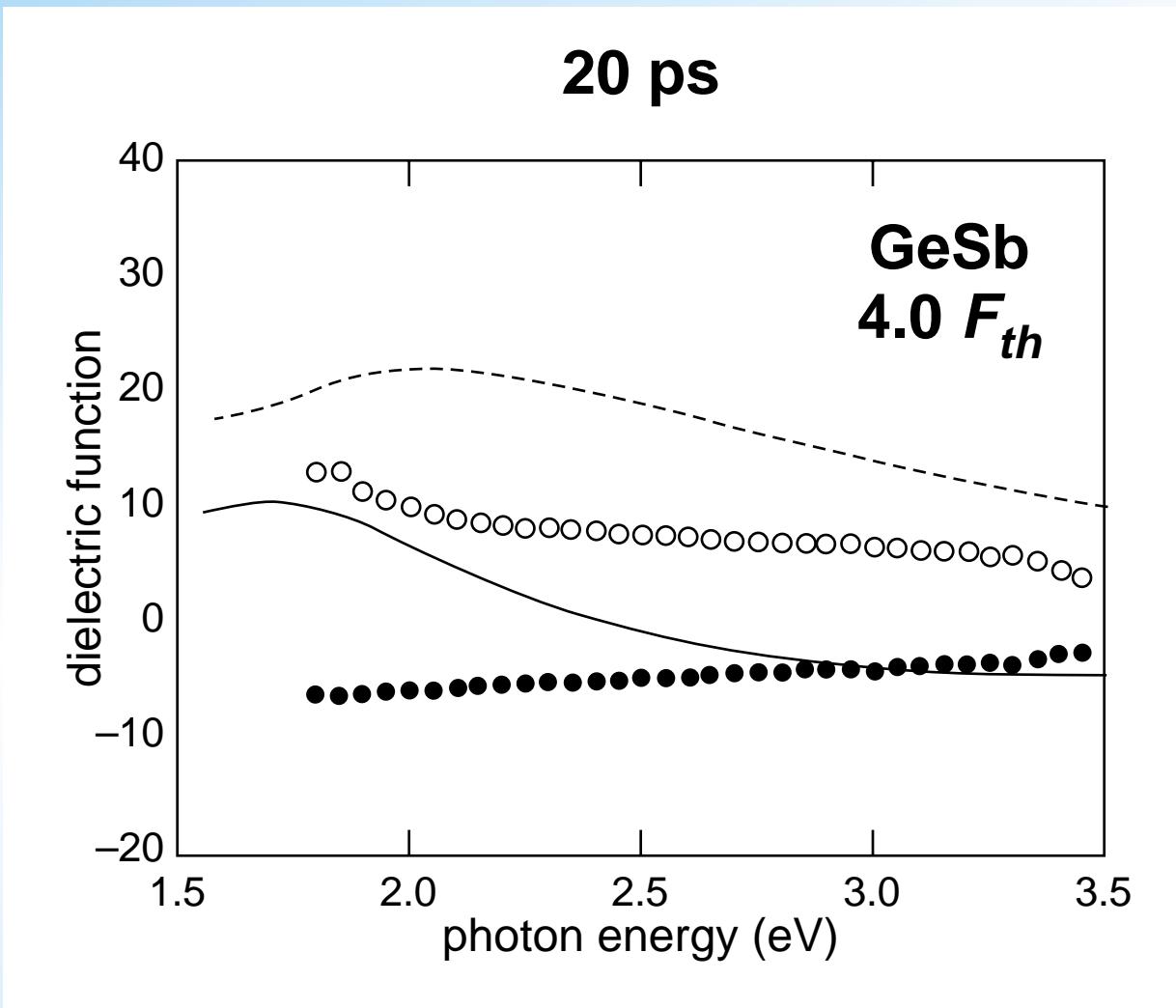
Results



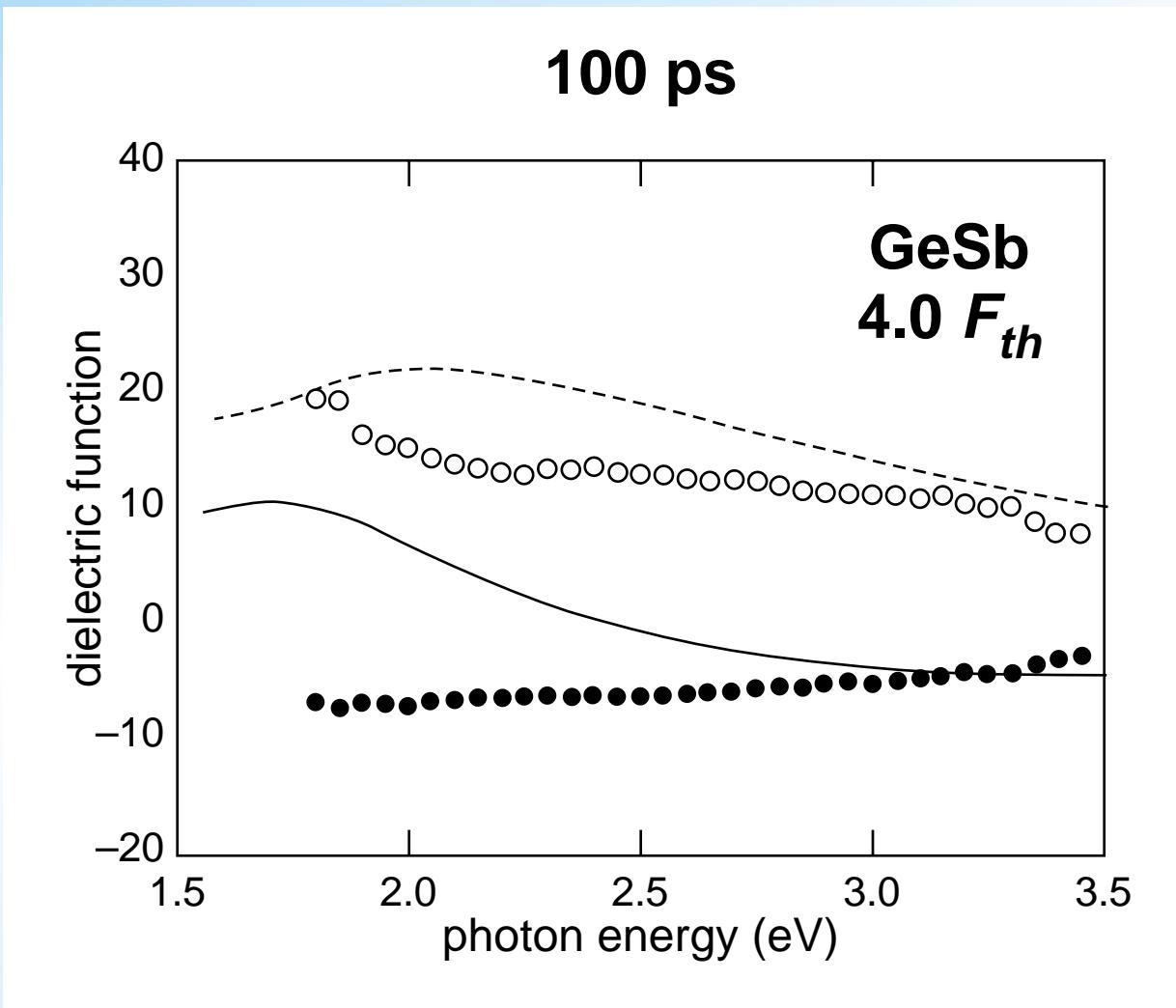
Results



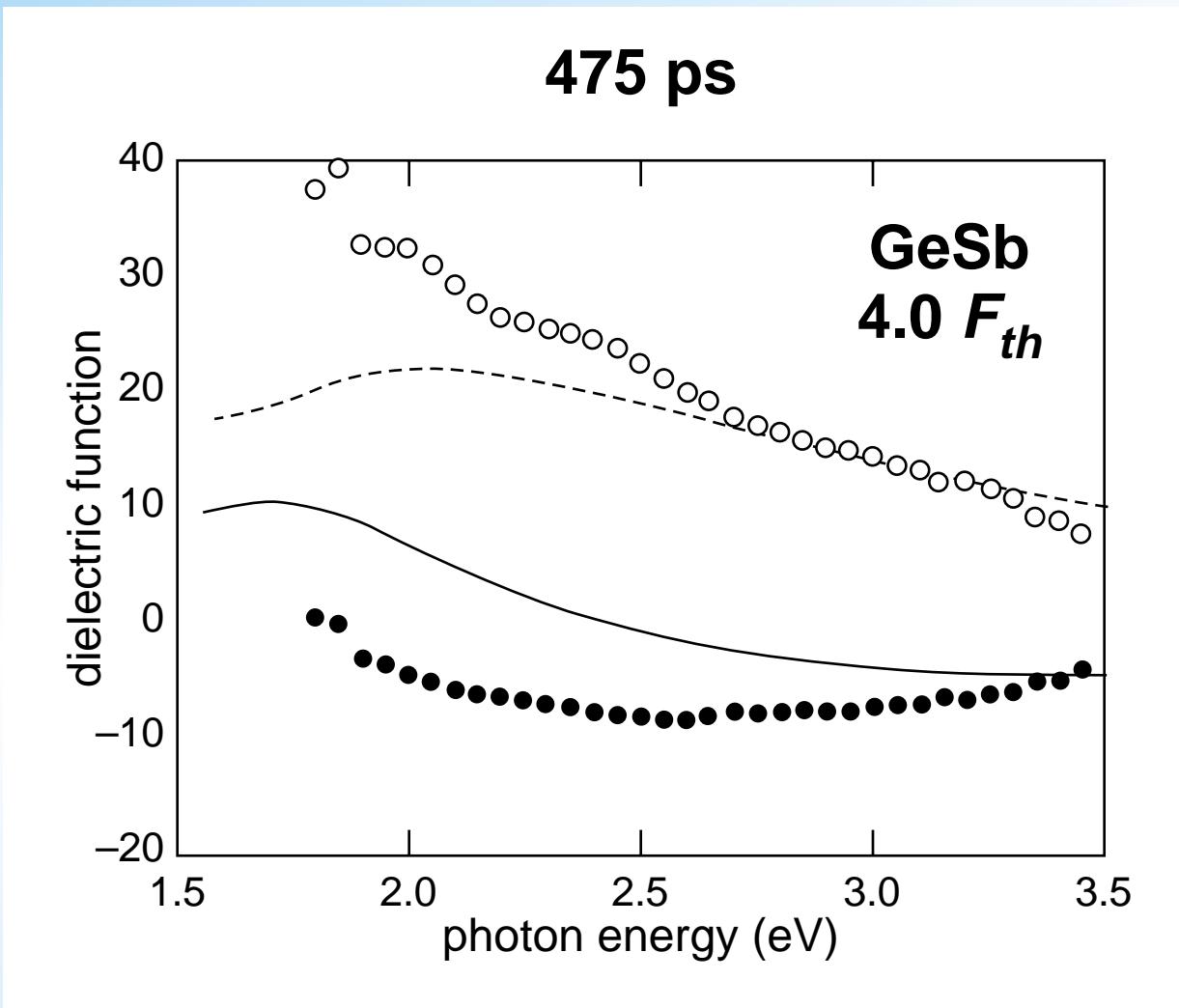
Results



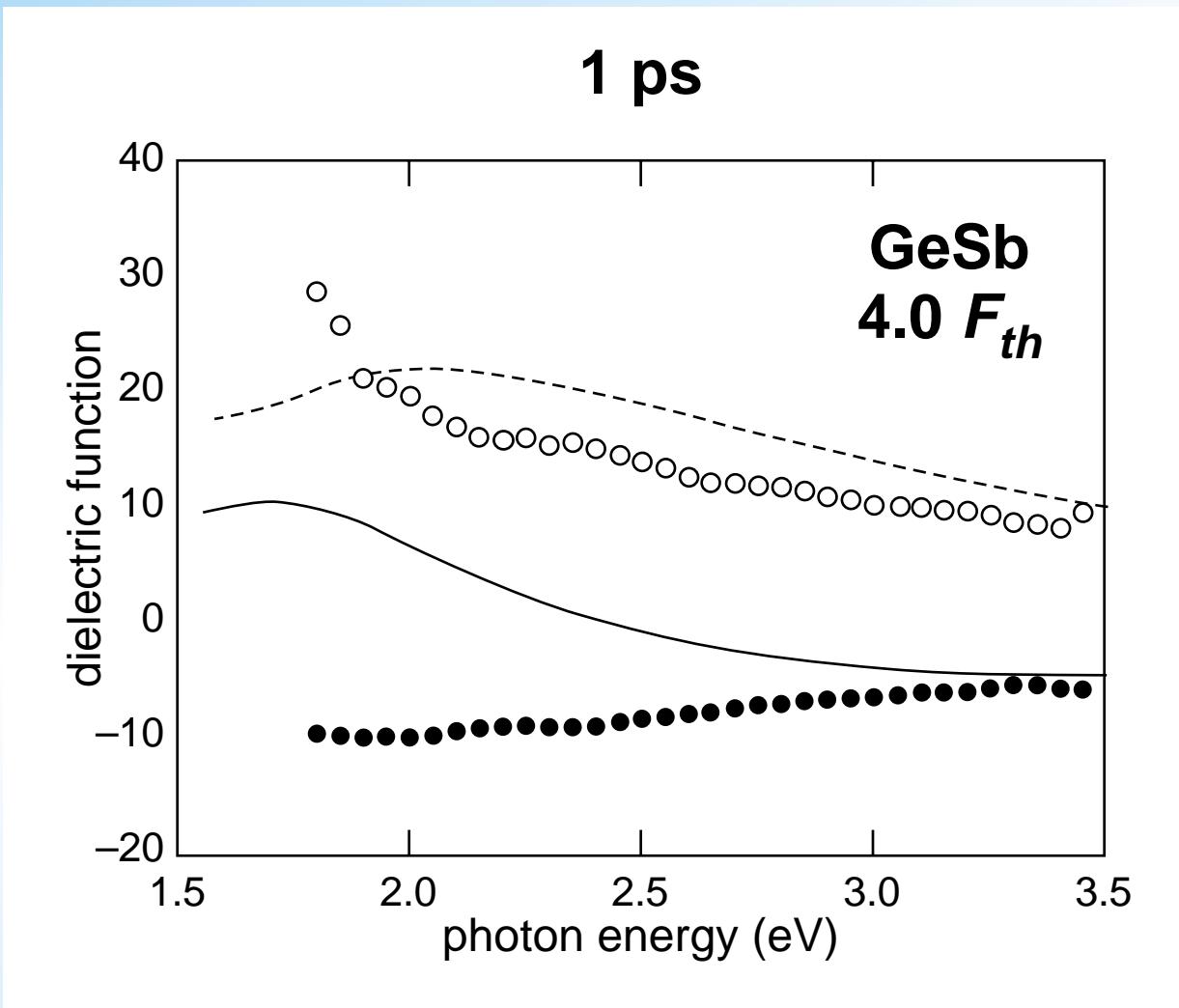
Results



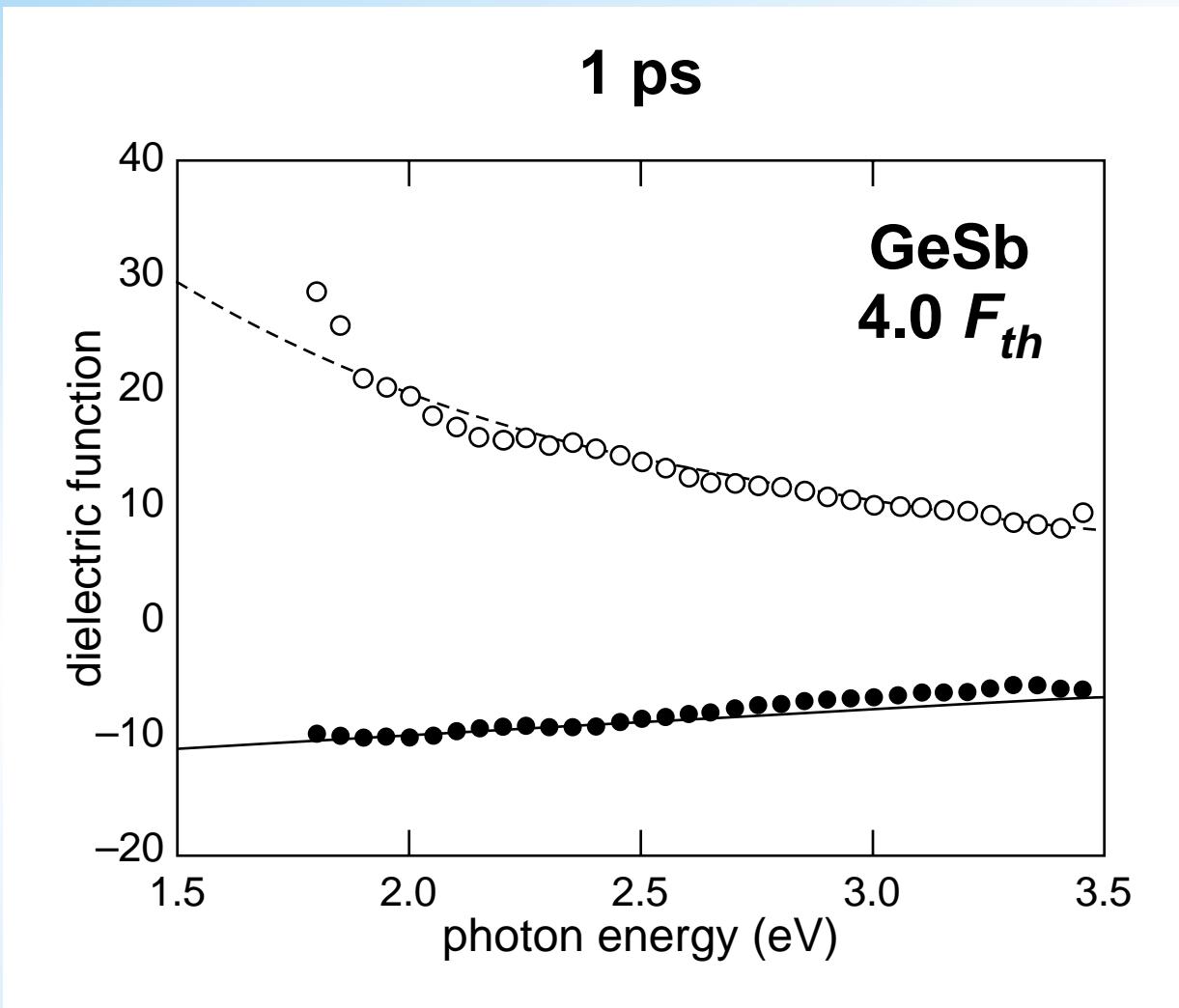
Results



Results



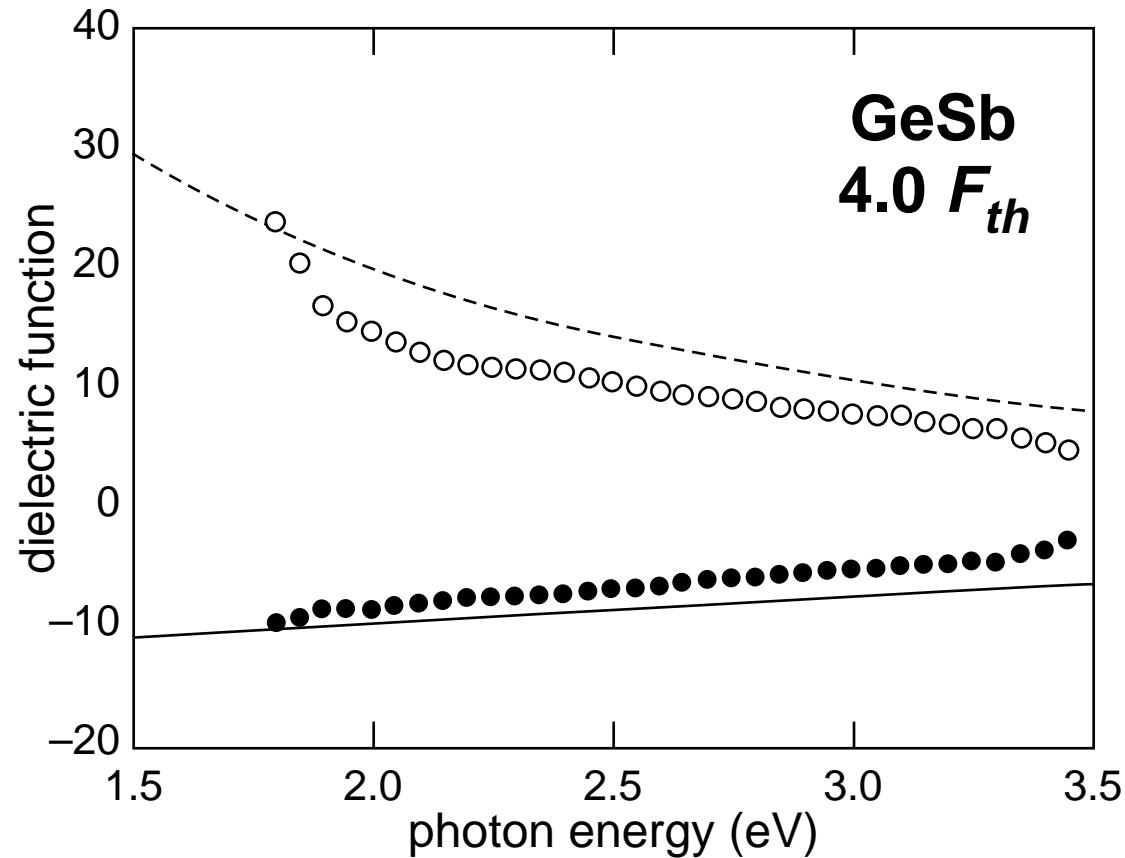
Results



Drude-like after 1 ps

Results

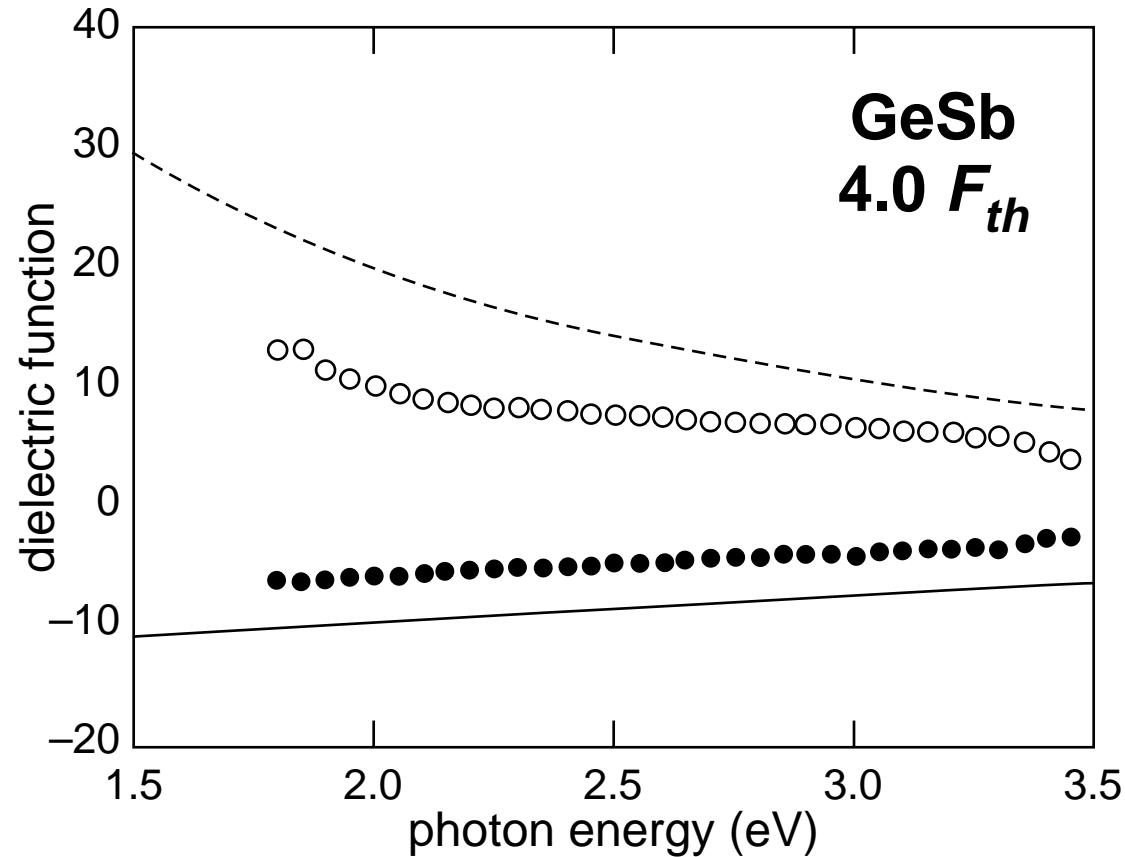
5 ps



plasma frequency decreases

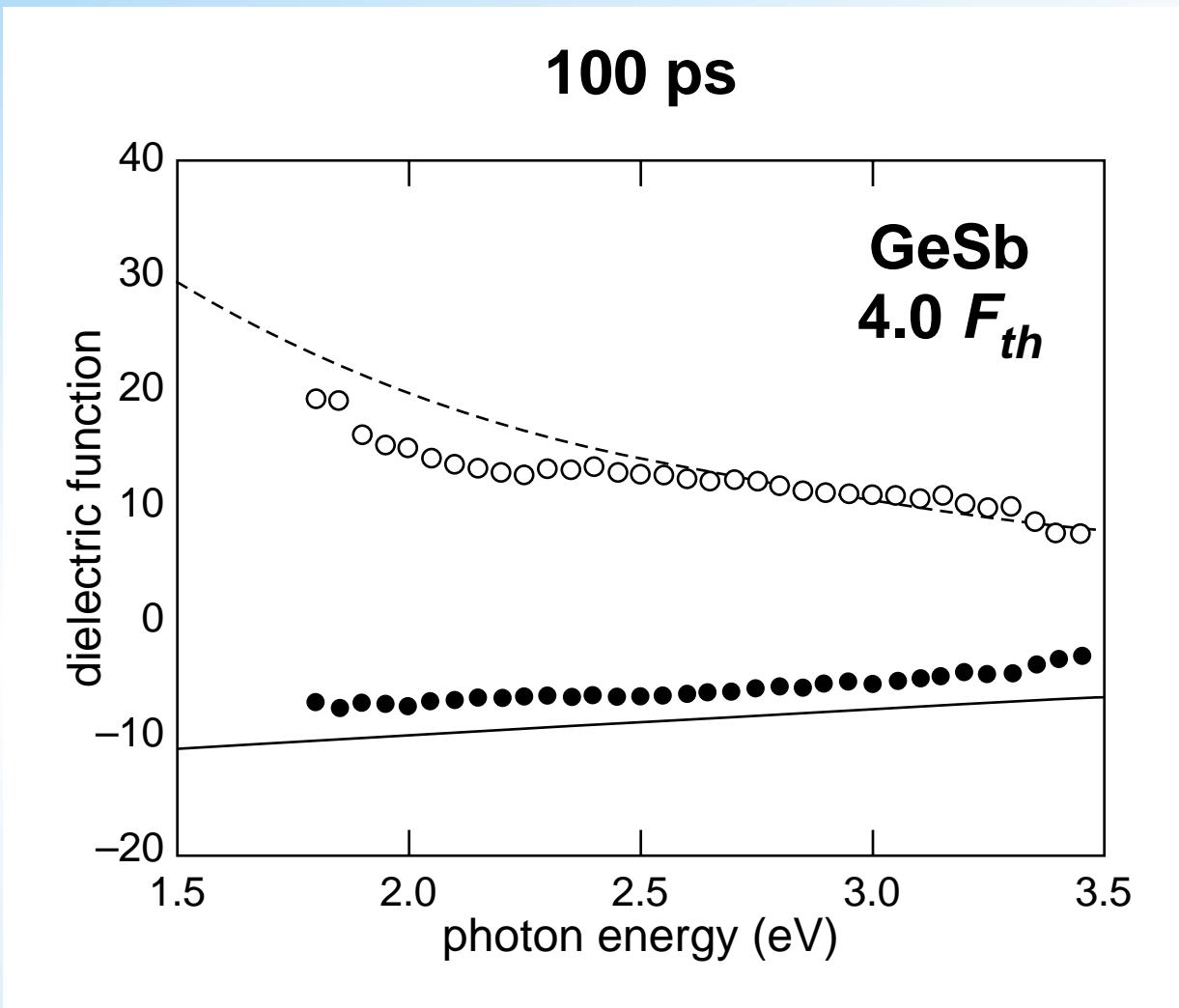
Results

20 ps

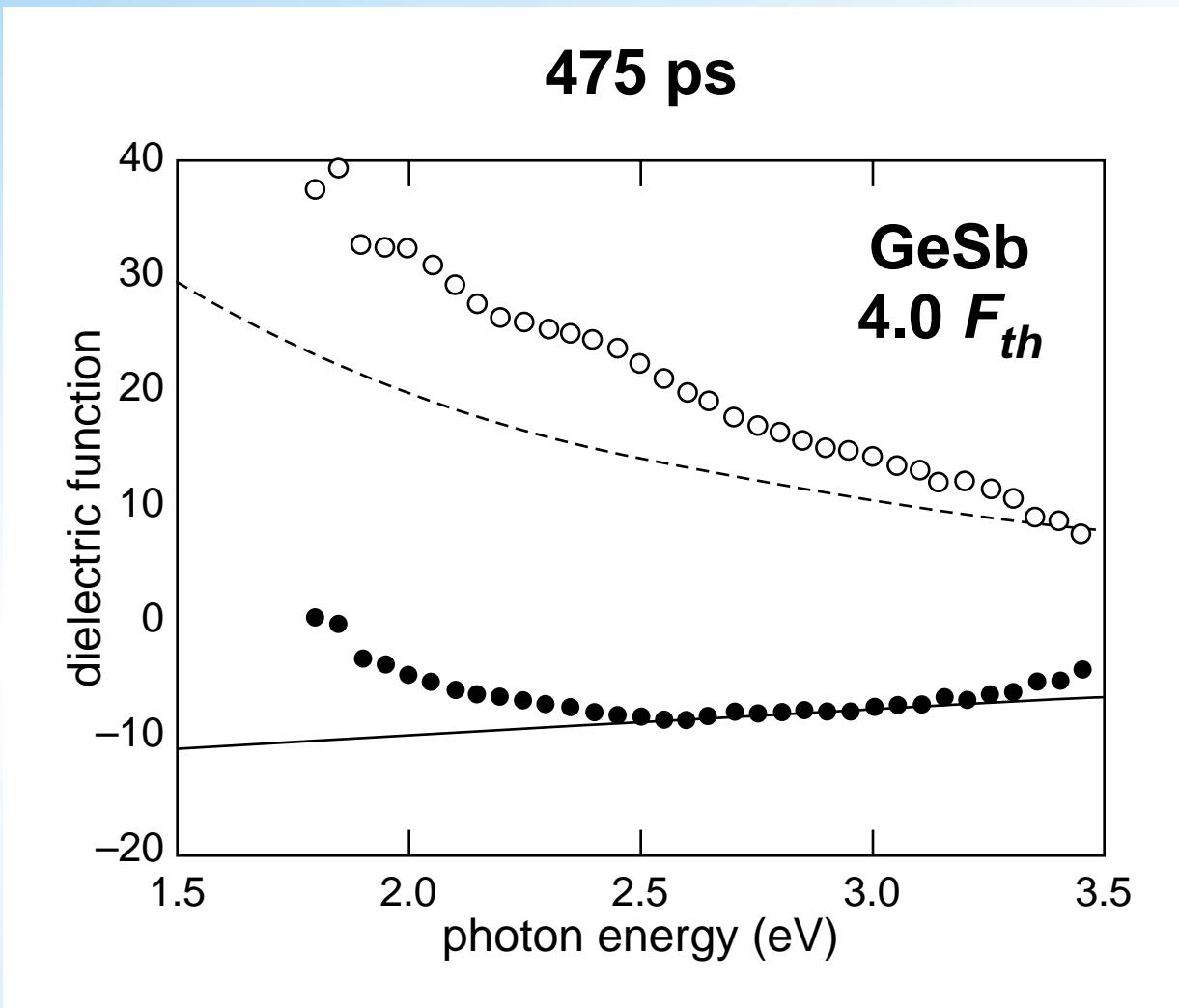


plasma frequency decreases

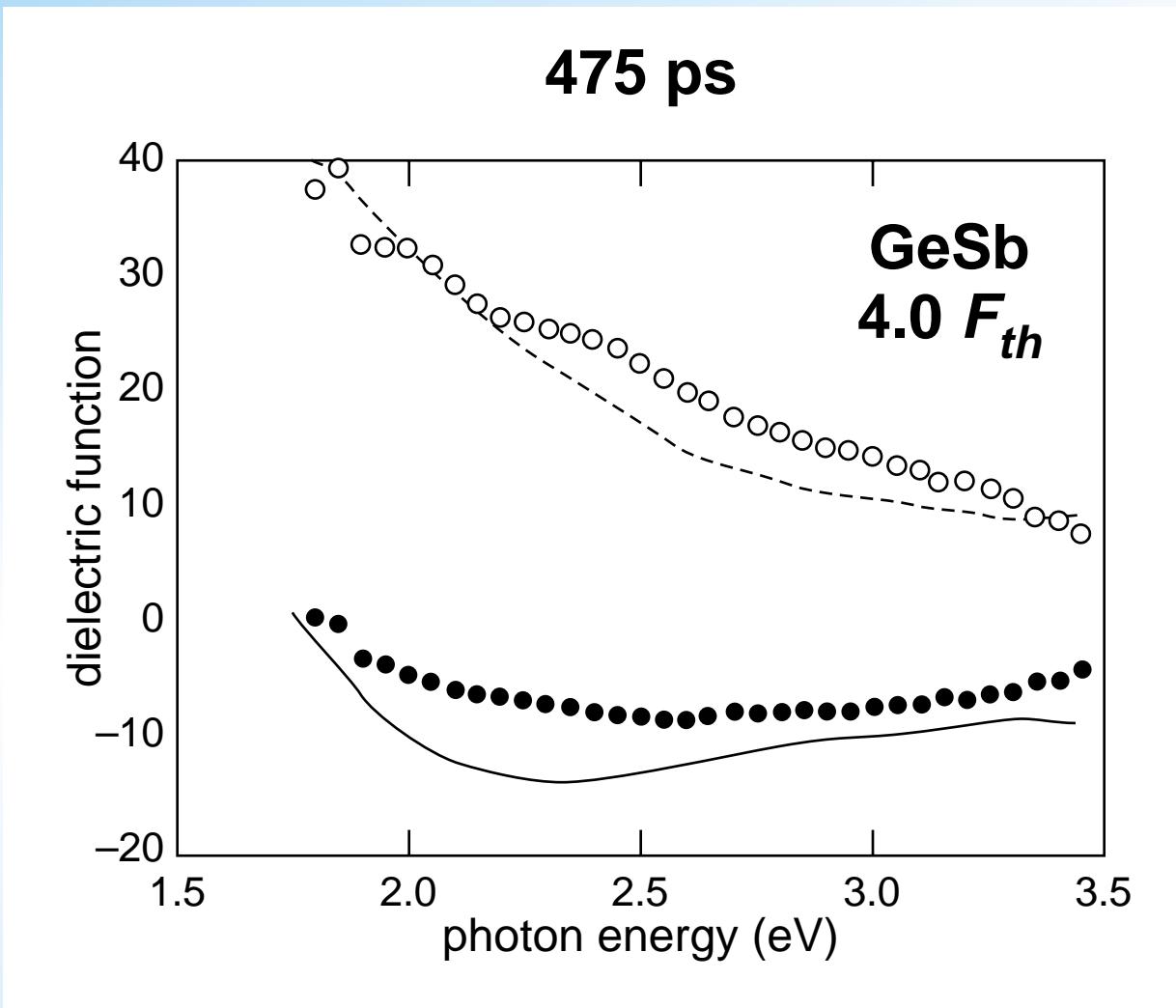
Results



Results



Results



laser-induced recrystallization

Summary

Universal features:

Summary

Universal features:

- ▶ semiconductor-to-metal transition

Summary

Universal features:

- ▶ semiconductor-to-metal transition
- ▶ decrease in bonding-antibonding splitting takes picoseconds

Summary

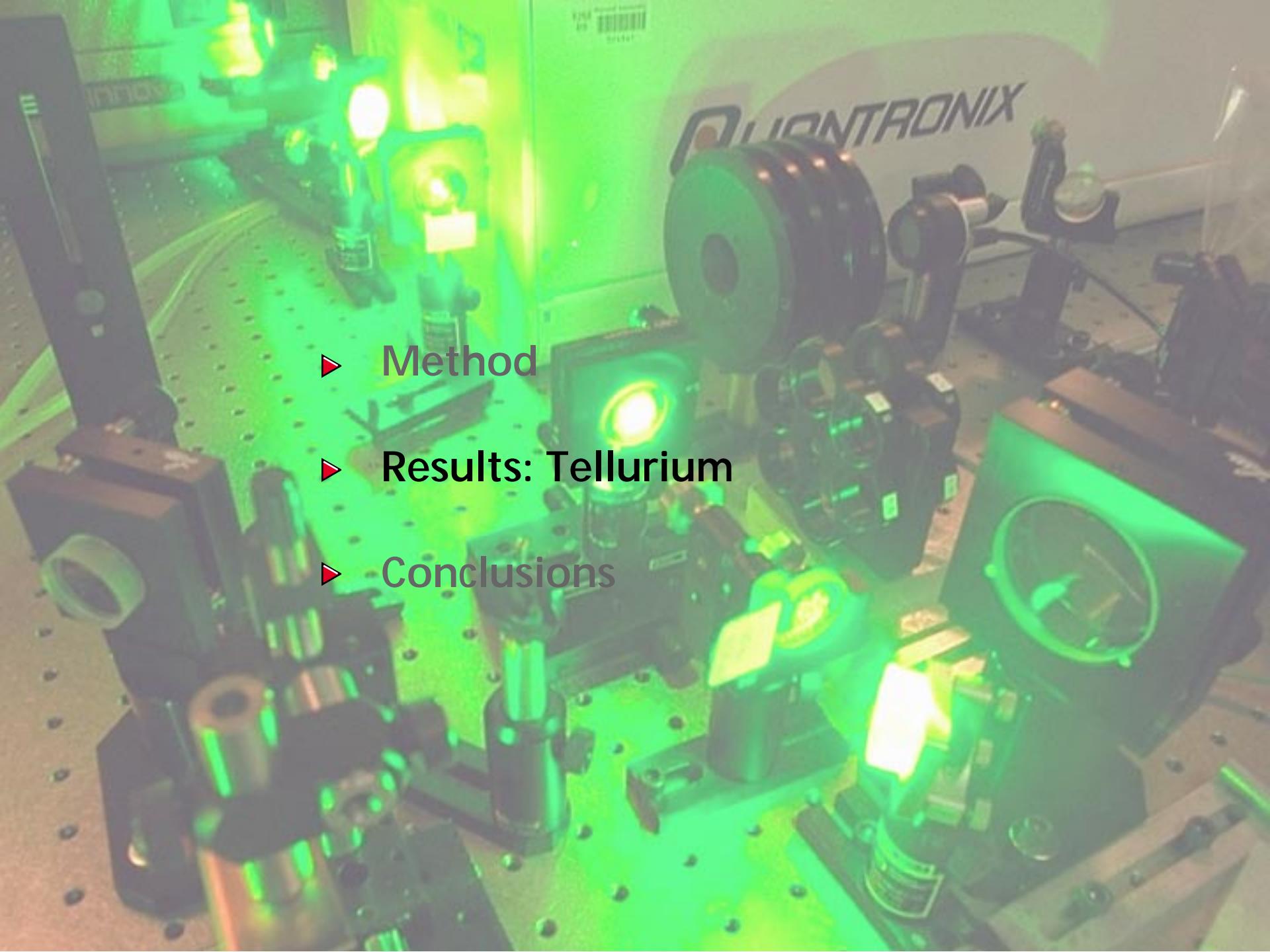
Universal features:

- ▶ semiconductor-to-metal transition
- ▶ decrease in bonding-antibonding splitting takes picoseconds
- ▶ plasma frequency decreases with time

Summary

Universal features:

- ▶ **semiconductor-to-metal transition**
- ▶ **decrease in bonding-antibonding splitting takes picoseconds**
- ▶ **plasma frequency decreases with time**
- ▶ **plasma frequency decreases with increasing fluence**

- 
- ▶ Method
 - ▶ Results: Tellurium
 - ▶ Conclusions

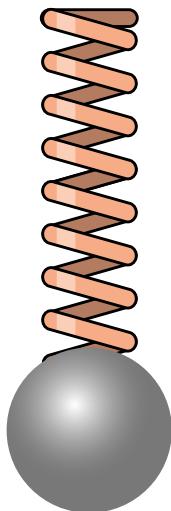
Why Tellurium?

- ▶ semimetal (0.3 eV bandgap)
- ▶ large ΔR due to A_1 phonons
- ▶ A_1 phonons at 3.6 THz

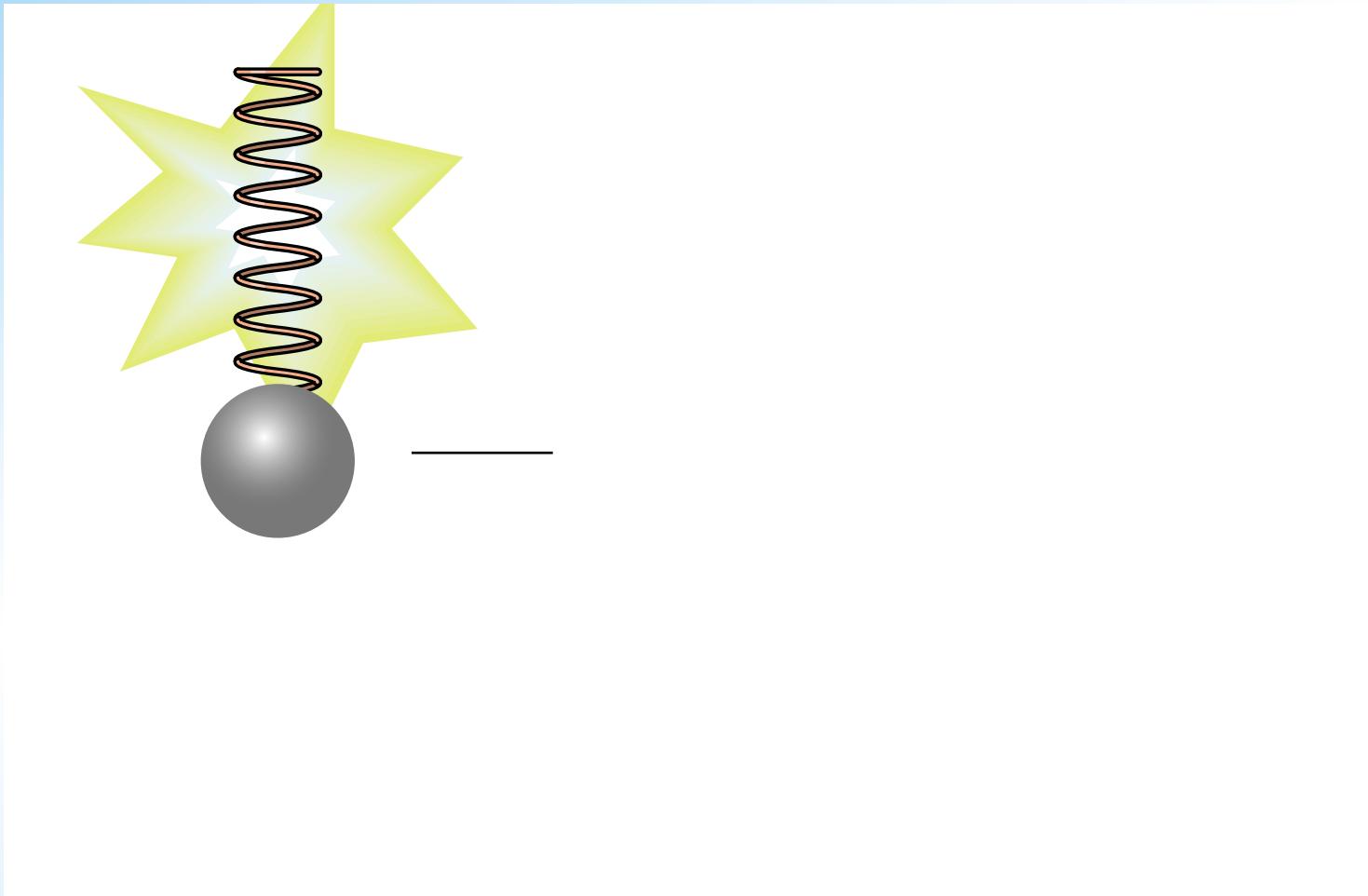
Why Tellurium?

- ▶ semimetal (0.3 eV bandgap)
 - ▶ large ΔR due to A_1 phonons
 - ▶ A_1 phonons at 3.6 THz
- ... but A_1 -mode is not IR-active

Displacive excitation

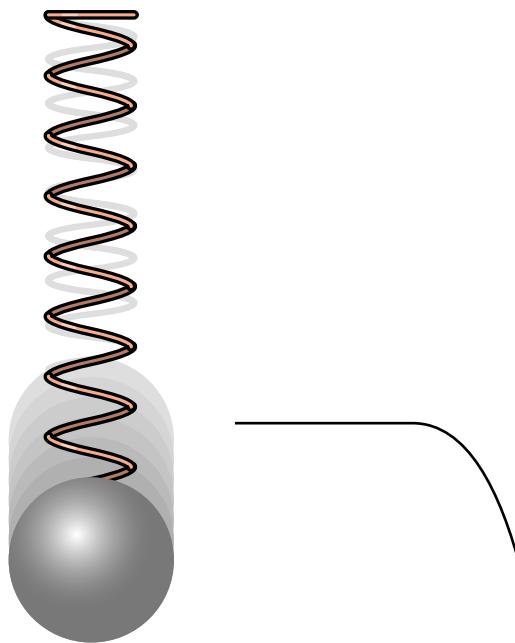


Displacive excitation



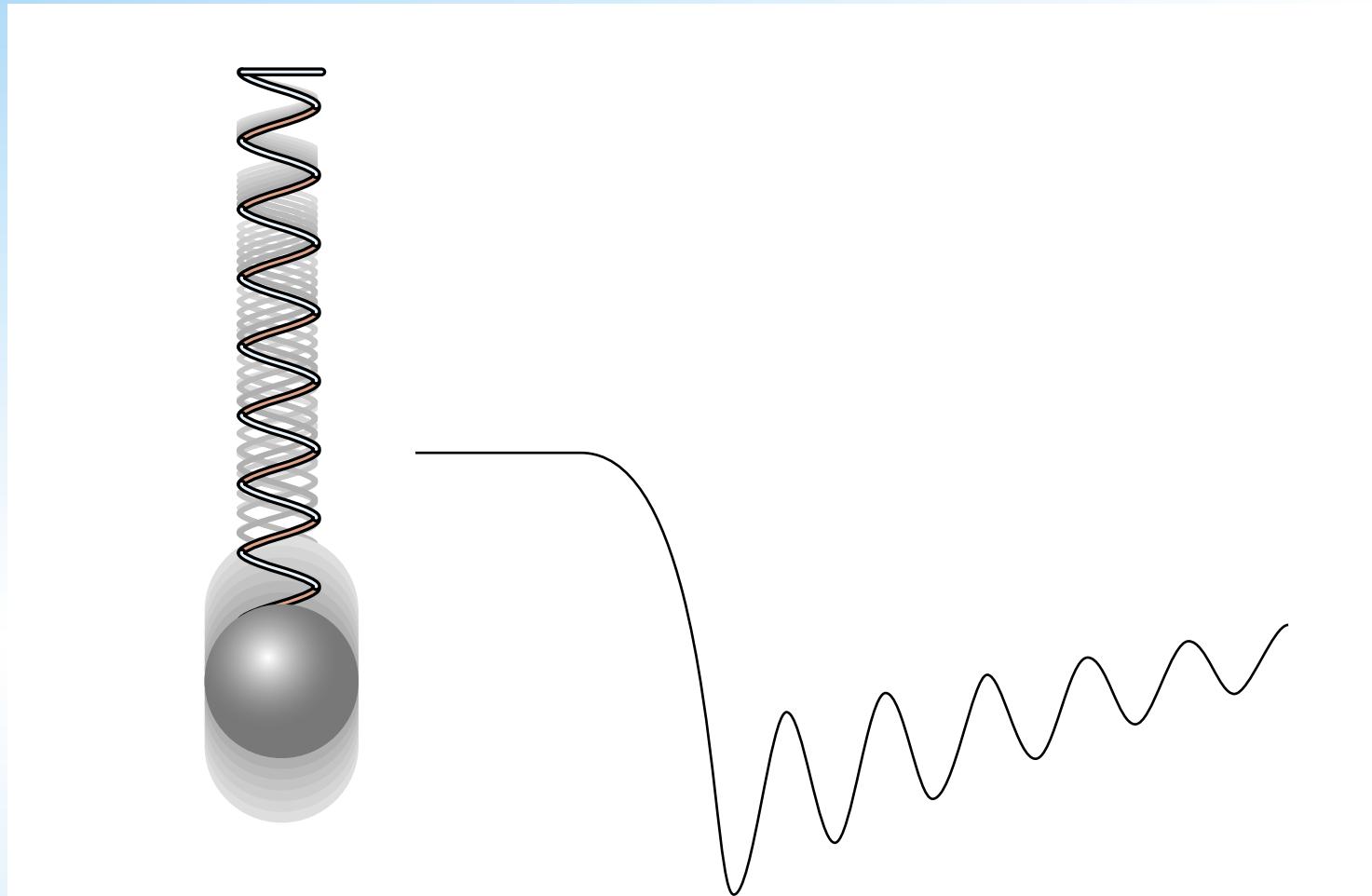
excitation weakens binding force

Displacive excitation



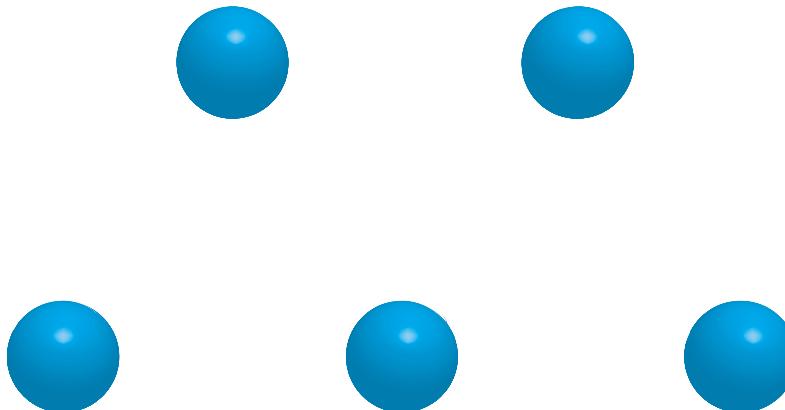
ion moves to new equilibrium position

Displacive excitation



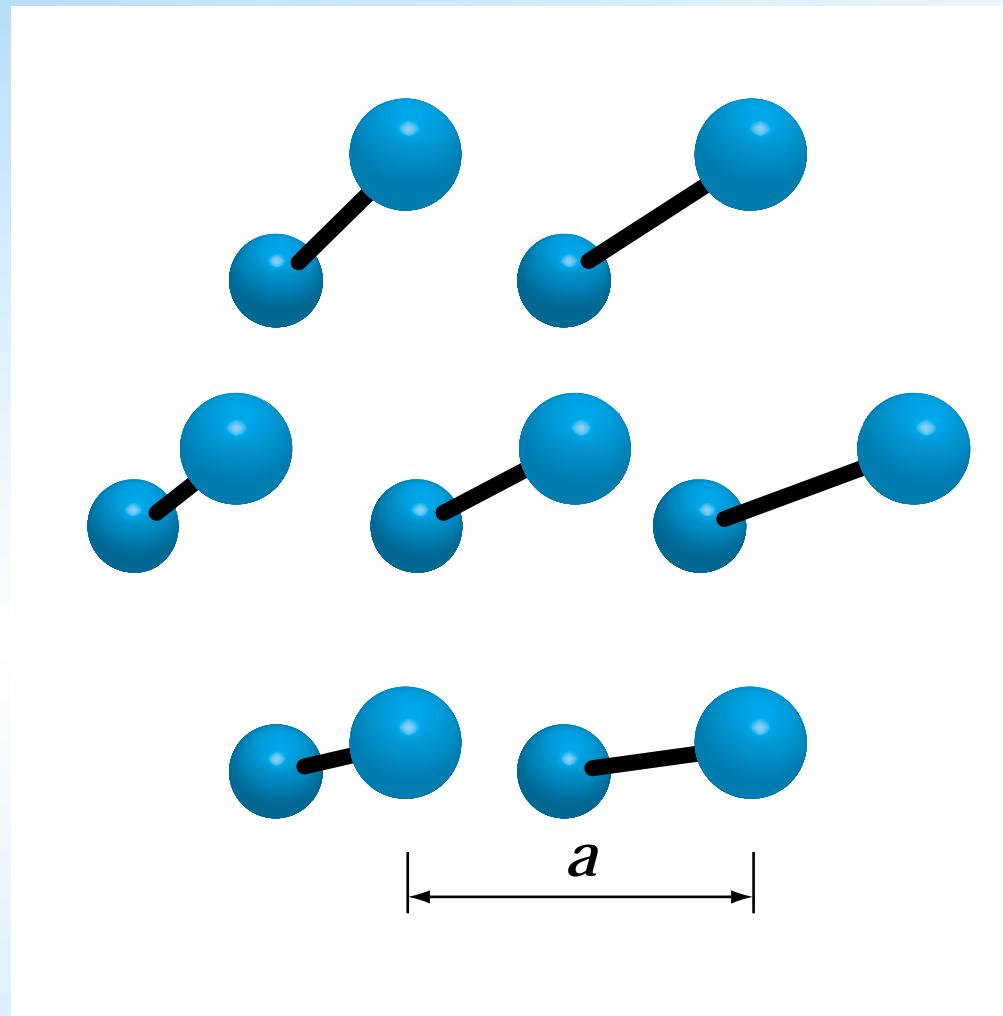
oscillation damps out as force is restored

Tellurium structure

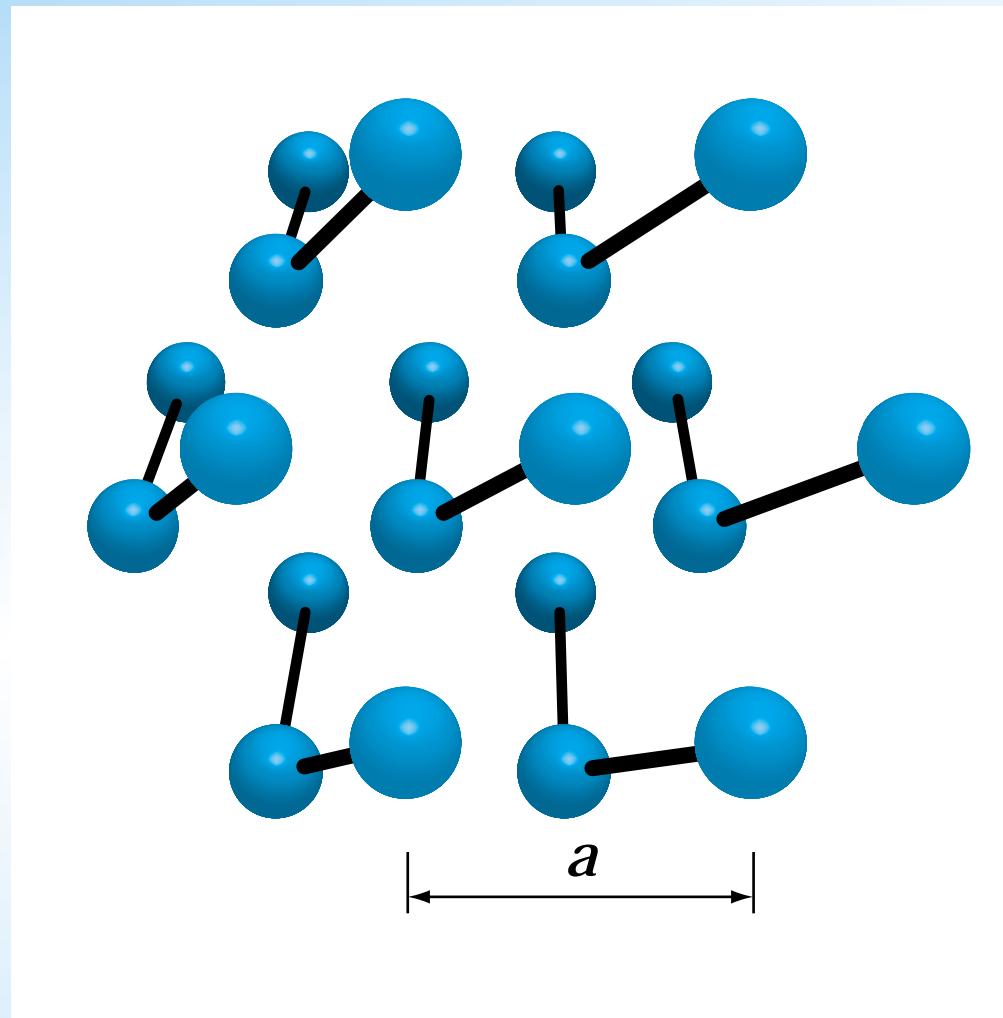


a

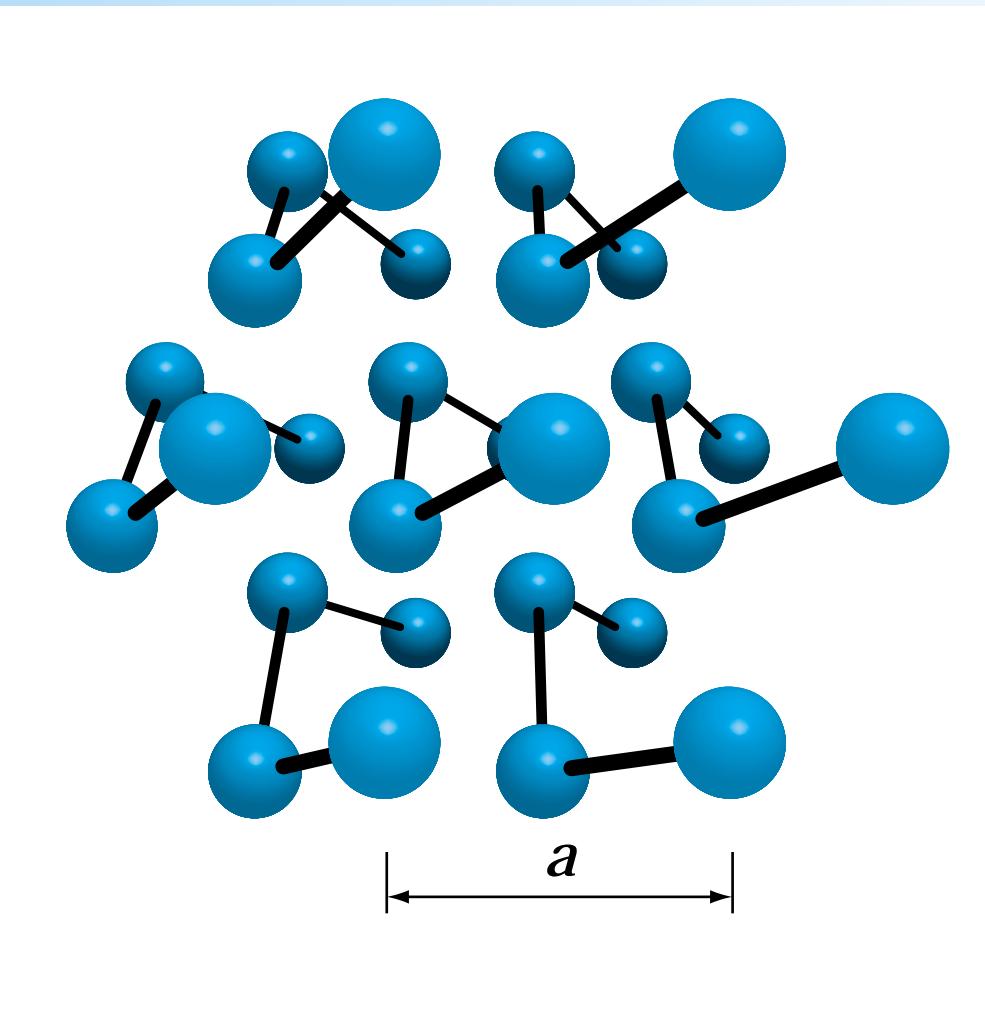
Tellurium structure



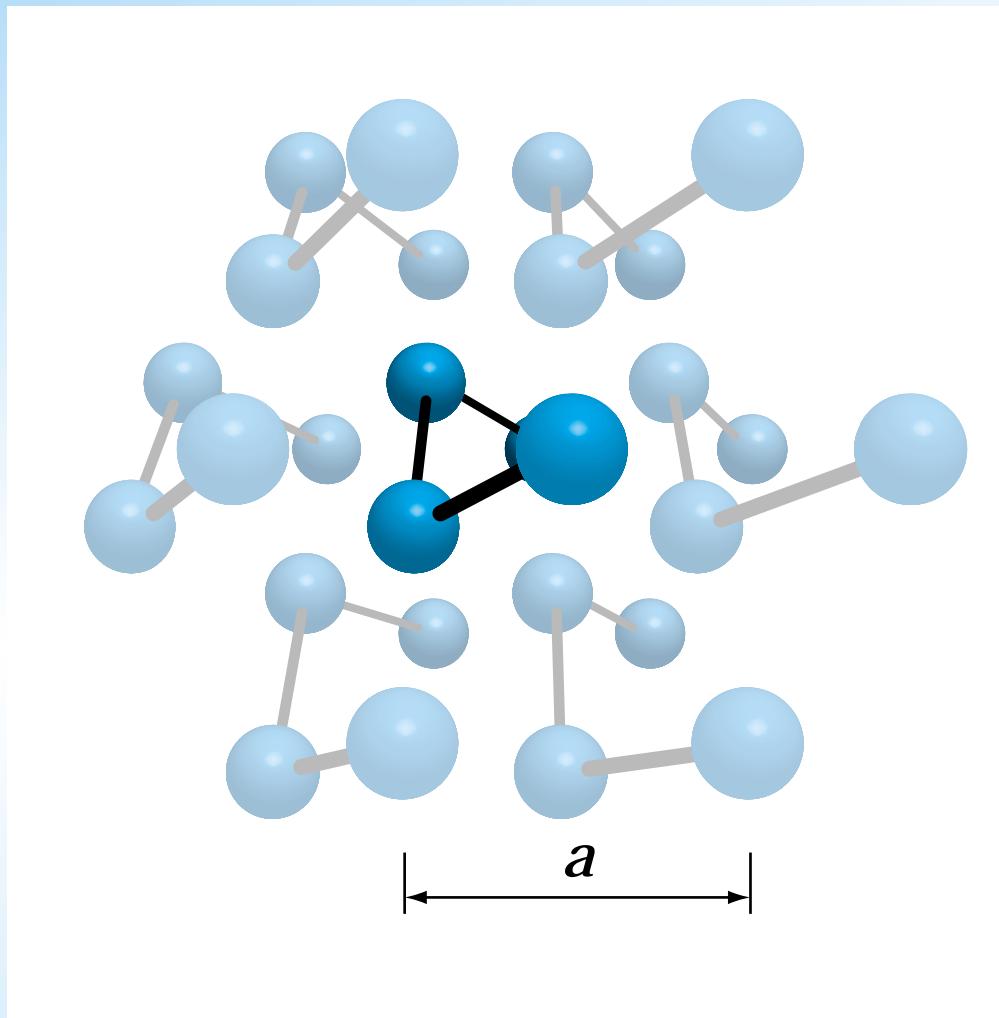
Tellurium structure



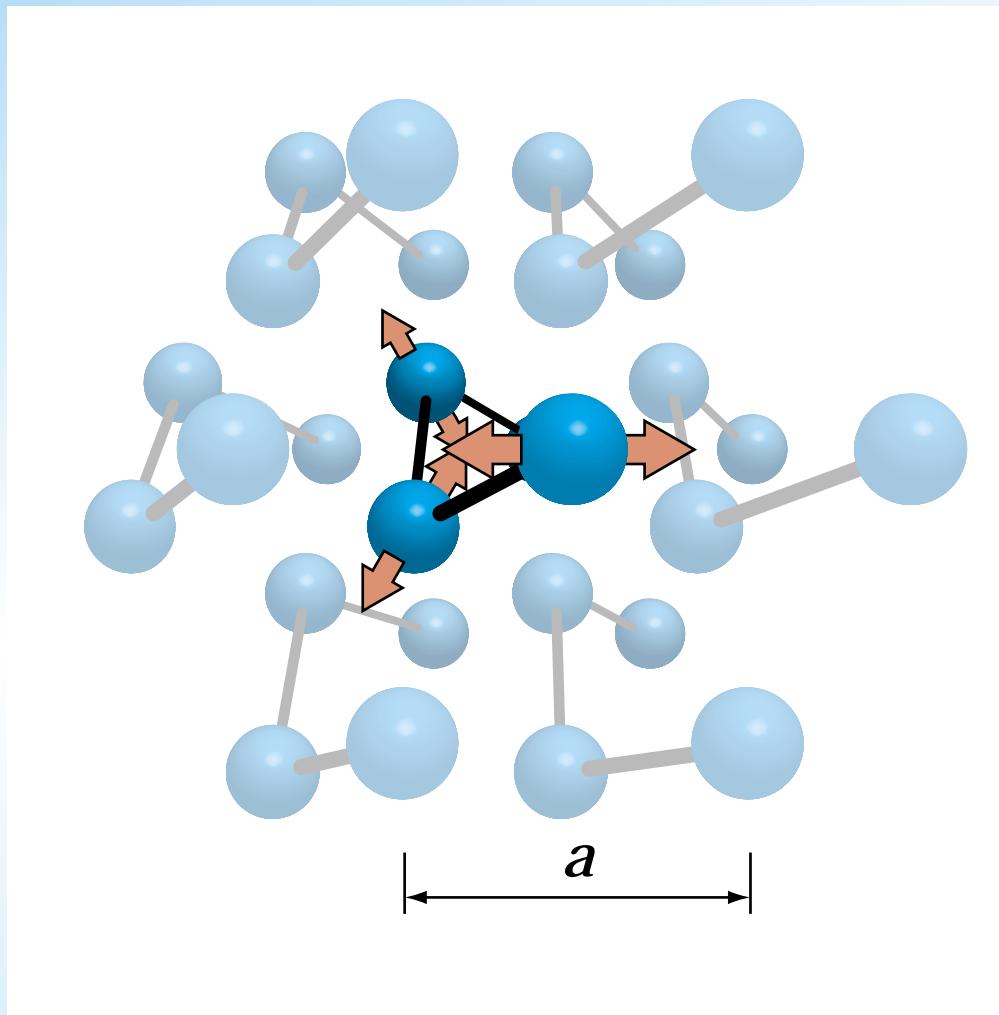
Tellurium structure



Tellurium structure

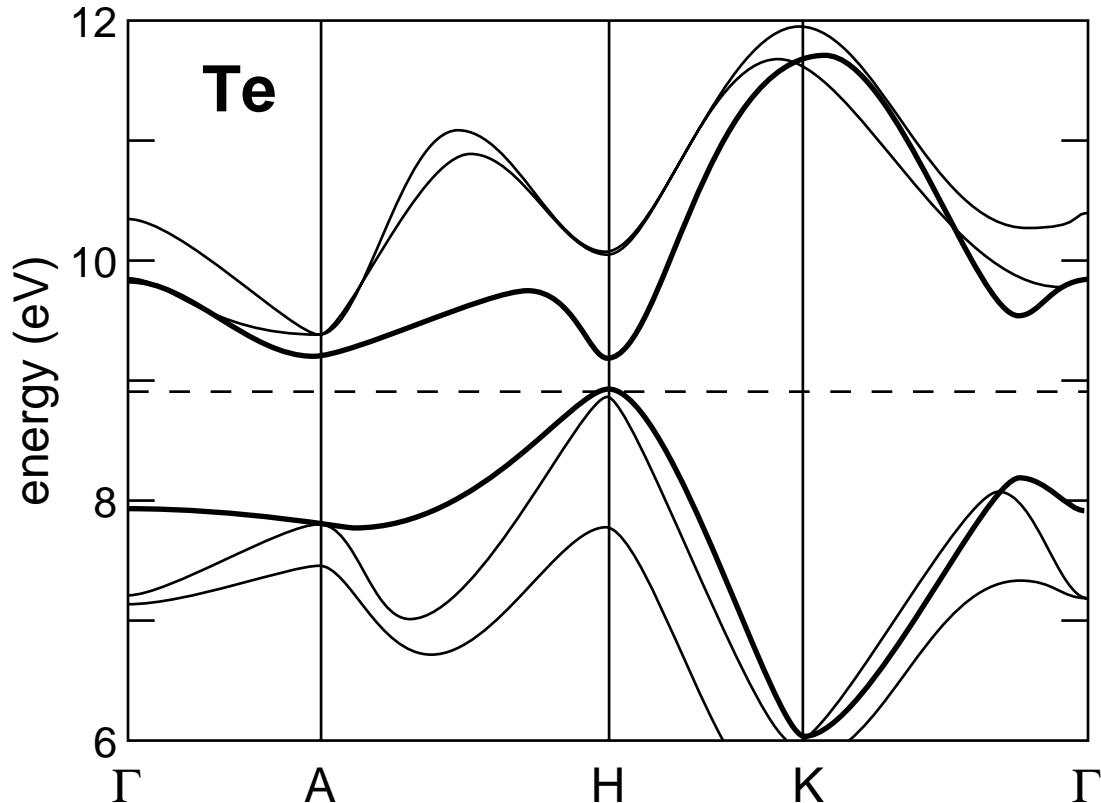


Tellurium structure



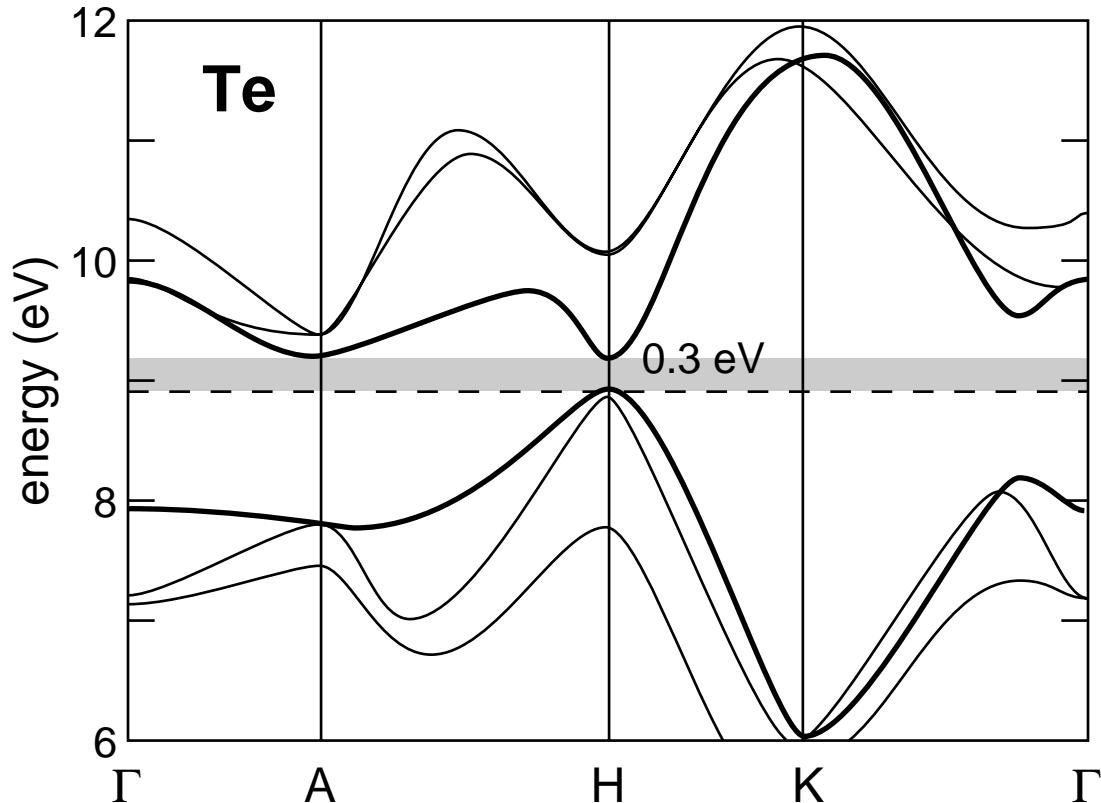
A_1 mode

Tellurium bandstructure



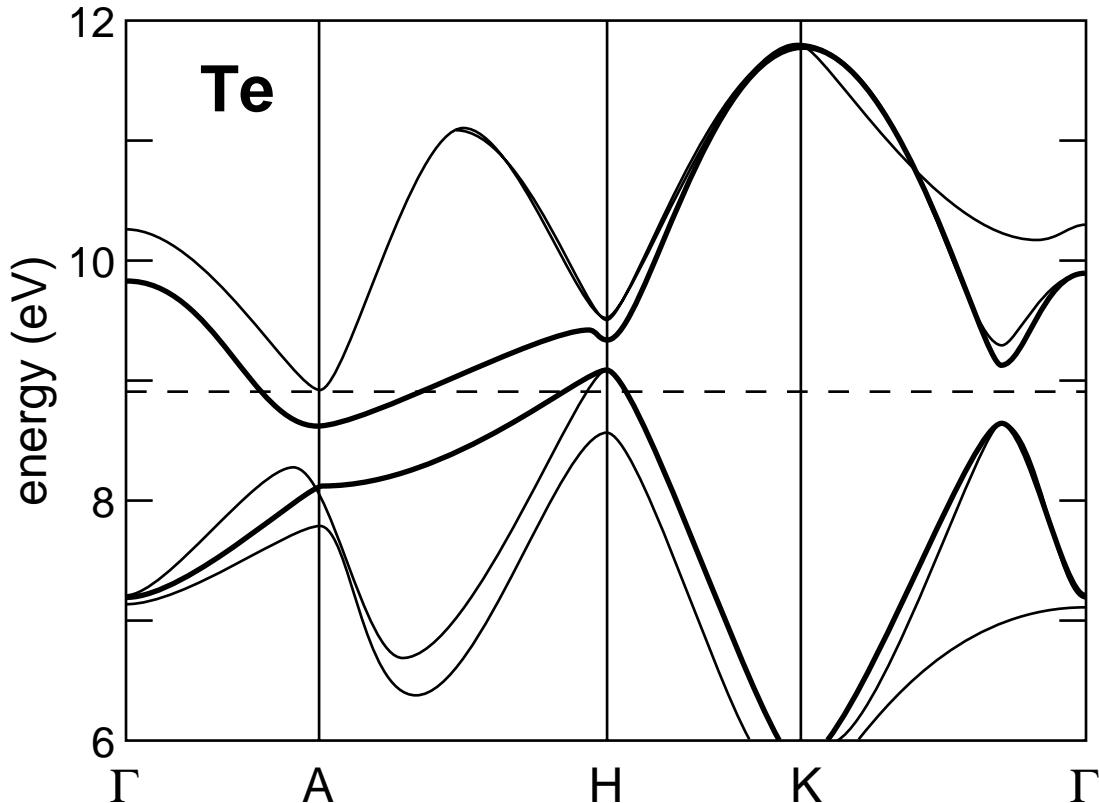
P. Tangney (Princeton) and S. Fahey (Cork), *private communication*

Tellurium bandstructure



P. Tangney (Princeton) and S. Fahey (Cork), *private communication*

Tellurium bandstructure

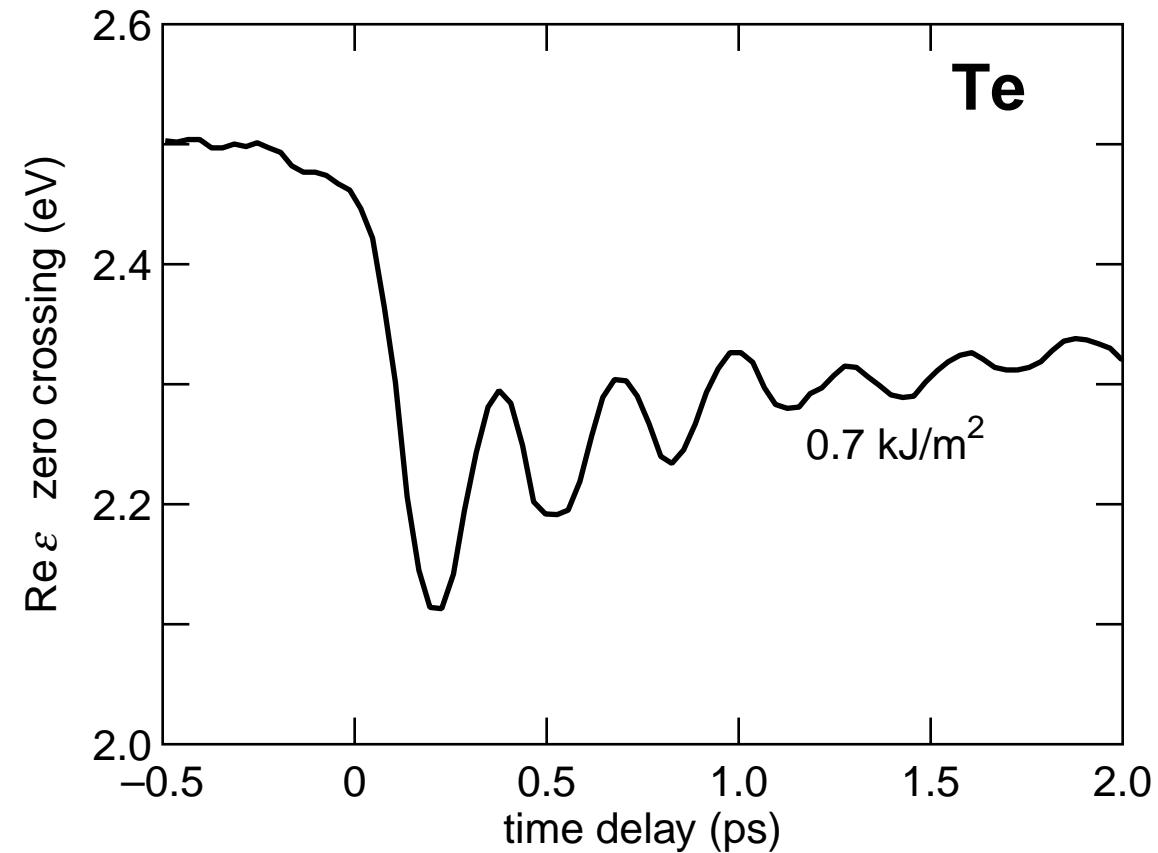


P. Tangney (Princeton) and S. Fahey (Cork), *private communication*

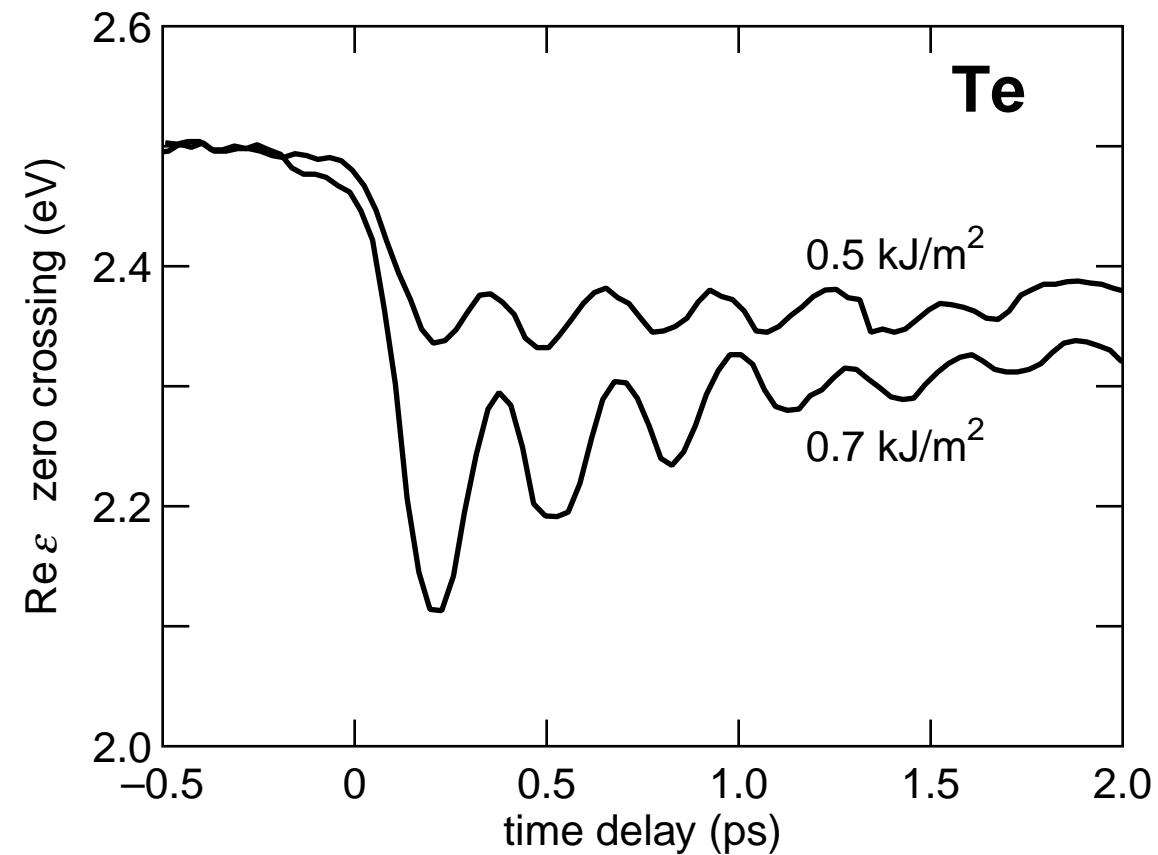
Results



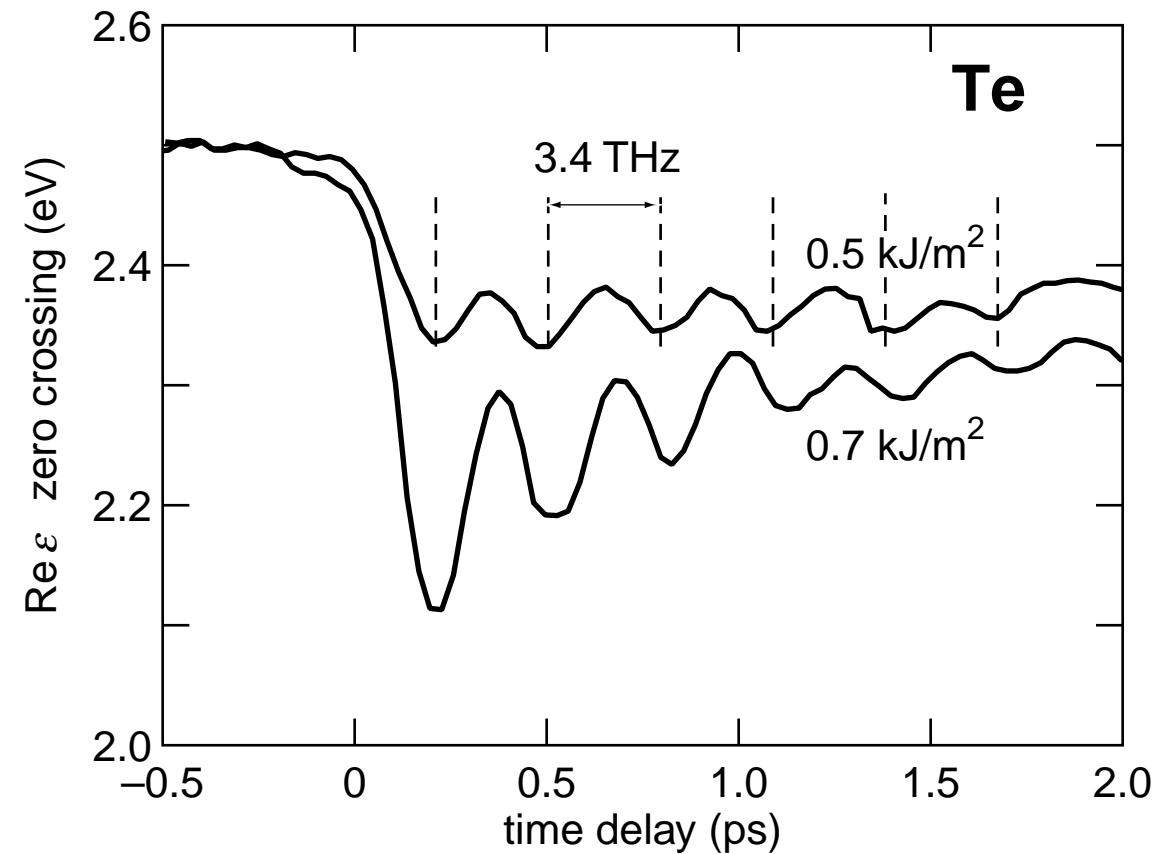
Results



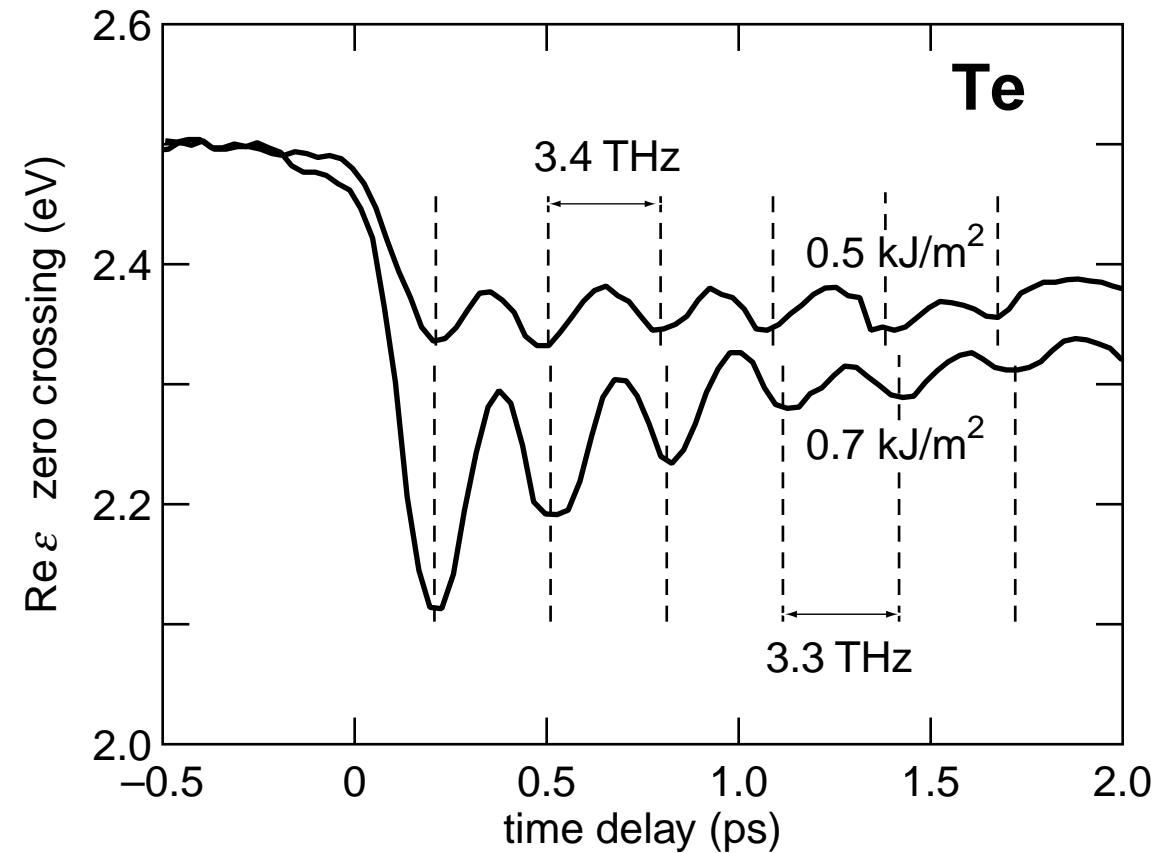
Results



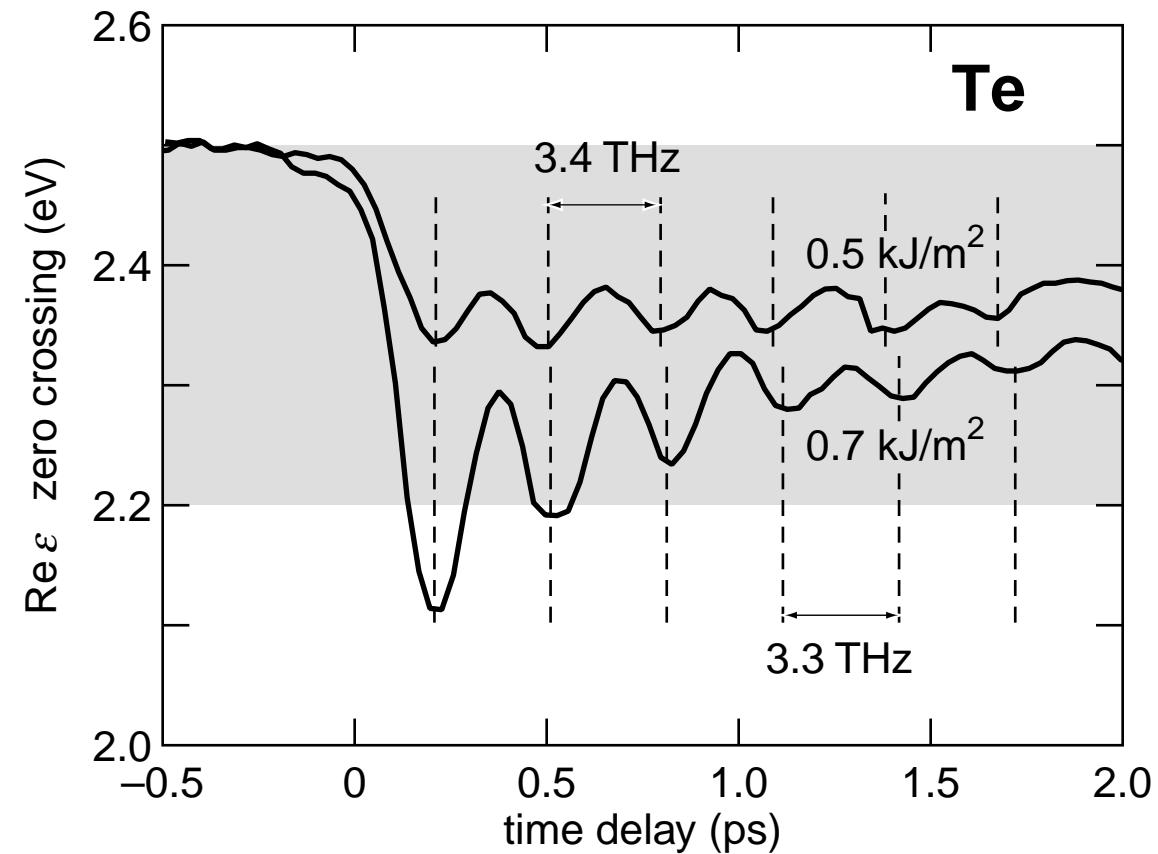
Results

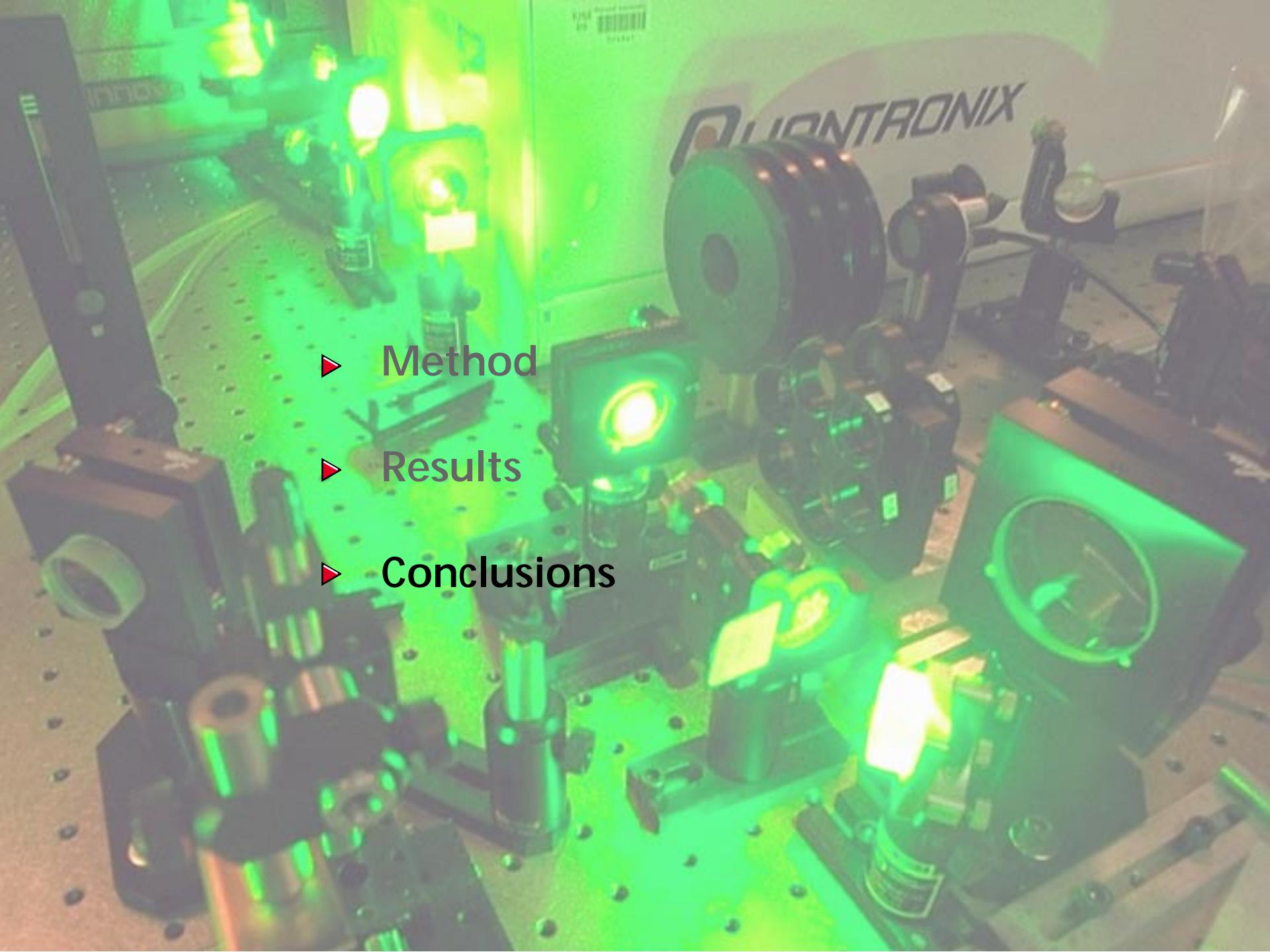


Results



Results

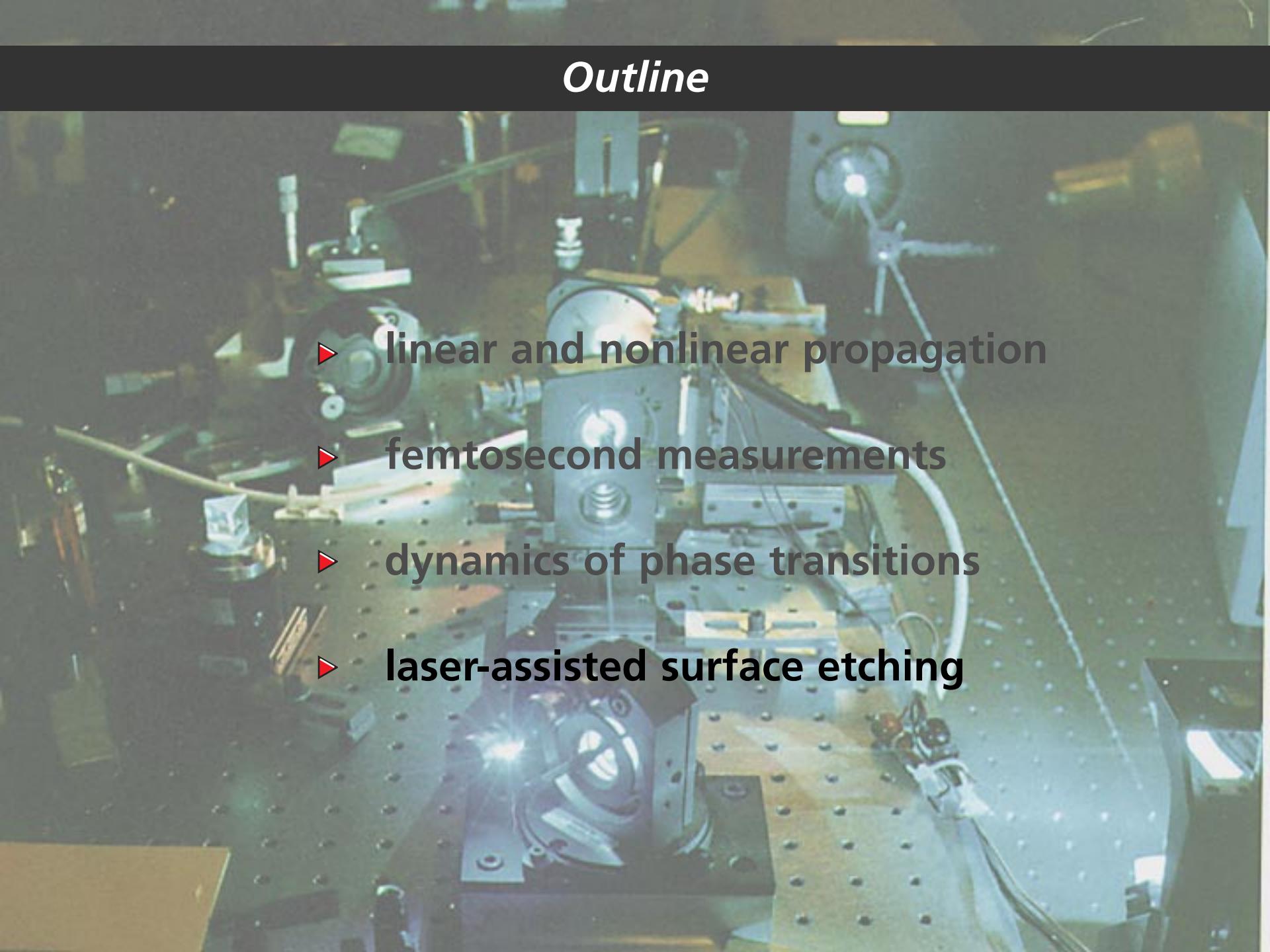


- 
- ▶ Method
 - ▶ Results
 - ▶ Conclusions

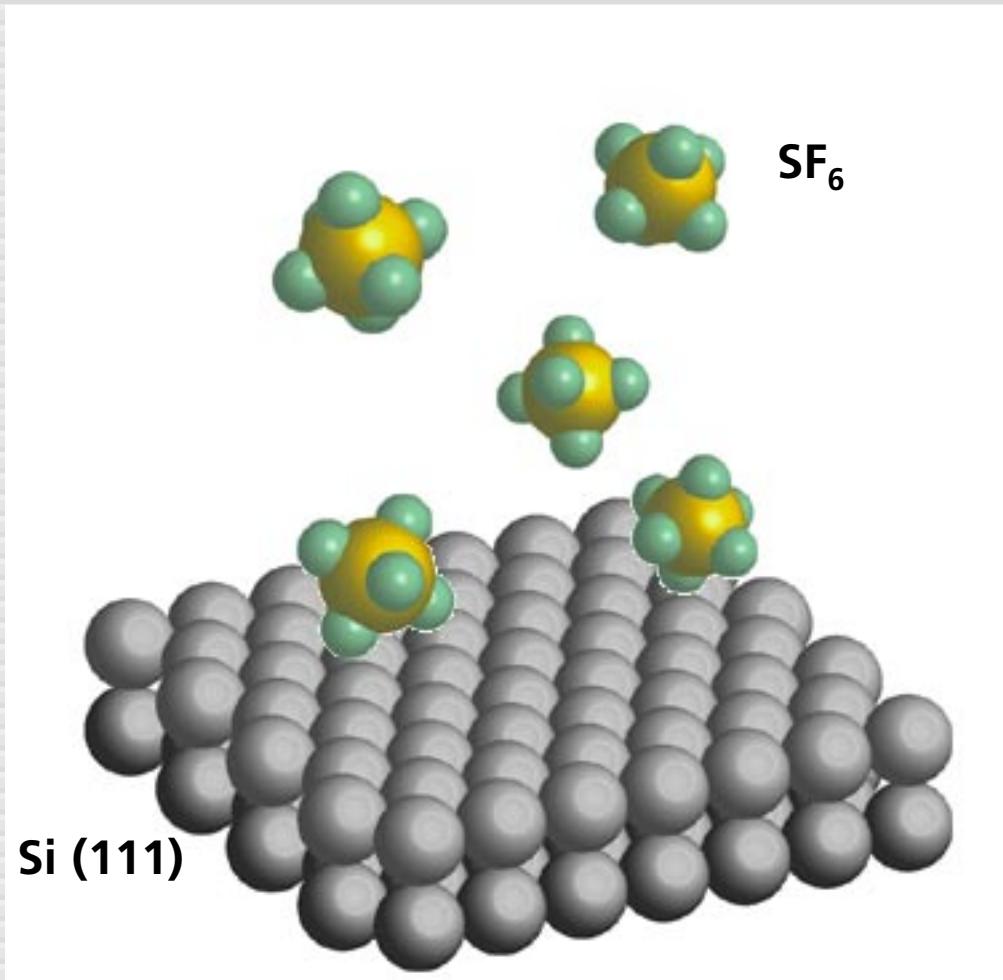
Conclusions

- ▶ Femtosecond time-resolved ellipsometry
powerful tool for tracking ultrafast electron
and lattice dynamics in highly excited solids
- ▶ Strong electronic excitation can drive a
structural transition
- ▶ Direct observation of coherent phonons

Outline

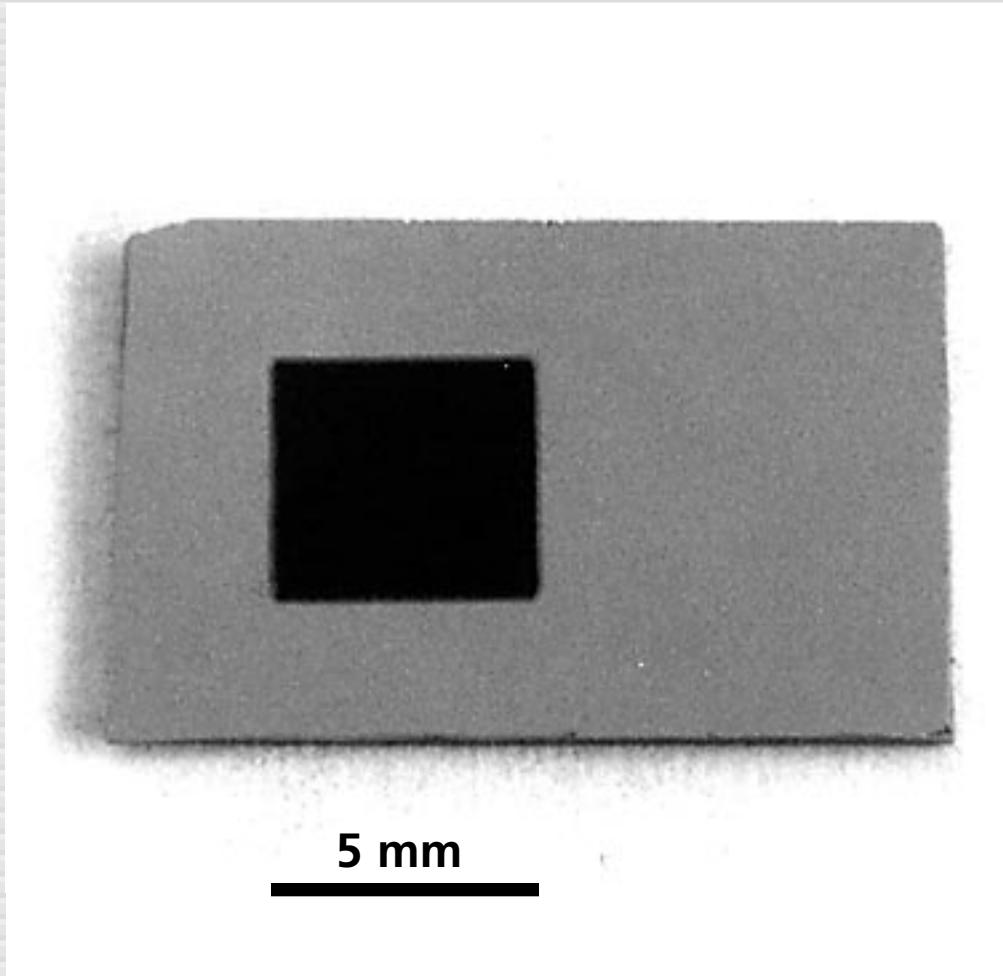
- 
- ▶ linear and nonlinear propagation
 - ▶ femtosecond measurements
 - ▶ dynamics of phase transitions
 - ▶ laser-assisted surface etching

Introduction



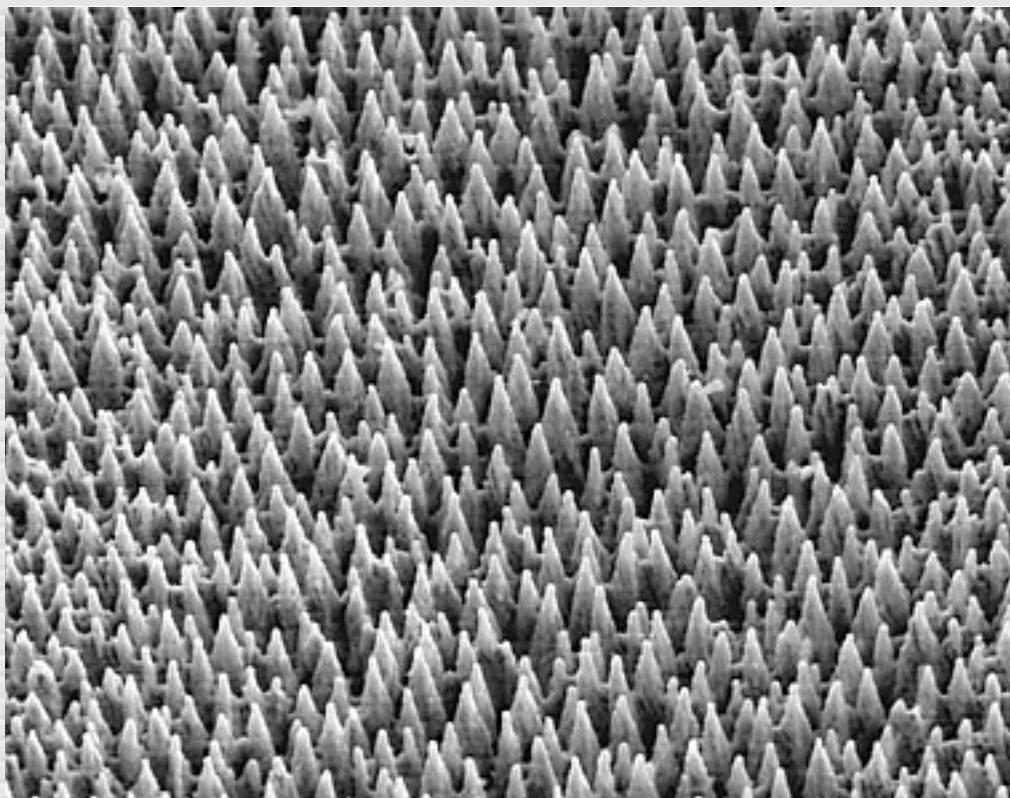
irradiate with 100-fs 10 kJ/m² pulses

Introduction



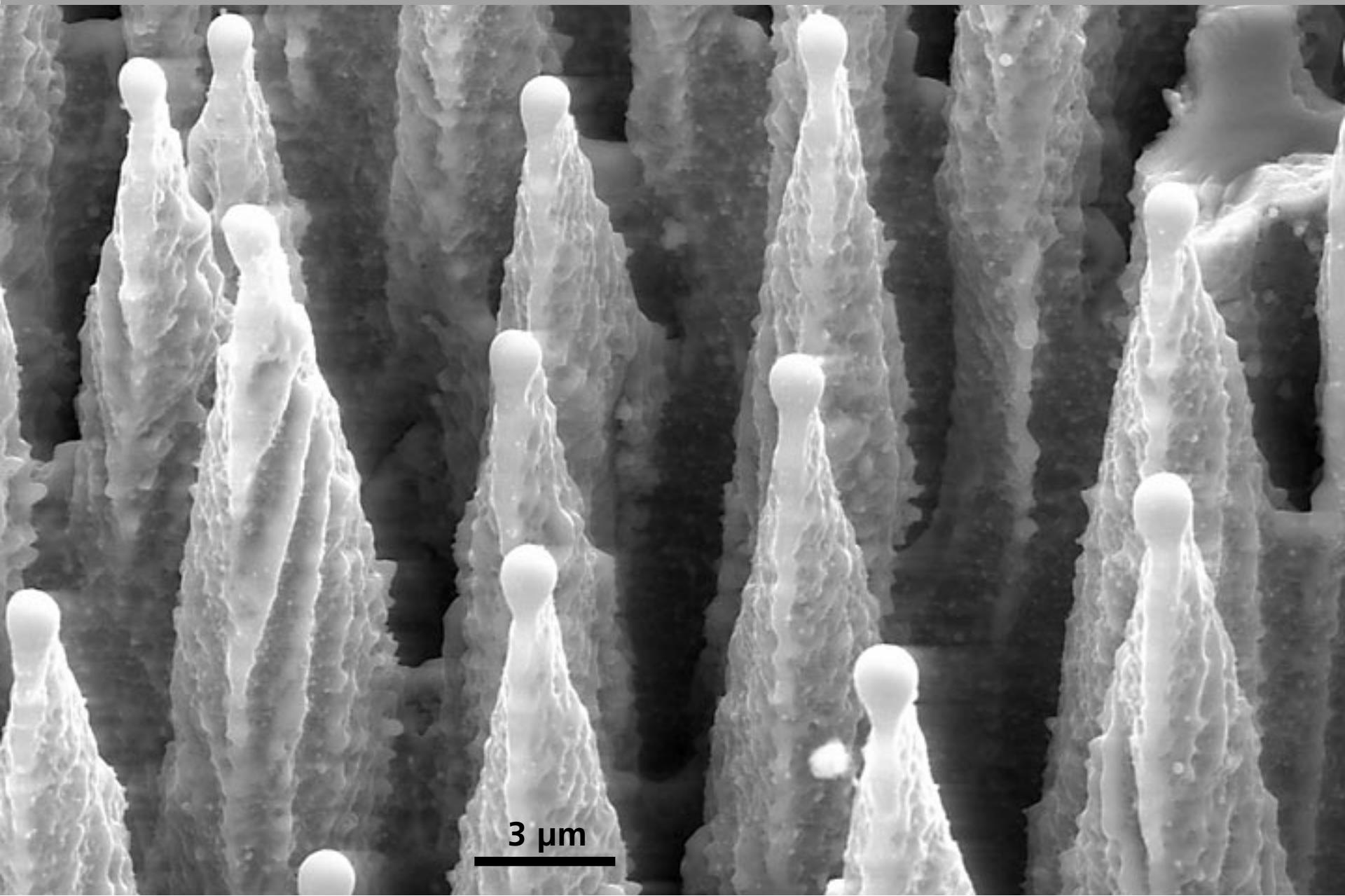
"black silicon"

Introduction

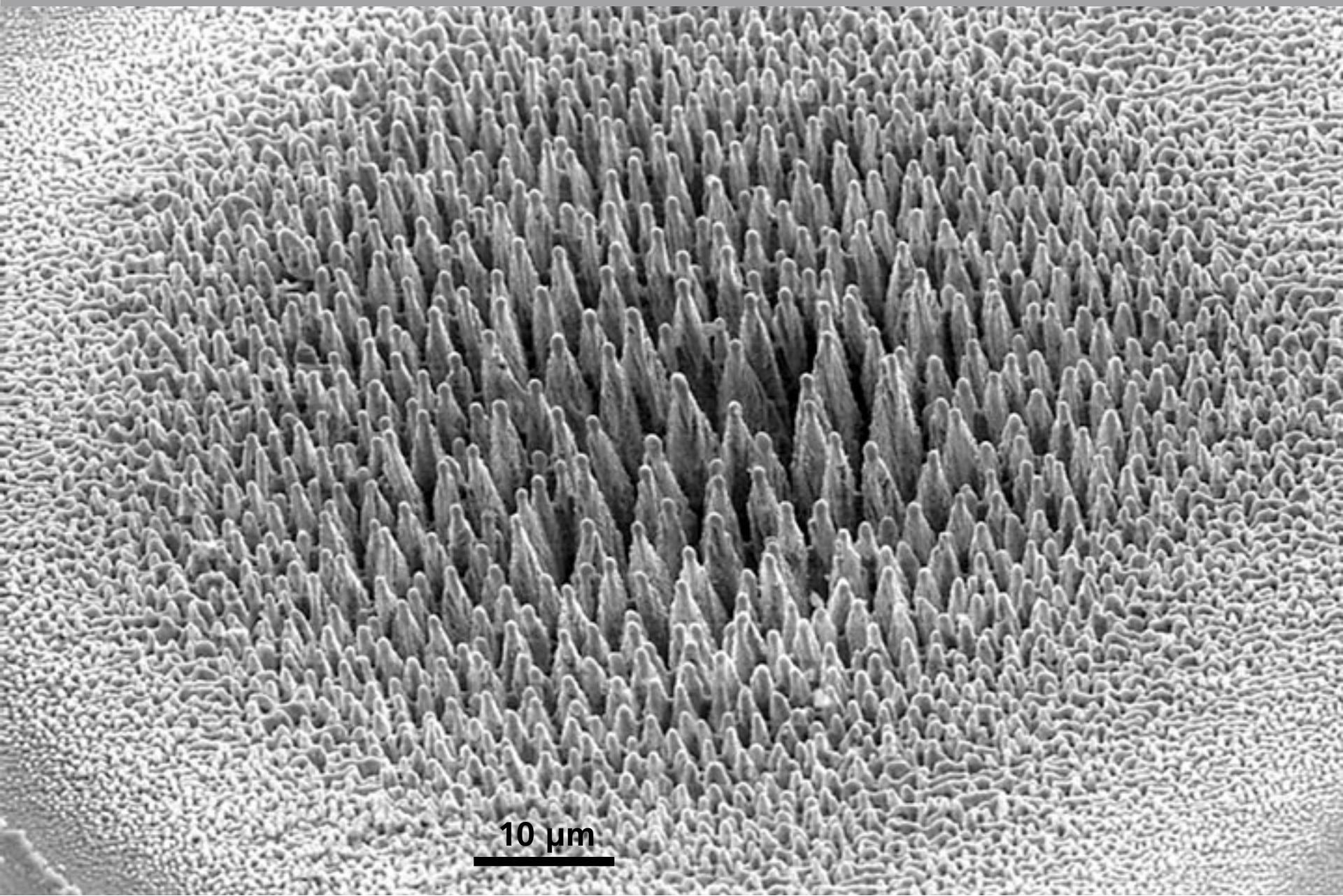


20 μm

Introduction



Introduction



10 μm

Outline

- ▶ **Background**
- ▶ **Results**
- ▶ **Discussion**

x2000

#3548

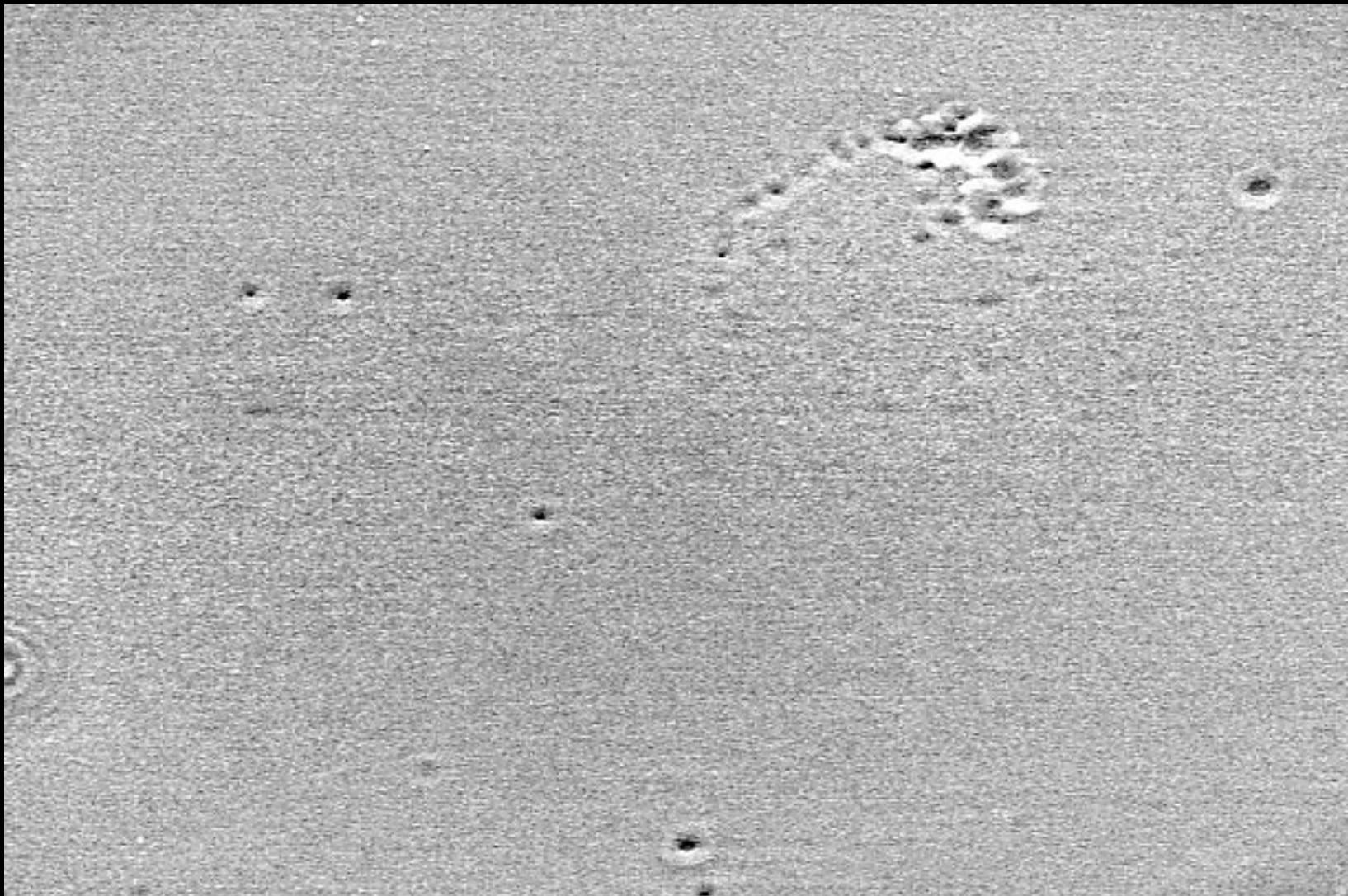
512 x 480

20 μ m

10kV

15mm

0000



x2000

#3548

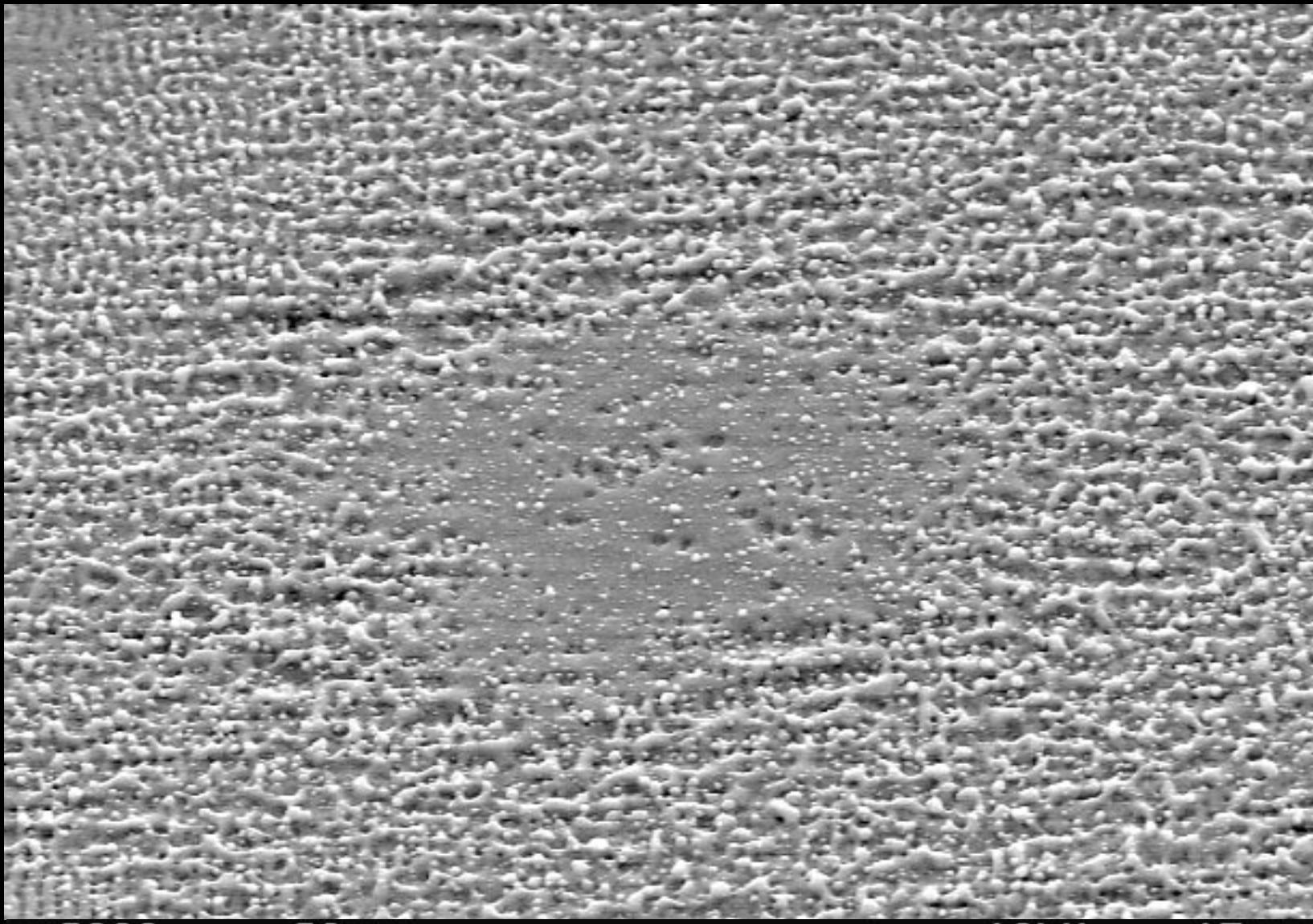
512 x 480

20 μm

10kV

15mm

0001



x2000

#3548

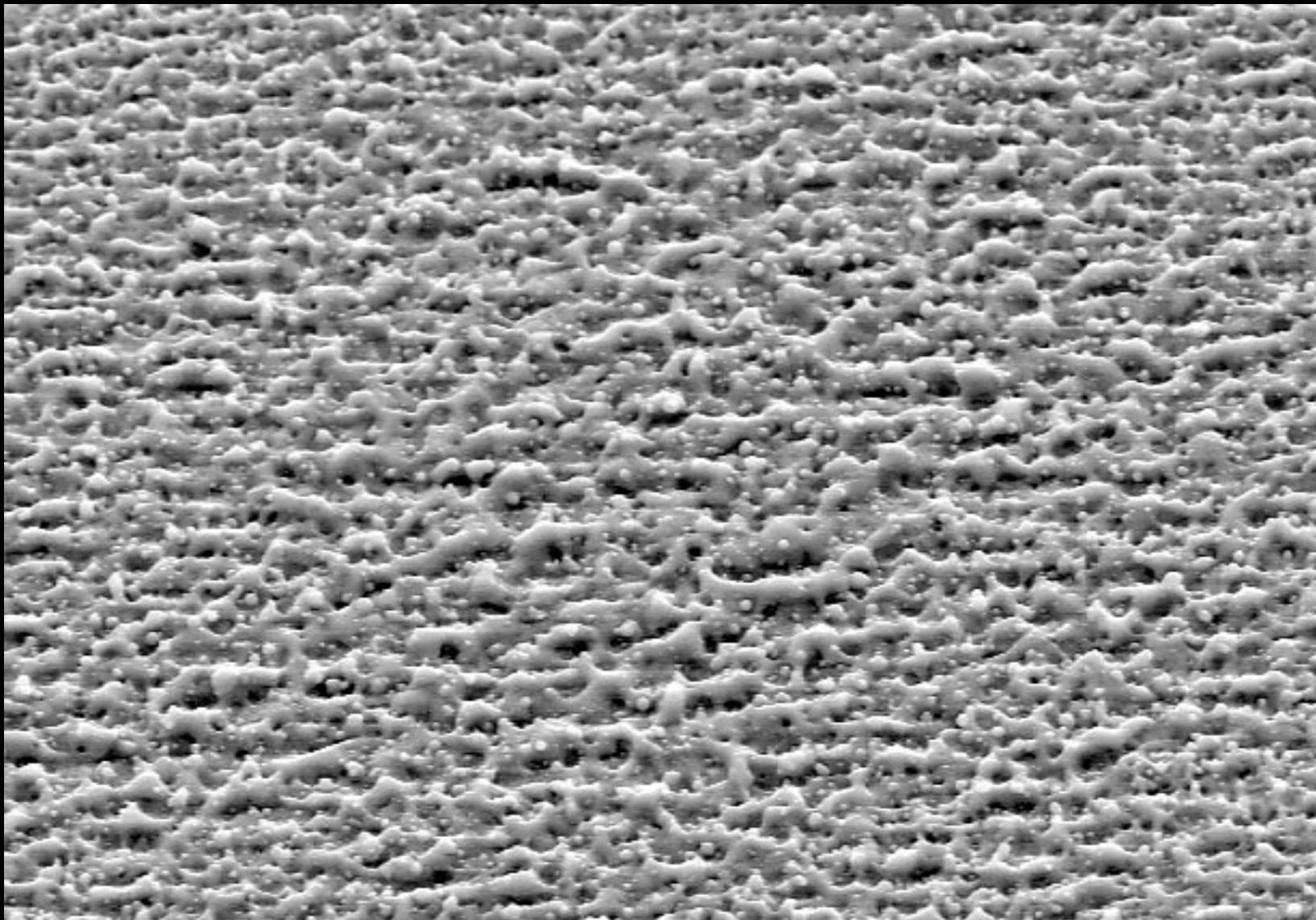
512 x 480

20 μm

10kV

15mm

0005



x2000

#3548

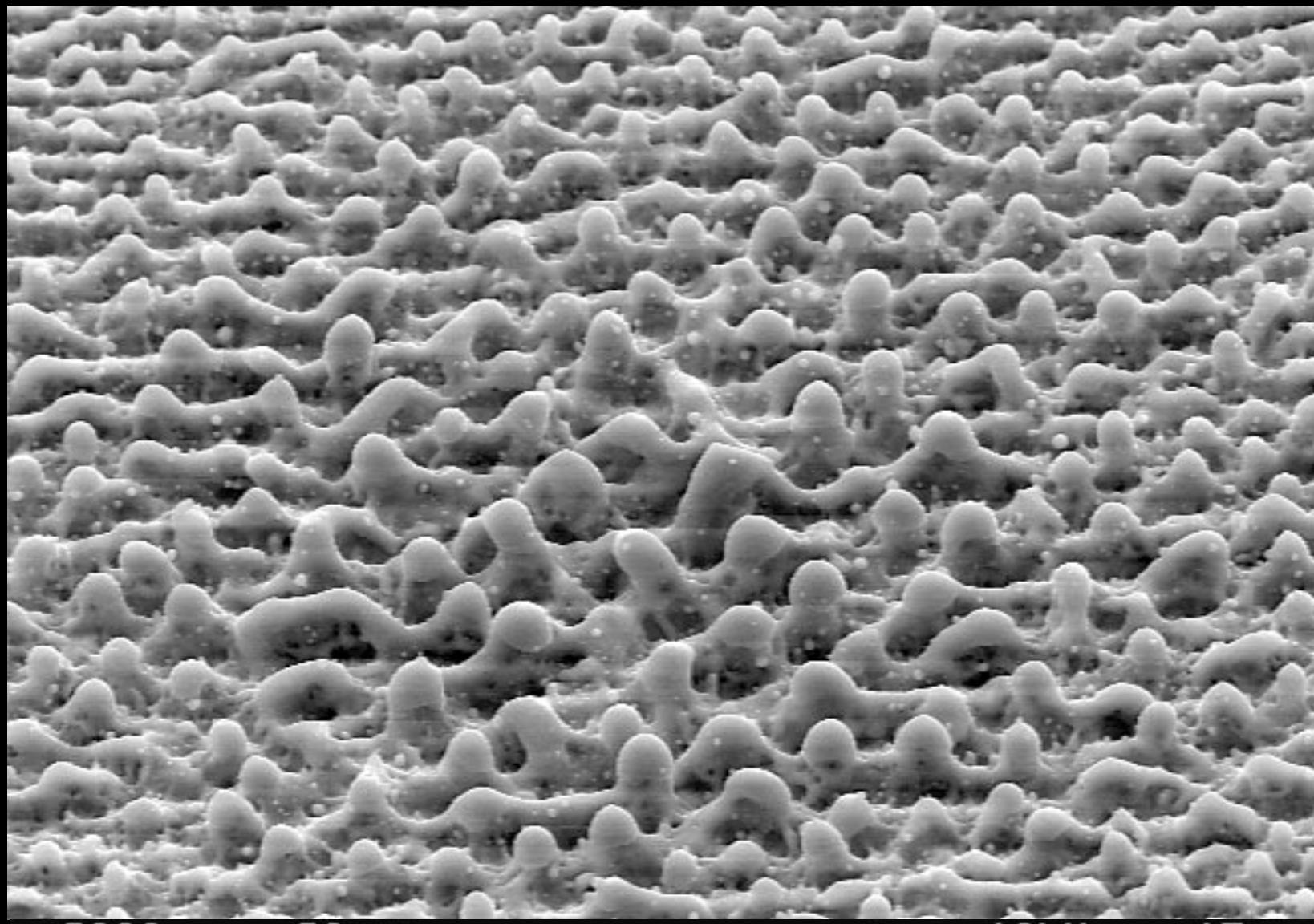
512 x 480

20 μm

10kV

15mm

0010



x2000

#3548

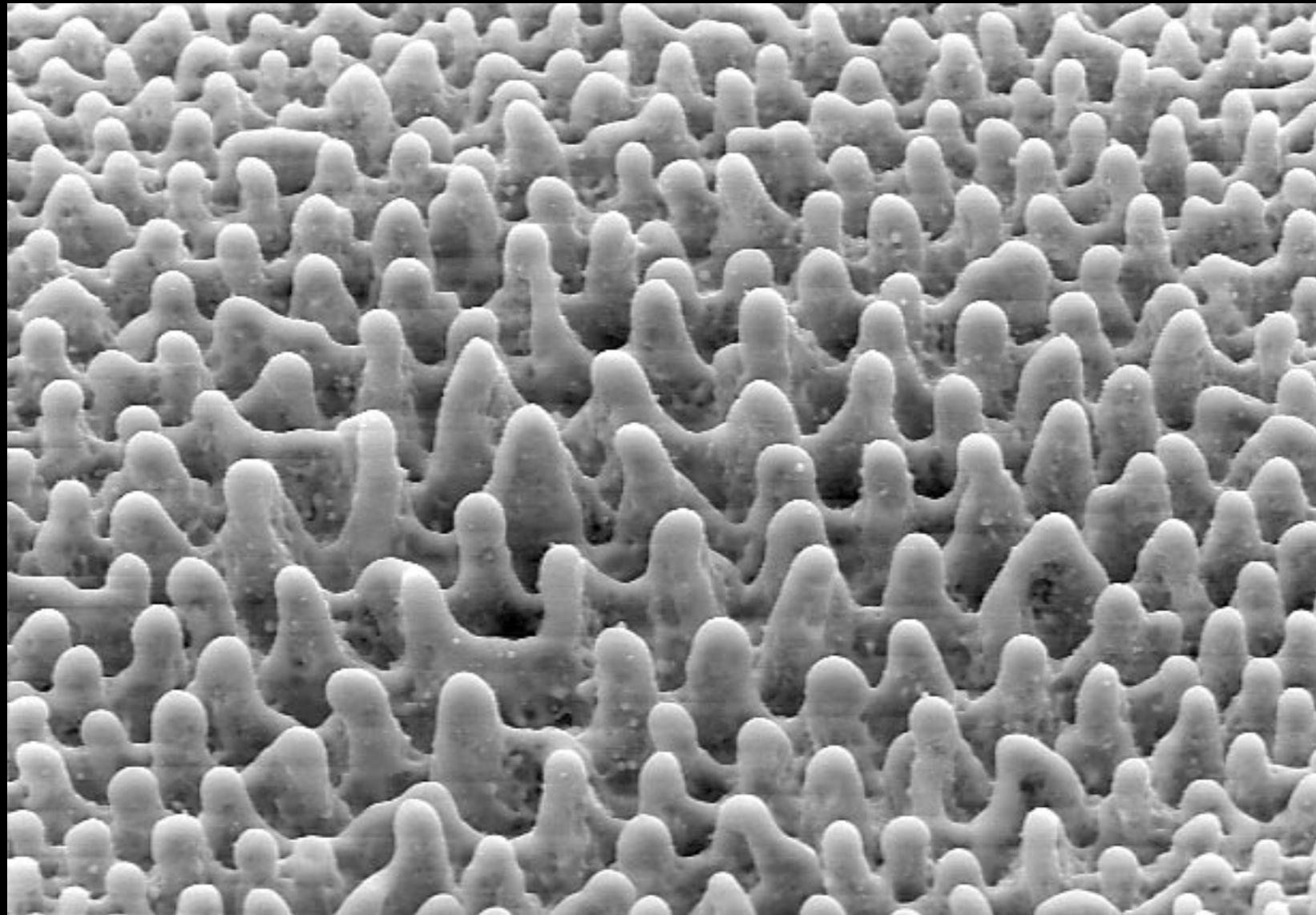
512 x 480

20 μm

10kV

15mm

0025



x2000

#3548

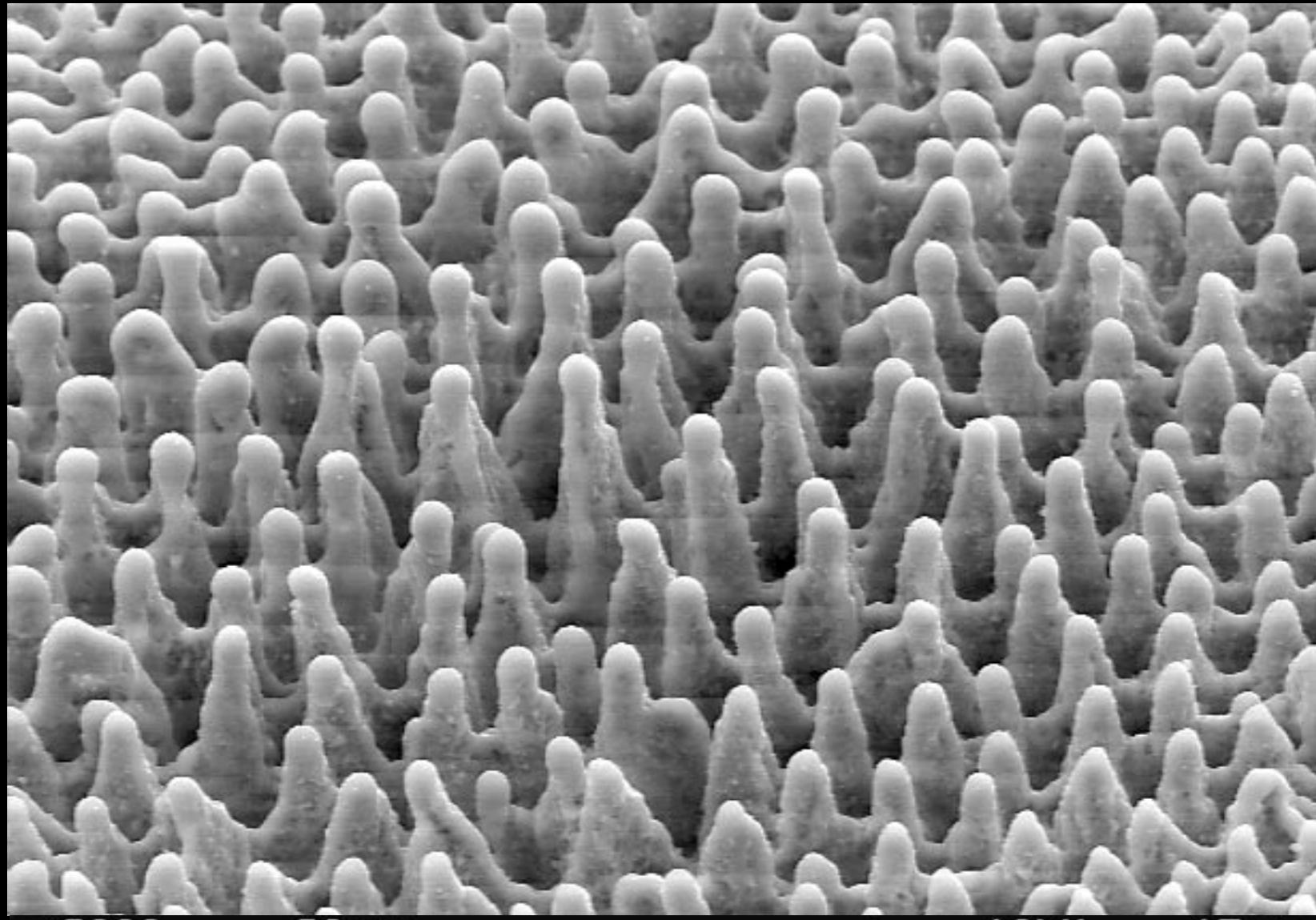
512 x 480

20 μm

10kV

15mm

0050



x2000

20 μm

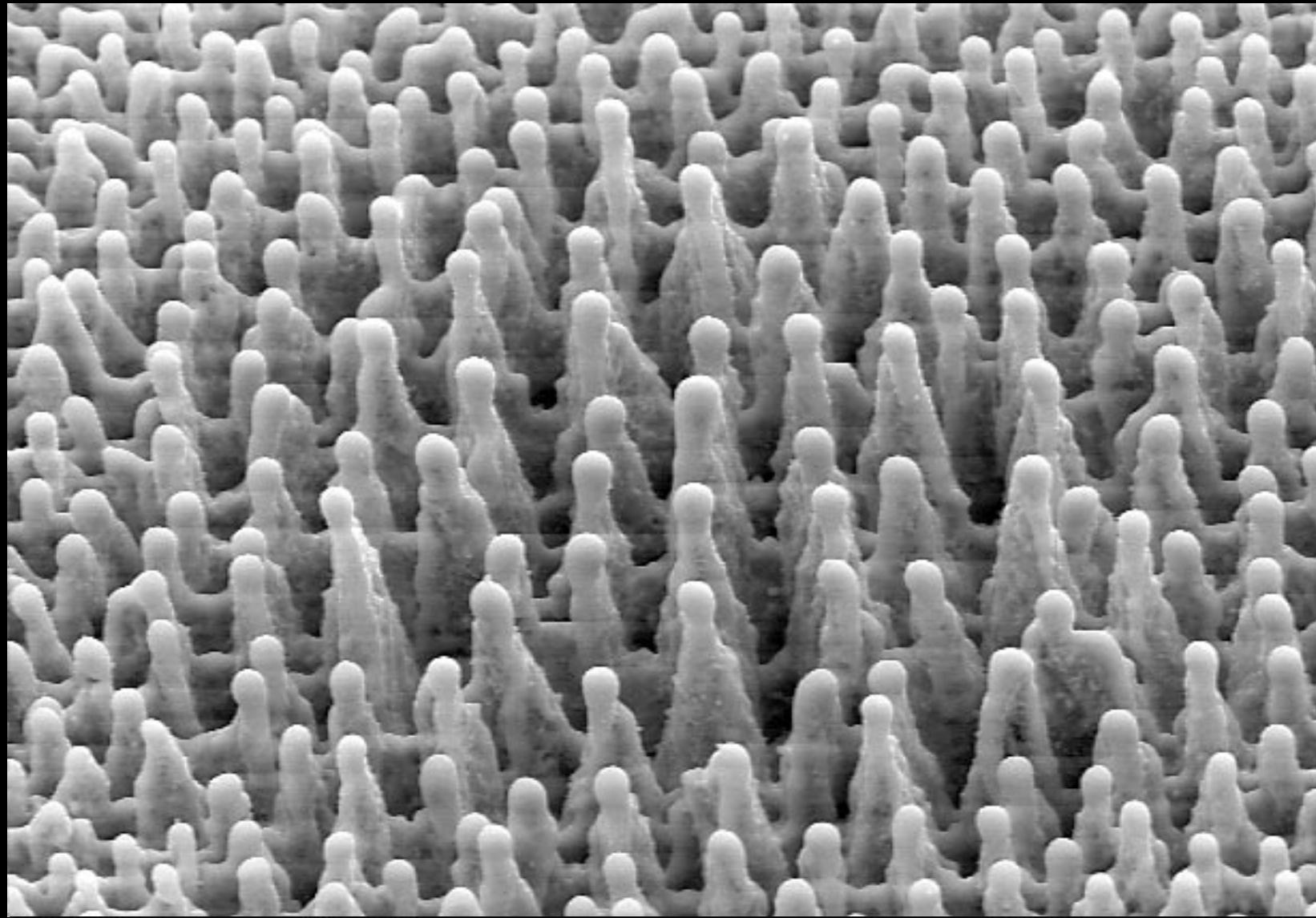
10kV

15mm

#3548

512 x 480

0075



x2000

20 μm

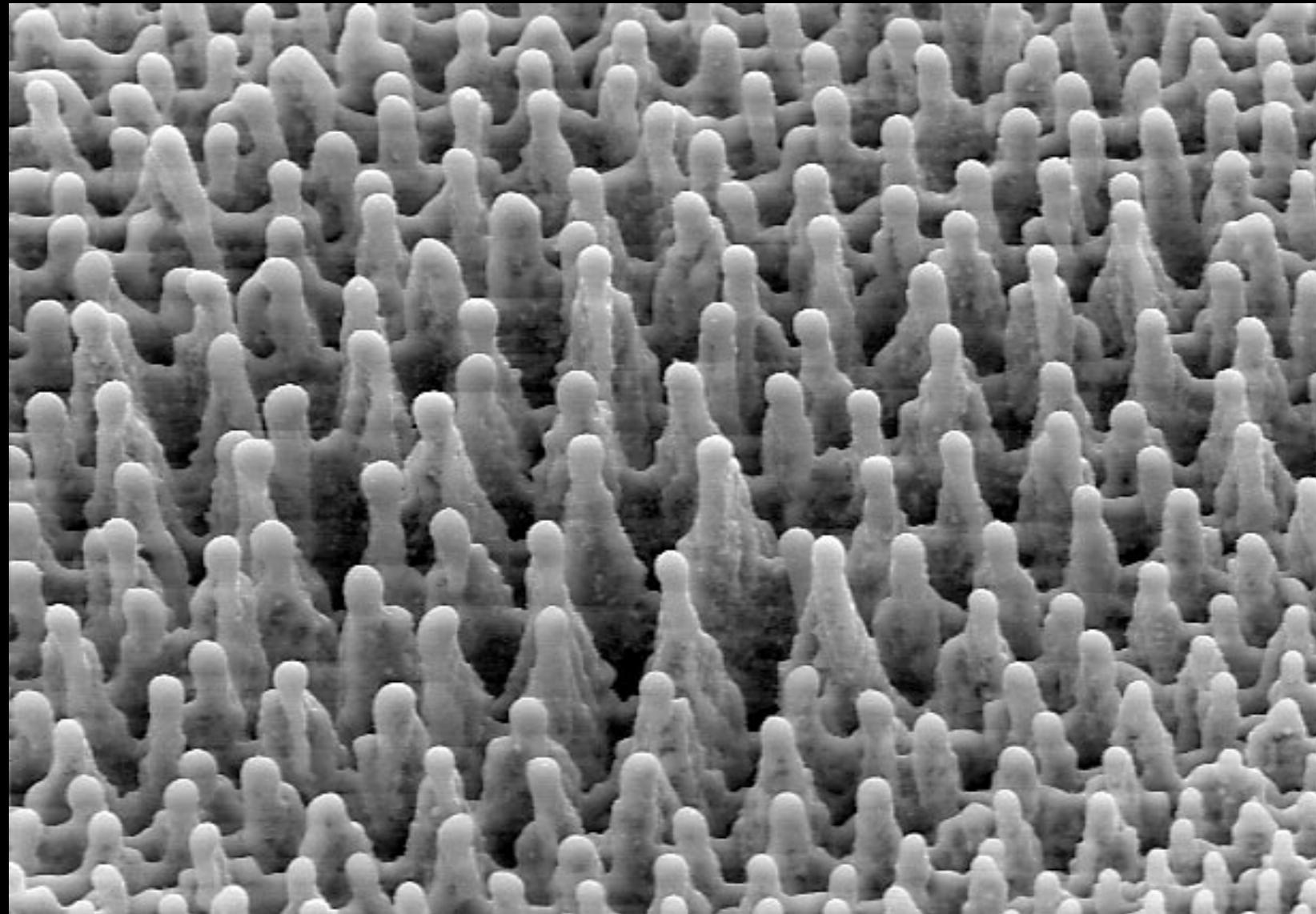
10kV

15mm

#3548

512 x 480

0100



x2000

#3548

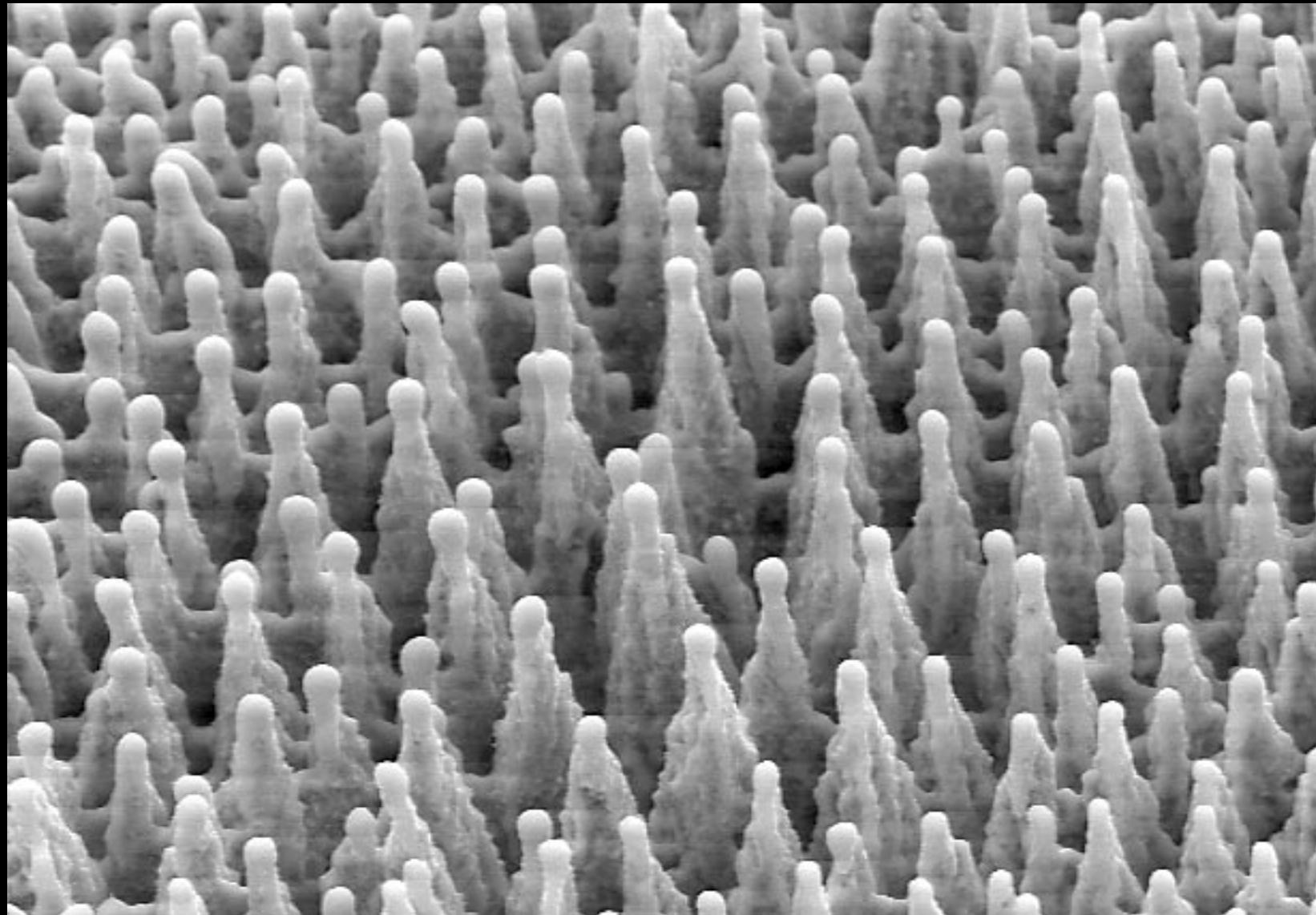
512 x 480

20 μm

10kV

15mm

0125



x2000

#3548

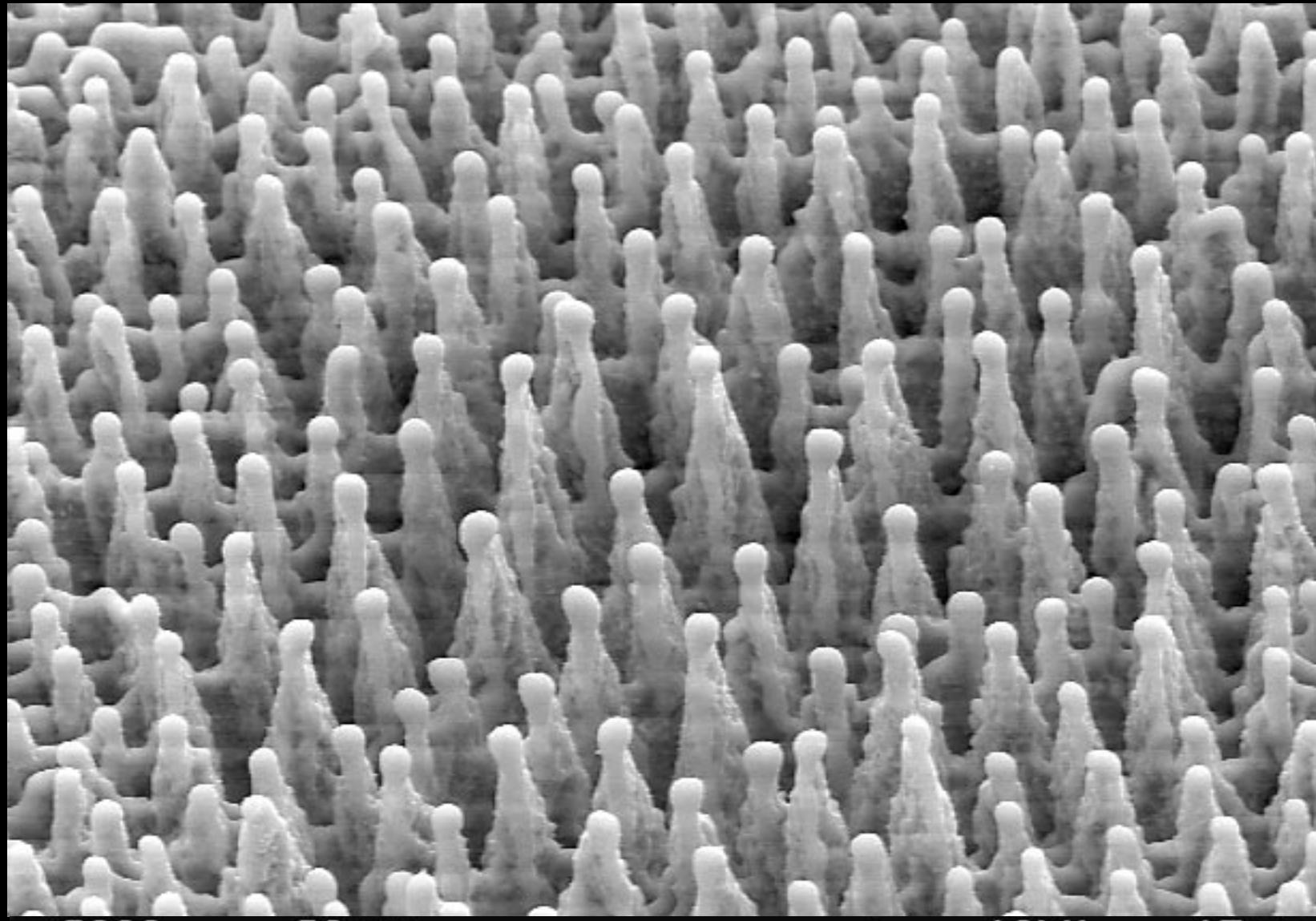
512 x 480

20 μm

10kV

15mm

0250



x2000

#3548

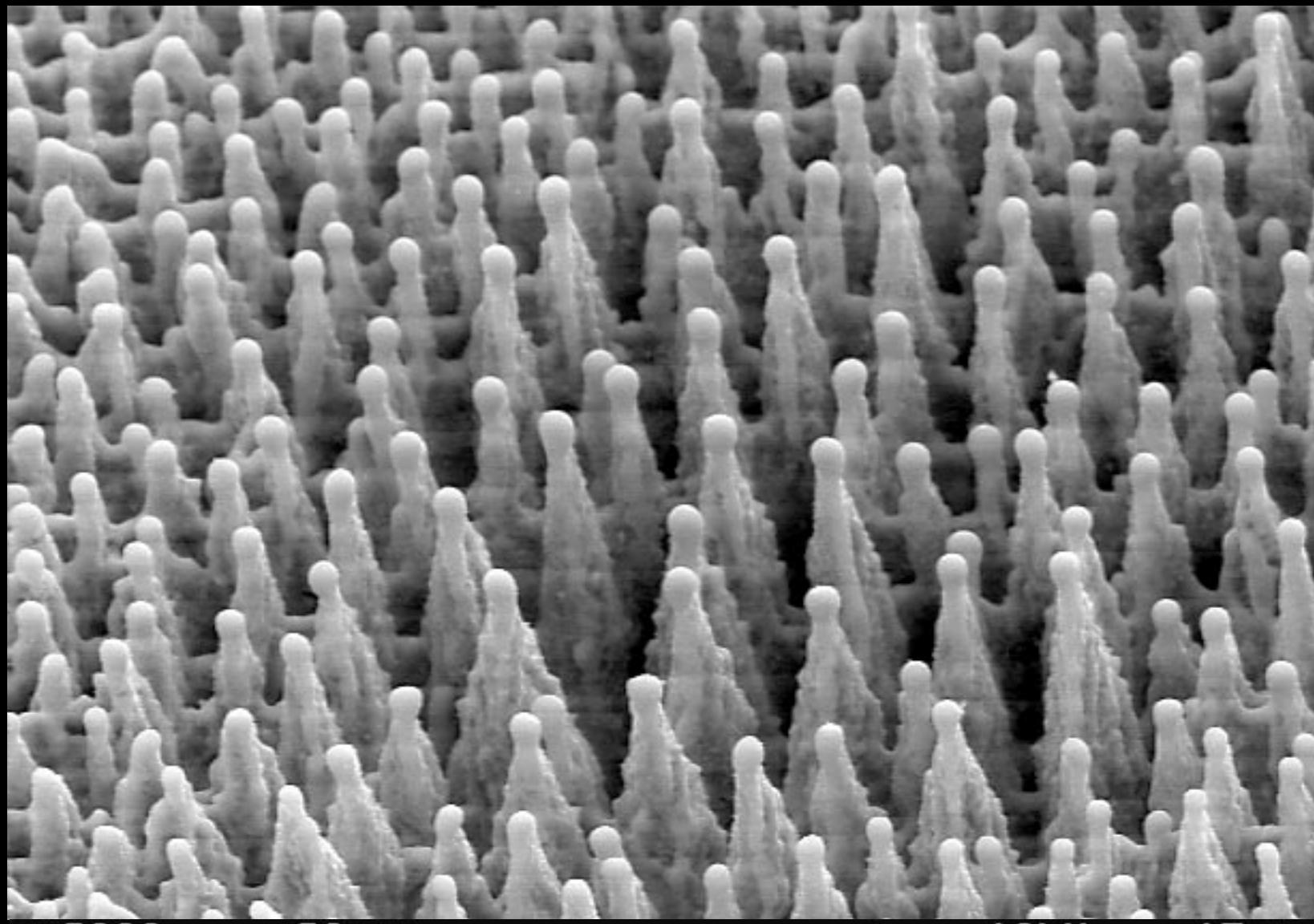
512 x 480

20 μm

10kV

15mm

0300



x2000

20 μm

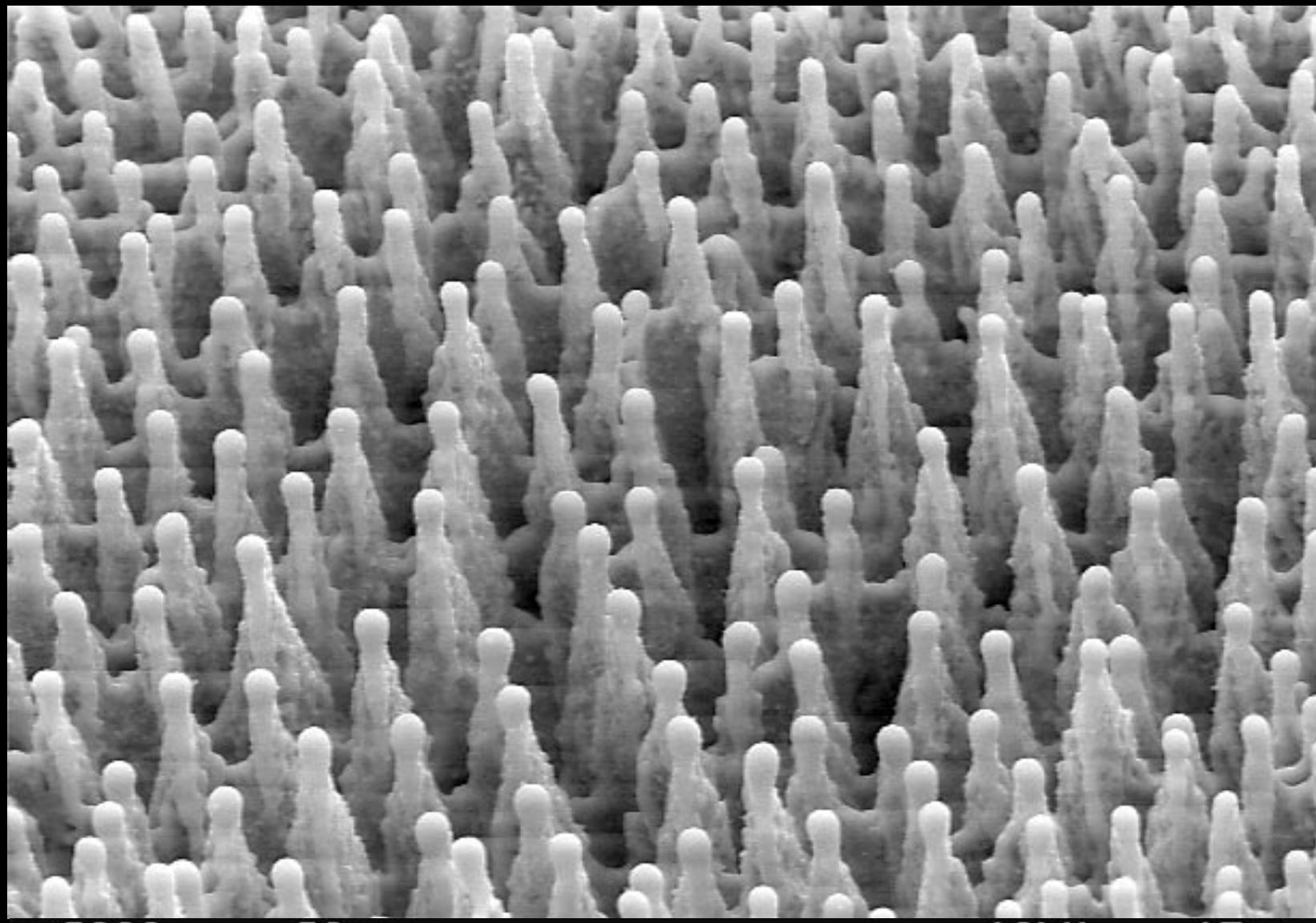
10kV

15mm

#3548

512 x 480

0400



x2000

20 μm

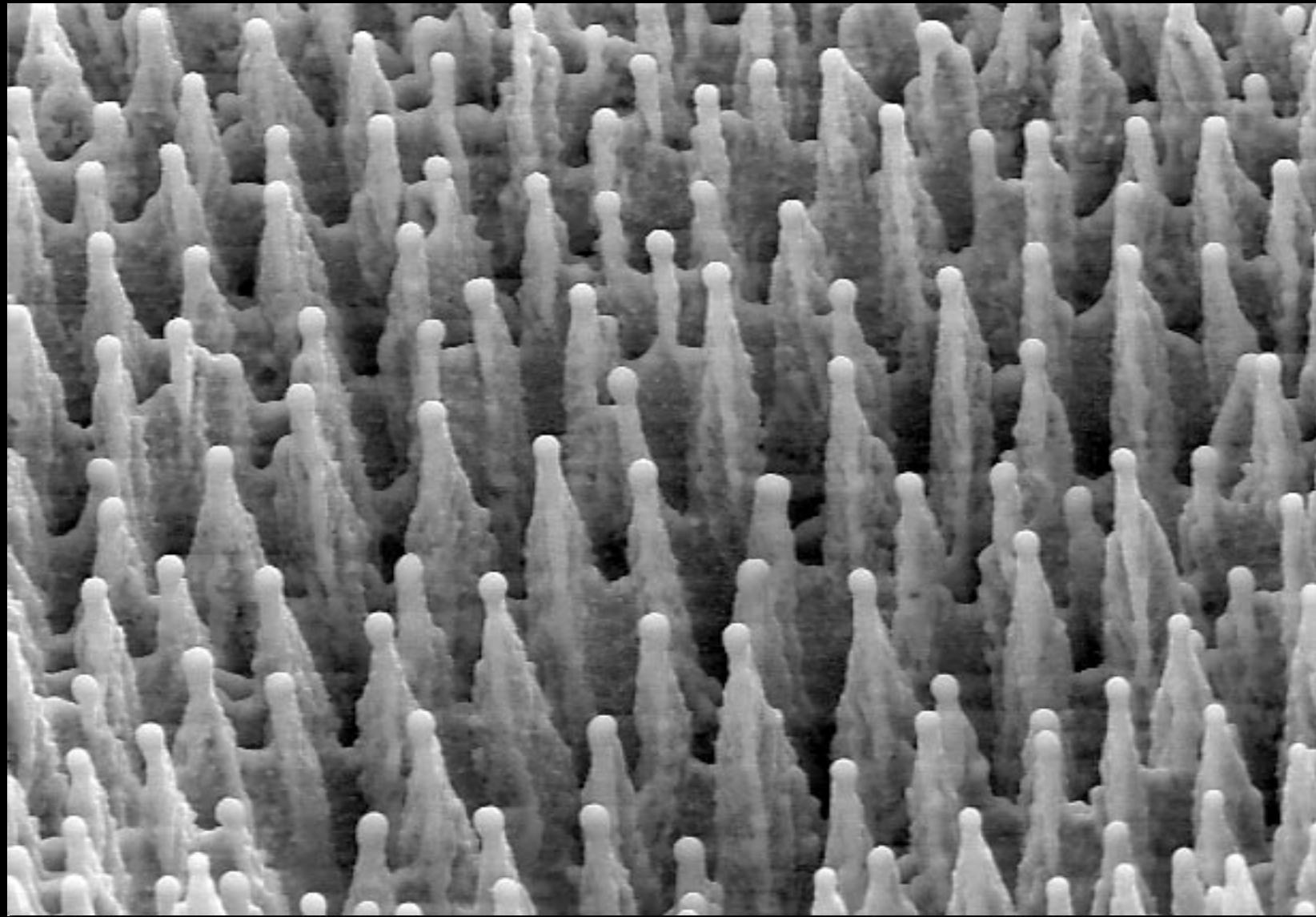
10kV

15mm

#3548

512 x 480

0500



x2000

20 μm

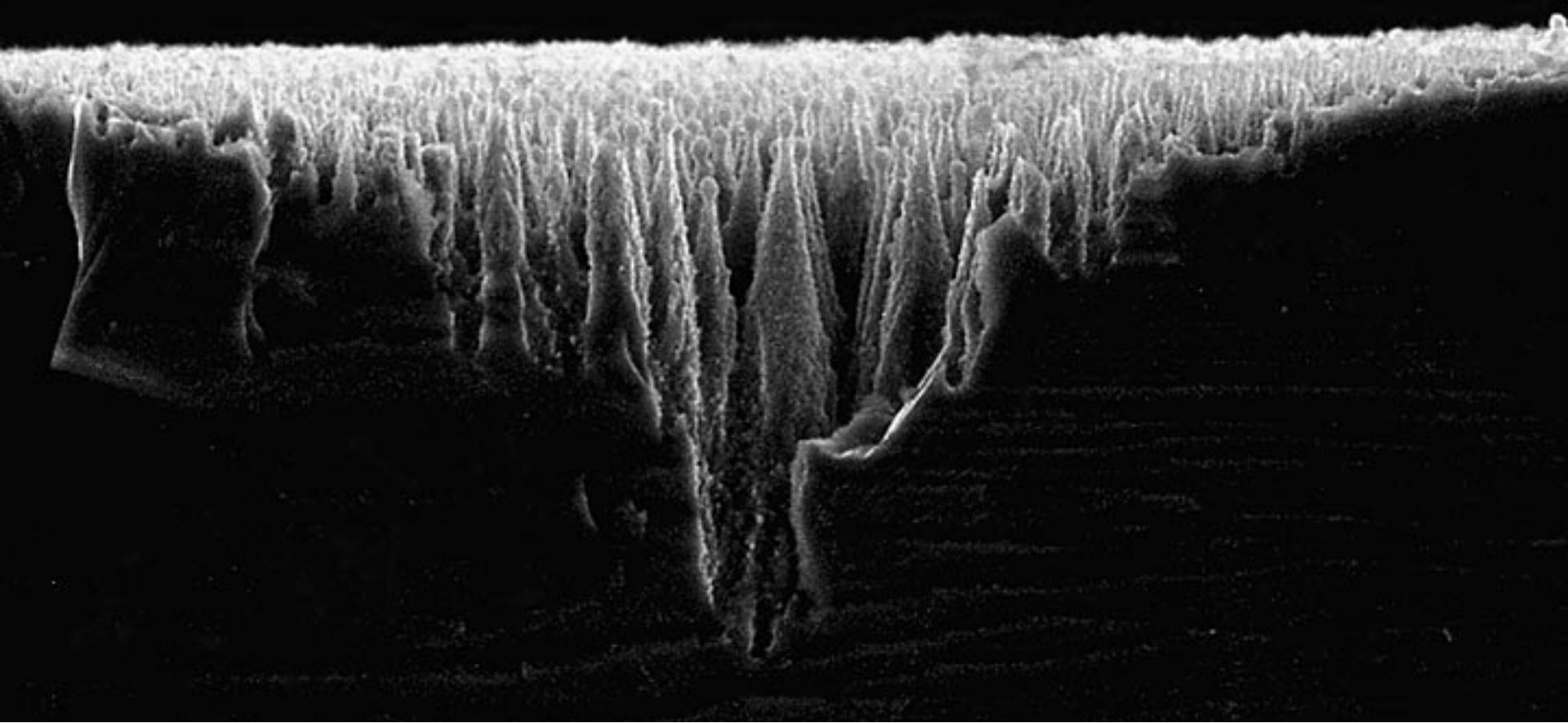
#3548

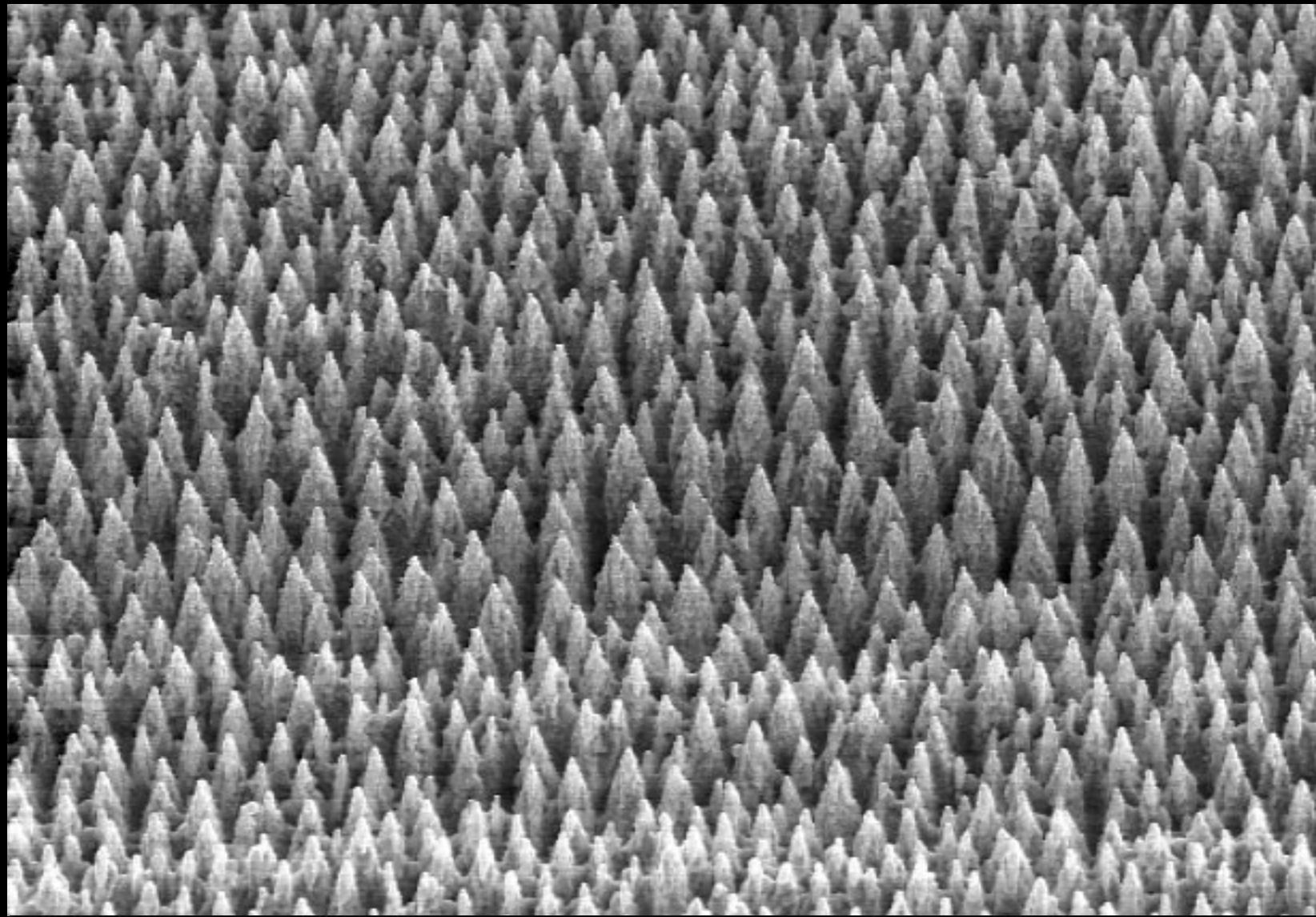
512 x 480

10kV

15mm

1000





x2000

20 μ m

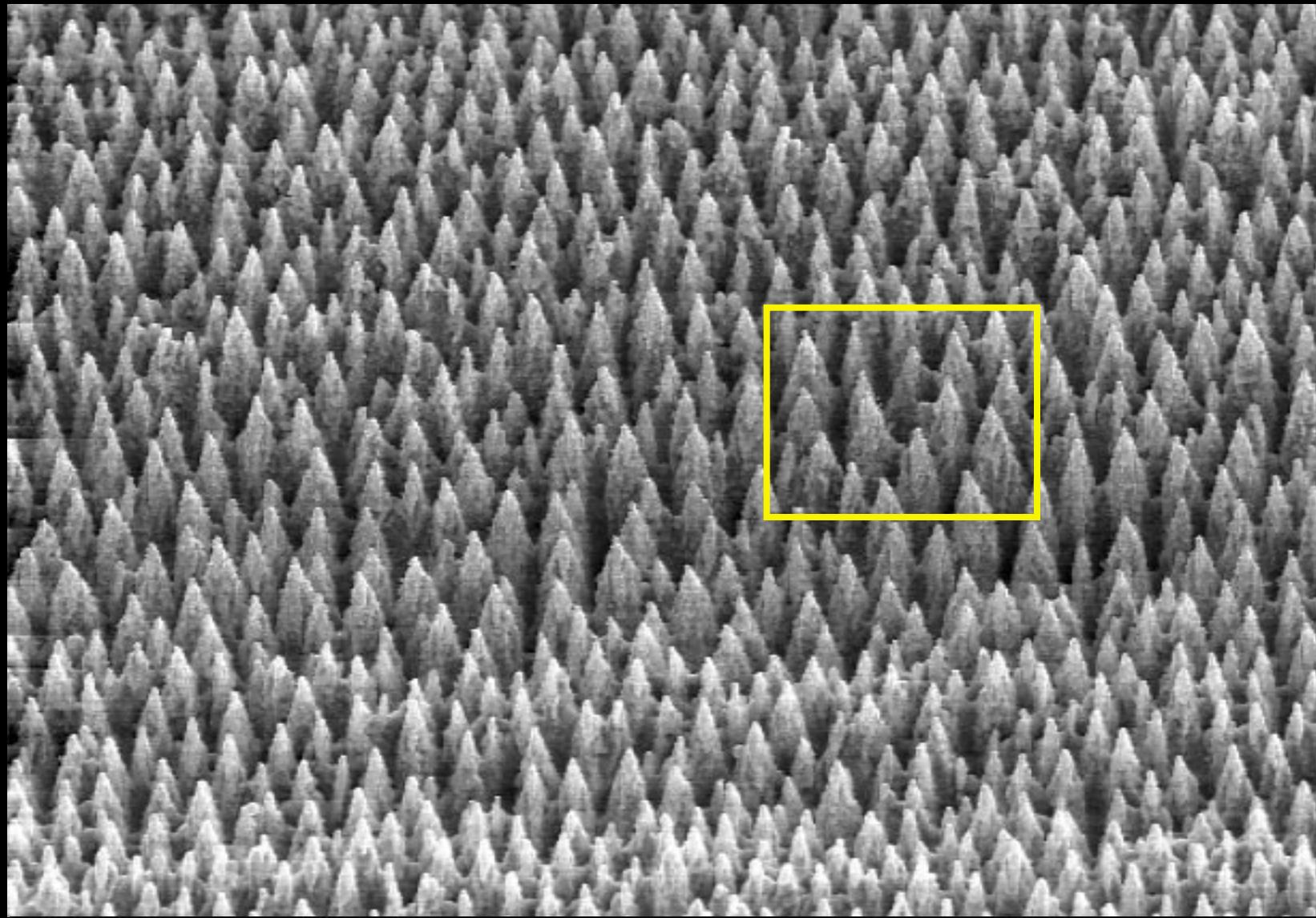
#3548

512 x 480

10kV

15mm

scanned



x2000

#3548

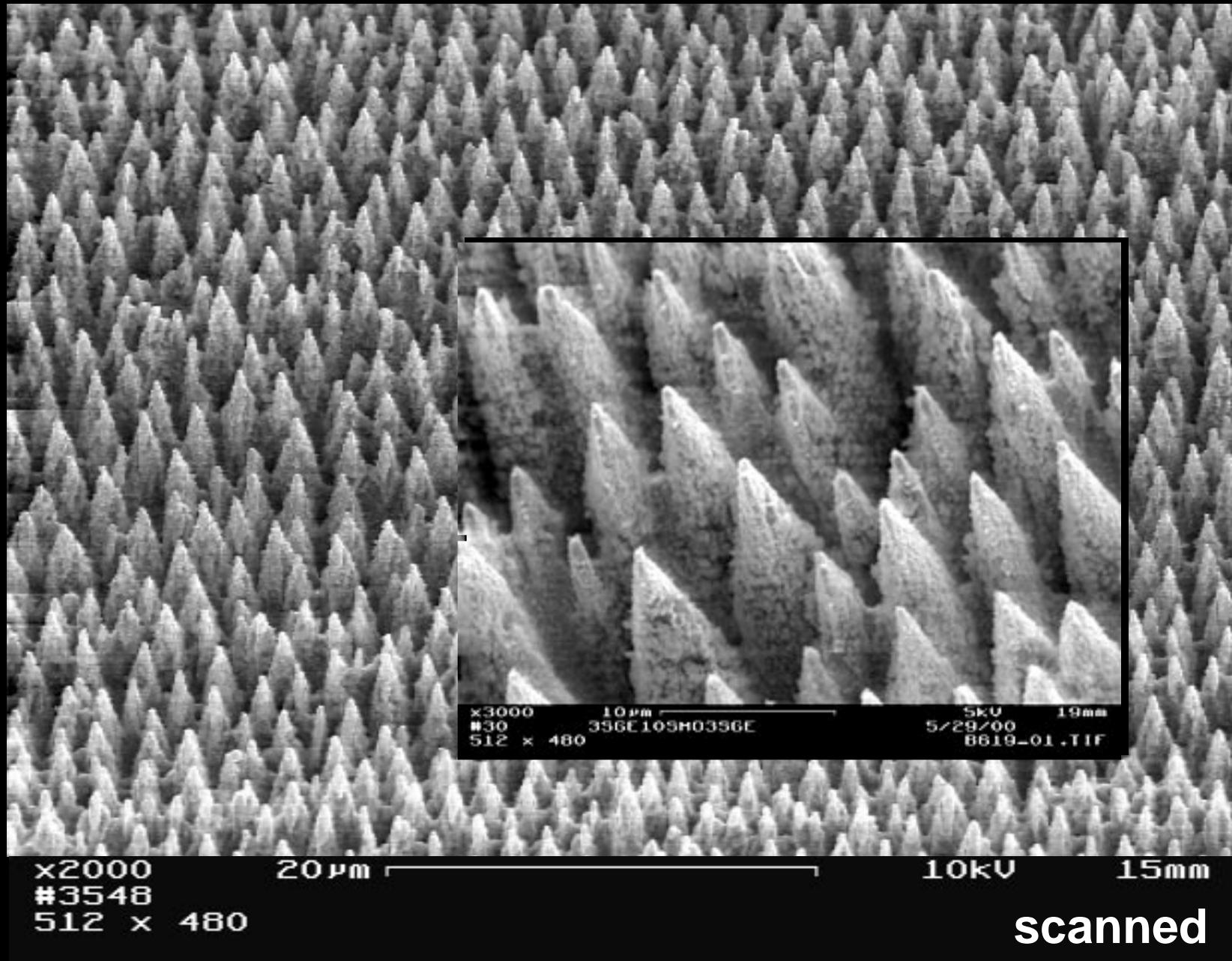
512 x 480

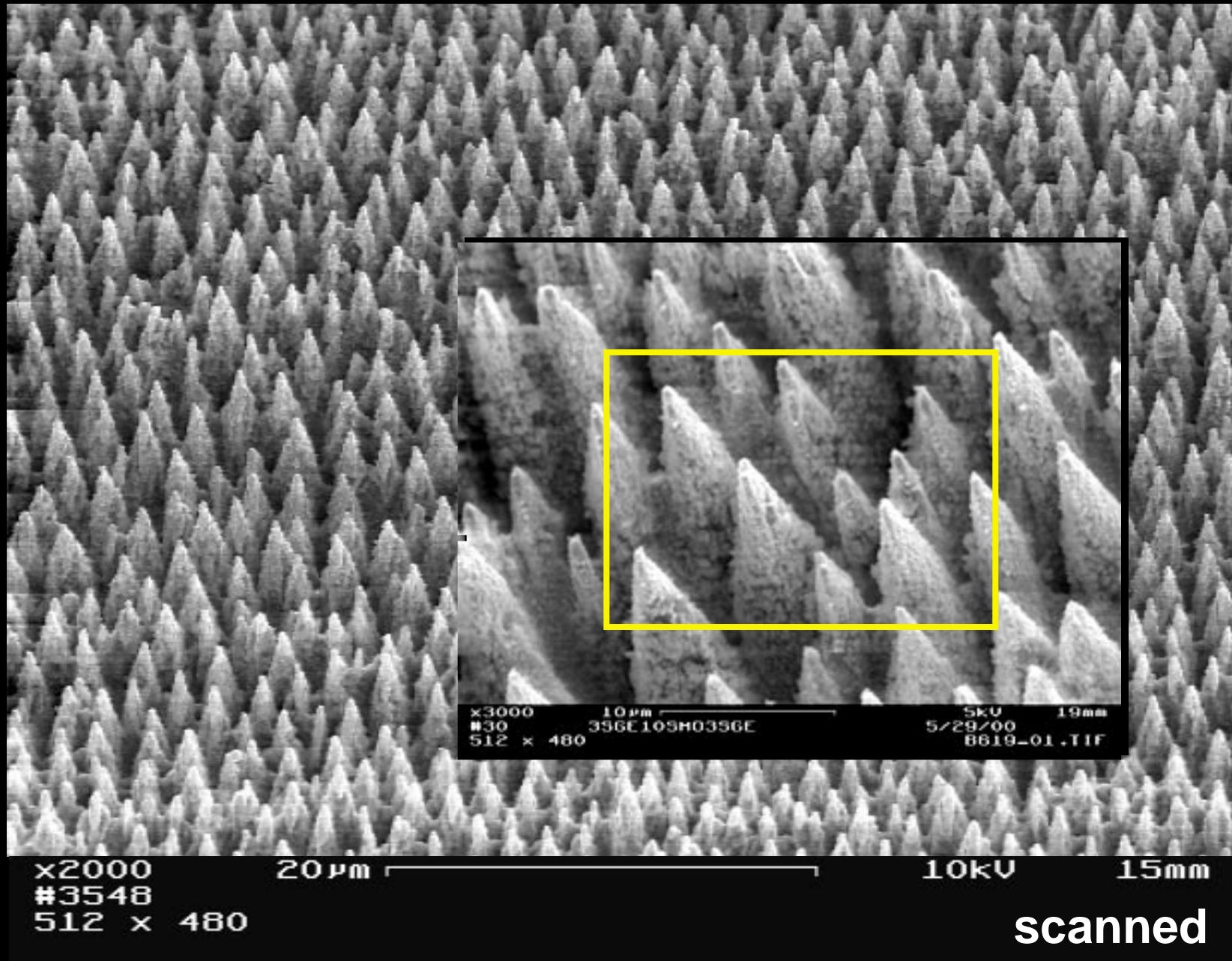
20 μm

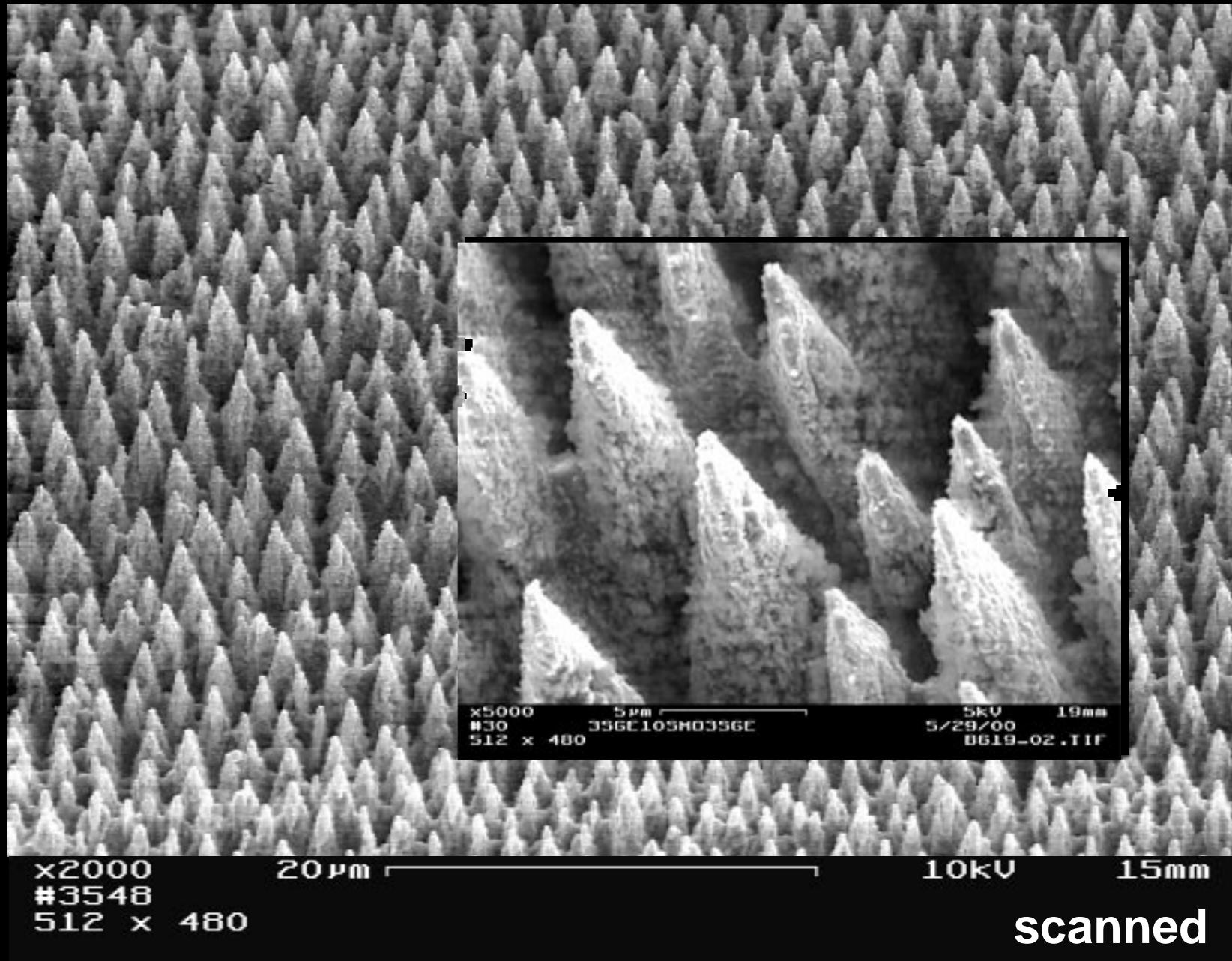
10kV

15mm

scanned

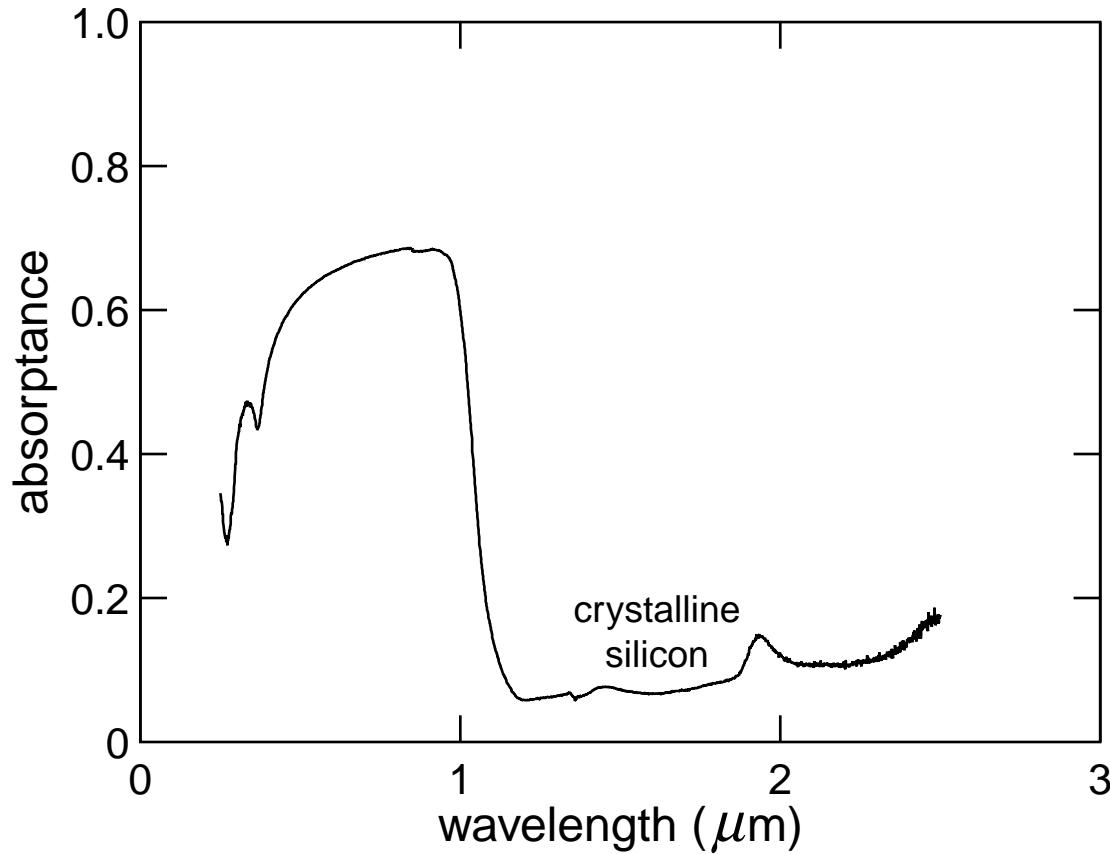






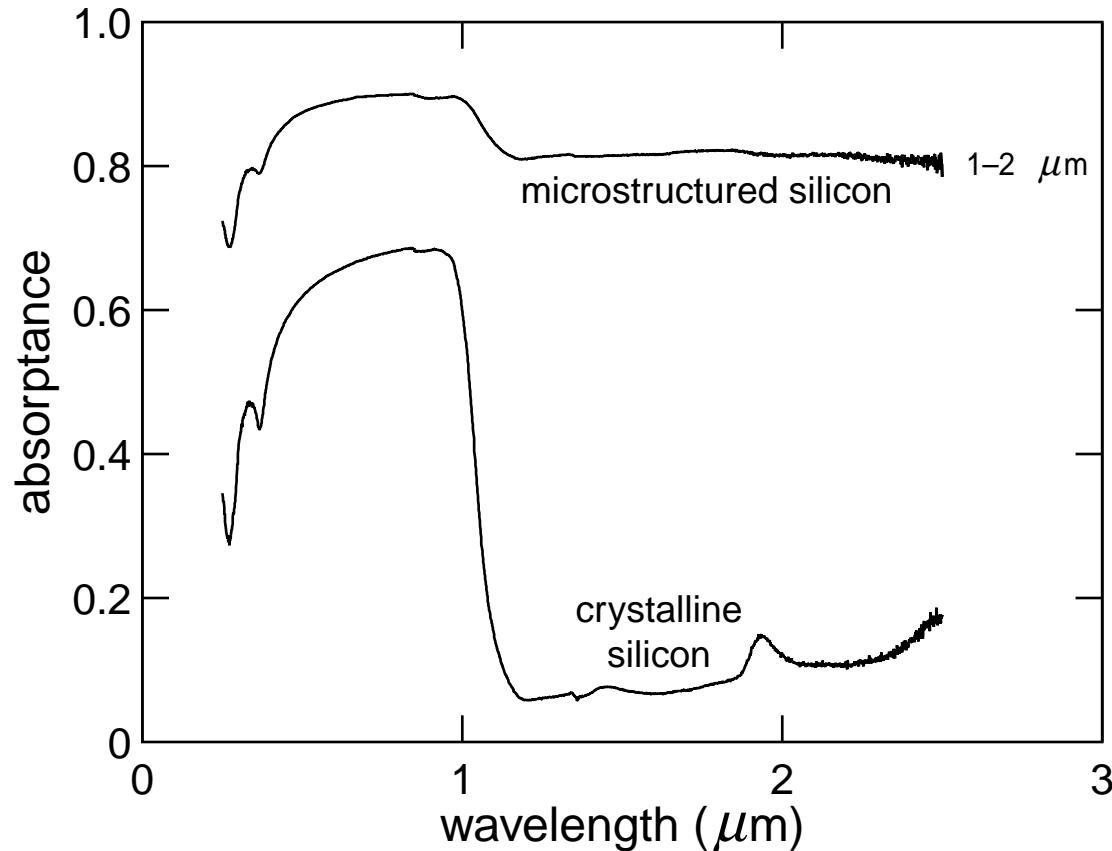
Background

absorptance



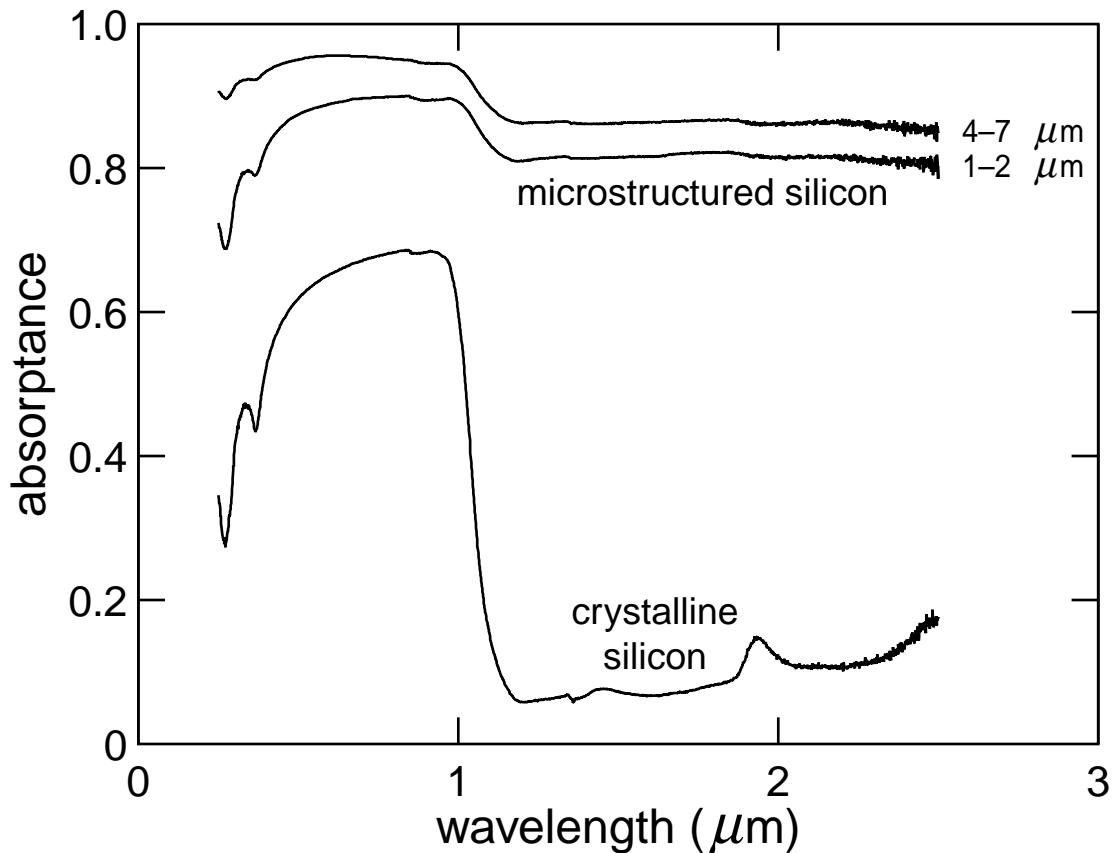
Background

absorptance



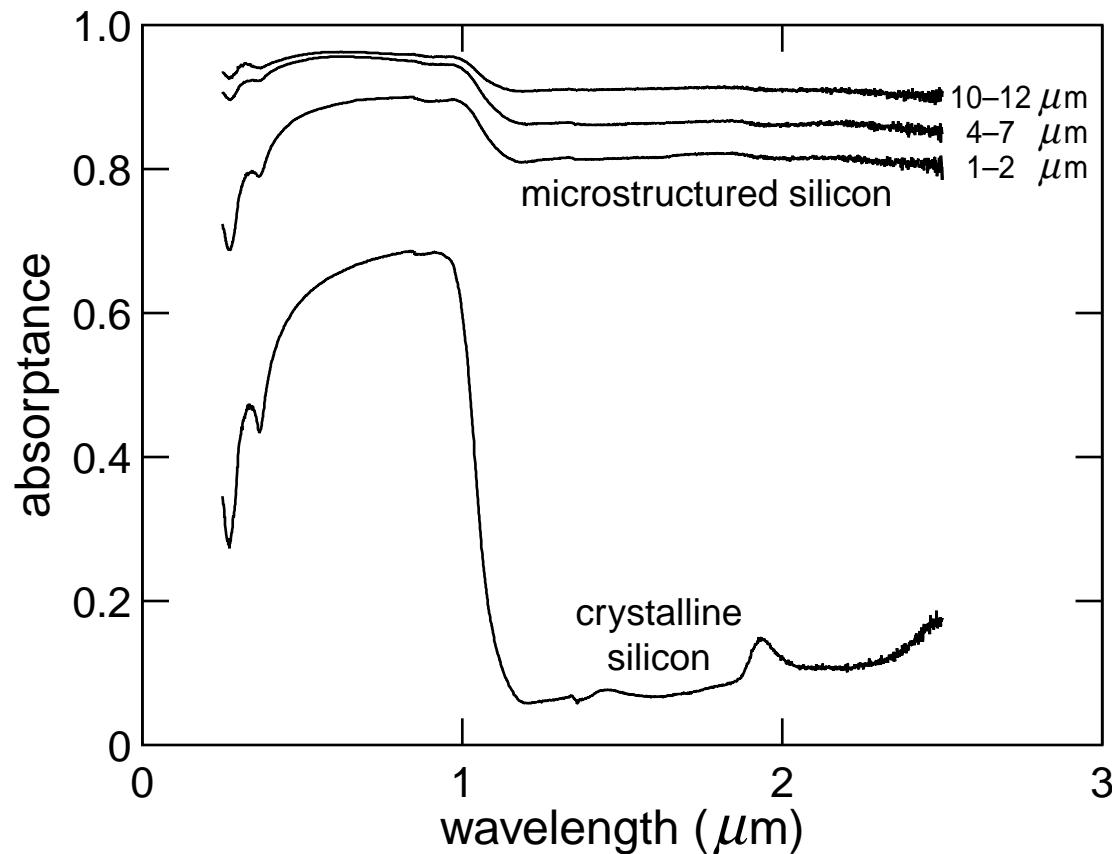
Background

absorptance



Background

absorptance



Background

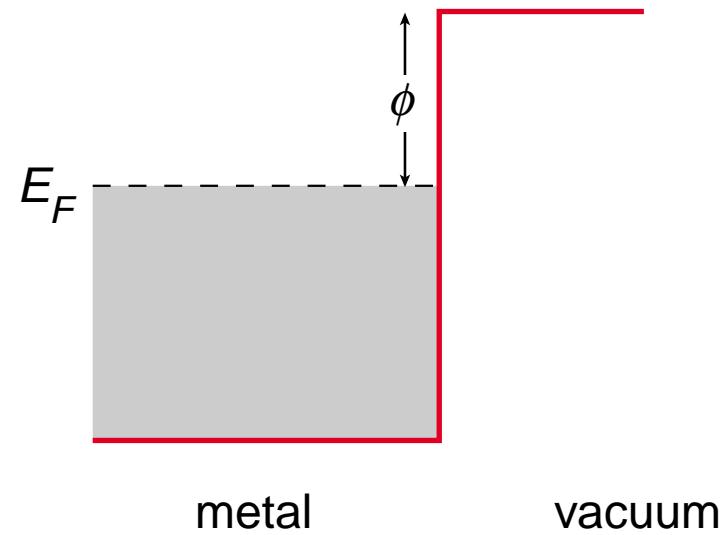
Points to keep in mind:

- ▶ **one-step, maskless process**
- ▶ **large area with uniform high density of spikes**
- ▶ **band structure change**

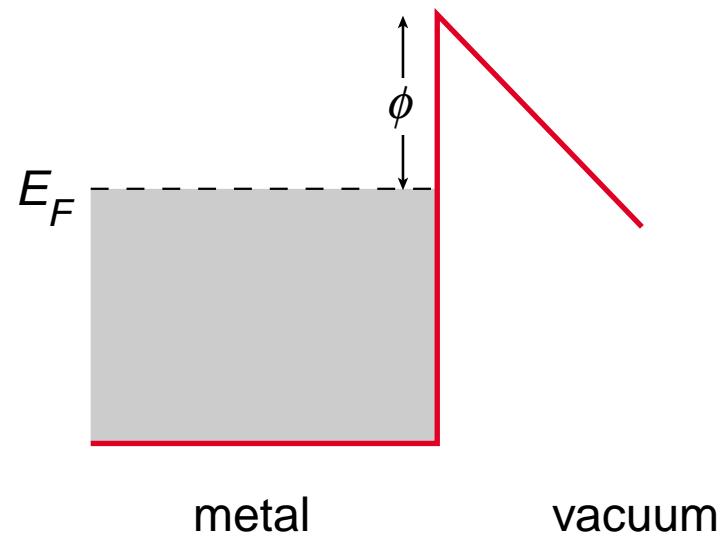
Outline

- ▶ **Background**
- ▶ **Results**
- ▶ **Discussion**

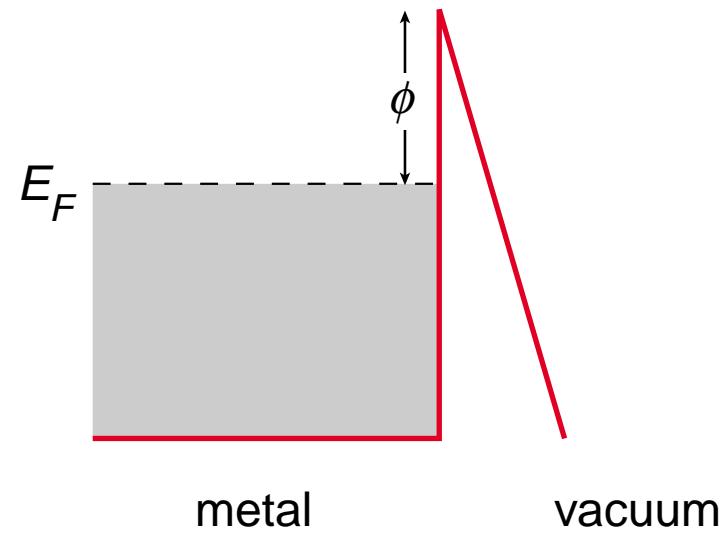
Field emission



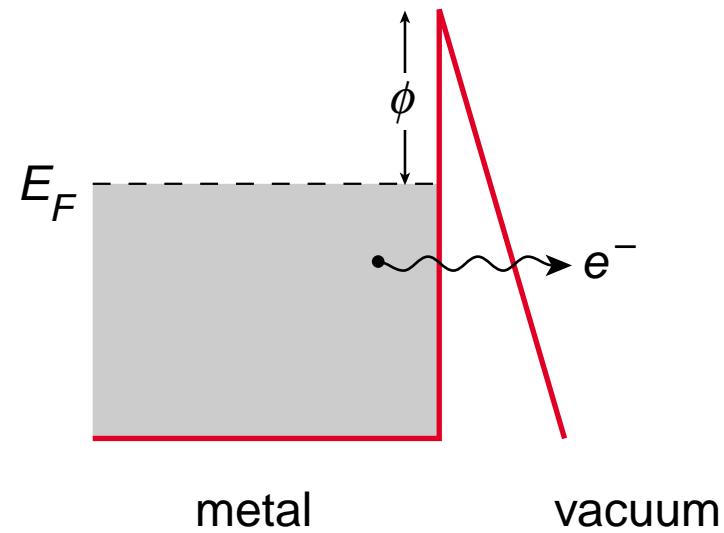
Field emission



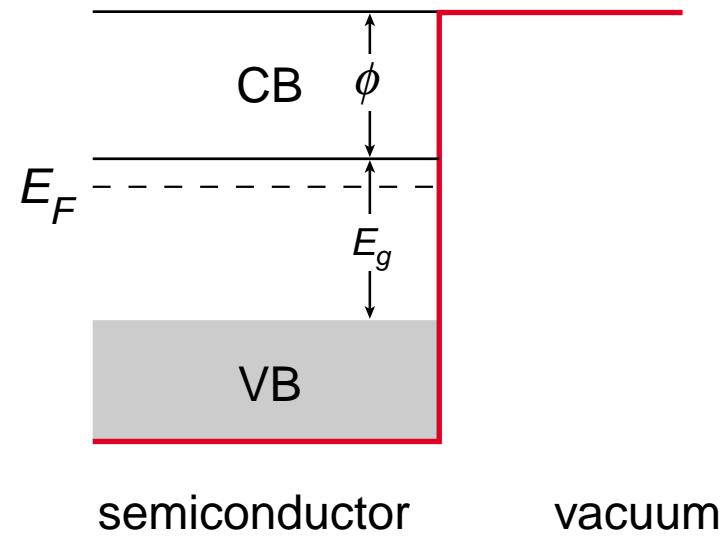
Field emission



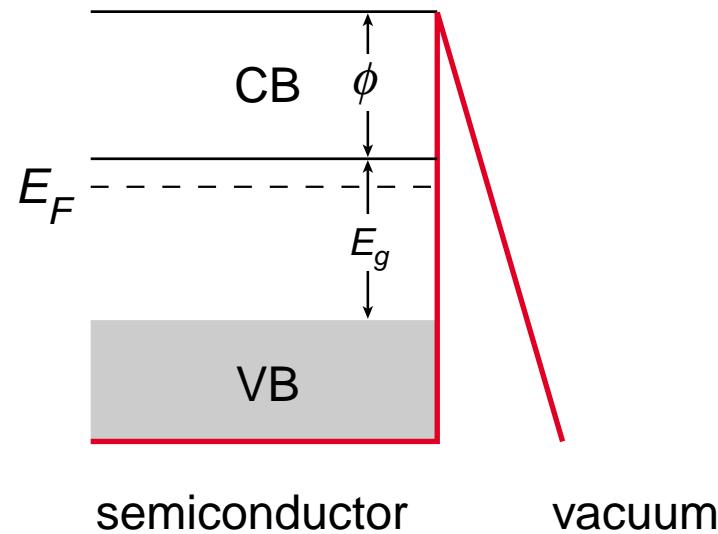
Field emission



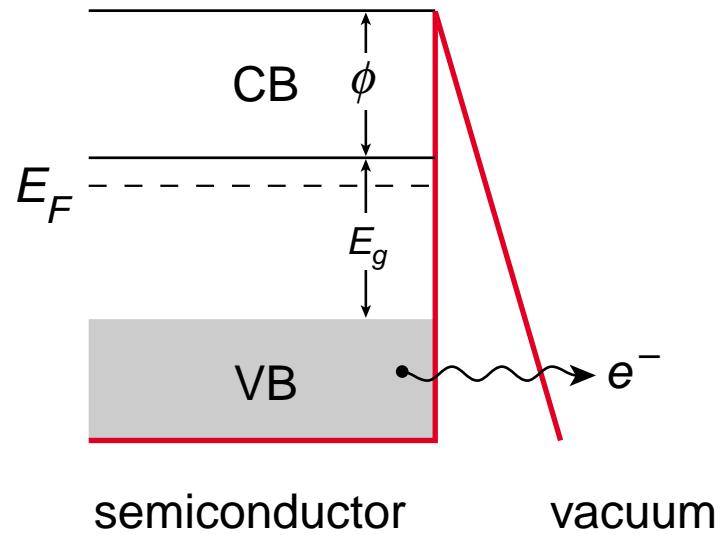
Field emission



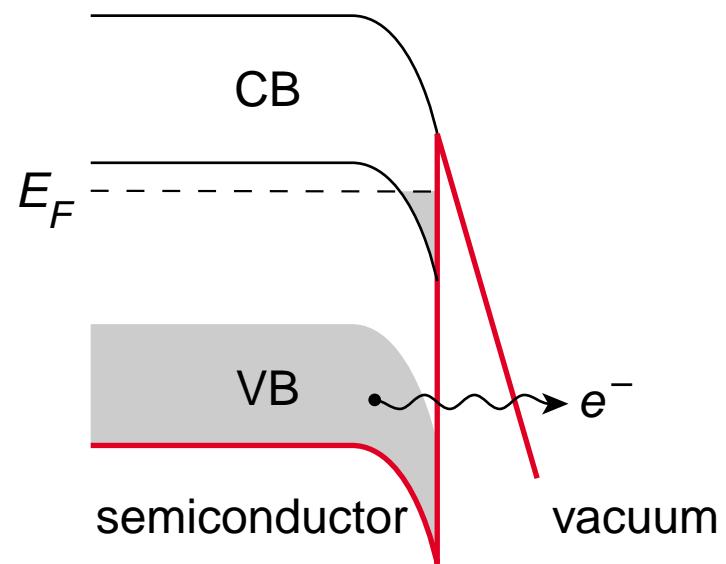
Field emission



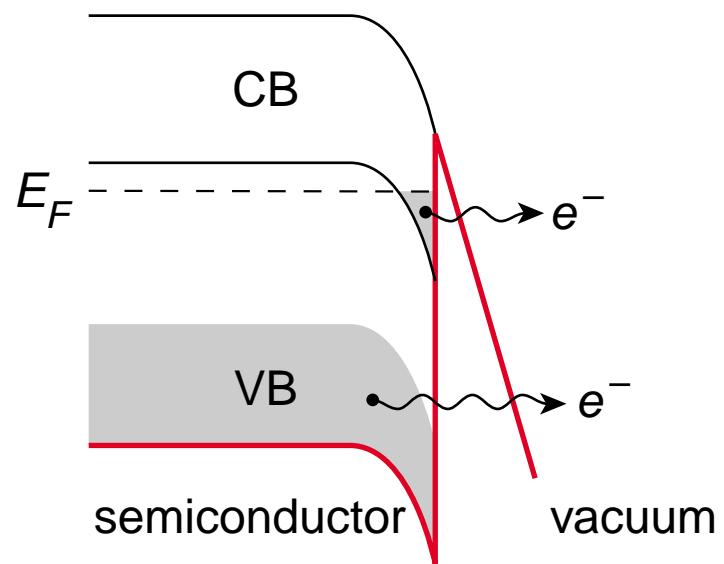
Field emission



Field emission

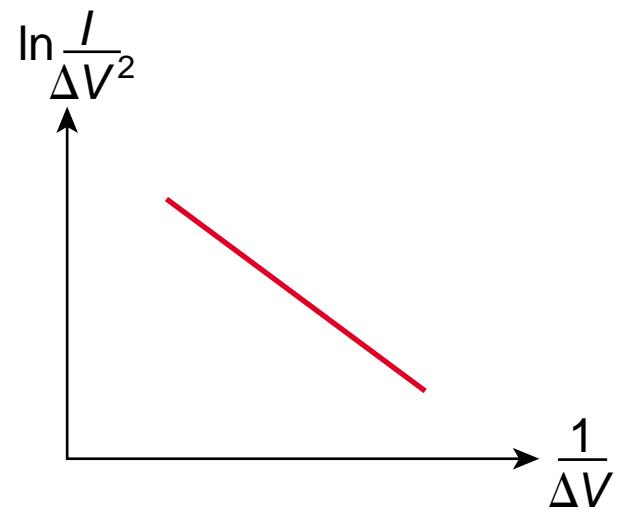


Field emission



Field emission

$$\ln \frac{I}{\Delta V^2} = \ln a - b \frac{1}{\Delta V}$$



R.H. Fowler and L. Nordheim, *Proc. R. Soc. Lond. A* (1928)

Setup



Setup



gold coating

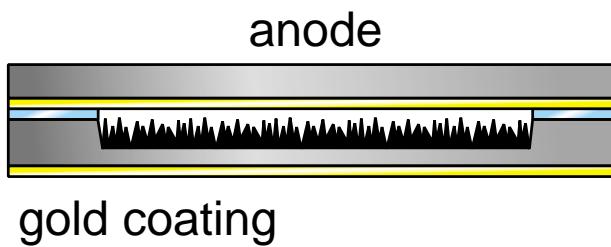
Setup

20 μm mica spacers

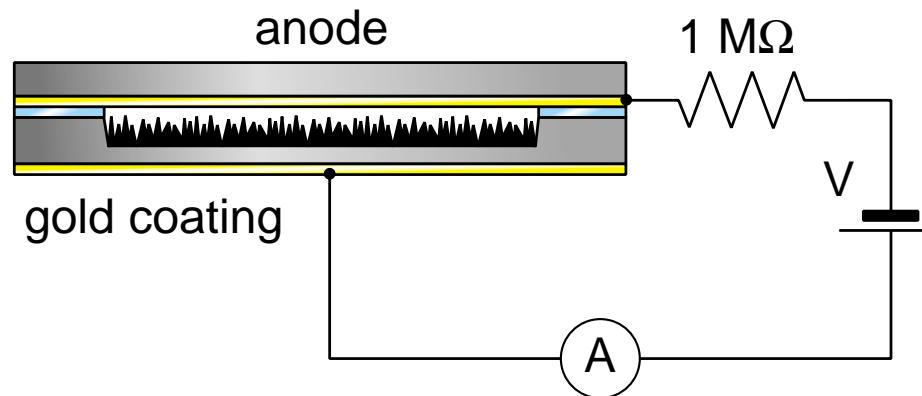


gold coating

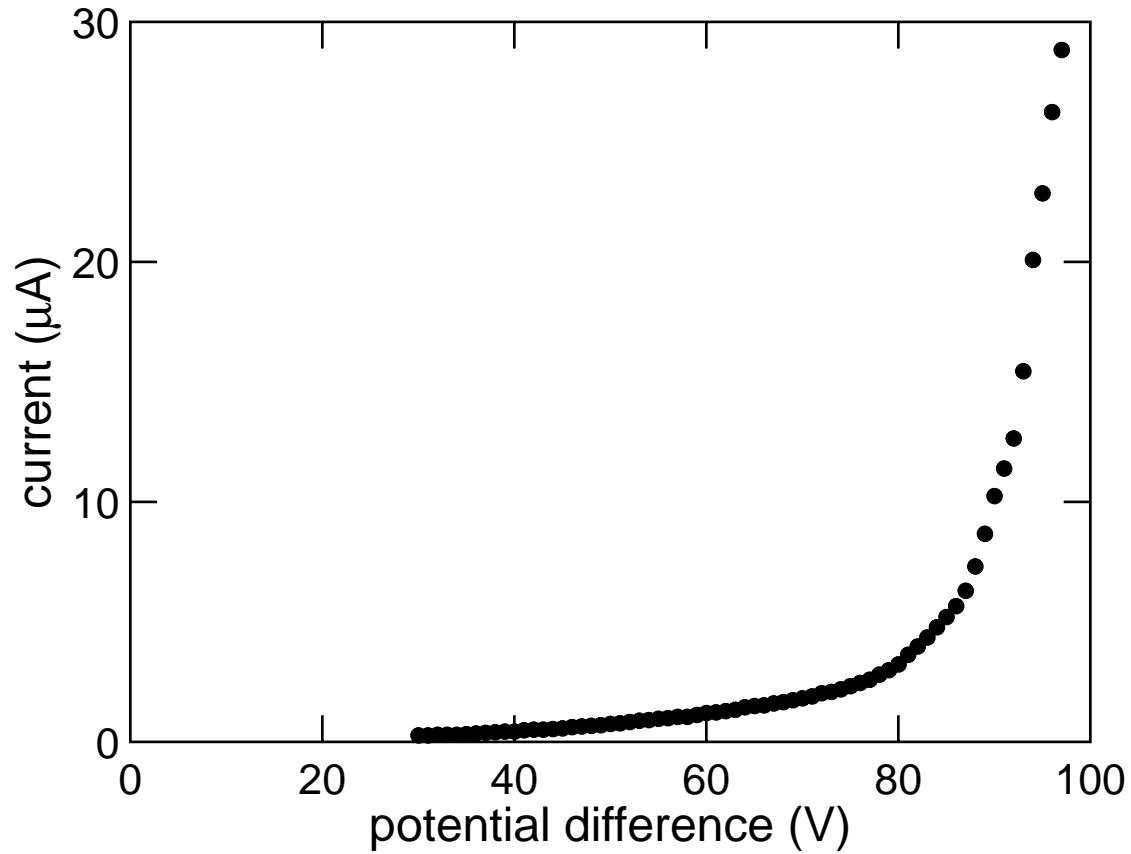
Setup



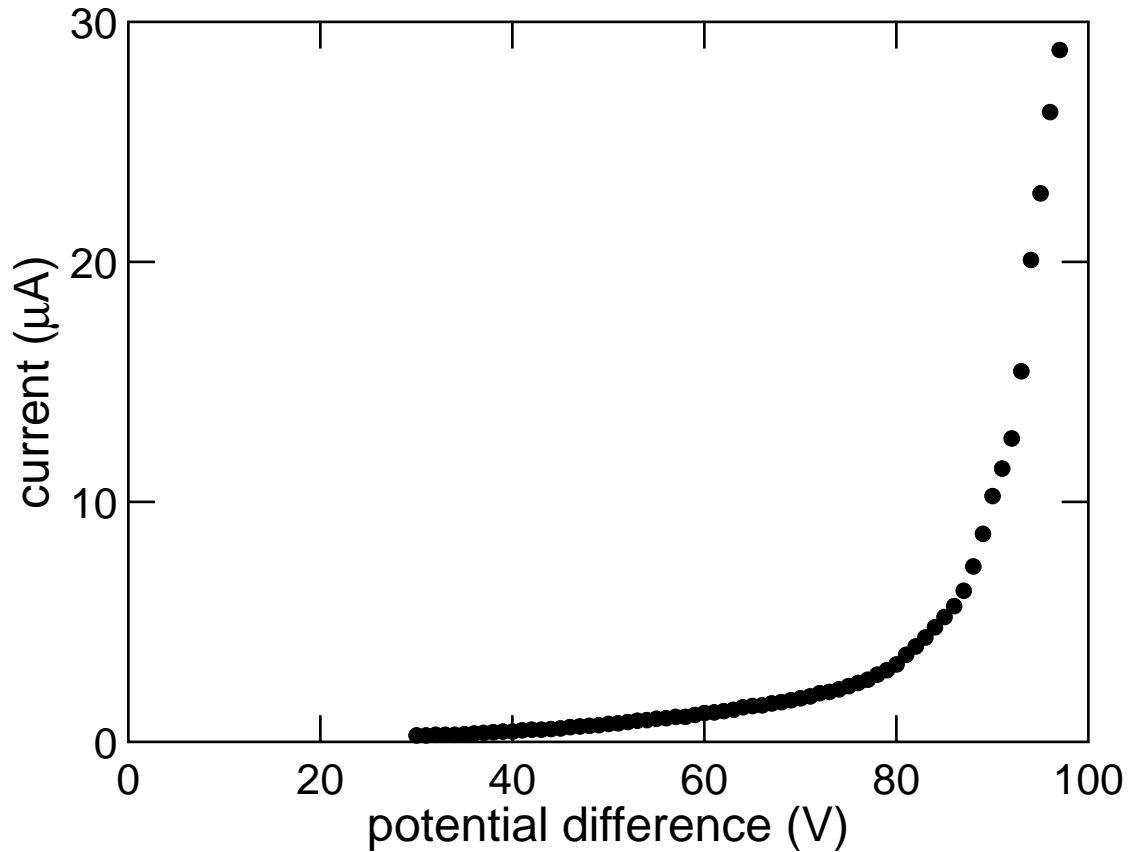
Setup



Results

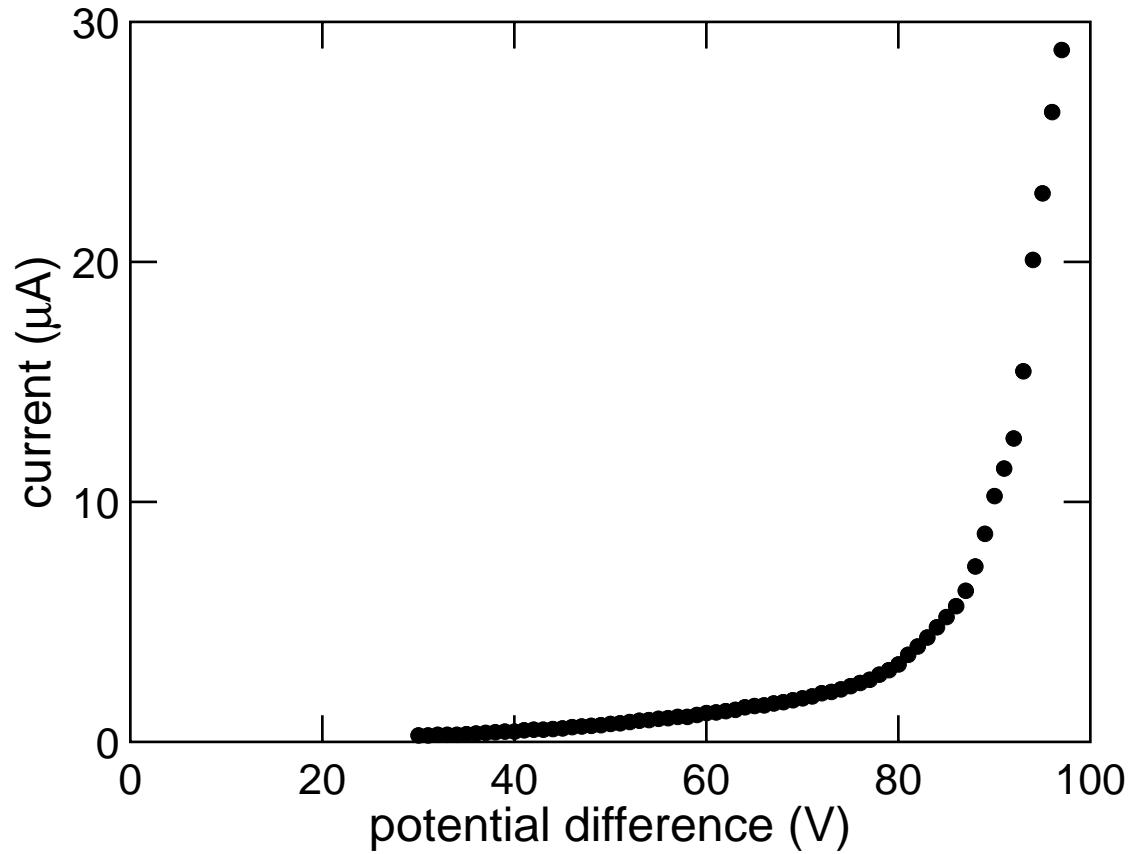


Results



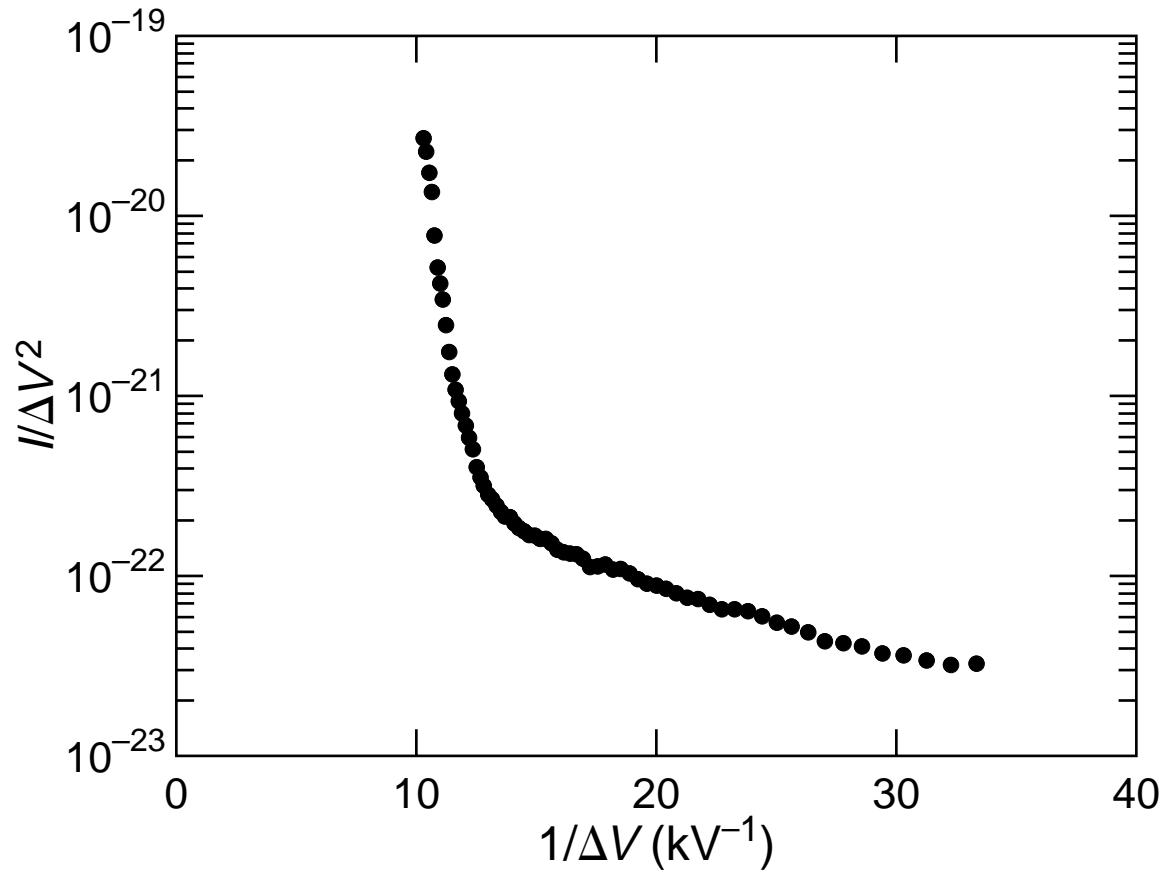
turn-on field (1 $\mu\text{A}/\text{cm}^2$): 1.3 V/ μm

Results

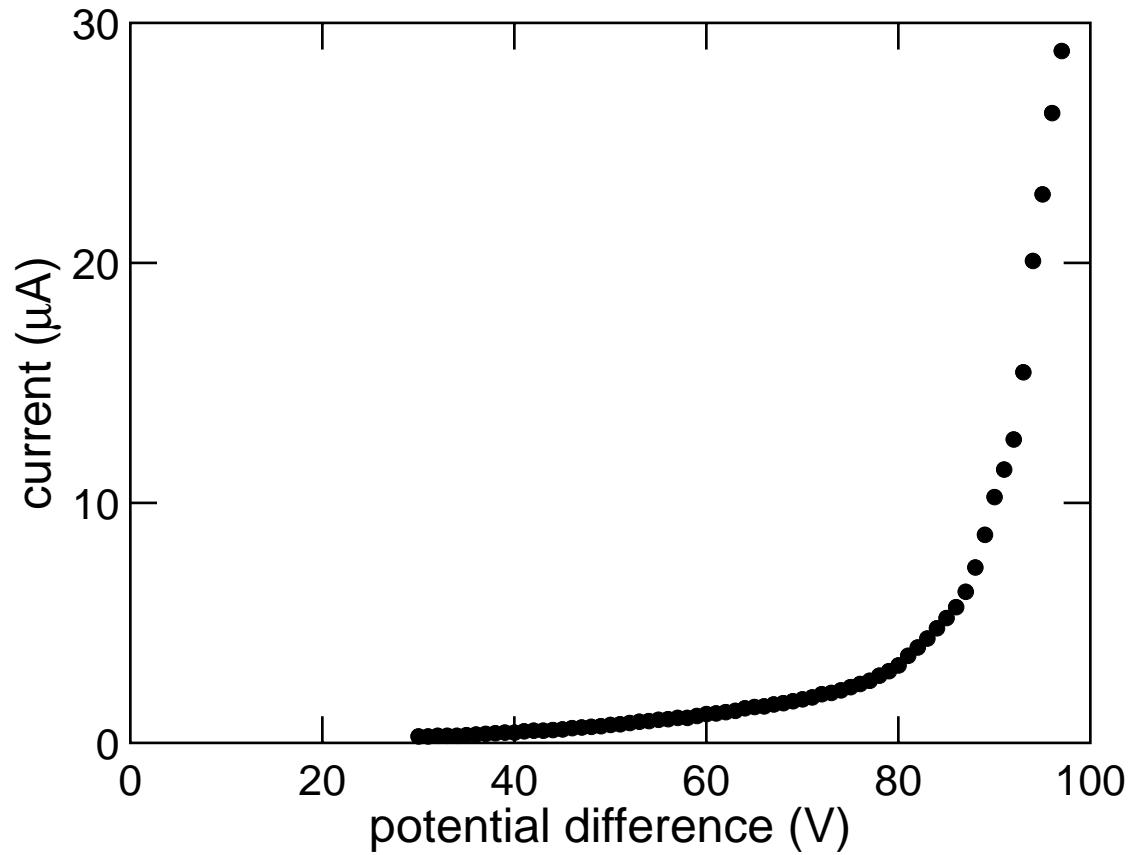


threshold field ($10 \mu\text{A}/\text{cm}^2$): $2.15 \text{ V}/\mu\text{m}$

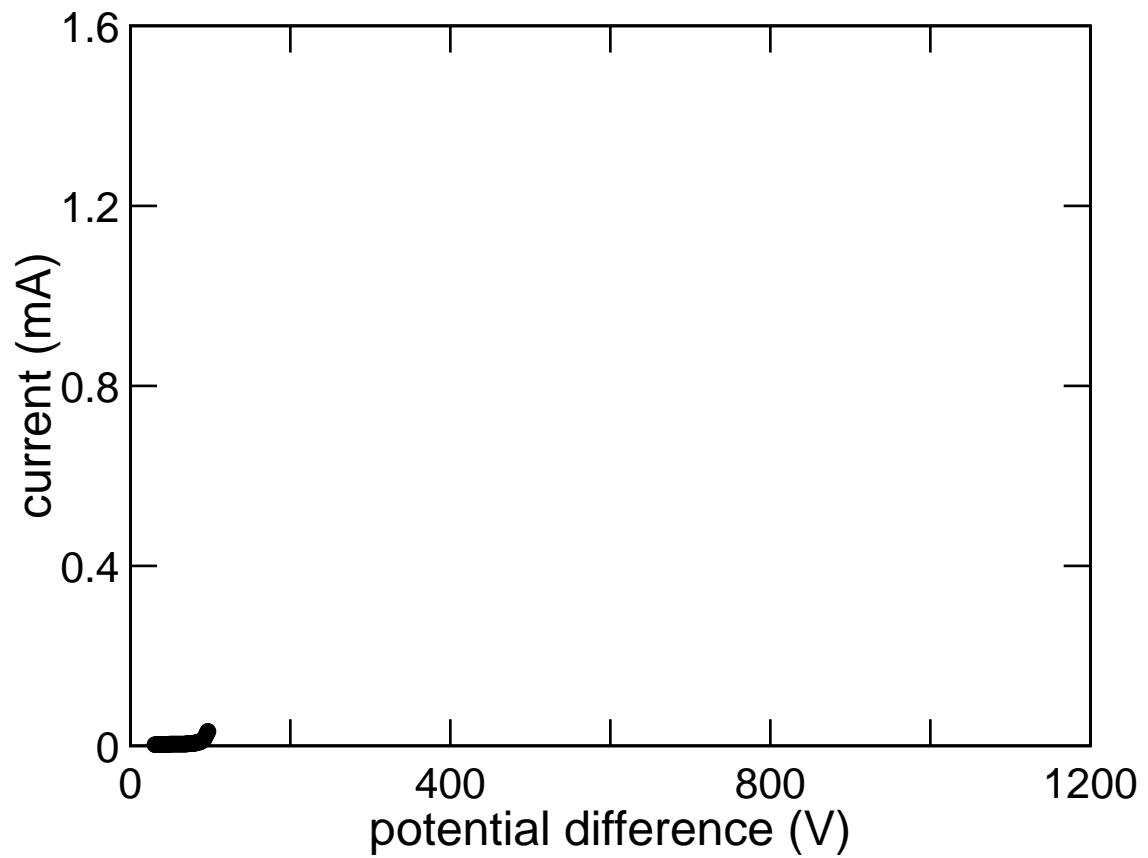
Results



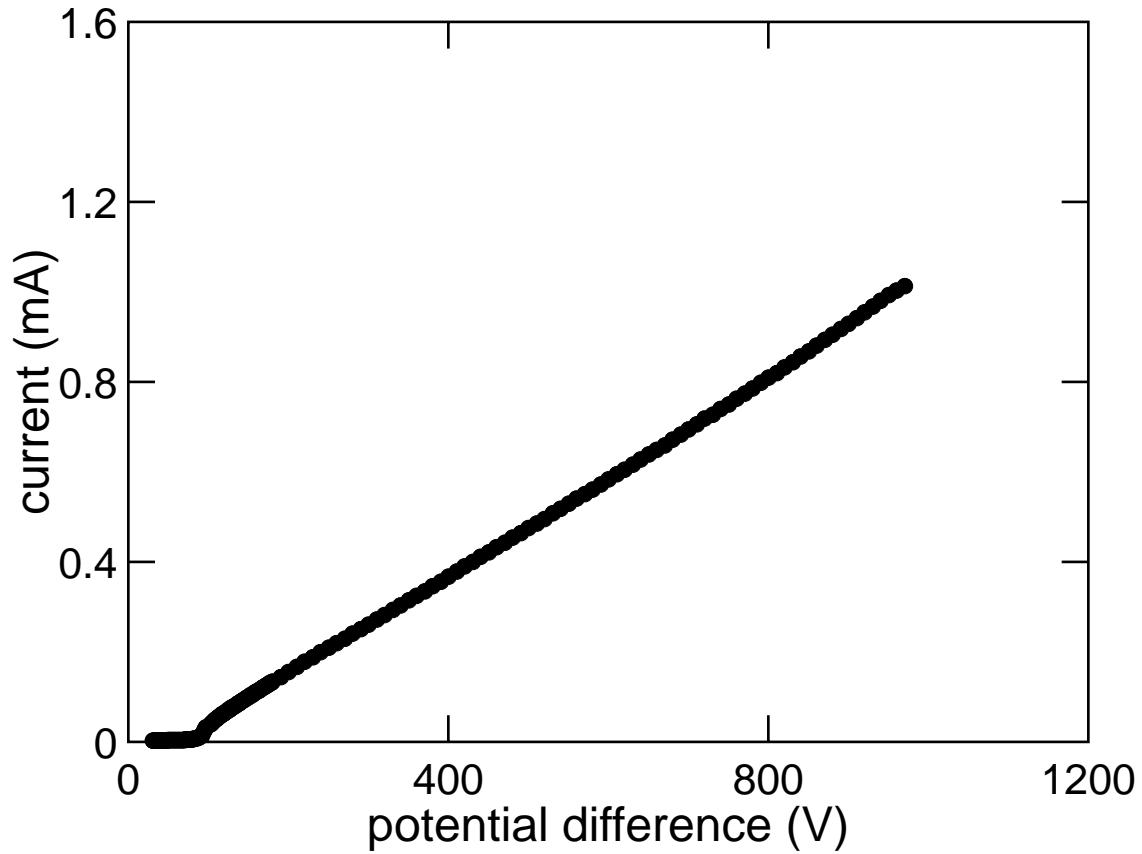
Results



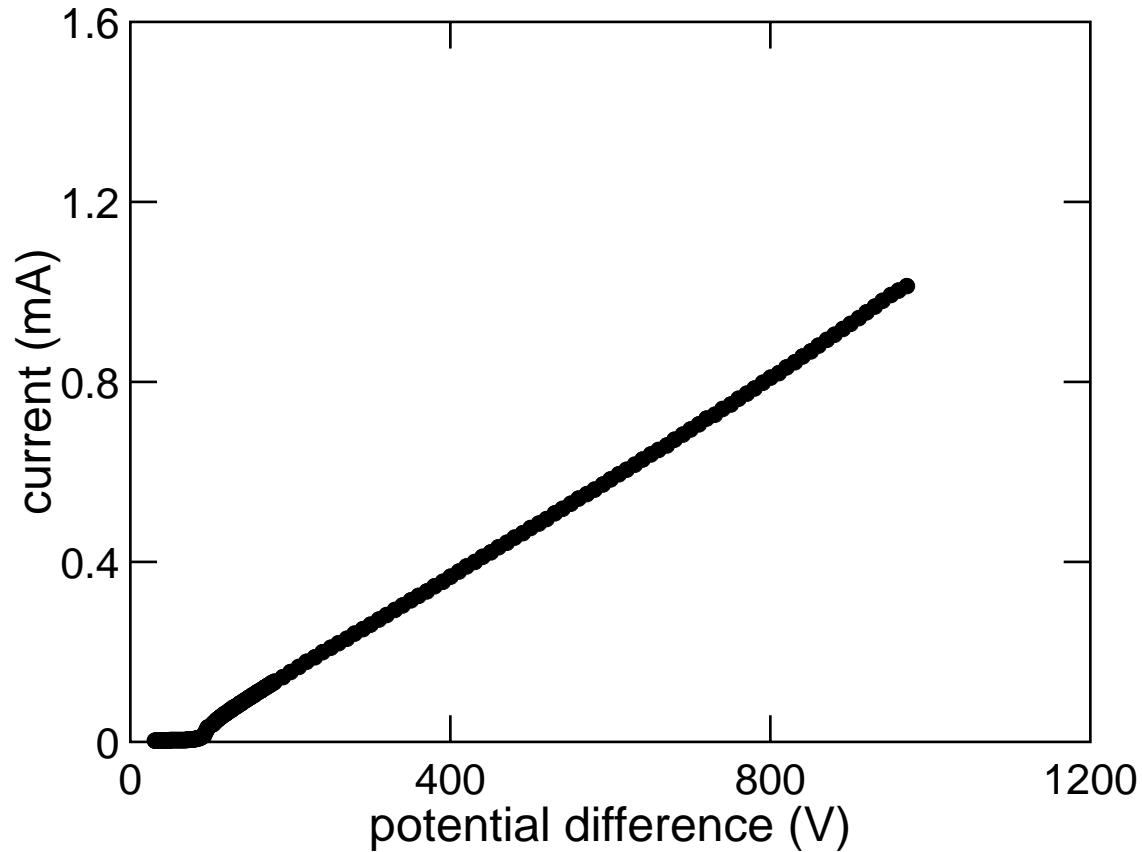
Results



Results

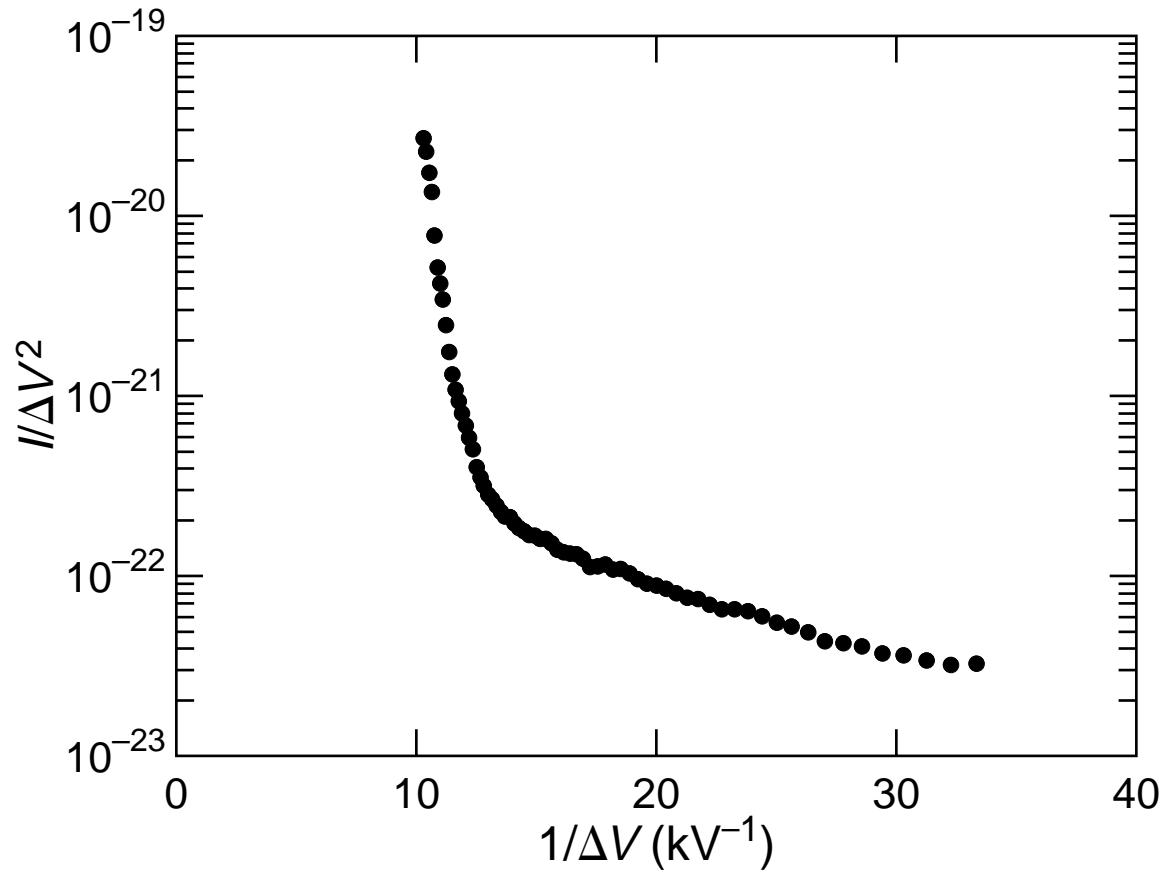


Results

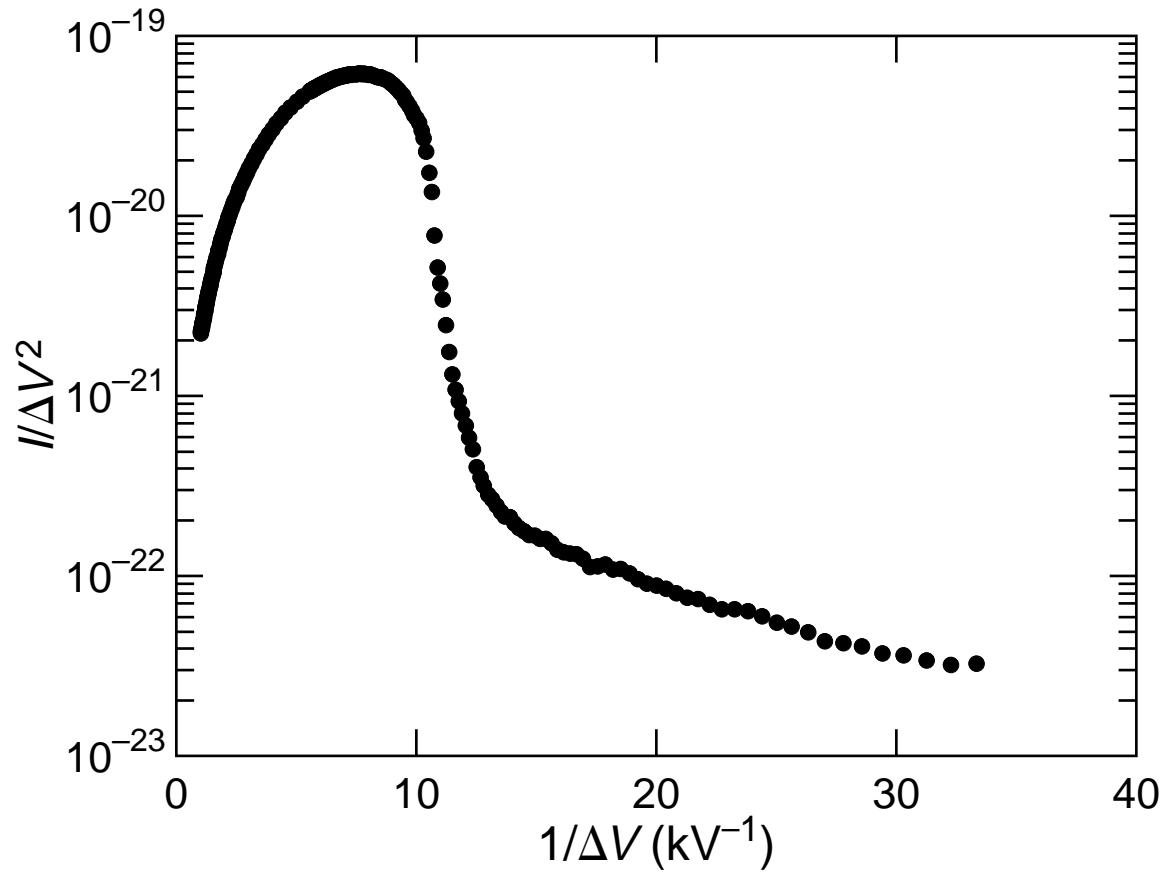


maximum current: 2 mA (4 mm² sample)

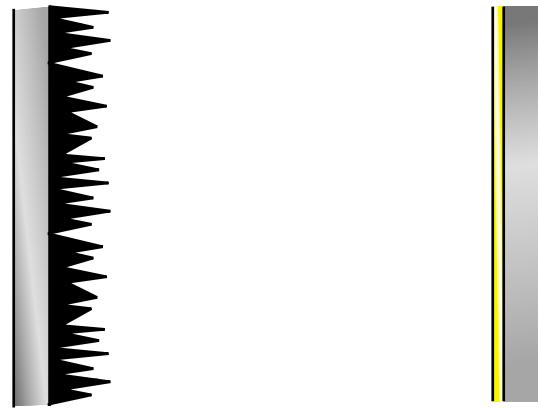
Results



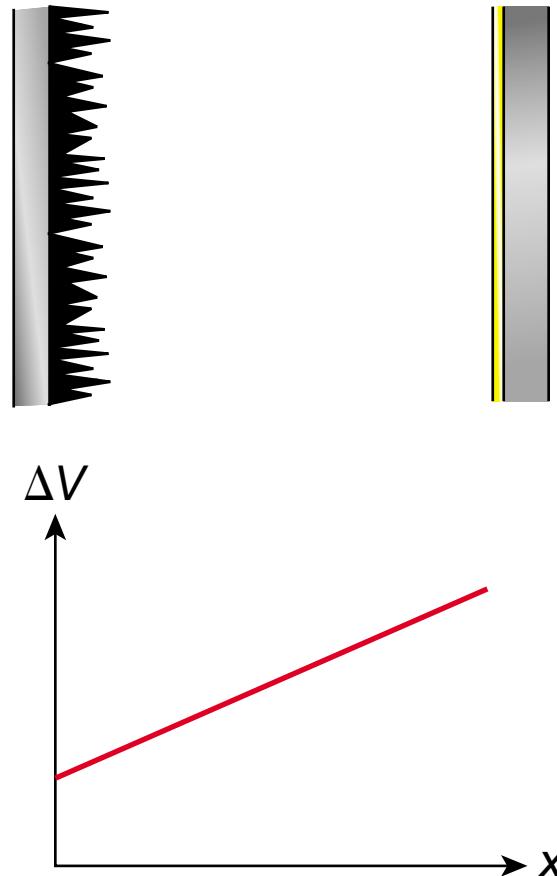
Results



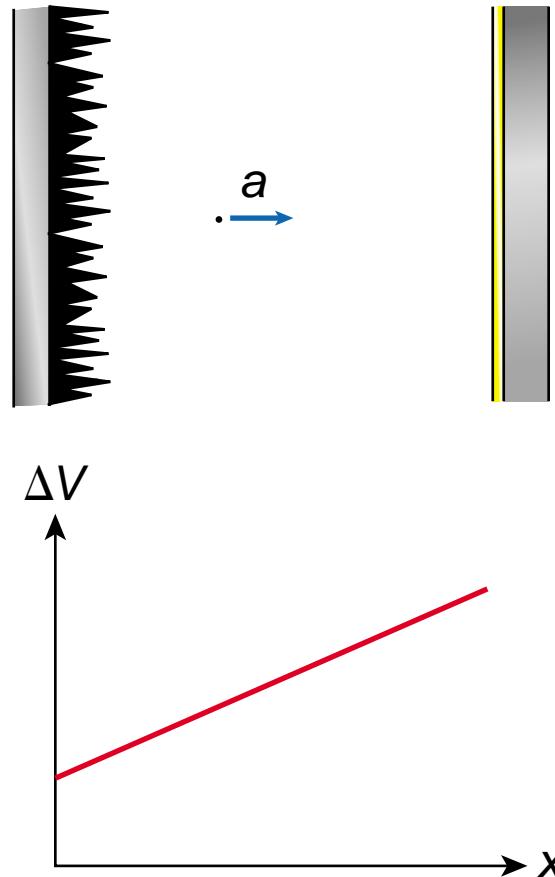
Space charge effect



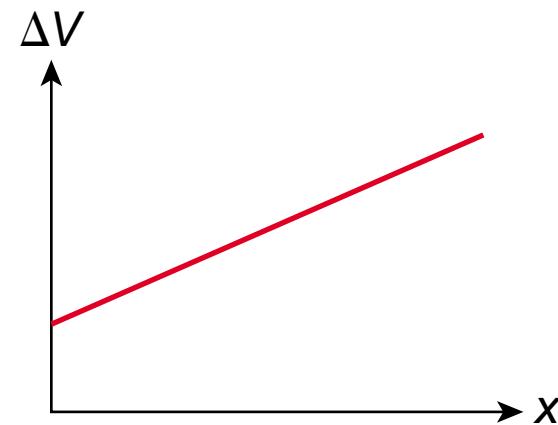
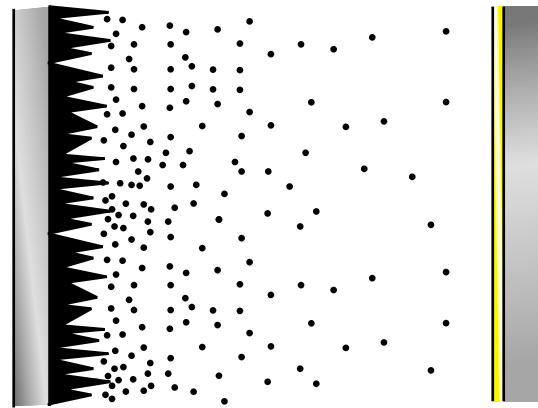
Space charge effect



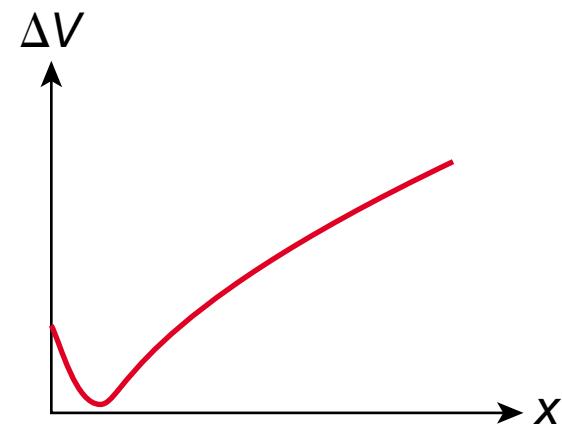
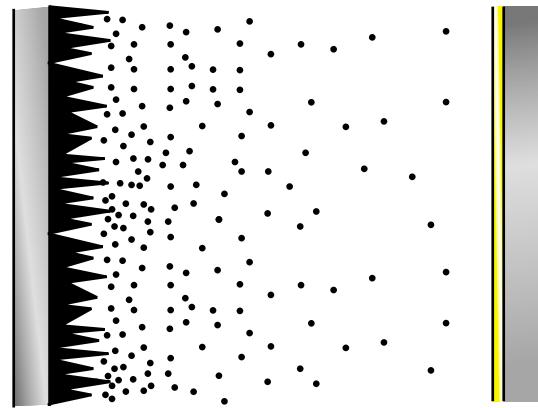
Space charge effect



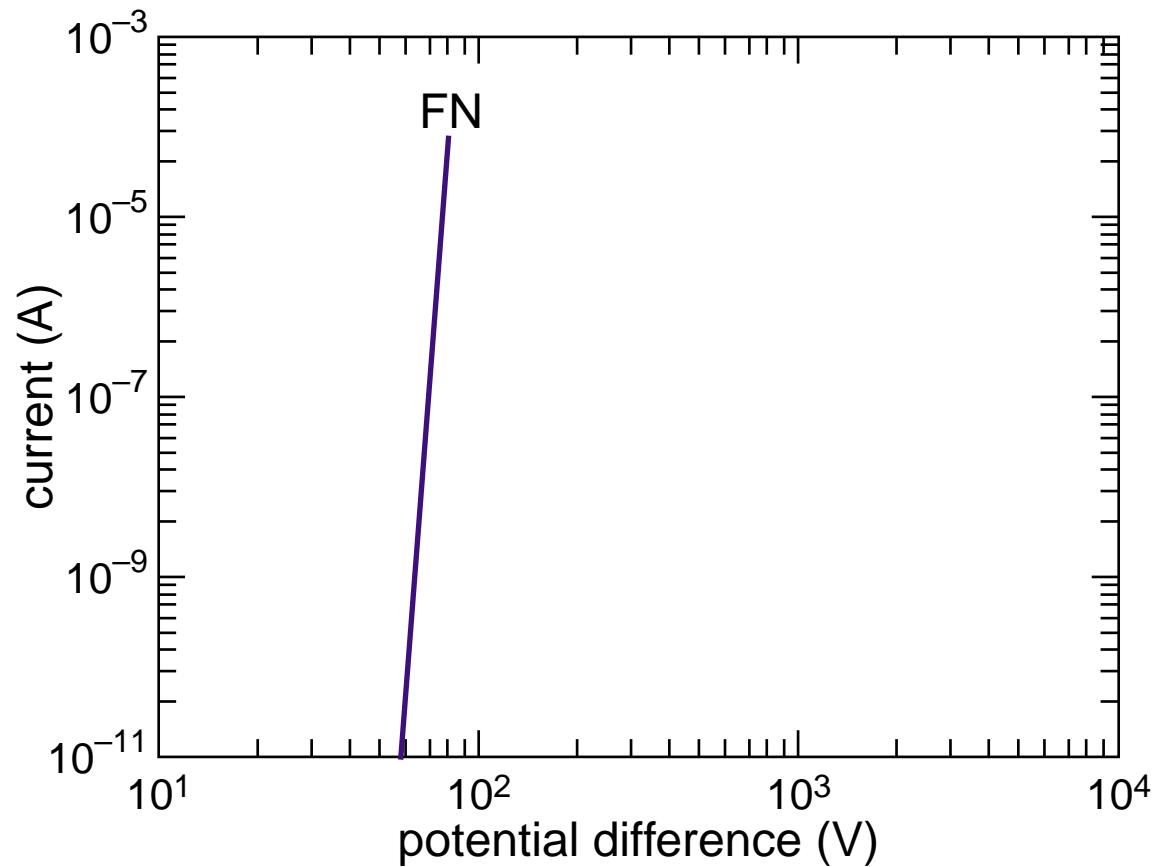
Space charge effect



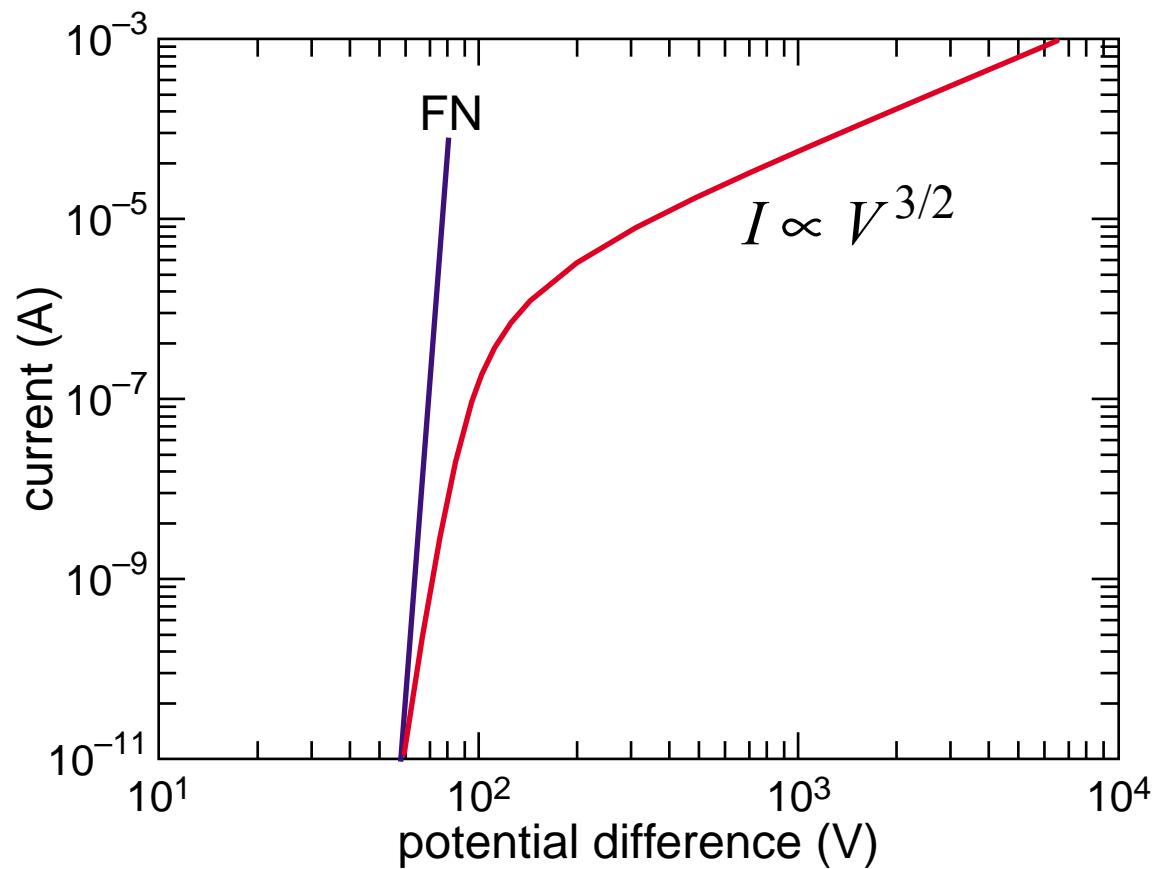
Space charge effect



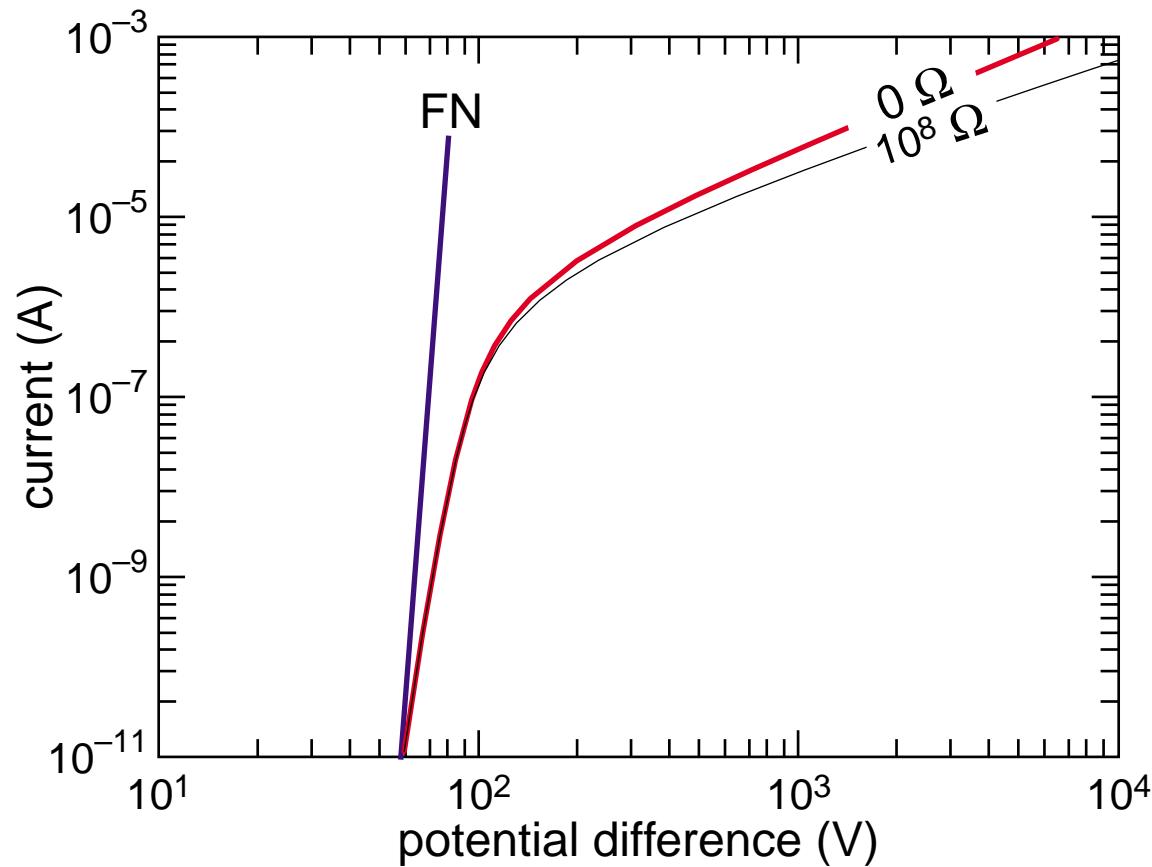
Space charge effect



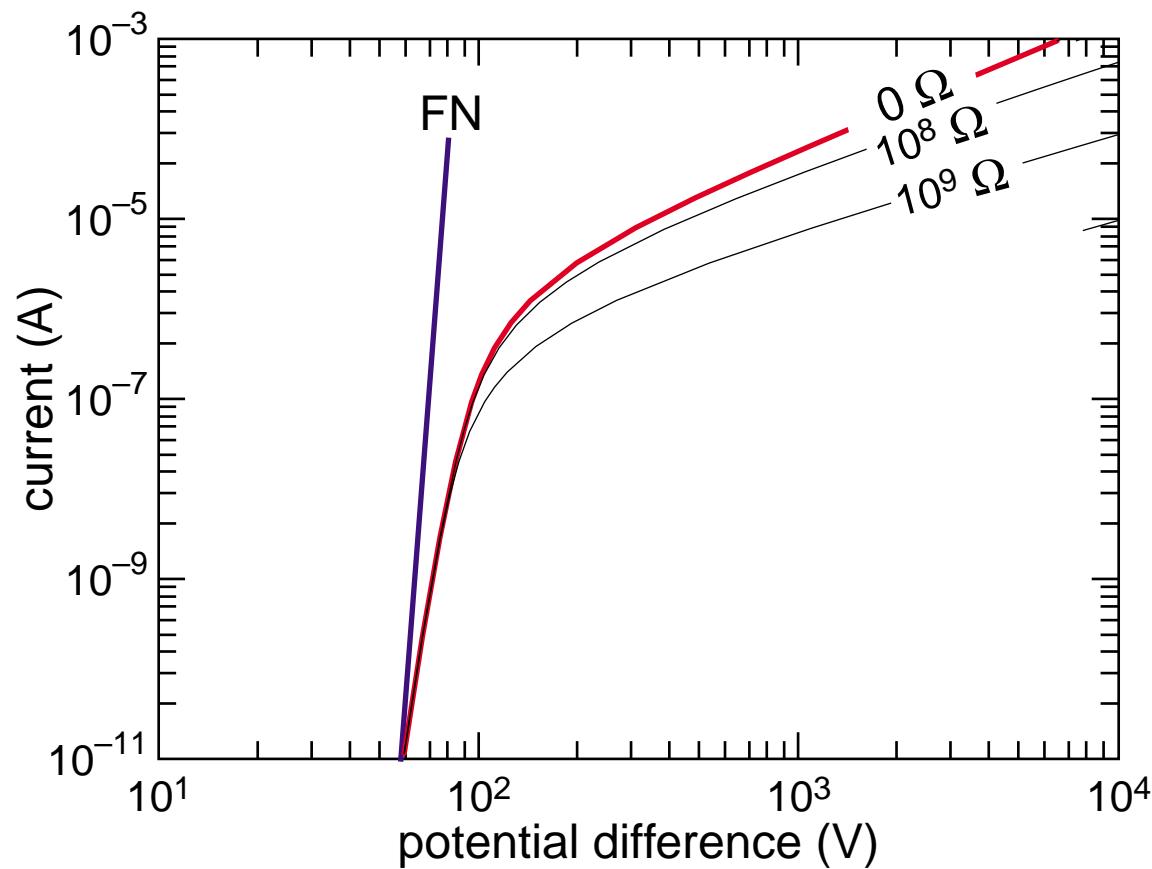
Space charge effect



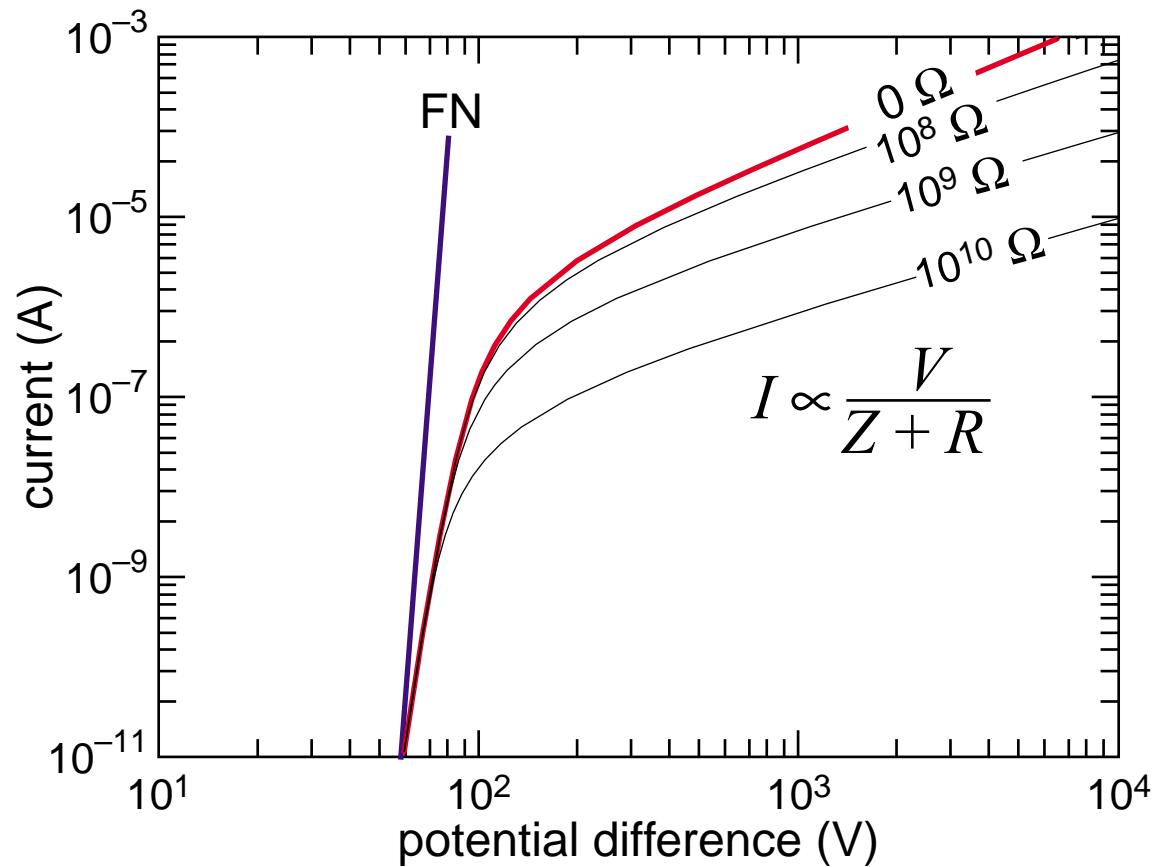
Space charge effect



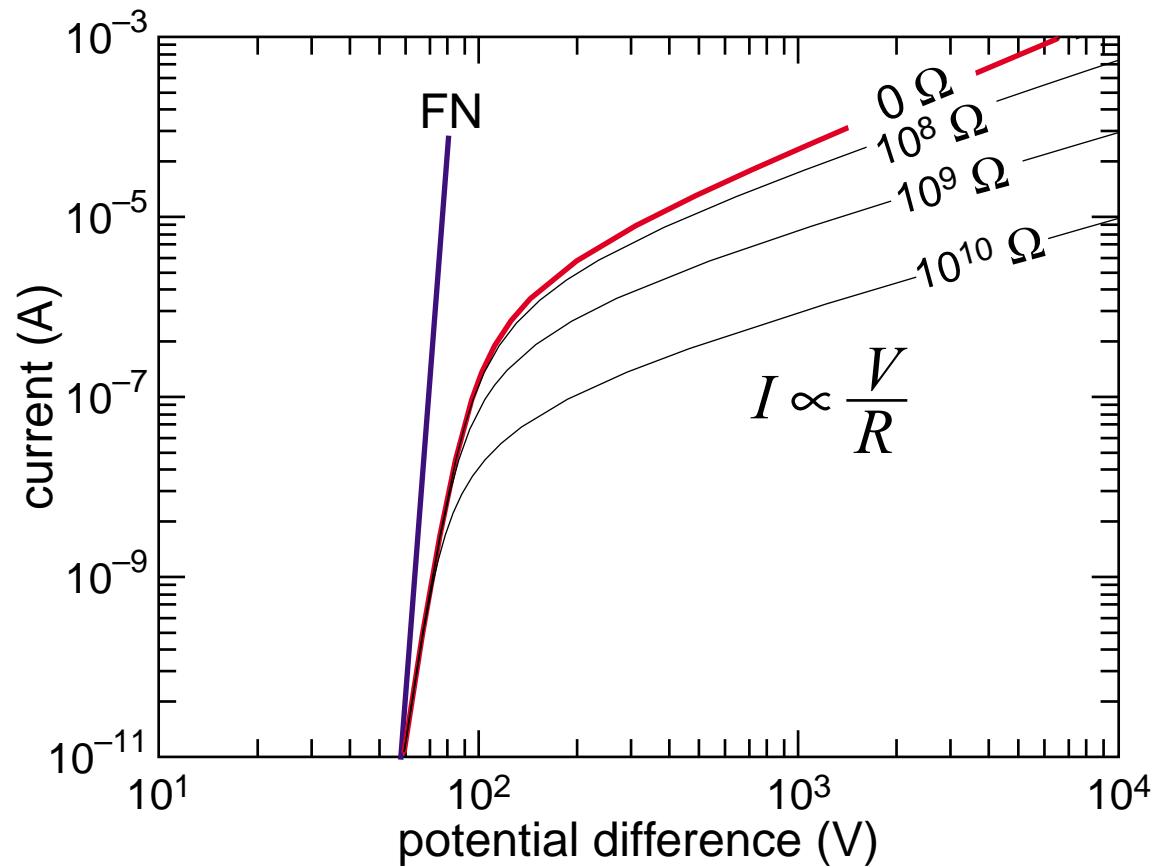
Space charge effect



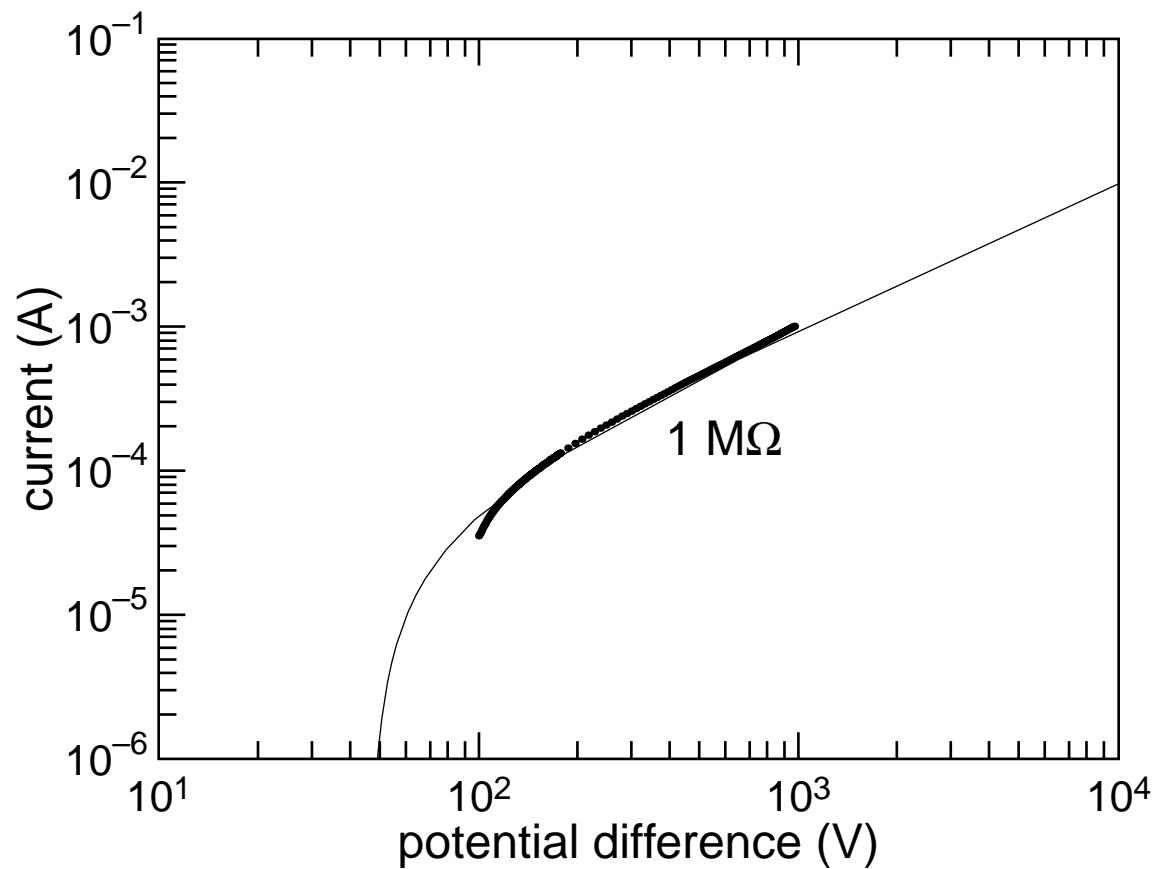
Space charge effect



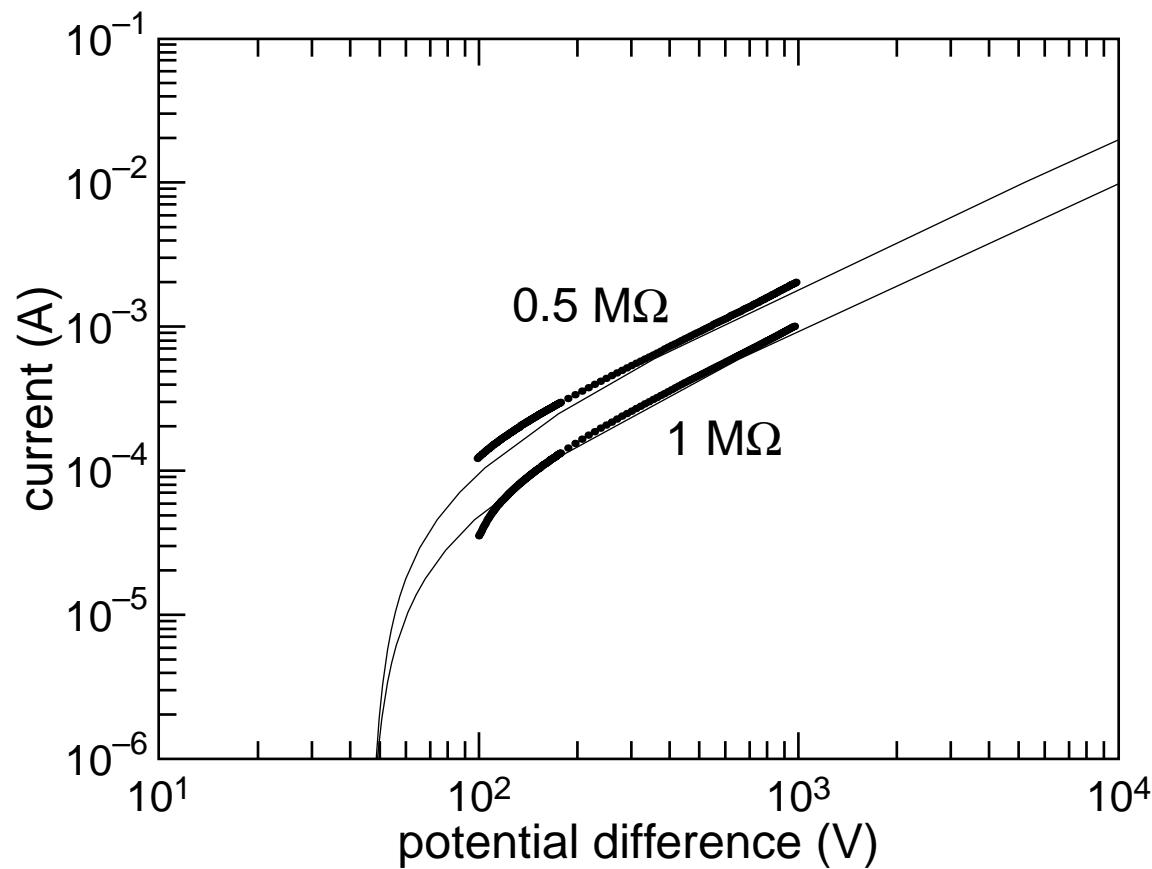
Space charge effect



Space charge effect



Space charge effect



Outline

- ▶ **Background**
- ▶ **Results**
- ▶ **Discussion**

Discussion

Ion channeling and electron backscattering

- ▶ **spikes retain crystalline order**
- ▶ **high density of defects**

Discussion

Secondary ion mass spectrometry:

- ▶ 10^{20} cm^{-3} sulfur

- ▶ 10^{17} cm^{-3} fluorine

Discussion

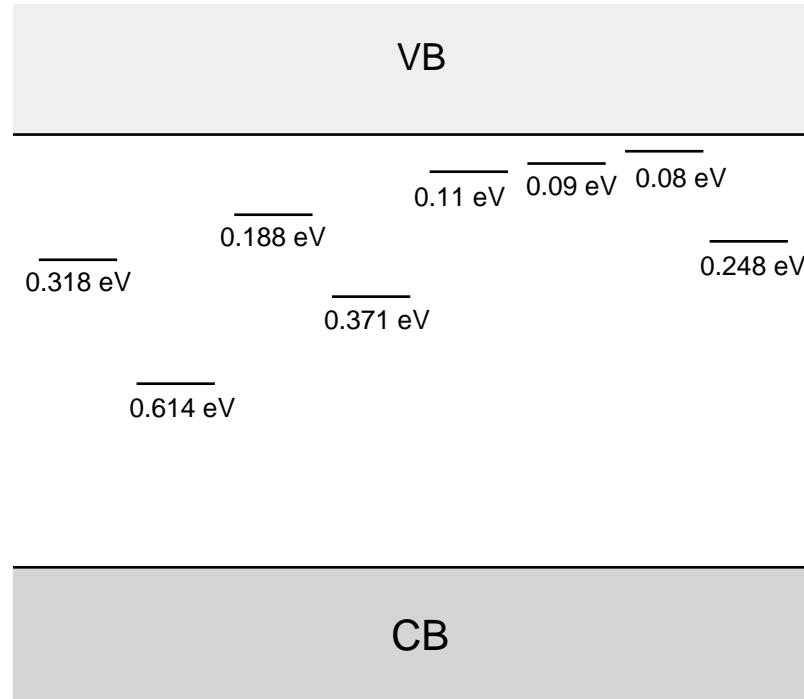
sulfur introduces states in the gap

VB

CB

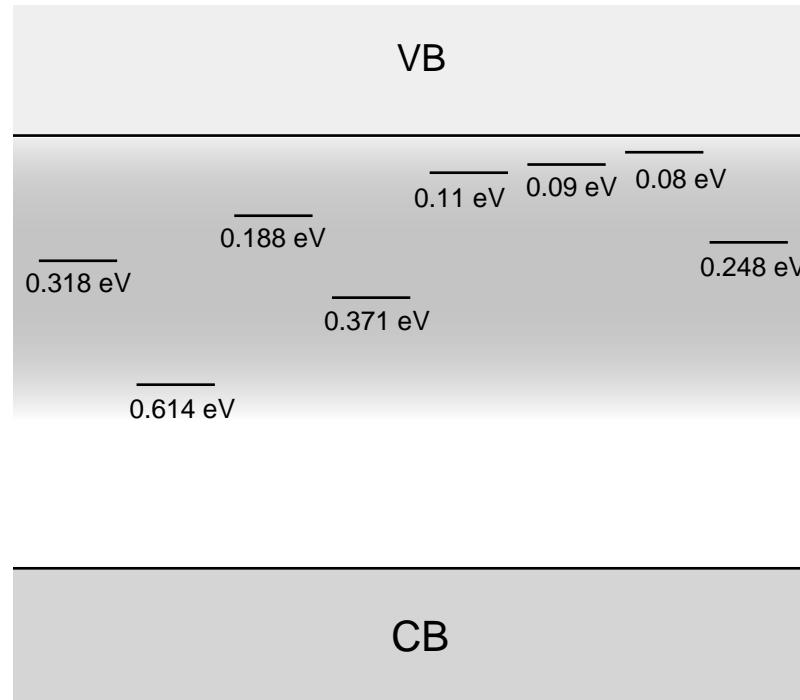
Discussion

sulfur introduces states in the gap

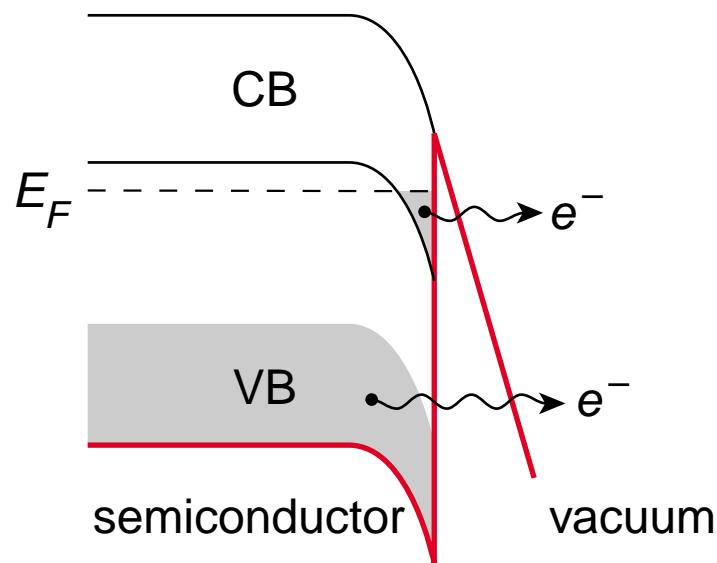


Discussion

states broaden into a band

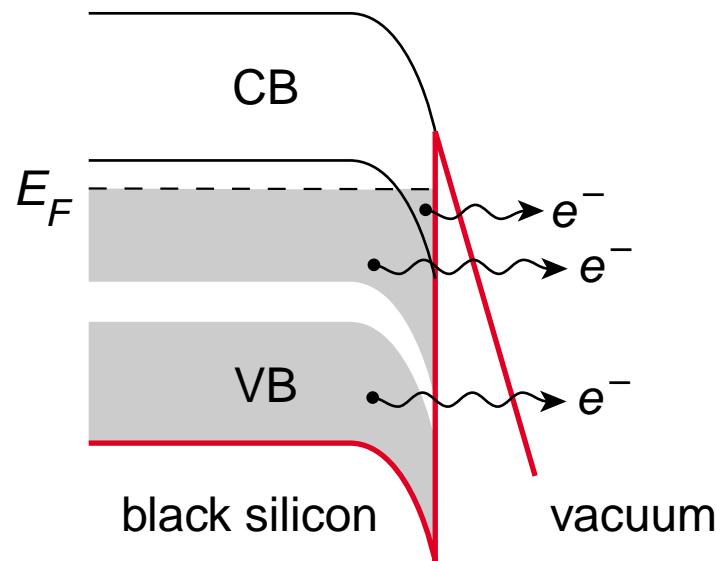


Discussion



Discussion

sulfur band provides additional electrons



Summary

Microstructured silicon

- ▶ **fabricated by simple, maskless process**

Summary

Microstructured silicon

- ▶ **fabricated by simple, maskless process**
- ▶ **can be integrated with microelectronics**

Summary

Microstructured silicon

- ▶ **fabricated by simple, maskless process**
- ▶ **can be integrated with microelectronics**
- ▶ **provides stable, high field-emission current**

Summary

Microstructured silicon

- ▶ **fabricated by simple, maskless process**
- ▶ **can be integrated with microelectronics**
- ▶ **provides stable, high field-emission current**
- ▶ **is durable**

Summary

New Scientist 13, 34 (2001)

A forest of silicon spikes could revolutionise solar cells and give you painless injections. **Bruce Schechter** peers into the mysterious world of black silicon

TALL, DARK AND STRANGER

WE ALL love stories of serendipity. They seem to hark back to a time when a fogged-up Petri dish or a filthy Petri dish

semiconductors with a powerful laser. In the early 1990s, Mazur's was the first academic lab in the world to get its hands on a femtosecond laser. This device produces pulses of light that are hundreds of times brighter than the Sun.

around the laboratory," he claims. "Well, it was almost the only reason short laser pulse will break down into sulphur and fluorine radicals, which will attack a silicon substrate. Hydrogen fluoride is used to etch silicon. I thought maybe the SF₆ would decompose and then the fluorine would decompose the silicon." Mazur would soon

Applications

- ▶ **display technology**
- ▶ **detector technology**
- ▶ **solar cells**

A forest of silicon spikes could revolutionise solar cells and give you painless injections. **Bruce Schechter** peers into the mysterious world of black silicon

TALL, DARK AND STRANGER

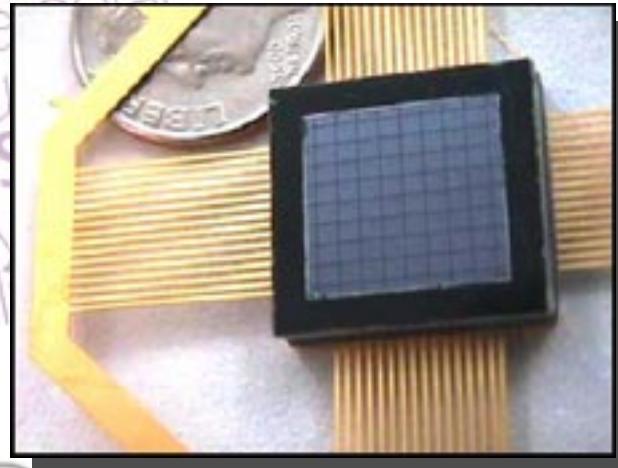
We ALL know stories of werewolves. The most horrific tale is when a werewolf turns into a tiny, tiny devil when he's full moon.

Silicon has a powerful laser. In the early 1990s, Marantz was the first to make use of the 400 nm to 250 nm band on a femtosecond laser. This device can produce several times more than the Sun's energy in just one second, and extremely

short pulses. These very bright pulses can cut and thicken materials, will attack specific substances. Hydrofluoride is used to etch silicon, though it's also used to etch glass, and then the fluorine attacks

Applications

- ▶ **display technology**
- ▶ **detector technology**
- ▶ **solar cells**

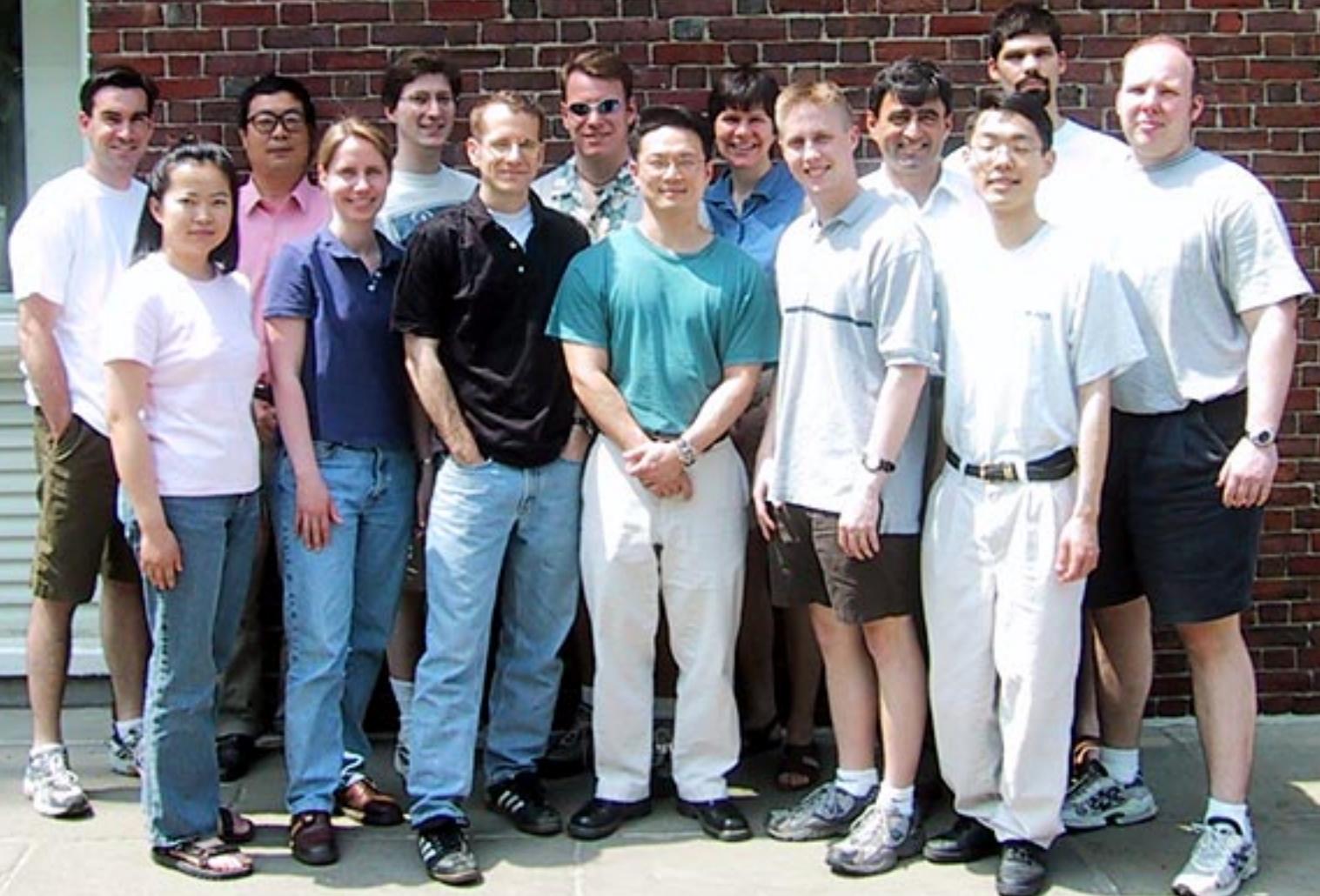


**TALL, DARK
AND STRANGER**

We all have stories of encounters with a tall, dark figure in the night when a friend or family member has been missing. Now there's a new one: a tiny, thin device that can peer into the world of black silicon.

Silicon is a semiconductor with a powerful laser. In the early 1990s, Marantz was the first to demonstrate the use of the 400 nm to 250 nm band in a terahertz laser. This device, based on a terahertz laser, can peer into the world of black silicon 100 times faster than the Sun's eye. It is extremely sensitive and can detect even the tiniest changes in the environment. Marantz esti-

CORDON MCKAY
LABORATORY OF
APPLIED SCIENCE





**Funding: National Science Foundation
US Department of Energy
Army Research Office**

**Acknowledgments:
Prof. N. Bloembergen
Prof. H. Ehrenreich
Prof. T. Kaxiras**

**For a copy of this talk and
additional information, see:**

<http://mazur-www.harvard.edu>