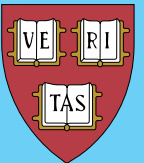
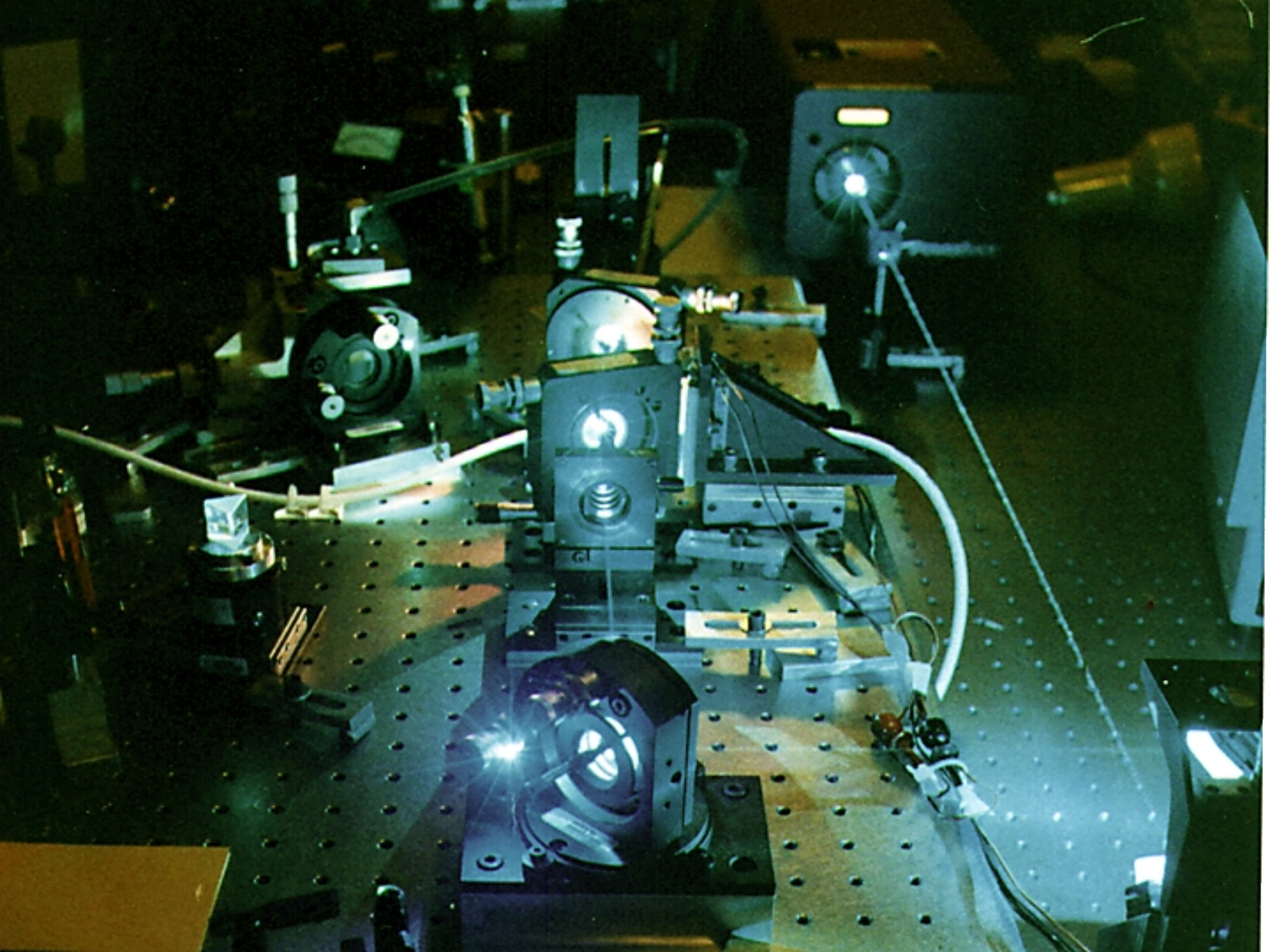


An introduction to femtosecond laser science

**Eric Mazur
Harvard University**

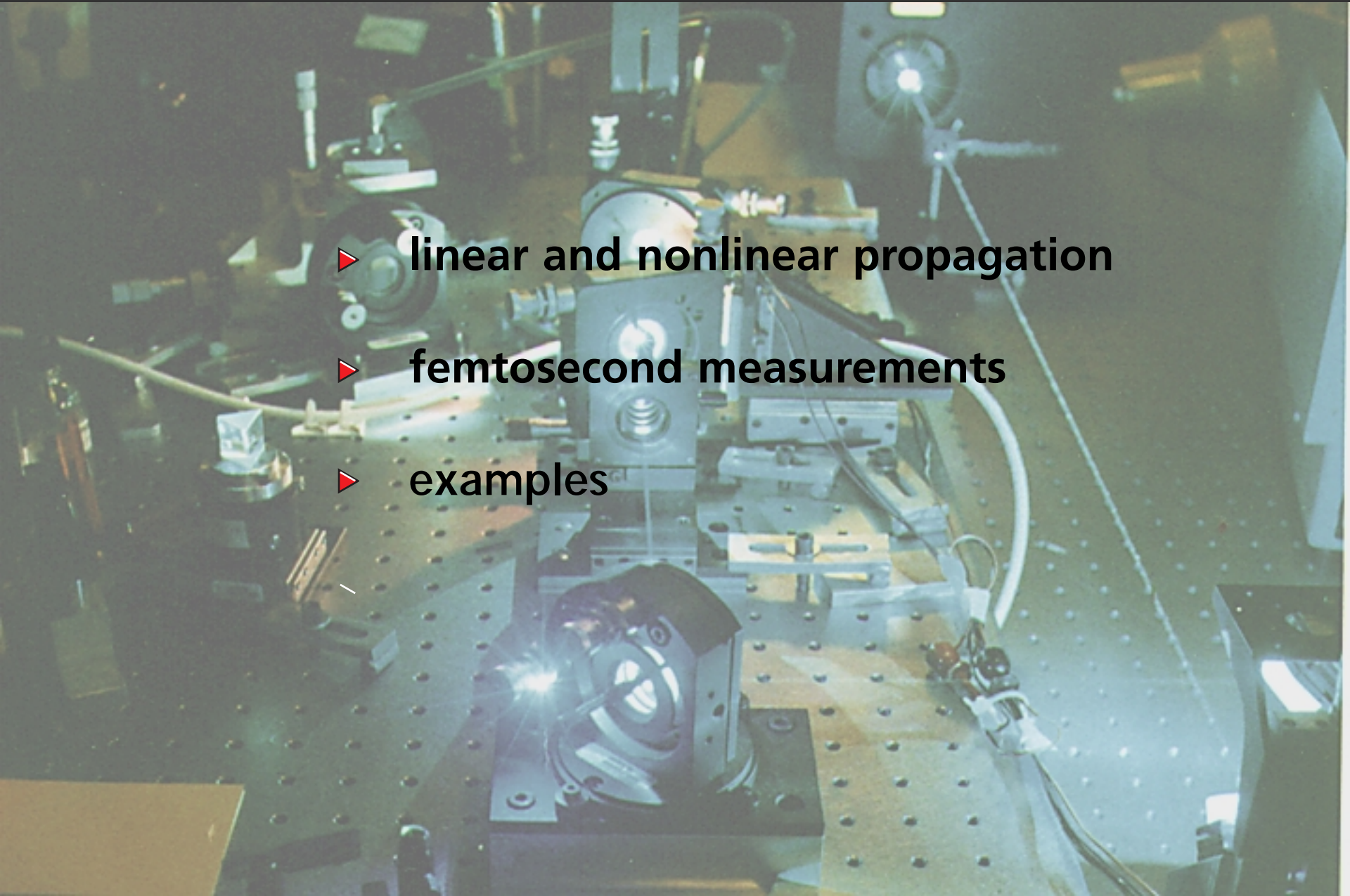
**Short course 541
Photonics West 2003
San Jose, 28 January 2003**





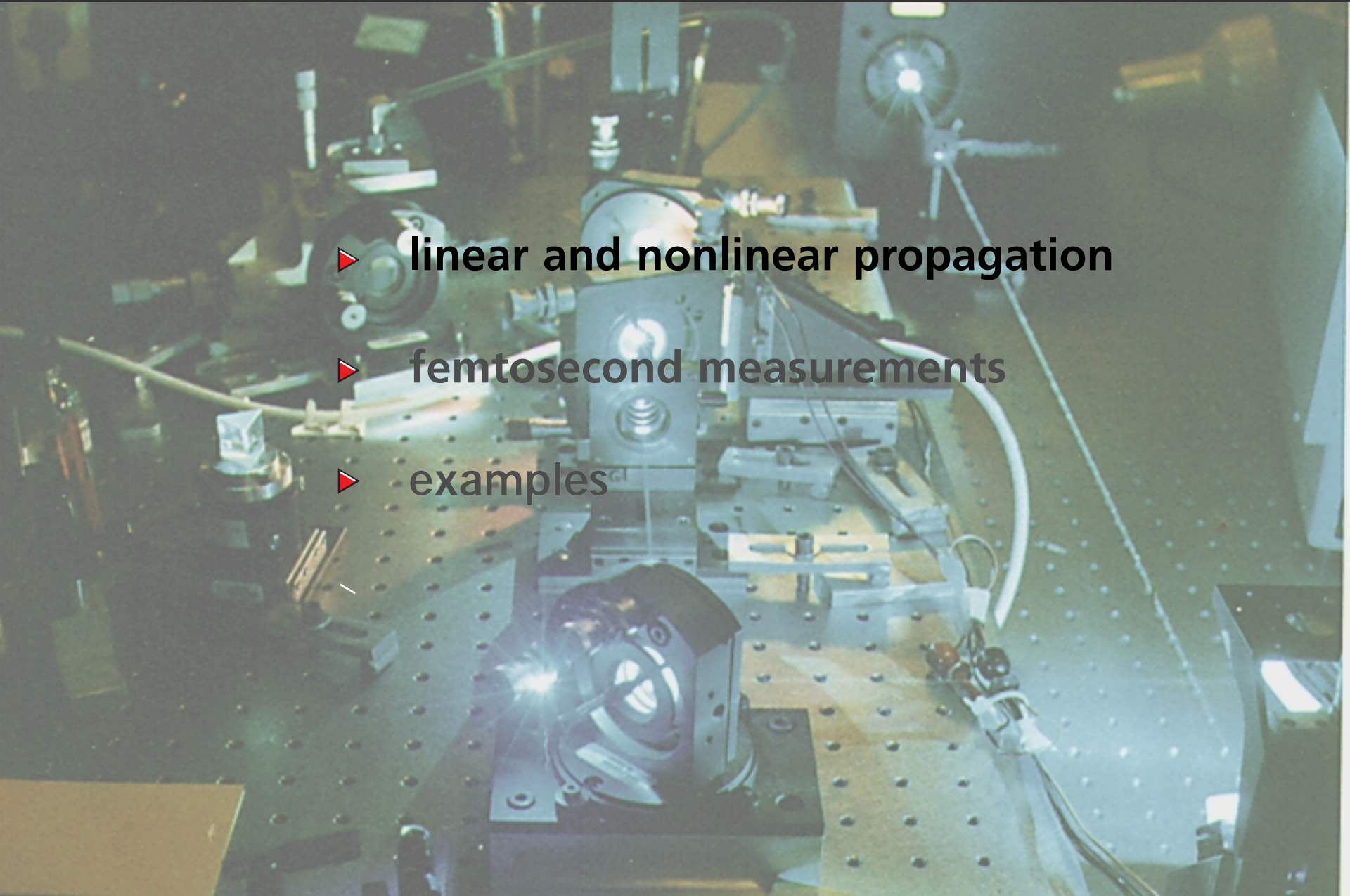
Outline

- ▶ linear and nonlinear propagation
- ▶ femtosecond measurements
- ▶ examples

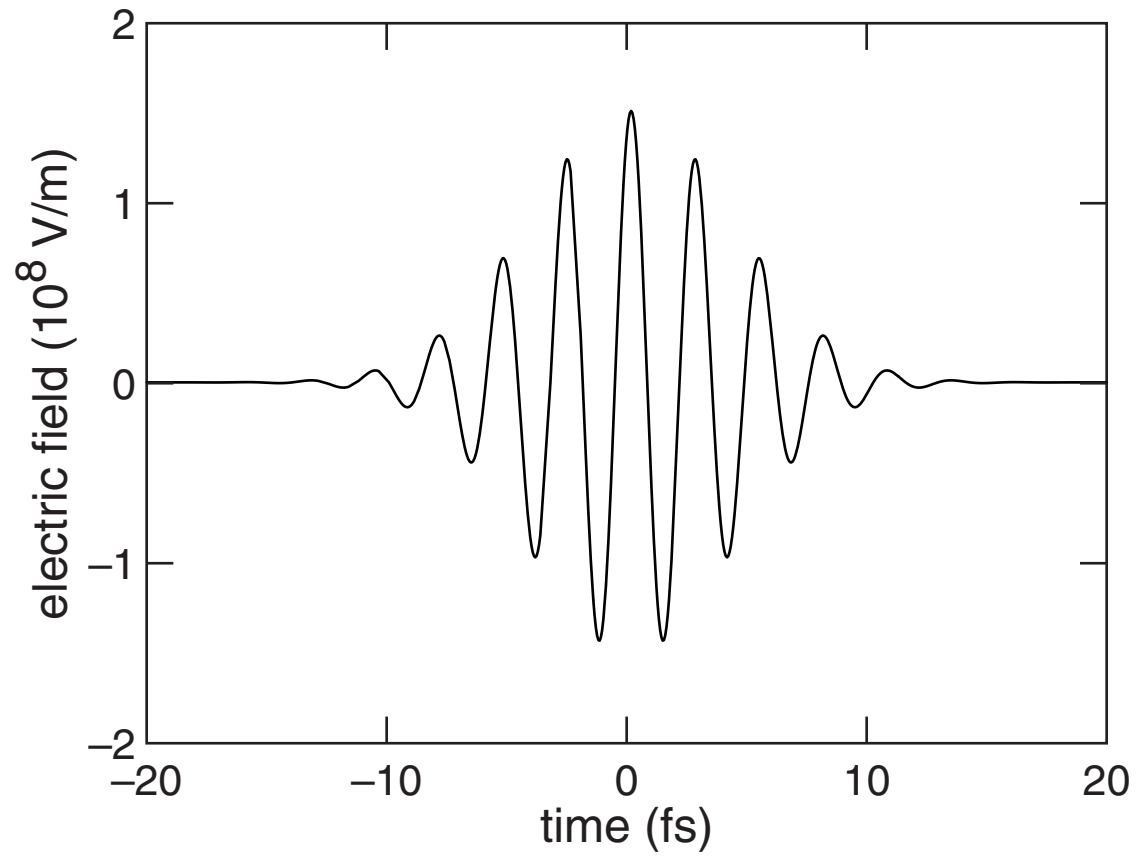


Outline

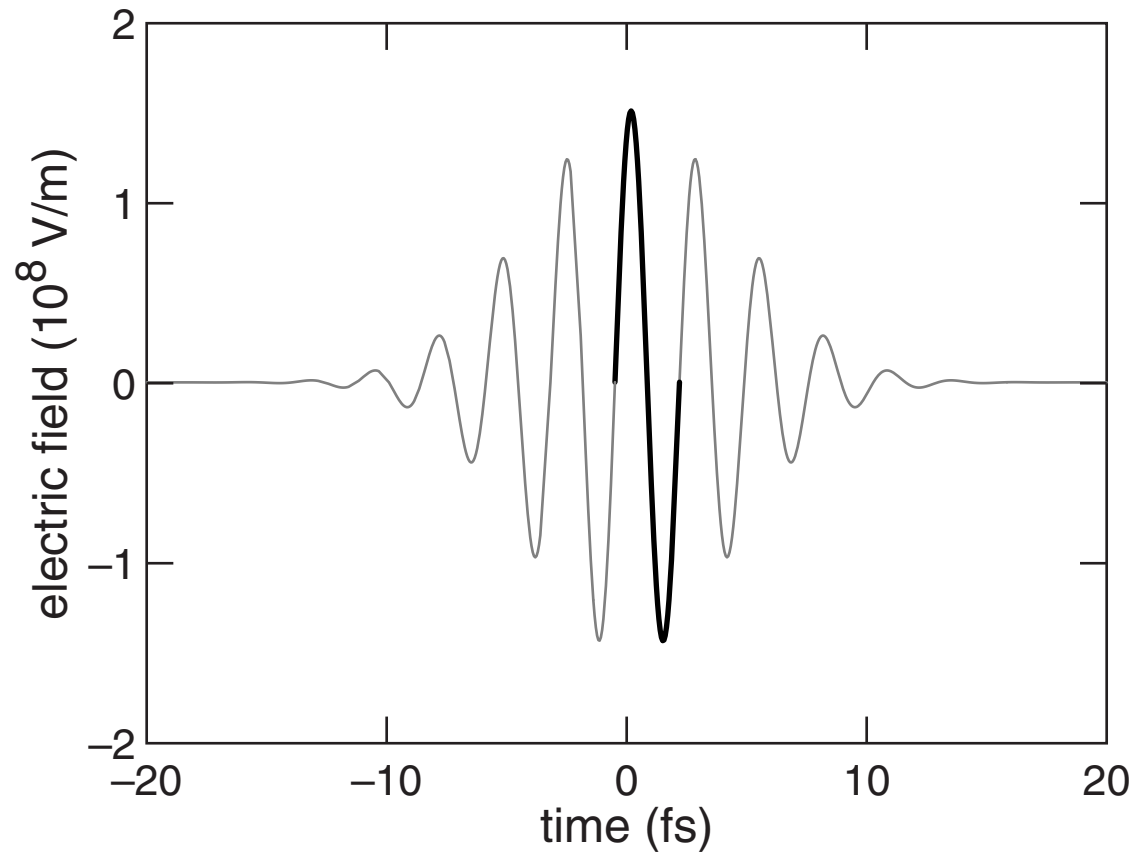
- ▶ **linear and nonlinear propagation**
- ▶ **femtosecond measurements**
- ▶ **examples**



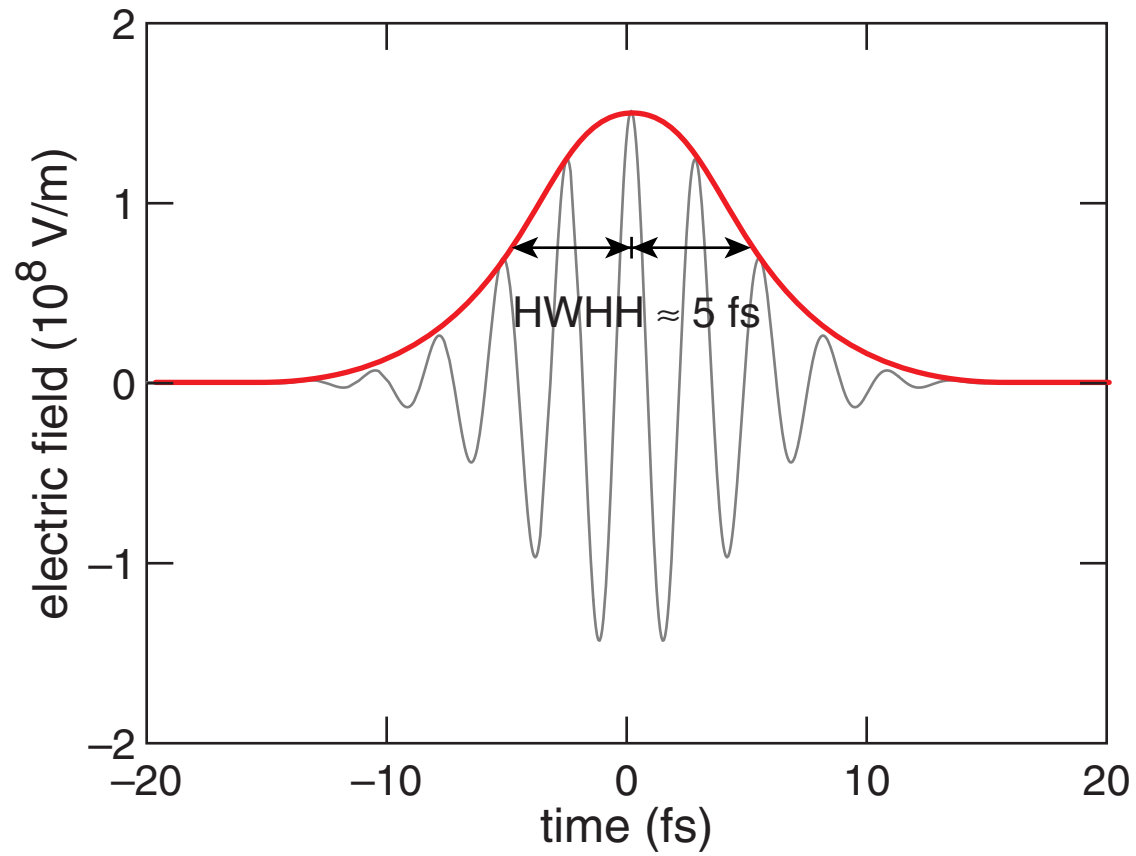
Introduction



Introduction



Introduction



Introduction

- ▶ **time resolution**
- ▶ **high intensity**
- ▶ **nonlinear optics**
- ▶ **new physics**

Propagation of EM waves through medium

Governed by wave equation

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

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where

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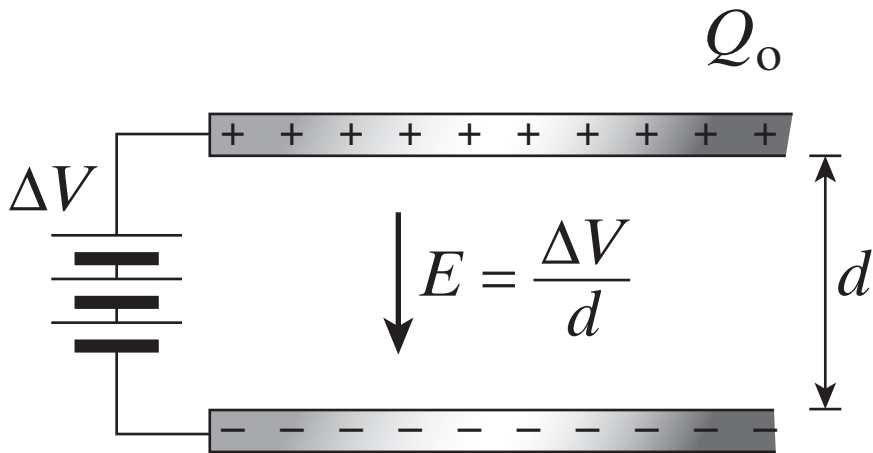
In non-ferromagnetic media $\mu \approx 1$, and so $n \approx \sqrt{\epsilon}$.

In dispersive media $n = n(\omega)$.

Propagation of EM waves through medium

Dielectric constant measures increase in capacitance

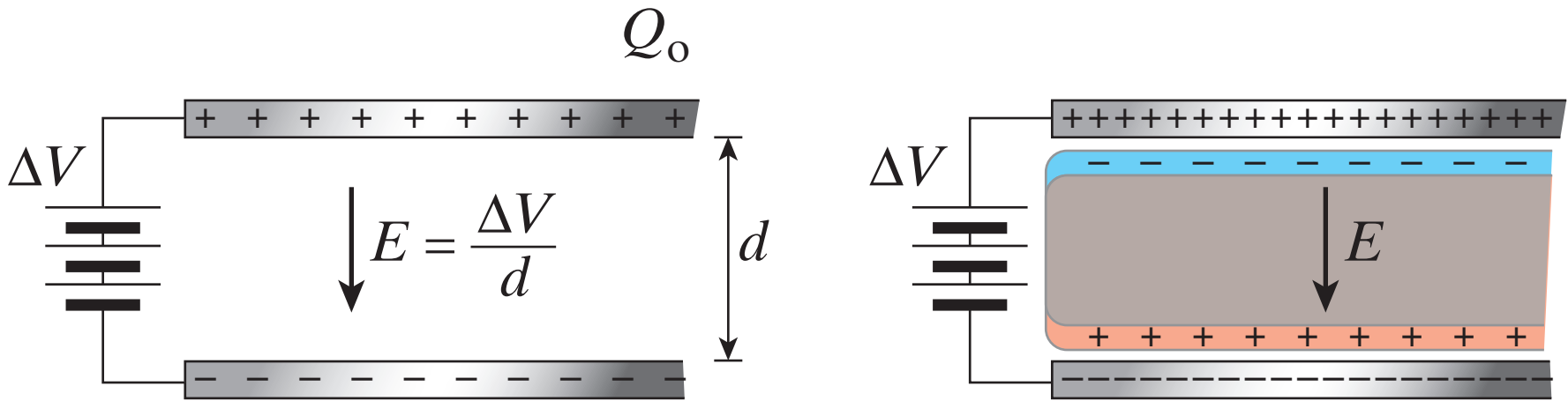
$$\epsilon = \frac{C_d}{C_o}$$



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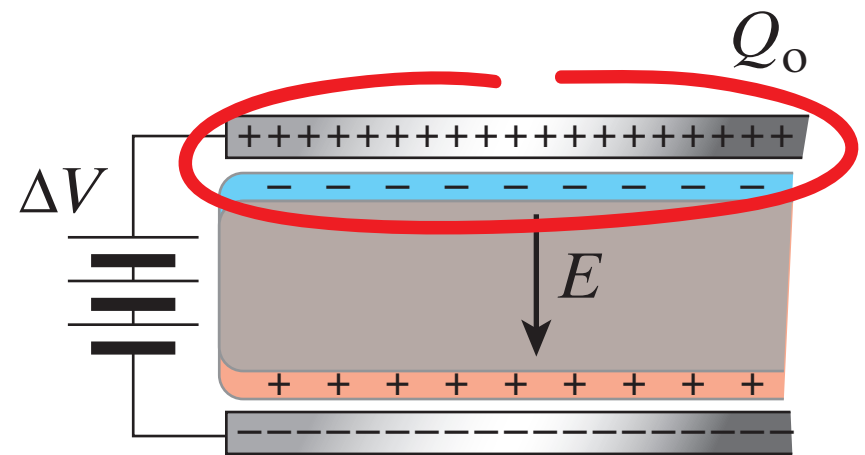
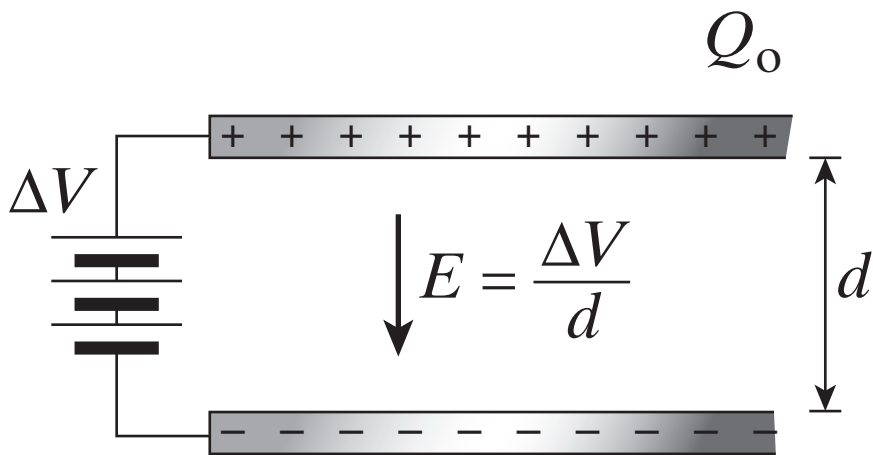
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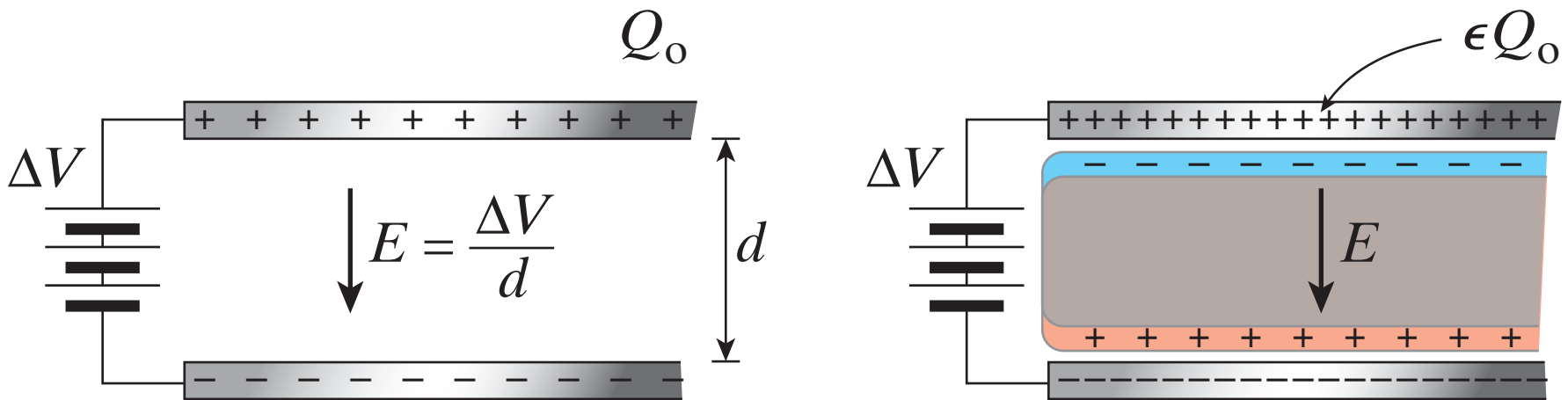
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Propagation of EM waves through medium

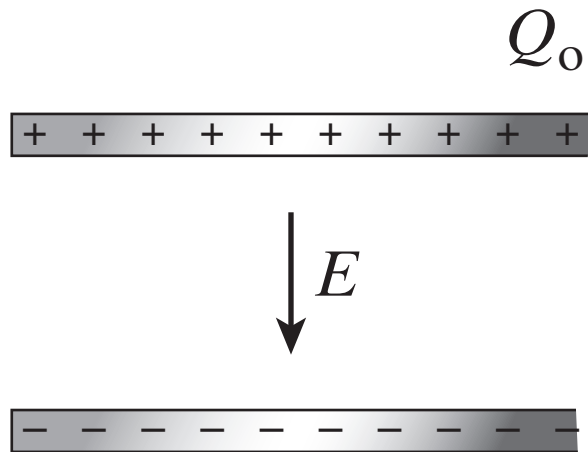
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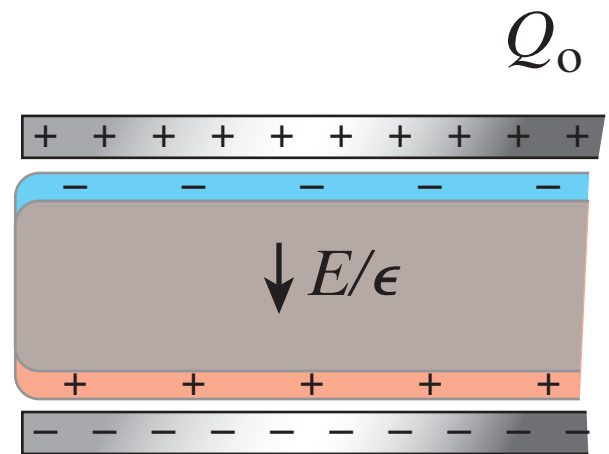
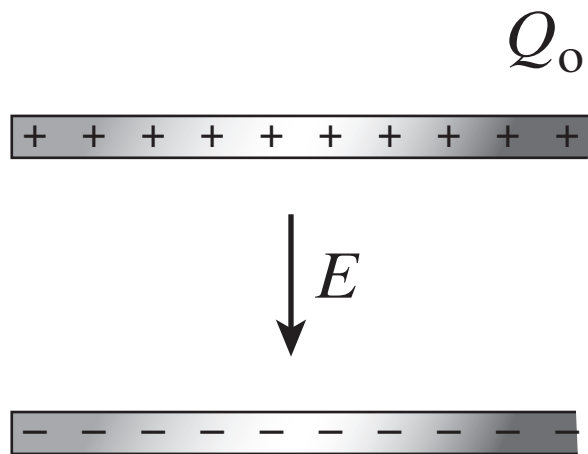
Propagation of EM waves through medium

Alternatively, ϵ is measure of the attenuation of the field



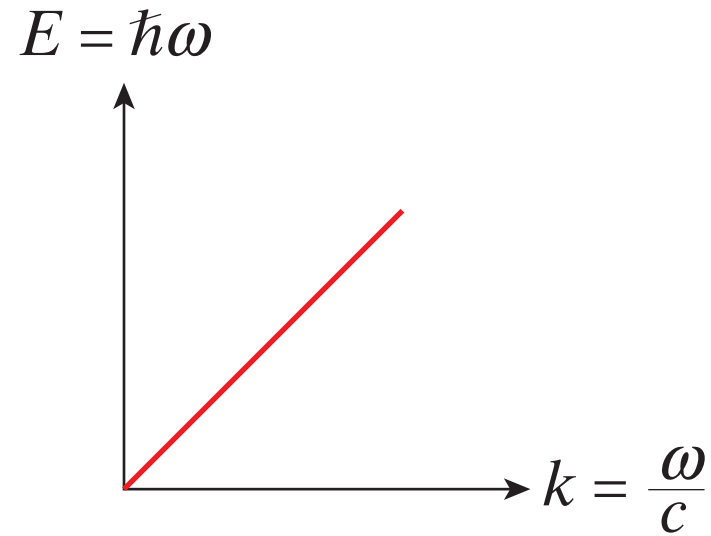
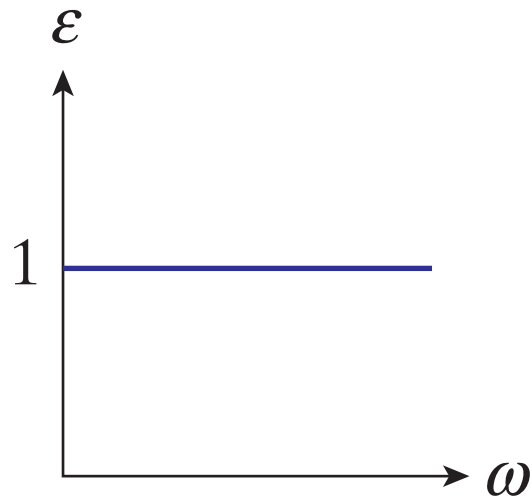
Propagation of EM waves through medium

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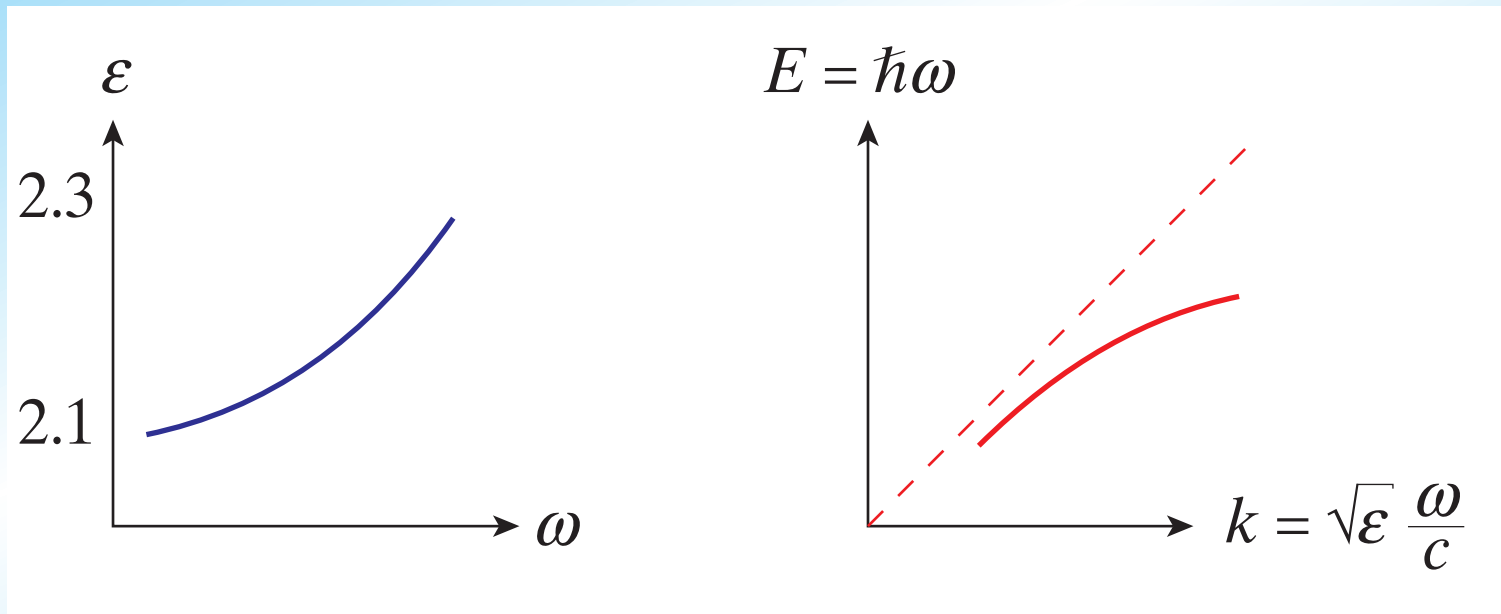
Propagation of EM waves through medium

In vacuum: $f\lambda = \frac{\omega}{k} = c \quad \Rightarrow \quad \omega = c k$

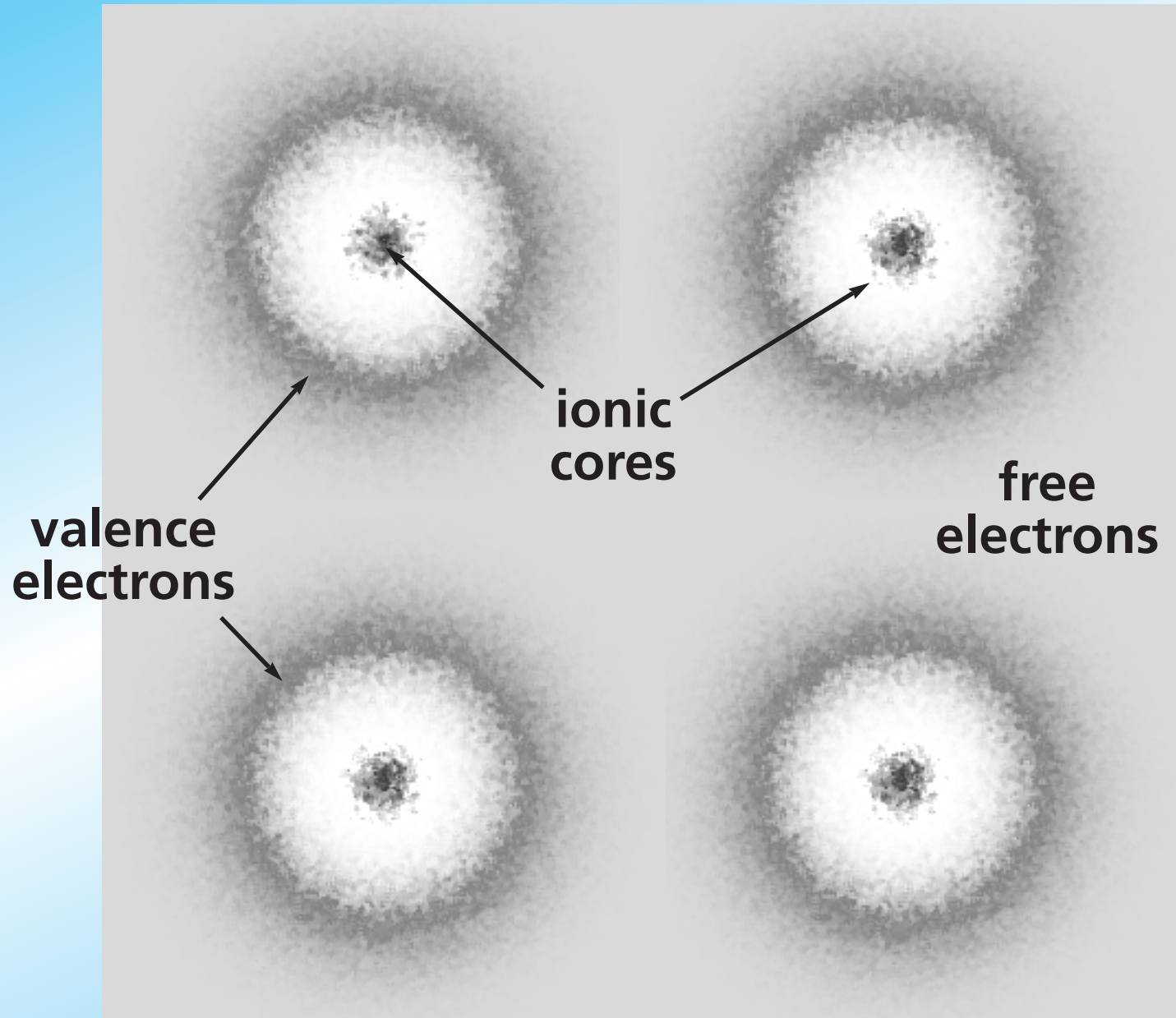


Propagation of EM waves through medium

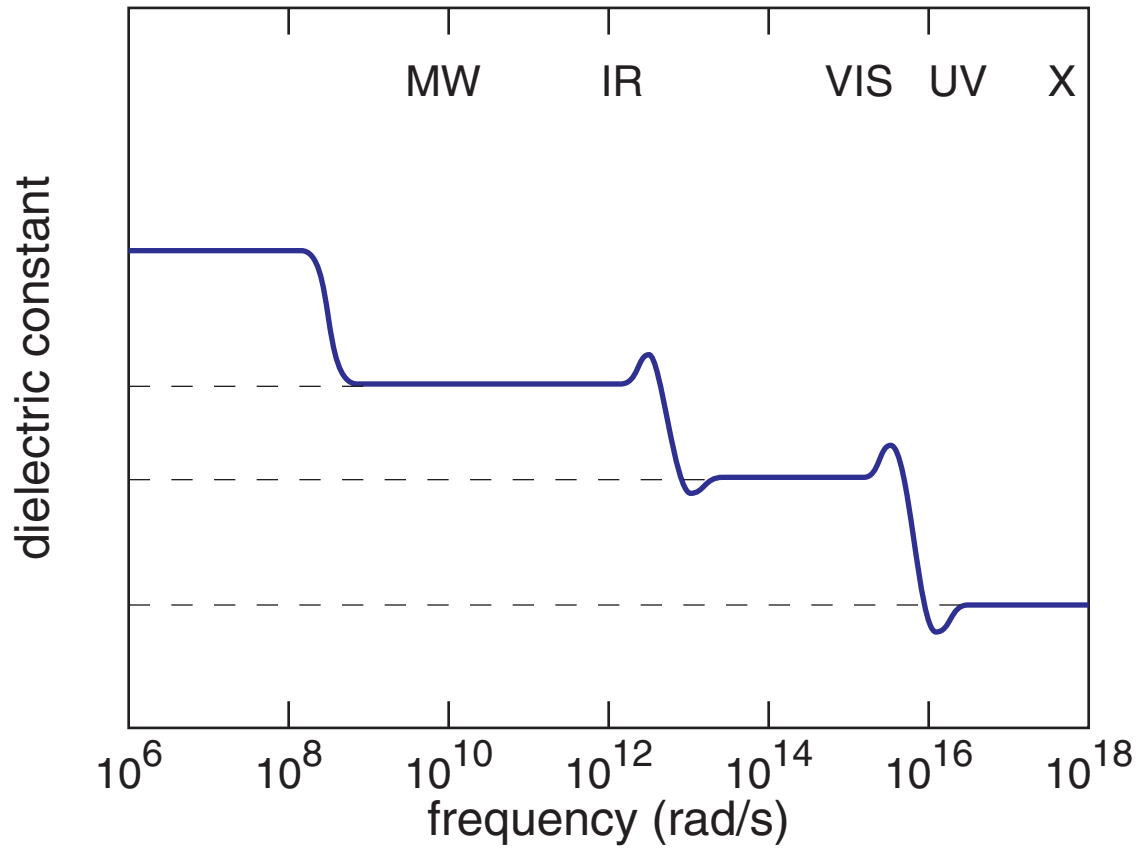
In medium: $v = \frac{c}{\sqrt{\epsilon(\omega)}} = \frac{c}{n} \quad \Rightarrow \quad \omega = \frac{c}{\sqrt{\epsilon}} k$



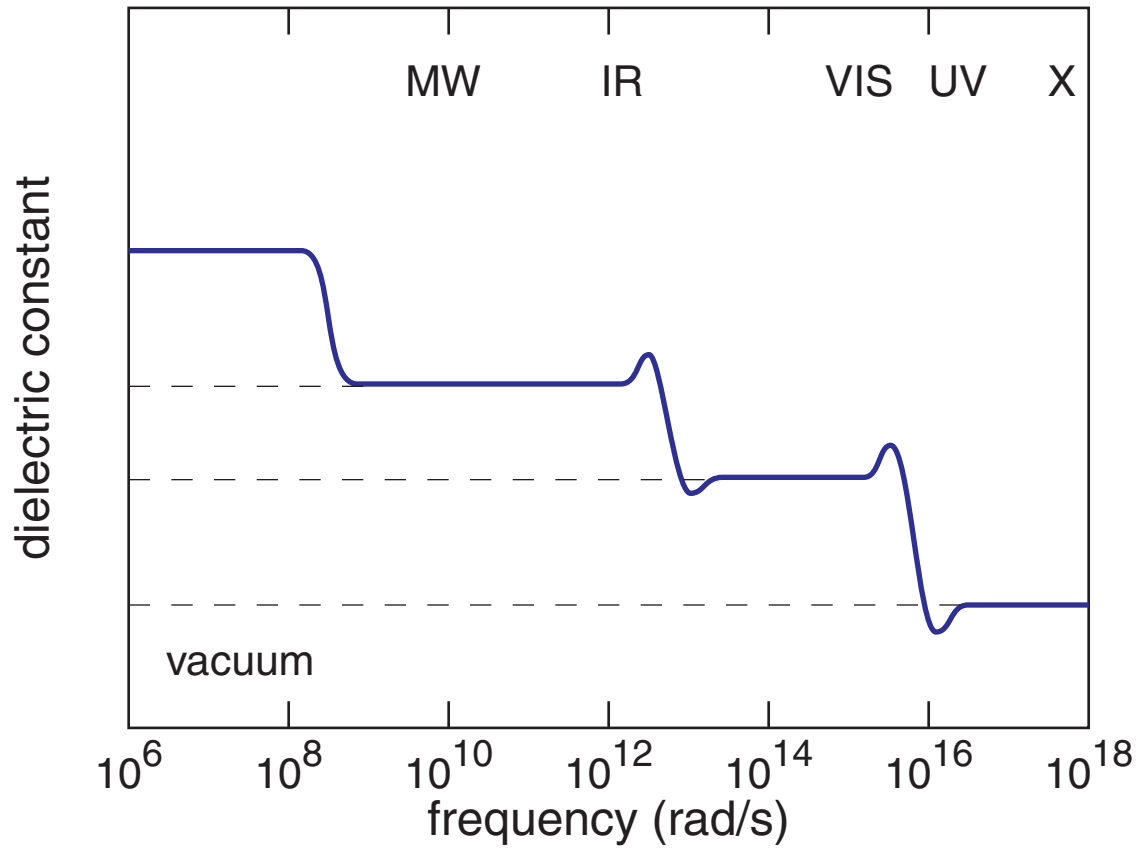
Which charges participate?



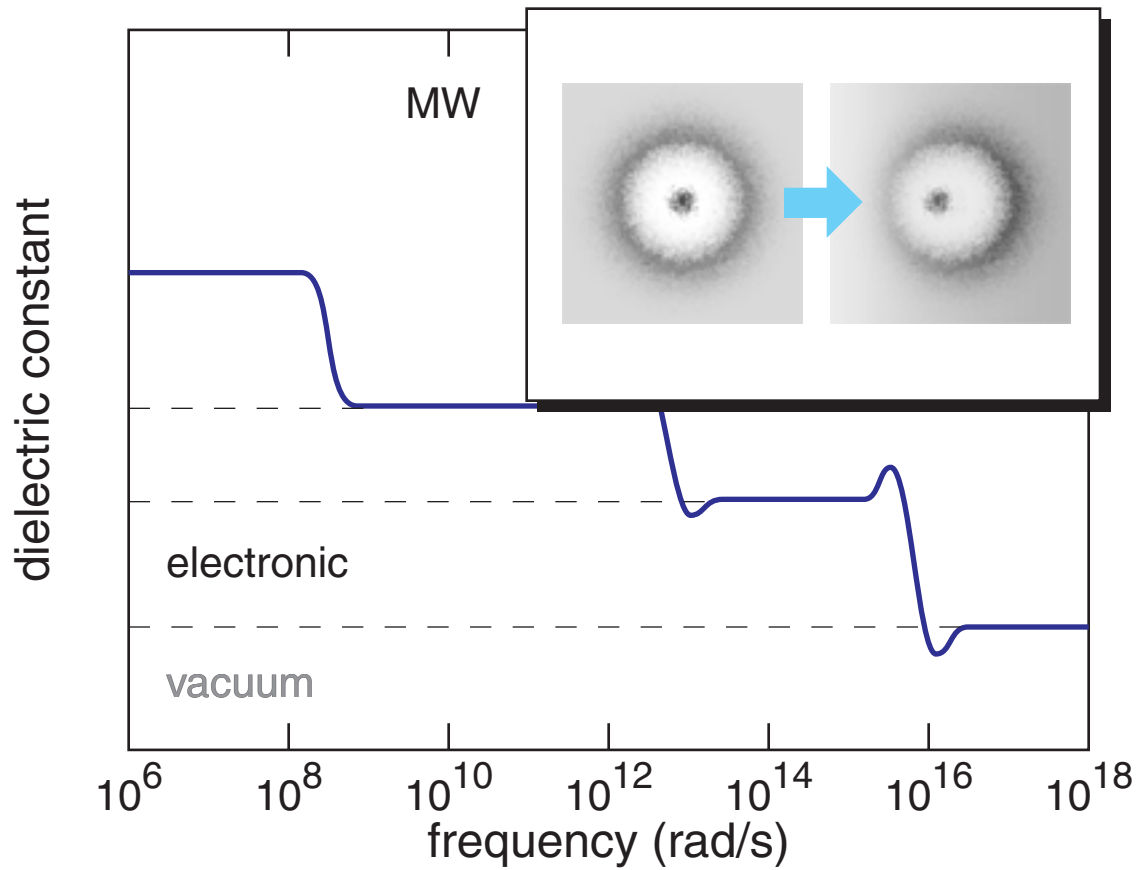
Dielectric function



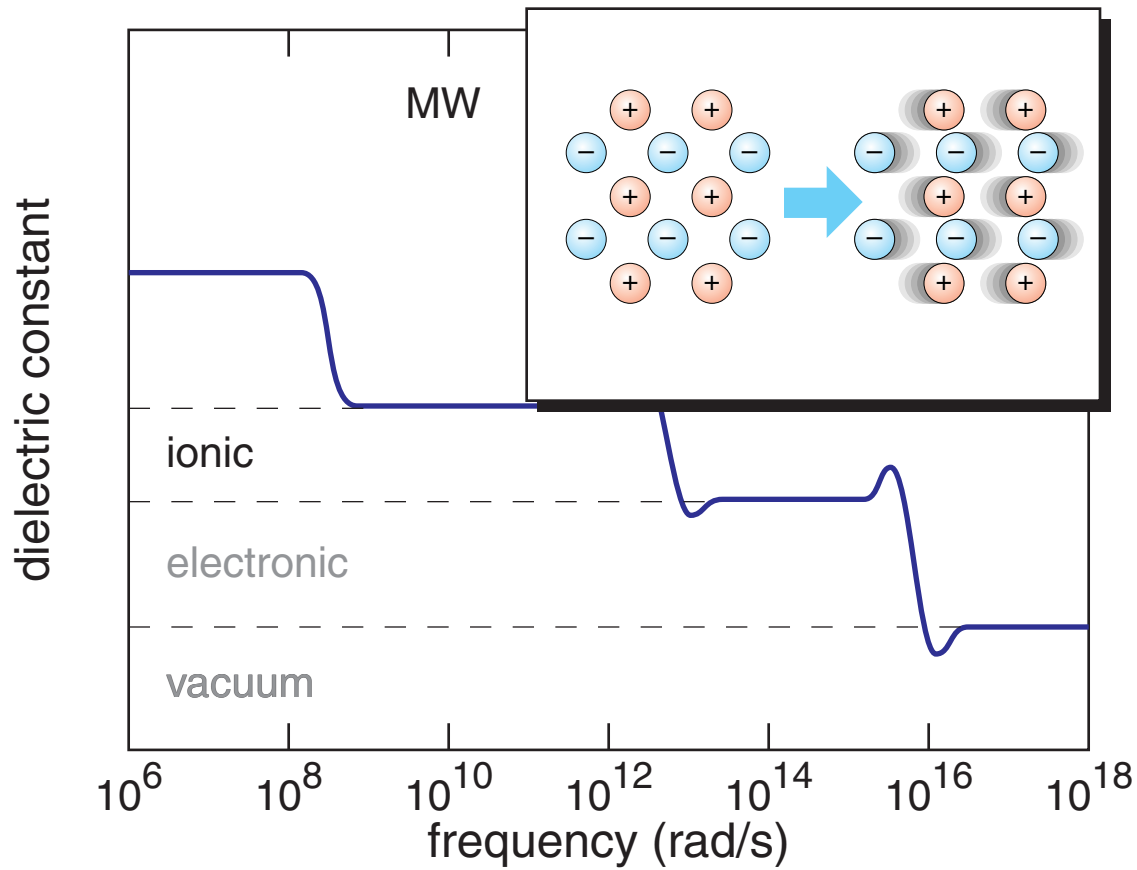
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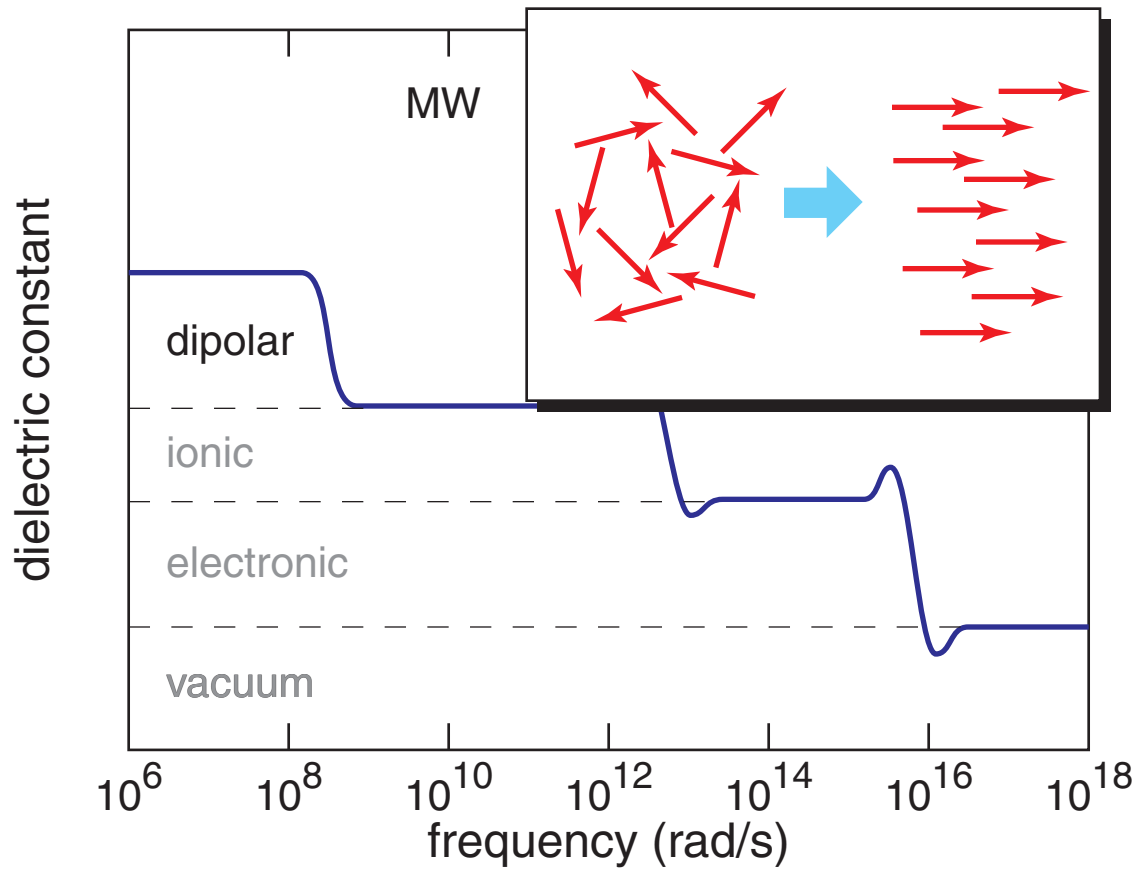
Dielectric function



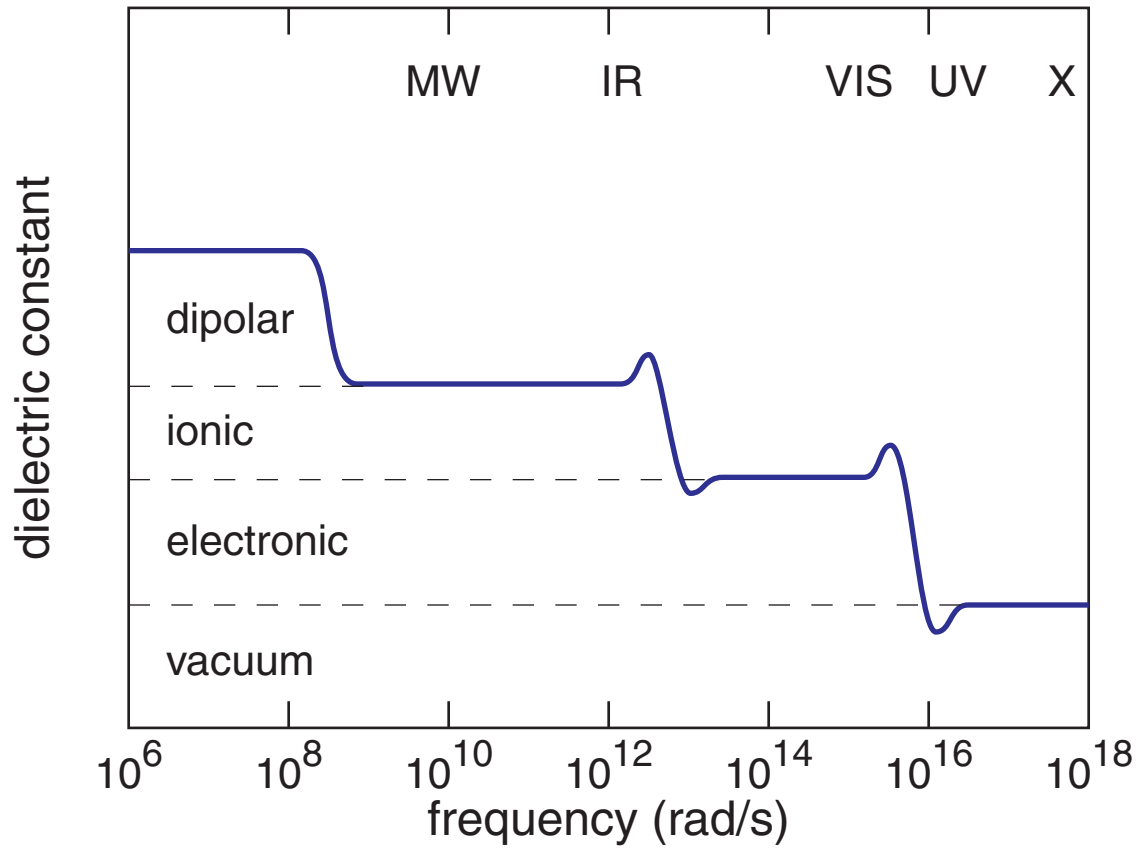
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Bound electrons

Electron on a string:

$$F_{binding} = - m_e \omega_o^2 x$$

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$$m \frac{d^2 x}{dt^2} + m \gamma \frac{dx}{dt} + m \omega_o^2 x = -eE$$

Bound electrons

Steady state: electron oscillates at driving frequency

$$x(t) = x_o e^{-i\omega t} \quad x_o = -\frac{e}{m} \frac{1}{(\omega_o^2 - \omega^2) - i\gamma\omega} E_o$$

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$$P(t) = \left(\frac{Ne^2}{m} \right) \sum_j \frac{f_j}{(\omega_j^2 - \omega^2) - i\gamma_j\omega} E(t) \equiv \epsilon_o \chi_e E(t)$$

Bound electrons

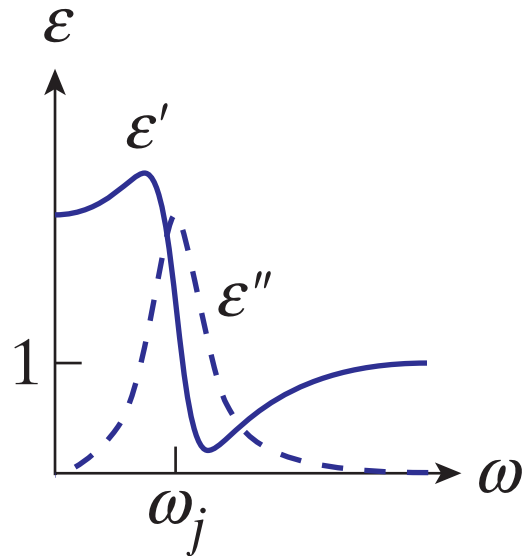
Dielectric function

$$\epsilon(\omega) \equiv 1 + \chi_e = 1 + \frac{Ne^2}{\epsilon_0 m} \sum_j \frac{f_j}{(\omega_j^2 - \omega^2) - i\gamma_j \omega}$$

Bound electrons

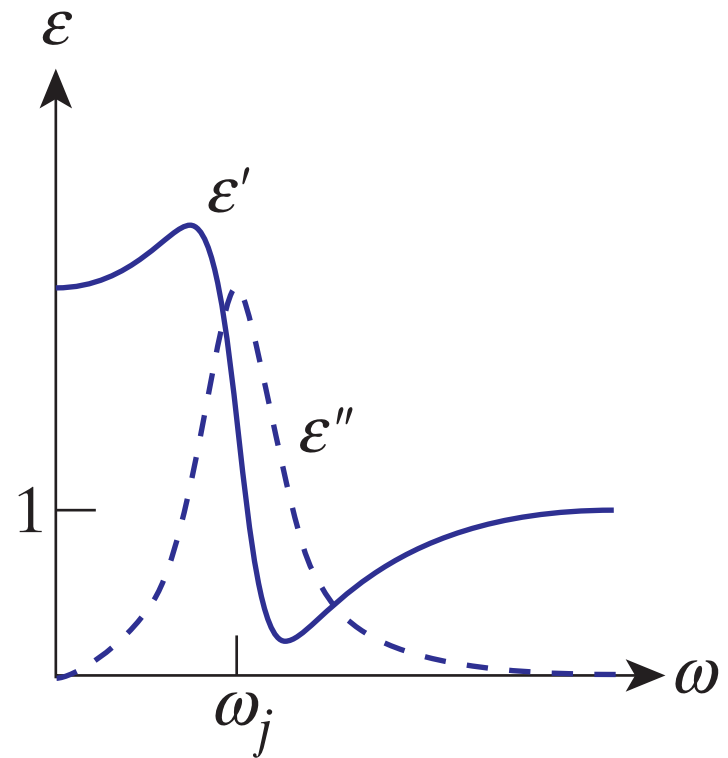
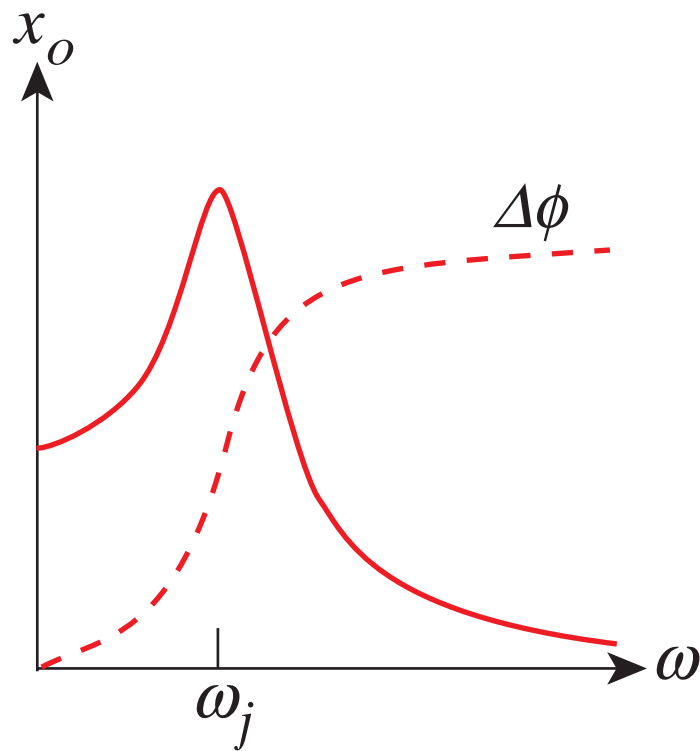
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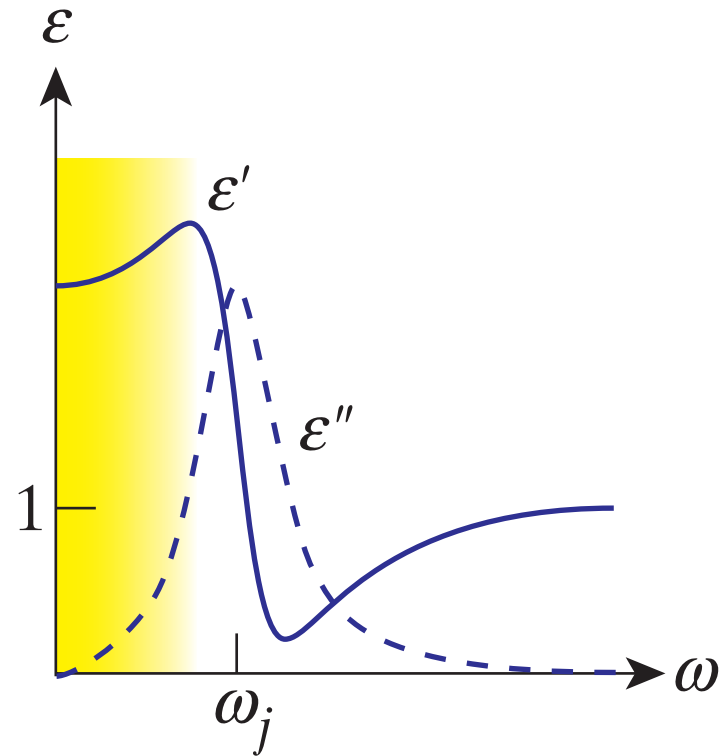
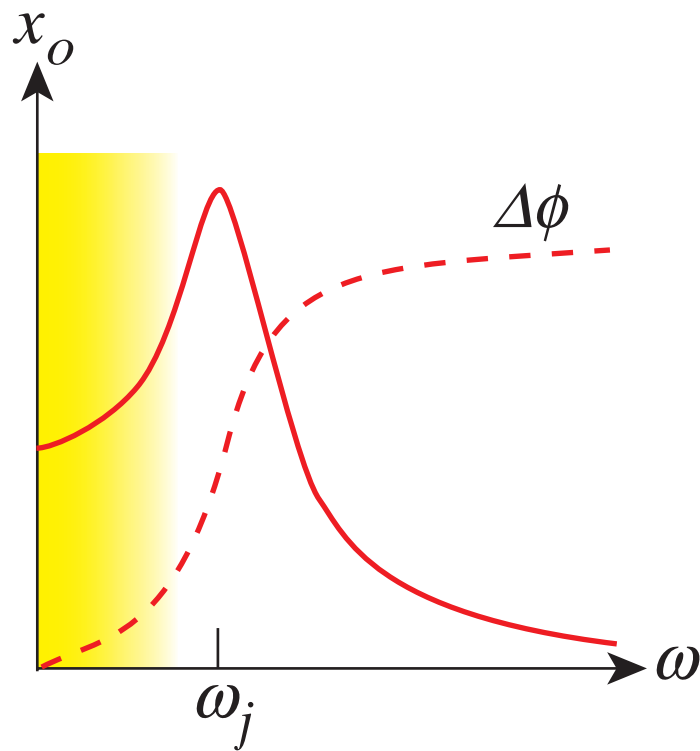
Bound electrons

amplitude of bound charge oscillation



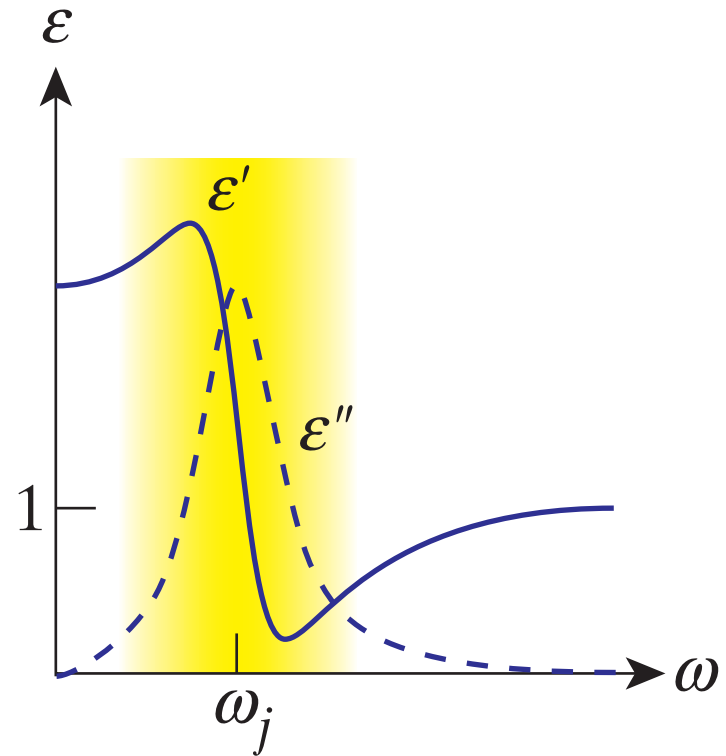
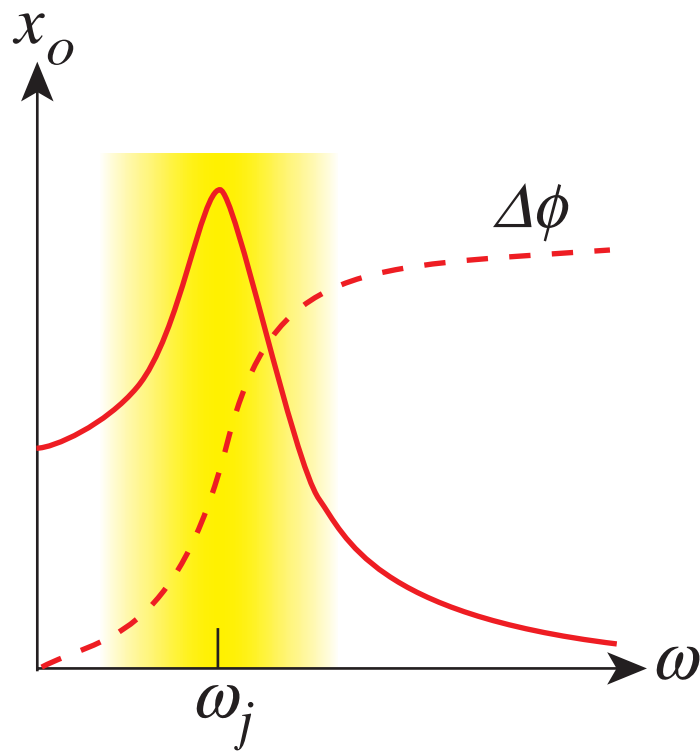
Bound electrons

Below resonance: bound charges keep up with driving field \Rightarrow field attenuated, wave propagates more slowly



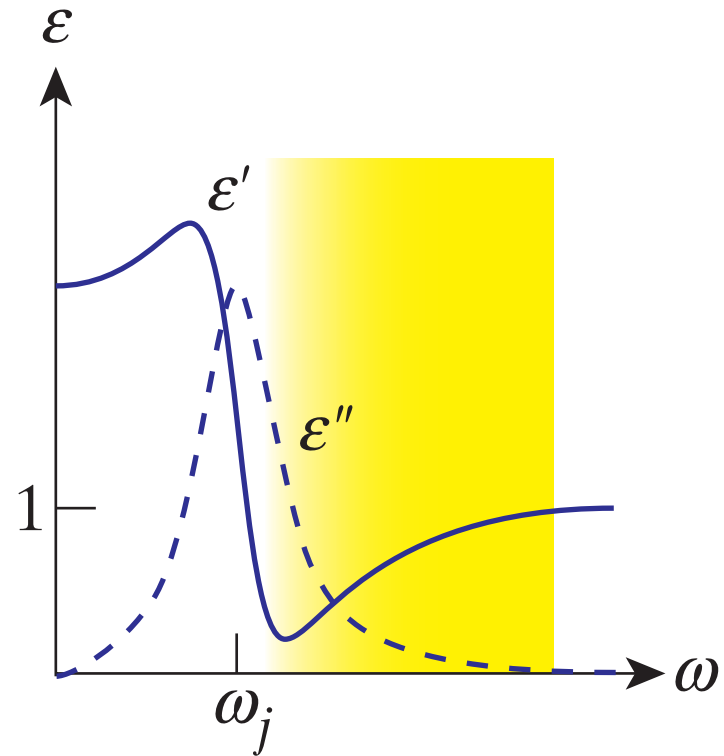
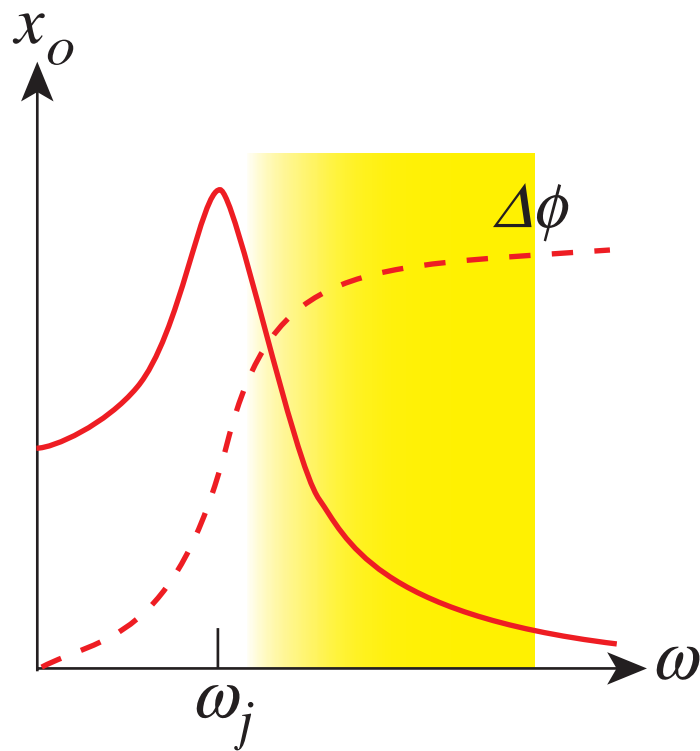
Bound electrons

At resonance: energy transfer from wave to bound charges \Rightarrow wave attenuates (absorption)



Bound electrons

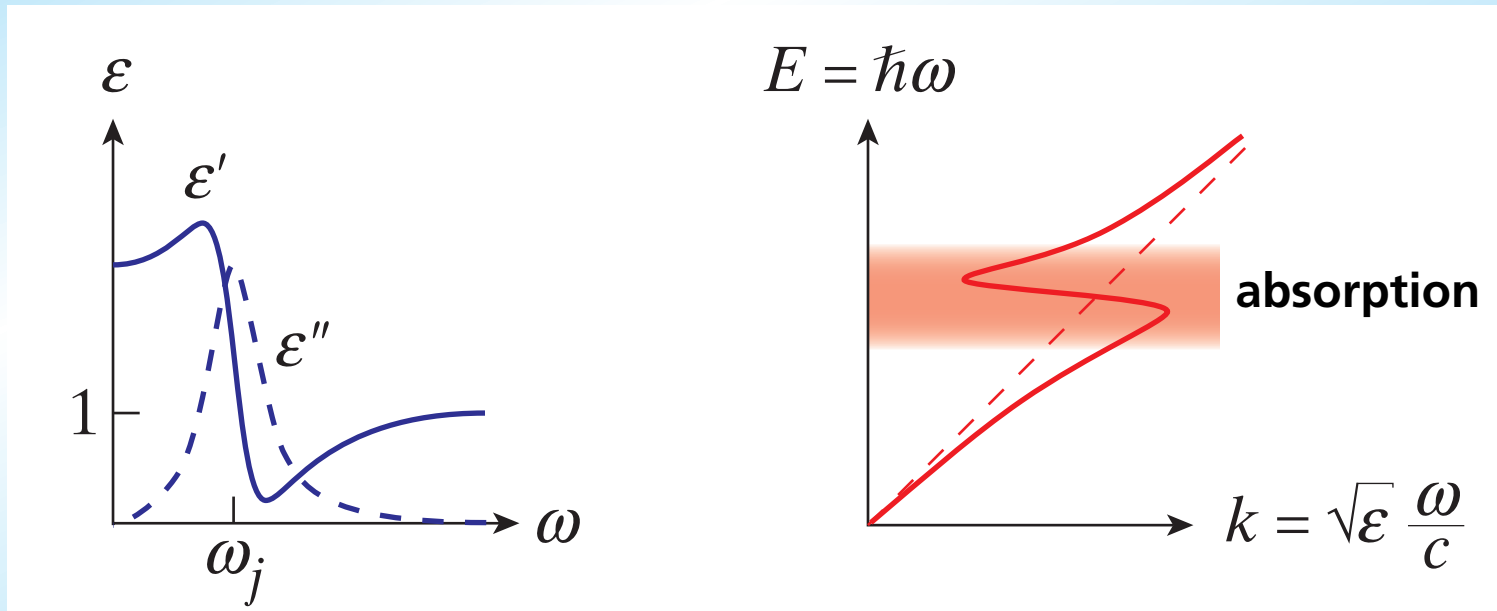
Above resonance: bound charges cannot keep up with driving field \Rightarrow dielectric like a vacuum



Bound electrons

Dielectric function

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Low frequency ($\omega \ll 1$) \Rightarrow current generated

$$J = -Ne \frac{dx}{dt} = \frac{Ne^2}{m} \frac{1}{\gamma - i\omega} E \approx \frac{Ne^2}{m\gamma} E \equiv \sigma E$$

Free electrons

$\omega \gg \gamma$: σ complex $\Rightarrow J$ out of phase with E

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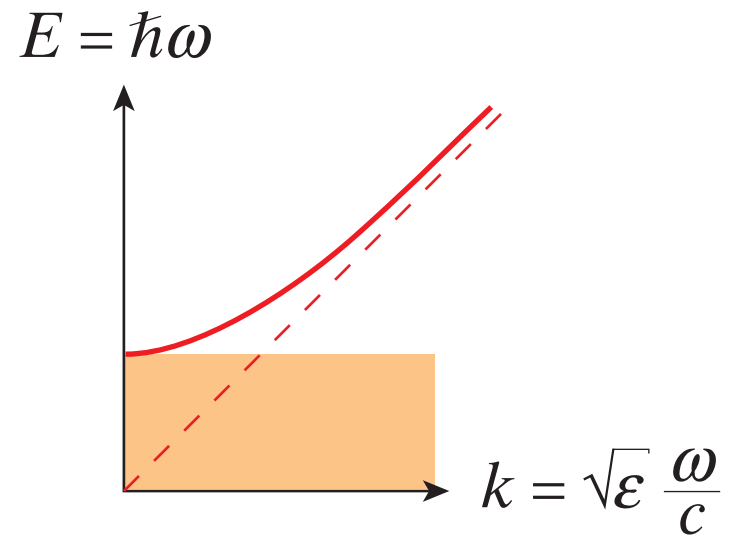
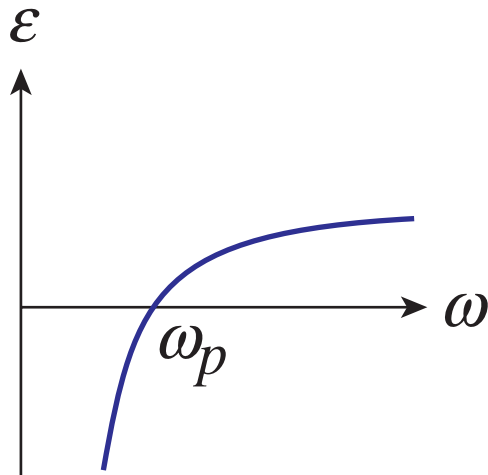
Dielectric function

$$\epsilon(\omega) \equiv 1 + \chi_e = 1 - \frac{Ne^2}{m\epsilon_0} \frac{1}{\omega^2 + i\gamma\omega} = \epsilon'(\omega) + i\epsilon''(\omega)$$

Plasma

$$\gamma \approx 0 \quad \Rightarrow \quad \epsilon'' = 0$$

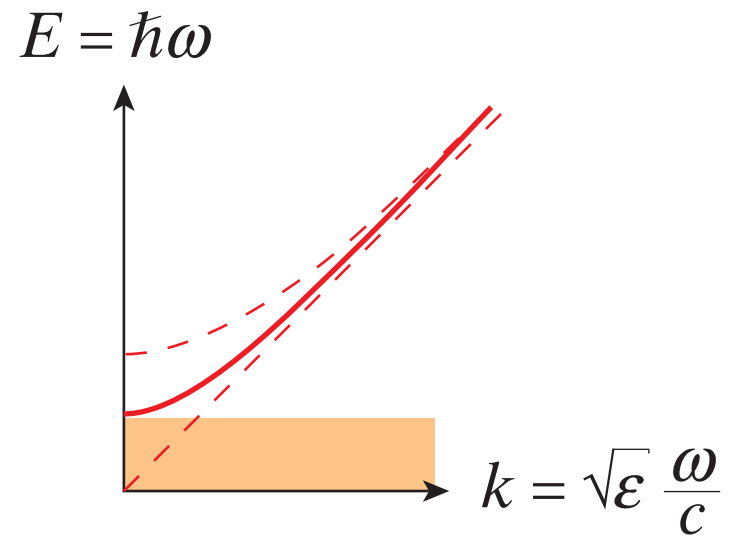
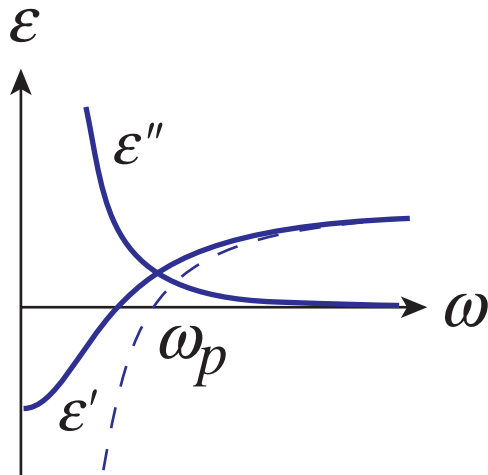
$$\epsilon'(\omega) = 1 - \frac{Ne^2}{m\epsilon_0} \frac{1}{\omega^2} \equiv 1 - \frac{\omega_p^2}{\omega^2}$$



Plasma

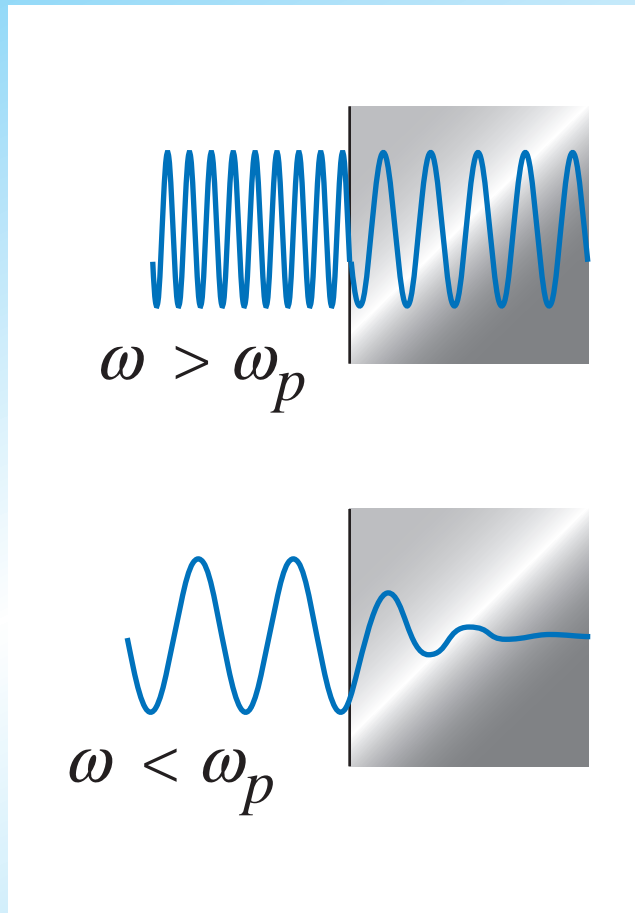
Add damping

$$\gamma \lesssim \omega_p$$



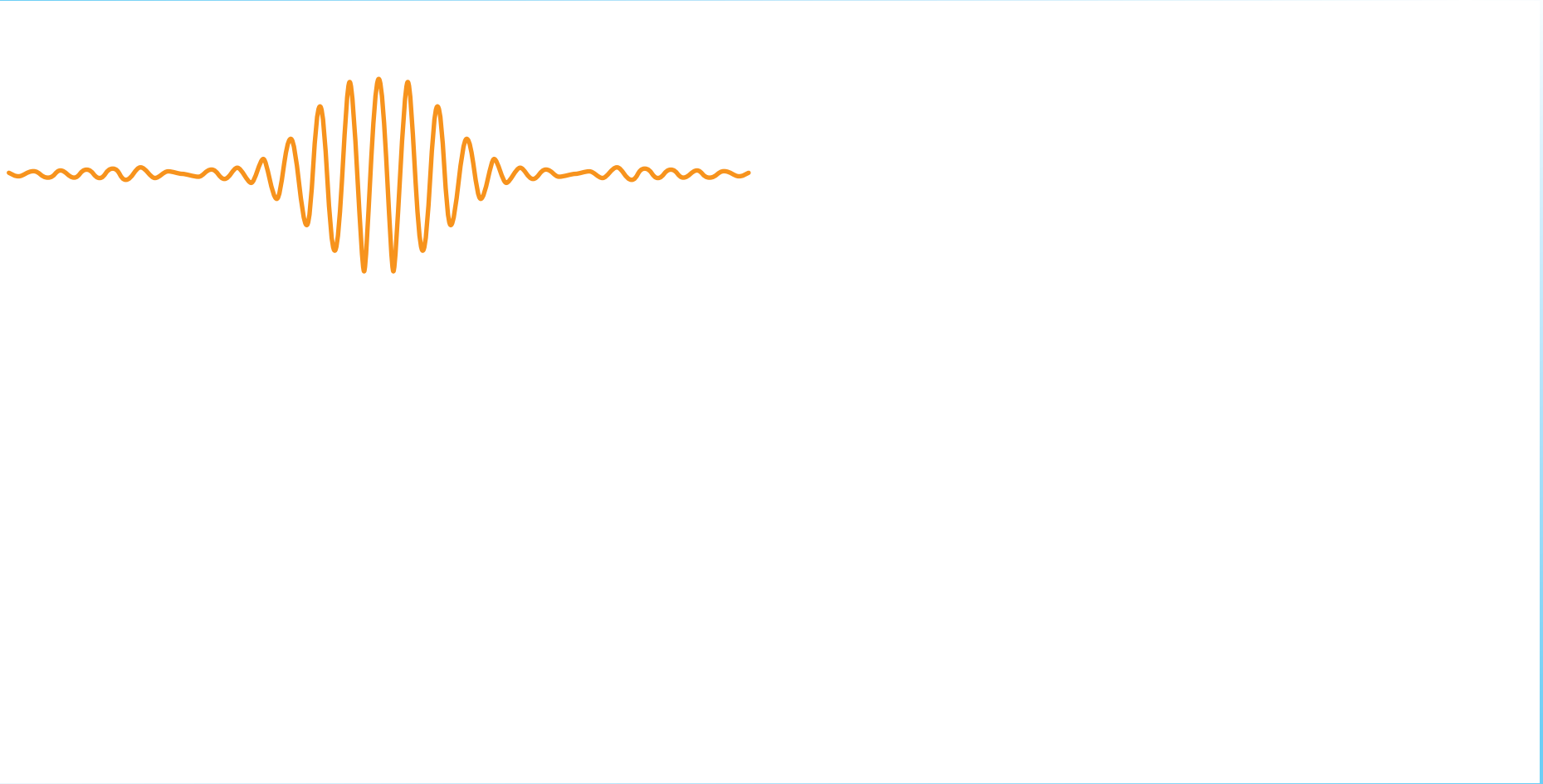
Plasma

Plasma acts like a high-pass filter:

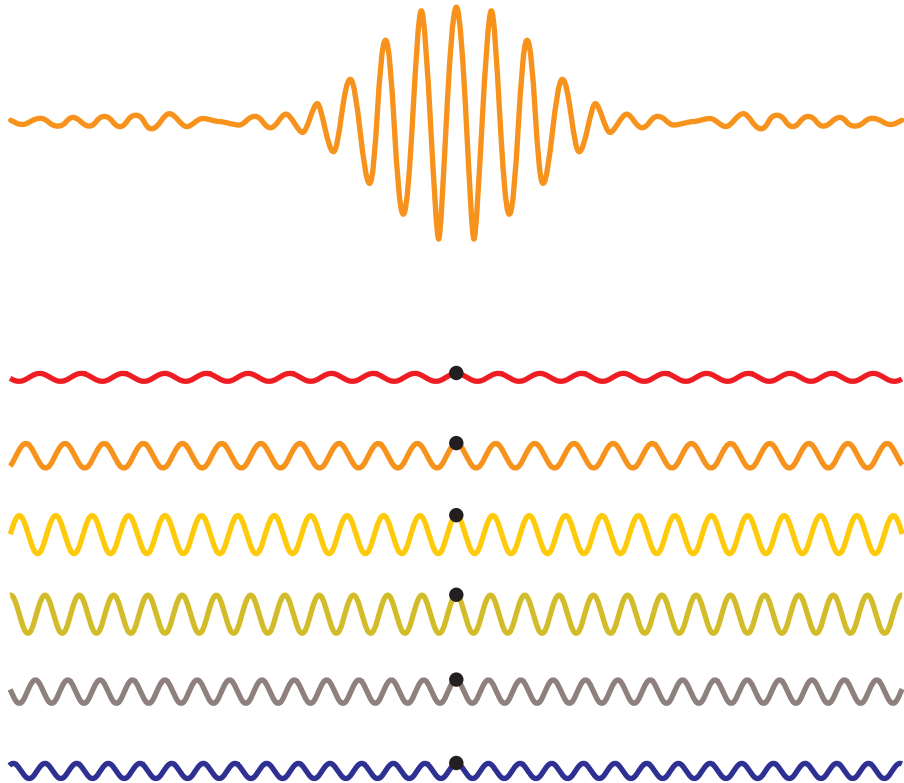


$\log N$ (cm^{-3})	ω_p (rad s^{-1})	λ_p
22	6×10^{15}	330 nm
18	6×10^{13}	33 μm
14	6×10^{11}	3.3 mm
10	6×10^9	0.33 m

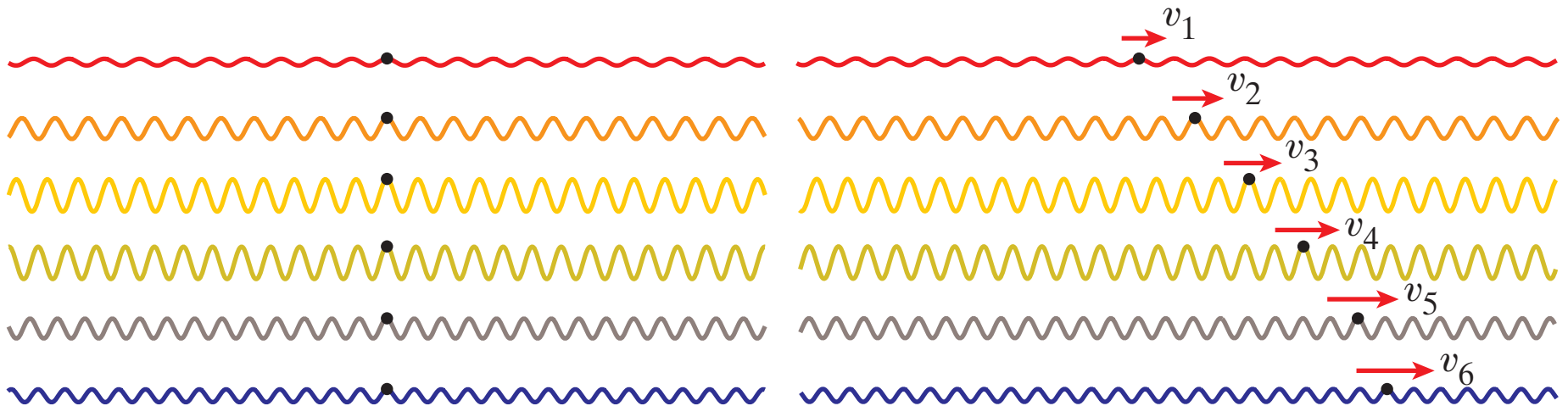
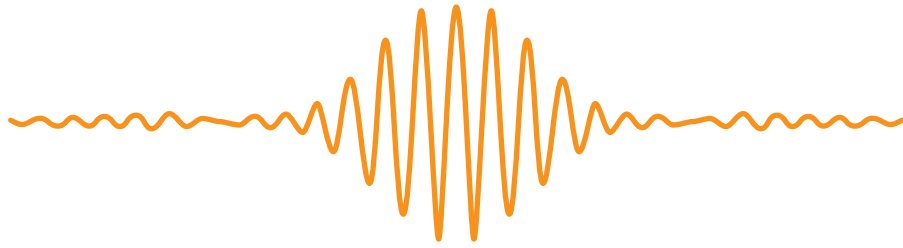
Pulse dispersion



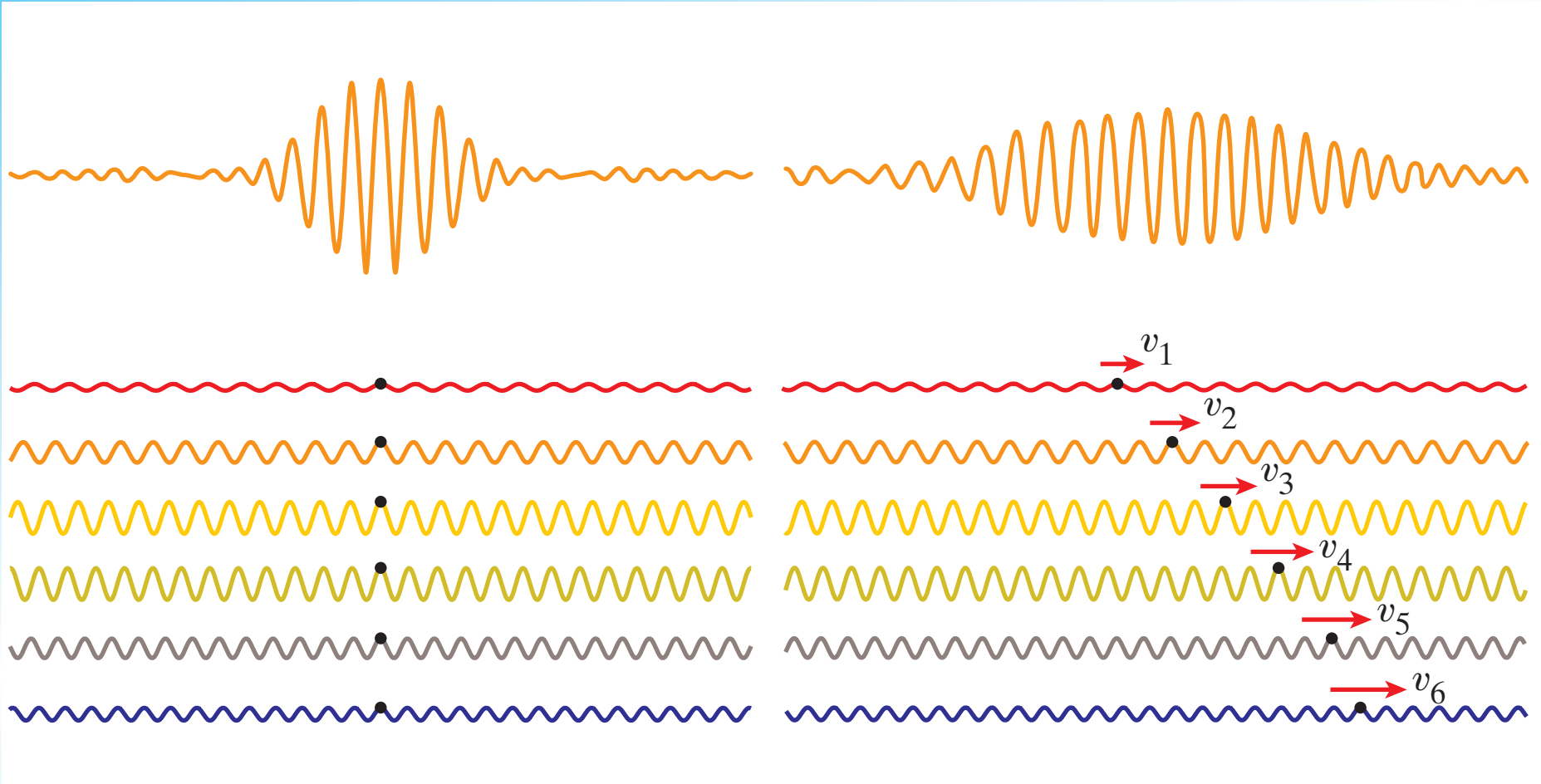
Pulse dispersion



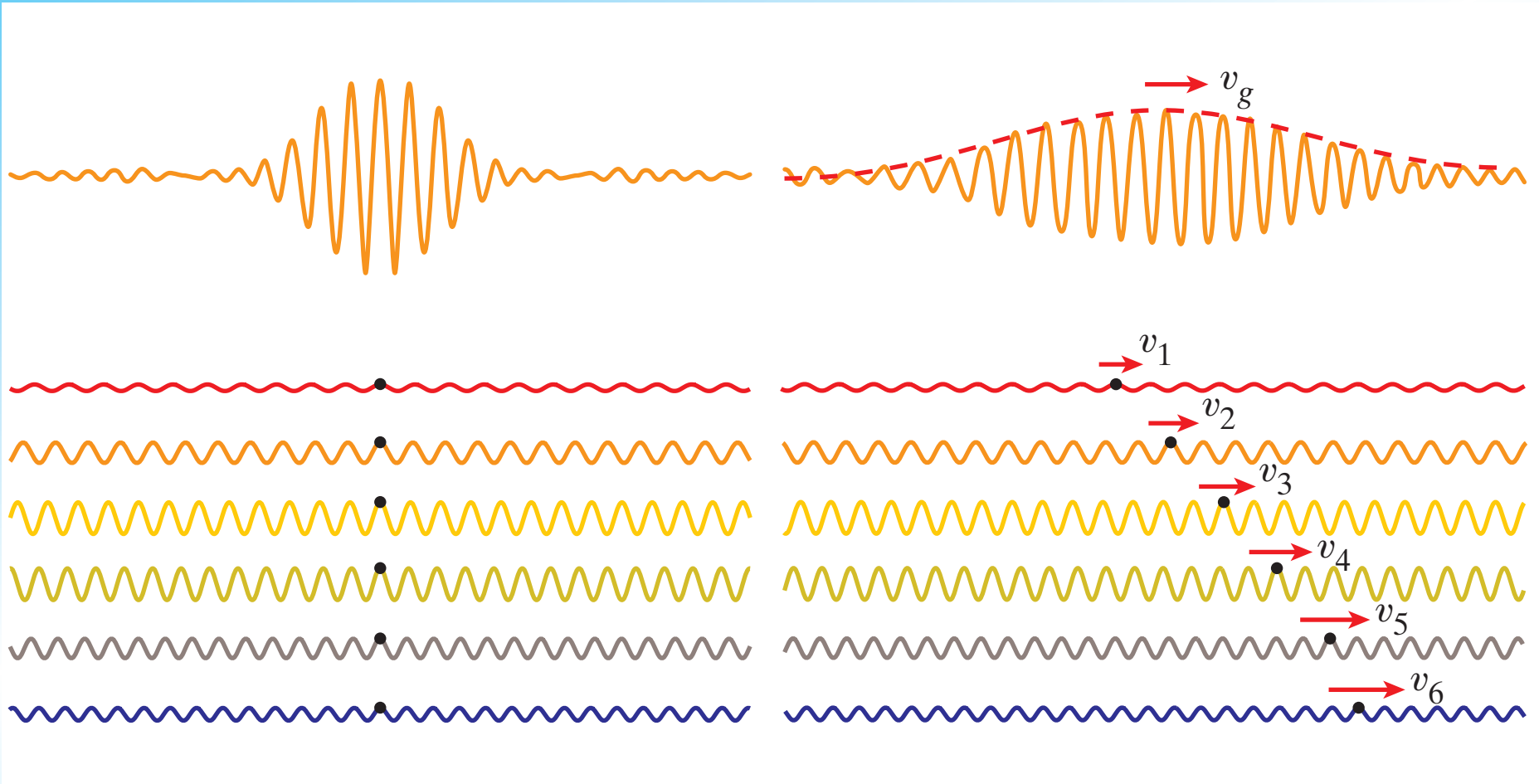
Pulse dispersion



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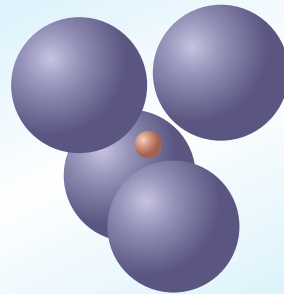
Pulse dispersion



Nonlinear optics

Linear response

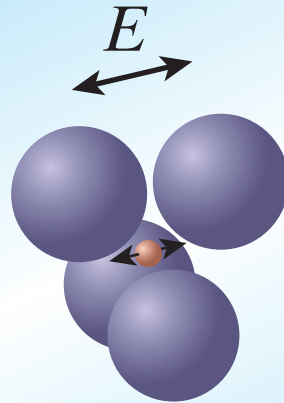
$$P(t) = \epsilon_0 \chi_e E(t)$$



Nonlinear optics

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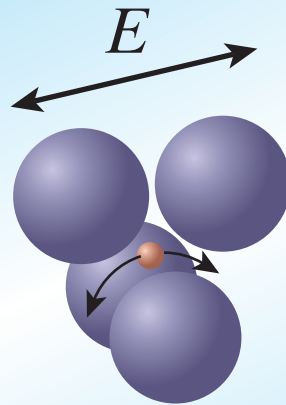
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Nonlinear optics

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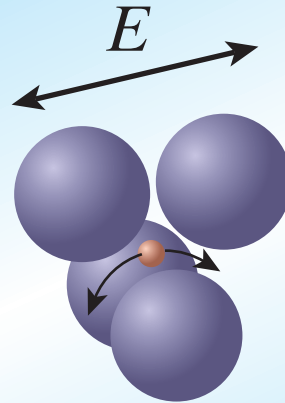
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Nonlinear optics

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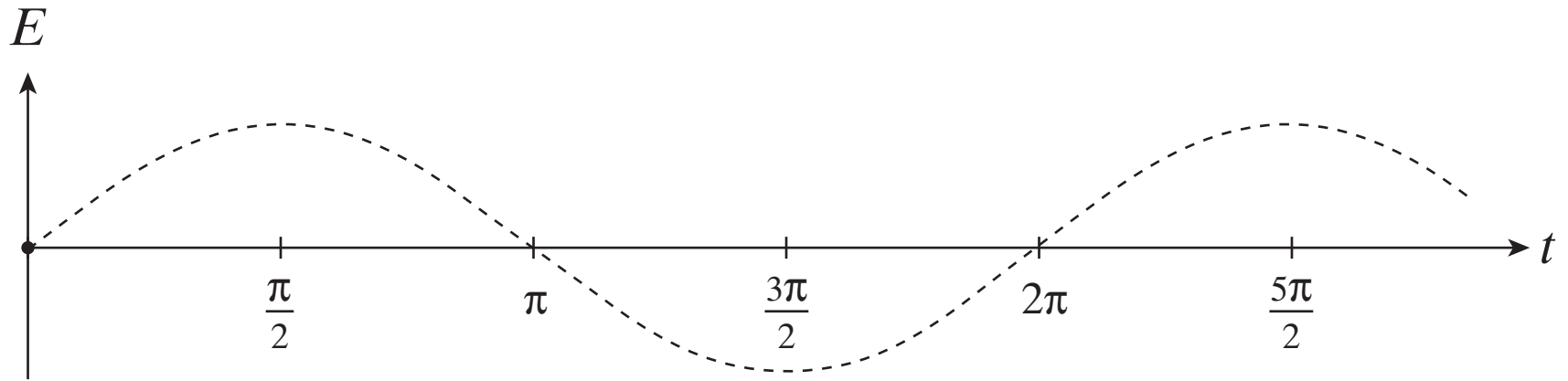


Nonlinear polarization:

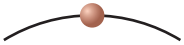
$$P = \chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \dots$$

$$P = P^{(1)} + P^{(2)} + P^{(3)} + \dots$$

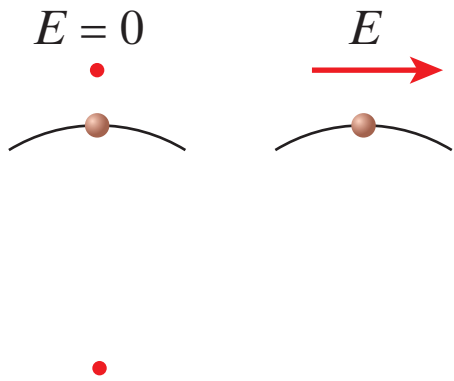
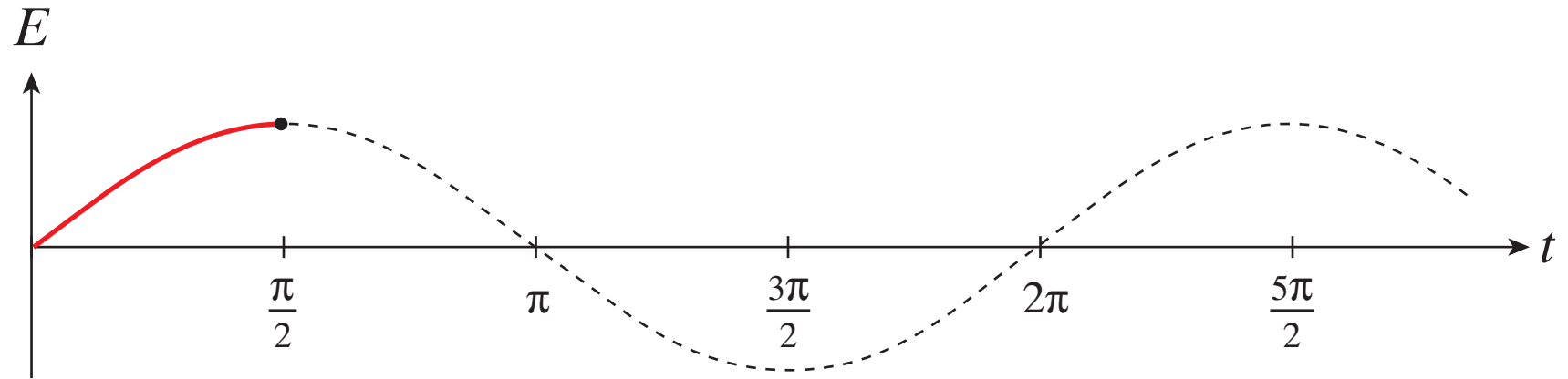
Nonlinear optics



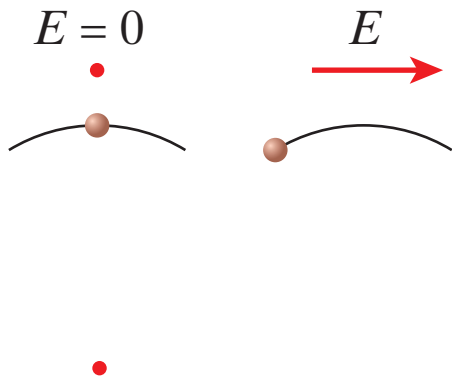
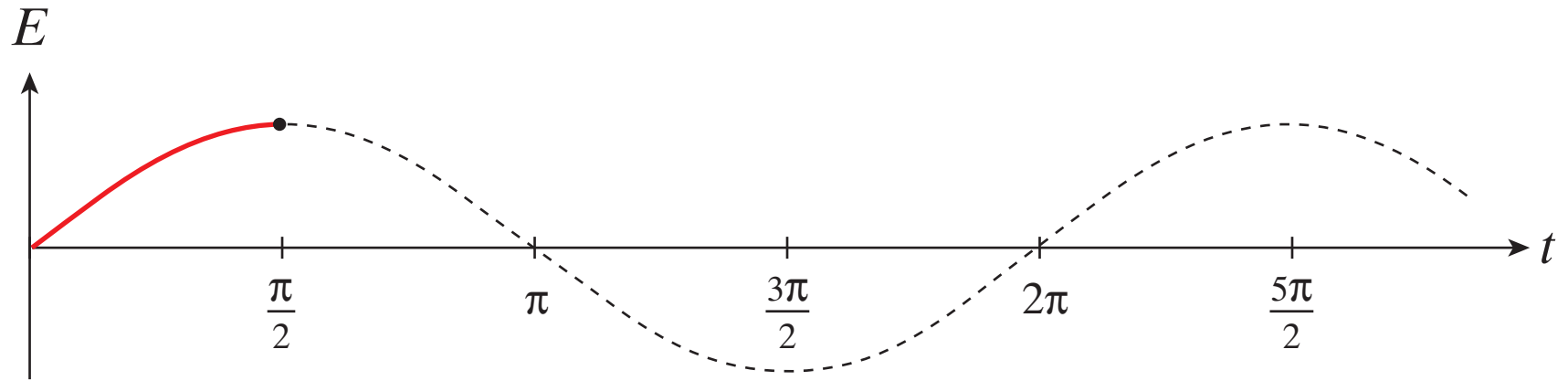
$E = 0$



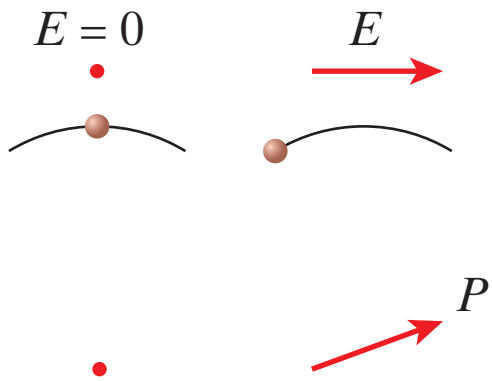
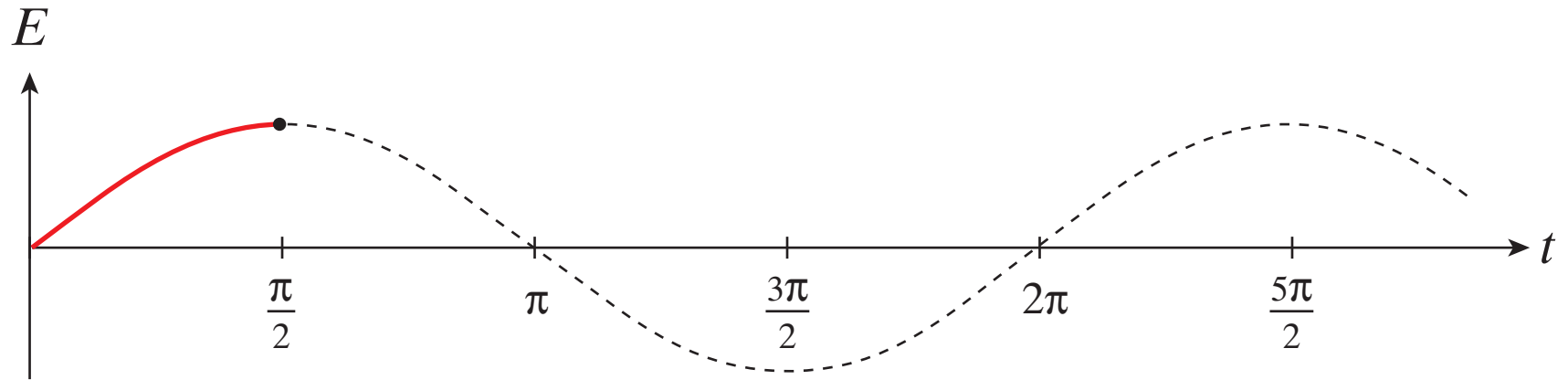
Nonlinear optics



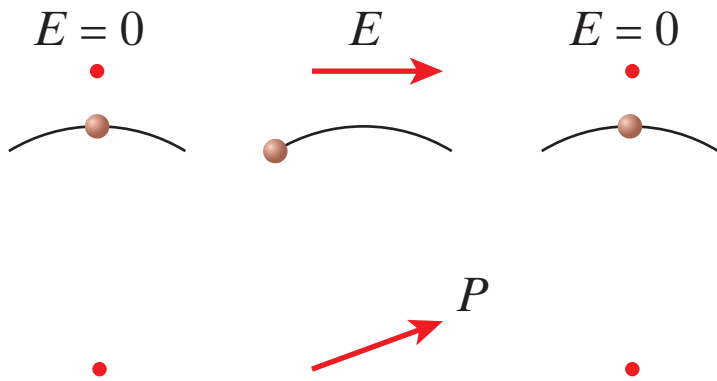
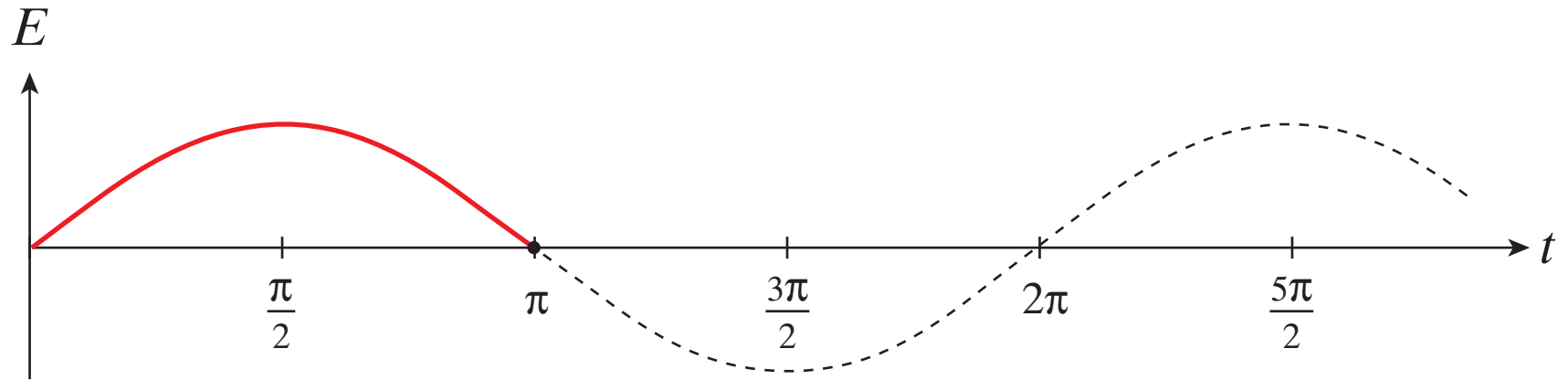
Nonlinear optics



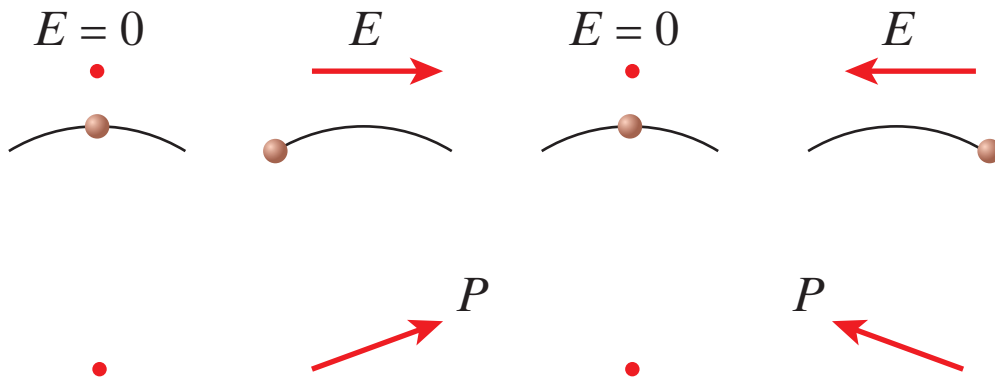
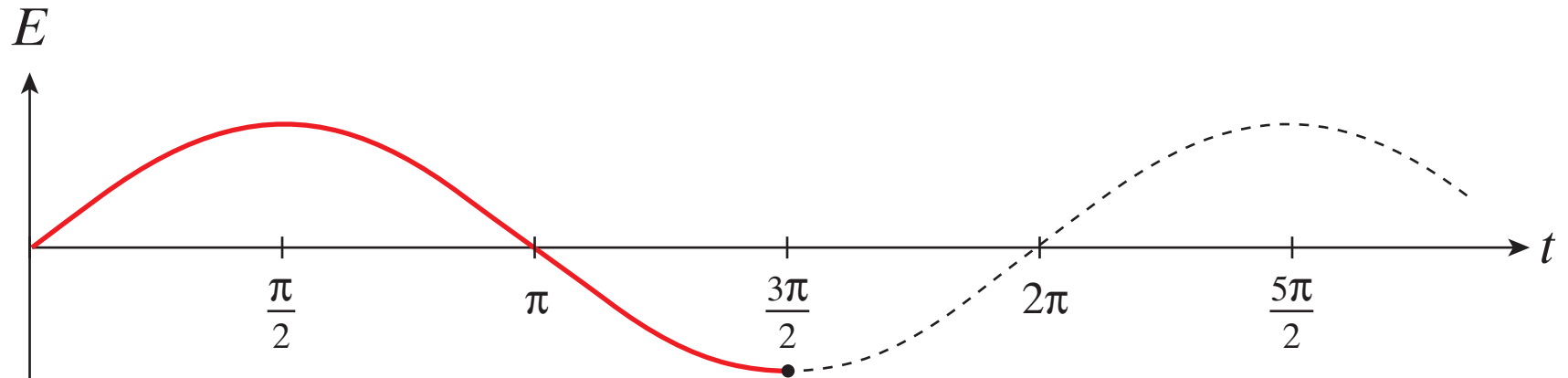
Nonlinear optics



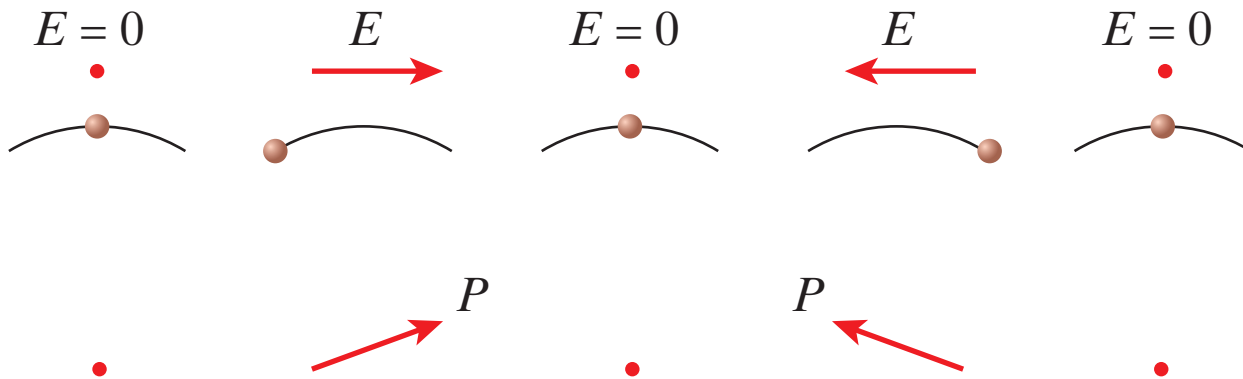
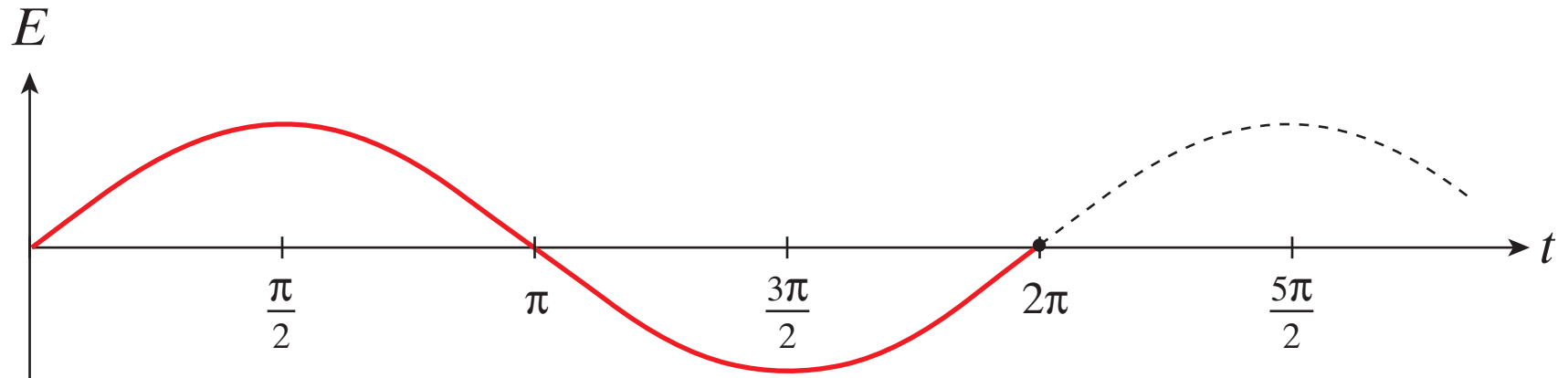
Nonlinear optics



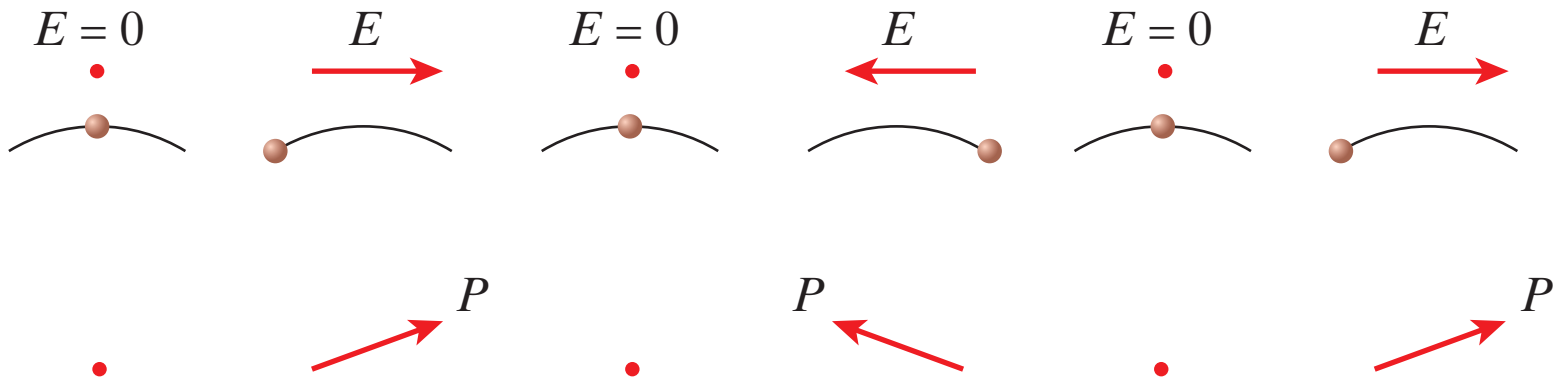
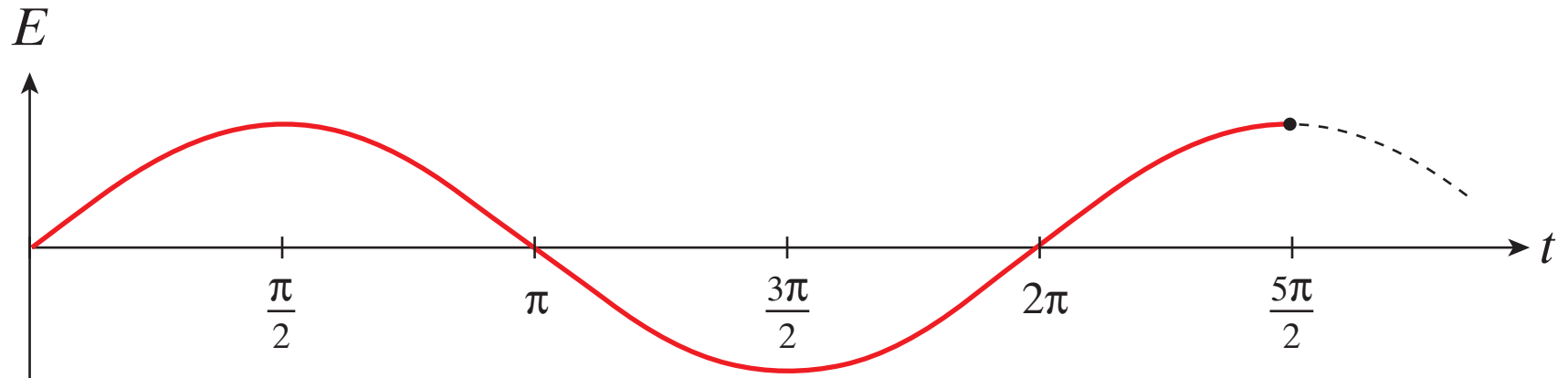
Nonlinear optics



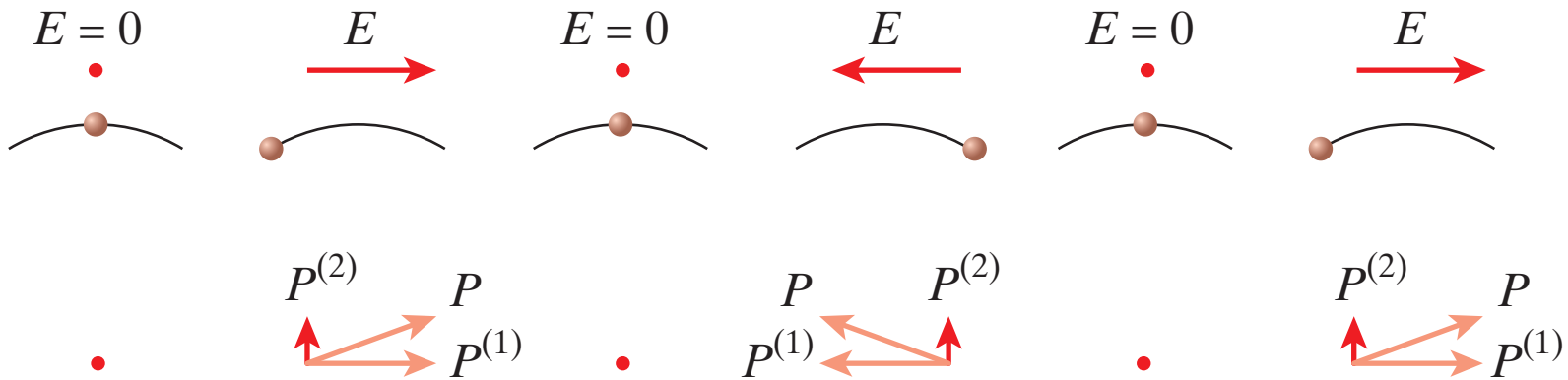
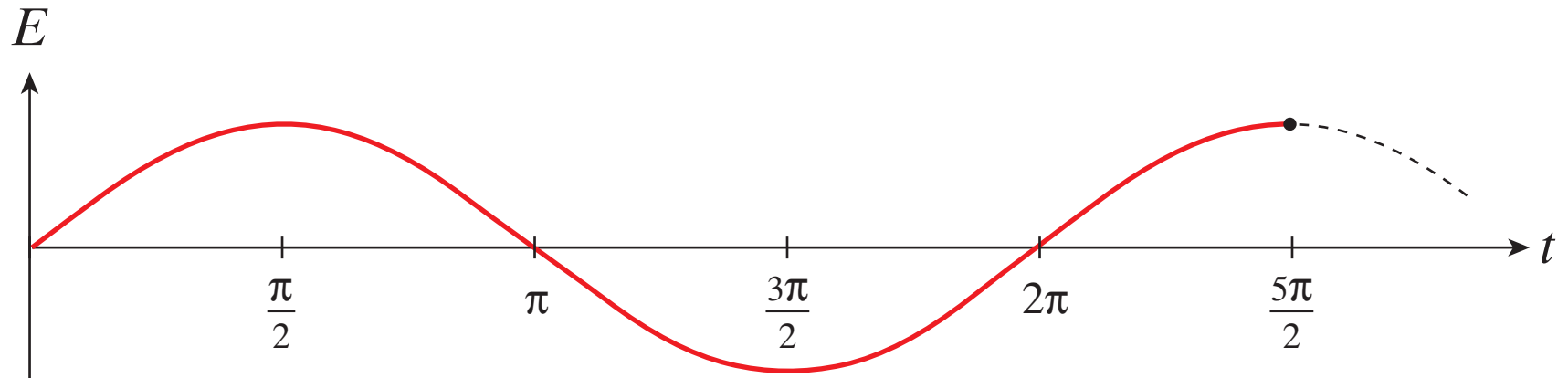
Nonlinear optics



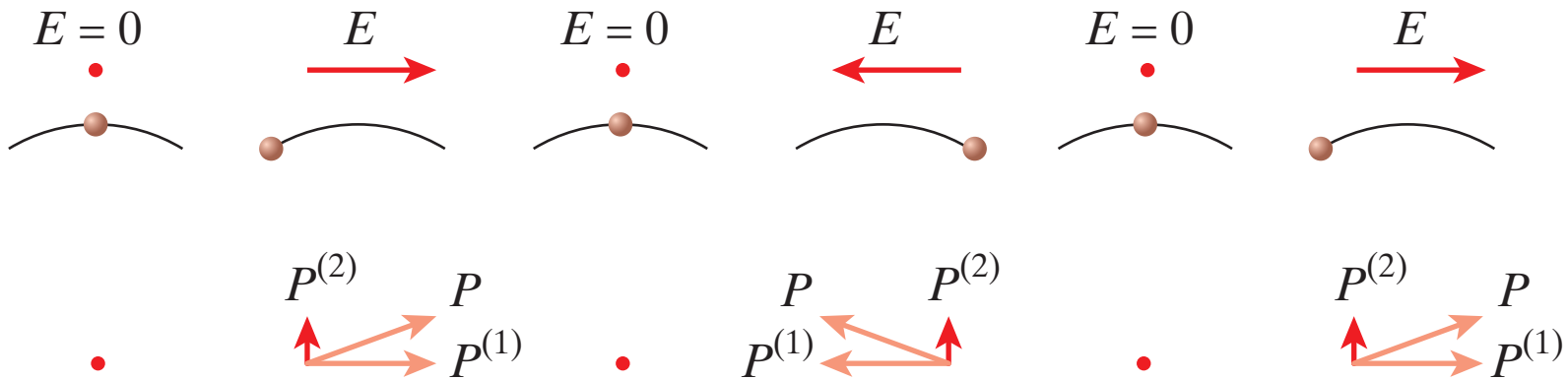
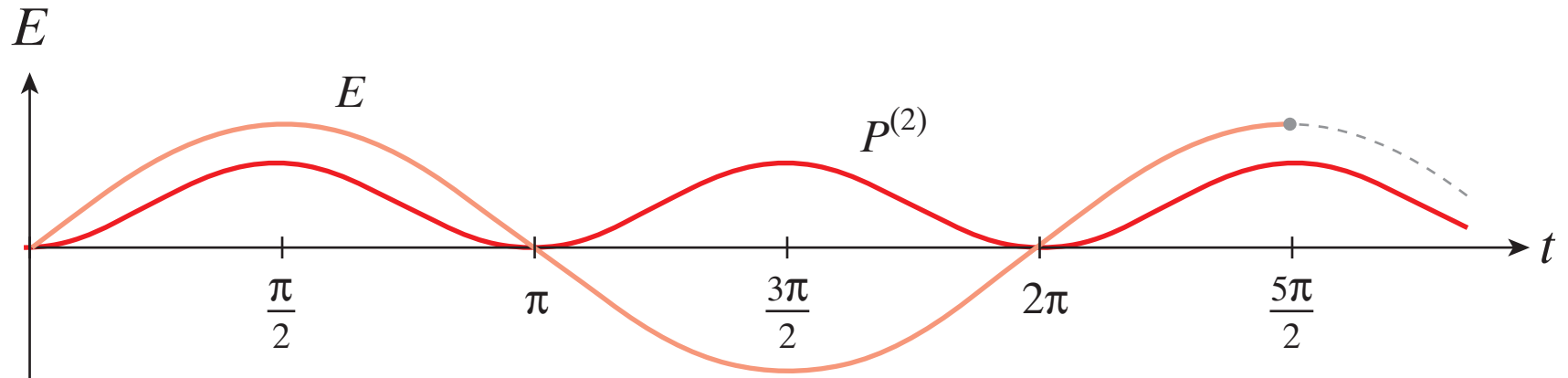
Nonlinear optics



Nonlinear optics



Nonlinear optics



Nonlinear optics

In medium with inversion symmetry

$$\vec{P}^{(2)} = \vec{\chi}^{(2)} : \vec{E} \vec{E} \quad \Rightarrow \quad -\vec{P}^{(2)} = \vec{\chi}^{(2)} : (-\vec{E})(-\vec{E})$$

Nonlinear optics

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and so

$$\chi^{(2)} = -\chi^{(2)} = 0$$

Nonlinear optics

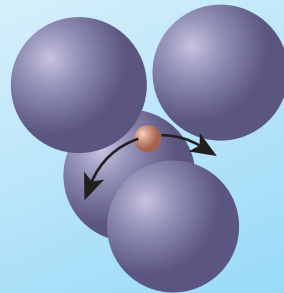
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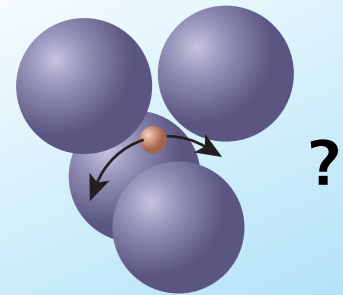
$$\chi^{(2)} = -\chi^{(2)} = 0$$

... but ...

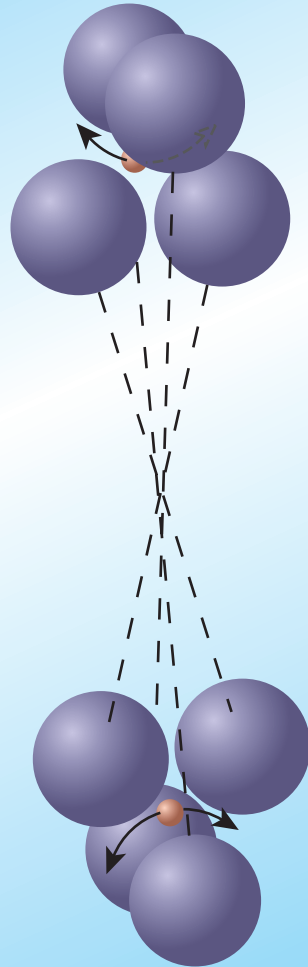


Nonlinear optics

How to reconcile $\chi^{(2)} = -\chi^{(2)} = 0$ with



Nonlinear optics



Nonlinear optics

Nonlinear polarization:

$$P = \chi^{(1)}E + \chi^{(3)}E^3 + \dots$$

Nonlinear optics

Nonlinear polarization:

$$P = \chi^{(1)}E + \chi^{(3)}E^3 + \dots$$

Third order polarization

$$P^{(3)}(t) = \chi^{(3)}E(t)E^*(t)E(t) = \chi^{(3)}I(t)E(t)$$

Nonlinear optics

Nonlinear polarization:

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Nonlinear optics

Nonlinear polarization:

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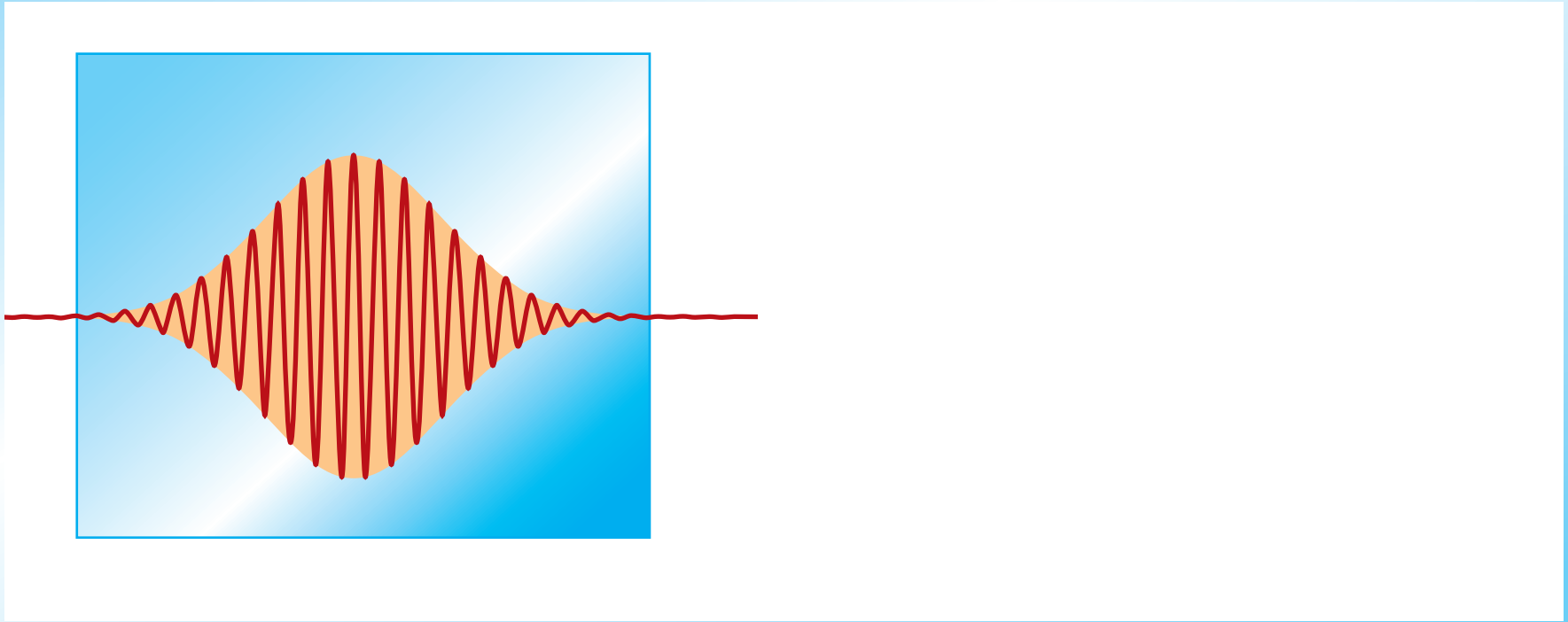
and so
$$P = P^{(1)} + P^{(3)} = (\chi^{(1)} + \chi^{(3)}I)E \equiv \chi_{eff}E$$

$$n = \sqrt{\epsilon} = \sqrt{1 + \chi_{eff}} \approx \sqrt{1 + \chi^{(1)}} + \frac{1}{2} \frac{\chi^{(3)}I}{\sqrt{1 + \chi^{(1)}}} = n_o + n_2I$$

Nonlinear optics

Intensity dependent index of refraction:

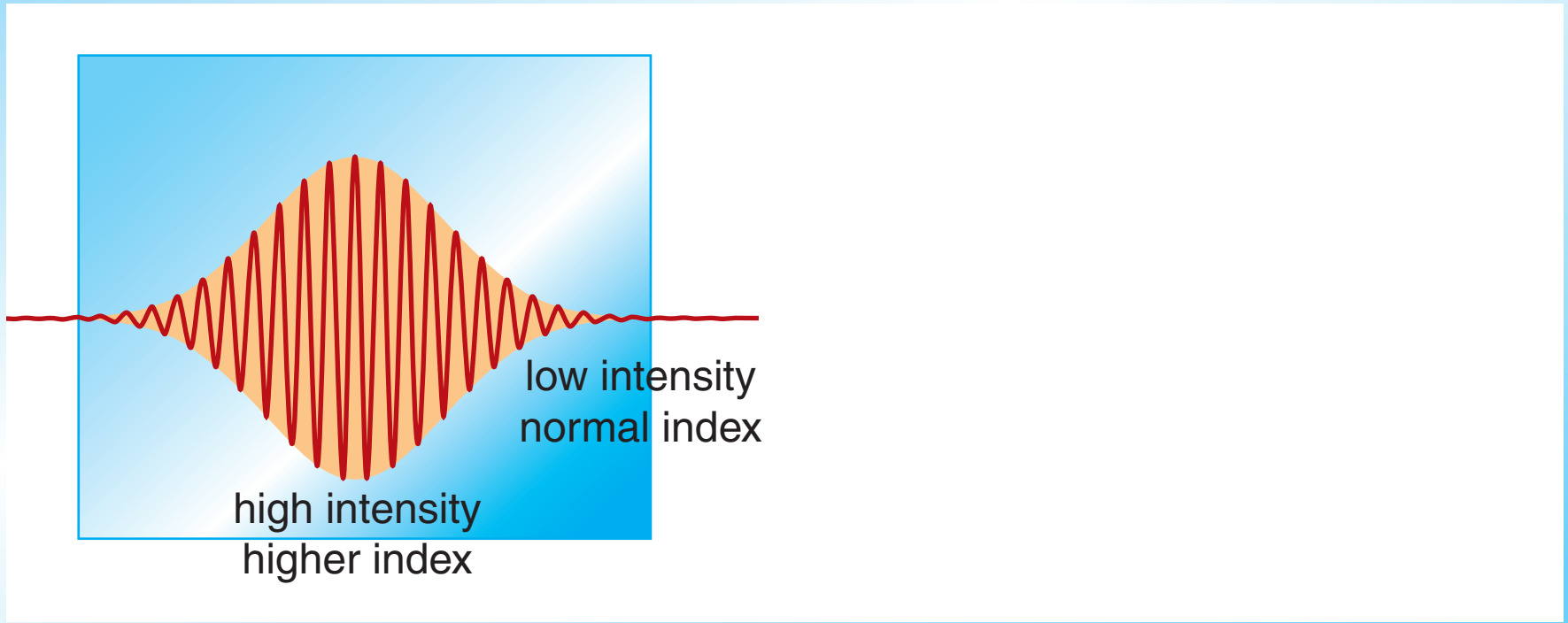
$$n = n_o + n_2 I$$



Nonlinear optics

Intensity dependent index of refraction:

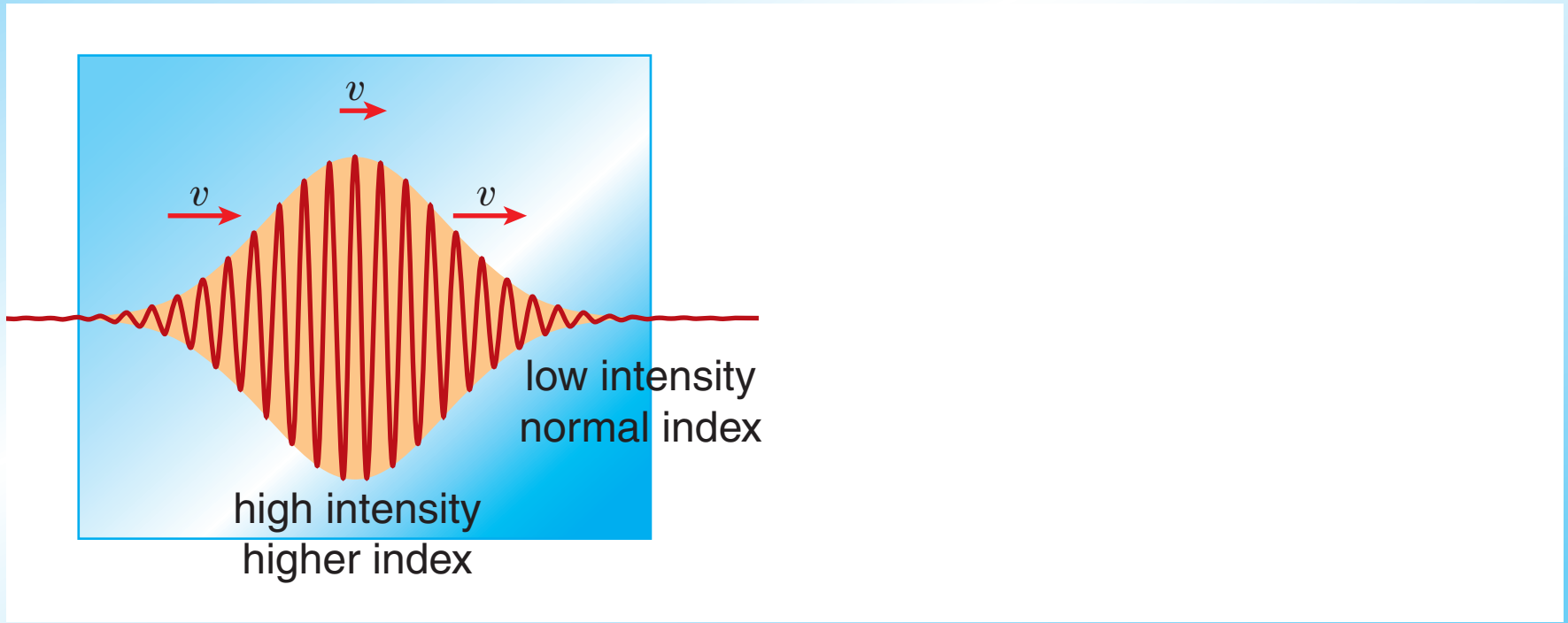
$$n = n_o + n_2 I$$



Nonlinear optics

Intensity dependent index of refraction:

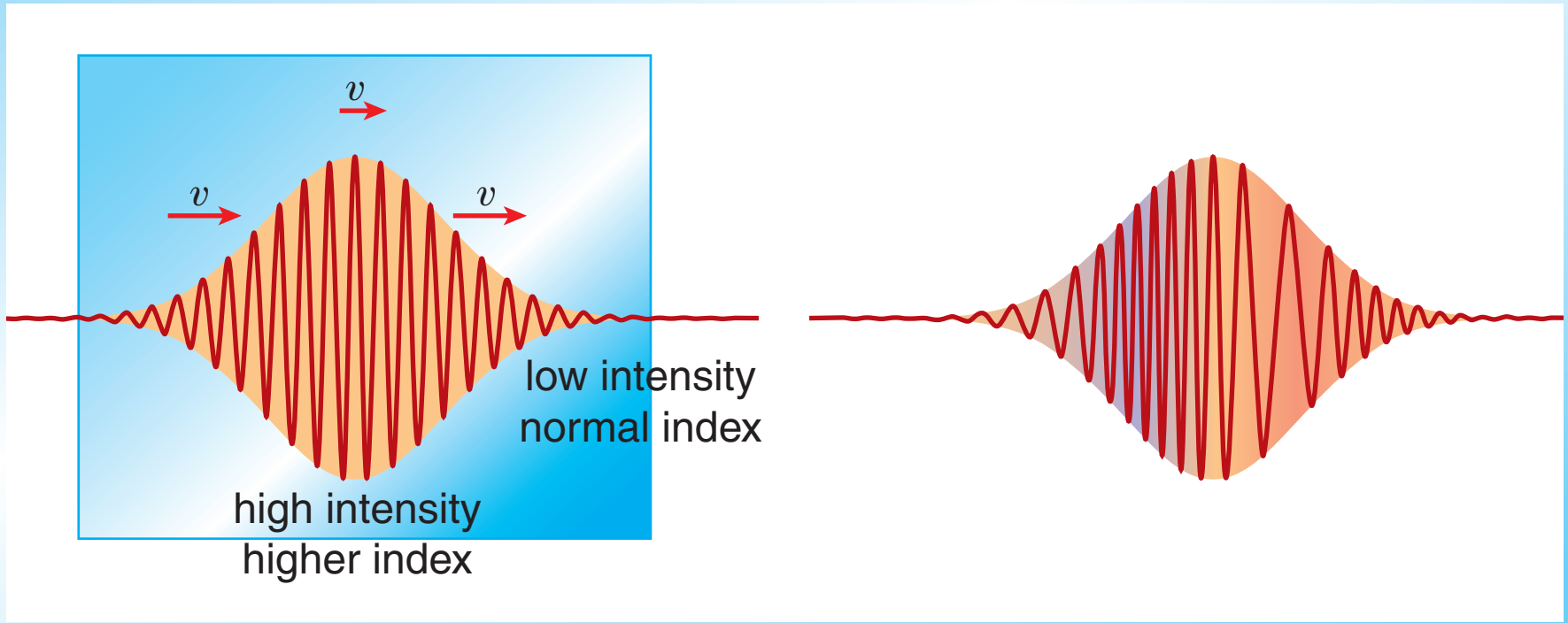
$$n = n_o + n_2 I$$



Nonlinear optics

Intensity dependent index of refraction:

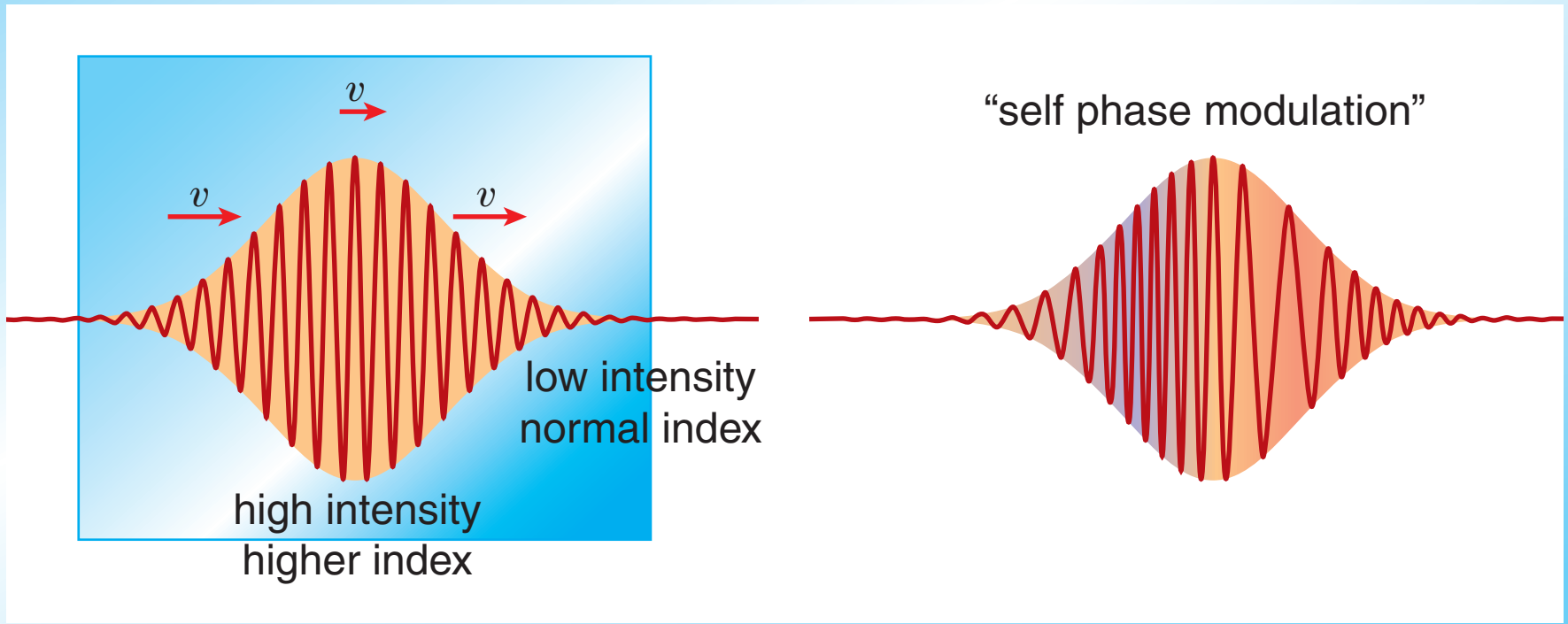
$$n = n_o + n_2 I$$



Nonlinear optics

Intensity dependent index of refraction:

$$n = n_o + n_2 I$$



Nonlinear optics

Phase:

$$\frac{\phi}{2\pi} = \frac{nL}{\lambda}$$

Nonlinear optics

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Nonlinear optics

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Frequency change:

$$\Delta\omega = -\frac{d\phi}{dt} = \frac{-2\pi}{\lambda} L n_2 \frac{dI}{dt}$$

Nonlinear optics

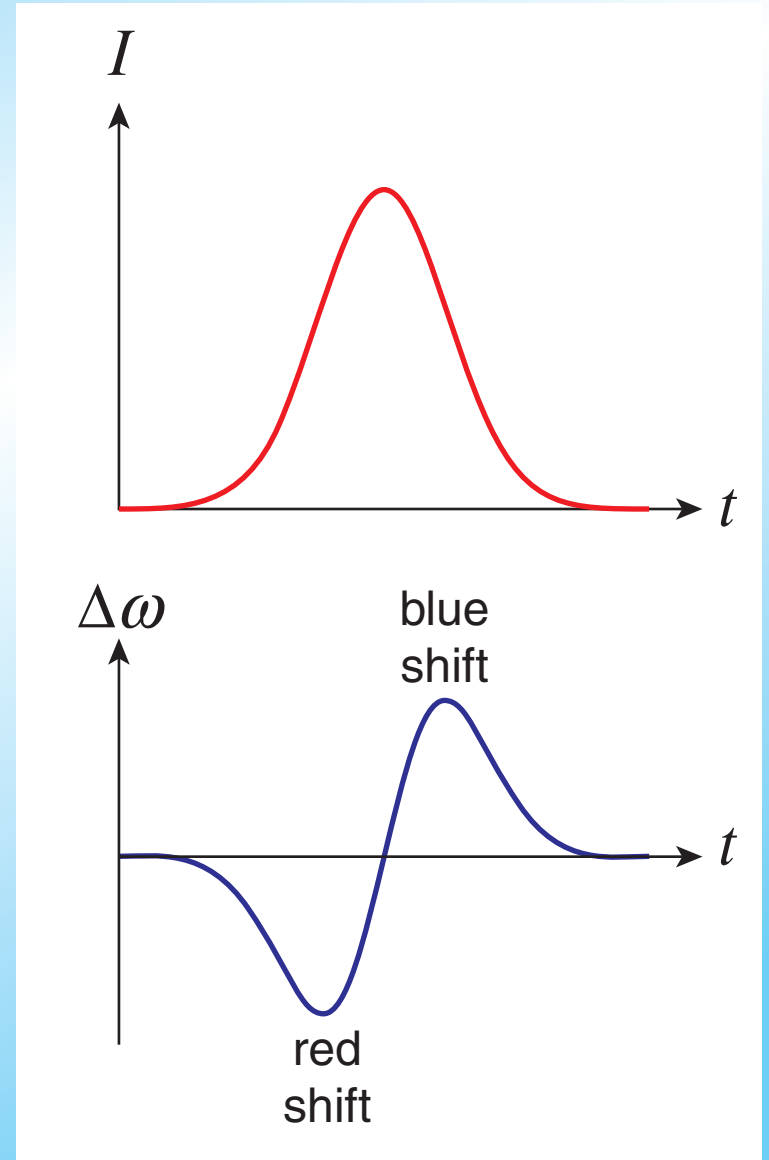
Phase:

$$\frac{\phi}{2\pi} = \frac{nL}{\lambda}$$

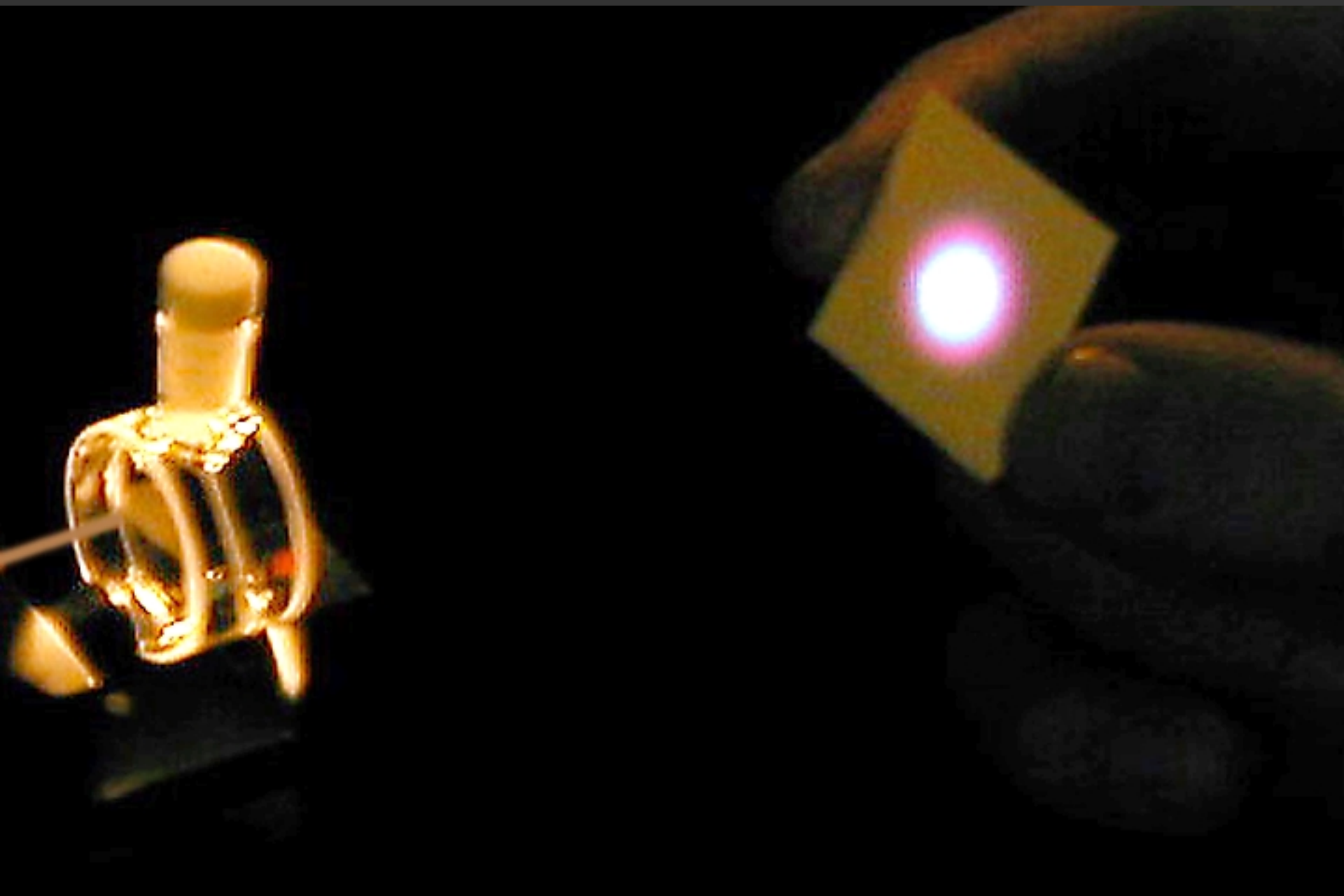
$$\phi = \frac{2\pi}{\lambda} L(n_o + n_2 I)$$

Frequency change:

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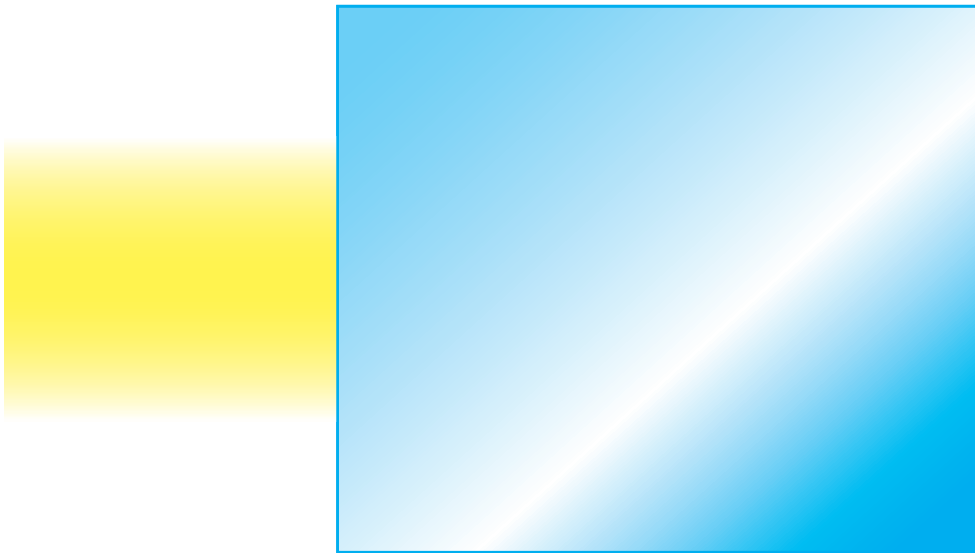


Nonlinear optics



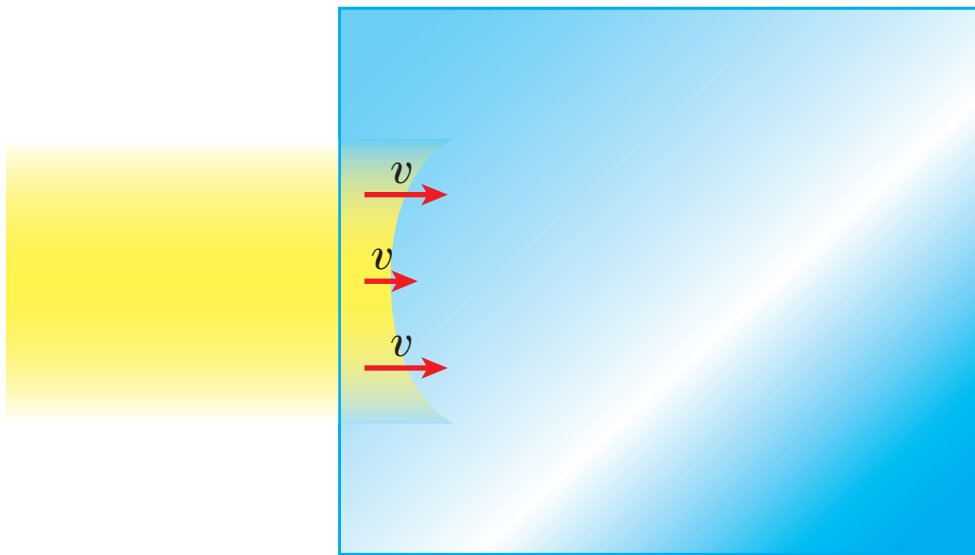
Nonlinear optics

Spatial intensity profile...



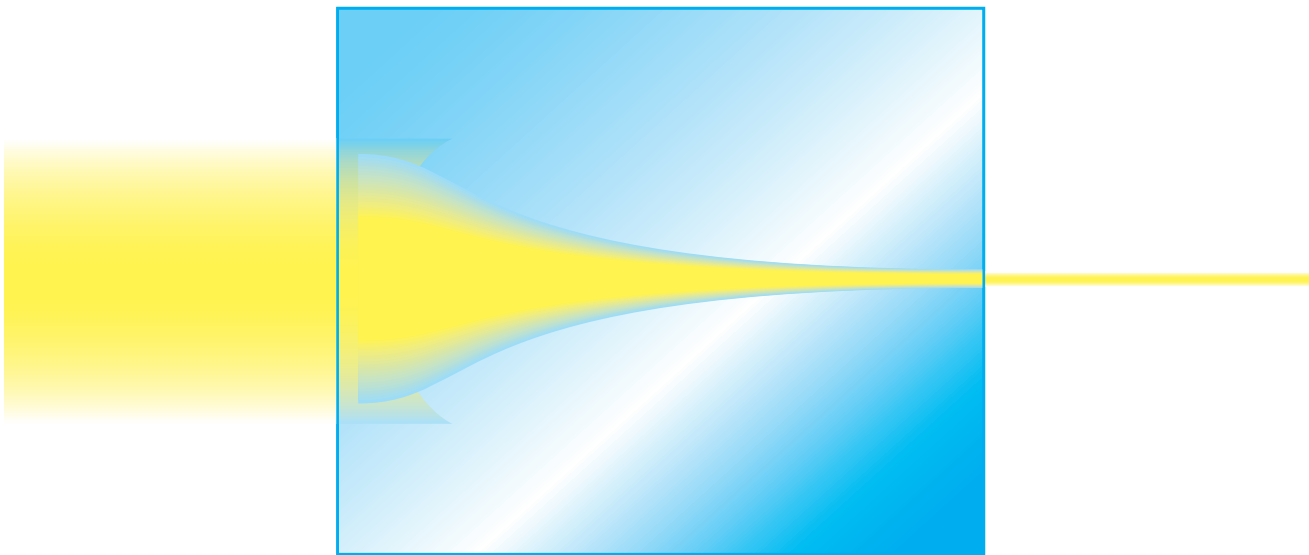
Nonlinear optics

Spatial intensity profile...



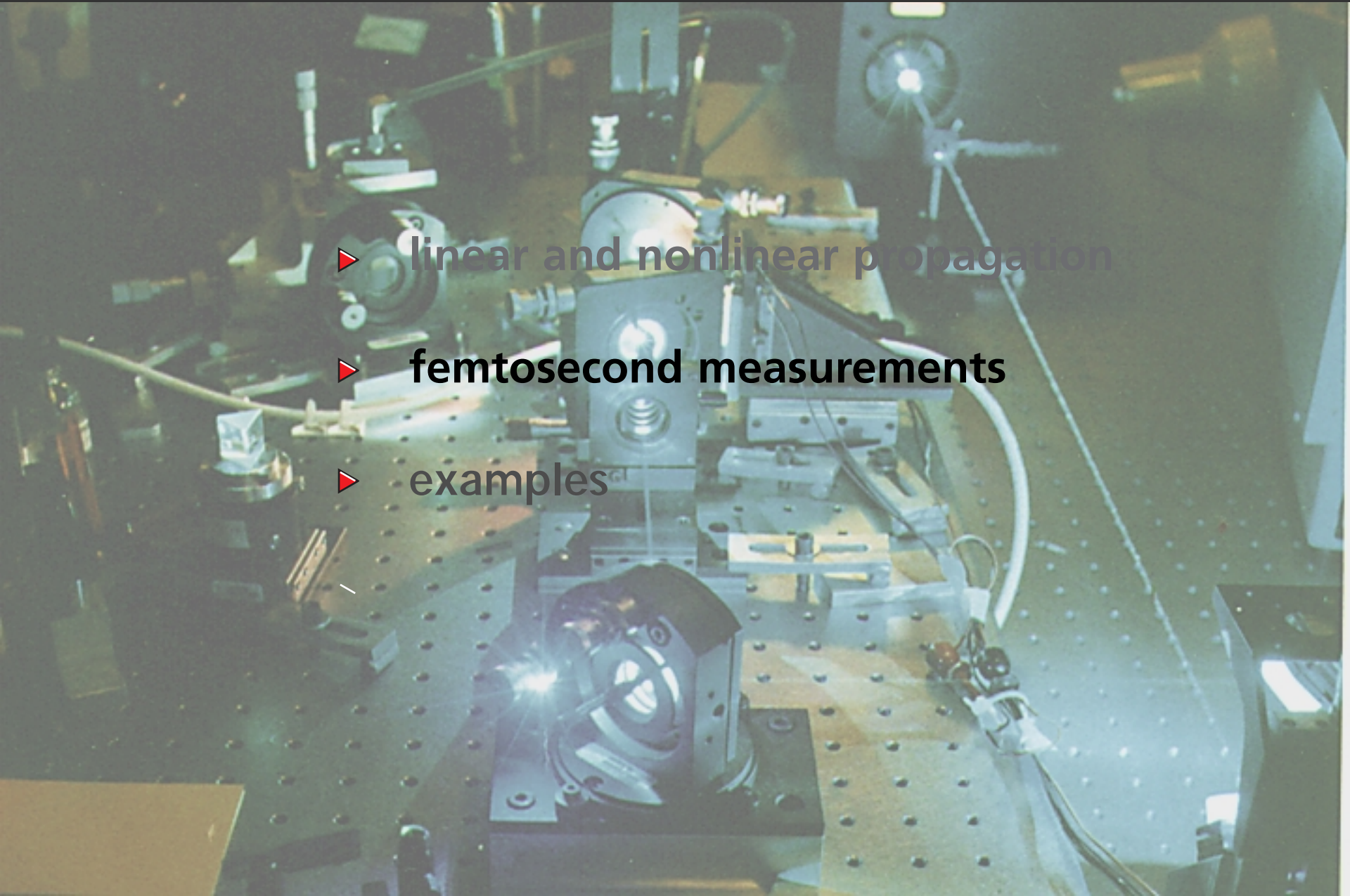
Nonlinear optics

...causes self-focusing



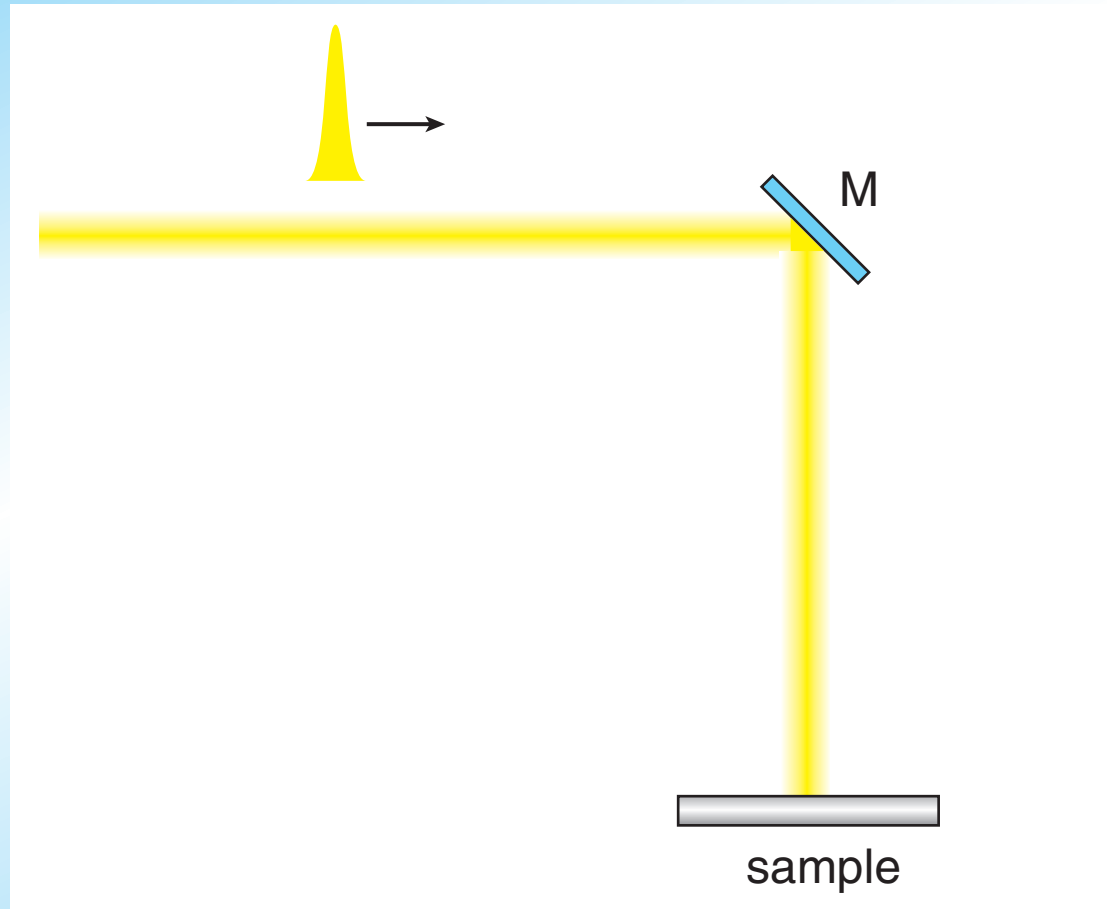
Outline

- ▶ linear and nonlinear propagation
- ▶ femtosecond measurements
- ▶ examples



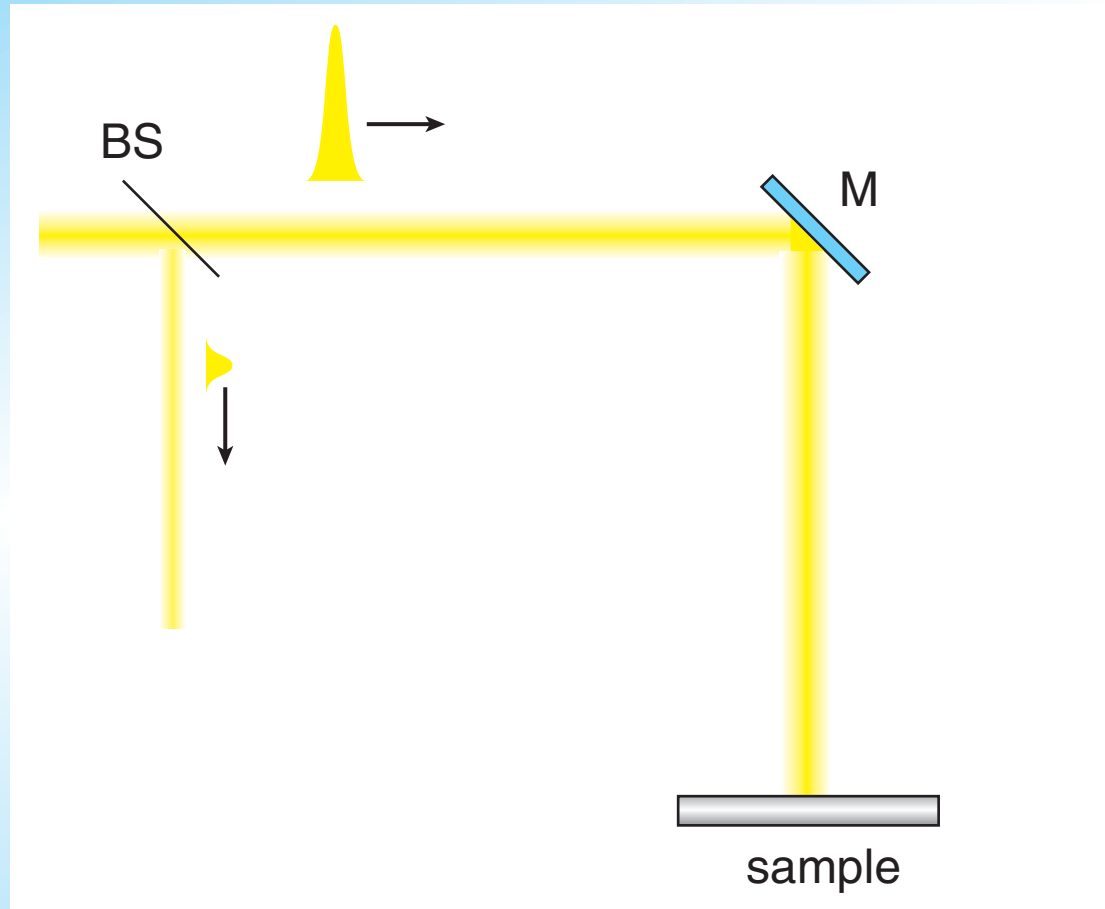
Introduction

How to measure on the femtosecond time scale?



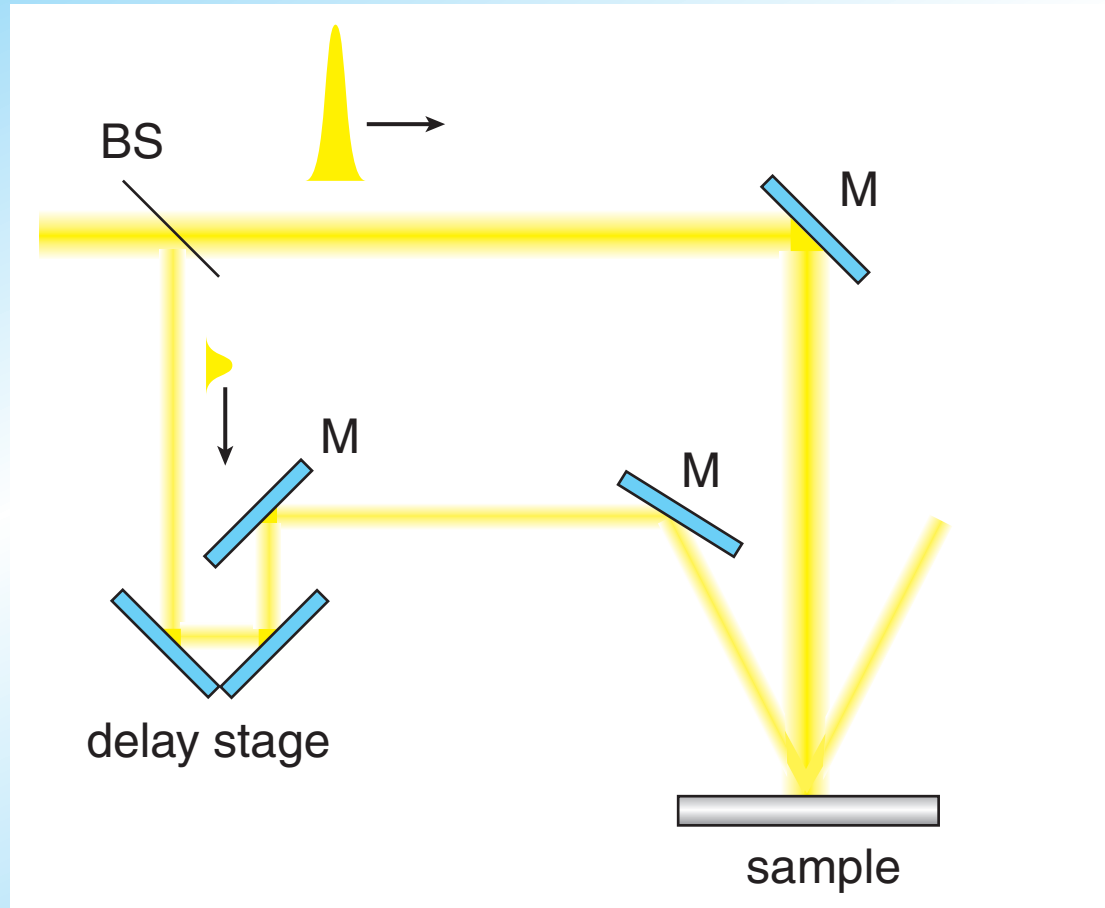
Introduction

Use pump-probe technique



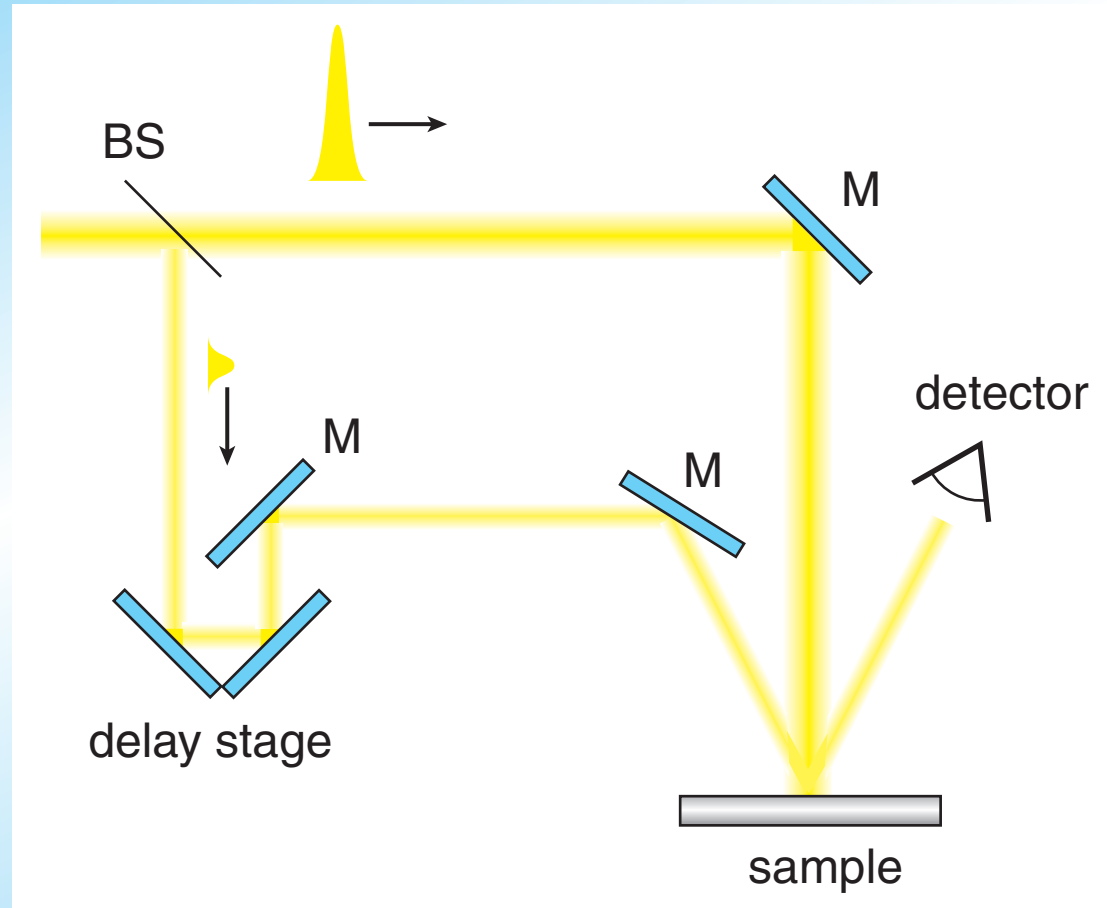
Introduction

Use pump-probe technique



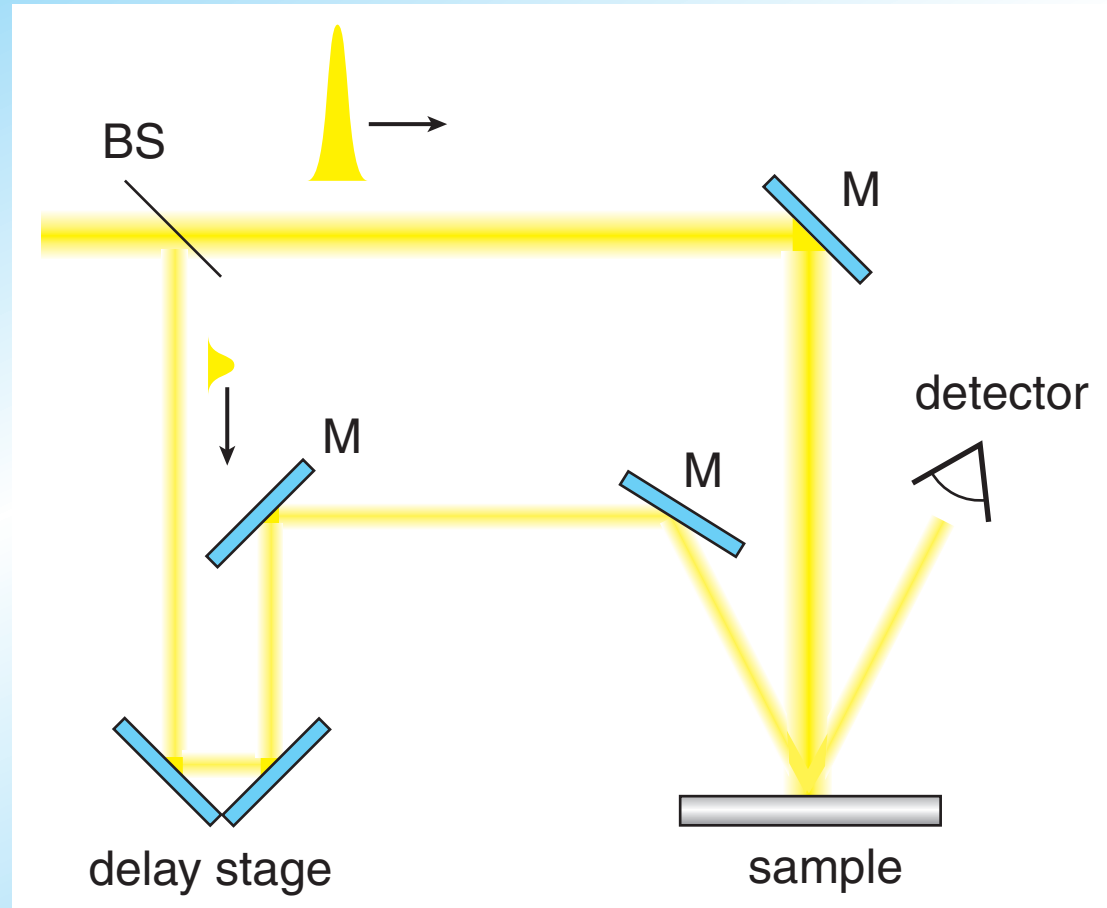
Introduction

Use pump-probe technique



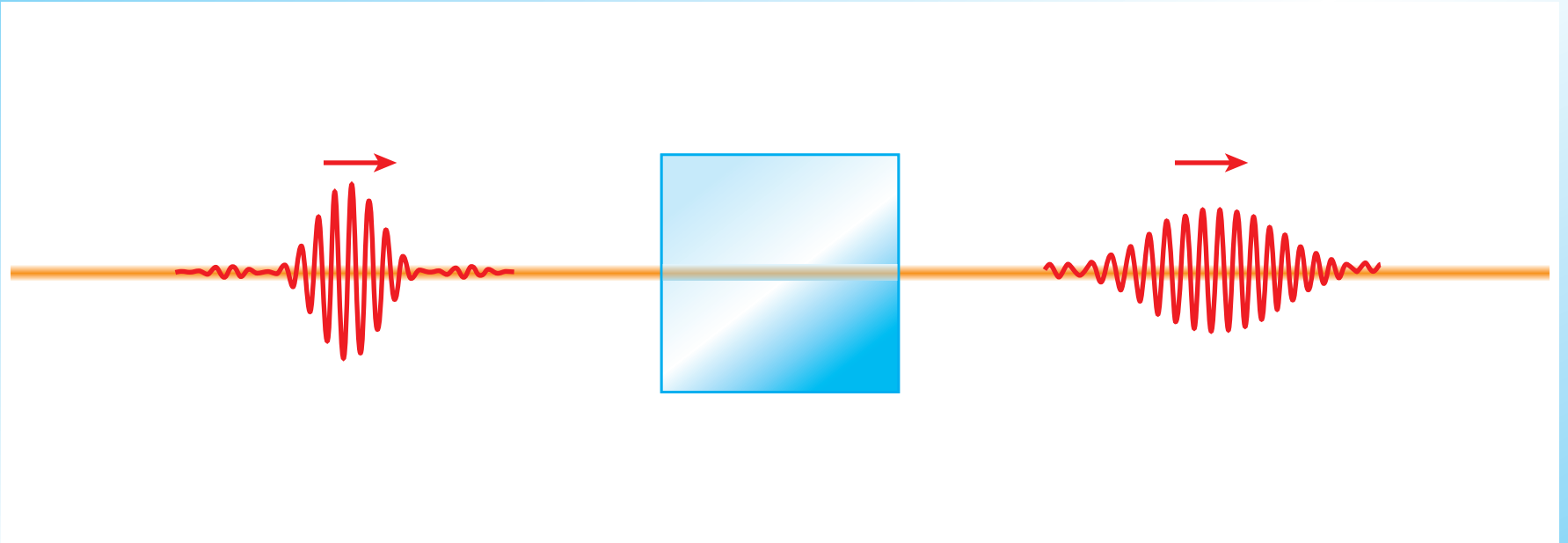
Introduction

Vary delay to get time resolution



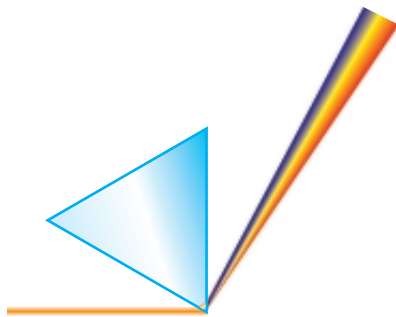
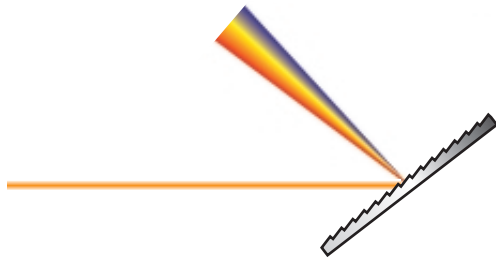
Dispersion compensation

Dispersion stretches the pulse

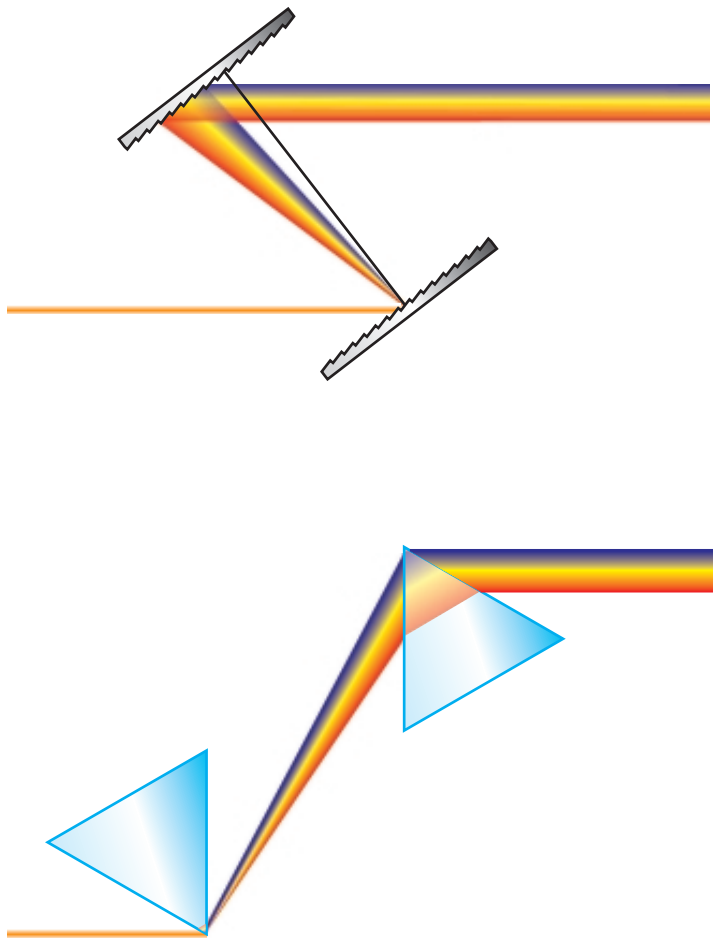


Compensate by rearranging spectral components!

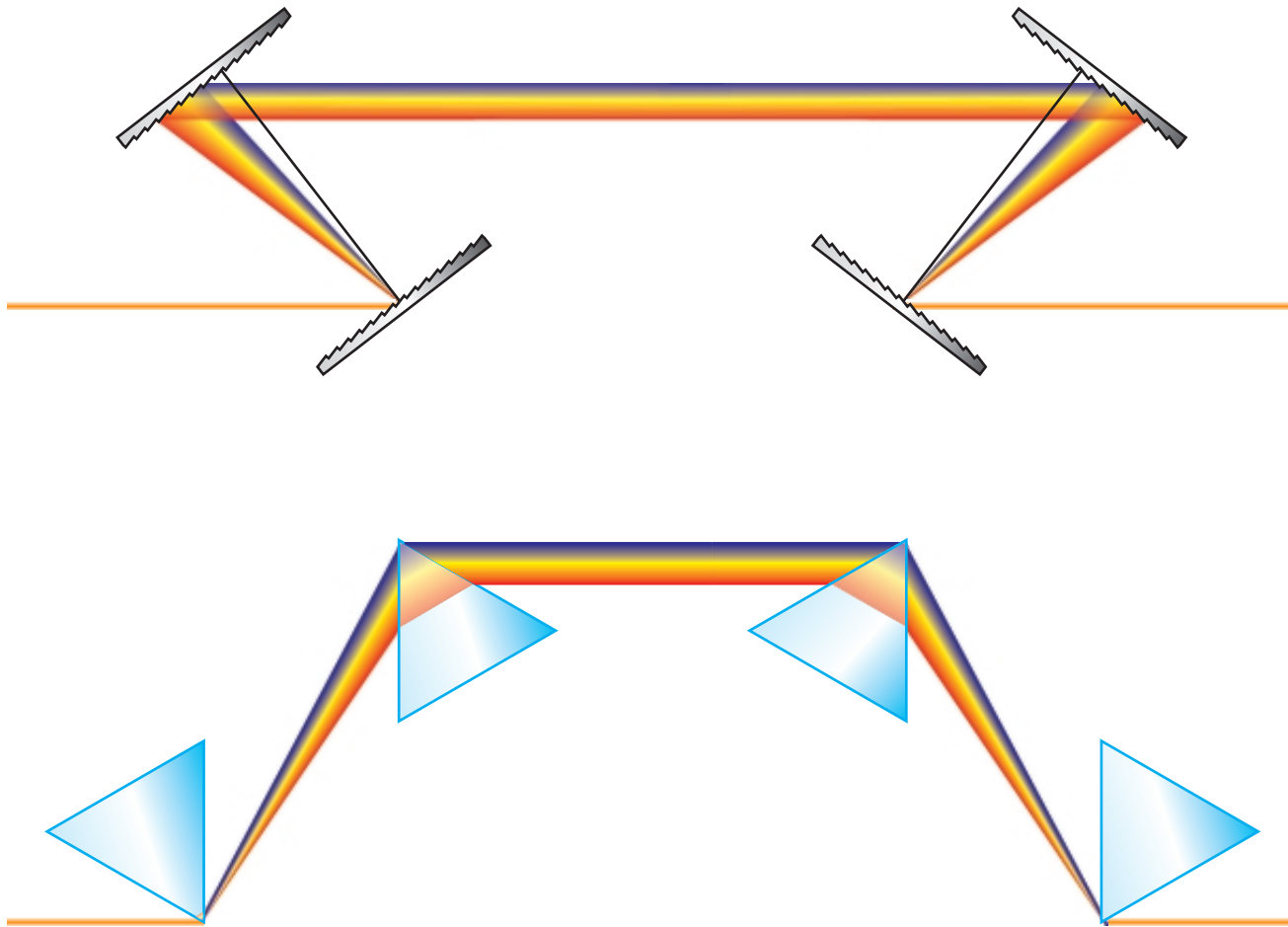
Dispersion compensation



Dispersion compensation



Dispersion compensation

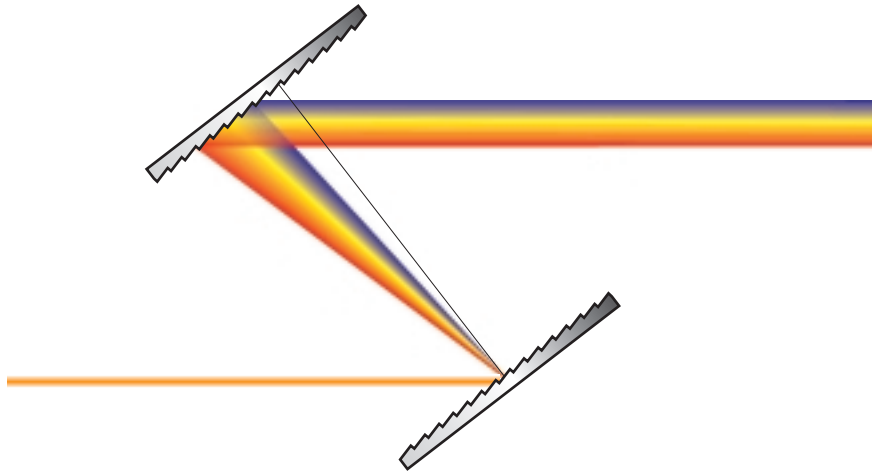


Dispersion compensation

How do these arrangements work?

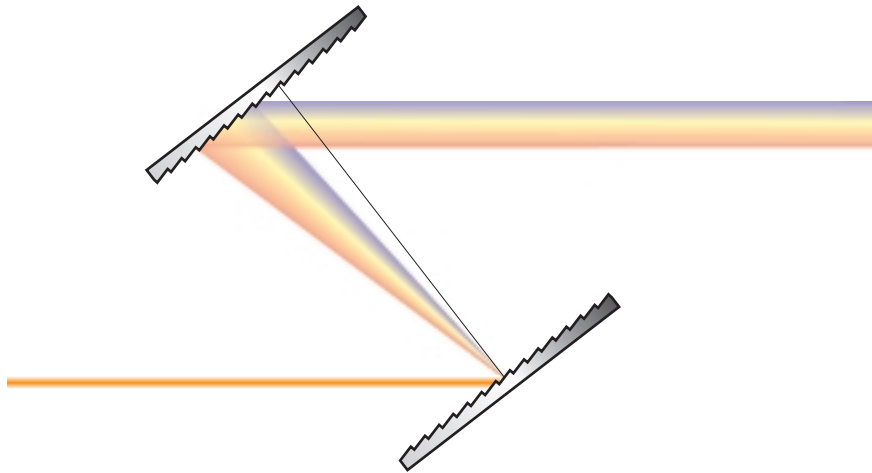
Dispersion compensation

Does path length difference compensate?



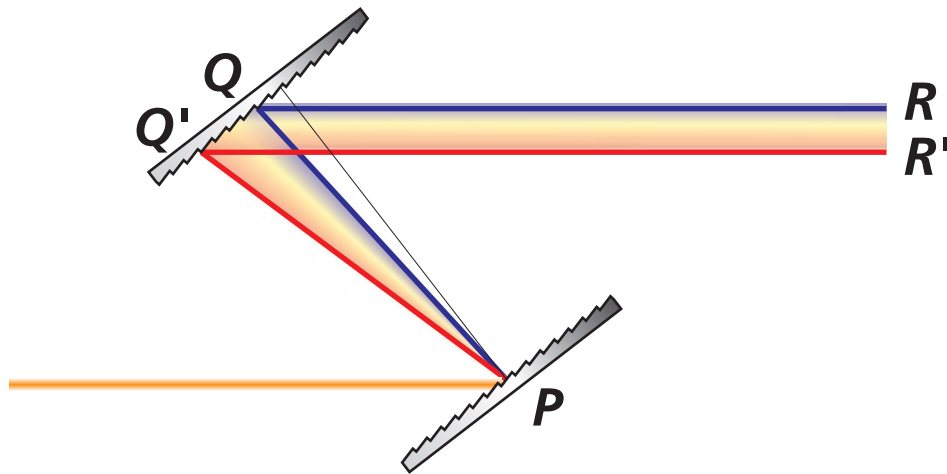
Dispersion compensation

Does path length difference compensate?



Dispersion compensation

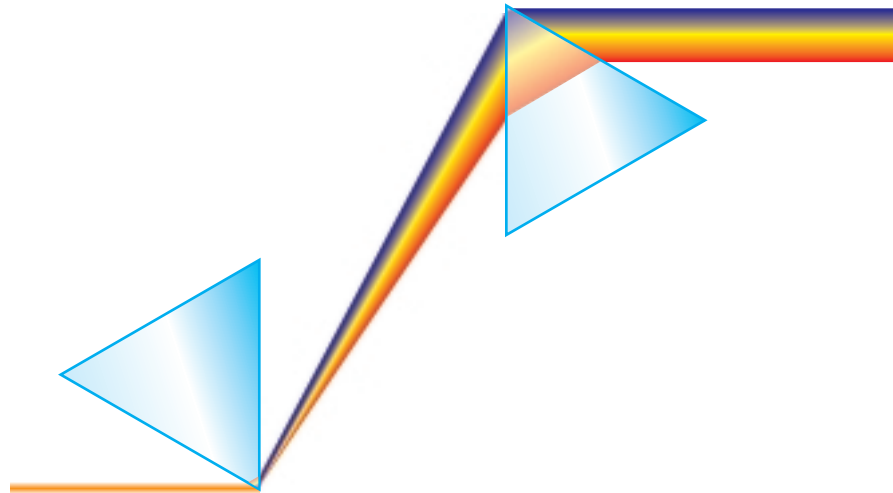
Does path length difference compensate?



Grating gives low frequency longer path length...

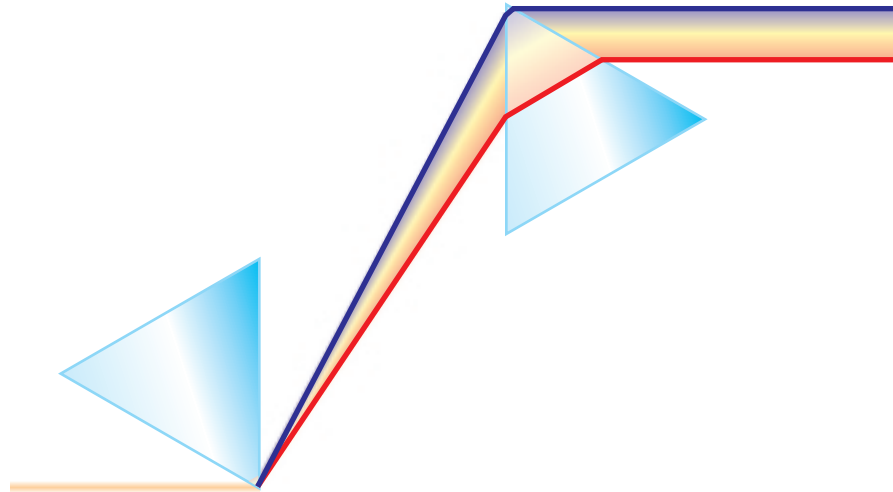
Dispersion compensation

Does path length difference compensate?



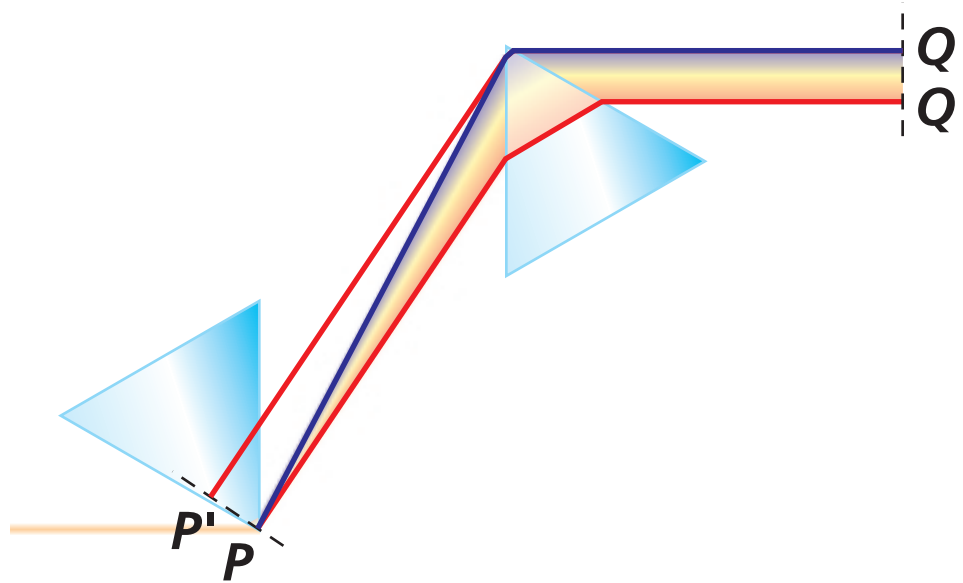
Dispersion compensation

Does path length difference compensate?



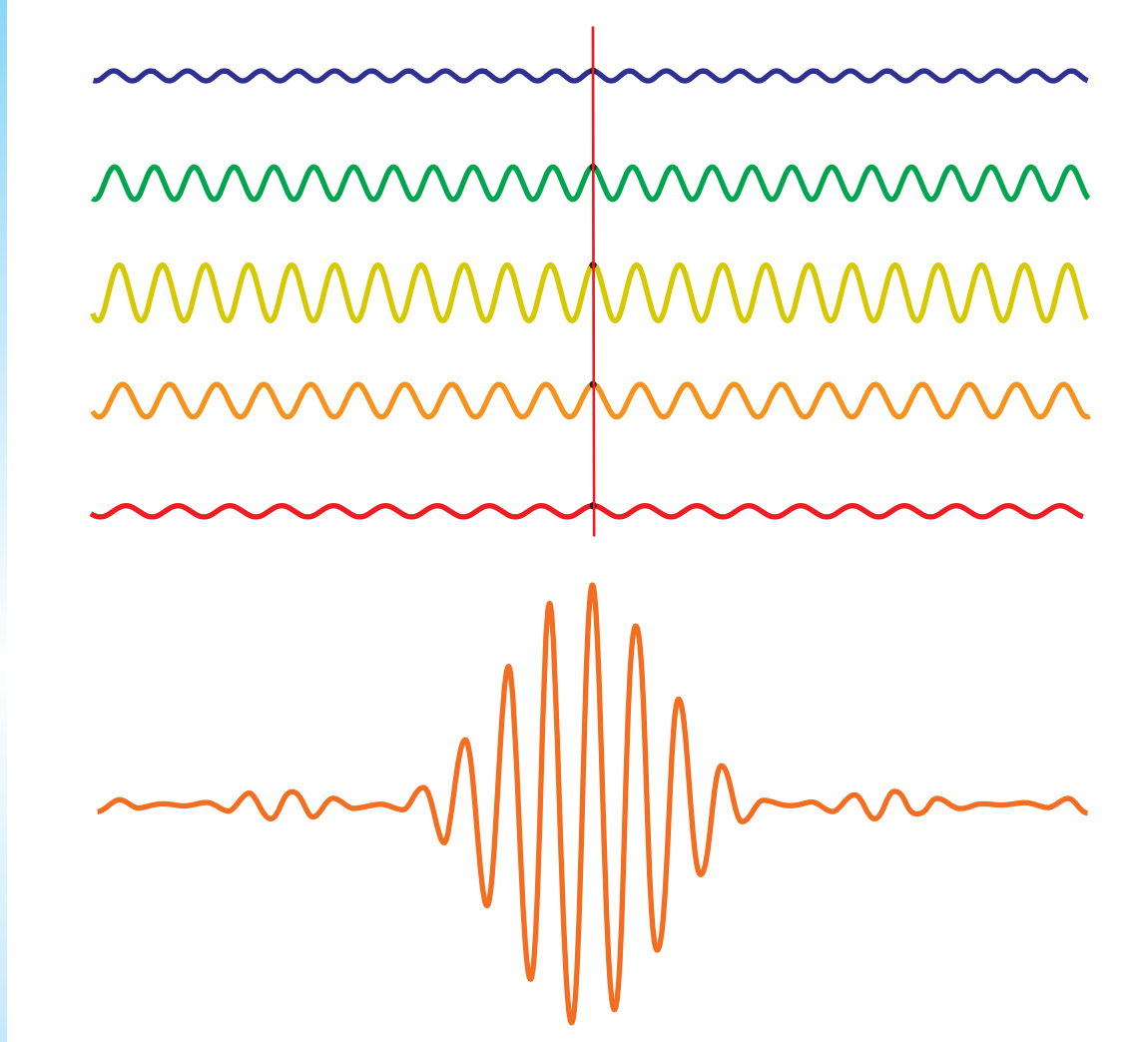
Dispersion compensation

Does path length difference compensate?

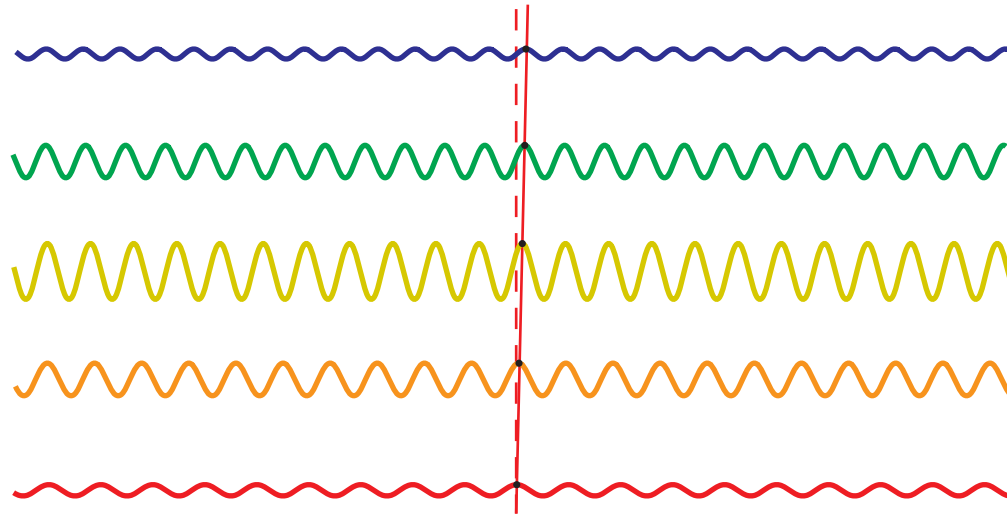


...so prism gives low frequency *shorter* path length...

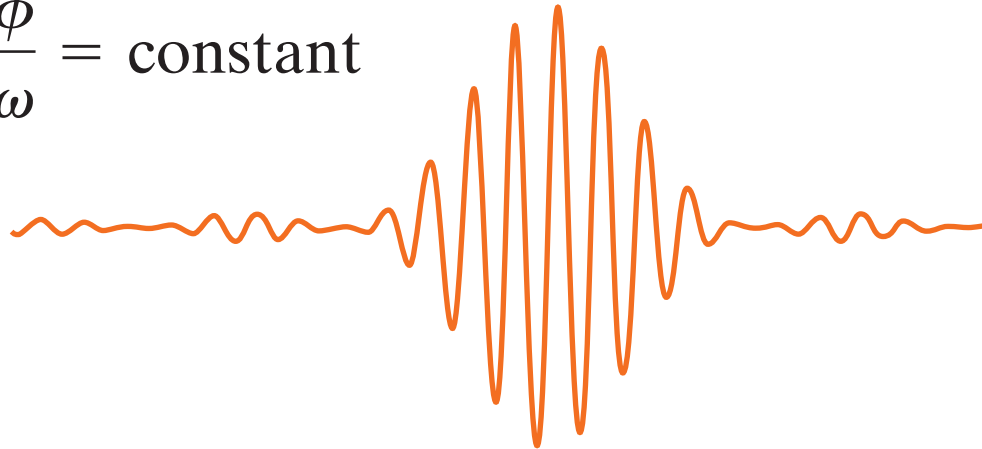
Dispersion compensation



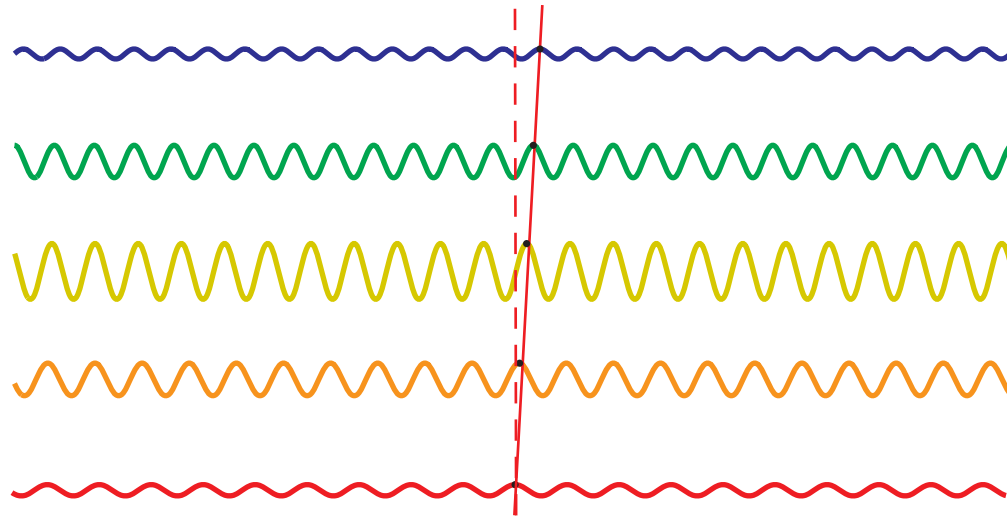
Dispersion compensation



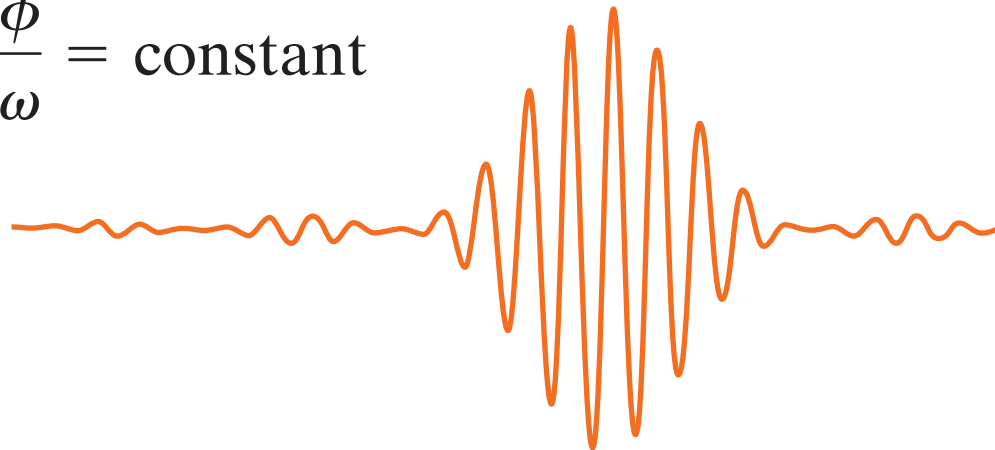
$$\frac{d\phi}{d\omega} = \text{constant}$$



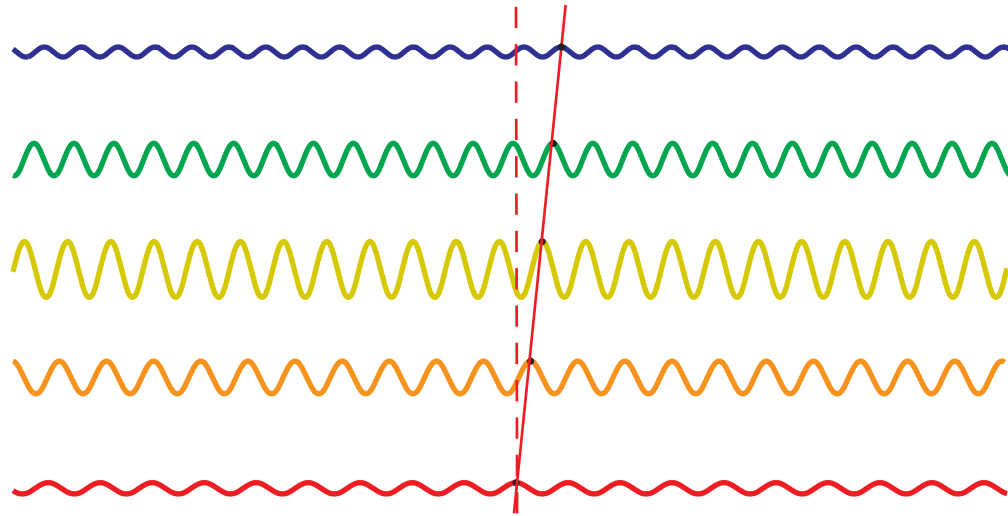
Dispersion compensation



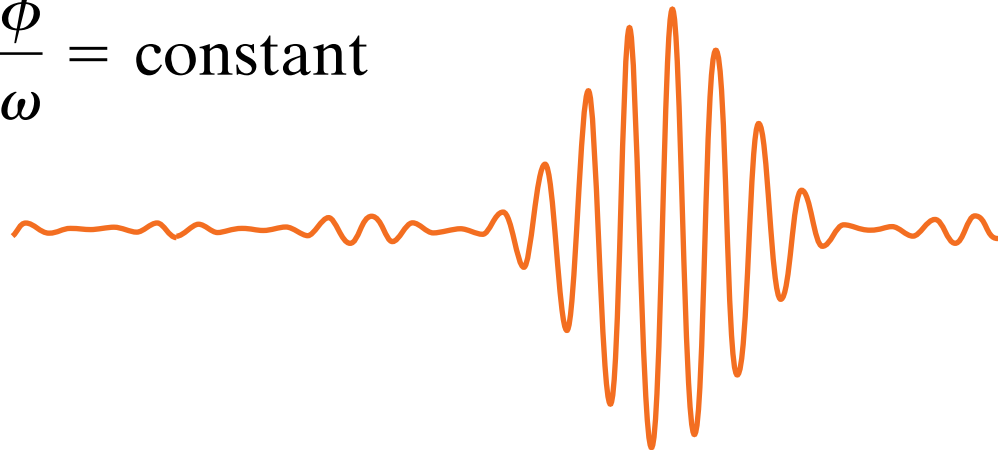
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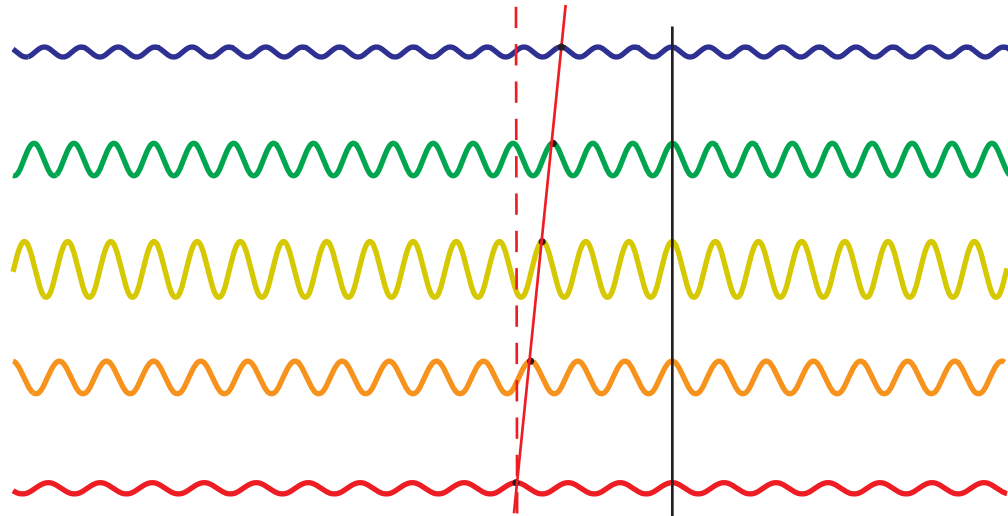
Dispersion compensation



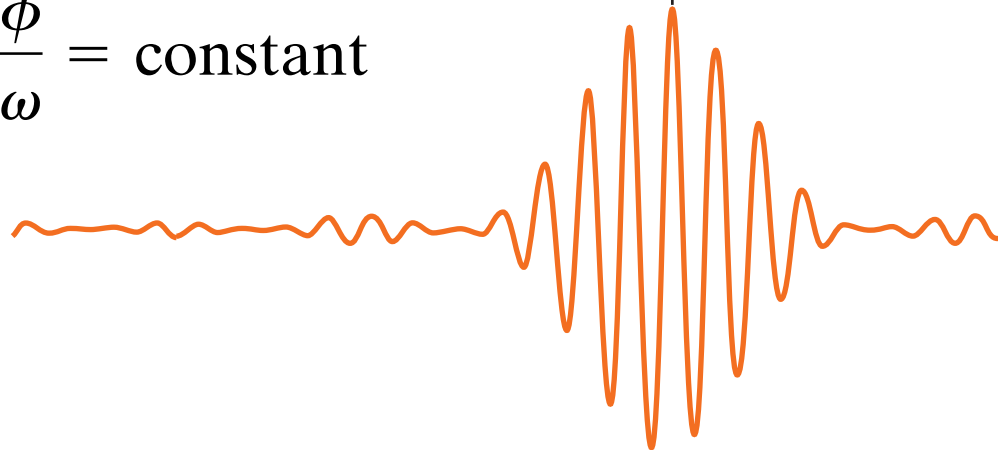
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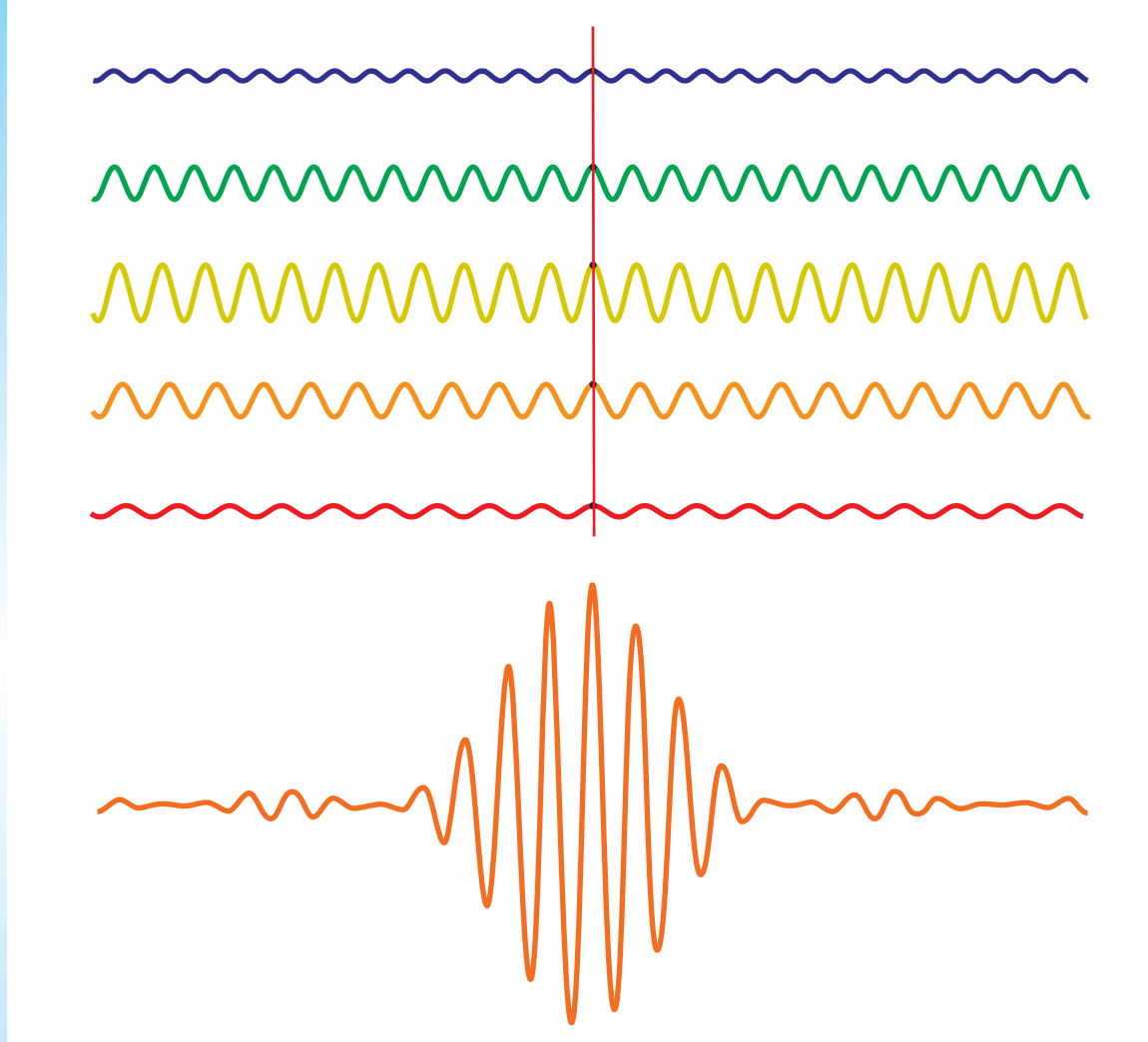
Dispersion compensation



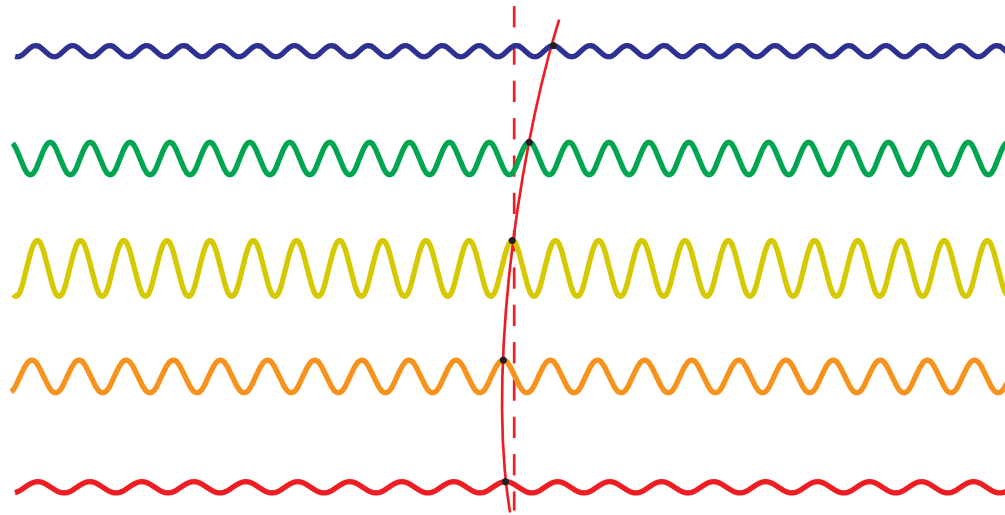
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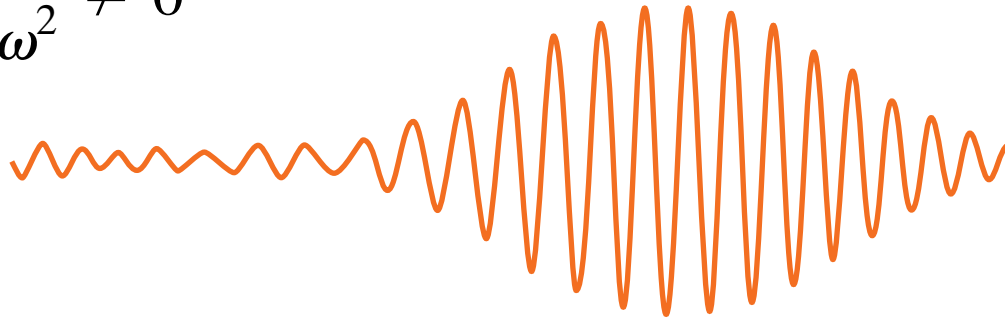
Dispersion compensation



Dispersion compensation



$$\frac{d^2\phi}{d\omega^2} \neq 0$$

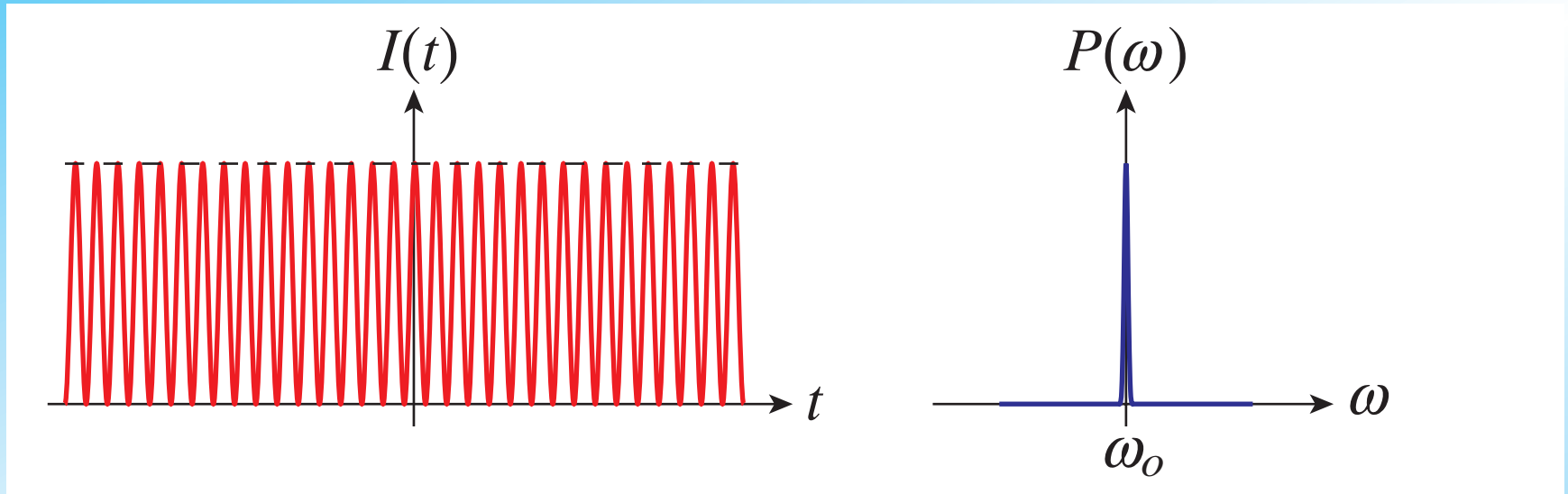


Dispersion compensation

So not path length but $\frac{d^2\phi}{d\omega^2}$ matters!

	$\frac{dl_{eff}}{d\omega}$	$\frac{d^2\phi}{d\omega^2}$
dispersion	+	+
gratings	-	-
prisms	+	-

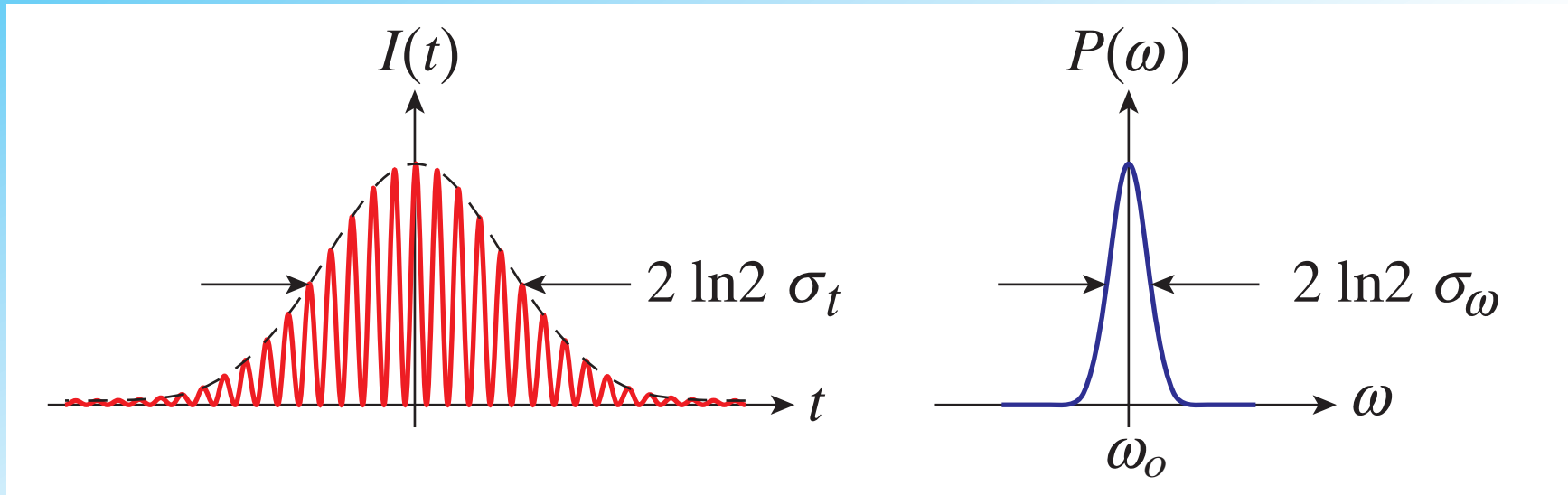
Representation of pulses



Spectrum of sinusoidal intensity is a delta function

$$I(t) = \cos^2(\omega_0 t) \quad \Rightarrow \quad P(\omega) = \delta(\omega - \omega_0)$$

Representation of pulses



Modulate amplitude

$$I(t) = \exp\left[-\frac{t^2}{\sigma_t^2}\right] \cos^2(\omega_0 t)$$

Representation of pulses

Fourier relations:

$$E(t) = \exp\left[-\frac{t^2}{2\sigma_t^2} - i\omega_0 t\right]$$

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$$E(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{t^2}{2\sigma_t^2} + i(\omega - \omega_o)t\right] dt =$$

Representation of pulses

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$$= \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{\sigma_t^2(\omega - \omega_o)^2}{2}\right] \int_{-\infty}^{\infty} \exp\left[-\left[\frac{t}{\sqrt{2}\sigma_t} - i\frac{(\omega - \omega_o)\sigma_t}{\sqrt{2}}\right]^2\right] dt =$$

Representation of pulses

Fourier relations:

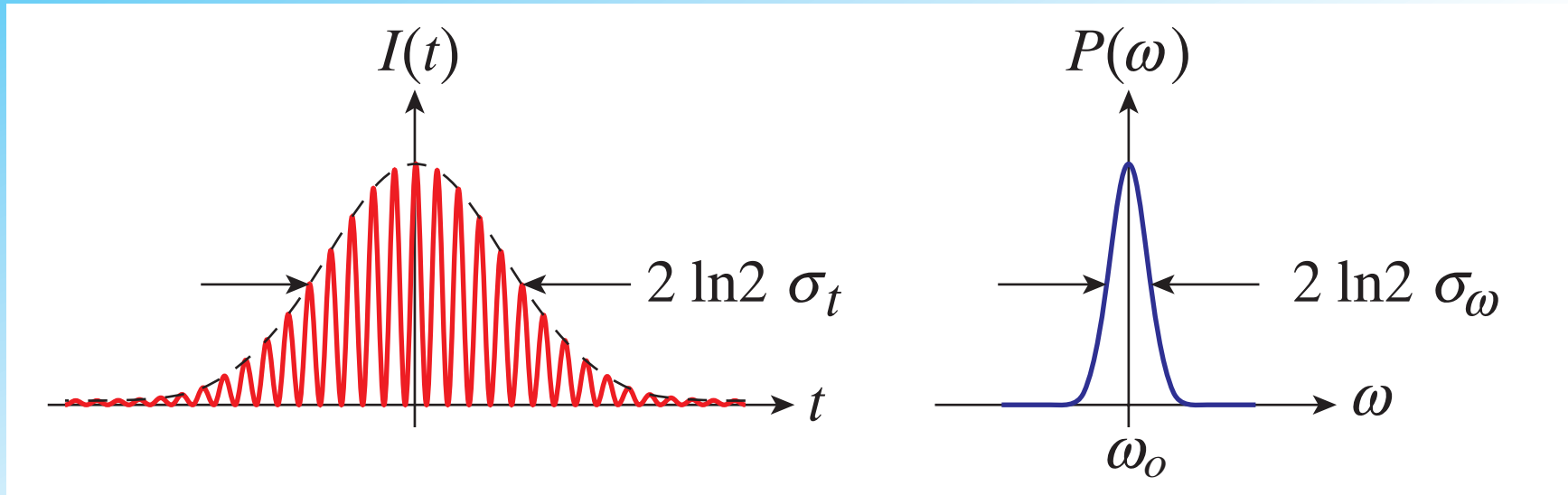
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$$= \sigma_t \exp\left[-\frac{\sigma_t^2(\omega - \omega_0)^2}{2}\right] \equiv \sigma_t \exp\left[-\frac{(\omega - \omega_0)^2}{2\sigma_\omega^2}\right]$$

Representation of pulses

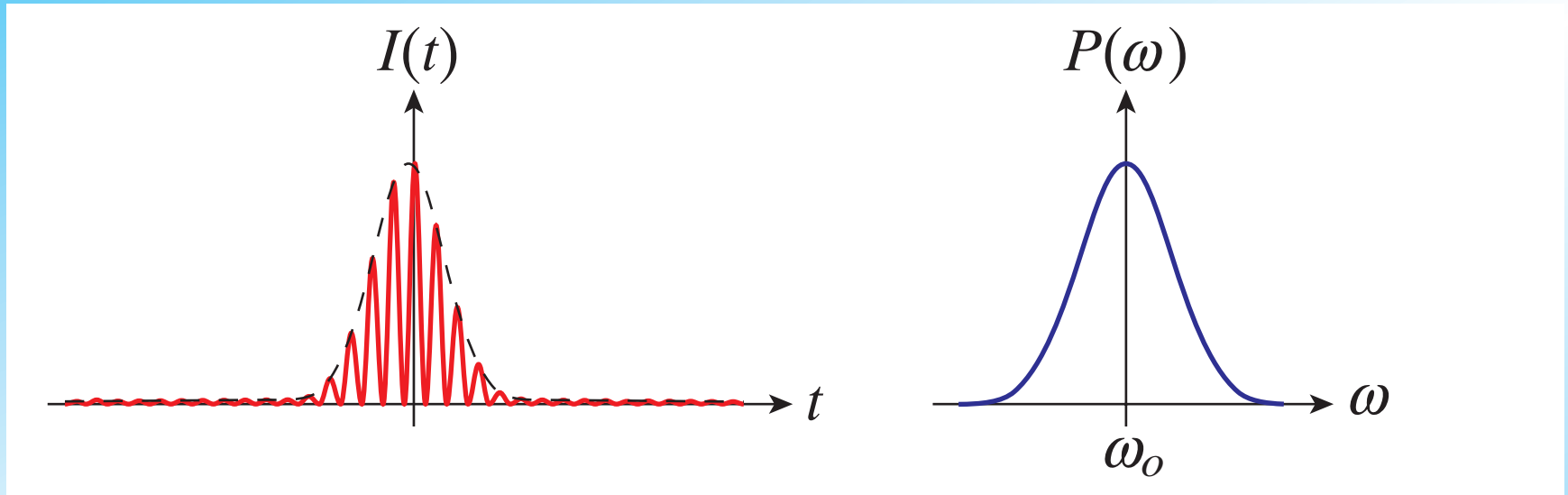


Pulse duration-bandwidth product: $\sigma_t \sigma_\omega = 1$

$$I(t) = [\text{Re } E(t)]^2 \propto \exp\left[-\frac{t^2}{\sigma_t^2}\right] \cos^2(\omega_0 t)$$

$$P(\omega) = E(\omega)E^*(\omega) = \exp\left[-\frac{(\omega - \omega_0)^2}{\sigma_\omega^2}\right]$$

Representation of pulses



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Joint time-frequency representation

Wigner representation:

$$\begin{aligned} W(t, \omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} E\left(\omega + \frac{\omega'}{2}\right) E^*\left(\omega - \frac{\omega'}{2}\right) e^{-i\omega't} d\omega' = \\ &= \int_{-\infty}^{\infty} E\left(t + \frac{t'}{2}\right) E^*\left(t - \frac{t'}{2}\right) e^{-i\omega t'} dt' \end{aligned}$$

Joint time-frequency representation

Wigner representation:

$$W(t, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E\left(\omega + \frac{\omega'}{2}\right) E^*\left(\omega - \frac{\omega'}{2}\right) e^{-i\omega't} d\omega' =$$

$$= \int_{-\infty}^{\infty} E\left(t + \frac{t'}{2}\right) E^*\left(t - \frac{t'}{2}\right) e^{-i\omega t'} dt'$$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} W(t, \omega) d\omega = |E(t)|^2 = I(t)$$

Joint time-frequency representation

Wigner representation:

$$W(t, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E\left(\omega + \frac{\omega'}{2}\right) E^*\left(\omega - \frac{\omega'}{2}\right) e^{-i\omega't} d\omega' =$$

$$= \int_{-\infty}^{\infty} E\left(t + \frac{t'}{2}\right) E^*\left(t - \frac{t'}{2}\right) e^{-i\omega t'} dt'$$

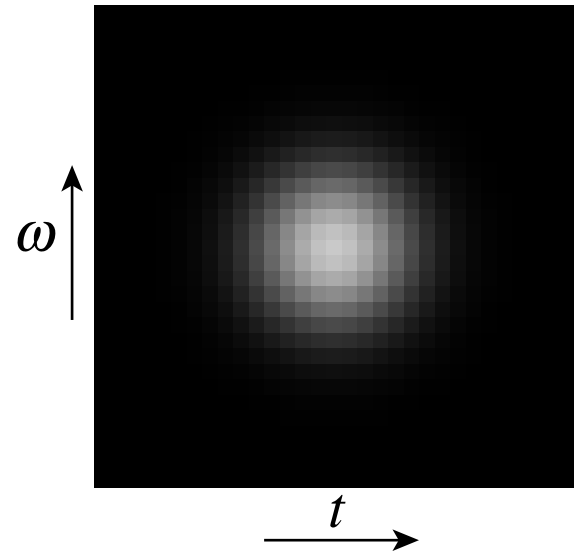
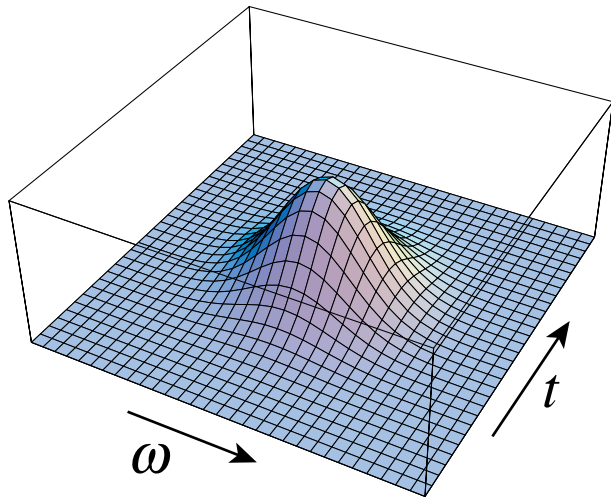
$$\frac{1}{2\pi} \int_{-\infty}^{\infty} W(t, \omega) d\omega = |E(t)|^2 = I(t)$$

$$\int_{-\infty}^{\infty} W(t, \omega) dt = |E(\omega)|^2 = I(\omega)$$

Joint time-frequency representation

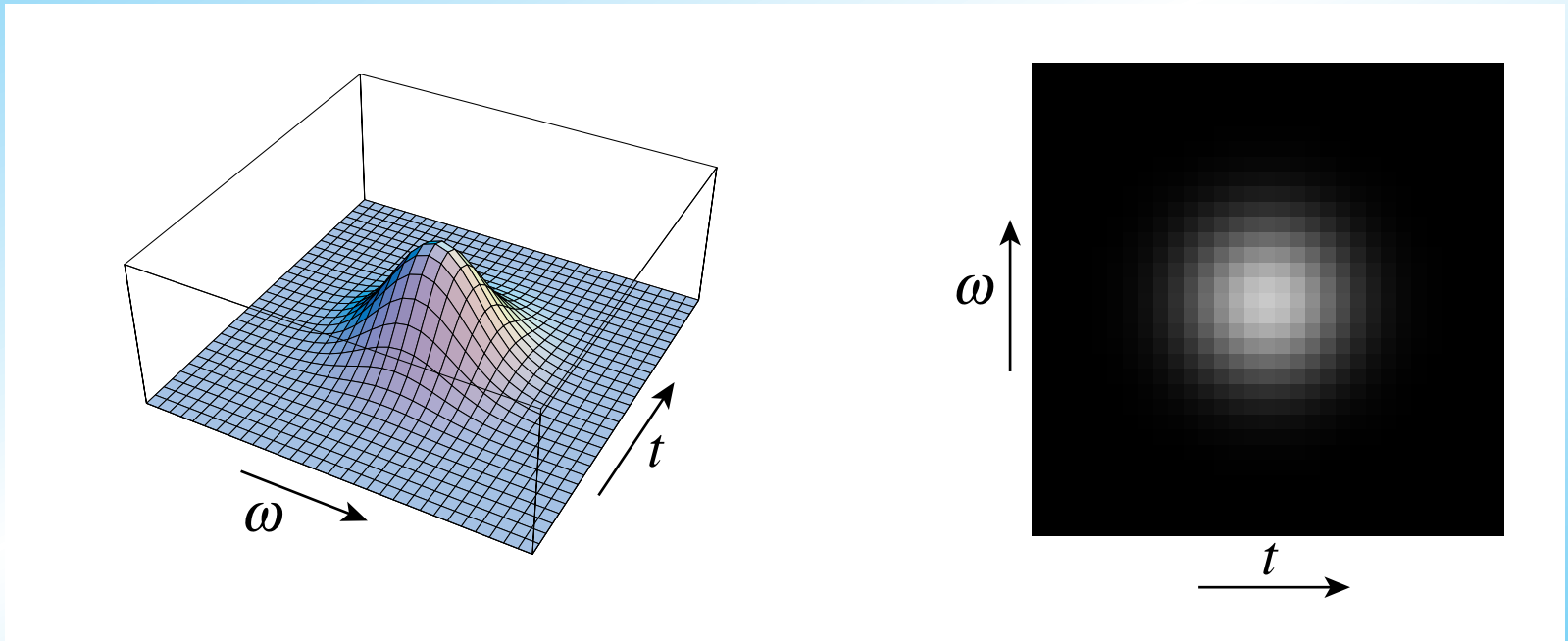
Energy:

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(t, \omega) dt d\omega$$



Joint time-frequency representation

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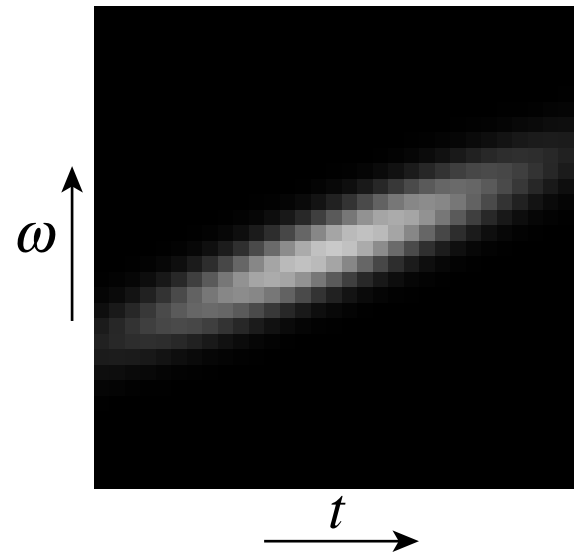


$W(t, \omega)$ must be nonzero in phase-space area larger than π

Joint time-frequency representation

Energy:
$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(t, \omega) dt d\omega$$

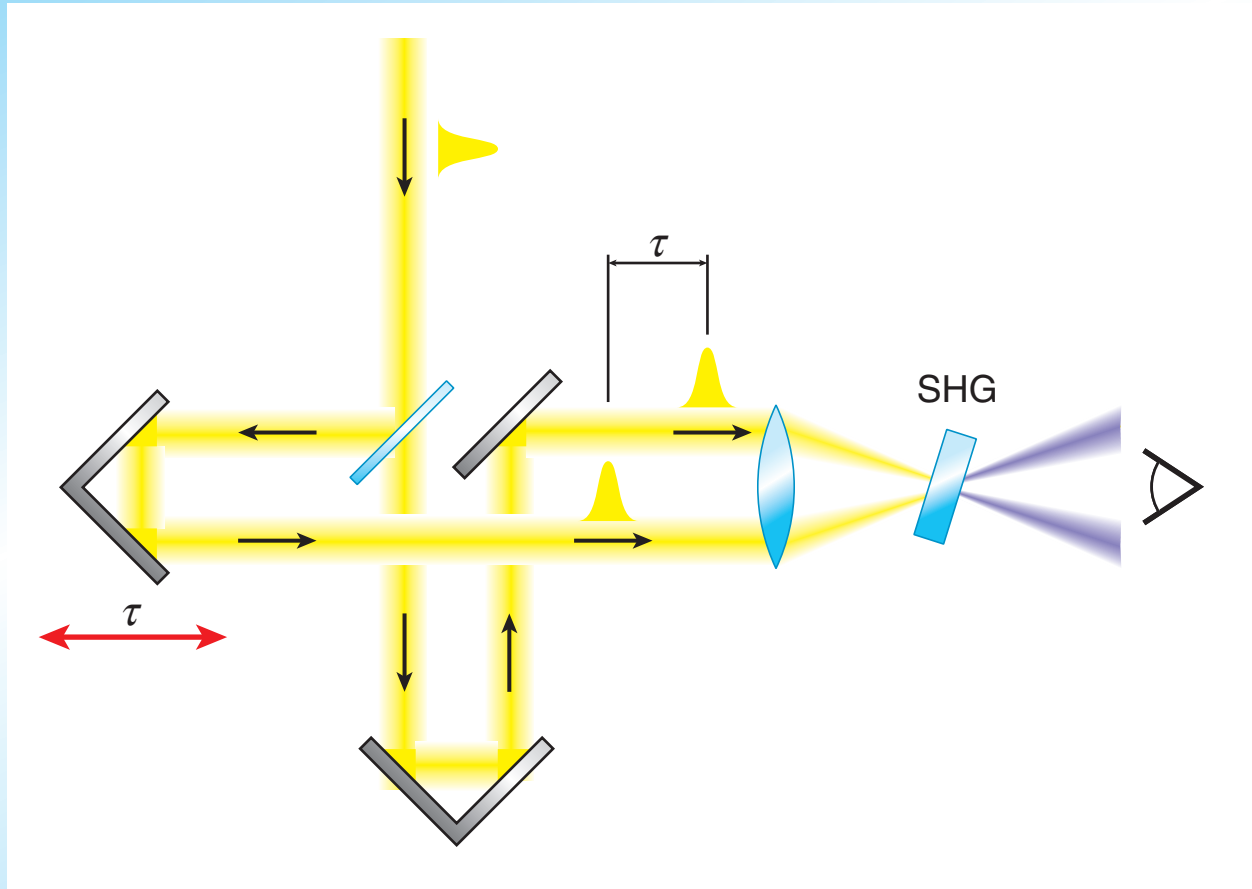
chirped pulse



$W(t, \omega)$ must be nonzero in phase-space area larger than π

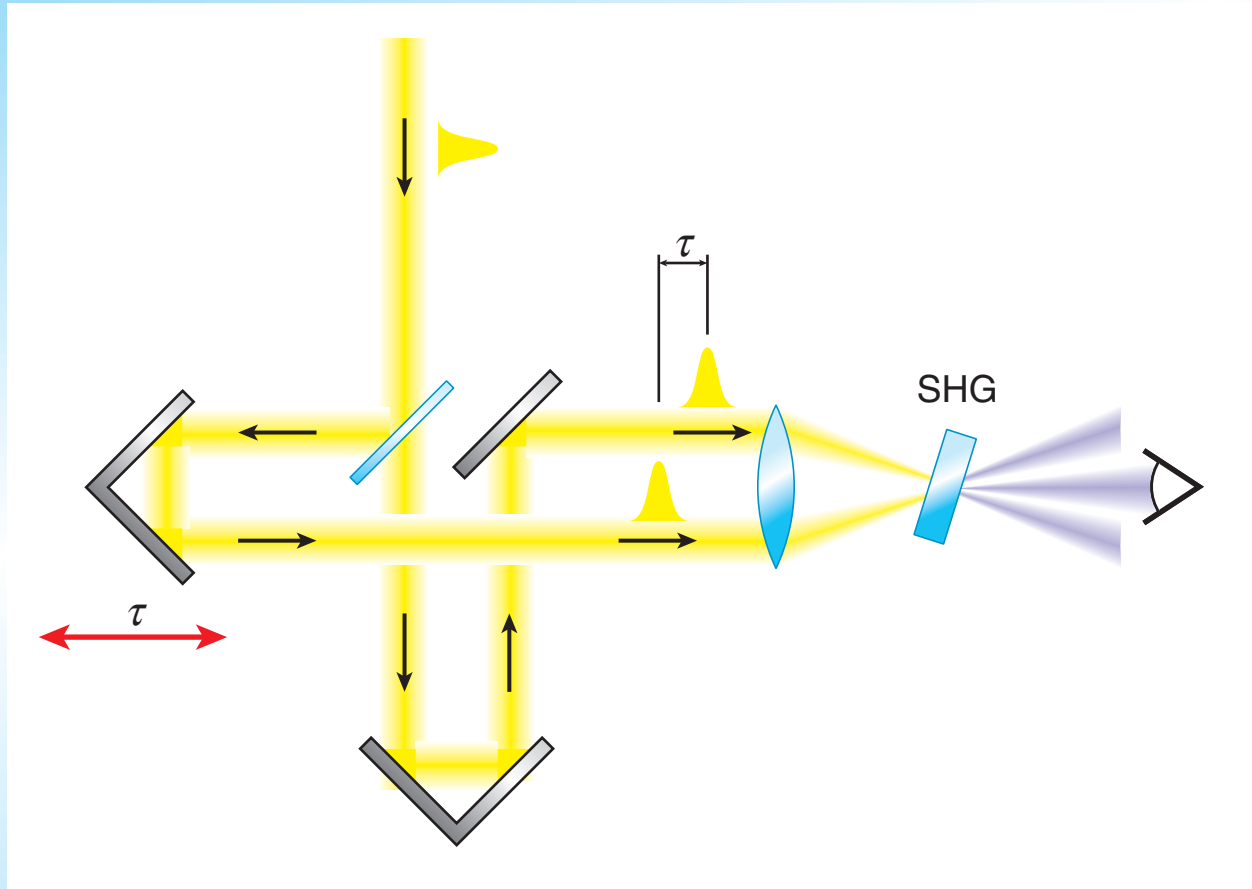
Temporal characterization

Use pulse to measure itself...



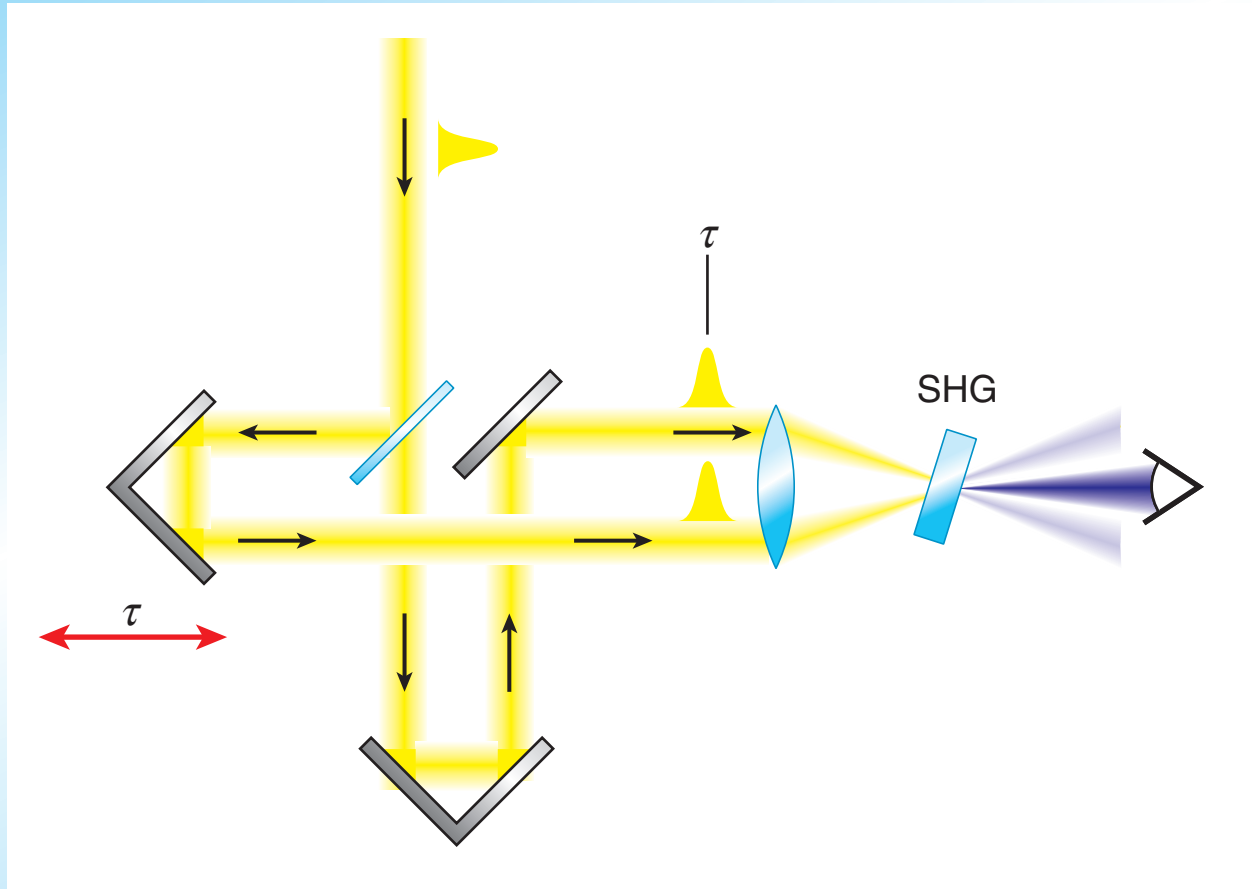
Temporal characterization

Use pulse to measure itself...



Temporal characterization

Use pulse to measure itself...



Temporal characterization

Electric field at SHG crystal

$$E_{tot}(t, \tau) = \frac{1}{\sqrt{2}}E_1(t) + \frac{1}{\sqrt{2}}E_2(t + \tau)$$

Temporal characterization

Electric field at SHG crystal

$$E_{tot}(t, \tau) = \frac{1}{\sqrt{2}}E_1(t) + \frac{1}{\sqrt{2}}E_2(t + \tau)$$

Second harmonic field

$$E_{2\omega} \propto \chi^{(2)} E_{tot}^2$$

Temporal characterization

Electric field at SHG crystal

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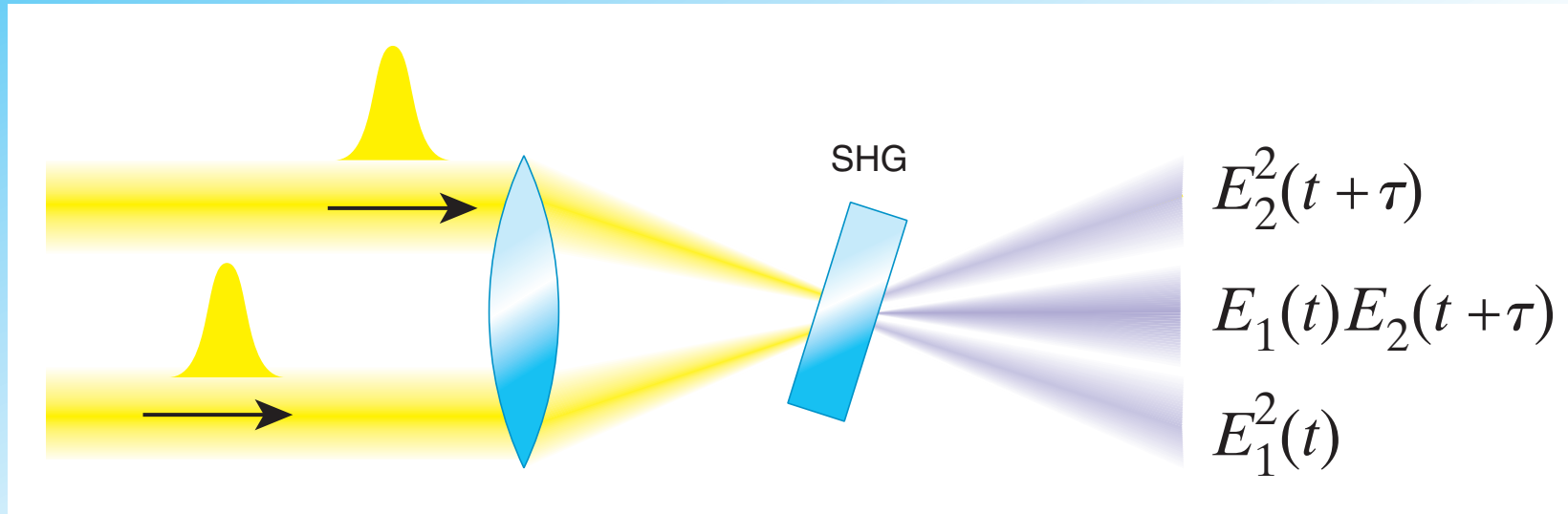
Second harmonic field

$$E_{2\omega} \propto \chi^{(2)} E_{tot}^2$$

Second harmonic intensity

$$I_{2\omega}(t, \tau) \propto |\chi^{(2)}|^2 |E_{tot}^2|^2 = |\chi^{(2)}|^2 |E_1^2(t) + 2E_1(t)E_2(t + \tau) + E_2^2(t + \tau)|^2$$

Temporal characterization



Second harmonic intensity

$$I_{2\omega}(t, \tau) \propto |\chi^{(2)}|^2 |E_{tot}^2|^2 = |\chi^{(2)}|^2 |E_1^2(t) + 2E_1(t)E_2(t + \tau) + E_2^2(t + \tau)|^2$$

detector selects middle term

Temporal characterization

Integrated detector signal yields intensity autocorrelation

$$A(\tau) = \int I_{2\omega}(t, \tau) dt \propto \int |\chi^{(2)}|^2 4|E_1(t)|^2 |E_2(t + \tau)|^2 dt$$

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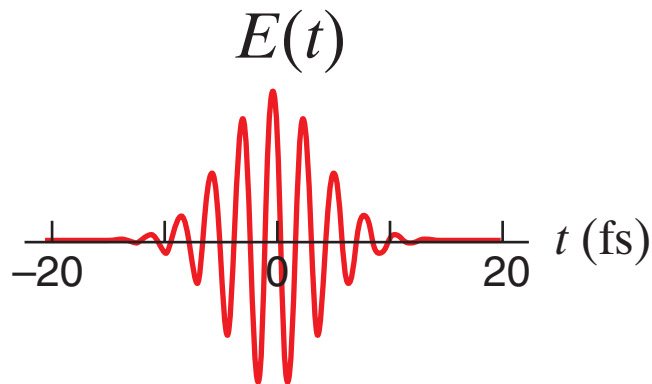
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Temporal characterization

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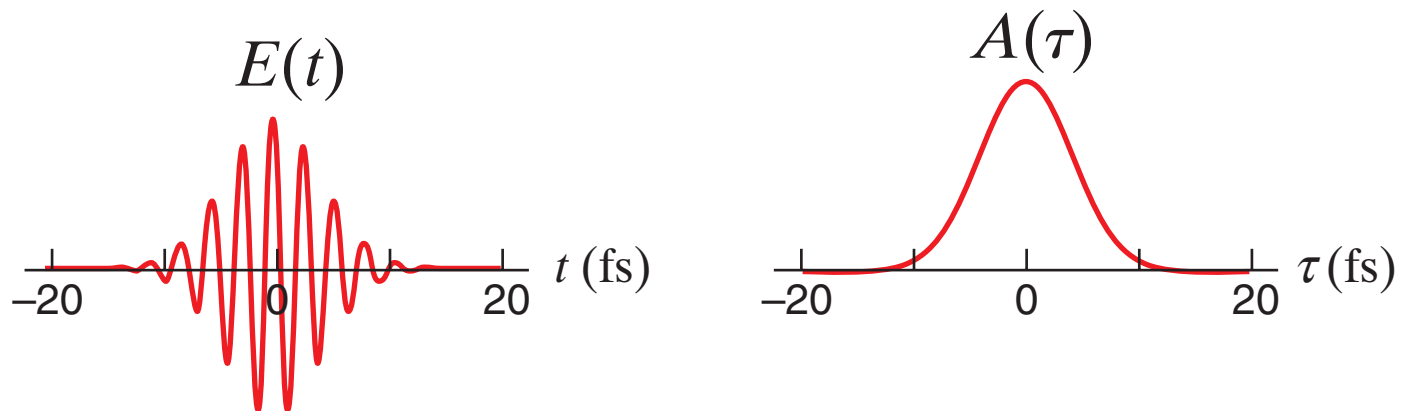


Temporal characterization

Integrated detector signal yields intensity autocorrelation

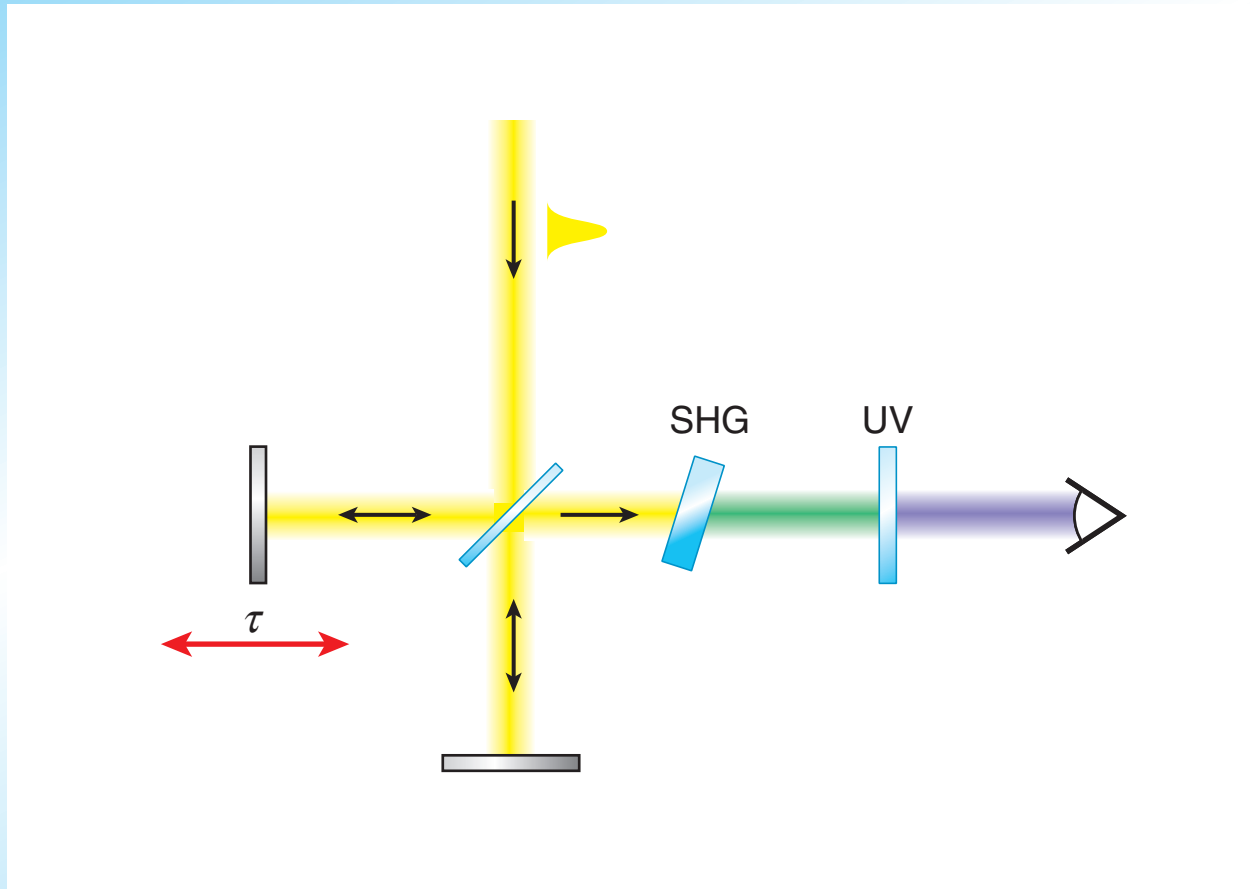
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Temporal characterization

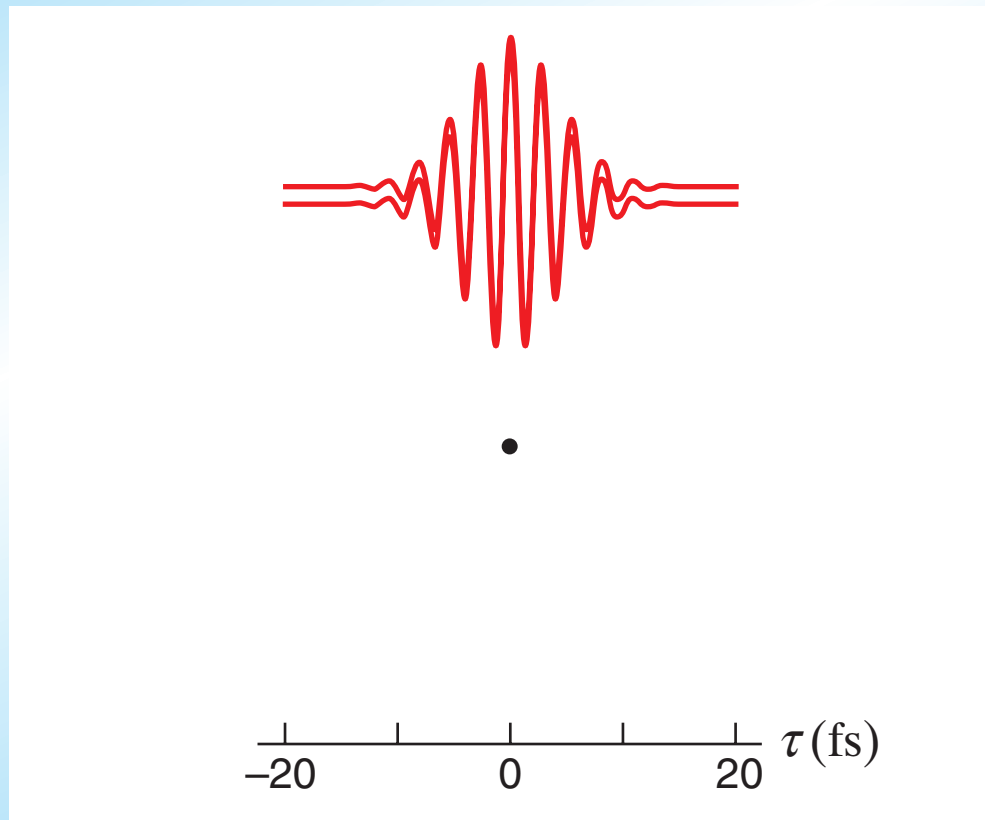
Alternative colinear geometry



Temporal characterization

All terms now contribute:

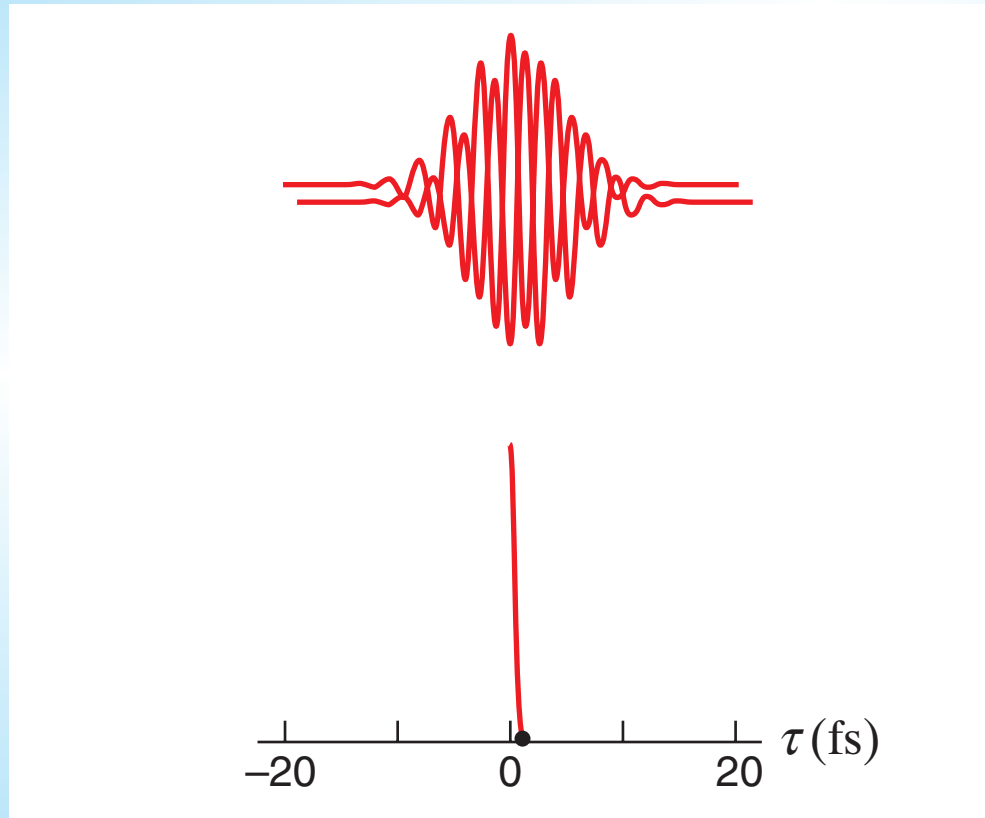
$$I_{2\omega}(t, \tau) \propto |\chi^{(2)}|^2 |E_{tot}^2|^2 = |\chi^{(2)}|^2 |E_1^2(t) + 2E_1(t)E_2(t+\tau) + E_2^2(t+\tau)|^2$$



Temporal characterization

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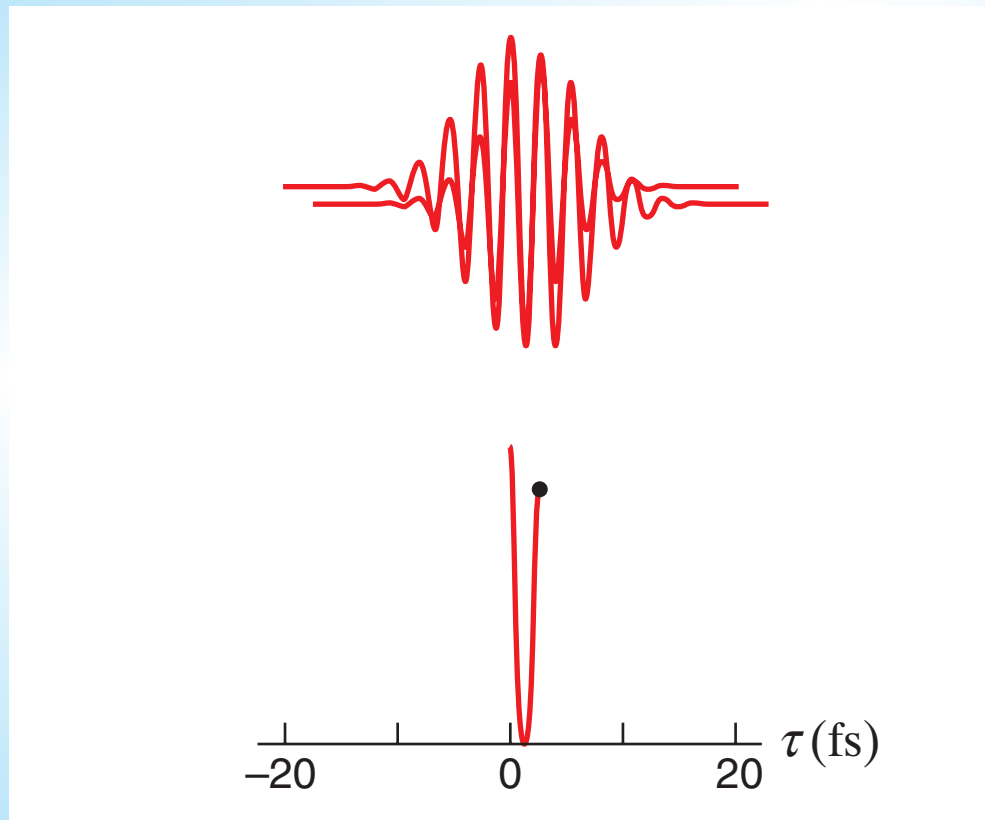
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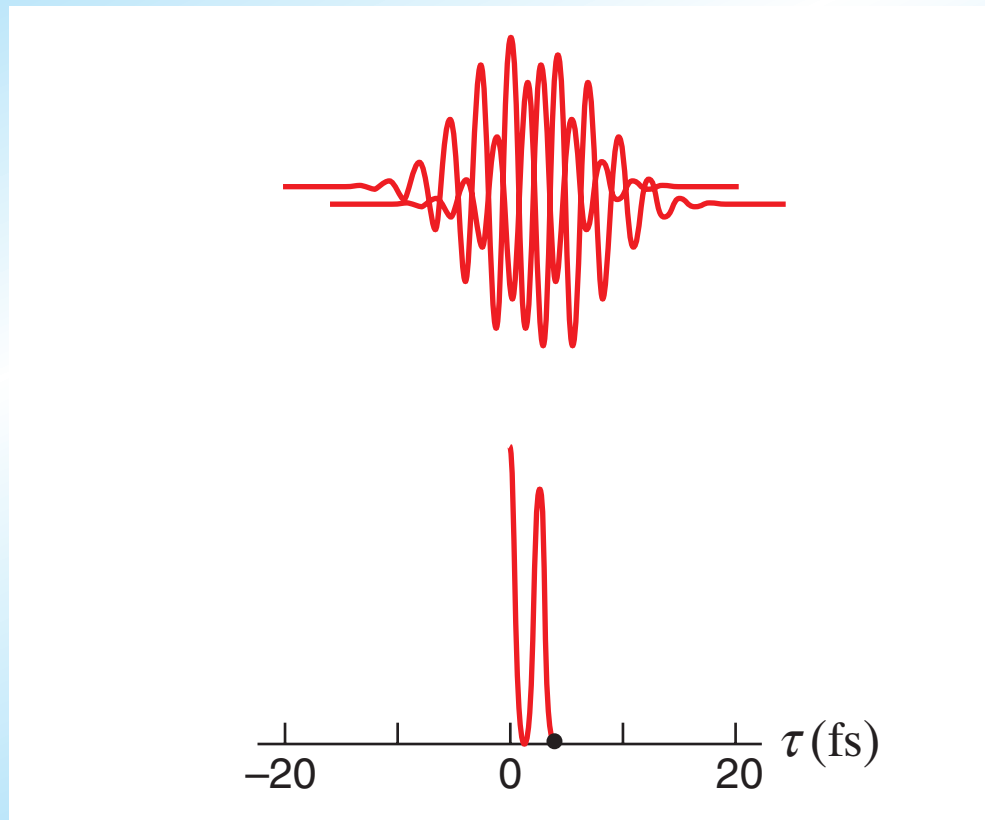
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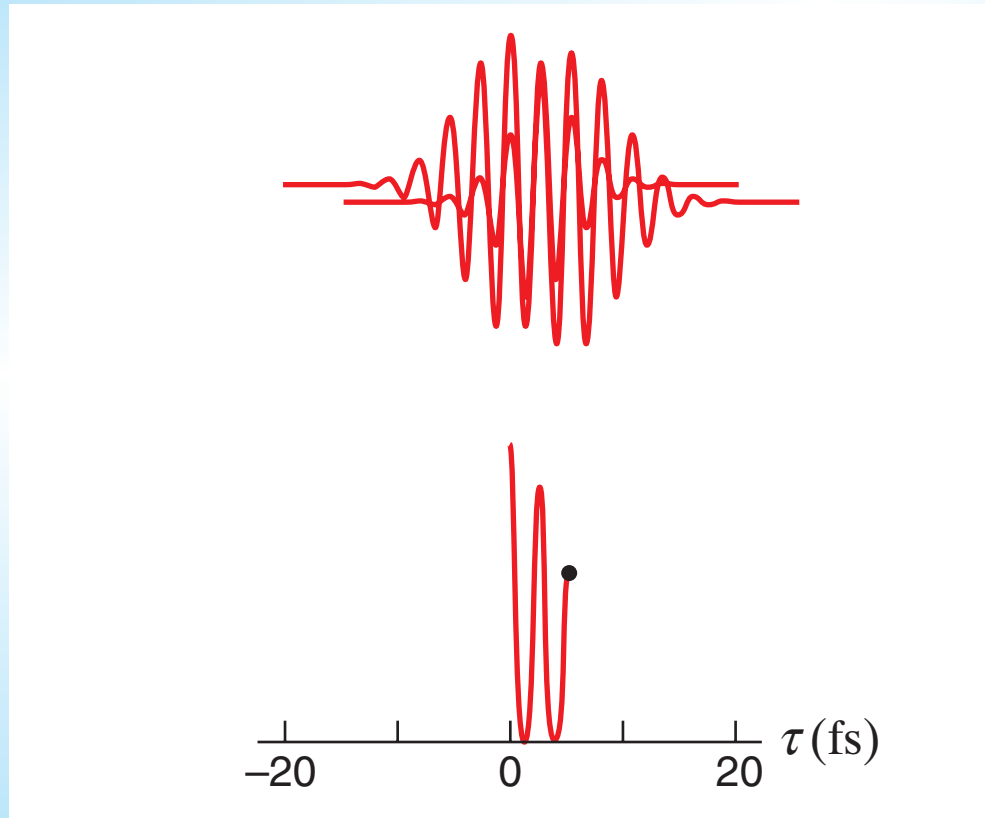
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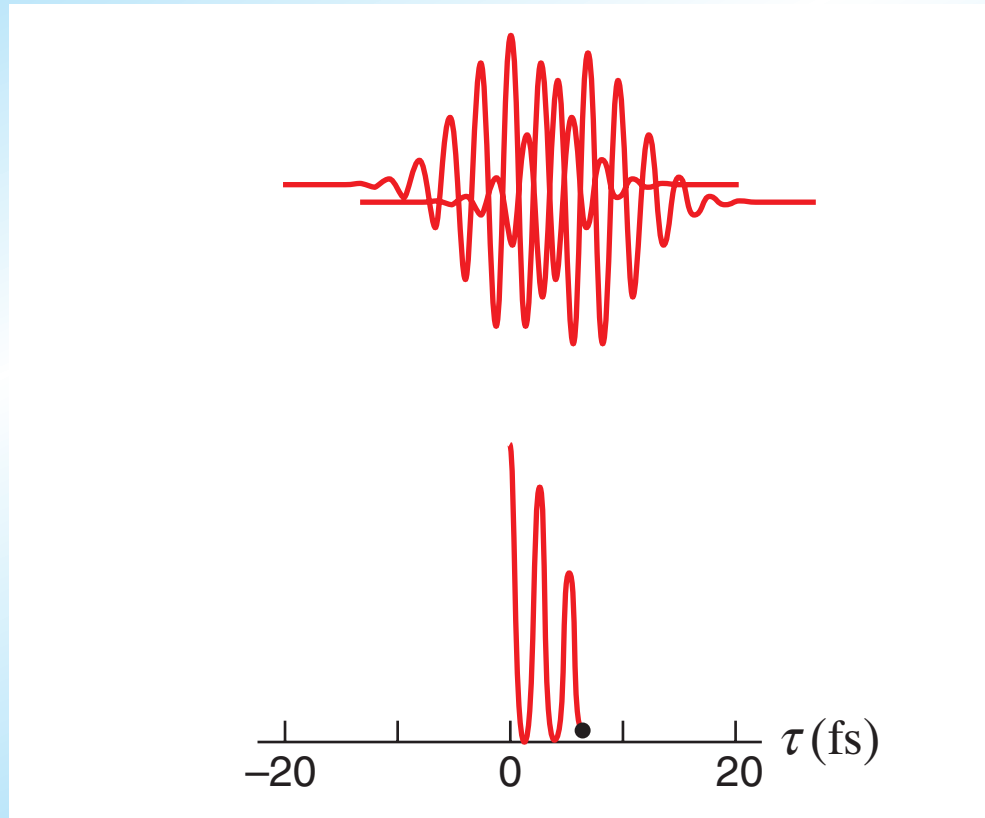
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Temporal characterization

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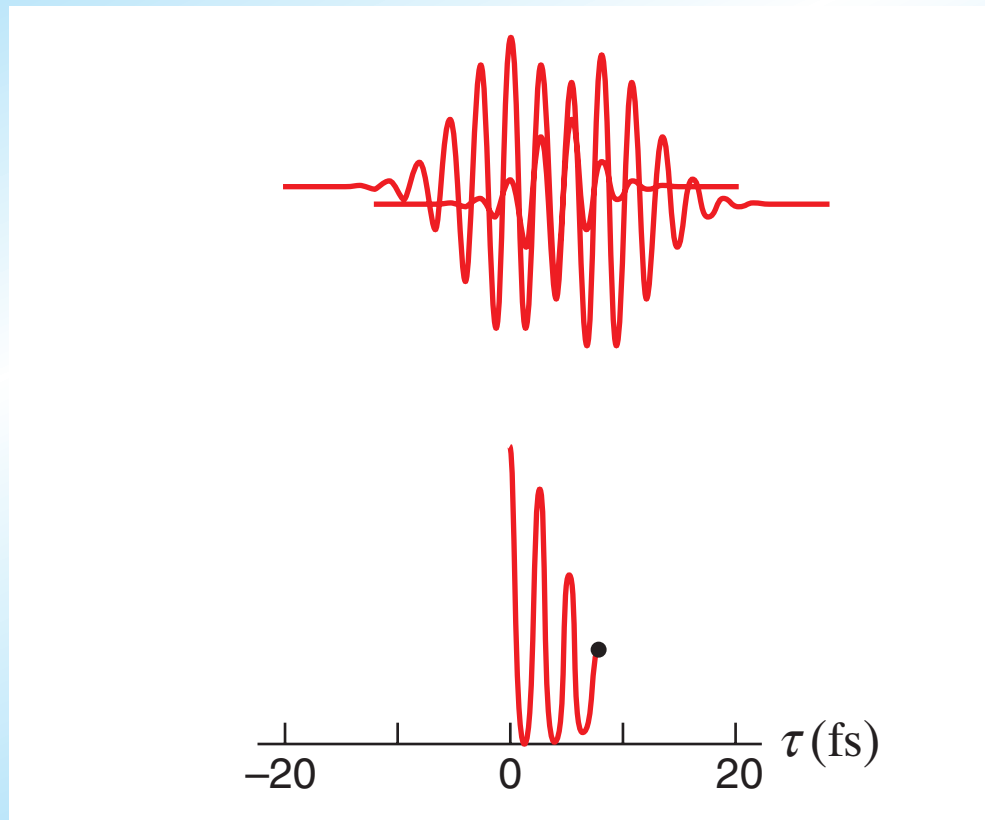
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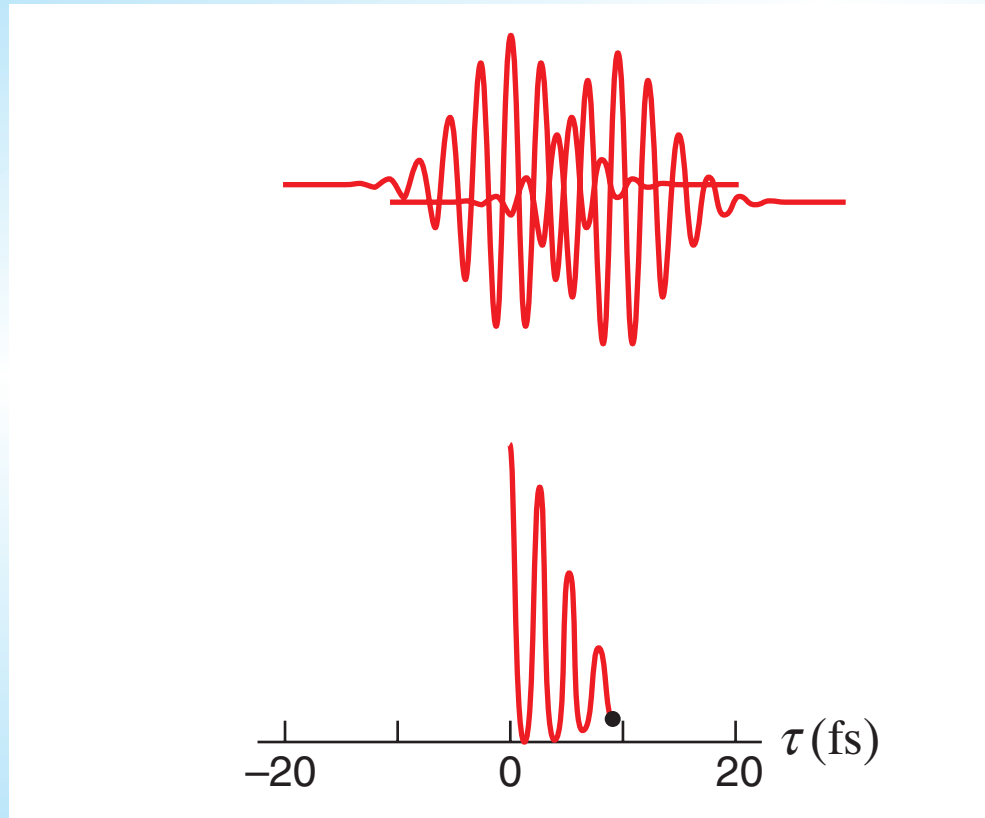
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Temporal characterization

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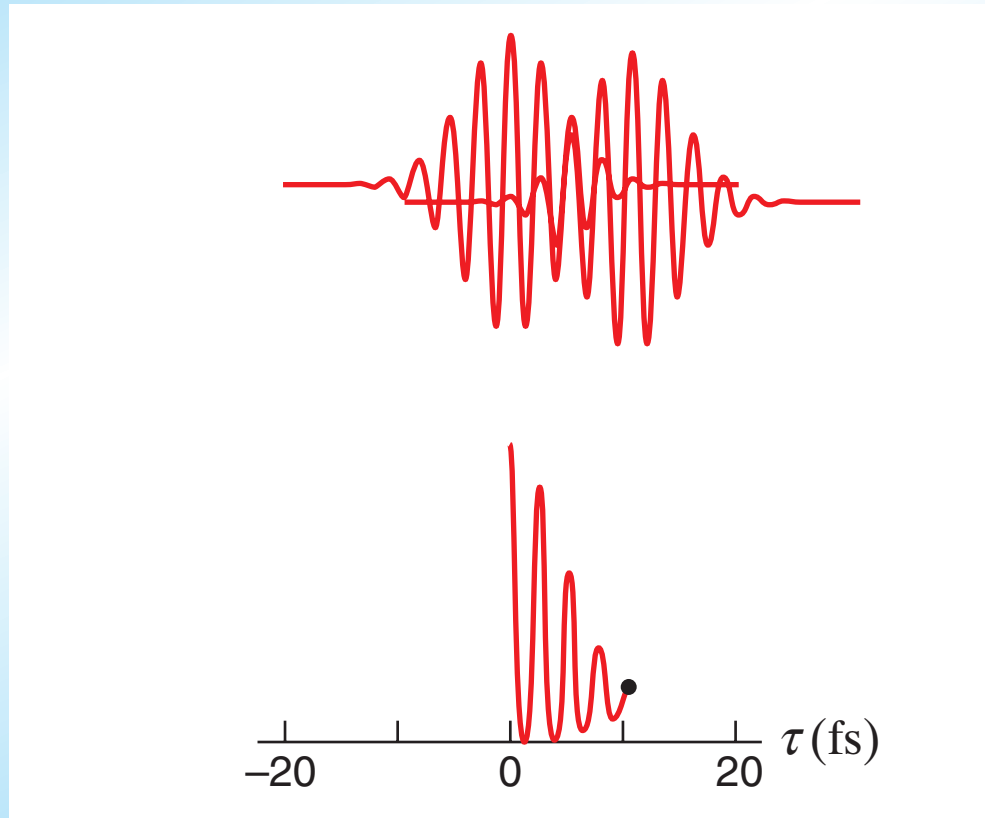
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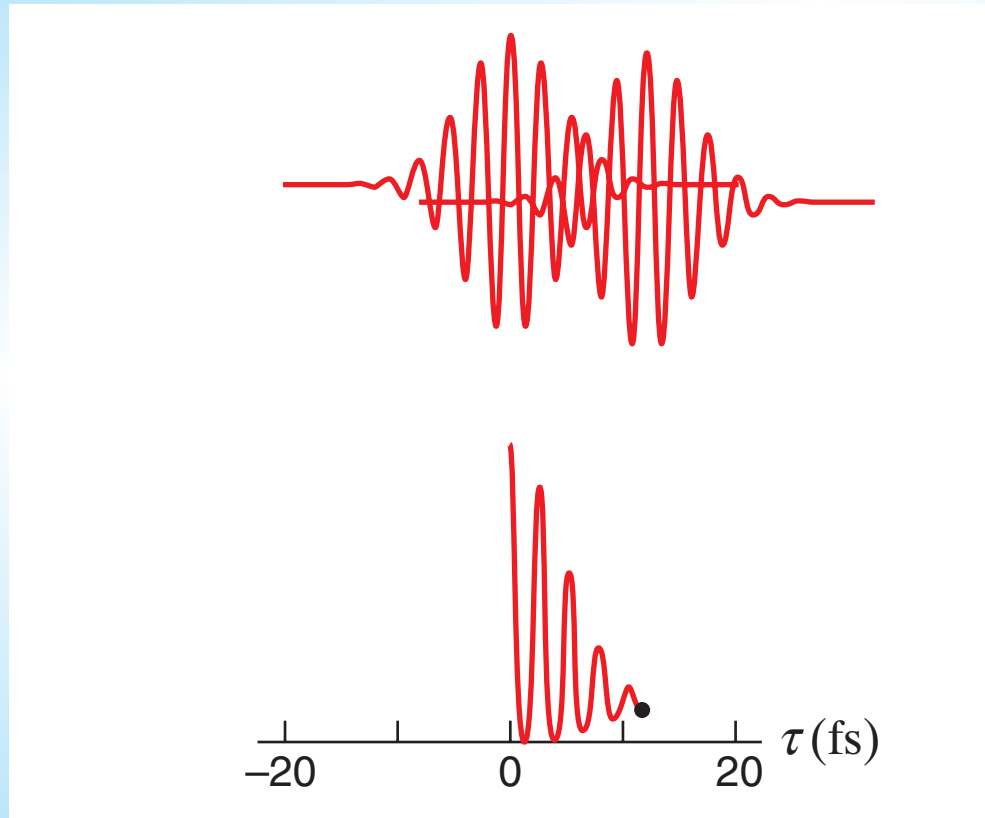
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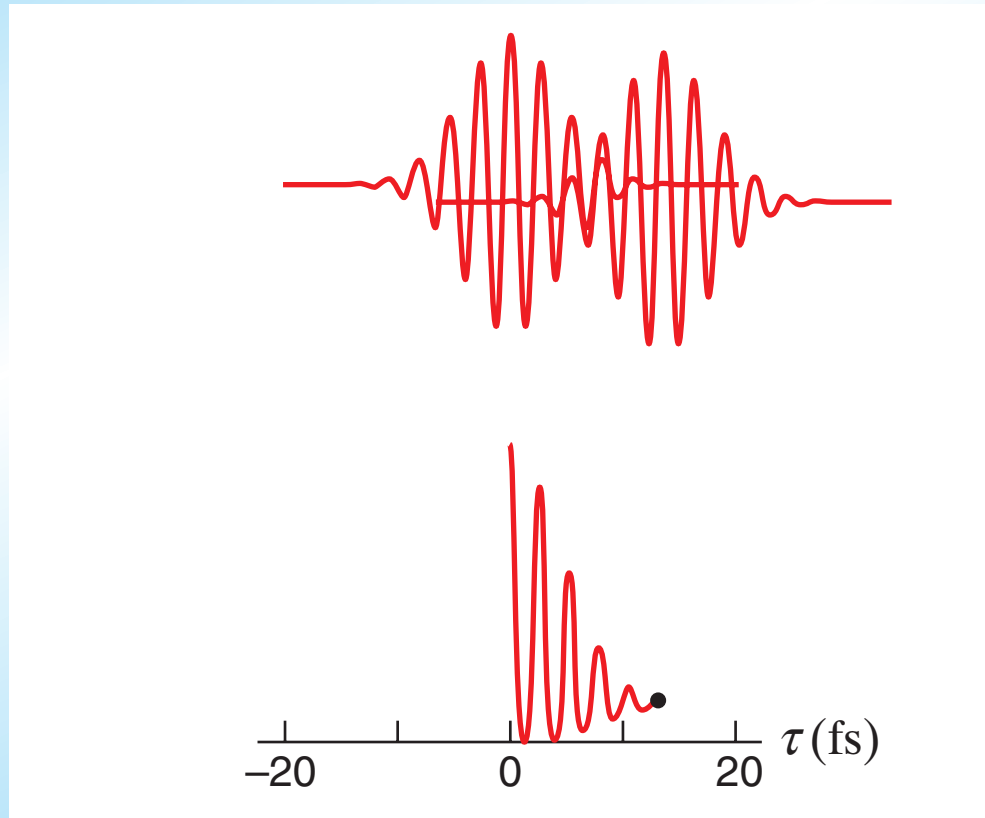
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Temporal characterization

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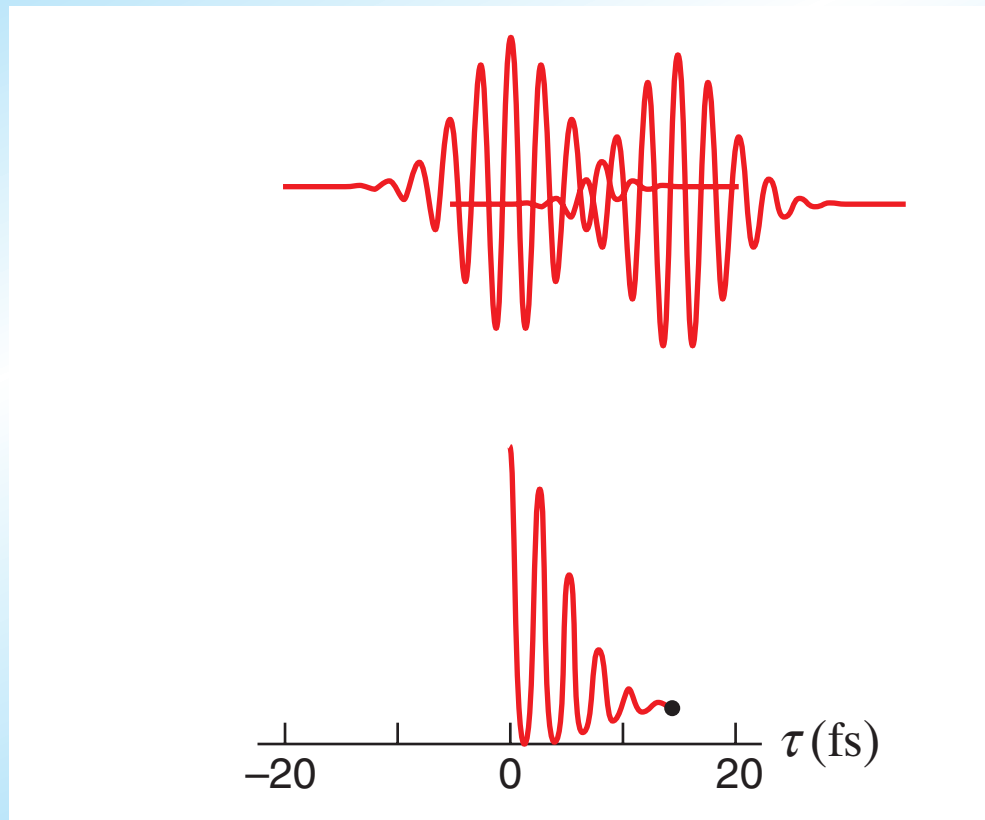
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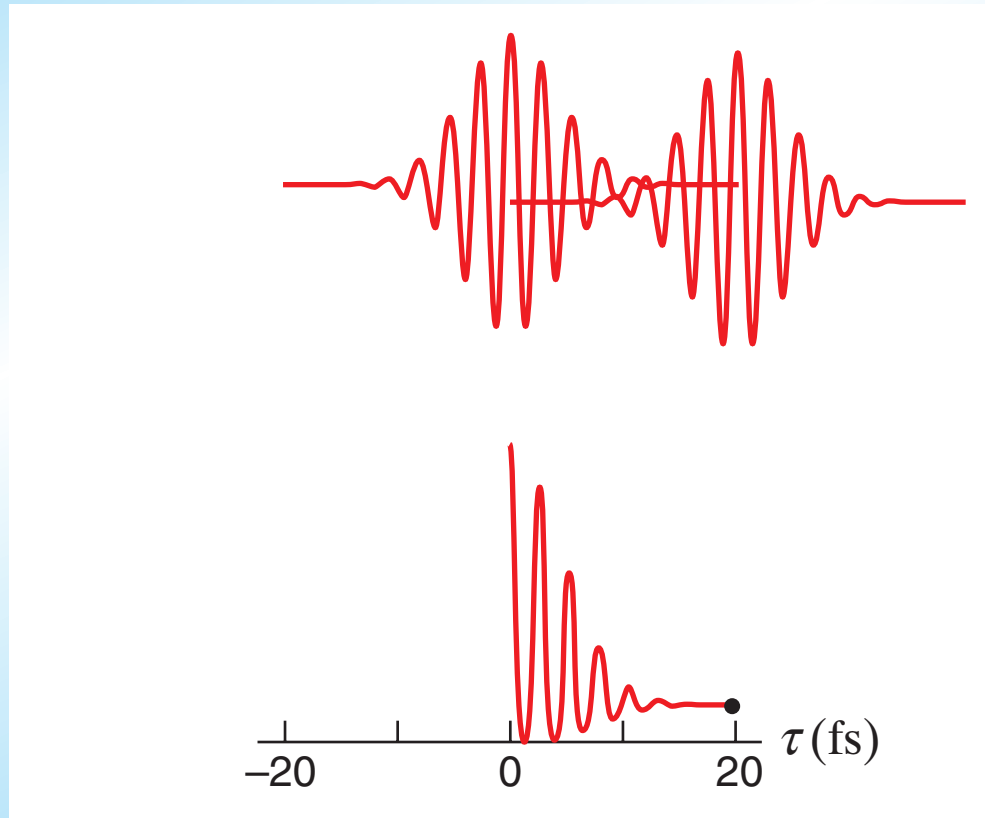
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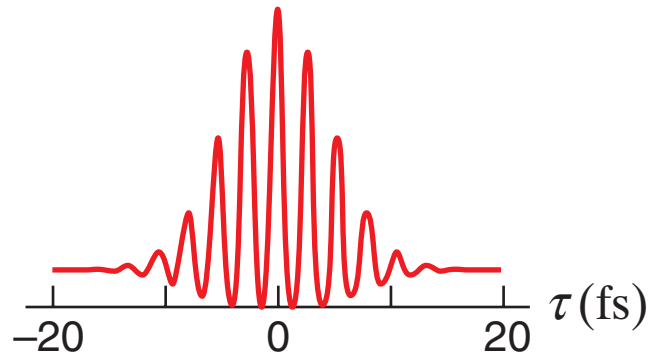
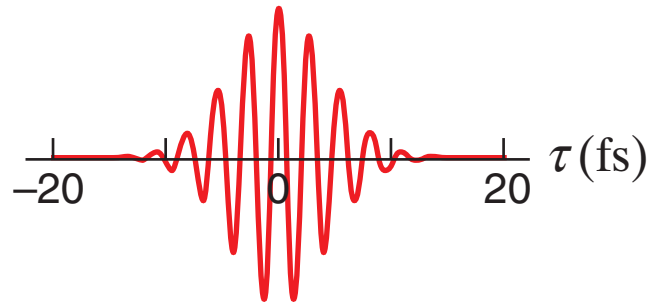
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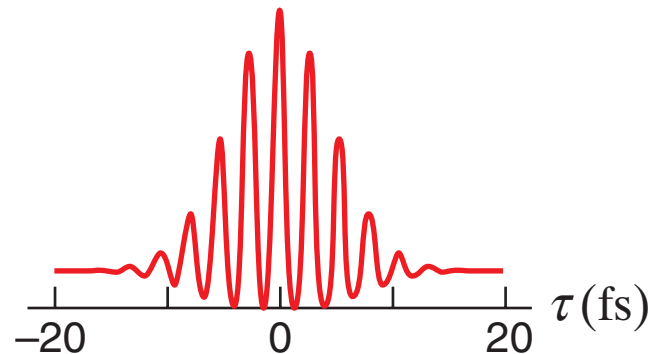
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at $\tau = 0$:

$$I_{2\omega}(t, \tau) \propto 16E^4(t)$$



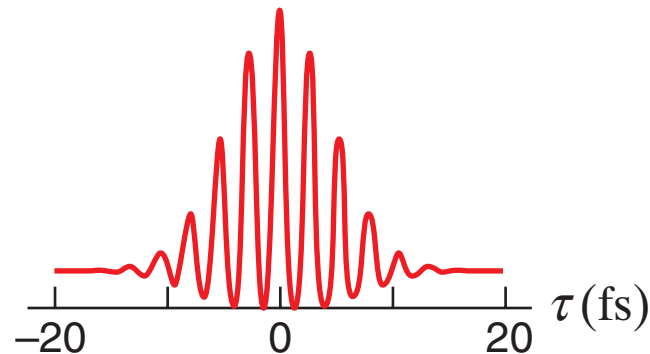
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at $\tau = 0$: $I_{2\omega}(t, \tau) \propto 16E^4(t)$

as $\tau \rightarrow \pm \infty$: $I_{2\omega}(t, \tau) \propto 2E^4(t)$



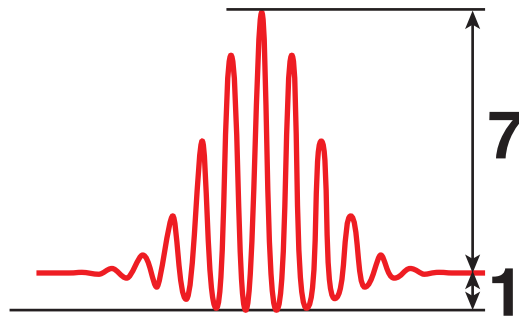
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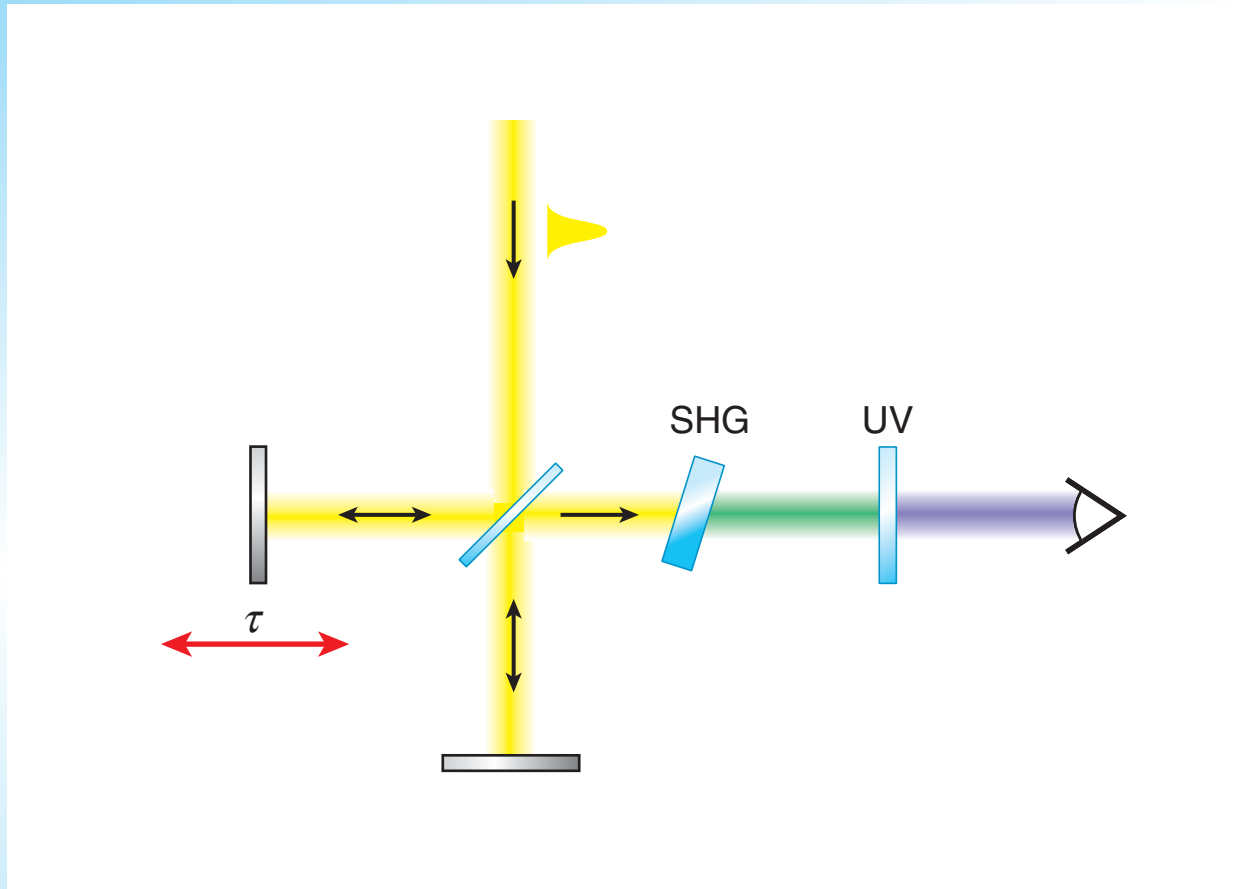
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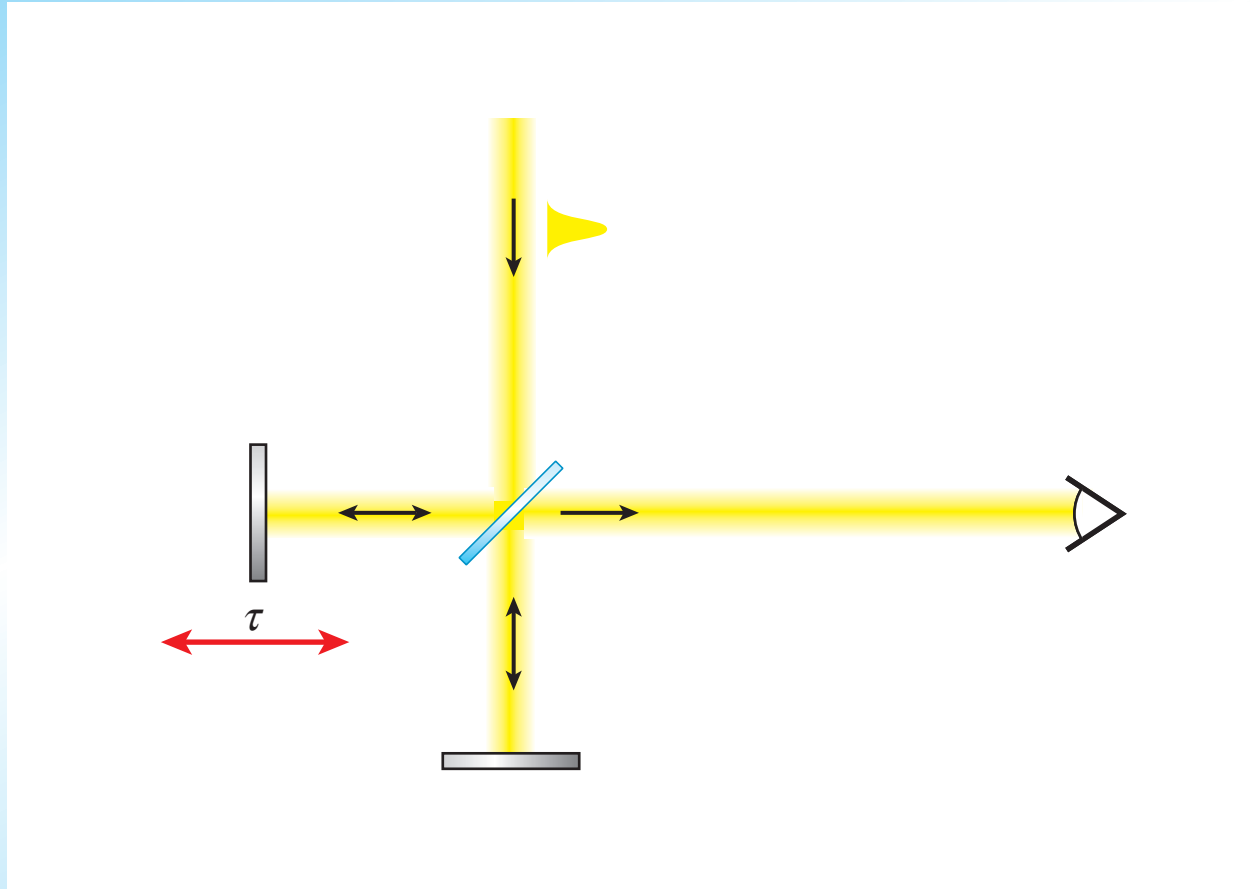
Temporal characterization

Do we really need the second-harmonic crystal...?



Temporal characterization

Would this work?



Temporal characterization

Intensity at detector

$$I_{\omega}(t, \tau) \propto |E_1(t) + E_2(t + \tau)|^2$$

Temporal characterization

Intensity at detector

$$I_{\omega}(t, \tau) \propto |E_1(t) + E_2(t + \tau)|^2$$

Detected signal

$$S_{\omega}(\tau) = \int I_{\omega}(t, \tau) dt$$

Temporal characterization

Intensity at detector

$$I_{\omega}(t, \tau) \propto |E_1(t) + E_2(t + \tau)|^2$$

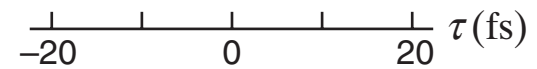
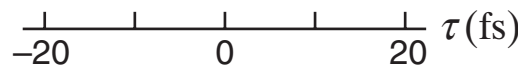
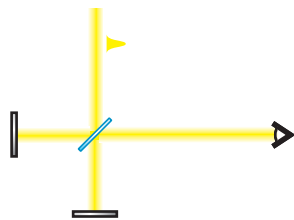
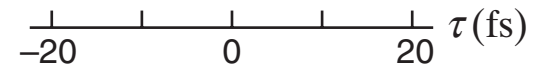
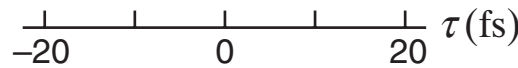
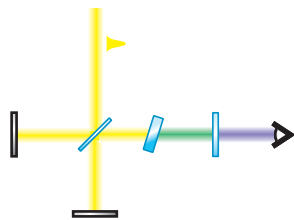
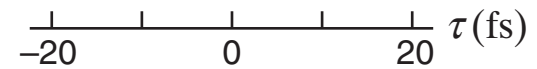
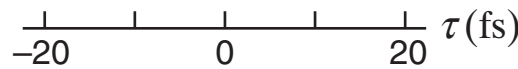
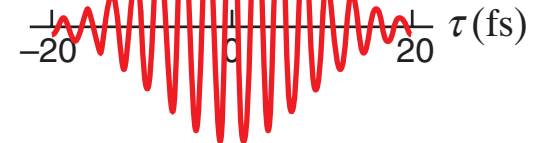
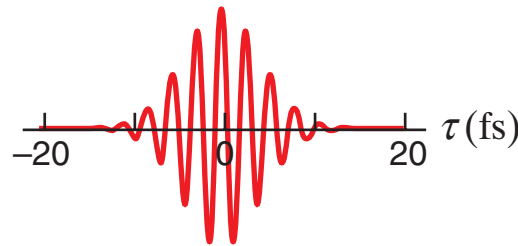
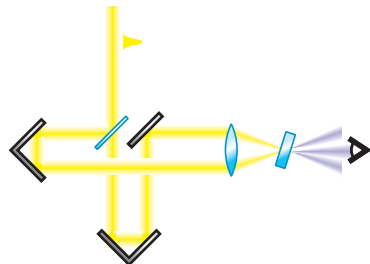
Detected signal

$$S_{\omega}(\tau) = \int I_{\omega}(t, \tau) dt$$

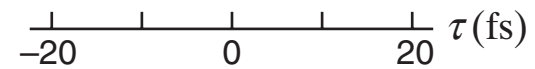
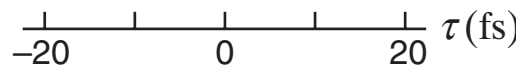
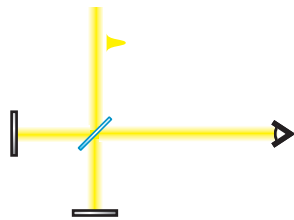
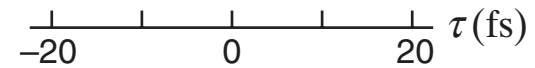
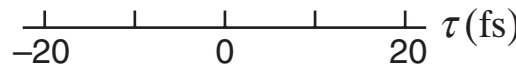
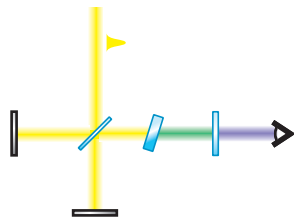
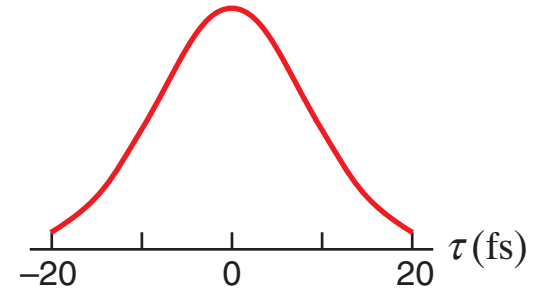
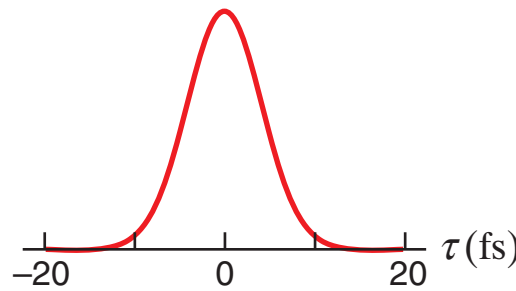
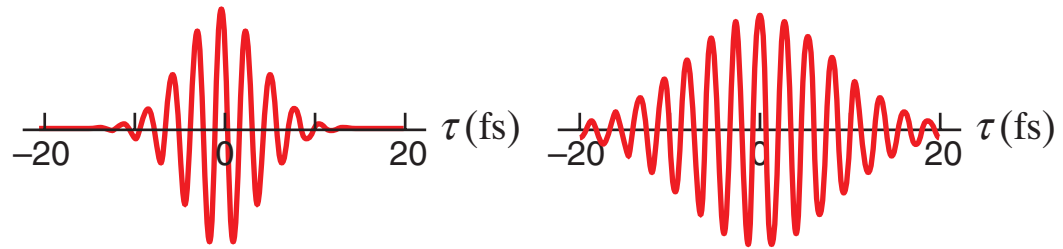
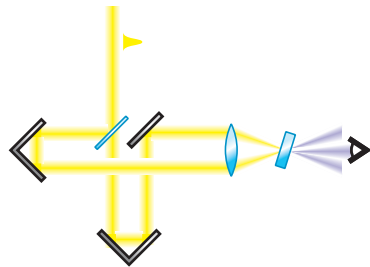
so

$$S_{\omega}(\tau) \propto \int \{|E_1(t)|^2 + |E_2(t + \tau)|^2 + E_1(t)E_2^*(t + \tau) + E_1^*(t)E_2(t + \tau)\} dt$$

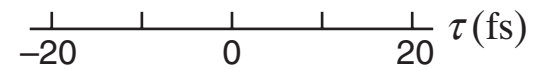
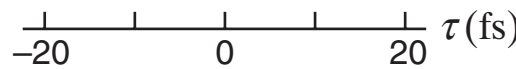
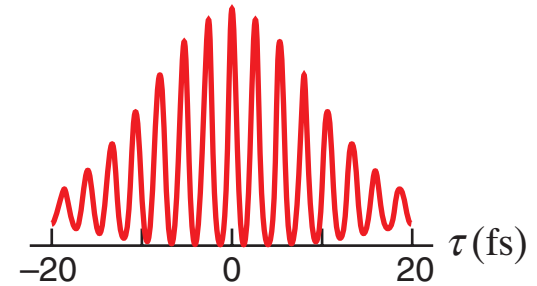
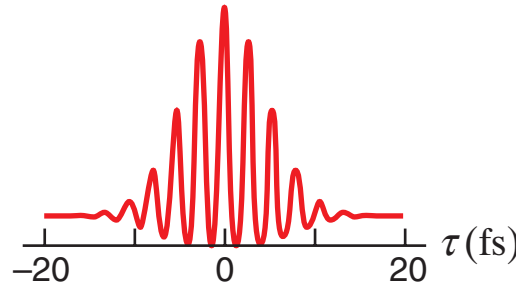
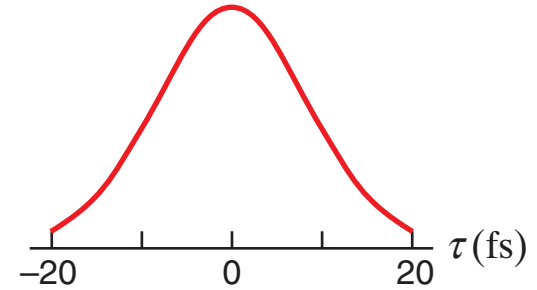
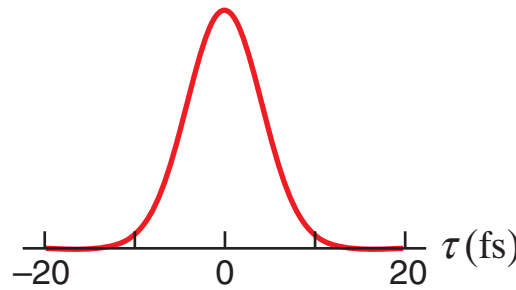
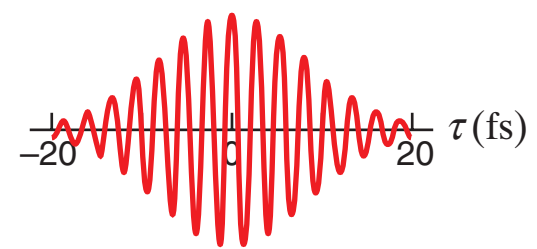
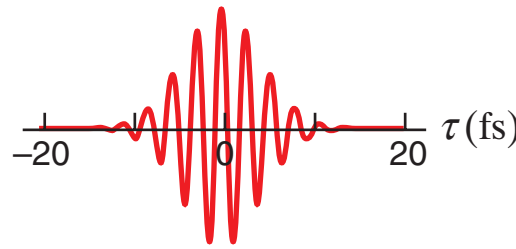
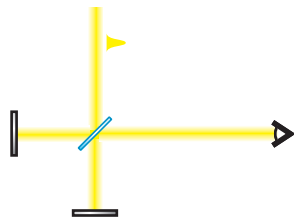
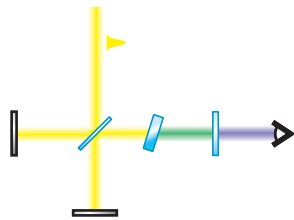
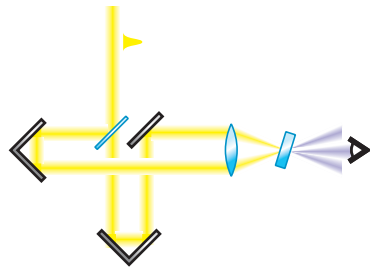
Temporal characterization



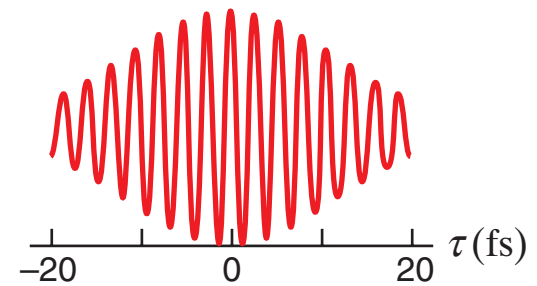
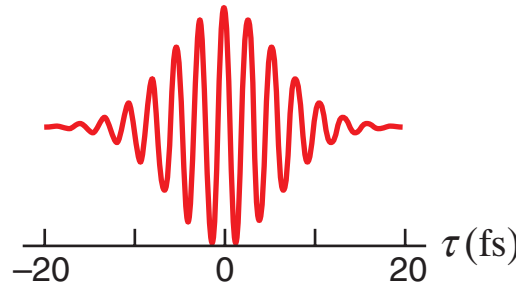
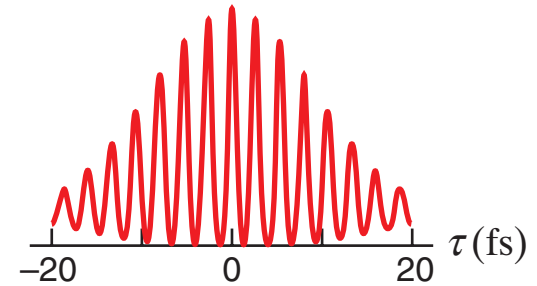
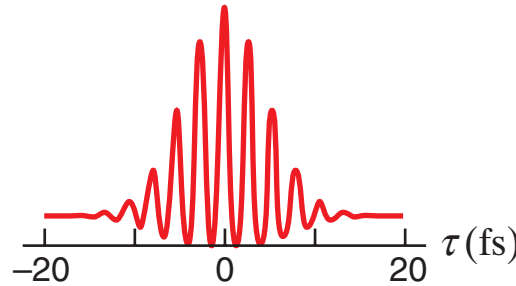
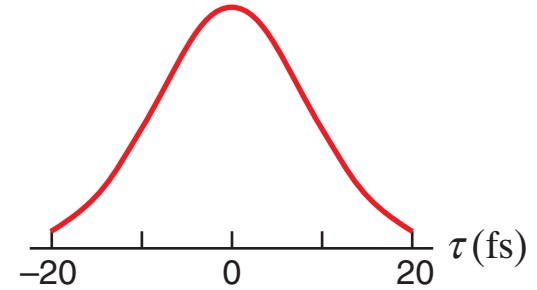
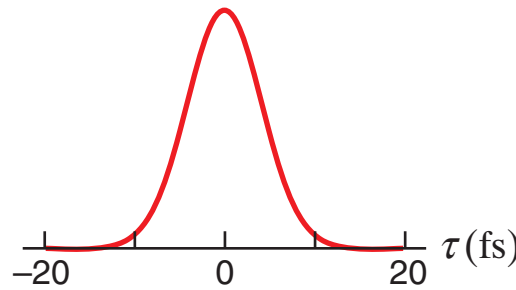
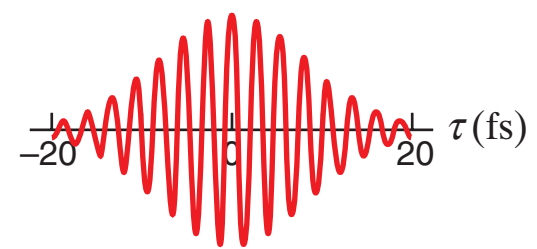
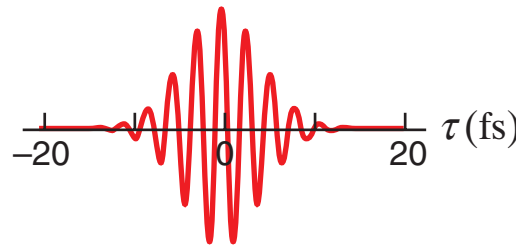
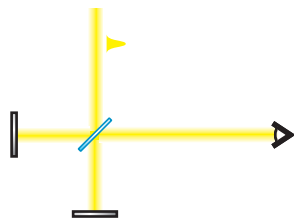
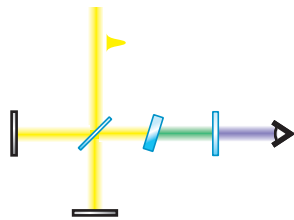
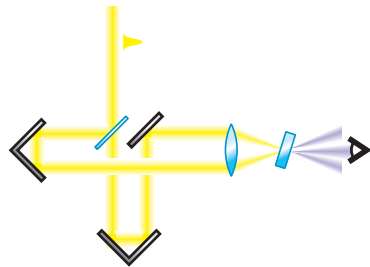
Temporal characterization



Temporal characterization



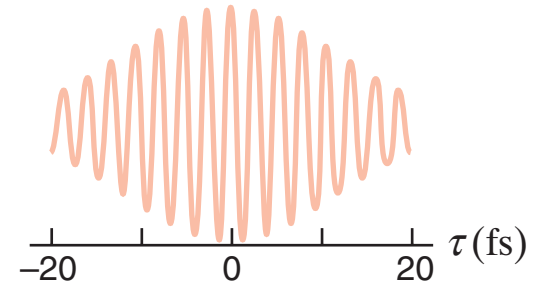
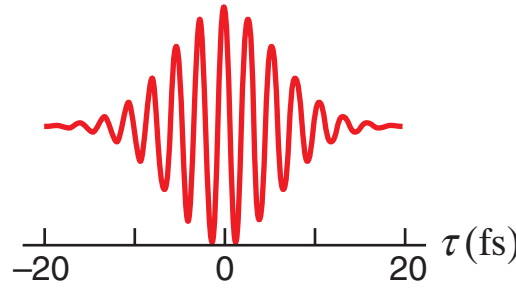
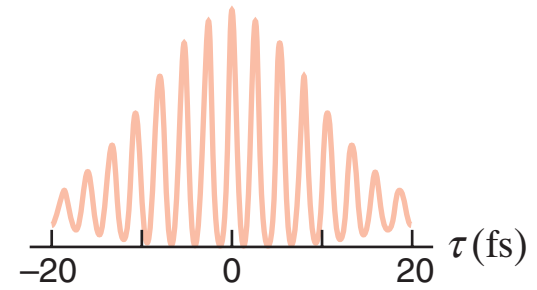
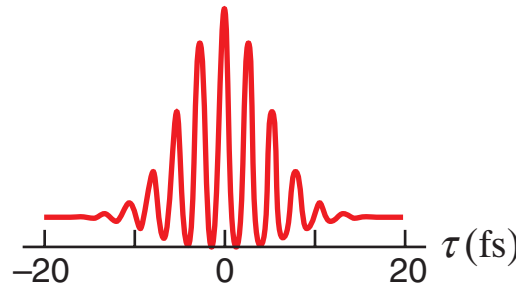
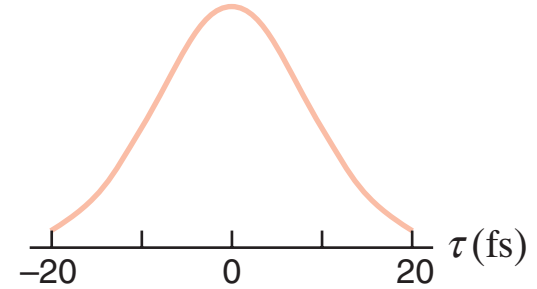
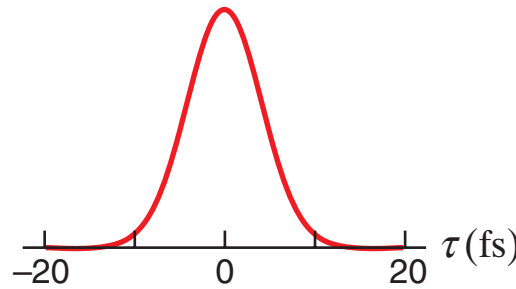
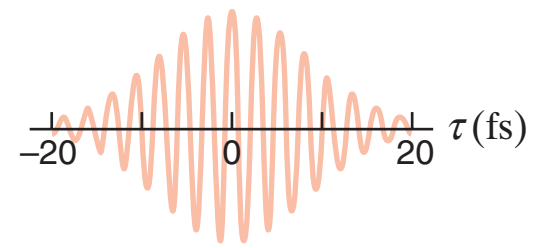
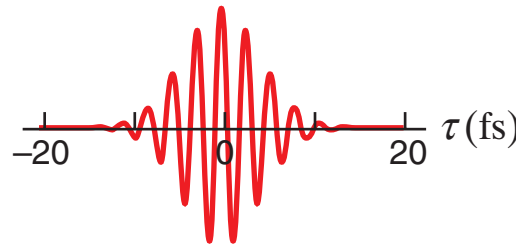
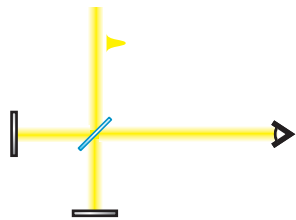
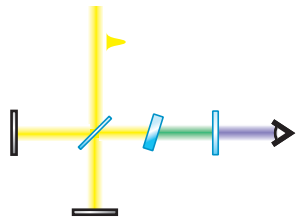
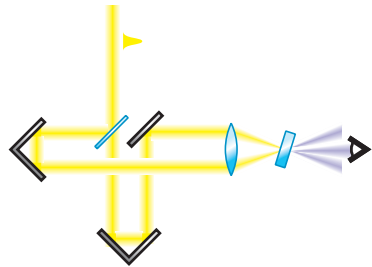
Temporal characterization



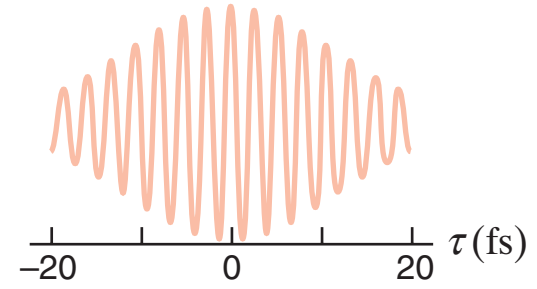
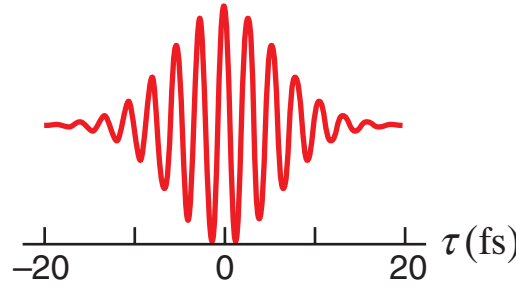
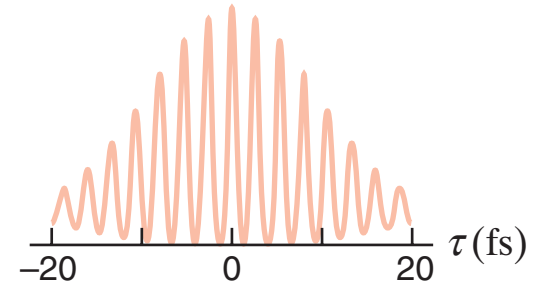
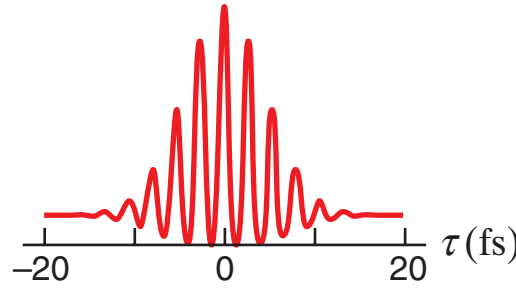
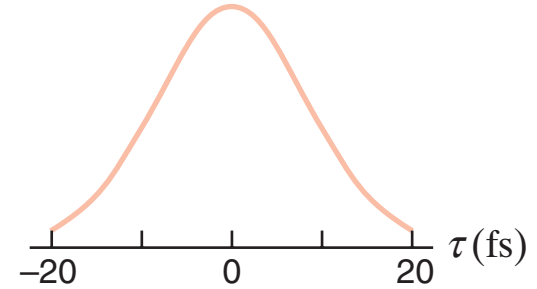
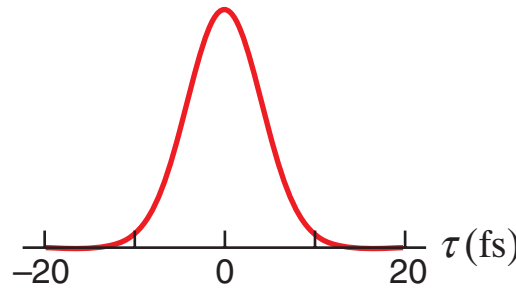
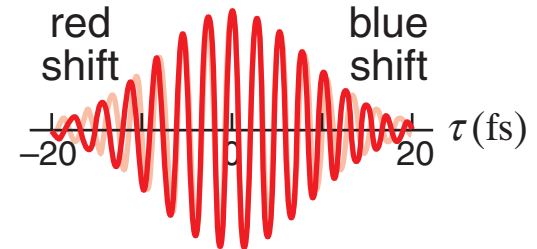
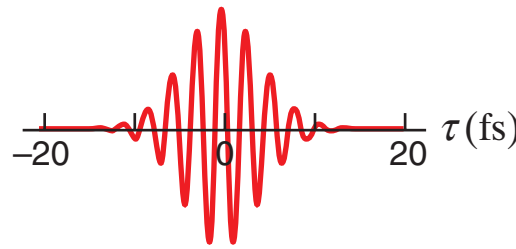
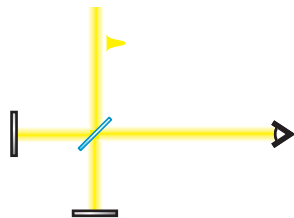
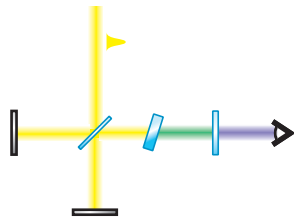
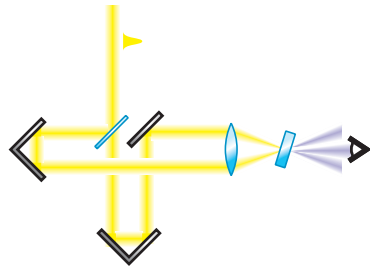
Temporal characterization

But what about dispersion?

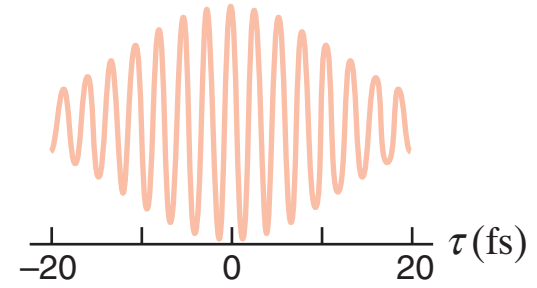
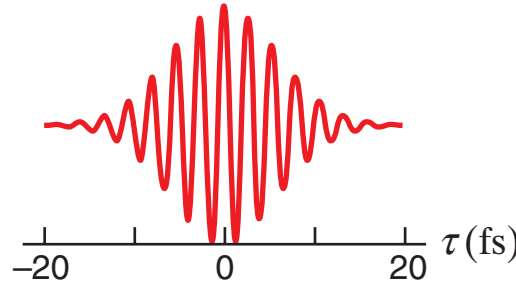
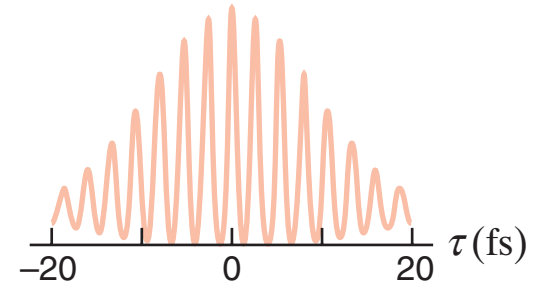
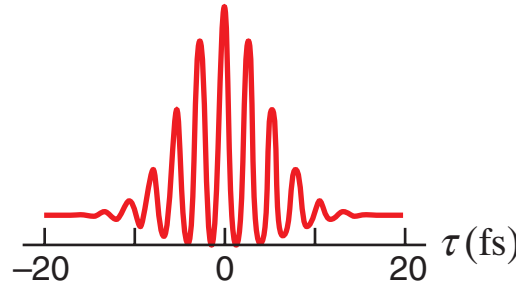
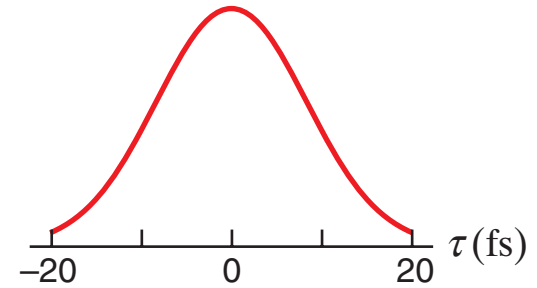
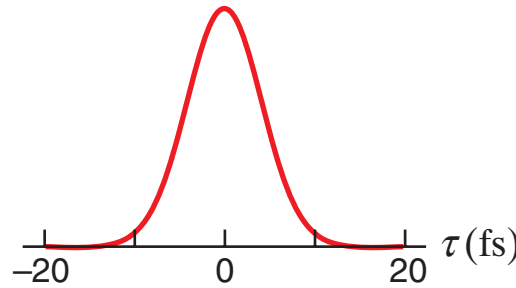
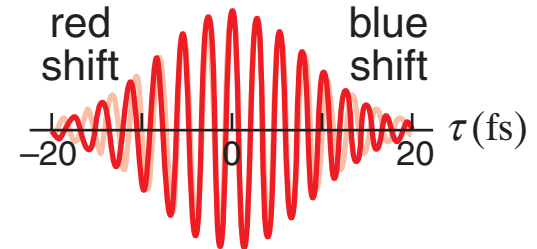
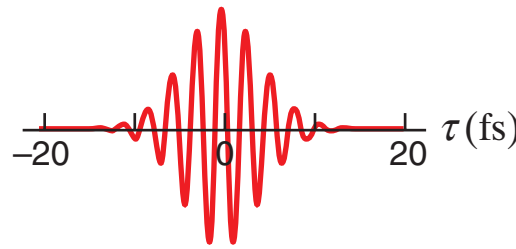
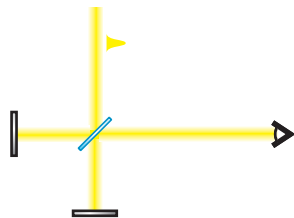
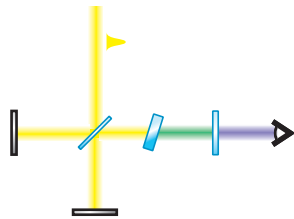
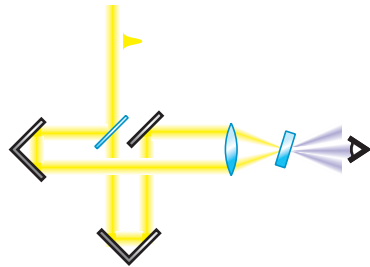
Temporal characterization



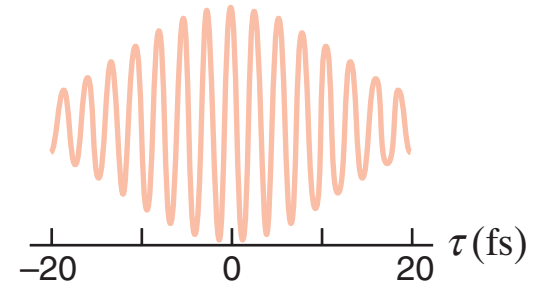
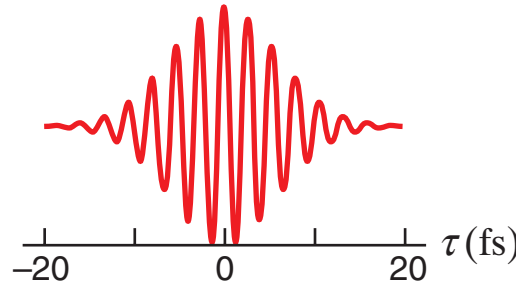
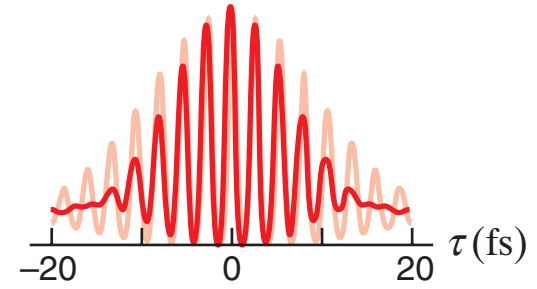
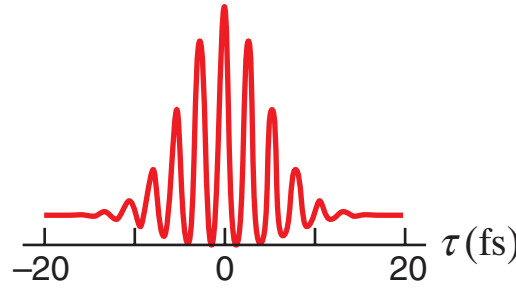
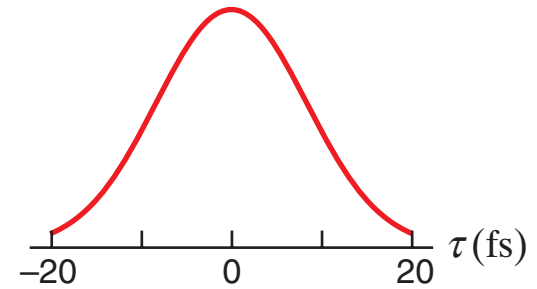
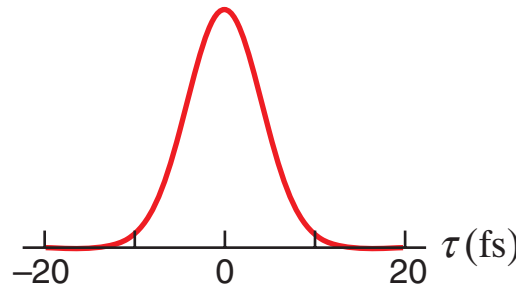
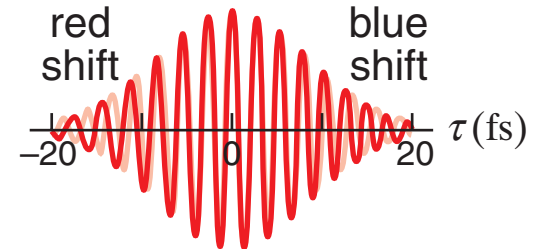
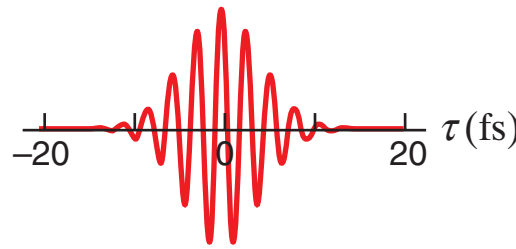
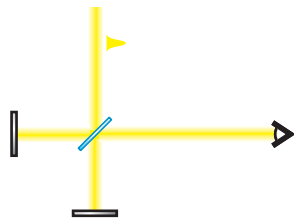
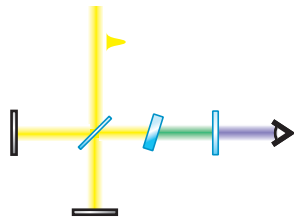
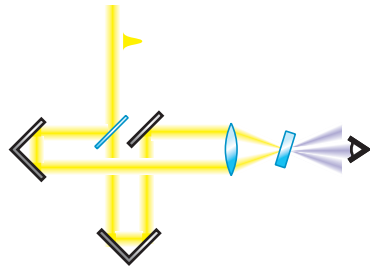
Temporal characterization



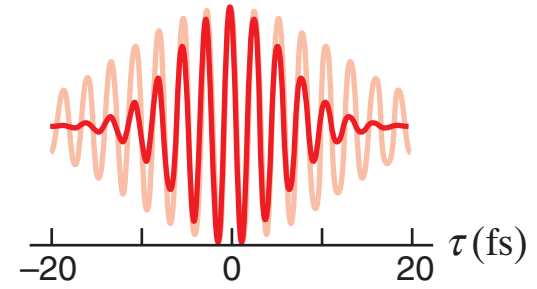
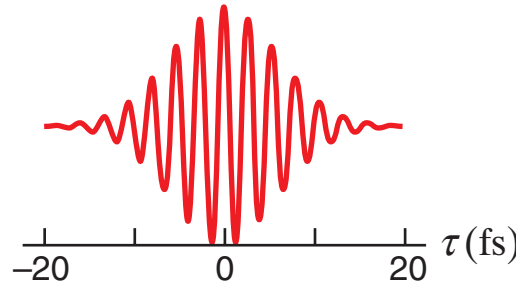
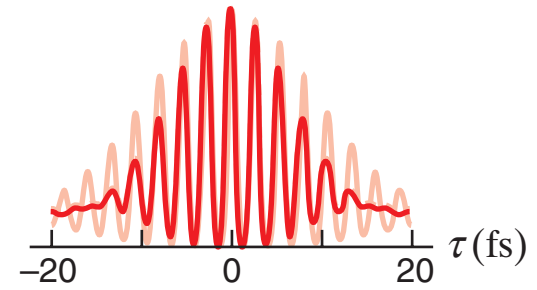
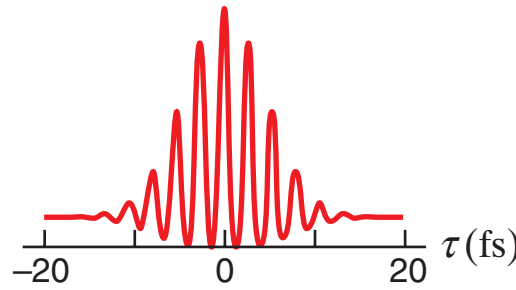
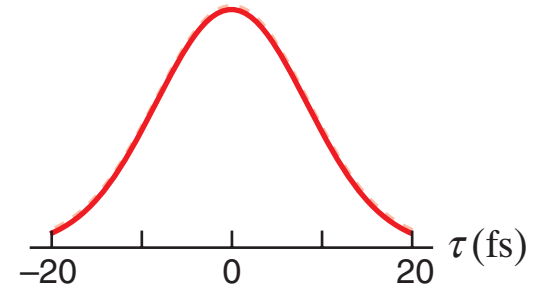
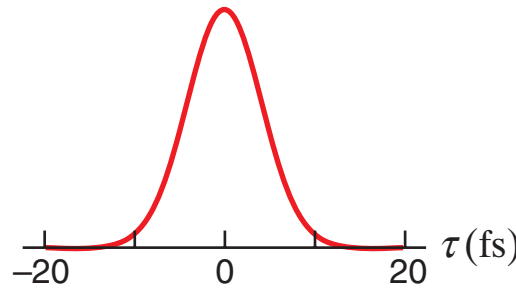
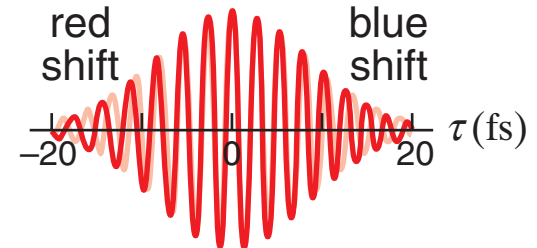
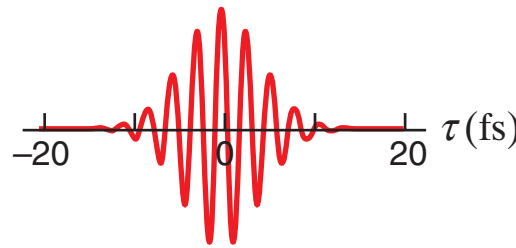
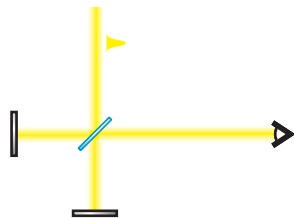
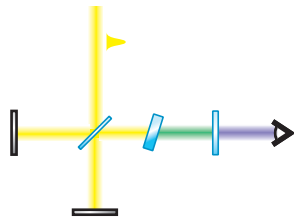
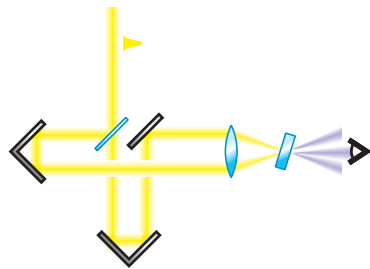
Temporal characterization



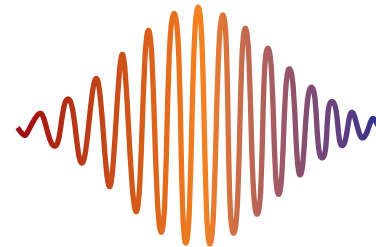
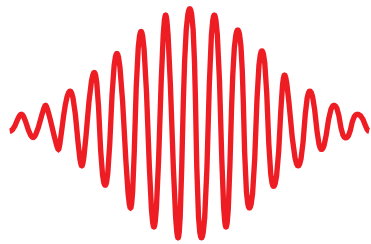
Temporal characterization



Temporal characterization

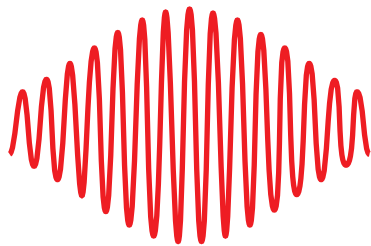
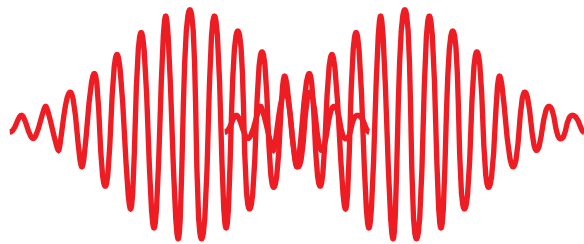


Temporal characterization

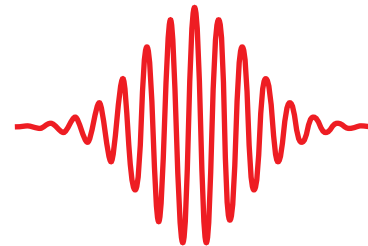
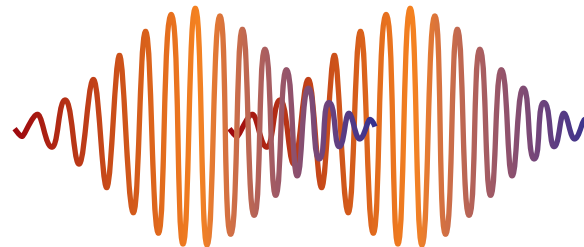


Temporal characterization

good
interference



poor
interference



Temporal characterization

Let $E_{disp}(\omega) = E_{orig}(\omega)e^{-i\phi(\omega)}$.

Temporal characterization

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Interference term in linear autocorrelation:

$$\int E_{disp}(t+\tau)E_{disp}^*(t) dt = \mathcal{F}^{-1}\{E_{disp}(\omega)E_{disp}^*(\omega)\} =$$

Temporal characterization

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Temporal characterization

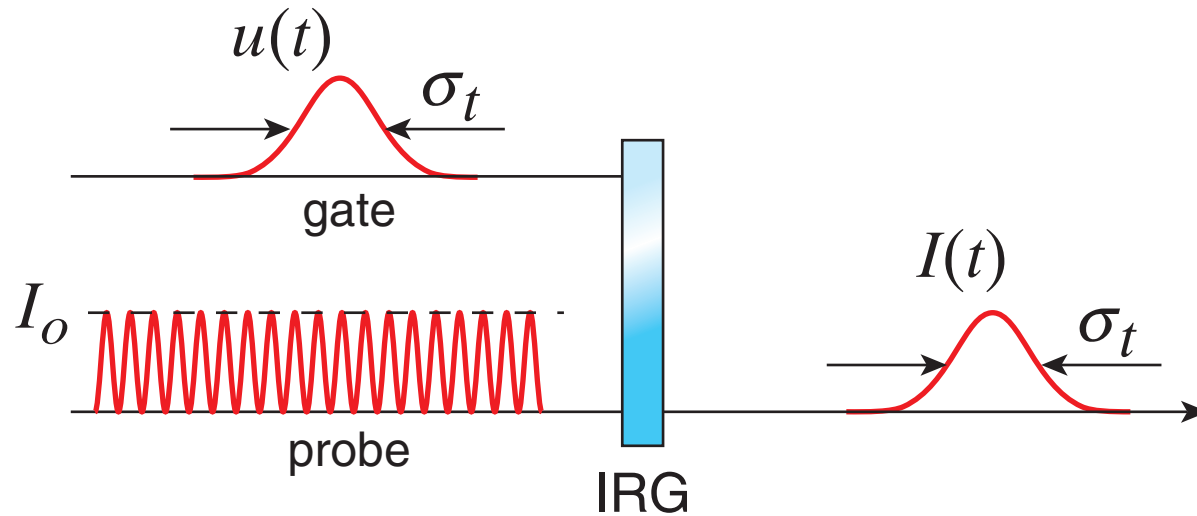
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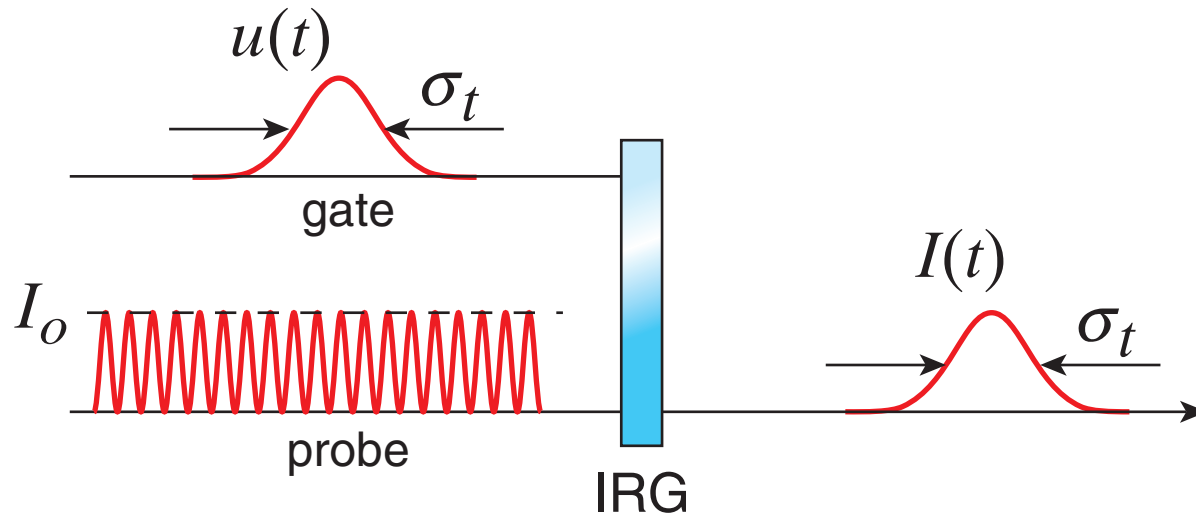
Joint time-frequency measurements



IRG ("instantaneous response gate"): device whose transmittance of a weak probe pulse is proportional to the intensity envelope of the pump ("gate")

$$T(t) = u(t)$$

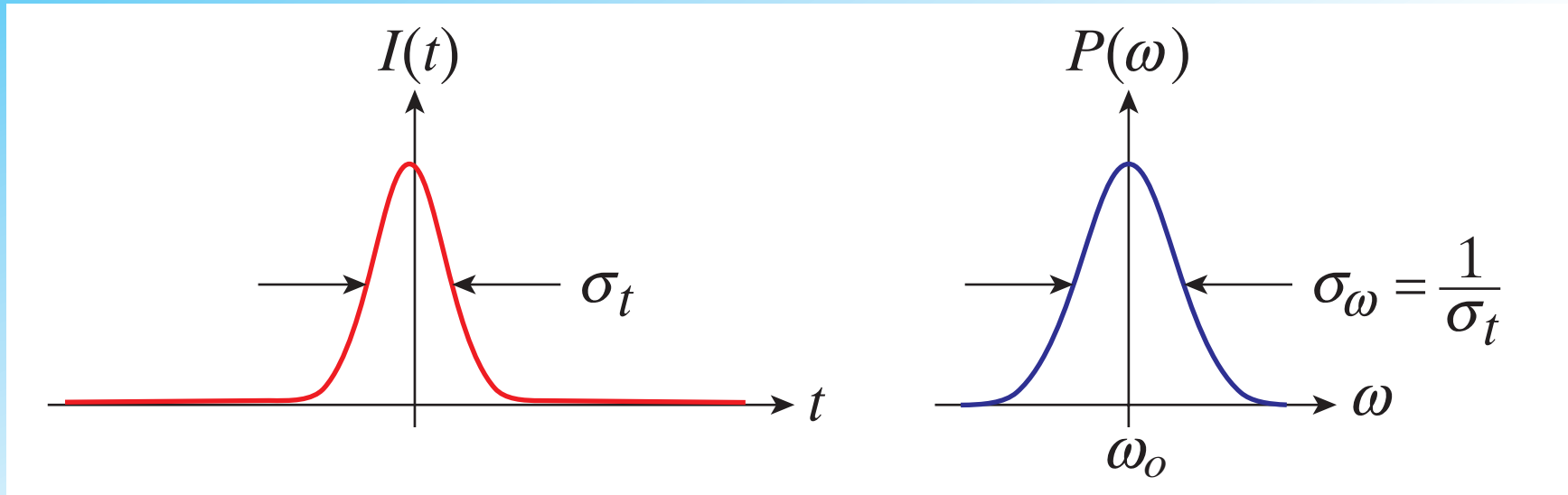
Joint time-frequency measurements



Transmitted intensity

$$I(t) = I_o T(t) = I_o u(t) = I_o \exp\left[-\frac{t^2}{\sigma_t^2}\right]$$

Joint time-frequency measurements

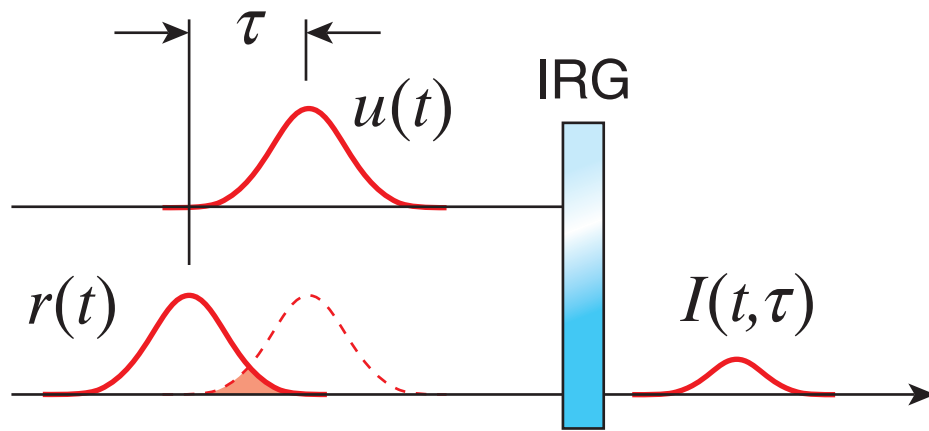


Transmitted intensity

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$$\sigma_t \sigma_\omega = 1$$

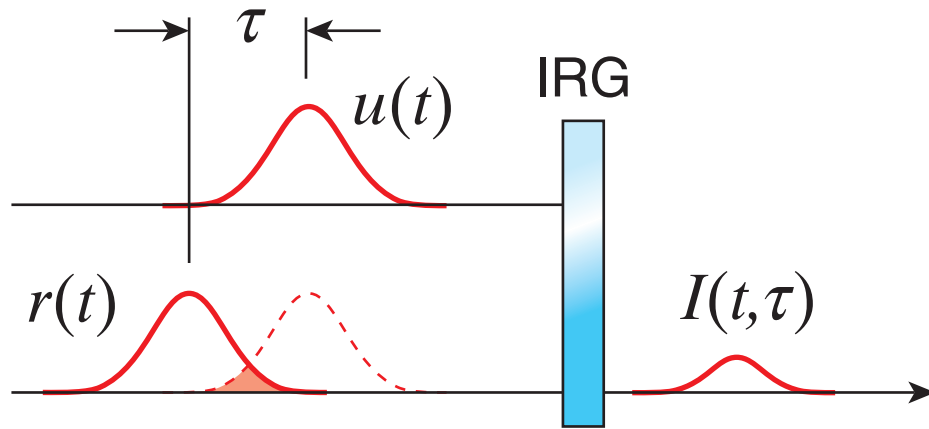
Joint time-frequency measurements



Transmitted intensity

$$\begin{aligned} I(t, \tau) &= u(t)r(t+\tau) = \exp\left[-\frac{t^2}{\sigma^2}\right] \exp\left[-\frac{(t+\tau)^2}{\sigma^2}\right] = \\ &= \exp\left[-\frac{2t^2 + 2t\tau + \tau^2}{\sigma^2}\right] = \exp\left[-\frac{2t^2 + 2t\tau + \tau^2/2}{\sigma^2}\right] \exp\left[-\frac{\tau^2}{2\sigma^2}\right] = \end{aligned}$$

Joint time-frequency measurements

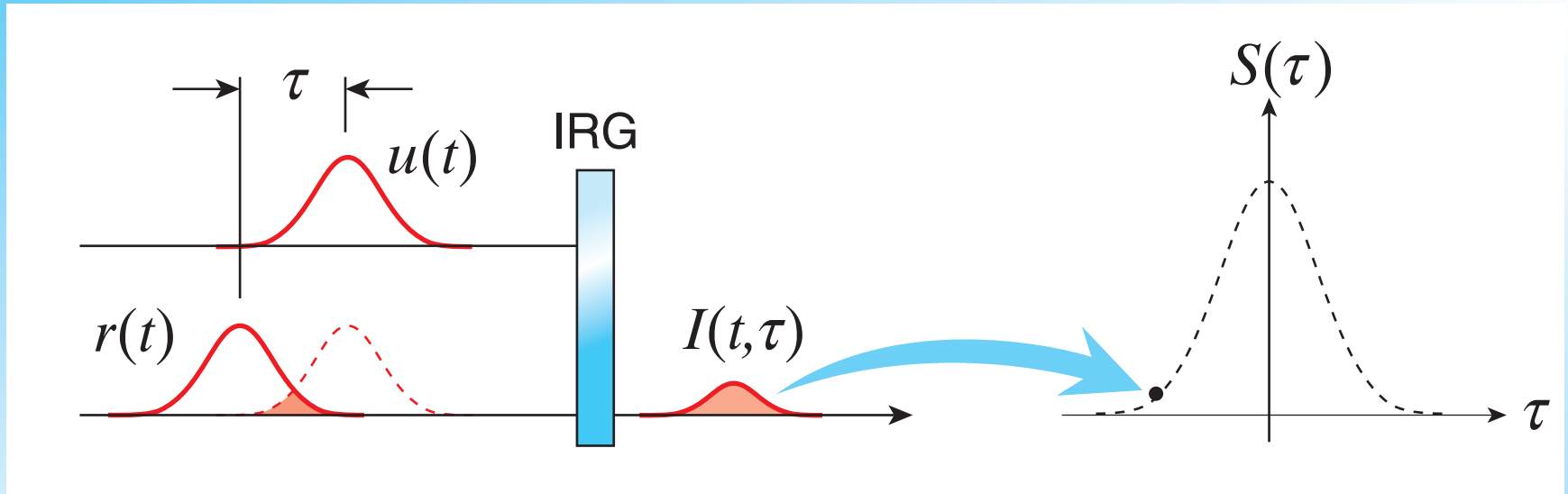


Transmitted intensity

$$I(t, \tau) = \exp\left[-\frac{\tau^2}{2\sigma^2}\right] \exp\left[-\left(\frac{t + \tau/2}{\sigma/\sqrt{2}}\right)^2\right]$$

so $I(t, \tau)$ narrowed by $\sqrt{2}$

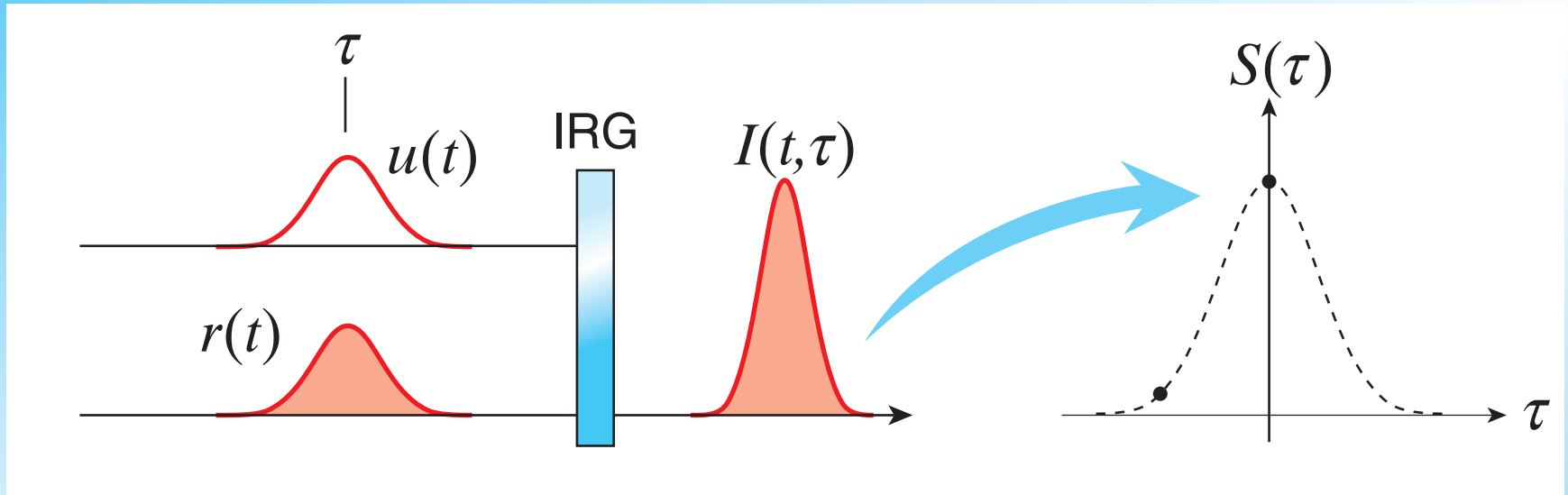
Joint time-frequency measurements



...but detector integrates $I(t, \tau)$:

$$S(\tau) = \int_{-\infty}^{\infty} I(t, \tau) dt = \int_{-\infty}^{\infty} \exp\left[-\frac{\tau^2}{2\sigma^2}\right] \exp\left[-2\left(\frac{t + \tau/2}{\sigma}\right)^2\right] dt$$

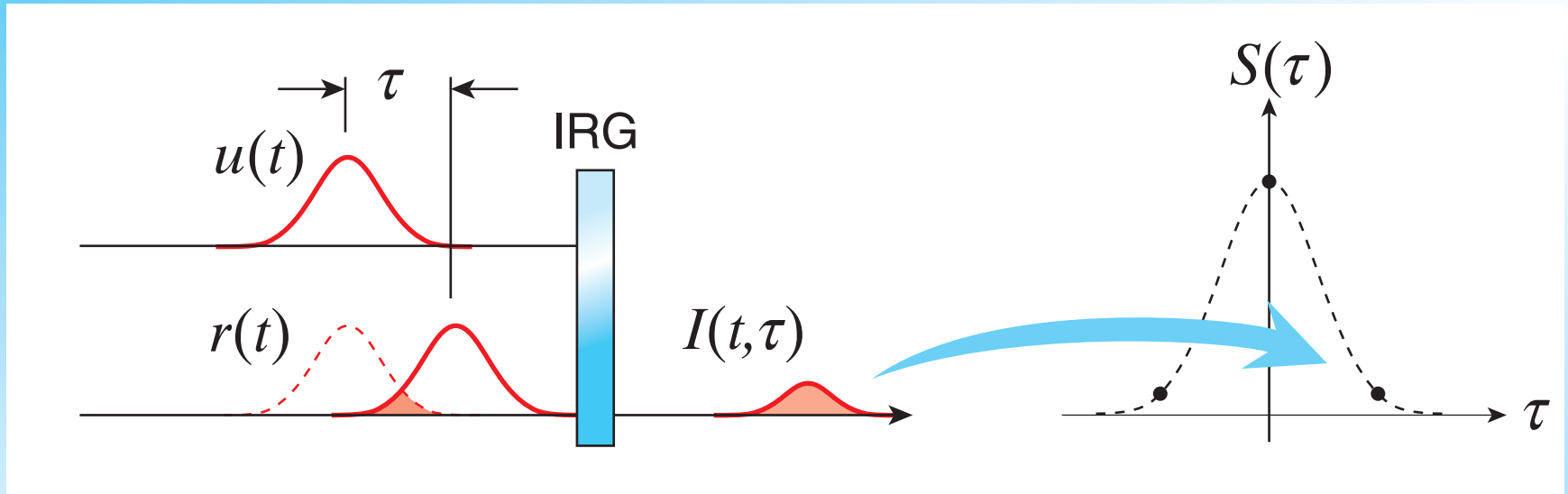
Joint time-frequency measurements



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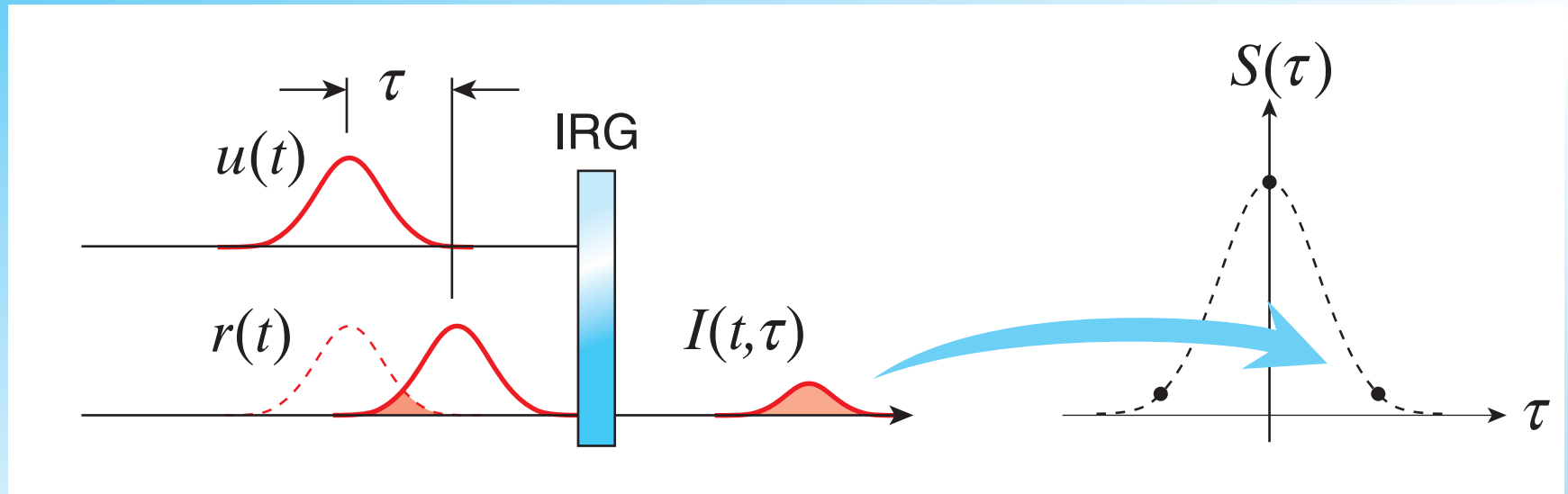
Joint time-frequency measurements



...but detector integrates $I(t, \tau)$:

$$S(\tau) = \int_{-\infty}^{\infty} I(t, \tau) dt = \int_{-\infty}^{\infty} \exp\left[-\frac{\tau^2}{2\sigma^2}\right] \exp\left[-2\left(\frac{t + \tau/2}{\sigma}\right)^2\right] dt$$

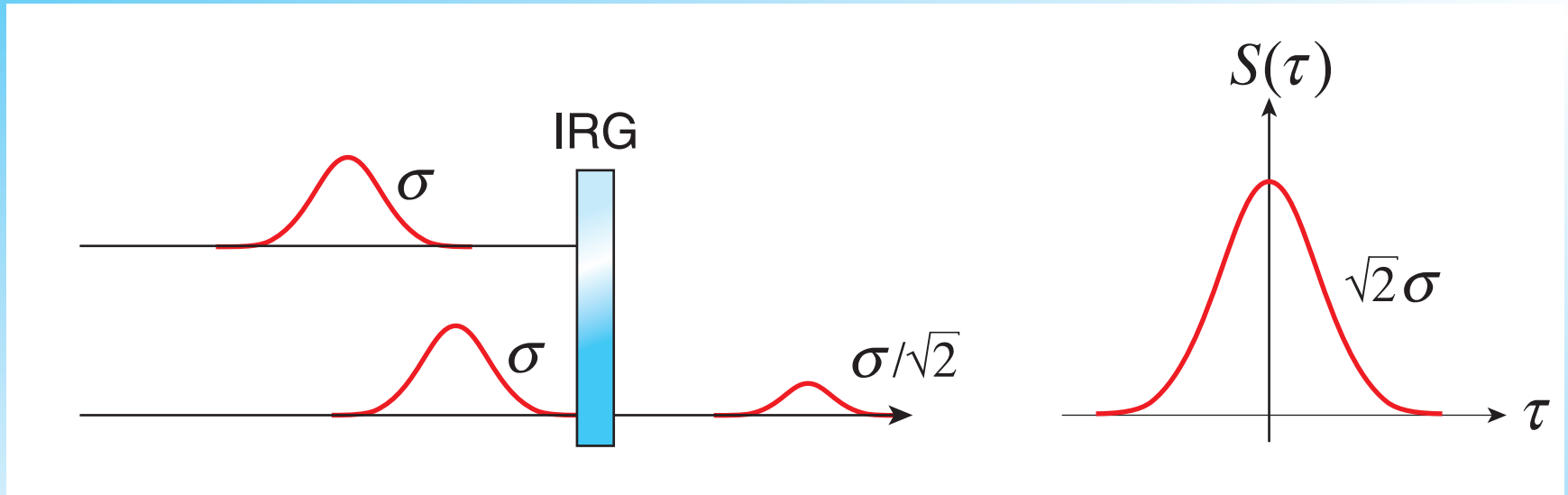
Joint time-frequency measurements



...but detector integrates $I(t, \tau)$:

$$\begin{aligned} S(\tau) &= \int_{-\infty}^{\infty} I(t, \tau) dt = \int_{-\infty}^{\infty} \exp\left[-\frac{\tau^2}{2\sigma^2}\right] \exp\left[-2\left(\frac{t + \tau/2}{\sigma}\right)^2\right] dt \\ &= \exp\left[-\frac{\tau^2}{2\sigma^2}\right] \int_{-\infty}^{\infty} \exp\left[-\frac{2u^2}{\sigma^2}\right] du = \sqrt{\frac{\pi}{2}} \sigma \exp\left[-\frac{\tau^2}{(\sigma\sqrt{2})^2}\right] \end{aligned}$$

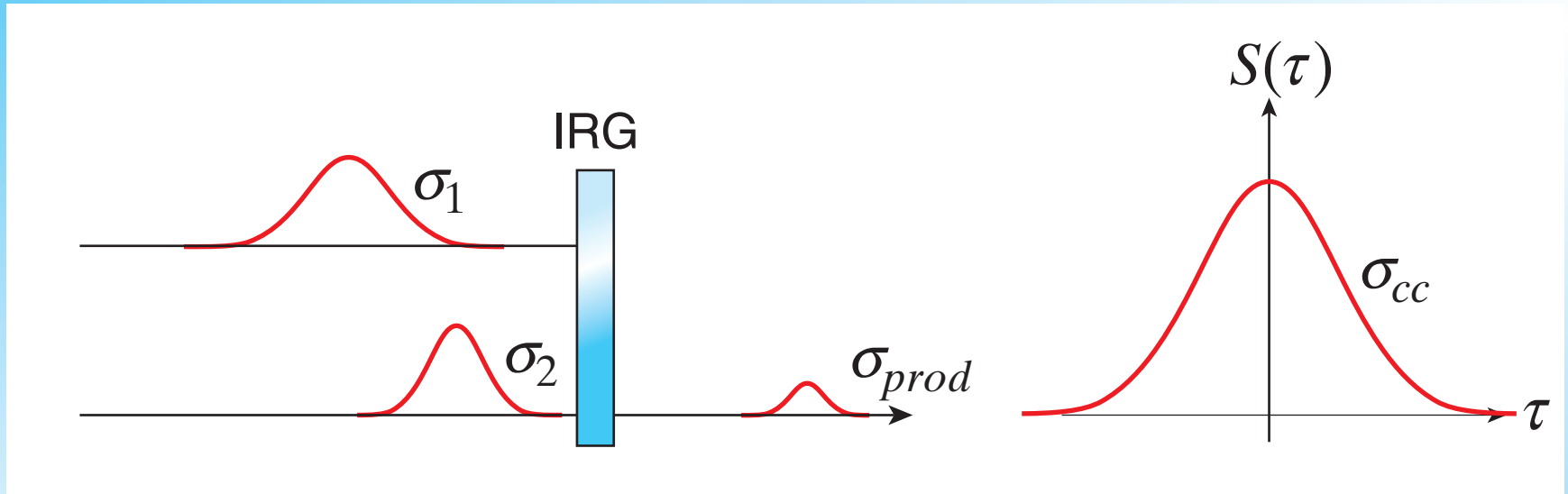
Joint time-frequency measurements



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$$\begin{aligned} S(\tau) &= \int_{-\infty}^{\infty} I(t, \tau) dt = \int_{-\infty}^{\infty} \exp\left[-\frac{\tau^2}{2\sigma^2}\right] \exp\left[-2\left(\frac{t + \tau/2}{\sigma}\right)^2\right] dt \\ &= \exp\left[-\frac{\tau^2}{2\sigma^2}\right] \int_{-\infty}^{\infty} \exp\left[-\frac{2u^2}{\sigma^2}\right] du = \sqrt{\frac{\pi}{2}} \sigma \exp\left[-\frac{\tau^2}{(\sigma\sqrt{2})^2}\right] \end{aligned}$$

Joint time-frequency measurements

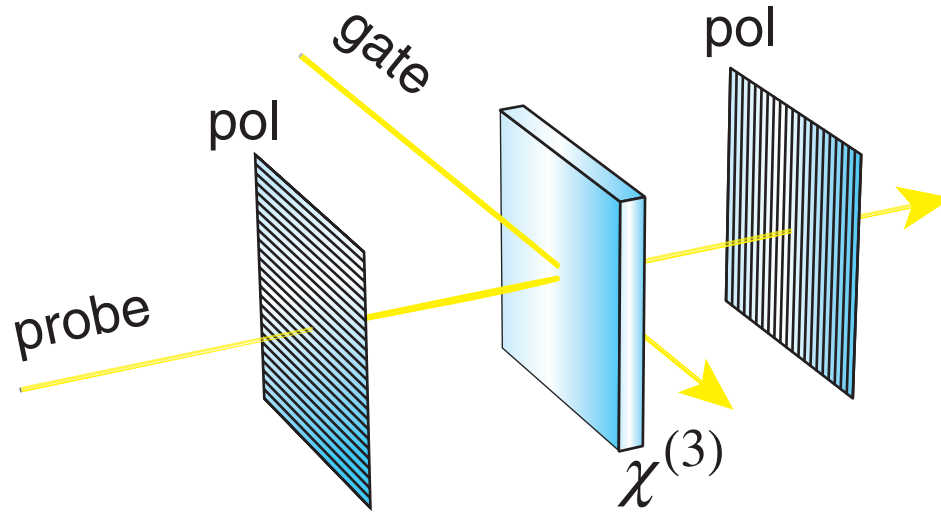


If gate and probe unequal:

$$\sigma_{prod}^2 = \frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 + \sigma_2^2} \quad \text{(narrower than both)}$$

$$\sigma_{cc}^2 = \sigma_1^2 + \sigma_2^2 \quad \text{(wider than both)}$$

Joint time-frequency measurements

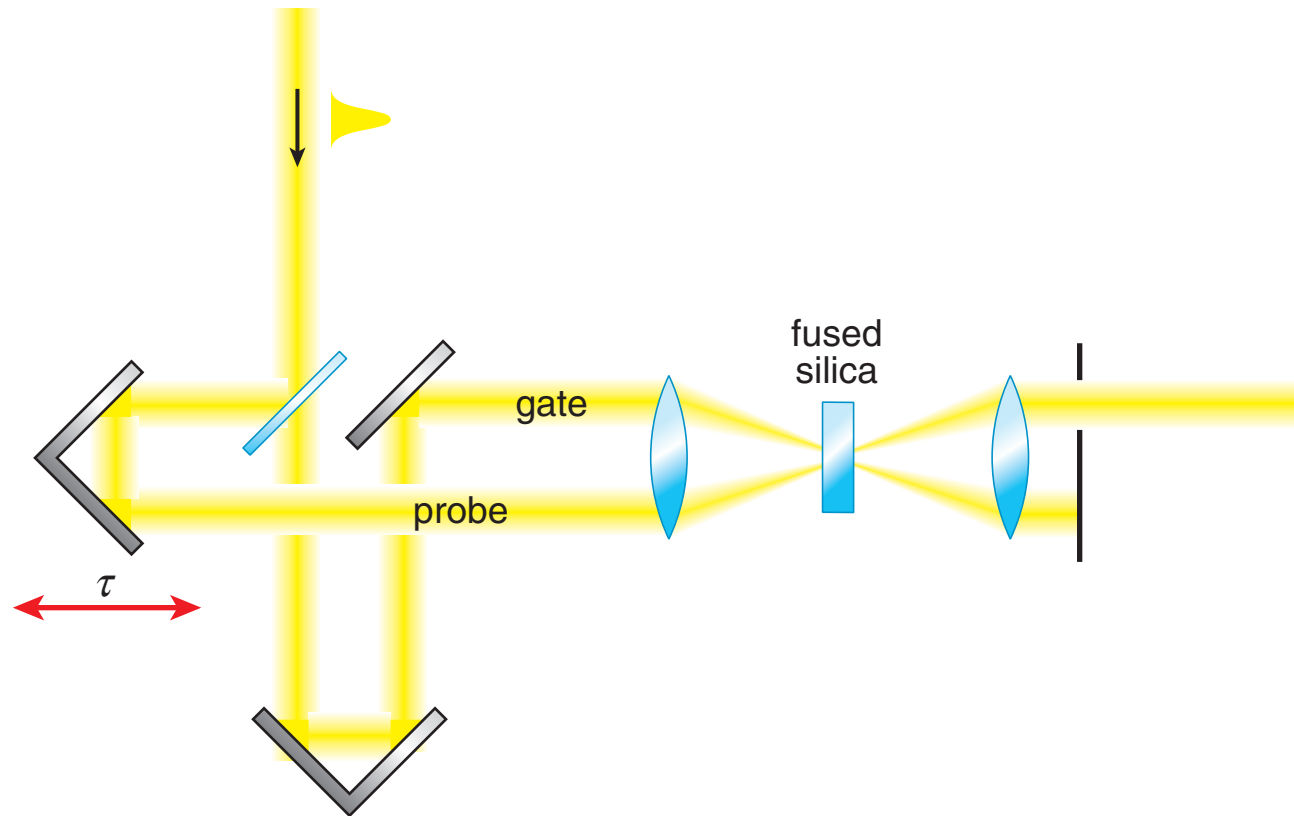


Transmitted field:

$$E_{trans}(t, \tau) \propto \chi^{(3)} E_{probe}(t) |E_{gate}(t + \tau)|^2$$

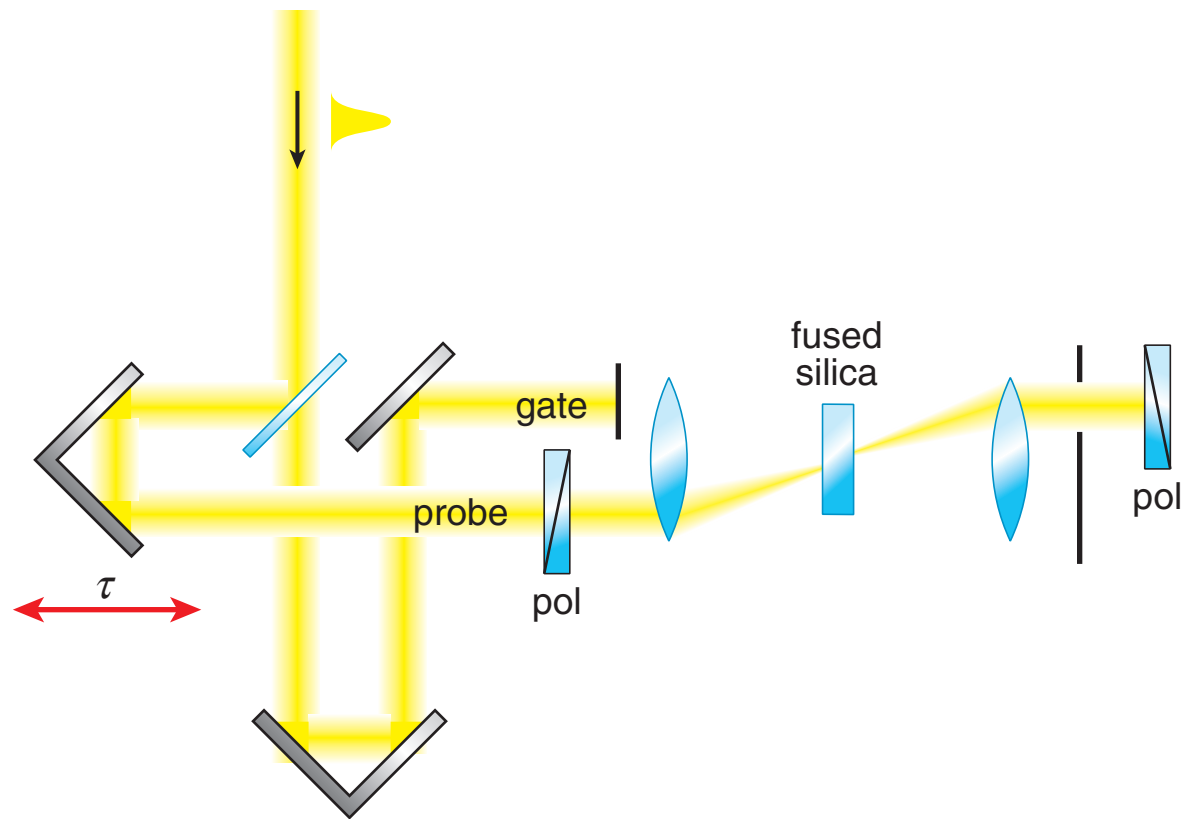
Joint time-frequency measurements

FROG: frequency-resolved optical gating



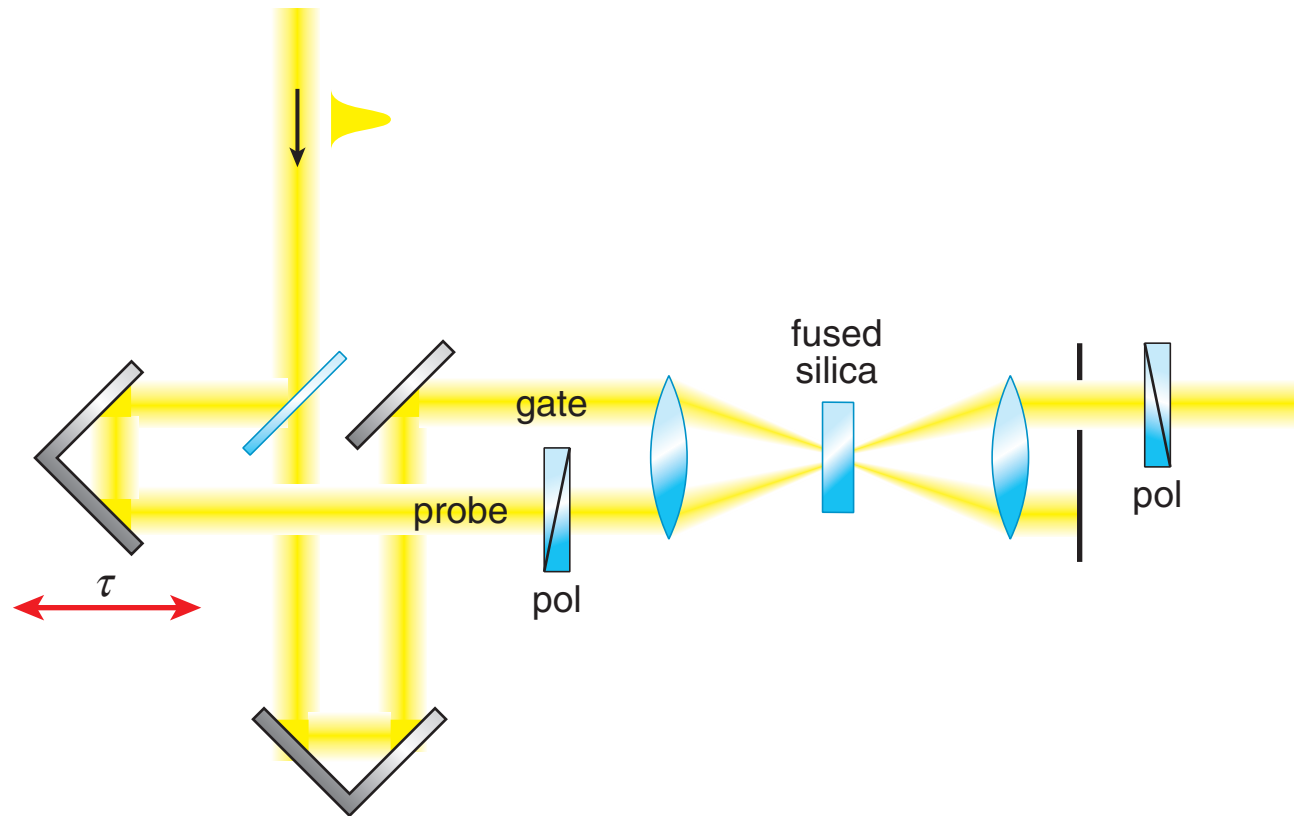
Joint time-frequency measurements

FROG: frequency-resolved optical gating



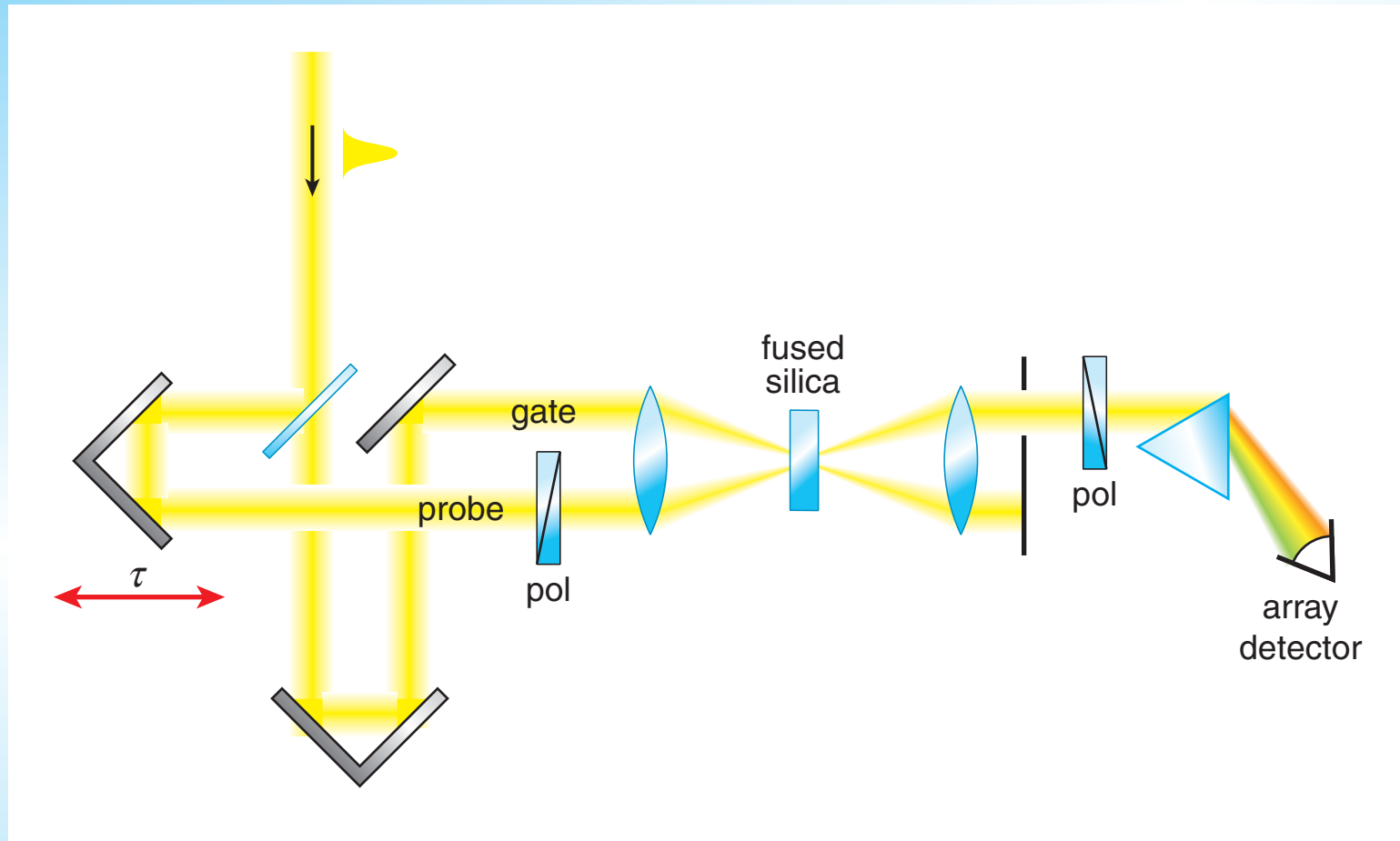
Joint time-frequency measurements

FROG: frequency-resolved optical gating

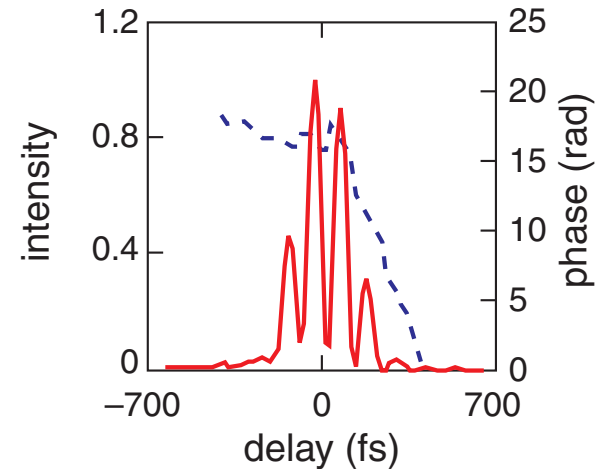
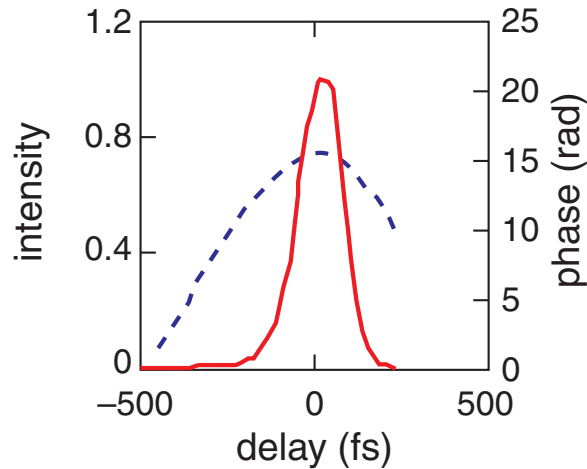
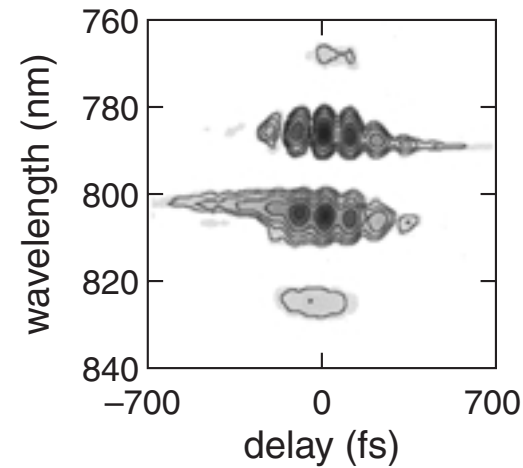
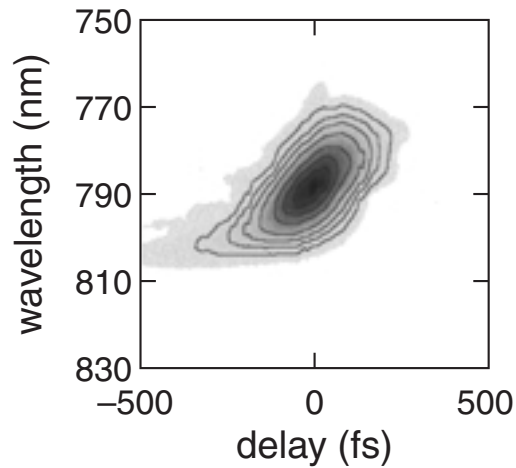


Joint time-frequency measurements

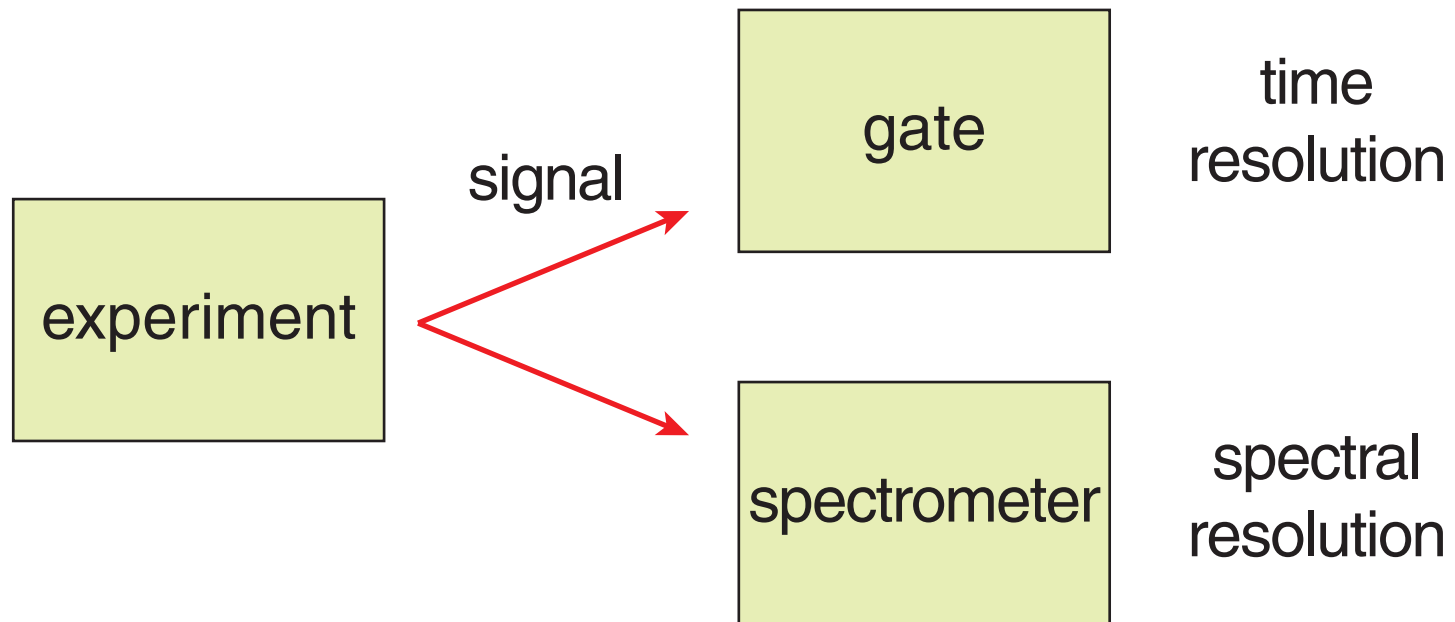
FROG: frequency-resolved optical gating



Joint time-frequency measurements



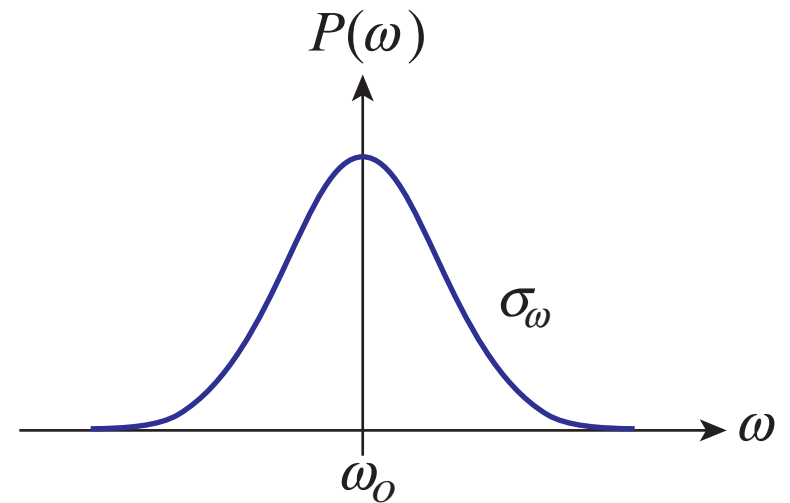
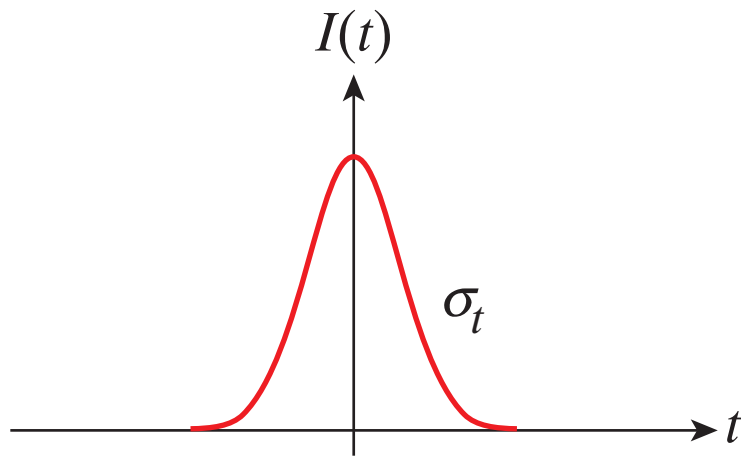
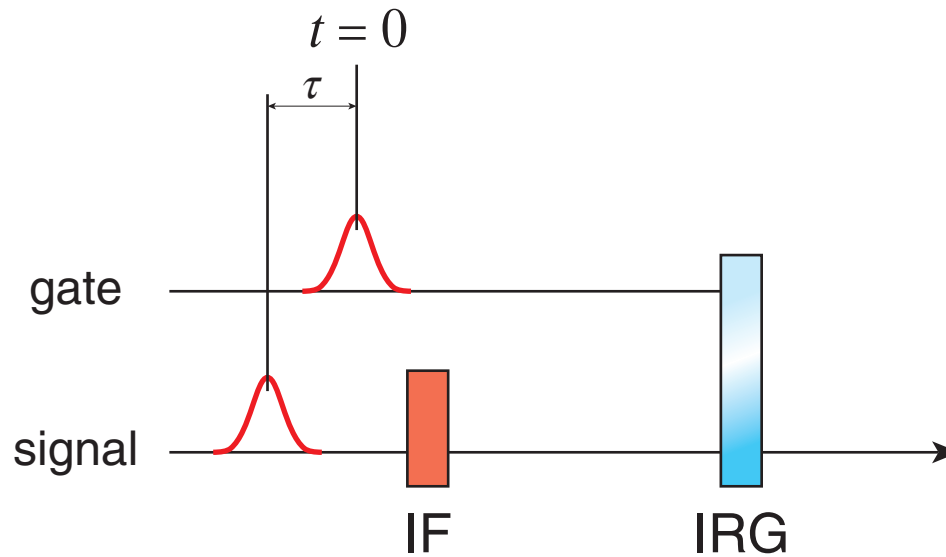
Joint time-frequency measurements



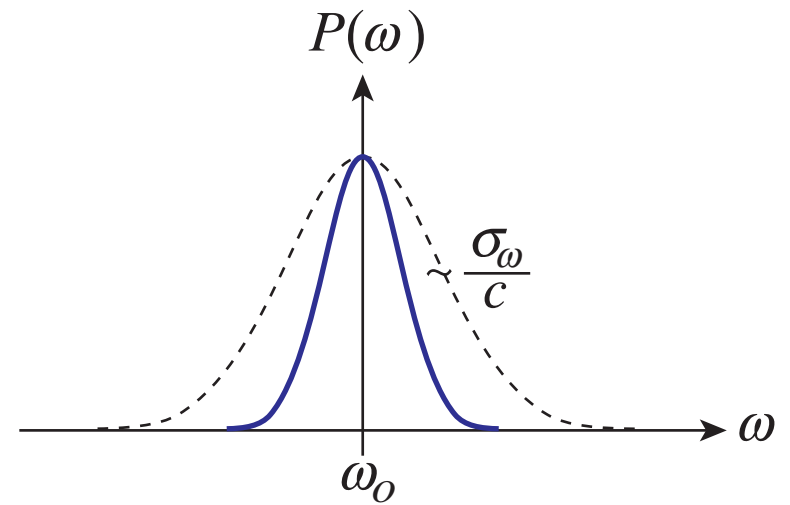
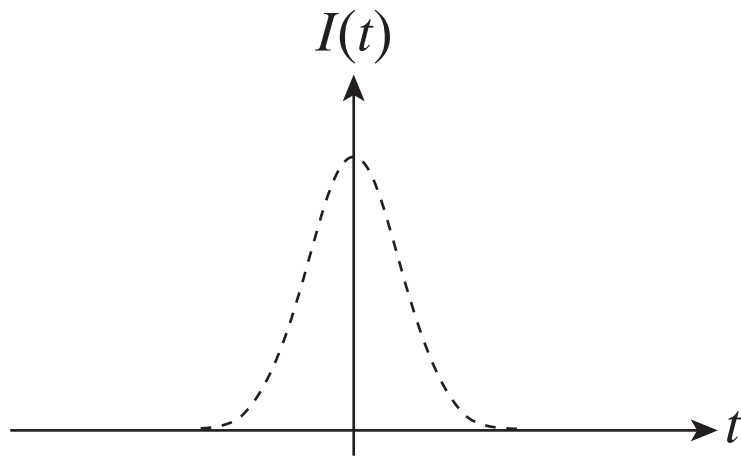
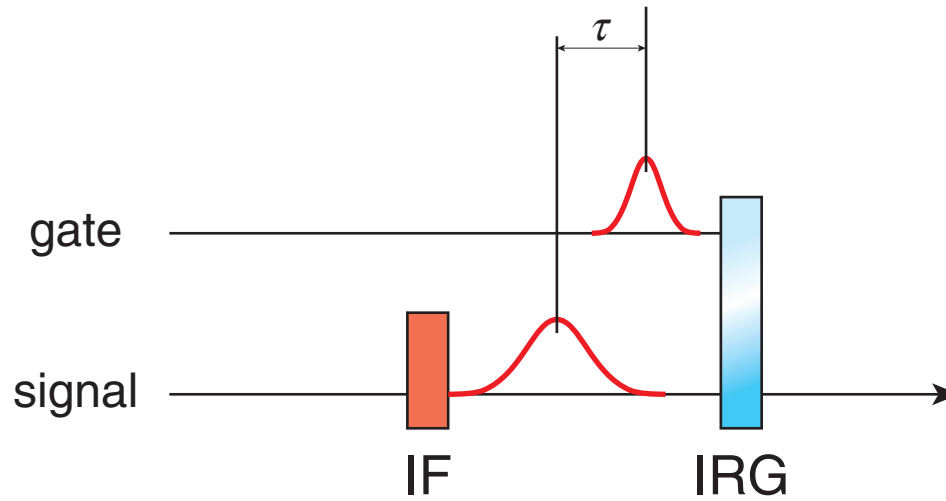
Joint time-frequency measurements

What are the resolution limits?

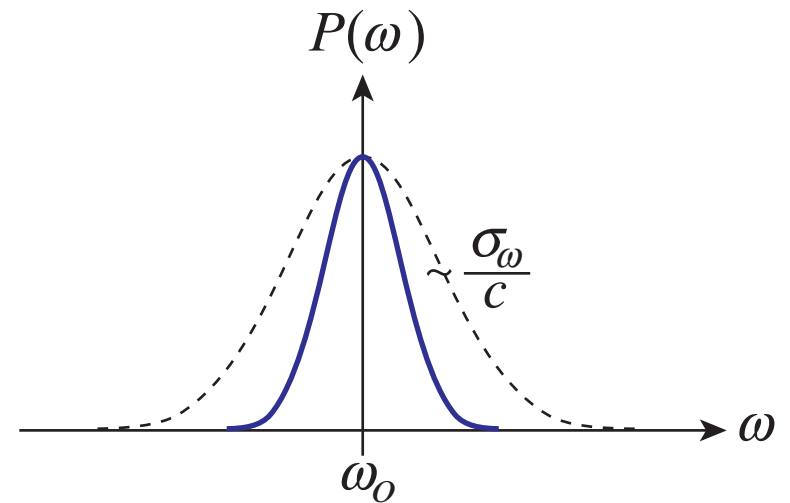
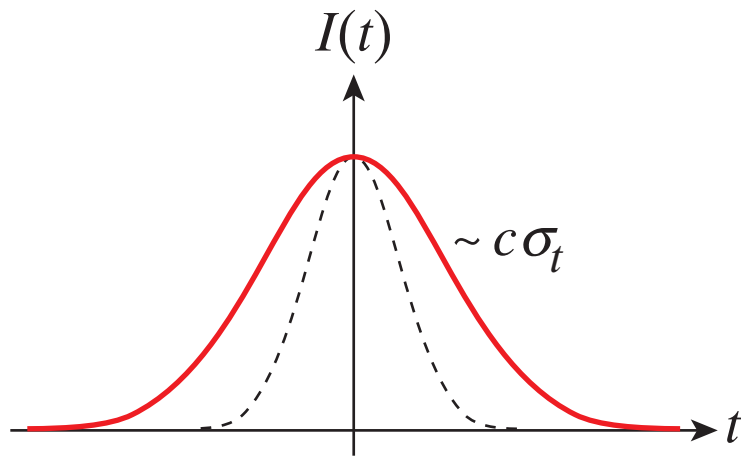
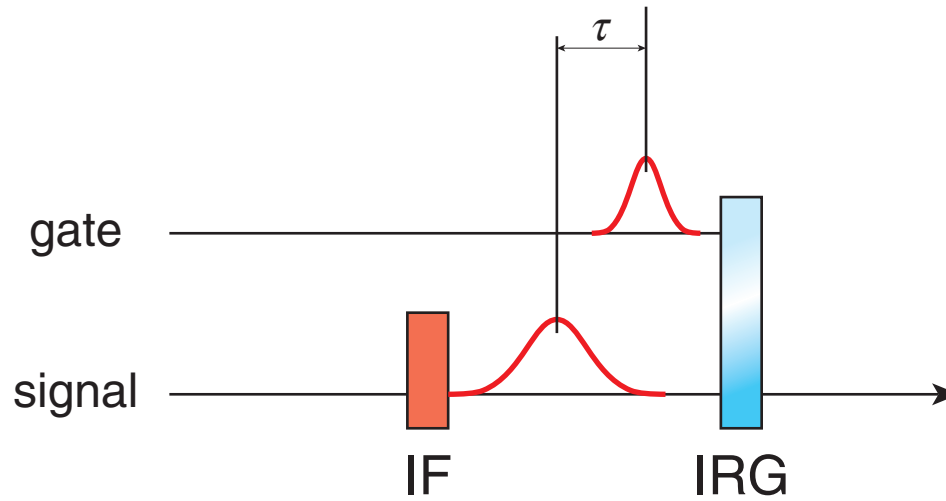
Experiment 1



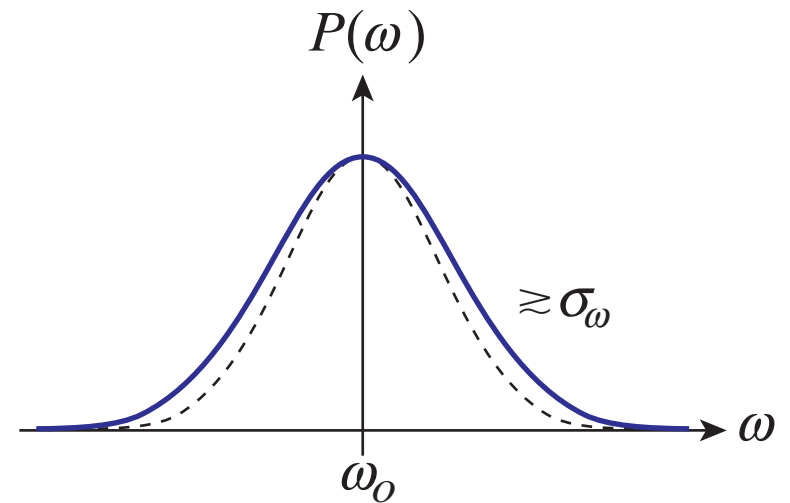
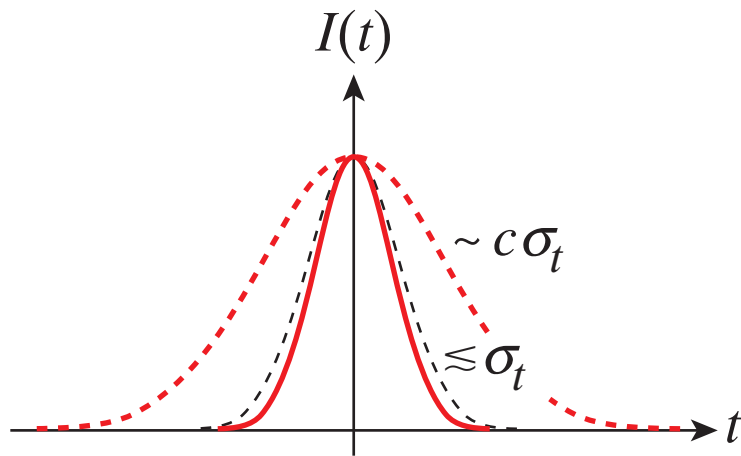
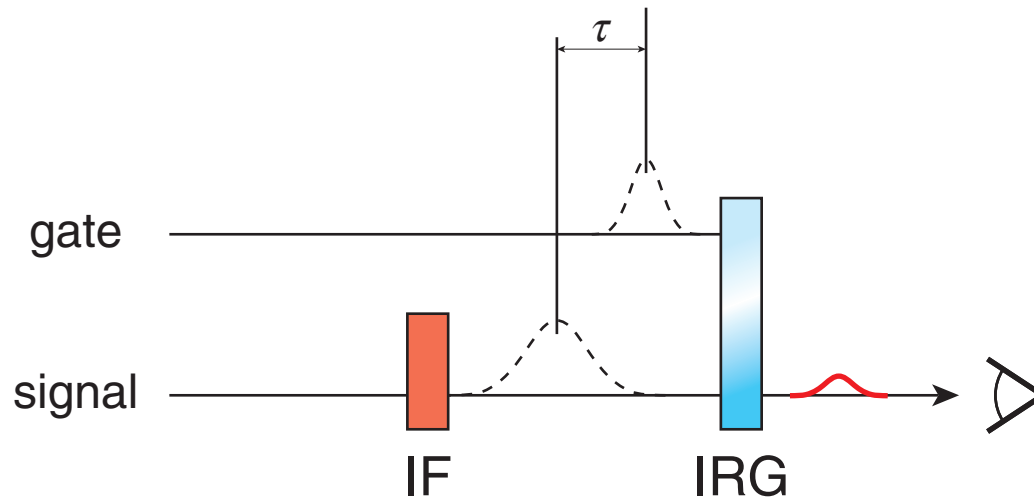
Experiment 1



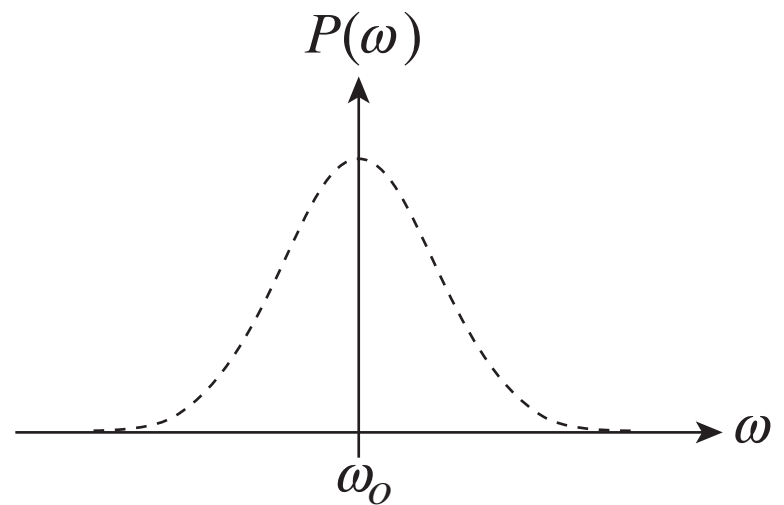
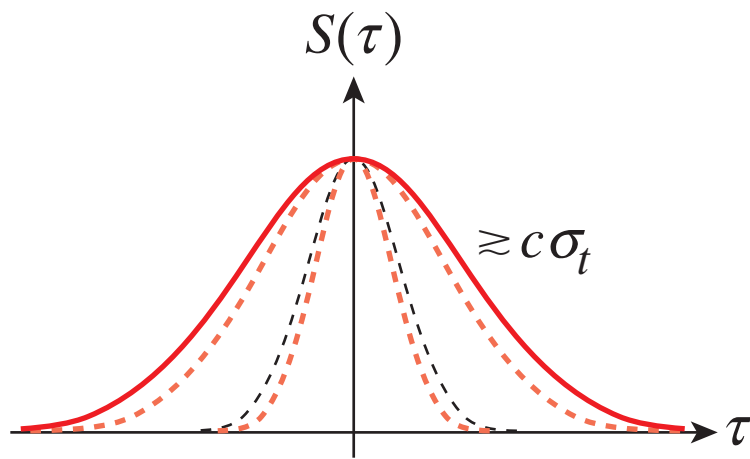
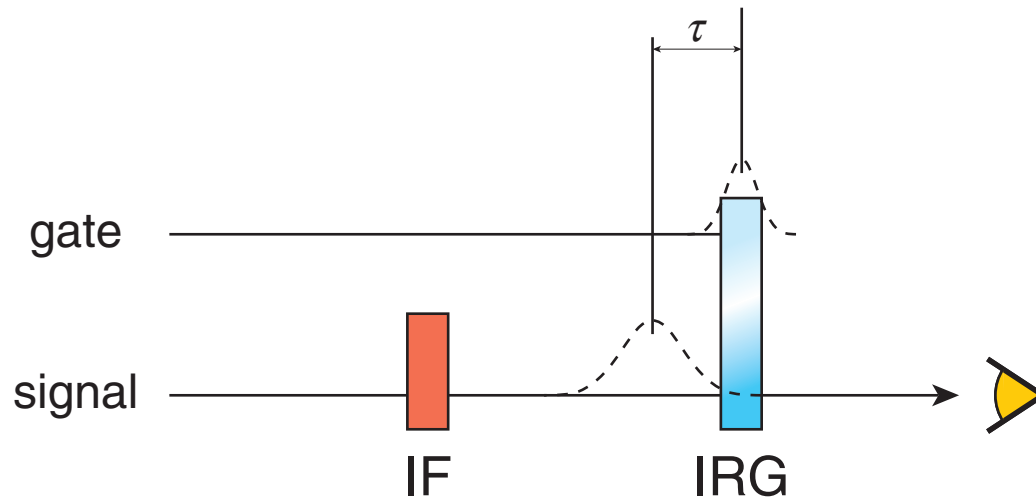
Experiment 1



Experiment 1

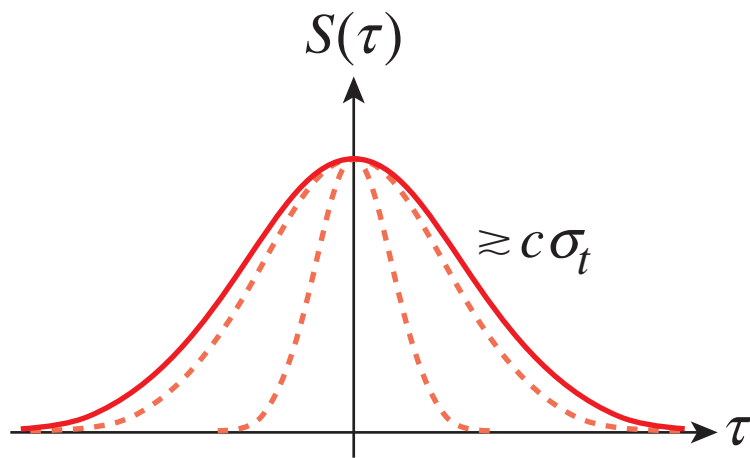
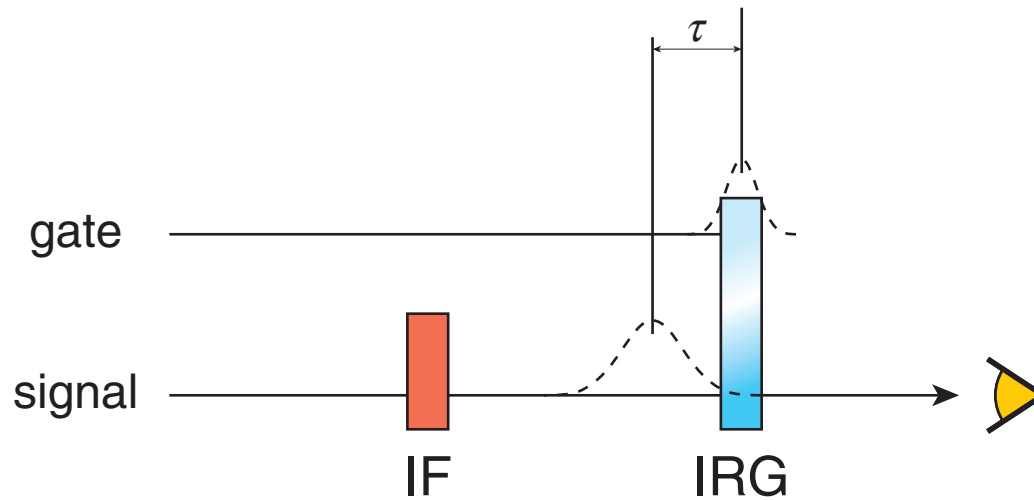


Experiment 1

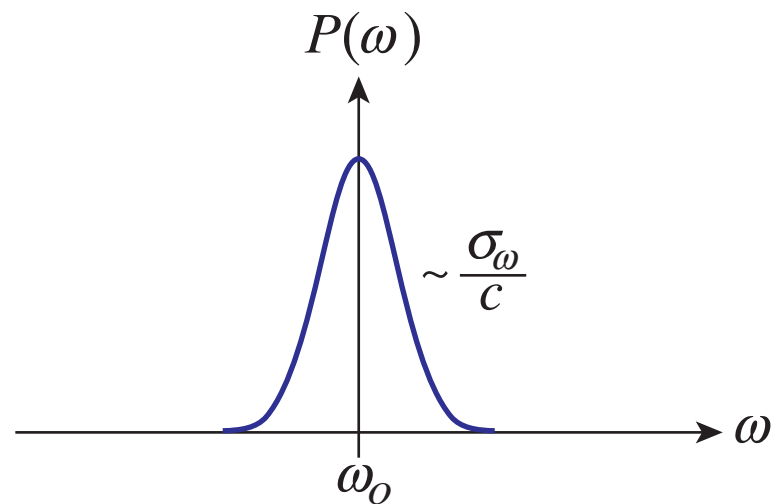


time resolution

Experiment 1

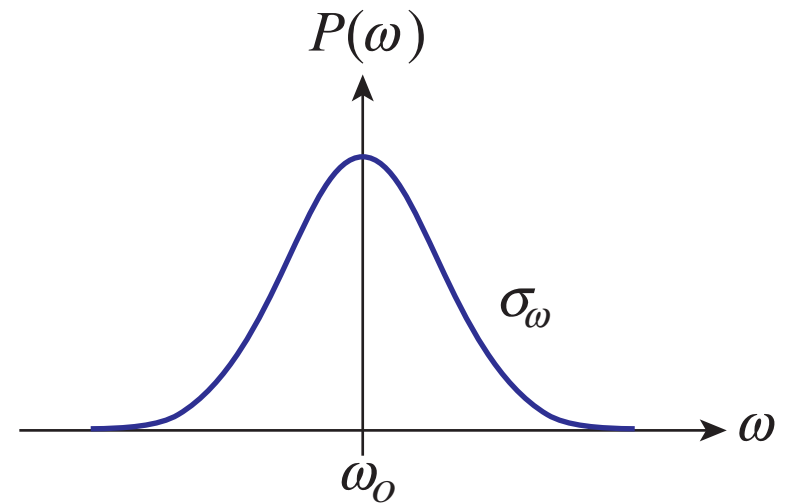
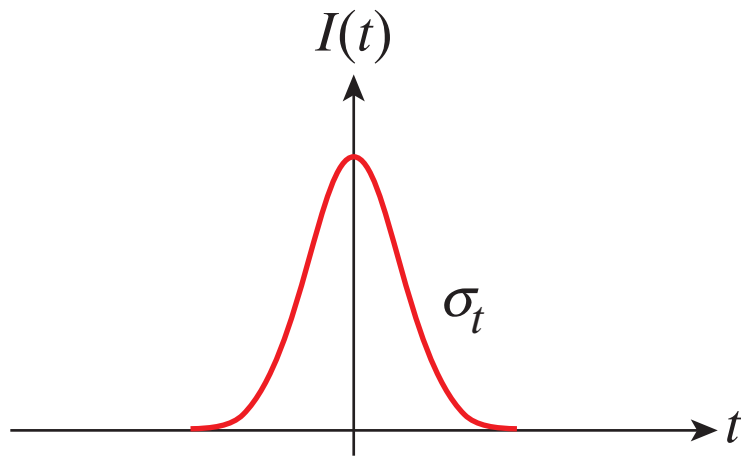
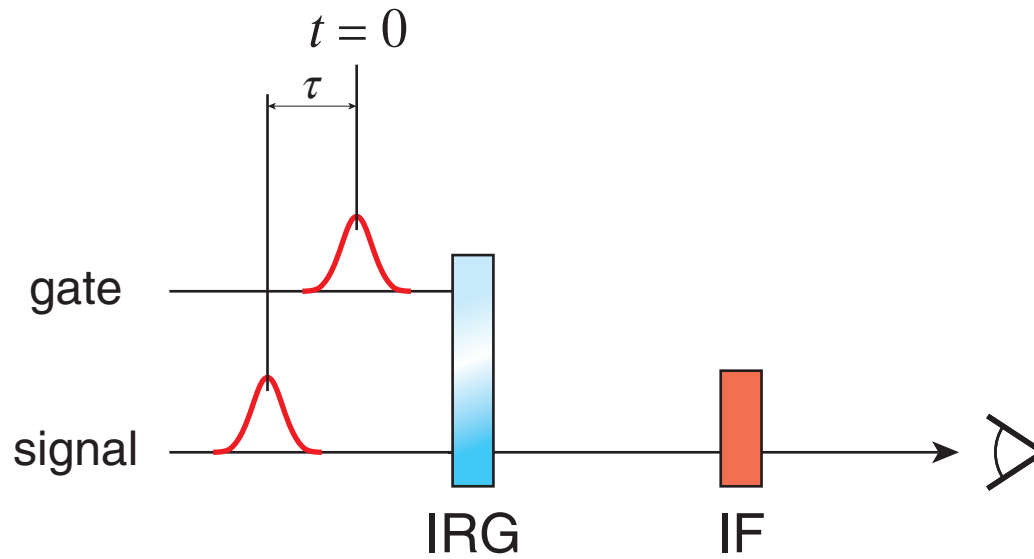


time resolution

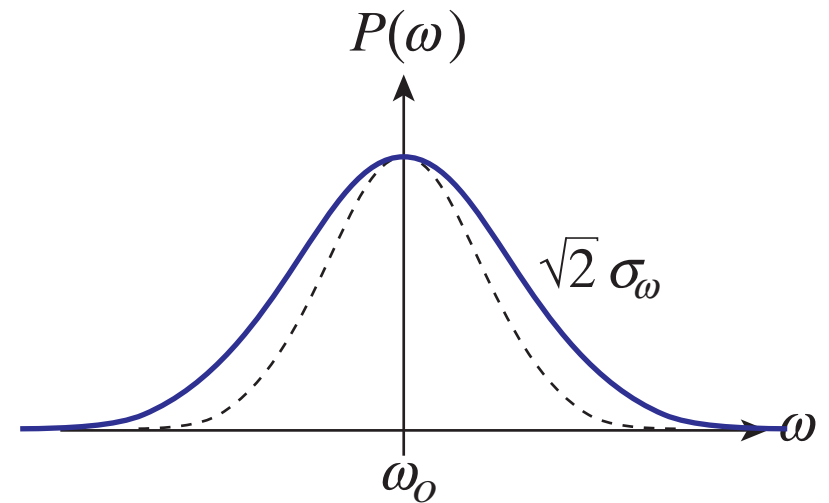
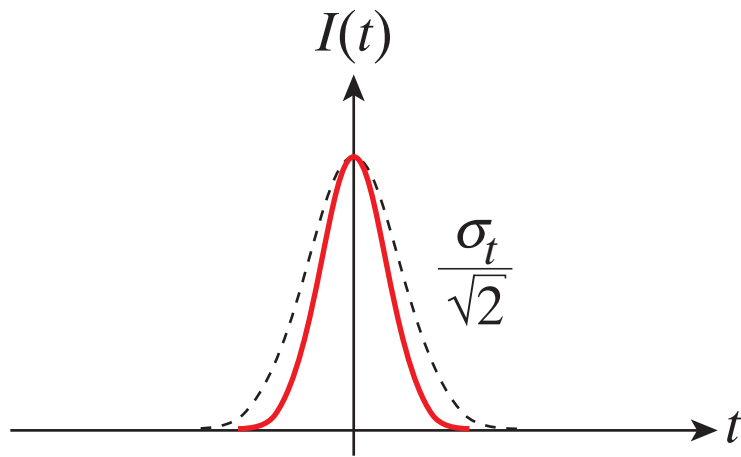
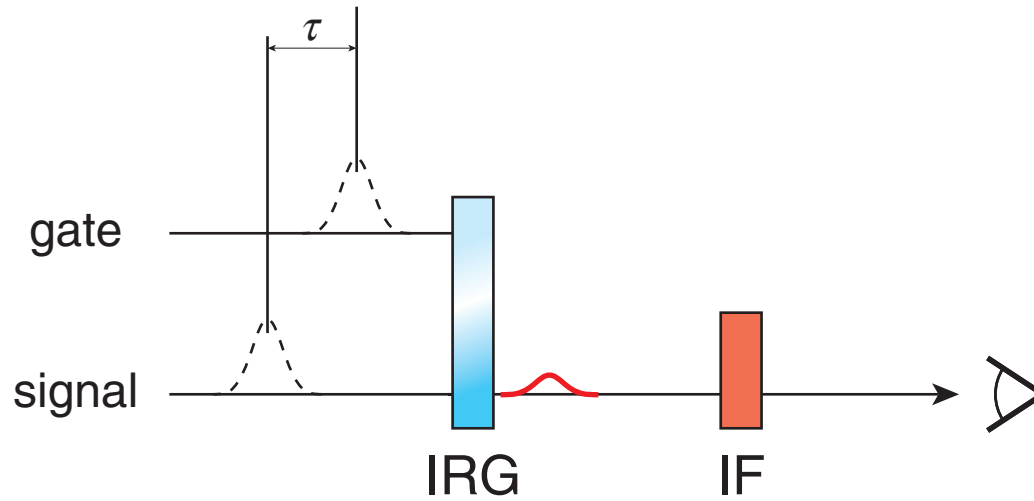


frequency resolution

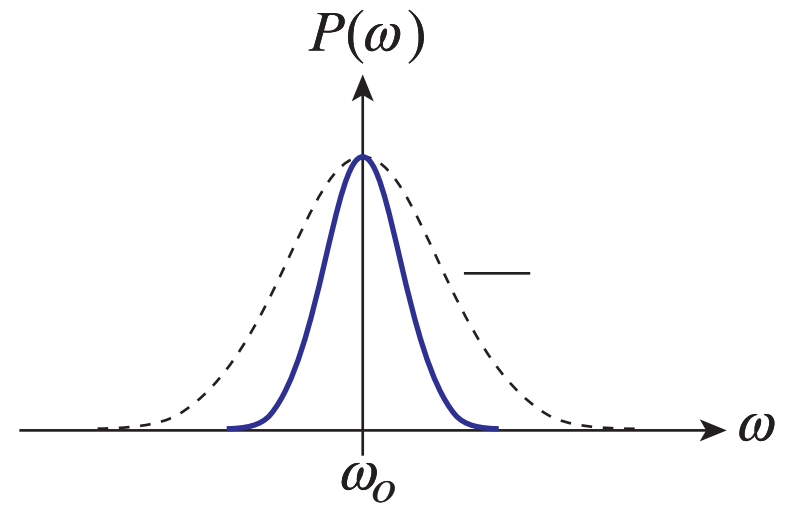
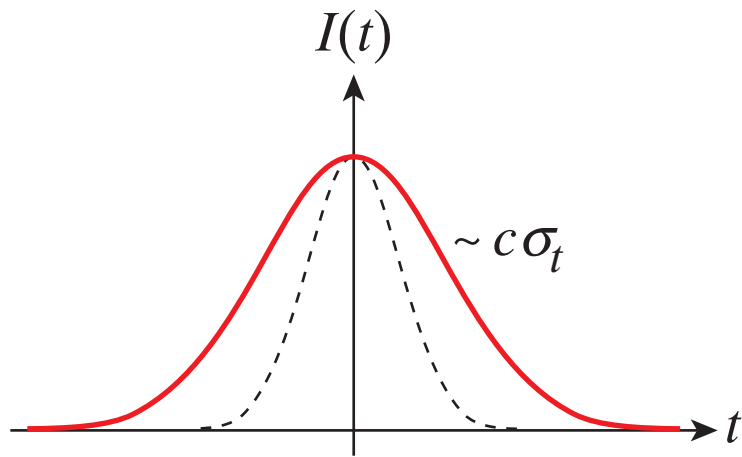
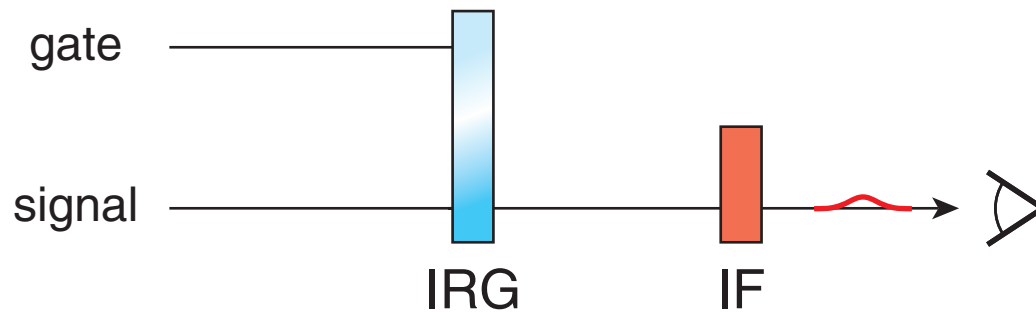
Experiment 2



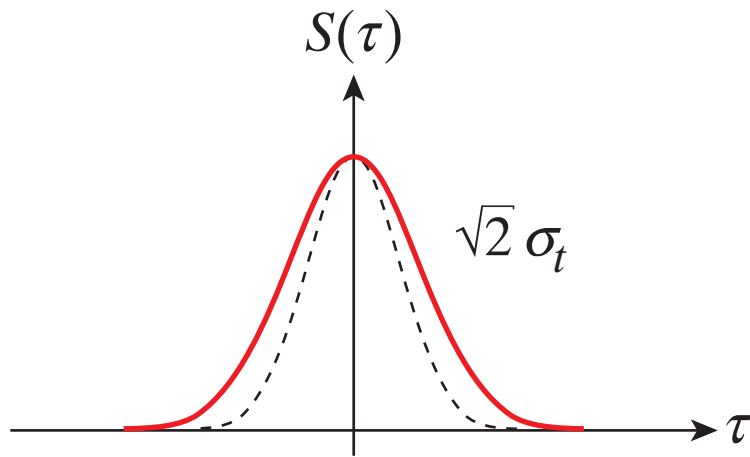
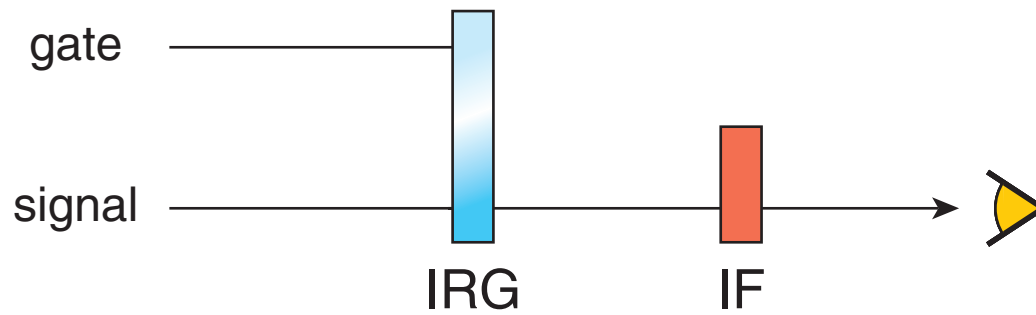
Experiment 2



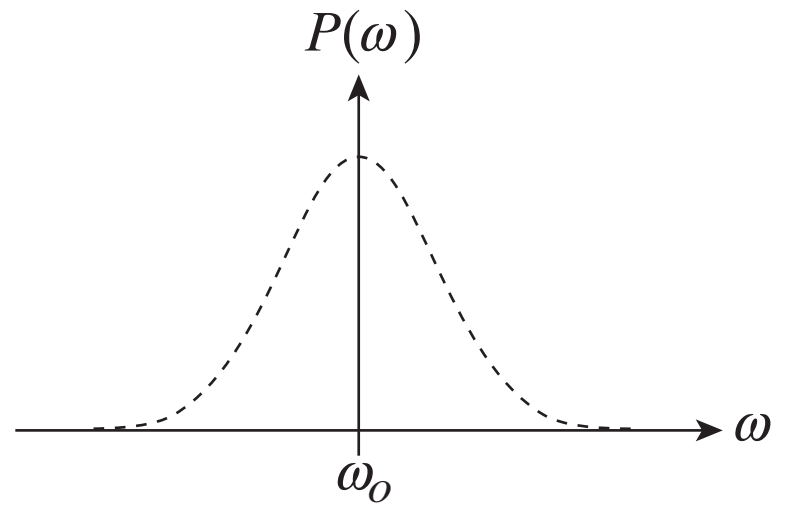
Experiment 2



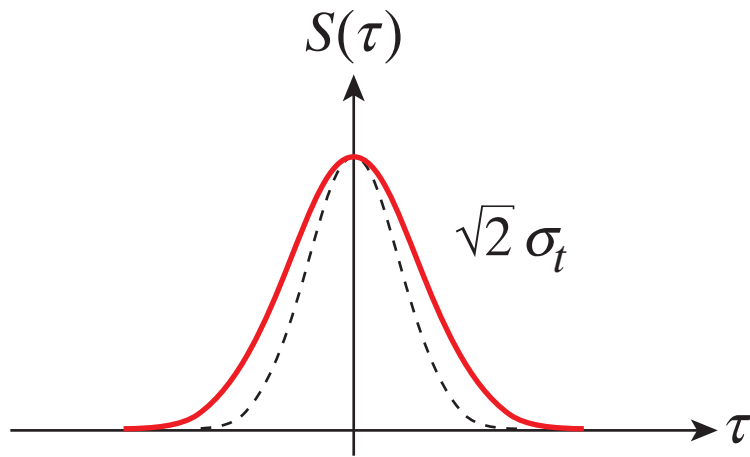
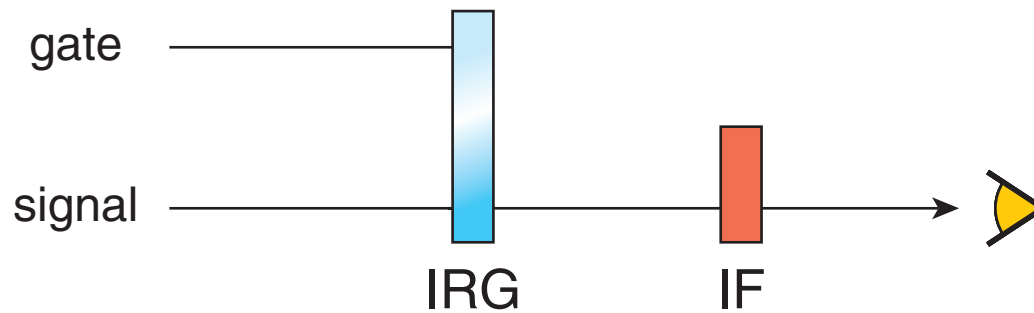
Experiment 2



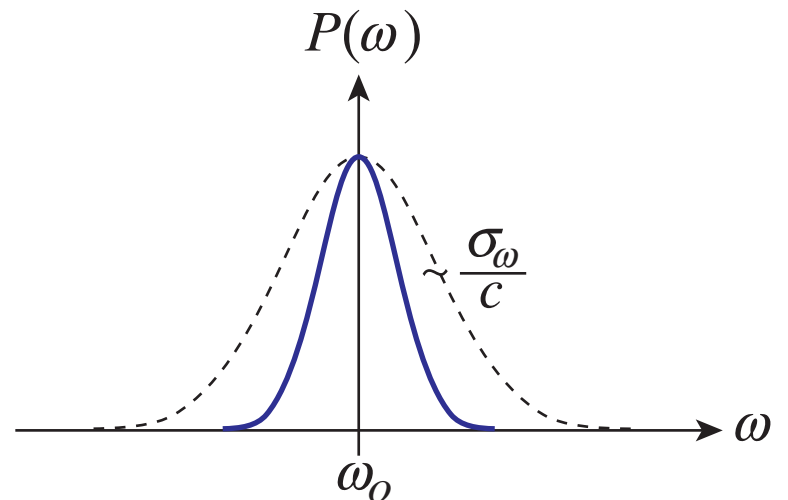
time resolution



Experiment 2



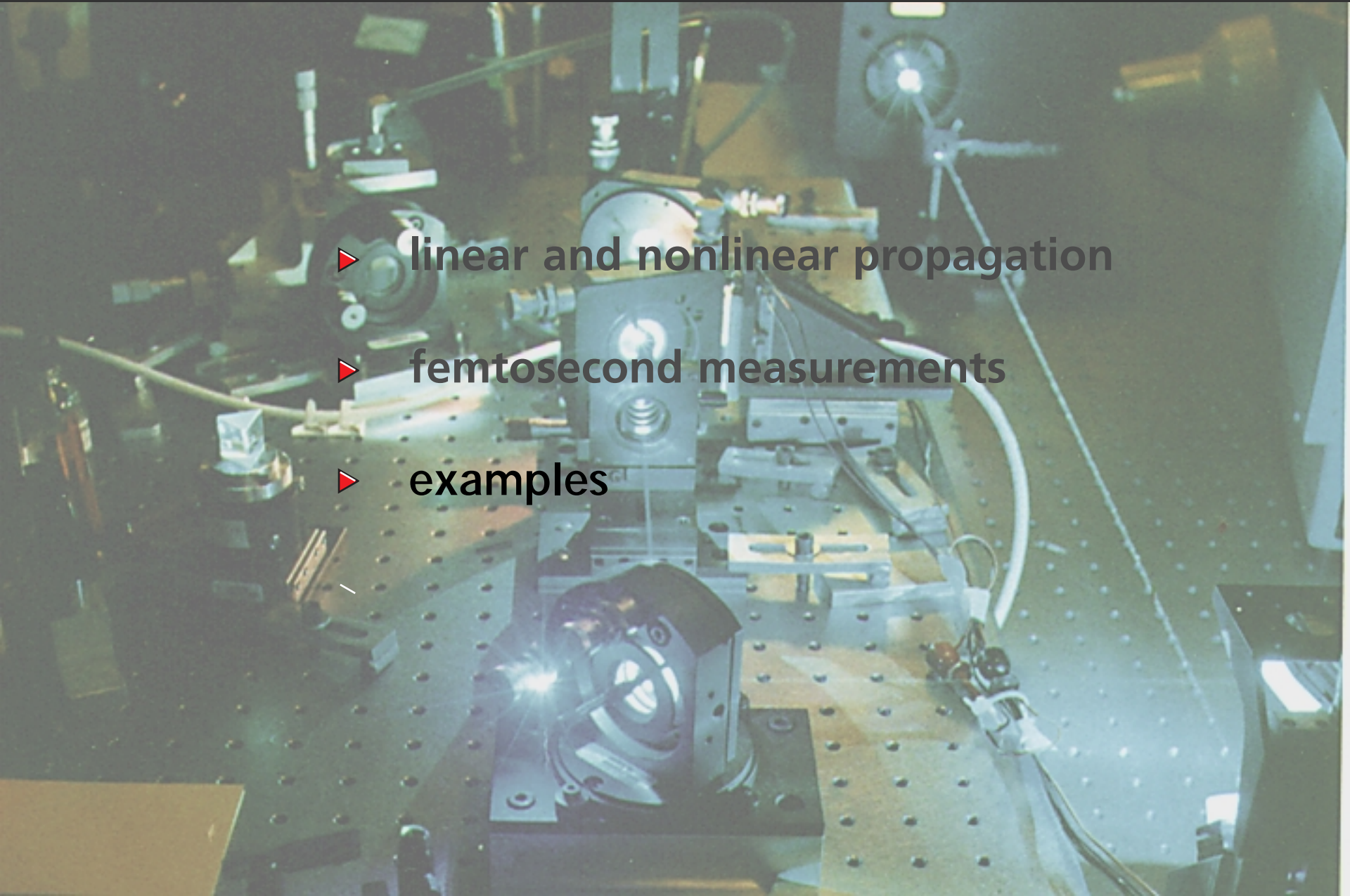
time resolution



spectral resolution

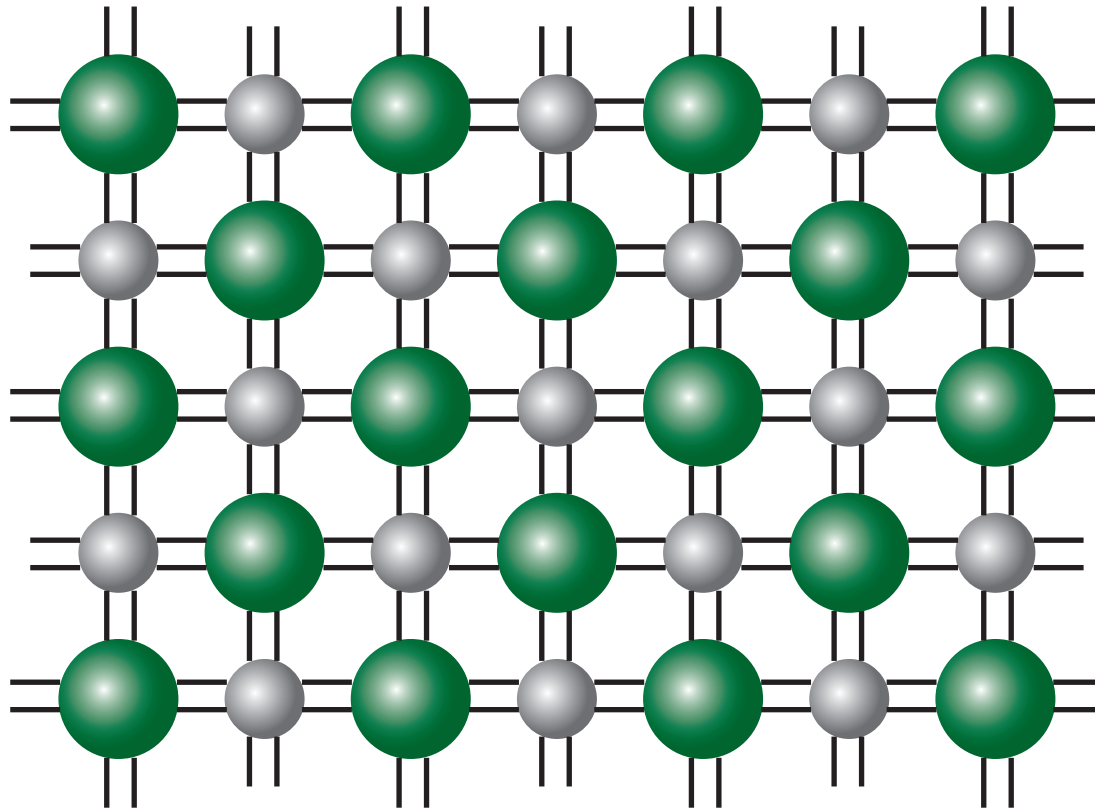
Outline

- ▶ linear and nonlinear propagation
- ▶ femtosecond measurements
- ▶ examples



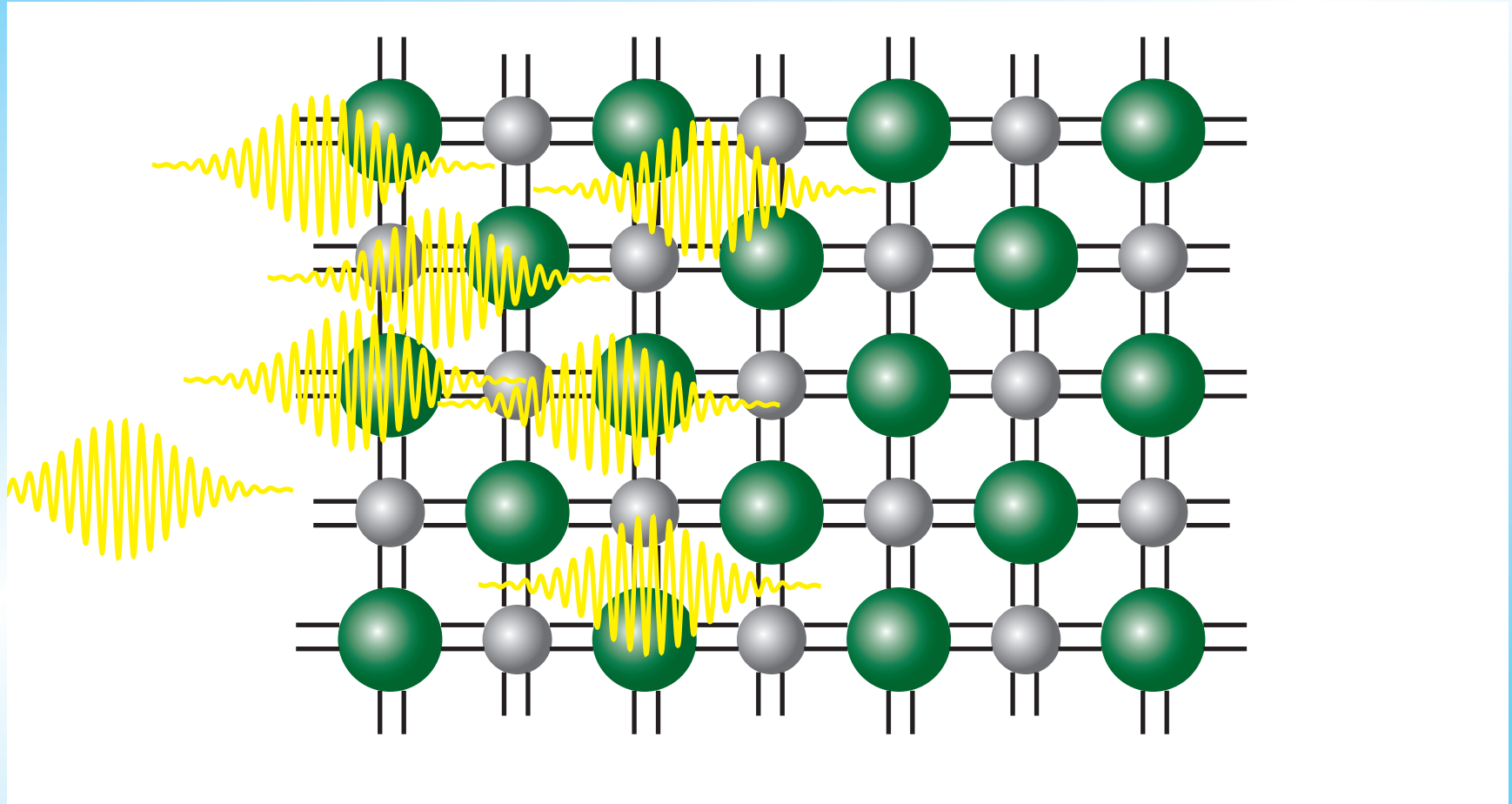
Dielectric function measurements

how do femtosecond laser pulses alter a solid?



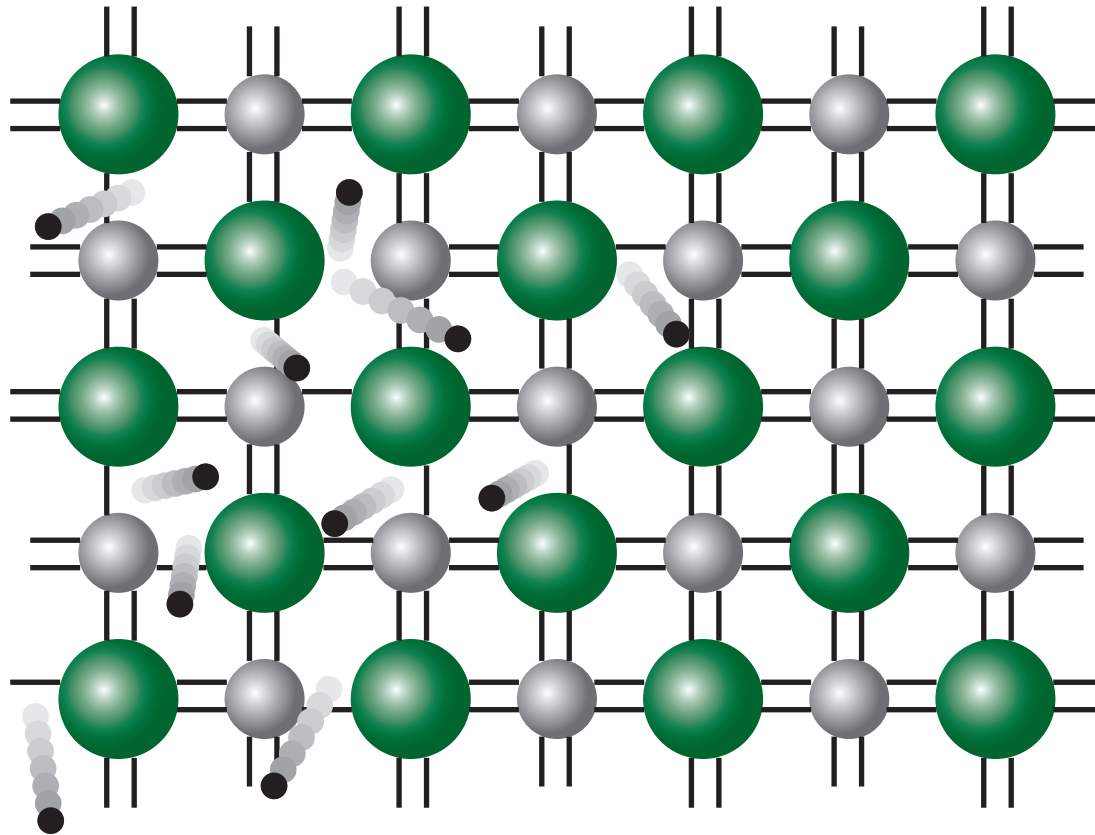
Dielectric function measurements

photons excite valence electrons...



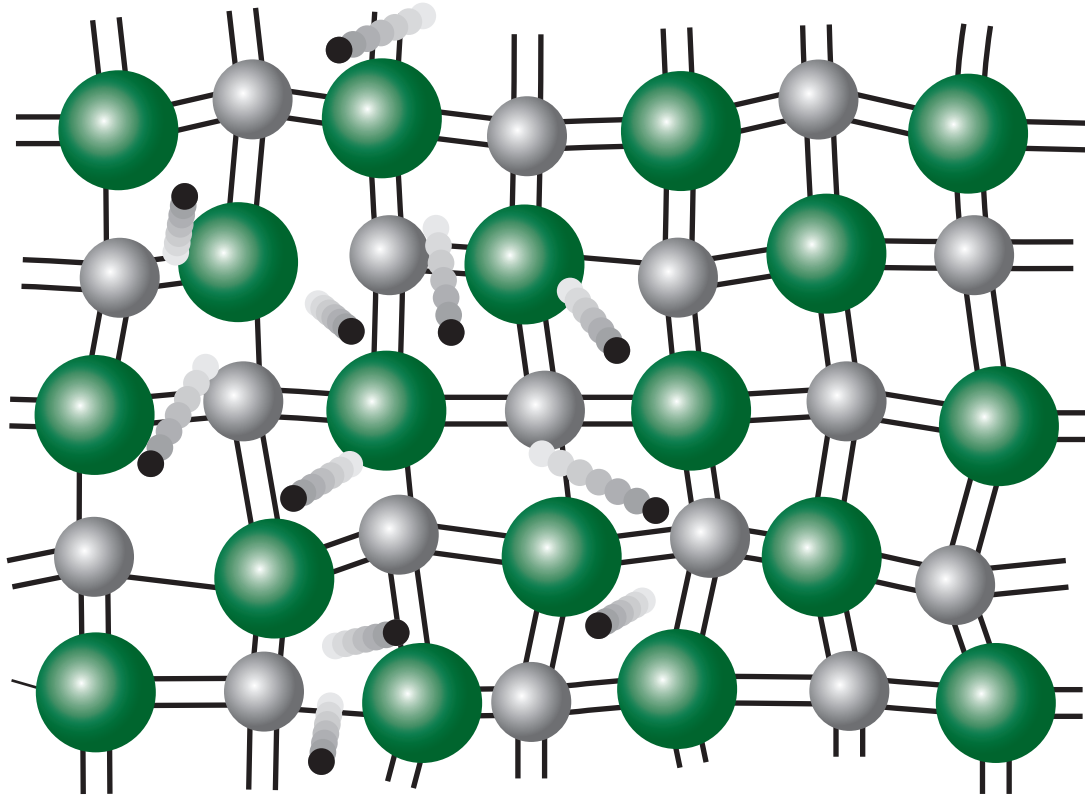
Dielectric function measurements

...and create free electrons...



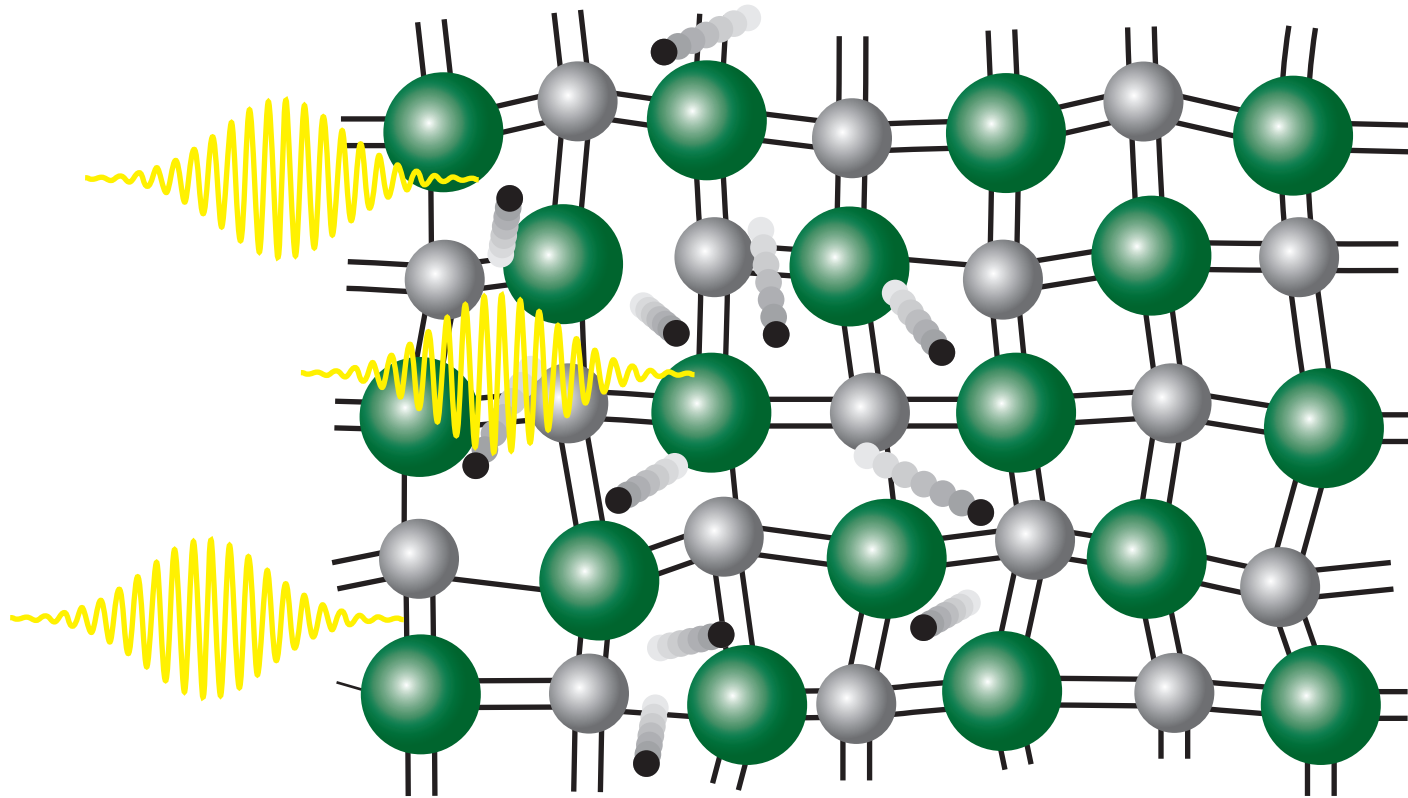
Dielectric function measurements

...causing electronic and structural changes...

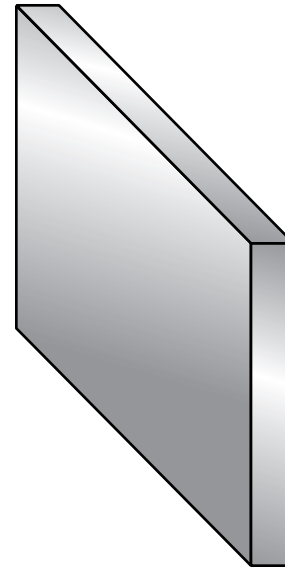


Dielectric function measurements

...which we measure with another pulse

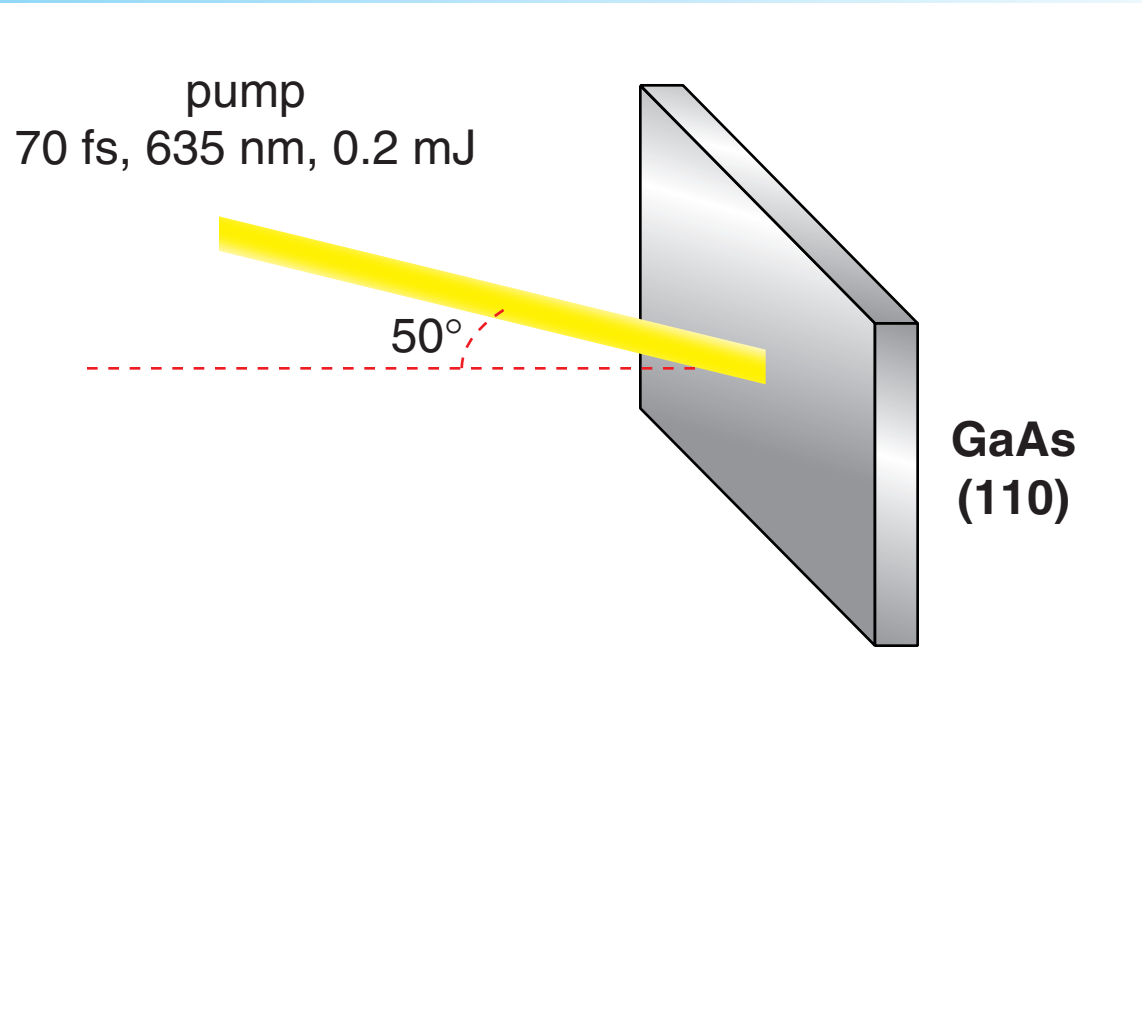


Dielectric function measurements

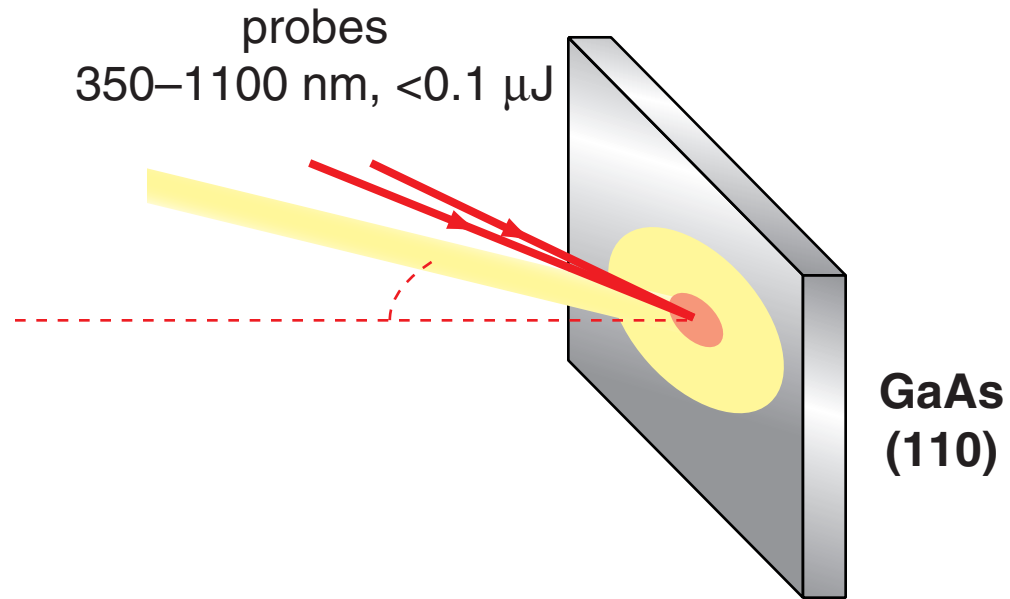


**GaAs
(110)**

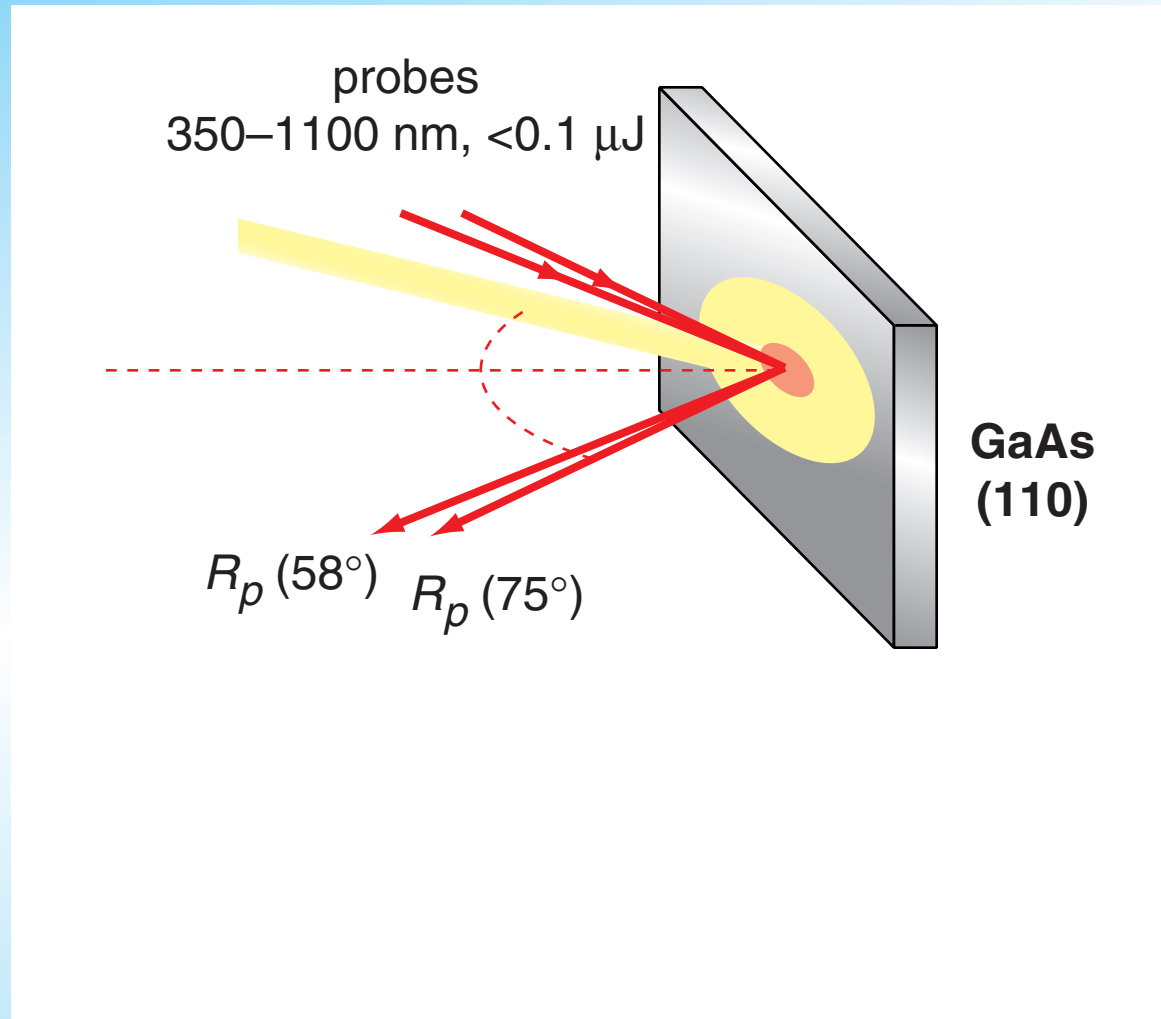
Dielectric function measurements



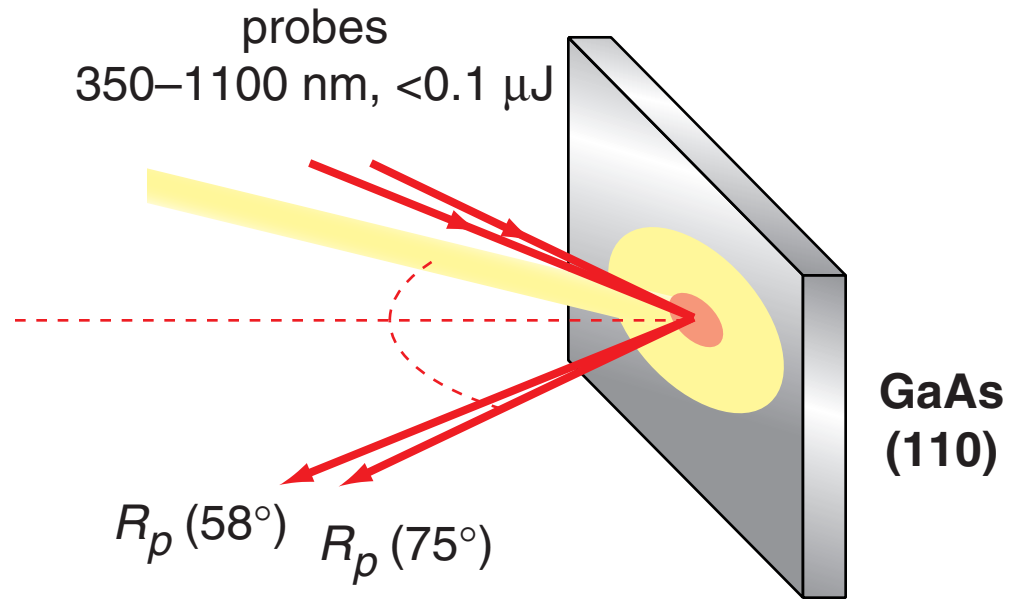
Dielectric function measurements



Dielectric function measurements

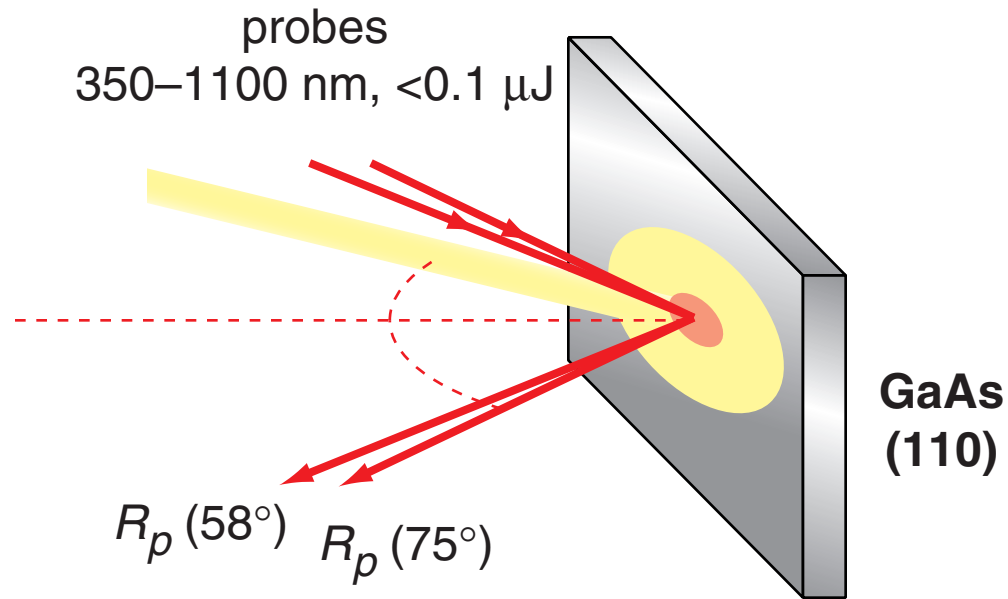


Dielectric function measurements

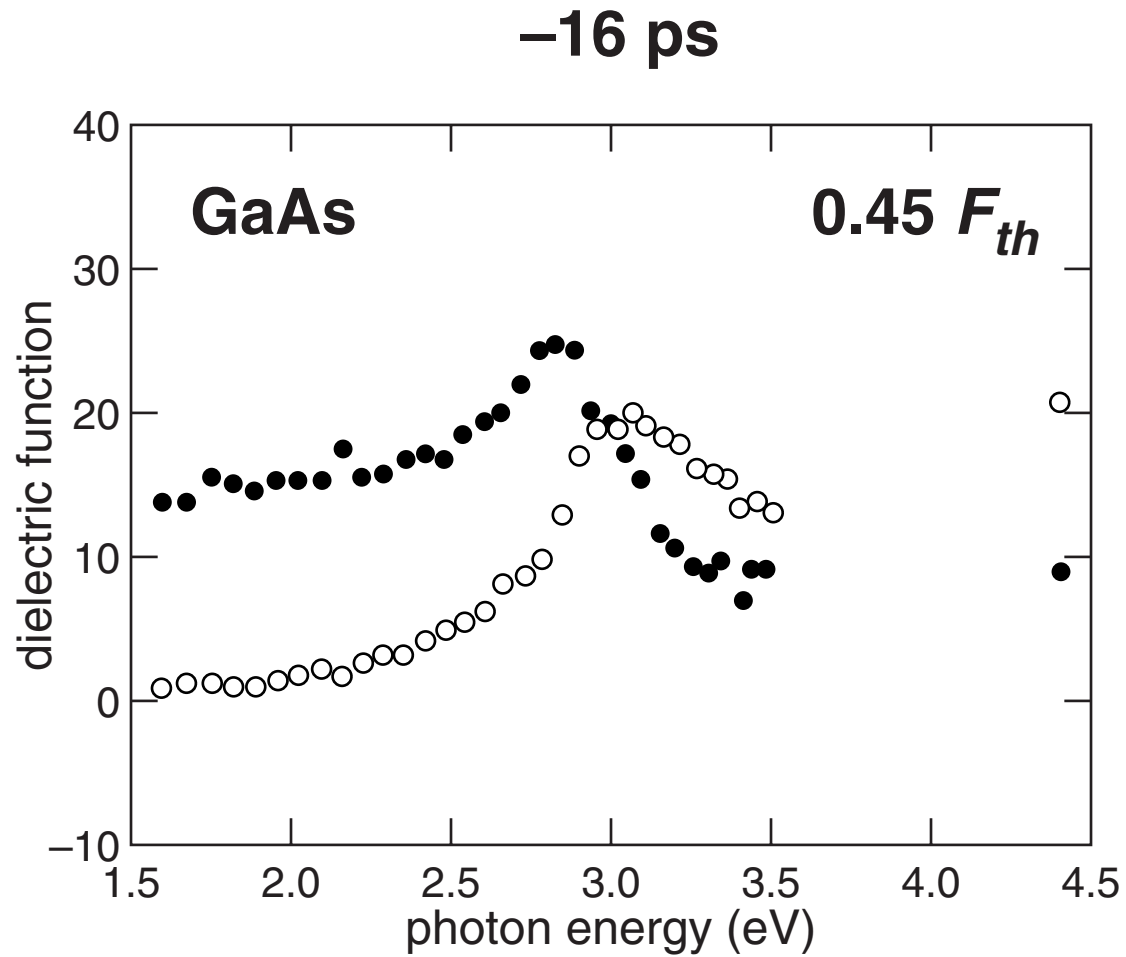


Fresnel
equations

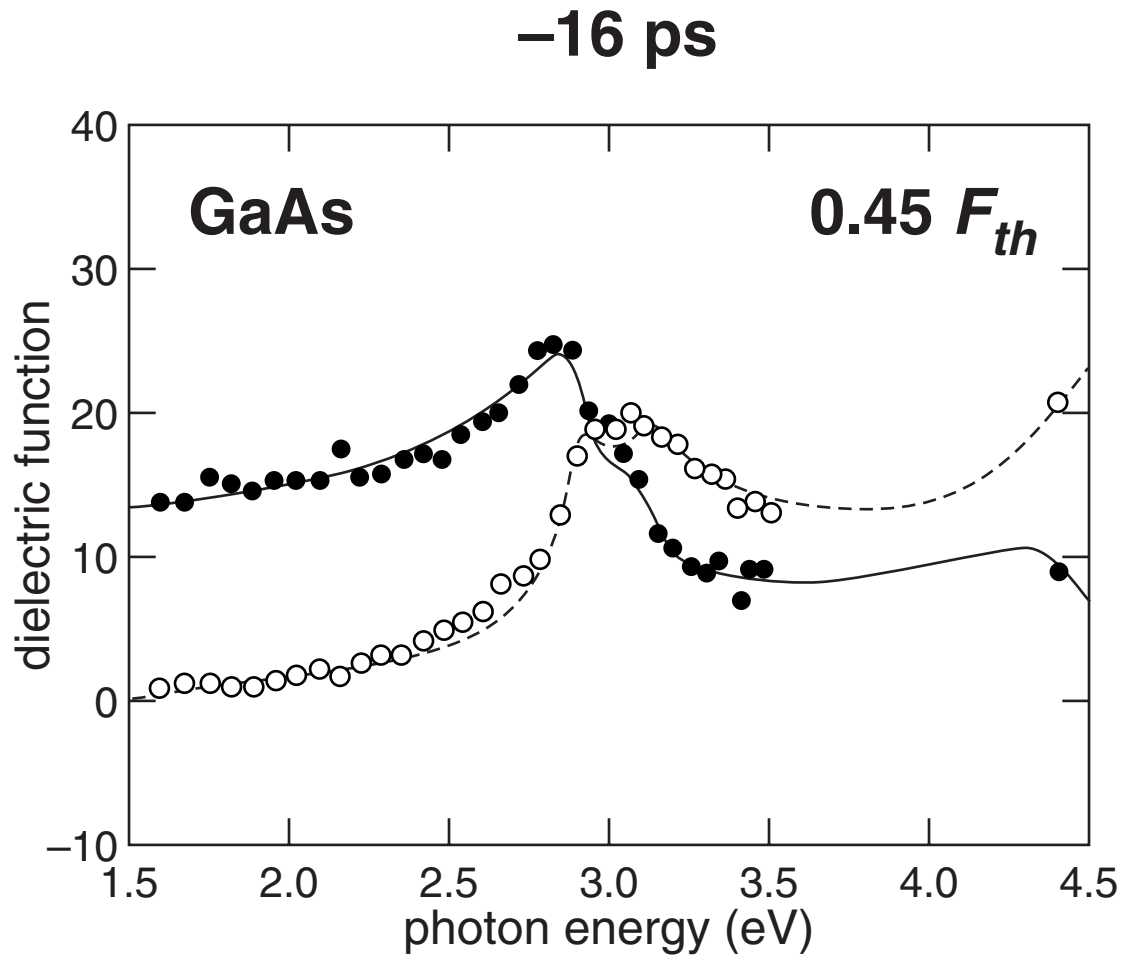
Dielectric function measurements



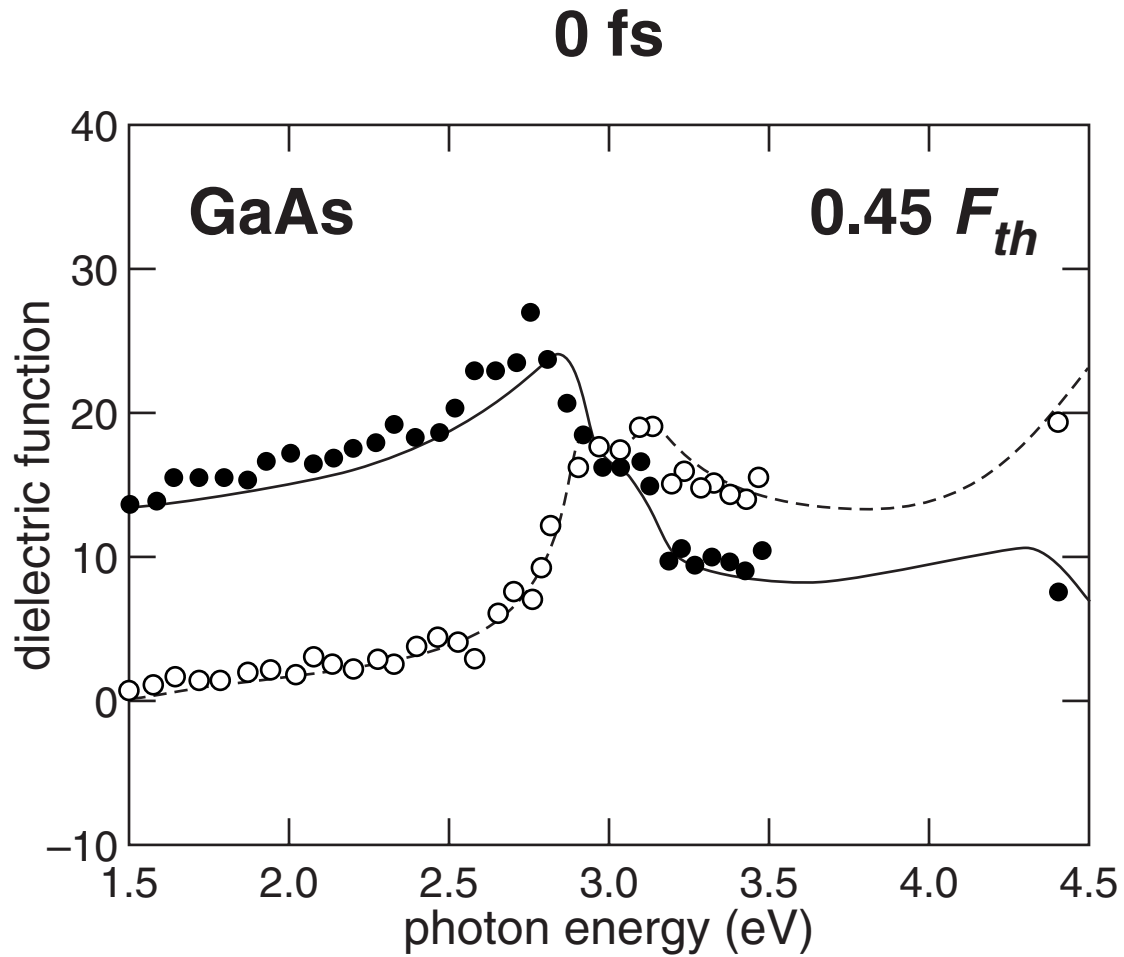
Dielectric function measurements



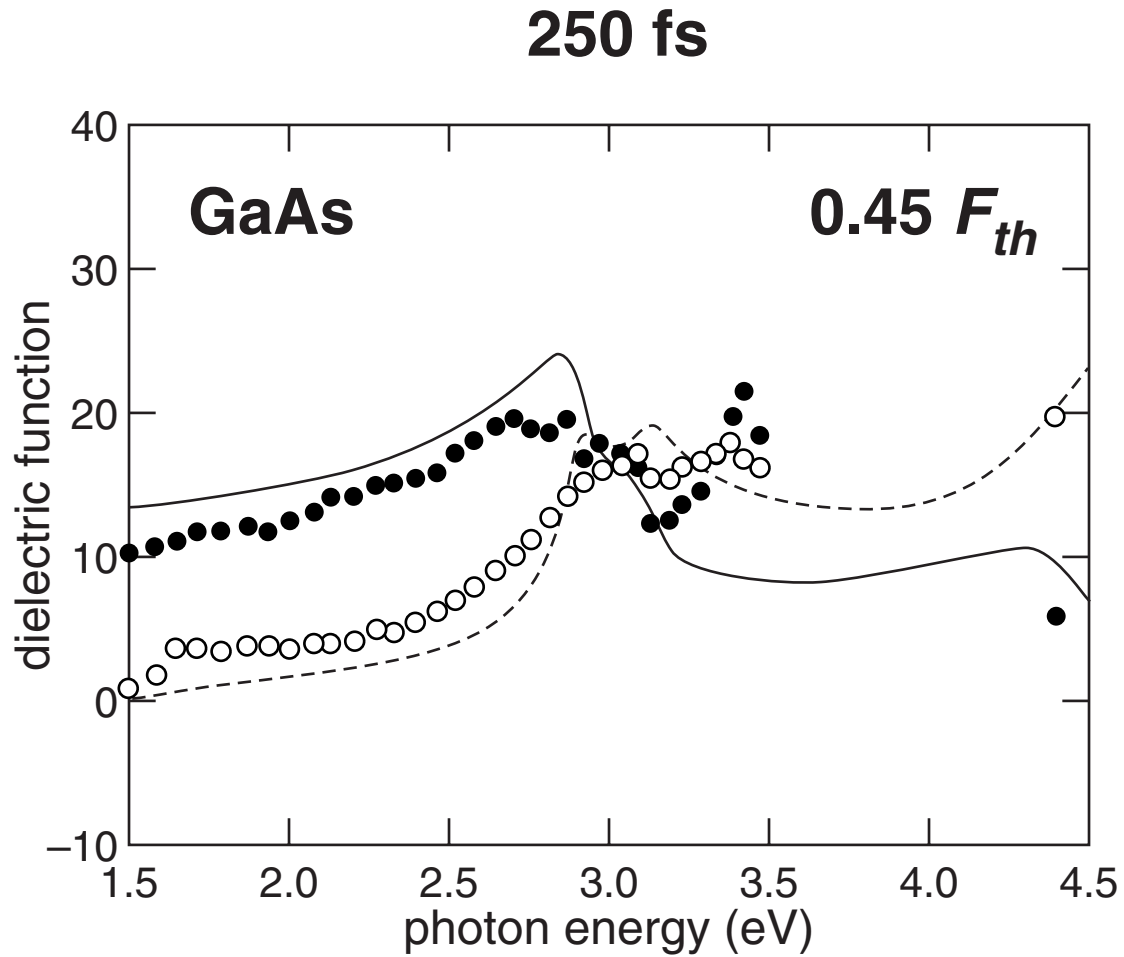
Dielectric function measurements



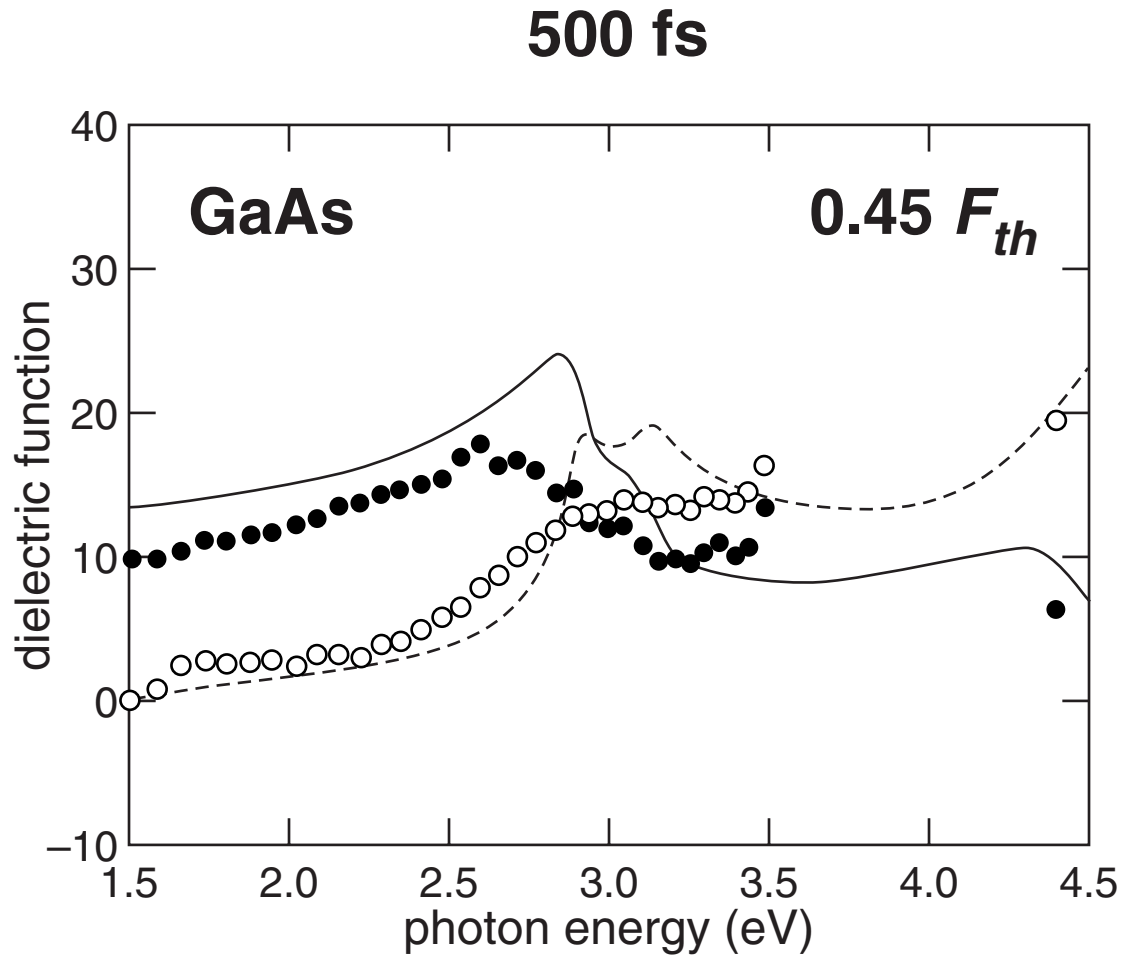
Dielectric function measurements



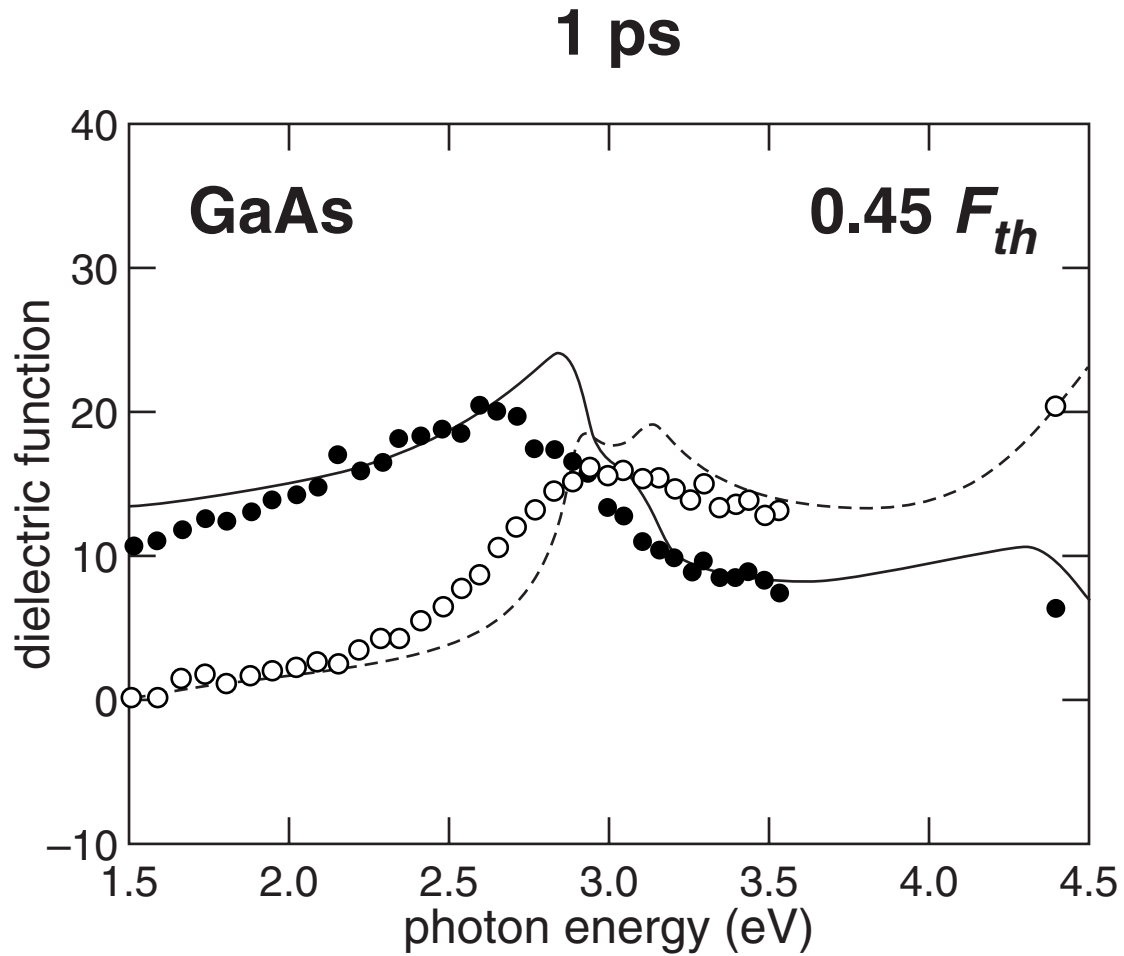
Dielectric function measurements



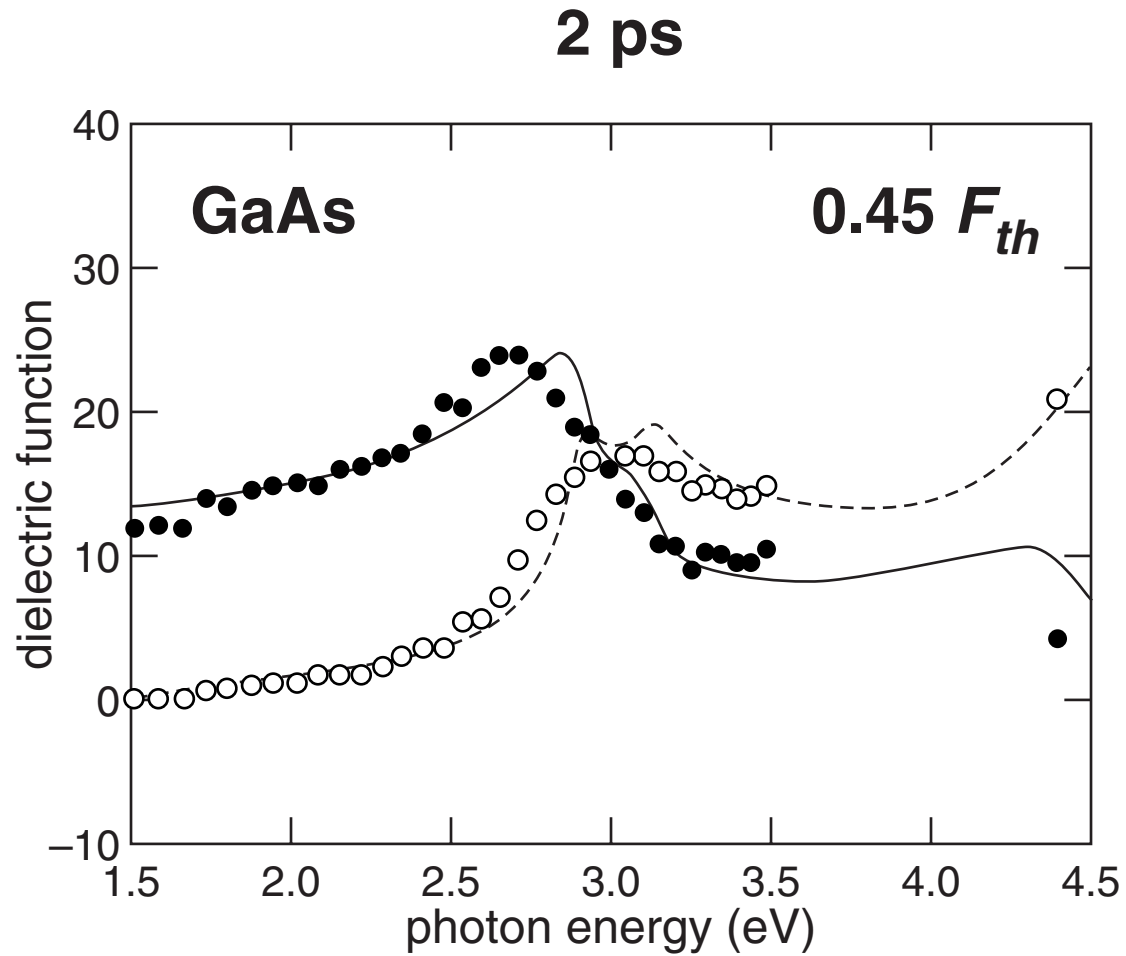
Dielectric function measurements



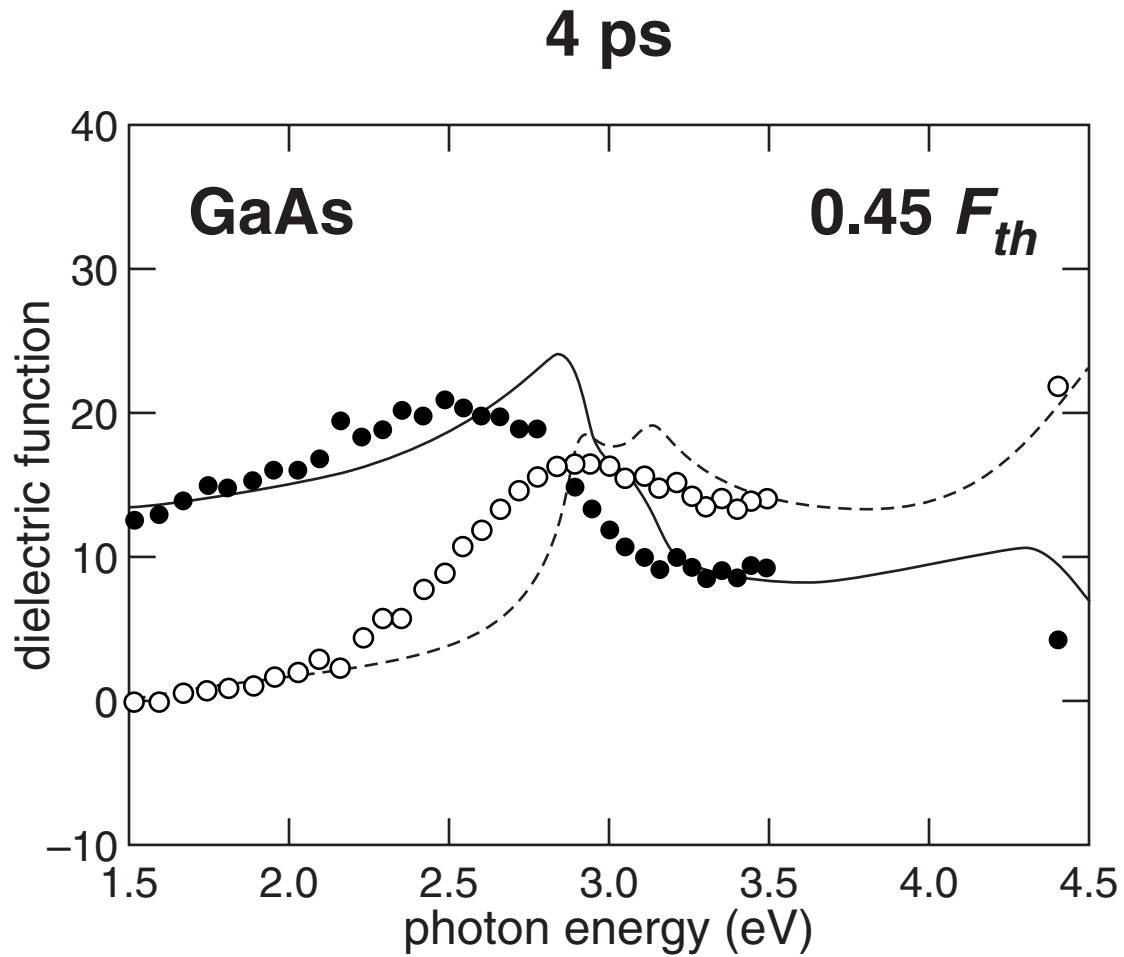
Dielectric function measurements



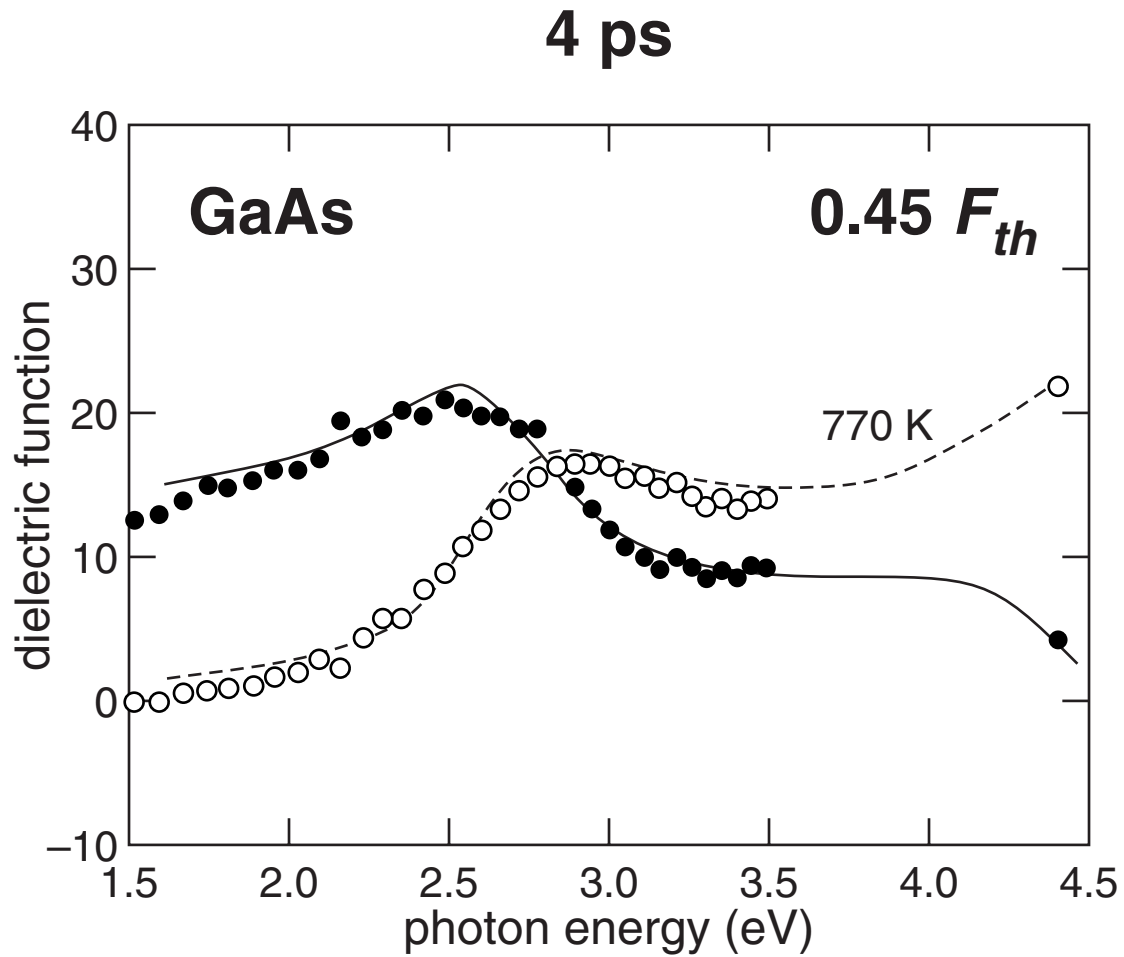
Dielectric function measurements



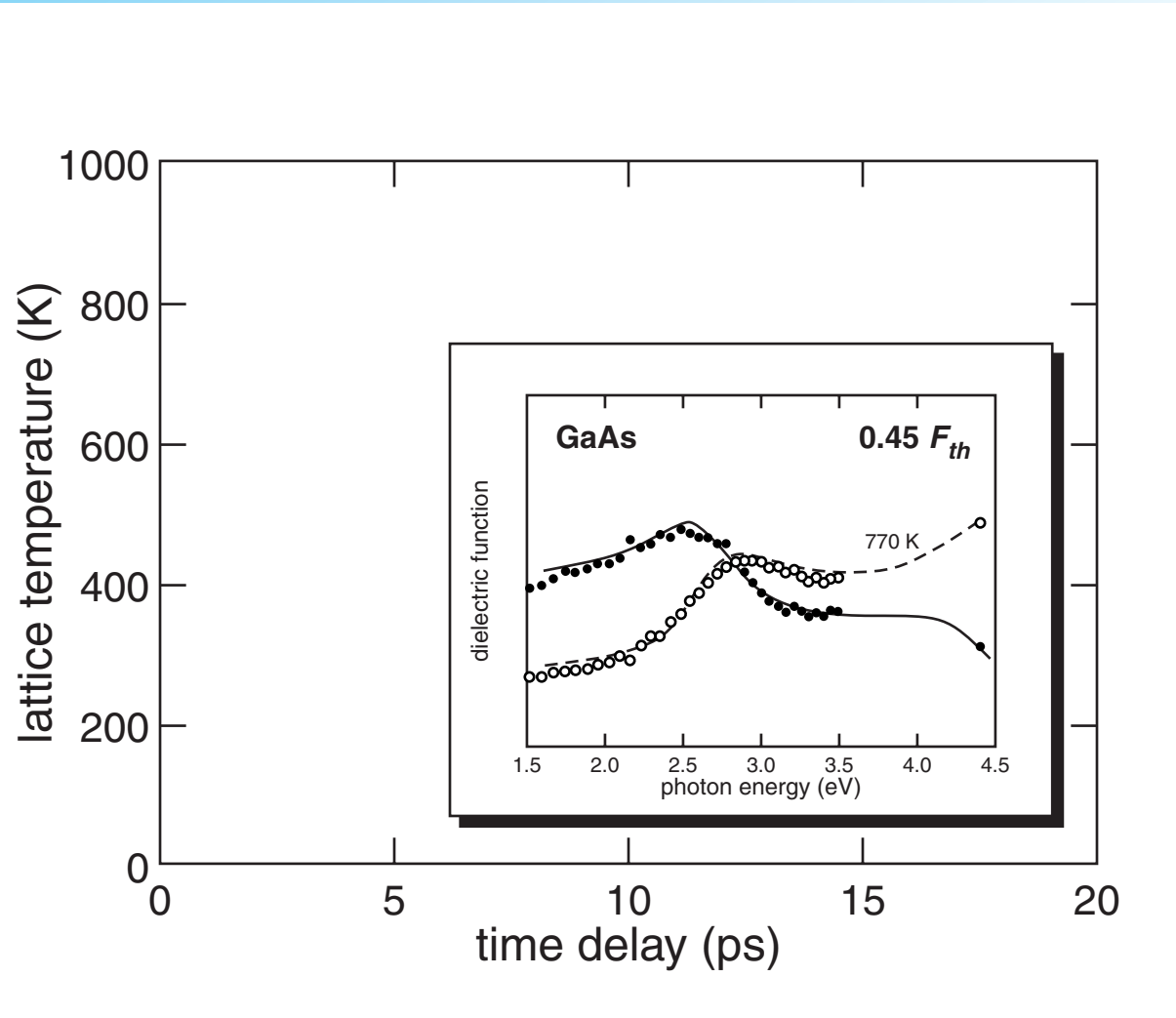
Dielectric function measurements



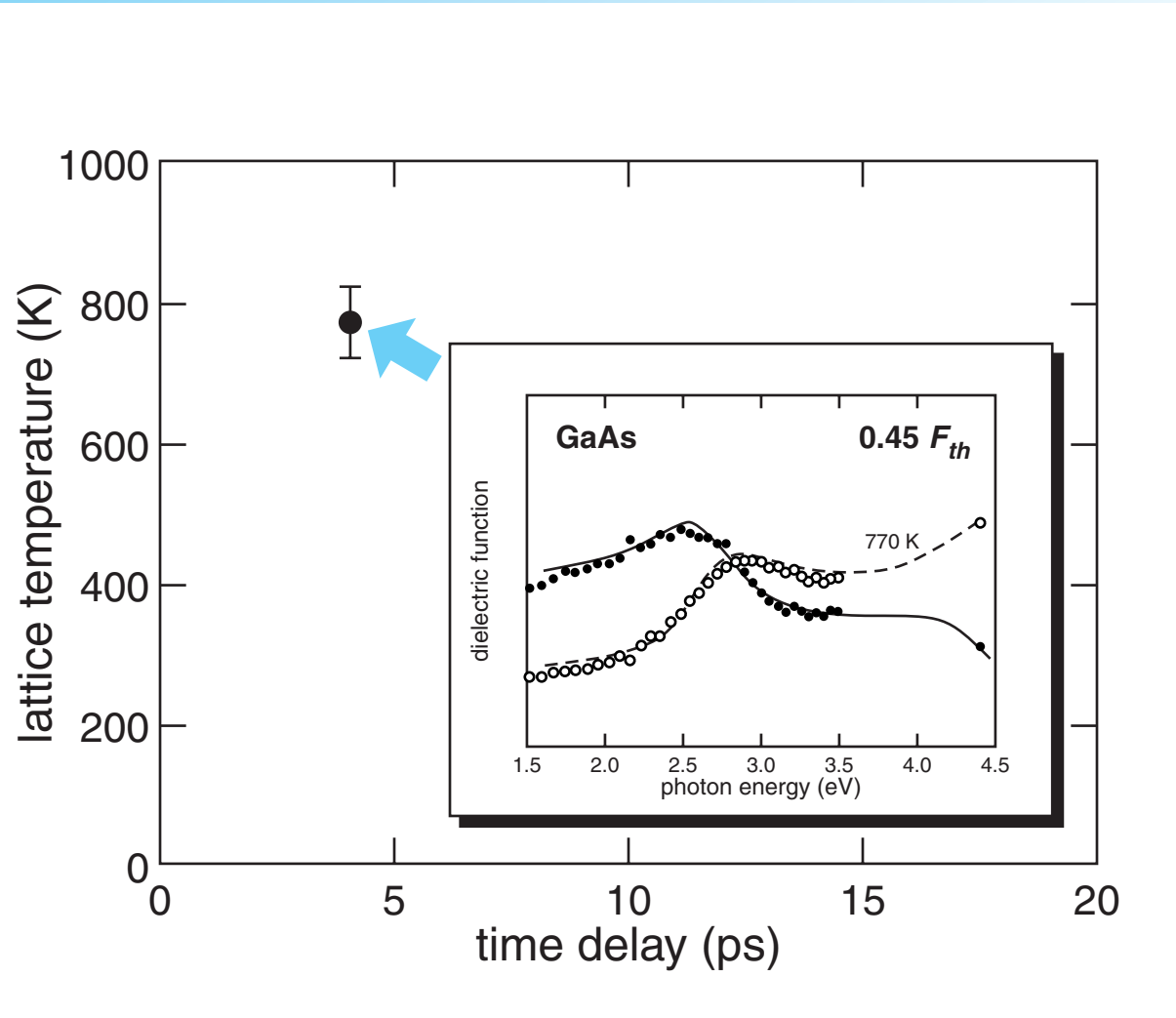
Dielectric function measurements



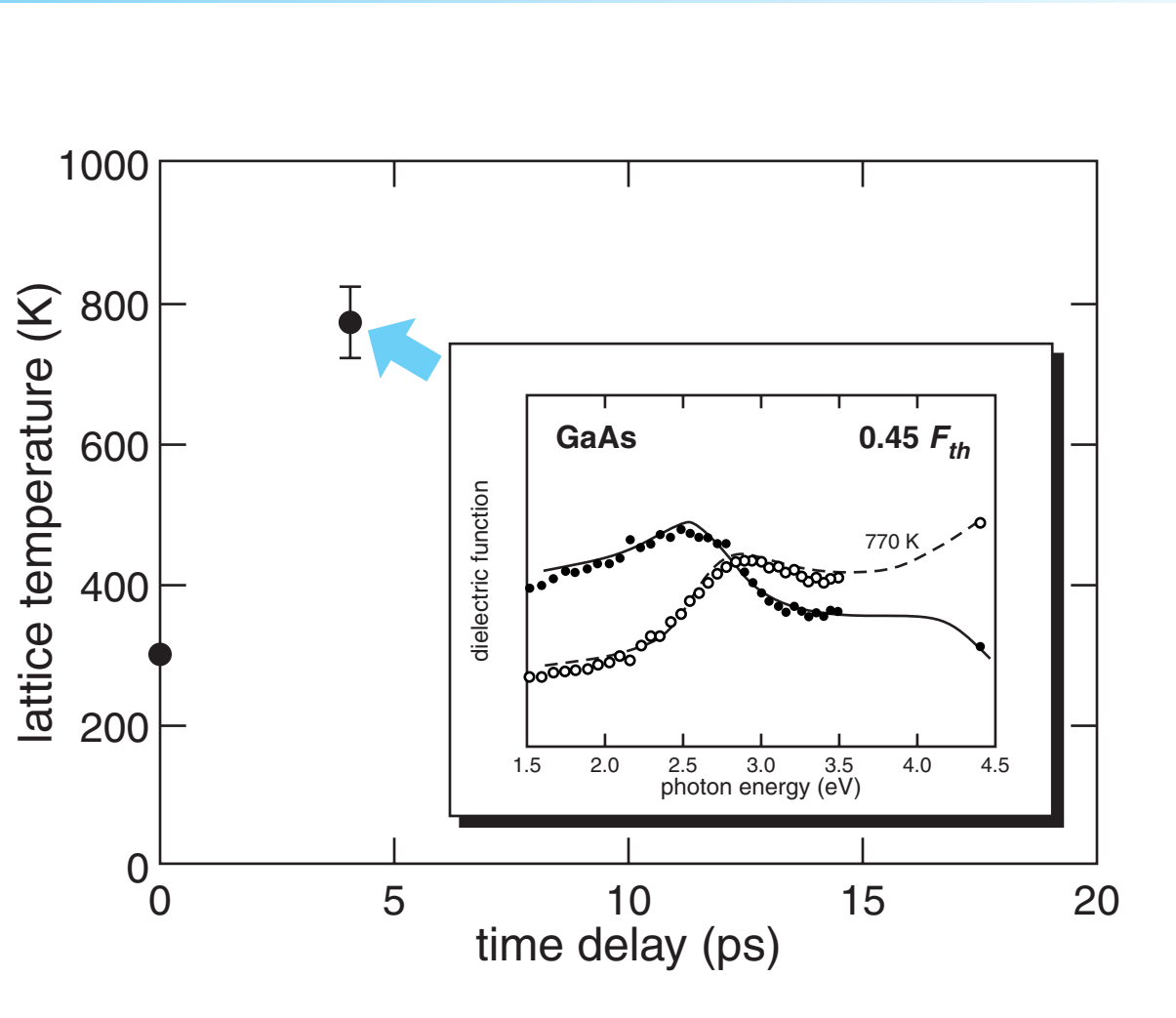
Dielectric function measurements



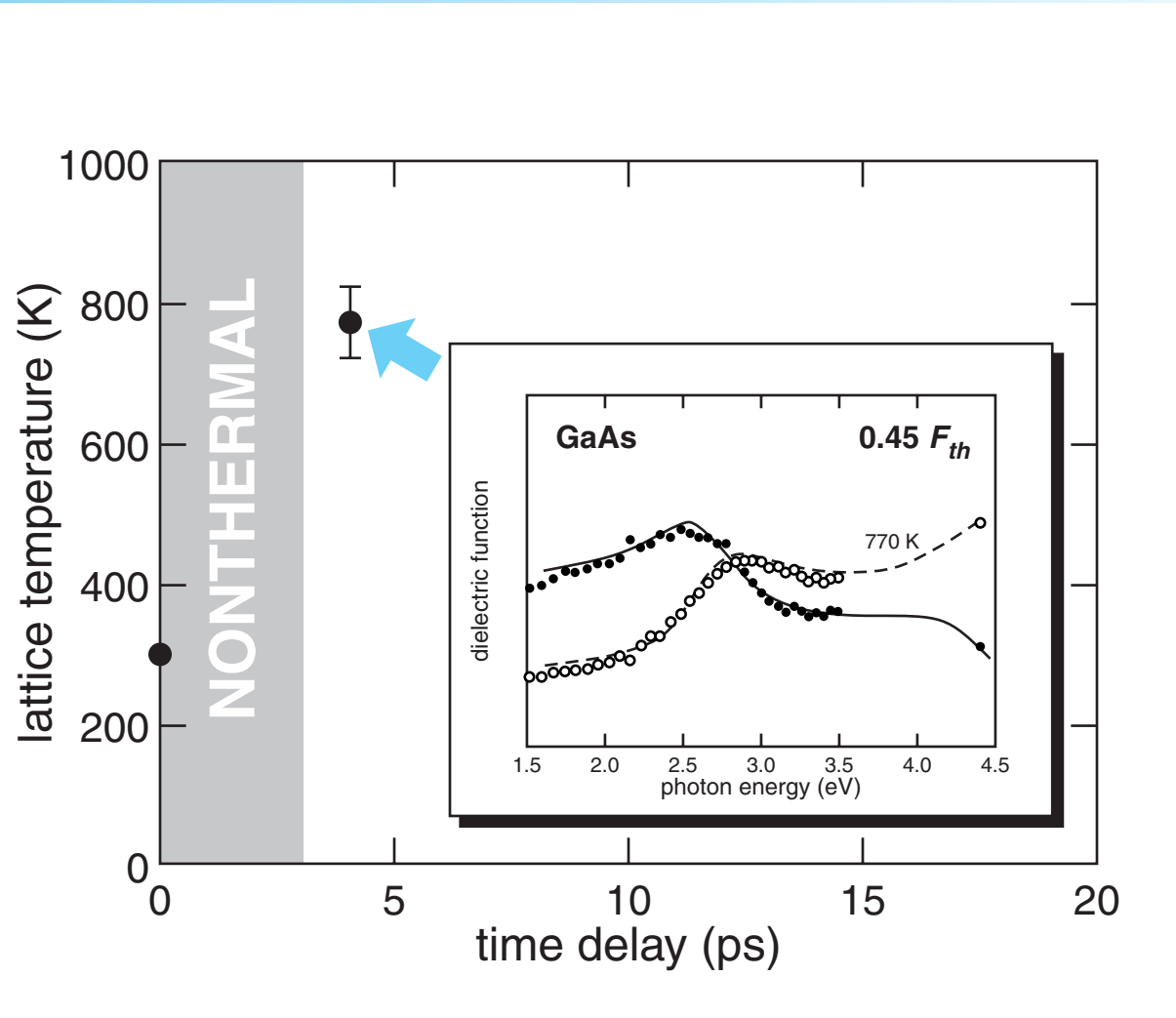
Dielectric function measurements



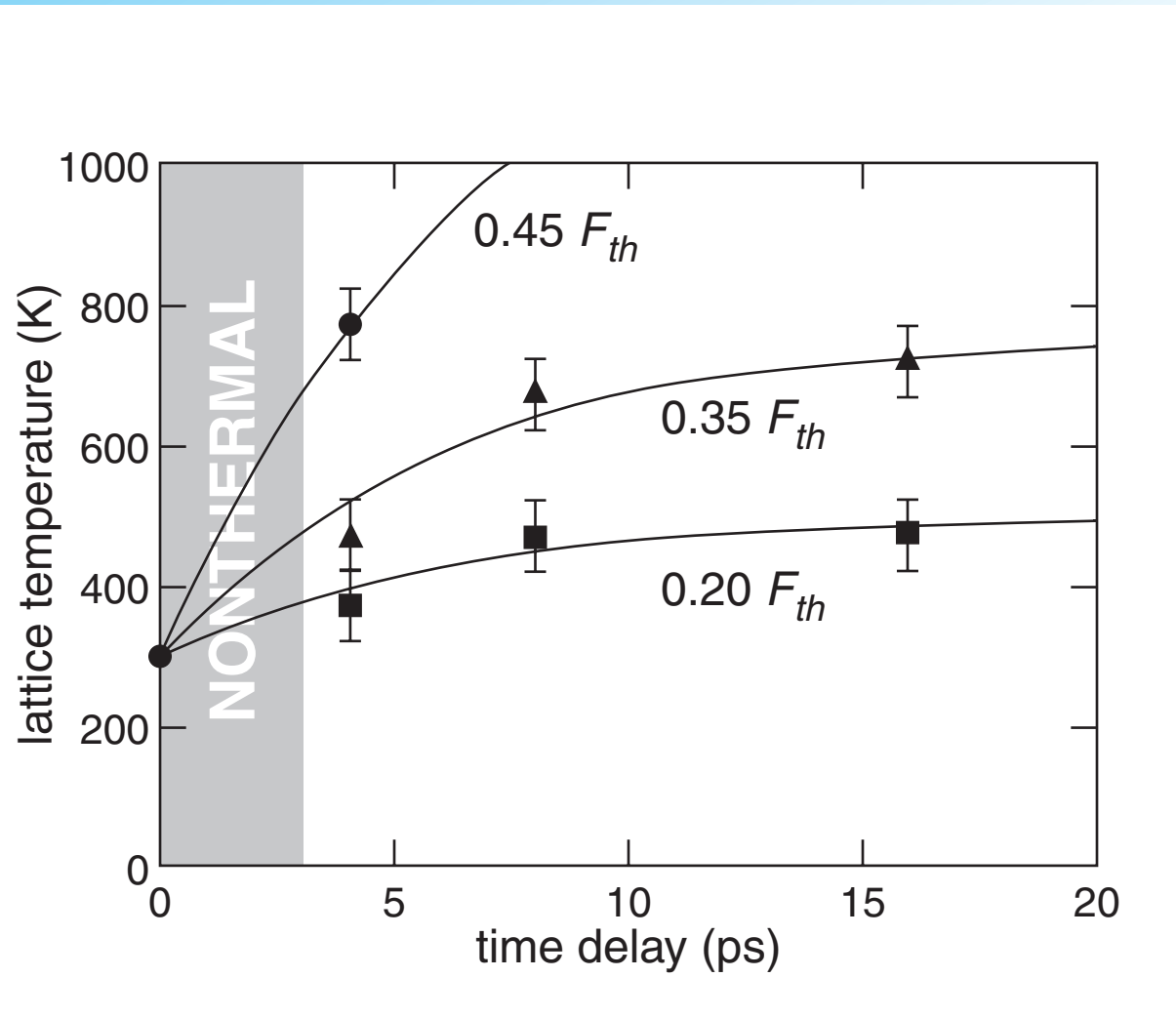
Dielectric function measurements



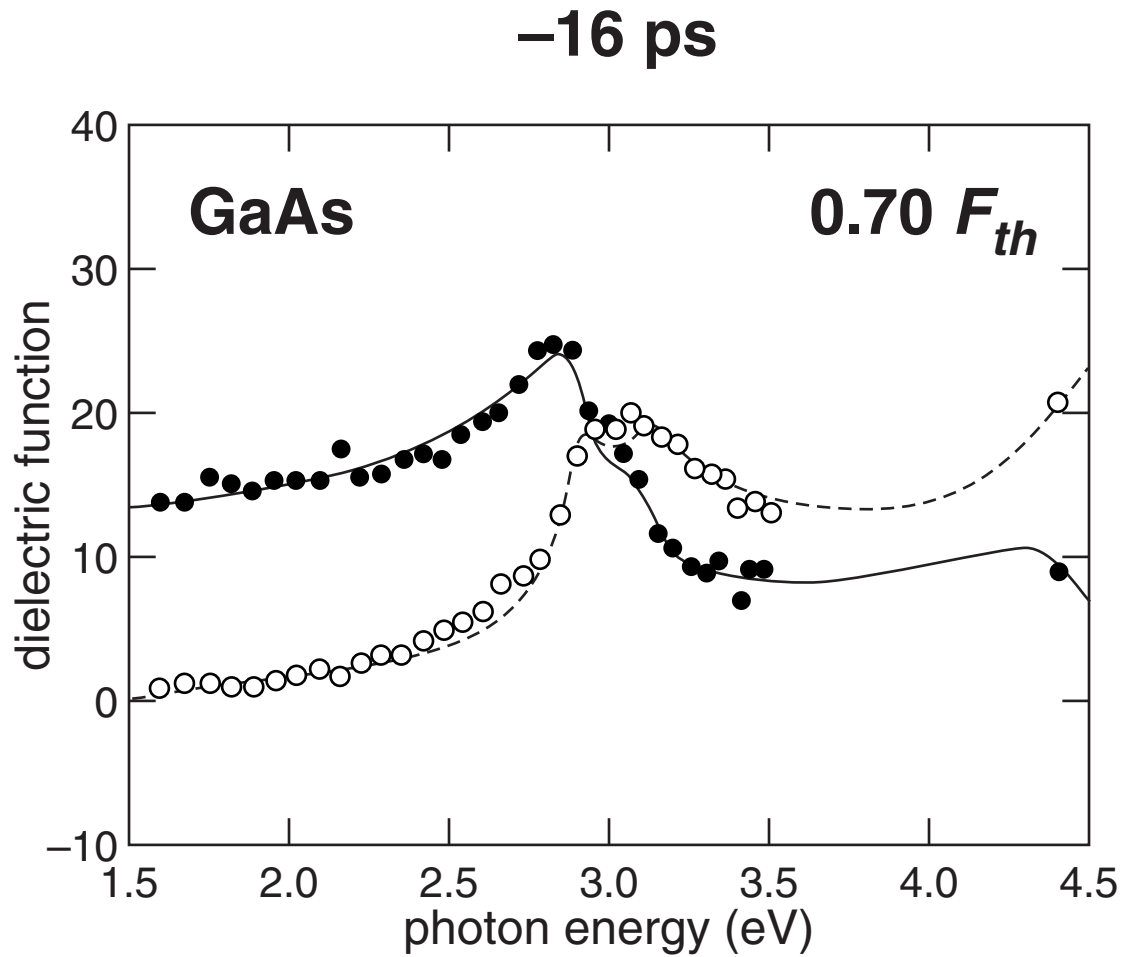
Dielectric function measurements



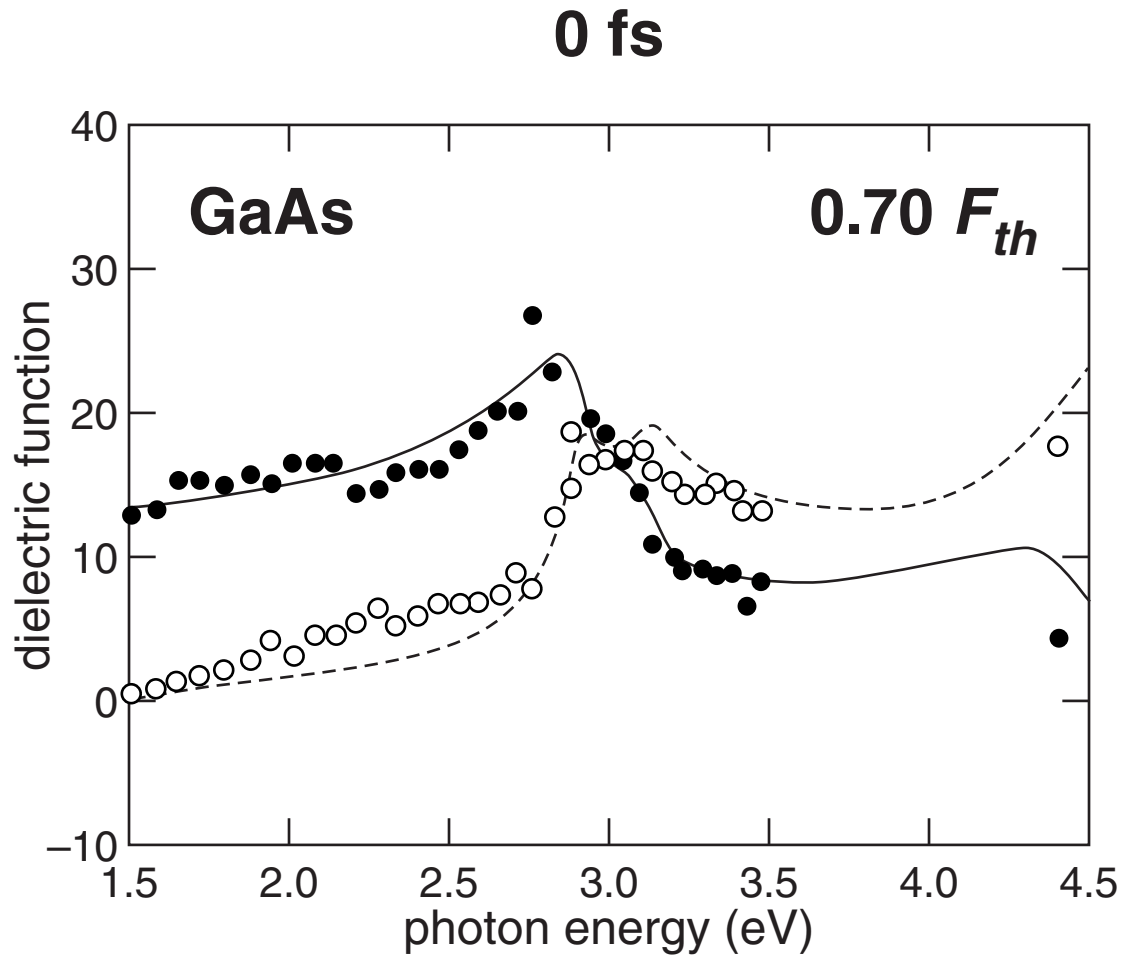
Dielectric function measurements



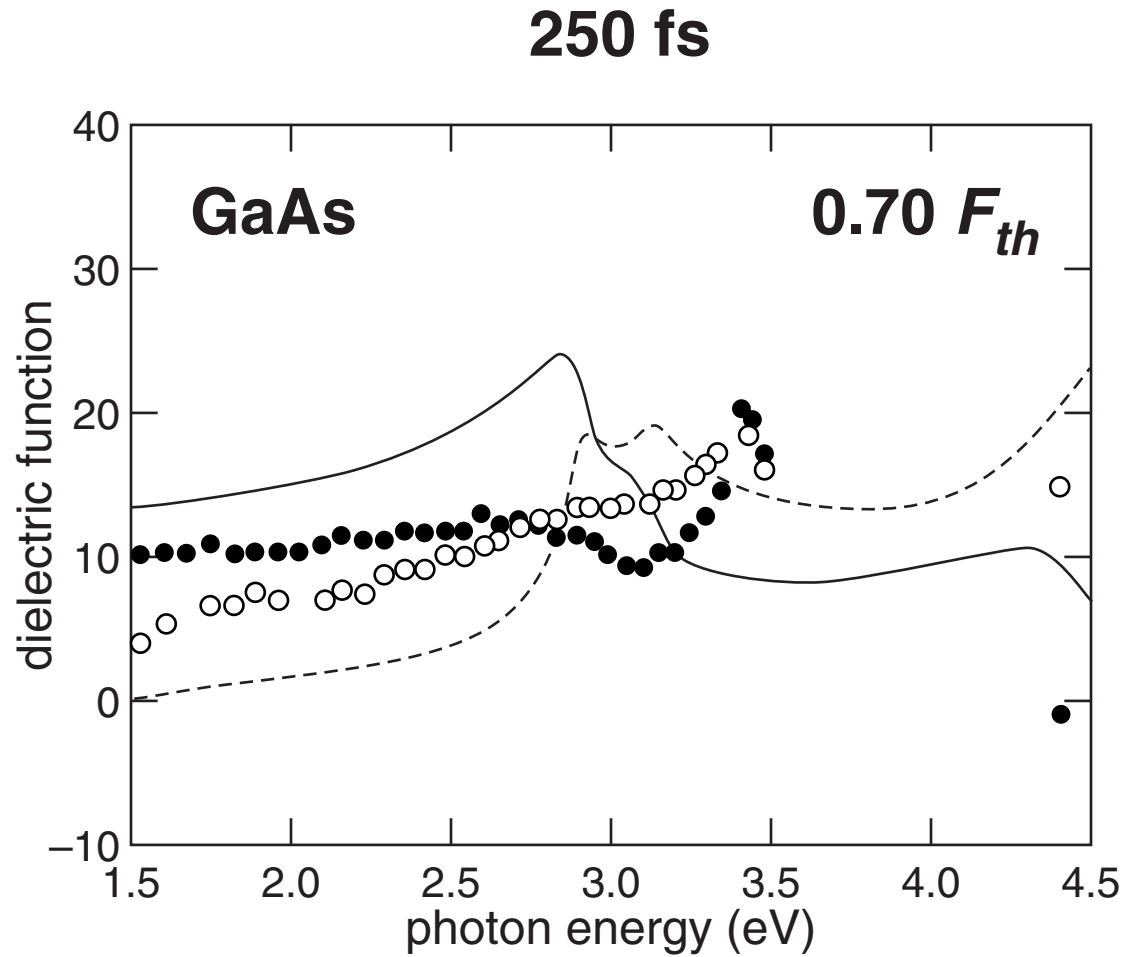
Dielectric function measurements



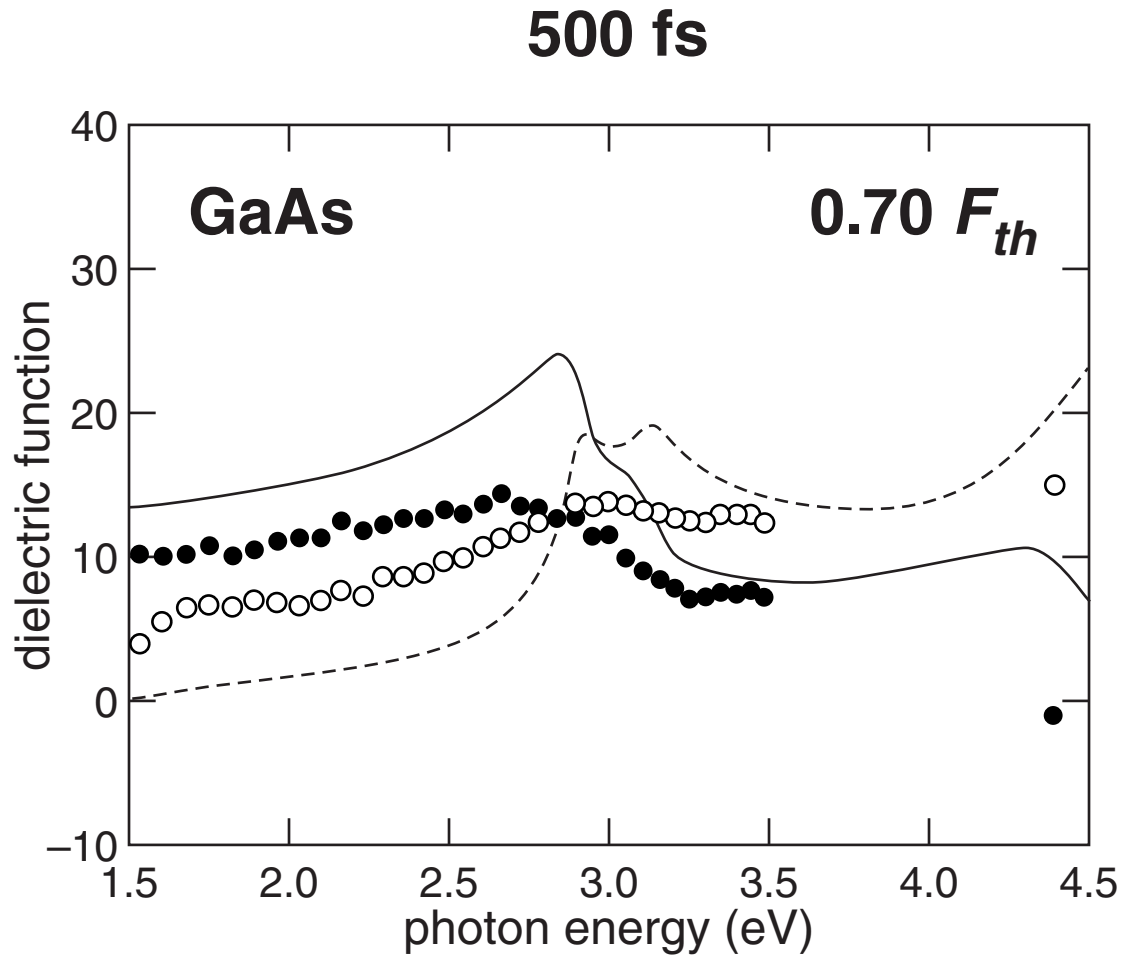
Dielectric function measurements



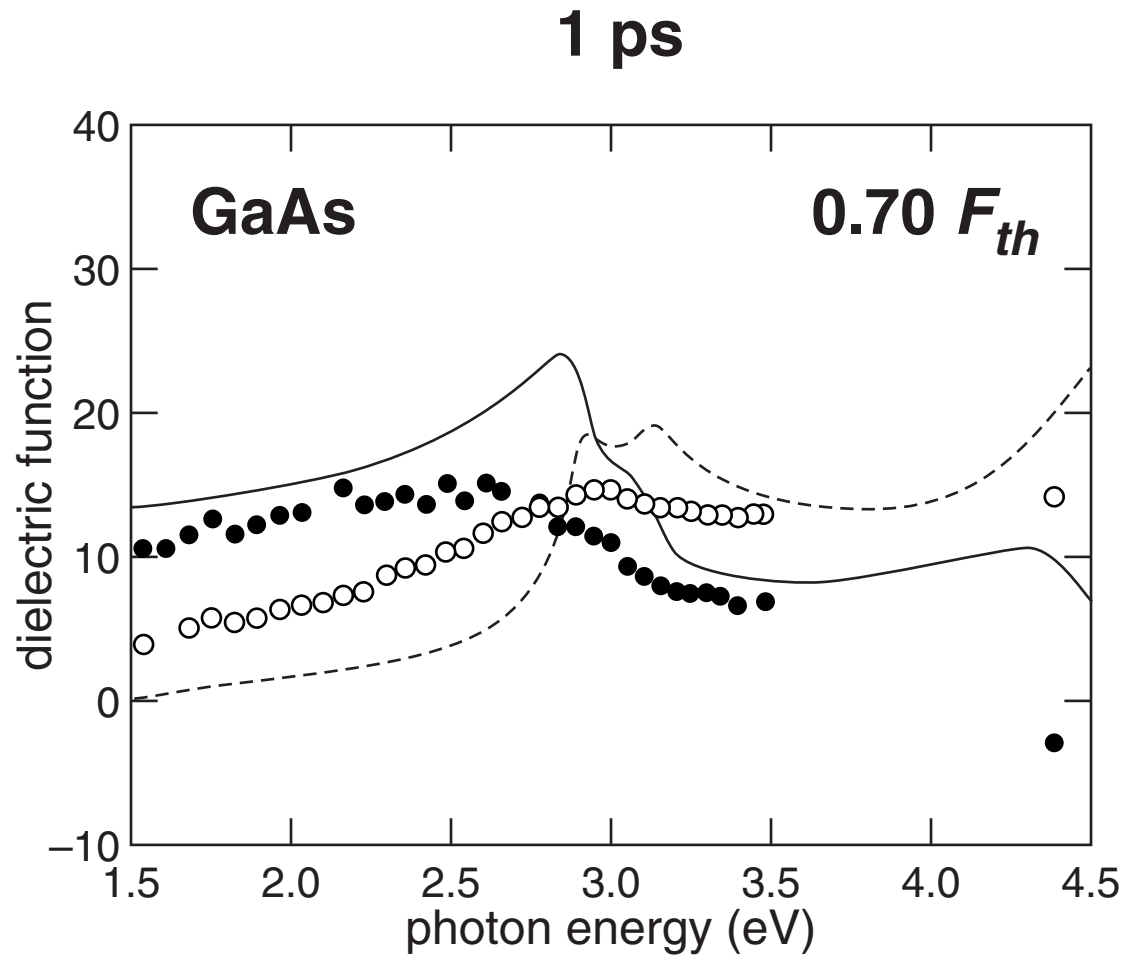
Dielectric function measurements



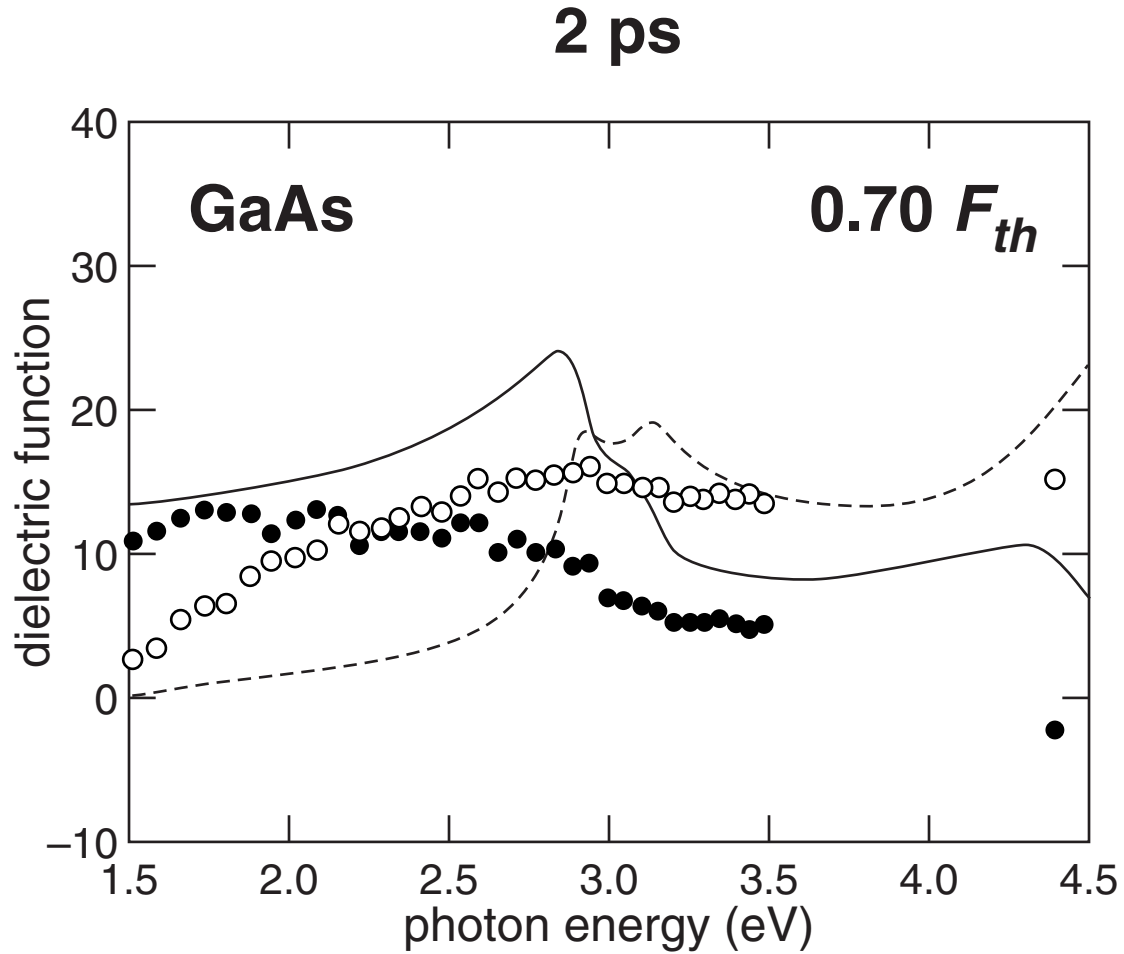
Dielectric function measurements



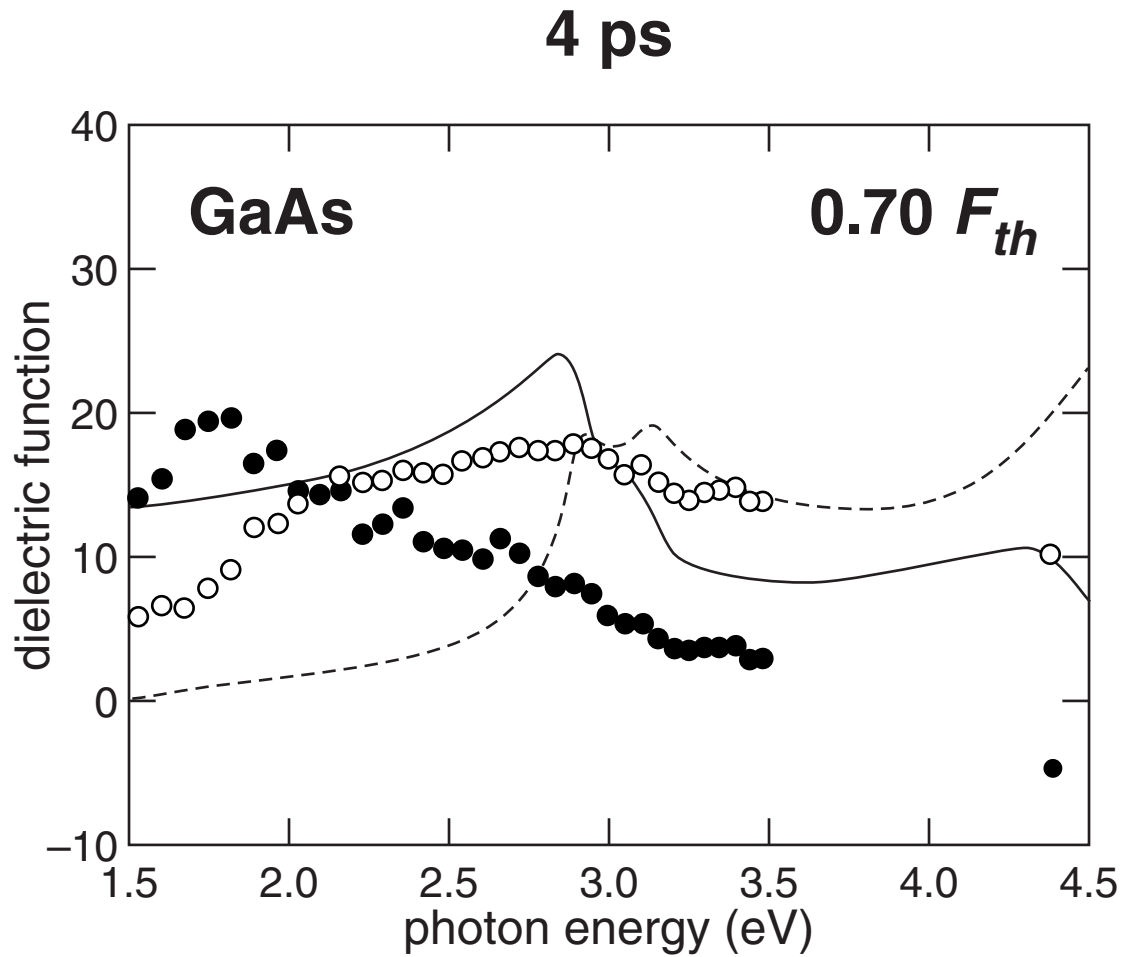
Dielectric function measurements



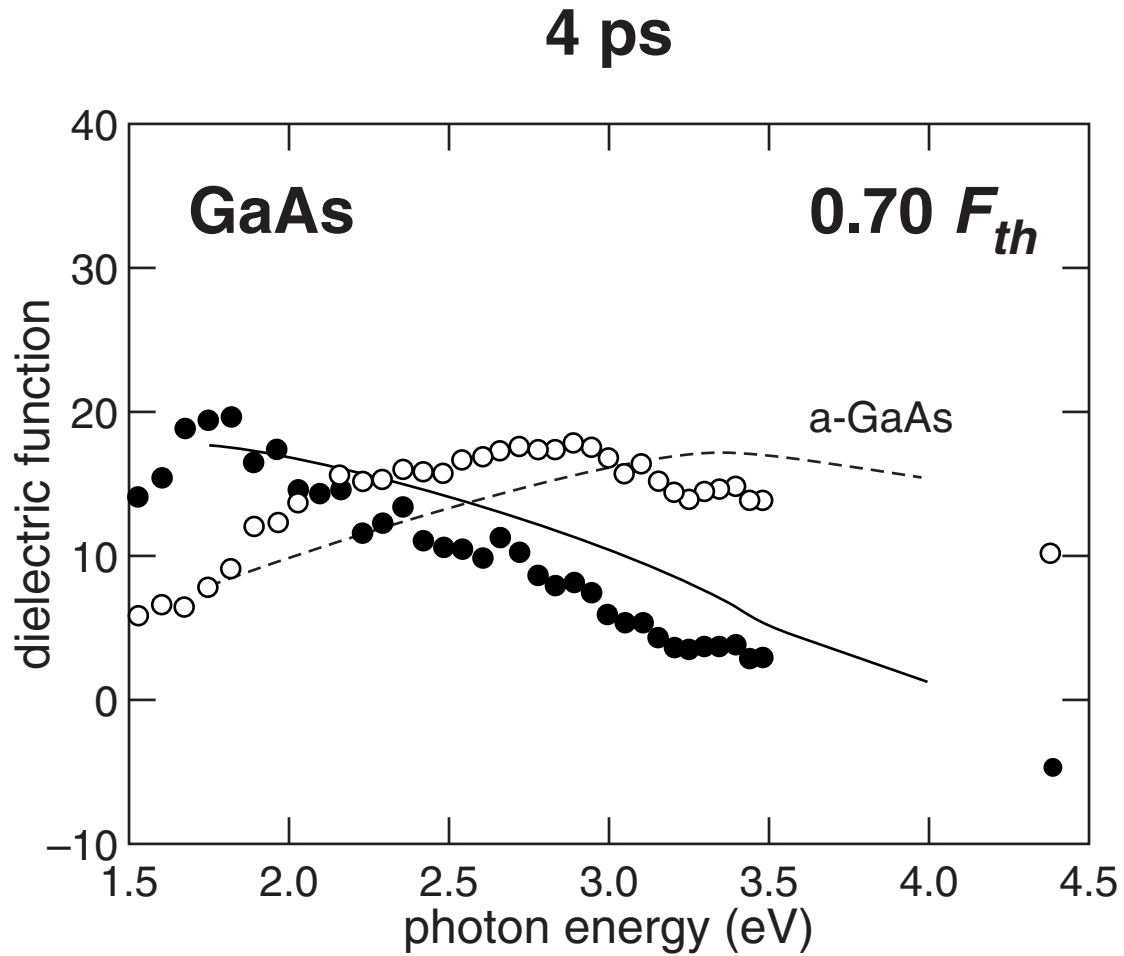
Dielectric function measurements



Dielectric function measurements

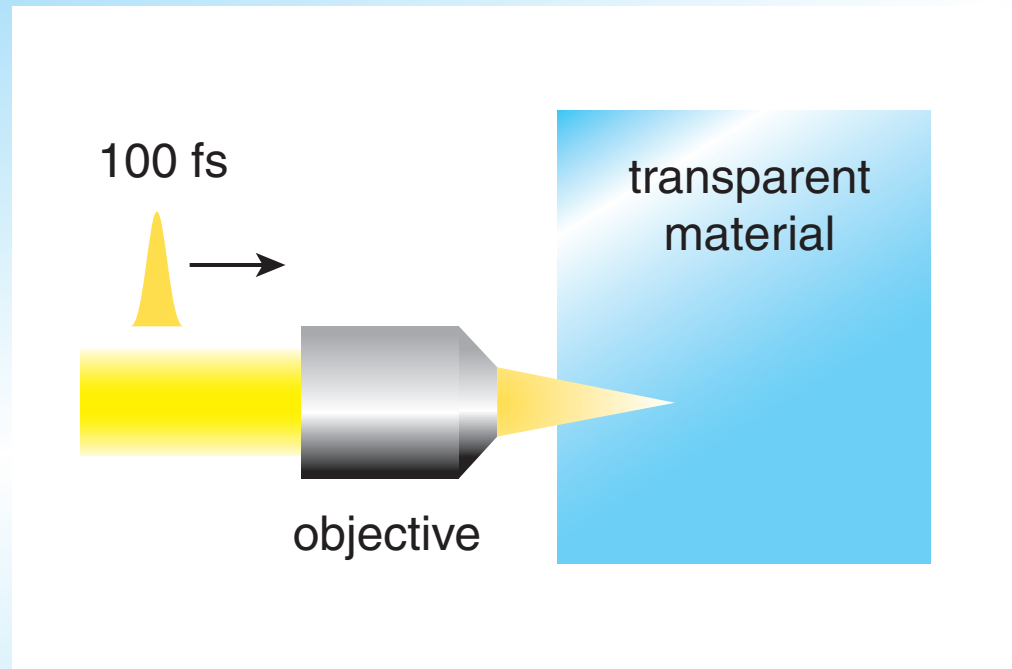


Dielectric function measurements



Extreme nonlinear optics

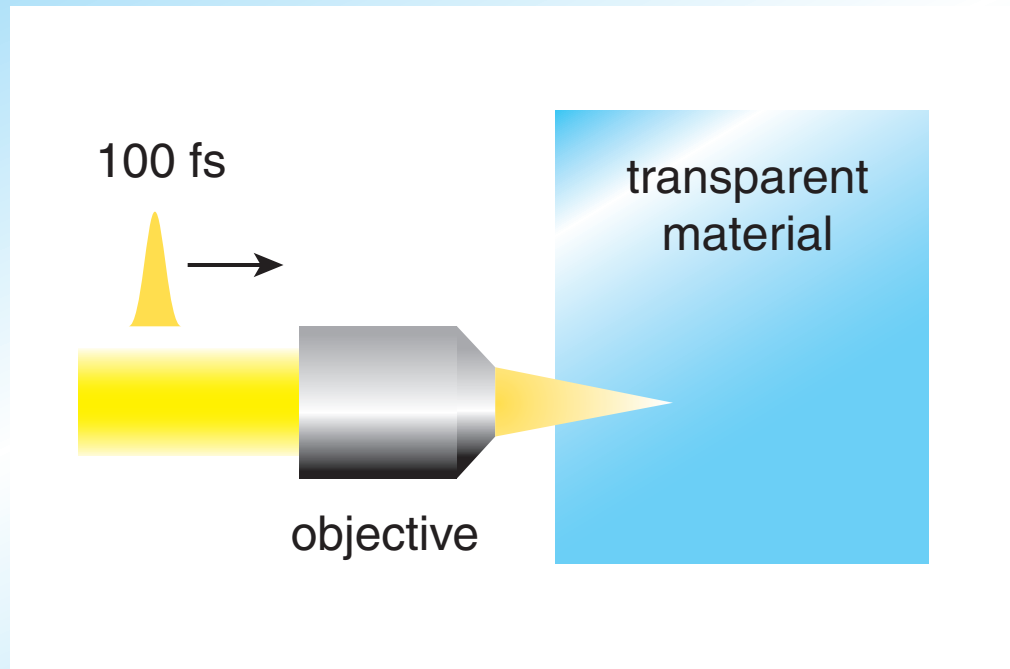
focus laser beam inside material...



Glezer, *et al.*, *Opt. Lett.* 21, 2023 (1996)

Extreme nonlinear optics

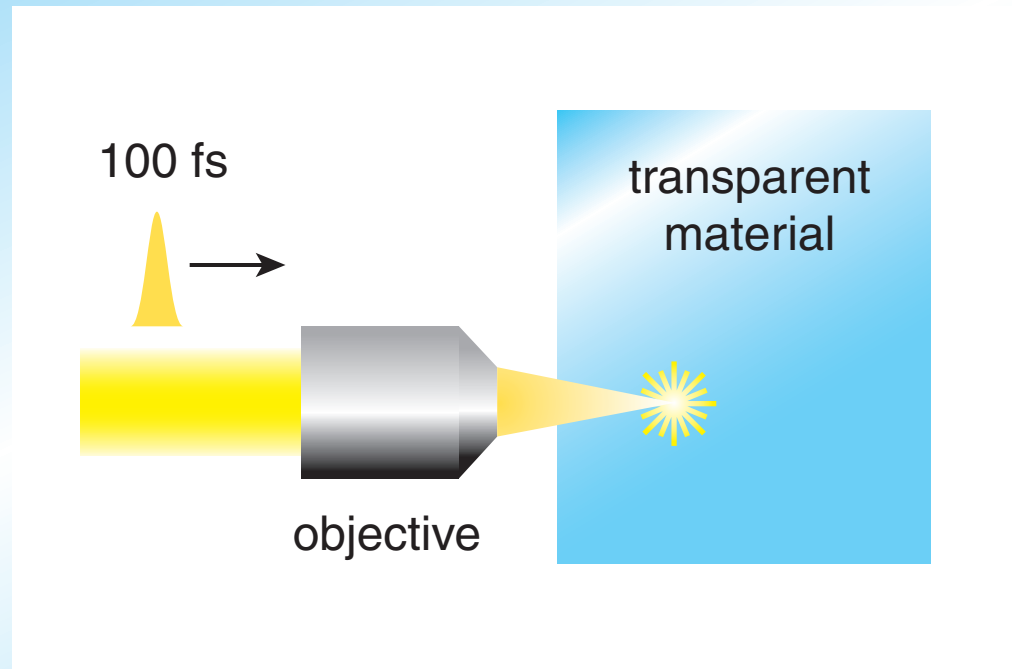
high intensity at focus...



Glezer, *et al.*, *Opt. Lett.* 21, 2023 (1996)

Extreme nonlinear optics

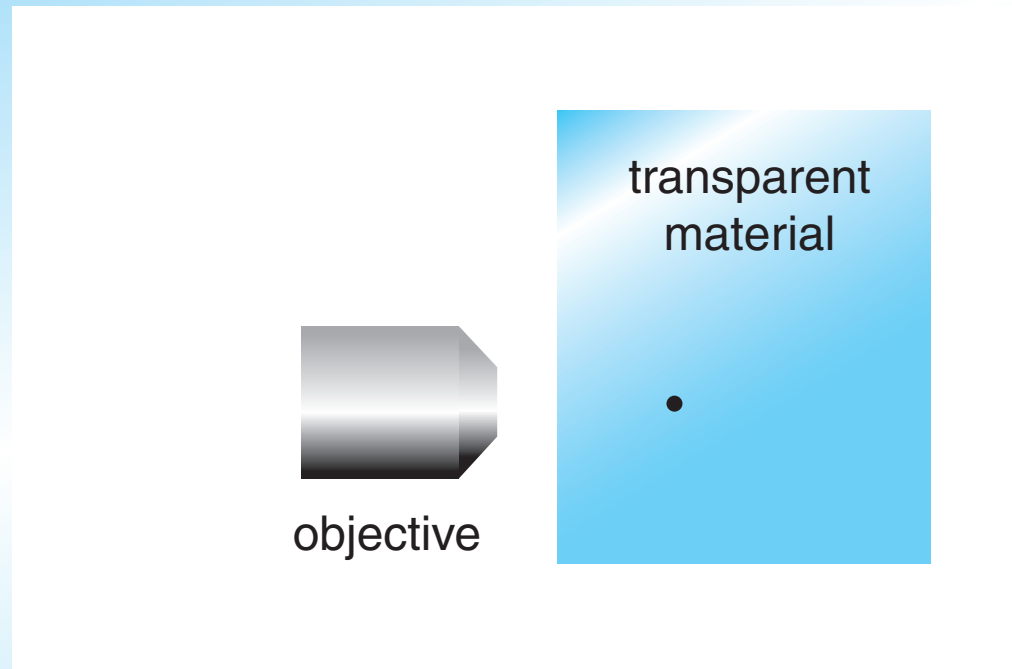
... causes nonlinear ionization...



Glezer, *et al.*, *Opt. Lett.* 21, 2023 (1996)

Extreme nonlinear optics

and microscopic bulk damage



Glezer, et al., *Opt. Lett.* 21, 2023 (1996)

Extreme nonlinear optics

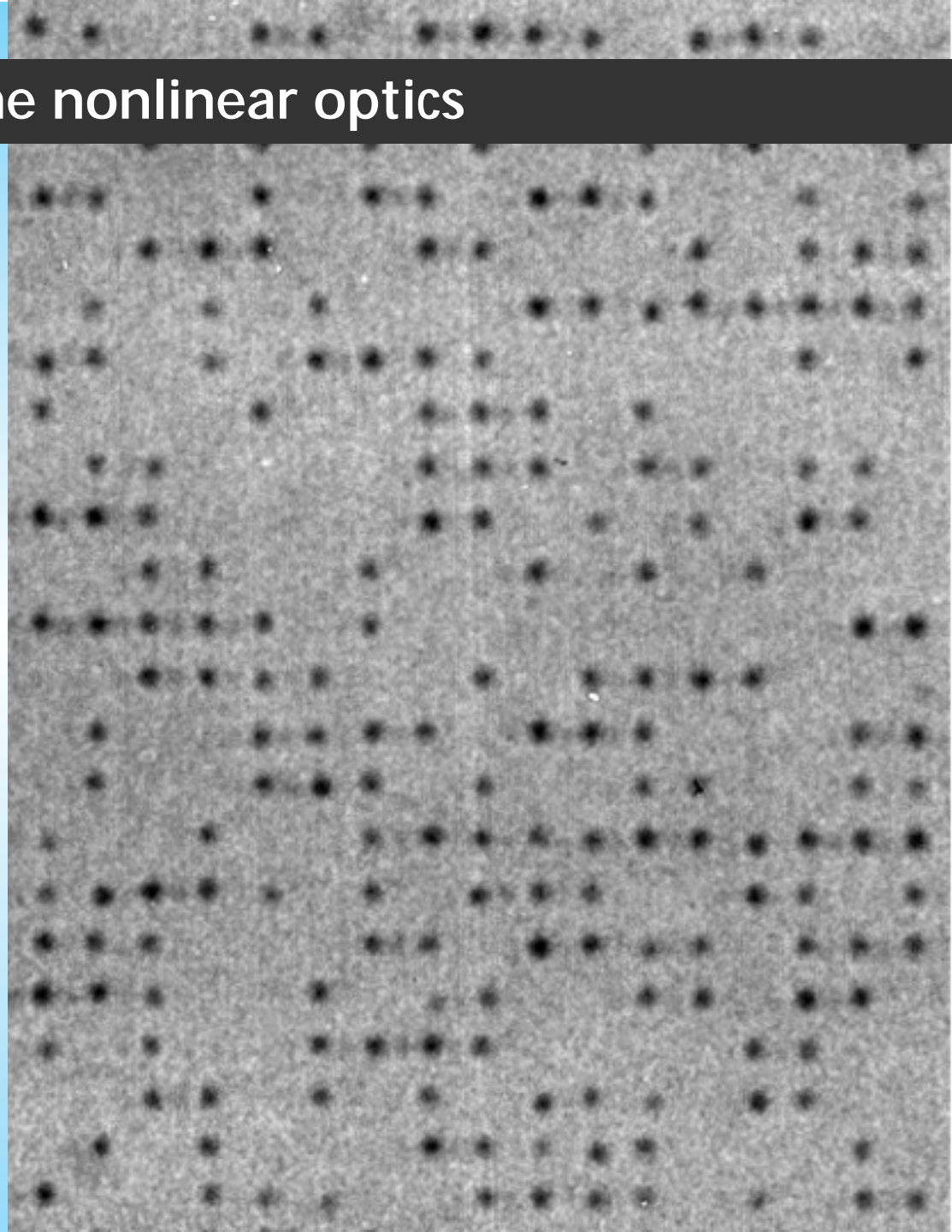
optical microscopy

2 x 2 μm array

fused silica

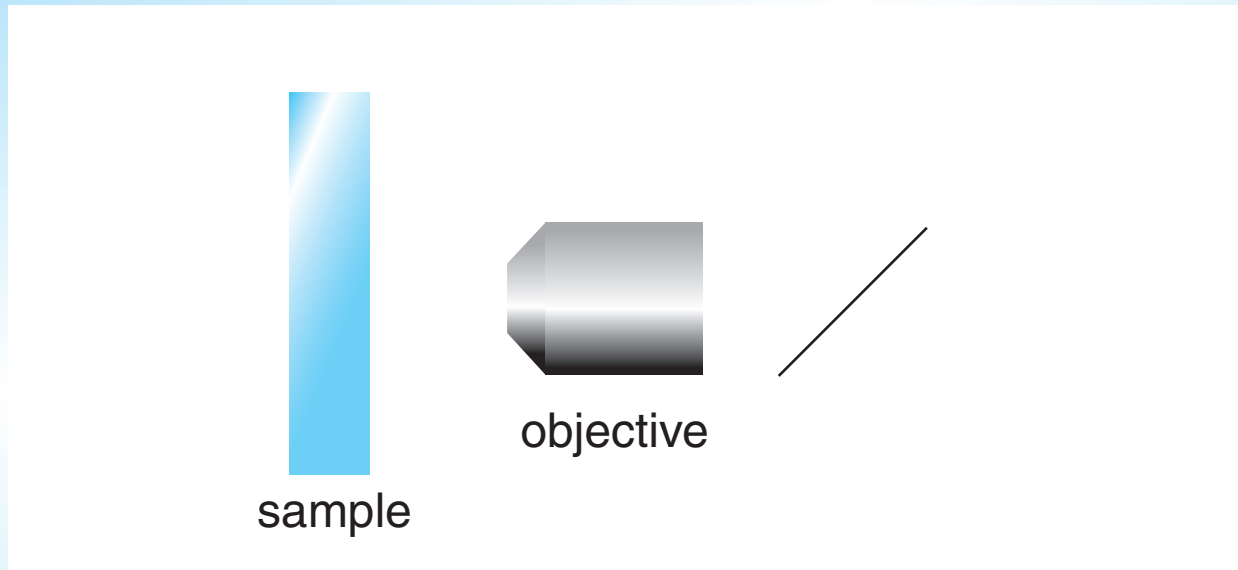
0.5 μJ , 100 fs, 800 nm

Opt. Lett. 21, 2023 (1996)



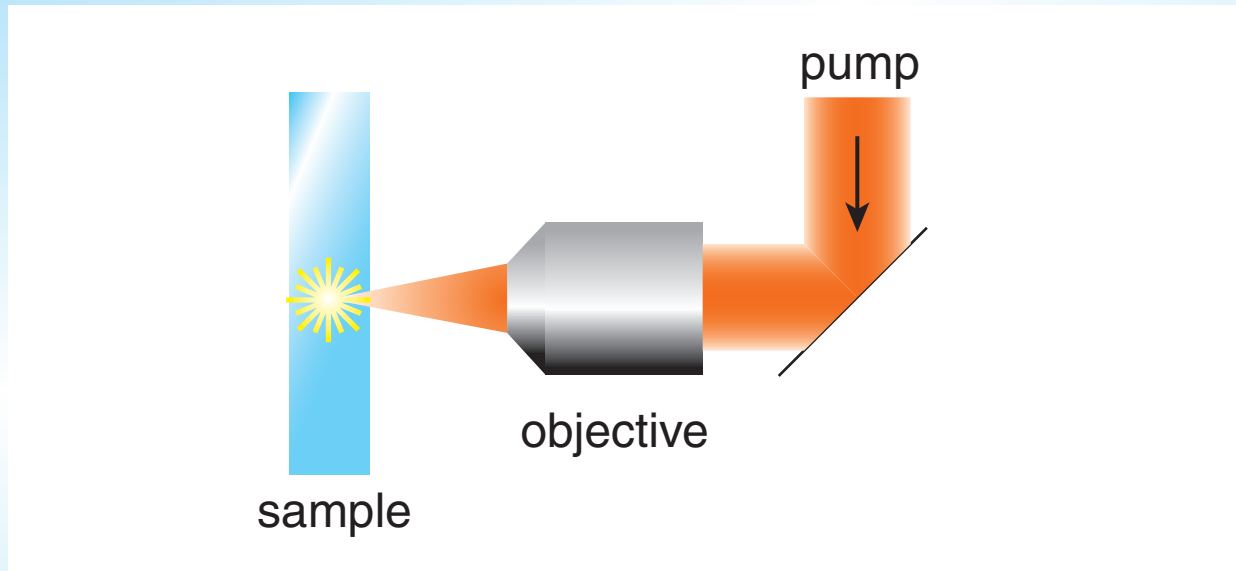
Extreme nonlinear optics

imaging setup



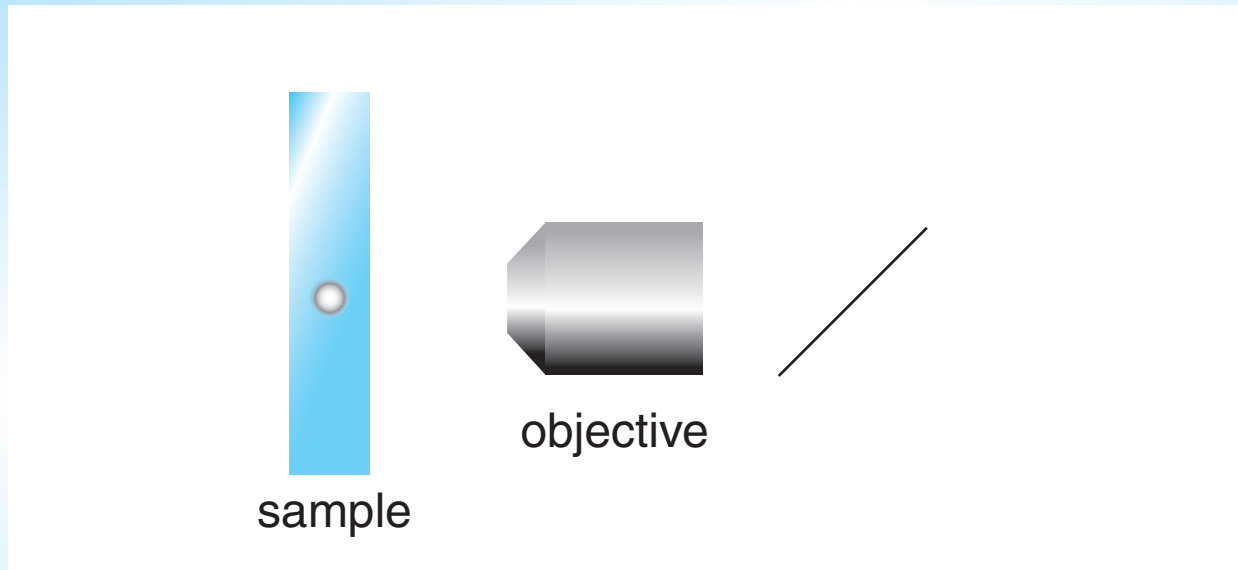
Extreme nonlinear optics

imaging setup



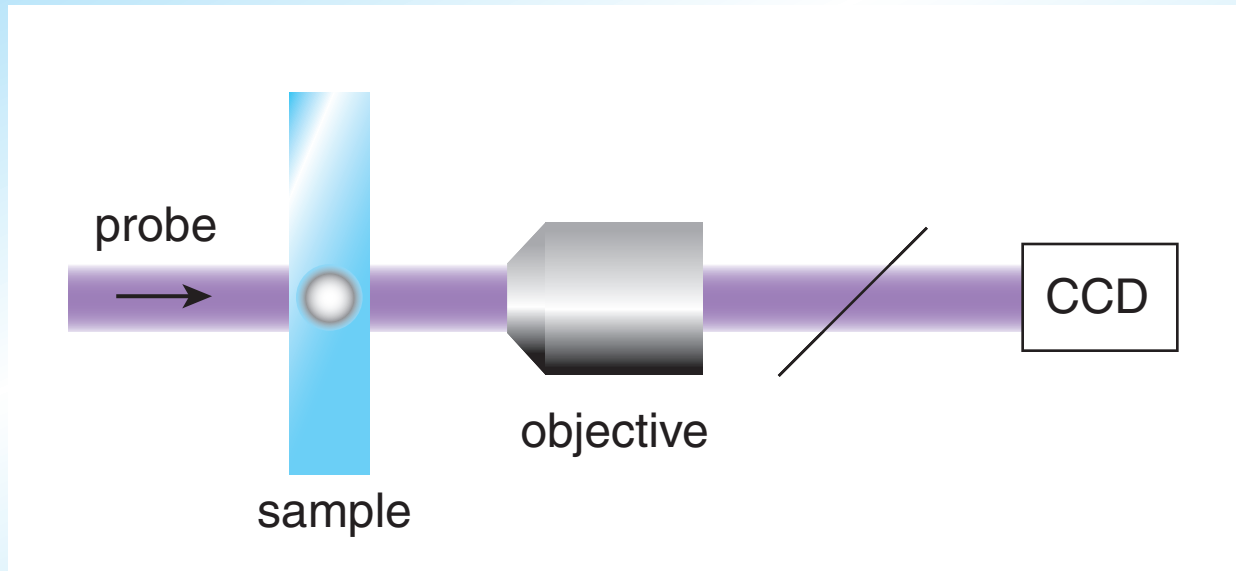
Extreme nonlinear optics

imaging setup



Extreme nonlinear optics

imaging setup



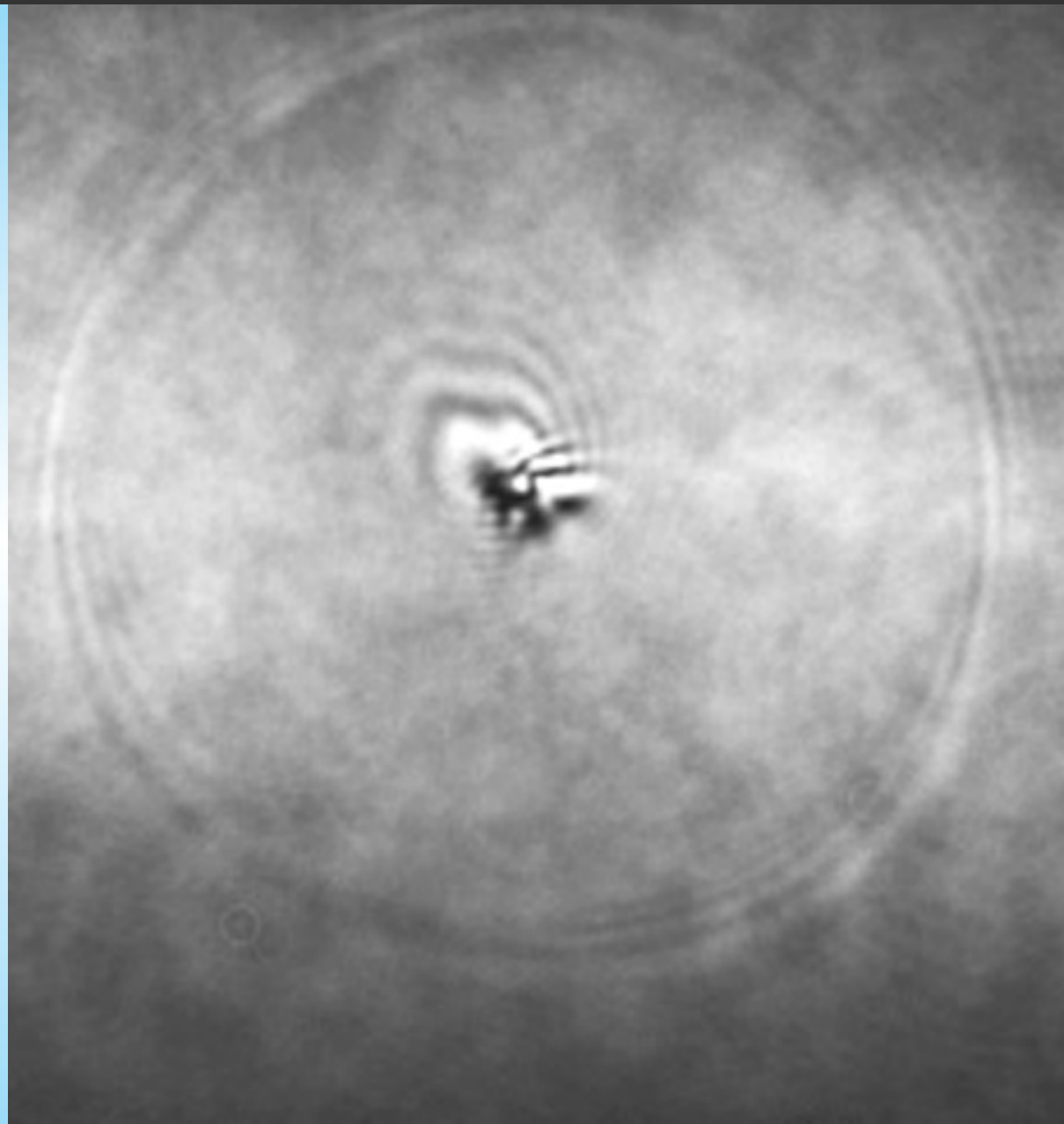
Extreme nonlinear optics

sapphire

3 μJ pulse

3.8 ns delay

40 μm radius



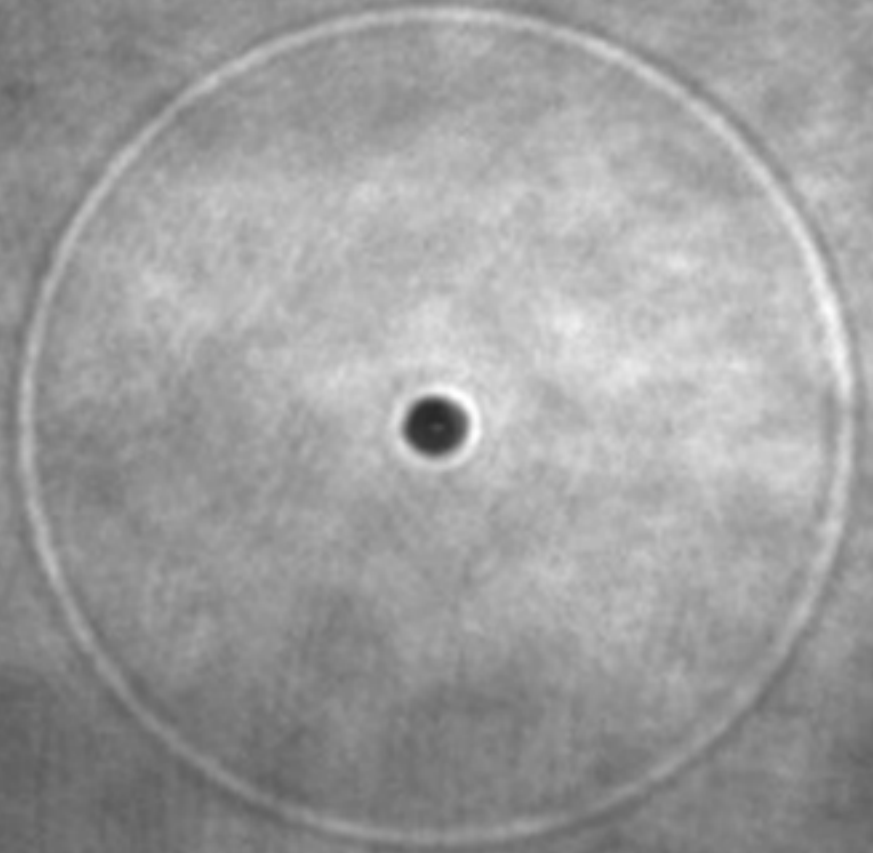
Extreme nonlinear optics

water

1.0 μJ pulse

35 ns delay

58 μm radius



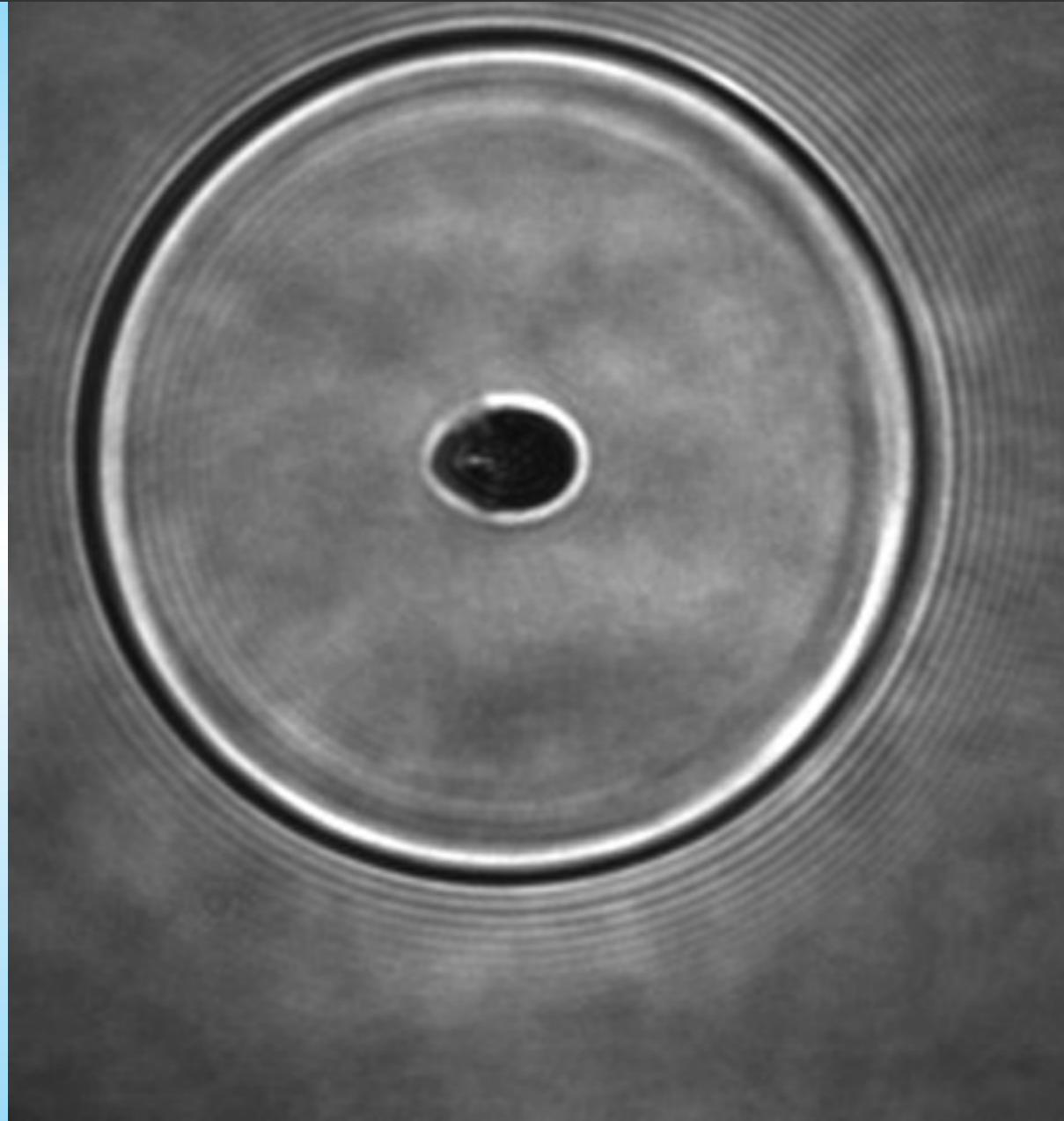
Extreme nonlinear optics

water

14 μJ pulse

35 ns delay

64 μm radius



Extreme nonlinear optics



Summary

Femtosecond lasers offer:

- ▶ **unprecedented view into dynamics**
- ▶ **extreme conditions with very little energy**
- ▶ **new research in materials science**

Many exciting talks in Symposium Q!

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APPLIED SCIENCE





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Prof. T. Kaxiras**

**For a copy of this talk and
additional information, see:**

<http://mazur-www.harvard.edu>